Power and Sample Size Analysis

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Topics

- Intro to power and sample size concepts
- Calculate power and sample size for various statistical tests using the `pwr` package in R and a few built-in R functions

Hello, my name is. . .

- I suspect people place sticky-back name tags on the left side of their chest about 75% of the time (probably because most people are right handed). I create an experiment to verify this.
- I randomly sample *n* people and determine the proportion *p* of people who place a name tag on the left.
- I conduct a one-sample proportion test to see if the sample proportion is significantly greater than random chance (0.50).
- I will reject the null hypothesis of random chance if the p-value is below 0.05.
- *How many people should I sample?*
- Or, *I can only sample 30 people. Do I have sufficient power?*

Determining sample size and power

A sufficient sample size for a statistical test is determined from:

1. Power
2. Effect size
3. Significance level
4. Alternative direction (*only for certain tests*)

Determining the power of a statistical test is determined from:

1. Sample Size
2. Effect size
3. Significance level
4. Alternative direction (*only for certain tests*)

What is power?

▶ Power is the probability a statistical test will correctly detect a hypothesized effect (if it really exists).

▶ In a hypothesis test, we assume two possible realities:

1. Null Hypothesis: No effect (eg, random chance, 0.50)
2. Alternative Hypothesis: *some* effect (eg, 0.75)

▶ At the conclusion, we decide whether to reject or fail to reject #1, usually based on a *p-value* falling below a threshold such as 0.05.

▶ We would like to have a high probability (or high power) of rejecting #1 if #2 is true. The usual desired power is at least 0.80.

What is effect size?

- One definition is "the degree to which the null hypothesis is false."
- Estimating 90% of people place name tags on the left is much larger than estimating 55% put name tags on the left.
- In the former scenario (90%), I don't have to sample that many people to confirm my suspicion. In the latter scenario (55%), I probably need to sample quite a few people to get a proportion that is significantly different from random chance (50%)
- In sample size and power analyses, we have to pick an effect size. We usually pick the *smallest effect we don't want to miss*.

What is significance level?

- This is the cut-off for determining whether or not our p-value is significant.
- Typical values are 0.05, 0.01 and 0.001.
- Recall, a p-value is the probability under the null hypothesis that a statistical summary of data would be equal to or more extreme than its observed value.

What is Alternative Direction?

- ▶ This refers to how we think our alternative hypothesis differs from the null.
- ▶ If I think the left preference is greater than (or less than) 50%, then I have a *one-sided* alternative.
- ▶ If I think the left preference is simply different from 50%, then I have a *two-sided* alternative.
- ▶ The one-sided alternative is a stronger assumption. Most power and sample size analyses will play it safe by assuming a two-sided alternative.
- ▶ Some hypothesis tests only have a two-sided alternative, such as ANOVA and chi-square.

Type I and Type II errors

- If I conclude people prefer left when they actually don't I have made a *Type I error*. (Rejecting the null hypothesis in error)
- If I conclude people have no preference when they really do prefer left I have made a *Type II error*. (Failing to reject null hypothesis in error)
- We usually never know if we have made these errors.
- Our tolerance for a Type I error is the significance level. Usually 0.05, 0.01 or lower.
- Our tolerance for a Type II error is 1 - Power. Usually 0.20 or lower.

Visualizing a one-sample proportion test

The following web app allows us to see how power is affected by sample size, effect size and significance level:

https://clayford.shinyapps.io/power_nhst/

Let's take a look.

Calculating power and sample size

Power and sample size formulas have been derived for many statistical tests that allows us to. . .

▶ calculate **sample size** given power, effect size and significance level
▶ calculate **power** given sample size, effect size and significance level

The parameters in the formulas are related such that one is determined given the others.

The pwr package

Today we'll use three base R functions and the pwr package.

```
install.packages("pwr")
library(pwr)
```

The pwr package implements power and sample size analyses as described in *Statistical Power Analysis for the Behavioral Sciences (2nd ed.)*, Cohen (1988).

One of the tricks to using the pwr package is understanding how it defines *effect size*.

Effect size in the `pwr` package

- ▶ Cohen defines "effect size" as "the degree to which the null hypothesis is false."
- ▶ Example: If our null is 50%, and the alternative 75%, the effect size is 25%.
- ▶ But the functions in the `pwr` package require the effect size to be metric-free (unitless).
- ▶ **This means you need to calculate effect size before using `pwr` functions. Entering the wrong effect size leads to incorrect power and sample size estimates!**
- ▶ Fortunately the `pwr` package provides a few functions for this.

The `pwr` functions and associated statistical tests (1)

▶ `pwr.p.test`: one-sample test for proportions (ES=h)
▶ `pwr.2p.test`: two-sample test for proportions (ES=h)
▶ `pwr.2p2n.test`: two-sample test for proportions, unequal sample sizes (ES=h)
▶ `pwr.t.test`: one-sample and two-sample t-tests for means (ES=d)
▶ `pwr.t2n.test`: two-sample t-test for means, unequal sample sizes (ES=d)

Notice the effect sizes: h and d. We'll define these shortly.

The `pwr` functions and associated statistical tests (2)

- `pwr.chisq.test`: chi-squared tests; goodness of fit and association (ES=w)
- `pwr.r.test`: correlation test (ES=r)
- `pwr.anova.test`: test for one-way balanced anova (ES=f)
- `pwr.f2.test`: test for the general linear model (multiple regression) (ES=f2)

Notice the effect sizes: w, r, f and f2. We'll define these shortly.

The ES functions

Functions to compute effect size:

- `ES.h`: compute effect size h for proportion tests
- `ES.w1`: compute effect size w1 for chi-squared test for goodness of fit
- `ES.w2`: compute effect size w2 for chi-squared test for association
- `cohen.ES`: return conventional effect size (small, medium, large) for all tests available in `pwr`

We will use these functions as needed in the examples that follow.

Other effect sizes (d, r, f, and f2) must be calculated by hand.

Conventional effect size

- Sometimes we don't know the precise effect size we expect or hope to find. In this case we can resort to conventional effect sizes of "small", "medium", or "large".

- The `cohen.ES` function returns these for us according to the statistical test of interest.

- For example, a "medium" effect size for a proportion test:
  `cohen.ES(test="p", size="medium")`

- This returns 0.5.

Base R power and sample size functions

Base R includes three functions for calculating power and sample size:

- ▶ `power.prop.test`: two-sample test for proportions
- ▶ `power.t.test`: one-sample and two-sample t tests for means
- ▶ `power.anova.test`: one-way analysis of variance tests

These functions **do not** require calculating a unitless effect size and assume equal sample sizes across groups.

Leave one out

- The `pwr` functions and base R functions have `n` and `power` arguments.
- To calculate `power`, you **leave it out** of the function.
- To calculate sample size (`n`), you **leave it out** of the function.
- For example, to calculate the sample size needed for a one-sample proportion test to have 80% power assuming a "small" effect size of $h = 0.2$, significance level of 0.05 and a one-sided "greater" alternative:

```
pwr.p.test(h=0.2, power = 0.8, sig.level=0.05,
           alternative = "greater")
```

- This returns a sample size of 155. If there really is a "small" effect in the population, a sample size of $n = 155$ gives us a 80% chance of rejecting the null of no effect.

Let's get started!

- ▶ We'll go through each function available to us in the `pwr` package and base R.
- ▶ We'll go to R and demonstrate how to use it.
- ▶ I'll give you a quick opportunity to practice.
- ▶ As we'll see, understanding power and sample size analyses requires understanding the statistical test we're using.

one-sample test for proportions

- ▶ Test if a proportion is equal to some hypothesized value versus a null value, such as random chance, or 0.5.
- ▶ `pwr.p.test`
- ▶ Requires effect size h, which is the arcsine transformation. Use `ES.h` function.
- ▶ Why h? Observe 0.65 - 0.50 and 0.16 - 0.01 both equal 0.15. But 0.16 is 16 times larger than 0.01, while 0.65 is only 1.3 times larger than 0.50. The arcsine transformation basically captures these differences.
- ▶ Conventional effect sizes: 0.2 (small), 0.5 (medium) and 0.8 (large)
- ▶ Remember, effect size *h* is not a proportion. It ranges in practical value from about 0.02 to 3.

one-sample test for proportions - example

We think people place name tags on the left side of their chest 75% percent of the time versus random chance (50%). What sample size do we need to show this assuming a significance level (Type I error) of 0.05 and a desired power of 0.80?

one-sample test for proportions - code

```
library(pwr) # do this once per session
h <- ES.h(p1 = 0.75, p2 = 0.50)
pwr.p.test(h = h, sig.level = 0.05, power = 0.80,
           alternative = "greater")
```

```
##
##        proportion power calculation for binomial distribut
##
##              h = 0.5235988
##              n = 22.55126
##      sig.level = 0.05
##          power = 0.8
##    alternative = greater
```
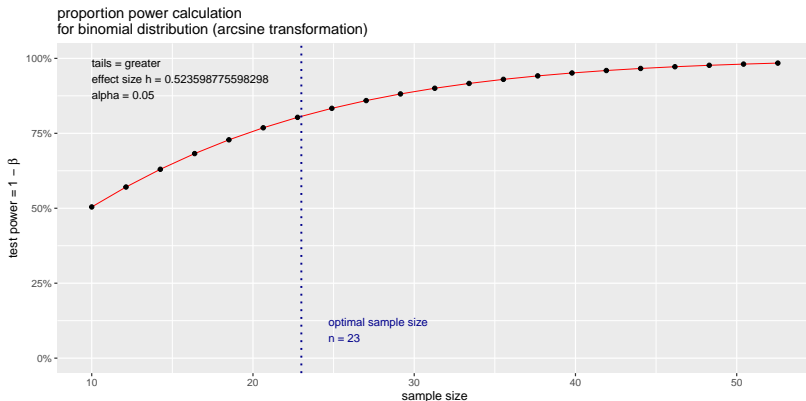
one-sample test for proportions - plot

```
plot(pwr.p.test(h = h, sig.level = 0.05, power = 0.80,
                alternative = "greater"))
```


proportion power calculation
for binomial distribution (arcsine transformation)

tails = greater
effect size h = 0.523598775598298
alpha = 0.05

optimal sample size
n = 23

one-sample test for proportions - results

- ▶ Always round up n. In our example, that gives us 23.
- ▶ Notice the argument `alternative = "greater"`. That was because we hypothesized greater than random chance (75% > 50%)
- ▶ A safer and more common approach is to accept the default alternative: `alternative = "two.sided"`
- ▶ The `two.sided` alternative says we're not sure which direction the effect is in. It results in a larger sample size.
- ▶ For the remainder of the workshop we'll almost always use the default `alternative = "two.sided"`

How effect size affects sample size

Sample size vs Effect size h
for 80% power and 0.05 significance level
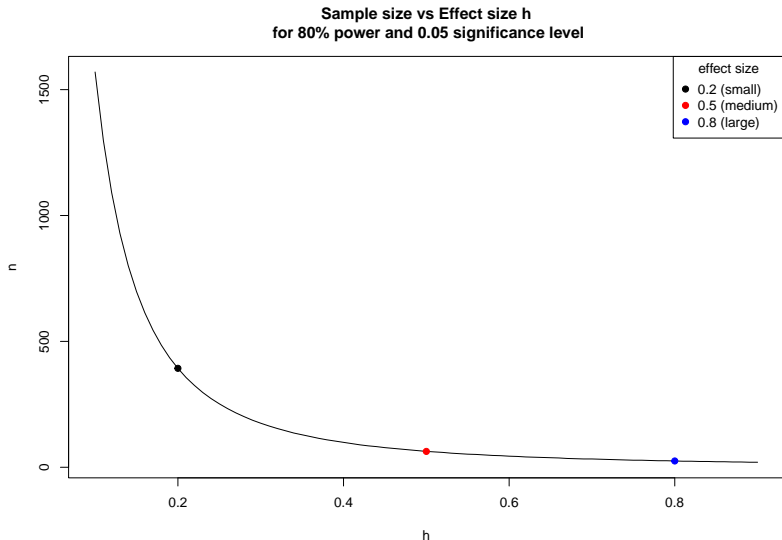
Let's go to R!

two-sample test for proportions

- Test if two proportions are equal. The Null is no difference.
- `pwr.2p.test` or `power.prop.test`
- `pwr.2p.test` requires effect size `h`. Use `ES.h` function. (Effect size depends on the two proportions we compare.)
- `power.prop.test` allows you to use the raw proportions in the function.
- Both return sample size *per group*.

two-sample test for proportions - example

We want to randomly sample male and female UVa undergrad students and ask them if they consume alcohol at least once a week. Our null hypothesis is no difference in the proportion that answer yes. Our alternative hypothesis is that there is a difference. (two-sided; one gender has higher proportion, I don't know which.) I'd like to detect a difference as small as 5%. How many students do I need to sample in each group if we want 80% power and a significance level of 0.05?

two-sample test for proportions - code

These return different sample sizes!

```r
# 55% vs. 50%
pwr.2p.test(h = ES.h(p1 = 0.55, p2 = 0.50),
            sig.level = 0.05, power = .80)
# 35% vs. 30%
pwr.2p.test(h = ES.h(p1 = 0.35, p2 = 0.30),
            sig.level = 0.05, power = .80)
# 15% vs. 10%
pwr.2p.test(h = ES.h(p1 = 0.15, p2 = 0.10),
            sig.level = 0.05, power = .80)
```

two-sample test for proportions - code

The base R function is perhaps a little easier to use:

```
power.prop.test(p1 = 0.55, p2 = 0.50,
                sig.level = 0.05, power = .80)
power.prop.test(p1 = 0.35, p2 = 0.30,
                sig.level = 0.05, power = .80)
power.prop.test(p1 = 0.15, p2 = 0.10,
                sig.level = 0.05, power = .80)
```

two-sample test for proportions - conventional effect size

▶ We may just want to use a conventional effect size if we're not comfortable specifying proportions

▶ Again, those are 0.2, 0.5, and 0.8

▶ Example

```
pwr.2p.test(h = 0.2, sig.level = 0.05, power = 0.8)
```

▶ We can only use conventional effect sizes with `pwr` functions

Let's go to R!

two-sample test for proportions, unequal sample sizes

- Test if two proportions are equal with unequal sample sizes. The Null is no difference.
- `pwr.2p2n.test`
- Requires effect size h. Use `ES.h` function.
- It has two `n` arguments: n1 and n2. Can be used to find a sample size for one group when we already know the size of the other.

two-sample test for proportions, unequal sample sizes - example

Let's return to our undergraduate survey of alcohol consumption. It turns out we were able to survey 543 males and 675 females. What's the power of our test with a significance level of 0.05? Let's say we're interested in being able to detect a "small" effect size (0.2).

two-sample test for proportions, unequal sample sizes - code

```
pwr.2p2n.test(h = 0.2,
              n1 = 543, n2 = 675,
              sig.level = 0.05)
```

Let's go to R!

one-sample, two-sample and paired t tests for means

- Test if a mean is equal to specific value (one-sample), test if means of two different groups are equal (two-sample), or test if "paired" means are equal
- `pwr.t.test` requires effect size d. d is the difference in population means divided by the standard deviation of either population (since they are assumed equal). Now we have to make a guess at the standard deviation.
- There is no function for effect size d. We have to calculate this ourselves if we wish to use `pwr.t.test`.
- Conventional effect sizes: 0.2, 0.5 and 0.8
- Specify type of test with `type` argument: `"two.sample"`, `"one.sample"`, `"paired"`

one-sample, two-sample and paired t tests for means

- ▶ The base R function `power.t.test` calculates effect size automatically given `delta` and `sd` arguments.
- ▶ `delta` is difference in means; `sd` is standard deviation
- ▶ Specify type of test with `type` argument: `"two.sample"`, `"one.sample"`, `"paired"`

two-sample t test - example 1

I'm interested to know if there is a difference in the mean price of what male and female students pay at the library coffee shop. Let's say I randomly observe 30 male and 30 female students check out from the coffee shop and note their total purchase price. How powerful is this experiment if I want to detect a "medium" effect in either direction with a 0.05 significance level?

two-sample t test - code

```
pwr.t.test(n = 30, d = 0.5, sig.level = 0.05)
```

```
##
##          Two-sample t test power calculation
##
##               n = 30
##               d = 0.5
##       sig.level = 0.05
##           power = 0.4778965
##     alternative = two.sided
##
## NOTE: n is number in *each* group
```

two-sample t test - example 2

- Let's say we want to be able to detect a difference of at least 75 cents in the mean purchase price. How can we convert that to an effect size?
- We need to make a guess at the population standard deviation. If we have absolutely no idea, one rule of thumb: take the difference between the maximum and minimum values and divide by 4 (or 6).
- Let's say max is \$10 and min is \$1. So our guess at a standard deviation is (10 - 1)/4 = 2.25.
- $d = 0.75/2.25 \approx 0.333$

two-sample t test - code

```
# requires d
pwr.t.test(d = 0.333, power = 0.80, sig.level = 0.05)

# does not require d
power.t.test(delta = 0.75, sd = 2.25,
             power = 0.80, sig.level = 0.05, )
```

one-sample and paired t test

- To calculate power and sample size for one-sample t test, set the `type` argument to `"one.sample"`
- A paired t-test is basically the same as a one-sample t test. Instead of one sample of individual observations, you have one sample of `pairs` of observations, where you take the difference between each pair to get a single sample of differences. These are commonly before and after measures on the same person.
- To calculate power and sample size for paired t test, set the `type` argument to `"paired"`

one-sample t test - example

I think the average purchase price at the Library coffee shop is over $3 per student. My null is $3 or less; my alternative is greater than $3. If the true average purchase price is $3.50, I would like to have 90% power to declare my estimated average purchase price is greater than $3. How many transactions do I need to observe assuming a significance level of 0.05?

Let's say max purchase price is $10 and min is $1. So our guess at a standard deviation is $9/4 = 2.25$.

one-sample t test - code

```
d <- 0.50/2.25
pwr.t.test(d = d, sig.level = 0.05, power = 0.90,
           alternative = "greater",
           type = "one.sample")

# or with power.t.test:
power.t.test(delta = 0.50, sd = 2.25, power = 0.90,
             sig.level = 0.05,
             alternative = "one.sided",
             type = "one.sample")
```

Let's go to R!

two-sample t test for means, unequal sample sizes

- Test if means from different groups are equal with unequal sample sizes. The Null is no difference.
- `pwr.t2n.test`
- Requires effect size d.
- It has two n arguments: n1 and n2. Can be used to find a sample size for one group when we already know the size of the other.

two-sample t test for means, unequal sample sizes - example

Let's say we have data on 35 male customers and estimated a mean purchase price. How many females do we need to sample to detect a medium gender effect of 0.5 with a desired power of 0.80 and a significance level is 0.05?

two-sample t test for means, unequal sample sizes - code

```
pwr.t2n.test(n2 = 35, d = 0.5, power = 0.8)
```

```
##
##          t test power calculation
##
##                  n1 = 321.7572
##                  n2 = 35
##                   d = 0.5
##           sig.level = 0.05
##               power = 0.8
##         alternative = two.sided
```

Let's go to R!

chi-squared tests

Two kinds of chi-squared tests:

1. goodness of fit test
2. test for association

- ▶ `pwr.chisq.test`
- ▶ Uses effect size `w`, which differs depending on the test.
- ▶ Use `ES.w1` for goodness of fit and `ES.w2` for test for association
- ▶ Also requires degrees of freedom: `df`
- ▶ conventional effect sizes: 0.1, 0.3, 0.5

chi-squared tests - goodness of fit

- ▶ A single dimension of proportions is tested against a prespecified set of proportions which constitutes the null hypothesis.
- ▶ Example: $H_0 : \frac{1}{3}, \frac{1}{3}, \frac{1}{3}$ vs $H_a : \frac{1}{2}, \frac{1}{4}, \frac{1}{4}$
- ▶ Rejecting the null means we have sufficient evidence to conclude the data don't appear to "fit" the prespecified set of proportions.
- ▶ If we were hoping to show our data "fit" the prespecified set of proportions, then failure to reject the Null is a good thing.
- ▶ df = number of categories - 1

chi-squared tests - test of association

- A table of counts classified by two variables is tested against the expected table of counts given the two variables are independent.
- Rejecting the null means the data appear to be associated in some way.
- df = (Var1 number of categories - 1) $\times$ (Var2 number of categories - 1)
- This test doesn't tell you anything about the strength or direction of association.

chi-square goodness of fit test - example

A market researcher is seeking to determine preference among 4 package designs. He arranges to have a panel of 100 consumers rate their favorite package design. He wants to perform a chi-square goodness of fit test against the null of equal preference (25% for each design) with a significance level of 0.05. What's the power of the test if 3/8 of the population actually prefers one of the designs and the remaining 5/8 are split over the other 3 designs? *(From Cohen, example 7.1)*

chi-square goodness of fit test - code

```
# To calculate effect size, we need to create vectors
# of null and alternative proportions:
null <- rep(0.25, 4)
alt <- c(3/8, rep((5/8)/3, 3))
pwr.chisq.test(w=ES.w1(P0 = null,P1 = alt),
               N=100, df=(4-1), sig.level=0.05)
```

Let's go to R!

Correlation test

- Test whether there is any linear relationship between two continuous variables. Null is correlation coefficient $r = 0$.
- `pwr.r.test`
- Testing if correlation is 0 is the same as testing if the slope in simple linear regression is 0.
- Correlation is already unitless, so we don't require a formula to calculate effect size.
- Conventional effect sizes: 0.1, 0.3, 0.5
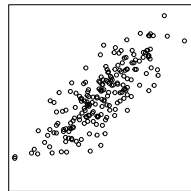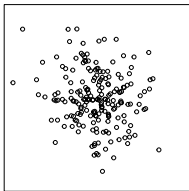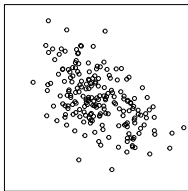
Correlation review
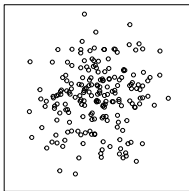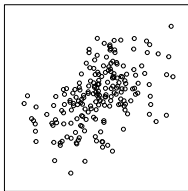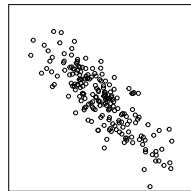
Correlation test - example

I'm a web developer and I want to conduct an experiment with one of my sites. I want to randomly select a group of people, ranging in age from 18 - 65, and time them how long it takes them to complete a task, say locate some piece of information. I suspect there may be a "small" positive linear relationship between time it takes to complete the task and age. How many subjects do I need to detect this positive (ie, $r > 0$) relationship with 80% power and the usual 0.05 significance level?

Correlation test - code

```
pwr.r.test(r = 0.1, sig.level = 0.05, power = 0.8,
           alternative = "greater")
```

```
##
##      approximate correlation power calculation (arctangh
##
##              n = 616.1032
##              r = 0.1
##      sig.level = 0.05
##          power = 0.8
##    alternative = greater
```

Let's go to R!

balanced one-way analysis of variance test

- ▶ ANOVA, or Analysis of Variance, tests whether or not means differ between more than 2 groups.
- ▶ "One-way" means one explanatory variable.
- ▶ "Balanced" means we have equal sample size in each group.
- ▶ The null hypothesis is that the means are all equal.
- ▶ `pwr.anova.test` or `power.anova.test`
- ▶ The `power.anova.test` function that comes with base R is easier to use than `pwr.anova.test` and does not require calculating an effect size.

balanced one-way analysis of variance test

- ▶ The `power.anova.test` function requires you to specify the number of groups, the between group variance (`between.var`), and the within group variance (`within.var`), which we assume is the same for all groups.
- ▶ The `pwr.anova.test` function requires you to provide an effect size, f.
- ▶ The effect size, f, for k groups is calculated as SD_{means} / $SD_{populations}$ (Translation: standard deviation of the k means divided by the common standard deviation of the populations involved.)
- ▶ conventional effect sizes: 0.1, 0.25, 0.4

balanced one-way analysis of variance test - example

I'm a web developer and I'm interested in 3 web site designs for a client. I'd like to know which design(s) help users find information fastest, or which design requires the most time. I design an experiment where I have 3 groups of randomly selected people use one of the designs to find some piece of information and I record how long it takes. (All groups look for the same information.) How many people do I need in each group if I believe two of the designs will take 30 seconds and one will take 25 seconds? Assume population standard deviation is 5 and that I desire power and significance levels of 0.8 and 0.05.

balanced one-way analysis of variance test - code

```r
# The between group variance: var(c(30, 30, 25)) = 8.3
# The within group variance: 5^2
power.anova.test(groups = 3, between.var = 8.3,
                 within.var = 5^2, power = 0.8)
```

Let's go to R!

test for the general linear model

- By "general linear model" we mean multiple regression.
- Test that the proportion of variance explained by the model predictors is 0. Equivalently, test whether all the model coefficients (except the intercept) are 0.
- `pwr.f2.test`
- This is a little tricky to use because not only do we have to supply an "effect size" ($f2$), we also have to supply numerator (u) and denominator (v) degrees of freedom instead of sample size.
- numerator (u) and denominator (v) degrees of freedom refer to the F test that tests whether all the model coefficients (except the intercept) are 0.
- conventional effect sizes: 0.02, 0.15, 0.35
- There is currently no built-in plot method.

test for the general linear model - effect size

- The f2 effect size is $R^2/(1 - R^2)$, where R^2 is the coefficient of determination, aka the "proportion of variance explained".
- To determine effect size you hypothesize the proportion of variance your model explains, or the R^2. For example, 0.45. This leads to an effect size of $0.45/(1 - 0.45) \approx 0.81$
- We can reverse this. Given an effect size, we can determine R^2: $ES/(1 + ES)$. For example, $0.81/(1 + 0.81) \approx 0.45$
- There is no function for this.

test for the general linear model - degrees of freedom

- The numerator degrees of freedom, u is the number of coefficients you'll have in your model (minus the intercept).
- The denominator degrees of freedom v is the number of error degrees of freedom. v = n - u - 1.
- if we want to determine sample size for a given power and effect size, we have to find v, which we then use to solve n = v + u + 1. (!)
- There is no n argument!

test for the general linear model - example

I'm hired to survey a company's workforce about job satisfaction. I ask employees to rate their satisfaction on a scale from 1 (hating life) to 10 (loving life). I know there will be variability in the answers, but I think two variables that will explain this variability are salary and age. In fact I think it will explain at least 30% ($R^2 = .30$) of the variance. How powerful is my "experiment" if I randomly recruit 40 employees and accept a 0.05 significance level?

test for the general linear model - code

```
# Two predictors, so u = 2
# 40 subjects, so v = 40 - 2 - 1 = 37
# R^2 = .30, so effect size f2 = 0.3/(1 - 0.3)

pwr.f2.test(u = 2, v = 37, f2 = 0.3/(1 - 0.3),
            sig.level = 0.05)
```

Let's go to R!

Other Software

Software

- ▶ PASS. http://www.ncss.com/software/pass/ ($395/year or $795 perpetual)
- ▶ nQuery. http://www.statsols.com/products/nquery-advisor-nterim/ ($440/year)
- ▶ PROC POWER in SAS. Power and sample size analyses for a variety of statistical analyses
- ▶ G*Power. http://www.gpower.hhu.de/en.html (Free)

R packages

- ▶ `TrialSize`. Functions and examples from the book *Sample Size Calculation in Clinical Research*
- ▶ `samplesize`. Computes sample size for Student's t-test and for the Wilcoxon-Mann-Whitney test for categorical data
- ▶ `clinfun`. Functions for both design and analysis of clinical trials.

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Thanks for coming today!

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