Aerial Maneuvering for UAV via Feedback Control System

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Abstract

In 2012, the Federal Aviation Administration Modernization and Reform act layed the foundation for Unmanned Aerial Vehicles to be owned and operated within the United States. This prospect opens up a vast array of different applications for these drones, many of which require navigation in an urban environment. This paper is concerned with designing a control system for a small, quadcopter implementation of a UAV. The application described in this study is that of package delivery, but any number of other applications are feasible for the design suggested. The scope of this paper is limited to controlling the altitude component of the system, as well as touch on some other components such direction control and collision detection.

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Introduction

This paper attempts to create a feedback control system to control the movement of an UAV (Unmanned Aerial Vehicle). Because the discussion of an entire system to control the UAV would be too broad, I will limit my control system to the controlling the altitude component of the system by controlling the thrust, as well as touch on some other components such direction control and collision detection.

Relevance

In the past year, legislations in California and its border states have placed a significant amount of emphasis on the legality of owning and operating UAVs (Unmanned Aerial Vehicles) [1]. While the specifics about safety and privacy for the public are not agreed upon, the commercial and scientific benefits of such systems are widely accepted. However, because they have been illegal to use without permission from the government, little research has been done on the commercial applications of UAVs.

In 2011, the Federal Aviation Administration Modernization and Reform act [1, 2] was signed into federal law, which states in Title III Subtitle B: "the Secretary of Transportation, in consultation with representatives of the aviation industry, Federal agencies that employ unmanned aircraft systems technology in the national airspace system, and the unmanned aircraft systems industry, shall develop a comprehensive plan to safely accelerate the integration of civil unmanned aircraft systems into the national airspace system." Operational and certification requirements for UAVs are to be established no later than December 31, 2015. On February 22, 2013, California Assembly members Gorell and Bradford were the first to introduce legislation that attempted to implement this plan [2]. California bills AB1326 and AB1327 aimed to make the commercial and personal ownership of UAVs legal, as well as provide tax benefits to those that chose to invest in such progressive ventures.

The delivery of packages by UAVs would be more efficient because of the amount of time spent on this repetitive task by humans would be greatly diminished. An UAV could deliver a single package more quickly than a human could, because it can take a more direct route from the post office without having to reroute because of traffic or wait at stopping lights. Lastly, the one time investment in the UAV would soon pay for itself and be a much more affordable option than paying a human by the hour. These combined benefits prove that UAVs that could deliver packages in the United States airspace would be a very desirable product.

Literary Analysis

One particularly useful paper is primarily concerned with the implementation of a control system for a ducted fan to pilot a UAV so that it may physically interact with an unknown environment [3]. The author begins by briefly discusses force feedback for robotic systems. To provide a better understanding of the forces involved for the proposed design, the author goes into detail about the mechanisms of a ducted fan. This model describes the thrust generated by the propeller in the vertical direction [3]:

$$V_i = \sqrt{\frac{T}{\rho S_{disk}}} \tag{1}$$

Where ρ is the density of air and S_{disk} is the area of the disk. The paper also presents the formula for lift and drag, namely

$$F_L = \frac{1}{2}\rho {V_i}^2 C_L S \tag{2}$$

and

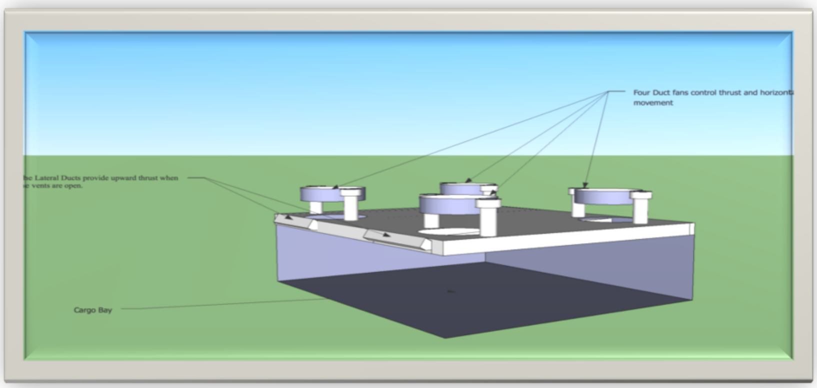
$$F_D = \frac{1}{2}\rho {V_i}^2 C_D S \tag{3}$$

These equations, along with the exact mechanisms involved in the force analysis, are further explained in the dynamical equations analysis portion of this paper. For use in this paper, these equations have been simplified to keep calculations concise.

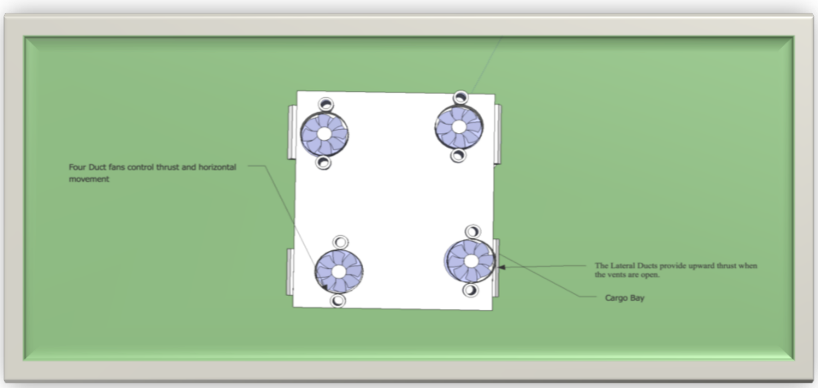
Design Specifications

Movement

Figures 1a and 1b show a initial mockup of the UAV created in Google Sketchup. The control system contains four duct fans placed in each corner of the rectangular craft. For simplicity, I have left key areas open in the diagram, such as the remaining walls of the cargo bay and the four air channels connecting the fans to the lateral ducts. The lateral and horizontal movement will be controlled completely by a series of channels inside the ducts that will push the air evenly through the exit vents. As you can see from the diagram, forcing the air downwards through the vents placed on the side rather than exiting directly beneath the cargo bay achieves thrust. This design exerts force in the optimal position to maintain balance while maximizing thrust. The exit vents on the side promote rotational and horizontal movement depending on how you force the air.



(a) UAV created in Google Sketchup (Side View)



(b) UAV created in Google Sketchup (Top View)

Parts and Dimensions

For this design, I chose four ducted fans (effectively a quadcopter/ducted fan hybrid) because it provides a maximum thrust to weight ratio while maintaining maneuverability. Maneuverability is highly dependent upon the rate at which your fan can adjust thrust (or rotations per minute), so the conciseness of the fans control system is key to creating a craft that is agile [3]. Furthermore, the VTOL (vertical takeoff or landing) design is essential to a successful package delivery application because of tightly packed residential and commercial buildings we find in cities today. One of the major tasks of this project will be finding a ducted fan that maintains an acceptable balance of thrust and weight.

Because I have not tested any ducted fans for optimal performance and the lack of detailed online specifications, the dimensions for this project cannot be accurately projected. I can, however, make an educated guess and determine a reasonable range for these values. Through various articles and papers on quad motors thrust to weight ratio, I have determined that a reasonable value for thrust to weight ratio is 2:1. A commonly used EDF system (complete with motor) from Dr. Mad provides 2.35 kg of thrust while only adding 313 g to the craft per motor [4]. If we were to add four of these to our system, we would have 8148 g of thrust remaining, or the ability for our payload to be 4074 g, or about 9 lb. for materials to make up the craft and the payload. I am anticipating the craft to be about 1.75 ft. x 1.25 ft x 1 ft., since we have relatively small fans. If one wanted to design a system to carry larger packages, more powerful fans must be used.

Control System

In this design, sensors would include accelerometers for pitch and yaw calculations, as well as GPS sensors for locational data. The controller would be a small microprocessor that makes calculations based on the sensor data and provides data to the plant on how to direct the fans. The plant will be comprised of the duct fan controls as well as the position on the ventilation shafts directing air either horizontally or vertically. Disturbance could come from quickly changing wind speeds, humidity, air pollution, and collisions. The output and desired variable would be dependent upon the state of the craft, which include loading, takeoff, transport, landing, and delivery.

Block Diagram

The block diagram for this design is described in Figure 2. The controller consists of a microcontroller that makes mathematical calculations based on sensor data about pitch, yaw, and altitude and adjusts the thrust. The actuator contains the plant and output signal. These create the output signal which is fed back into the control system. The disturbance comes from high wind speeds and objects in the path.

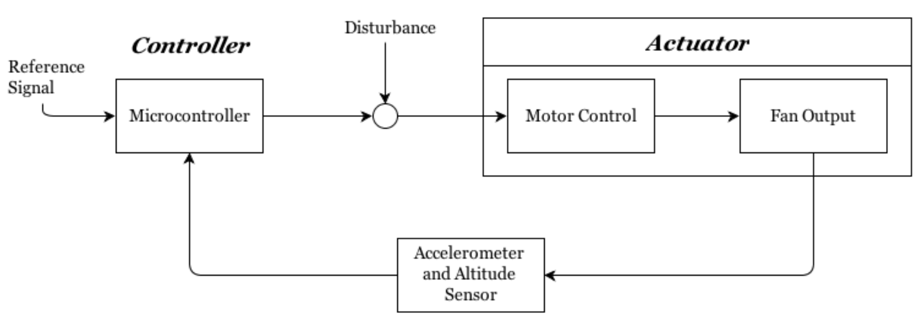


Figure 2: Block Diagram of the System

Dynamical Equations Analysis

A complete analysis of our UAV requires an in depth analysis of the forces applied to the aircraft in flight. We accomplish this by deriving a set of differential equations, called *dynamical equations* that reduce the behavior of the craft to a function of its parameters. The following describes the dynamical equations acting on the craft in flight, while ignoring the minor forces that would complicate our simple model. In our analysis, we will assume that F_x and F_y are some scable force applied by our thrusters in the x and y directions respectively. To gain intuition on how our craft might react based on the force applied by the thrusters, let's assume a simple model for our thrusters, shown in Figure 3.

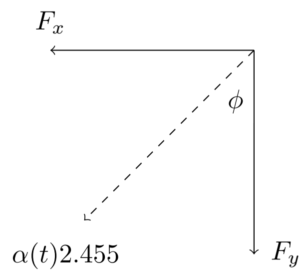


Figure 3: Simple Model of Thruster Forces

This model assumes that the thrusters are pointed in the downward direction, are tilted at the angle ϕ, and that there exists some constant $\alpha(t)$ with the range of 0 to 1 by which the force applied by the thrusters are scaled. Thrust is determined by Fraud's momentum theorm [3] given in Equation 1. Although this formula is useful in general, the maximum thrust of each duct fan is given in the specifications as 2.455 kg [4]. Upon inspection of Figure 3, it is clear that

$$\sin(\phi) = \frac{F_x}{\alpha(t)2.455} \Rightarrow \alpha(t)2.455\sin(\phi) = F_x. \tag{4}$$

$$\cos(\phi) = \frac{F_y}{\alpha(t)2.455} \Rightarrow \alpha(t)2.455\cos(\phi) = F_y. \tag{5}$$

With an accurate estimation of the forces being applied in both the x and y directions, a free-body diagram can now be constructed for the UAV in operation, shown in figures 4 and 5.

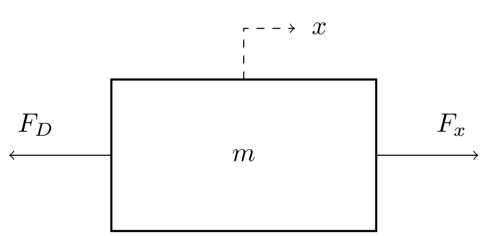


Figure 4: Horizontal Forces, Side View

In Figure 4, F_D and F_L are the forces due to drag in the x and y directions. These forces can be modelled by the following equation:

$$F_D = \frac{1}{2}\rho\dot{x}^2 C_D A \text{ and } F_L = \frac{1}{2}\rho\dot{y}^2 C_L A \tag{6}$$

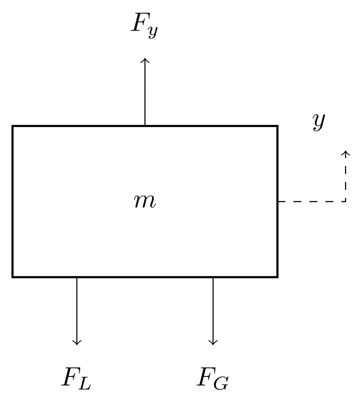


Figure 5: Vertical Forces, Side View

where ρ is the density of the fluid, $\dot{x}$ is the veloctiy, A is the cross-sectional area, and C_D/C_L are the drag coefficients in the x and y directions.

$$\rho = 287.058 \frac{J}{kg \cdot k} \Rightarrow 1.1839 \frac{kg}{m^3} \text{ at } 77°\text{F} \tag{7}$$

$$A = 1.75ft \cdot 1.25ft = 2.1875ft^2 = 0.2m^2 \tag{8}$$

$$C_D \Rightarrow \text{similar to a model rocket} \Rightarrow 0.75 \tag{9}$$

The force of gravity on the craft (F_G) remains constant within our altitude limit, namely:

$$F_G = 9.80665 \frac{m}{s^2} \tag{10}$$

We can solve for F_D in terms of velocity

$$F_D = \frac{1}{2} \cdot 1.1839 \frac{kg}{m^3} \cdot \dot{x}^2 \cdot 0.75 \cdot 0.2m^2 \Rightarrow 0.089\dot{x}^2 \tag{11}$$

Because F_D is the same as F_L when in the y-plane (accepting that C_L is proportional to C_D:

$$F_L = 0.089\dot{y}^2 \tag{12}$$

Figure 5 is concerned with the forces in the y direction. As previously stated, F_y is the force of the thrusters propelling the craft up or down.

To determine the force of F_D and F_L, one must first examine the generalized formula for drag in Equation 3. Clearly, the relationship between the force applied by the drag is not linearly related to the velocity of the object. It is common practice to relate the speed v to nonlinear drag force by some coefficient b to make the system linear [5]. This is accomplished by Taylor series analysis of Formula 6, when all constants are ignored:

$$\text{Let } w(t) = \dot{y}^2 \tag{13}$$

Therefore, it follows that

$$w(t) = w(t_0) + w'(t_0)(t - t_0) + \frac{1}{2!} w''(t_0)(t - t_0)^2 + ... \tag{14}$$

So plugging in $w(t) = v^2$ we get

$$v(t)^2 = v(t_0)^2 + v'(t_0)^2(t_0)(t-t_0) + (2!)v''(t_0)^2(t_0)(t-t_0)^2 + ... \tag{15}$$

and by evaluating this series at infinity, we get

$$v(t)^2 = K + 2v(t_0)(t_0)(t-t_0) + 0 + ... = 2\dot{y}(t_0) \tag{16}$$

So, we can substitute Equation 16 into Equation 11 to get

$$F_L = 0.089\dot{y}^2 \Rightarrow 0.0178\dot{y} \tag{17}$$

From these identities, we can develop a generalized formula for our dynamical equations in the horizontal and the vertical directions:

$$m\ddot{x} = F_x - F_D \tag{18}$$

$$m\ddot{y} = F_y - F_G - F_L \tag{19}$$

Substituting our known values in, we get our final dynamical equations:

$$m\ddot{x} + 0.0178\dot{x} = \alpha(t)0.602\sin(\phi) \tag{20}$$

$$m\ddot{y} + 0.0178\dot{y} = \alpha(t)0.602\cos(\phi) - 9.8 \tag{21}$$

Since this paper is only concerned with the vertical direction, let's find our transfer function $U(s)$. Dividing through by the mass, we get

$$\ddot{y} + \frac{0.0178}{m}\dot{y} = \frac{0.602\cos(\phi)}{m}\alpha(t) - \frac{9.8}{m} \tag{22}$$

Extracting constants b_1, ..., b_n and assuming we only generate upward thrust ($\phi = 0$), we get:

$$\ddot{y} + b_1\dot{y} = b_2\alpha(t) - 9.8 \tag{23}$$

From transforming Equation 23 into the laplace domain, assuming $y(0) = 0$ and $\dot{y}(0) = 0$, and allowing b_1 and b_2 to be realistics values of our system based upon our design specifications. $b_1 = 2$ because of Equation 16. b_2 is near 1 because when $\phi = 0$, all of the force is in the y-plane, so we get:

$$Y(s)[s^2 + 2s + 9.8] = A(s) \tag{24}$$

Therefore

$$U(S) = \frac{Y(s)}{A(s)} = \frac{1}{s^2 + 2s + 9.8} \tag{25}$$

Therefore, the roots of this system are

$$s^2 + 2s + 9.8 = 0 \Rightarrow s = -1 \pm j2.966 \tag{26}$$

Control System Analysis

Loop Gain

To control the plant output, given by Equation 25, we will design a PID controller such that our controller equation is as follows:

$$w(s) = k_p + k_d s + \frac{k_i}{s} \Rightarrow \frac{k_d s^2 + k_p s + k_i}{s} \tag{27}$$

Therefore, our forward loop gain function becomes

$$G_{OL}(s) = \frac{k_d s^2 + k_p s + k_i}{s(s^2 + 2 * s + 9.8)} \Rightarrow \frac{k_d s^2 + k_p s + k_i}{s^3 + 2s^2 + 9.8s} \tag{28}$$

Since we have a feedback coefficient of 1, our closed-loop gain formula becomes

$$G_{CL}(s) = \frac{\frac{k_d s^2 + k_p s + k_i}{s^3 + 2s^2 + 9.8s}}{1 + \frac{k_d s^2 + k_p s + k_i}{s^3 + 2s^2 + 9.8s}} \tag{29}$$

Root Locus

To determine a root locus plot for Equation 28, we must assign two of the PID coefficients to a acceptable value and see how the poles vary with respect to the third parameter. We should model our equation to look like so:

$$c(s) = a(s) + kb(s) \Rightarrow 1 + k\frac{b(s)}{a(s)} \tag{30}$$

where k is the varying parameter. Let's begin with $k_d = k_i = 1$ and solve for k_p. By subsituting these values into equation 28, we get:

$$\frac{s^2 + k_p s + 1}{s^3 + 2s^2 + 9.8s} \Rightarrow \frac{s^2 + 1}{s^3 + 2s^2 + 9.8s} + \frac{k_p s}{s^3 + 2s^2 + 9.8s} \Rightarrow 1 + k_p \frac{s}{s^2 + 1} \tag{31}$$

Figure 6a shows the root locus of this function, which describes how the poles are varied as k_p is varied. Let's assume with the data from Figure 6a that $k_p = 0.75, k_d = 1$ and solve for k_i. Similarly to Equation 31:

$$\frac{s^2 + 075s + k_i}{s^3 + 2s^2 + 9.8s} \Rightarrow \frac{s^2 + 0.75s}{s^3 + 2s^2 + 9.8s} + \frac{k_i}{s^3 + 2s^2 + 9.8s} \Rightarrow 1 + k_i \frac{1}{s^2 + 0.75s} \tag{32}$$

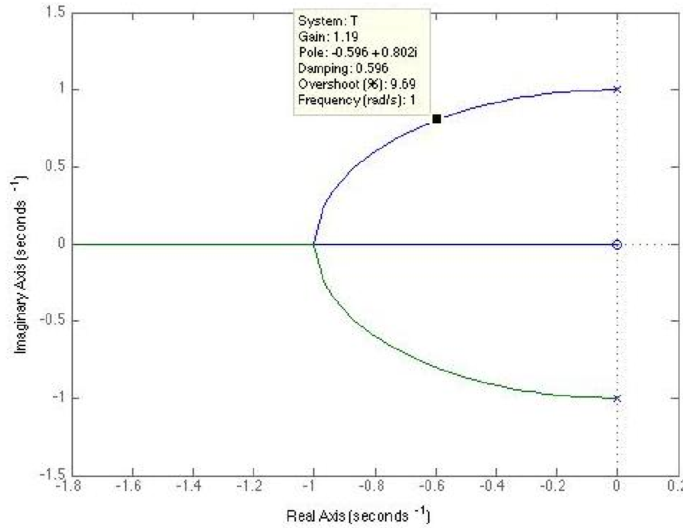
Figure 6b shows how the poles react as k_i is varied. Continuing on, lets take the root locus with respect to k_d when $k_p = 0.75, k_i = 18$.

$$\frac{k_d s^2 + 0.75s + 18}{s^3 + 2s^2 + 9.8s} \Rightarrow \frac{0.75s + 18}{s^3 + 2s^2 + 9.8s} + \frac{k_d s^2}{s^3 + 2s^2 + 9.8s} \Rightarrow 1 + k_d \frac{s^2}{0.75s + 18}. \tag{33}$$

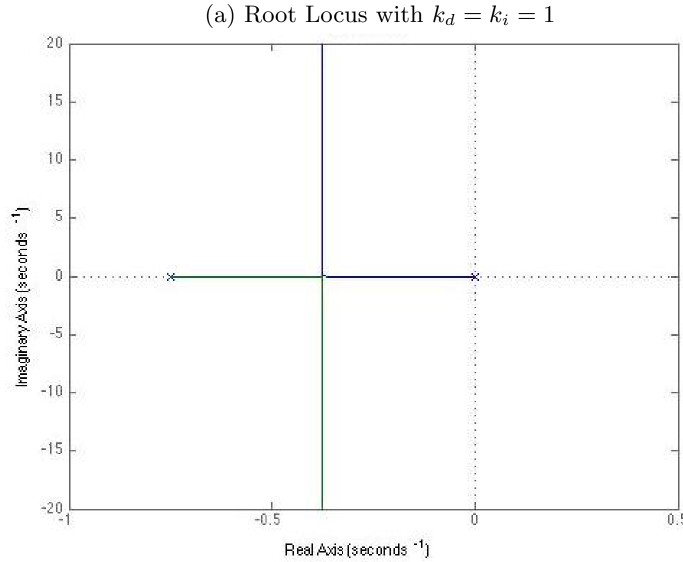
Figure 6c shows how the poles react as k_d is varied.

Final PID Parameters

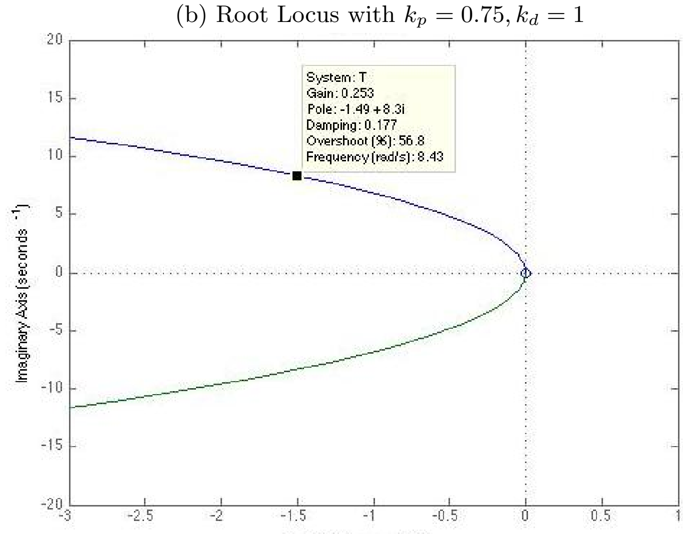
The root locus method was iterated several times with the same technique as above. The final optimized parameters for this system are as follows: $k_p = 19$, $k_i = 18$, $k_d = 1$. From simulations done on the control system, the rise time was found to be 0.3238 seconds, the overshoot was found to be 9.1706%, and the settling time was found to be 4.1116 seconds. Figure 7 shows the begining vs. the final step response of the system.



(a) Root Locus with $k_d = k_i = 1$



(b) Root Locus with $k_p = 0.75, k_d = 1$



(c) Root Locus with $k_p = 0.75, k_i = 18$

Figure 6: Root Locus Graphs for varying k_d, k_i, k_p

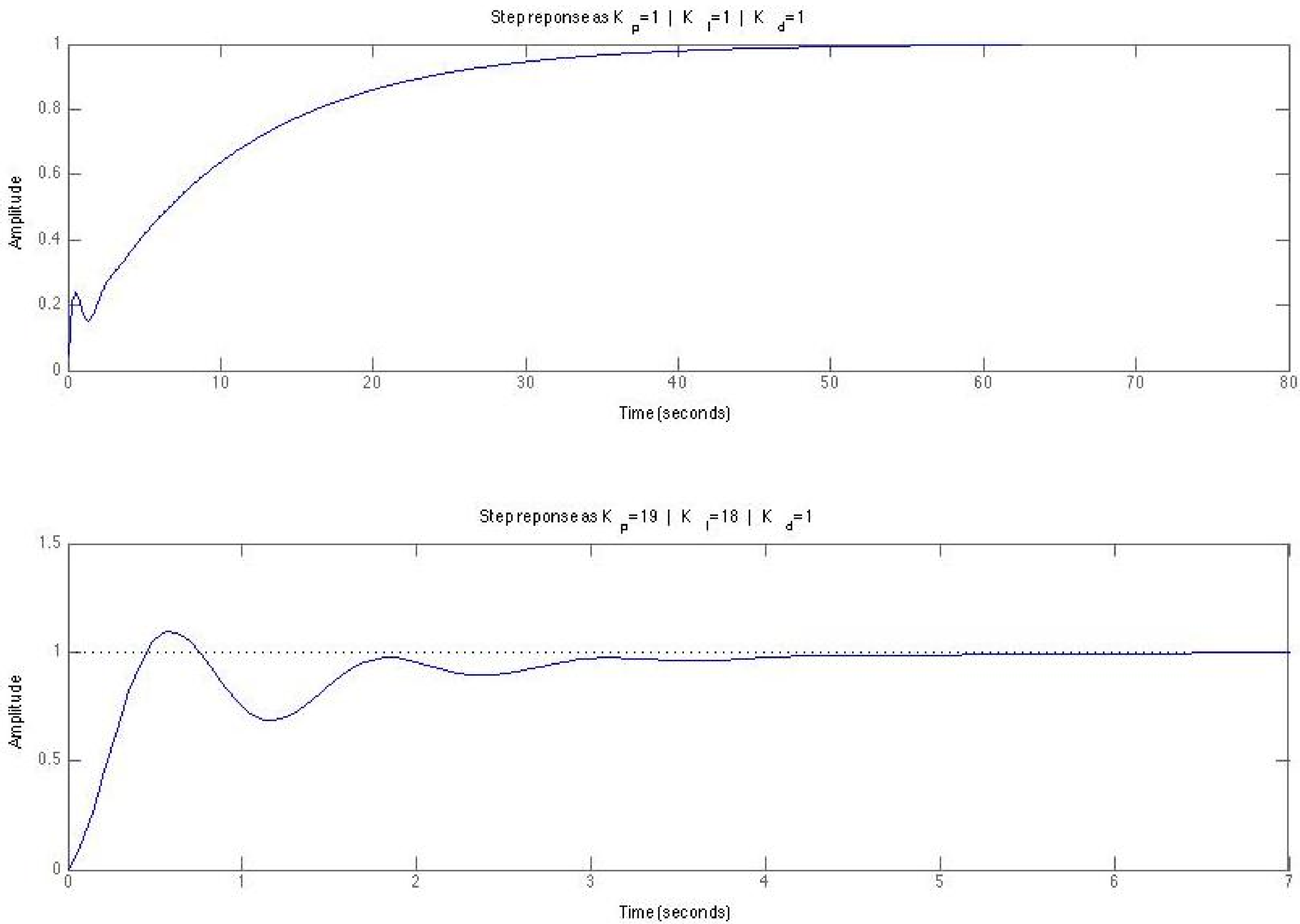


Figure 7: Beginning vs. Final step response for the system

Conclusion

This design has been optimized for maneuverability and accessibility as defined in the specifics section of this paper. All of the parts included in this design are readily available for anyone to purchase and build, and thus the design of such a control system is very practical and could be extremely beneficial. Through the use of a PID controller, the control system described will accurately control the lift of a UAV that delivers packages to customers. Such a product is not so impractical as a consumer might think at first glance, and might be available on the market much sooner than we had imagined just a few years ago.

References

[1] Rep. John Mica, "FAA Modernization and Reform Act of 2012," *H.R. 658*, vol. 112, 2011.

[2] Rep. Jeff Gorell; Rep. Steven Bradford, "AB-1327 Unmanned aircraft systems," *California Assembly Bill*, 2013.

[3] Marconi, L.; Naldi, R., "Control of Aerial Robots: Hybrid Force and Position Feedback for a Ducted Fan," *Control Systems, IEEE*, vol. 32, pp. 43,65, August 2012.

[4] HobbyKing.com, "Dr. Mad Thrust 70mm 10-Blade Alloy EDF 2200kv Motor - 1900watt (6s)," November 2013.

[5] G. Franklin;, *Feedback Control of Dynamical Systems*, vol. I. Pearson, 6th ed., 2009.

Appendicies

A. MATLAB Code

Listing 1: Example of plotting root locus

```
s = tf('s');
T = 1/(s^2 + 0.75*s);
figure;
rlocus(T);
axis([-1 .5 -20 20]);
```

Listing 2: Beginning and final step response

```
s = tf('s');
Kp = 1;
Ki = 1;
Kd = 1;
controller = pid(Kp,Ki,Kd)
transfer = 1/(s^2+2*s+9.8);
T = feedback(controller*transfer,1)
figure;
subplot(2, 1, 1);
step(T)
title(sprintf('Step reponse as K_p=%d   |   K_i=%d   |   K_d=%d', Kp, Ki, Kd));

Kp = 19;
Ki = 18;
Kd = 1;
controller = pid(Kp,Ki,Kd)
T = feedback(controller*transfer,1)
subplot(2, 1, 2);
step(T)
stepinfo(T)
title(sprintf('Step reponse as K_p=%d   |   K_i=%d   |   K_d=%d', Kp, Ki, Kd))
```