Binary Fuse Filters: Fast and Tiny Immutable Filters

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Probabilistic filters?

- Is x in the set S?
- Maybe or *definitively not*

Usage scenario?

- We have this expensive *database*. Querying it cost you.
- Most queries should not end up in the data.
- We want a small 'filter' that can prune out queries.

Theoretical bound

- Given N elements in the set
- Spend k bits per element
- Get a false positive rate of $1/2^k$

Usual constraints

- Fixed initial capacity
- Difficult to update safely without access to the set
- To get a 1% false-positive rate: ≈ 8 bits?

Hash function

- From any objet in the *universe* to a *word* (e.g., 64-bit word)
- Result looks random

```c
uint64_t murmur64(uint64_t h) {
  h ^= h >> 33;
  h *= UINT64_C(0xff51afd7ed558ccd);
  h ^= h >> 33;
  h *= UINT64_C(0xc4ceb9fe1a85ec53);
  h ^= h >> 33;
  return h;
}
```

Conventional Bloom filter

- Start with a bitset B.
- Using `k` hash functions $f_1, f_2, \ldots$.

Adding an element

- Given an object x from the set, set up to `k` bits to 1
- $B[f_1(x)] \leftarrow 1, B[f_2(x)] \leftarrow 1, \ldots$

Checking an element

- Given an object x from the universe, set up to `k` bits to 1
- $(B[f_1(x)] = 1) \text{ AND } (B[f_2(x)] = 1) \text{ AND } \ldots$

Checking an element: implementation

- Typical implementation is *branchy*
- If not $(B[f_1(x)] = 1)$, return false
- If not $(B[f_2(x)] = 1)$, return false
- ...
- return true

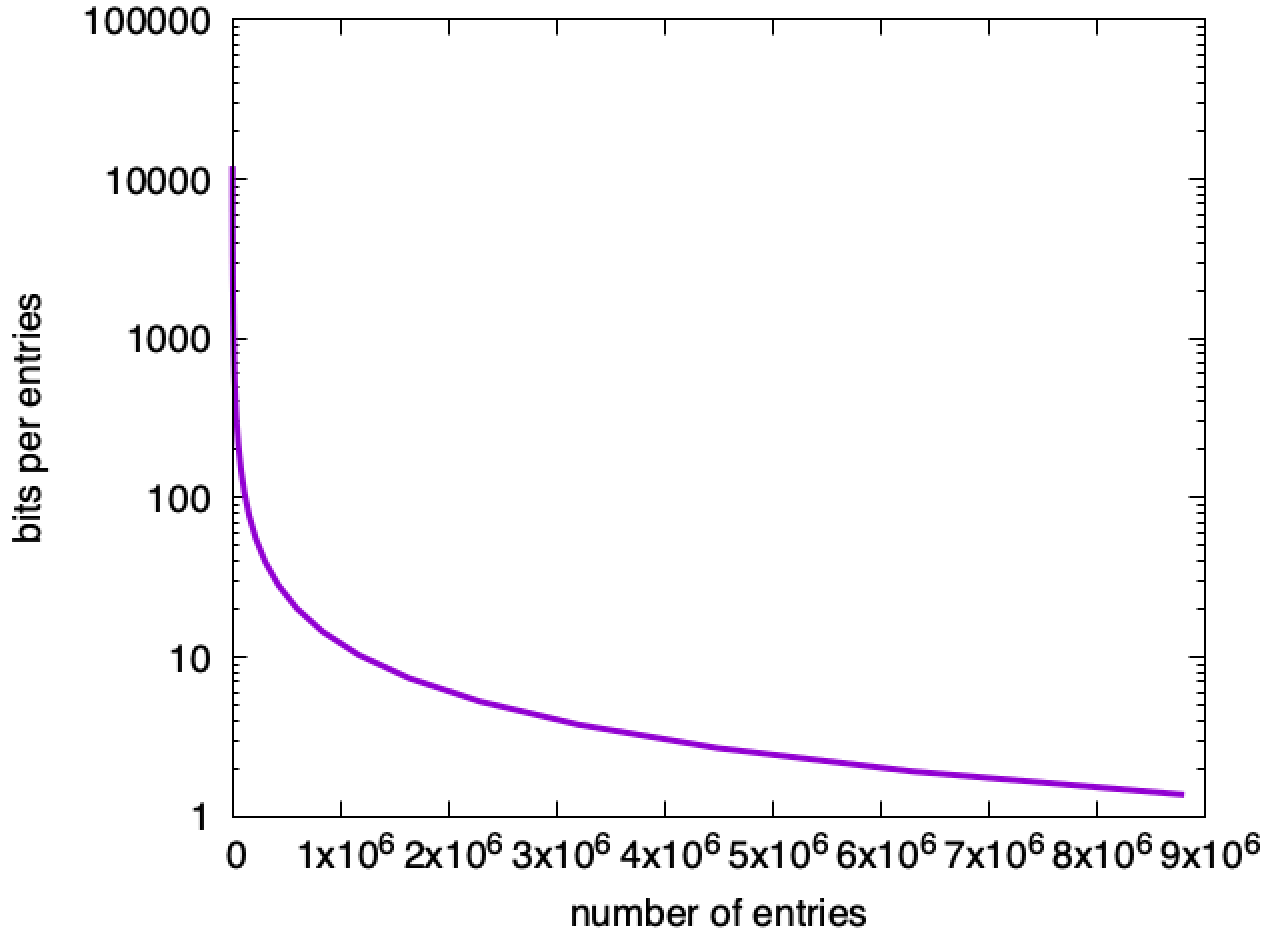
```
uint64_t hash = hasher(key);
uint64_t a = (hash >> 32) | (hash << 32);
uint64_t b = hash;
for (int i = 0; i < k; i++) {
  if ((data[reduce(a, length)] & getBit(a)) == 0) {
    return NotFound;
  }
  a += b;
}
return Found;
```

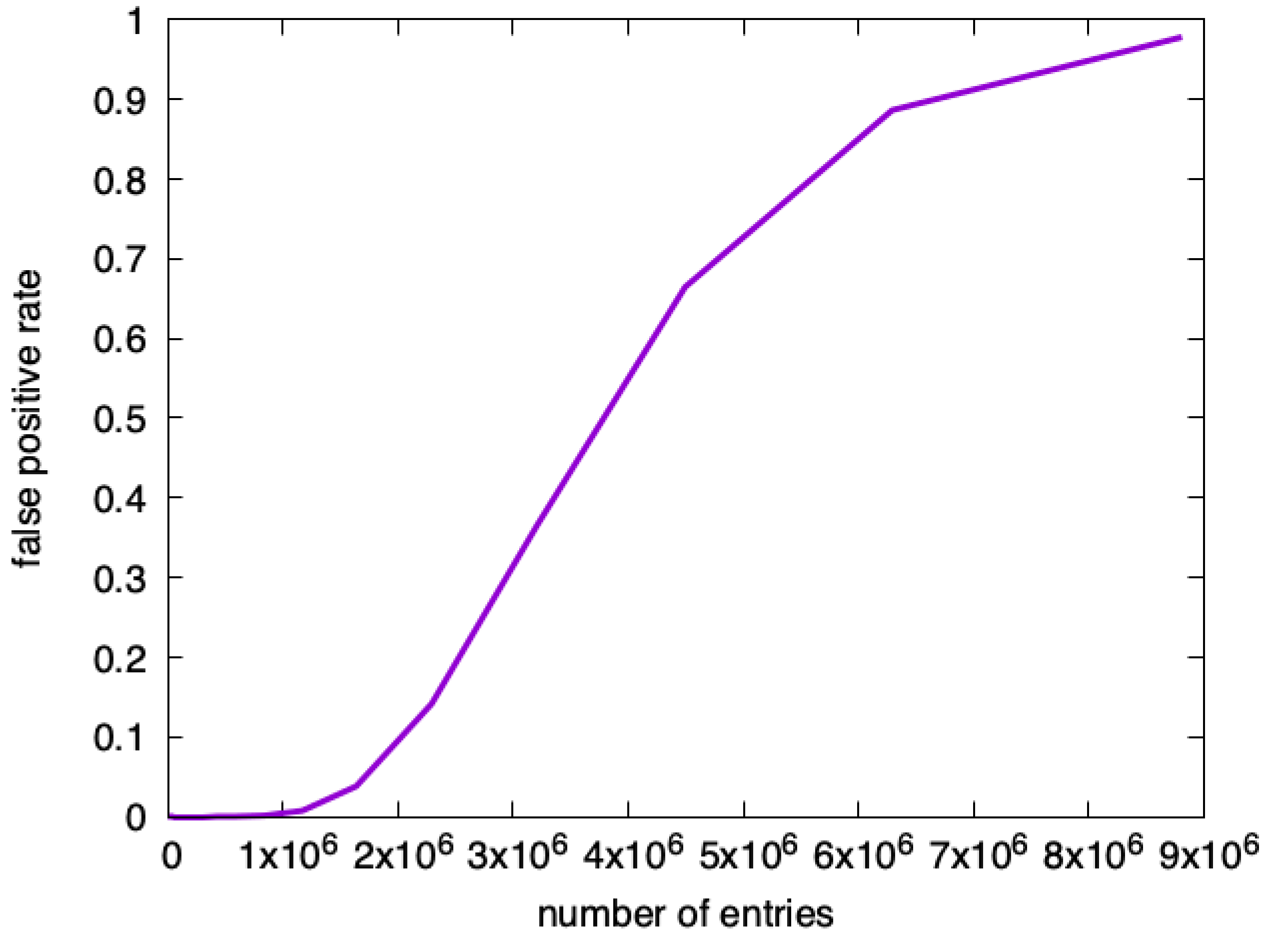
False positive rate

bits per element	hash functions	fpp
9	6	1.3%
10	7	0.8%
12	8	0.3%
13	9	0.2%
15	10	0.07%
16	11	0.04%

Bloom filters: upsides

- Fast construction
- Flexible: excess capacity translates into lower false positive rate
- Degrades smoothly to a useless but 'correct' filter

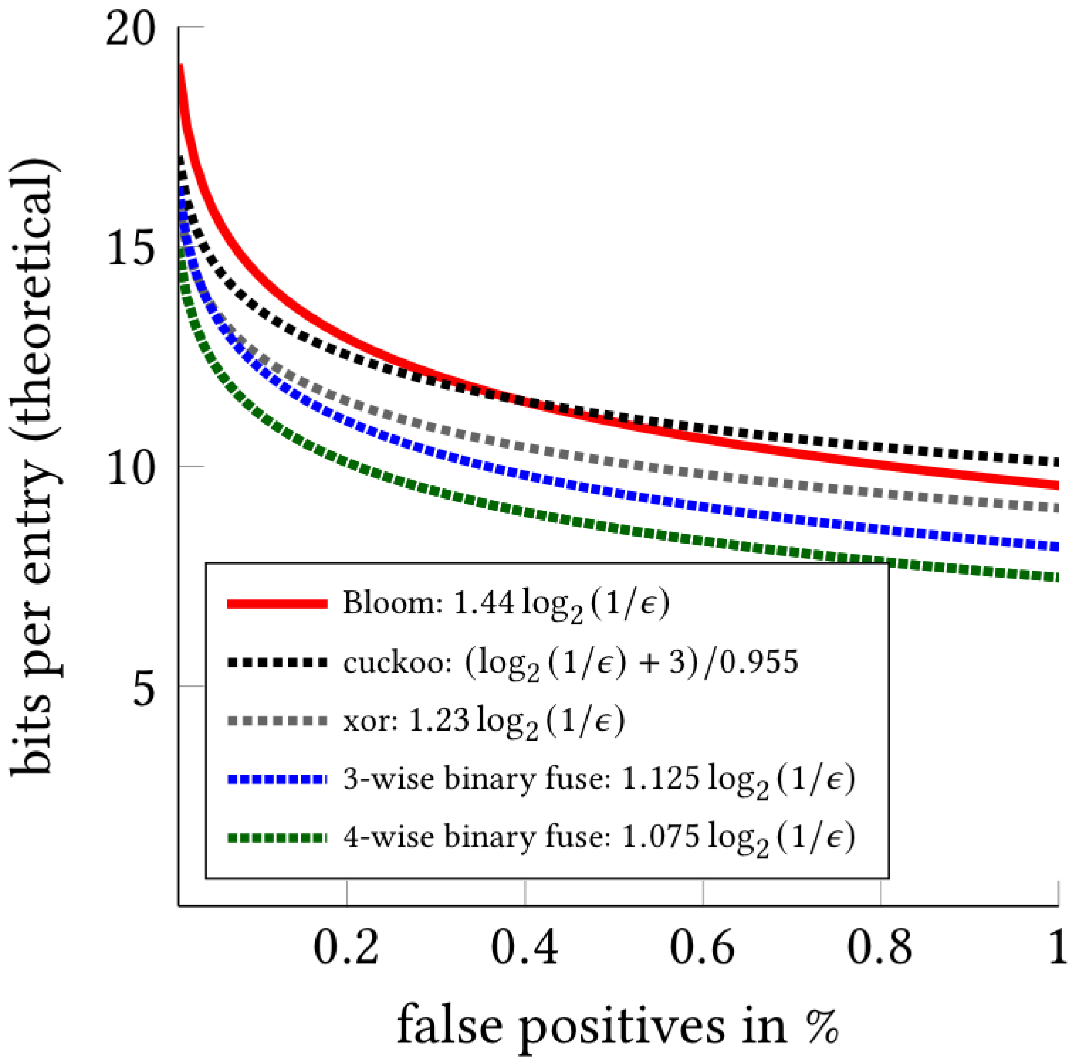

100000
10000
1000
100
10
1
0
1x10^6
2x10^6
3x10^6
4x10^6
5x10^6
6x10^6
7x10^6
8x10^6
9x10^6
number of entries
bits per entries



1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
false positive rate
0
1x10^6
2x10^6
3x10^6
4x10^6
5x10^6
6x10^6
7x10^6
8x10^6
9x10^6
number of entries

Bloom filters: downsides

- 44% above the theoretical minimum in storage
- Slower than alternatives (lots of memory accesses)


20
15
10
5
bits per entry (theoretical)
0.2
0.4
0.6
0.8
1
false positives in %
Bloom: $1.44 \log_2(1/\epsilon)$
cuckoo: $(\log_2(1/\epsilon) + 3)/0.955$
xor: $1.23 \log_2(1/\epsilon)$
3-wise binary fuse: $1.125 \log_2(1/\epsilon)$
4-wise binary fuse: $1.075 \log_2(1/\epsilon)$

Memory accesses

number of hash functions	cache misses (miss)	cache misses (hit)
8	3.5	7.5
11	3.8	10.5

(Intel Ice Lake processor, out-of-cache filter)

Mispredicted branches

number of hash functions	all out	all in
8	0.95	0.0
11	0.95	0.0

(Intel Ice Lake processor, out-of-cache filter)

Performance

number of hash functions	always out (cycles/entry)	always in (cycles/entry)
8	135	170
11	140	230

(Intel Ice Lake processor, out-of-cache filter)

Blocked Bloom filters

- Same as a Bloom filters, but for a given object, put all bits in one cache line
- Optional: Use SIMD instructions to reduce instruction count

Blocked Bloom filters: pros/cons

- Stupidly fast in both construction and queries
- ~56% above the theoretical minimum in storage

```
auto hash = hasher_(key);
uint32_t bucket_idx = reduce(rotl64(hash, 32), bucketCount);
__m256i mask = MakeMask(hash);
__m256i bucket = directory[bucket_idx];
return _mm256_testc_si256(bucket, mask);
```

Binary fuse filters

- Based on theoretical work by Dietzfelbinger and Walzer
- Immutable datastructure: build it once
- Fill it to capacity
- Fast construction
- Fast and simple queries

Arity : 3-wise, 4-wise

- 3-wise version has three hits, 12% overhead
- 4-wise version has four hits, 8% overhead

Queries are silly

- Have an array of *fingerprints* (e.g., 8-bit words)
- Compute 3 (or 4) hash functions: $f_1(x), f_2(x), f_3(x)$
- Compute fingerprint function ($f(x) \to$ 8-bit word)
- Compute XOR and compare with fingerprint:
  $B[f_1(x)]\ \mathrm{XOR}\ B[f_2(x)]\ \mathrm{XOR}\ B[f_3(x)] = f(x)$

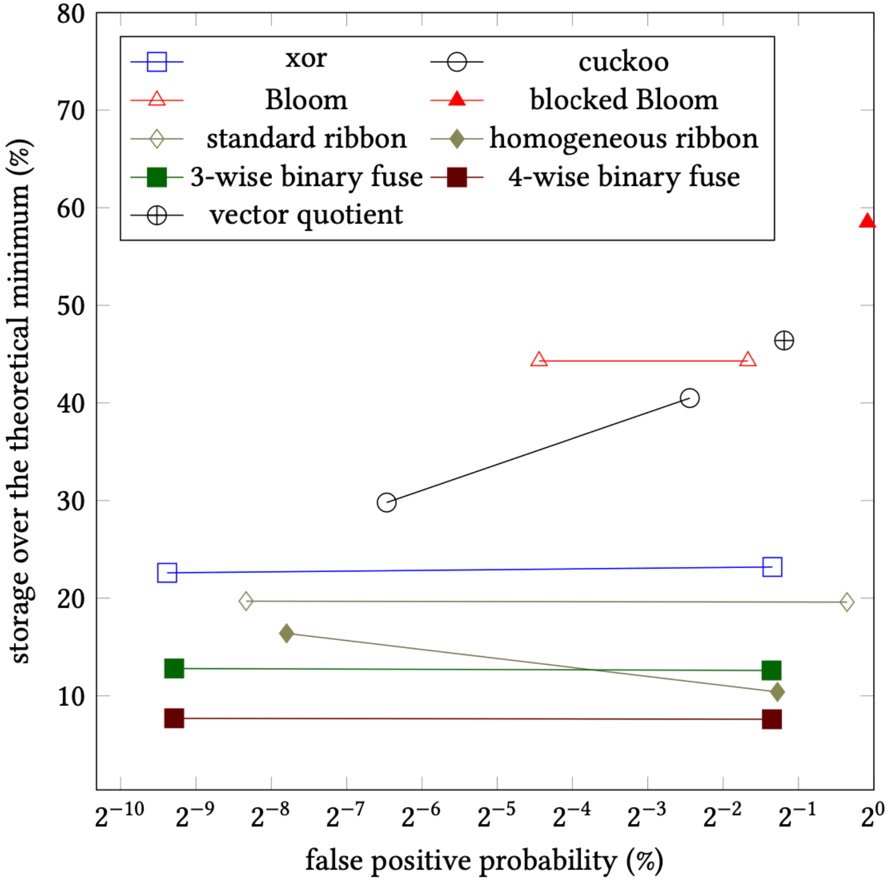
```c
bool contain(uint64_t key, const binary_fuse_t *filter) {
  uint64_t hash = mix_split(key, filter->Seed);
  uint8_t f = fingerprint(hash);
  binary_hashes_t hashes = hash_batch(hash, filter);
  f ^= filter->Fingerprints[hashes.h0] ^ filter->Fingerprints[hashes.h1] ^
       filter->Fingerprints[hashes.h2];
  return f == 0;
}
```

	cache misses	mispredictions
3-wise binary fuse	2.8	0.0
4-wise binary fuse	3.7	0.0

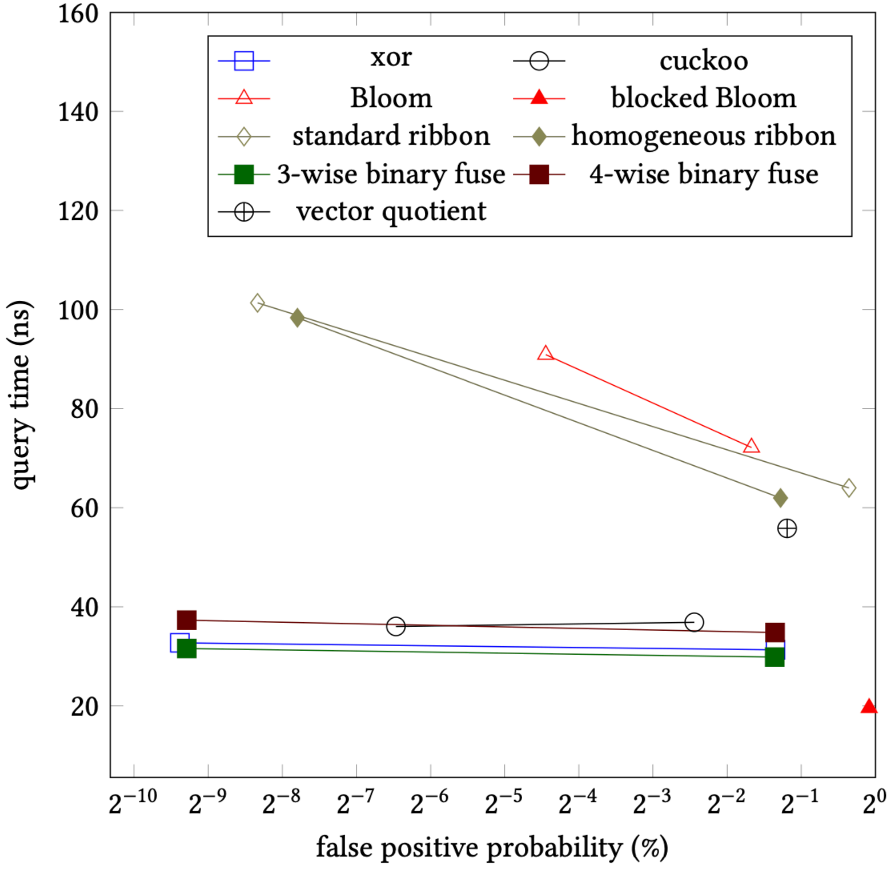
(Intel Ice Lake processor, out-of-cache filter)

	always out (cycles/entry)	always in (cycles/entry)	bits per entry
Bloom $k = 8$	135	170	12
3-wise bin. fuse	85	85	9.0
4-wise bin. fuse	100	100	8.6

(Intel Ice Lake processor, out-of-cache filter)



(a) Relative space usage



(b) Query time

Construction 1

- Start with array for fingerprints containing slightly more fingerprints than you have elements in the set
- Divide the array into segments (e.g., 300 disjoint)
- Number of fingerprints in segment: power of two (hence *binary*)

Construction 2

- Map each object x in set, to locations $B[f_1(x)]$, $B[f_2(x)]$, $B[f_3(x)]$
- The locations should be in three consecutive segments (so relatively nearby in memory).

Construction 3

- At the end, each location $B[i]$ is associated with some number of objects from the set

Construction 4

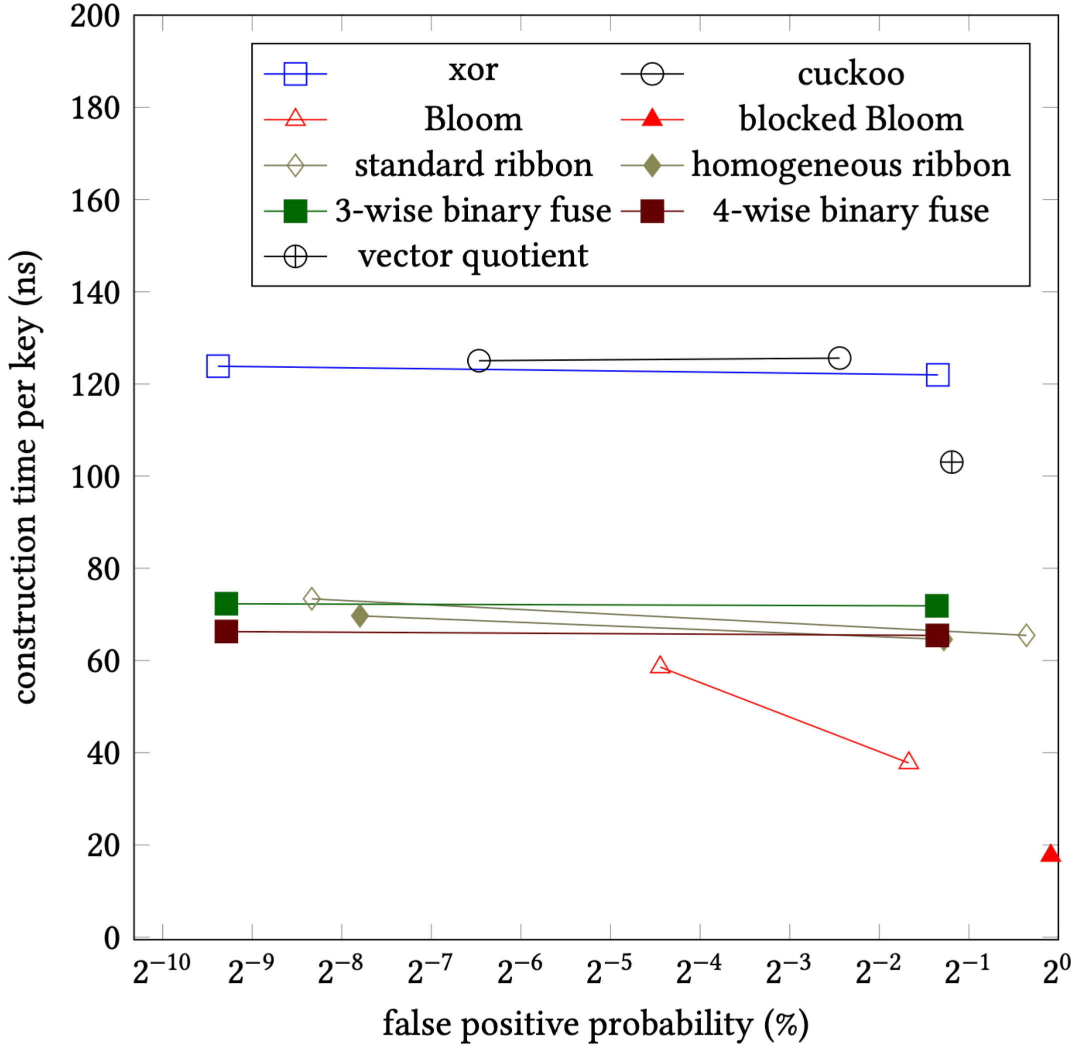
- Find a location mapped from a single set element x, e.g., $B[f_1(x)]$
- Record this location which is owned by x
- Remove the mapping of x to locations $B[f_1(x)]$, $B[f_2(x)]$, $B[f_3(x)]$
- Repeat

Construction 5

- Almost always, the construction terminates after one trial
- Go through the matched keys, in reverse order, adn set (e.,g.) $B[f_1(x)] = f(x) \text{ XOR } B[f_2(x)] \text{ XOR } B[f_3(x)]$

Construction: Performance

- Implemented naively: terrible performance (random access!!!)
- Before the construction begins, sort the elements of the sets according to the segments they are mapped to.
- This greatly accelerates the construction



xor
cuckoo
Bloom
blocked Bloom
standard ribbon
homogeneous ribbon
3-wise binary fuse
4-wise binary fuse
vector quotient
construction time per key (ns)
false positive probability (%)
0
20
40
60
80
100
120
140
160
180
200
2^{-10}
2^{-9}
2^{-8}
2^{-7}
2^{-6}
2^{-5}
2^{-4}
2^{-3}
2^{-2}
2^{-1}
2^{0}

How does the performance scale with size?

For warm small filters, number of access is less important.

Becomes more computational.

For large cold filters, accesses are costly.

10M entries

	ns/query (all out)	ns/query (all in)	fpp	bits per entry
Bloom	17	14	0.32%	12.0
Blocked Bloom (NEON)	3.8	3.8	0.6%	12.8
3-wise bin. fuse	3.5	3.5	0.39%	9.0
4-wise bin. fuse	4.0	4.0	0.39%	8.6

(Apple M2)

100M entries

	ns/query (all out)	ns/query (all in)	fpp	bits per entry
Bloom	38	33	0.32%	12.0
Blocked Bloom (NEON)	11	11	0.6%	12.8
4-wise bin. fuse	17	17	0.39%	9.0
4-wise bin. fuse	20	20	0.39%	8.6

(Apple M2)

Compressibility (zstd)

	bits per entry (raw)	bits per entry (zstd)
Bloom $k = 8$	12.0	12.0
3-wise bin. fuse	9.0	8.59
4-wise bin. fuse	8.60	8.39
theory	8.0	8.0

Sending compressed filters

Compressed (zstd) binary fuse filters can be within 5% of the theoretical minimum.

Some links

- Bloom filters in Go: https://github.com/bits-and-blooms/bloom
- Binary fuse filters in Go: https://github.com/FastFilter/xorfilter
- Binary fuse filters in C: https://github.com/FastFilter/xor_singleheader
- Binary fuse filters in Java: https://github.com/FastFilter/fastfilter_java
- Giant benchmarking platform: https://github.com/FastFilter/fastfilter_cpp

Other Links

- Blog https://lemire.me/blog/
- Twitter: @lemire
- GitHub: https://github.com/lemire