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A Turning Point in Mathematical Learning Theory

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This target article by Estes (1950) sparked the mathematical learning theory movement, which took seriously the goal of predicting quantitative details of behavioral data from standard learning experiments. The central constructs of Estes's theory were stimulus variability, stimulus sampling, and stimulus–response association by contiguity, all cast within a framework enabling predictions of response probabilities and latencies. The math models enterprise flourished during the period 1950–1975 and provided successful quantitative accounts of data from Pavlovian and instrumental conditioning, probability learning, verbal learning, concept identification, and other standard learning paradigms. The techniques have been assimilated into the armamentarium of theoretical psychology. Stimulus sampling theory has faded away as it has been transformed into modern descendants such as connectionism and information-processing models of cognition.

The designation of the 1950 article by Estes as a major event in the history of theoretical psychology is assuredly warranted. So few of the thousands of articles we toil to publish and read cause a major difference in how we think, what we believe, and how we behave as scientists. Estes's 1950 article marked a major turning point in my career as well as the careers of many other quantitatively oriented psychologists. I am pleased to have this opportunity to publicly acknowledge the influence that Estes's writings, beginning with this article, had on the careers of so many psychologists.

A Personal Note

I begin with a personal note regarding this article's impact on me. As an undergraduate student I had been persuaded that to become more scientific, psychological theory had to become more quantitative and mathematical, emulating theories in the physical sciences. So in preparation for that eventuality, I took many courses in mathematics, logic, and philosophy of science. I had also tried to formalize Clark Hull's theories of learning but had become disillusioned with the aridity of those efforts.

While attending a course, "Applications of Mathematics in the Social Sciences" at the University of Minnesota in Winter 1955, I heard my math teacher describe the gist of Estes's 1950 article to illustrate the use of differential equations in psychology. It struck a responsive chord in me. I hurried off to the university library and read this Estes article with mounting excitement and pleasure; here at last was a kind of theorizing whose depth and eloquent simplicity resonated to my esthetic ideal of what powerful theories in science should be. I was instantly hooked. Over the next several weeks, I read related articles by Estes as well as articles by Bush and Mosteller (e.g., 1951a, 1951b). I also organized an informal seminar with Minnesota graduate students to work through the Bush–Mosteller book, *Stochastic Models for Learning* (1955), which had just appeared. It was an exhilarating time in my youthful intellectual life.

Rereading Estes's article today, 39 years later, evokes vestiges of the same pleasure and exhilaration I felt originally, with a tinge of nostalgia for lost youth and enthusiasm. I suppose most scientists and scholars can identity similar "critical moments" in their lives when they decided to pursue a particular career path. My retrospection is also colored by my continued respect and affection for this man who has been my mentor and friend since we met in 1957. It is therefore a pleasure for me to honor Estes by commenting on the significance of this article in the history of our field.

The Estes 1950 Article

Historical Context

The field of psychology in the 1930s and 1940s had been dominated by the "global theories of learning" associated with Clark Hull (1943), Edward Tolman (1932/1949), Edwin Guthrie (1935), B. F. Skinner (1938), and Gestalt psychologists such as Kurt Lewin (1942) and Wolfgang Köhler (1940). Although considerable controversy and disagreement raged among theorists, the general hope had developed that the disputes would be resolved by adhering to the methodological constraints of operationalism along with trenchant analysis and comparison of the conflicting theories according to the rigorous canons of philosophy of science. A volume of detailed critiques of the major learning theories using such criteria found all of them to be woefully inadequate, often embarrassingly so (Estes et al., 1954). So, by the mid-1950s the field was clearly searching for a new paradigm, direction, or method to help resolve disputes and advance the science. It is probably not a historical accident that the mathematical learning theory movement began and acquired a following during this period of disillusionment with the global theories.

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Orienting Attitudes

In many respects, the orienting attitudes in Estes's article reflected the assumptions of his times. The orientation was that psychological phenomena were to be understood in terms of some version of associationism; more specifically, behavior was viewed as analyzable into the way that situational stimuli (Ss) were connected to and controlled behavioral responses (Rs). A major concern was how and why these connections were modified through the action of reinforcers and punishers. The arguments for S–R associationism as a foundation for psychological theories had been around for centuries, and the behaviorists of the 1940s largely differed from one another only in what they emphasized when implementing that framework.

A second orienting attitude was that hypotheses about psychological phenomena were to be rigorously checked out in the laboratory, if possible with nonverbal organisms such as rats, cats, or monkeys. This test bed was preferred to block the then-reviled tendency to theorize using mentalistic constructs or anecdotes with subjective interpretations.

A third orienting attitude was a firm belief in biological continuity of different species. Concepts of Darwinian evolution were widely taken to imply that fundamental behavioral mechanisms were fairly constant across mammalian species and were merely elaborated incrementally in ascending the phylogenetic scale. These beliefs provided the rationale for the almost exclusive reliance of theorists on animal research for grounding and testing the competing global theories. These beliefs also justified the cavalier generalization of the findings across species. Moreover, the allegedly basic behavioral processes were also assumed to be laid bare for the most convenient inspection in just a few simple, animal-learning situations, namely, Pavlovian conditioning, instrumental conditioning in the runway or operant chamber, and discrimination learning in choice situations such as the T maze, Wisconsin General Test Apparatus, or the two-key pigeon chamber.

These paradigm assumptions comprised the background for Estes's 1950 article. His article opened with general assertions to the effect that all fundamental behavioral laws would be of the form R = *f*(S); then, after introducing his views on how to characterize stimuli, response classes, and associative connections of stimuli to responses, Estes applied his theory to standard experiments in which (a) an animal learns to associate a buzzer to an electric shock, (b) hungry rats reduce their latencies while learning to traverse a runway to a food reward, and (c) a thirsty rat learns to press a lever in an operant chamber to obtain a water reward. In retrospect, such data hardly seem to comprise the volatile material or gunpowder needed to spark a profound revolution of tradition; rather, the revolutionary aspect of Estes's enterprise was the way in which he formulated and applied S–R theory to these examples. Please consider a few of the unique aspects of this article.

Unique Aspects of Estes's 1950 Theory

A first important feature is that Estes believed that advances in understanding "may be maximized by defining concepts in terms of experimentally manipulable variables, and developing the consequences of assumptions by strict mathematical reasoning" (Estes, 1950, p. 94). This approach contrasted with that of the popular theories of Hull (1943) or Tolman (1932/1949) that began by postulating factors affecting hypothetical constructs such as habit strengths, expectancies, or cognitive maps but then later struggled to link up these central constructs to measures of behavior.

Second, Estes cut through a debate that was then occupying learning theorists, namely, at what level the theorist should define *the response* the subject is learning: Is it an achievement-oriented action such as lever pressing (the so-called "molar" view), or is it a patterned sequence of minute, muscular movements that just happen coincidentally to produce some measurable effect (the "molecular" view)? Estes followed his mentor, B. F. Skinner (1938), in opting for the molar view, which proposes that investigators should define response classes at whatever level yields most lawful regularities and that theories should aggregate over (and classify together) behavioral sequences that are not treated differently by the environment or reinforcing agent. Thus, for theoretical purposes, all behaviors that get the lever depressed enough to activate the recorder and the feeder will be counted as equivalent instances of the "lever-pressing" response class. The issue has still not been fully settled to every one's satisfaction, since the molecular and micromolar approaches continue to have strong proponents (e.g., Logan, 1954, 1960; Rosenbaum, 1991).

A second move was to define response classes for a given organism in a given situation that were exhaustive and mutually exclusive. That is, response classes were so defined that any of the organism's possible behaviors in a given situation could be assigned unambiguously to one or another of the classes and so that at any given moment in time the organism could be conceived as emitting an instance of one, and only one, response class. These response assumptions require that when investigators identify a reference-conditioned response (e.g., lever pressing) of special interest, they must also define at least its complement class, of "everything else" other than the reference response.

Although these theoretical assumptions are common within S–R behaviorism, psychologists rarely appreciate how much they diverge from the way laypeople talk about everyday actions. In lay language, people typically describe themselves as doing a number of things more or less at the same time, like sitting upright while eating breakfast (itself a series of complex moves) while reading the morning newspaper and intermittently carrying on a casual conversation with a companion.

The advantage of having a closed set of responses is that the organism's behavioral state in a given situation can be fully characterized in terms of the *probabilities* of its emitting each of the N response classes. Within such a system, learning has the consequence of increasing the probability of some reference ("correct") response while decreasing the probabilities of competing alternatives in that situation. Extinction or unlearning of the reference response reduces its probability while increasing the probability of competing responses. By defining response classes whose probabilities always sum to unity, one is committed to an "interference" or competing response view of extinction. This view, largely adopted from Guthrie (1935) and Skinner (1938), may be contrasted with the views of Pavlov (1927), Hull (1943), and Rescorla (1969), that extinction introduces actively negative, antiresponse tendencies that inhibit or suppress the reference response.

Other Response Measures

An advantage of directly formulating a behavior theory in terms of situational response probabilities and their changes with experience is that the theory is put into immediate contact with observables in experiments. No elaborate measuring instruments are required; one merely has to count occurrences of the different response classes and tote up their relative frequencies in the data.

Besides relative frequencies, however, psychologists also record response-time (*latency*) measures and, in the free operant situation, the *rate* of a given response (Rs/min). However, these measures can be related to the basic response probabilities. Estes used a model of response latency to derive an expression for mean latency from response probabilities. Conceiving of each short time period of length h as an opportunity during which the reference response may occur (with probability p) or not, the number of time-periods (n) that elapse before the reference response occurs (thus terminating the trial) is geometrically distributed, with mean $L = \bar{n}h = h/p$. Thus, having a theory of how p changes over reinforced trials permits one to predict mean response latency by this elementary formula. Estes did this with his Equation 4, which he fit to the runway latency data plotted in his Figure 1.

The momentary response rate in a free operant situation was similarly related to the interresponse time (which is like latency), so the h/p formula describes the mean interresponse time. Thus, the rate is just the reciprocal, or p/h. Estes's main concern was to calculate how p would change with time in the free operant situation because the subjects (by their responses) control when reinforcements and learning events occur. He accomplished this derivation in his Equations 5 and 6, leading up to Equation 7, which is the response-rate function fitted in Figure 3 to the bar-pressing rate of a rat. It is important that with a theory about how momentary *probability* of the reference response changes with reinforcement, Estes then fitted the trial-by-trial changes in response *latency* (in a runway) or in momentary response *rate* in a free operant situation.

Although this latency assumption permitted Estes to apply the theory to these particular data, later, more careful examination by Estes and others soon indicated shortcomings of both the latency model and the free operant model. The geometric distribution of latencies implied by the simple model was clearly wrong. However, Bush and Mosteller (1955) showed that this flaw could be rectified by assuming that the recorded latency required the successful completion of k smaller steps in a response chain, each of which took a geometrically distributed time. The resulting sum of k such small time steps has a gamma distribution with mean kh/p, again a simple reciprocal function of probability. Because the gamma distribution more closely approximates observed latency distributions (see Bush & Mosteller, 1955), this particular difficulty was overcome. More serious challenges to Estes's views of the response are posed by Logan's (1954, 1960) arguments for the micromolar approach as well as later research by Shimp (1975, 1978) and others indicating that animals in free operant situations are learning to emit bursts of sequential interresponse times as "response units."

Stimulus Sampling

Perhaps the most salient assumption in Estes's article was his representation of the stimulus situation identified with a given experimental arrangement. The stimulus situation was represented as a population of independently variable components or aspects of the total environment, called *stimulus elements.* At any moment in an experiment, only a sample of these elements from the total population is active or effective. This stimulus variability can arise from either incidental changes in the environment or changes in the subjects' orientation, sensory processes, posture, or internal milieu. This view of stimulus variability and its importance for performance was emphasized in Guthrie's (1935) writings. There was no commitment to any fixed amount of stimulus variability: that was to be estimated in part from the data. The total number of elements that might be potentially active in a given situation for a given subject is denoted as N (or S in Estes's article), from which is drawn on each trial a random sample of size s.

It takes little analysis to realize that stimulus elements serve as hypothetical constructs in Estes's theory, ones that bear only probabilistic relationships to actual sources of experimental stimulation. Thus, even in a simple conditioning situation in which the same stimulus (say, the sound of a tone) is presented and paired with food on each trial, the theory makes no commitment as to what is N, the size of the population of tone-elements: it could be 1, or 10, or 100. That is to be estimated from the data.

State of the System

The theory assumes that each stimulus element is connected (conditioned) to one and only one of the response classes. This connection is all-or-none, at full strength. Accordingly, in principle one could characterize the subject's behavioral state with respect to each situation by listing all N stimulus elements and the response-connection of each. Throughout the course of learning, the elements will be sampled and perhaps change their connections according to what response was reinforced when they were last sampled. If the elements are each sampled with the same probability, then to characterize the behavioral dispositions of the organism we need not know which stimulus elements are associated with each response, but only the overall proportion of the elements that are associated with the reference response. In this case, we can describe the state of the learning system on Trial n simply by a single number, p_n, the proportion of elements in the population connected to the reference response.

Performance Rules

Performance on any trial is determined by the stimulus elements that are sampled or are experienced on that trial. The probability of any response on a given trial was assumed to equal the proportion of elements in that trial's sample connected to that response. If the sample size is large enough, then by the law of large numbers the expected proportion of sampled elements connected to a given response will nearly equal the proportion of such connected elements in the overall population.

Different Sampling Schemes

Several schemes for sampling elements have been investigated. In the first, each of the N elements has probability Θ of being sampled independently of other elements; this leads to a binomial distribution of sample sizes, with a mean sample size of $N\Theta$ (see Estes & Burke, 1953). In the second scheme, a fixed sample of size s is drawn from the N elements, so that the fraction s/N plays somewhat the same role as does Θ in the first scheme. In this 1950 article, Estes wrote as though he had the first conception in mind, but the derivation of the mean learning curve is the same regardless of which sampling scheme is used (replace s/N with Θ).

In later writings, Estes (1959b) developed models based on the single element ($s = 1$) sampling scheme, dubbing them *pattern* models. This move proved to be quite productive, leading in particular to the one-element and two-element models that had modest runs of success in the 1960s (see Bower, 1961a, 1961b; Bower & Theios, 1964; Suppes & Atkinson, 1960; Theios, 1963; Theios & Brelsford, 1966).

Reinforcement Rule

Having drawn a stimulus sample and responded, the subject receives some kind of reinforcement. In the simplest version, all of the elements sampled on that trial become thereby fully connected to the reinforced response. It is these reinforcing events that cause the changes over trials in the conditional status of the stimulus elements. Successive practice trials cause the progressive sampling of more elements from the stimulus population, causing them to become attached to the reinforced (adaptive, "correct") response and lose their previous connection to competing responses.

Figure 1 illustrates this sampling and conditioning process. The population of stimulus elements is analogous to a large bowl of white marbles (25 here) connected initially to a competing (A_2) response. On each trial, a sample (5 here) of stimulus marbles is drawn and the reinforcement causes them to switch their connection, from A_2 to the reference response, A_1. In Figure 1, this switchover is depicted by coloring the sampled marbles black. Following reinforcement, the sampled elements are returned to the bowl, scrambled up with the others, and then a new random sample is drawn for the next trial, containing some more black (A_1) marbles and fewer white (A_2) ones than before.

Estes described the trial-by-trial changes in the number of conditioned elements (x) in his differential Equations 1 and 2. When reinforcement conditions remain constant over a series of trials, these trial-by-trial changes can be summed (or integrated), yielding the exponential growth function in Estes's Equation 3. Dividing x by the size of the stimulus population, the ratio x/S in Equation 3′ is interpreted as p, the probability of the reference response on trial T of the experiment. Equation 3′ then plays a central role in fitting learning curves in the article.

This early statement of the learning axioms in terms of differential equations was soon replaced (in Estes & Burke, 1953) by the use of finite difference equations more appropriate to modeling trial-by-trial changes in conditioning at discrete points of time. In fact, it was eventually realized that the trial-by-trial changes in conditional status of each stimulus element could be represented as a special stochastic process called a *Markov chain* (see Estes, 1959b). Thereafter, the mathematics of Markov processes played an increasing prominent role in the derivation of consequences from mathematical models for learning (see, e.g., Atkinson, Bower, & Crothers, 1965).

Although Estes was not the first psychologist to offer an equation for "the learning curve" (see, e.g., Hovland, 1951; Thurstone, 1930), he was the first to state his mathematical description in terms of difference equations that (a) interpreted the parameters directly in terms of experimental observables, and (b) expressed the trial-by-trial changes in response tendencies in terms of momentary conditions of stimulation or reinforcement, or both. These features made Estes's proposal far more flexible than were earlier proposals regarding the equation for "the learning curve" because Estes's approach enabled him to interpret (or predict) the influence of many more variations in experimental conditions (e.g., see Estes, 1961).

Interpretation of Reinforcement

In this 1950 statement, Estes was clearly trying to follow Edwin Guthrie's interpretation of how reinforcing events operated to produce conditioning. Guthrie's theoretical claim was that empirical reinforcers were events that somehow insured that the correct response was the last one to occur to the critical stimuli on a given trial. For example, when a rat turns left in a T maze to escape a painful foot shock, by removing the shock, the left-turning was the last response to occur to the shock and choice-point stimuli on that trial; therefore, those stimuli will become (or remain) associated to that response. In positive reward situations such as a runway, the animal approaches and consumes the food or water in the goal box, thereby removing itself from the runway cues.

This Guthrian theory of reinforcement was much debated in the 1940s but was slowly forced to make various revisions and implausible, ad hoc assumptions to interpret the full range of reinforcing events. Eventually the amended theory became overstrained and insupportable, and lost adherents, Estes among them. In his later writings, Estes (1969b, 1976, 1982) switched to a more cognitive interpretation of reinforcement (see later).

Later Developments

Developments in Theory

Following this 1950 article, Estes began a series of theoretical and empirical investigations that greatly expanded the scope and application of his earlier theory. Major theoretical changes were introduced in an article with his long-time collaborator, Cletus J. Burke (Estes & Burke, 1953); they made more explicit the stimulus representation, the independent-element sampling process, the analysis of stimulus generalization and discrimination experiments, and introduced the use of finite difference equations to describe trial-by-trial changes in learning. Then, in a highly original pair of articles, Estes (1955a, 1955b) elaborated a time-dependent notion of stimulus sampling to explain forgetting and spontaneous recovery, viewing them as arising from the gradual, time-dependent exchange of stimulus elements between an active and an inactive state as time passed between experimental sessions. In another article, Estes (1958) interpreted the influence of drive level on learning and performance in terms of variations in the sampling probability

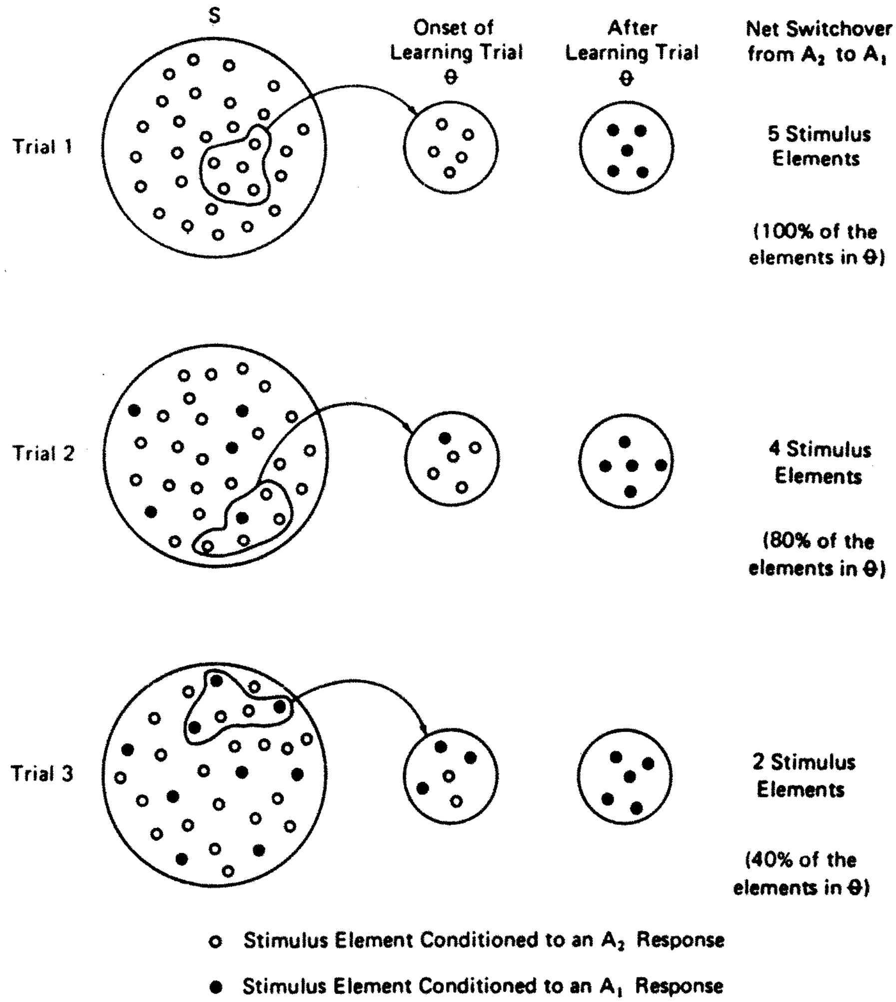

Figure 1. Estes's model of how stimulus elements are sampled from the population of elements (left-hand circle) and are then changed from being connected to response A_2 to being connected to the reinforced response, A_1. From *An Introduction to Theories of Learning* (4th ed., p. 230) by B. R. Hergenhahn and M. H. Olson, 1993, Englewood Cliffs, NJ: Prentice Hall. Copyright 1993 by Prentice Hall. Reprinted by permission.

(weight and importance) of drive-related stimuli relative to other stimuli.

In later articles, Estes (1969a, 1969b) modified his reinforcement theory to interpret both positive reward and punishment effects within the same framework. In that interpretation, an organism experiencing a stimulus–response–outcome (*S–R–O*) sequence in a given context (*C*) forms a four-way, interassociated data-structure or memory trace. That data structure may be retrieved by later presentations of the stimulus in the context, thus causing anticipation of the outcome. If the outcome is a positive, desired outcome, then it provides facilitatory feedback to potentiate performance of the response; if the anticipated outcome is negative, then inhibitory feedback prevents the response. In many respects, Estes's later theory of reinforcement and incentive was similar to Tolman's expectancy theory or the Hull–Spence incentive theory of how anticipated rewards or punishments selected responses. These psychological theories about drive, reinforcement, and punishment are still very much on the active research agenda of today.

Changing the Experimental Base

One of the most obvious changes over the years was that Estes and his followers concentrated to an increasing extent on applying stimulus sampling theory (henceforth SST) to human learning experiments while lessening attention to conditioning experiments with lower animals. Estes began his career as a "rat runner" (to use psychologists' vernacular) in Skinner's laboratory and conducted no studies of human learning until he began tests of his theory in the mid-1950s with his students and collaborators at Indiana University. The experimental situations studied were probability learning (adapted from earlier studies by Egon Brunswick and Lloyd Humphreys), paired-associate learning, discrimination learning, and concept identification. These experimental situations provided the primary test bed for SST during the 1953–1970 period when it expanded and flourished.

In addition, in the mid-1960s the cognitive revolution in human memory research was in full swing; studies of short-term memory, free recall, and concept learning were very fashionable, so several theorists influenced by Estes developed models for those experimental paradigms. Included here would be the hypothesis-testing models of concept learning (Bower & Trabasso, 1964; Restle, 1962; Trabasso & Bower, 1968), the author's queuing models for short-term memory (Bower, 1967), the Atkinson–Shiffrin buffer model for short-term memory (1968), Murdock's (1974) fluctuation model of short-term memory, and Shiffrin's search of associative memory (SAM) model of free recall (Raaijmakers & Shiffrin, 1980, 1981; Shiffrin, 1970). Indicative of the shift from animal to human learning in applications of SST, a recent two-volume *Festschrift* in honor of Estes (Healy, Kosslyn, & Shiffrin, 1992a, 1992b) contained 21 chapters written by authors whose works were inspired by Estes's theories, and none of them referred in any substantial manner to animal learning experiments.

Probability Learning

A large portion of the second stage of SST research was conducted with human adults in probability learning situations. In its simplest arrangement, the subject's task is to predict which one of two outcomes, E_1 or E_2, will occur on each trial in a constant situation. The common feature of such experiments is that the events occur in a random sequence and no information besides the base rate is available to help subjects predict which event will occur on a given trial. Letting p_n denote the probability of an A_1 response of predicting outcome E_1 on trial n of the experiment, the changes in p_n produced by an E_1 or E_2 outcome are given by Equations 1a and 1b:

$$p_{n+1} = \begin{cases} (1-\Theta)p_n + \Theta, & \text{if } E_1 \text{ on trial } n. \quad (1a) \\ (1-\Theta)p_n, & \text{if } E_2 \text{ on trial } n. \quad (1b) \end{cases}$$

If the probability of an E_1 reinforcement is π and of an E_2 reinforcement is $1 - \pi$, then the expected value of p_{n+1} can be obtained by multiplying Equation 1a by π and Equation 1b by $1 - \pi$, yielding:

$$p_{n+1} = (1-\Theta)p_n + \Theta\pi. \quad (2)$$

Equation 2 is a linear difference equation that has the solution

$$p_n = \pi - (\pi - p_1)(1-\Theta)^{n-1}. \quad (3)$$

That is, the mean A_1 response probability should move from p_1 on Trial 1 to an asymptote of π, which is the probability that the A_1 response has been reinforced over the series of trials.

Equation 3 is the probability-matching prediction. It has been tested and confirmed many times in many experimental variants with different subject populations (e.g., rats, pigeons, monkeys, children, retardates, amnesiacs, and normal adults). The result has been extended in several directions by experiments in which the E_1 reinforcements are scheduled in novel or unusual ways. For example, the probability of an E_1 event could change systematically or cyclically over a long series of trials; Equation 1 implies that subjects' A_1 probability will "track" the changing π_n function but with a lag (Estes, 1957). Another variant is to have the probability of an E_1 event depend on the response made on that trial, or on the response made several trials before. Still another variant has 2 subjects, in a game-like situation, make a series of responses in lock-step, in which the probability of an E_1 event for each subject depends jointly on both their responses (Suppes & Atkinson, 1960). The theory predicts (and experiments find) eventual probability matching for each subject in all such learning situations; the only novel wrinkle in such experiments is to use the theory to figure out what that asymptotic level would be. The theory has been almost uniformly successful in predicting mean learning curves and asymptotes in such experiments.

Sequential Statistics

Beyond predicting the mean learning curve, the model affords predictions of many descriptive statistics of most data. Chief among these are sequential statistics. For instance, in a two-choice probability learning situation wherein some response A_i and outcome E_j occur on each trial, the theory predicts the average probability of response A_i on trial $n + 1$ given a particular sequence of A_is and E_js over one or more preceding trials. Thus, one trial back provides 4 such sequences (A_1E_1, A_1E_2, A_2E_1, A_2E_2), two trials back provides 16 (4×4) such sequences, and so on. The number of sequential statistics available to test the theory grows exponentially the more previous trials over which one conditionalizes.

These sequential statistics (or response probabilities conditionalized on prior trial events) are not just idle curiosities, ancillary to the learning curve. On the contrary, they reflect the fundamental trial-by-trial changes in contingent response probabilities that the statistical theory allegedly describes; they are, in some sense, more basic than the mean learning curve because the latter is derived by aggregating over these sequential probabilities.

In light of their central role, theorists were embarrassed to discover (surprisingly early on) that sequential probabilities in human probability learning experiments often deviated markedly from the SST predictions. One of the more prominent of these discrepancies was the so-called "gambler's fallacy," whereby during a series of consecutive E_1 reinforcements, subjects became increasingly likely to predict the opposite, E_2 event, as though it became increasingly "due." Such tendencies clearly violate the predictions of SST. Other documented tendencies were for subjects to be looking for some kind of systematic pattern in the E_1/E_2 event series, such as double alternation, or a preponderance of runs of reinforcements of particular lengths.

Some reviewers (e.g., N. H. Anderson, 1964, 1966) considered these discrepancies sufficiently serious to reject the basic SST as an inadequate description of probability learning. Such critiques motivated the search for theoretical alternatives such as viewing subjects as testing hypotheses regarding run-lengths of given reinforcing events (see Myers, 1970; Restle, 1961, chap. 6). Estes's reaction to such sequential discrepancies (Estes, 1972b) was to argue that (a) the gambler's fallacy and similar response biases reflect human subjects' maladaptive prior beliefs that would be extinguished given sufficient exposure to a true probability learning situation, or (b) under some circumstances, the *previous sequence of responses or reinforcing* events (e.g., 2 back) provide distinct stimulus traces that can become connected differently to responses. Thus, in an alternating event series, E_1 E_2 E_1 E_2, and so on, the event trace continuing into the next trial would uniformly predict the correct response. This general approach was fairly successful (Myers, 1970; Restle, 1961), and most investigators accepted this version of the theory. The field then moved on to investigating the learning of probabilistic contingencies or base rates in many different domains such as category learning or multitrial decision making.

Paired-Associate Learning

A second area of significant applications of SST was paired-associate learning. The stimulus term of each pair in a list of pairs was conceived as comprising a stimulus element to which the correct response was becoming associated during study trials. In a series of articles (Estes, 1960, 1964; Estes, Hopkins, & Crothers, 1960), Estes argued that in simplified cases, individual pairs appeared to be learned in an all-or-none fashion rather than by gradual increments in S–R strength with each study trial. These papers kicked off the *incremental versus all-or-none* debate that occupied some verbal learning researchers in the 1960–1965 period. That debate was conducted in three domains, each assessing whether learning of an individual S–R pair could be "partial," or could only be either nearly complete or none at all.

Domain #1. Irvin Rock (1957) reported the startling finding that overall rate of learning a list of verbal paired associates was not slowed by replacing any pair on which subjects erred on a test trial by a new pair during the next study trial. Such an outcome appears contrary to the incremental viewpoint, which would expect the benefits of partial learning to be wiped out if items were replaced by new ones (resetting learning to zero) whenever an error occurred. The result suggested the absence of subthreshold or partial learning of specific S–R associations. The incremental theorists (Postman, 1963; Underwood & Keppel, 1962) countered that Rock's drop-out method capitalized on selection artifacts because difficult items from a pool (leading to errors) would on average be replaced by easier items. Later attempts to equate pair difficulty showed that the replacement method did indeed cause significant impairment in overall learning. However, the all-or-none theorists had a counter, namely, that if paired associates involved two or three all-or-none subcomponents such as response learning along with stimulus–response association, then one would expect the replacement method to lead to some slowing. Kintsch (1963) elegantly showed that the appropriate amount of slowing due to the replacement method could be predicted closely by such a two-stage, all-or-none model.

Domain #2. A second test of incremental theory was provided by an exceedingly simple arrangement, namely, one study trial of a list of pairs followed by two or more test trials without intervening study trials, the so-called STT paradigm. Estes (1960, 1964) noted that if each pair i has its recall probability incremented to some intermediate level p_i by the study trial, then according to the incremental theory, correct and incorrect responses over the two closely spaced test trials should occur independently. For example, the probabilities of correct responses on both T_1 and T_2 should be p_i^2, whereas the probability of a correct-then-incorrect sequence or an incorrect-then-correct sequence should each be $p_i(1 - p_i)$. In a large number of experiments, begun by Estes but confirmed by many others, these incremental predictions failed badly. In fact, the results were almost always much closer to the predictions of the all-or-none theory, which expects the conditional probability of a correct response on T_2 following a correct response on T_1 to be very high (in the simplest case, unity) but following an error on T_1 to be very low, near chance guessing.

The incremental theorists (e.g., Underwood & Keppel, 1962) counterargued that the huge differences in conditional probabilities reflected several artifacts—learning on the first test trial, item-selection, or subject-selection biases. Although Estes had tried to anticipate and answer most such counterarguments, the answers required assumptions about how learning or forgetting on test trials might occur. The issue ended in a stalemate over whether selection artifacts had been properly estimated and removed.

Domain #3. A third approach to testing all-or-none theory of paired associates was to compare the fit of the incremental versus all-or-none models to a large number of descriptive statistics obtained from conventional, multi-trial paired-associate learning experiments. This approach was illustrated in articles by Bower (1961a, 1961b) and Bower and Theios (1964). Such goodness-of-fit comparisons always favored the all-or-none model by a hefty margin, with the all-or-none model sometimes fitting data with spectacular accuracy.

However, when the overall learning process involved significant amounts of either stimulus discrimination or response learning, the simple all-or-none (or one-stage) model failed to account well for all the data. Those cases led Bower and Theios (1964) and Restle (1964) to formulate two-or-three-stage models to describe paired-associate learning, each stage of which was conceived to be learned in an all-or-none fashion (see McGuire, 1961). The first stage might be learning the responses as integrated, available units; a second stage might be discriminating a given stimulus term from confusable others in a list of pairs; and the final stage would be associating the integrated response as a unit to the discriminative aspects of the stimulus of the pair. A series of experiments showed the plausibility of such analyses as well as the quantitative accuracy of such multistage models in fitting paired-associate data (e.g., Bower, 1967; Bower & Theios, 1964; Kintsch, 1963; Polson, Restle, & Polson, 1965; Restle, 1964). In addition, a number of powerful statistical procedures were developed and used in successfully fitting multistage models to data from a variety of learning experiments (e.g., Restle & Greeno, 1970; Theios, 1968; Wickens, 1982).

Although these multistage, all-or-none theories were adequate for fitting considerable learning data, they greatly blurred the dramatically large discrepancies in *qualitative* predictions that had attracted so many investigators into the debate between all-or-none and incremental theories. It was conceded that in many situations, learning of a complex memory could sometimes be partial rather than completely all-or-none. The issue then became translated into deciding what determined the number of significant stages. In addition, investigators became more interested in the types of cognitive mediators that human subjects might use to accelerate particular stages of the associative process.

After this period, investigators of human memory became far more attracted to investigating the processes underlying short-term memory, organizational factors in memory, and mnemonic coding rather than providing detailed quantitative descriptions of standard, multitrial paired-associate data. The all-or-none learning proposal continued to surface periodically in the ensuing years: for example, Kintsch (1974) and Goetz, Anderson, and Schallert (1981) proposed it for people's learning of simple propositions; Jones (1976, 1978) and Ross and Bower (1981) proposed versions of it for complex memory structures. However, the topic essentially lost its ability to arouse much further interest. This does not mean that the multistage models were wrong; rather, the field simply became interested in a new set of research questions.

As mentioned earlier, some of these newer questions concerned models for experimental situations that had become popular in the cognitive revolution, including short-term memory, recognition memory, memory scanning, retrieval speed, recency judgments, and frequency judgments (Bower, 1972; Murdock, 1974); models of free recall such as those of Shiffrin (1970), J. R. Anderson (1972), and Raaijmakers and Shiffrin (1981); models regarding memory for propositions and sentences, such as those of J. R. Anderson and Bower (1973), Goetz et al. (1981), and Jones (1976, 1978); and models of memory for coherent texts, such as those of Kintsch and van Dijk (1978) and Schank and Abelson (1977).

Whatever Became of Stimulus Sampling Theory?

If one adheres to a strict definition of SST, then very few investigators are still working actively with that theory. The basic ideas and findings of that approach have been assimilated into the expanding wisdom of the learning-theory tradition. Experimental scientists are restless and place a premium on originality, so they continually search for new topics and novel domains to explore. Therefore, it is hardly surprising that the field has moved on to other questions, issues, experimental paradigms, and theoretical methods (primarily computer simulation).

As a formulation of laws of elementary conditioning, the contiguity proposal of SST has been eclipsed by the Rescorla–Wagner theory of conditioning (Rescorla, 1972; Rescorla & Wagner, 1972). In that theory, the amount of learning accorded a stimulus on a reinforced trial depends on the discrepancy between the actual outcome versus what the organism expected given the full complex of stimuli on that trial. The primary evidence for the Rescorla–Wagner theory arose from studies of blocking and stimulus-compounding. The simple contiguity proposal has been proven inadequate for that class of conditioning experiments.

The declining reference to SST has come about for various reasons. One reason is that few theoreticians have continued to study the experimental arrangements (e.g., rats in runways or multitrial paired associates) that were popular in the 1950s and 1960s and that were the focus of SST's modeling efforts. Even if those arrangements are studied today, current investigators tend to ask somewhat different questions regarding the behavioral processes revealed by them.

Second, SST was formulated within an S–R framework. However, that framework was largely replaced in the 1970s by the concepts and vocabulary of information processing. In fact, in his later writings Estes has often formulated his ideas in terms of information processing, encoding, storage, and retrieval of memory representations (see Estes, 1972a, 1976, 1986; Lee & Estes, 1981). However, as fate would have it, John Watson is getting his revenge: Liberalized versions of S–R concepts have made a strong comeback in the modern disguise of condition–action production models of cognition (see J. R. Anderson, 1983, 1993; Newell, 1973, 1991).

Third, the mathematical techniques of SST were largely appropriate when the models were simple and were being applied to uncomplicated behavioral situations. But more complex cognitive models became fashionable in the 1970s. The cognitive revolution was inspired in part by artificial intelligence theories of cognition that dealt with quite complex behaviors (e.g., reasoning, problem solving, and language understanding) and using theories that involved quite complex symbolic representations. The approach lent itself to representing theories of behavior in terms of lengthy computer programs, with complex data structures undergoing cognitive operations according to a program designed to simulate thinking (Newell, 1991; Newell & Simon, 1972; Schank & Abelson, 1977). Typically, such programs are too complex for their users to derive mathematically their general implications for behavior. Rather, their implications are obtained by conducting Monte Carlo simulations, running the computer program to notice how it performs under different sets of parameter settings. In this manner, computer simulation has often replaced mathematical derivations as a means for checking implications of a set of assumptions about psychological processes. Although general solutions are preferred to limited simulations regarding implications of a theory, scientists are realistic and learn how to make do with less than optimal methods to explore the implications of a given theory.

Connectionism and SST

Stimulus sampling theory bears an obvious resemblance to some versions of connectionist or adaptive network models of learning (Rumelhart & McClelland, 1986). In standard network models, the experimental stimuli are usually represented by activation over a distributed pattern of N sensory (input) elements, each taking on values 1 or 0 (or possibly -1) depending on which external stimulus is presented. In the simplest one-layer network model, each sensory unit, i, is connected to each response (output) unit, j, with a weight, w_{ij}, that reflects the amount of evidence that sensory feature i provides toward response j as the desired (or correct) output for this stimulus. For a given input pattern, the network's choice is selected by comparing the summed activation levels of the alternative response units, choosing A_i according to its strength relative to that of the alternative responses. Learning consists of changing the weights of units active on a trial so that on the next trial those sensory units will more strongly activate the correct response unit. The learning rule in such network models is typically the Rescorla–Wagner conditioning axiom, also dubbed the "delta rule" (see Gluck & Bower, 1988a, 1988b; Sutton & Barto, 1981).

More powerful network models arise when one or more hidden layers of interassociation units are interposed between the sensory input layer and response output layer. These interassociation units are trainable and can be tuned by training experiences to encode and transform input stimuli into complex representations that are most adaptive for the S–R mapping problems to be solved. These intermediate units thus provide all the prospective explanatory power (as well as the identifiability difficulties) as did "mediating processes" in older neoassociation theories (e.g., Osgood, 1953).

If one confines the discussion to the one-layer network framework, however, its correspondence to SST is quite direct. First, experimentally distinct stimuli are represented by a population of (often hypothetical) sensory units, each of which may be activated or not on a given trial. Second, the similarity of two external stimuli is equal to the number of feature-values they share in common. This provides the basis for generalization of learning from one stimulus to the other depending on their common elements, just as in SST. The shared elements also provide the substrate for abstracting (or extracting) the common or invariant features of diverse exemplars of a single category. In addition, as in SST, adaptive network models compute response tendencies by simply summing the associations to different responses from the sensory units active on a given trial. These correspondences of connectionism to SST have been described in greater detail by Gluck (1992).

As noted earlier, the adaptive network model uses the Rescorla–Wagner conditioning rule to modify weights. Besides enabling such theories to deal with stimulus-compounding and blocking in conditioning, a secondary benefit is that the delta rule enables these models to solve the "overlap problem" encountered in most SST accounts of discrimination learning. The shared elements that lead to initial generalization between two stimuli must somehow be rendered ineffective as subjects learn to discriminate between the two stimuli. The basic SST had no way to accomplish this, which failing led to alternative models incorporating special assumptions or additional processes (e.g., attentional responses and adaptation of irrelevant cues).

The Rescorla–Wagner conditioning rule has an inherent "cue competition" property whereby relatively more valid cues (in a cue compound) drive to zero the associative weights of less valid cues that accompany them. Cue competition implies that sensory elements common to two overlapping stimulus populations will be rendered ineffective during the course of discrimination training. This outcome is expected because the shared cues always appear accompanied by the distinctive cues that more validly predict the correct response. The implication is important because evidence continues to accumulate that such a cue-competition principle provides an adequate account of human as well as animal learning (e.g., Busemeyer, Myung, & McDaniel, 1993; Chapman, 1991; Chapman & Robbins, 1990; Gluck & Bower, 1988a, 1988b; Shanks, 1989, 1991).

Therefore, in a broad historical perspective, although the specific assumptions and experimental paradigms of SST have been superseded and modified over the years, the current enthusiasm for parallel distributed, connectionist models of learning and cognition may be viewed as a partial outgrowth and legacy of the SST framework. It should come as no surprise that in the current scene one of the more creative and vigorous investigators of adaptive network models is that inimitable and indefatigable theorist, William K. Estes (e.g., Estes, 1986, 1993b; Estes, Campbell, Hatsopoulos, & Hurwitz, 1989). His most recent production (in the Paul Fitts lectures given at the University of Michigan) is an elegantly simple model that mimics a vast range of data on human memory and categorization by combining an exemplar memory process with an adaptive learning network (Estes, 1993a).

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