

Relativistic Heavy Ions II - Soft physics

RHI Physics

*The US National Nuclear
Physics Summer School &
TRIUMF Summer Institute*

Vancouver, Canada

Helen Caines - Yale University

June 2010

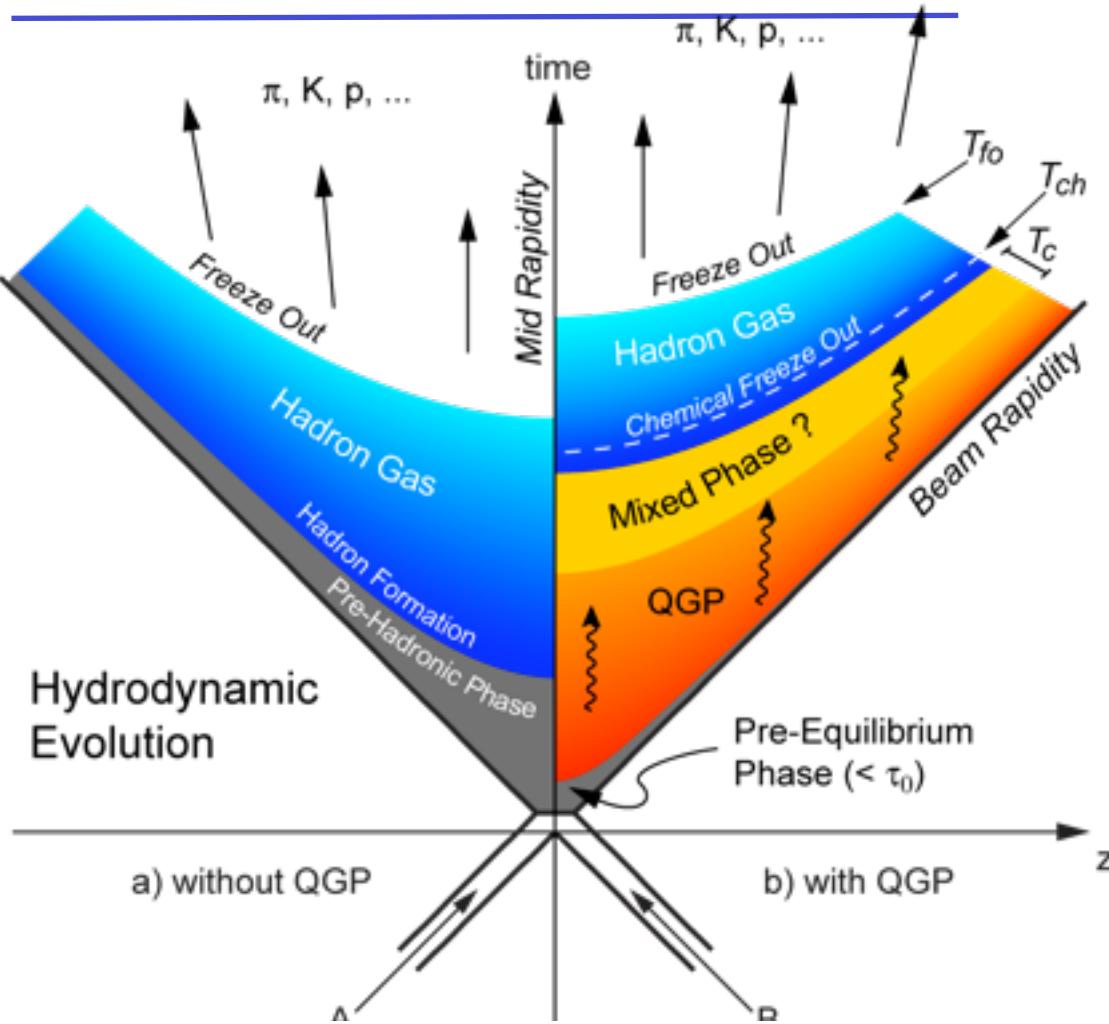
Outline:
The Energy Density
The Temperature
Fluid and Flow



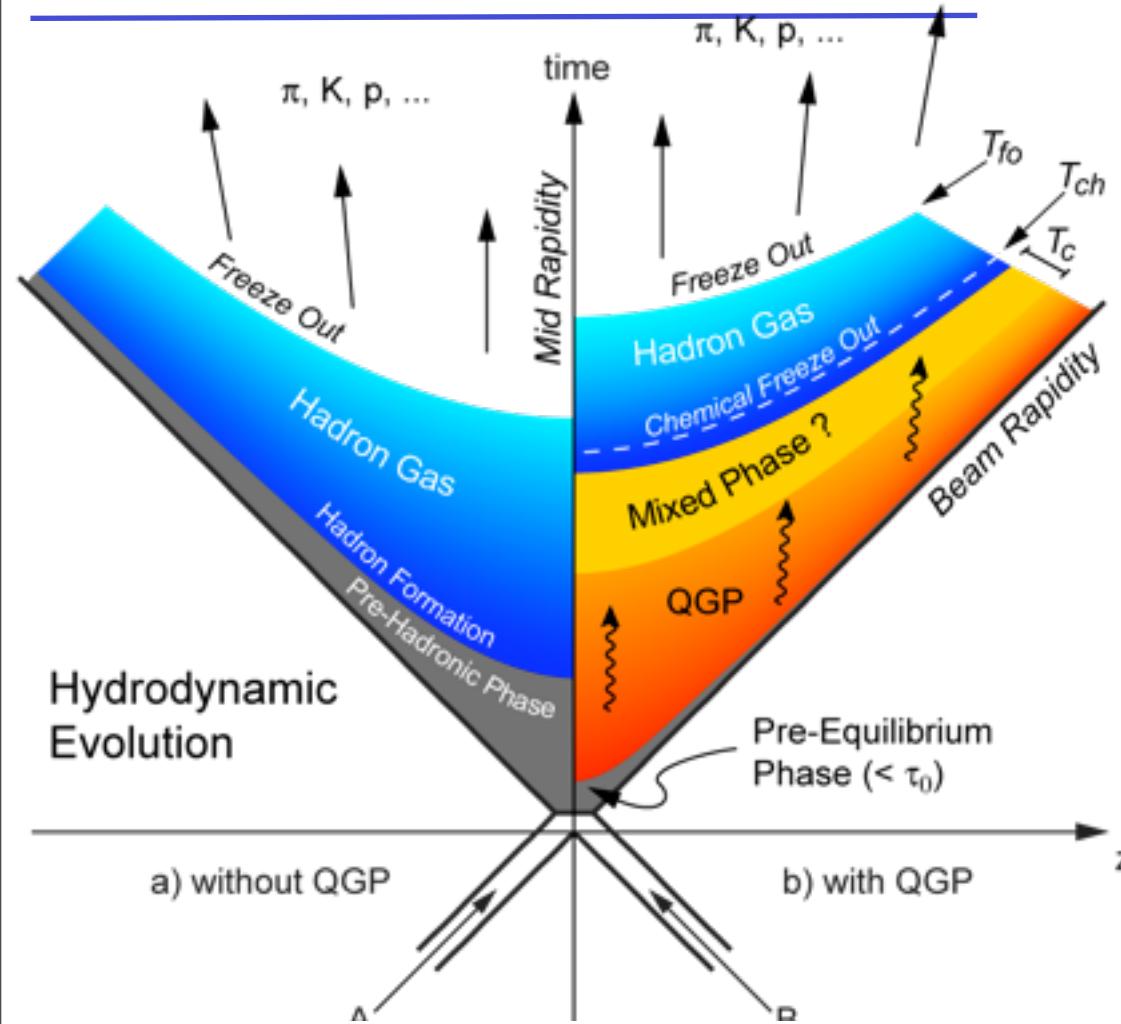
Recap of first lecture

- Looking for evidence of a new state of matter → QGP
- Predicted by QCD to occur, due to screening of colour charge, at high T and/or density
 - $T_c \sim 160$ MeV
- Create in laboratory by colliding ultra-relativistic heavy-ions
- Large multi-purpose experiments necessary to sift through all the data produced

The phase transition in the laboratory



The phase transition in the laboratory



Lattice (2-flavor):

$$T_c \approx 173 \pm 8 \text{ MeV}$$

$$\varepsilon_c \approx (6 \pm 2) T^4 \approx 0.70 \text{ GeV/fm}^3$$

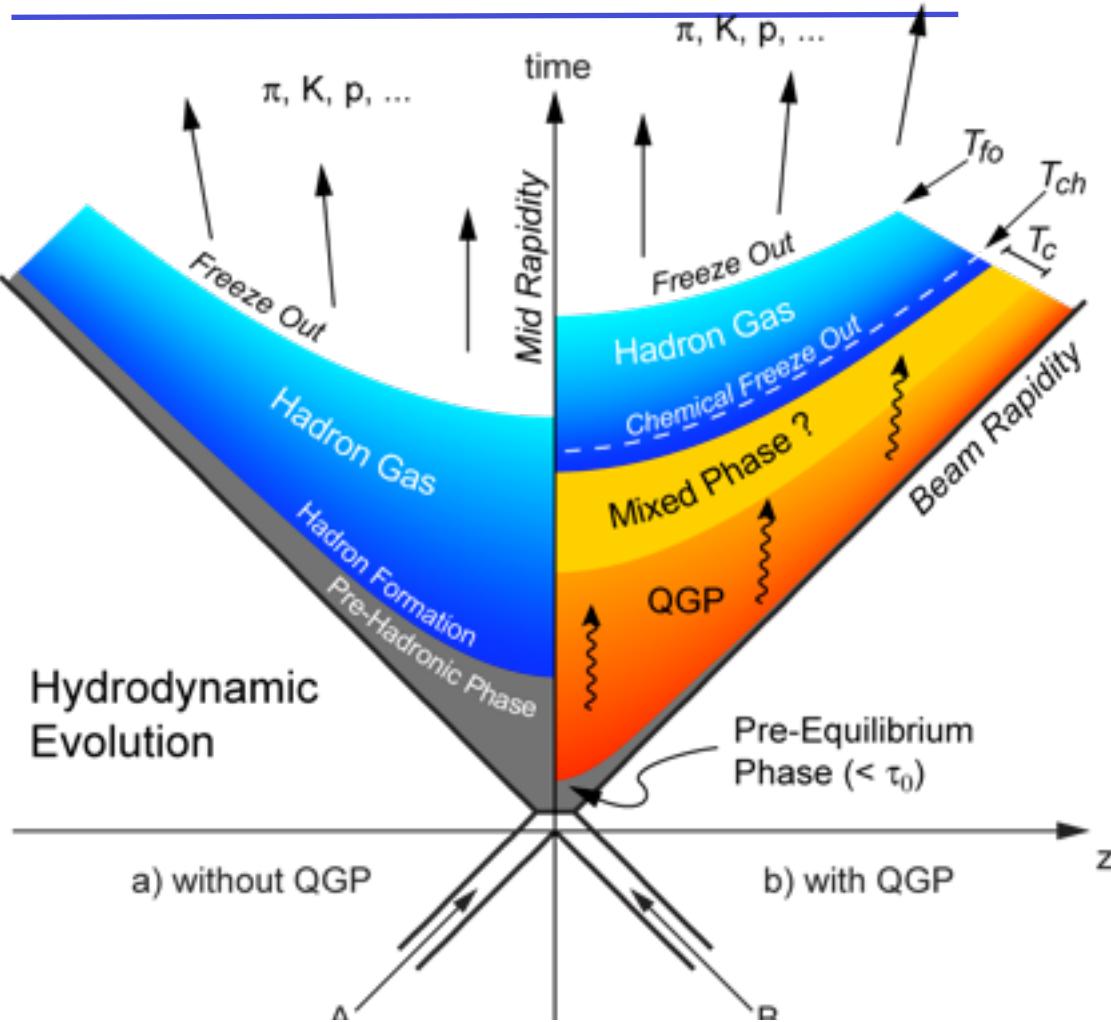
Remember: cold nuclear matter

$$\varepsilon_{cold} \approx u / 4 / _3 \pi r_0^3 \approx 0.13 \text{ GeV/fm}^3$$

Chemical freeze-out ($T_{ch} \leq T_c$): inelastic scattering ceases

Kinetic freeze-out ($T_{fo} \leq T_{ch}$): elastic scattering ceases

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Necessary but **not** sufficient condition

Tevatron (Fermilab)

$$\epsilon(\sqrt{s} = 1.8 \text{ TeV pp}) \gg$$

$\epsilon(\sqrt{s} = 200 \text{ GeV Au+Au RHIC})$

Thermal Equilibrium \Rightarrow
many constituents

Chemical freeze-out ($T_{ch} \leq T_c$): inelastic scattering ceases

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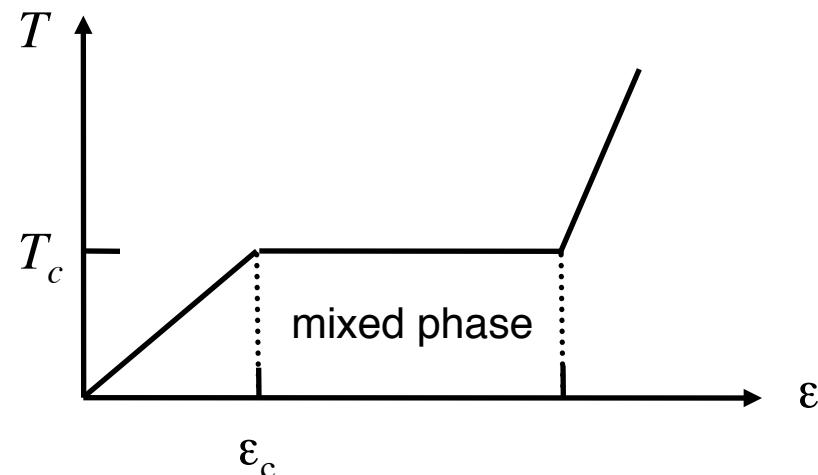
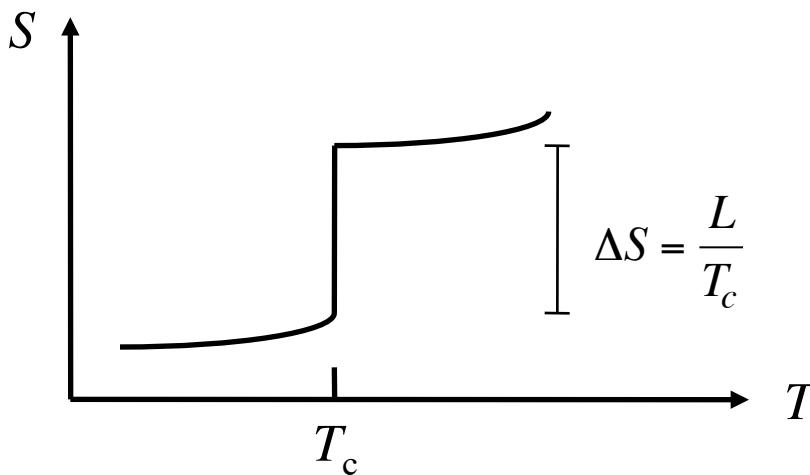
Size matters !!!

Thermodynamics - phase transitions

Phase transition or a crossover?

Signs of a phase transition:

1st order: discontinuous in entropy at T_c → Latent heat, a mixed phase



Higher order: discontinuous in higher derivatives of $\delta^n S / \delta T^n$ → no mixed phase - system passed smoothly and uniformly into new state (ferromagnet)

Temperature \Leftrightarrow transverse momentum

$$T \propto \langle p_T \rangle$$

Energy density \Leftrightarrow transverse energy

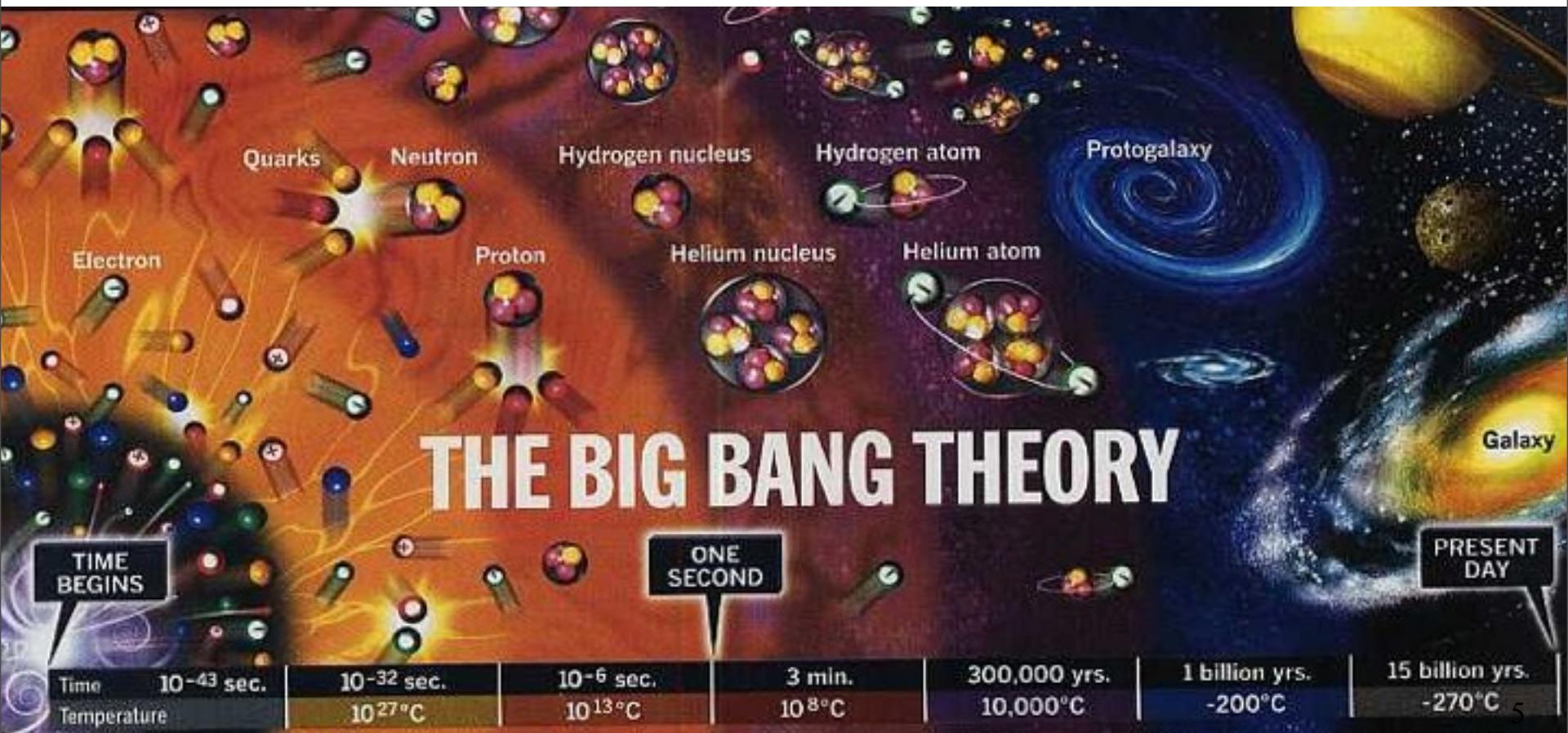
$$\varepsilon \propto dE_T/dy \cong \langle m_T \rangle dN/dy$$

Entropy \Leftrightarrow multiplicity

$$S \propto dN/dy$$

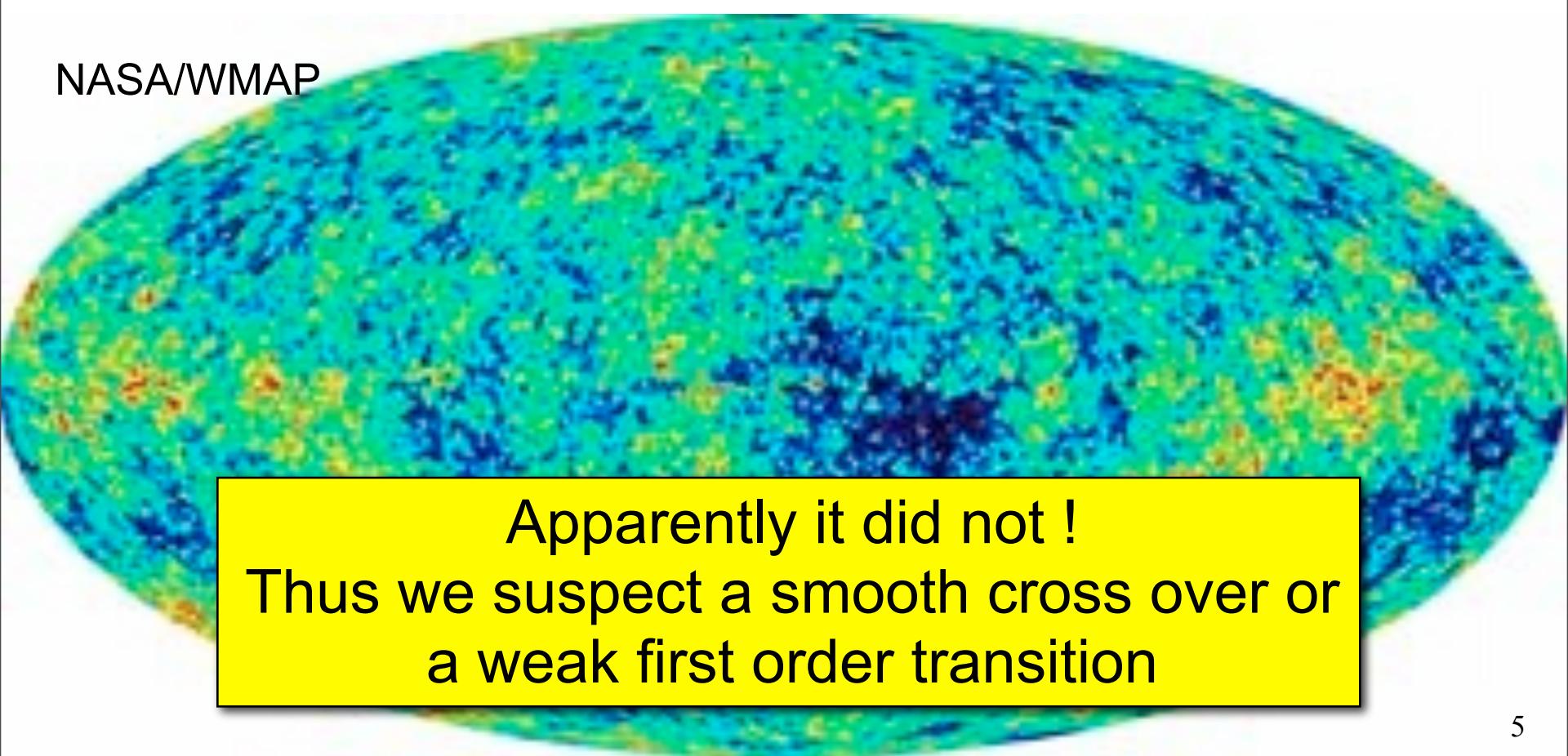
The order of the phase transition

“A first-order QCD phase transition that occurred in the early universe would lead to a surprisingly rich cosmological scenario.” Ed Witten, Phys. Rev. D (1984)



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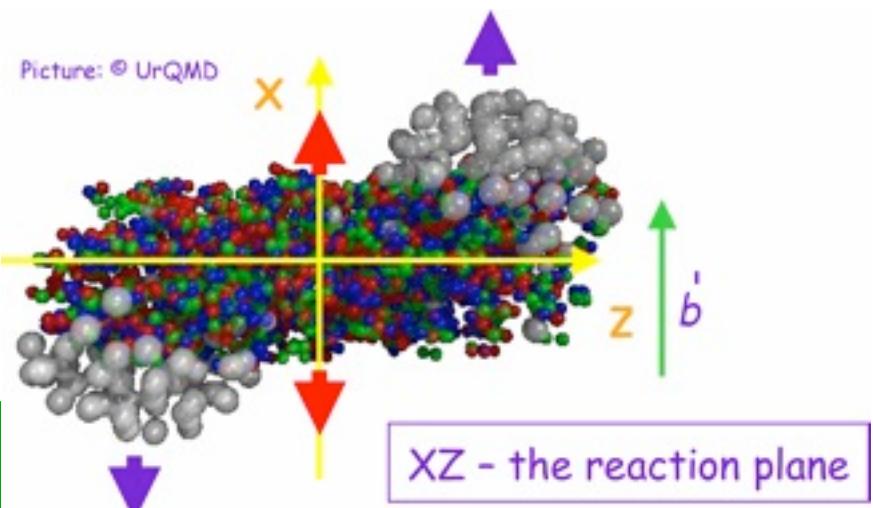
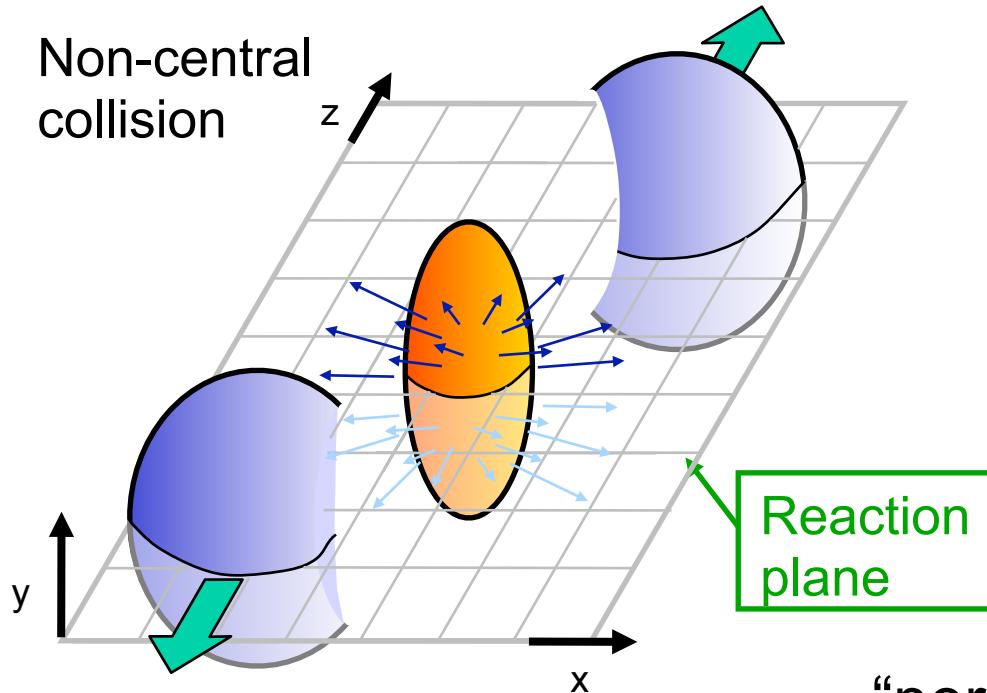


The language of RHI collisions

- Before starting, we need to know some specific terminology used in RHI collisions.
- Relativity:
Energy: $E^2 = p^2 + m^2$ or $E = T + m$ or $E = \gamma m c^2$
where: $\gamma = \frac{1}{\sqrt{1 - \beta^2}}$ and $\beta = \frac{v}{c} = \frac{p}{E}$
- Lorentz Transformations:
$$E' = \gamma(E + \beta p_z)$$
$$p'_z = \gamma(p_z + \beta E)$$
- Kinematics:
 $p_L = p_z$
 $p_T = \sqrt{(p_x^2 + p_y^2)}$
 $m_T = \sqrt{(p_T^2 + m^2)}$
Transverse mass
- $y = \frac{1}{2} \ln \frac{E + p_L}{E - p_L}$ Rapidit
- $y' = y + \tanh^{-1} \beta$
- $\eta = \frac{1}{2} \ln \frac{p + p_L}{p - p_L}$ Psuedo-Rapidity
(no particle id required)

Geometry of a heavy-ion collision

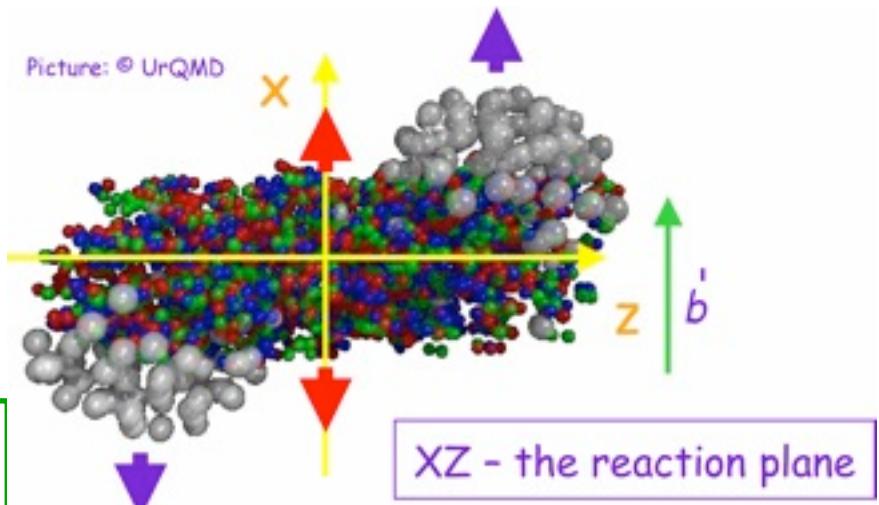
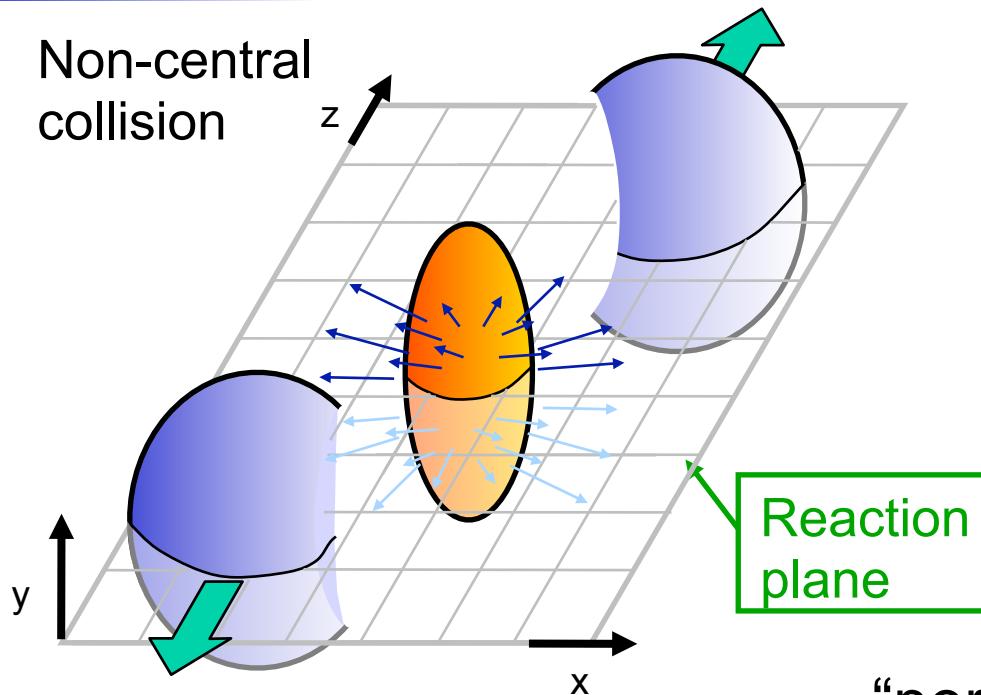
Non-central
collision



“peripheral” collision ($b \sim b_{\max}$)
“central” collision ($b \sim 0$)

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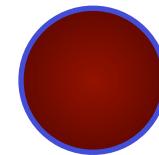
Number of participants (N_{part}): number of incoming nucleons (participants) in the overlap region

Number of binary collisions (N_{bin}): number of equivalent inelastic nucleon-nucleon collisions

$$N_{\text{bin}} \geq N_{\text{part}}$$

Quantifying the geometry

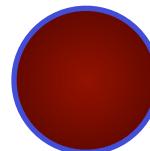
p+p: 2 Participants, 1 Binary Collision



Participants: those nucleons that have interacted at least once
Binary collisions: the number of 1+1 collisions

Quantifying the geometry

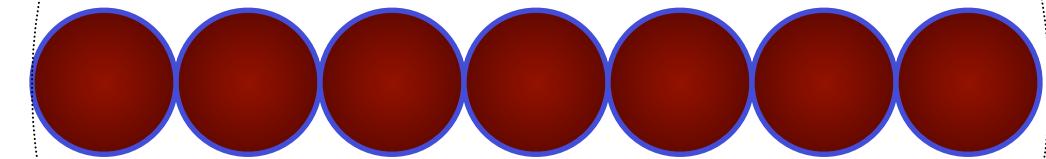
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Quantifying the geometry

p+A: 8 Participants, 7 Binary Collisions

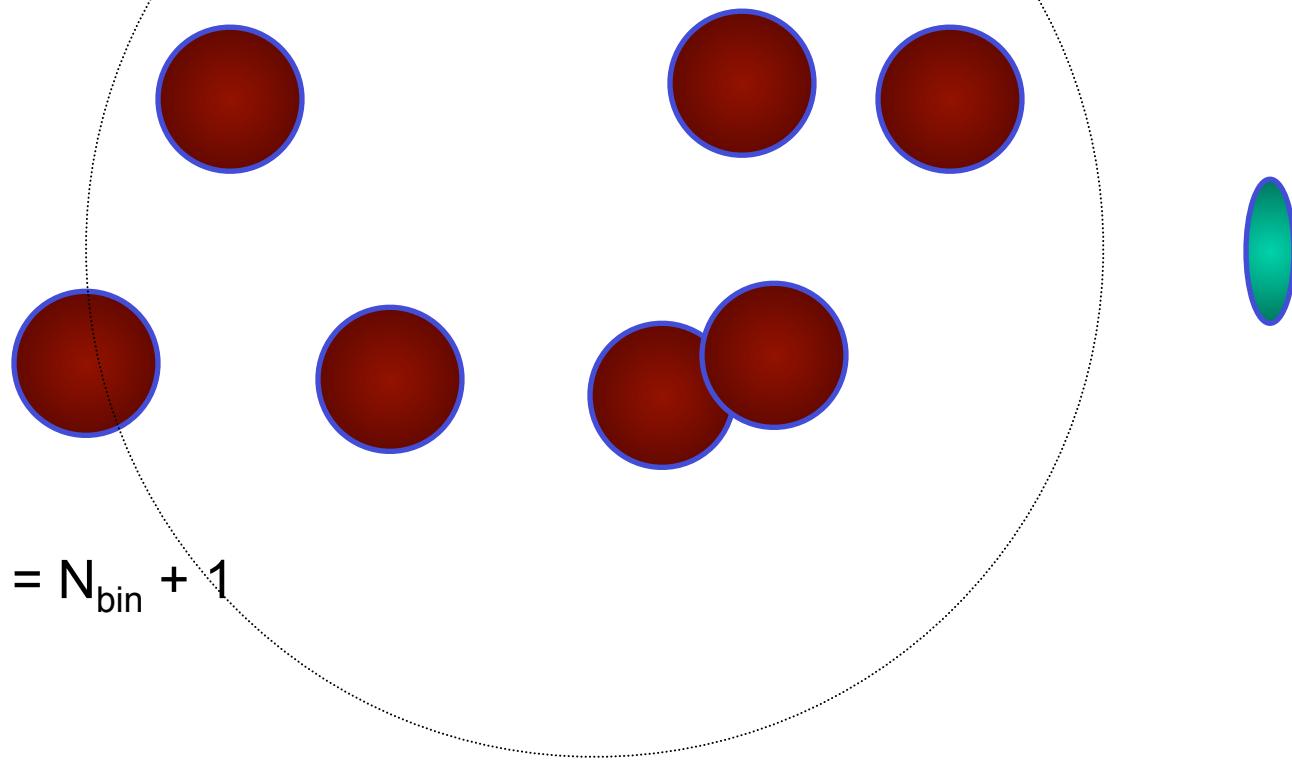


$$\text{Generically: } N_{\text{part}} = N_{\text{bin}} + 1$$

Participants: those nucleons that have interacted at least once
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Quantifying the geometry

p+A: 8 Participants, 7 Binary Collisions

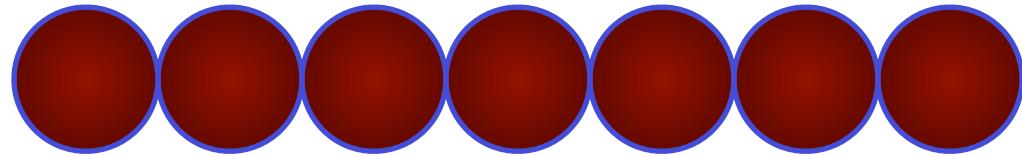
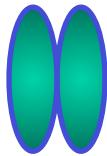


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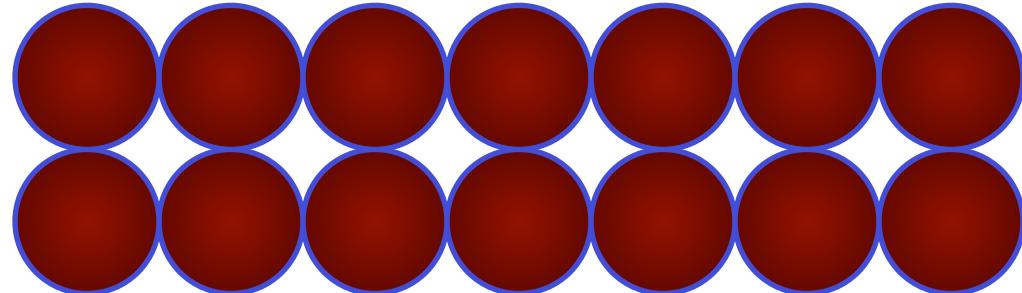
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Binary collisions: the number of 1+1 collisions

Quantifying the geometry

A+A: 9 Participants, 14 Binary Collisions



A+A: 16 Participants, 14 Binary Collisions

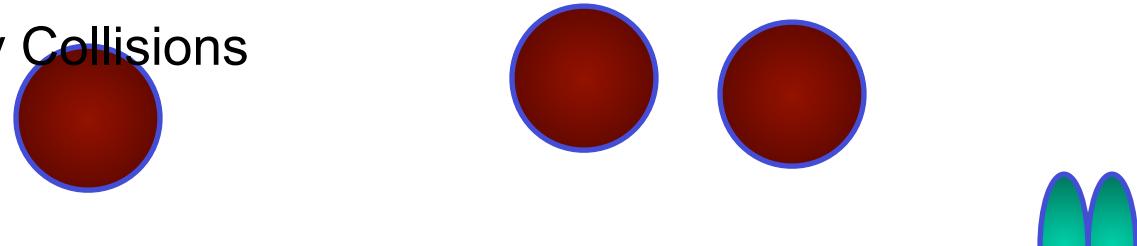


Participants: those nucleons that have interacted at least once

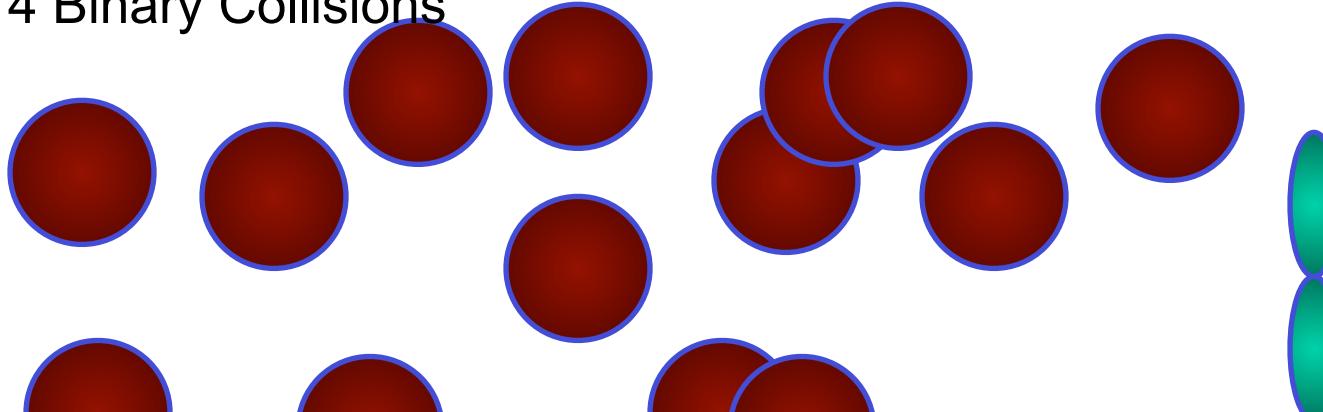
Binary collisions: the number of 1+1 collisions

Quantifying the geometry

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A+A: 16 Participants, 14 Binary Collisions



Participants: those nucleons that have interacted at least once

Binary collisions: the number of 1+1 collisions

Glauber calculations

Use a Glauber calculation to estimate N_{bin} and N_{part}

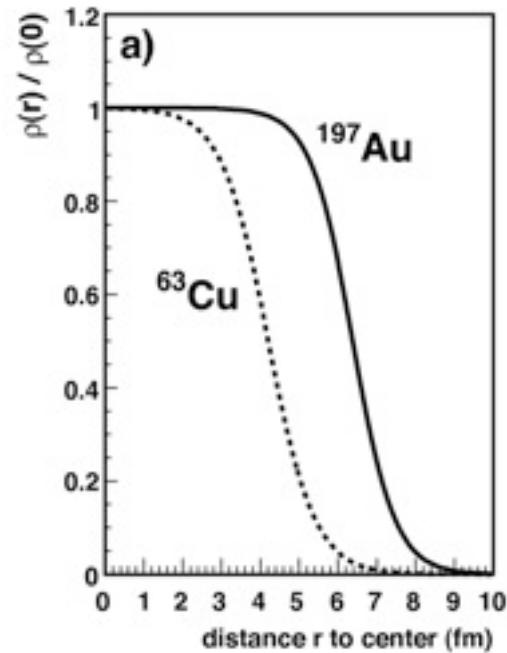
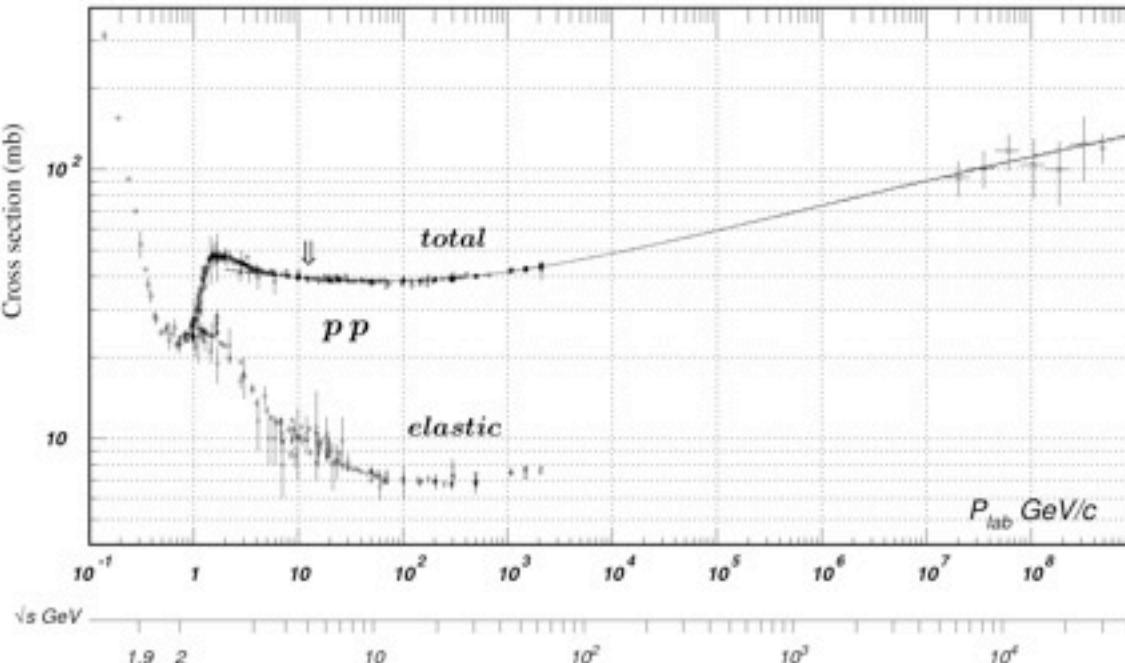
- Roy Glauber: Nobel prize in physics 2005 for “his contribution to the quantum theory of optical coherence”
- Application of Glauber theory to heavy ion collisions does not use the full sophistication of these methods. Two simple assumptions:
 - Eikonal: constituents of nuclei proceed in straight-line trajectories
 - Interactions determined by initial-state shape of overlapping nuclei



Ingredients for Glauber calculations

Particle Data Book: W.-M. Yao et al., J. Phys. G 33,1 (2006) Fig 40.11

M. Miller et al, nucl-ex/0701025



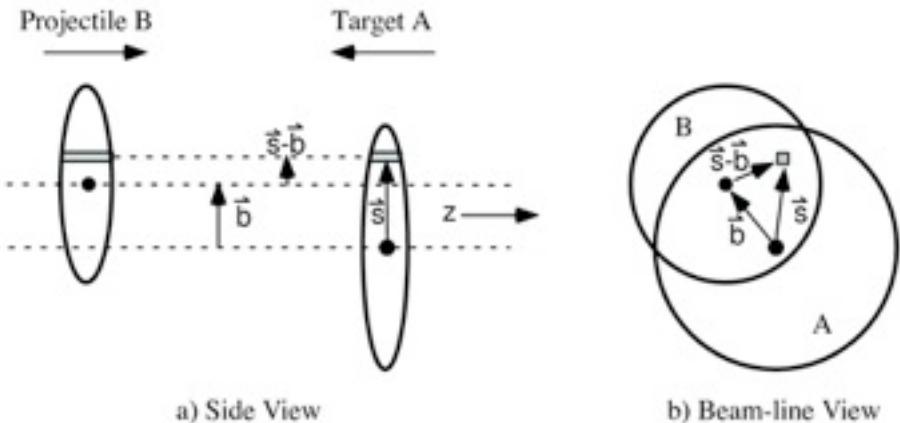
- Assumptions: superposition of straight-line interactions of colliding nucleons
- Need nucleon-nucleon interaction cross section
 - Most use inelastic: 42 mb at $\sqrt{s}=200$ GeV
 - Other choices: Non-singly-diffractive, 30 mb at $\sqrt{s} = 200$ GeV
- Need probability density for nucleons:
'Wood-Saxon' from electron scattering experiments

Implementations of Glauber

M. Miller et al, nucl-ex/0701025

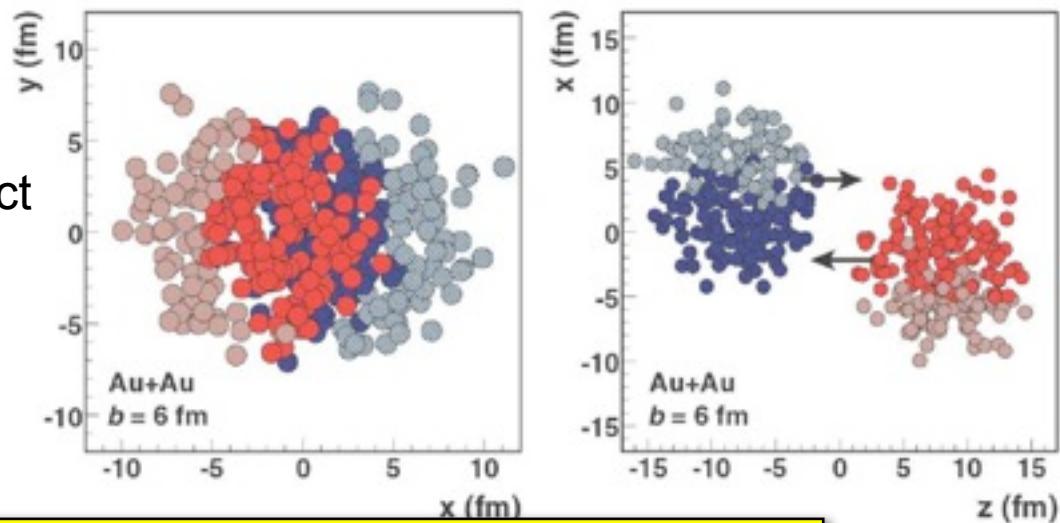
- **Optical Glauber**

- Smooth distribution assumed
- Analytic overlap calculation from integration over nuclear shape functions, weighted with appropriate N-N cross-section



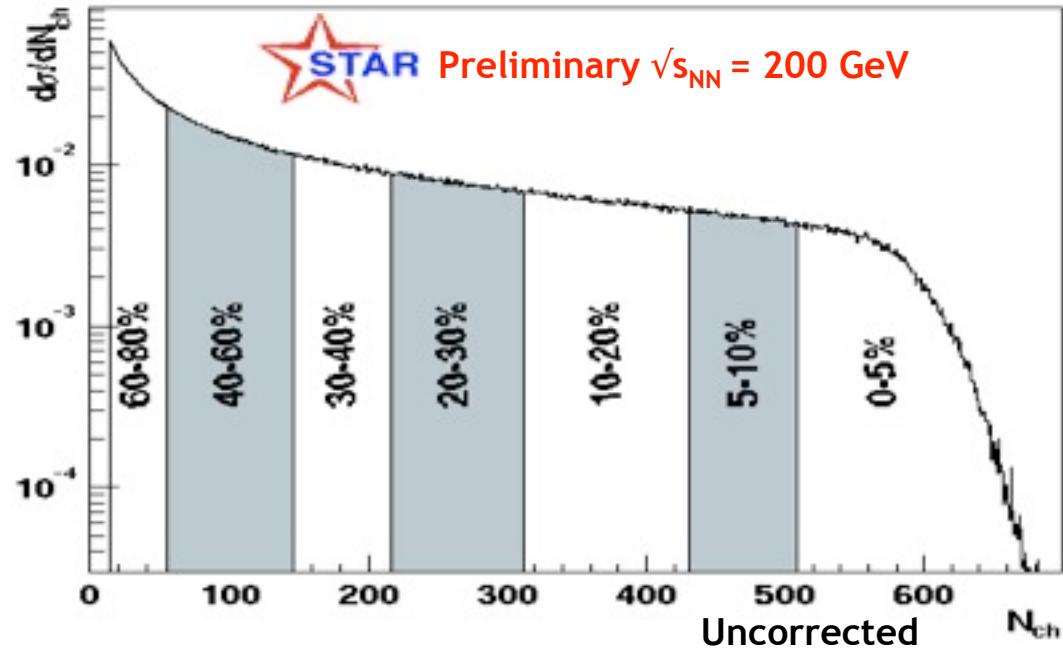
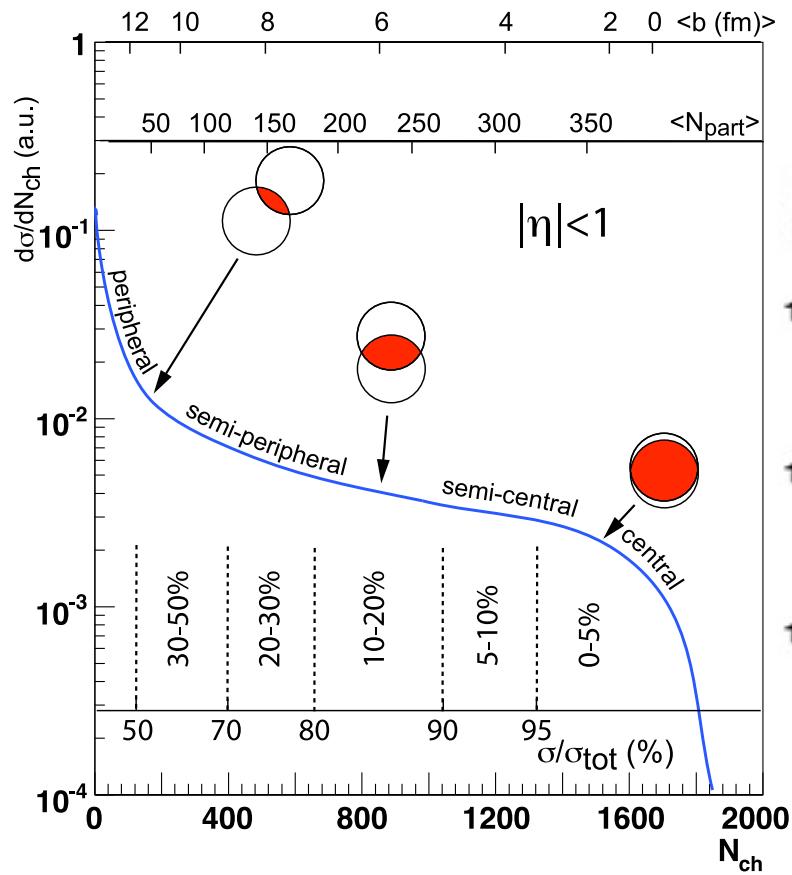
- **Monte Carlo Glauber**

- Randomly initialize nucleons sampling nuclear shape
- At randomly selected impact parameter, allow nuclei to interact
- Randomly sample probability of nucleons to interact from interaction cross-section
 - e.g. if distance d between nucleons is $< \sqrt{\sigma_{\text{int}}/\pi}$



Calculate probability that N_{part} or N_{bin} occurs per event

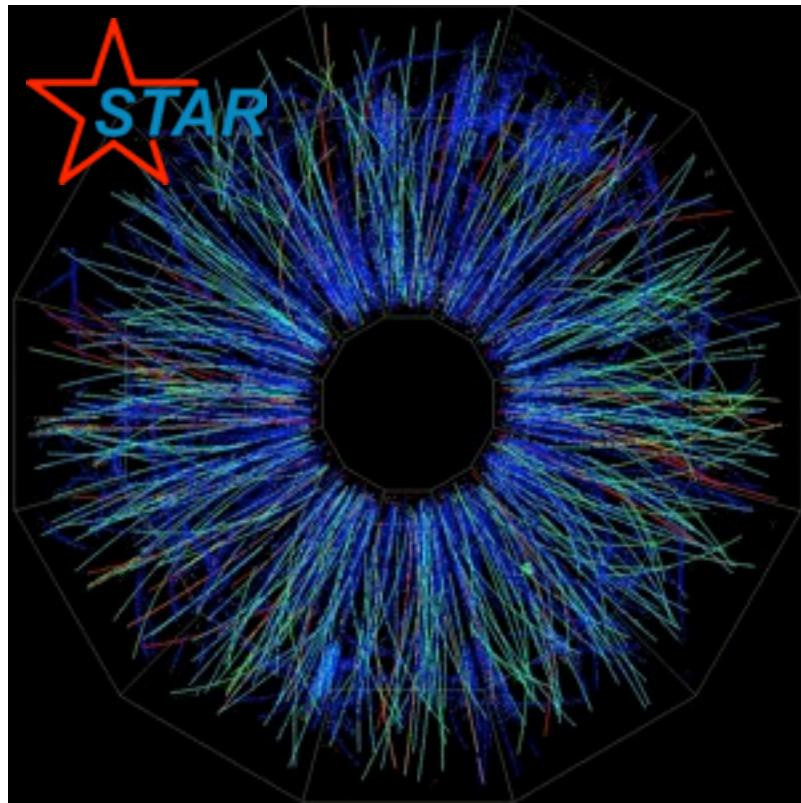
Comparing to data heavy-ion collision



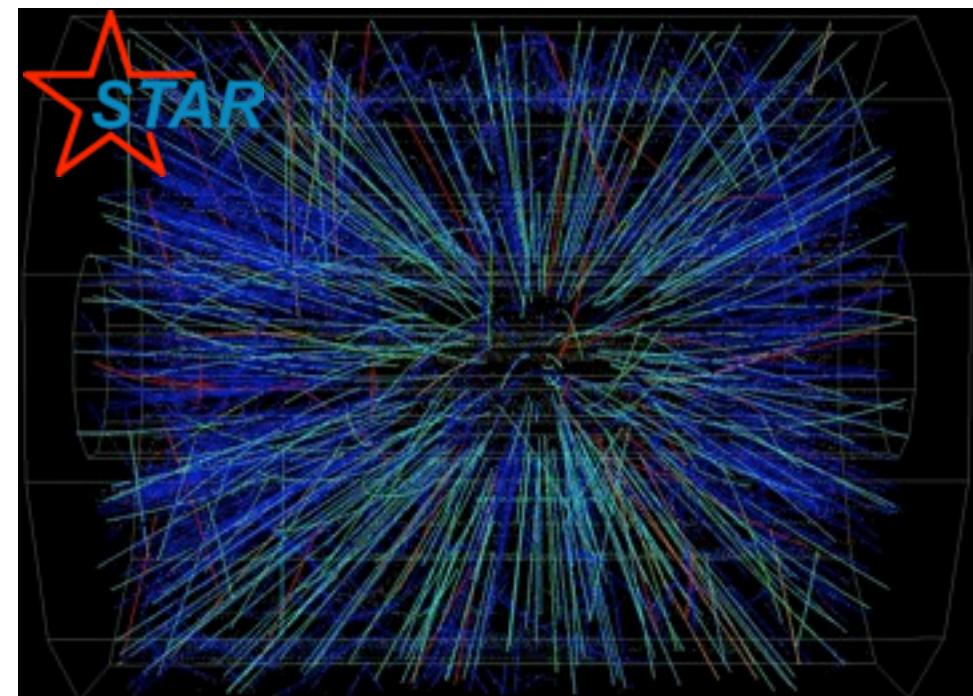
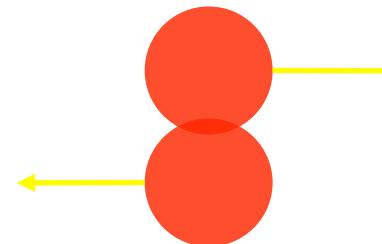
Good agreement between data and calculation

Measured mid-rapidity particle yield can be related to size of overlap region

A peripheral Au-Au collision

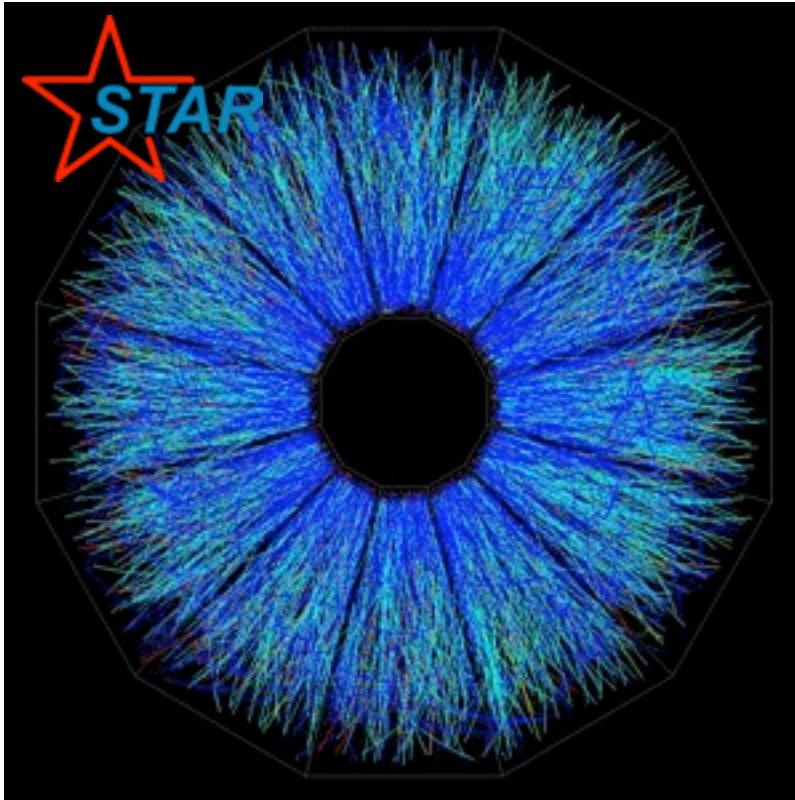


Peripheral Collision



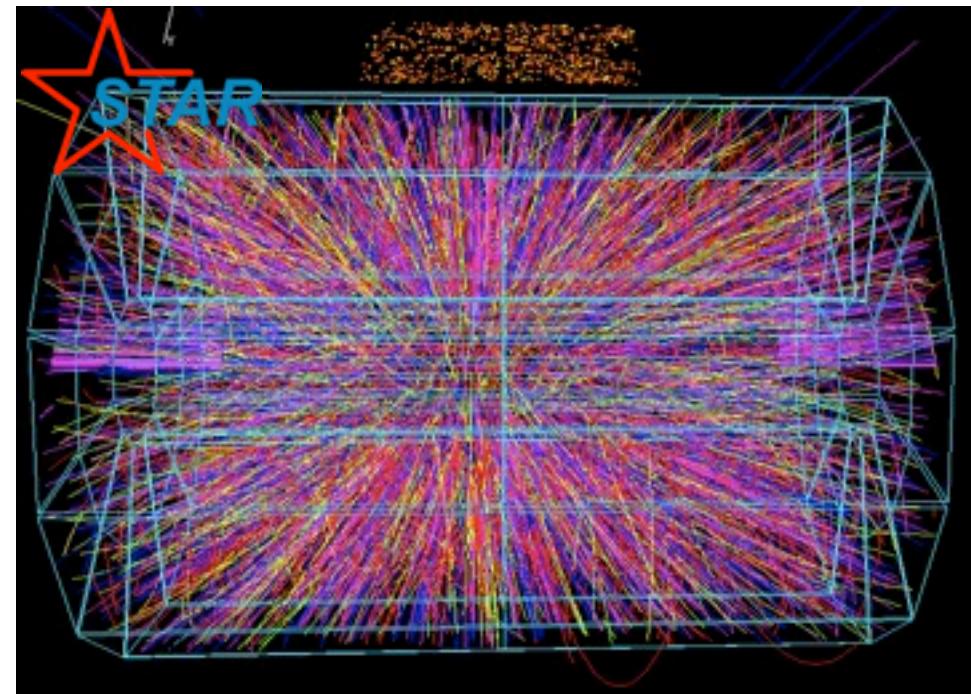
Color \Rightarrow Energy loss in TPC gas

39.4 TeV in central Au-Au collision

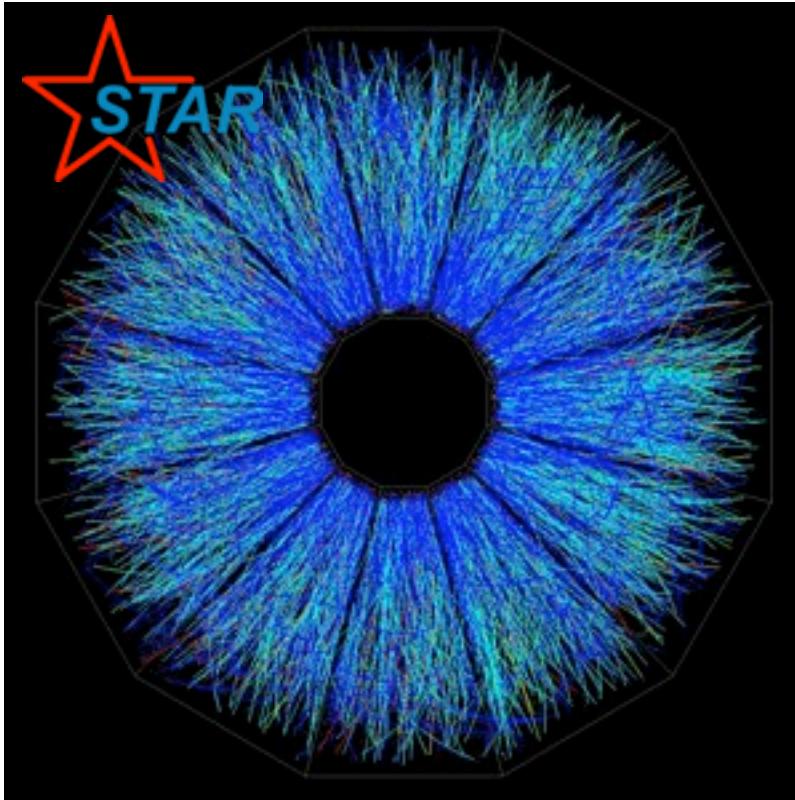


>5000 hadrons and leptons

- Only **charged** particles shown
- Neutrals don't ionise the TPC's gas so are not "seen" by this detector.



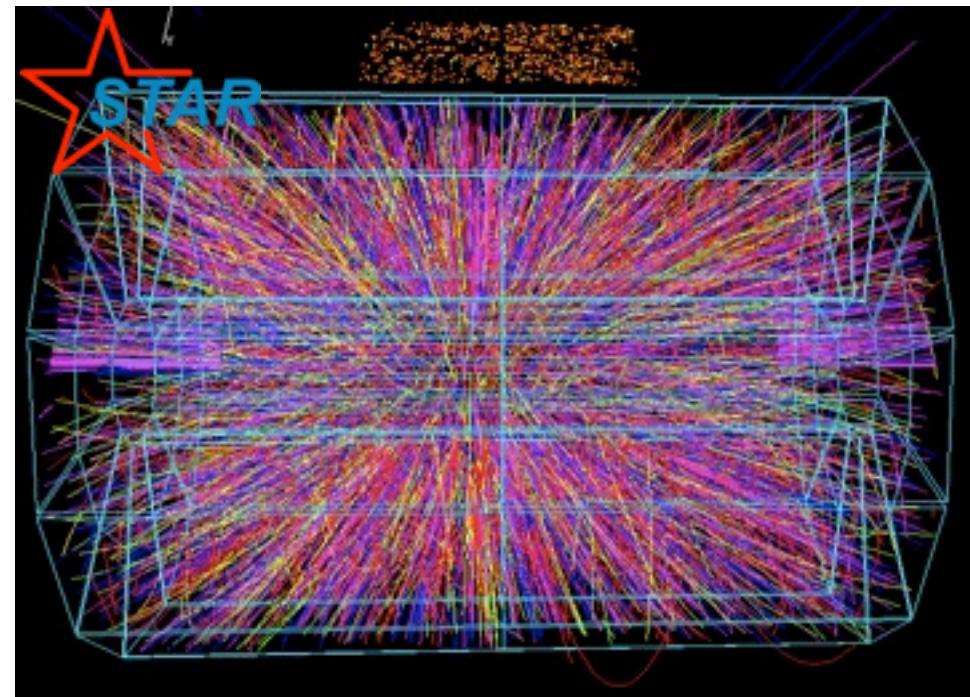
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>5000 hadrons and leptons

26 TeV is removed
from colliding beams.

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The energy is contained in one collision

Central Au+Au Collision:
26 TeV ~ 6 μ Joule



The energy is contained in one collision

Central Au+Au Collision:
26 TeV \sim 6 μ Joule

Sensitivity of human ear:
 10^{-11} erg = 10^{-18} Joule = 10^{-12} μ Joule
A Loud “Bang” if E \Rightarrow Sound



The energy is contained in one collision

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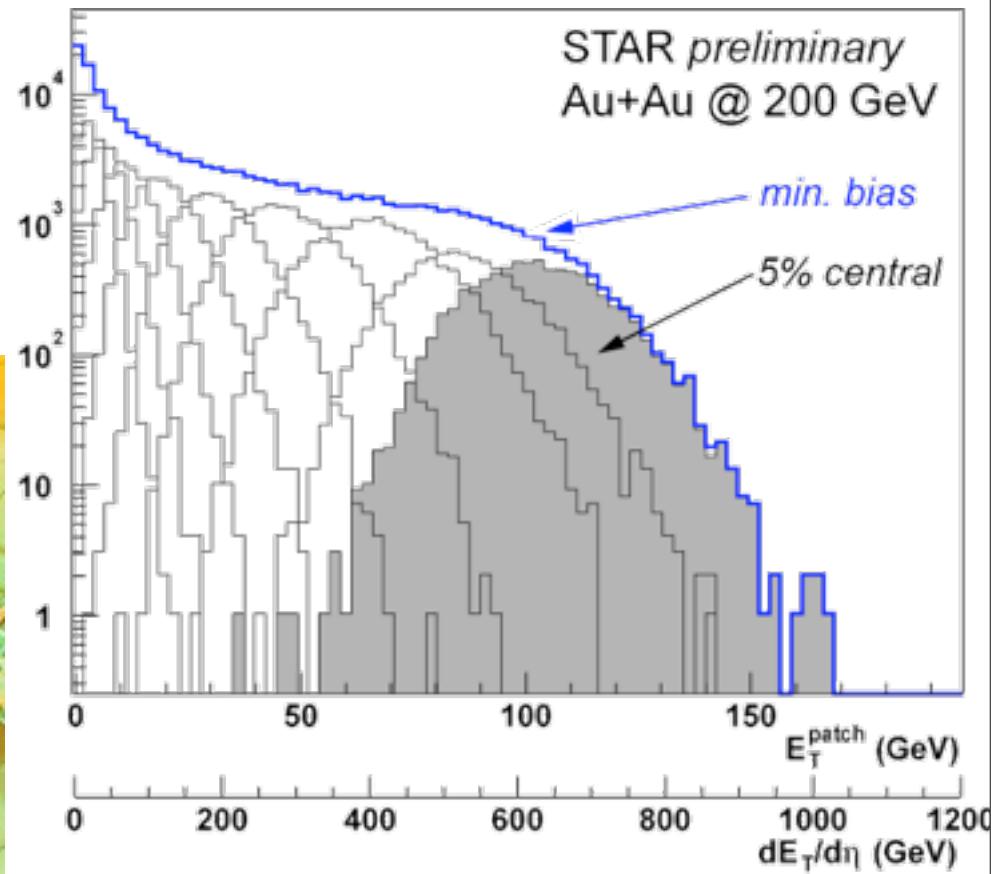
Most goes into particle creation

Energy density in central Au-Au collisions

- use calorimeters to measure total energy



The
PHENIX
Calorimeter



Energy density in central Au-Au collisions

- use calorimeters to measure total energy
- estimate volume of collision

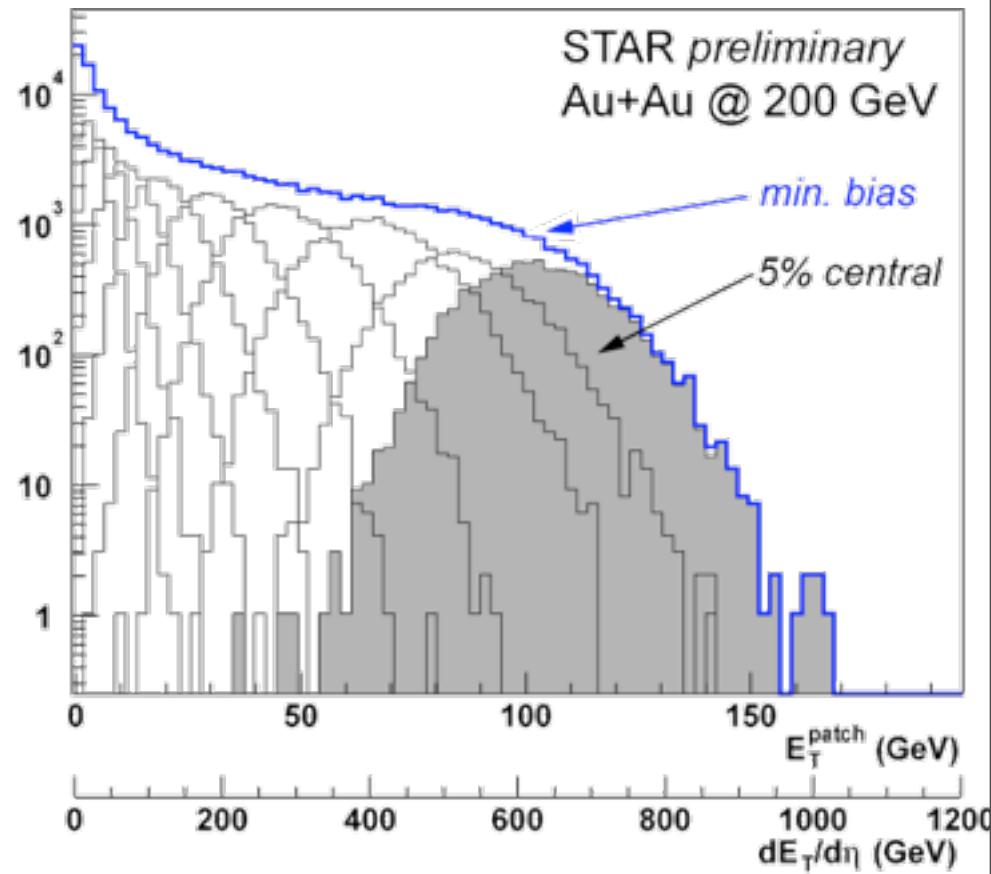
Bjorken-Formula for Energy Density:

$$\varepsilon_{Bj} = \frac{\Delta E_T}{\Delta V} = \frac{1}{\pi R^2} \frac{1}{\tau_0} \frac{dE_T}{dy}$$

$R \sim 6.5 \text{ fm}$

Time it takes to thermalize system ($t_0 \sim 1 \text{ fm/c}$)

A diagram of a cylindrical volume element. The cylinder has a radius labeled πR^2 and a thickness labeled $dz = \tau_0 dy$. Arrows indicate the radial extent of the cylinder.



Energy density in central Au-Au collisions

- use calorimeters to measure total energy
- estimate volume of collision

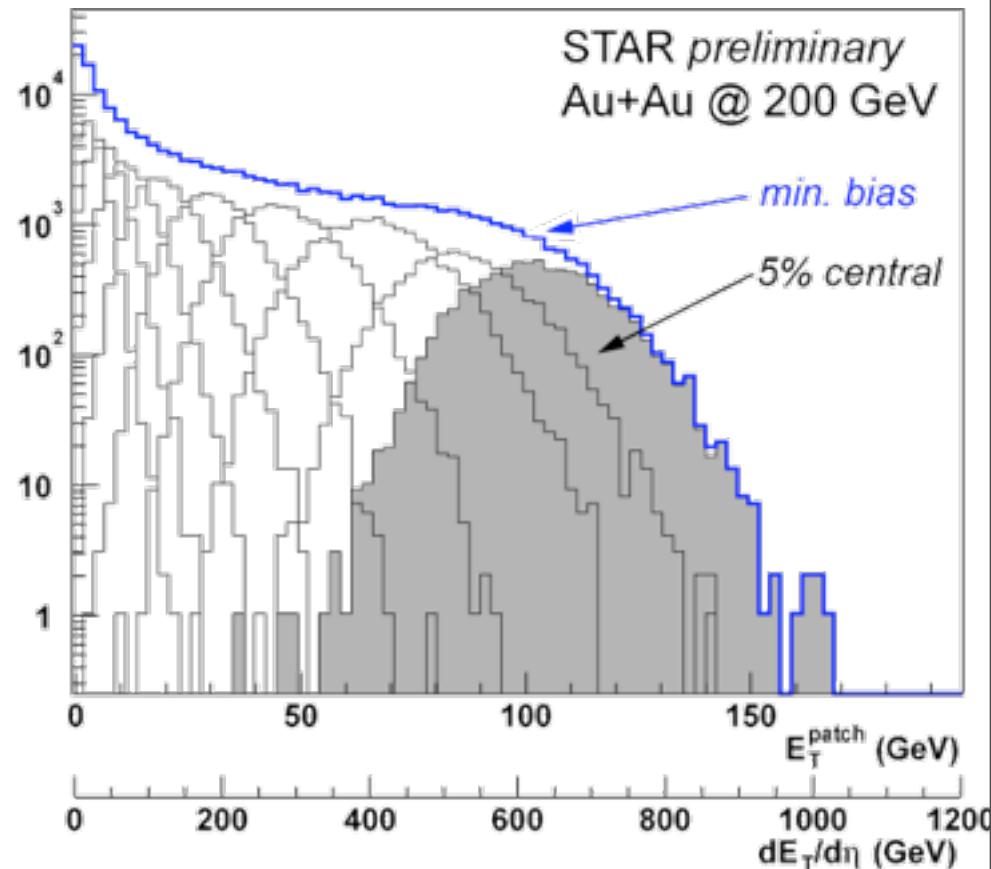
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Time it takes to thermalize system ($t_0 \sim 1 \text{ fm/c}$)

$$dz = \tau_0 dy$$
$$\pi R^2$$



$\varepsilon_{BJ} \approx 5.0 \text{ GeV/fm}^3$
~30 times normal nuclear density
~ 5 times $> \varepsilon_{\text{critical}}$ (lattice QCD)

5 GeV/fm³. Is that a lot?

In a year, the U.S. uses \sim 100 quadrillion BTUs of energy
(1 BTU = 1 burnt match):

$$100 \times 10^{15} BTU \times \frac{1060J}{BTU} \times \frac{1eV}{1.6 \times 10^{-19}J} = 6.6 \times 10^{38} eV$$

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At 5 GeV/fm³, this would fit in a volume of:

$$6.6 \times 10^{38} eV \div \frac{5 \times 10^9 eV}{fm^3} = 1.3 \times 10^{29} fm^3$$

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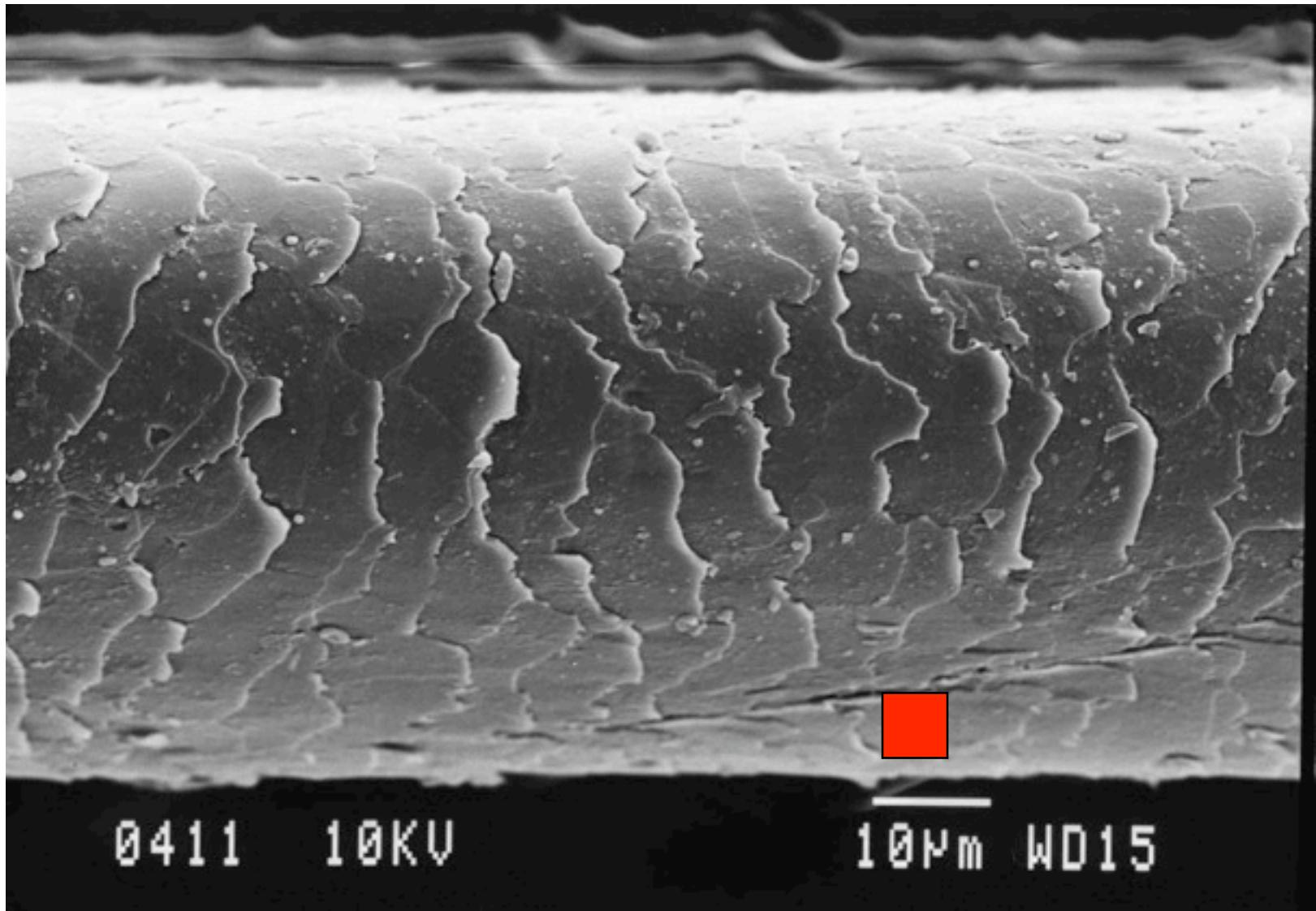
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Or, in other words, in a box of the following dimensions:

$$\sqrt[3]{1.3 \times 10^{29} fm^3} = 5 \times 10^9 fm = 5 \mu m$$

A human hair



What is the temperature of the medium?

- Statistical Thermal Models:
 - Assume a system that is **thermally** (constant T_{ch}) and **chemically** (constant n_i) equilibrated
 - System composed of non-interacting hadrons and resonances
 - Obey conservation laws: Baryon Number, Strangeness, Isospin
- Given T_{ch} and μ 's (+ system size), n_i 's can be calculated in a grand canonical ensemble

$$n_i = \frac{g}{2\pi^2} \int_0^\infty \frac{p^2 dp}{e^{(E_i(p) - \mu_i)/T} \pm 1}, \quad E_i = \sqrt{p^2 + m_i^2}$$

Fitting the particle ratios

Number of particles of a given species related to temperature

$$dn_i \sim e^{-(E-\mu_B)/T} d^3 p$$

- Assume all particles described by same temperature T and μ_B
- one ratio (e.g., \bar{p} / p) determines μ / T :

$$\frac{\bar{p}}{p} = \frac{e^{-(E+\mu_B)/T}}{e^{-(E-\mu_B)/T}} = e^{-2\mu_B/T}$$

- A second ratio (e.g., K / π) provides $T \rightarrow \mu$

$$\frac{K}{\pi} = \frac{e^{-E_K/T}}{e^{-E_\pi/T}} = e^{-(E_K - E_\pi)/T}$$

- Then all other hadronic ratios (and yields) defined

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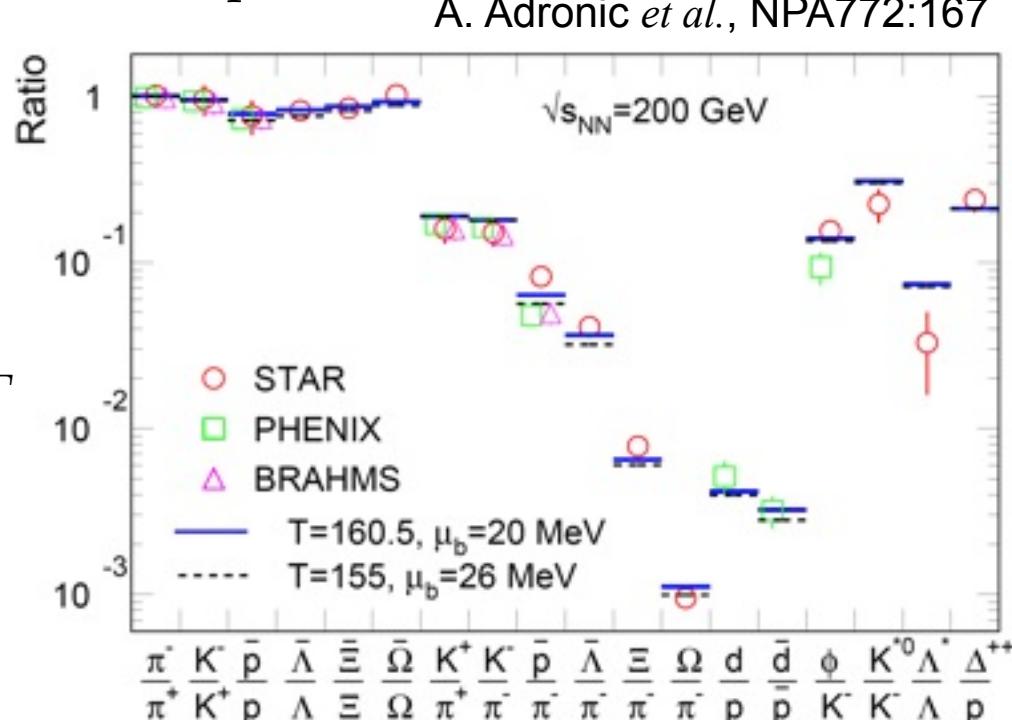
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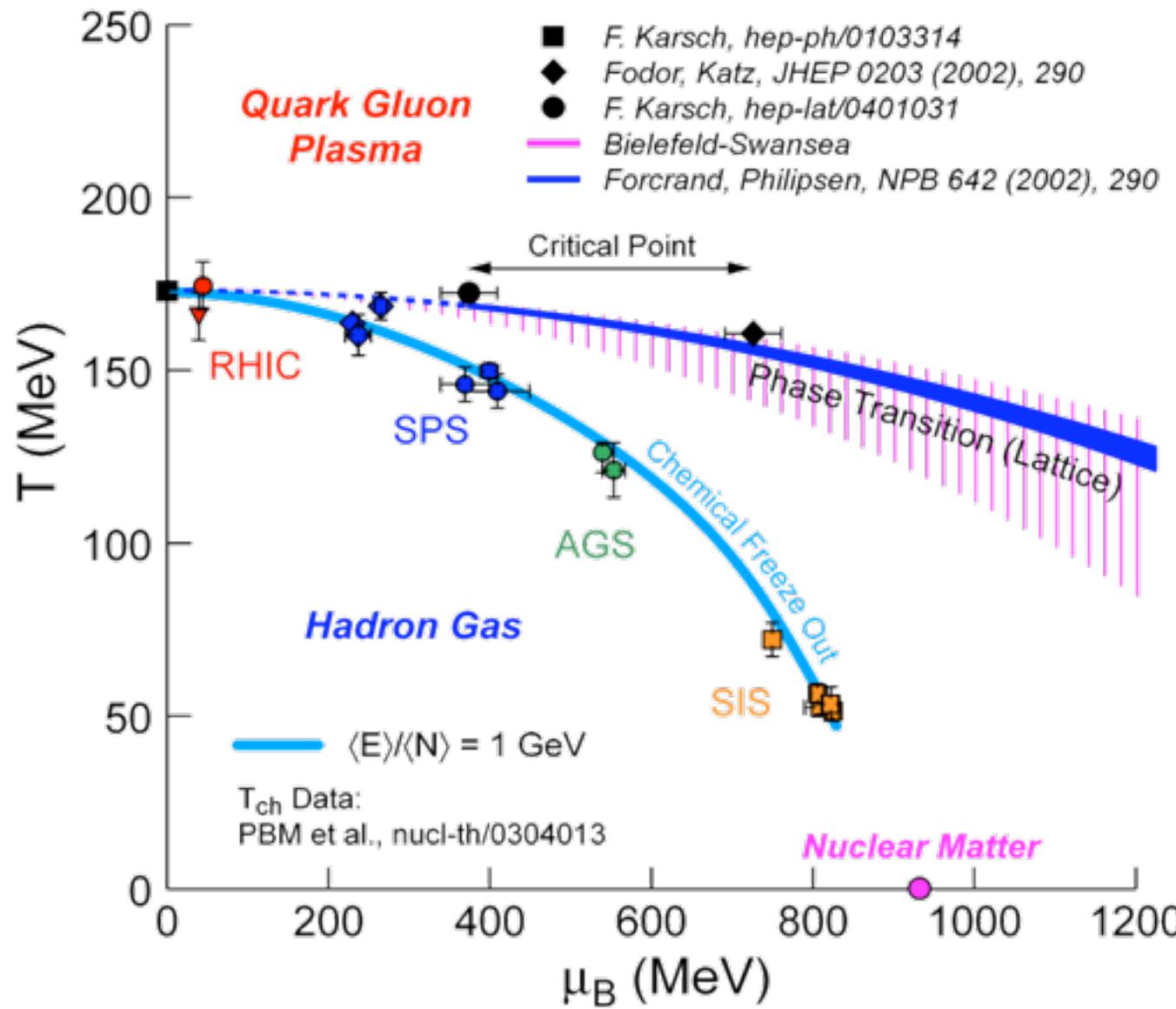
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$T \sim 160 \text{ MeV}, \mu_b \sim 20 \text{ MeV}$

Initial Temperature
probably much higher

Where RHIC sits on the phase diagram



Off on a tangent

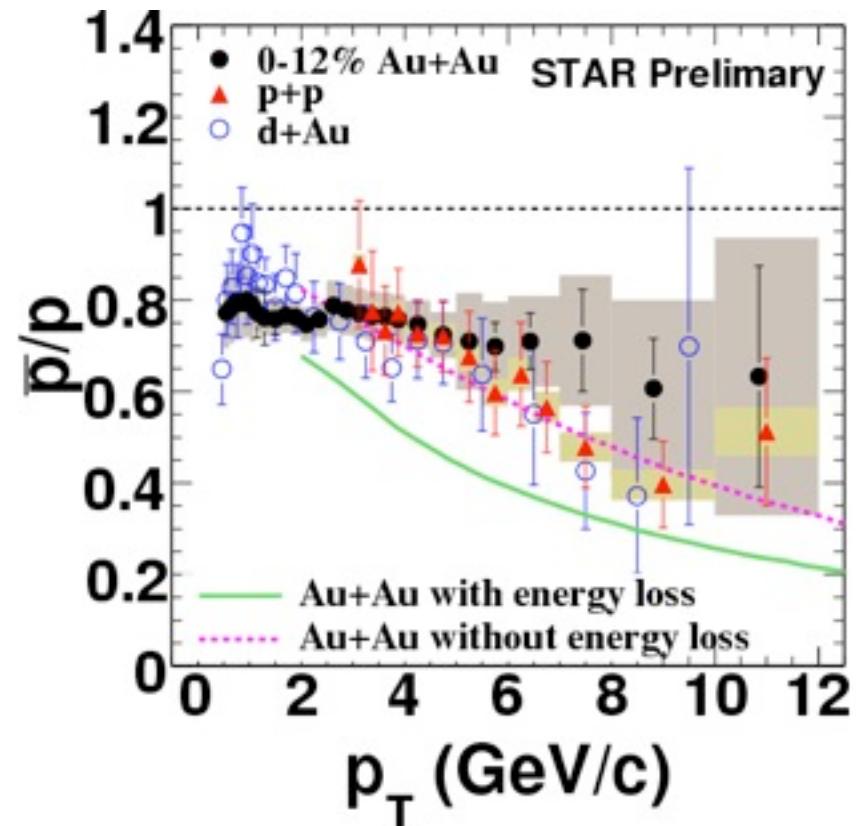
Take a second look at the anti-proton/proton ratio

$$\bar{p}/p \sim 0.8$$

There is a net baryon number at mid-rapidity!!

Baryons number is being transported over 6 units of rapidity from the incoming beams to the collision zone!

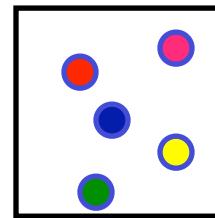
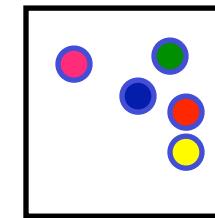
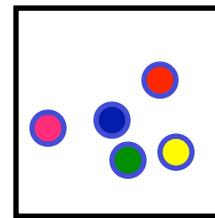
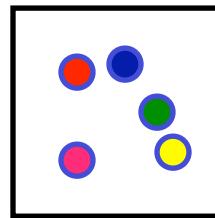
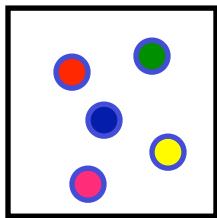
Consider what impulse that must be



Baryon number not carried by quarks
- baryon junctions postulated

Statistics \neq thermodynamics

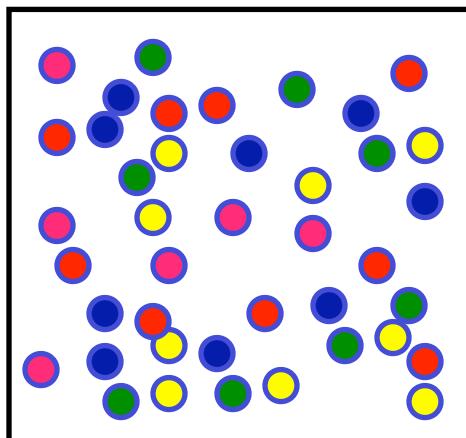
$p+p$



Ensemble of events constitutes a statistical ensemble

T and μ are simply Lagrange multipliers
“Phase Space Dominance”

$A+A$



One (1) system is already statistical !

- We can talk about pressure
- T and μ are more than Lagrange multipliers

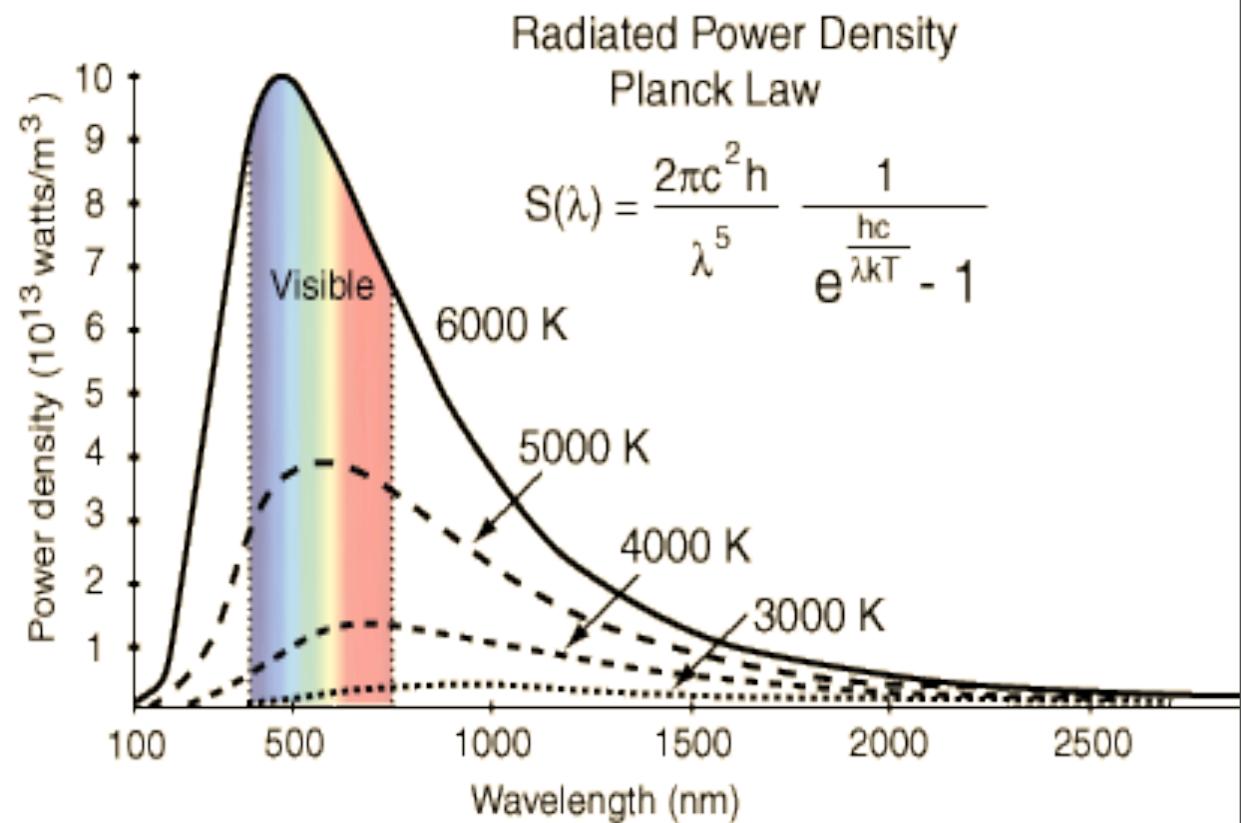
Evidence for thermalization

- Not all processes which lead to multi-particle production are thermal - elementary collisions
- Any mechanism for producing hadrons which evenly populates the free particle phase space will mimic a microcanonical ensemble.
- Relative probability to find n particles is the ratio of the phase-space volumes $P_n/P_{n'} = \varphi_n(E)/\varphi_{n'}(E) \Rightarrow$ given by statistics only.
- Difference between MCE and CE vanishes as the size of the system N increases.
- Such a system is NOT in thermal equilibrium - to thermalize need interactions/re-scattering

Need to look for evidence of collective motion

Blackbody radiation

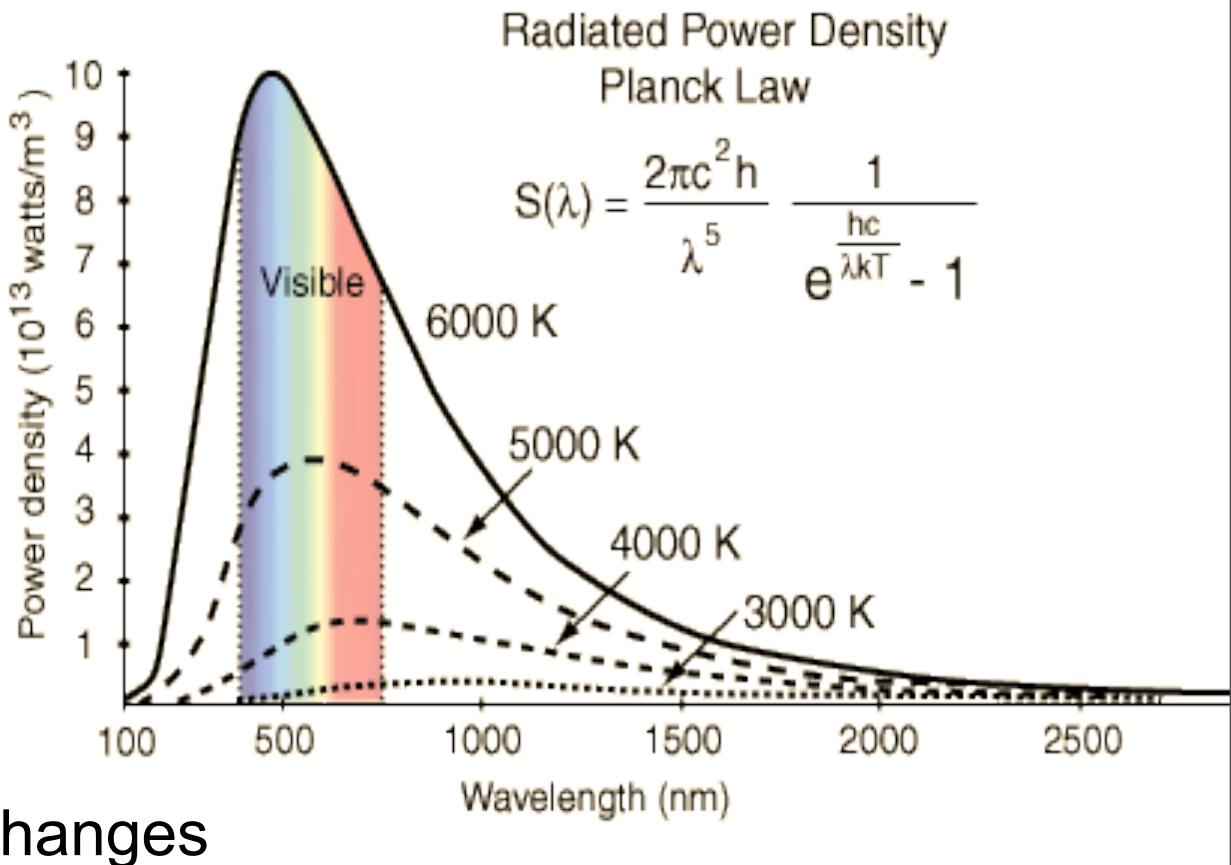
Planck distribution describes intensity as a function of the wavelength of the emitted radiation



Blackbody radiation

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“Blackbody” radiation is the spectrum of radiation emitted by an object at temperature T

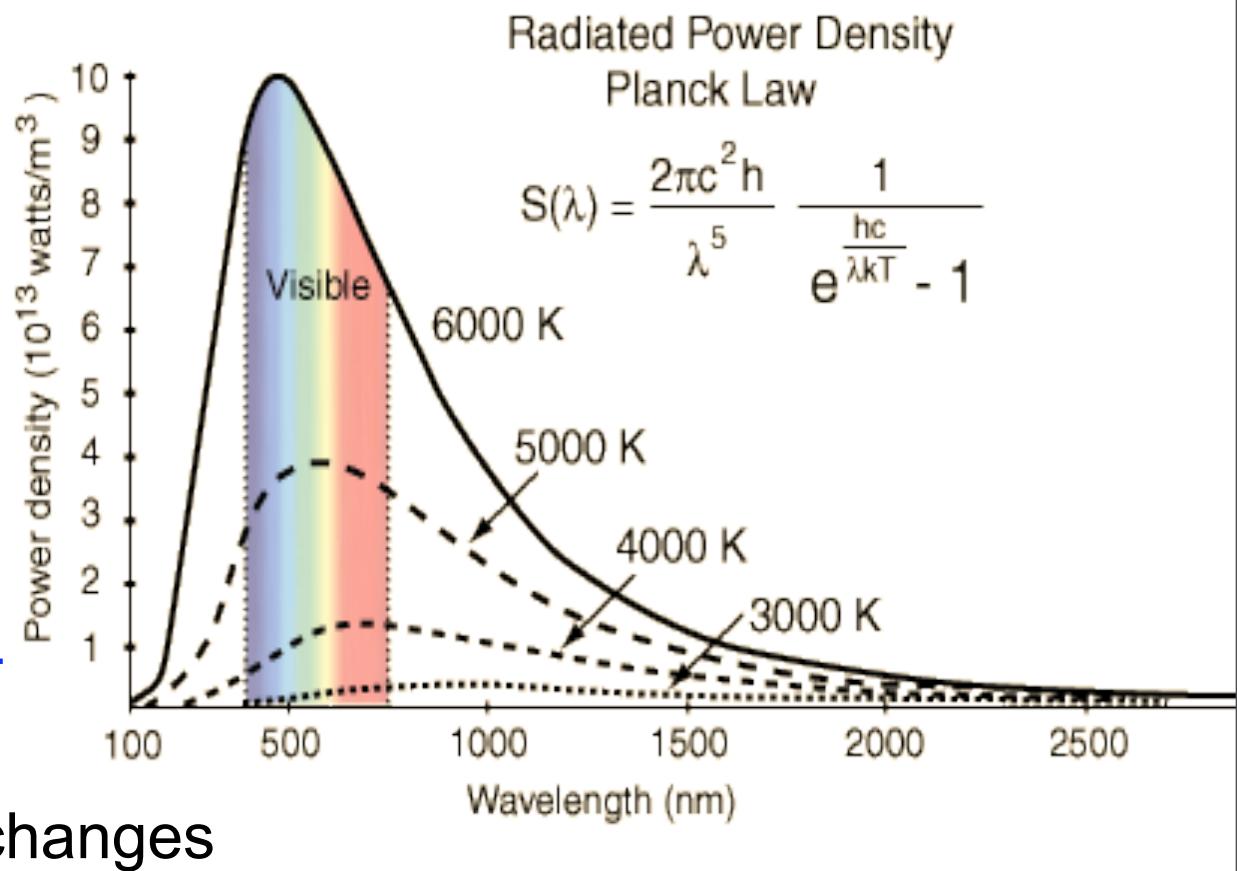


As T increases curve changes

Blackbody radiation

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As T increases curve changes

$$1/\text{Wavelength} \propto \text{Frequency} \propto E \propto p$$

Determining the temperature

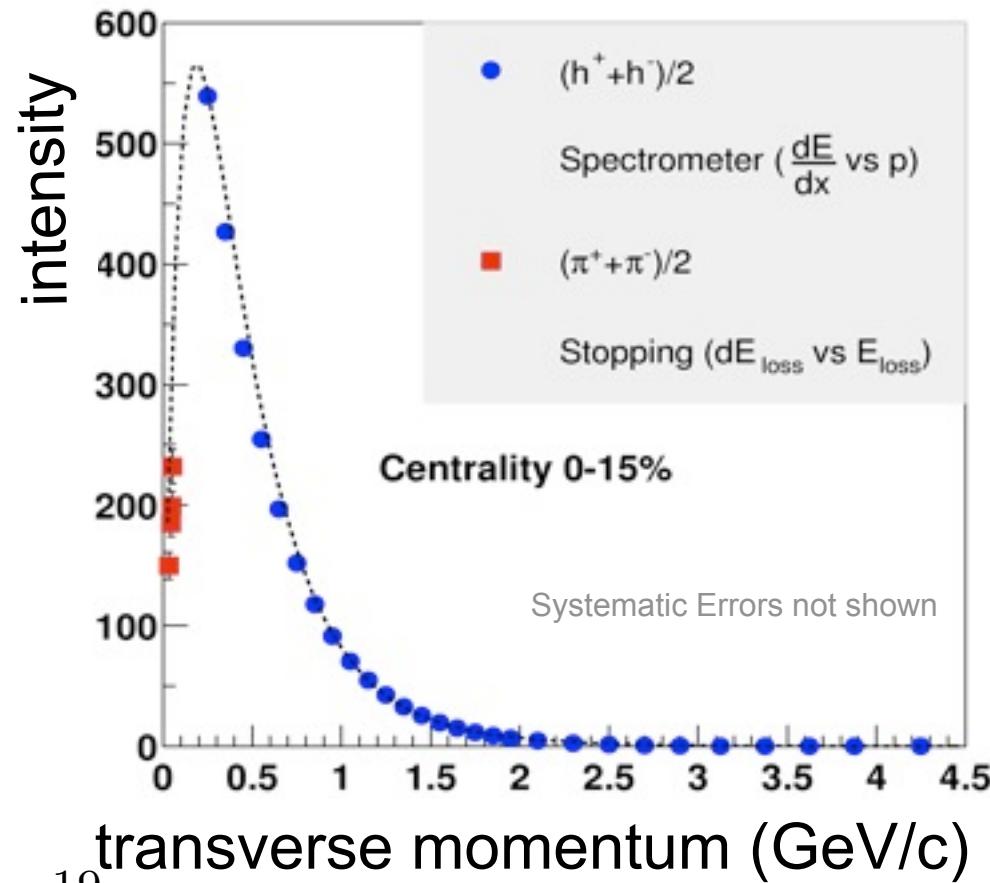
From transverse momentum distribution of pions deduce temperature ~ 120 MeV

$$E = \frac{3}{2}kT$$

$$T = \frac{2E}{3k}$$

$$= \frac{2 \times 120 \times 10^6}{3 \times 1.4 \times 10^{-23}} \times 1.6 \times 10^{-19}$$

$$\sim 9 \times 10^{11} K$$



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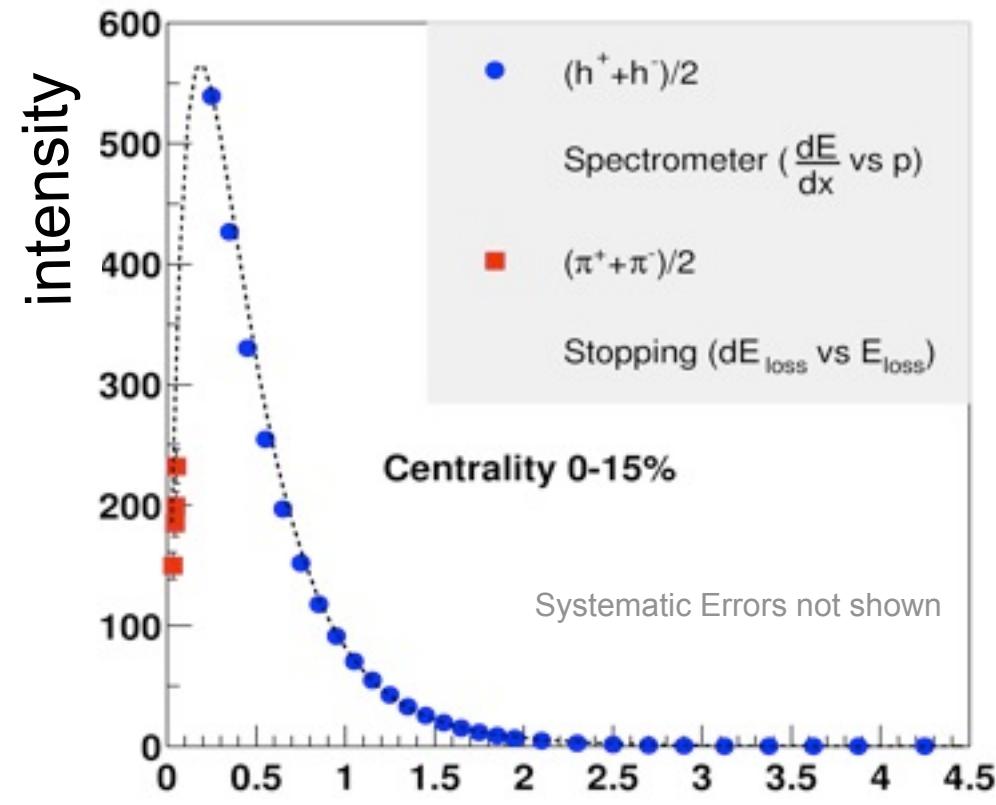
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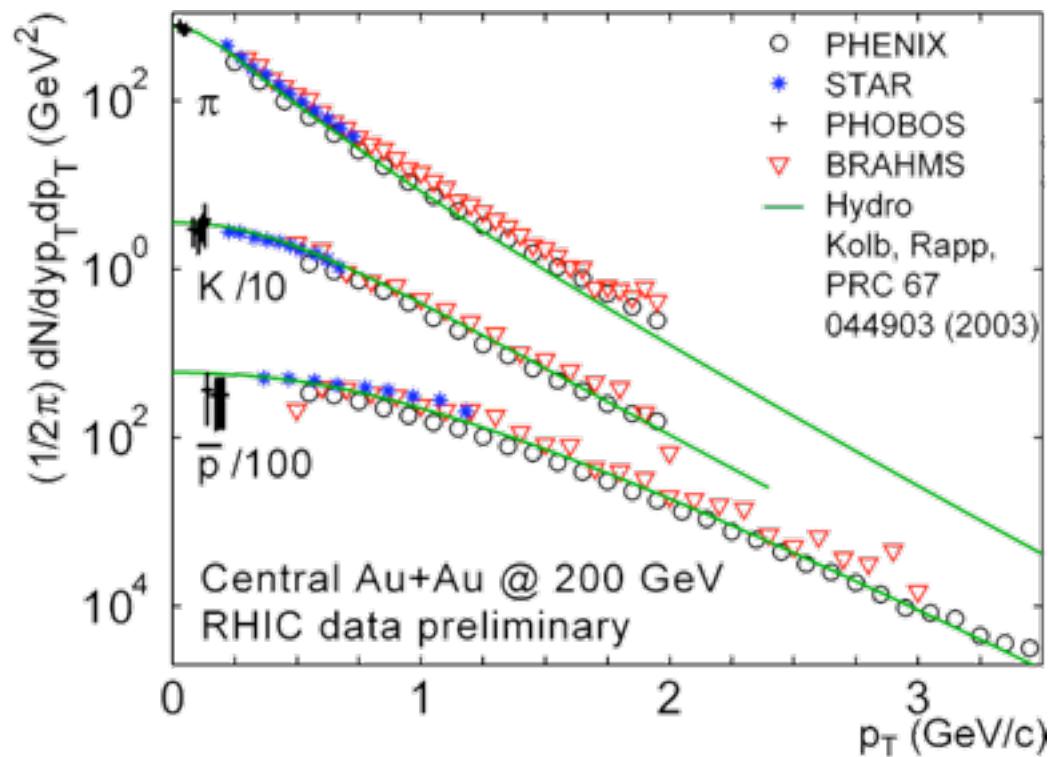
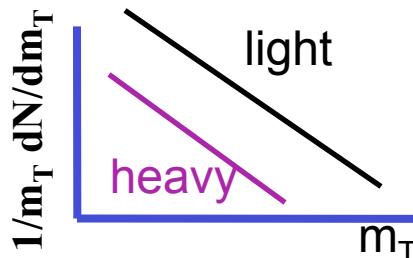
$$T_{ch} > T_{fo}$$

System exist for time in hadronic phase

Strong collective radial expansion



purely thermal
source

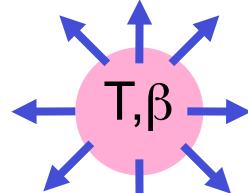


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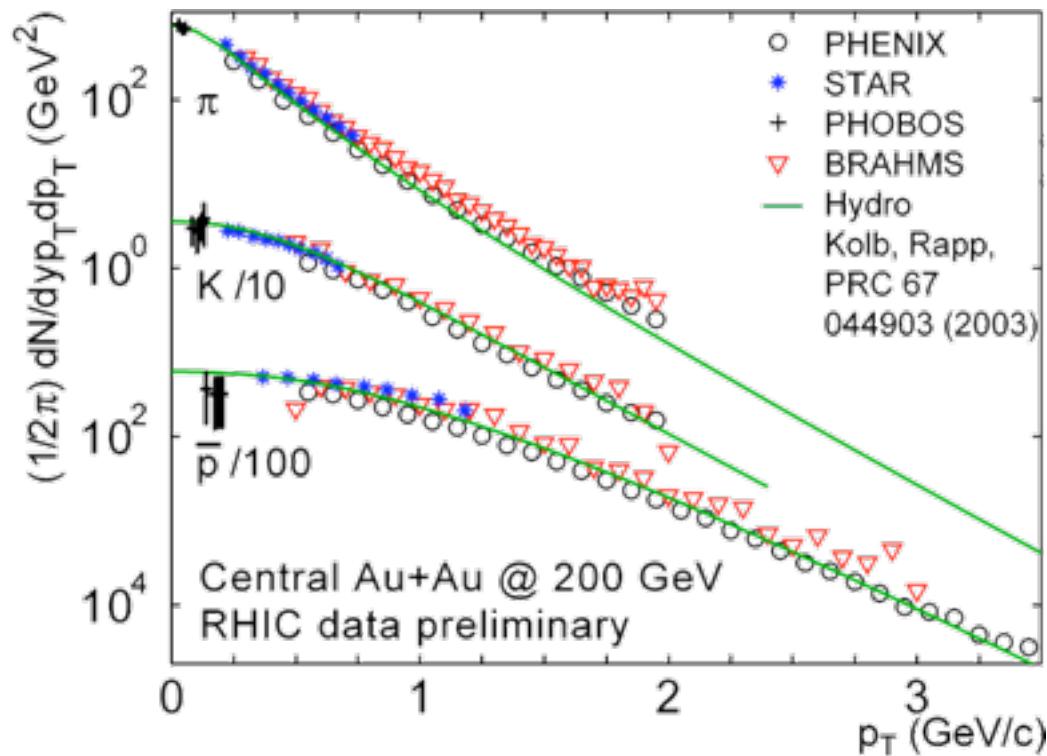
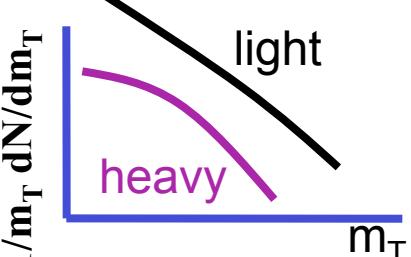
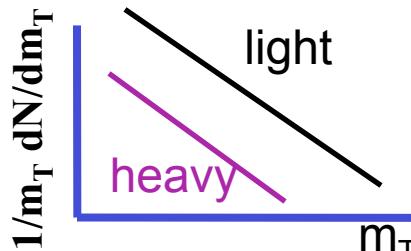


purely thermal source

explosive source



$$m_T = (p_T^2 + m^2)^{1/2}$$



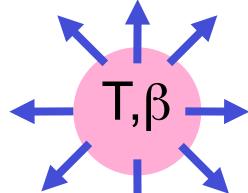
- Different spectral shapes for particles of differing mass
→ strong **collective radial flow**

Strong collective radial expansion

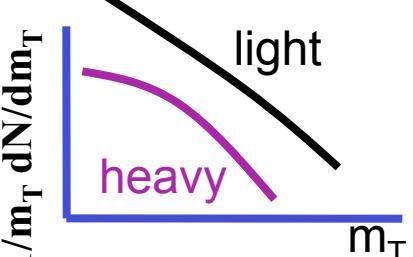
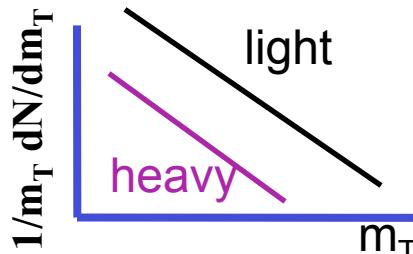


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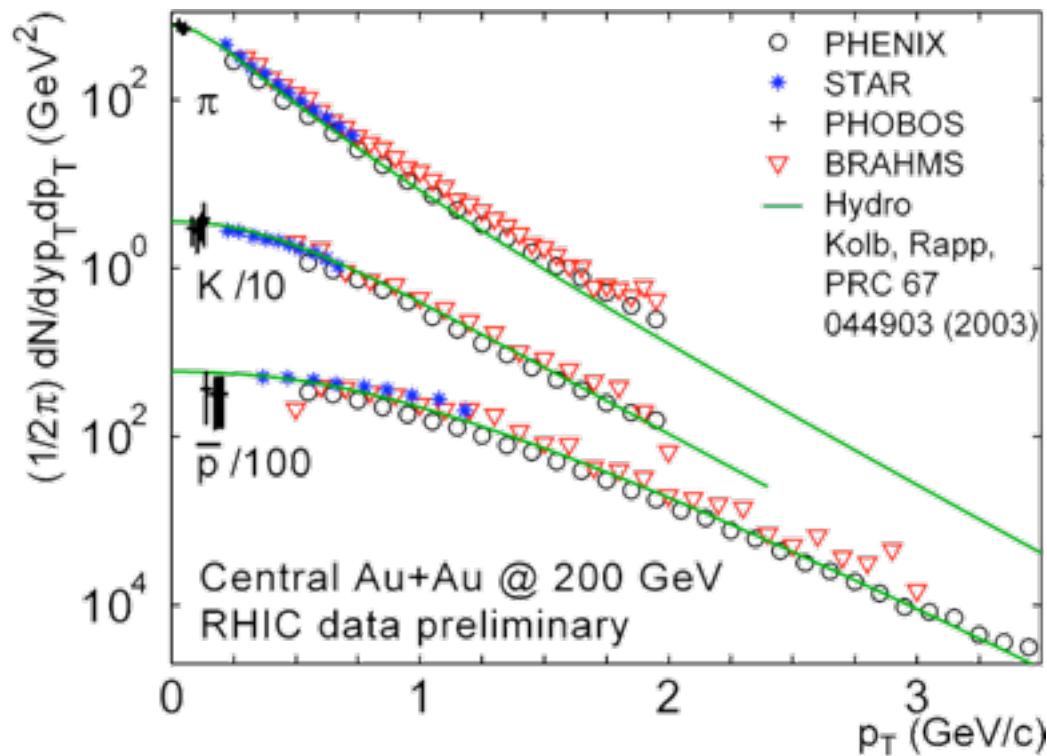
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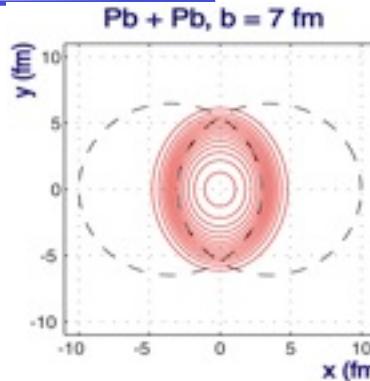
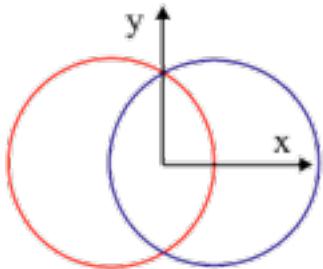


$$T_{fo} \sim 100 \text{ MeV}$$

$$\langle \beta_T \rangle \sim 0.55 c$$

Good agreement with hydrodynamic prediction for soft EOS (QGP+HG)

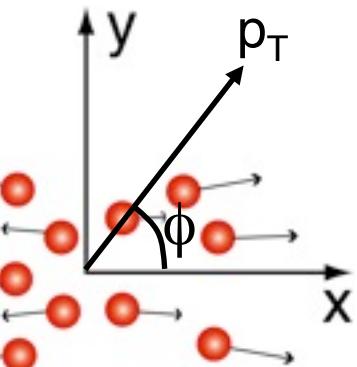
Anisotropic/Elliptic flow



Almond shape overlap
region in coordinate space



Interactions/
Rescattering



Anisotropy in
momentum
space

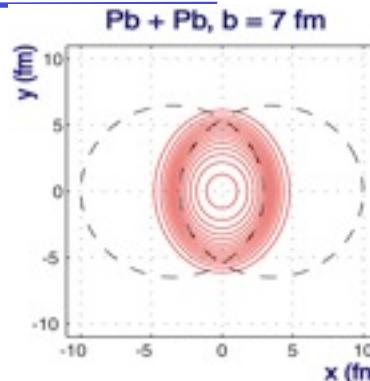
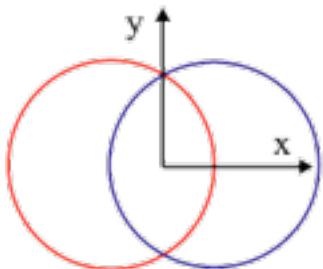
$$dN/d\phi \sim 1 + 2 v_2(p_T) \cos(2\phi) + \dots$$

$$\phi = \tan^{-1}(p_y/p_x)$$

$$v_2 = \langle \cos 2\phi \rangle$$

v_2 : 2nd harmonic Fourier coefficient in $dN/d\phi$ with respect to the reaction plane

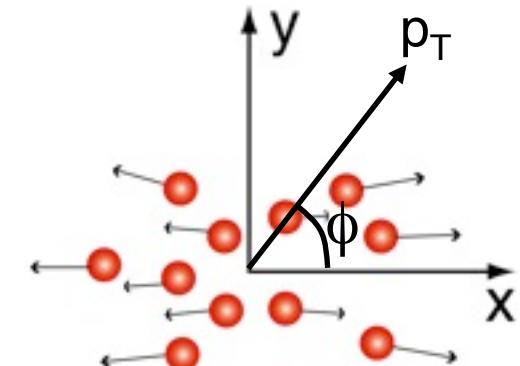
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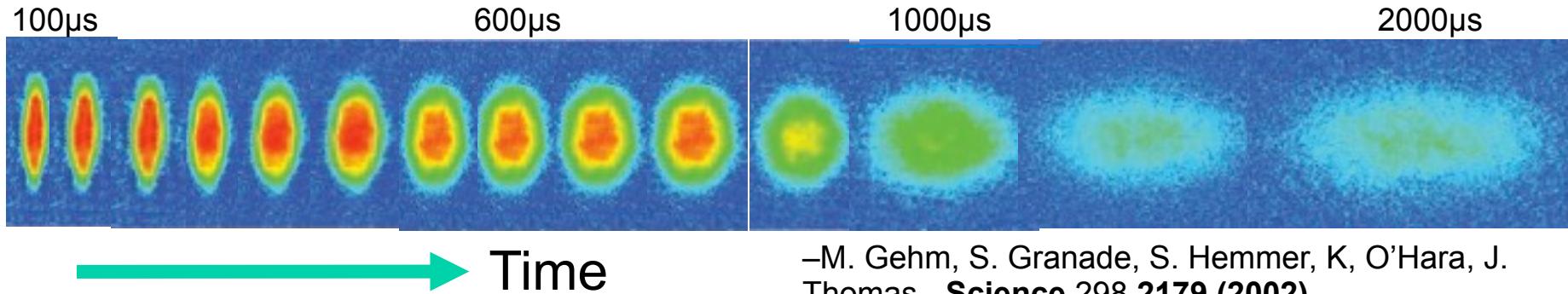
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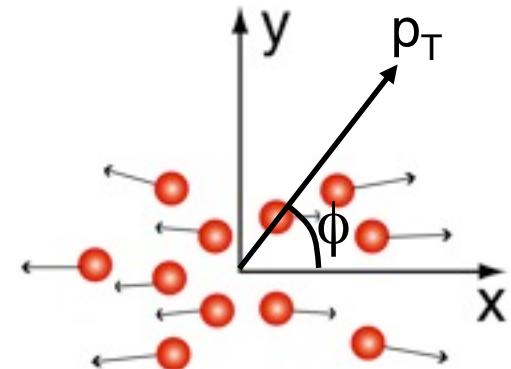
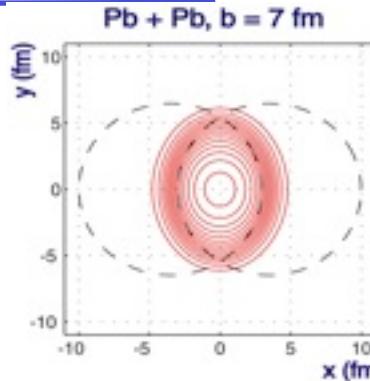
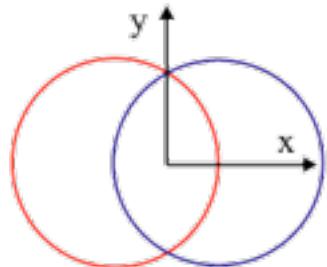
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M. Gehm, S. Granade, S. Hemmer, K. O'Hara, J. Thomas - **Science 298 2179 (2002)**

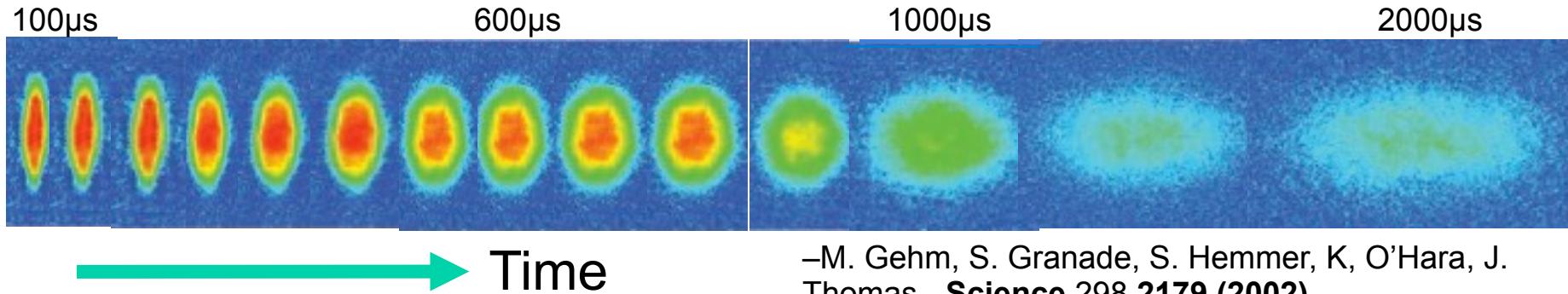
Anisotropic/Elliptic flow



Elliptic flow observable sensitive to early evolution of system

Mechanism is self-quenching

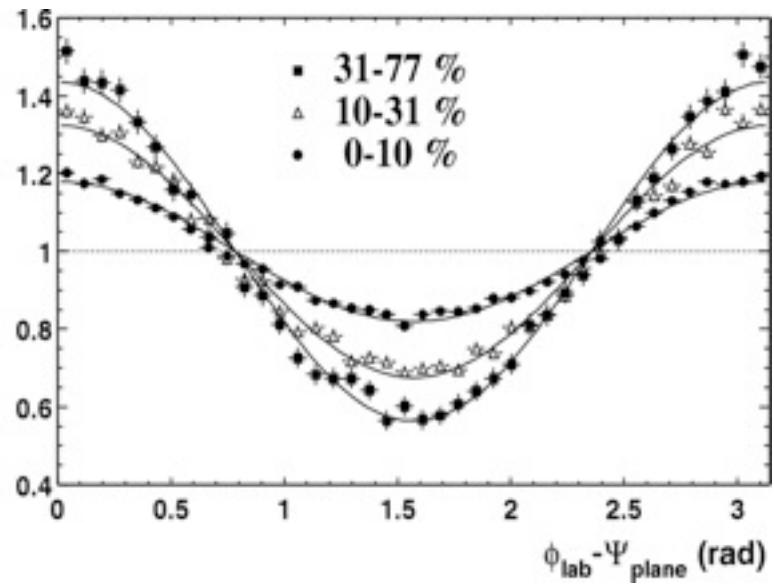
Large v_2 is an indication of **early** thermalization



—M. Gehm, S. Granade, S. Hemmer, K. O'Hara, J. Thomas - **Science 298 2179 (2002)**

Elliptic flow

Distribution of particles with respect to event plane, $\phi - \psi$, $p_t > 2$ GeV; STAR PRL 90 (2003) 032301

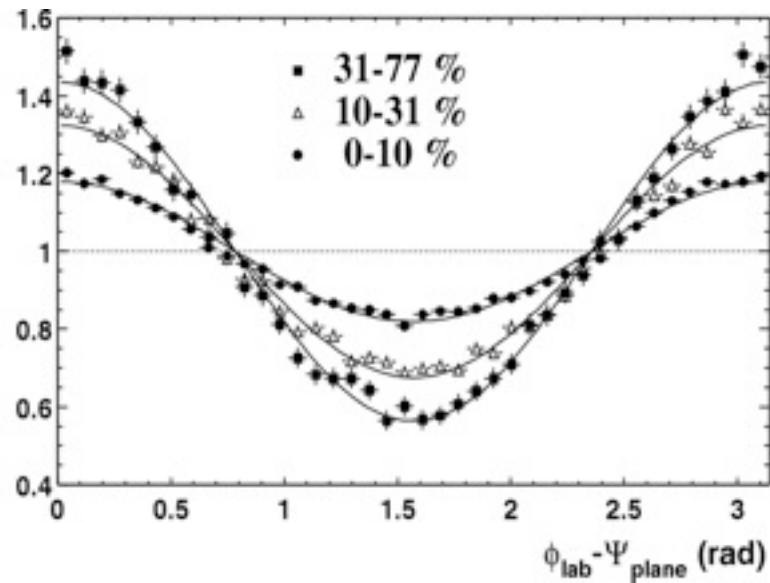


- Very strong elliptic flow → early equilibration

Factor 3:1 peak to valley

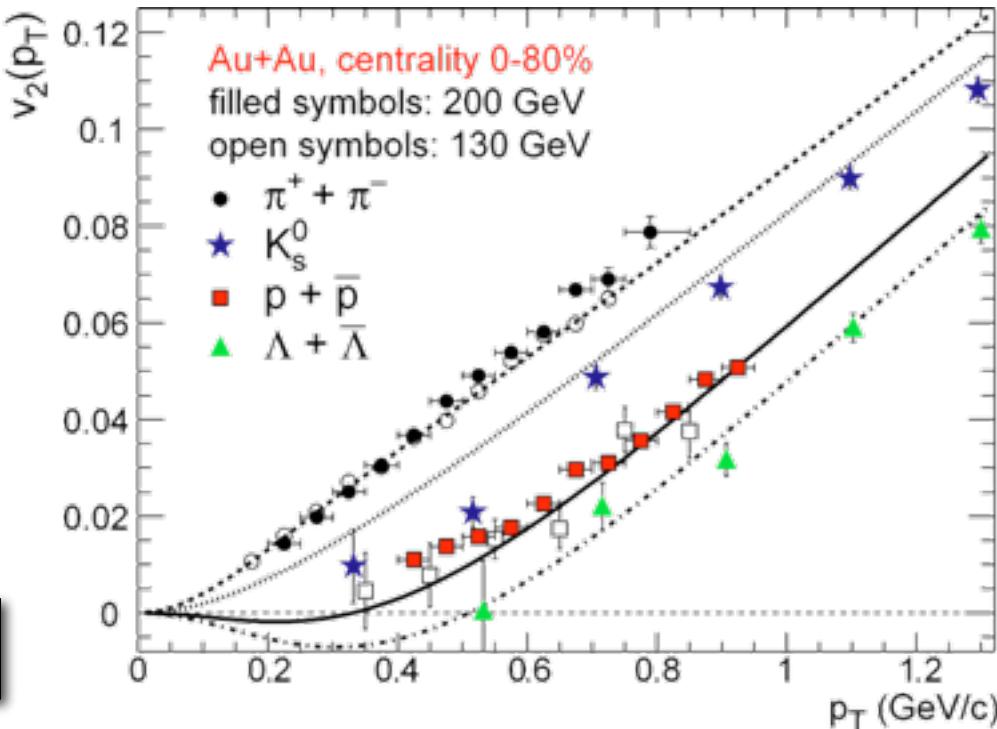
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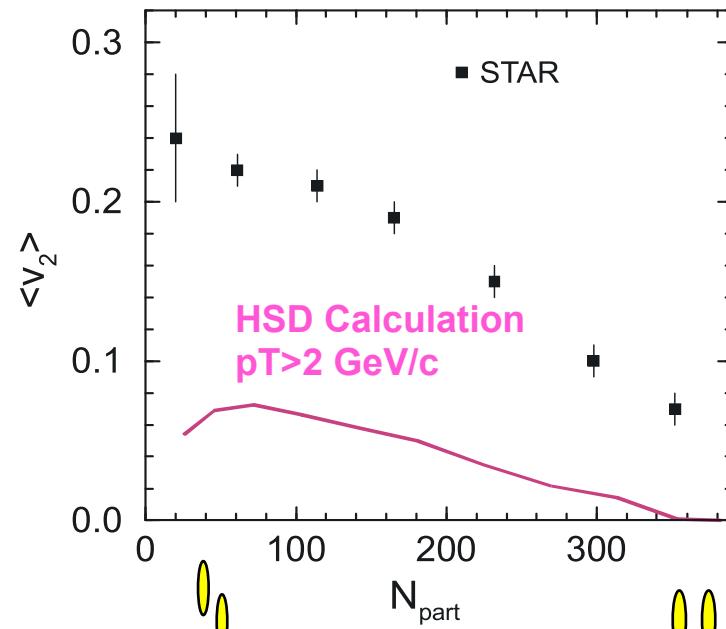
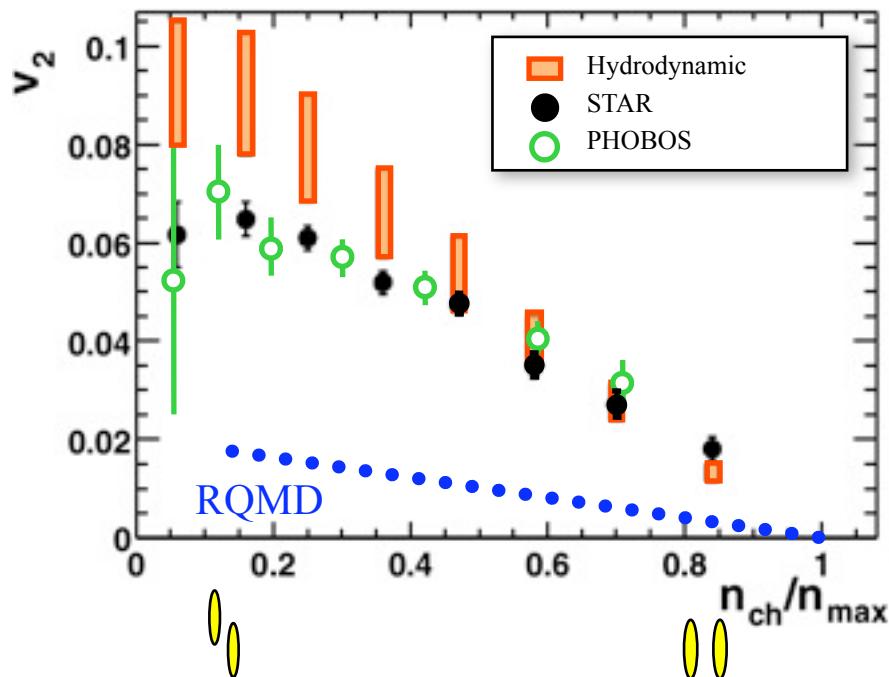


QGP → almost perfect fluid

Just a gas of hadrons?

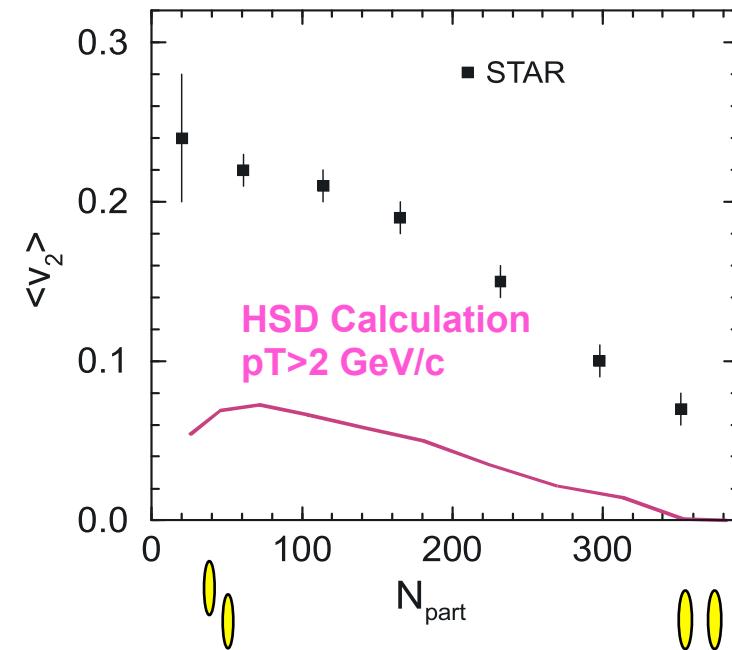
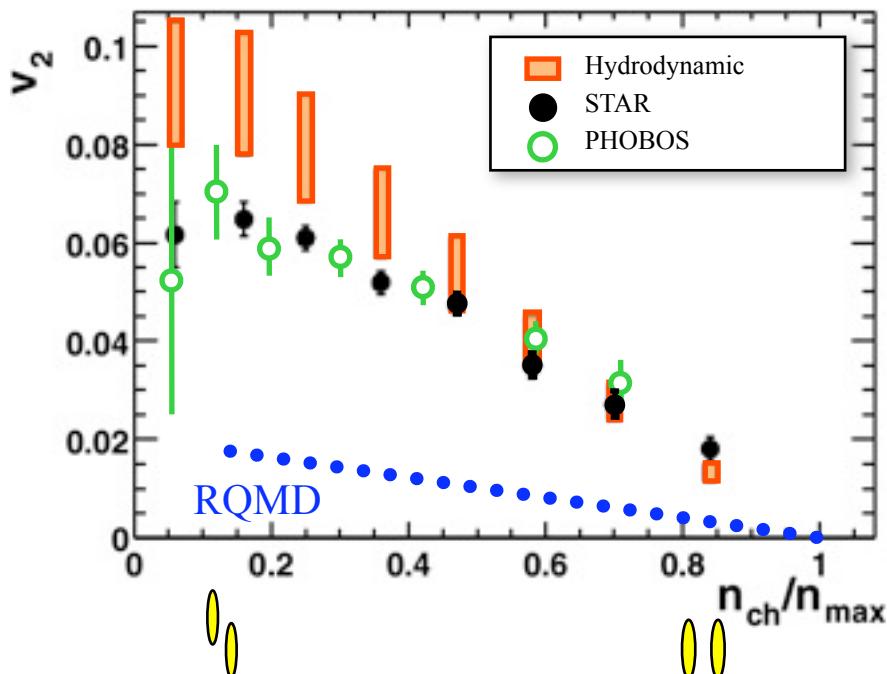
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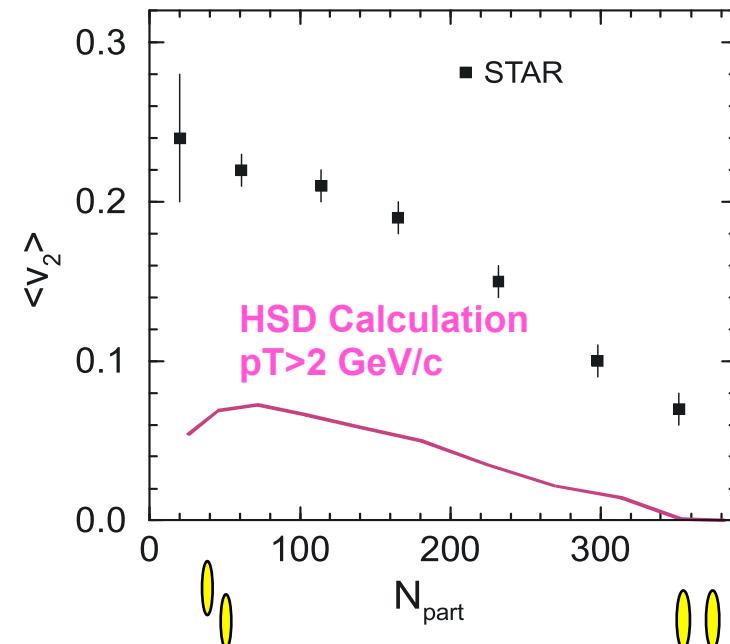
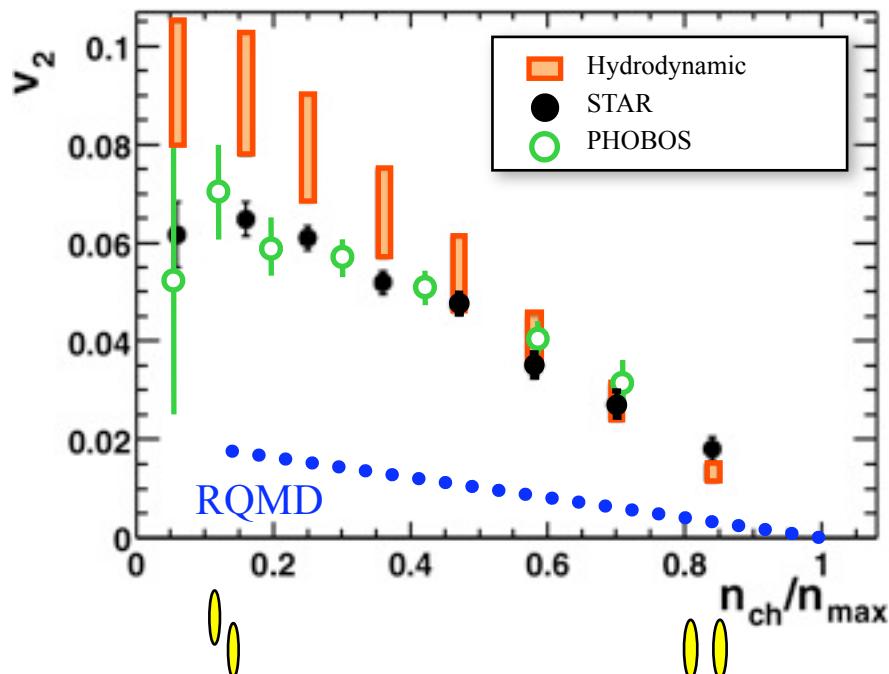
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Hydrodynamical calculations: thermalization time t=0.6 fm/c

What interactions can lead to equilibration in < 1 fm/c?

The constituents “flow”

- Elliptic flow is additive.
- If partons are flowing the *complicated* observed flow pattern in $v_2(p_T)$ for hadrons

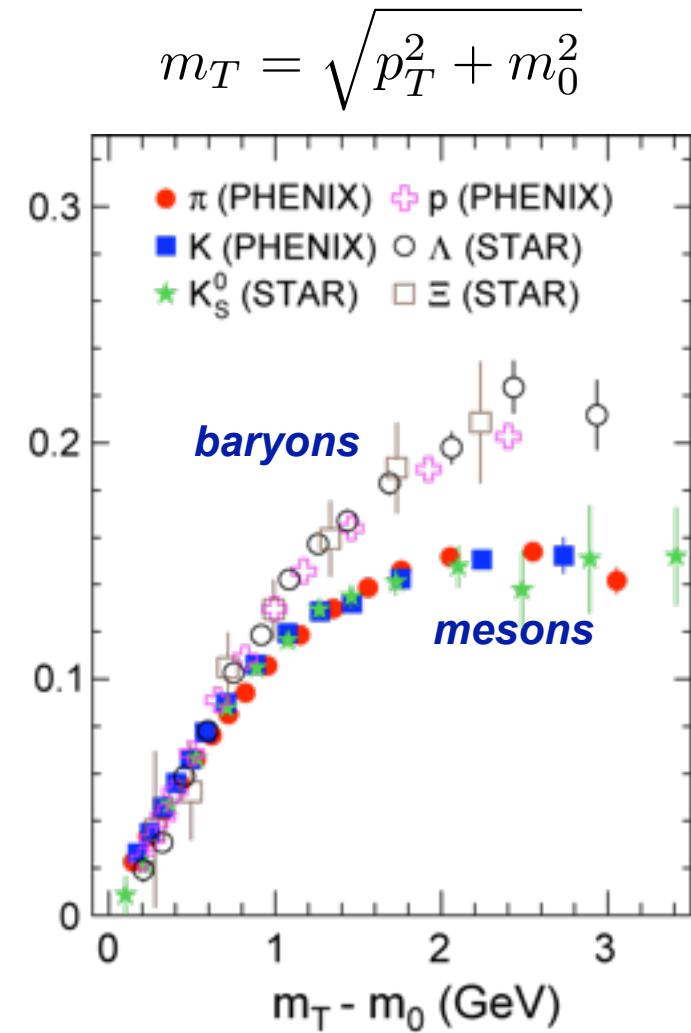
$$\frac{d^2N}{dp_T d\phi} \propto 1 + 2 v_2(p_T) \cos(2\phi)$$

should become *simple* at the quark level

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$$n = (2, 3) \text{ for (meson, baryon)}$$



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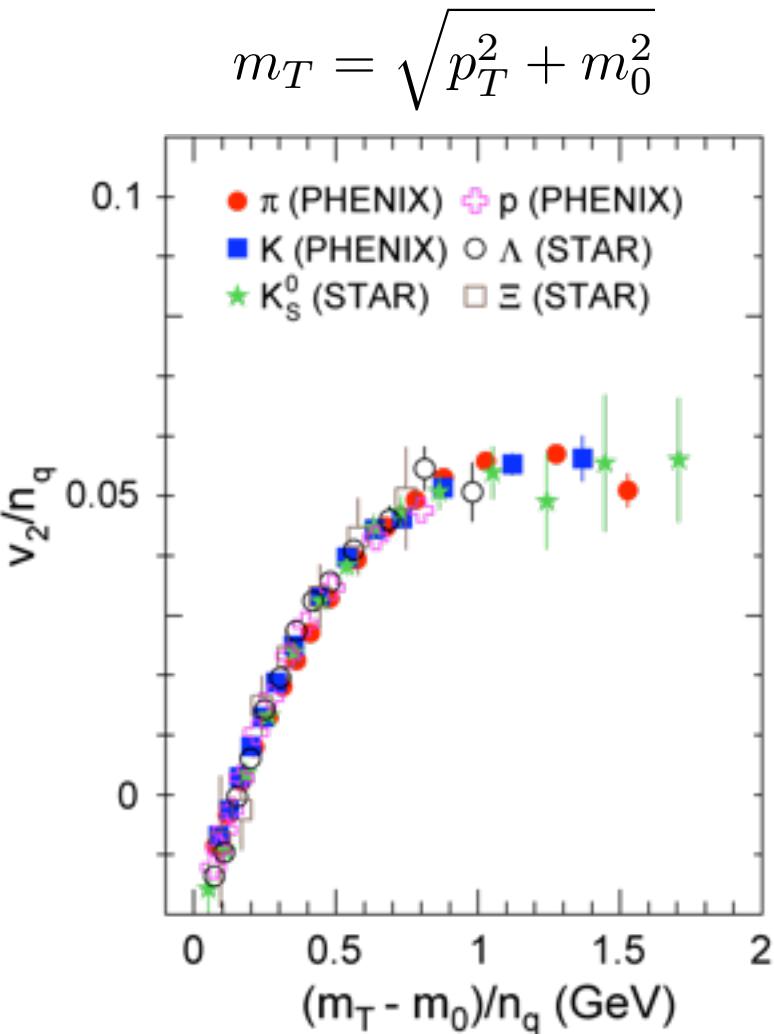
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Works for p , π , K_s^0 , Λ , Ξ ..

$$v_2^s \sim v_2^{u,d} \sim 7\%$$



Constituents of QGP are partons

Summary of what we learned so far

- Energy density in the collision region is way above that where hadrons can exist
- The initial temperature of collision region is way above that where hadrons can exist
- The medium has quark and gluon degrees of freedom in initial stages

We have created a new state of matter at RHIC
- the QGP

- The QGP is flowing like an almost “perfect” liquid