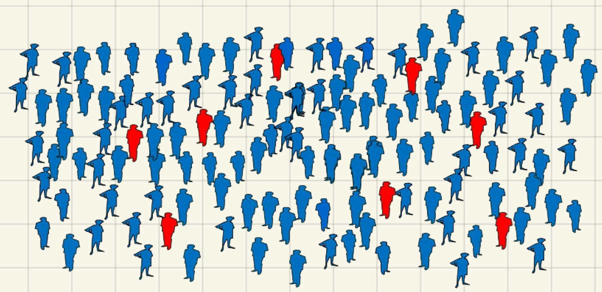
Lecture 7: Network Epidemiology

epidemiology: the study of disease

infectious disease epi: study of transmissible diseases

network epi: the study network-mediated disease transmission → structured populations ↓ shapes who gets sick



← our focus

↑ populations

- individuals
- internal systems
- tissues
- cells

↓ molecules

all scales

SARS-CoV-2 spike protein

ID epi warm up: the compartmental model

- lets first ignore the network → assume all-to-all connections (also called "well mixed")

- each node has a state variable $x_i \in \{S, I, R\}$ or $x_i \in \{S, I\}$

  metadata

  S: susceptible → can get sick

  I: infected → is sick

  multi-stage infections

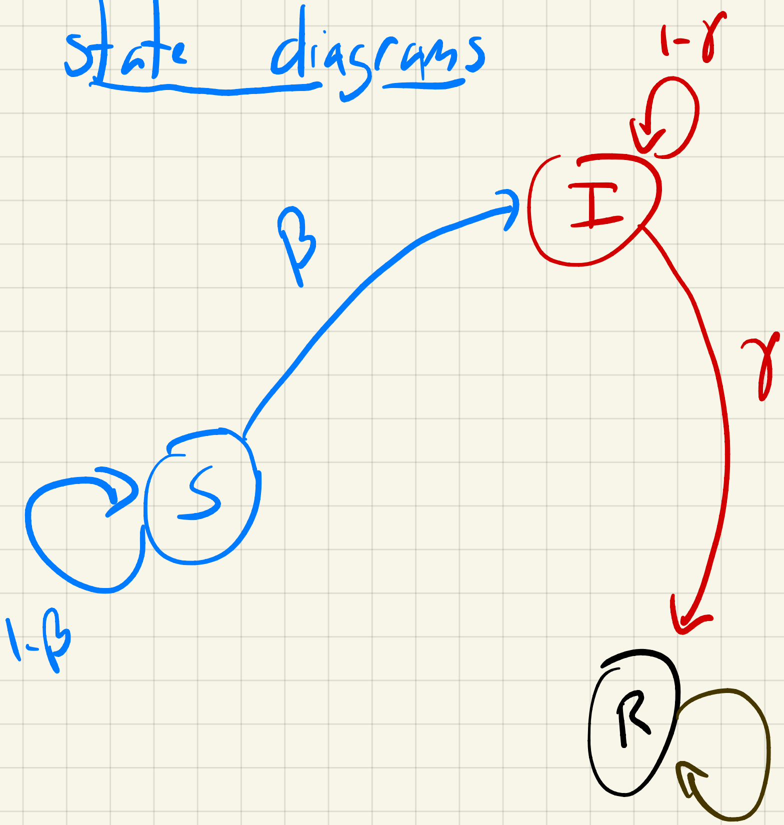
  $\to I_1 \to I_2 \to$

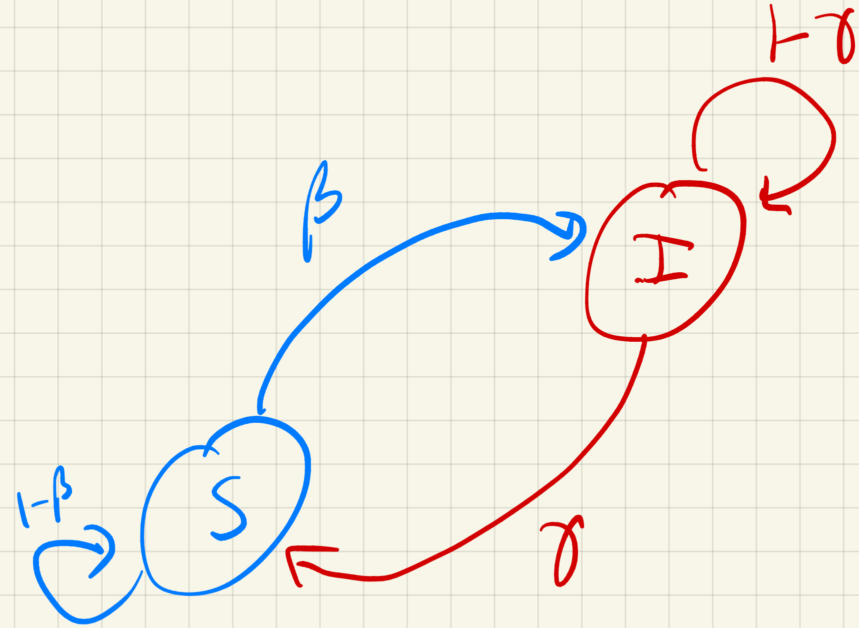
  R: recovered → used to be sick (immune)

- define state transition rules to update each x_i

  these rules are a model of the disease

state diagrams





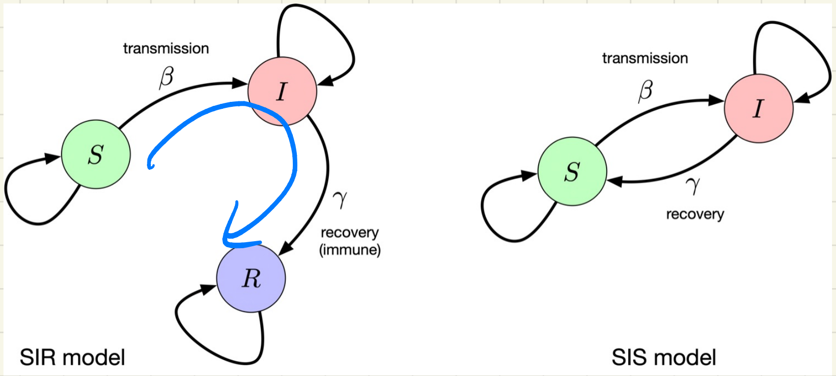
SIR model

chickenpox
measles
HPV
* anything you can get a childhood vaccine for

SIS model

influenza
rhinovirus
coronavirus?
* anything for which infection does not confer long-lasting immunity

Specifying SIR / SIS



β: rate (probability) of transmission

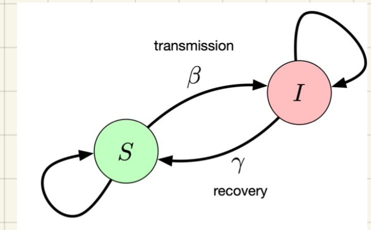
higher β: more infected, more "velocity"

lower β: fewer infected, more "friction"

γ: rate (probability) of recovery

higher γ: faster recovery, fewer secondary infections

lower γ: slower recovery, more infected people around



SIS dynamics

- let N be total population (constant)
- an infected person transmits disease to a susceptible person w/ prob $\bar{\beta}$ per unit time
- an infected person recovers ($I \to S$) w/ prob γ per unit time
- let $S(t)$ and $I(t)$ denote #S and #I   note: $N = S(t) + I(t)$

  at $t=0$ $S(0) = N-1$ $I(0) = 1$
- how $S(t)$ evolves

$$S(t+\Delta t) = \underbrace{S(t)}_{} - \underbrace{S(t) \times \bar{\beta}\,\Delta t\, I(t)}_{\text{flow from } S \to I} + \underbrace{I(t) \times \gamma\,\Delta t}_{\text{flow from } I \to S}$$

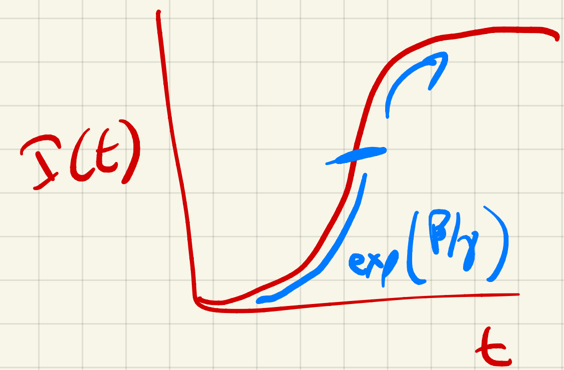
convert difference eqtn to differential eqtn:

$$s = \frac{S}{N} \qquad i = \frac{I}{N} \qquad \beta = \bar{\beta} N$$

divide by Δt

rate equation

$$\frac{di}{dt} = \overbrace{\beta i}^{S\to I}\overbrace{(1-i)}^{S} - \overbrace{\gamma i}^{I \to S}$$

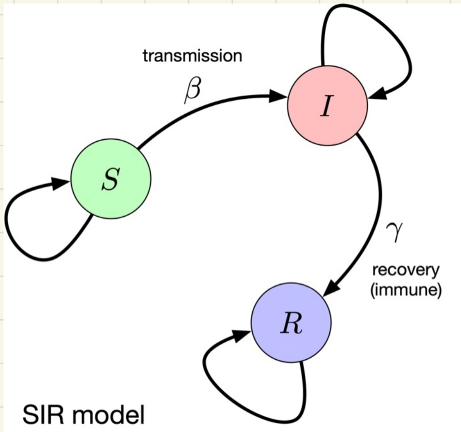


simple question: will an epidemic spread?

- let i be very small (relative to N) to approx: $(1-i) \simeq 1$

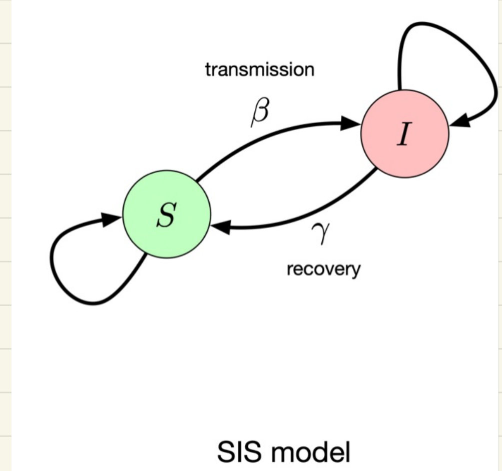
$$\frac{di}{dt} \simeq i(\beta - \gamma)$$

- simple ODE with logistic function as solution to $\frac{\beta}{\gamma}$: $\begin{cases} > 1 \\ = 1 \\ < 1 \end{cases}$ ↑ ···· epidemic threshold ···· ↓

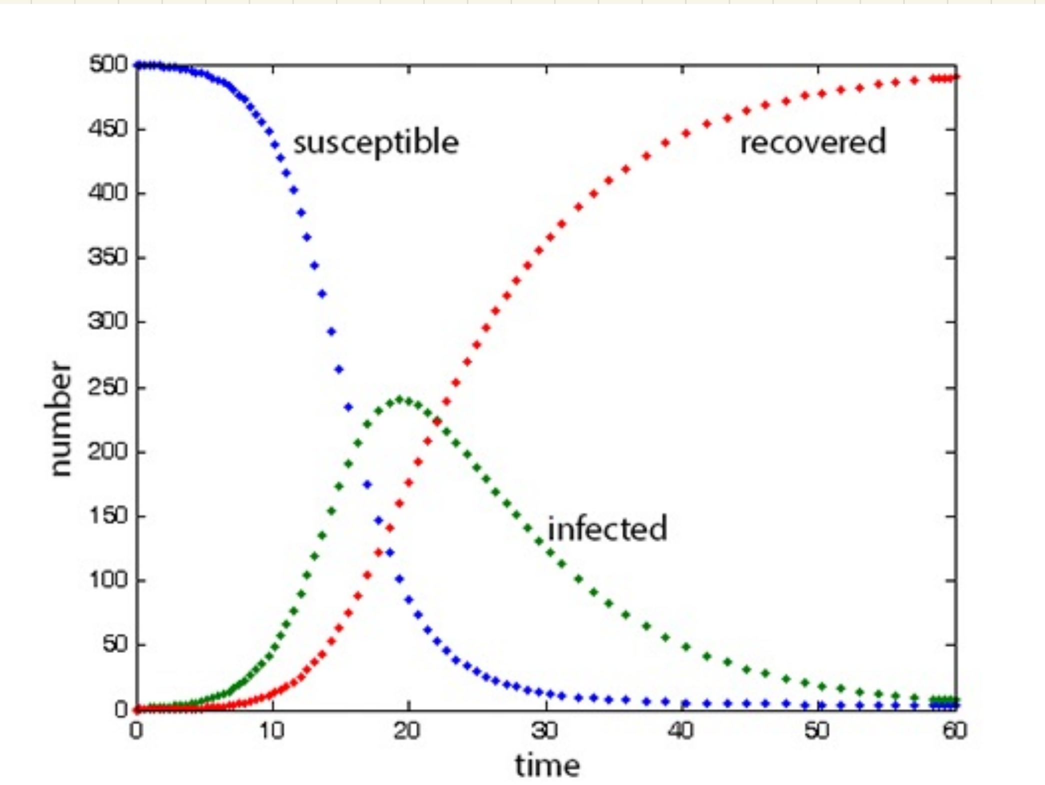


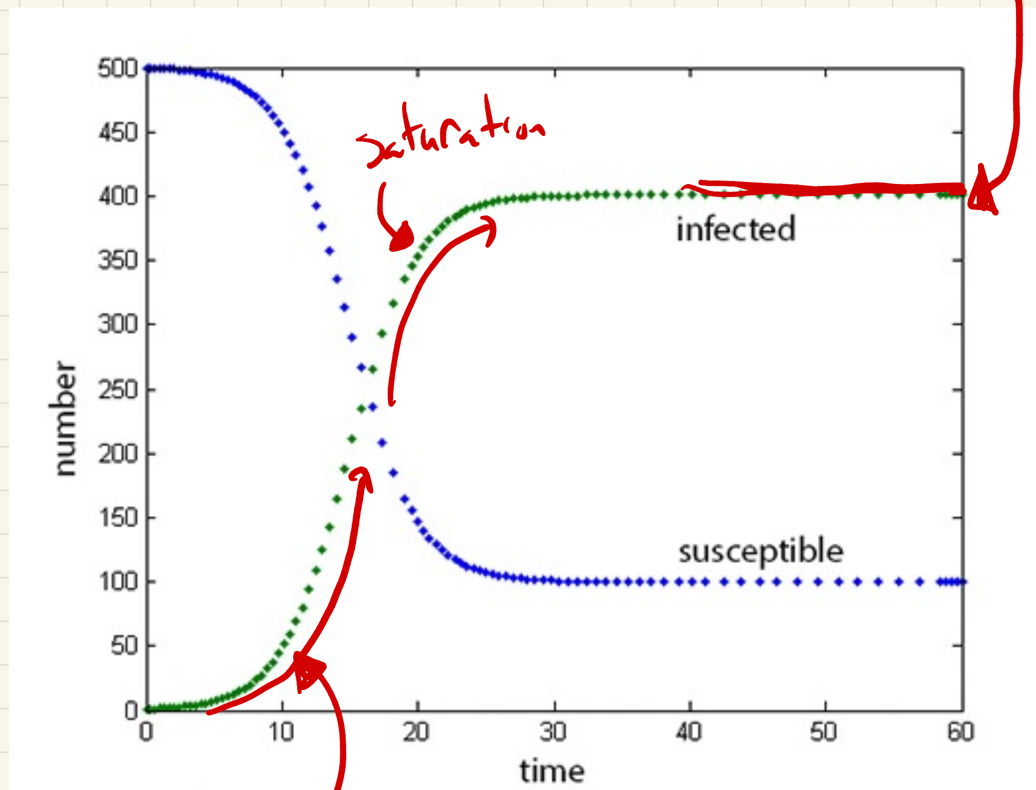
$R_0 \equiv \beta/\gamma$

exponential rate when $i \ll 1$



$1-\beta/\gamma$



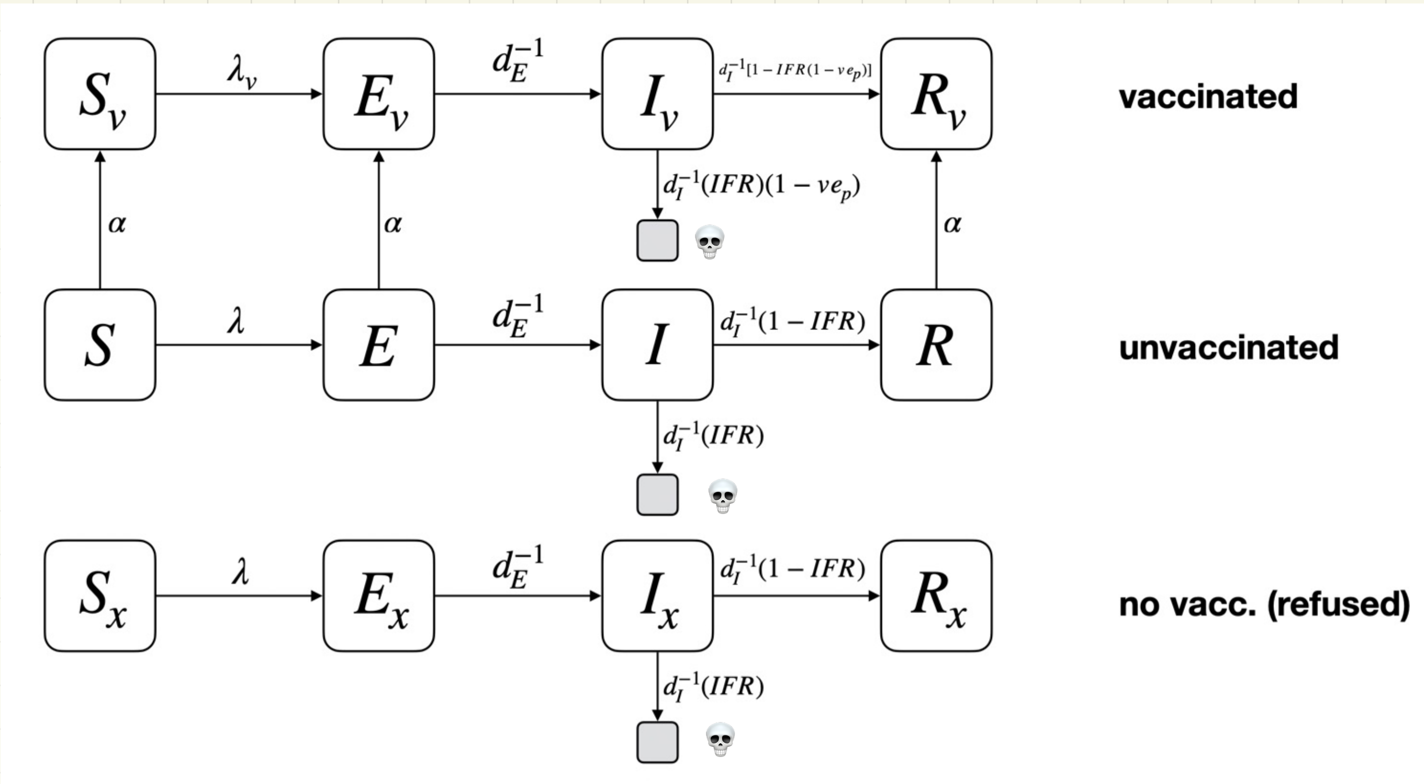


saturation

exponential spread

lets get crazy (covid edition)

SEIR model with death and vaccinations



α: vaccination rate

λ: our β

IFR: infected fatality rate

Science — RESEARCH ARTICLES

Cite as: K. M. Bubar *et al.*, *Science* 10.1126/science.abe6959 (2021).

Model-informed COVID-19 vaccine prioritization strategies by age and serostatus

Kate M. Bubar[1,2]*, Kyle Reinholt[3], Stephen M. Kissler[4], Marc Lipsitch[4,5], Sarah Cobey[6], Yonatan H. Grad[4], Daniel B. Larremore[3,7]*

[1]Department of Applied Mathematics, University of Colorado Boulder, Boulder, CO 80303, USA. [2]IQ Biology Program, University of Colorado Boulder, Boulder, CO 80309, USA. [3]Department of Computer Science, University of Colorado Boulder, Boulder, CO 80309, USA. [4]Department of Immunology and Infectious Diseases, Harvard T.H. Chan School of Public Health, Boston, MA 02115, USA. [5]Center for Communicable Disease Dynamics, Harvard T.H. Chan School of Public Health, Boston, MA 02115, USA. [6]Department of Ecology and Evolution, University of Chicago, Chicago, IL 60637, USA. [7]BioFrontiers Institute, University of Colorado Boulder, Boulder, CO 80303, USA.

CU authors

exponential growth: covid-19

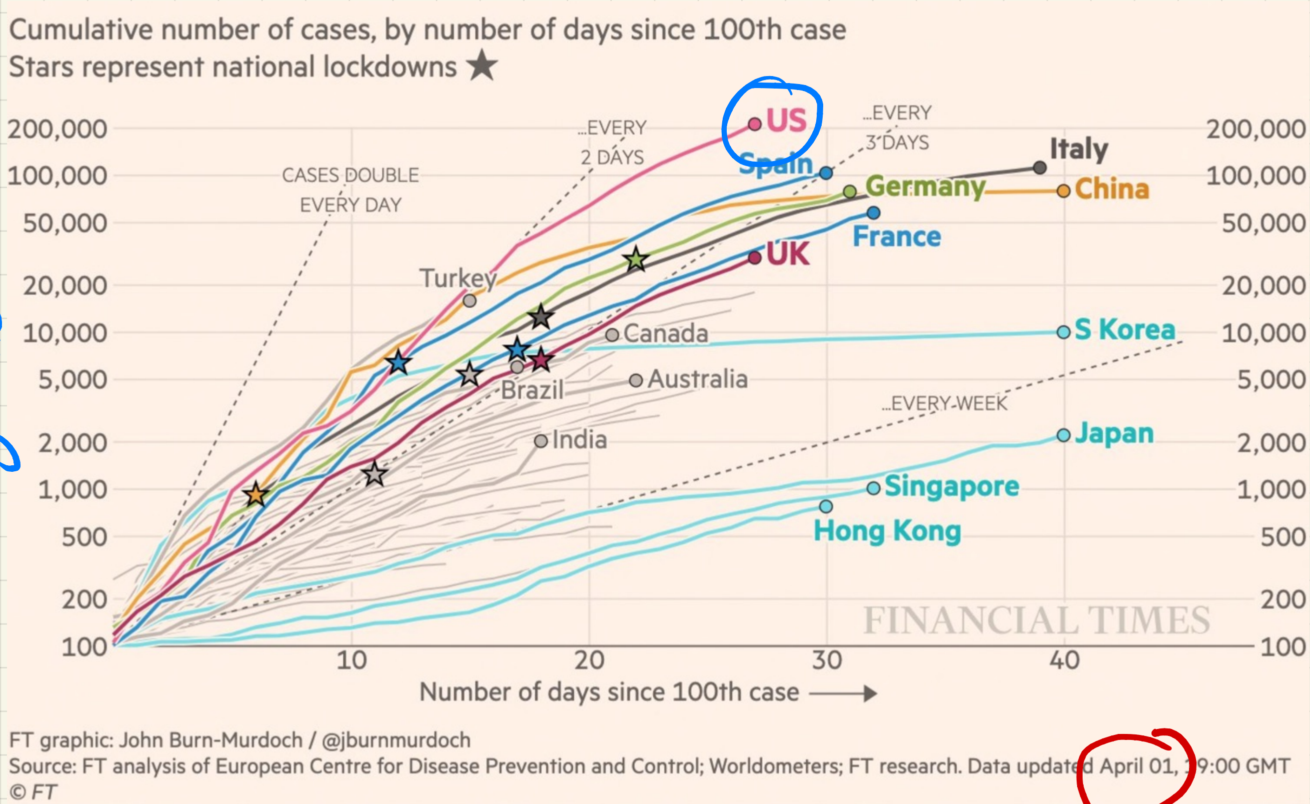
it's very fast!

$$I(t) \approx (R_0)^t$$

note • 1 April 2020

• dashed lines are doubling rates

log axis



Italy cases in 2020 (WHO data)

24 Feb.	27 Feb.	1 Mar.	4 Mar.	8 Mar.	11 Mar.	15 Mar.
124	400	1128	2502	7375	10149	21157

$$\frac{\text{15 Mar}}{\text{24 Feb}} = 170 \times \text{ growth}$$

→ doubles every 3.64 days

9 weeks of this → 14,000,000 cases → 84,000 dead (0.6% IFR) — vs. 17,000 flu deaths per year

- Key point: exponential growth cannot go on forever

$\rightsquigarrow$ once $i = \Theta(1)$, R_0 not representative of growth

why?

flatten the curve (like an epidemiologist)

$$R_0 = \beta/\gamma$$

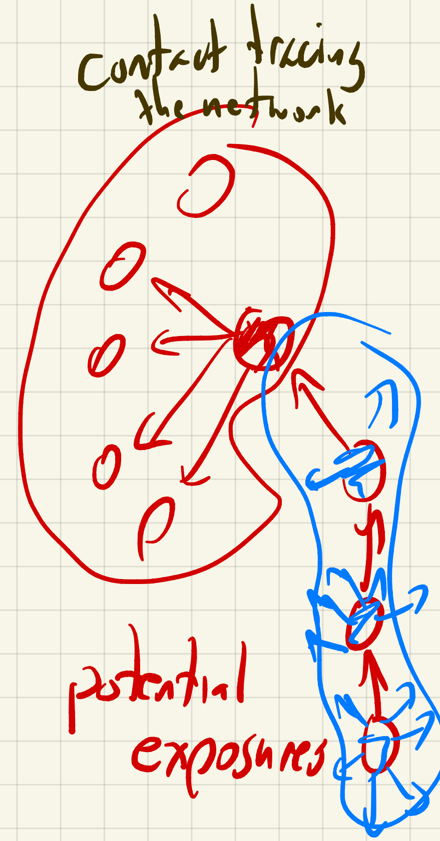
$$S \xrightarrow{\beta} I \xrightarrow{\gamma} R$$

1) β:
   - masks
   - social distancing / avoid gatherings
   - hand washing

2) γ:
   - steroids (?)
   - ventilators
   - antivirals

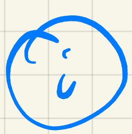
Epidemics on networks

- not every node can infect every other node
- infections go across edges



What are nodes? individuals

what are edges? potential exposures

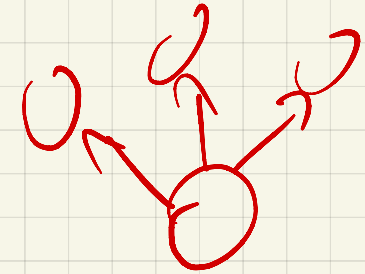


all possible potential exposures

the exposure network $G=(V,E)$ weighted, undirected

vs the transmission network $G_T=(V,E_T)$ $E_T \subseteq E$ directed, always a tree for SIR

the meaning of R_0 in a network



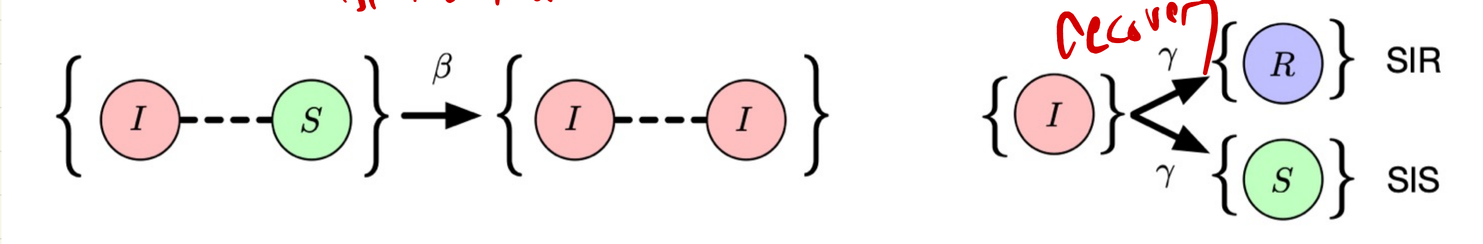
- avg # of secondary infections → $\langle k^{out} \rangle$ in G_T
- not an intrinsic property of pathogen
- function of $Pr(k)$ in G and β

[modify R_0 → modify $\langle k \rangle$ in G]

dynamics on the network

- β, γ are now probabilities
- each time step, loop over infected nodes →
  1) apply edge rules
  2) apply node rule

transmission



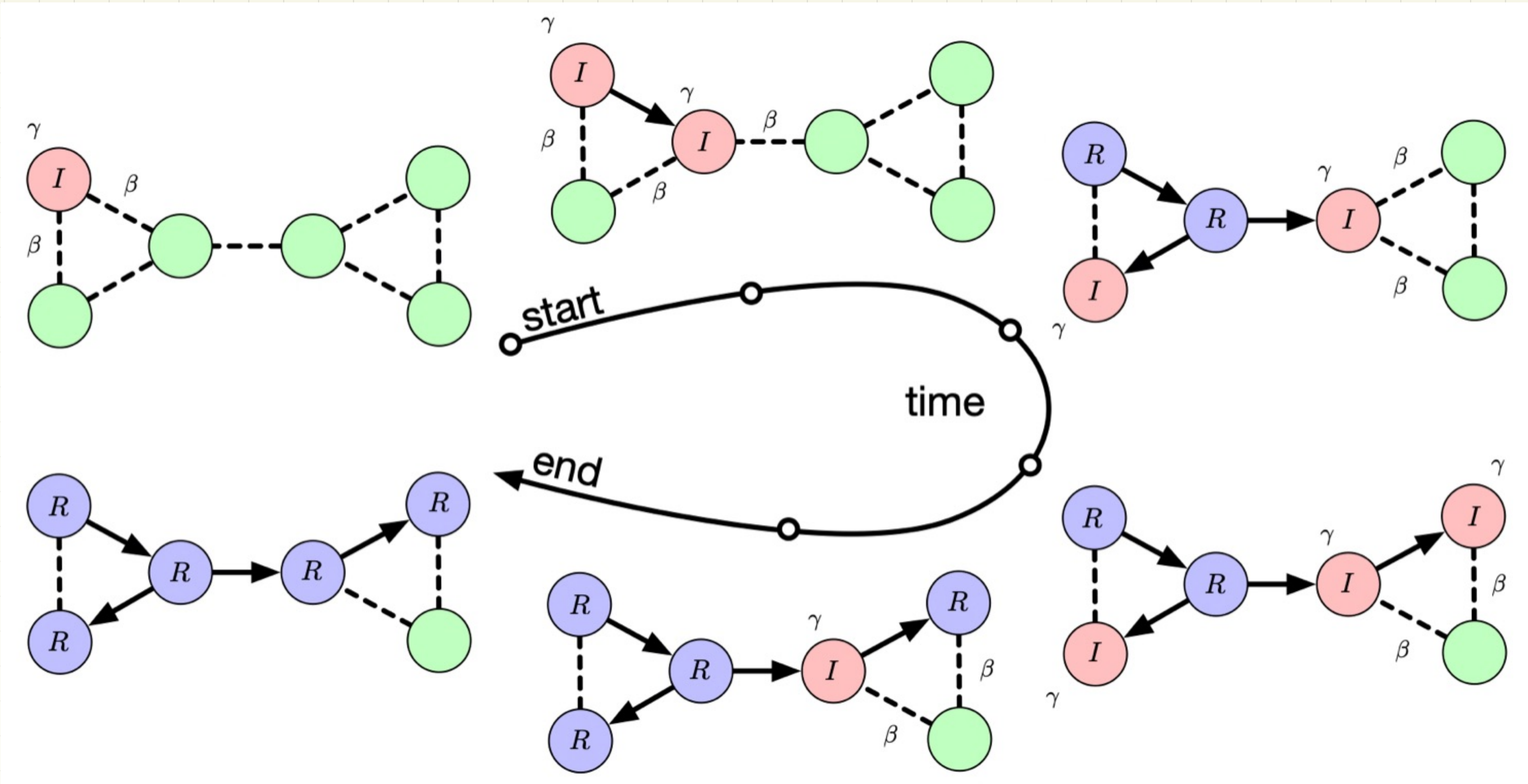
edge rule

node rule

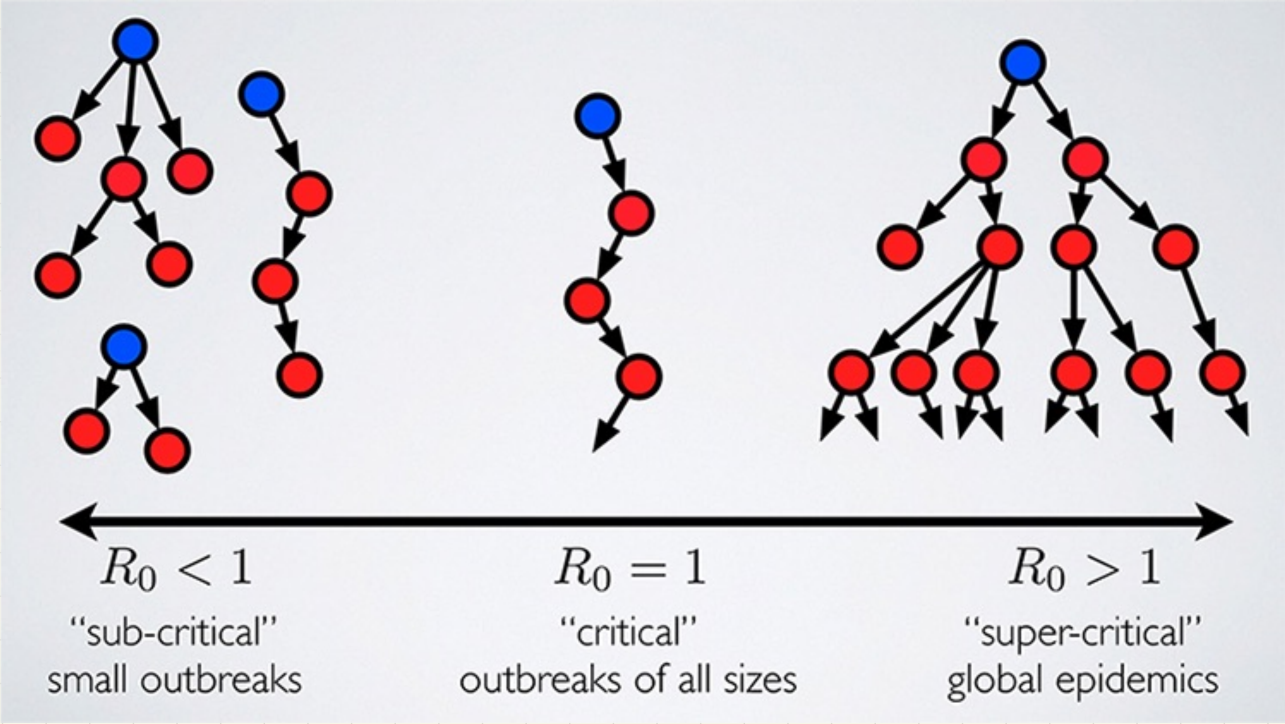
example

order of ops
1) all edges first
2) all nodes

SIR



the role of R_0



2^n

3^n

disease	R_0	transmission	vax.	data from
measles	12 – 18	airborne	90–95%	1912-1928 in US + 1944-1979 in UK
chickenpox	7 – 12		85–90%	
polio	5 – 7	fecal-oral route	82–87%	
small pox	1.5 – 20+	airborne droplet	70–80%	
H1N1 flu	1 – 3	airborne droplet		
ebola	1.5 – 2.5	bodily fluids		
zika	2			
covid-19	2.4	airborne	70–85%	

Q: SARS-CoV-2 variant B.1.1.7

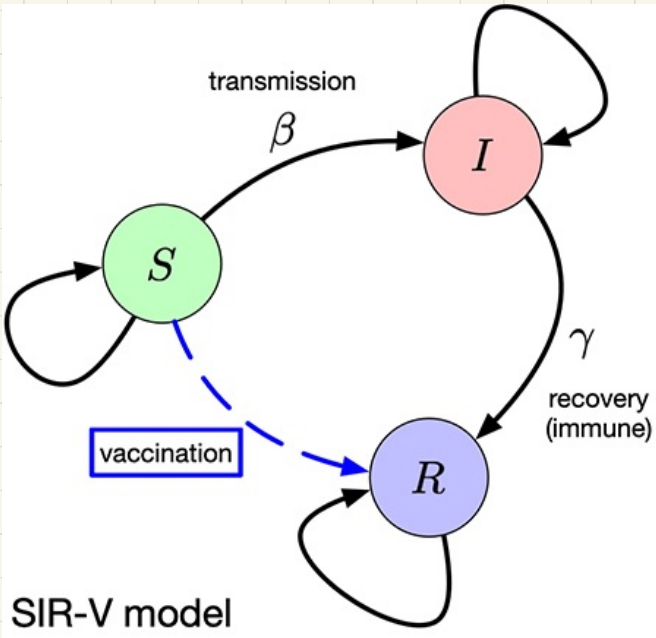
- which is worse?

50% more transmissible

OR

50% more deadly

Stopping an epidemic (on a network)



"herd immunity" → when S is very small → $R<1$

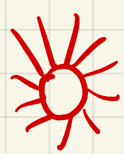
naturally acquired $I \to R$

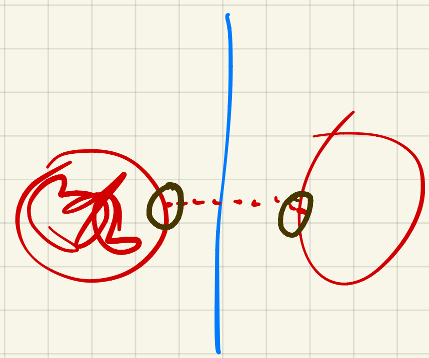
vs

vaccinations $S \to R$

network strategies

1) high degree nodes
2) bridge between communities
3) randomly
4) based on attributes (age, health status)





Try your hand

https://vax.herokuapp.com/