You may want to embed an image. If so, use a figure code block.

$$
\begin{aligned}
T(n) &= T(n-1) + O(1) && (16)\\
&= T(n-2) + 2 * O(1) && (17)\\
&= T(n-3) + 3 * O(1) && (18)\\
&= T(n-4) + 4 * O(1) && (19)\\
&\cdots && (20)\\
&= T(n-n) + n * O(1) && (21)\\
&= T(0) + n * O(1) && (22)\\
&= 1 + n * O(1) && (23)\\
&= O(n) && (24)
\end{aligned}
$$

4. (15 points) What is the best, worst and average running time of quick sort algorithm?

Best case:

$$O(nlog(n)) \tag{25}$$

Average case:

$$O(nlog(n)) \tag{26}$$

Worst case:

$$O(n^2) \tag{27}$$

(a) Provide recurrence relations. For the average case, you may assume that quick sort partitions the input into two halves proportional to c and $1-c$ on each iteration.

Best:

$$T(n) = \begin{cases} 2T(n/2) + O(n), & \text{if } n > 1 \\ 0, & \text{if } n = 1 \end{cases} \tag{28}$$

Average:

$$T(n) = \begin{cases} T(cn) + T((1-c)n) + n, & \text{if } n > 1 \\ 0, & \text{if } n = 1 \end{cases} \tag{29}$$

Worst:

$$T(n) = \begin{cases} T(n-1) + n, & \text{if } n > 1 \\ 0, & \text{if } n = 1 \end{cases} \tag{30}$$

(b) Solve each recurrence relation you provided in part (a)

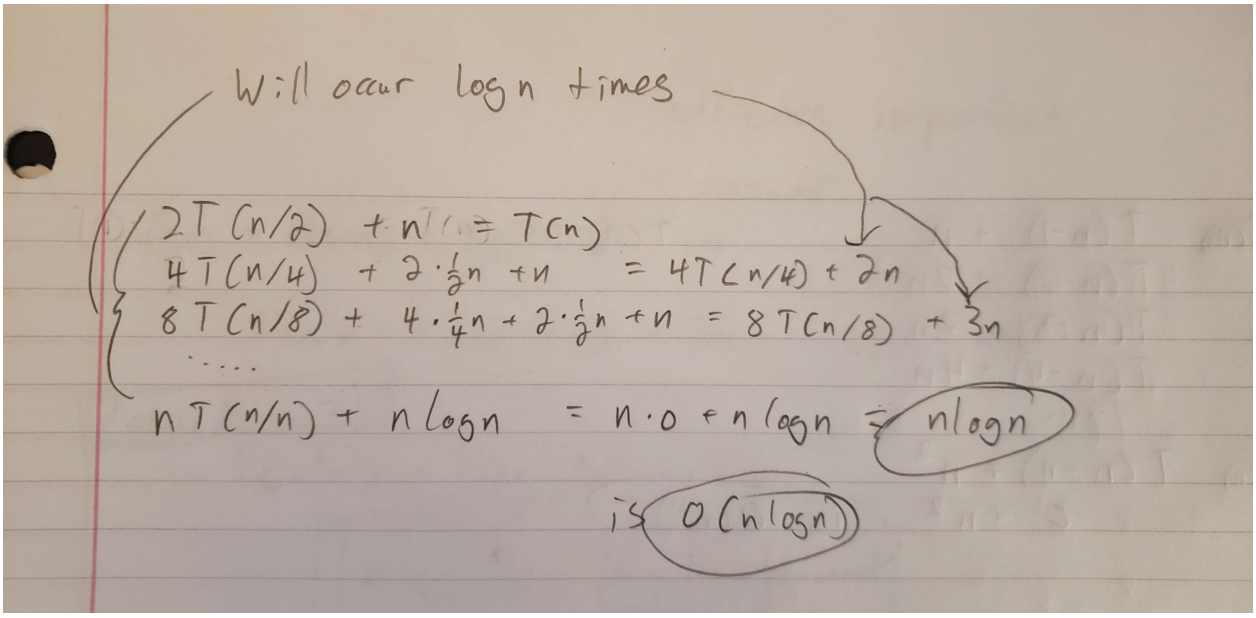

Figure 4: Solution to best case recurrence relation

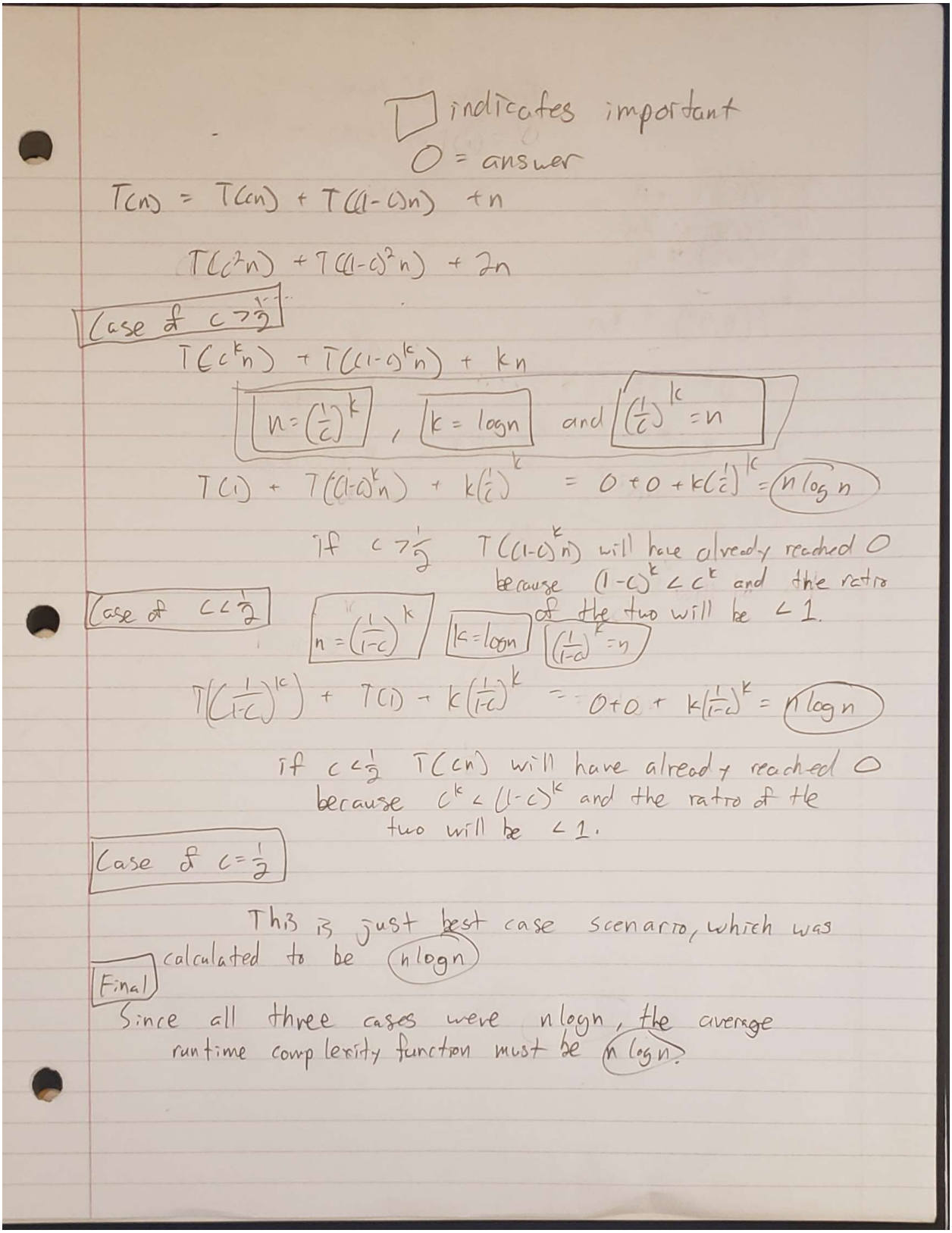

Figure 5: Solution to average case recurrence relation

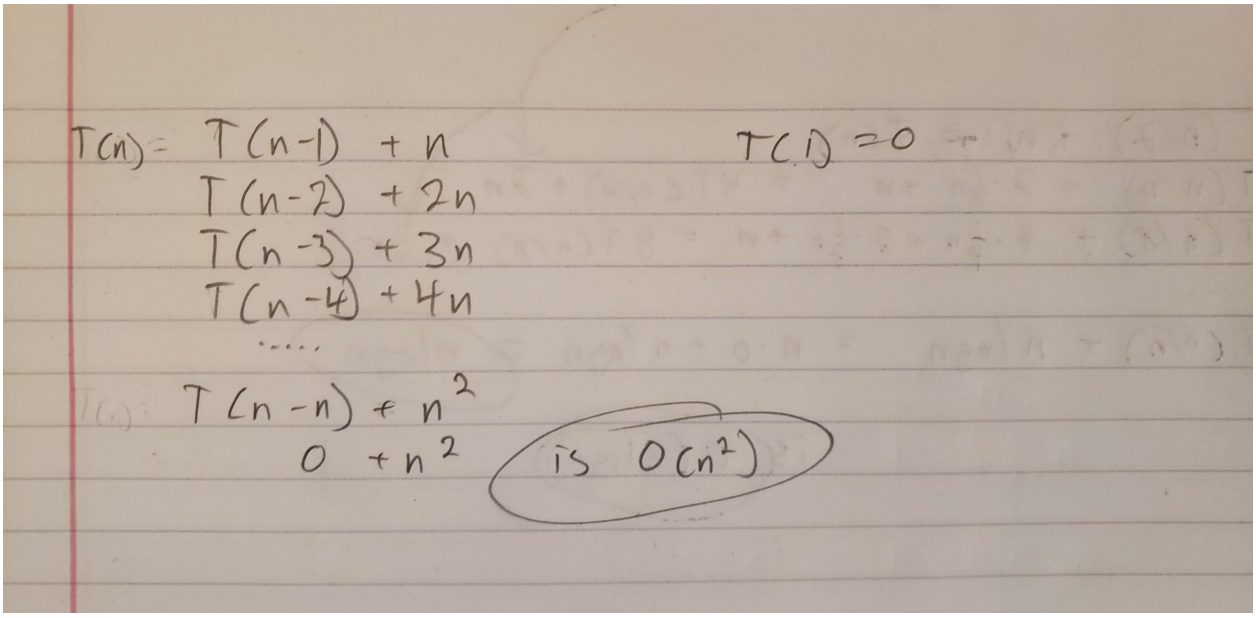

Figure 6: Solution to worst case recurrence relation

(c) Provide an arrangement of the input array which results in each case. Assume the first item is always chosen as the pivot for each iteration.

Best	$\{5, 2, 3, 4, 1, 7, 8, 9, 6\}$
Average	$\{9, 6, 7, 4, 5, 8, 3, 1, 2\}$
Worst	$\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

5. (15 points) Write a C++ function that counts the total number of nodes with two children in a binary tree (do not count nodes with one or none child). You can use a STL container if you need to use an additional data structure to solve this problem.

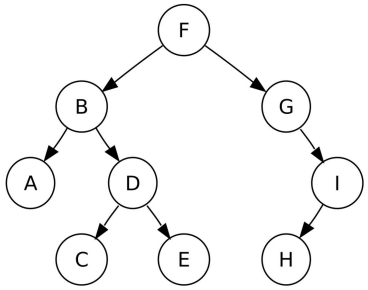

Figure 7: Calling `count_filled_nodes` on the root node F returns 3

(a) Complete the function below. The function will be called with the root node (e.g. `count_filled_nodes(root)`). The tree may be empty. Do not modify the tree.

```
1 #include <vector>
2
```