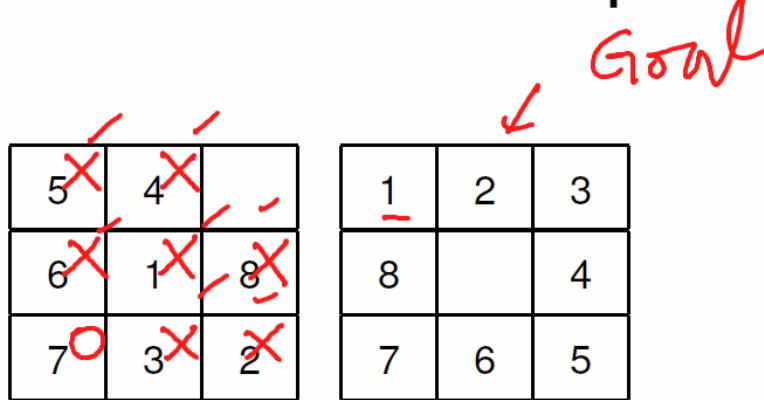




Heuristic Functions: Example

Eight puzzle



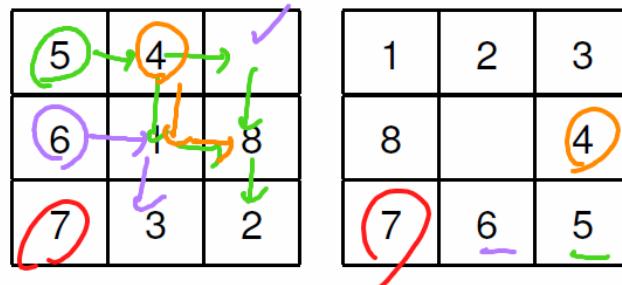
- $h_1(n)$ = number of misplaced tiles
 - $h_2(n)$ = total **Manhattan** distance (city block distance)
- $h_1(n) = 7$ (not counting the blank tile)
- $h_2(n) = 2+3+3+2+4+2+0+2 = 18$

* Both are admissible heuristic functions.



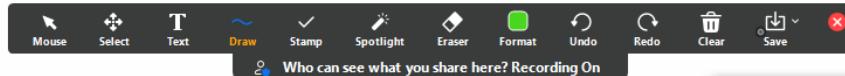
Heuristic Functions: Example

Eight puzzle



- $h_1(n)$ = number of misplaced tiles
 - $h_2(n)$ = total **Manhattan** distance (city block distance)
- $$h_1(n) = 7 \text{ (not counting the blank tile)}$$
- $$h_2(n) = 2+3+3+2+4+2+0+2 = 18$$

* Both are admissible heuristic functions.



Dominance

If $h_2(n) \geq h_1(n)$ for all n and both are admissible, then we say that $h_2(n)$ **dominates** $h_1(n)$, and is better for search.

Typical search costs for depth $d = 14$:

- Iterative Deepening : 3,473,941 nodes visited
- $A^*(h_1)$: 539 nodes
- $A^*(h_2)$: 113 nodes

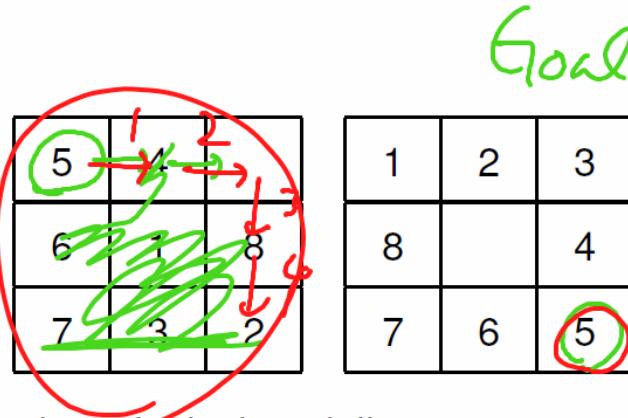
Observe that in A^* , every node with $f < f^*$ is visited . Since $f = g + h$, nodes with $h(n) < f^* - g(n)$ will be visited, so larger h will result in less nodes being visited.

- f^* is the f value for the optimal solution path.



Heuristic Functions: Example

Eight puzzle



Goal

- $h_1(n)$ = number of misplaced tiles
- $h_2(n)$ = total **Manhattan** distance (city block distance)

$h_1(n) = 7$ (not counting the blank tile)

$$h_2(n) = 2+3+3+2+4+2+0+2 = 18$$

Tile 1 2 3 4 5

* Both are admissible heuristic functions.

So for each state you can compute the sum of and I think
distance which becomes a heuristic function



Heuristic Functions: Example

Eight puzzle

$$h_1(n) = 7$$

5	4	
8	1	8
7	3	2

1	2	3
8		4
7	6	5

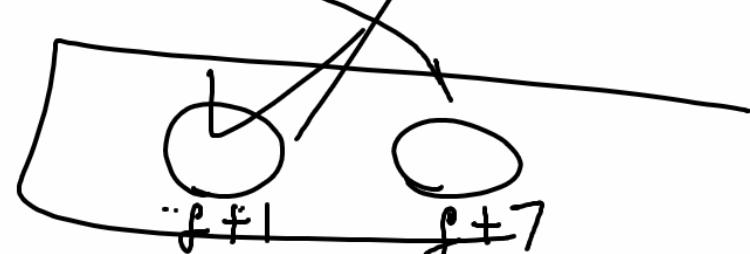
- $h_1(n)$ = number of misplaced tiles
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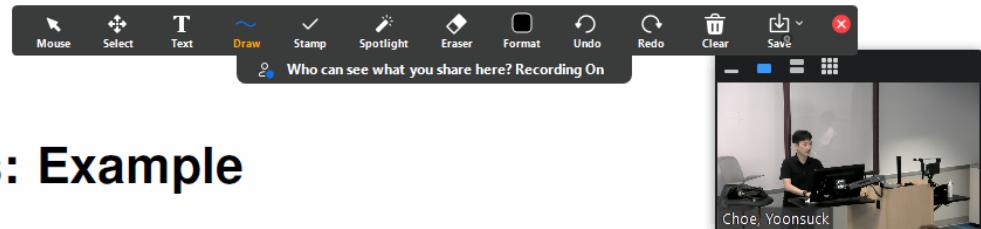
$$h_1(n) = 7 \text{ (not counting the blank tile)}$$

$$h_2(n) = 2+3+3+2+4+2+0+2 = 18$$

* Both are admissible heuristic functions.

$$\underline{h_1(n)=1}$$





Eight puzzle

5	4	
6	1	8
7	3	2

1	2	3
8		4
7	6	5

- $h_1(n)$ = number of misplaced tiles
- $h_2(n)$ = total **Manhattan** distance (city block distance)

$h_1(n) = 7$ (not counting the blank tile)

$$\underline{h_2(n)} = 2+3+3+2+4+2+0+2 = 18$$

* Both are admissible heuristic functions.



Heuristic Functions: Example

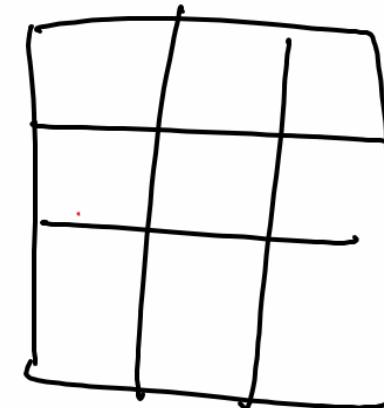


Eight puzzle



5	4	
6	1	8
7	3	2
8		4

1	2	3
8		4
7	6	5



- $h_1(n)$ = number of misplaced tiles
- $h_2(n)$ = total **Manhattan** distance (city block distance)

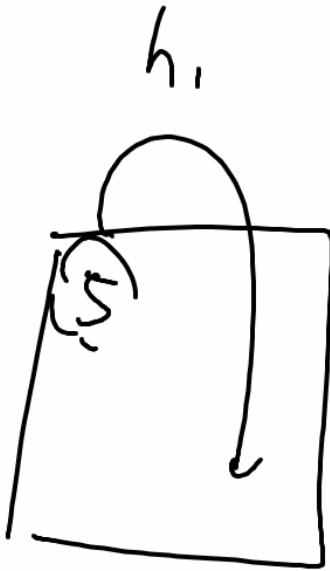
$$h_1(n) = 7 \text{ (not counting the blank tile)}$$

$$h_2(n) = 2+3+3+2+4+2+0+2 = 18$$

* Both are admissible heuristic functions.



Heuristic Functions: Example

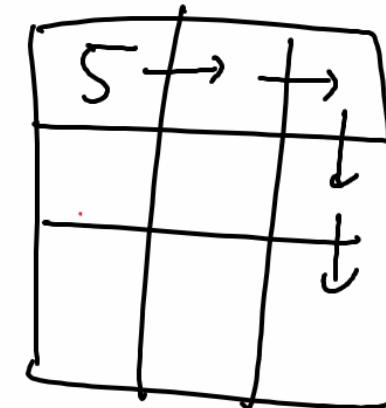
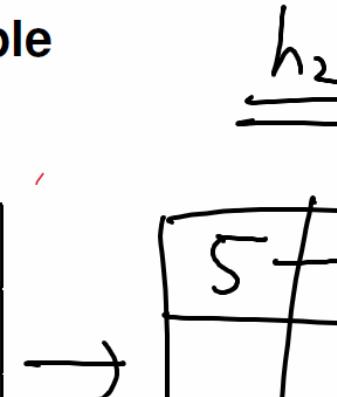


Eight puzzle



5	4	
6	1	8
7	3	2

1	2	3
8		4
7	6	5



- $h_1(n)$ = number of misplaced tiles
- $h_2(n)$ = total **Manhattan** distance (city block distance)

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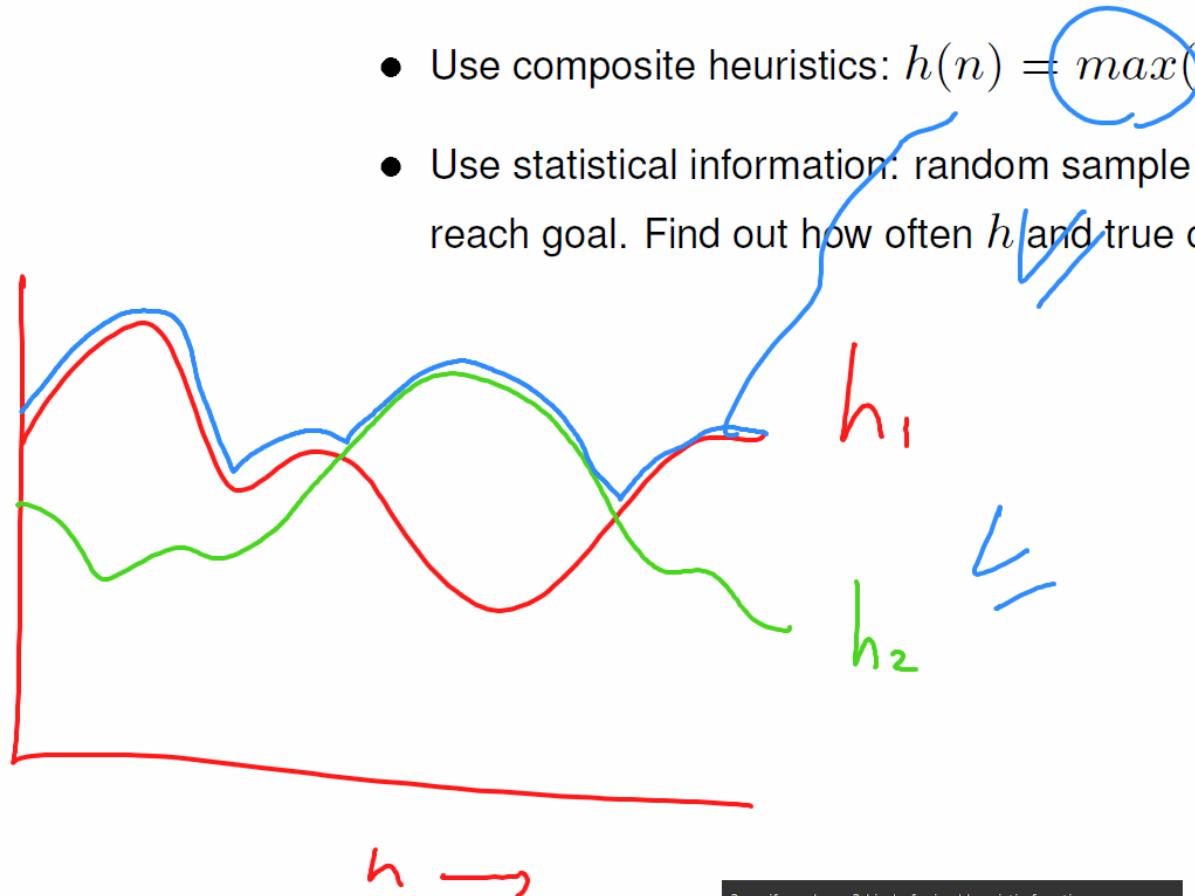
* Both are admissible heuristic functions.

Then that's how you get this the second heuristic. So in general, to get a misunderstanding heuristic, you can always relax the constraint where you can easily



Other Heuristic Design

- Use composite heuristics: $h(n) = \max(h_1(n), \dots, h_m(n))$
- Use statistical information: random sample h and true cost to reach goal. Find out how often h and true cost is related.



2. so if you have 2 kind of mixed hereistic functions you can make just find the Max of those basically use both to get the strictly domain. And here's



Iterative Deepening A*: IDA*

IDS

A* is complete and optimal, but the performance is limited by the available space.



Depth = 0

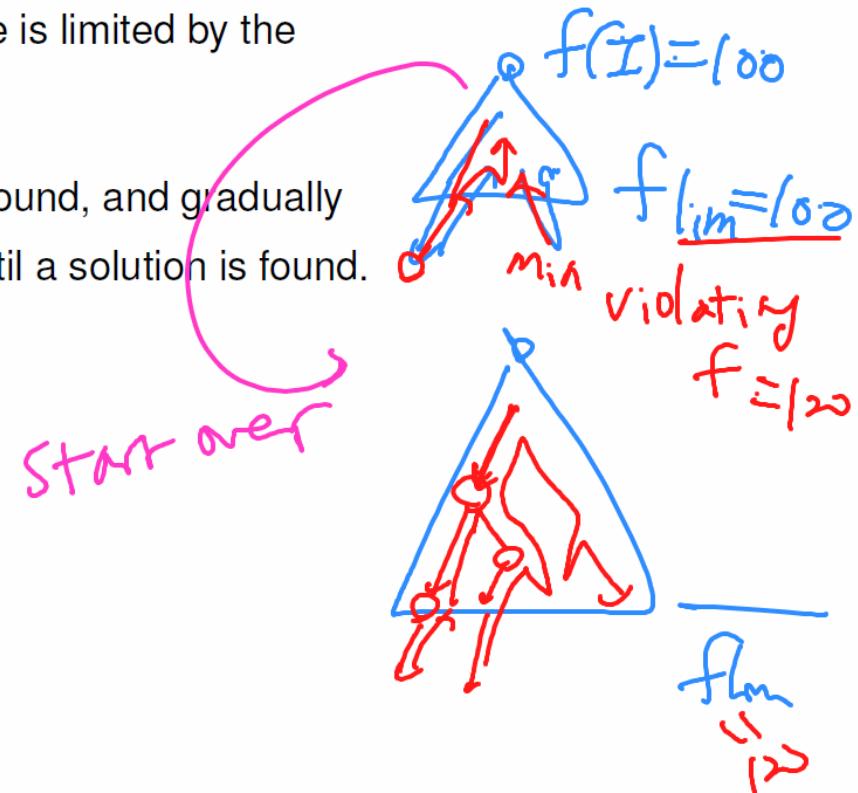
- Basic idea: only search within a certain f bound, and gradually increase the f bound and restart search until a solution is found.



Depth = 1



Depth = 2



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Problem 5 (Written; 10 pts): Consider iterative deepening search. When the goal is ⑨, how many nodes are visited before reaching that node?

2 Informed Search

A search tree diagram with root node 'a'. Node 'a' has three children: 'b', 'c', and 'd'. Node 'b' has two children: 'e' and 'f'. Node 'c' has one child: 'g'. Node 'd' has one child: 'h'. Handwritten annotations show red circles around nodes 'a', 'b', 'c', and 'd', and a green circle around node 'g'.

Figure 2: Informed Search.

Problem 6 (Written; 10 pts): For the problem shown in Fig. 2, show that the heuristic is admissible ($h(n) \leq h^*(n)$ for all n). Note: You have to compute $h^*(n)$ for each n and compare to the $h(n)$ table.

Hint: It is best to work backwards from the goal, where $h^*(h) = 0$ (already at goal), $h^*(f) = 17$ (true minimum cost from \textcircled{f} to \textcircled{h}), $h^*(b) = 40 + 24 = 64$ (true minimum cost from \textcircled{b} to \textcircled{h} : note that there are multiple paths and this path has the minimum cost!), etc.

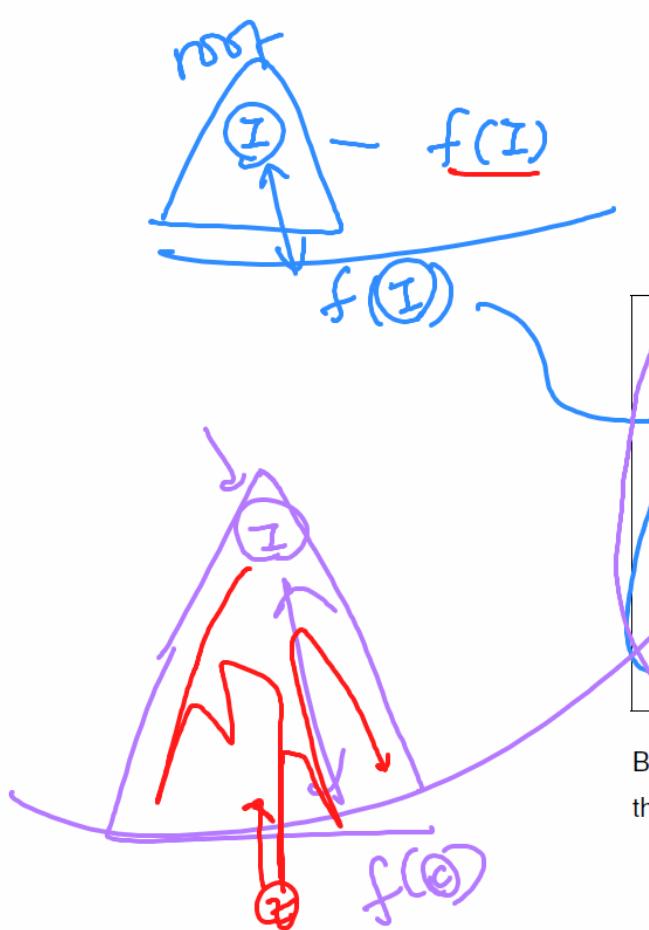
Problem 7 (Written; 20 pts): Manually conduct greedy best-first search on the graph below (Fig. 2), with initial node \textcircled{a} and goal \textcircled{h} . Actual cost from node to node are shown as edge labels. Yeah.

Node	$h(n)$
a	22
b	22
c	10
d	23
e	14
f	16
g	10
h	0

Handwritten notes on the right side of the screen show the following calculations:

- Top: $a \rightarrow f = 22$, $f = 0 \rightarrow 22$
- Middle: $a = 14 + 22 = 36$, $b = 43$, $c = 24$, $d = 23$
- Bottom: $c = 23$, $e = 24$, $b = 36$, $d = 43$, $g = 5$
- Final result: $g = 43 + 10 = 53$

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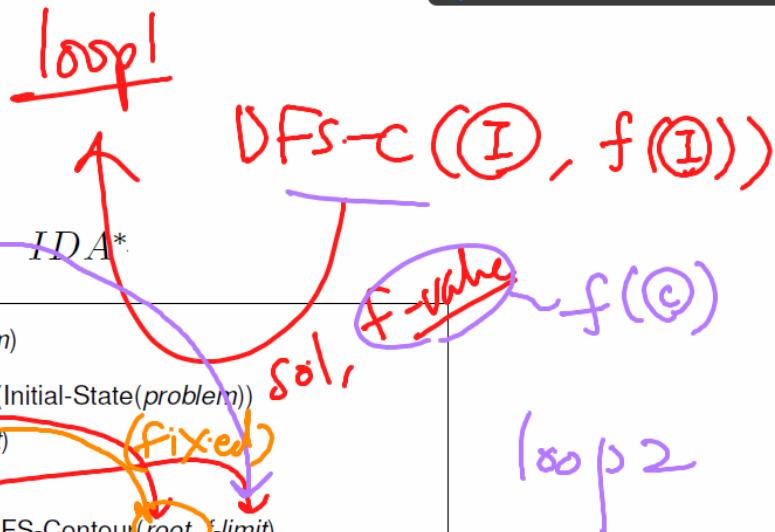


```

function IDA*(problem)
    root ← Make-Node(Initial-State(problem))
    f-limit ← f-Cost(root)
    loop do
        solution, f-limit ← DFS-Contour(root, f-limit)
        if solution != NULL then return solution
        if f-limit == ∞ then return failure
    end loop
  
```

Basically, iterative deepening depth-first-search with depth defined as the f -cost ($f = g + n$):

60



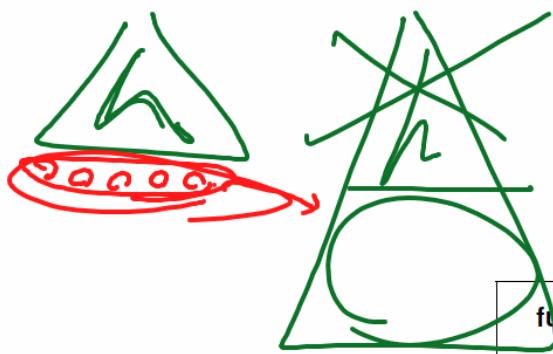
$DFS-c(I, f(I))$

loop 2

$DFS-c(I, f(c))$

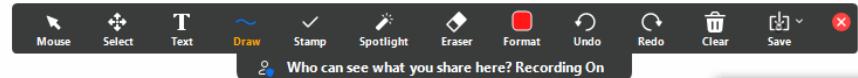
loop 3

$DFS-c(I, f(\Theta))$

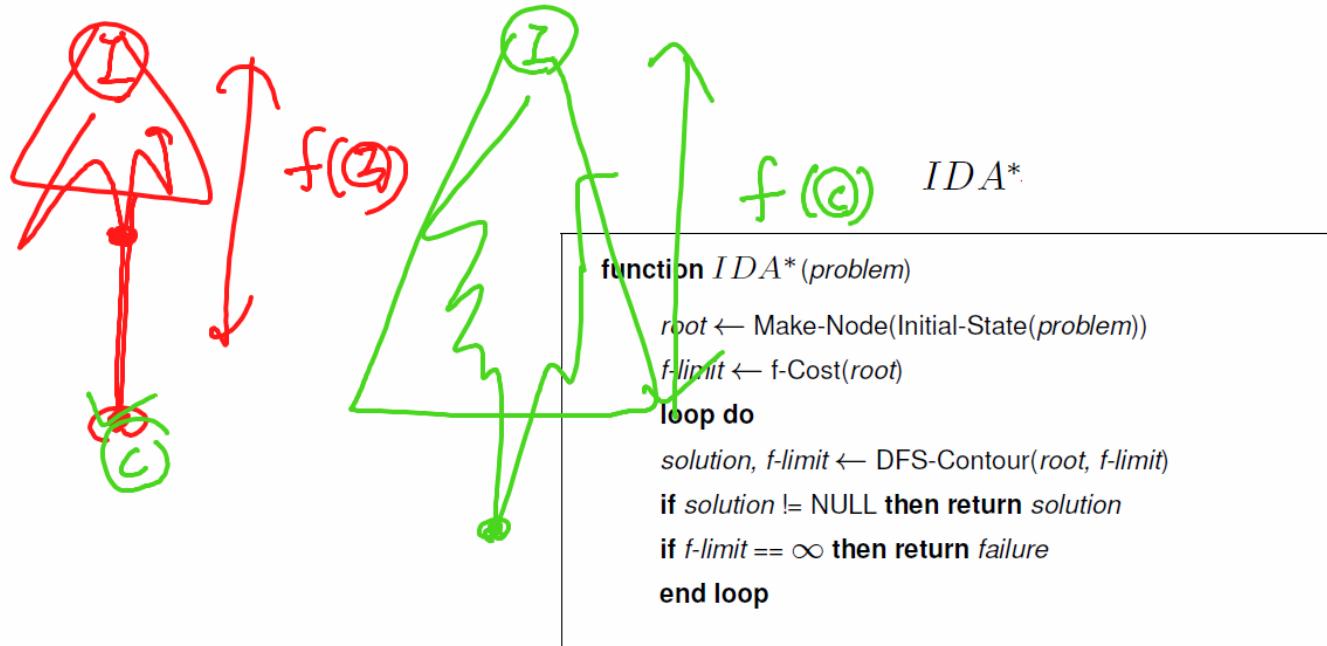


*IDA**

```
function IDA*(problem)
    root ← Make-Node(Initial-State(problem))
    f-limit ← f-Cost(root)
    loop do
        solution, f-limit ← DFS-Contour(root, f-limit)
        if solution != NULL then return solution
        if f-limit == ∞ then return failure
    end loop
```



Basically, iterative deepening depth-first-search with depth defined as the *f*-cost ($f = g + n$):



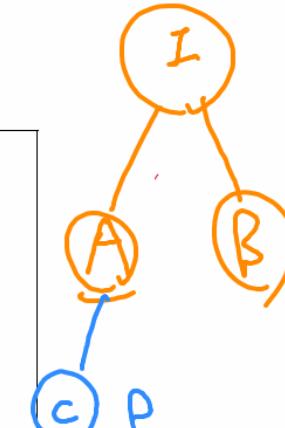
Basically, iterative deepening depth-first-search with depth defined as the f -cost ($f = g + n$):



IDA^*

```

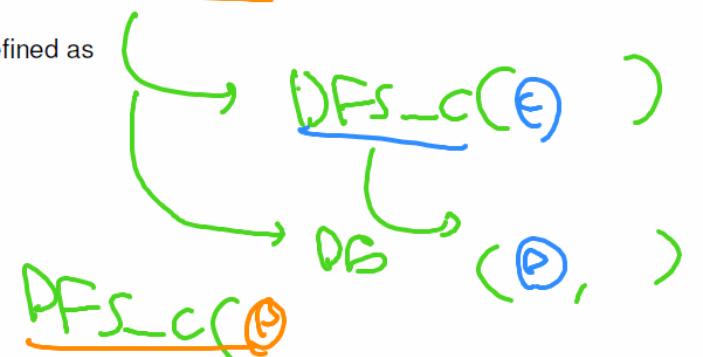
function  $IDA^*$ (problem)
    root ← Make-Node(Initial-State(problem))
    f-limit ← f-Cost(root)
    loop do
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        if solution != NULL then return solution
        if f-limit ==  $\infty$  then return failure
    end loop
  
```



DFS-c(A)

Basically, iterative deepening depth-first-search with depth defined as the f -cost ($f = g + n$):

60

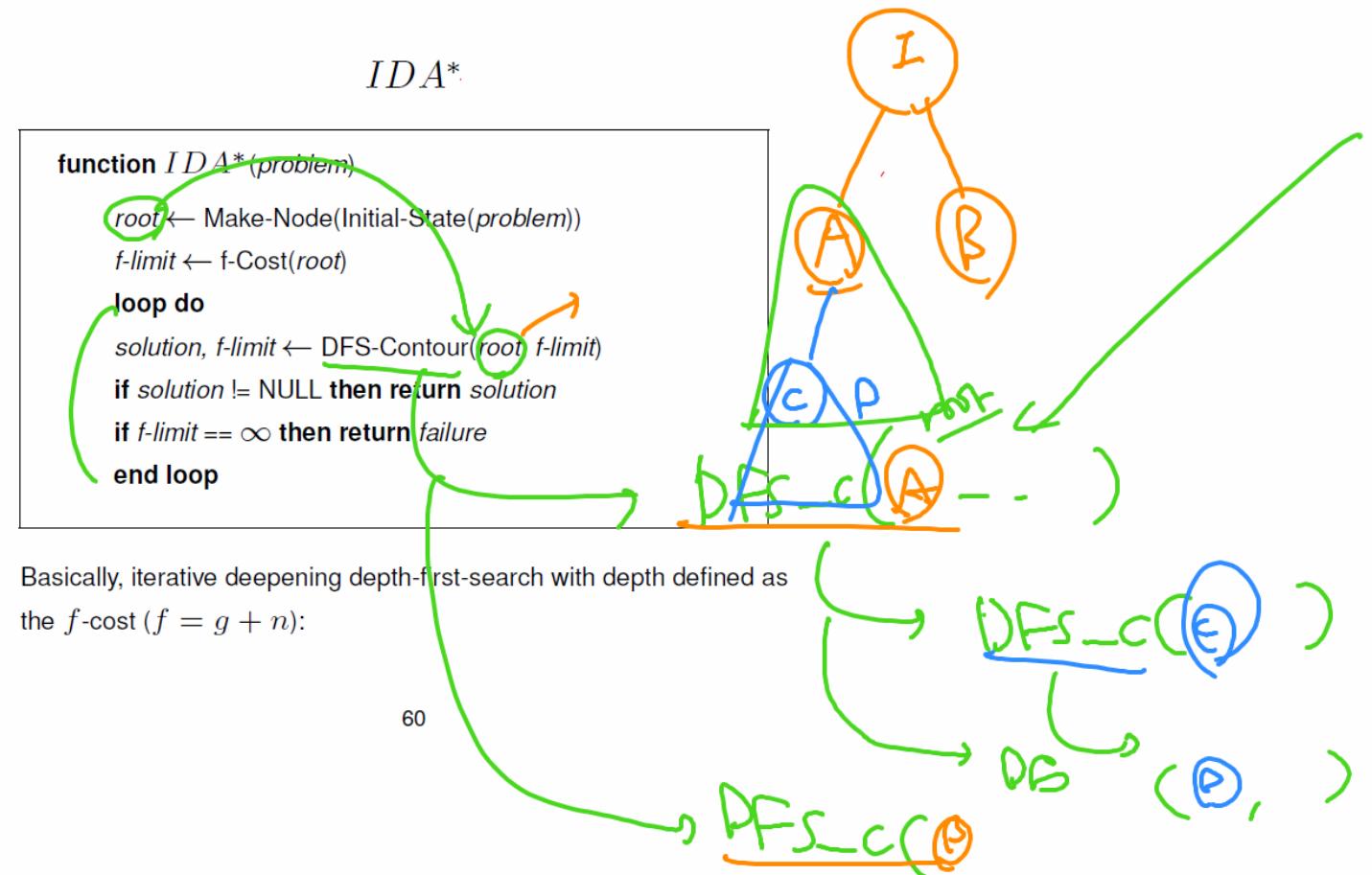




IDA^*

```

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        if solution != NULL then return solution
        if f-limit ==  $\infty$  then return failure
    end loop
  
```



Then this. What this would be the root of this subtree, and

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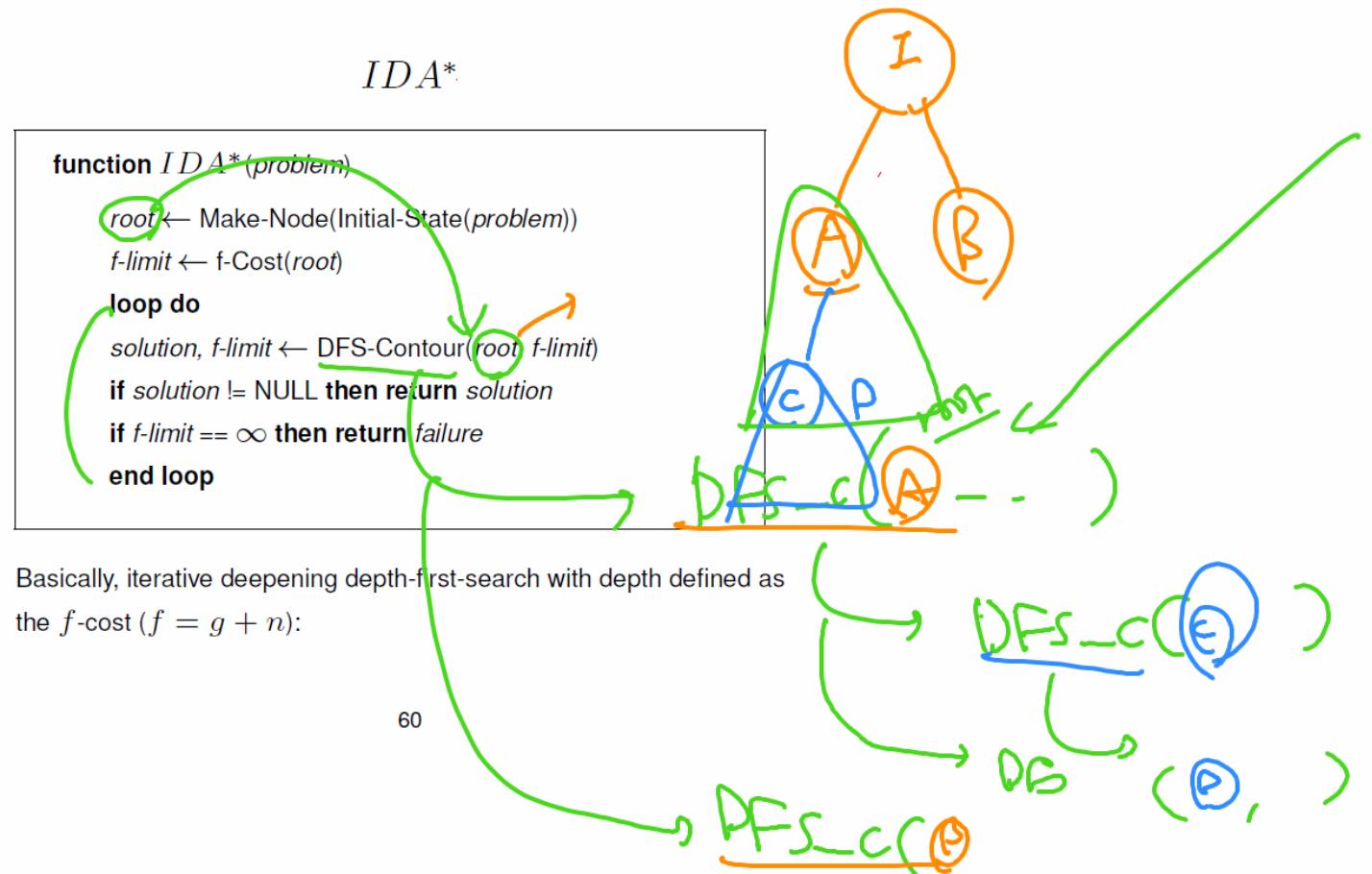


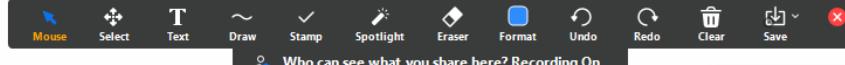
IDA^*

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```

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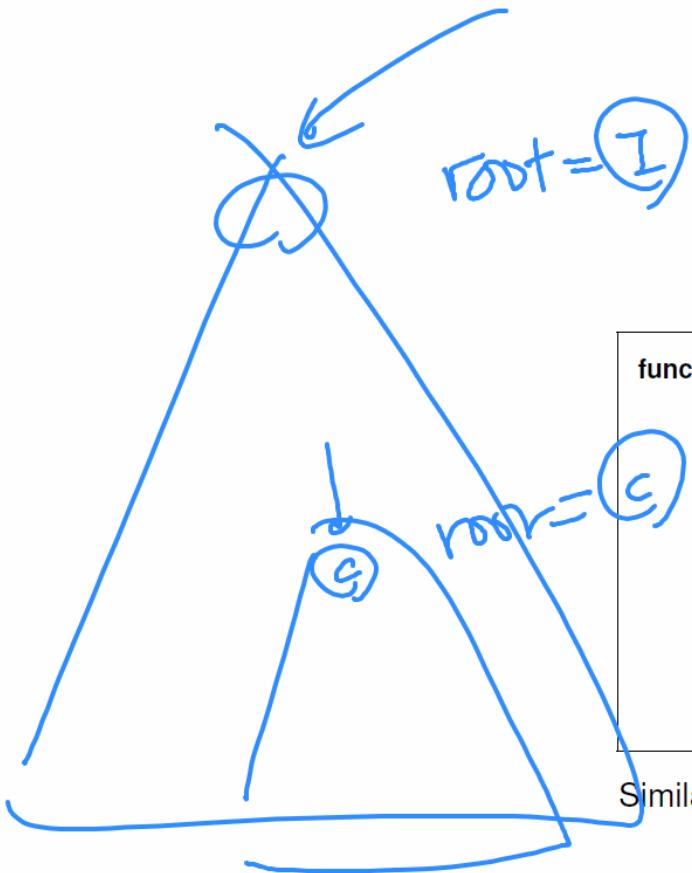
DFS-Contour(*root*, *f-limit*)

```
function DFS-Contour (root, f-limit): returns solution, f-limit
```

Find solution from node **root**, within the *f*-cost limit of *f-limit*. DFS-Contour returns **solution sequence** and new *f*-cost limit.

- if $f\text{-cost}(\text{root}) > \text{f-limit}$, return **null solution** and the $f\text{-cost}(\text{root})$.
- if **root** is a goal node, return solution and its $f\text{-cost}(\text{root})$.
- **recursively call** on all successors, and find successor with minimum *f*-cost. Return its solution and its *f*-cost.

Similar to the recursive implementation of DFS.

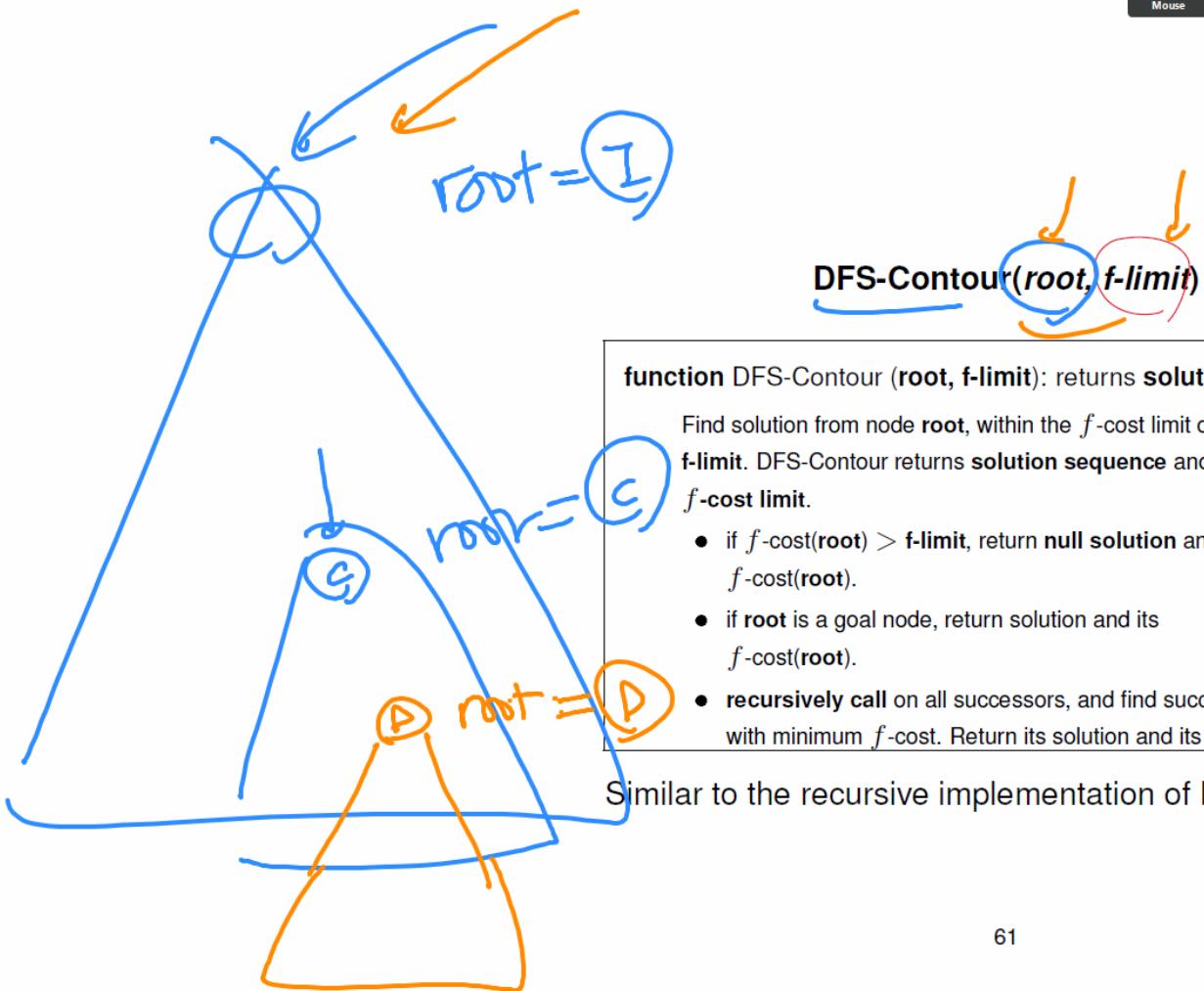


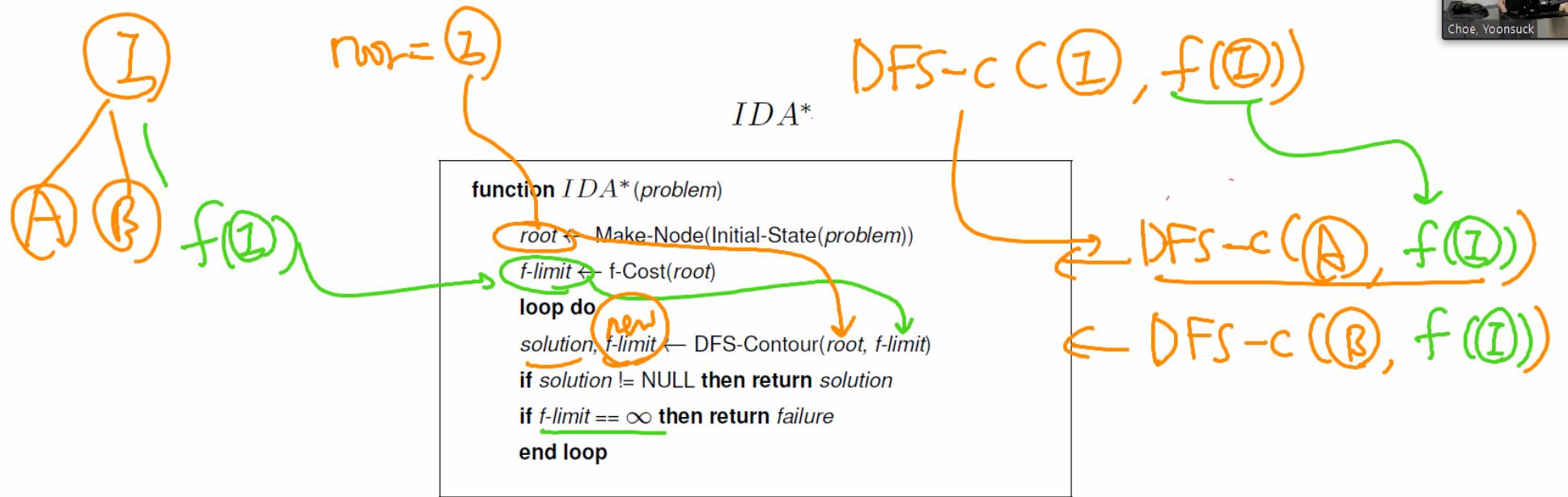
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Find solution from node **root**, within the *f*-cost limit of **f-limit**. DFS-Contour returns **solution sequence** and new *f*-cost limit.

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Basically, iterative deepening depth-first-search with depth defined as the f -cost ($f = g + n$):

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Problem 5 (Written; 10 pts): Consider iterative deepening search. When the goal is ⑨, how many nodes are visited before reaching that node?

2 Informed Search

Figure 2: Informed Search.

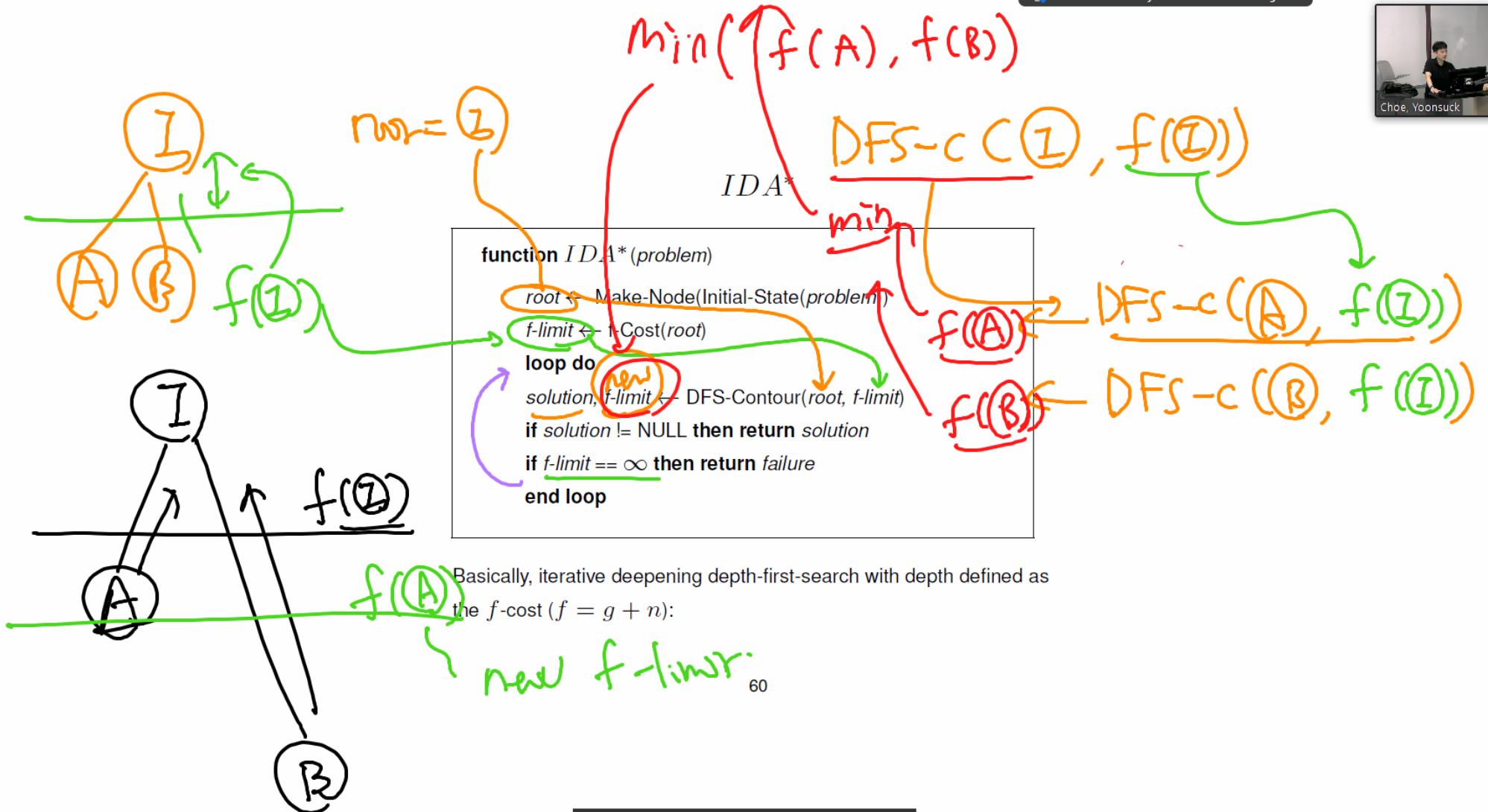
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c	10
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h	0

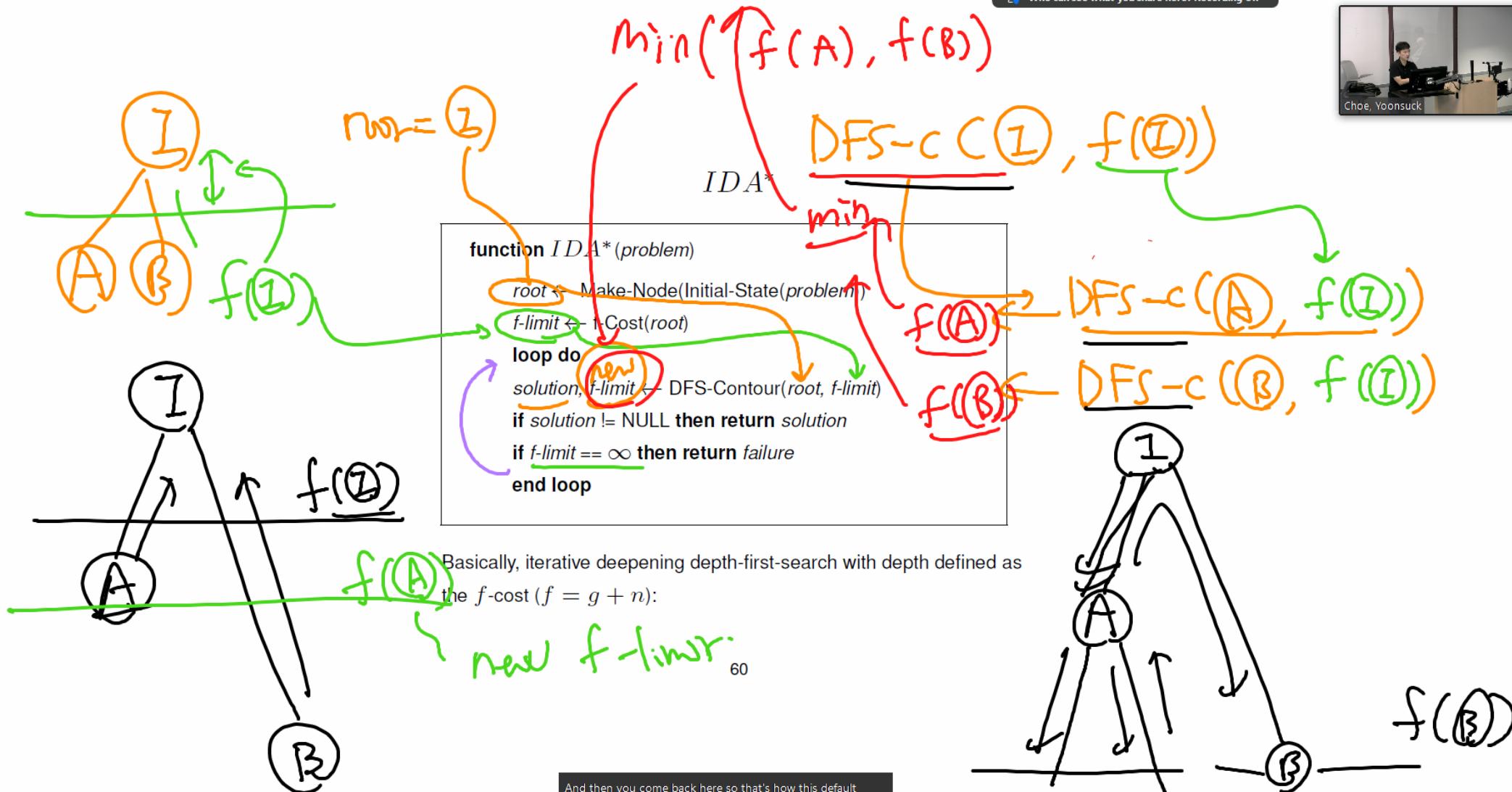
Handwritten annotations on the right side of the slide:

- (Q) $h^*(d)$
- ↓
- (Q) 24
- ↓
- (Q) $25 + 50$
- $h^*(n)$
- $h^*(h) = 0$

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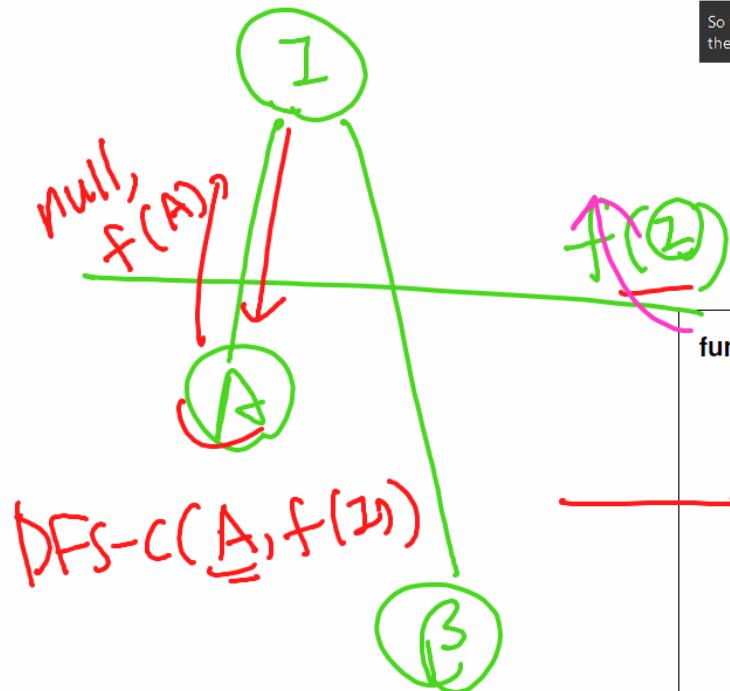




And then you come back here so that's how this default strategy works so probably

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So that answer would be 120, because you want to return the minimum cost

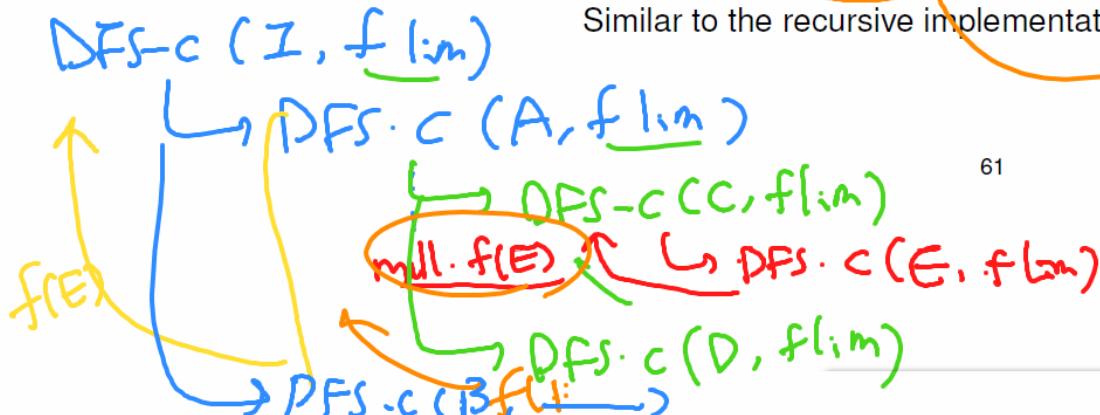


$\text{DFS}(I, \dots)$
 $\text{DFS-Contour}(\text{root}, f\text{-limit})$

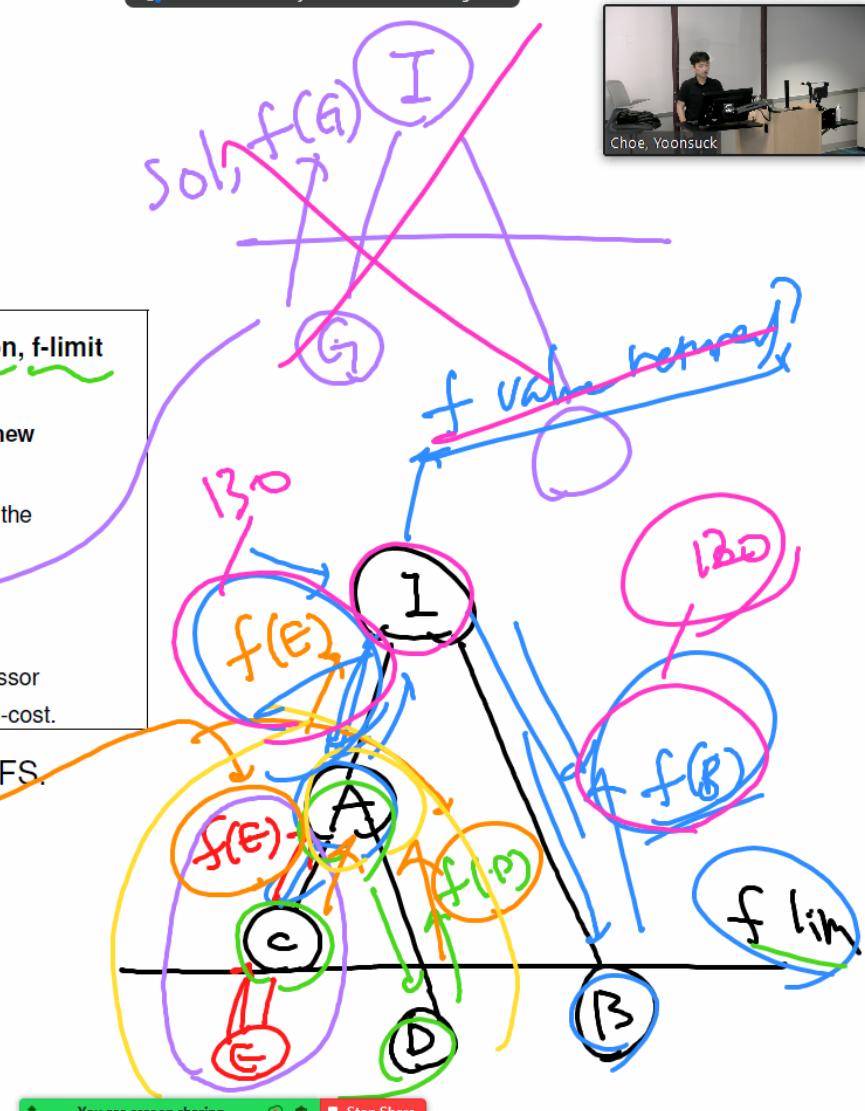
function $\text{DFS-Contour}(\text{root}, f\text{-limit})$: returns **solution**, $f\text{-limit}$
 Find solution from node **root**, within the f -cost limit of $f\text{-limit}$. **DFS-Contour** returns **solution sequence** and new f -cost limit.

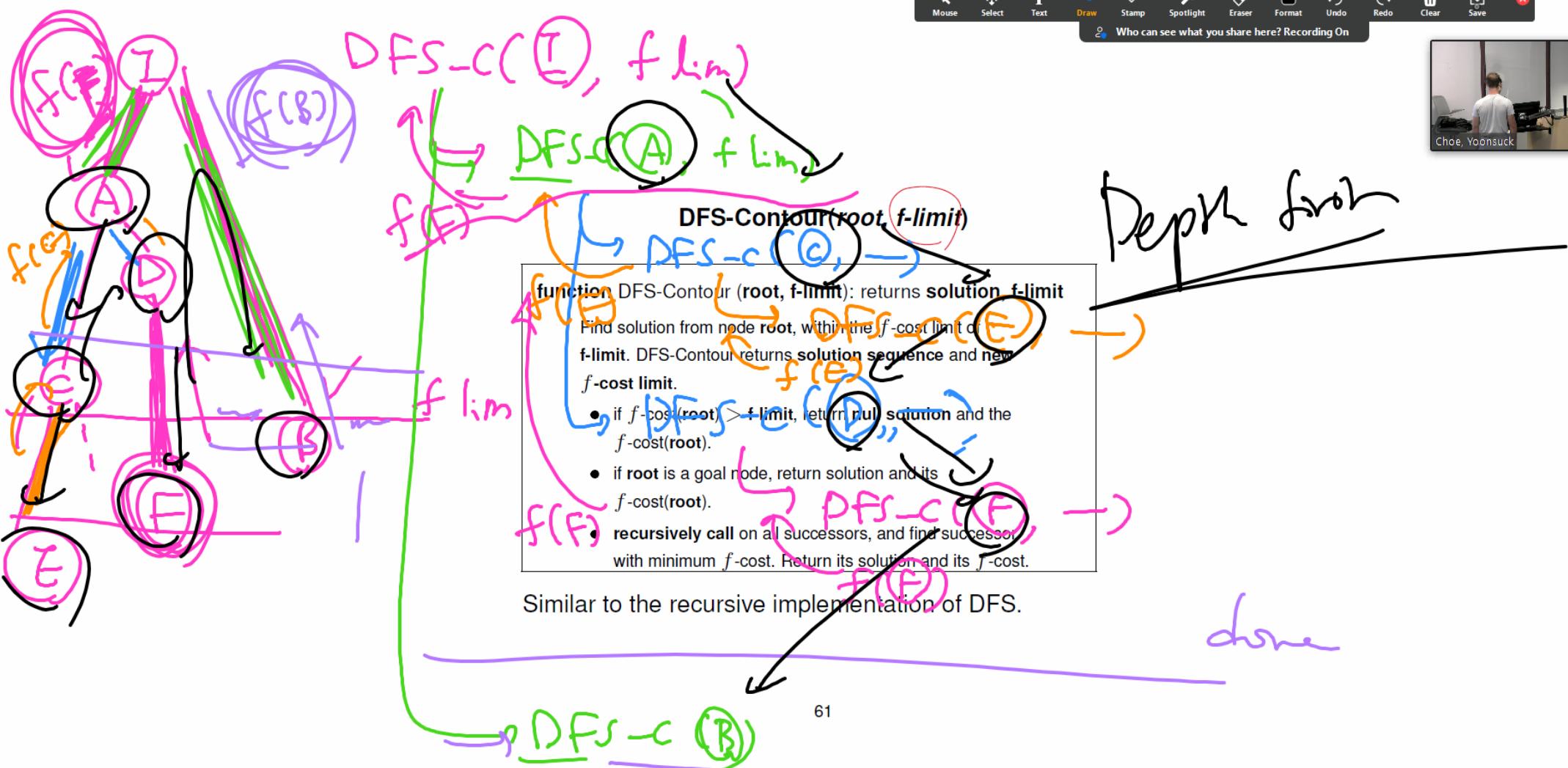
- if $f\text{-cost}(\text{root}) > f\text{-limit}$, return **null solution** and the $f\text{-cost}(\text{root})$.
- if **root** is a goal node, return **solution** and its $f\text{-cost}(\text{root})$.
- **recursively call** on all successors, and find successor with minimum f -cost. Return its **solution** and its f -cost.

Similar to the recursive implementation of DFS.



61





Oh, I'm I was out of town for a wedding did you already email Me? Oh, okay, just send me an email. And then you know, that's fine. Yeah, just send me an email

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Problem 5 (Written; 10 pts): Consider iterative deepening search. When the goal is ⑨, how many nodes are visited before reaching that node?

2 Informed Search

Figure 2: Informed Search.

Node	$h^*(n)$
a	22
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Handwritten notes on the right side of the screen:

- (Q) $h^*(d)$
- ↓
- ① 24
- ↓
- ② 25 + 50
- $h^*(n)$
- $h^*(h) = 0$
- $h^*(f) = 17$
- $h^*(b) = 40 + 24 = 64$

Problem 6 (Written; 10 pts): For the problem shown in Fig. 2, show that the heuristic is admissible ($h(n) \leq h^*(n)$ for all n). Note: You have to compute $h^*(n)$ for each n and compare to the $h(n)$ table.

Hint: It is best to work backwards from the goal, where $h^*(h) = 0$ (already at goal), $h^*(f) = 17$ (true minimum cost from ① to ④), $h^*(b) = 40 + 24 = 64$ (true minimum cost from ⑤ to ④; note that there are multiple paths and this path has the minimum cost!), etc.

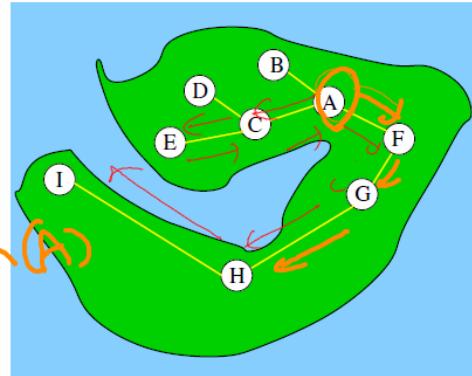
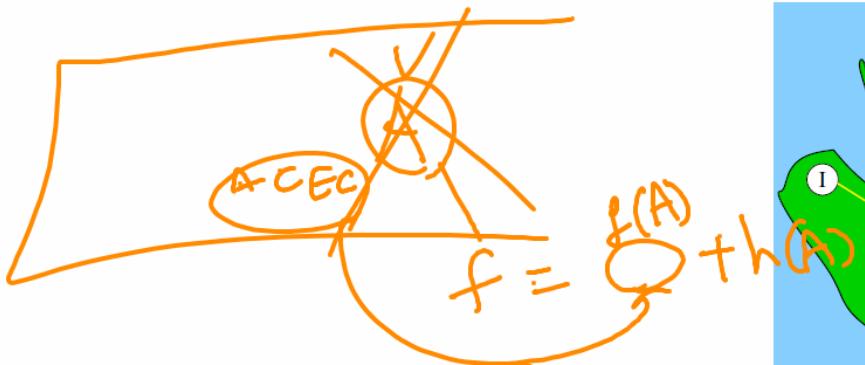
Problem 7 (Written; 20 pts): Manually conduct greedy best-first search on the graph below (Fig. 2), with initial node ① and goal ④. Actual cost from node to node are shown as edge labels. Get any sales.

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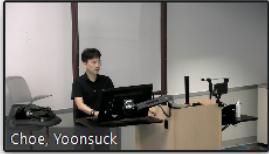
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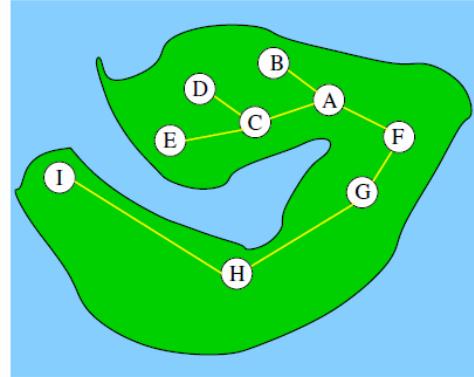
Optimality of A*: Example



1. **Visit of parent disallowed:** search fails at nodes **B**, **D**, and **E**.
2. **Visit of parent allowed:** paths through nodes **B**, **D**, and **E** will have an inflated path cost $g(n)$, thus will become nonoptimal.
- $\checkmark \quad A \rightarrow C \rightarrow E \rightarrow C \xrightarrow{\text{inflated path cost}} A \rightarrow F \rightarrow \dots$
- $A \rightarrow F \rightarrow G \rightarrow H$



Optimality of A*: Example



1. **Visit of parent disallowed:** search fails at nodes **B**, **D**, and **E**.
2. **Visit of parent allowed:** paths through nodes **B**, **D**, and **E** will have an inflated path cost $g(n)$, thus will become nonoptimal.

$A \rightarrow C \rightarrow E \rightarrow C \rightarrow A \rightarrow F \rightarrow \dots$

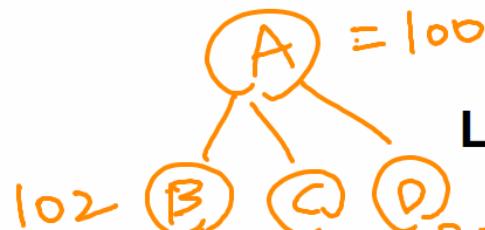
inflated path cost

So this won't be selected at all so this is called the inflated pass coast

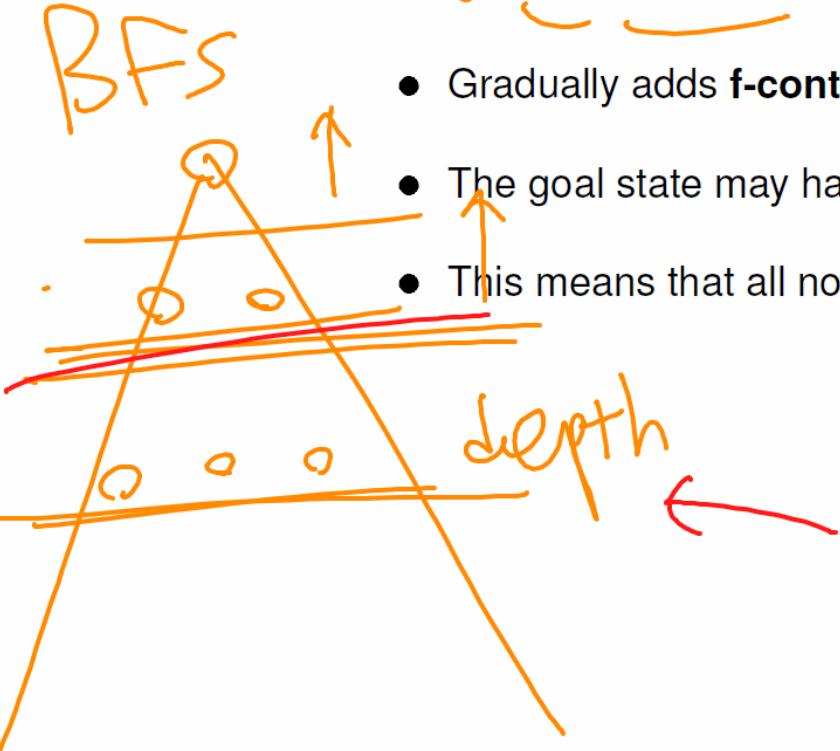


Choe, Yoonsuck

Lemma to the Optimality of A*

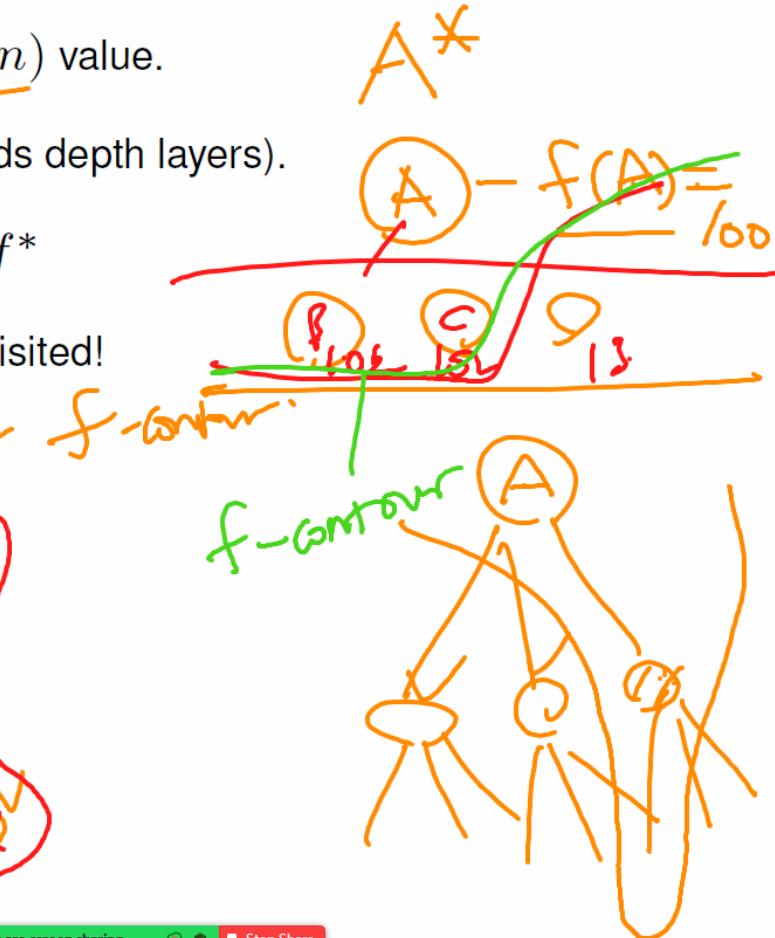
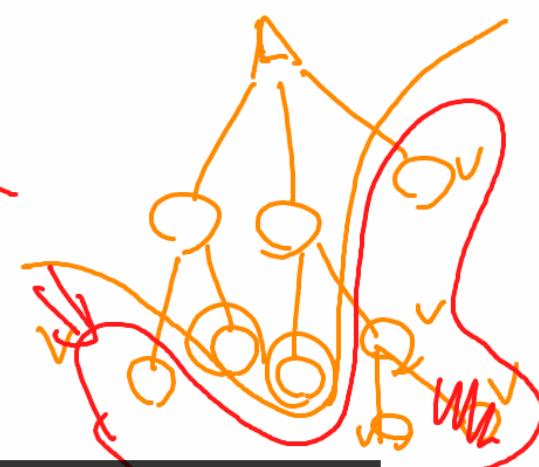


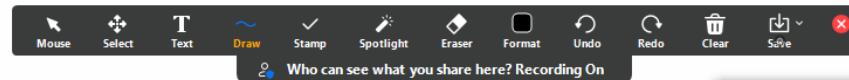
Lemma: A* visits nodes in the order of increasing $f(n)$ value.



- Gradually adds **f-contours** of nodes (cf. BFS adds depth layers).
- The goal state may have an f value: let's call it f^*
- This means that all nodes with $f < f^*$ will be visited!

So you'll have to find that the number of these nodes that are tangling outside of your control will become exponential





$h^*(n)$

Complexity of A*

$h_1(n)$

A^* is complete and optimal, but space complexity can become exponential if the heuristic is not good enough.

$h_2(n)$

- condition for **subexponential** growth:

$$|h(n) - h^*(n)| \leq O(\log h^*(n)),$$

where $h^*(n)$ is the **true** cost from n to the goal.

(Q) Which heuristic is better?

① h_1

② h_2

$$h^*(n) = 100$$

98

$$\sim \leq 2$$

$$h^*(n) = 1000$$

≤ 3 997

Unfortunately, with most heuristics, error is at least proportional with the true cost, i.e. $\geq O(h^*(n)) > O(\log h^*(n))$.



Problem with A^*

Space complexity is usually **exponential!**

- we need a memory bounded version
- one solution is: Iterative Deepening A^* , or IDA^*



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\approx IDS
 \approx for A^* ?