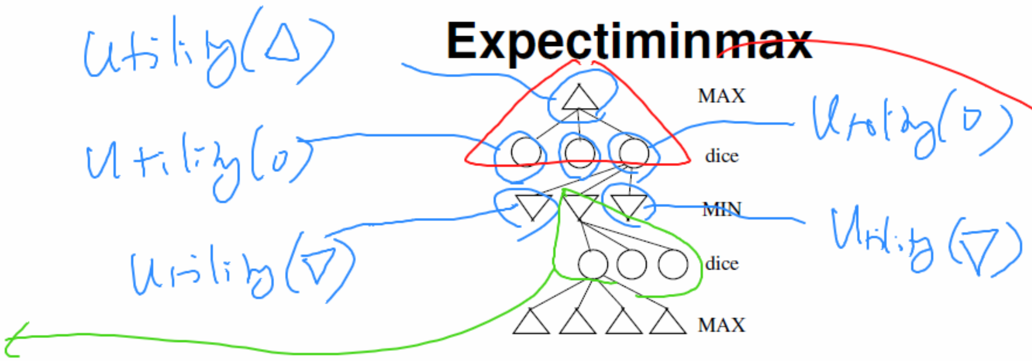
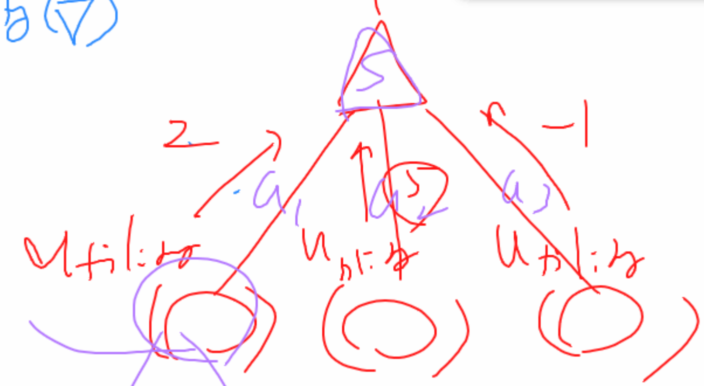
Expectiminmax





Expected value of random variable X:

$$E(X) = \sum_{x \in X} x P(x)$$

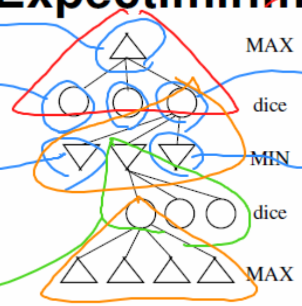
Expectiminmax: a is action, r is random outcome

$$Expectiminmax(s) =$$

$$\begin{array}{ll} Utility(s, MAX_{player}) & \text{if } s \text{ is terminal} \\ max_a Expectiminmax(Result(s, a)) & \text{if MAX node's turn} \\ min_a Expectiminmax(Result(s, a)) & \text{if MIN node's turn} \\ \sum_r P(r) Expectiminmax(Result(s, r)) & \text{if Chance node} \end{array} \quad (1)$$

The only thing that differs is the the chance

Expectiminmax



Utility(△)
Utility(○)
Utility(▽)

Expected value of random variable X:

$E(X) = \sum_{x \in X} xP(x)$

Expectiminmax: a is action, r is random outcome

$$Expectiminmax(s) = \begin{cases} Utility(s, MAX_{player}) & \text{if } s \text{ is terminal} \\ max_a Expectiminmax(Result(s, a)) & \text{if MAX node's turn} \\ min_a Expectiminmax(Result(s, a)) & \text{if MIN node's turn} \\ \sum_r P(r) Expectiminmax(Result(s, r)) & \text{if Chance node} \end{cases} \quad (1)$$

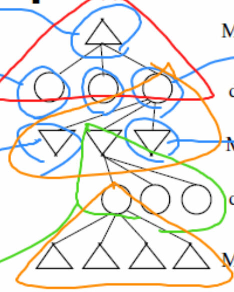
Case1
dice prob.
outcome
① random outcome
② prob.

Expectiminmax

Utility(△)

Utility(○)

Utility(▽)



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\sum_r P(r) Expectiminmax(Result(s, r)) & \text{if Chance node}
\end{cases}
\tag{1}
$$

Case 1

dice prob.

outcome

① random outcome

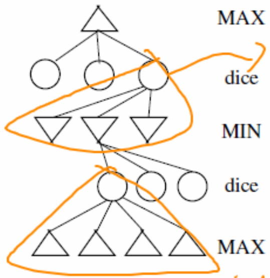
② prob.

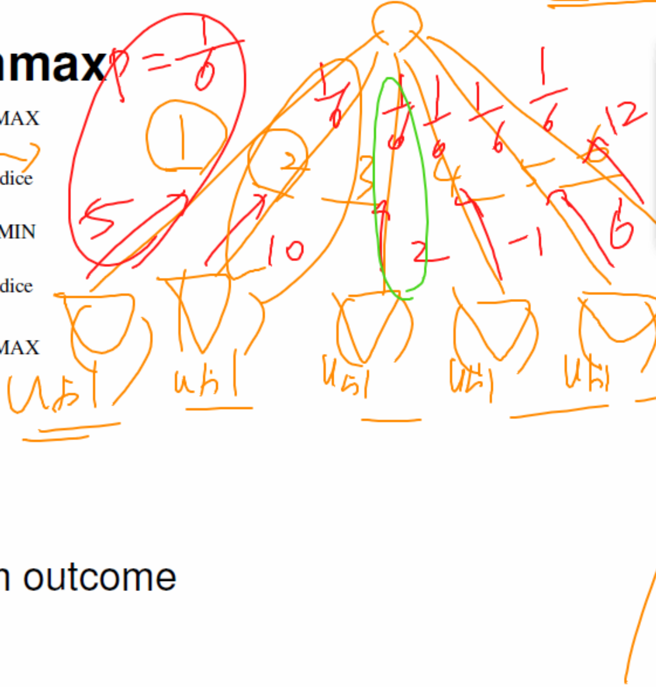
$uti = E[uti]$

Expectiminmax

$E[\nabla\, uti]$

$= \frac{1}{6} \times 5 + \frac{1}{6} \times 10 + \cdots + \frac{1}{6} \times 2 + \cdots$





Expected value of random variable X:

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Expectiminmax: a is action, r is random outcome

$$Expectiminmax(s) =$$

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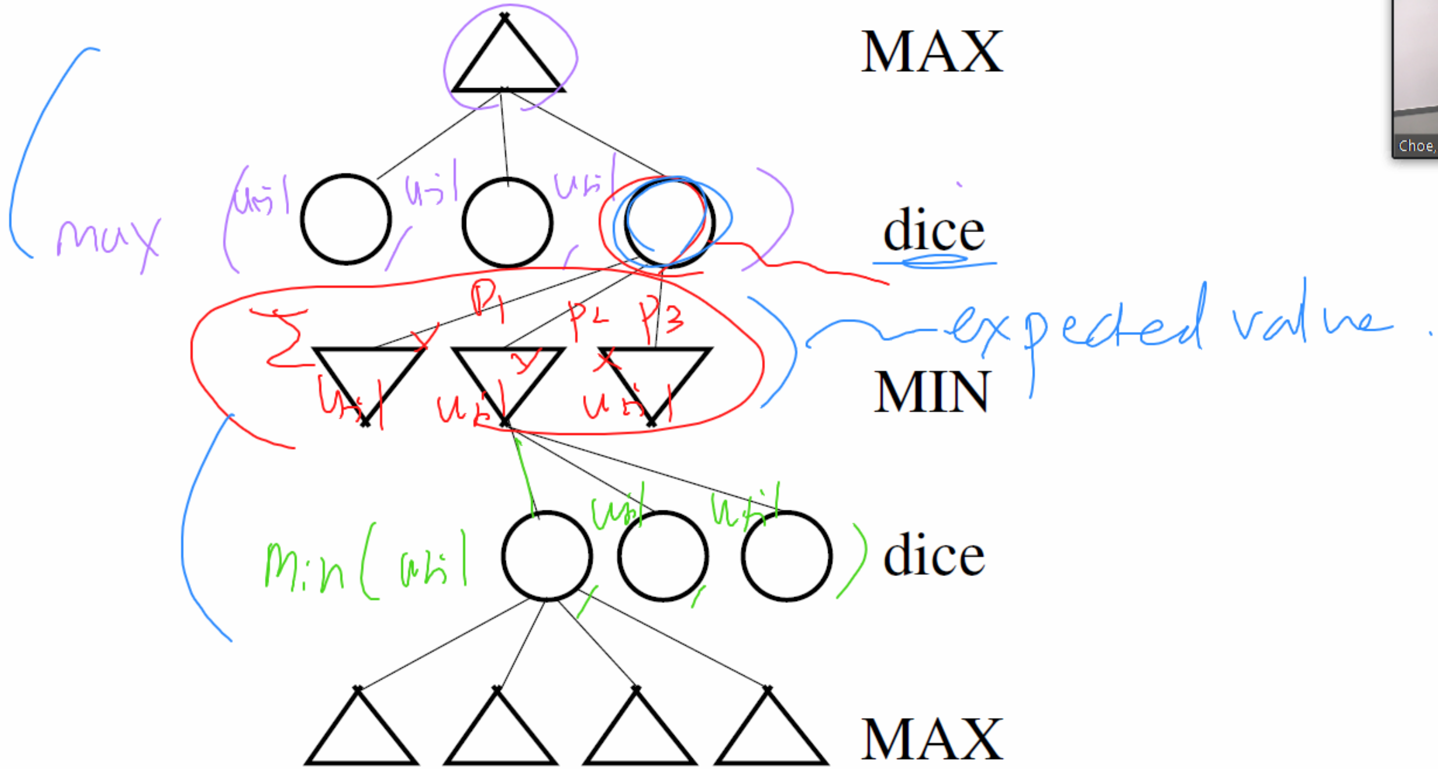
$max_a Expectiminmax(Result(s,a))$	if MAX node's turn
$min_a Expectiminmax(Result(s,a))$	if MIN node's turn
$\sum_r P(r) \underbrace{Expectiminmax(Result(s,r))}_{uti}$	if Chance node

random outcome

(1)

Game Tree With Chance Element

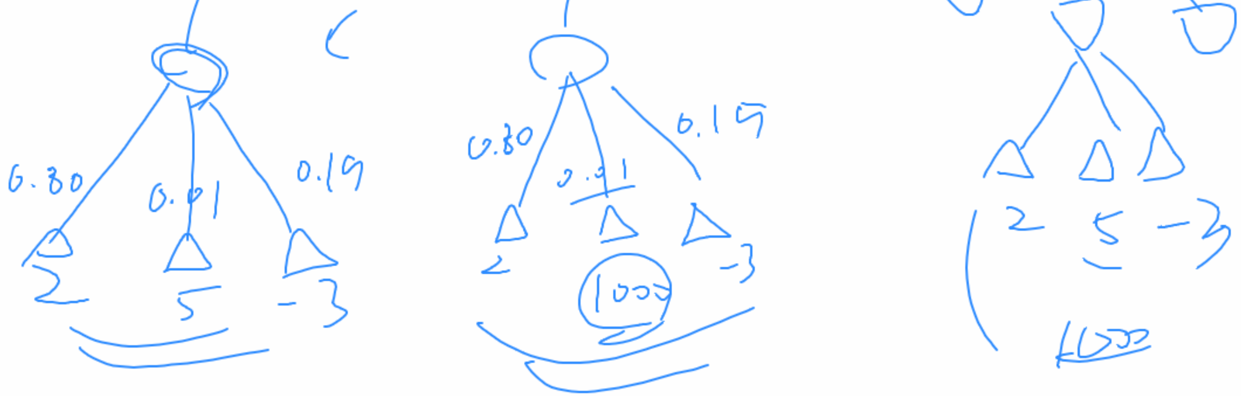


Rolling the dice, shuffling the deck of card and drawing, etc.

- chance element forms a new ply (e.g. dice, shown above)

Design Considerations for Probabilistic Games

- the **value** of evaluation function, not just the **scale** matters now! (think of what expected value is)
- time complexity: $b^m n^m$, where n is the number of distinct dice rolls
- pruning can be done if we are careful



Design Considerations for Probabilistic Games

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- pruning can be done if we are careful

Well-Formed Formulas in Propositional Logic

Components of well-formed formulas (sentences):

- propositional symbols (atoms): **P**, **Q**, **R**
- parentheses: **( )**
- connectives: $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$
- constants: **True, False**

And R and parentheses, and connectives like these logical operators, the negation and conjunction, disjunction, implication, and and equivalence

Well-Formed Formulas (Cont'd)

Well-Formed Formulas (wff): Syntax

True, False

$$wff \Rightarrow atom | constant$$

≈ Boolean variables

$$wff \Rightarrow (\neg wff)$$

wff

$$
\begin{aligned}
wff \Rightarrow\ & (wff \vee wff) \\
& | (wff \wedge wff) \\
& | (wff \rightarrow wff) \\
& | (wff \leftrightarrow wff)
\end{aligned}
$$

$\neg P \vee True$

Operator precedence: $\neg$; $\wedge, \vee$; $\rightarrow, \leftrightarrow$ (decreasing order)

$\neg P \rightarrow Q \vee \neg R \wedge S$ is evaluated as $(\neg P) \rightarrow ((Q \vee (\neg R)) \wedge S)$

Propositional Logic: Semantics

$P \vee Q$

- atoms can take on **True** or **False** values.
- an interpretation assigns specific truth values to the atoms.
- for a formula with n atoms, there are 2^n possible truth assignments.
- a formula is true **under an interpretation** iff the formula evaluates to **True** with the assignment of truth values within the interpretation.
- a formula is **valid** iff it is **True** under all interpretations.
- a formula is **inconsistent (unsatisfiable)** iff it is **False** under all interpretations.
- a formula G is **valid** iff $\neg G$ is **inconsistent**

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

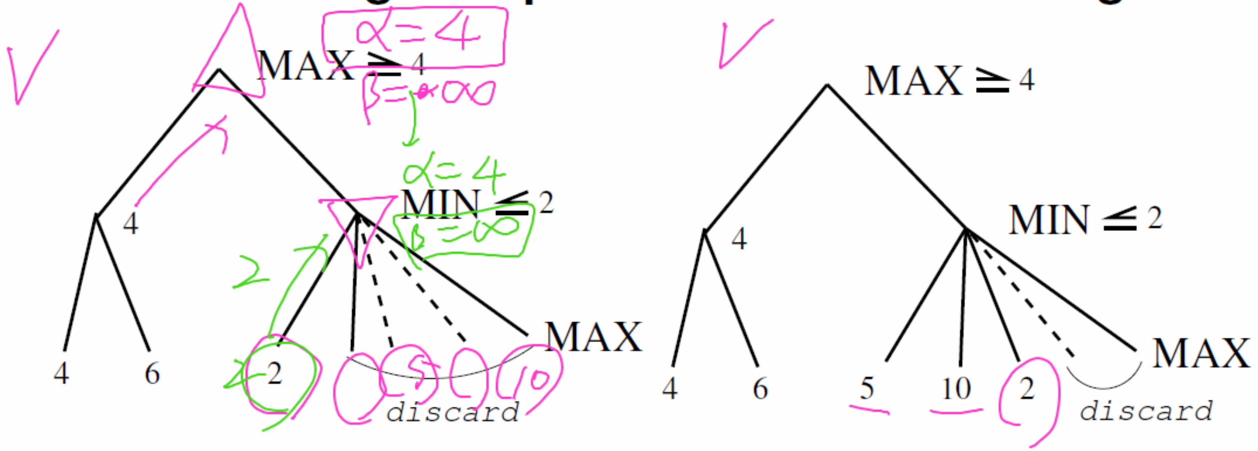
4 $= 2^2$

$(P \vee Q) \wedge \neg R$

$2^3 = 8$

So on. if, on the old different interpretations if the result is true there's called valid

Ordering is Important for Good Pruning



- For MIN, sorting successor's utility in an **increasing** order is better (shown above; left).
- For MAX, sorting in **decreasing** order is better.

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$\neg(P \vee \neg P) = \neg P \wedge P = P \wedge \neg P$

Propositional Logic: Semantics

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P	Q	?
T	T	T
T	F	T
F	T	T
F	F	T

valid

P	$P \vee \neg P$
T	$T \vee \neg T = T \vee F = T$
F	$F \vee \neg F = F \vee T = T$

P	$P \wedge \neg P$
T	$T \wedge \neg T = T \wedge F = F$
F	$F \wedge \neg F = F \wedge T = F$

So the valid sentences and consistent sentences are

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Propositional Logic: Semantics (cont'd)

P Q R ..	F
T F F T ..	T
F T T T	F

sat

I

falsifies

- if a formula F is **True** under an interpretation I, then we say I satisfies F. We also say I is a **model** for F
- if formula F is **False** under interpretation I, then we say I falsifies F
- two formulas F and G are equivalent iff F and G have the same truth values under every interpretation I:

$$F \leftrightarrow G$$

- there can be many models (at least one) of a formula F if F is satisfiable.

Caution: Do not confuse formula F with constant **False**.

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- there can be many models (at least one) of a formula F if F is satisfiable.

Caution: Do not confuse formula F with constant **False**.

P Q R.. | F
T F F T | T sat
F T T T | F falsifies
I

P Q R..	F	G
	T	T
	F	F
	T	T
	T	T
	F	F
	⋮	⋮

Propositional Logic: Semantics (cont'd)

P	Q	R	..	F
T	F	F	T	T
F	T	T	T	F

sat

I

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equiv

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P Q R ..	F	G
	T	T
	F	F
	T	T
	T	T
	F	F
	:	:

Caution: Do not confuse formula F with constant **False**.

Where, if it's satisfiable so to look at here this, this, this, So these are the models, all the models where this F is satisfiable.

Propositional Logic: Semantics (cont'd)

- if a formula F is **True** under an interpretation I, then we say I satisfies F. We also say I is a **model** for F
- if formula F is **False** under interpretation I, then we say I falsifies F
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$$F \leftrightarrow G$$

- there can be many models (at least one) of a formula F if F is satisfiable.

Caution: Do not confuse formula F with constant **False**.

Basic Laws of Propositional Logic

- $F \vee G = G \vee F$,
  $F \wedge G = G \wedge F$ (commutative)
- $(F \vee G) \vee H = F \vee (G \vee H)$,
  $(F \wedge G) \wedge H = F \wedge (G \wedge H)$ (associative)
- $F \vee (G \wedge H) = (F \vee G) \wedge (F \vee H)$,
  $F \wedge (G \vee H) = (F \wedge G) \vee (F \wedge H)$ (distributive)
- $F \vee \mathbf{False} = F$,
  $F \wedge \mathbf{True} = F$ (identity)
- $F \vee \mathbf{True} = \mathbf{True}$
  $F \wedge \mathbf{False} = \mathbf{False}$ (universal bound)
- $F \vee \neg F = \mathbf{True}$
  $F \wedge \neg F = \mathbf{False}$ (negation)

Distributed over that, and then the same

Basic Laws of Propositional Logic

- $F \vee G = G \vee F$,
  $F \wedge G = G \wedge F$ (commutative) ✓
- $(F \vee G) \vee H = F \vee (G \vee H)$,
  $(F \wedge G) \wedge H = F \wedge (G \wedge H)$ (associative)
- $F \vee (G \wedge H) = (F \vee G) \wedge (F \vee H)$,
  $F \wedge (G \vee H) = (F \wedge G) \vee (F \wedge H)$ (distributive)
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  $F \wedge \mathbf{False} = \mathbf{False}$ (universal bound)
- $F \vee \neg F = \mathbf{True}$
  $F \wedge \neg F = \mathbf{False}$ (negation)

$\vee$: $+$

$\wedge$: $*$

$F + (G * H)$

$= (F+G) * (F+H)$

arithmetically a nonsense, but perfectly fine for logic

So this is mathematically nonsense, but perfectly fine, or logic

Basic Laws of Propositional Logic

- $F \vee G = G \vee F$,
  $F \wedge G = G \wedge F$ (commutative)
- $(F \vee G) \vee H = F \vee (G \vee H)$,
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  $F \wedge \mathbf{False} = \mathbf{False}$ (universal bound)
- $F \vee \neg F = \mathbf{True}$
  $F \wedge \neg F = \mathbf{False}$ (negation)

And also we saw this: Oh, yeah, formula and not formula is false.

Basic Formulas (cont'd)

- $\neg(\neg F) = F$ (double negation)
- $\neg(F \vee G) = \neg F \wedge \neg G$
  $\neg(F \wedge G) = \neg F \vee \neg G$ (De Morgan's Law)
- $F \leftrightarrow G = (F \rightarrow G) \wedge (G \rightarrow F)$ (equivalence)
- $F \rightarrow G = \neg F \vee G$ (implication)
- $F \wedge F = F$
  $F \vee F = F$ (idempotent)

Inference Rules

Notations: $\frac{P,Q}{R}$ means $(P \wedge Q) \rightarrow R$.

Given two sentences P and Q, you can generate a new sentence R.

then

- Modus Ponens:

$$\frac{F \rightarrow G, F}{G}$$

- Unit Resolution:

$$\frac{F \vee G, \neg G}{F}$$

- Resolution:

$$\frac{F \vee G, \neg G \vee H}{F \vee H} \quad or\ equivalently \quad \frac{\neg F \rightarrow G, G \rightarrow H}{\neg F \rightarrow H}$$

$P = \text{false} \wedge Q = ?$

$\rightarrow R$

False

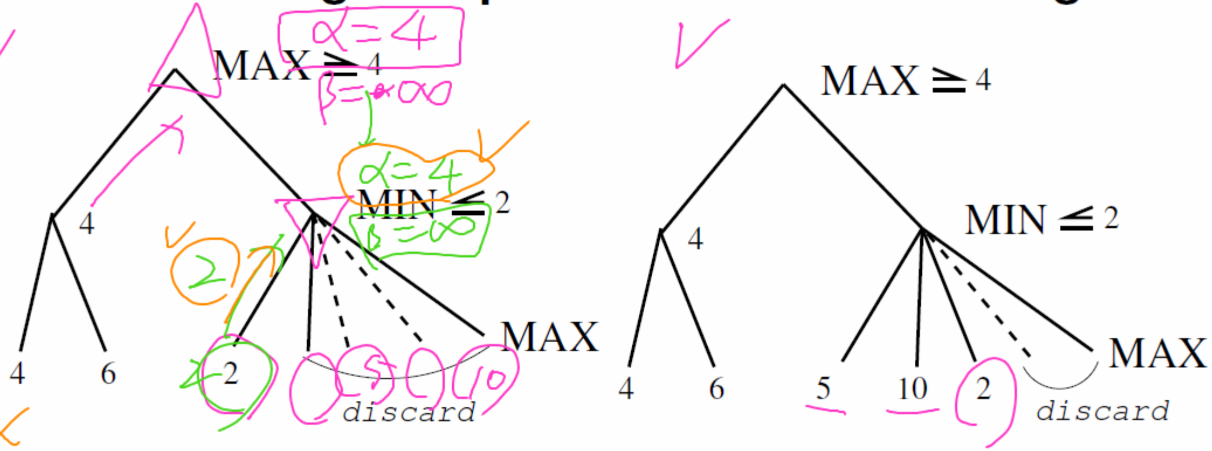
$(P \wedge Q) \rightarrow R$

$\text{False} \rightarrow R = \text{true}$

P	Q	P→Q
T	T	T
T	F	F
F	T	T
F	F	T

When these 2 are both true.

Ordering is Important for Good Pruning



(Q) Is 2 worse than $\alpha=4$ as a max

① Yes

② No

$4 = \max(2, \alpha=4)$: not worse.

- For MIN, sorting successor's utility in an **increasing** order is better (shown above; left).
- For MAX, sorting in **decreasing** order is better.

Inference Rules

valid.

Notations: $\frac{P,Q}{R}$ means $(P \wedge Q) \rightarrow R$.

Given two sentences P and Q, you can generate a new sentence R.

- Modus Ponens:

$$\frac{F \rightarrow G, F}{G}$$

- Unit Resolution:

$$\frac{F \vee G, \neg G}{F}$$

same

$$\frac{F \vee G, \neg G}{F}$$

$$\frac{\neg G \rightarrow F, \neg G}{F}$$

$$= \frac{G \vee F, \neg G}{F}$$

$$\frac{\neg(\neg G) \vee F, \neg G}{F.}$$

step 1

- Resolution:

$$\frac{F \vee G, \neg G \vee H}{F \vee H} \quad \textit{or equivalently} \quad \frac{\neg F \rightarrow G, G \rightarrow H}{\neg F \rightarrow H}$$

And then you have this in the conclusion pot. So basically, these 2 are the same

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Inference Rules

Notations: $\frac{P,Q}{R}$ means $(P \wedge Q) \rightarrow R$.

Given two sentences P and Q, you can generate a new sentence R.

- Modus Ponens:

$$\frac{F \rightarrow G, F}{G}$$

- Unit Resolution:

$$\frac{F \vee G, \neg G}{F}$$

- Resolution:

$$\frac{F \vee G, \neg G \vee H}{F \vee H} \quad \textit{or equivalently} \quad \frac{\neg F \rightarrow G, G \rightarrow H}{\neg F \rightarrow H}$$

$F \vee G = \neg(\neg F) \vee G = \neg F \rightarrow G$

$\neg G \vee H = G \rightarrow H$

transitivity of implication

$\neg F \rightarrow G \rightarrow H$

$\neg(\neg F) \vee H = F \vee H$

G. and G. implies H. So as you get not if implies H, which is when you remove the implication, then it becomes F or H, which is, basically what do you want

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Inference Rules

Notations: $\frac{P,Q}{R}$ means $(P \wedge Q) \rightarrow R$.

Given two sentences P and Q, you can generate a new sentence R.

- Modus Ponens:

$$\frac{F \rightarrow G, F}{G}$$

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Inference Rules

Notations: $\frac{P,Q}{R}$ means $(P \wedge Q) \rightarrow R$.

Given two sentences P and Q, you can generate a new sentence R.

- Modus Ponens:

$$\frac{F \rightarrow G, F}{G}$$

K.B. ① $F \rightarrow G$
② F

modus ponens

Query G ? $\Leftarrow$ G

- Unit Resolution:

$$\frac{F \vee G, \neg G}{F}$$

- Resolution:

$$\frac{F \vee G, \neg G \vee H}{F \vee H} \quad \textit{or equivalently} \quad \frac{\neg F \rightarrow G, G \rightarrow H}{\neg F \rightarrow H}$$

Inference Rules

Notations: $\frac{P,Q}{R}$ means $(P \wedge Q) \rightarrow R$.

Given two sentences P and Q, you can generate a new sentence R.

- Modus Ponens:

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- Unit Resolution:

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- Resolution:

$$\frac{F \vee G, \neg G \vee H}{F \vee H} \quad \textit{or equivalently} \quad \frac{\neg F \rightarrow G, G \rightarrow H}{\neg F \rightarrow H}$$

KB
① $F \vee G$ (rule)
② $\neg G$ (fact)

Query: F ?

K.B.
① $F \rightarrow G$
② F

modus ponens

Query G ? $\Leftarrow G$

Normal Forms (I)

- **literals**: $atom | \neg atom$

$P, \neg P, Q, \neg Q$

- **clauses**: disjunction of 1 or more **literals**

$\neg(P \wedge R)$ not an atom

not a literal

$$literal \vee literal \vee \ldots$$

- **terms**: conjunction of 1 or more **literals**

$$literal \wedge literal \wedge \ldots$$

R sounds like that. So this is not a the role, because this is not an atom not a literal

$P \vee \neg Q \wedge R$

$P \vee (\neg Q \wedge R)$

Normal Forms (I)

- **literals**: $atom \mid \neg atom$

$P, \neg P, Q, \neg Q$

- **clauses**: disjunction of 1 or more **literals**

$P \vee \neg Q$

~~$P \wedge Q$~~

$P \vee Q \vee \neg R$

$$literal \vee literal \vee \ldots$$

$\neg (P \wedge R)$ not an atom

not a literal

- **terms**: conjunction of 1 or more **literals**

$$literal \wedge literal \wedge \ldots$$

$P \wedge Q$ $\quad$ $P \wedge \neg Q \wedge R$ $\quad$ $P \wedge (\neg Q \vee R)$

not a literal

not a literal

$P \vee (\neg Q \wedge R)$

$P \vee \neg Q \wedge R$

Normal Forms (I)

- **literals**: $atom \mid \neg atom$

$P, \quad \neg P, \quad Q, \quad \neg Q$

- **clauses**: disjunction of 1 or more **literals**

$$literal \vee literal \vee \ldots$$

$P \vee \neg Q$

~~$P \wedge Q$~~

$P \vee Q \vee \neg R$

$\neg(P \wedge R)$

not an atom.

not a literal

- **terms**: conjunction of 1 or more **literals**

$$literal \wedge literal \wedge \ldots$$

$P \wedge Q$

$P \wedge \neg Q \wedge R$

$P \wedge (\neg Q \vee R)$

not a literal

This is not a Tom, because this is not a literal, and so on.

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any wff can be converted

Normal Forms (II)

one or more

Choe, Yoonsuck

- **Conjunctive Normal Form (CNF)**: conjunction of **clauses**

clause

$$C1 \wedge C2 \wedge C3 \ldots$$

$(l_1 \vee l_2 \vee l_3)$

$$e.g. \ (\neg F \vee G \vee H) \wedge (\neg G) \wedge (K \vee L)$$

- **Disjunctive Normal Form (DNF)**: disjunction of **terms**

one or more

term

$$T1 \vee T2 \vee T3 \ldots$$

$(l_1 \wedge l_2 \wedge l_3)$

$$e.g. \ (\neg F \wedge G \wedge H) \vee (\neg G) \vee (K \wedge L)$$

- Exercise: $P \vee Q$ is in both CNF and DNF. Why?

So Now, one very important thing is that conjunction of one or more closes, then this function of one or more

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any wff can be converted

Normal Forms (II)

one or more

- **Conjunctive Normal Form (CNF)**: conjunction of **clauses**

clause

$C1 \wedge C2 \wedge C3 \ldots$

$e.g.\ (\neg F \vee G \vee H) \wedge (\neg G) \wedge (K \vee L)$

$(l_1 \vee l_2 \vee l_3)$

- **Disjunctive Normal Form (DNF)**: disjunction of **terms**

one or more

term

$T1 \vee T2 \vee T3 \ldots$

$e.g.\ (\neg F \wedge G \wedge H) \vee (\neg G) \vee (K \wedge L)$

$(l_1 \wedge l_2 \wedge l_3)$

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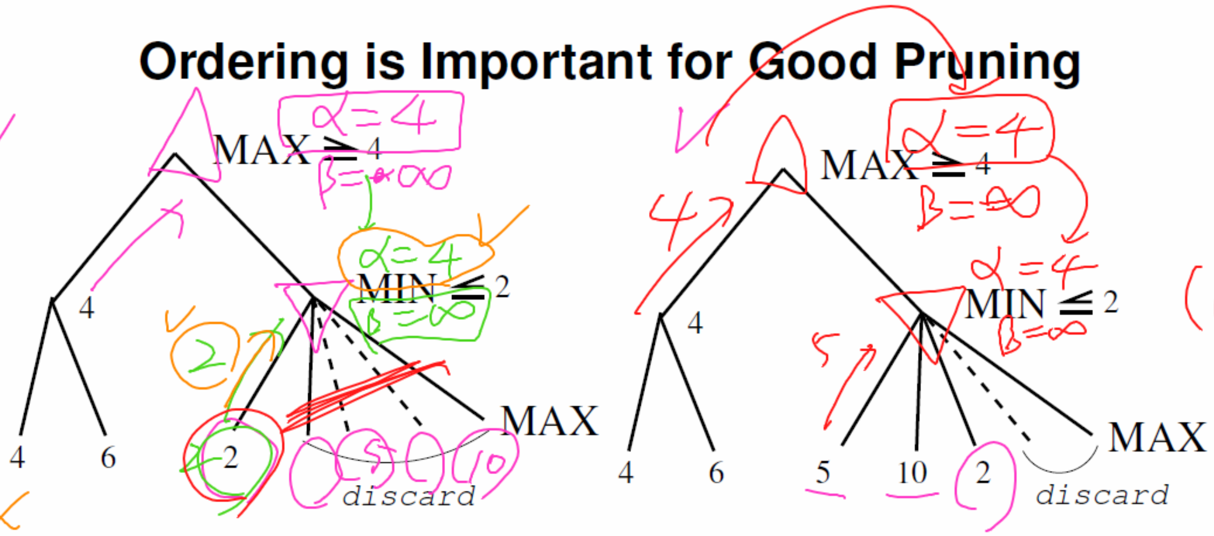
Ordering is Important for Good Pruning



(Q) Is 2 worse than α=4 as a max

① Yes

② No

wrong answer

4 = max(2, α=4) : ~~not worse.~~

is worse

(Q) Is 5 worse than α=4 as a max

- For MIN, sorting successor's utility in an **increasing** order is better (shown above; left).
- For MAX, sorting in **decreasing** order is better.

Normal Forms (II)

(Q)

(P ∨ Q)

- **Conjunctive Normal Form (CNF)**: conjunction of **clauses**

1 or more

$$C1 \wedge C2 \wedge C3 \ldots$$

① CNF

P∨Q clause

$$e.g.\ (\neg F \vee G \vee H) \wedge (\neg G) \wedge (K \vee L)$$

P

$C_1 = (P \vee Q)$

- **Disjunctive Normal Form (DNF)**: disjunction of **terms**

1 or more

② DNF

$$T1 \vee T2 \vee T3 \ldots$$

P ∨ Q

$$e.g.\ (\neg F \wedge G \wedge H) \vee (\neg G) \vee (K \wedge L)$$

③ both

term term

- Exercise: $P \vee Q$ is in both CNF and DNF. Why?

Normal Forms (II)

$P \wedge Q$

- **Conjunctive Normal Form (CNF)**: conjunction of **clauses**

① CNF: $(P) \wedge (Q)$

C_1 C_2

$$C1 \wedge C2 \wedge C3 \ldots$$

$$e.g.\ (\neg F \vee G \vee H) \wedge (\neg G) \wedge (K \vee L)$$

② DNF: $(P \wedge Q)$

T_1

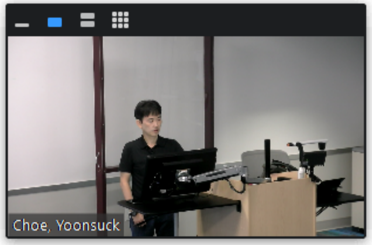
- **Disjunctive Normal Form (DNF)**: disjunction of **terms**

$$T1 \vee T2 \vee T3 \ldots$$

$$e.g.\ (\neg F \wedge G \wedge H) \vee (\neg G) \vee (K \wedge L)$$

③ Both?

- Exercise: $P \vee Q$ is in both CNF and DNF. Why?



Normal Forms (II)

$P \wedge Q$

① CNF: $\underbrace{P}_{C_1} \wedge \underbrace{Q}_{C_2}$

② DNF: $\underbrace{P \wedge Q}_{T_1}$

③ Both?

- **Conjunctive Normal Form (CNF)**: conjunction of **clauses**

  $C1 \wedge C2 \wedge C3 \ldots$

  $e.g.\ (\neg F \vee G \vee H) \wedge (\neg G) \wedge (K \vee L)$

- **Disjunctive Normal Form (DNF)**: disjunction of **terms**

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  $e.g.\ (\neg F \wedge G \wedge H) \vee (\neg G) \vee (K \wedge L)$

- Exercise: $P \vee Q$ is in both CNF and DNF. Why?

distrib.

$\underbrace{(P \wedge Q)}_{T_1} \vee \underbrace{R}_{T_2}$ DNF

$= \underbrace{(P \vee R)}_{C_1} \wedge \underbrace{(Q \vee R)}_{C_2} \rightarrow$ CNF

Normal Forms (II)

- **Conjunctive Normal Form (CNF)**: conjunction of **clauses**

  $C1 \wedge C2 \wedge C3 \ldots$

  $e.g.\ (\neg F \vee G \vee H) \wedge (\neg G) \wedge (K \vee L)$

- **Disjunctive Normal Form (DNF)**: disjunction of **terms**

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- Exercise: $P \vee Q$ is in both CNF and DNF. Why?

Good. So what most of the case will only deal with Cnf
because it's the most convenient

$(G \wedge H) \to F$ only one

Horn Clauses

Horn clauses

- clauses that contain ≤ 1 **positive** literal:
  $F \vee \neg G \vee \neg H$, $\quad \neg F \vee G$

$\neg G \vee \neg K$

H. C.
$F \vee \neg G$

$F \vee G \vee (\neg H)$
not H. C.

Horn Normal Form: conjunction of horn clauses

- for example, $(F \vee \neg G \vee \neg H) \wedge (\neg F \vee G)$
- it is the same as: $((G \wedge H) \to F) \wedge (F \to G)$
- Easier to do inference (computationally less intensive) than other normal forms.
- Restrictive, so not all formulas can be represented in horn normal form.

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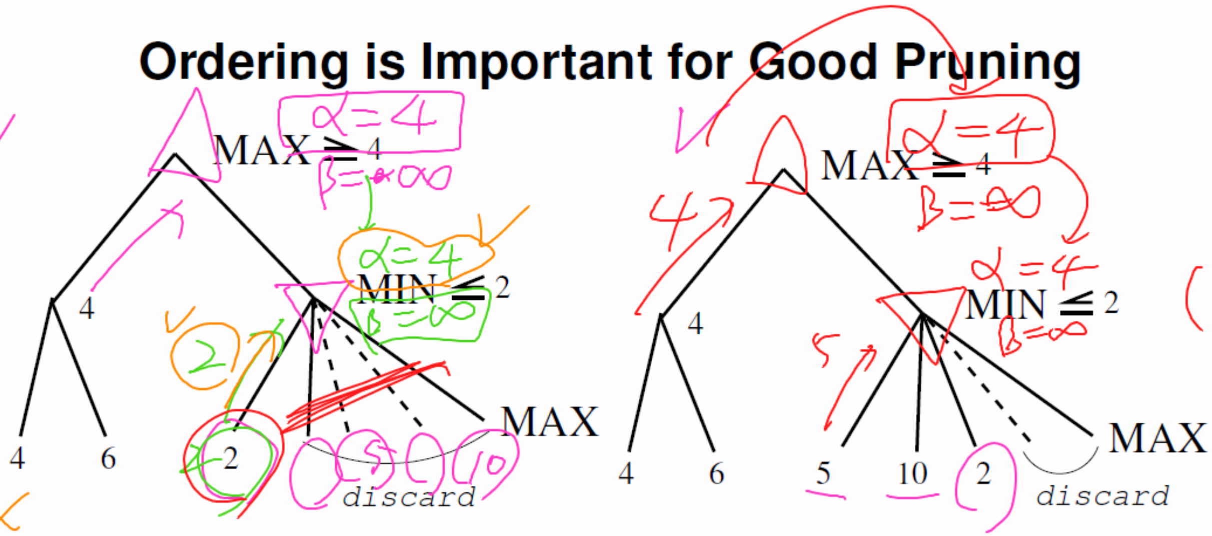
Who can see what you share here? Recording On

Ordering is Important for Good Pruning



(Q) Is 2 worse than $\alpha=4$ as a max

① Yes

② ~~No~~

wrong answer

$4 = \max(2, \alpha=4)$: ~~not worse.~~

is worse

- For MIN, sorting successor's utility in an **increasing** order is better (shown above; left).
- For MAX, sorting in **decreasing** order is better.

(Q) Is 5 worse than $\alpha=4$ as a max

① Yes

② No

$5 = \max(5, \alpha=4)$ better.

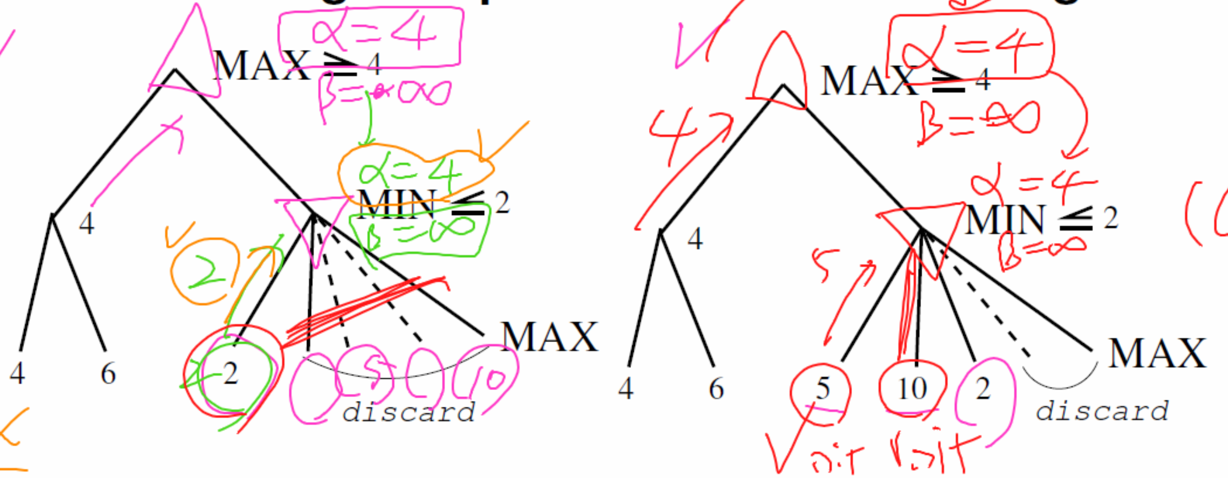
= not worse

So it's not was so that says no

You are screen sharing

Stop Share

Ordering is Important for Good Pruning



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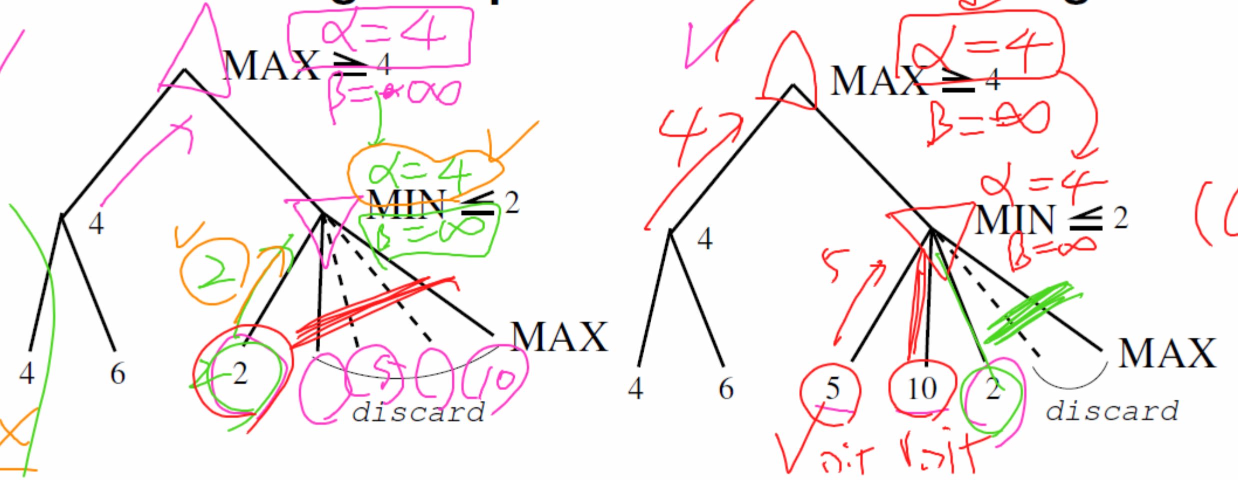

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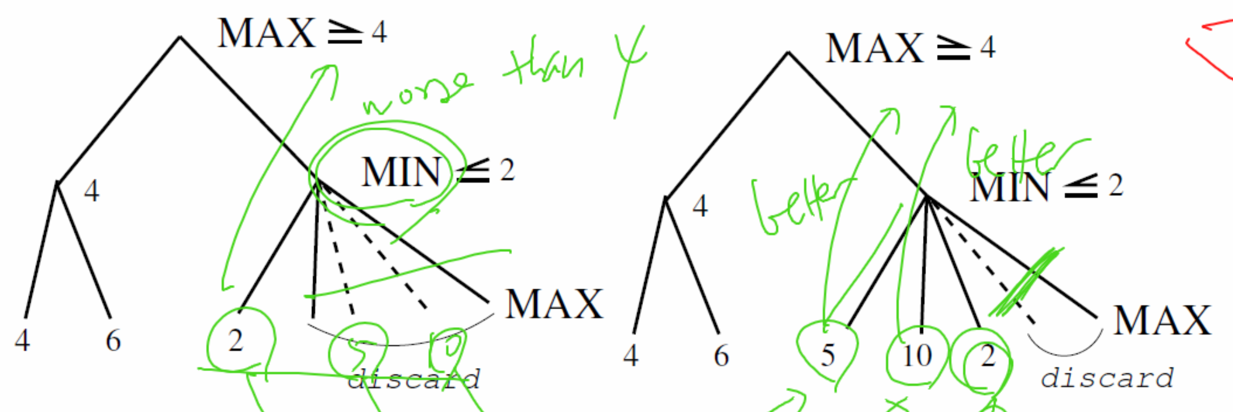

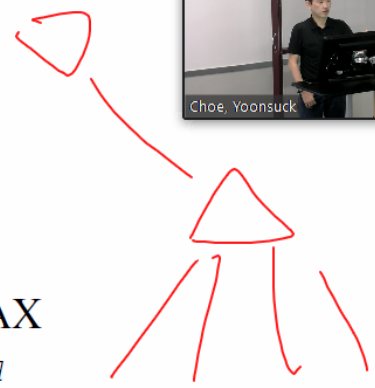

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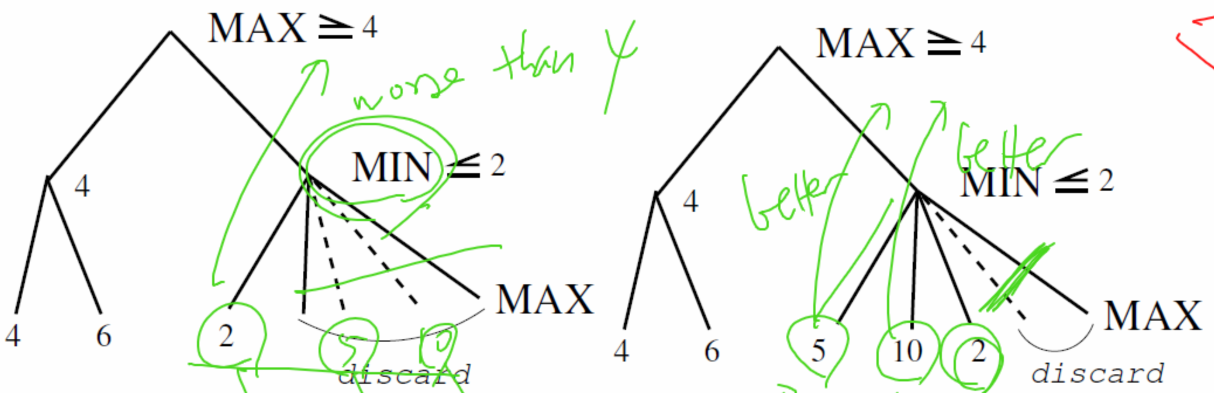
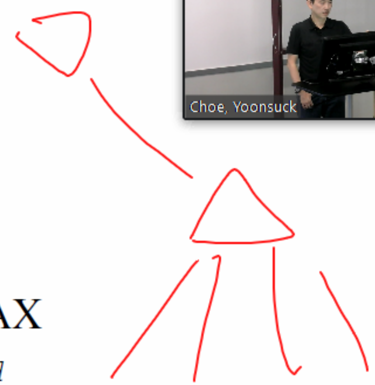
MIN : increasing order

MAX : decreasing order

Ordering is Important for Good Pruning





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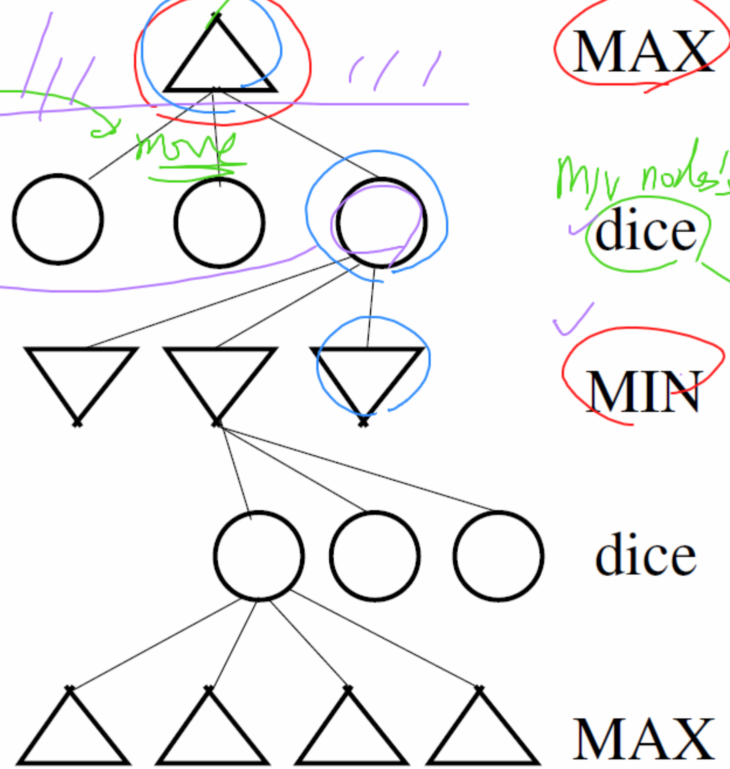
MIN: increasing order

MAX: decreasing order

(Q) Utility value

Game Tree With Chance Element



MIN node's dice roll!

players.

chance element, not controlled by the players

Rolling the dice, shuffling the deck of card and drawing, etc.

- chance element forms a new ply (e.g. dice, shown above)