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Exercise: Solution Using Resolution

1. $\neg M \vee I$
2. (a) $M \vee \neg I$, (b) $M \vee L$
3. (a) $\neg I \vee H$, (b) $\neg L \vee H$
4. $\neg H \vee G$
5. $\neg H$ (negated conclusion)

KB.

3a,5	$\neg I$	(6)
3b,5	$\neg L$	(7)
2b,7	M	(8)
1,6	$\neg M$	(9)
8,9	**False**	(10)

Newly generated one

Set (KB) expands

Right for the purpose of this serum. This becomes something like your know

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$\neg M$

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$\neg M$

$$\frac{M, \neg M}{\cancel{\text{False}}\ \{\ \}}$$

empty

$M \wedge \neg M = \text{False}$

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8,9	**False**	(10)

(1) easy to write computer code to automatically derive false.

(2) human readability is bad.

Very Clumbsy!

Is The Unicorn Horned?

Don't do this in the homework.

Given:

1. $M \to I$
2. $\neg M \to (\neg I \wedge L)$
3. $(I \vee L) \to H$
4. $H \to G$

Prove: H

1,	$\neg I \to \neg M$	(5)
2 and 5,	$\neg I \to \neg M \to (\neg I \wedge L)$	(6)
6,	$I \vee (\neg I \wedge L)$	(7)
7,	$(I \vee \neg I) \wedge (I \vee L) = \textbf{True} \wedge (I \vee L) = (I \vee L)$	(8)
8 and 3,	$\frac{(I \vee L), (I \vee L) \to H}{H}$ (modus ponens), so H	(9)

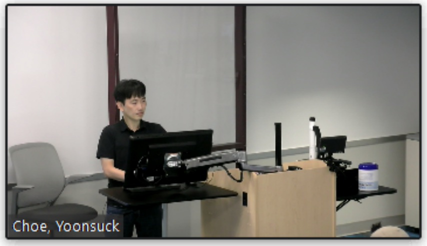
distribute

$\neg I \to (\neg I \wedge L)$

$(I \vee L)$

$(I \vee L) \to H$

H



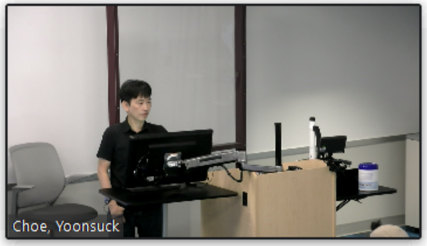
Limitation of Propositional Logic

Limited expressive power:

proposition

- P: All men are mortal
- Q: Socrates is a man
- R: Socrates is mortal

Can you prove $(P \wedge Q) \rightarrow R$ using propositional logic?



Predicate Calculus (First-Order Logic)

Propositional logic does not allow us to perform any reasoning based on the use of general rules, so its usefulness is limited. Predicate Calculus generalizes Propositional Calculus to allow the expression and use of **general rules**.

- objects
- relations
- properties
- functions

For all x ...

There exists x ...

Quantifiers

Objects - - Stuff

Terms in Predicate Calculus

A **Term** is:

- constant: $a, b, c, \ldots$
- variable: $x, y, z, \ldots$
- $f(t_1, \ldots, t_n)$, where f is a function symbol and $t_1, t_2, \ldots, t_n$ are terms.

Terms refer to objects in a domain.

term

Predicate Calculus Constructs

Terms

~ objects

- Variables: $x, y, z, \ldots$
- Constants: John, Mary, 3
- Functions: $f(x), g(y), h(z), father(\text{John}), \ldots$
  - maps **term(s)** to a **term**

term term

$x > 3$ function

Relation & l.of

Properties

- Predicates:
  $P(x, y), GREATER(x, 3), LOVE(father(\text{John}), \text{John})$
  - function whose value is **True** or **False**

term

Max

T/F

- Quantifiers: $\forall$ (for all), $\exists$ (there exists)

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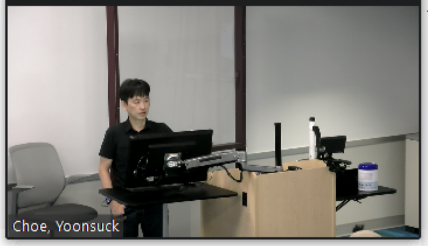
mid term exam materials.

very important.

but, no programming in hw2



So this being midtown, e

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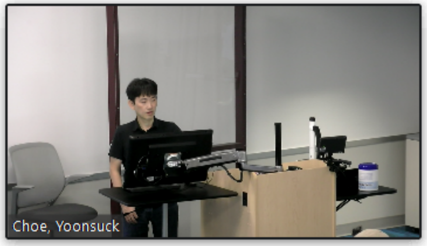
  Max

T/F

terms

- Quantifiers: $\forall$ (for all), $\exists$ (there exists)

Univ. Existential



Mortality Revisited

John Mary Max Socrates

Modus Ponens

Propositional logic

- P: All men are mortal
- Q: Socrates is a man
- R: Socrates is mortal

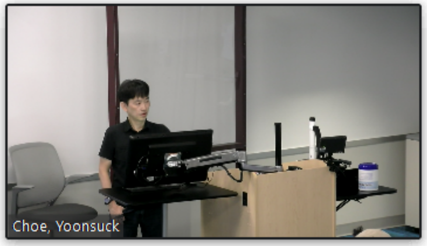
Man(Socrates) → Mortal(Socrates)
Man(Socrates)

Mortal(Socrates)

First-order logic

- P: All men are mortal $\forall x\ MAN(x) \rightarrow MORTAL(x)$
- Q: Socrates is a man $MAN(\mathrm{Socrates})$
- R: Socrates is mortal $MORTAL(\mathrm{Socrates})$

Man(John) → Mortal(John)
Man(Max) → Mortal(Max)
Man(Socrates) → Mortal(Socrates)



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1
2
3

Mortality Revisited

John Mary Max Socrates

Modus Ponens

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Man(Socrates)

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1 2 3

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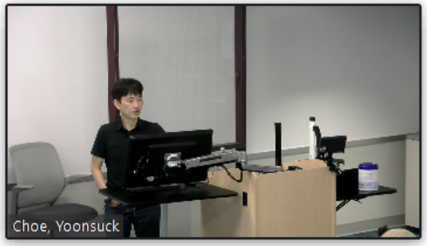

T/F

A Formal Definition

T/F

Well Formed Formula (or Sentence):

WFF	= Atomic-Formula | WFF Connective WFF | Quantifier Variable, WFF | ¬WFF | (WFF)
Atomic-Formula	= Predicate(Term, ...) | Term = Term
Term	= Function(Term, ...) | Constant | Variable
Connective	= $\rightarrow$ | $\wedge$ | $\vee$ | $\leftrightarrow$
Quantifier	= $\forall$ | $\exists$
Constant	= A | X_1 | John | ...
Variable	= a | x | s | ...
Predicate	= Before | HasColor | Raining | ...
Function	= Mother | LocationOf | ...

equivalence test.

T/F

rel. prop.

54

So these something that returns a true false value. Okay. So this is about relation or p

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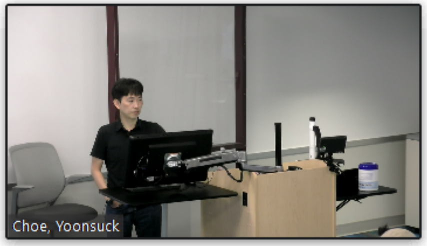

T/F

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Atomic-Formula	= Predicate(Term, ...) | Term = Term
Term	= Function(Term, ...) | Constant | Variable
Connective	= → | ∧ | ∨ | ↔
Quantifier	= ∀ | ∃
Constant	= A | X_1 | John | ...
Variable	= a | x | s | ...
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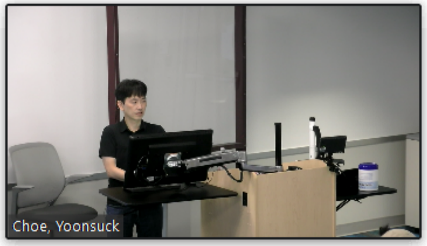
equivalence test.

T/F

rel. prp.



term T/F

Functions vs. Predicates

const

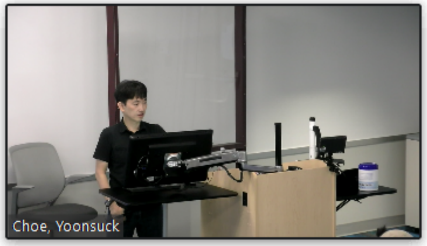
- Functions: returns a single object (term); relations
  - $FatherOf(GeorgeJr) = GeorgeSr,$
  - $DistanceBetween(MilkyWay, Andromeda) =$

  2-million light years, ...

- Predicates: returns a truth value; properties
  - $IsFather(GeorgeSr, GeorgeJr) = \mathbf{True}$
  - $HeavierThan(Earth, Sun) = \mathbf{False}$

Must disambiguate: $Brother(x, y)$ could be

- $AreBrothers(x, y)$:predicate, or
- $BrotherOf(x, y)$:function, i.e. a common brother of x and y.



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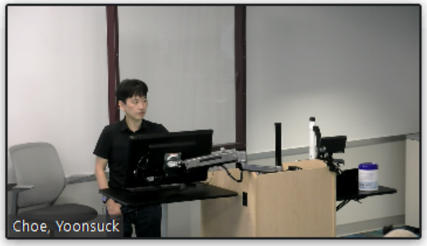

term

T/F

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→ term.

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mid term exam materials.

very important.

but, no programming in hw2

Yeah, any questions, righ

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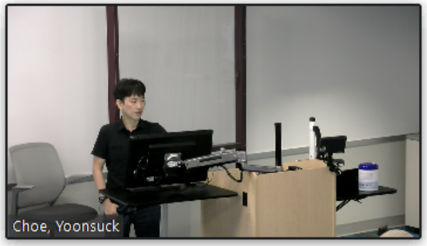


Quantifiers

$\forall$var wff

- Universal quantifier $\forall$:
  - Every $Skunk$ is $Stinky$: translates into $\forall x\, Skunk(x) \to Stinky(x)$
  - *note that the main connective is* $\to$

$\forall$

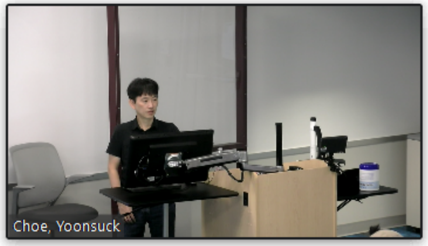
$\exists$var, wff

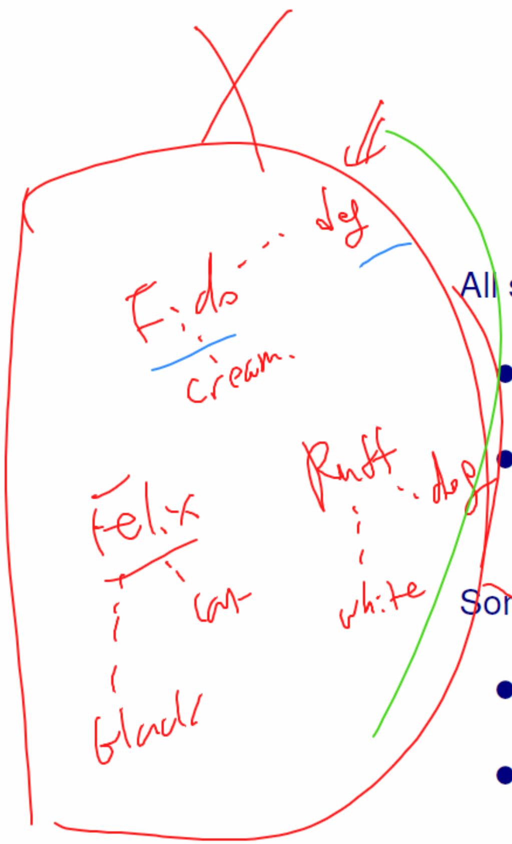
- Existential quantifier $\exists$:
  - There exists a Cat that is $White$: translates into $\exists x\, Cat(x) \wedge White(x)$
  - Same as: Some Cat is $White$
  - *note that the main connective is* $\wedge$

$\exists$

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

Common Mistakes With Quantifiers





All skunks are stinky:

- Correct: $\forall x\, Skunk(x) \rightarrow Stinky(x)$ ✓
- Wrong: $\forall x\, Skunk(x) \wedge Stinky(x)$ ✗
  - this means: everything is a skunk **and** it is stinky.

Some cats are white:

- Correct: $\exists x\, Cat(x) \wedge White(x)$ ✓
- Wrong: $\exists x\, Cat(x) \rightarrow White(x)$
  - this is true if there is something that is **not** a cat!

Fido Fido Fido

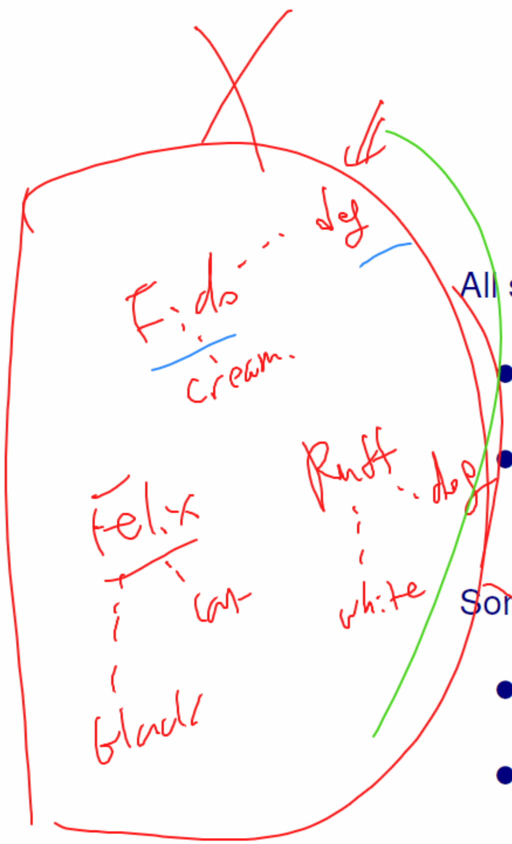
False

(Q) Cat(Fido)
① True
② False

(Q) Cat(Fido) → White(Fido)
False → White(Fido)
① True
② False ?

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

Common Mistakes With Quantifiers



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Fido Fido Fido

True

False

(Q) Cat(Fido)
① True
② False

(Q) Cat(Fido) → White(Fido)
False → White(Fido)
① True
② False ?

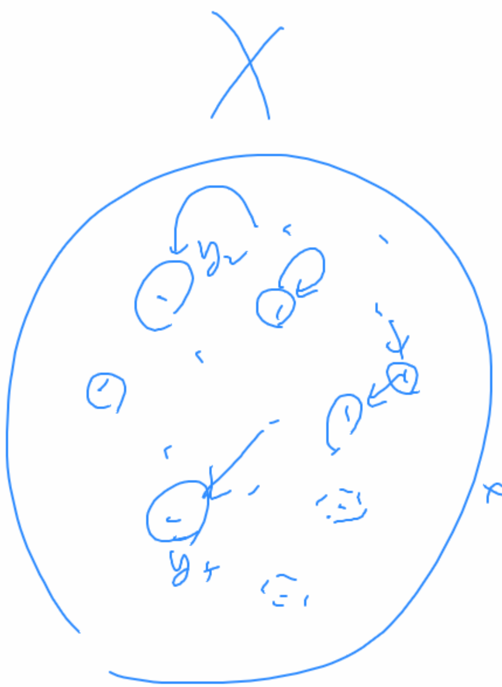
Properties of Quantifiers

x y ∴ y x

Same kind of quantifier.

- $\forall x\ \forall y = \forall y\ \forall x$
- $\exists x\ \exists y = \exists y\ \exists x$
- $\forall x\ \exists y \neq \exists y\ \forall x$
  $\exists x\ \forall y\ Loves(x, y)$ vs.
  $\forall y\ \exists x\ Loves(x, y)$
- quantifiers can be translated using each other:
  $\forall x\ Likes(x, Coffee)$ $\qquad \neg\exists x\ \neg Likes(x, Coffee)$
  $\exists x\ Likes(x, Broccoli)$ $\qquad \neg\forall x\ \neg Likes(x, Broccoli)$

Properties of Quantifiers



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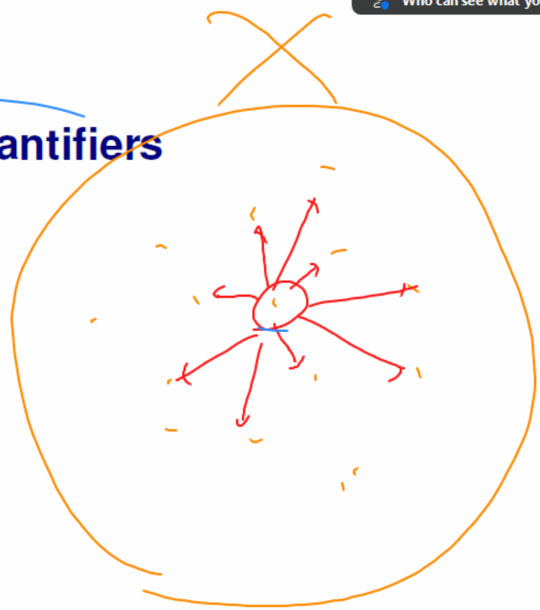

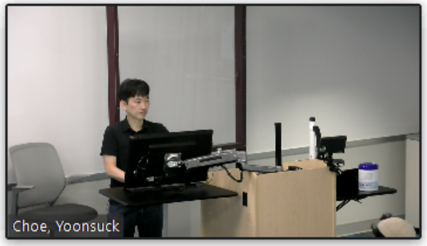

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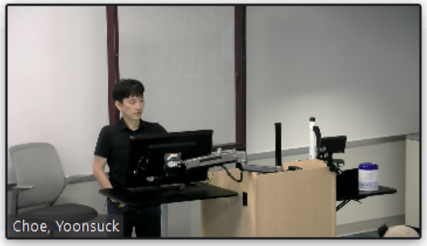
  dislike



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- quantifiers can be translated using each other:
  $\forall x\, Likes(x, Coffee) \qquad \neg\exists x\, \neg Likes(x, Coffee)$
  $\exists x\, Likes(x, Broccoli) \qquad \neg\forall x\, \neg Likes(x, Broccoli)$

why needed?

needed to convert to a standard form.

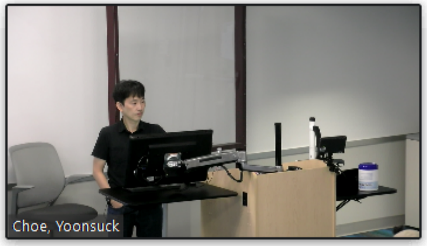
Semantics of Predicate Calculus

Formulas are true with respect to a **model** and and **interpretation**.

Models contains **objects** and **relations**:

- objects: constants
- relations and properties: predicates
- functional relations: functions

An atomic formula $Predicate(term_1, term_2, ..., term_n)$ is true iff the **objects** referred to by $term_1, term_2, ..., term_n$ are in the **relation** referred to by $Predicate$.



Semantics of Predicate Calculus

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Example: Howling Hounds

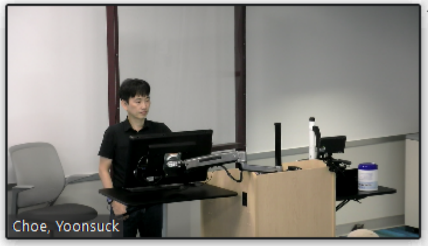
1. All hounds howl at night.
2. Anyone who has any cats will not have any mice.
3. Light sleepers do not have anything which howls at night.
4. John has either a cat or a hound.
5. **Prove**: If John is a light sleeper, then John does not have any mice.

4	9/13/2022	9/15/2022	Game playing	Ch 5	Ch 5			slide02-search-and-gameplaying.pdf	
5	9/20/2022	9/22/2022	Propositional logic and theorem proving	Ch 7	Ch 7	hw2 announced	hw1 due 9/25 11:59pm	slide03-logic	
6	9/27/2022	9/29/2022	First-order logic and theorem proving	Ch 8, 9	Ch 8, 9			slide03-logic	
7	10/4/2022	10/6/2022	First-order logic and theorem proving; Prolog ; Planning;	Ch 8, 9	Ch 8, 9			slide03-logic, slide04-planning	
8	**no class**	10/13/2022	**Midterm Exam (Thu, in class)**	Ch 10	Ch 11	hw3 announced	hw2 due: 10/11 Tuesday 11:59pm!!		10/10: midsemester grades due
9	10/18/2022	10/20/2022	Uncertainty and probabilistic reasoning	Ch 13, 14	Ch 12, 13				
10	10/25/2022	10/27/2022	Uncertainty and probabilistic reasoning	Ch 13, 14	Ch 12, 13				
11	11/1/2022	11/3/2022	Machine learning intro	Ch 18.1-18.2	Ch 19.1-19.2	hw4 announced	hw3 due		
12	11/8/2022	11/10/2022	Machine learning : decision tree	Ch 18.3	Ch 19.3				
13	11/15/2022	11/17/2022	Machine learning: neural	Ch 18.7	Ch 21.1-21.2				11/18: Q-

mid term exam materials.

very important

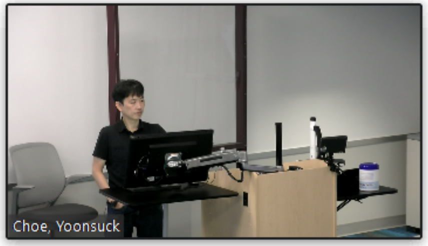
but, no programming in hw2



Example: Howling Hounds (cont'd)

1. $\forall x\,(HOUND(x) \rightarrow HOWL(x))$
2. $\forall x \forall y((HAVE(x,y) \wedge CAT(y)) \rightarrow$ $\neg\exists z(HAVE(x,z) \wedge MOUSE(z)))$
3. $\forall x\,(LS(x) \rightarrow \neg\exists y(HAVE(x,y) \wedge HOWL(y)))$
4. $\exists x\,(HAVE(John,x) \wedge (CAT(x) \vee HOUND(x)))$
5. **Prove:**

   $LS(John) \rightarrow \neg\exists x\,(HAVE(John,x) \wedge Mouse(x))$



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The last premise

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Example: Howling Hounds (cont'd)

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2. $\forall x \forall y((HAVE(x,y) \wedge CAT(y)) \rightarrow$
   $\neg\exists z(HAVE(x,z) \wedge MOUSE(z)))$

   $\forall x \forall y(\ldots \rightarrow \neg\exists z(\ldots))$
3. $\forall x\,(LS(x) \rightarrow \neg\exists y(HAVE(x,y) \wedge HOWL(y)))$
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5. **Prove:**
   $LS(John) \rightarrow \neg\exists x\,(HAVE(John,x) \wedge Mouse(x))$

$\forall x \forall y (\ldots \exists z (\ldots))$

$\forall x \forall y \exists z$

Canonical Forms of Predicate Calculus

1. Prenex Normal Form: arranged all quantifiers at the front of the formula : use De Morgan's rules (p. 193)
2. Convert the non-quantifier part (called the **matrix**) into Conjunctive Normal Form

CNF.

3. Skolemization: eliminate existential quantifiers by introducing **Skolem constants** or **Skolem functions**.

$\forall x \forall y \cancel{\exists z} (\ldots)$

Result:

$\forall x_1 \forall x_2 \ldots \forall x_n (\mathbf{CNF})$

$z \to f(x,y)$ $\quad z \to f(x,y)$

$\forall x \forall y (\cdots \exists z (\cdots))$

$\forall x \forall y \exists z$

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z z

Result:

$$\forall x_1 \forall x_2 \ldots \forall x_n (\mathbf{CNF})$$

$f(x,y)$ $f(x,y)$

all universally quantified. !

$(CNF \; x \; y \; z)$

assumed to be $\forall$

$(P \vee Q) \wedge \neg R \rightarrow T/F$

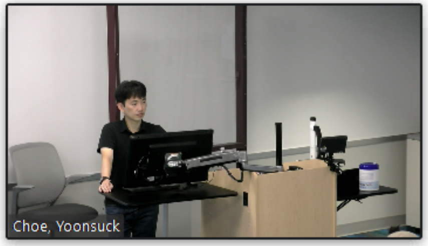
T F F

Interpretation in Predicate Calculus

An **interpretation** of a formula F in first-order logic consists of a nonempty domain D, and an assignment of values to each constant, function, and predicate occurring in F as follows:

1. **constant**:
   assign an element of D (e.g. an integer)
2. **function** with n arguments:
   assign a mapping from D^n to D
3. **predicate** with n arguments:
   assign a mapping from D^n to $\{\textbf{True}, \textbf{False}\}$

$D^n = \{(x_1, ..., x_n) | x_i \in D \text{ for } i = 1, ..., n\}$ Similar to assigning truth values in propositional logic.



Side Note: Bound vs. Free Variables

- **Scope** of a quantifier:
  the range (parentheses) over which the associated variable takes effect
- **Bound variable**:
  an occurrence of a variable in a formula is **bound** iff the occurrence is within the scope of a quantifier employing the variable.
- **Free variable**: Skip
  an occurrence of a variable in a formula is **free** iff the occurrence is not bound.

Bound: $\forall x \forall y P(x, y)$; Free: $\forall x\ P(x, y)$;

Both Free and Bound: $(\forall x\ P(x, y)) \wedge (\forall y\ Q(y))$

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Who can see what you share here? Recording On

proposition: M

The unicorn is immortal : I

Exercise

Given:

Premise

If the unicorn is mythical, then it is immortal($M \rightarrow I$), but if it is not mythical, then it is a mortal mammal ($\neg M \rightarrow (\neg I \wedge L)$). If the unicorn is either immortal or a mammal, then it is horned ($(I \vee L) \rightarrow H$). The unicorn is magical if it is horned ($H \rightarrow G$).

mammal

Horned

magical

Mythical : M

Mortal : $\neg I$

Mammal : L

Magical : G

Prove or disprove:

1. The unicorn is mythical (M).
2. The unicorn is magical (G).
3. The unicorn is horned (H). ⟵ Let's prove this.

Goal:

https://aimacode.github.io/aima-exercises/knowledge-logic-exercises/ex_2/

So this is the rule, and th

You are screen sharing Stop Share

Example: Interpretation and Evaluation

Given the **interpretation**:

- **Domain**: $D = \{Bob, Carol, Ted, Alice\}$

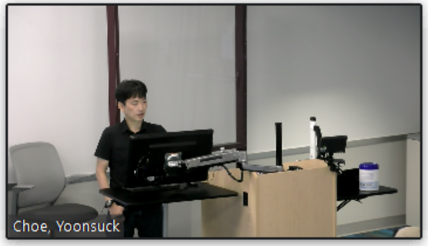
  $= T$ $= T$

- **Predicates**: $Woman(Carol), Woman(Alice)$
  $Man(Bob), Man(Ted)$
  $Loves(Bob, Carol), Loves(Ted, Alice), Loves(Carol, Ted)$

- **Functions**:
  $Brother(Bob) = Ted, Boss(Alice) = Carol$

Evaluate:

$$\forall x\, (Man(x) \rightarrow \exists y (Woman(y) \wedge Loves(x, y)))$$



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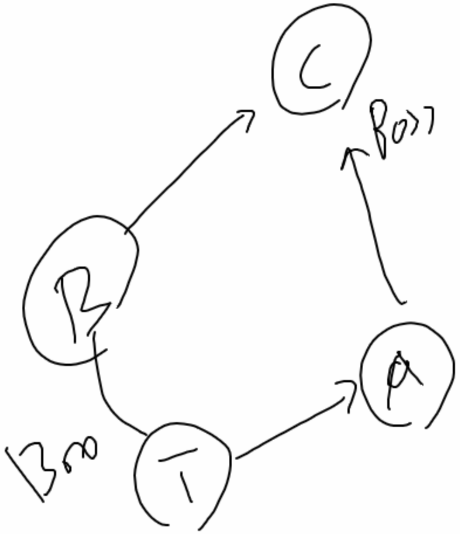

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T

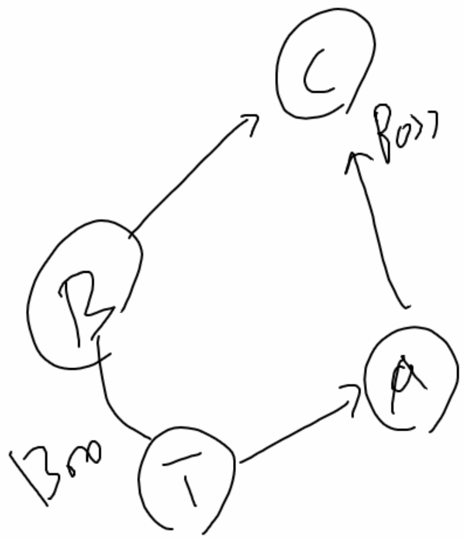
Bob   Man(Bob)

Carol

Ted

Alice

Example: Interpretation and Evaluation



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$$\forall x (Man(x) \to \exists y (Woman(y) \wedge Loves(x, y)))$$

T

Bob Man(Bob)

Carol False Man(Carol)

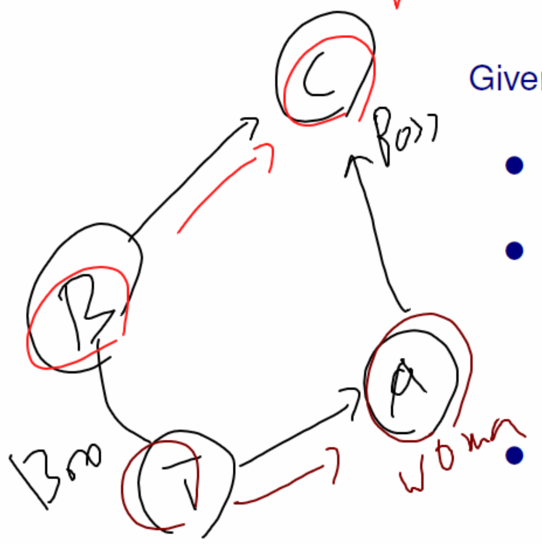
Ted Man(Ted) True T

Alice True

$\exists y\, Woman(y) \wedge Loves(Bob, y)$

$\exists y\, Woman(y) \wedge Loves(Ted, y)$

Example: Interpretation and Evaluation



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Evaluate:

$$\forall x(Man(x) \to \exists y(Woman(y) \wedge Loves(x, y)))$$

True

Bob	$Man(Bob)$ T
Carol	False $Man(Carol)$
Ted	$Man(Ted)$ T
Alice	True

Standard Forms of Predicate Calculus

Need this for automated theorem proving:

1. Prenex Normal Form: arranged all quantifiers at the front of the formula : use De Morgan's rules (p. 193)

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Result:

$$\forall x_1 \forall x_2 ... \forall x_n (\mathbf{CNF})$$

Quantifier Equivalences: Converting to Prenex Normal Form

Equivalence formulas ($Q = \forall$ or $\exists$):

- $(Qx\ F(x)) \vee G = Qx\ (F(x) \vee G)$
  $(Qx\ F(x)) \wedge G = Qx\ (F(x) \wedge G)$
- $\neg(\forall x\ F(x)) = \exists x\ (\neg F(x))$
  $\neg(\exists x\ F(x)) = \forall x\ (\neg F(x))$
- $(\forall x\ F(x)) \wedge (\forall x\ G(x)) = \forall x\ (F(x) \wedge G(x))$
  $(\exists x\ F(x)) \vee (\exists x\ G(x)) = \exists x\ (F(x) \vee G(x))$
- $(Q_1 x\ F(x)) \vee (Q_2 x\ H(x)) = Q_1 x\ Q_2 z\ (F(x) \vee H(z))$
  $(Q_1 x\ F(x)) \wedge (Q_2 x\ H(x)) = Q_1 x\ Q_2 z\ (F(x) \wedge H(z))$

$\forall x\ F(x) = \neg\exists x \neg F(x)$

$\neg$

$\boxed{\neg\forall x\ F(x)} = \neg\neg\exists x \neg F(x)$

$= \exists x \neg F(x)$

Quantifier Equivalences: Converting to Prenex Normal Form

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  $\neg(\exists x\ F(x)) = \forall x\ (\neg F(x))$
- $(\forall x\ F(x)) \wedge (\forall x\ G(x)) = \forall x\ (F(x) \wedge G(x))$
  $(\exists x\ F(x)) \vee (\exists x\ G(x)) = \exists x\ (F(x) \vee G(x))$
- $(Q_1 x\ F(x)) \vee (Q_2 x\ H(x)) = Q_1 x\ Q_2 z\ (F(x) \vee H(z))$
  $(Q_1 x\ F(x)) \wedge (Q_2 x\ H(x)) = Q_1 x\ Q_2 z\ (F(x) \wedge H(z))$

$\forall x\ F(x) = \neg \exists x \neg F(x)$

$\boxed{\neg \forall x\ F(x)} = \neg\neg \exists x \neg F(x)$

$\boxed{= \exists x \neg F(x)}$

Exercise: Solution Using Resolution

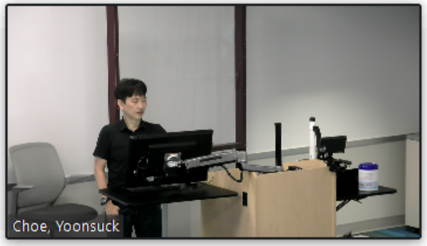
1. $\neg M \vee I$
2. (a) $M \vee \neg I$, (b) $M \vee L$
3. (a) $\neg I \vee H$, (b) $\neg L \vee H$
4. $\neg H \vee G$
5. $\neg H$ (negated conclusion)

3a,5	$\neg I$	(6)
3b,5	$\neg L$	(7)
2b,7	M	(8)
1,6	$\neg M$	(9)
8,9	**False**	(10)

Resolution
: two clauses
that has exactly
one pair of
opposing literals

(1) p

(2) $\neg p$



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2b,7	M	(8)
1,6	$\neg M$	(9)
8,9	**False**	(10)

(Q) Can you resolve?
(1) Yes
(2) No

$$\frac{\neg I \vee H, \quad \neg H}{\neg I}$$

Exercise: Solution Using Resolution

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2. (a) $M \vee \neg I$, (b) $M \vee L$
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3a,5	$\neg I$	(6)
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Newly generated ones

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KB.

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Newly generated ones

set (KB) expands