School of Rock

Mathematics Department

Specialist Mathematics

2022

Task 1

Question booklet

- Answer ***all*** questions
- Write your answers in this question booklet
- Allow approximately 120 minutes minutes
- Approved calculators may be used

Examination information

Materials

- Question booklet
- Formula sheet

Instructions

- Show appropriate working and steps of logic in the question booklets
- State all answers correct to three significant figures, unless otherwise instructed
- Use black or blue pen
- You may use a sharp dark pencil for diagrams

Total time: 120 minutes
Total marks: 12

Student Name: ..... Class: .....

Question 1 (6 marks)

(a) Write $-1 + i\sqrt{3}$ in $r \operatorname{cis} \theta$ form.

Solution: $2 \operatorname{cis}\left(\frac{2\pi}{3}\right)$

(1 mark)

(b) Consider the complex number $z_1 = x + iy$, where $x > 0$, $y > 0$, and $x > y$.

The complex number z_1, which lies in the first quadrand of the Argand diagram, is shown in Figure 1.

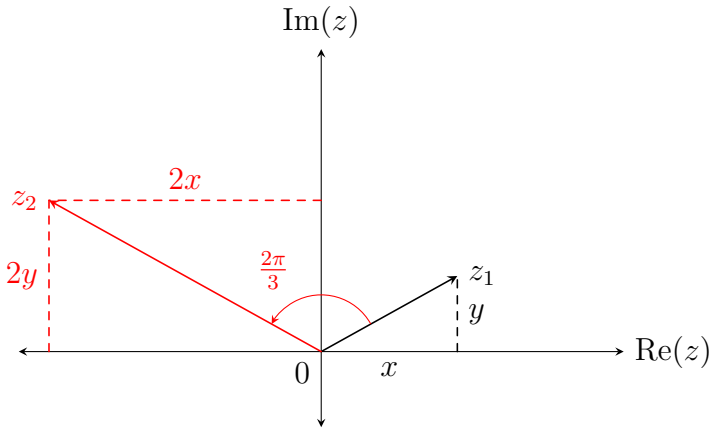

Figure 1

(i) Let $z_2 = (-1 + i\sqrt{3})z_1$.

Using part (a), show that $|z_2| = 2|z_1|$.

Solution:

$$\begin{aligned} |z_2| &= |-2 + i\sqrt{3}||z_1| \\ &= |2 \operatorname{cis}\left(\frac{2\pi}{3}\right)||z_1| \\ &= 2|z_1| \end{aligned}$$

(1 mark)

(ii) On the Argand diagram in Figure 1, draw z_2.

(2 marks)

(c) Use the triangle inequality to show that $|z_1 - z_2| < 3|z_1|$.

(2 marks)

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Question 2 (6 marks)

Figure 2 shows a diagram of an elliptical-shaped oil spill that is expanding in area on the ocean surface.

The area of an ellipse is $A = \pi ab$, where a and b are measurements on the axes of symmetry, as shown in Figure 2.

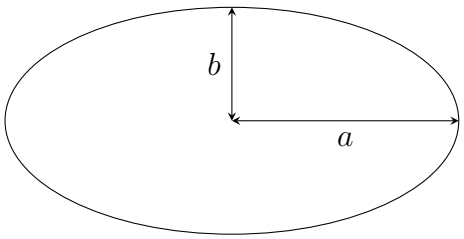

Figure 2

The rate of change of the area of the lliptical oil spill is given by $\frac{\mathrm{d}A}{\mathrm{d}t}$. It may be assumed that the elliptical shape is maintained as the oil spill expands.

(a) Show that $a\frac{\mathrm{d}b}{\mathrm{d}t} = \frac{1}{\pi}\frac{\mathrm{d}A}{\mathrm{d}t} - b\frac{\mathrm{d}a}{\mathrm{d}t}$.

(2 marks)

(b) Consider the instant when the area A is $12\ \mathrm{m}^2$.

(i) Show that $a = \frac{12}{\pi b}$

(1 mark)

(ii) The area of the oil spill is expanding at a rate of $2\ \mathrm{m}^2\,\mathrm{s}^{-1}$ at the instant when

$$A = 12\ \mathrm{m}^2,\ b = 2\ \mathrm{m},\ \text{and}\ \frac{\mathrm{d}a}{\mathrm{d}t} = 0.5\ \mathrm{m}\,\mathrm{s}^{-1}.$$

Find the ***exact*** value of $\dfrac{\mathrm{d}b}{\mathrm{d}t}$ at this instant.

(3 marks)