Algebraic Fractions*

Learning Outcome
MA-F1.1: Algebraic techniques
Students:

- manipulate complex algebraic expressions involving algebraic fractions

Important note

- Obtain a ..common.. ..denominator.. prior to adding.
- Only multiply up by as much as required.

Example 1
Fully simplify:

(a) $\dfrac{1}{x-4} - \dfrac{1}{x}$

(b) $\dfrac{2}{x^2-x} - \dfrac{5}{x^2-1}$

*Taken from Hubert Lam's *Algebraic Techniques, Functions and Graphs* booklet and altered slightly for demonstration purposes.

Exercises

1. Simplify fully. Also state any values of pronumeral(s) for which the simplification is not valid.

   (a) $\dfrac{4a+6}{4}$

   (b) $\dfrac{4a+6}{6a+9}$

   (c) $\dfrac{x^2}{5x-x^2}$

   (d) $\dfrac{x^2-2x-3}{x+1}$

   (e) $\dfrac{x+1}{x^2-2x-3}$

   (f) $\dfrac{1-x}{x-1}$

   (g) $\dfrac{4a-8}{12-6a}$

   (h) $\dfrac{a^2-4}{a+2}$

   (i) $\dfrac{a^2+a-6}{a^2-3a+2}$

   (j) $\dfrac{1-b^2}{b^3-1}$

   (k) $\dfrac{x^4-5x^2+4}{x^2-x-2}$

   (l) $\dfrac{x^3+2x^2-4x-8}{x^2+4x+4}$

2. Write as a single fraction in simplest form.

   (a) $\dfrac{a}{2}+\dfrac{2a}{3}$

   (b) $\dfrac{m-1}{2}-\dfrac{2m-3}{5}$

   (c) $x+\dfrac{1}{x}$

   (d) $a-\dfrac{a+b}{3}$

   (e) $\dfrac{4}{a}-\dfrac{3}{b}$

   (f) $\dfrac{1}{a^2}+\dfrac{2}{a}$

   (g) $\dfrac{a+1}{2a}-\dfrac{a-2}{3a}$

   (h) $\dfrac{1}{(x-3)^2}+\dfrac{1}{x-3}$

   (i) $\dfrac{3}{a^3b}-\dfrac{2}{a^2b^4}$

   (j) $\dfrac{2}{x-2}+\dfrac{3}{x+1}$

   (k) $\dfrac{2x}{x+3}-\dfrac{x-2}{x+1}$

   (l) $\dfrac{1}{x^2+x}+\dfrac{1}{x^2-x}$

   (m) $\dfrac{x}{(x-2)(x+2)}+\dfrac{x+1}{(x-2)(x-1)}$

   (n) $\dfrac{2}{y+2}-\dfrac{1}{y+3}+\dfrac{5}{y-1}$

   (o) $\dfrac{k}{k+1}+\dfrac{1}{k^2+3k+2}$

   (p) $\dfrac{1}{x^2-x-2}-\dfrac{1}{x^2+5x+4}-\dfrac{1}{x^2+2x-8}$

   (q) $\dfrac{a^2}{a^2+3a+2}-\dfrac{2a}{a+2}$

3. Simplify fully:

   (a) $\dfrac{4x^2}{3y^2}\times\dfrac{6y}{15x^4}$

   (b) $\dfrac{3a^3}{7b^2}\div\dfrac{9a}{7b}$

   (c) $\dfrac{2}{a}\div a$

   (d) $\dfrac{x+1}{2}\times\dfrac{4x}{(x+1)^2}$

   (e) $\dfrac{1}{x+2}\times\dfrac{4x+8}{3}$

   (f) $\dfrac{9}{x^3+64}\div\dfrac{6}{x+4}$

   (g) $\dfrac{x+y}{a-b}\times\dfrac{b-a}{y+x}$

   (h) $\dfrac{a^2-a}{6a^3+6a^2}\div\dfrac{a^2-1}{8a}$

   (i) $\dfrac{a^2-2a-3}{a^2+3a}\times\dfrac{3a^2+18a+27}{a^2-9}$

4. x is the smallest of three consecutive integers.

   (a) Find as a single fraction in simplest form, an expression for the sum of the reciprocals of these integers.

   (b) Three fractions are formed by dividing each of these integers by the integer following it. Find an expression, in simplest form, for the product of these fractions.

5. Find the reciprocal of $\dfrac{1}{a}+\dfrac{1}{b}$.

6. Simplify:

(a) $\dfrac{\frac{1}{m}+\frac{1}{n}}{m+n}$ (b) $\dfrac{1}{1-\frac{m}{n}}+\dfrac{1}{1-\frac{n}{m}}$ (c) $\dfrac{1-\frac{2}{t+1}}{t-\frac{2}{t+1}}$

7. (a) If $f(n)=n(n+1)(n+2)$, simplify $\dfrac{f(n)}{f(n+1)}$.

(b) If $f(n)=\dfrac{n^2}{n-1}$, prove that $f\left(\dfrac{t}{t-1}\right)=f(t)$.

Answers

1. (a) $\frac{2a+3}{2}$ (b) $\frac{2}{3}$ $\left[a\neq-\frac{3}{2}\right]$ (c) $\frac{x}{5-x}$ $[x\neq0,5]$ (d) $x-3$ $[x\neq1]$ (e) $\frac{1}{x-3}$ $[x\neq-1,3]$ (f) -1 $[x\neq1]$ (g) $-\frac{2}{3}$ $[a\neq2]$ (h) $a-2$ $[a\neq-2]$ (i) $\frac{a+3}{a-1}$ $[a\neq1,2]$ (j) $-\frac{b+1}{b^2+b+1}$ $[b\neq1]$ (k) $(x+2)(x-1)$ $[x\neq-1,2]$ (l) $x-2$ $[x\neq-2]$ **2.** (a) $\frac{a}{6}$ (b) $\frac{m+1}{10}$ (c) $\frac{x^2+1}{x}$ (d) $\frac{2a-b}{3}$ (e) $\frac{4b-3a}{ab}$ (f) $\frac{2a+1}{a^2}$ (g) $\frac{a+7}{6a}$ (h) $\frac{x-2}{(x-3)^2}$ (i) $\frac{3b^3-2a}{a^3b^4}$ (j) $\frac{5x-4}{(x-2)(x+1)}$ (k) $\frac{x^2+x+6}{(x+3)(x+1)}$ (l) $\frac{2}{x^2-1}$ (m) $\frac{2\left(x^2+x+1\right)}{(x-2)(x+1)(x+2)}$ (n) $\frac{2\left(3y^2+14y+13\right)}{(y+2)(y+3)(y-1)}$ (o) $\frac{k+1}{k+2}$ (p) $\frac{-x+5}{(x-2)(x+1)(x+4)}$ (q) $-\frac{a}{a+1}$ **3.** (a) $\frac{8}{15x^2y}$ (b) $\frac{a^2}{3b}$ (c) $\frac{2}{a^2}$ (d) $\frac{2x}{x+1}$ (e) $\frac{4}{3}$ (f) $\frac{3}{2(x^2-4x+16)}$ (g) -1 (h) $\frac{4}{3(a+1)^2}$ (i) $\frac{3(a+1)}{a}$ **4.** (a) $\frac{3x^2+6x+2}{x(x+1)(x+2)}$ (b) $\frac{x}{x+3}$ **5.** $\frac{ab}{a+b}$ (NOT $a+b$!!!) **6.** (a) $\frac{1}{mn}$ (b) 1 (c) $\frac{1}{t+2}$ **7.** (a) $\frac{n}{n+3}$

Further exercises
Ex 1D* Q2-12 last column

*Legacy textbooks