SYNTHESIS MODULE #4

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1. Due Date

A hard-copy of this assignment is due before 6pm on February 20th, 2020 in the ESE 448 homework bin or in class. An electronic copy of your work and matlab code is due in Canvas at the same time. Late-work will be graded but receives no credit.

2. Academic Integrity

This is an individual assignment. I expect everyone to turn in their own Matlab code, their own Simulink models, and their own writing.

If you are struggling with this assignment, please reach out to me. Post any and all questions on canvas, and I will answer them as soon as possible.

You are bright and brilliant and more than capable of mastering this material. I absolutely believe in each you. I want you to be the best engineers.

3. Assignment

This assignment further develops the UAV's nonlinear and linear dynamics and adds signal processing for our 1-g hover feedback control system.

By now, you have developed a linear system for the UAV. Each of you selected controls and states that describe your aircraft's linear dynamics. These states describe a nonlinear system

$$\dot{x} = f(x, u),$$

linearized around the reference state and control, x_0 and u_0, which yields a significantly simpler representation of the dynamics:

$$\delta\dot{x} = A\delta x + B\delta u.$$

There are four virtual inputs δT, δE, δA, and δR of the linear system that drive the aircraft's states.

In this assignment, we will leverage the work from the previous assignment and develop an even simpler system. We will decouple the full, 12 state linear system into 4 linear systems. One system should correspond to one of these virtual inputs.

Apply a similarity transformation to your full linear system: I want you to practice a simple change of coordinates.

(1) Change the state vector to have the following ordering of states:

$$x = \begin{bmatrix} Z_{NED} \\ w \\ \theta \\ q \\ \phi \\ p \\ \psi \\ r \\ X_{NED} \\ u \\ Y_{NED} \\ v \end{bmatrix} \tag{3.1}$$

(2) Write out the transformation matrix R that accomplishes this and the resulting A, B,C,D matrices

Change your full linear system's input to use the virtual control inputs: Assuming that you may have originally defined your control input v for the nonlinear system as the motor speeds, and you want to transform to u as the virtual inputs, i.e.,

$$v = \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \end{bmatrix}, \quad u = \begin{bmatrix} T \\ E \\ A \\ R \end{bmatrix}.$$

You can easily switch between the two linear representations by using the chain rule. This is because the motor outputs are a linear function of the the virtual inputs, i.e.,

$$v = Mu + u_0, \tag{3.2}$$

where M is your motor mixing matrix and u_0 was a bias vector that sets the motors to their equilibrium value.

To apply the chain rule, take a partial derivative of (3.2) with respect to the motor inputs,

$$\frac{\partial v}{\partial u} = M.$$

If you defined around reference values x_0, v_0 with your B defined as

$$\bar{B} := \frac{\partial}{\partial v}\left(f(x_0, v_0)\right)$$

Then by the chain-rule, your new B matrix corresponding to the virtual inputs is

$$B := \frac{\partial f}{\partial u} = \left(\frac{\partial f}{\partial v}\right)\left(\frac{\partial v}{\partial u}\right) = \bar{B}M.$$

This yields a linear system around the equilibrium:

$$\delta\dot{x} = A\delta x + B\delta u.$$

It is essential that you change your representation so that the linear control is

$$\delta u = \begin{bmatrix} \delta T \\ \delta E \\ \delta A \\ \delta R \end{bmatrix}.$$

(1) Report your A, B, C, and D matrices after this transformation.

Decouple the remaining dynamics into 4 linear systems: The states of the UAV are:

$$\delta u, \delta v, \delta w, \delta p, \delta q, \delta r, \delta\theta, \delta\phi, \delta\psi, \delta X_{ned}, \delta Y_{ned}, \delta Z_{ned}.$$

There are four outputs:

$$\delta h, \delta a_x, \delta a_y, \delta a_z.$$

There are four inputs:

$$\delta T, \delta E, \delta A, \delta R$$

(1) Decouple the linear, 12-state, 4-output, 4-input, multi-input, multi-output (MIMO) linear system into 4 single-input, multi-output (SIMO) linear systems. Each virtual input only influences a subset of the states and outputs of the linear system; for each of the four linear systems only include this subset that corresponds to one of the virtual inputs. For convenience, output all states in the linear system.
(2) Present the A, B, C, D, states, inputs, and outputs matrix for these four linear systems.

For example, the linear SIMO system corresponding to throttle δT might be:

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} \delta Z_{ned} \\ \delta w \end{bmatrix} = A \begin{bmatrix} \delta Z_{ned} \\ \delta w \end{bmatrix} + B\delta T,$$

$$\begin{bmatrix} \delta Z_{ned} \\ \delta w \\ \delta h \\ \delta a_z \end{bmatrix} = C \begin{bmatrix} \delta Z_{ned} \\ \delta w \end{bmatrix} + D\delta T.$$

Analyze the four open-loop control systems: For each of these four linear systems, do the following (use Matlab):

(1) Construct a phase-portrait using only two-states. If there are more than two states in the linear system, just choose two states that produce the most interesting plots.
(2) Determine all of the eigenvalues and eigenvectors of the A matrix.
(3) Determine the stability of the open-loop system
(4) Determine the state-transition matrix and the mode functions
(5) Determine which states are controllable
(6) Determine which states are observable. Please add realism to this question by considering the outputs available on your UAV.
(7) Add the decoupled version of the linear dynamics to your simulation of the UAV and create plots that demonstrate your decoupling assumptions are valid.

4. What to turn in

Please submit files individually into Canvas (no-more Zips please).

(1) All matlab files pertaining to this work
(2) All Simulink models pertaining to this work
(3) A report that includes all numbered items from the assignment section.

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