Assignment 8: System ID

Jockabeth Ponce

Group members: Ethan, Chris, Clay

ESE 448

Systems Lab

Introduction

The data below was collected by TA Max Barnett and Professor Bhan:

- A symmetric bang-bang command, one virtual input at a time (Throttle, Aileron, Elevator, and Rudder).
- A logarithmic chirp ranging from 0 Hz to 100 Hz was injected over a period of 10 seconds, and it was applied to all four motors simultaneously.

We constructed four parametric representations of our UAV's linear dynamics using the bang-bang data, then identified transfer functions for each individual motor using the chirp data.

Part 1: Parametric Representations

We assumed that the parameters in the model are as follows:

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & b \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ c \end{bmatrix} u$$

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The parameters for the UAV model were grouped as triplets to fit with the model above.

Triplet #	x1	x2	u
1	Zned	Vertical speed (w)	Throttle
2	Pitch angle (theta)	Pitch rate (q)	Elevator

3	Roll angle (phi)	Roll rate (p)	Aileron
4	Yaw angle (psi)	Yaw rate (r)	Rudder

We identified the b and c values for each triplet model and represented these dynamics discretely.

$$x_{i+1} = \bar{A}x_i + \bar{B}u_i,$$
$$\bar{A} = e^{A\Delta t}, \quad \bar{B} = \int_0^{\Delta t} e^{A(\Delta t - \sigma)} B \mathrm{d}\sigma$$

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In the bang bang data, we were given the motor speeds with the sign positive or negative depending on the direction of the motor. The motor speeds for motors 2 and 4 are negative since they rotate counterclockwise.

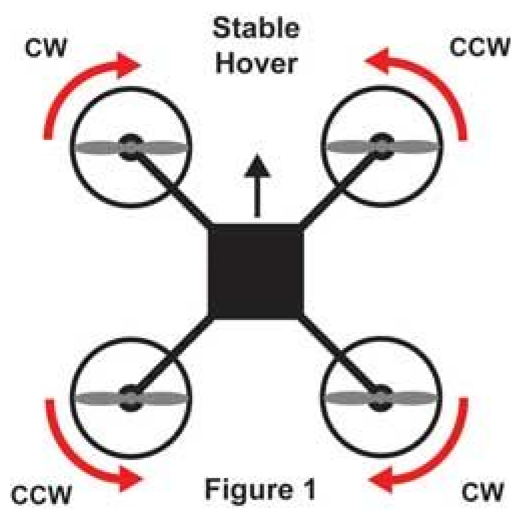


Figure 1

Drone HD Wallpaper (http://www.regimage.org/drone-reverse-motor-direction/)

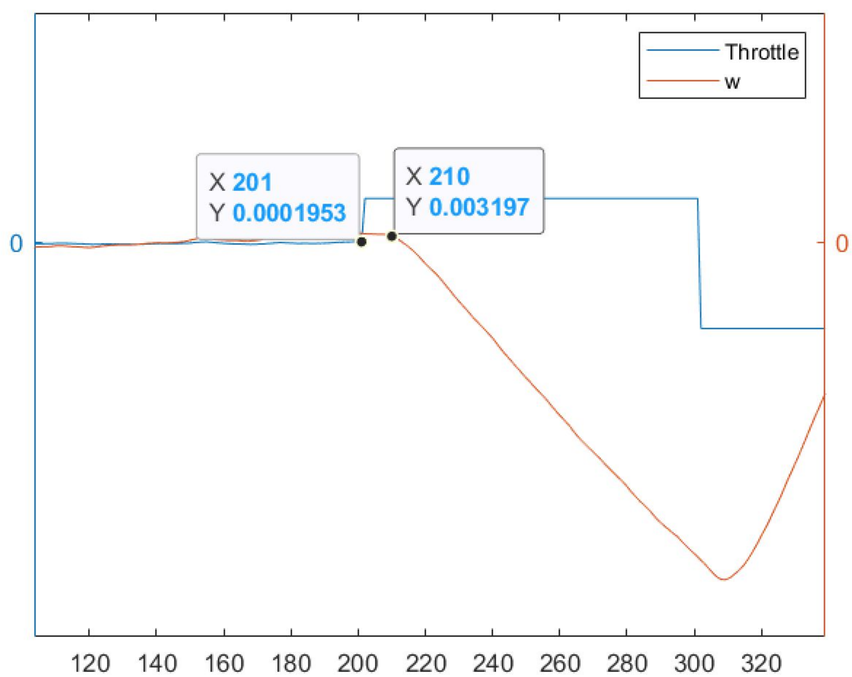
This was important to calculate the vectors T, E, A, and R since our motor mix matrix changes.

Our original motor matrix was:

```
MotorMix = [...%T  E  A  R
               1  1  1 -1 ;... %n1  T+ E+ A+ R-
               1  1 -1  1 ;... %n2  T+ E+ A- R+
               1 -1 -1 -1 ;... %n3  T+ E- A- R-
               1 -1  1  1 ;... %n4  T+ E- A+ R+
]';
```

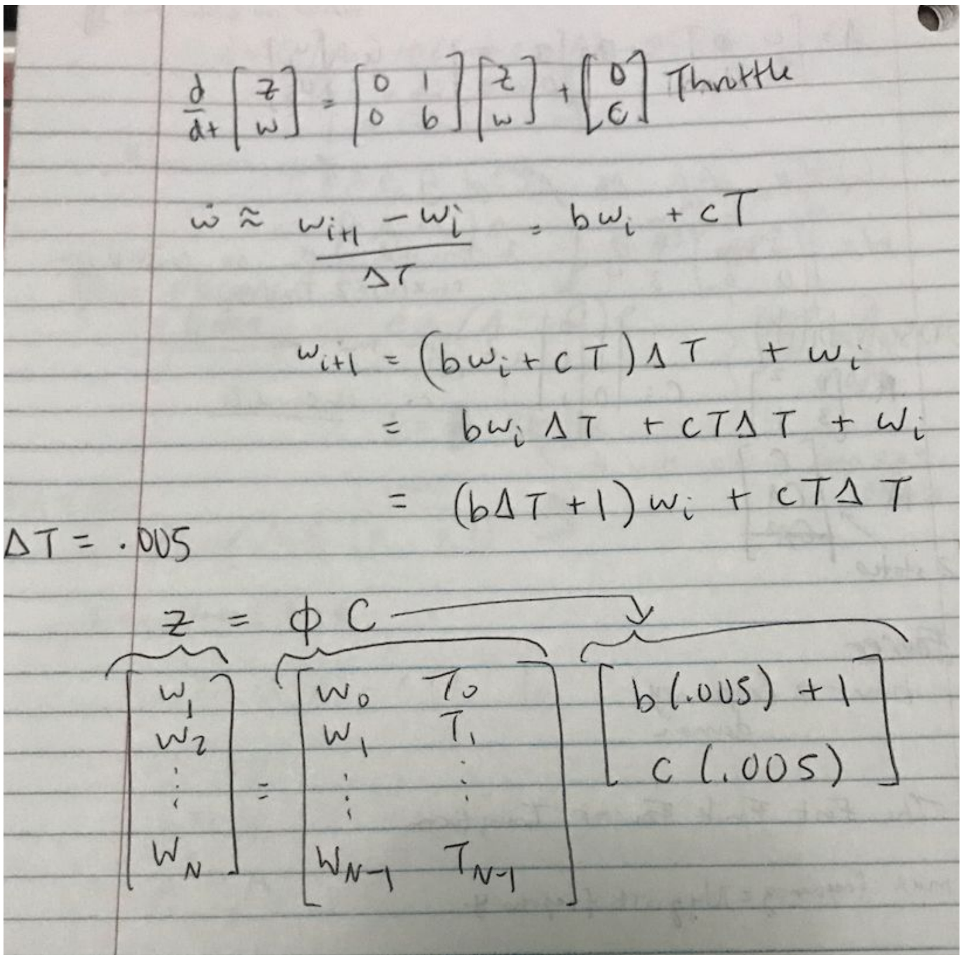
The 2nd and 4th columns were multiplied by -1 to get the new motor mix matrix.

Using the bang bang w data and the T we just solved for, they were both plotted below:



Because there is a delay between our Throttle and w, our system identification won't be accurate. There is a delay of about 9 data points which is about 9*.005= .045 seconds= 45 milliseconds.

We used the method in lecture, C=PHI\Z, to backwards solve for vector C which contains our b and c values using some arithmetic.



$$\frac{d}{dt}\begin{bmatrix} z \\ w \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & b \end{bmatrix}\begin{bmatrix} z \\ w \end{bmatrix} + \begin{bmatrix} 0 \\ c \end{bmatrix} \text{Throttle}$$

$$\dot{w} \approx \frac{w_{i+1} - w_i}{\Delta T} = b w_i + cT$$

$$\begin{aligned} w_{i+1} &= (b w_i + cT)\Delta T + w_i \\ &= b w_i \Delta T + cT\Delta T + w_i \\ &= (b\Delta T + 1) w_i + cT\Delta T \end{aligned}$$

$\Delta T = .005$

$$\underbrace{\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{bmatrix}}_{Z} = \underbrace{\begin{bmatrix} w_0 & T_0 \\ w_1 & T_1 \\ \vdots & \vdots \\ w_{N-1} & T_{N-1} \end{bmatrix}}_{\phi} \underbrace{\begin{bmatrix} b(.005)+1 \\ c(.005) \end{bmatrix}}_{C}$$

Handwritten work: Jockabeth Ponce

Once solving for vector C, to get b we used the first element and subtracted 1 then divided by dT=.005 and to get c we used the second element and divided by dT=.005. (b is the damping term for x2 (w,q,p,r respectively) and c is the coefficient for T,E,A, and R respectively).

The b and c values calculated for each triplet is shown below:

Throttle	b= .4972 c= -.0411
Elevator	b= -.0981 c= 4.8964
Aileron	b= 4.8073 c= 65.7956
Rudder	b= -.45 c= 2.7482

Using the b and c values and the equation [w_i; T_i] = [b c]^-1*az, we were able to solve for the actual motor speeds by solving for T and multiplying by the motor mix matrix.

We were given logarithmic chip data at 1Hz, 5Hz, 10Hz, 20Hz, and 50Hz. We chose to use the 50Hz data since the signal to noise ratio will be the largest at higher frequencies. The commanded motor speeds were extracted from the chirp data from lines to 2000 to 4000 and we added the delay of 9 lines (45 ms).

Using the vector of the actual and commanded motor speeds, we used some matlab tools to get the transfer function.

Part 2: Transfer Functions

Using non-parametric identification, we identified transfer functions that represent the parrot's actuator, each individual motor.

We identified a transfer function in the form below, where $n_{a,i}$ is the achieved angular velocity of the ith motor and $n_{c,i}$ is the commanded angular velocity out of the control system.

$$\frac{n_{a,i}}{n_{c,i}} = H_i(s)$$

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First we used the Matlab function iddata, which creates an iddaata object that contains the output, input, and sample time. Here the output y is actual motor speeds n_a, and the input u is commanded motor speeds n_c, and Ts is .005 seconds which is the sample time.

iddata

Input-output data and its properties for system identification in the time or frequency domain

```
>>data = iddata(y,u,Ts)
```

https://www.mathworks.com/help/ident/ref/iddata.html

Next, we used the Matlab function tfest, which takes in the data object created in the previous step and the number of poles, and estimates a transfer function.

tfest

Transfer function estimation

```
sys = tfest(data,np)
```

https://www.mathworks.com/help/ident/ref/tfest.html

Our matlab code used was:

obj=iddata(n_a, n_c, deltaT);

sys=tfest(obj,1);

And the estimated transfer function was:

$$\frac{27.89}{s + 23.13}$$

This transfer function does not take into account the delay, which is why we must add a delay in the numerator. A delay in the time domain is e^{-sT} in the Laplace domain (https://lpsa.swarthmore.edu/LaplaceXform/FwdLaplace/LaplaceProps.html). Our delay was T= 9*.005= .045 seconds= 45 milliseconds so we can add this term in the numerator to get a more accurate transfer function.

New transfer function:

$$\frac{27.89\, e^{-0.045\, s}}{s + 23.13}$$

Conclusion

This assignment was super helpful in learning system identification techniques since real data was used. If we had been given perfect data then this assignment would have been easier, but not realistic. We had to think like engineers and think about the factors that may be affecting our data. Originally, we had not thought about the time delay between w and throttle so the b and c values were leading us to get transfer functions that did not make sense.

We also had assumed that we would be using the same motor mixing matrix, but then we realized that the motor speed data had negative speeds for motors 2 and 4 so we had to correct this data as well. Using the corrected data, we were able to get a more accurate function that represented our actuator.