ASSIGNMENT #3

DR. SANKALP K. BHAN

Due Date: 02/11/20 @ 6 pm. Electronic submissions of all models, code, and writing should be included in Canvas before the deadline. A written copy of the report is due in class or in the course box before the deadline. Late work is graded but receives no credit. **Due to its importance to the rest of the course, assignment #3 is worth two assignments**.

1. Academic Integrity

This is an individual assignment. I expect everyone to turn in their own Matlab code, their own Simulink models, and their own writing.

If you are struggling with this assignment, please reach out to me or the T.A., Max (mtbbarnett@gmail.com); please do not make the mistake of cheating. We are here to teach you. We want to see you succeed. We want you to learn. We want you to become incredible engineers.

2. Assignment

Using Simulink, develop a nonlinear and linear simulation of the Parrot Mambo starting in a 1-g hover. You should compare the simulink.

This reuses the work from your previous assignment. Using your trim-to-1g thrust, develop A, B, C, D matrices, and integrating the linear and nonlinear equations.

The simulations should have four scenarios:

(1) A 6s duration simulation of all states and outputs during a throttle step input of 4s duration that engages at 1s and disengages at 5s.
(2) A 6s simulation of all states and outputs during an elevator step input of 4s duration that engages at 1s and disengages at 5s.
(3) A 6s simulation of all states and outputs during an aileron step input of 4s duration that engages at 1s and disengages at 5s.
(4) A 6s simulation of all states and outputs during an rudder step input of 4s duration that engages at 1s and disengages at 5s.

Choose the size of these inputs so that Euler angles do not rotate more than 90 degrees in the simulation. Plot the simulation results from the nonlinear simulation against the linear simulations and make observations.

3. What to turn in

Please turn in the simulink model with a nonlinear and linear version of the dynamics and all m files that simulate the nonlinear and linear version of the dynamics. Also please write and submit a document that includes the following sections:

(1) An introduction that describes the purpose of the assignment in your words.

(2) An explanation of assumptions embedded into your simulation of the UAV, (e.g. earth is flat.)
(3) Documentation of the linearization of the equations, the state matrices (A,B,C,D), and definitions of the state, control, and output vectors
(4) A results and discussion section that presents graphs and discusses the trajectories. Be sure to explain the following:
  (a) Does the linear system adequately capture the changes in the nonlinear system?
  (b) Which states in the nonlinear system and linear system are stable?
  (c) Are your results correct? How well do you think these simulations match reality?
  (d) What states are coupled? Do these grouping match your intuition?
(5) Write a conclusions section that summarizes your findings.

The relationship between motor speeds, n_1, n_2, n_3, n_4 and thrust is given by:

$$T_i = C_T \rho n_i^2 D^4, \quad i = 1 \ldots 4.$$

The relationship between motor speeds, n_1, n_2, n_3, n_4 and aileron is given by:

$$L_{T,i} = bC_l \rho n_i^2 D^4, \quad i = 1 \ldots 4,$$

where C_l is the coefficient of rolling moment derived from your previous assignment. The relationship between motor speeds, n_1, n_2, n_3, n_4 and elevator is given by:

$$M_{T,i} = bC_m \rho n_i^2 D^4, \quad i = 1 \ldots 4,$$

where C_l is the coefficient of pitching moment derived from your previous assignment. The relationship between motor speeds, n_1, n_2, n_3, n_4 and rudder is given by:

$$N_{T,i} = bC_n \rho n_i^2 D^4, \quad i = 1 \ldots 4,$$

where C_n is the coefficient of yawing moment derived from your previous assignment.

4. Simulation Equations and Parameters

4.1. **Parameters.** Use the following parameters:
$g = 9.81$; % mps2
$m = 0.068$; % kg
$J_{xx} = 0.0000582857$; % kgm2
$J_{yy} = 0.0000716914$; % kgm2
$J_{zz} = 0.0001$; % kgm2
$h = b = 0.047625$; % distance from CoM to center of rotor, y-axis in meters

4.2. **Notation.** The notation discussed in class was:

- p,q,r: body-axis components of angular velocity ω w.r.t. to x,y,z axes
- u,v,w: body-axis components of velocity V w.r.t to flat-earth x,y,z axes
- X_A,Y_A,Z_A: body-axis components of aerodynamic forces $\vec{F}_A$
- L_A,M_A,N_A: body-axis components of aerodynamic moments $\vec{M}_A$
- X_T,Y_T,Z_T: body-axis components of propulsive forces $\vec{F}_T$
- L_T,M_T,N_T: body-axis components of propulsive moments $\vec{M}_T$
- L_{gyro},M_{gyro},N_{gyro}: body-axis components of gyroscopic moments $\vec{M}_{gyro}$
- X_{NED}, Y_{NED}, Z_{NED}: inertial axis components of position
- X_g,Y_g,Z_g: body axis components of gravity $\vec{F}_g$

- T: Thrust
- ϕ, θ, ψ: roll-pitch-yaw Euler angles
- $R(\phi)$, $R(\theta)$, $R(\psi)$: positive roll, pitch, and yaw rotation matrices

4.3. **State Equations.** The state equations are:

$$m\dot{V} + \omega \times mV = \vec{F}_G(\phi, \theta, \psi) + \vec{F}_T + \vec{F}_A(V, \omega, \theta), \tag{4.1}$$

$$J\dot{\omega} + \omega \times J\omega = \vec{M}_T + \vec{M}_{gyro} + \vec{M}_A\left(V, \omega, \theta\right), \tag{4.2}$$

$$\dot{\phi} = p + \tan\theta\left(q\sin\phi + r\cos\phi\right), \tag{4.3}$$

$$\dot{\theta} = q\cos\phi - r\sin\phi, \tag{4.4}$$

$$\dot{\psi} = \frac{q\sin\phi + r\cos\phi}{\cos\theta}, \tag{4.5}$$

$$\begin{bmatrix} \dot{X}_{NED} \\ \dot{Y}_{NED} \\ \dot{Z}_{NED} \end{bmatrix} = R^T(\psi)R^T(\theta)R^T(\phi)\begin{bmatrix} u \\ v \\ w \end{bmatrix}. \tag{4.6}$$

4.4. **Output Equations.** The output equations (altitude and accelerations) are:

$$h = -Z_{NED}, \tag{4.7}$$

$$\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \dot{V} + \omega \times V - \frac{\vec{F}_g}{m} = \frac{1}{m}\left(\vec{F}_a + \vec{F}_t\right). \tag{4.8}$$

We assumed that aerodynamic terms are negligible.

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