# LA MODELISATION ARCH (EN FINANCE)

## Section 1 : Rappels sur le SKEWNESS et le KURTOSIS

Le test de la distribution de la normalité (ou non normalité) se concentre sur le coefficient d'asymétrie (skewness) et sur le coefficient d'aplatissement (kurtosis). Le skewness multiplié par racine (N/6) et le kurtosis multiplié par racine (N/24) suivent une N(0,1).

Concrètement :

$$sk \rightarrow N\left(0, \sqrt{\frac{6}{n}}\right) \qquad kurt \rightarrow N\left(3, \sqrt{\frac{24}{n}}\right)$$

On va construire les statistiques suivantes :

$$u1 = \frac{|sk|}{\sqrt{\frac{6}{n}}} \qquad u2 = \frac{|kurt - 3|}{\sqrt{\frac{24}{n}}}$$

Les hypothèses sont les suivantes :

$$\begin{cases} H_0 : u1 = 0 \\ H_1 : u1 \neq 0 \end{cases} \qquad \begin{cases} H_0 : u2 = 0 \\ H_1 : u2 \neq 0 \end{cases}$$

Si on accepte $H_0$ alors l'hypothèse de normalité est vérifiée. Le critère du test est :

Si $u1 \leq 1.96$ alors on accepte H0
Si $u2 \leq 1.96$ alors on accepte H0

1.96 est le seuil critique lu dans une Normale(0,1) à 5%

On peut tester la normalité d'une autre façon :

- Si le Kurtosis est inférieure à 3 on dit que la distribution est platikurtique,

- Si le Kurtosis est supérieure à 3 on dit que la distribution est leptokurtique,

- Si le Kurtosis est égale à 3 on dit que la **distribution est plate donc normale**.

Pour une loi normale le skewness doit être égale à 0 (hypothèse de symétrie).

## Section 2 : Qu'entend-t-on exactement par modèle ARCH ?

### 2.1. Les différentes formes de spécifications

L'objectif des modèles ARCH (et extension) est de mieux décrire et prévoir la volatilité des séries temporelles, notamment financières (que ne le faisait les modèles ARIMA).

Les modèles ARCH ont été introduits par Engle en 1982, puis généralisés en modèles GARCH en 1986 par Bollerslev et en 1990 par Nelsen (développement des modèles à volatilité stochastique).

Pour le moment soyons plus précis sur les termes « modèles ARCH ». Cette écriture peut se décliner sous deux formes :

Forme 1 : Les processus ARCH(1) à proprement parlé

Le processus $Y_t$ est un processus ARCH(1) si

$$\boxed{Y_t = \varepsilon_t \sqrt{\alpha_0 + \alpha_1 Y_{t-1}^2}}$$

Avec $\varepsilon_t \sim N(0,1)$

**Forme 2 : Les modèles avec erreurs ARCH(1)**

$$\boxed{\begin{array}{l} y_t = \text{var } exogenes + \varepsilon_t \\ \varepsilon_t = u_t \sqrt{\alpha_0 + \alpha_1 \varepsilon_{t-1}^2} \\ \left(\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2\right) \end{array}}$$

Avec $u_t \sim N(0,1)$

NB : var exogènes : veut dire variables exogènes et n'est pas précisé dans un premier temps.

Dans notre exposé on s'intéressera exclusivement à la forme n°2.

Déterminons maintenant les différentes extensions, les généralisations de l'approche ARCH.

### 2.1.1. Spécification d'un ARCH(1) :

$$
\begin{aligned}
&y_t = \beta X_t + \varepsilon_t \\
&\varepsilon_t \approx N(0, h_t) \quad \text{i.e.} \quad \varepsilon_t = v_t \sqrt{h_t} \\
&h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 \\
&v_t \approx N(0,1)
\end{aligned}
$$

NB : $X_t$ peut aussi bien représenter des variables exogènes que des variables endogènes retardées ($y_{t-i}$), les deux ensembles aussi. $X_t$ peut ne pas exister, dans ce cas $y_t$ ne dépendra que de la constante.

### ➲ Spécification d'un ARCH(q) :

$$
\begin{aligned}
&y_t = \beta X_t + \varepsilon_t \\
&\varepsilon_t = v_t \sqrt{h_t} \\
&h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2
\end{aligned}
$$

Ce processus est un processus de moyenne mobile (MA(q)) d'ordre q.

## 2.1.2. Spécification d'un modèle GARCH(1,1) :

Generalised Autoregressive Conditionally Heteroscedastic

$$\boxed{\begin{aligned} y_t &= \beta X_t + \varepsilon_t \\ \varepsilon_t &= \nu_t \sqrt{h_t} \\ h_t &= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} \end{aligned}}$$

## ➲ Spécification d'un modèle GARCH(p,q) :

$$\boxed{\begin{aligned} y_t &= \beta X_t + \varepsilon_t \\ \varepsilon_t &= \nu_t \sqrt{h_t} \\ h_t &= \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-i} \end{aligned}}$$

### 2.1.3. Spécification d'un GARCH(1,1)-M (GARCH en moyenne) :

Modèle proposé par Engle, Lilien et Robins en 1987.

δ serait sous certaines conditions, le coefficient d'aversion au risque relatif. Ce modèle a été appliqué dans diverses études sur la volatilité des rendements des actifs. Ici la volatilité est un déterminant de la rentabilité.

Forme linéaire :

$$\boxed{\begin{aligned} y_t &= \beta X_t + \delta h_t + \varepsilon_t \\ \varepsilon_t &= \nu_t \sqrt{h_t} \\ h_t &= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} \end{aligned}}$$

Forme log linéaire :

$$\boxed{\begin{aligned} y_t &= \beta X_t + \delta \log h_t + \varepsilon_t \\ \varepsilon_t &= \nu_t \sqrt{h_t} \\ h_t &= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} \end{aligned}}$$

Forme racine carrée :

$$\boxed{\begin{aligned} y_t &= \beta X_t + \delta \sqrt{h_t} + \varepsilon_t \\ \varepsilon_t &= \nu_t \sqrt{h_t} \\ h_t &= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} \end{aligned}}$$

**➲ Spécification d'un GARCH(p,q)-M (GARCH en moyenne) :**

Forme linéaire :

$$\boxed{\begin{aligned} y_t &= \beta X_t + \delta h_t + \varepsilon_t \\ \varepsilon_t &= \nu_t \sqrt{\mathbf{h}_t} \\ h_t &= \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-i} \end{aligned}}$$

Idem pour les autres formes.

### 2.1.4. Spécification d'un IGARCH(1,1) :

Le processus IGARCH permet une persistance infinie de la volatilité. L'effet d'un choc se répercute sur toutes les prévisions. Le modèle IGARCH est un modèle GARCH avec une contrainte sur la somme des coefficients.

$$\boxed{\begin{aligned} y_t &= \beta X_t + \varepsilon_t \\ \varepsilon_t &= v_t \sqrt{h_t} \\ h_t &= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} \\ sc \quad & \alpha_1 + \beta_1 = 1 \end{aligned}}$$

**➲ Spécification d'un IGARCH(p,q) :**

$$\boxed{\begin{aligned} y_t &= \beta X_t + \varepsilon_t \\ \varepsilon_t &= v_t \sqrt{h_t} \\ h_t &= \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-i} \\ sc \quad & \sum \alpha_i + \sum \beta_i = 1 \end{aligned}}$$

### 2.1.5. Spécification d'un EGARCH(1,1) :

Le modèle EGARCH permet de prendre en considération que de bonnes nouvelles et de mauvaises nouvelles peuvent avoir un impact différent sur la volatilité. Il y a donc une évolution asymétrique de la volatilité (cf. Nelson 1990-1991-1993 puis Nelson et Ng (1993)).

$$y_t = \beta X_t + \varepsilon_t$$

$$\varepsilon_t = \nu_t \sqrt{h_t}$$

$$\log(h_t) = \alpha_0 + a_1 \nu_{t-1} + b_1\left(|\nu_{t-1}| - E[|\nu_{t-1}|]\right) + \beta_1 \log(h_{t-1})$$

Le coefficient $b_1$ mesure l'effet d'amplitude du terme d'erreur passé.
Le coefficient $a_1$ capte l'effet du signe de l'erreur.

La valeur de l'espérance E(.) dépend de la loi supposée de ν : soit une loi gaussienne, soit une loi de Student, soit une loi GED (Generalized Error Distribution), soit une loi de Student dissymétrique (avec loi gama)…

### ➲ Spécification d'un EGARCH(p,q) :

$$y_t = \beta X_t + \varepsilon_t$$

$$\varepsilon_t = \nu_t \sqrt{h_t}$$

$$\log(h_t) = \alpha_0 + \sum_{i=1}^{q} a_i \nu_{t-i} + \sum_{i=1}^{q} b_i\left(|\nu_{t-i}| - E[|\nu_{t-i}|]\right) + \sum_{i=1}^{p} \beta_i \log(h_{t-i})$$

### 2.1.6. Spécification d'un TGARCH(1,1) :

Les processus TGARCH, TARCH (T pour threshold) permettent aussi de modéliser les asymétries en retenant des modélisations à seuils de la forme des modèles TAR de Tong (1990). Les modèles TARCH sont dus à Zakoian (1991) et les modèles TGARCH à Zakoian en 1994.

$$y_t = \beta X_t + \varepsilon_t$$

$$\varepsilon_t = \nu_t \sqrt{h_t}$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} + \gamma_1 \Omega \varepsilon_{t-1}^2$$

$$avec \begin{cases} \Omega = 1 si \varepsilon_{t-1} < 0 \\ \Omega = 0 si \varepsilon_{t-1} \geq 0 \end{cases}$$

### ➲ Spécification d'un TGARCH(p,q) :

$$y_t = \beta X_t + \varepsilon_t$$

$$\varepsilon_t = \nu_t \sqrt{h_t}$$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-i} + \gamma \varepsilon$$

Nous avons aussi : les modèles GJR-GARCH, APARCG, VSGARCH, QGARCH, LSTGARCH, ANSTGARCH, FIGARCH, HYGARCH, FAPARCH…

Nous venons de voir les différentes modélisations de la variance conditionnelle. Dans les spécifications précédentes la partie variance conditionnelle est spécifique pour chaque processus i.e. fixe mais la forme de l'équation que nous voulons modéliser est, elle, infinie !!!. Prenons un exemple.

Soit l'estimation d'un AR(2)-ARCH(1). La première partie (AR(2)) correspond à notre équation et la deuxième partie (ARCH(1)) à la variance conditionnelle, de la forme :

$$\boxed{\begin{aligned} y_t &= \beta_1 y_{t-1} + \beta_2 y_{t-2} + \varepsilon_t \\ \varepsilon_t &= \nu_t \sqrt{\mathrm{h}_t} \\ h_t &= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 \end{aligned}}$$

On peut aussi avoir un ARMA(1,2)-GARCH(2,3). On remarque que les combinaisons sont multiples. On peut aussi avoir un modèle de régression multiple standard….

## 2.2. Les méthodes d’estimation

Aucun problème : les méthodes adéquates sont le maximum de vraisemblance ou le pseudo maximum de vraisemblance. Voir un cours d’Econométrie pour plus de détails.

## 2.3. Les tests

Aucun problème : ils existent de nombreux tests permettant de tester l’effet ARCH. Citons le test de Ljung-Box et le test ARCH. Voir un cours d’Econométrie pour plus de détails.

## 2.4. Les prévisions

Aucun problème : comme pour les modèles ARIMA les prévisions ont obtenues par récurrence et ne posent aucun problème. Toutefois avant de prévoir le rendement ou le cours du titre il faut prévoir la variance conditionnelle, ce qui fait un double travail.

## Section 3 : Caractéristiques des séries financières

Soient $P_t$ le prix d'un titre (actif) à la date t et $r_t$ le rendement i.e. :

$$\boxed{r_t = \frac{p_t - p_{t-1}}{p_t}}$$

En s'aidant de Mackinlay, Campbell et Lo (1997) puis de Cuthbertson (2000) on peut faire un résumé rapide des caractéristiques des séries financières :

Caractéristique n°1 : Les processus $P_t$ sont généralement non stationnaires tandis que les processus $r_t$ sont (ou semblent) compatibles avec la propriété de stationnarité.

Caractéristique n°2 : La série $r^2_t$ associée aux carrés des rendements présente généralement de fortes auto-corrélations (cf. ACF) [absence de bruits blancs] tandis que la série $r_t$ présente généralement de faibles auto-corrélations (cf. ACF) [hypothèse de bruit blanc].

Caractéristique n°3 : L'hypothèse de normalité des rendements est généralement rejetée. Les queues des distributions empiriques des rendements sont généralement plus épaisses que celles d'une loi gaussienne. On parle alors de distribution leptokurtique.

Caractéristique n°4 : On observe empiriquement que de fortes variations de rendements sont généralement suivies par d'autres fortes variations (hypothèse de regroupement des extrêmes (clustering volatility)).

Caractéristique n°5 : Même une fois corrigée de la volatilité clustering, la distribution des résidus demeure leptokurtique (mais plus faible que dans le cas non conditionnelle).

Caractéristique n°6 : Il existe une asymétrie entre l'effet des valeurs passées négatives et l'effet des valeurs passées positives sur la volatilité des cours ou des rendements. Les baisses de cours tendent à engendrer une augmentation de la volatilité supérieure à celle induite par une hausse des cours de même ampleur. (Effet de levier).

Caractéristique n°7 : La volatilité tend à augmenter lorsque les marchés ferment (week-end, jours fériés…). Il y a donc un phénomène de saisonnalité.

Caractéristique n°8 : La distribution des cours est généralement asymétrique : il y a plus de mouvements forts à la baisse qu'à la hausse. On teste cette hypothèse (asymétrie ou symétrie) par le coefficient skweness.

## Section 4 : Stratégies de modélisation (application)

Nous arrivons maintenant à la partie application de notre étude. Cela ne poserait aucun problème si une stratégie (du style Box-Jenkins) était en place.

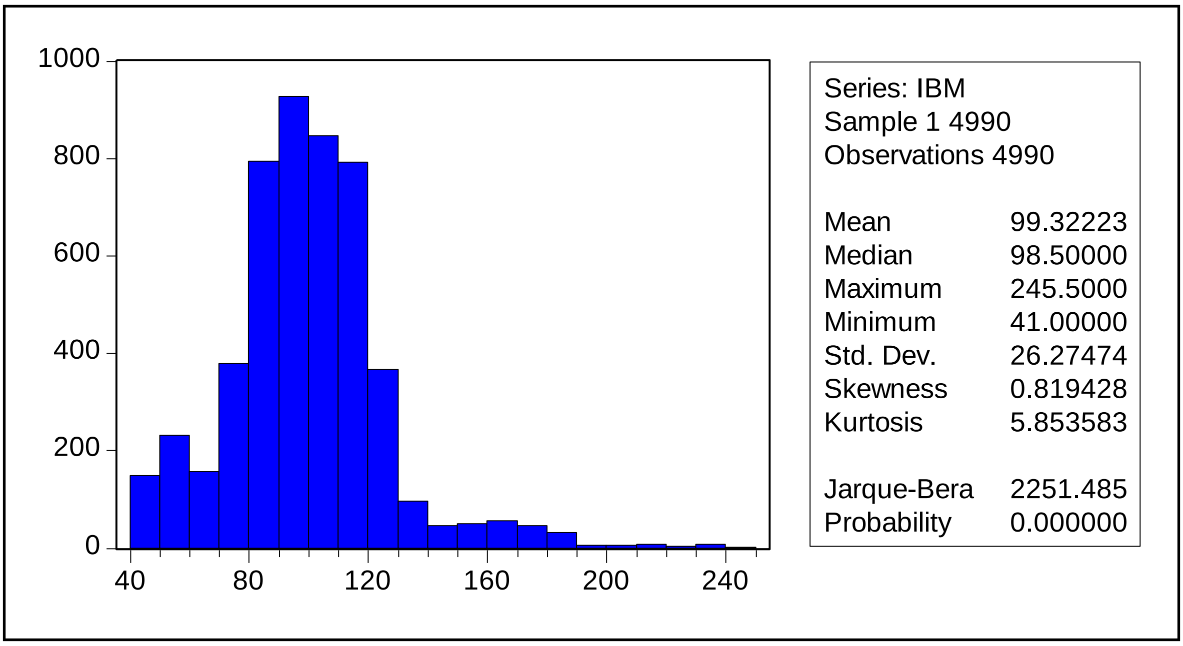
Malheureusement il en est rien. Nous allons estimer toutes les formes des processus définies dans la section 2 (estimation-test-prévision-étude comparative).

Base de données : prix de l'action IBM de 1987 à 2007 en jours soit 4990 données. Source FinInfo.

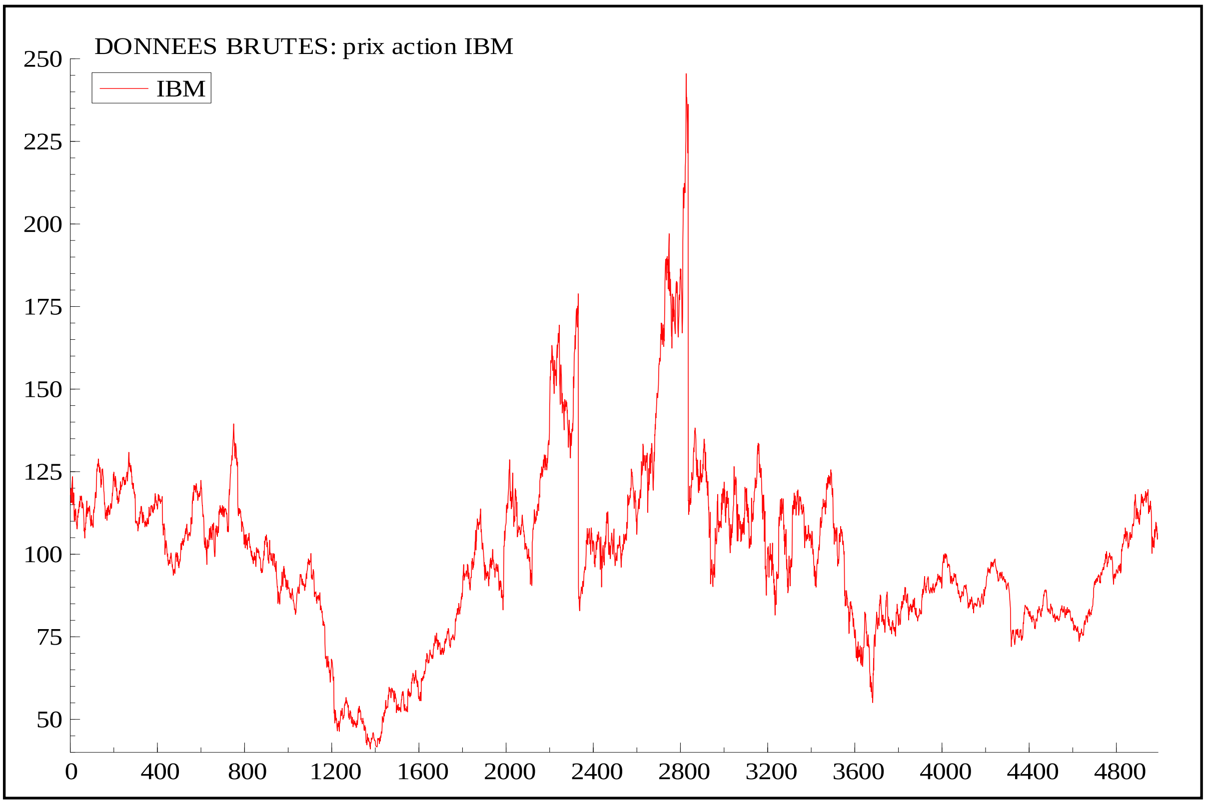
L'étude économétrique a été réalisée sur les logiciels TSP 5.1, EVIEWS 8.0. et SAS V9.4.

NB : on fera l'estimation de 1 à 4989, on se garde la dernière observation pour vérifier l'exactitude de notre prévision. Soit la valeur 107.14 pour la $4990^{\text{ième}}$ observation.

# 1. Les estimations sans stratégie
## 1.1. Statistiques et graphiques
### 1.1.1. Série prix des actions

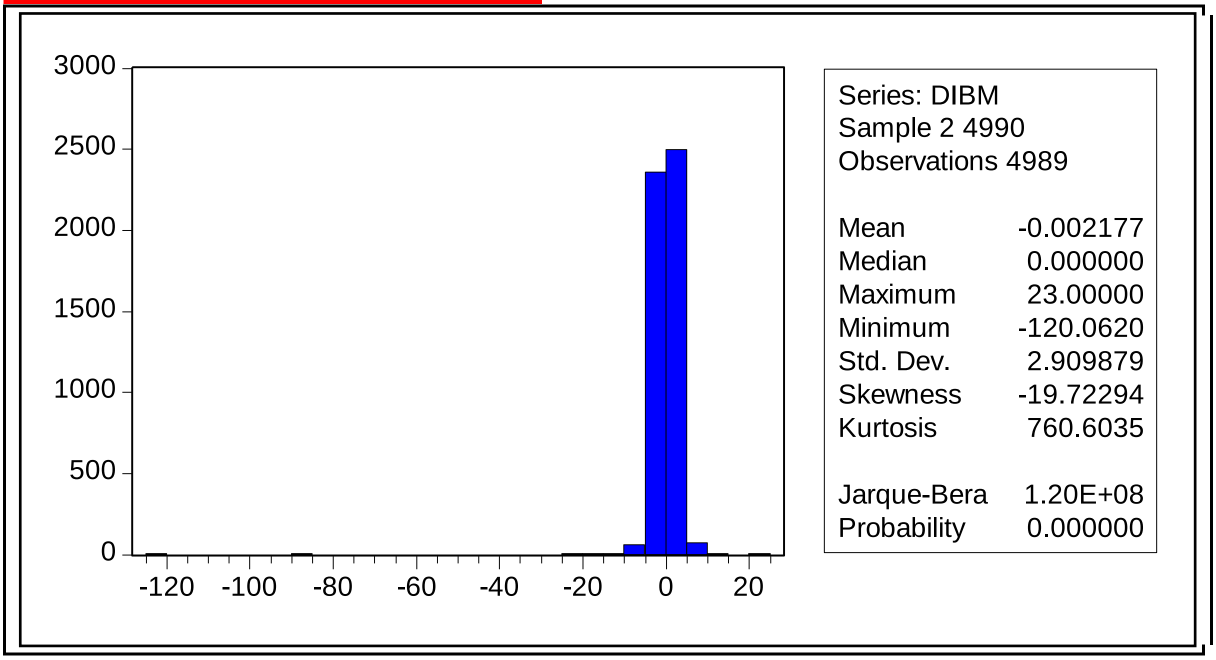



Graphique n°

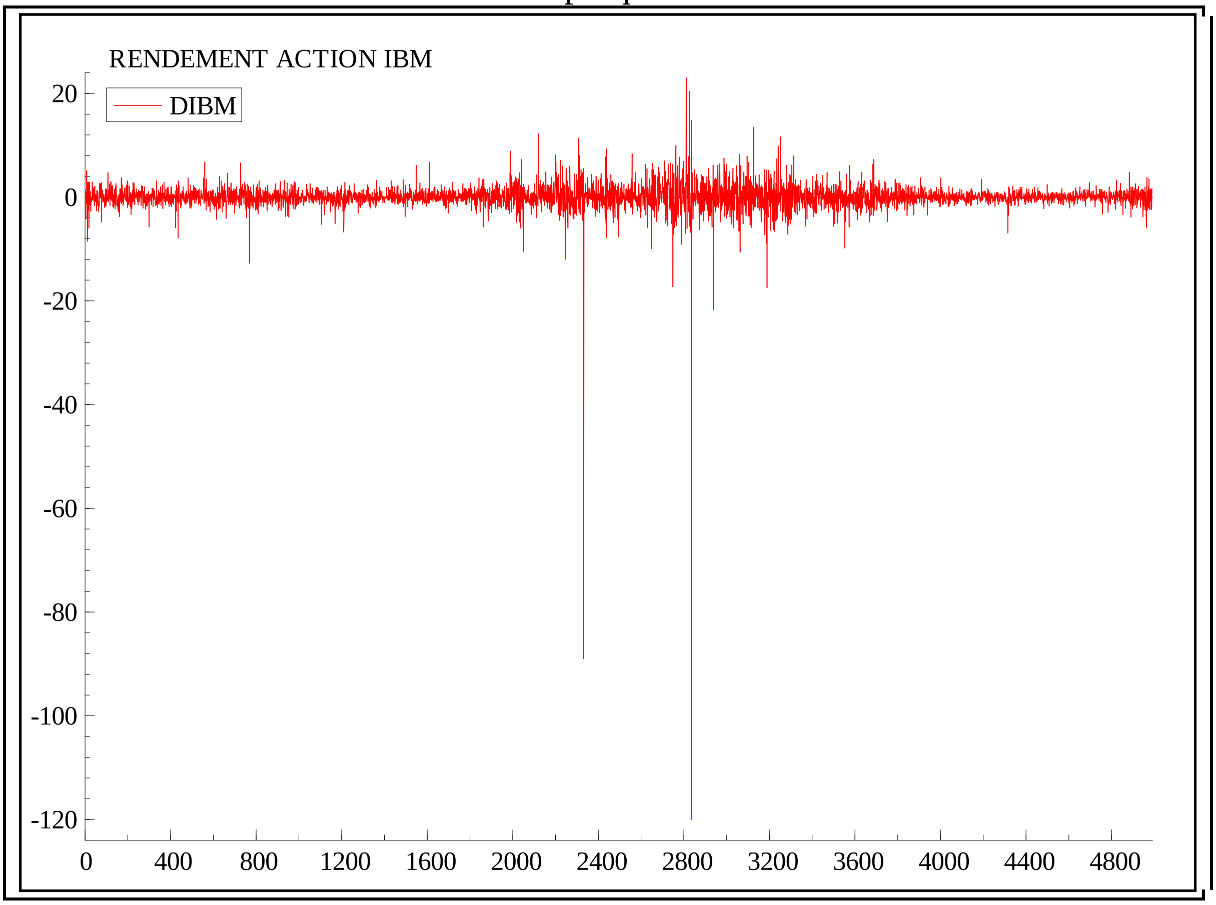



Le graphique des données brutes met en évidence une non stationnarité de la série prix. En examinant les statistiques fournies on peut dire que la série est de type leptokurtique .

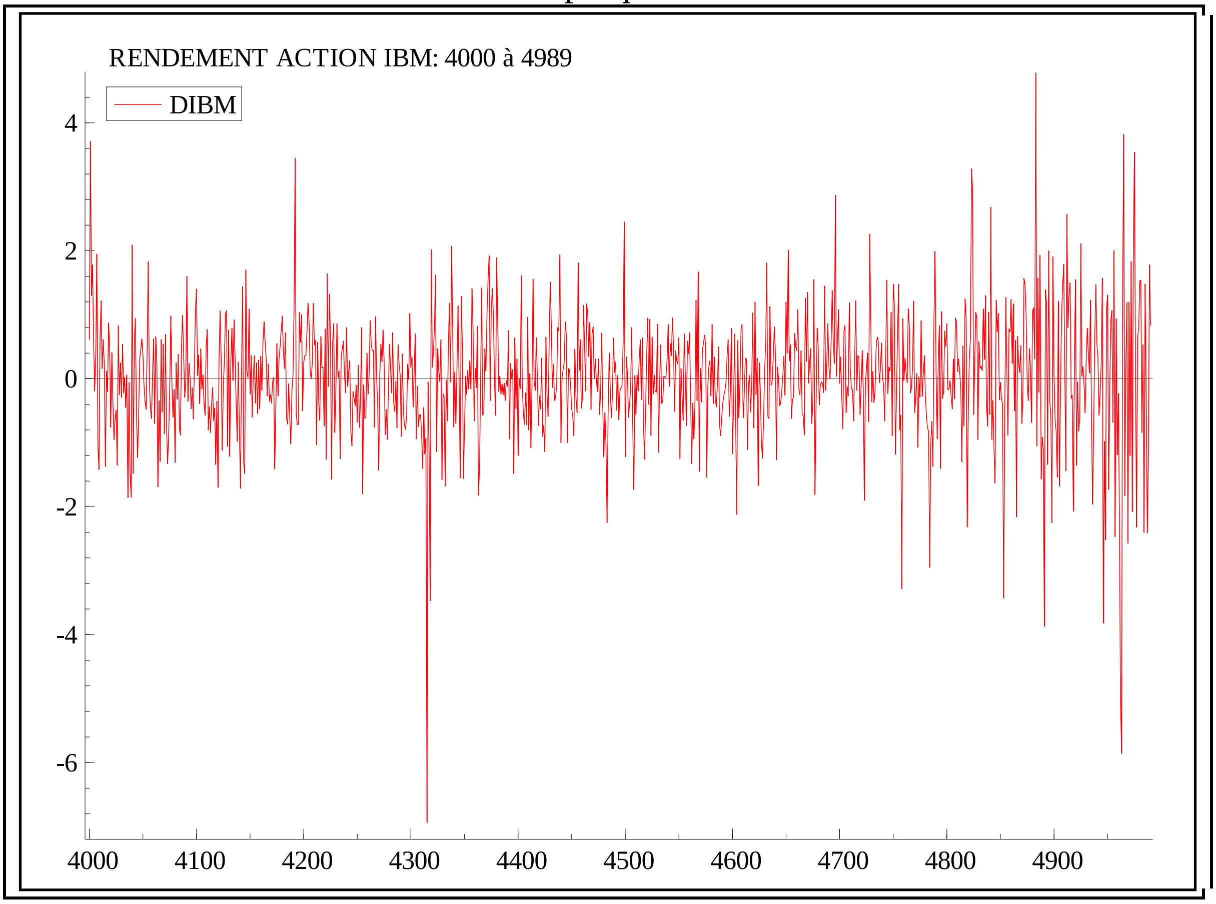
## 1.1.2. Série rendement des actions




Graphique n°



Graphique n°




Nous retrouvons bien ici les caractéristiques 1 et 3 de la section 3 à savoir que les processus $P_t$ sont généralement non stationnaires tandis que les processus $r_t$ sont (ou semblent) compatibles avec la propriété de stationnarité (cf. les deux derniers graphiques).

Ici l'hypothèse_de normalité des rendements est rejetée : la kurtosis est de 760.60 au lieu de 3 !!. On parle alors de distribution leptokurtique.

## 1.2. Spécification d'un ARCH(1) :

On va travailler sur le prix de l'action IBM.

Nous allons estimer le modèle suivant :

$$
\boxed{\begin{aligned}
IBM_t &= cste + \varepsilon_t \\
\varepsilon_t &= v_t \sqrt{h_t} \\
h_t &= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2
\end{aligned}}
$$

Il y a aura deux estimations, deux prévisions à réaliser : celles pour $h_t$ (la variance conditionnelle) puis celles de $IBM_t$ (prix de l'action).

Les résultats sont les suivant avec TSP puis EVIEWS :

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```
       Mean of dep. var. = 97.7914
    Std. dev. of dep. var. = 5.72741
Sum of squared residuals = 163592.
    Variance of residuals = 32.8102
Std. error of regression = 5.72802
                R-squared = .176459E-06
       Adjusted R-squared = .176459E-06
            Durbin-Watson = .144859 [<1.00]
         Jarque-Bera test = 340.601 [.000]
(Statistics based on original data)
       Mean of dep. var. = 99.3129
    Std. dev. of dep. var. = 26.2795
Sum of squared residuals = .345625E+07
    Variance of residuals = 693.191
Std. error of regression = 26.3285
                R-squared = .352402E-08
       Adjusted R-squared = .352402E-08
            Durbin-Watson = .012219
           Schwarz B.I.C. = 19934.2
Akaike Information Crit. = 19924.5
           Log likelihood = -19921.5
             Number of observations in LogL =   4987
               Initial observations dropped =      0
               Est. initial values for H(t) =      0
                    Initial values for H(t) =   693.05

                          Standard
Parameter  Estimate         Error        t-statistic    P-value
C          97.7076        1.08864       89.7519        [.000]
ALPHA0     3.21162        .552171       5.81635        [.000]
ALPHA1     .992531        .011509       86.2431        [.000]
```

EViews - [Equation: UNTITLED Workfile: IBMARCH]

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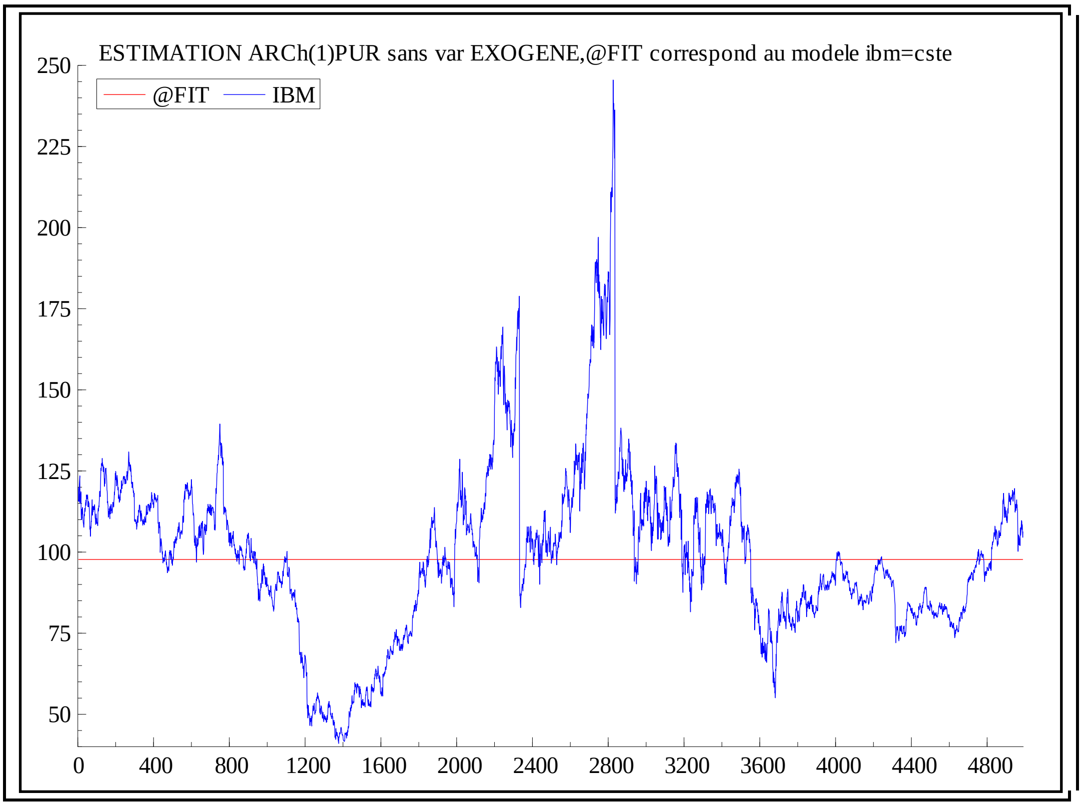
View | Procs | Objects | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

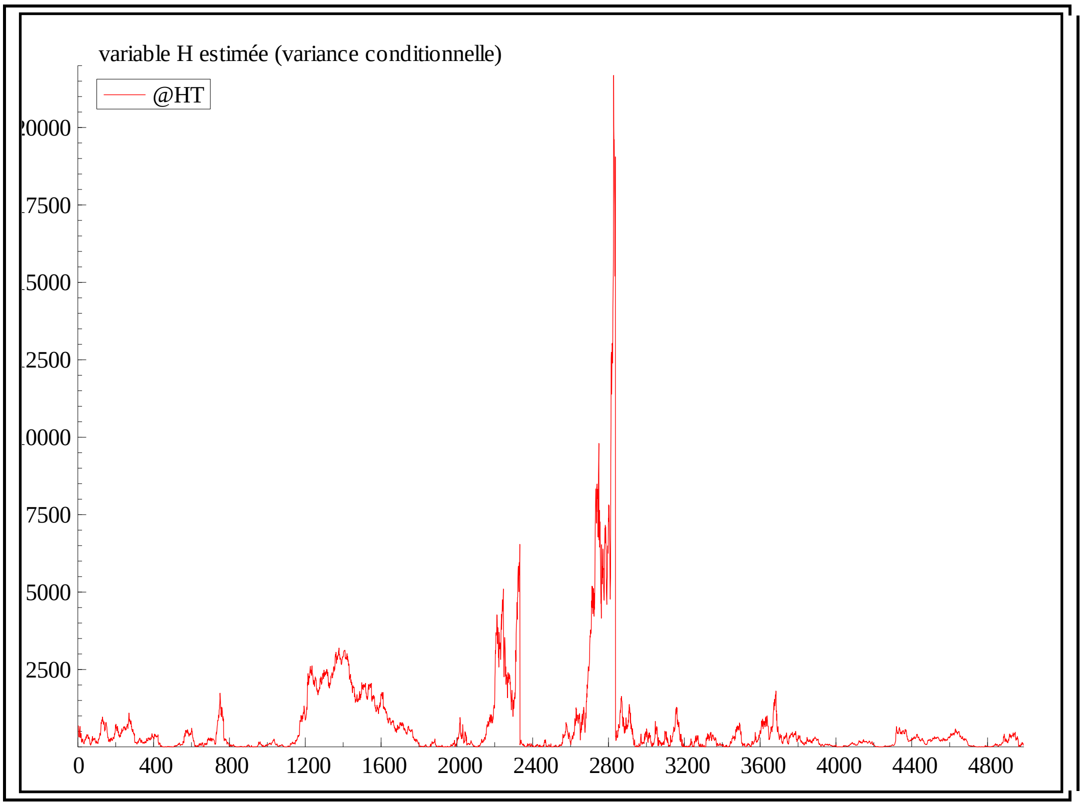
Dependent Variable: IBM
Method: ML - ARCH (Marquardt)
Date: 03/11/08 Time: 15:26
Sample: 1 4989
Included observations: 4989
Convergence achieved after 213 iterations
Variance backcast: ON

| | Coefficient | Std. Error | z-Statistic | Prob. |
|---|---|---|---|---|
| C | 97.73042 | 0.070002 | 1396.101 | 0.0000 |
| | Variance Equation | | | |
| C | 3.208092 | 0.150460 | 21.32182 | 0.0000 |
| ARCH(1) | 0.992789 | 0.081282 | 12.21409 | 0.0000 |

| | | | |
|---|---|---|---|
| R-squared | -0.003663 | Mean dependent var | 99.32066 |
| Adjusted R-squared | -0.004066 | S.D. dependent var | 26.27714 |
| S.E. of regression | 26.33050 | Akaike info criterion | 7.990926 |
| Sum squared resid | 3456770. | Schwarz criterion | 7.994844 |
| Log likelihood | -19930.37 | Durbin-Watson stat | 0.012218 |

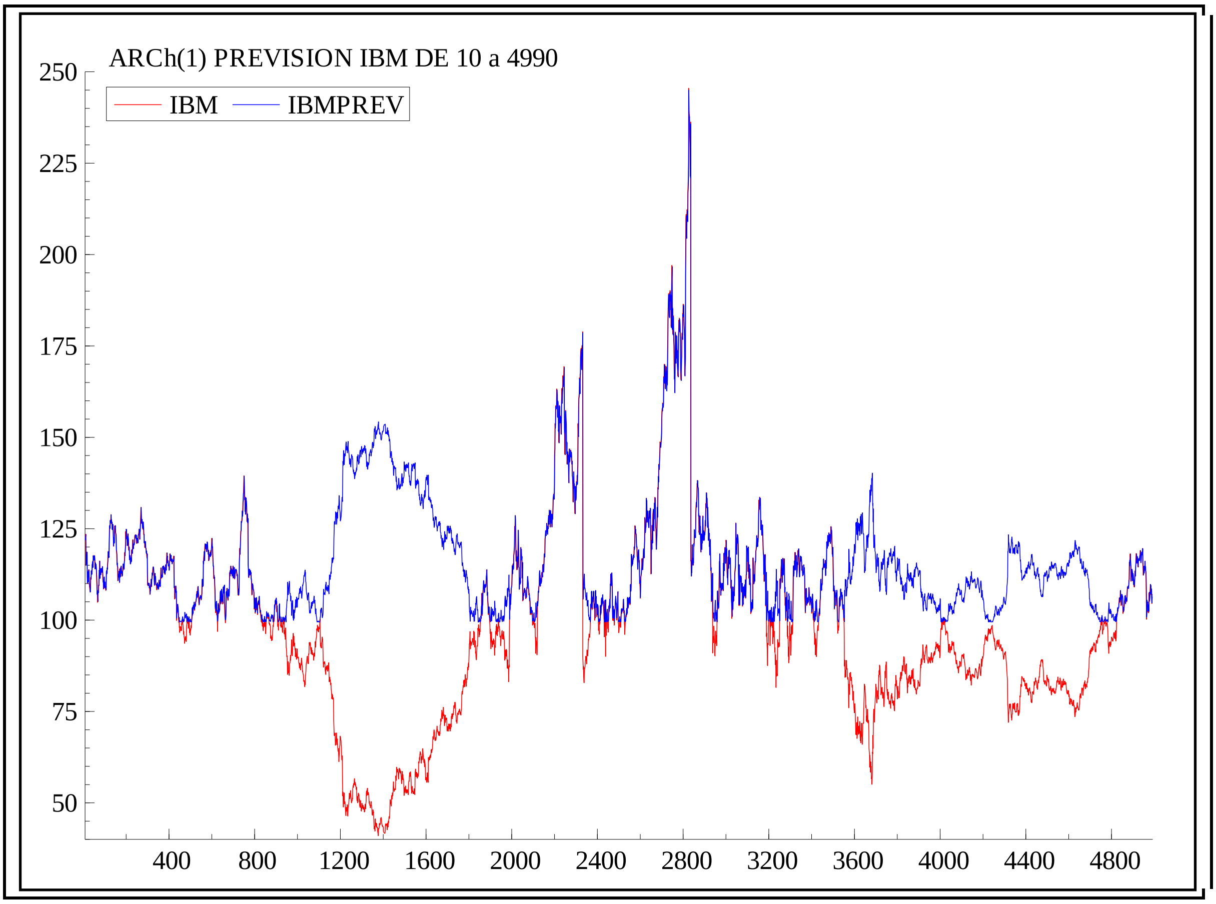



Ce graphique estime le prix de l'action IBM. On remarque que c'est sans intérêt car on obtient une droite [c'est normale car on régresse sur une constante]. Par contre ce qui est plus intéressant c'est l'estimation de la variance conditionnelle :


variable H estimée (variance conditionnelle)
@HT
20000
17500
15000
12500
10000
7500
5000
2500
0
400
800
1200
1600
2000
2400
2800
3200
3600
4000
4400
4800

En combinant les deux modèles nous obtenons une estimation du prix de l'action plus réaliste :

$$\boxed{IBM_t = cste + \nu_t \sqrt{\alpha_0 + \alpha_1 \varepsilon_{t-1}^2}}$$




Finalement la prévision pour 4990 est : 106.46317 que nous devons comparer à 107.14. Ce qui n'est pas trop mal compte tenu de la faiblesse des variables exogènes. Pouvons-nous faire mieux ?

## 1.3. Spécification d'un modèle GARCH(1,1) :

Le modèle global à estimer est le suivant :

$$\boldsymbol{IBM}_t = cste + \nu_t \sqrt{\alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}}$$

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```
  Std. dev. of dep. var. = 5.31351
Sum of squared residuals = 140811.
   Variance of residuals = 28.2412
Std. error of regression = 5.31425
               R-squared = .241591E-06
      Adjusted R-squared = .241591E-06
           Durbin-Watson = .136162 [<1.00]
        Jarque-Bera test = 420.628 [.000]
(Statistics based on original data)
      Mean of dep. var. = 99.3129
  Std. dev. of dep. var. = 26.2795
Sum of squared residuals = .345800E+07
   Variance of residuals = 693.541
Std. error of regression = 26.3352
               R-squared = .168496E-07
      Adjusted R-squared = .168496E-07
           Durbin-Watson = .012213
          Schwarz B.I.C. = 19922.2
Akaike Information Crit. = 19909.1
          Log likelihood = -19905.1
            Number of observations in LogL =   4987
              Initial observations dropped =      0
              Est. initial values for H(t) =      0
                   Initial values for H(t) =   693.40

                          Standard
Parameter  Estimate        Error        t-statistic    P-value
C          97.6018        .654013       149.235        [.000]
ALPHA0     1.60435        .341423       4.69900        [.000]
ALPHA1     .799940        .058611       13.6482        [.000]
BETA1      .206444        .059823       3.45093        [.001]
```

EViews - [Equation: UNTITLED Workfile: IBMARCH]

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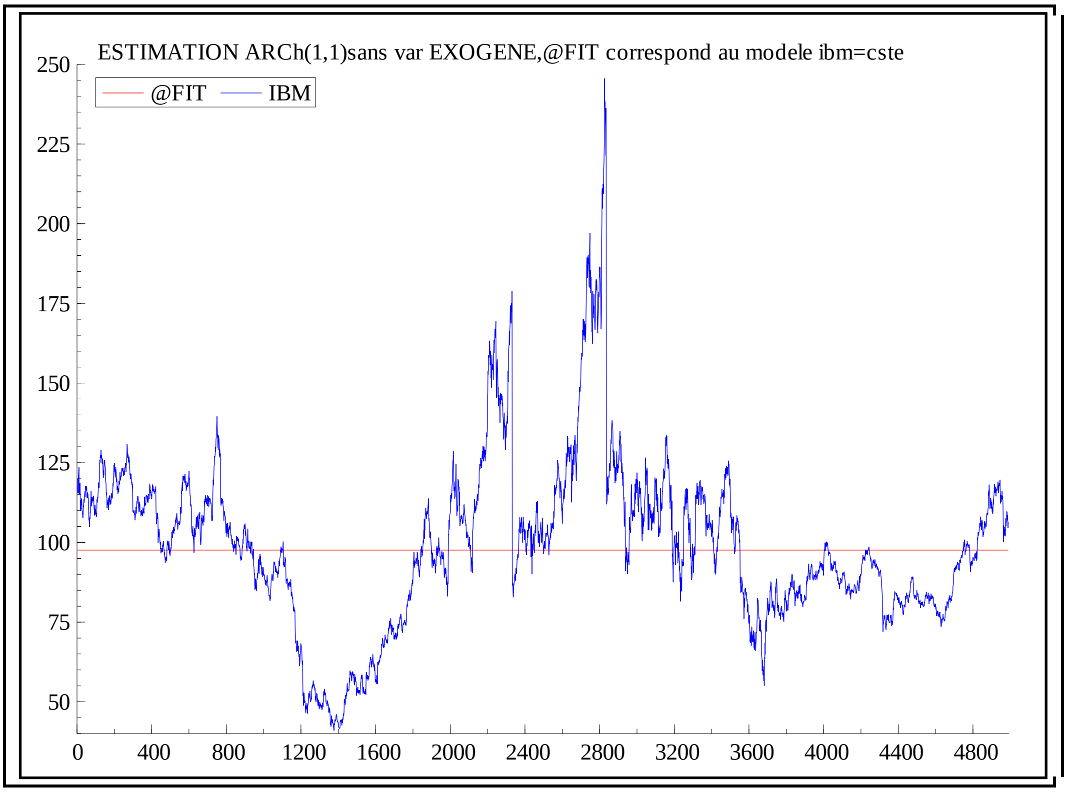
View | Procs | Objects | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Dependent Variable: IBM
Method: ML - ARCH (Marquardt)
Date: 03/11/08 Time: 18:25
Sample: 1 4990
Included observations: 4990
Convergence achieved after 205 iterations
Variance backcast: ON

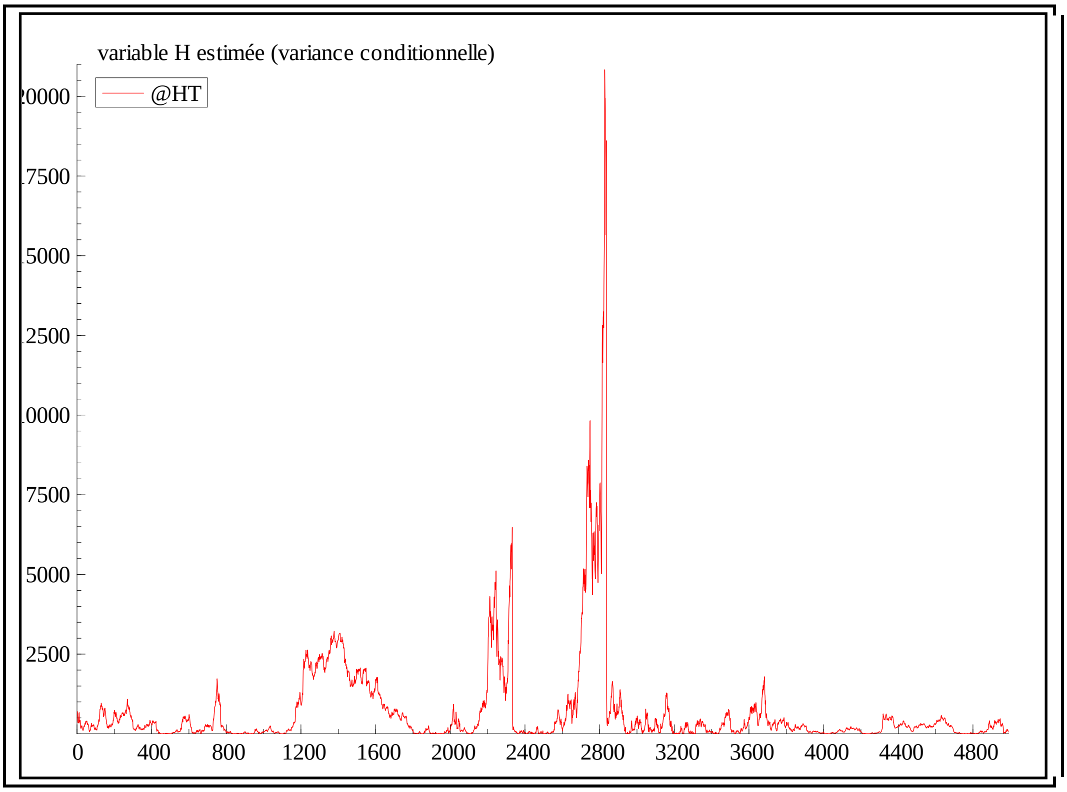
| | Coefficient | Std. Error | z-Statistic | Prob. |
|---|---|---|---|---|
| C | 97.61883 | 0.066813 | 1461.085 | 0.0000 |
| Variance Equation | | | | |
| C | 1.601577 | 0.206459 | 7.757343 | 0.0000 |
| ARCH(1) | 0.799545 | 0.072914 | 10.96555 | 0.0000 |
| GARCH(1) | 0.206996 | 0.027140 | 7.627009 | 0.0000 |

| | | | |
|---|---|---|---|
| R-squared | -0.004204 | Mean dependent var | 99.32223 |
| Adjusted R-squared | -0.004808 | S.D. dependent var | 26.27474 |
| S.E. of regression | 26.33783 | Akaike info criterion | 7.984660 |
| Sum squared resid | 3458694. | Schwarz criterion | 7.989882 |
| Log likelihood | -19917.73 | Durbin-Watson stat | 0.012211 |

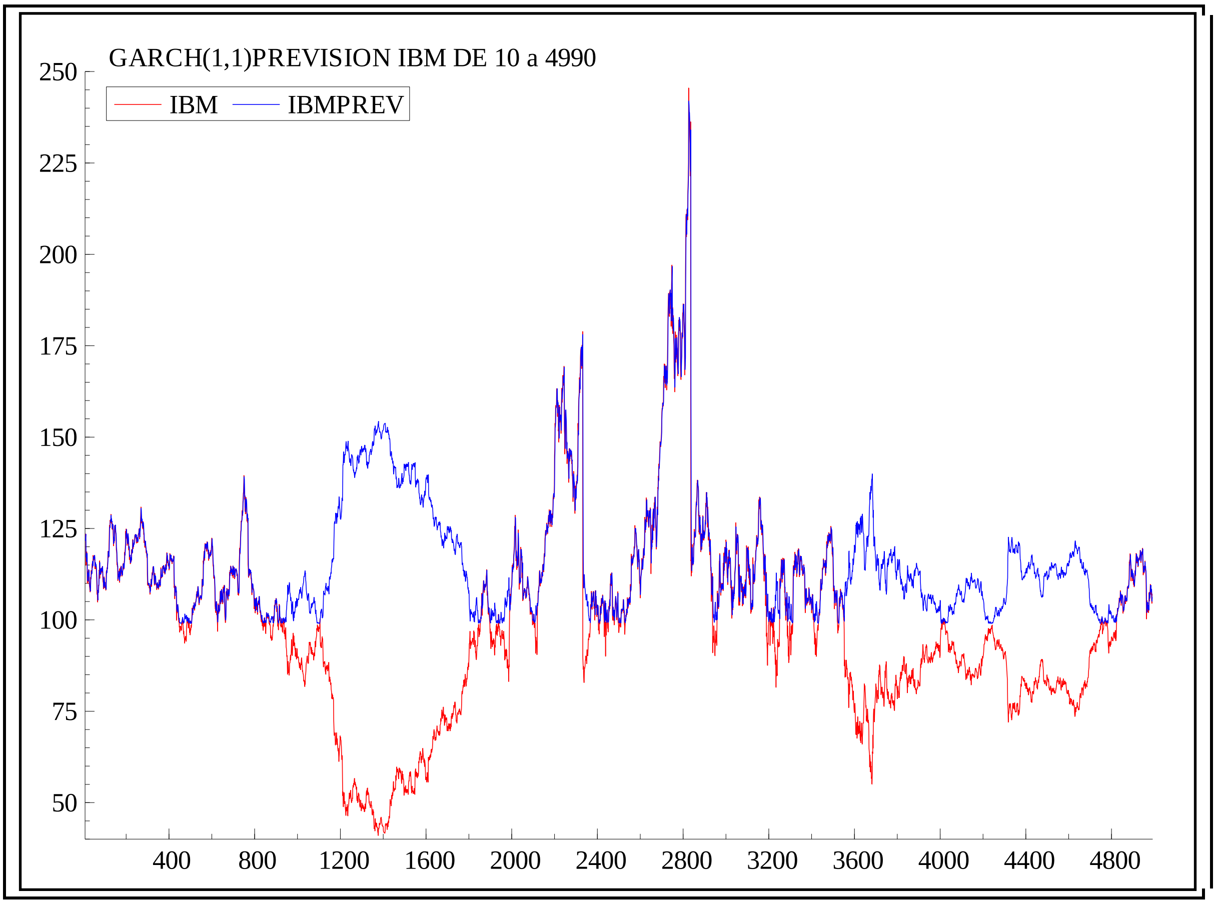
Estimation de la première équation :




Estimation de la variance conditionnelle :



Estimation des deux modèles :




Nous obtenons une prévision pour 4990 de 106.19722.

NB : Il y a très peu de différence entre les deux approches.

## 1.3. Spécification d'un GARCH(1,1)-M (GARCH en moyenne) :

Forme linéaire :

$$IBM_t = cste + \delta h_t + \varepsilon_t$$

$$\varepsilon_t = \nu_t \sqrt{h_t}$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}$$

Les estimations ont été réalisées sur SAS V9 :

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The AUTOREG Procedure

Dependent Variable IBM
IBM

Ordinary Least Squares Estimates

| | | | |
|---|---|---|---|
| SSE | 3444214.8 | DFE | 4989 |
| MSE | 690.36176 | Root MSE | 26.27474 |
| SBC | 46789.2282 | AIC | 46782.713 |
| Regress R-Square | 0.0000 | Total R-Square | 0.0000 |
| Durbin-Watson | 0.0123 | | |

| Variable | DDL | Estimation | Standard Error | t Value | Approx Pr > \|t\| |
|---|---|---|---|---|---|
| Intercept | 1 | 99.3222 | 0.3720 | 267.03 | <.0001 |

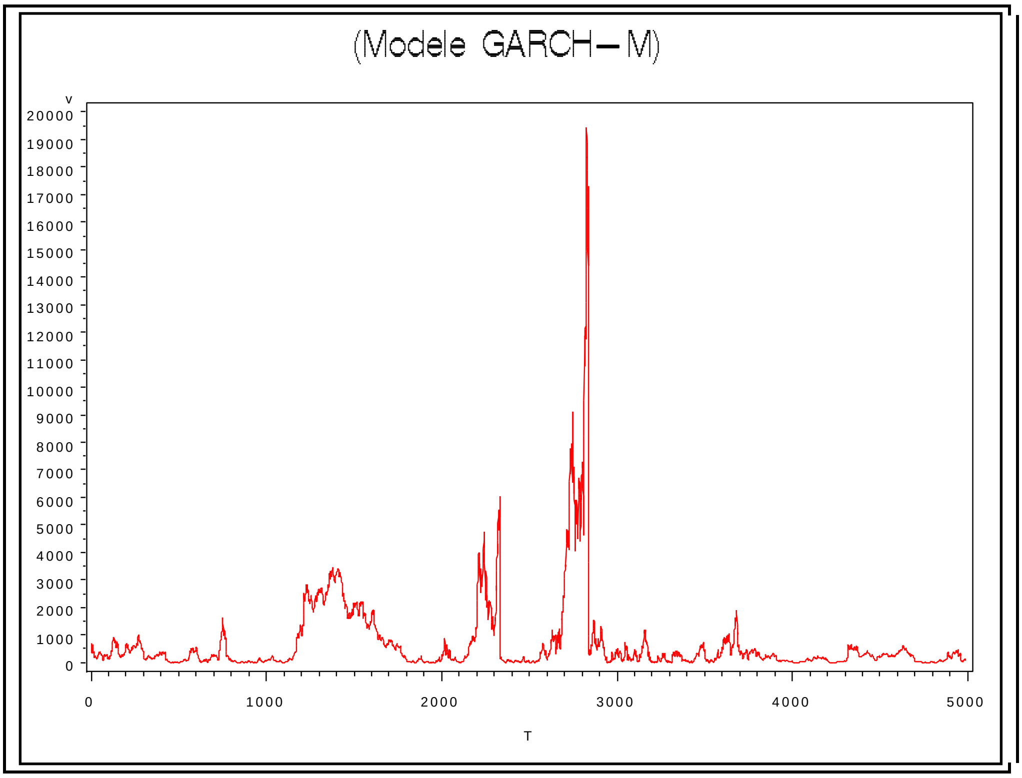
Algorithm converged.

GARCH Estimates

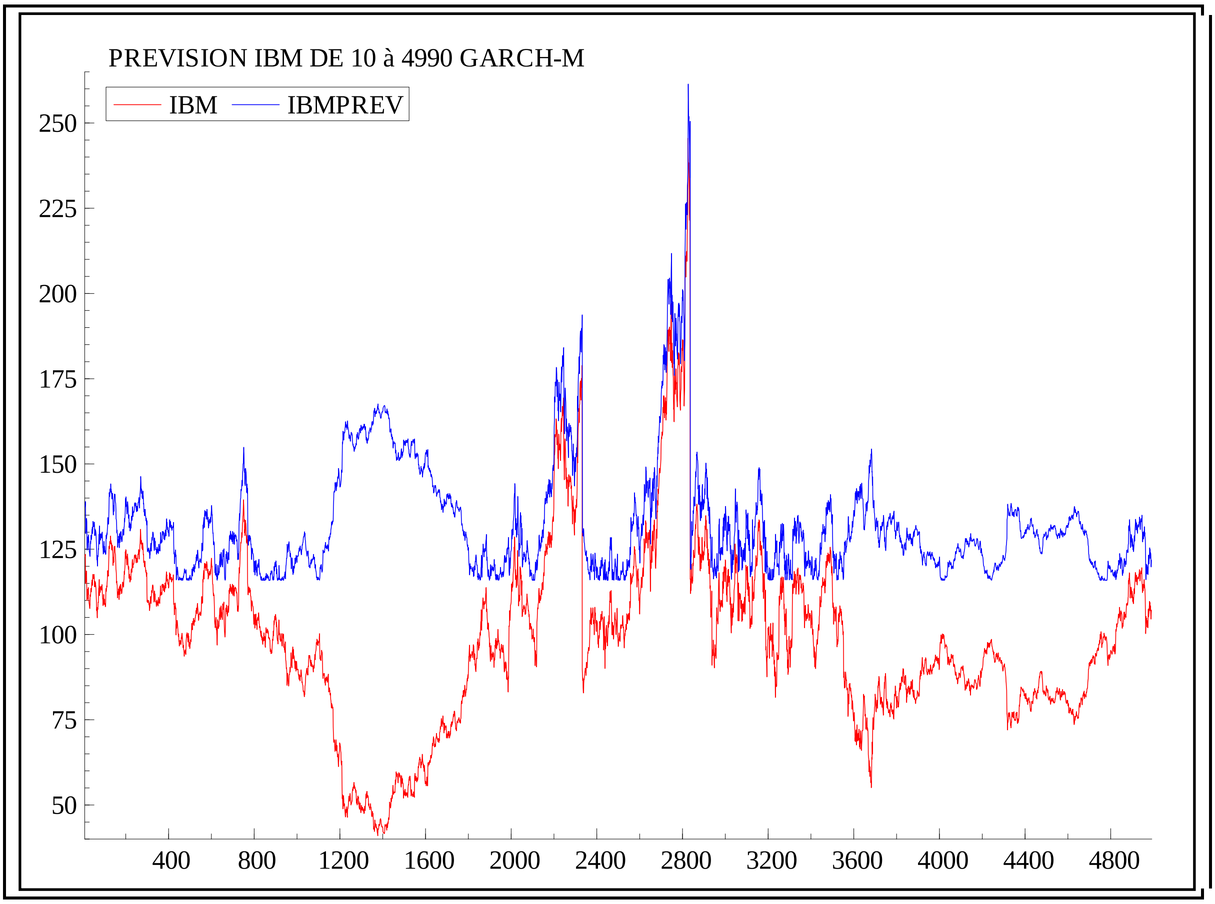
| | | | |
|---|---|---|---|
| SSE | 3408482.7 | Observations | 4990 |
| MSE | 683.06267 | Uncond Var | . |
| Log Likelihood | -19914.533 | Total R-Square | 0.0104 |
| SBC | 39871.6413 | AIC | 39839.0653 |
| Normality Test | 434.4025 | Pr > ChiSq | <.0001 |

| Variable | DDL | Estimation | Standard Error | t Value | Approx Pr > \|t\| |
|---|---|---|---|---|---|
| Intercept | 1 | 97.2123 | 0.0952 | 1020.76 | <.0001 |
| ARCH0 | 1 | 1.6405 | 0.2259 | 7.26 | <.0001 |
| ARCH1 | 1 | 0.8035 | 0.0730 | 11.01 | <.0001 |
| GARCH1 | 1 | 0.2026 | 0.0277 | 7.31 | <.0001 |
| DELTA | 1 | 0.0437 | 0.0187 | 2.34 | 0.0193 |

Variance conditionnelle :




Nous obtenons maintenant les prévisions des 2 modèles :




Nous obtenons une prévision pour 4990 de 121.80366 assez éloignée de 107.14.

## 1.4. Spécification d'un IGARCH(1,1) :

Le processus IGARCH permet une persistance infinie de la volatilité. L'effet d'un choc se répercute sur toutes les prévisions. Le modèle IGARCH est un modèle GARCH avec une contrainte sur la somme des coefficients.

$$IBM_t = cste + \varepsilon_t$$

$$\varepsilon_t = \nu_t \sqrt{h_t}$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}$$

$$sc \quad \alpha_1 + \beta_1 = 1$$

Estimations obtenues :

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(Modele GARCH-M) 13:25 Wedn

The AUTOREG Procedure

Dependent Variable IBM
IBM

Ordinary Least Squares Estimates

| | | | |
|---|---|---|---|
| SSE | 3444214.8 | DFE | 4989 |
| MSE | 690.36176 | Root MSE | 26.27474 |
| SBC | 46789.2282 | AIC | 46782.713 |
| Regress R-Square | 0.0000 | Total R-Square | 0.0000 |
| Durbin-Watson | 0.0123 | | |

| Variable | DDL | Estimation | Standard Error | t Value | Approx Pr > \|t\| |
|---|---|---|---|---|---|
| Intercept | 1 | 99.3222 | 0.3720 | 267.03 | <.0001 |

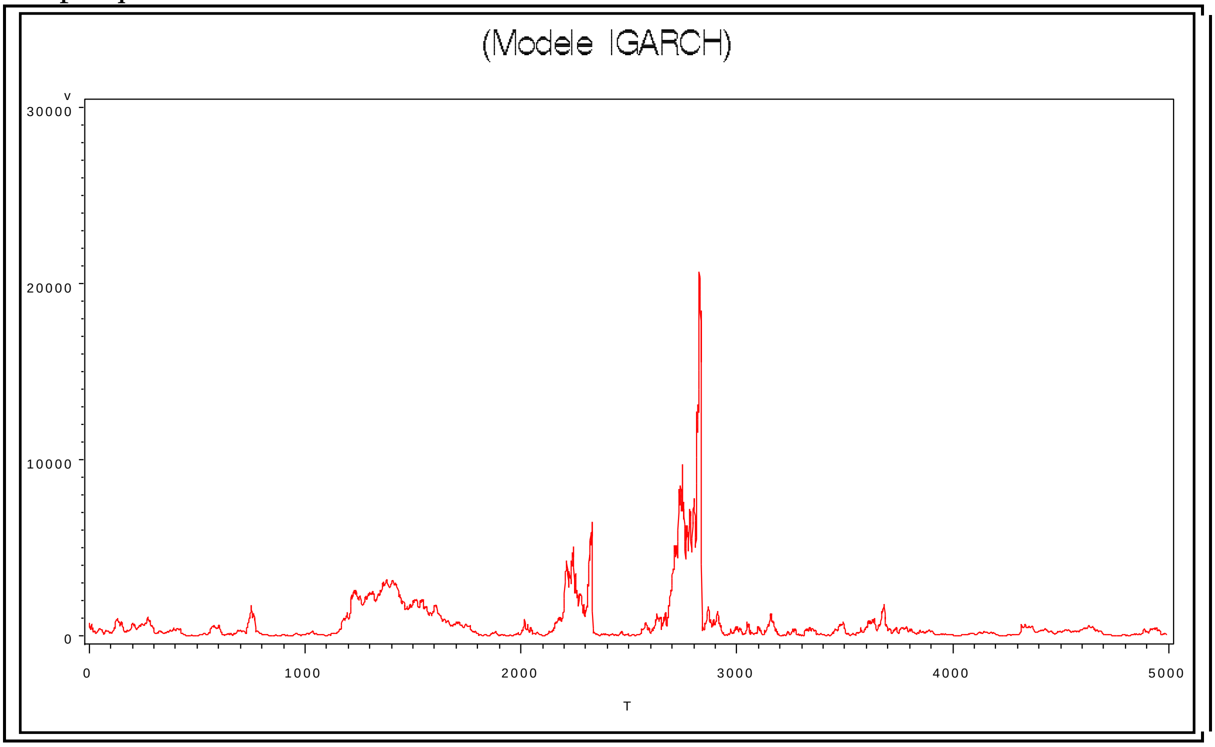
Algorithm converged.

Integrated GARCH Estimates

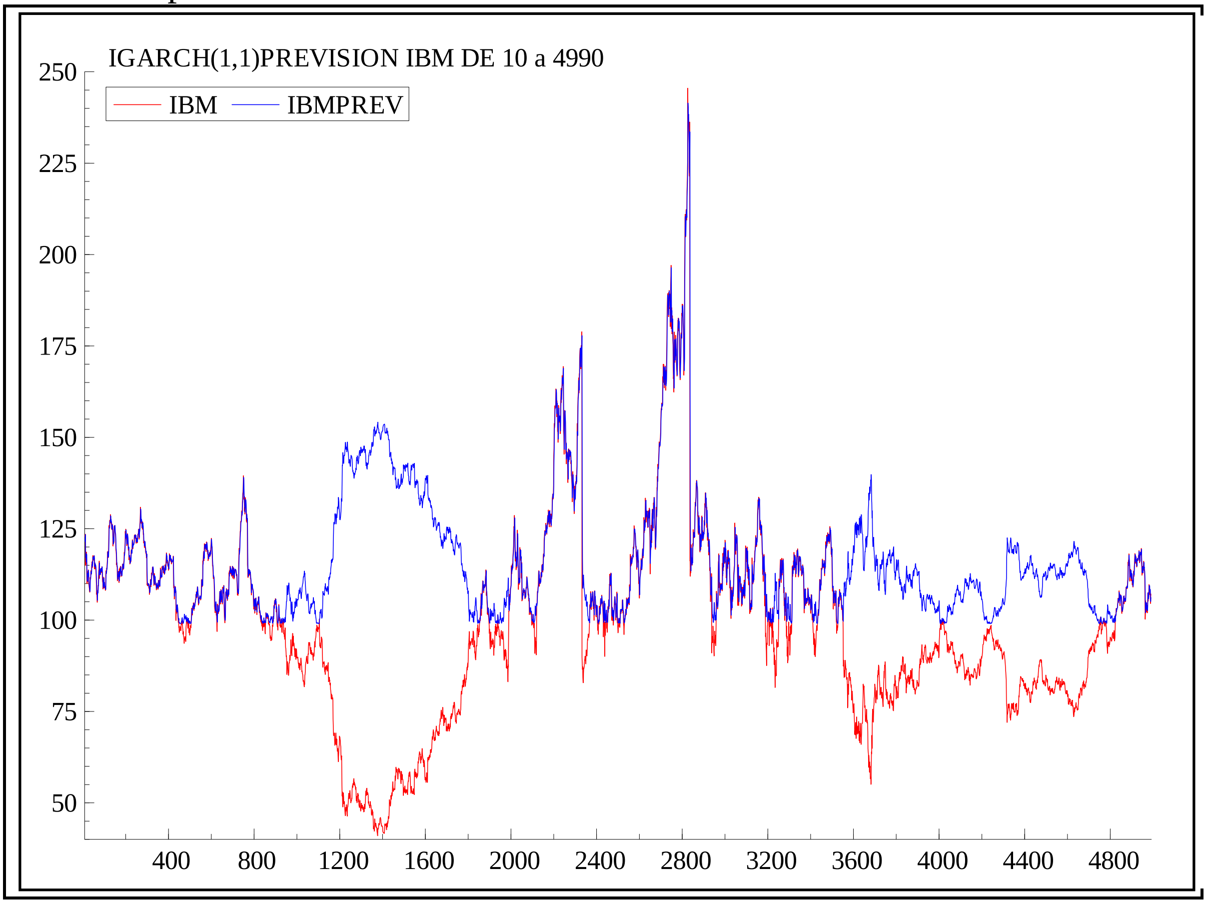
| | | | |
|---|---|---|---|
| SSE | 3458861.97 | Observations | 4990 |
| MSE | 693.15871 | Uncond Var | . |
| Log Likelihood | -19917.845 | Total R-Square | . |
| SBC | 39861.2358 | AIC | 39841.6903 |
| Normality Test | 433.4808 | Pr > ChiSq | <.0001 |

| Variable | DDL | Estimation | Standard Error | t Value | Approx Pr > \|t\| |
|---|---|---|---|---|---|
| Intercept | 1 | 97.6090 | 0.0669 | 1458.31 | <.0001 |
| ARCH0 | 1 | 1.6246 | 0.2024 | 8.03 | <.0001 |
| ARCH1 | 1 | 0.7940 | 0.0264 | 30.12 | <.0001 |
| GARCH1 | 1 | 0.2060 | 0.0264 | 7.81 | <.0001 |

Graphique de la variance conditionnelle :




Prévisions pour les deux modèles :




Prévision obtenue pour 4990 : 106.17787.

## 1.5. Spécification d'un EGARCH(1,1) :

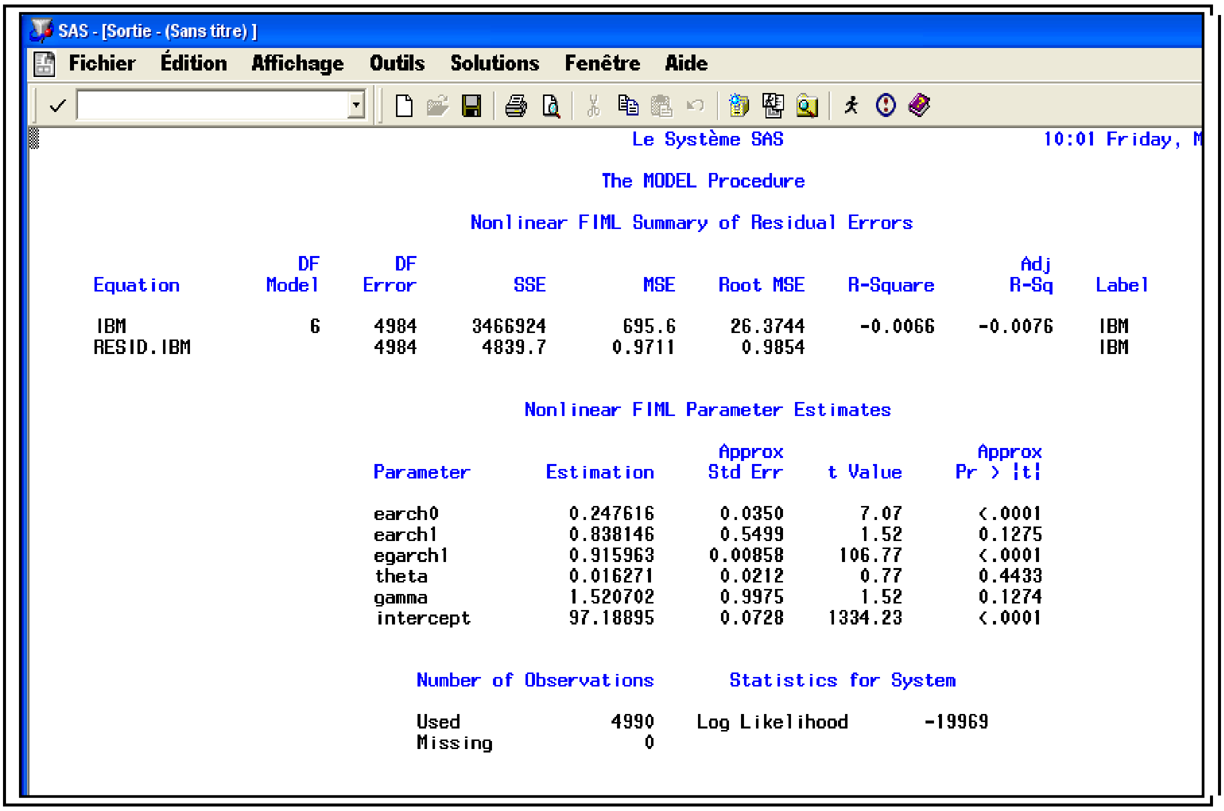
Le modèle EGARCH permet de prendre en considération que de bonnes nouvelles et de mauvaises nouvelles peuvent avoir un impact différent sur la volatilité. Il y a donc une évolution asymétrique de la volatilité.

$$IBM_t = cste + \varepsilon_t$$

$$\varepsilon_t = \nu_t \sqrt{h_t}$$

$$\log(h_t) = \alpha_0 + a_1 \nu_{t-1} + b_1\left(|\nu_{t-1}| - E\left[|\nu_{t-1}|\right]\right) + \beta_1 \log(h_{t-1})$$

Estimations obtenues :


SAS - [Sortie - (Sans titre) ]

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Le Système SAS 10:01 Friday, M

The MODEL Procedure

Nonlinear FIML Summary of Residual Errors

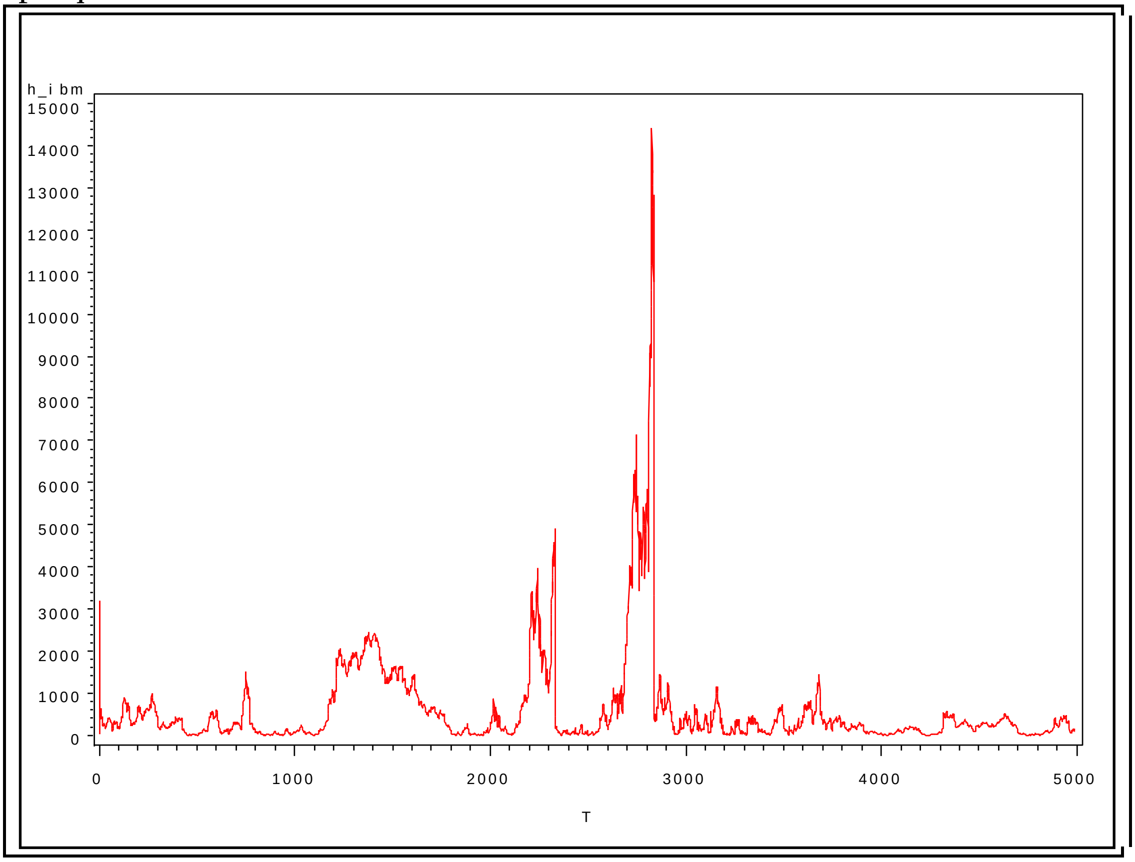
| Equation | DF Model | DF Error | SSE | MSE | Root MSE | R-Square | Adj R-Sq | Label |
|---|---|---|---|---|---|---|---|---|
| IBM | 6 | 4984 | 3466924 | 695.6 | 26.3744 | -0.0066 | -0.0076 | IBM |
| RESID.IBM | | 4984 | 4839.7 | 0.9711 | 0.9854 | | | IBM |

Nonlinear FIML Parameter Estimates

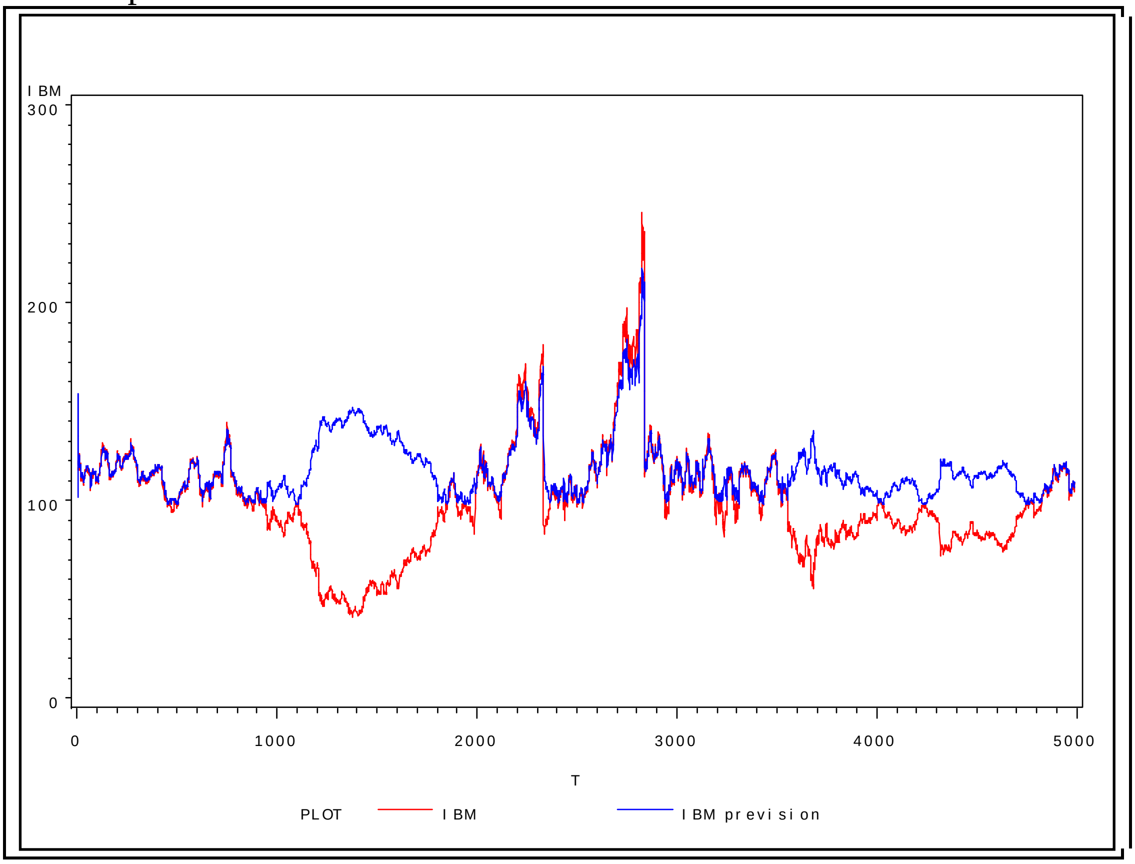
| Parameter | Estimation | Approx Std Err | t Value | Approx Pr > \|t\| |
|---|---|---|---|---|
| earch0 | 0.247616 | 0.0350 | 7.07 | <.0001 |
| earch1 | 0.838146 | 0.5499 | 1.52 | 0.1275 |
| egarch1 | 0.915963 | 0.00858 | 106.77 | <.0001 |
| theta | 0.016271 | 0.0212 | 0.77 | 0.4433 |
| gamma | 1.520702 | 0.9975 | 1.52 | 0.1274 |
| intercept | 97.18895 | 0.0728 | 1334.23 | <.0001 |

| Number of Observations | | Statistics for System | |
|---|---|---|---|
| Used | 4990 | Log Likelihood | -19969 |
| Missing | 0 | | |

Graphique de la variance conditionnelle :




Prévisions pour les deux modèles :




Prévision obtenue pour 4990 : 107.03.

## 1.6. Spécification d'un TGARCH(1,1) :

$$IBM_t = cste + \varepsilon_t$$

$$\varepsilon_t = \nu_t \sqrt{h_t}$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} + \gamma_1 \Omega \varepsilon_{t-1}^2$$

$$avec \quad \begin{cases} \Omega = 1 \quad si \quad \varepsilon_{t-1} < 0 \\ \Omega = 0 \quad si \quad \varepsilon_{t-1} \geq 0 \end{cases}$$

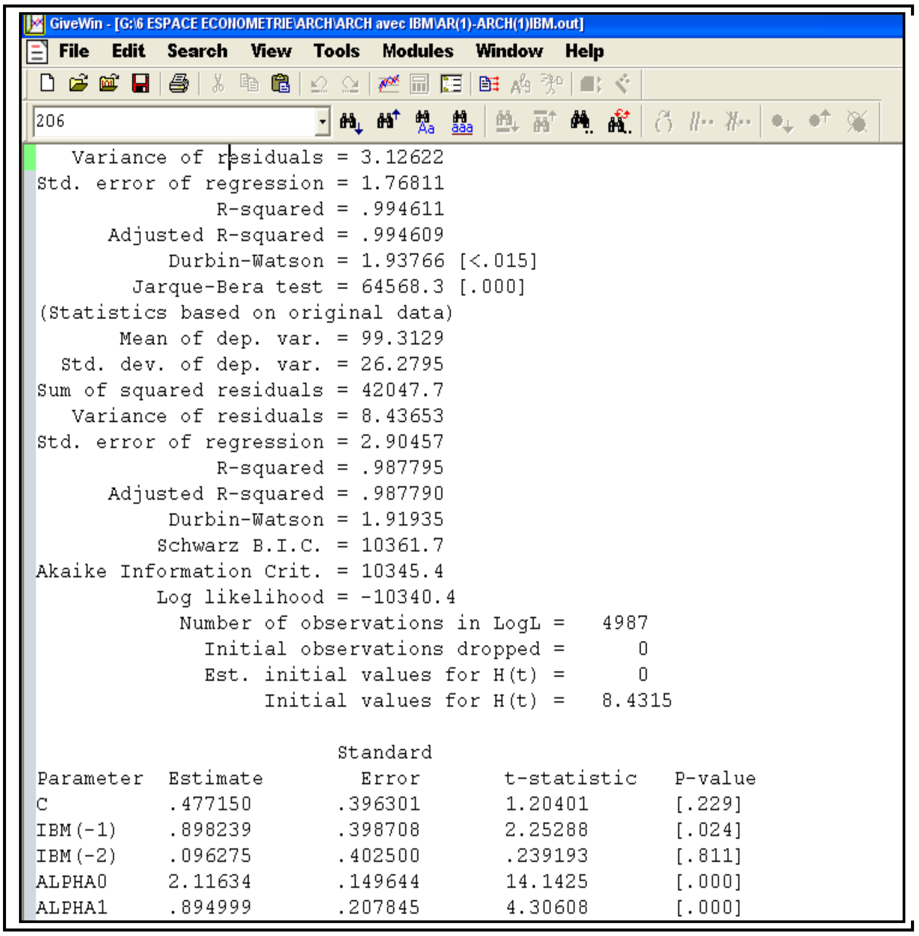
Malheureusement nous n'obtenons aucun résultat, il y a un problème de convergence des estimateurs….

## 1.7. Spécification d’un AR(2)-ARCH(1) :

Soit l’estimation d’un AR(2)-ARCH(1). La première partie (AR(2)) correspond à notre équation et la deuxième partie (ARCH(1)) à la variance conditionnelle, de la forme :

$$\boxed{\begin{aligned} &IBM_t = \beta_1 IBM_{t-1} + \beta_2 IBM_{t-2} + \varepsilon_t \\ &\varepsilon_t = \nu_t \sqrt{h_t} \\ &h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 \end{aligned}}$$

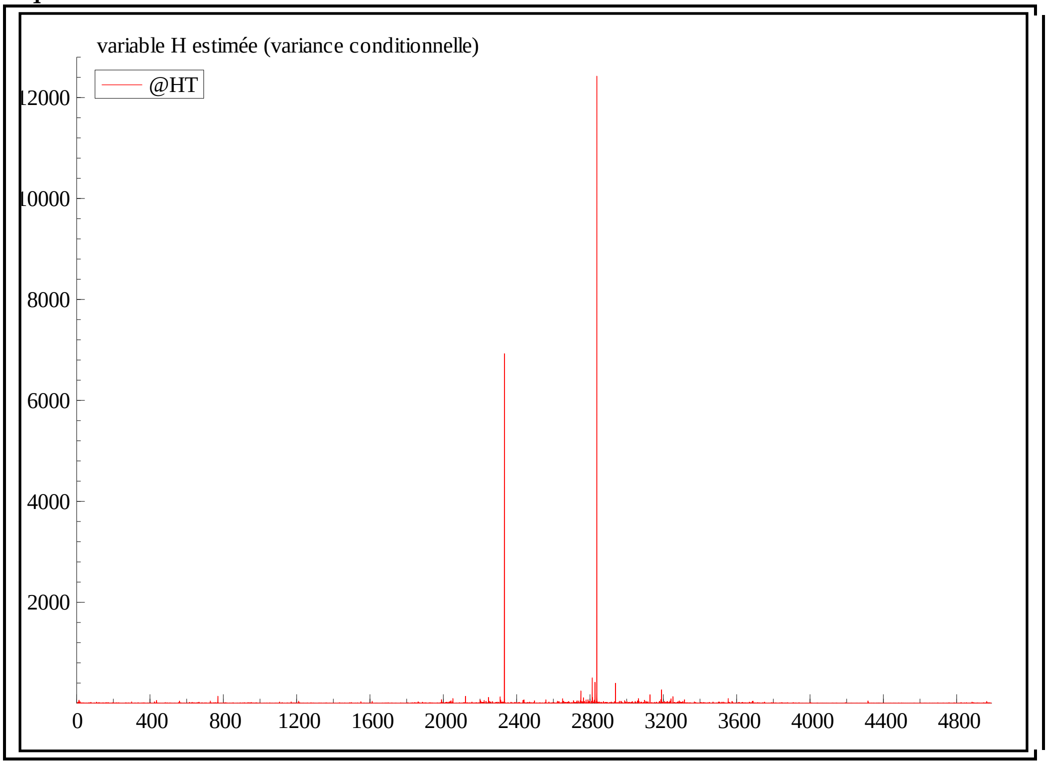
Nous obtenons les estimations suivantes :



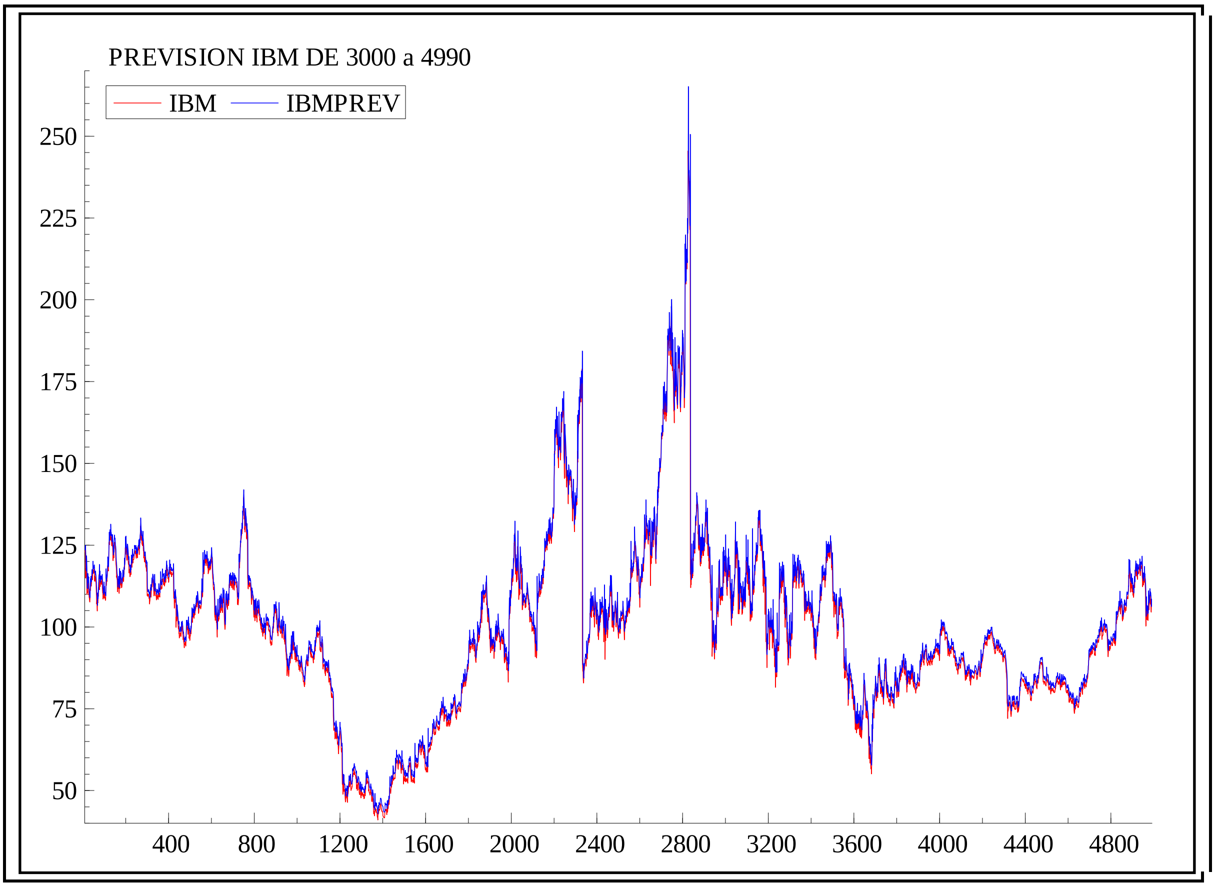

```
   Variance of residuals = 3.12622
Std. error of regression = 1.76811
               R-squared = .994611
      Adjusted R-squared = .994609
           Durbin-Watson = 1.93766 [<.015]
        Jarque-Bera test = 64568.3 [.000]
(Statistics based on original data)
        Mean of dep. var. = 99.3129
  Std. dev. of dep. var. = 26.2795
Sum of squared residuals = 42047.7
   Variance of residuals = 8.43653
Std. error of regression = 2.90457
               R-squared = .987795
      Adjusted R-squared = .987790
           Durbin-Watson = 1.91935
          Schwarz B.I.C. = 10361.7
Akaike Information Crit. = 10345.4
          Log likelihood = -10340.4
            Number of observations in LogL =   4987
              Initial observations dropped =      0
              Est. initial values for H(t) =      0
                   Initial values for H(t) =   8.4315

                           Standard
Parameter  Estimate          Error        t-statistic    P-value
C          .477150         .396301        1.20401        [.229]
IBM(-1)    .898239         .398708        2.25288        [.024]
IBM(-2)    .096275         .402500        .239193        [.811]
ALPHA0     2.11634         .149644        14.1425        [.000]
ALPHA1     .894999         .207845        4.30608        [.000]
```

Graphique de la variance conditionnelle :




Prévisions pour les deux modèles :




Prévision obtenue pour 4990 : 108.24143

## 2. Une stratégie de modélisation ?

Il n’existe aucune stratégie de modélisation à l’heure actuelle…

## CONCLUSION

Les modèles ARCH et associés sont utilisés dans la modélisation de l’incertitude notamment la volatilité des séries financières. Rappelons que la volatilité est une mesure de l’instabilité du cours d’un actif financier….Elle mesure donc l’amplitude des variations d’une action, d’un produit dérivé….Il existe des périodes de forte volatilité qui alternent avec des périodes de faible volatilité. Ce phénomène est appelé l’hétéroscédasticité conditionnelle. En d’autres termes la variance conditionnelle varie avec le temps. Cette condition, nous l’avons vu, ne peut pas être prise en compte par la modélisation ARIMA et même si l’apport ARCH n’est à l’heure actuelle que dans une phase expérimentale on peut dire, sans se tromper, que les modèles ARCH sont promis à un bel avenir…économétrique.

Bibliographie simple

Bollerslev, Tim (1986). "Generalized Autoregressive Conditional Heteroskedasticity," Journal of Econometrics, 31, 307-327.

Box, George E. P. and Gwilym M. Jenkins (1976). Time Series Analysis: Forecasting and Control, Revised Edition, Holden-Day.

Greene, W. Econométrie. 5e éd., Pearson Education, Paris, 2005. p 231-242

Engle, Robert F. (1982). "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of U.K. Inflation," Econometrica, 50, 987-1008.

Engle, Robert F. and K. F. Kroner (1995). "Multivariate Simultaneous Generalized ARCH," Econometric Theory, 11, 122-150.

Engle, Robert F., David M. Lilien, and Russell P. Robins (1987). "Estimating Time Varying Risk Premia in the Term Structure: The ARCH-M Model," Econometrica, 55, 391-407.

Engle, Robert F. and Victor K. Ng (1993). "Measuring and Testing the Impact of News on Volatility," Journal of Finance, 48, 1022-1082.

Droesbeke, J.J., Fichet, B. & Tassi, P. (1994). Modélisaton ARCH : théorie statistique et applications dans le domaine de la finance. Association pour la Statistique et ses Utilisations. Ellipses.

Bibliographie complète

Anderson, T. W. and Mentz, R. P. (1980), "On the Structure of the Likelihood Function of Autoregressive and Moving Average Models," *Journal of Time Series*, 1, 83–94.

Ansley, C. F., Kohn, R., and Shively, T. S. (1992), "Computing *p*-Values for the Generalized Durbin-Watson and Other Invariant Test Statistics," *Journal of Econometrics*, 54, 277–300.

Baillie, R. T. and Bollerslev, T. (1992), "Prediction in Dynamic Models with Time-Dependent Conditional Variances," *Journal of Econometrics*, 52, 91–113.

Balke, N. S. and Gordon, R. J. (1986), "Historical Data," in R. J. Gordon, ed., *The American Business Cycle*, 781–850, Chicago: The University of Chicago Press.

Bartels, R. (1982), "The Rank Version of von Neumann's Ratio Test for Randomness," *Journal of the American Statistical Association*, 77(377), 40–46.

Beach, C. M. and MacKinnon, J. G. (1978), "A Maximum Likelihood Procedure for Regression with Autocorrelated Errors," *Econometrica*, 46, 51–58.

Bhargava, A. (1986), "On the Theory of Testing for Unit Roots in Observed Time Series," *The Review of Economic Studies*, 53, 369–384.

Bollerslev, T. (1986), "Generalized Autoregressive Conditional Heteroskedasticity," *Journal of Econometrics*, 31, 307–327.

Bollerslev, T. (1987), "A Conditionally Heteroskedastic Time Series Model for Speculative Prices and Rates of Return," *The Review of Economics and Statistics*, 69, 542–547.

Box, G. E. P. and Jenkins, G. M. (1976), *Time Series Analysis: Forecasting and Control*, Revised Edition, San Francisco: Holden-Day.

Breitung, J. (1995), "Modified Stationarity Tests With Improved Power in Small Samples," *Statistical Papers*, 36, 77–95.

Breusch, T. S. and Pagan, A. R. (1979), "A Simple Test for Heteroscedasticity and Random Coefficient Variation," *Econometrica*, 47, 1287–1294.

Broock, W. A., Dechert, W. D., and Scheinkman, J. A. (1987), "A Test for Independence Based on the Correlation Dimension," Department of Economics, University of Wisconsin at Madison, University of Houston, and University of Chicago.

Broock, W. A., Scheinkman, J. A., Dechert, W. D., and LeBaron, B. (1996), "A Test for Independence Based on the Correlation Dimension," *Econometric Reviews*, 15(3), 197–235.

Campbell, J. Y. and Perron, P. (1991), "Pitfalls and Opportunities: What Macroeconomists Should Know About Unit Roots," *NBER Macroeconomics Annual*, 141–201.

Caner, M. and Kilian, L. (2001), "Size Distortions Of Tests Of The Null Hypothesis Of Stationarity: Evidence And Implications For The PPP Debate," *Journal of International Money and Finance*, 20, 639–657.

Chipman, J. S. (1979), "Efficiency of Least Squares Estimation of Linear Trend When Residuals Are Autocorrelated," *Econometrica*, 47, 115–128.

Chow, G. (1960), "Tests of Equality between Sets of Coefficients in Two Linear Regressions," *Econometrica*, 28, 531–534.

Cochrane, D. and Orcutt, G. H. (1949), "Application of Least Squares Regression to Relationships Containing Autocorrelated Error Terms," *Journal of the American Statistical Association*, 44, 32–61.

Cribari-Neto, F. (2004), "Asymptotic Inference under Heteroskedasticity of Unknown Form," *Computational Statistics and Data Analysis*, 45, 215–233.

Cromwell, J. B., Labys, W. C., and Terraza, M. (1994), *Univariate Tests for Time Series Models*, Thousand Oaks, CA: Sage.

Davidson, R. and MacKinnon, J. G. (1993), *Estimation and Inference in Econometrics*, New York: Oxford University Press.

Davies, R. B. (1973), "Numerical Inversion of a Characteristic Function," *Biometrika*, 60, 415–417.

DeJong, D. N., Nankervis, J. C., Savin, N. E., and Whiteman, C. H. (1992), "The Power Problems of Unit Root Rest in Time Series with Autoregressive Errors," *Journal of Econometrics*, 53, 323–343.

Dickey, D. A. and Fuller, W. A. (1979), "Distribution of the Estimators for Autoregressive Time Series with a Unit Root," *Journal of the American Statistical Association*, 74, 427–431.

Ding, Z., Granger, C. W. J., and Engle, R. (1993), "A Long Memory Property of Stock Market Returns and a New Model," *Journal of Empirical Finance*, 1, 83–106.

Duffie, D. (1989), *Futures Markets*, Englewood Cliffs, NJ: Prentice Hall.

Durbin, J. (1969), "Tests for Serial Correlation in Regression Analysis Based on the Periodogram of Least-Squares Residuals," *Biometrika*, 56, 1–15.

Durbin, J. (1970), "Testing for Serial Correlation in Least-Squares Regression When Some of the Regressors Are Lagged Dependent Variables," *Econometrica*, 38, 410–421.

Edgerton, D. and Wells, C. (1994), "Critical Values for the CUSUMSQ Statistic in Medium and Large Sized Samples," *Oxford Bulletin of Economics and Statistics*, 56, 355–365.

Elliott, G., Rothenberg, T. J., and Stock, J. H. (1996), "Efficient Tests for an Autoregressive Unit Root," *Econometrica*, 64, 813–836.

Engle, R. F. (1982), "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation," *Econometrica*, 50, 987–1007.

Engle, R. F. and Bollerslev, T. (1986), "Modelling the Persistence of Conditional Variances," *Econometric Review*, 5, 1–50.

Engle, R. F. and Granger, C. W. J. (1987), "Co-Integration and Error Correction: Representation, Estimation, and Testing," *Econometrica*, 55, 251–276.

Engle, R. F., Lilien, D. M., and Robins, R. P. (1987), "Estimating Time Varying Risk in the Term Structure: The ARCH-M Model," *Econometrica*, 55, 391–407.

Engle, R. F. and Ng, V. K. (1993), "Measuring and Testing the Impact of News on Volatility," *Journal of Finance*, 48, 1749–1778.

Engle, R. F. and Yoo, B. S. (1987), "Forecasting and Testing in Co-integrated Systems," *Journal of Econometrics*.

Fuller, W. A. (1976), *Introduction to Statistical Time Series*, New York: John Wiley & Sons.

Gallant, A. R. and Goebel, J. J. (1976), "Nonlinear Regression with Autoregressive Errors," *Journal of the American Statistical Association*, 71, 961–967.

Glosten, L., Jaganathan, R., and Runkle, D. (1993), "Relationship between the Expected Value and Volatility of the Nominal Excess Returns on Stocks," *Journal of Finance*, 48, 1779–1802.

Godfrey, L. G. (1978a), "Testing against General Autoregressive and Moving Average Error Models When the Regressors Include Lagged Dependent Variables," *Econometrica*, 46, 1293–1301.

Godfrey, L. G. (1978b), "Testing for Higher Order Serial Correlation in Regression Equations When the Regressors Include Lagged Dependent Variables," *Econometrica*, 46, 1303–1310.

Godfrey, L. G. (1988), *Misspecification Tests in Econometrics*, Cambridge: Cambridge University Press.

Golub, G. H. and Van Loan, C. F. (1989), *Matrix Computations*, Second Edition, Baltimore: Johns Hopkins University Press.
Greene, W. H. (1993), *Econometric Analysis*, Second Edition, New York: Macmillan.

Hamilton, J. D. (1994), *Time Series Analysis*, Princeton, NJ: Princeton University Press.

Hannan, E. J. and Quinn, B. G. (1979), "The Determination of the Order of an Autoregression," *Journal of the Royal Statistical Society, Series B*, 41, 190–195.

Harvey, A. C. (1981), *The Econometric Analysis of Time Series*, New York: John Wiley & Sons.

Harvey, A. C. (1990), *The Econometric Analysis of Time Series*, Second Edition, Cambridge: MIT Press.

Harvey, A. C. and McAvinchey, I. D. (1978), "The Small Sample Efficiency of Two-Step Estimators in Regression Models with Autoregressive Disturbances," Discussion Paper No. 78-10, University of British Columbia.

Harvey, A. C. and Phillips, G. D. A. (1979), "Maximum Likelihood Estimation of Regression Models with Autoregressive-Moving Average Disturbances," *Biometrika*, 66, 49–58.

Hayashi, F. (2000), *Econometrics*, Princeton University Press.

Hildreth, C. and Lu, J. Y. (1960), *Demand Relations with Autocorrelated Disturbances*, Technical Report 276, Michigan State University Agricultural Experiment Station, East Lansing, MI.

Hobijn, B., Franses, P. H., and Ooms, M. (2004), "Generalization of the KPSS-Test for Stationarity," *Statistica Neerlandica*, 58, 483–502.

Hurvich, C. M. and Tsai, C.-L. (1989), "Regression and Time Series Model Selection in Small Samples," *Biometrika*, 76, 297–307.

Inder, B. A. (1984), "Finite-Sample Power of Tests for Autocorrelation in Models Containing Lagged Dependent Variables," *Economics Letters*, 14, 179–185.

Inder, B. A. (1986), "An Approximation to the Null Distribution of the Durbin-Watson Statistic in Models Containing Lagged Dependent Variables," *Econometric Theory*, 2, 413–428.

Jarque, C. M. and Bera, A. K. (1980), "Efficient Tests for Normality, Homoskedasticity and Serial Independence of Regression Residuals," *Economics Letters*, 6, 255–259.

Johansen, S. (1988), "Statistical Analysis of Cointegration Vectors," *Journal of Economic Dynamics and Control*, 12, 231–254.

Johansen, S. (1991), "Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models," *Econometrica*, 59, 1551–1580.

Johnston, J. (1972), *Econometric Methods*, Second Edition, New York: McGraw-Hill.

Jones, R. H. (1980), "Maximum Likelihood Fitting of ARMA Models to Time Series with Missing Observations," *Technometrics*, 22, 389–396.

Judge, G. G., Griffiths, W. E., Hill, R. C., Lutkepohl, H., and Lee, T. C. (1985), *The Theory and Practice of Econometrics,* Second Edition, New York: John Wiley & Sons.

Kanzler, L. (1999), "Very Fast and Correctly Sized Estimation of the BDS Statistic," Christ Church and Department of Economics, University of Oxford.

King, M. L. and Wu, P. X. (1991), "Small-Disturbance Asymptotics and the Durbin-Watson and Related Tests in the Dynamic Regression Model," *Journal of Econometrics,* 47, 145–152.

Kwiatkowski, D., Phillips, P. C. B., Schmidt, P., and Shin, Y. (1992), "Testing the Null Hypothesis of Stationarity against the Alternative of a Unit Root," *Journal of Econometrics,* 54, 159–178.

Lee, J. H. and King, M. L. (1993), "A Locally Most Mean Powerful Based Score Test for ARCH and GARCH Regression Disturbances," *Journal of Business and Economic Statistics*, 11(1), 17–27.

L'Esperance, W. L., Chall, D., , and Taylor, D. (1976), "An Algorithm for Determining the Distribution Function of the Durbin-Watson Test Statistic," *Econometrica*, 44, 1325–1326.

MacKinnon, J. G. (1991), "Critical Values for Cointegration Tests," in *Long-Run Economic Relationships,* Oxford University Press.

Maddala, G. S. (1977), *Econometrics*, New York: McGraw-Hill.

Maeshiro, A. (1976), "Autoregressive Transformation, Trended Independent Variables and Autocorrelated Disturbance Terms," *Review of Economics and Statistics,* 58, 497–500.

McLeod, A. I. and Li, W. K. (1983), "Diagnostic Checking ARMA Time Series Models Using Squared-Residual Autocorrelations," *Journal of Time Series Analysis,* 4, 269–273.

Nelson, C. R. and Plosser, C. I. (1982), "Trends and Random Walks in Macroeconomic Time Series: Some Evidence and Implications," *Journal of Monetary Economics,* 10, 139–162.

Nelson, D. B. (1990), "Stationarity and Persistence in the GARCH(1,1) Model," *Econometric Theory,* 6, 318–334.

Nelson, D. B. (1991), "Conditional Heteroskedasticity in Asset Returns: A New Approach," *Econometrica*, 59, 347–370.

Nelson, D. B. and Cao, C. Q. (1992), "Inequality Constraints in the Univariate GARCH Model," *Journal of Business & Economic Statistics*, 10, 229–235.

Newey, W. K. and West, D. W. (1987), "A Simple, Positive Semi-definite, Heteroscedasticity and Autocorrelation Consistent Covariance Matrix," *Econometrica*, 55, 703–708.

Ng, S. and Perron, P. (2001), "Lag Length Selection and the Construction of Unit Root Tests with Good Size and Power," *Econometrica*, 69, 1519–1554.

Park, R. E. and Mitchell, B. M. (1980), "Estimating the Autocorrelated Error Model with Trended Data," *Journal of Econometrics*, 13, 185–201.
Phillips, P. (1997), "Time Series Regression with a Unit Root," *Econometrica*, 55, 277–301.

Phillips, P. C. and Ouliaris, S. (1990), "Asymptotic Properties of Residual Based Tests for Cointegration," *Econometrica*, 58, 165–193.

Phillips, P. C. and Perron, P. (1988), "Testing for a Unit Root in Time Series Regression," *Biometrika*, 75, 335–346.

Prais, S. J. and Winsten, C. B. (1954), "Trend Estimators and Serial Correlation," Cowles Commission Discussion Paper No. 383.

Ramsey, J. B. (1969), "Tests for Specification Errors in Classical Linear Least Squares Regression Analysis," *Journal of Royal Statistical Society*.

Said, S. E. and Dickey, D. A. (1984), "Testing for Unit Roots in ARMA Models of Unknown Order," *Biometrika*, 71, 599–607.

Savin, N. E. and White, K. J. (1978), "Testing for Autocorrelation with Missing Observations," *Econometrica*, 46, 59–67.

Schwarz, G. (1978), "Estimating the Dimension of a Model," *Annals of Statistics*, 6, 461–464.

Schwert, G. (1989), "Tests for Unit Roots: A Monte Carlo Investigation," *Journal of Business and Economic Statistics*, 7, 147–159.

Shively, T. S. (1990), “Fast Evaluation of the Distribution of the Durbin-Watson and Other Invariant Test Statistics in Time Series Regression,” *Journal of the American Statistical Association,* 85, 676–685.

Spitzer, J. J. (1979), “Small-Sample Properties of Nonlinear Least Squares and Maximum Likelihood Estimators in the Context of Autocorrelated Errors,” *Journal of the American Statistical Association*, 74, 41–47.

Stock, J. (1994), *Unit Roots, Structural Breaks and Trends,* chapter 46, 2739–2841, Elsevier.

Stock, J. H. and Watson, M. W. (1988), “Testing for Common Trends,” *Journal of the American Statistical Association*.

Theil, H. (1971), *Principles of Econometrics,* New York: John Wiley & Sons.
Vinod, H. D. (1973), “Generalization of the Durbin-Watson Statistic for Higher Order Autoregressive Process,” *Communication in Statistics*, 2, 115–144.

Wallis, K. F. (1972), “Testing for Fourth Order Autocorrelation in Quarterly Regression Equations,” *Econometrica,* 40, 617–636.

White, H. (1980), “A Heteroskedasticity-Consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticity,” *Econometrica*, 48, 817–838.

White, H. (1982), “Maximum Likelihood Estimation of Misspecified Models,” *Econometrica*, 50, 1–25.

White, K. J. (1992), “The Durbin-Watson Test for Autocorrelation in Nonlinear Models,” *Review of Economics and Statistics,* 74, 370–373.

Wong, H. and Li, W. K. (1995), “Portmanteau Test for Conditional Heteroscedasticity, Using Ranks of Squared Residuals,” *Journal of Applied Statistics*, 22(1), 121–134.

Zakoian, J. M. (1994), “Threshold Heteroscedastic Models,” *Journal of Economic Dynamics and Control*, 18, 931–955.

# EXTENSION

## Modèles univariés :

| | |
|---|---|
| ARCH | |
| GARCH | |
| saarch | simple asymmetric ARCH |
| tarch | threshold ARCH |
| aarch | asymmetric ARCH |
| narch | nonlinear ARCH |
| narchk | nonlinear ARCH with single shift |
| abarch | absolute value ARCH |
| atarch | absolute threshold ARCH |
| sdgarch | lags of st |
| earch | new in Nelson's EGARCH model |
| egarch | lags of $\ln(st^2)$ |
| parch | power ARCH |
| tparch | threshold power ARCH |
| aparch | asymmetric power ARCH |
| nparch | nonlinear power ARCH |
| nparchk | nonlinear power ARCH with single shift |
| pgarch | power GARCH |

| Common term | Options to specify |
|---|---|
| ARCH (Engle 1982) | arch() |
| GARCH (Bollerslev 1986) | arch() garch() |
| ARCH-in-mean (Engle, Lilien, and Robins 1987) | archm arch() [garch()] |
| GARCH with ARMA terms | arch() garch() ar() ma() |
| EGARCH (Nelson 1991) | earch() egarch() |
| TARCH, threshold ARCH (Zakoian 1994) | abarch() atarch() sdgarch() |
| GJR, form of threshold ARCH (Glosten, Jagannathan, and Runkle 1993) | arch() tarch() [garch()] |
| SAARCH, simple asymmetric ARCH (Engle 1990) | arch() saarch() [garch()] |
| PARCH, power ARCH (Higgins and Bera 1992) | parch() [pgarch()] |
| NARCH, nonlinear ARCH | narch() [garch()] |
| NARCHK, nonlinear ARCH with one shift | narchk() [garch()] |
| A-PARCH, asymmetric power ARCH (Ding, Granger, and Engle 1993) | aparch() [pgarch()] |
| NPARCH, nonlinear power ARCH | nparch() [pgarch()] |

## Modèles multivariés :

| | | |
|---|---|---|
| Mgarch | ccc | constant conditional correlation |
| Mgarch | dcc | dynamic conditional correlation |
| Mgarch | vcc | varying conditional correlation |
| Mgarch | dvech | diagonal vech |

**<u>Descriptions :</u>**

**mgarch ccc** (p 355) estimates the parameters of constant conditional correlation (CCC) multivariate generalized autoregressive conditionally heteroskedastic (MGARCH) models in which the conditional variances are modeled as univariate generalized autoregressive conditionally heteroskedastic (GARCH) models and the conditional covariances are modeled as nonlinear functions of the conditional variances. The conditional correlation parameters that weight the nonlinear combinations of the conditional variance are constant in the CCC MGARCH model.

❑ Technical note

Formally, the CCC MGARCH model derived by Bollerslev (1990) can be written as

$$\begin{aligned} \mathbf{y}_t &= \mathbf{C}\mathbf{x}_t + \boldsymbol{\epsilon}_t \\ \boldsymbol{\epsilon}_t &= \mathbf{H}_t^{1/2}\boldsymbol{\nu}_t \\ \mathbf{H}_t &= \mathbf{D}_t^{1/2}\mathbf{R}\mathbf{D}_t^{1/2} \end{aligned}$$



where

$\mathbf{y}_t$ is an $m \times 1$ vector of dependent variables;

$\mathbf{C}$ is an $m \times k$ matrix of parameters;

$\mathbf{x}_t$ is a $k \times 1$ vector of independent variables, which may contain lags of $\mathbf{y}_t$;

$\mathbf{H}_t^{1/2}$ is the Cholesky factor of the time-varying conditional covariance matrix $\mathbf{H}_t$;

$\boldsymbol{\nu}_t$ is an $m \times 1$ vector of normal, independent, and identically distributed innovations;

$\mathbf{D}_t$ is a diagonal matrix of conditional variances,

$$\mathbf{D}_t = \begin{pmatrix} \sigma_{1,t}^2 & 0 & \cdots & 0 \\ 0 & \sigma_{2,t}^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_{m,t}^2 \end{pmatrix}$$

in which each $\sigma_{i,t}^2$ evolves according to a univariate GARCH model of the form

$$\sigma_{i,t}^2 = s_i + \sum_{j=1}^{p_i} \alpha_j \epsilon_{i,t-j}^2 + \sum_{j=1}^{q_i} \beta_j \sigma_{i,t-j}^2$$

The CCC MGARCH model is less flexible than the dynamic conditional correlation MGARCH model (see [TS] mgarch dcc) and varying conditional correlation MGARCH model (see [TS] mgarch vcc), which specify GARCH-like processes for the conditional correlations. The conditional correlation MGARCH models are more parsimonious than the diagonal vech MGARCH model (see [TS] mgarch dvech).

**mgarch dcc** (p 375) estimates the parameters of dynamic conditional correlation (DCC) multivariate generalized autoregressive conditionally heteroskedastic (MGARCH) models in which the conditional variances are modeled as univariate generalized autoregressive conditionally heteroskedastic (GARCH) models and the conditional covariances are modeled as nonlinear functions of the conditional variances. The conditional quasicorrelation parameters that weight the nonlinear combinations of the conditional variances follow the GARCH-like process specified in Engle (2002).

The DCC MGARCH model is about as flexible as the closely related varying conditional correlation MGARCH model (see [TS] mgarch vcc), more flexible than the conditional correlation MGARCH model (see [TS] mgarch ccc), and more parsimonious than the diagonal vech MGARCH model (see [TS] mgarch dvech).

❑ Technical note

The DCC GARCH model proposed by Engle (2002) can be written as

$$
\begin{aligned}
\mathbf{y}_t &= \mathbf{C}\mathbf{x}_t + \boldsymbol{\epsilon}_t \\
\boldsymbol{\epsilon}_t &= \mathbf{H}_t^{1/2}\boldsymbol{\nu}_t \\
\mathbf{H}_t &= \mathbf{D}_t^{1/2}\mathbf{R}_t\mathbf{D}_t^{1/2} \\
\mathbf{R}_t &= \text{diag}(\mathbf{Q}_t)^{-1/2}\mathbf{Q}_t\text{diag}(\mathbf{Q}_t)^{-1/2} \\
\mathbf{Q}_t &= (1-\lambda_1-\lambda_2)\mathbf{R} + \lambda_1\widetilde{\boldsymbol{\epsilon}}_{t-1}\widetilde{\boldsymbol{\epsilon}}'_{t-1} + \lambda_2\mathbf{Q}_{t-1} \qquad (1)
\end{aligned}
$$

where

$\mathbf{y}_t$ is an $m \times 1$ vector of dependent variables;

$\mathbf{C}$ is an $m \times k$ matrix of parameters;

$\mathbf{x}_t$ is a $k \times 1$ vector of independent variables, which may contain lags of $\mathbf{y}_t$;

$\mathbf{H}_t^{1/2}$ is the Cholesky factor of the time-varying conditional covariance matrix $\mathbf{H}_t$;

$\nu_t$ is an $m \times 1$ vector of normal, independent, and identically distributed innovations;

$\mathbf{D}_t$ is a diagonal matrix of conditional variances,

$$\mathbf{D}_t = \begin{pmatrix} \sigma_{1,t}^2 & 0 & \cdots & 0 \\ 0 & \sigma_{2,t}^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_{m,t}^2 \end{pmatrix}$$

in which each $\sigma_{i,t}^2$ evolves according to a univariate GARCH model of the form

$\sigma_{i,t}^2 = s_i + \sum_{j=1}^{p_t} \alpha_j \epsilon_{i,t-j}^2 + \sum_{j=1}^{q_t} \beta_j \sigma_{i,t-j}^2$

by default, or

$\sigma_{i,t}^2 = \exp(\gamma_i \mathbf{z}_{i,t}) + \sum_{j=1}^{p_t} \alpha_j \epsilon_{i,t-j}^2 + \sum_{j=1}^{q_t} \beta_j \sigma_{i,t-j}^2$

when the `het()` option is specified, where $\gamma_t$ is a $1 \times p$ vector of parameters, $\mathbf{z}_i$ is a $p \times 1$ vector of independent variables including a constant term, the $\alpha_j$'s are ARCH parameters, and the $\beta_j$'s are GARCH parameters;

$\mathbf{R}_t$ is a matrix of conditional quasicorrelations,

$$\mathbf{R}_t = \begin{pmatrix} 1 & \rho_{12,t} & \cdots & \rho_{1m,t} \\ \rho_{12,t} & 1 & \cdots & \rho_{2m,t} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{1m,t} & \rho_{2m,t} & \cdots & 1 \end{pmatrix}$$

$\widetilde{\epsilon}_t$ is an $m \times 1$ vector of standardized residuals, $\mathbf{D}_t^{-1/2}\epsilon_t$; and

$\lambda_1$ and $\lambda_2$ are parameters that govern the dynamics of conditional quasicorrelations. $\lambda_1$ and $\lambda_2$ are nonnegative and satisfy $0 \leq \lambda_1 + \lambda_2 < 1$.

When $\mathbf{Q}_t$ is stationary, the $\mathbf{R}$ matrix in (1) is a weighted average of the unconditional covariance matrix of the standardized residuals $\widetilde{\epsilon}_t$, denoted by $\overline{\mathbf{R}}$, and the unconditional mean of $\mathbf{Q}_t$, denoted by $\overline{\mathbf{Q}}$. Because $\overline{\mathbf{R}} \neq \overline{\mathbf{Q}}$, as shown by Aielli (2009), $\mathbf{R}$ is neither the unconditional correlation matrix nor the unconditional mean of $\mathbf{Q}_t$. For this reason, the parameters in $\mathbf{R}$ are known as quasicorrelations; see Aielli (2009) and Engle (2009) for discussions.

❑

**mgarch dvech** (p 395) estimates the parameters of diagonal vech (DVECH) multivariate generalized autoregressive conditionally heteroskedastic (MGARCH) models in which each element of the conditional correlation matrix is parameterized as a linear function of its own past and past shocks. DVECH MGARCH models are less parsimonious than the conditional correlation models discussed in [TS] mgarch ccc, [TS] mgarch dcc, and [TS] mgarch vcc because the number of parameters in DVECH MGARCH models increases more rapidly with the number of series modeled.

❑ Technical note

The general vech MGARCH model developed by Bollerslev, Engle, and Wooldridge (1988) can be written as

$$\mathbf{y}_t = \mathbf{C}\mathbf{x}_t + \boldsymbol{\epsilon}_t \tag{1}$$

$$\boldsymbol{\epsilon}_t = \mathbf{H}_t^{1/2}\boldsymbol{\nu}_t \tag{2}$$

$$\mathbf{h}_t = \mathbf{s} + \sum_{i=1}^{p} \mathbf{A}_i \text{vech}(\boldsymbol{\epsilon}_{t-i}\boldsymbol{\epsilon}'_{t-i}) + \sum_{j=1}^{q} \mathbf{B}_j \mathbf{h}_{t-j} \tag{3}$$

where

$\mathbf{y}_t$ is an $m \times 1$ vector of dependent variables;

$\mathbf{C}$ is an $m \times k$ matrix of parameters;

$\mathbf{x}_t$ is a $k \times 1$ vector of independent variables, which may contain lags of $\mathbf{y}_t$;

$\mathbf{H}_t^{1/2}$ is the Cholesky factor of the time-varying conditional covariance matrix $\mathbf{H}_t$;

$\boldsymbol{\nu}_t$ is an $m \times 1$ vector of independent and identically distributed innovations;

$\mathbf{h}_t = \text{vech}(\mathbf{H}_t)$;

the vech() function stacks the lower diagonal elements of a symmetric matrix into a column vector, for example,

$$\text{vech}\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix} = (1,\ 2,\ 3)'$$

$\mathbf{s}$ is an $m(m+1)/2 \times 1$ vector of parameters;

each $\mathbf{A}_i$ is an $\{m(m+1)/2\} \times \{m(m+1)/2\}$ matrix of parameters; and

each $\mathbf{B}_j$ is an $\{m(m+1)/2\} \times \{m(m+1)/2\}$ matrix of parameters.

**mgarch vcc** (p 414) estimates the parameters of varying conditional correlation (VCC) multivariate generalized autoregressive conditionally heteroskedastic (MGARCH) models in which the conditional variances are modeled as univariate generalized autoregressive conditionally heteroskedastic (GARCH) models and the conditional covariances are modeled as nonlinear functions of the conditional variances. The conditional correlation parameters that weight the nonlinear combinations of the conditional variance follow the GARCH-like process specified in Tse and Tsui (2002).
The VCC MGARCH model is about as flexible as the closely related dynamic conditional correlation MGARCH model (see [TS] mgarch dcc), more flexible than the conditional correlation MGARCH model (see [TS] mgarch ccc), and more parsimonious than the diagonal vech model (see [TS] mgarch dvech).

❑ Technical note

The VCC GARCH model proposed by Tse and Tsui (2002) can be written as

$$\begin{aligned} \mathbf{y}_t &= \mathbf{C}\mathbf{x}_t + \boldsymbol{\epsilon}_t \\ \boldsymbol{\epsilon}_t &= \mathbf{H}_t^{1/2}\boldsymbol{\nu}_t \\ \mathbf{H}_t &= \mathbf{D}_t^{1/2}\mathbf{R}_t\mathbf{D}_t^{1/2} \\ \mathbf{R}_t &= (1-\lambda_1-\lambda_2)\mathbf{R} + \lambda_1\boldsymbol{\Psi}_{t-1} + \lambda_2\mathbf{R}_{t-1} \end{aligned} \tag{1}$$

where

$\mathbf{y}_t$ is an $m \times 1$ vector of dependent variables;

$\mathbf{C}$ is an $m \times k$ matrix of parameters;

$\mathbf{x}_t$ is a $k \times 1$ vector of independent variables, which may contain lags of $\mathbf{y}_t$;

$\mathbf{H}_t^{1/2}$ is the Cholesky factor of the time-varying conditional covariance matrix $\mathbf{H}_t$;

$\nu_t$ is an $m \times 1$ vector of independent and identically distributed innovations;

$\mathbf{D}_t$ is a diagonal matrix of conditional variances,

$$\mathbf{D}_t = \begin{pmatrix} \sigma_{1,t}^2 & 0 & \cdots & 0 \\ 0 & \sigma_{2,t}^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_{m,t}^2 \end{pmatrix}$$

in which each $\sigma_{i,t}^2$ evolves according to a univariate GARCH model of the form

$$\sigma_{i,t}^2 = s_i + \sum_{j=1}^{p_i} \alpha_j \epsilon_{i,t-j}^2 + \sum_{j=1}^{q_i} \beta_j \sigma_{i,t-j}^2$$

by default, or

$$\sigma_{i,t}^2 = \exp(\gamma_i \mathbf{z}_{i,t}) + \sum_{j=1}^{p_i} \alpha_j \epsilon_{i,t-j}^2 + \sum_{j=1}^{q_i} \beta_j \sigma_{i,t-j}^2$$

when the `het()` option is specified, where $\gamma_t$ is a $1 \times p$ vector of parameters, $\mathbf{z}_i$ is a $p \times 1$ vector of independent variables including a constant term, the $\alpha_j$'s are ARCH parameters, and the $\beta_j$'s are GARCH parameters;

$\mathbf{R}_t$ is a matrix of conditional correlations,

$$\mathbf{R}_t = \begin{pmatrix} 1 & \rho_{12,t} & \cdots & \rho_{1m,t} \\ \rho_{12,t} & 1 & \cdots & \rho_{2m,t} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{1m,t} & \rho_{2m,t} & \cdots & 1 \end{pmatrix}$$

$\mathbf{R}$ is the matrix of means to which the dynamic process in (1) reverts;

$\mathbf{\Psi}_t$ is the rolling estimator of the correlation matrix of $\widetilde{\epsilon}_t$, which uses the previous $m+1$ observations; and

$\lambda_1$ and $\lambda_2$ are parameters that govern the dynamics of conditional correlations. $\lambda_1$ and $\lambda_2$ are nonnegative and satisfy $0 \leq \lambda_1 + \lambda_2 < 1$.