Assignment 4: Random Forests and Gradient Boosting

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https://github.com/clboetticher/practicalML/blob/master/Boetticher_MSDS%20Assignment4.ipynb

Data and assumptions

The Boston Housing Study is a market response study for 506 census tracts in the Boston metropolitan area, with data on 13 house attributes, community/society attributes, and environmental attributes. This study examines whether the median sale price of a home can be more reliably predicted as a function of these attributes via linear regression methods, random forests and ensemble methods. This report assumes this study's data represents the larger Boston metropolitan market sufficiently enough given the city's various social, economic, and political circumstances at the time of the study and that the recommended model will be applied in the same period.

Study design and initial findings

Five regression modeling methods are initially evaluated across three separately-transformed Boston housing datasets: Linear, Ridge, Lasso, ElasticNet, Random Forest, and Extra Trees. A k-fold cross-validation design with five modeling methods is employed so that all 506 samples can be used for training and testing, which can help reduce model bias and provide a more rigorous approach to examining how models perform on unseen data. Root mean-squared error is used as an index of prediction error, indicating how closely the observed data points from the original study are to each model's predicted MV values. The Random Forest and Extra Trees methods performed best on Box Cox-transformed data, likely given their handling of outliers (see Table 2 for outlier data on each feature) and lower bias and moderate variance, in general. Extra Trees regression was selected for further fine-tuning using grid search, then re-fitted to the full dataset. Performance on the full dataset improved, with R squared of 0.895 and mean squared error of 0.003. Figure 15 shows the goodness of fit with the predictions versus actual mv values, showing a close linear relationship (Figure 15). LSTAT (percentage of homeowners considered "lower class"), (average number of rooms among homes) , CRIM (per capita crime rate by town), and PTRATIO (ratio of students to teachers in schools) are the most important features calculated for Extra Trees (Figure 16); feature importance could be more considered with random forest models, which will be taken into account in final recommendations, as well as the higher bias and lower variance.

Random Forest and ensemble method evaluation

Decision tree regressor learners are tested on individual predictor features (LSTAT, ROOMS, and PTRATIO, Figures 17-19) then on the larger set of 12 predictor features (Figure 20) to assess optimal depths. This provides a useful introduction to ensemble method evaluations, which will likely generalize better than individual decision trees due to the randomness that helps decrease model variance. Gradient boosting (Figure 21) and stochastic gradient boosting (Figure 22) are then tested as combined options of multiple weak learners across a range of learning rates for the dataset. A model with a slower learning rate of 0.3 features gives the best performance on the test set and good performance on the training set. As a final ensemble method evaluation, a stacked model approach blending Random Forest, Lasso, and Gradient Boosting learners and a Ridge regressor to combine outputs is tested to determine potential to outperform the prediction performance of individual models from the study (Figure 23). Performance metrics for training each regressor on the full dataset are as follows:

Metric	Extra Trees	Decision Tree	Gradient Boosting	Stochastic Gradient Boosting	Stacked
R squared	0.895	0.827	0.864	0.757	.72 +- .07
MSE	0.003	0.005	0.004	0.007	.01

Recommendation and caveats

The Extra Trees and Gradient Boosting regressors exhibited better performance and more reliable predictions on the test data (i.e., lower variance) after fine-tuning through grid search and other parameter comparisons. The Extra Trees model benefits from faster training and overfitting from individual trees averaging out, which could prove useful if the dataset grows or evolves in complexity with new features. This model performs well except at the high and low end of the MV scale, so caution is advised as the scale of median prices changes with new data added. The Gradient Boosting regressor has the added benefit of predictions that may generalize better to unseen housing data with the shrinkage regularization technique (learning rate lowered) applied. Beyond the utility of the models themselves, the feature importance values could also serve as a useful means for identifying compelling subsets of housing features to employ in additional predictive models (all predictor features except neighborhood are included in this study's model). This measure of how much each feature reduces impurity on average across all trees in the forest shows four predictor features consistently ranking highest across all tested techniques: ROOMS, LSTAT, CRIM, and PTRATIO. As an overall caveat to any recommended use of these models and current feature predictive potential, real estate circumstances change rapidly with time. Sale prices in this 1978 dataset are primarily in the $20-24,000 range, capped at $50,000. Depending on when models based on this data are applied, the distribution of home prices will have likely changed so caution is advised making assumptions of home value using this dataset due to the rapidly-evolving factors that feed into how a home is priced.

Assignment 4: Random Forests and Gradient Boosting

Use all explanatory variables (except neighborhood) and all 506 census tract observations from the Boston Housing Study. Use one of two response variables: (1) the median value of homes in thousands of 1970 dollars or (2) the log median value of homes in thousands of 1970 dollars. Employ at least two regression modeling methods selected from those discussed in Chapter 4 of the Géron (2017) textbook: linear regression, stochastic gradient descent, ridge regression, lasso regression, and elastic net. Also employ random forests to the regression problem, following methods described in Géron (2017) Chapter 7. Evaluate these methods within a cross-validation design, using root mean-squared error (RMSE) as an index of prediction error. Python scikit-learn should be your primary environment for conducting this research.

Try alternative versions of random forests and gradient boosting. Select a best modeling method for the Boston Housing Study. Employ that method on the full data set, obtaining results that you can report to management.

Regarding the management problem, imagine that you again are advising a real estate brokerage firm in its attempt to employ machine learning methods. The firm wants to use machine learning to complement conventional methods for assessing the market value of residential real estate. Of the modeling methods examined in your study, which would you recommend to management and why? Reviewing the results of the random forests and gradient boosting model you have selected to present to management, which explanatory variables are most important in predicting home prices?

Boston house prices dataset

The Boston Housing Study is a market response study of sorts, with the market being 506 census tracts in the Boston metropolitan area. The objective of the study was to examine the effect of air pollution on housing prices, controlling for the effects of other explanatory variables. The response variable is the median price of homes in the census track.

Data Set Characteristics:

```
:Number of Instances: 506

:Number of Attributes: 13 numeric/categorical predictive. Median Value (attribute 14) is usually the target.

:Attribute Information (in order):
    - CRIM     per capita crime rate by town
    - ZN       proportion of residential land zoned for lots over 25,000 sq.ft.
    - INDUS    proportion of non-retail business acres per town
    - CHAS     Charles River dummy variable (= 1 if tract bounds river; 0 otherwise)
    - NOX      nitric oxides concentration (parts per 10 million)
    - RM       average number of rooms per dwelling
    - AGE      proportion of owner-occupied units built prior to 1940
    - DIS      weighted distances to five Boston employment centres
    - RAD      index of accessibility to radial highways
    - TAX      full-value property-tax rate per 10,000 (USD)
    - PTRATIO  pupil-teacher ratio by town
    - B        1000(Bk - 0.63)^2 where Bk is the proportion of blacks by town
    - LSTAT    % lower status of the population
    - MEDV     Median value of owner-occupied homes in 1000's (USD)
```

In [22]:
```
# Import dependencies for analysis/data prep
import numpy as np
import pandas as pd
import os
from math import sqrt  # for root mean-squared error calculation
import itertools
from scipy import stats as st
import random
import time

# Modeling routines from Scikit Learn packages
from sklearn.metrics import mean_squared_error, r2_score
from sklearn import preprocessing  # feature transformations
from sklearn.compose import ColumnTransformer   # for scaling particular features
from sklearn.linear_model import LinearRegression, Ridge, RidgeCV, Lasso, LassoCV, ElasticNet
from sklearn.svm import LinearSVC
from sklearn.experimental import enable_hist_gradient_boosting  # noqa
from sklearn.ensemble import RandomForestRegressor, ExtraTreesRegressor, BaggingRegressor, HistGradientBoostingRegress
or, GradientBoostingRegressor, AdaBoostRegressor, StackingRegressor
from sklearn.neighbors import KNeighborsRegressor
from sklearn.tree import DecisionTreeRegressor, export_graphviz, plot_tree
from sklearn.model_selection import KFold, train_test_split, cross_val_score, GridSearchCV, cross_val_predict
# import xgboost

# Plotting and visualization
from yellowbrick.model_selection import CVScores, FeatureImportances
from yellowbrick.regressor import PredictionError, ResidualsPlot
from yellowbrick.regressor.alphas import AlphaSelection
import matplotlib.pyplot as plt  # static plotting
import seaborn as sns  # pretty plotting, including heat map
%matplotlib inline
```

Data preparation and initial inspection

In [3]:
```
# seed value for random number generators to obtain reproducible results
RANDOM_SEED = 1

# although we standardize X and y variables on input,
# we will fit the intercept term in the models
# Expect fitted values to be close to zero
SET_FIT_INTERCEPT = True
```

In [4]:
```
# Read in Boston housing data to a pandas DataFrame
boston_input = pd.read_csv('boston.csv')
```

In [5]:
```
# Inspect first 5 rows of data
boston_input.head()
```

Out[5]:

	neighborhood	crim	zn	indus	chas	nox	rooms	age	dis	rad	tax	ptratio	lstat	mv
0	Nahant	0.00632	18.0	2.31	0	0.538	6.575	65.2	4.0900	1	296	15.3	4.98	24.0
1	Swampscott	0.02731	0.0	7.07	0	0.469	6.421	78.9	4.9671	2	242	17.8	9.14	21.6
2	Swanpscott	0.02729	0.0	7.07	0	0.469	7.185	61.1	4.9671	2	242	17.8	4.03	34.7
3	Marblehead	0.03237	0.0	2.18	0	0.458	6.998	45.8	6.0622	3	222	18.7	2.94	33.4
4	Marblehead	0.06905	0.0	2.18	0	0.458	7.147	54.2	6.0622	3	222	18.7	5.33	36.2

In [6]:
```
# Drop neighborhood attribute
boston_input = boston_input.drop('neighborhood', 1)
```

In [7]:
```
# Check for missing values
boston_input.isnull().sum()
```

Out[7]:
```
crim       0
zn         0
indus      0
chas       0
nox        0
rooms      0
age        0
dis        0
rad        0
tax        0
ptratio    0
lstat      0
mv         0
dtype: int64
```

Descriptive statistics and visualizations of original data

In [7]:
```
# Inspect data shape: 506 rows, 13 columns/features/variables/attributes
boston_input.shape
```

Out[7]:
```
(506, 13)
```

Table 1: Initial descriptive statistics on original data

In [29]:
```
# Inspect data
boston_input.describe(include='all')
```

Out[29]:

	crim	zn	indus	chas	nox	rooms	age	dis	rad	tax	ptratio	lstat	
count	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.00
mean	3.613524	11.363636	11.136779	0.069170	0.554695	6.284634	68.574901	3.795043	9.549407	408.237154	18.455534	12.653063	22.52
std	8.601545	23.322453	6.860353	0.253994	0.115878	0.702617	28.148861	2.105710	8.707259	168.537116	2.164946	7.141062	9.18
min	0.006320	0.000000	0.460000	0.000000	0.385000	3.561000	2.900000	1.129600	1.000000	187.000000	12.600000	1.730000	5.00
25%	0.082045	0.000000	5.190000	0.000000	0.449000	5.885500	45.025000	2.100175	4.000000	279.000000	17.400000	6.950000	17.02
50%	0.256510	0.000000	9.690000	0.000000	0.538000	6.208500	77.500000	3.207450	5.000000	330.000000	19.050000	11.360000	21.20
75%	3.677082	12.500000	18.100000	0.000000	0.624000	6.623500	94.075000	5.188425	24.000000	666.000000	20.200000	16.955000	25.00
max	88.976200	100.000000	27.740000	1.000000	0.871000	8.780000	100.000000	12.126500	24.000000	711.000000	22.000000	37.970000	50.00

Figure 1: Pair plot of feature relationships

In [12]:
```
# Create pair plot for initial inspection of relationships
sns.pairplot(boston_input, diag_kind='hist')

# mv increases as the value of rooms increases
# mv decreases with increase in lstat
```

Out[12]: <seaborn.axisgrid.PairGrid at 0x11ef74cf8>

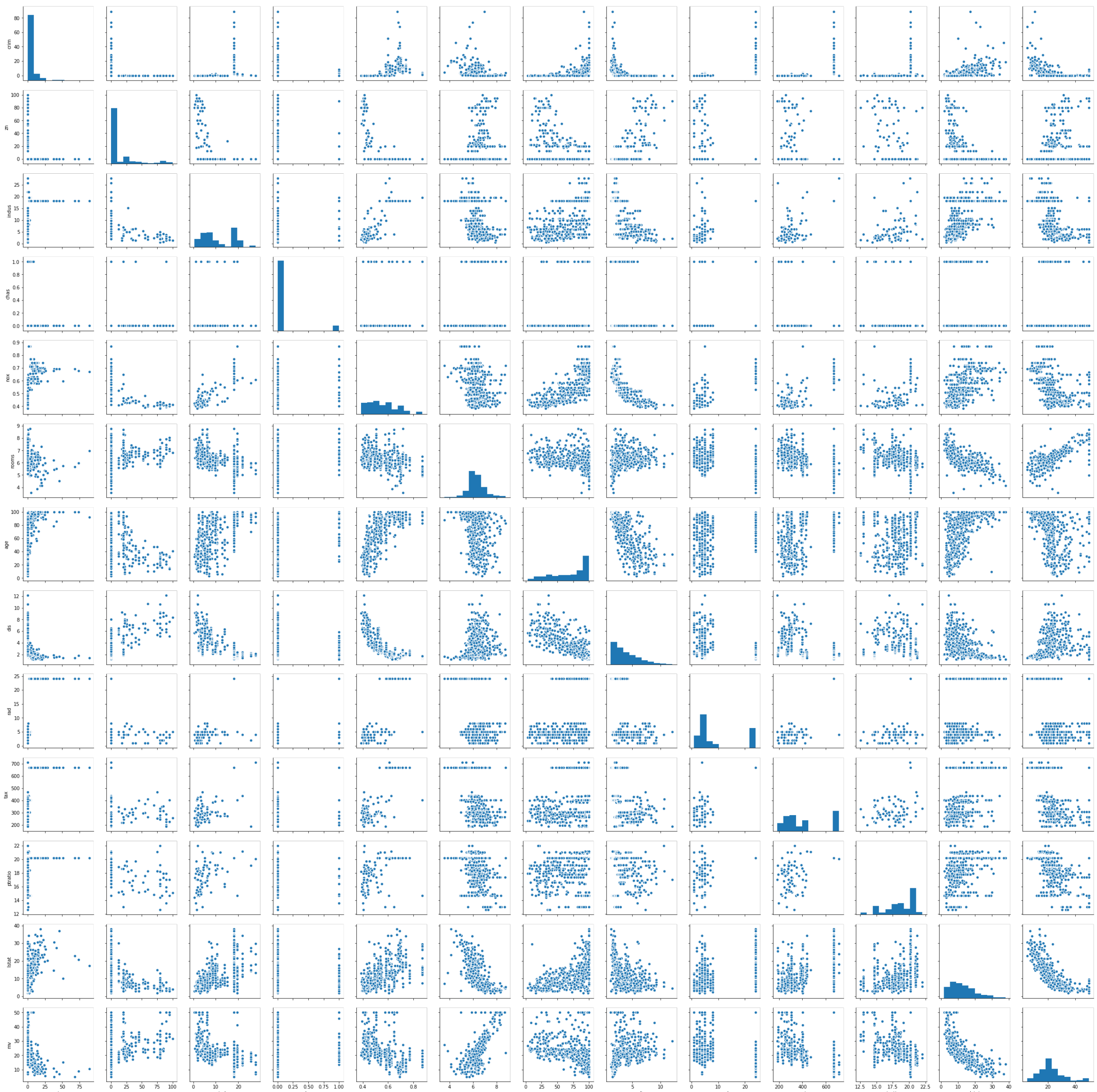

Figure 2: Box plots of original data

In [7]:
```
# Create boxplot of raw data to examine values, ranges, and measures of centrality
boston_input.boxplot(vert=False, figsize=(10,10), grid=False)
```

Out[7]: <matplotlib.axes._subplots.AxesSubplot at 0x11c0ba630>

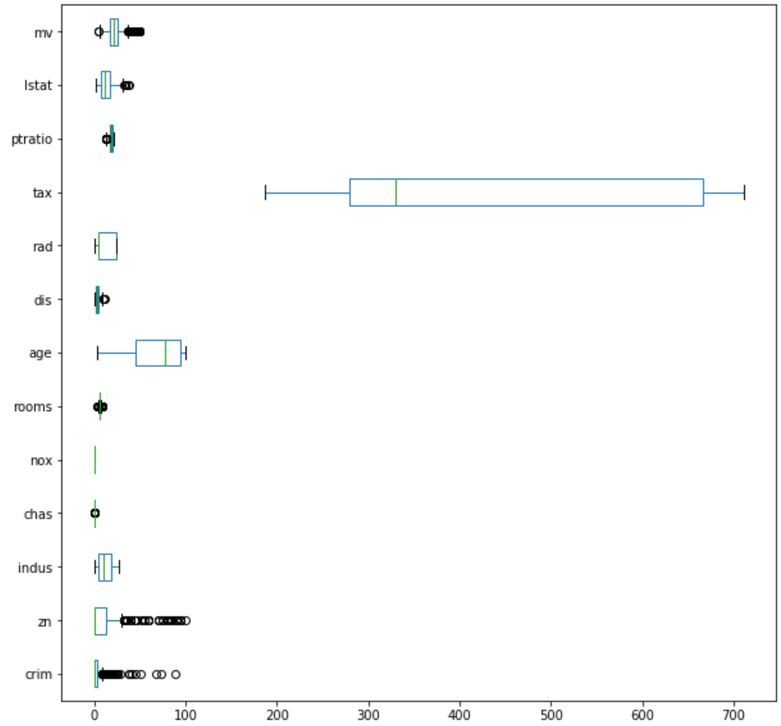

Figure 3: Alternate box plots of original data

In [9]:
```
# Alternate look at features with boxplot
fig, axs = plt.subplots(ncols=7, nrows=2, figsize=(20, 10))
index = 0
axs = axs.flatten()
for k,v in boston_input.items():
    sns.boxplot(y=k, data=boston_input, ax=axs[index])
    index += 1
plt.tight_layout(pad=0.4, w_pad=0.5, h_pad=5.0)
```

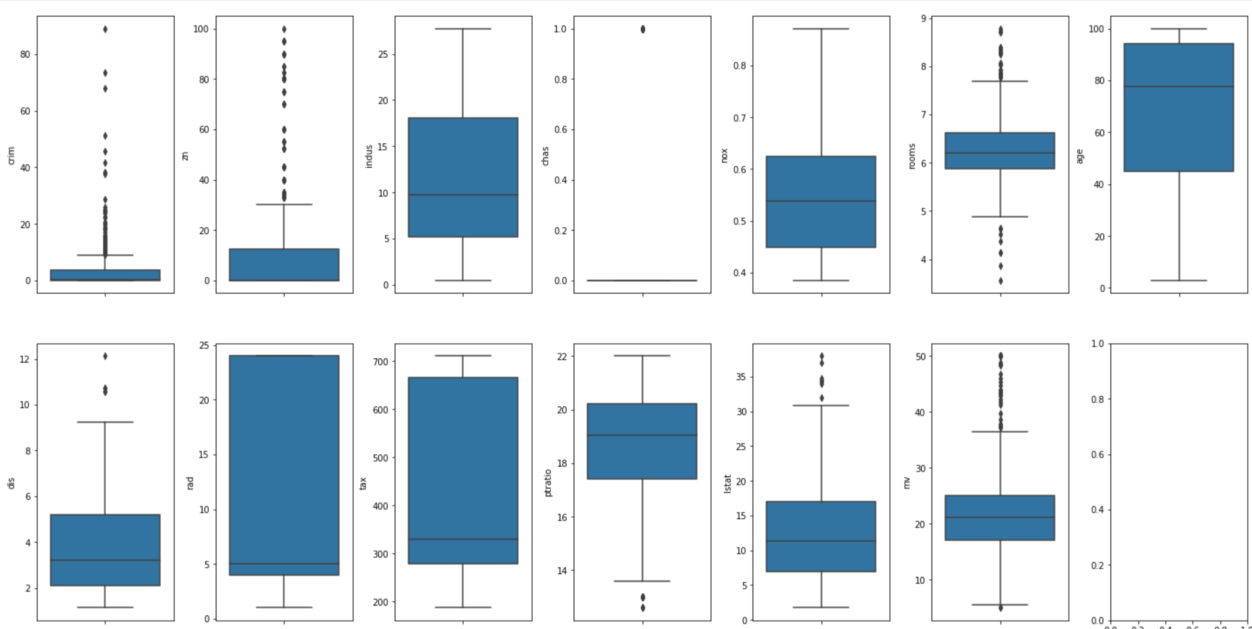

Table 2: Outliers

In [12]:
```
# Examine outliers for each feature
for k, v in boston_input.items():
    q1 = v.quantile(0.25)
    q3 = v.quantile(0.75)
    irq = q3 - q1
    v_col = v[(v <= q1 - 1.5 * irq) | (v >= q3 + 1.5 * irq)]
    perc = np.shape(v_col)[0] * 100.0 / np.shape(boston_input)[0]
    print("Column %s outliers = %.2f%%" % (k, perc))
```

```
Column crim outliers = 13.04%
Column zn outliers = 13.44%
Column indus outliers = 0.00%
Column chas outliers = 100.00%
Column nox outliers = 0.00%
Column rooms outliers = 5.93%
Column age outliers = 0.00%
Column dis outliers = 0.99%
Column rad outliers = 0.00%
Column tax outliers = 0.00%
Column ptratio outliers = 2.96%
Column lstat outliers = 1.38%
Column mv outliers = 7.71%
```

Figure 4: Distributions of features

In [13]:
```
# Examine shape of distributions of features
fig, axs = plt.subplots(ncols=7, nrows=2, figsize=(20, 10))
index = 0
axs = axs.flatten()
for k,v in boston_input.items():
    sns.distplot(v, ax=axs[index])
    index += 1
plt.tight_layout(pad=0.4, w_pad=0.5, h_pad=5.0)

# The histogram also shows that columns CRIM, ZN, B has highly skewed distributions.
# Also MEDV looks to have a normal distribution (the predictions) and other colums seem to have
# normal or bimodel ditribution of data except CHAS (which is a discrete variable).
```

```
/anaconda3/lib/python3.6/site-packages/matplotlib/axes/_axes.py:6462: UserWarning: The 'normed' kwarg is deprecated, and has been replaced by the 'density' kwarg.
  warnings.warn("The 'normed' kwarg is deprecated, and has been "
/anaconda3/lib/python3.6/site-packages/matplotlib/axes/_axes.py:6462: UserWarning: The 'normed' kwarg is deprecated, and has been replaced by the 'density' kwarg.
  warnings.warn("The 'normed' kwarg is deprecated, and has been "
/anaconda3/lib/python3.6/site-packages/matplotlib/axes/_axes.py:6462: UserWarning: The 'normed' kwarg is deprecated, and has been replaced by the 'density' kwarg.
  warnings.warn("The 'normed' kwarg is deprecated, and has been "
/anaconda3/lib/python3.6/site-packages/matplotlib/axes/_axes.py:6462: UserWarning: The 'normed' kwarg is deprecated, and has been replaced by the 'density' kwarg.
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  warnings.warn("The 'normed' kwarg is deprecated, and has been "
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  warnings.warn("The 'normed' kwarg is deprecated, and has been "
/anaconda3/lib/python3.6/site-packages/matplotlib/axes/_axes.py:6462: UserWarning: The 'normed' kwarg is deprecated, and has been replaced by the 'density' kwarg.
  warnings.warn("The 'normed' kwarg is deprecated, and has been "
/anaconda3/lib/python3.6/site-packages/matplotlib/axes/_axes.py:6462: UserWarning: The 'normed' kwarg is deprecated, and has been replaced by the 'density' kwarg.
  warnings.warn("The 'normed' kwarg is deprecated, and has been "
/anaconda3/lib/python3.6/site-packages/matplotlib/axes/_axes.py:6462: UserWarning: The 'normed' kwarg is deprecated, and has been replaced by the 'density' kwarg.
  warnings.warn("The 'normed' kwarg is deprecated, and has been "
/anaconda3/lib/python3.6/site-packages/matplotlib/axes/_axes.py:6462: UserWarning: The 'normed' kwarg is deprecated, and has been replaced by the 'density' kwarg.
  warnings.warn("The 'normed' kwarg is deprecated, and has been "
/anaconda3/lib/python3.6/site-packages/matplotlib/axes/_axes.py:6462: UserWarning: The 'normed' kwarg is deprecated, and has been replaced by the 'density' kwarg.
  warnings.warn("The 'normed' kwarg is deprecated, and has been "
```
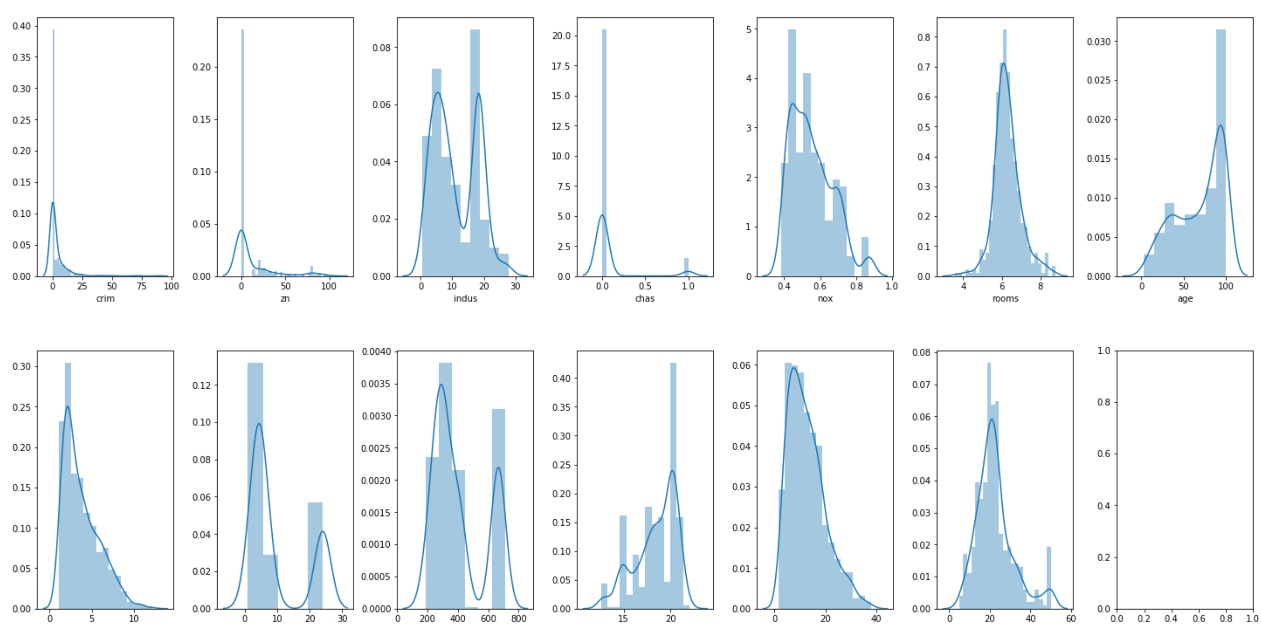



Figure 5: Distribution of median sale price (mv)

In [17]:
```
# Examine target variable (mv) specifically
sns.set(rc={'figure.figsize':(11.7,8.27)})
plt.hist(boston_input['mv'], bins=30)
plt.xlabel("Median house prices in $1000")
plt.show

# Prices are distributed mostly normally, with a few outliers on the higher end
# Most houses are around the 20-24k range
```

Out[17]:
```
<function matplotlib.pyplot.show(*args, **kw)>
```

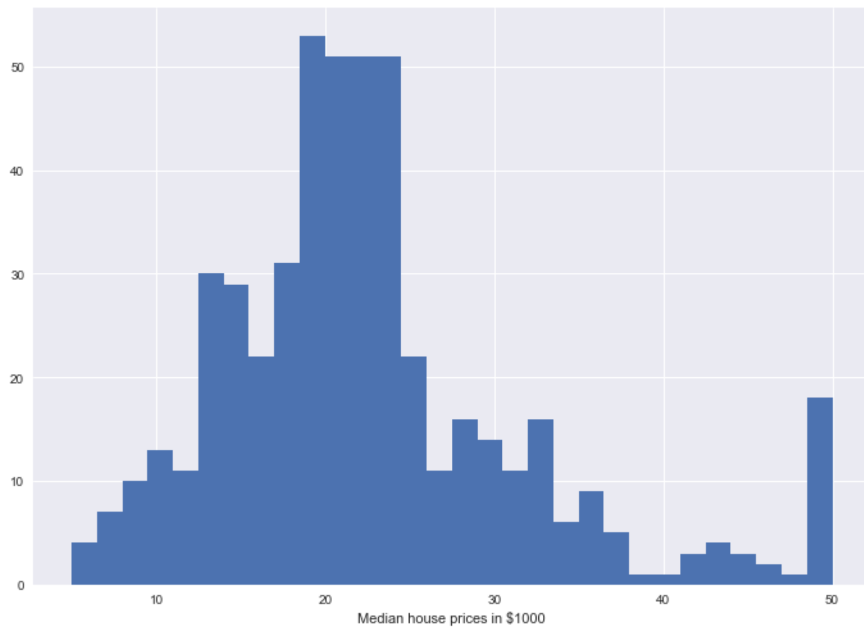
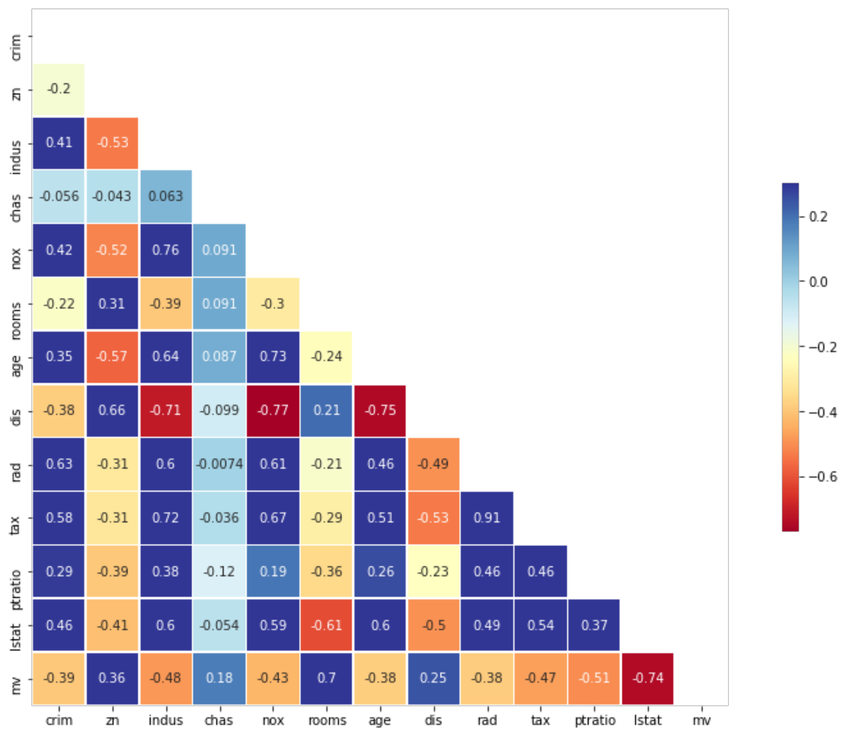


Figure 6: Correlation matrix for original data

In [81]:
```
# Create correlation matrix for original data
plt.figure(figsize=(15,10))
corr = boston_input.corr(method='pearson')
mask = np.zeros_like(corr, dtype=np.bool)
mask[np.triu_indices_from(mask)] = True
sns.heatmap(corr, mask=mask, annot=True, vmax=.3, cmap="RdYlBu", square=True, linewidths = .5,
            cbar_kws={"shrink": .5})

# rm (average # of rooms per dwelling) has a strong positive correlation with mv (price)
# lstat (% lower status of the population) has a strong negative correlation with mv

# Examine multicolinearity, where features have a strong correlation
# rad (index of accessibility to radial highways) and tax (full-value property-tax rate per $10000)
# highly correlated
# dis (weighted distances to 5 Boston employment centers) and age (proportion of owner-occupied units built
# prior to 1940) highly correlated

# From correlation matrix, we see TAX and RAD are highly correlated features.
# The columns LSTAT, INDUS, RM, TAX, NOX, PTRATIO has a correlation score above 0.5 with mv
# which is a good indication of using as predictors.
```

Out[81]: <matplotlib.axes._subplots.AxesSubplot at 0x131adc748>



Data transformation - log transformation of median value

In [7]:
```
# Log transform mv values
boston_log_mv = boston_input.copy()
boston_log_mv['mv'] = boston_input['mv'].apply(np.log)
```

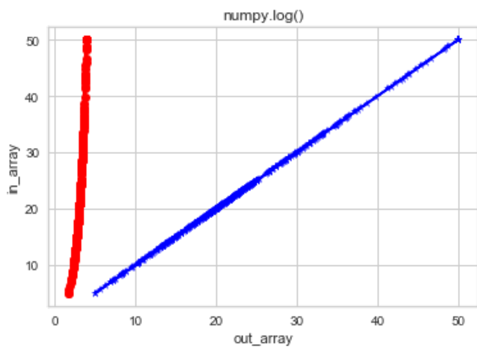
In [8]:
```
# Inspect first 5 rows
boston_log_mv.head()
```

Out[8]:

	crim	zn	indus	chas	nox	rooms	age	dis	rad	tax	ptratio	lstat	mv
0	0.00632	18.0	2.31	0	0.538	6.575	65.2	4.0900	1	296	15.3	4.98	3.178054
1	0.02731	0.0	7.07	0	0.469	6.421	78.9	4.9671	2	242	17.8	9.14	3.072693
2	0.02729	0.0	7.07	0	0.469	7.185	61.1	4.9671	2	242	17.8	4.03	3.546740
3	0.03237	0.0	2.18	0	0.458	6.998	45.8	6.0622	3	222	18.7	2.94	3.508556
4	0.06905	0.0	2.18	0	0.458	7.147	54.2	6.0622	3	222	18.7	5.33	3.589059

Figure 7: MV versus MV log

In [11]:
```
# Create plot for visual comparison
plt.plot(boston_input['mv'], boston_input['mv'],
         color = 'blue', marker = "*")

# Red for log transformed md, blue for original mv values
plt.plot(boston_log_mv['mv'], boston_input['mv'],
         color = 'red', marker = "o")

plt.title("numpy.log()")
plt.xlabel("out_array")
plt.ylabel("in_array")
plt.show()
```



In [9]:
```
# Data prep
# Prepare transformed data and inspect
cols_scaled = boston_log_mv.columns.tolist()
cols_scaled = cols_scaled[-1:] + cols_scaled[:-1]
boston_log_mv_model = boston_log_mv[cols_scaled]

# Inspect scaled data
boston_log_mv_model.describe(include='all')
```

Out[9]:

	mv	crim	zn	indus	chas	nox	rooms	age	dis	rad	tax	ptratio	
count	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.00
mean	3.034558	3.613524	11.363636	11.136779	0.069170	0.554695	6.284634	68.574901	3.795043	9.549407	408.237154	18.455534	12.65
std	0.408275	8.601545	23.322453	6.860353	0.253994	0.115878	0.702617	28.148861	2.105710	8.707259	168.537116	2.164946	7.14
min	1.609438	0.006320	0.000000	0.460000	0.000000	0.385000	3.561000	2.900000	1.129600	1.000000	187.000000	12.600000	1.73
25%	2.834680	0.082045	0.000000	5.190000	0.000000	0.449000	5.885500	45.025000	2.100175	4.000000	279.000000	17.400000	6.95
50%	3.054001	0.256510	0.000000	9.690000	0.000000	0.538000	6.208500	77.500000	3.207450	5.000000	330.000000	19.050000	11.36
75%	3.218876	3.677082	12.500000	18.100000	0.000000	0.624000	6.623500	94.075000	5.188425	24.000000	666.000000	20.200000	16.95
max	3.912023	88.976200	100.000000	27.740000	1.000000	0.871000	8.780000	100.000000	12.126500	24.000000	711.000000	22.000000	37.97

Data transformation - standard scaler

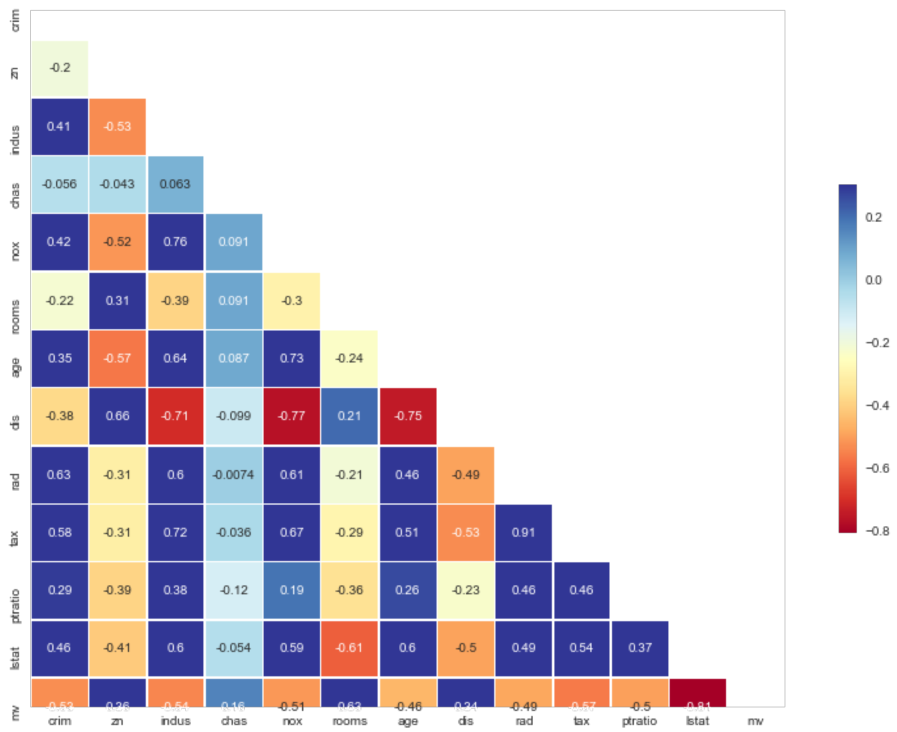
In [16]:
```
# Create copy of the original DataFrame
boston_scaled_standard = boston_log_mv.copy()

# Scaling
col_names = ['crim', 'zn', 'indus', 'chas', 'nox', 'rooms', 'age', 'dis', 'rad', 'tax', 'ptratio', 'lstat', 'mv']
features = boston_scaled_standard[col_names]
scaler = preprocessing.StandardScaler().fit(features.values)
features = scaler.transform(features.values)

# Assign result to columns
boston_scaled_standard[col_names] = features
print(boston_scaled_standard)
```

```
         crim        zn     indus      chas       nox     rooms       age  \
0   -0.419782  0.284830 -1.287909 -0.272599 -0.144217  0.413672 -0.120013
1   -0.417339 -0.487722 -0.593381 -0.272599 -0.740262  0.194274  0.367166
2   -0.417342 -0.487722 -0.593381 -0.272599 -0.740262  1.282714 -0.265812
3   -0.416750 -0.487722 -1.306878 -0.272599 -0.835284  1.016303 -0.809889
4   -0.412482 -0.487722 -1.306878 -0.272599 -0.835284  1.228577 -0.511180
..        ...       ...       ...       ...       ...       ...       ...
501 -0.413229 -0.487722  0.115738 -0.272599  0.158124  0.439316  0.018673
502 -0.415249 -0.487722  0.115738 -0.272599  0.158124 -0.234548  0.288933
503 -0.413447 -0.487722  0.115738 -0.272599  0.158124  0.984960  0.797449
504 -0.407764 -0.487722  0.115738 -0.272599  0.158124  0.725672  0.736996
505 -0.415000 -0.487722  0.115738 -0.272599  0.158124 -0.362767  0.434732

          dis       rad       tax   ptratio     lstat        mv
0    0.140214 -0.982843 -0.666608 -1.459000 -1.075562  0.351817
1    0.557160 -0.867883 -0.987329 -0.303094 -0.492439  0.093498
2    0.557160 -0.867883 -0.987329 -0.303094 -1.208727  1.255744
3    1.077737 -0.752922 -1.106115  0.113032 -1.361517  1.162127
4    1.077737 -0.752922 -1.106115  0.113032 -1.026501  1.359501
..        ...       ...       ...       ...       ...       ...
501 -0.625796 -0.982843 -0.803212  1.176466 -0.418147  0.182663
502 -0.716639 -0.982843 -0.803212  1.176466 -0.500850 -0.022720
503 -0.773684 -0.982843 -0.803212  1.176466 -0.983048  0.341580
504 -0.668437 -0.982843 -0.803212  1.176466 -0.865302  0.138486
505 -0.613246 -0.982843 -0.803212  1.176466 -0.669058 -0.220950

[506 rows x 13 columns]
```

Figure 8: Correlation matrix for standard scaled data

In [17]:
```
# Create correlation matrix for scaled data for comparison
plt.figure(figsize=(15,10))
corr = boston_scaled_standard.corr(method='pearson')
mask = np.zeros_like(corr, dtype=np.bool)
mask[np.triu_indices_from(mask)] = True
sns.heatmap(corr, mask=mask, annot=True, vmax=.3, cmap="RdYlBu", square=True, linewidths = .5,
            cbar_kws={"shrink": .5})
```

Out[17]: <matplotlib.axes._subplots.AxesSubplot at 0x1c19e42d30>



In [18]:
```
# Data prep
# Prepare scaled data and inspect
cols_scaled = boston_scaled_standard.columns.tolist()
cols_scaled = cols_scaled[-1:] + cols_scaled[:-1]
boston_scaled_standard_model = boston_scaled_standard[cols_scaled]

# Inspect scaled data
boston_scaled_standard_model.describe(include='all')
```

Out[18]:

	mv	crim	zn	indus	chas	nox	rooms	age	dis	rad	
count	5.060000e+02	5.060000e+02	5.060000e+02	5.060000e+02	5.060000e+02	5.060000e+02	5.060000e+02	5.060000e+02	5.060000e+02	5.060000e+02	50
mean	-4.353127e-16	-1.123388e-16	7.898820e-17	2.106352e-16	-3.510587e-17	-2.808469e-16	-4.563763e-17	-1.474446e-16	-8.425408e-17	-1.123388e-16	
std	1.000990e+00	1.000990e+00	1.000990e+00	1.000990e+00	1.000990e+00	1.000990e+00	1.000990e+00	1.000990e+00	1.000990e+00	1.000990e+00	
min	-3.494045e+00	-4.197819e-01	-4.877224e-01	-1.557842e+00	-2.725986e-01	-1.465882e+00	-3.880249e+00	-2.335437e+00	-1.267069e+00	-9.828429e-01	-
25%	-4.900528e-01	-4.109696e-01	-4.877224e-01	-8.676906e-01	-2.725986e-01	-9.130288e-01	-5.686303e-01	-8.374480e-01	-8.056878e-01	-6.379618e-01	-
50%	4.766992e-02	-3.906665e-01	-4.877224e-01	-2.110985e-01	-2.725986e-01	-1.442174e-01	-1.084655e-01	3.173816e-01	-2.793234e-01	-5.230014e-01	-
75%	4.519022e-01	7.396560e-03	4.877224e-02	1.015999e+00	-2.725986e-01	5.986790e-01	4.827678e-01	9.067981e-01	6.623709e-01	1.661245e+00	
max	2.151329e+00	9.933931e+00	3.804234e+00	2.422565e+00	3.668398e+00	2.732346e+00	3.555044e+00	1.117494e+00	3.960518e+00	1.661245e+00	

Data transformation - Min-Max scaler

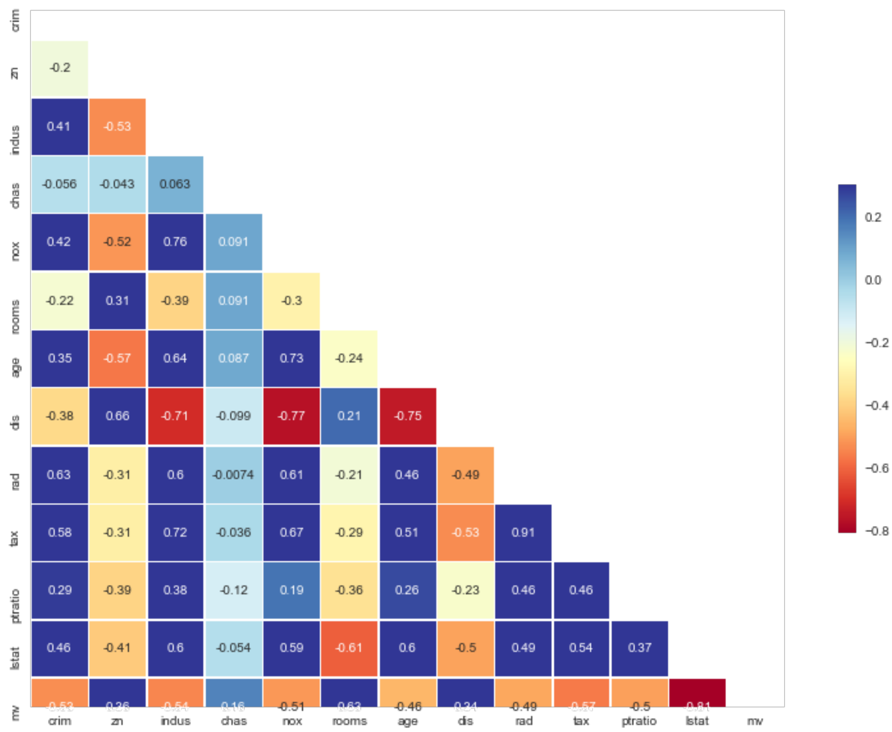
In [21]:
```
# Create copy of the original DataFrame
boston_scaled_minmax = boston_log_mv.copy()

# Scaling
col_names = ['crim', 'zn', 'indus', 'chas', 'nox', 'rooms', 'age', 'dis', 'rad', 'tax', 'ptratio', 'lstat', 'mv']
features = boston_scaled_minmax[col_names]
scaler = preprocessing.MinMaxScaler().fit(features.values)
features = scaler.transform(features.values)

# Assign result to columns
boston_scaled_minmax[col_names] = features
print(boston_scaled_minmax)
```

```
         crim    zn     indus  chas       nox     rooms       age       dis  \
0    0.000000  0.18  0.067815   0.0  0.314815  0.577505  0.641607  0.269203
1    0.000236  0.00  0.242302   0.0  0.172840  0.547998  0.782698  0.348962
2    0.000236  0.00  0.242302   0.0  0.172840  0.694386  0.599382  0.348962
3    0.000293  0.00  0.063050   0.0  0.150206  0.658555  0.441813  0.448545
4    0.000705  0.00  0.063050   0.0  0.150206  0.687105  0.528321  0.448545
..        ...   ...       ...   ...       ...       ...       ...       ...
501  0.000633  0.00  0.420455   0.0  0.386831  0.580954  0.681771  0.122671
502  0.000438  0.00  0.420455   0.0  0.386831  0.490324  0.760041  0.105293
503  0.000612  0.00  0.420455   0.0  0.386831  0.654340  0.907312  0.094381
504  0.001161  0.00  0.420455   0.0  0.386831  0.619467  0.889804  0.114514
505  0.000462  0.00  0.420455   0.0  0.386831  0.473079  0.802266  0.125072

          rad       tax   ptratio     lstat        mv
0    0.000000  0.208015  0.287234  0.089680  0.681241
1    0.043478  0.104962  0.553191  0.204470  0.635484
2    0.043478  0.104962  0.553191  0.063466  0.841359
3    0.086957  0.066794  0.648936  0.033389  0.824776
4    0.086957  0.066794  0.648936  0.099338  0.859739
..        ...       ...       ...       ...       ...
501  0.000000  0.164122  0.893617  0.219095  0.651278
502  0.000000  0.164122  0.893617  0.202815  0.614897
503  0.000000  0.164122  0.893617  0.107892  0.679428
504  0.000000  0.164122  0.893617  0.131071  0.643453
505  0.000000  0.164122  0.893617  0.169702  0.579784

[506 rows x 13 columns]
```

Figure 9: Correlation matrix for min-max scaled data

In [22]:
```
# Create correlation matrix for scaled data for comparison
plt.figure(figsize=(15,10))
corr = boston_scaled_minmax.corr(method='pearson')
mask = np.zeros_like(corr, dtype=np.bool)
mask[np.triu_indices_from(mask)] = True
sns.heatmap(corr, mask=mask, annot=True, vmax=.3, cmap="RdYlBu", square=True, linewidths = .5,
            cbar_kws={"shrink": .5})
```

Out[22]: <matplotlib.axes._subplots.AxesSubplot at 0x1c1825d128>



In [23]:
```
# Data prep
# Prepare scaled data and inspect
cols_scaled = boston_scaled_minmax.columns.tolist()
cols_scaled = cols_scaled[-1:] + cols_scaled[:-1]
boston_scaled_minmax_model = boston_scaled_minmax[cols_scaled]

# Inspect scaled data
boston_scaled_minmax_model.describe(include='all')
```

Out[23]:

	mv	crim	zn	indus	chas	nox	rooms	age	dis	rad	tax	ptratio	
count	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.00
mean	0.618922	0.040544	0.113636	0.391378	0.069170	0.349167	0.521869	0.676364	0.242381	0.371713	0.422208	0.622929	0.30
std	0.177311	0.096679	0.233225	0.251479	0.253994	0.238431	0.134627	0.289896	0.191482	0.378576	0.321636	0.230313	0.19
min	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.00
25%	0.532116	0.000851	0.000000	0.173387	0.000000	0.131687	0.445392	0.433831	0.088259	0.130435	0.175573	0.510638	0.14
50%	0.627366	0.002812	0.000000	0.338343	0.000000	0.314815	0.507281	0.768280	0.188949	0.173913	0.272901	0.686170	0.26
75%	0.698970	0.041258	0.125000	0.646628	0.000000	0.491770	0.586798	0.938980	0.369088	1.000000	0.914122	0.808511	0.42
max	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.00

Data transformation - Box Cox transformation

In [8]:
```
# Data transformations and inspection

# Apply transformation
boston_input2 = boston_input.apply(lambda x: x+.01)

# Inspect first 5 rows of transformed dataset
boston_input2.head()
```

Out[8]:

	crim	zn	indus	chas	nox	rooms	age	dis	rad	tax	ptratio	lstat	mv
0	0.01632	18.01	2.32	0.01	0.548	6.585	65.21	4.1000	1.01	296.01	15.31	4.99	24.01
1	0.03731	0.01	7.08	0.01	0.479	6.431	78.91	4.9771	2.01	242.01	17.81	9.15	21.61
2	0.03729	0.01	7.08	0.01	0.479	7.195	61.11	4.9771	2.01	242.01	17.81	4.04	34.71
3	0.04237	0.01	2.19	0.01	0.468	7.008	45.81	6.0722	3.01	222.01	18.71	2.95	33.41
4	0.07905	0.01	2.19	0.01	0.468	7.157	54.21	6.0722	3.01	222.01	18.71	5.34	36.21

Figure 10: Pair plot of Box Cox transformed features

In [9]:
```
# Box Cox transformation
boston_input3 = boston_input2.apply(lambda x: st.boxcox(x)[0])

# Inspect first 5 rows of transformed dataset
boston_input3.head()

# Visualize relationships again with pair plots
# sns.pairplot(boston_input3, diag_kind='hist')
```

Out[9]:

	crim	zn	indus	chas	nox	rooms	age	dis	rad	tax	ptratio	lstat	mv
0	-5.752617	1.708418	1.014663	-587728.314092	-0.810467	2.960401	201.842701	1.264160	0.009943	1.807153	33009.396839	1.939868	4.589092
1	-4.286098	-13.373080	3.075151	-587728.314092	-1.064124	2.905657	260.910666	1.416765	0.664175	1.796475	63758.135934	2.876120	4.380581
2	-4.286990	-13.373080	3.075151	-587728.314092	-1.064124	3.170577	184.938977	1.416765	0.664175	1.796475	63758.135934	1.642857	5.357762
3	-4.076712	-13.373080	0.932774	-587728.314092	-1.111280	3.107229	125.396307	1.568513	1.018925	1.791545	79016.524332	1.226204	5.275244
4	-3.107491	-13.373080	0.932774	-587728.314092	-1.111280	3.157778	157.365843	1.568513	1.018925	1.791545	79016.524332	2.038263	5.450026

Figure 11: Histogram of distributions of Box Cox transformed features

In [10]:
```
# Scale predictors using normalization
boston_input_boxcox = boston_input3.transform(lambda x: (x - x.min()) / (x.max() - x.min()))

# Create pair plots of scaled data to examine attribute relationships again
boston_input_boxcox.hist(figsize=(10,10))
```

Out[10]:
```
array([[<matplotlib.axes._subplots.AxesSubplot object at 0x1c19595860>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x1c196a6eb8>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x1c19643898>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x1c195dd278>],
       [<matplotlib.axes._subplots.AxesSubplot object at 0x1c19999c18>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x1c199d45f8>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x1c19a03f98>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x1c19a3a9b0>],
       [<matplotlib.axes._subplots.AxesSubplot object at 0x1c19a3a9e8>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x1c19aa2cf8>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x1c19adb6d8>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x1c19b180b8>],
       [<matplotlib.axes._subplots.AxesSubplot object at 0x1c19b43a58>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x1c19b7d438>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x1c19bacdd8>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x1c19be47b8>]],
      dtype=object)
```

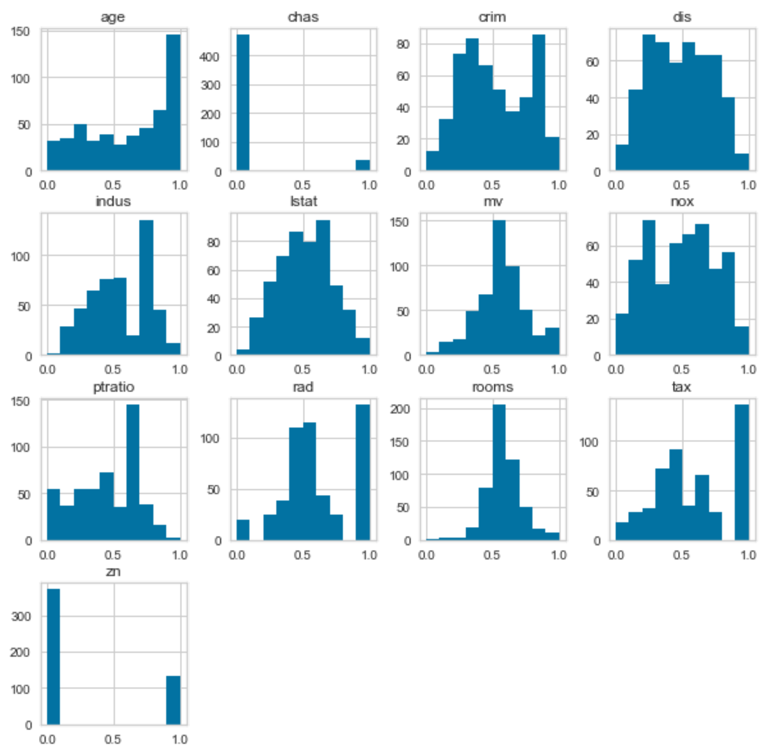
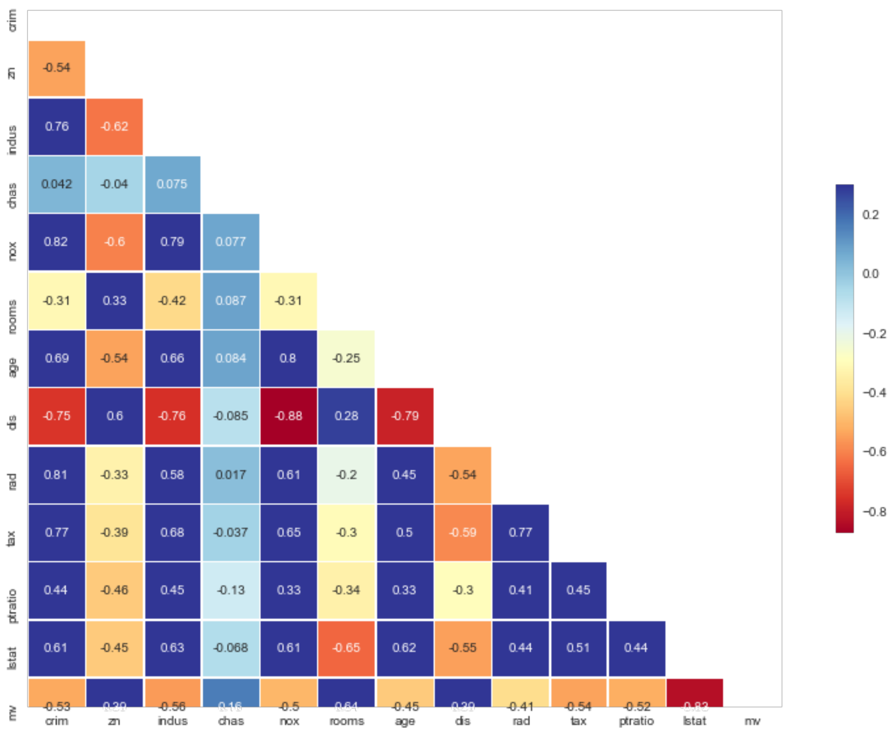


Figure 12: Correlation matrix for Box Cox transformed data

In [29]:
```python
# Create correlation matrix for scaled data
plt.figure(figsize=(15,10))
corr = boston_input_boxcox.corr(method='pearson')
mask = np.zeros_like(corr, dtype=np.bool)
mask[np.triu_indices_from(mask)] = True
sns.heatmap(corr, mask=mask, annot=True, vmax=.3, cmap="RdYlBu", square=True, linewidths = .5,
            cbar_kws={"shrink": .5})
```

Out[29]: <matplotlib.axes._subplots.AxesSubplot at 0x1c19c58b38>



In [11]:
```python
# Data prep
# Prepare scaled data and inspect
cols_scaled = boston_input_boxcox.columns.tolist()
# cols_scaled = cols_scaled[:-1] + cols_scaled[:-1]
# cols_scaled = cols_scaled[:-1] + cols_scaled[:-1]
boston_boxcox_model = boston_input_boxcox[cols_scaled]

# Inspect scaled data
boston_boxcox_model.describe(include='all')
```

Out[11]:

	crim	zn	indus	chas	nox	rooms	age	dis	rad	tax	ptratio	lstat	
count	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.0C
mean	0.518606	0.261061	0.562812	0.069170	0.490420	0.577607	0.625279	0.485270	0.625027	0.581265	0.461663	0.517794	0.56
std	0.247164	0.435430	0.232825	0.253994	0.251621	0.129442	0.315506	0.229684	0.260145	0.280933	0.236275	0.200937	0.18
min	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.0C
25%	0.319026	0.000000	0.378568	0.000000	0.249073	0.506588	0.337094	0.297601	0.491579	0.375989	0.298320	0.364902	0.46
50%	0.476717	0.000000	0.559586	0.000000	0.500909	0.567485	0.707846	0.485207	0.562195	0.511675	0.489371	0.524193	0.56
75%	0.771394	0.967068	0.796857	0.000000	0.678615	0.643218	0.920638	0.683853	1.000000	0.965644	0.659680	0.667866	0.64
max	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.0C

Model setup

Using Boxcox transformed: boston_boxcox_model

In [13]:
```python
# Seed value for random number generator to obtain reproducible results
RANDOM_SEED = 42
```

Original hyperparameters used

In [14]:
```
# Model method calls preparation
names = ['Linear_Regression', 'Ridge_Regression', 'Lasso_Regression', 'ElasticNet_Regression',
        'RandomForest_Regression', 'ExtraTrees_Regression']

# Specify set of regression models to evaluate
# Set normal=False because data was already transformed
regressors = [LinearRegression(fit_intercept = True, normalize = False),
             Ridge(alpha=75, solver='auto', fit_intercept=True, normalize=False,
                   random_state = RANDOM_SEED),
             Lasso(alpha=0.01, max_iter=10000, tol=0.01, fit_intercept=True, normalize=False,
                   random_state=RANDOM_SEED),
             ElasticNet(alpha=0.01, l1_ratio=0.5, max_iter=10000, tol=0.01, fit_intercept=True,
                        normalize=False, random_state=RANDOM_SEED),
             RandomForestRegressor(n_estimators=10, criterion='mse', max_features=10, bootstrap=True,
                                   n_jobs=-1, random_state=RANDOM_SEED),
             ExtraTreesRegressor(n_estimators=100, criterion='mse', max_features=10, bootstrap=True,
                                   n_jobs=-1, random_state=RANDOM_SEED)]
```

Hyperparameters used for fine-tuning with grid search

In [15]:
```
# Model method calls preparation with adjusted hyperparameters for Random Forest and Extra Trees Regression methods
names = ['Linear_Regression', 'Ridge_Regression', 'Lasso_Regression', 'ElasticNet_Regression',
        'RandomForest_Regression', 'ExtraTrees_Regression']

# Specify set of regression models to evaluate
# Set normal=False because data was already transformed
regressors = [LinearRegression(fit_intercept = True, normalize = False),
             Ridge(alpha=100, solver='auto', fit_intercept=True, normalize=False,
                   random_state = RANDOM_SEED),
             Lasso(alpha=0.1, max_iter=10000, tol=0.01, fit_intercept=True, normalize=False,
                   random_state=RANDOM_SEED),
             ElasticNet(alpha=0.1, l1_ratio=0.5, max_iter=10000, tol=0.01, fit_intercept=True,
                        normalize=False, random_state=RANDOM_SEED),
             RandomForestRegressor(n_estimators=100, criterion='mse', max_features=6, bootstrap=True,
                                   n_jobs=-1, random_state=RANDOM_SEED),
             ExtraTreesRegressor(n_estimators=100, criterion='mse', max_features=10, bootstrap=False,
                                   n_jobs=-1, random_state=RANDOM_SEED)]
```

Approach 1: Model evaluation with cross-validation design

- Log transformation of median sale price values
- Standard scaler

In [33]:
```
# Define model data
model_data = boston_scaled_standard_model.values
```

In [34]:
```
# Define number of cross folds to employ
N_FOLDS = 10

# Set up numpy array to store results
cv_results = np.zeros((N_FOLDS, len(names)))

# Initiate splitting process
# kf = KFold(n_splits=N_FOLDS, shuffle=False, random_state=RANDOM_SEED)
# Trying shuffle=True
kf = KFold(n_splits=N_FOLDS, shuffle=True, random_state=RANDOM_SEED)

# Check splitting process by looking at fold observation counts
index_for_fold = 0    # Fold count initiated
for train_index, test_index in kf.split(model_data):
    print('\nFold index:', index_for_fold, '--------------------------')

# Data modeling structure places response variable first and predictor variables following
# 1:model_data.shape[1] slices for predictor variables and 0 is the response variable index
    X_train = model_data[train_index, 1:model_data.shape[1]]
    X_test = model_data[test_index, 1:model_data.shape[1]]
    y_train = model_data[train_index, 0]
    y_test = model_data[test_index, 0]

    index_for_method = 0    # Initialize method cound
    for name, reg_model in zip(names, regressors):
        reg_model.fit(X_train, y_train)    # Fit on the train set for this fold

        # Evaluate on the test set for this fold
        y_test_predict = reg_model.predict(X_test)
        fold_method_result = sqrt(mean_squared_error(y_test, y_test_predict))
        cv_results[index_for_fold, index_for_method] = fold_method_result
        index_for_method += 1

    index_for_fold += 1

cv_results_df = pd.DataFrame(cv_results)
cv_results_df.columns = names

print('\n----------------------')
print('Average results from ', N_FOLDS, '-fold cross-validation\n',
      'in standardized units (mean 0, standard deviation 1)\n',
      '\nMethod               Root mean-squared error', sep = '')
print(cv_results_df.mean())
```

```
Fold index: 0 --------------------------

Fold index: 1 --------------------------

Fold index: 2 --------------------------

Fold index: 3 --------------------------

Fold index: 4 --------------------------

Fold index: 5 --------------------------

Fold index: 6 --------------------------

Fold index: 7 --------------------------

Fold index: 8 --------------------------

Fold index: 9 --------------------------

----------------------
Average results from 10-fold cross-validation
in standardized units (mean 0, standard deviation 1)

Method               Root mean-squared error
Linear_Regression          0.469137
Ridge_Regression           0.481631
Lasso_Regression           0.471980
ElasticNet_Regression      0.469064
RandomForest_Regression    0.381288
ExtraTrees_Regression      0.340040
dtype: float64
```

Approach 2: Model evaluation with cross-validation design

- Log transformation of median sale price values
- Min-Max scaler

In [35]:
```
# Define model data
model_data = boston_scaled_minmax_model.values
```

In [36]:
```
# Define number of cross folds to employ
N_FOLDS = 10

# Set up numpy array to store results
cv_results = np.zeros((N_FOLDS, len(names)))

# Initiate splitting process
# kf = KFold(n_splits=N_FOLDS, shuffle=False, random_state=RANDOM_SEED)
# Trying shuffle=True
kf = KFold(n_splits=N_FOLDS, shuffle=True, random_state=RANDOM_SEED)

# Check splitting process by looking at fold observation counts
index_for_fold = 0    # Fold count initiated
for train_index, test_index in kf.split(model_data):
    print('\nFold index:', index_for_fold, '--------------------------')

# Data modeling structure places response variable first and predictor variables following
# 1:model_data.shape[1] slices for predictor variables and 0 is the response variable index
    X_train = model_data[train_index, 1:model_data.shape[1]]
    X_test = model_data[test_index, 1:model_data.shape[1]]
    y_train = model_data[train_index, 0]
    y_test = model_data[test_index, 0]

    index_for_method = 0    # Initialize method cound
    for name, reg_model in zip(names, regressors):
        reg_model.fit(X_train, y_train)    # Fit on the train set for this fold

        # Evaluate on the test set for this fold
        y_test_predict = reg_model.predict(X_test)
        fold_method_result = sqrt(mean_squared_error(y_test, y_test_predict))
        cv_results[index_for_fold, index_for_method] = fold_method_result
        index_for_method += 1

    index_for_fold += 1

cv_results_df = pd.DataFrame(cv_results)
cv_results_df.columns = names

print('\n----------------------')
print('Average results from ', N_FOLDS, '-fold cross-validation\n',
      'in standardized units (mean 0, standard deviation 1)\n',
      '\nMethod               Root mean-squared error', sep = '')
print(cv_results_df.mean())
```

```
Fold index: 0 --------------------------

Fold index: 1 --------------------------

Fold index: 2 --------------------------

Fold index: 3 --------------------------

Fold index: 4 --------------------------

Fold index: 5 --------------------------

Fold index: 6 --------------------------

Fold index: 7 --------------------------

Fold index: 8 --------------------------

Fold index: 9 --------------------------

----------------------
Average results from 10-fold cross-validation
in standardized units (mean 0, standard deviation 1)

Method               Root mean-squared error
Linear_Regression          0.083101
Ridge_Regression           0.126469
Lasso_Regression           0.112074
ElasticNet_Regression      0.101857
RandomForest_Regression    0.066119
ExtraTrees_Regression      0.059416
dtype: float64
```

Approach 3: Model evaluation with cross-validation design

- Box Cox transformation

In [16]:
```
# Define model data
model_data = boston_boxcox_model.values
```

In [17]:

```
# Define number of cross folds to employ
N_FOLDS = 10

# Set up numpy array to store results
cv_results = np.zeros((N_FOLDS, len(names)))

# Initiate splitting process
# kf = KFold(n_splits=N_FOLDS, shuffle=False, random_state=RANDOM_SEED)
# Trying shuffle=True
kf = KFold(n_splits=N_FOLDS, shuffle=True, random_state=RANDOM_SEED)

# Check splitting process by looking at fold observation counts
index_for_fold = 0    # Fold count initiated
for train_index, test_index in kf.split(model_data):
    print('\nFold index:', index_for_fold, '--------------------------')

# Data modeling structure places response variable first and predictor variables following
# 1:model_data.shape[1] slices for predictor variables and 0 is the response variable index
    X_train = model_data[train_index, 1:model_data.shape[1]]
    X_test = model_data[test_index, 1:model_data.shape[1]]
    y_train = model_data[train_index, 0]
    y_test = model_data[test_index, 0]

    index_for_method = 0    # Initialize method cound
    for name, reg_model in zip(names, regressors):
        reg_model.fit(X_train, y_train)    # Fit on the train set for this fold

        # Evaluate on the test set for this fold
        y_test_predict = reg_model.predict(X_test)
        fold_method_result = sqrt(mean_squared_error(y_test, y_test_predict))
        cv_results[index_for_fold, index_for_method] = fold_method_result
        index_for_method += 1

    index_for_fold += 1

cv_results_df = pd.DataFrame(cv_results)
cv_results_df.columns = names

print('\n----------------------')
print('Average results from ', N_FOLDS, '-fold cross-validation\n',
      'in standardized units (mean 0, standard deviation 1)\n',
      '\nMethod                Root mean-squared error', sep = '')
print(cv_results_df.mean())
```

```
Fold index: 0 --------------------------

Fold index: 1 --------------------------

Fold index: 2 --------------------------

Fold index: 3 --------------------------

Fold index: 4 --------------------------

Fold index: 5 --------------------------

Fold index: 6 --------------------------

Fold index: 7 --------------------------

Fold index: 8 --------------------------

Fold index: 9 --------------------------

----------------------
Average results from 10-fold cross-validation
in standardized units (mean 0, standard deviation 1)

Method                Root mean-squared error
Linear_Regression          0.097324
Ridge_Regression           0.135320
Lasso_Regression           0.246740
ElasticNet_Regression      0.236236
RandomForest_Regression    0.052792
ExtraTrees_Regression      0.053582
dtype: float64
```

In [ ]:
```
# Results across all three approaches
Log transformed mv, standard scaler:
Average results from 10-fold cross-validation
in standardized units (mean 0, standard deviation 1)

Method                Root mean-squared error
Linear_Regression           0.469137
Ridge_Regression            0.481631
Lasso_Regression            0.471980
ElasticNet_Regression       0.469064
RandomForest_Regression     0.381288
ExtraTrees_Regression       0.340040

Log transformed mv, minmax scaler:
Average results from 10-fold cross-validation
in standardized units (mean 0, standard deviation 1)

Linear_Regression           0.083101
Ridge_Regression            0.126469
Lasso_Regression            0.112074
ElasticNet_Regression       0.101857
RandomForest_Regression     0.066119
ExtraTrees_Regression       0.059416

Box Cox scaler:
Average results from 10-fold cross-validation
in standardized units (mean 0, standard deviation 1)

Method                Root mean-squared error
Linear_Regression           0.097324
Ridge_Regression            0.135320
Lasso_Regression            0.246740
ElasticNet_Regression       0.236236
RandomForest_Regression     0.052792
ExtraTrees_Regression       0.053582
```

Modeling technique comparison

Approaches 2 and 3 have relatively close RMSE values for each modeling method and are lower than approach 1's values. Random Forest and Extra Trees Regression have the best performance for this data and learning approach.

Continue using Box Cox transformed Boston housing data (model_data) for remainder of study, using alternate techniques.

Random Forest and Extra Trees Fine-Tuning

In [198]:
```
# Assign model data
boston_model = boston_boxcox_model

X = boston_model.drop('mv', axis=1).values
y = boston_model.mv.values
```

In [199]:
```
# Split train and test data
X_train, X_test, y_train, y_test = train_test_split(boston_boxcox_model, y, test_size=0.2, random_state=42)
print(X_train.shape, y_train.shape)
print(X_test.shape, y_test.shape)
```

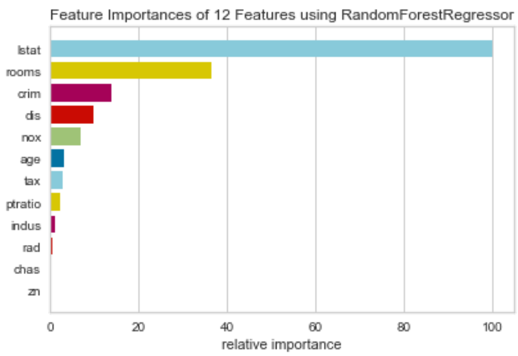
```
(404, 13) (404,)
(102, 13) (102,)
```

Feature importance

Figure 13: Feature importance for Random Forest model

In [160]:
```
# Evaluate feature importance for RF

# Create feature list
feat_list = ['crim', 'zn', 'indus', 'chas', 'nox', 'rooms', 'age', 'dis', 'rad', 'tax', 'ptratio', 'lstat']

# Visualize with feature names
model = RandomForestRegressor(n_estimators=100)
labels = feat_list
viz = FeatureImportances(model, labels = labels)
viz.fit(X, y)
viz.show()
```

```
/anaconda3/lib/python3.6/site-packages/sklearn/base.py:197: FutureWarning: From version 0.24, get_params will raise a
n AttributeError if a parameter cannot be retrieved as an instance attribute. Previously it would return None.
  FutureWarning)
```



Out[160]:
```
<matplotlib.axes._subplots.AxesSubplot at 0x1c1b32bf98>
```

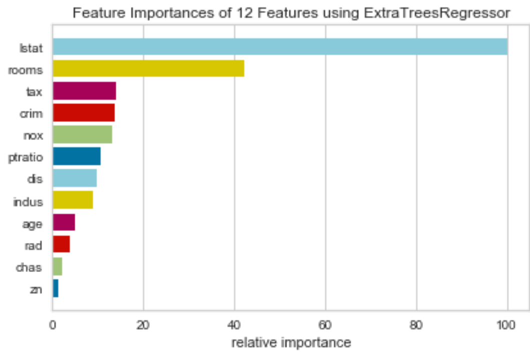
In [161]:
```
# Check numeric values of feature importance for RF
rnd_reg = RandomForestRegressor(n_estimators=500, n_jobs=-1)
rnd_reg.fit(X, y)
for name, score in zip(feat_list, rnd_reg.feature_importances_):
    print(name, score)
```

```
crim 0.07744619796871274
zn 0.0007692577049276498
indus 0.006501757647112281
chas 0.0009219150399438937
nox 0.03841870818712361
rooms 0.21307911470146207
age 0.016838805753805017
dis 0.053357736528015835
rad 0.0027578295378056076
tax 0.016349158050766175
ptratio 0.011917910201618068
lstat 0.5616416086787069
```

Figure 14: Feature importance for Extra Trees model

In [162]:
```
# Feature importance for ET

# Create feature list
feat_list = ['crim', 'zn', 'indus', 'chas', 'nox', 'rooms', 'age', 'dis', 'rad', 'tax', 'ptratio', 'lstat']

# Visualize with feature names
model = ExtraTreesRegressor(n_estimators=100)
labels = feat_list
viz = FeatureImportances(model, labels = labels)
viz.fit(X, y)
viz.show()
```

```
/anaconda3/lib/python3.6/site-packages/sklearn/base.py:197: FutureWarning: From version 0.24, get_params will raise a
n AttributeError if a parameter cannot be retrieved as an instance attribute. Previously it would return None.
  FutureWarning)
```



Out[162]: <matplotlib.axes._subplots.AxesSubplot at 0x1c1c28eb38>

In [163]:
```
# Check numeric values of feature importance for ET
rnd_reg = ExtraTreesRegressor(n_estimators=500, n_jobs=-1)
rnd_reg.fit(X, y)
for name, score in zip(feat_list, rnd_reg.feature_importances_):
    print(name, score)
```

```
crim 0.06335092087768143
zn 0.004693019663931 36
indus 0.0320328296368612
chas 0.008709967136886812
nox 0.056236232440205
rooms 0.19009940196937788
age 0.021577739139633 31
dis 0.045450030183720805
rad 0.024667626239400983
tax 0.05043597708787062
ptratio 0.054421614319203136
lstat 0.4483246413052274
```

Grid search for RF and ET

In [97]:
```
# Metric to use: 'neg_mean_squared_error'

# The best value is 0.0

# The actual MSE is simply the positive version of the number you're getting.

# The unified scoring API always maximizes the score, so scores which need to be minimized are
# negated in order for the unified scoring API to work correctly. The score that is returned is
# therefore negated when it is a score that should be minimized and left positive if it is a score
# that should be maximized.

# Another metric: R2
# R^2 (coefficient of determination) regression score function

# Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse).
# A constant model that always predicts the expected value of y, disregarding the input features,
# would get a R^2 score of 0.0.
```

In [79]:
```
# RF grid search with multiple parameters - R2

# Max_features is main parameter, try # of features for regression or around that number

rf_reg = RandomForestRegressor()

param_grid = {
    'bootstrap': [True],
    'max_depth': [80,90,100,110,120],
    'max_features': [2,4,6,8,10],
    'min_samples_leaf': [3,4,5],
    'min_samples_split': [8,10,12],
    'n_estimators': [100,200,300,800,1000]
}

# grid_search_rf = GridSearchCV(estimator=rf_reg, param_grid=param_grid, cv=3, scoring='neg_mean_squared_error',
#                               return_train_score=True, n_jobs=-1, verbose=2)

grid_search_rf = GridSearchCV(estimator=rf_reg, param_grid=param_grid, cv=3, scoring='r2',
                              return_train_score=True, n_jobs=-1, verbose=2, random_state=42)

grid_search_rf.fit(X_train, y_train)
```

```
Fitting 3 folds for each of 1125 candidates, totalling 3375 fits

[Parallel(n_jobs=-1)]: Using backend LokyBackend with 4 concurrent workers.
[Parallel(n_jobs=-1)]: Done  33 tasks      | elapsed:   10.5s
[Parallel(n_jobs=-1)]: Done 154 tasks      | elapsed:   45.1s
[Parallel(n_jobs=-1)]: Done 357 tasks      | elapsed:  1.8min
[Parallel(n_jobs=-1)]: Done 640 tasks      | elapsed:  3.4min
[Parallel(n_jobs=-1)]: Done 1005 tasks      | elapsed:  5.4min
[Parallel(n_jobs=-1)]: Done 1450 tasks      | elapsed:  7.8min
[Parallel(n_jobs=-1)]: Done 1977 tasks      | elapsed: 10.8min
[Parallel(n_jobs=-1)]: Done 2584 tasks      | elapsed: 13.9min
[Parallel(n_jobs=-1)]: Done 3273 tasks      | elapsed: 17.3min
[Parallel(n_jobs=-1)]: Done 3375 out of 3375 | elapsed: 17.8min finished
```

Out[79]:
```
GridSearchCV(cv=3, error_score=nan,
             estimator=RandomForestRegressor(bootstrap=True, ccp_alpha=0.0,
                                             criterion='mse', max_depth=None,
                                             max_features='auto',
                                             max_leaf_nodes=None,
                                             max_samples=None,
                                             min_impurity_decrease=0.0,
                                             min_impurity_split=None,
                                             min_samples_leaf=1,
                                             min_samples_split=2,
                                             min_weight_fraction_leaf=0.0,
                                             n_estimators=100, n_jobs=None,
                                             oob_score=False, rand...=None,
                                             verbose=0, warm_start=False),
             iid='deprecated', n_jobs=-1,
             param_grid={'bootstrap': [True],
                         'max_depth': [80, 90, 100, 110, 120],
                         'max_features': [2, 4, 6, 8, 10],
                         'min_samples_leaf': [3, 4, 5],
                         'min_samples_split': [8, 10, 12],
                         'n_estimators': [100, 200, 300, 800, 1000]},
             pre_dispatch='2*n_jobs', refit=True, return_train_score=True,
             scoring='neg_mean_squared_error', verbose=2)
```

In [80]:
```
# Display parameter recommendations
print('Test set score: ', grid_search_rf.score(X_test, y_test))
print('Best parameters: ', grid_search_rf.best_params_)
print('Best cross-validation score: ', grid_search_rf.best_score_)
```

```
Test set score:  -0.00037417350363380593
Best parameters:  {'bootstrap': True, 'max_depth': 100, 'max_features': 10, 'min_samples_leaf': 3, 'min_samples_split
': 8, 'n_estimators': 100}
Best cross-valisation score:  -0.00014726682311321917
```

In [ ]:
```
# RF grid search with multiple parameters - MSE

# Max_features is main parameter, try # of features for regression or around that number

rf_reg = RandomForestRegressor()

param_grid = {
    'bootstrap': [True],
    'max_depth': [80,90,100,110,120],
    'max_features': [2,4,6,8,10],
    'min_samples_leaf': [3,4,5],
    'min_samples_split': [8,10,12],
    'n_estimators': [100,200,300,800,1000]
}

grid_search_rf = GridSearchCV(estimator=rf_reg, param_grid=param_grid, cv=3, scoring='neg_mean_squared_error',
                              return_train_score=True, n_jobs=-1, verbose=2, random_state=42)

# grid_search_rf = GridSearchCV(estimator=rf_reg, param_grid=param_grid, cv=3, scoring='r2',
#                               return_train_score=True, n_jobs=-1, verbose=2, random_state=42)

grid_search_rf.fit(X_train, y_train)
```

In [ ]:
```
# Display parameter recommendations
print('Test set score: ', grid_search_rf.score(X_test, y_test))
print('Best parameters: ', grid_search_rf.best_params_)
print('Best cross-validation score: ', grid_search_rf.best_score_)
```

In [100]:
```
# RF grid search with one parameter - R2

param_grid = {'max_features':range(10, 15)}

grid_search_rf = GridSearchCV(estimator=rf_reg, param_grid=param_grid, cv=3, scoring='r2',
                              return_train_score=True, n_jobs=-1, verbose=2)

grid_search_rf.fit(X_train, y_train)
```

```
Fitting 3 folds for each of 5 candidates, totalling 15 fits

[Parallel(n_jobs=-1)]: Using backend LokyBackend with 4 concurrent workers.
[Parallel(n_jobs=-1)]: Done  15 out of  15 | elapsed:    1.2s finished
```

Out[100]:
```
GridSearchCV(cv=3, error_score=nan,
             estimator=RandomForestRegressor(bootstrap=True, ccp_alpha=0.0,
                                             criterion='mse', max_depth=None,
                                             max_features='auto',
                                             max_leaf_nodes=None,
                                             max_samples=None,
                                             min_impurity_decrease=0.0,
                                             min_impurity_split=None,
                                             min_samples_leaf=1,
                                             min_samples_split=2,
                                             min_weight_fraction_leaf=0.0,
                                             n_estimators=100, n_jobs=None,
                                             oob_score=False, random_state=None,
                                             verbose=0, warm_start=False),
             iid='deprecated', n_jobs=-1,
             param_grid={'max_features': range(10, 15)},
             pre_dispatch='2*n_jobs', refit=True, return_train_score=True,
             scoring='r2', verbose=2)
```

In [101]:
```
# Display parameter recommendations
print('Test set score: ', grid_search_rf.score(X_test, y_test))
print('Best parameters: ', grid_search_rf.best_params_)
print('Best cross-validation score: ', grid_search_rf.best_score_)
```

```
Test set score:  0.9968223271531906
Best parameters:  {'max_features': 13}
Best cross-validation score:  0.998811630568802
```

In [ ]:
```
# RF grid search with one parameter - MSE

param_grid = {'max_features':range(10, 15)}

grid_search_rf = GridSearchCV(estimator=rf_reg, param_grid=param_grid, cv=3, scoring='mean_squared_error',
                              return_train_score=True, n_jobs=-1, verbose=2)

grid_search_rf.fit(X_train, y_train)
```

In [ ]:
```
# Display parameter recommendations
print('Test set score: ', grid_search_rf.score(X_test, y_test))
print('Best parameters: ', grid_search_rf.best_params_)
print('Best cross-validation score: ', grid_search_rf.best_score_)
```

In [88]:
```
# ET grid search with multiple parameters

# Max_features is main parameter, try # of features for regression or around that number

et_reg = ExtraTreesRegressor()

param_grid = {
    'bootstrap': [True],
    'max_depth': [80,90,100,110,120],
    'max_features': [2,4,6,8,10],
    'min_samples_leaf': [3,4,5],
    'min_samples_split': [8,10,12],
    'n_estimators': [100,200,300,800,1000]
}

grid_search_et = GridSearchCV(estimator=et_reg, param_grid=param_grid, cv=3, scoring='r2',
                              return_train_score=True, n_jobs=-1, verbose=2)

grid_search_et.fit(X_train, y_train)
```

```
Fitting 3 folds for each of 1125 candidates, totalling 3375 fits

[Parallel(n_jobs=-1)]: Using backend LokyBackend with 4 concurrent workers.
[Parallel(n_jobs=-1)]: Done  33 tasks      | elapsed:    7.9s
[Parallel(n_jobs=-1)]: Done 154 tasks      | elapsed:   37.8s
[Parallel(n_jobs=-1)]: Done 357 tasks      | elapsed:  1.5min
[Parallel(n_jobs=-1)]: Done 640 tasks      | elapsed:  2.7min
[Parallel(n_jobs=-1)]: Done 1005 tasks      | elapsed:  4.2min
[Parallel(n_jobs=-1)]: Done 1450 tasks      | elapsed:  6.1min
[Parallel(n_jobs=-1)]: Done 1977 tasks      | elapsed:  8.4min
[Parallel(n_jobs=-1)]: Done 2584 tasks      | elapsed: 10.9min
[Parallel(n_jobs=-1)]: Done 3273 tasks      | elapsed: 13.8min
[Parallel(n_jobs=-1)]: Done 3375 out of 3375 | elapsed: 14.3min finished
```

Out[88]:
```
GridSearchCV(cv=3, error_score=nan,
             estimator=ExtraTreesRegressor(bootstrap=False, ccp_alpha=0.0,
                                           criterion='mse', max_depth=None,
                                           max_features='auto',
                                           max_leaf_nodes=None,
                                           max_samples=None,
                                           min_impurity_decrease=0.0,
                                           min_impurity_split=None,
                                           min_samples_leaf=1,
                                           min_samples_split=2,
                                           min_weight_fraction_leaf=0.0,
                                           n_estimators=100, n_jobs=None,
                                           oob_score=False, rando...=None,
                                           verbose=0, warm_start=False),
             iid='deprecated', n_jobs=-1,
             param_grid={'bootstrap': [True],
                         'max_depth': [80, 90, 100, 110, 120],
                         'max_features': [2, 4, 6, 8, 10],
                         'min_samples_leaf': [3, 4, 5],
                         'min_samples_split': [8, 10, 12],
                         'n_estimators': [100, 200, 300, 800, 1000]},
             pre_dispatch='2*n_jobs', refit=True, return_train_score=True,
             scoring='neg_mean_squared_error', verbose=2)
```

In [89]:
```
# Display parameter recommendations
print('Test set score: ', grid_search_et.score(X_test, y_test))
print('Best parameters: ', grid_search_et.best_params_)
print('Best cross-validation score: ', grid_search_et.best_score_)
```

```
Test set score:  -0.0006695978067477949
Best parameters:  {'bootstrap': True, 'max_depth': 80, 'max_features': 10, 'min_samples_leaf': 3, 'min_samples_split
': 8, 'n_estimators': 300}
Best cross-valisation score:  -0.0005772259617043201
```

In [102]:
```
# ET grid search with one parameter

param_grid = {'max_features':range(10, 15)}

grid_search_et = GridSearchCV(estimator=et_reg, param_grid=param_grid, cv=3, scoring='r2',
                              return_train_score=True, n_jobs=-1, verbose=2)

grid_search_et.fit(X_train, y_train)
```

```
Fitting 3 folds for each of 5 candidates, totalling 15 fits

[Parallel(n_jobs=-1)]: Using backend LokyBackend with 4 concurrent workers.
[Parallel(n_jobs=-1)]: Done  15 out of  15 | elapsed:    0.9s finished
```

Out[102]:
```
GridSearchCV(cv=3, error_score=nan,
             estimator=ExtraTreesRegressor(bootstrap=False, ccp_alpha=0.0,
                                           criterion='mse', max_depth=None,
                                           max_features='auto',
                                           max_leaf_nodes=None,
                                           max_samples=None,
                                           min_impurity_decrease=0.0,
                                           min_impurity_split=None,
                                           min_samples_leaf=1,
                                           min_samples_split=2,
                                           min_weight_fraction_leaf=0.0,
                                           n_estimators=100, n_jobs=None,
                                           oob_score=False, random_state=None,
                                           verbose=0, warm_start=False),
             iid='deprecated', n_jobs=-1,
             param_grid={'max_features': range(10, 15)},
             pre_dispatch='2*n_jobs', refit=True, return_train_score=True,
             scoring='r2', verbose=2)
```

In [103]:
```
# Display parameter recommendations
print('Test set score: ', grid_search_et.score(X_test, y_test))
print('Best parameters: ', grid_search_et.best_params_)
print('Best cross-validation score: ', grid_search_et.best_score_)
```

```
Test set score:  0.9981298410615217
Best parameters:  {'max_features': 13}
Best cross-validation score:  0.9987151045356225
```

In [ ]:
```
# refit the model that does best
```

In [297]:
```
# Refit Extra Trees

# Best parameters:  {'bootstrap': True, 'max_depth': 80, 'max_features': 10, 'min_samples_leaf': 3,
# 'min_samples_split': 8, 'n_estimators': 300}
X = boston_model.drop('mv', axis=1).values
y = boston_model.mv.values
refit_et = ExtraTreesRegressor(bootstrap=True, max_depth=80, max_features=10, min_samples_leaf=3,
                               min_samples_split=8, n_estimators=300, verbose=0, n_jobs=-1, random_state = 42)
```

In [298]:
```
refit_et.fit(X,y)
```

Out[298]:
```
ExtraTreesRegressor(bootstrap=True, ccp_alpha=0.0, criterion='mse',
                    max_depth=80, max_features=10, max_leaf_nodes=None,
                    max_samples=None, min_impurity_decrease=0.0,
                    min_impurity_split=None, min_samples_leaf=3,
                    min_samples_split=8, min_weight_fraction_leaf=0.0,
                    n_estimators=300, n_jobs=-1, oob_score=False,
                    random_state=42, verbose=0, warm_start=False)
```

In [299]:
```
pred = refit_et.predict(X)
```

In [300]:
```
# MSE
print("Mean squared error: %.3f"% mean_squared_error(pred, y))
print("Root MSE: %.2f"% sqrt(mean_squared_error(pred,y)))
# Explained variance score: 1 is perfect prediction
print('Test variance score: %.3f' % r2_score(pred,y))
```

```
Mean squared error: 0.003
Root MSE: 0.05
Test variance score: 0.895
```

Figure 15: Actual versus predicted values with fine-tuned Extra Trees model

In [261]:
```
# Run the model against the test data
fig, ax = plt.subplots()
ax.scatter(y, pred, edgecolors=(0, 0, 0))
ax.plot([y_test.min(), y_test.max()], [y_test.min(), y_test.max()], 'k--', lw=4)
ax.set_xlabel('Actual')
ax.set_ylabel('Predicted')
ax.set_title("Ground Truth vs Predicted")
plt.show()
```

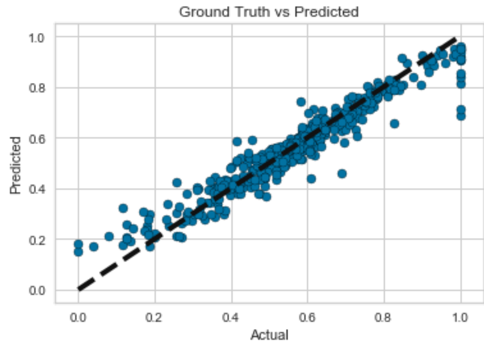
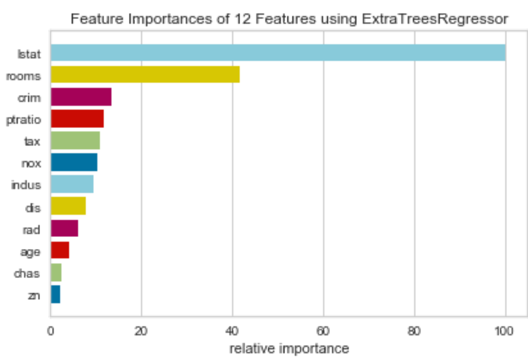


Figure 16: Feature importances for fine-tuned Extra Trees model

In [264]:
```
# Create feature list
feat_list = ['crim', 'zn', 'indus', 'chas', 'nox', 'rooms', 'age', 'dis', 'rad', 'tax', 'ptratio', 'lstat']

# Visualize with feature names
labels = feat_list
viz = FeatureImportances(refit_et, labels = labels)
viz.fit(X, y)
viz.show()
```

```
/anaconda3/lib/python3.6/site-packages/sklearn/base.py:197: FutureWarning: From version 0.24, get_params will raise a
n AttributeError if a parameter cannot be retrieved as an instance attribute. Previously it would return None.
  FutureWarning)
```



Out[264]: <matplotlib.axes._subplots.AxesSubplot at 0x1c1d0583c8>

In [263]:
```
# Check numeric values of feature importance for ET
refit_et.fit(X, y)
for name, score in zip(feat_list, refit_et.feature_importances_):
    print(name, score)
```

```
crim 0.0606357682887844
zn 0.009423805168847176
indus 0.04302208971157683
chas 0.01106309553651166
nox 0.04672843888329933
rooms 0.18919655666644386
age 0.01859028495760196
dis 0.035754935547659523
rad 0.027970767541390475
tax 0.05013300766040468
ptratio 0.05359476312959453
lstat 0.45388648690788547
```

Decision tree regressor

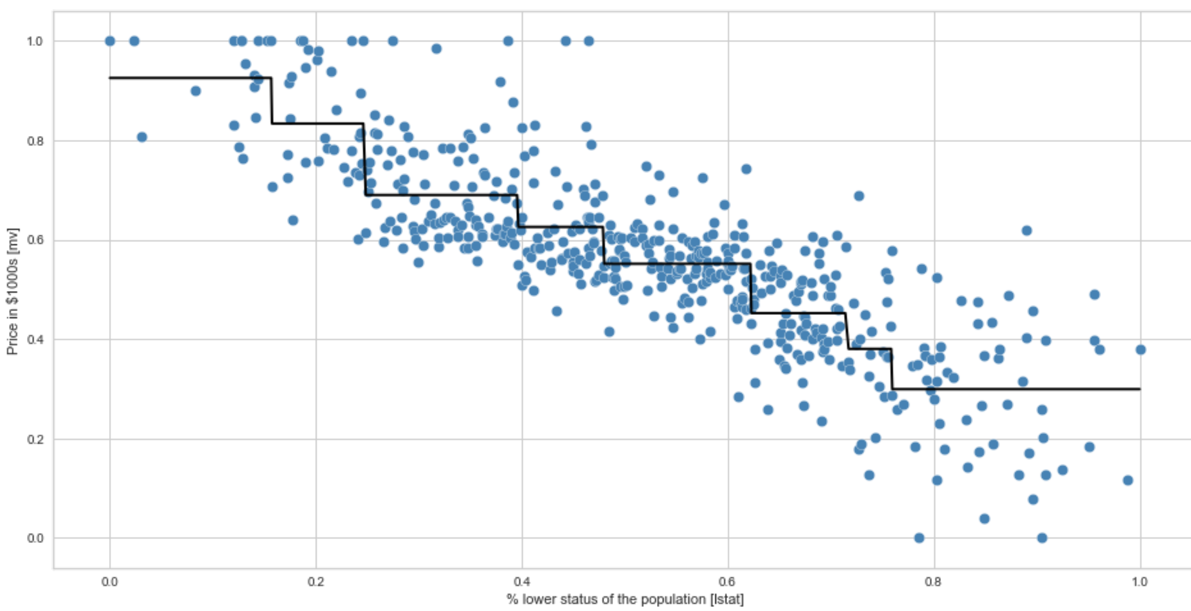
Decision trees with single predictor feature

In [114]:
```
# LSTAT

# Assign X matrix and y vector
X = boston_model[['lstat']].values
# X = X.reshape(-1,1)
y = boston_model['mv'].values

# Sort X and y by ascending values of X
sort_idx = X.flatten().argsort()
X = X[sort_idx]
y = y[sort_idx]

# Initialize and fit regressor
tree = DecisionTreeRegressor(criterion='mse', max_depth=3)
tree.fit(X, y)
```

Out[114]:
```
DecisionTreeRegressor(ccp_alpha=0.0, criterion='mse', max_depth=3,
                      max_features=None, max_leaf_nodes=None,
                      min_impurity_decrease=0.0, min_impurity_split=None,
                      min_samples_leaf=1, min_samples_split=2,
                      min_weight_fraction_leaf=0.0, presort='deprecated',
                      random_state=None, splitter='best')
```

Figure 17: Actual and predicted target against feature - LSTAT

In [115]:
```
# Plot actual versus predicted mv values
plt.figure(figsize=(16, 8))
plt.scatter(X, y, c='steelblue',                    # Plot actual target against features
            edgecolor='white', s=70)
plt.plot(X, tree.predict(X),                        # Plot predicted target against features
         color='black', lw=2)
plt.xlabel('% lower status of the population [lstat]')
plt.ylabel('Price in $1000s [mv]')
plt.show()
```

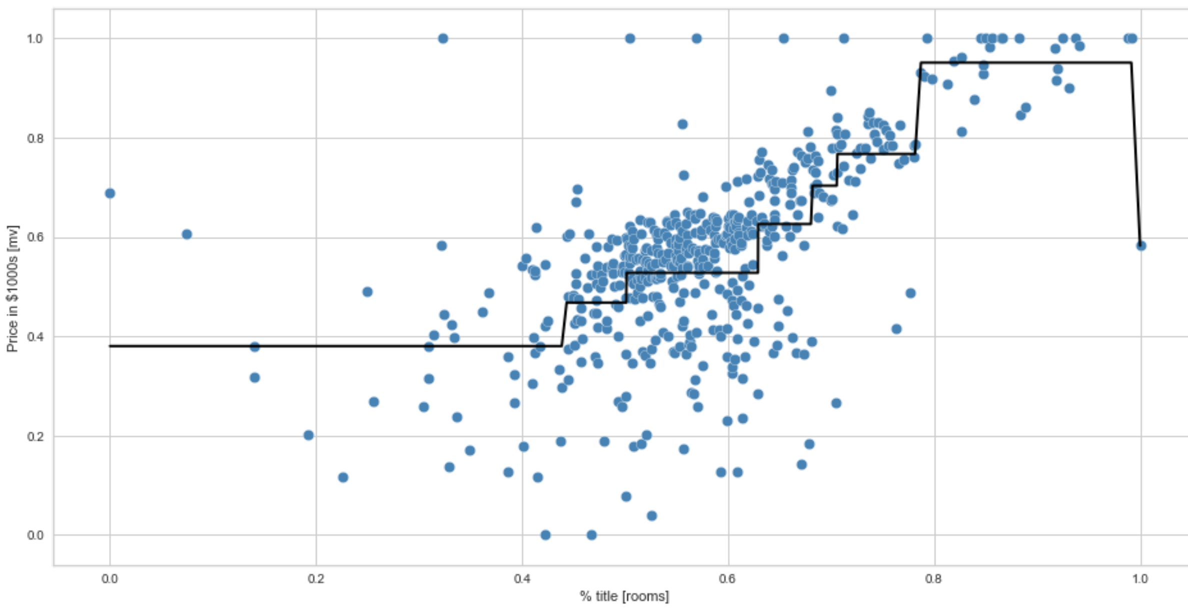


In [68]:
```
# ROOMS

# Assign X matrix and y vector
X = boston_model[['rooms']].values
# X = X.reshape(-1,1)
y = boston_model['mv'].values

# Sort X and y by ascending values of X
sort_idx = X.flatten().argsort()
X = X[sort_idx]
y = y[sort_idx]

# Initialize and fit regressor
tree = DecisionTreeRegressor(criterion='mse', max_depth=3)
tree.fit(X, y)
```

Out[68]:
```
DecisionTreeRegressor(ccp_alpha=0.0, criterion='mse', max_depth=3,
                      max_features=None, max_leaf_nodes=None,
                      min_impurity_decrease=0.0, min_impurity_split=None,
                      min_samples_leaf=1, min_samples_split=2,
                      min_weight_fraction_leaf=0.0, presort='deprecated',
                      random_state=None, splitter='best')
```

Figure 18: Actual and predicted target against feature - ROOMS

In [69]:
```
# Plot actual versus predicted mv values
plt.figure(figsize=(16, 8))
plt.scatter(X, y, c='steelblue',                  # Plot actual target against features
            edgecolor='white', s=70)
plt.plot(X, tree.predict(X),                      # Plot predicted target against features
         color='black', lw=2)
plt.xlabel('number of rooms [rooms]')
plt.ylabel('Price in $1000s [mv]')
plt.show()
```

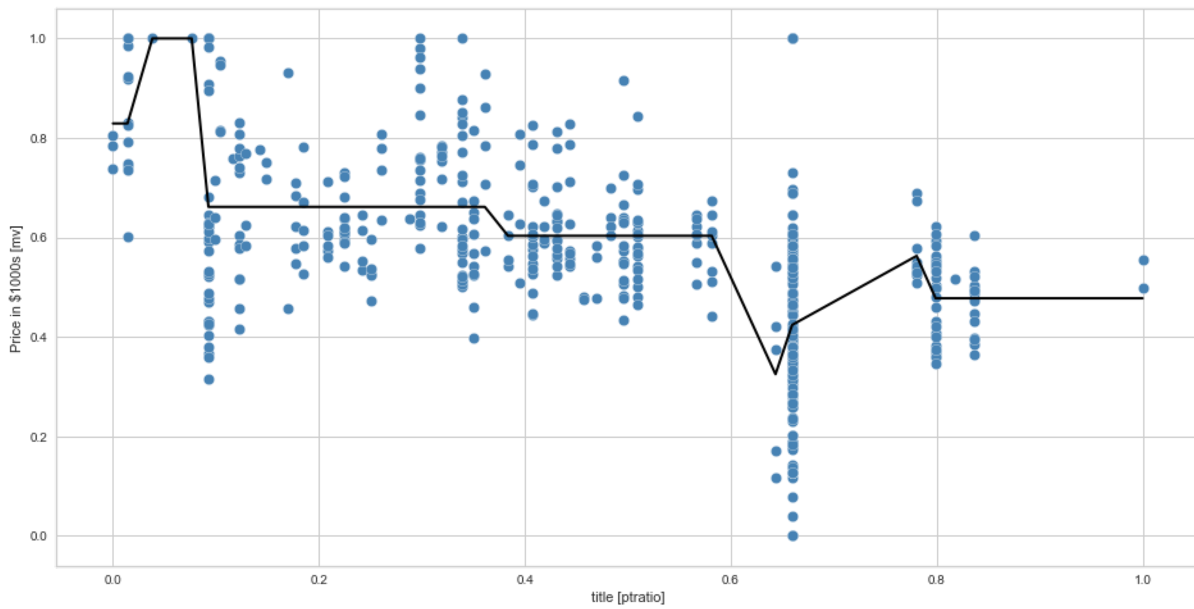


In [70]:
```
# PTRATIO
# Assign X matrix and y vector
X = boston_model[['ptratio']].values
# X = X.reshape(-1,1)
y = boston_model['mv'].values

# Sort X and y by ascending values of X
sort_idx = X.flatten().argsort()
X = X[sort_idx]
y = y[sort_idx]

# Initialize and fit regressor
tree = DecisionTreeRegressor(criterion='mse', max_depth=3)
tree.fit(X, y)
```

Out[70]:
```
DecisionTreeRegressor(ccp_alpha=0.0, criterion='mse', max_depth=3,
                      max_features=None, max_leaf_nodes=None,
                      min_impurity_decrease=0.0, min_impurity_split=None,
                      min_samples_leaf=1, min_samples_split=2,
                      min_weight_fraction_leaf=0.0, presort='deprecated',
                      random_state=None, splitter='best')
```

Figure 19: Actual and predicted target against feature - PTRATIO

In [71]:
```
# Plot actual versus predicted mv values
plt.figure(figsize=(16, 8))
plt.scatter(X, y, c='steelblue',                  # Plot actual target against features
            edgecolor='white', s=70)
plt.plot(X, tree.predict(X),                      # Plot predicted target against features
         color='black', lw=2)
plt.xlabel('pupil teacher ratio by town [ptratio]')
plt.ylabel('Price in $1000s [mv]')
plt.show()
```



Decision tree with multiple predictor features

In [150]:
```
# Split train and test set
X = boston_model.drop('mv', axis=1).values
y = boston_model.mv.values
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.20)
```

In [151]:
```
# Define model and fit
model =  DecisionTreeRegressor(max_depth=5, random_state=42)
model.fit(X_train, y_train)
```

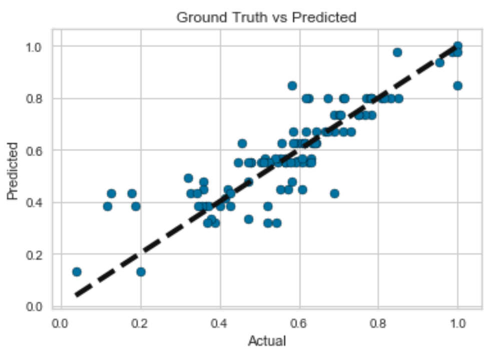
Out[151]:
```
DecisionTreeRegressor(ccp_alpha=0.0, criterion='mse', max_depth=5,
                      max_features=None, max_leaf_nodes=None,
                      min_impurity_decrease=0.0, min_impurity_split=None,
                      min_samples_leaf=1, min_samples_split=2,
                      min_weight_fraction_leaf=0.0, presort='deprecated',
                      random_state=42, splitter='best')
```

In [153]:
```
# Examine R sq to give an idea of the fit
# Explained variance score: 1 is perfect prediction
model_score = model.score(X_train, y_train)
print('Coefficient of determination R^2 of the prediction.: ',model_score)

# Predict values
y_pred = model.predict(X_test)

# MSE
print("Mean squared error: %.3f"% mean_squared_error(y_test, y_pred))
print("Root MSE: %.2f"% sqrt(mean_squared_error(y_test, y_pred)))
# Explained variance score: 1 is perfect prediction
print('Test Variance score: %.3f' % r2_score(y_test, y_pred))
```

```
Coefficient of determination R^2 of the prediction.:  0.9261765861164674
Mean squared error: 0.005
Root MSE: 0.07
Test Variance score: 0.827
```

Figure 20: Actual versus predicted target values (mv) with Decision Tree Regressor

In [119]:
```
# Run the model against the test data
fig, ax = plt.subplots()
ax.scatter(y_test, y_pred, edgecolors=(0, 0, 0))
ax.plot([y_test.min(), y_test.max()], [y_test.min(), y_test.max()], 'k--', lw=4)
ax.set_xlabel('Actual')
ax.set_ylabel('Predicted')
ax.set_title("Ground Truth vs Predicted")
plt.show()
```



In [ ]:

Ensemble methods

There are three main terms describing the ensemble (combination) of various models into one more effective model:

- Bagging to decrease the model's variance;
- Boosting to decreasing the model's bias, and;
- Stacking to increasing the predictive force of the classifier.

Gradient boosting

In [ ]:
```
# There's a trade-off between the learning rate and the number of trees needed, so you'll
# have to experiment to find the best values for each of the parameters, but small values
# less than 0.1 or values between 0.1 and 0.3 often work well.
```

In [313]:
```
# Split train and test set
X = boston_model.drop('mv', axis=1).values
y = boston_model.mv.values
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.20)
```

In [314]:
```
# Examine R sq to give an idea of the fit
# Explained variance score: 1 is perfect prediction
model = refit_et

model_score = model.score(X_train, y_train)
print('Coefficient of determination R^2 of the prediction: ',model_score)

# Predict values
y_pred = model.predict(X_test)

# # MSE
# print("Mean squared error: %.3f"% mean_squared_error(y_test, y_pred))
# print("Root MSE: %.2f"% sqrt(mean_squared_error(y_test, y_pred)))
# # Explained variance score: 1 is perfect prediction
# print('Test variance score: %.3f' % r2_score(y_test, y_pred))

# MSE
print("Mean squared error: %.3f"% mean_squared_error(y_pred, y_test))
print("Root MSE: %.2f"% sqrt(mean_squared_error(y_pred, y_test)))
# Explained variance score: 1 is perfect prediction
print('Test variance score: %.3f' % r2_score(y_pred, y_test))
```

```
Coefficient of determination R^2 of the prediction:  0.917365565235066
Mean squared error: 0.002
Root MSE: 0.05
Test variance score: 0.906
```

In [315]:
```
# Compare performance across different learning rates
# Mainly interested in the regressor's accuracy on the test set, but it looks like a learning
# rate of 0.3 with 12 max features gives us the best performance on the test set and good performance on the training
set.
lr_list = [0.05, 0.075, 0.1, 0.2, 0.3, 0.5, 0.75, 1]

for learning_rate in lr_list:
    gb_reg = GradientBoostingRegressor(n_estimators=100, learning_rate=learning_rate, max_features=10, max_depth=2, ra
ndom_state=42)
    gb_reg.fit(X_train, y_train)

    print("Learning rate: ", learning_rate)
    print("Accuracy score (training): {0:.3f}".format(gb_reg.score(X_train, y_train)))
    print("Accuracy score (test): {0:.3f}".format(gb_reg.score(X_test, y_test)))
    print('-------------------')
```

```
Learning rate:  0.05
Accuracy score (training): 0.917
Accuracy score (test): 0.850
-------------------
Learning rate:  0.075
Accuracy score (training): 0.935
Accuracy score (test): 0.862
-------------------
Learning rate:  0.1
Accuracy score (training): 0.943
Accuracy score (test): 0.870
-------------------
Learning rate:  0.2
Accuracy score (training): 0.965
Accuracy score (test): 0.878
-------------------
Learning rate:  0.3
Accuracy score (training): 0.977
Accuracy score (test): 0.872
-------------------
Learning rate:  0.5
Accuracy score (training): 0.986
Accuracy score (test): 0.857
-------------------
Learning rate:  0.75
Accuracy score (training): 0.991
Accuracy score (test): 0.868
-------------------
Learning rate:  1
Accuracy score (training): 0.994
Accuracy score (test): 0.800
-------------------
```

In [316]:

```
# Compare different  n_estimators
estimator_list = [20, 50, 100, 150, 200, 300, 400, 500]

for estimator in estimator_list:
    gb_reg = GradientBoostingRegressor(n_estimators=estimator, learning_rate=0.3, max_features=12, max_depth=2, random
_state=42)
    gb_reg.fit(X_train, y_train)

    print("n_estimators: ", estimator)
    print("Accuracy score (training): {0:.3f}".format(gb_reg.score(X_train, y_train)))
    print("Accuracy score (test): {0:.3f}".format(gb_reg.score(X_test, y_test)))
    print('-------------------')
```

```
n_estimators:  20
Accuracy score (training): 0.920
Accuracy score (test): 0.840
-------------------
n_estimators:  50
Accuracy score (training): 0.956
Accuracy score (test): 0.841
-------------------
n_estimators:  100
Accuracy score (training): 0.977
Accuracy score (test): 0.856
-------------------
n_estimators:  150
Accuracy score (training): 0.986
Accuracy score (test): 0.852
-------------------
n_estimators:  200
Accuracy score (training): 0.991
Accuracy score (test): 0.852
-------------------
n_estimators:  300
Accuracy score (training): 0.996
Accuracy score (test): 0.846
-------------------
n_estimators:  400
Accuracy score (training): 0.998
Accuracy score (test): 0.845
-------------------
n_estimators:  500
Accuracy score (training): 0.999
Accuracy score (test): 0.845
-------------------
```

In [317]:
```
# Experiment with warm start, adds to model already built
# Starts with 1 tree, adds trees til it has enough estimators
# Each iteration adds 5 more trees; after about 60, you're basically good with # of trees
for learning_rate in lr_list:
    gb_reg_ws = RandomForestRegressor(max_depth=2, random_state=42, warm_start=True, n_estimators=100)
    gb_reg_ws.fit(X_train, y_train)

    print("Learning rate: ", learning_rate)
    print("Accuracy score (training): {0:.3f}".format(gb_reg_ws.score(X_train, y_train)))
    print("Accuracy score (test): {0:.3f}".format(gb_reg_ws.score(X_test, y_test)))
    print('-------------------')
```

```
Learning rate:  0.05
Accuracy score (training): 0.784
Accuracy score (test): 0.759
-------------------
Learning rate:  0.075
Accuracy score (training): 0.784
Accuracy score (test): 0.759
-------------------
Learning rate:  0.1
Accuracy score (training): 0.784
Accuracy score (test): 0.759
-------------------
Learning rate:  0.2
Accuracy score (training): 0.784
Accuracy score (test): 0.759
-------------------
Learning rate:  0.3
Accuracy score (training): 0.784
Accuracy score (test): 0.759
-------------------
Learning rate:  0.5
Accuracy score (training): 0.784
Accuracy score (test): 0.759
-------------------
Learning rate:  0.75
Accuracy score (training): 0.784
Accuracy score (test): 0.759
-------------------
Learning rate:  1
Accuracy score (training): 0.784
Accuracy score (test): 0.759
-------------------
```

In [318]:
```
# Refit model with optimal learning rate and n_estimators
# Define model and fit
model =  GradientBoostingRegressor(max_depth=4, random_state=42, learning_rate=0.3, n_estimators=300,
                                   min_samples_split=2, loss='ls')


# 'n_estimators': 500, 'max_depth': 4, 'min_samples_split': 2,
#           'learning_rate': 0.03, 'loss': 'ls'


model.fit(X_train, y_train)
```

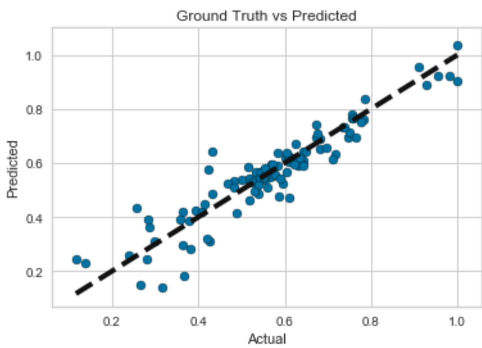
Out[318]:
```
GradientBoostingRegressor(alpha=0.9, ccp_alpha=0.0, criterion='friedman_mse',
                          init=None, learning_rate=0.3, loss='ls', max_depth=4,
                          max_features=None, max_leaf_nodes=None,
                          min_impurity_decrease=0.0, min_impurity_split=None,
                          min_samples_leaf=1, min_samples_split=2,
                          min_weight_fraction_leaf=0.0, n_estimators=300,
                          n_iter_no_change=None, presort='deprecated',
                          random_state=42, subsample=1.0, tol=0.0001,
                          validation_fraction=0.1, verbose=0, warm_start=False)
```

In [319]:
```
# Examine R sq to give an idea of the fit
# Explained variance score: 1 is perfect prediction
model_score = model.score(X_train, y_train)
print('Coefficient of determination R^2 of the prediction.: ',model_score)

# Predict values
y_pred = model.predict(X_test)

# MSE
print("Mean squared error: %.3f"% mean_squared_error(y_test, y_pred))
print("Root MSE: %.3f"% sqrt(mean_squared_error(y_test, y_pred)))
# Explained variance score: 1 is perfect prediction
print('Test Variance score: %.3f' % r2_score(y_test, y_pred))
```

```
Coefficient of determination R^2 of the prediction.:  0.9999971685206094
Mean squared error: 0.004
Root MSE: 0.064
Test Variance score: 0.864
```

Figure 21: Actual versus predicted target values (mv) with Gradient Boosting Regressor

In [320]:
```
# Run the model against the test data
fig, ax = plt.subplots()
ax.scatter(y_test, y_pred, edgecolors=(0, 0, 0))
ax.plot([y_test.min(), y_test.max()], [y_test.min(), y_test.max()], 'k--', lw=4)
ax.set_xlabel('Actual')
ax.set_ylabel('Predicted')
ax.set_title("Ground Truth vs Predicted")
plt.show()
```



In [321]:
```
# Check numeric values of feature importance for GB
for name, score in zip(feat_list, model.feature_importances_):
    print(name, score)
```

```
crim 0.07791579331161841
zn 0.00016215554361862273
indus 0.0063858107016804486
chas 0.0011613661056134453
nox 0.04278134546163636
rooms 0.2245846309264435
age 0.013178775564784563
dis 0.0549131329169467
rad 0.006653743710461629
tax 0.014875024875555124
ptratio 0.02314527510620031
lstat 0.534242945775441
```

In [ ]:
```
# Try with SGBD
# The GradientBoostibngRegressor class also supports a subsample hyperparameter, which specifies the
# fraction of training instances to be used for training each tree. For example, if subsample=0.25,
# then each tree is trained on 25% of the training instances, selected randomly. This technique trades
# a higher bias for a lower variance. It also speeds up training considerably.
# This is called Stochastic Gradient Boosting.
```

In [322]:
```
# Try Stochastic Gradient Boosting
# Define model and fit using subsample = 0.25
model =  GradientBoostingRegressor(max_depth=4, random_state=42, learning_rate=0.3, n_estimators=500,
                                   min_samples_split=2, loss='ls', subsample=0.25)

model.fit(X_train, y_train)
```

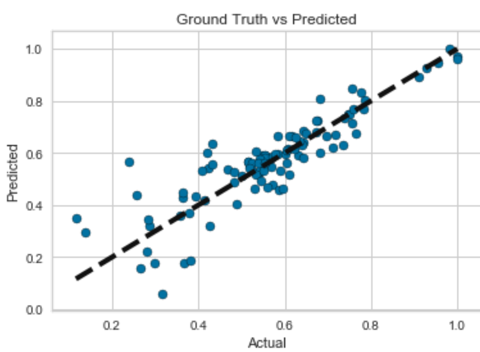
Out[322]:
```
GradientBoostingRegressor(alpha=0.9, ccp_alpha=0.0, criterion='friedman_mse',
                          init=None, learning_rate=0.3, loss='ls', max_depth=4,
                          max_features=None, max_leaf_nodes=None,
                          min_impurity_decrease=0.0, min_impurity_split=None,
                          min_samples_leaf=1, min_samples_split=2,
                          min_weight_fraction_leaf=0.0, n_estimators=500,
                          n_iter_no_change=None, presort='deprecated',
                          random_state=42, subsample=0.25, tol=0.0001,
                          validation_fraction=0.1, verbose=0, warm_start=False)
```

In [323]:
```
# Examine R sq to give an idea of the fit
# Explained variance score: 1 is perfect prediction
model_score = model.score(X_train, y_train)
print('Coefficient of determination R^2 of the prediction.: ',model_score)

# Predict values
y_pred = model.predict(X_test)

# MSE
print("Mean squared error: %.3f"% mean_squared_error(y_test, y_pred))
print("Root MSE: %.3f"% sqrt(mean_squared_error(y_test, y_pred)))
# Explained variance score: 1 is perfect prediction
print('Test Variance score: %.3f' % r2_score(y_test, y_pred))
```

```
Coefficient of determination R^2 of the prediction.:  0.9999419280494728
Mean squared error: 0.007
Root MSE: 0.085
Test Variance score: 0.757
```

Figure 22: Actual versus predicted target values (mv) with Stochastic Gradient Boosting

In [324]:
```
# Run the model against the test data
fig, ax = plt.subplots()
ax.scatter(y_test, y_pred, edgecolors=(0, 0, 0))
ax.plot([y_test.min(), y_test.max()], [y_test.min(), y_test.max()], 'k--', lw=4)
ax.set_xlabel('Actual')
ax.set_ylabel('Predicted')
ax.set_title("Ground Truth vs Predicted")
plt.show()
```



Stacking

Scikit-learn stacking was implemented as follows:

- Base learners are fitted on the full X while the final estimator is trained using cross-validated predictions of the base learners using cross_val_predict.

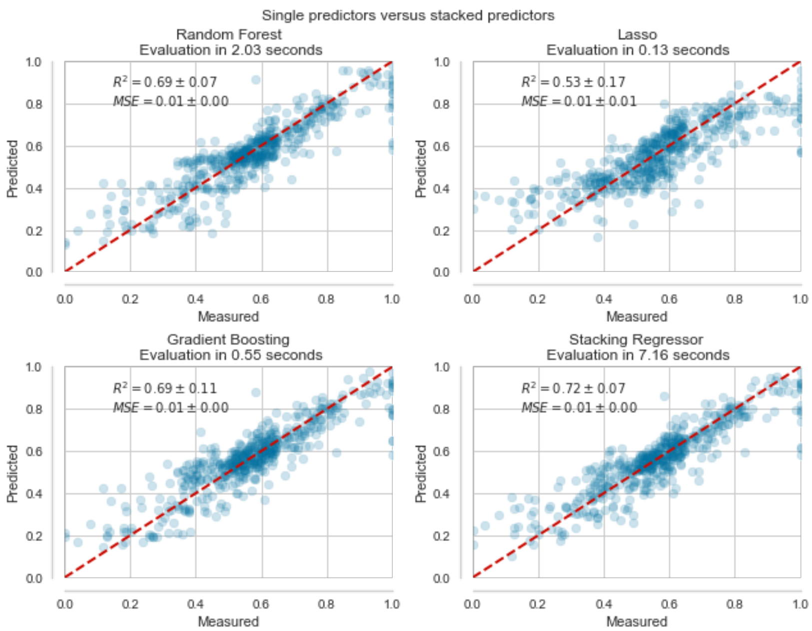
This means that the predictions of each individual base learner are stacked together and used as input to a meta learner to compute the prediction. This meta learner is then trained through cross-validation. Cross-validation is automatically set to 5-fold CV but can be adjusted manually.

In [129]:
```
#  Combine 3 learners (linear and non-linear) and use a ridge regressor to combine their outputs together
estimators = [
    ('Random Forest', RandomForestRegressor(random_state=42)),
    ('Lasso', LassoCV()),
    ('Gradient Boosting', HistGradientBoostingRegressor(random_state=0))
]
stacking_regressor = StackingRegressor(
    estimators=estimators, final_estimator=RidgeCV()
)
```

In [130]:
```
# Function for plotting
def plot_regression_results(ax, y_true, y_pred, title, scores, elapsed_time):
    """Scatter plot of the predicted vs true targets."""
    ax.plot([y_true.min(), y_true.max()],
            [y_true.min(), y_true.max()],
            '--r', linewidth=2)
    ax.scatter(y_true, y_pred, alpha=0.2)

    ax.spines['top'].set_visible(False)
    ax.spines['right'].set_visible(False)
    ax.get_xaxis().tick_bottom()
    ax.get_yaxis().tick_left()
    ax.spines['left'].set_position(('outward', 10))
    ax.spines['bottom'].set_position(('outward', 10))
    ax.set_xlim([y_true.min(), y_true.max()])
    ax.set_ylim([y_true.min(), y_true.max()])
    ax.set_xlabel('Measured')
    ax.set_ylabel('Predicted')
    extra = plt.Rectangle((0, 0), 0, 0, fc="w", fill=False,
                          edgecolor='none', linewidth=0)
    ax.legend([extra], [scores], loc='upper left')
    title = title + '\n Evaluation in {:.2f} seconds'.format(elapsed_time)
    ax.set_title(title)
```

Figure 23: Blended model results against individual models

In [131]:
```
# Plot results
fig, axs = plt.subplots(2, 2, figsize=(9, 7))
axs = np.ravel(axs)

for ax, (name, est) in zip(axs, estimators + [('Stacking Regressor',
                                               stacking_regressor)]):
    start_time = time.time()
    score = cross_validate(est, X, y,
                           scoring=['r2', 'neg_mean_squared_error'],
                           n_jobs=-1, verbose=0)
    elapsed_time = time.time() - start_time

    y_pred = cross_val_predict(est, X, y, n_jobs=-1, verbose=0)
    plot_regression_results(
        ax, y, y_pred,
        name,
        (r'$R^2={:.2f} \pm {:.2f}$' + '\n' + r'$MSE={:.2f} \pm {:.2f}$')
        .format(np.mean(score['test_r2']),
                np.std(score['test_r2']),
                -np.mean(score['test_neg_mean_squared_error']),
                np.std(score['test_neg_mean_squared_error'])),
        elapsed_time)

plt.suptitle('Single predictors versus stacked predictors')
plt.tight_layout()
plt.subplots_adjust(top=0.9)
plt.show()
```



In [ ]: