Assignment 1: Neural Networks for Computer Vision

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Google Colab link: https://colab.research.google.com/drive/1lRznCtJpwtEoHgyvuU_qJ82aFr9lhk3O

Abstract

Neural networks for image classification offer a powerful means of identifying and reliably classifying ever-varying handwritten digits, such as those that postal operations and financial institutions process at scale every day. Accurately deciphering digits ensures fast and accurate delivery of a variety of critical services, but given immense variation in handwriting styles, this is no small feat. The MNIST database offers a large labeled dataset of handwritten digits that has proven helpful in testing machine learning approaches for more accurate classification, enabling the kind of automation of processes that helps these operations and their employees work more effectively. This study explores neural networks for digit classification using MNIST for training and validation, with the goal of revealing more effective means for architecting and configuring effective models. Greater understanding of the intricacies of neural network architectures and the numerous ways they can be adjusted lead to more performant models more likely to work well in real-life classification scenarios.

Introduction

This study is a comparative exploration of simple neural networks for digit classification, with a two-fold goal: develop a neural network model for highly-accurate (99 percent-range) digit classification and gain an in-depth understanding of how the neurons in the simple single-hidden layer network have learned to represent features within the input data. Exploring and evaluating simple single hidden-layer networks – all trained with the backpropagation learning method with varying architectures with respect to width and dimensionality – helps determine their potential utility in multi-class classification problems on the MNIST dataset of hand-drawn digits and for future analogous Optical Character Recognition (OCR) efforts.

Literature review

The MNIST dataset of handwritten digits has formed the basis of many research efforts in classification, varying widely in terms of classifier type and preprocessing applied. Neural network architectures and convolutional neural nets have both been studied widely as a means to ever-higher classifier performance improvement. Findings by LeCun et al showed how a learning network could be fed the images directly (versus feature vectors) to demonstrate the capabilities of the backpropagation algorithm on digit classification (1989, 1990). Another oft-cited research is Kenneth Wilder, particularly his doctoral dissertation on decision tree algorithms for handwritten digit recognition (1998). Zhang and Srihari asses k-nearest neighbor (k-NN) algorithms' efficiency through extensive experiments using the MNIST database (2004). Arif, Siddique, Khan, and Oishe evaluate variations in classification accuracy for varying layer depths and epochs (2018). LeCun has also published extensively on CNNs more recently for digit classification. LeCun, Bottou, Bengio, and Haffner explore multilayer neural networks for handwritten digit processing while attempting to minimize preprocessing needed (1998). Additionally, Sermanet, Chintala, and LeCun experiment with different pooling techniques (2012).

Methods

The MNIST dataset contains gray-scale images of hand-drawn digits, from zero through nine. Each image is 28 pixels in height and 28 pixels in width, for a total of 784 pixels in total. Each pixel has a single pixel-value associated with it from 0 to 255, indicating the lightness or darkness of that pixel, with higher numbers meaning darker. The label distribution in the training and testing sets appears relatively balanced across zero to nine, which obviates the need for adjustment by up-sampling, down-sampling, or other methods. The 70,000 images are divided into a set of 60,000 training images (of which 5,000 are held back for validation) and 10,000 test images, then reshaped and normalized. The study evaluates 5 neural networks for performance, measured by classification accuracy, and processing time, with network architecture defined as follows:

Neural Network	Network Architecture	Trainable Parameters
Simple MLP 1	784 input nodes 1 Dense, ReLU activation, 1 node 1 Dense, Softmax activation, 10 output nodes	805
Simple MLP 2	784 input nodes 1 Dense, ReLU activation, 2 nodes 1 Dense, Softmax activation, 10 output nodes	1,600
"Best" MLP	784 input nodes 1 Dense, sigmoid activation, 700 nodes 1 Dense, Softmax activation, 10 output nodes	556,510
"Best" MLP with PCA	154 input nodes 1 Dense, sigmoid activation, 700 nodes 1 Dense, Softmax activation, 10 output nodes	556,510
"Best" MLP with dimension reduction via Random Forest	70 input nodes 1 Dense, sigmoid activation, 700 nodes 1 Dense, Softmax activation, 10 output nodes	556,510

All experimental networks consist of 784 input nodes and two fully-connected neural layers (a hidden layer with varying nodes and 10 output nodes). Simple MLP 1 serves as a baseline model under the assumption of reasonable performance on this proscribed dataset. The simple MLP 2 model extends the baseline with the addition of 1 additional neuron. Model experimentation in experiment 3 establishes the "best" assignment of 700 neurons via grid search, with the same layer depth. Given the potential of overfitting with this sizeable increase in layer width, and thus tunable parameters, attention is given to that scenario in model performance with both training and validation data. This architecture, with sigmoid activation instead of ReLU, is used for the remaining two experiments utilizing dimension reduction; for comparison principal components analysis (PCA) and feature prioritization with the Random Forest classifier are evaluated to further improve model performance. The following hyperparameters are employed:

- Learning rate: 0.01 / Batch size: 32 (default) / Epochs: 30 (all models)
- Activation: ReLU (models 1 and 2) and sigmoid (models 3, 4, 5)
- Optimizer: rmsprop (models 1 and 2) and Adam (models 3, 4, 5)

Results

The table below shows model fitting and evaluation results:

Type of Neural Net	Training time (in seconds)	Acc (Train)	Acc (Validation)
Simple MLP 1	84.26	0.436	0.433
Simple MLP 2	95.17	0.697	0.701
"Best" MLP 3	2421.23	0.999	0.982
"Best" MLP with PCA	275.36	0.999	0.979
"Best" MLP using RF	153.65	0.982	0.948

Simple MLP model 1 performs surprisingly well with just one hidden layer and 1 node – a jump from 10 percent accuracy at random to 43 percent on both training and validation data (Figure 1). The confusion matrix reveals that the model is not predicting any 5s, which is problematic considering the relative balance (thus known existence of 5s) within the dataset (Figure 2). 0s, 1s, and 7s are predicted relatively well; 8s and 9s prove challenging for the model. The boxplot of activation values (Figure 4) show that the model is lacking in discriminatory power: there is quite a bit of overlap across the 10 digits, so the neural network stands to improve in extracting essential features of the data. The extended architecture of MLP model 2, with only one more node, adds notable performance improvement with only a small increase in training time (Figure 5). The confusion matrix (Figure 6) shows noticeable improvement in prediction accuracy: 5s are now predicted and the correct convergence of predicted versus actual represented by the diagonal line has many more images correctly classified (except for 8s and 3s, both regularly and incorrectly predicted as 5s). The scatterplot of activation values and their grouping by predicted class (Figure 9) shows how well the model is discriminating; in a 100 percent accurate model, these clusters would have no overlap. This model discriminates more accurately than model 1, but the overlap shows a need for further experimentation. For the more complex neural networks, model 3 results in almost perfect performance on training data (likely a result of overfitting from the dramatic increase in layer width and tunable parameters) but still reliable performance on unseen validation data (Figure 10); one tradeoff is a sizeable increase in training time needed for this architecture. With a different activation function (sigmoid) and optimizer at model compilation (Adam, which adds momentum) the refined model 3 results in a slight performance increase over the model 3 baseline; this is most evident in the confusion matrices of the two models (Figures 11 and 13). Reducing dimensions via PCA (from 784 to 154 input nodes) results in comparable performance to model 3 (Figure 15). With features reduced via Random Forest classification (from 784 to 70 nodes), performance is slightly lower but still high for both training and testing data considering the drastic reduction in inputs (Figure 16). Both result in notable efficiencies in training time.

Conclusions

For this study's classification task, simple MLP models perform well on training and validation, processes efficiently, and generalize well on unseen data. For this dataset and comparable OCR problems with handwritten or typed digits, the simplicity of the MLP seems an appropriate baseline. However, given the management requirement for near-perfect classification accuracy as the measure of performance, a more complex network is likely required. Extending layer depth to 700 nodes provides this in both training and validation model performance, suggesting reliable generalization on new, unseen handwritten digits. With this complexity, though, comes higher numbers of tunable parameters and a much higher potential for overfitting. Future experiments on network architecture could test this layer width to decipher the extent of the overfitting problem with varying numbers of nodes and additional validation. This study did not experiment with layer depth, which could be an additional point of exploration for network architectures that would reach desired performance. Model compilation could be configured differently as well: this study experimented with rmsprop and Adam optimizers, however others may result in better performance. Moreover, learning rates, batch sizes, and epochs could also be adjusted numerous ways to explore optimal models for classification accuracy. Beyond the models evaluated in experiments 1 through 3, dimension reduction in experiments 4 and 5 using the "best" model provided a means to training efficiency and processing time; though this is not as much an issue with 784 pixels, it is worth considering with classification tasks on images of much higher dimensionality. Feature extraction with Random Forest could serve as a baseline for further experimentation: a simple model of 100 trees as default is used in this study, however numerous other hyperparameters (e.g., maximum tree depth, bootstrapping, impurity allowed, and/or warm start) could be tested via random or grid search to identify more promising models. Additionally, other ensemble methods beyond Random Forest might be explored, for example Extremely Randomized Trees or XGBoost. As for the data itself, a different train/validation split could be explored for further exploration of effect on training accuracy versus generalizability. Ultimately, though not offering a "perfect" solution to the digit classification task, the experiments in this study offer promising baselines and highlight important considerations for designing an optimal future approach.

References

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LeCun et al. (1990). "Handwritten Digit Recognition with a Back-propagation Network," in *Advances in Neural Information Processing Systems*, 396-404.

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Objective

This study is a comparative exploration of simple neural networks for digit classification, with a two-fold goal: develop a neural network model for highly-accurate (99 percent-range) digit classification and gain an in-depth understanding of how the neurons in the simple single-hidden layer network have learned to represent features within the input data. A comparison and evaluation of simple single hidden-layer networks with varying architectures (with respect to width and dimensionality) helps determine their potential utility in multi-class classification problems on the MNIST dataset of hand-drawn digits and for future analogous Optical Character Recognition (OCR) efforts.

In [1]:
```
# Import dependencies
import numpy as np
import pandas as pd
import os
import time

import tensorflow as tf
from tensorflow import keras
from tensorflow.keras import models
from tensorflow.keras import layers
from tensorflow.keras.wrappers.scikit_learn import KerasClassifier
from sklearn.metrics import confusion_matrix
from sklearn.model_selection import KFold, GridSearchCV
from sklearn.ensemble import RandomForestClassifier
from sklearn.decomposition import PCA

%matplotlib inline
import matplotlib.pyplot as plt
import seaborn as sns
```

In [2]:
```
# Check versions
print(keras.__version__)
print(tf.__version__)
```

```
2.2.4-tf
2.0.0
```

In [3]:
```
tf.compat.v1.disable_eager_execution()
```

In [4]:
```
# For consistent results
keras.backend.clear_session()
np.random.seed(42)
tf.random.set_seed(42)
```

Data import and pre-processing

In [5]:
```
# Import data from Keras
from tensorflow.keras.datasets import mnist
(train_images, train_labels), (test_images, test_labels) = mnist.load_data()
```

In [6]:
```
# Check shapes
train_images.shape, test_images.shape, test_labels.shape, test_labels.shape
```

Out[6]:
```
((60000, 28, 28), (10000, 28, 28), (10000,), (10000,))
```

Preprocess data by reshaping it into the shape that the network expects and scaling it so that all values are in the [0, 1] interval.

- Training images are stored in an array of shape (60000, 28, 28) of type uint8 with values in the [0, 255] interval.
- Transform images it into a float32 array of shape (60000, 28 * 28) with values between 0 and 1.

In [7]:
```
train_images = train_images.reshape((60000, 28 * 28))
train_images = train_images.astype('float32') / 255

test_images = test_images.reshape((10000, 28 * 28))
test_images = test_images.astype('float32') / 255
```

In [8]:
```
# Set apart 5,000 samples from training data for validation
val_images, train_images = train_images[:5000], train_images[5000:]
val_labels, train_labels = train_labels[:5000], train_labels[5000:]
```

EXPERIMENT 1:

This network will consist of 784 input nodes, a hidden layer with `1 node` and 10 output nodes (corresponding to the 10 digits). We use `mnist.load_data()` to get the 70,000 images divided into a set of 60,000 training images and 10,000 test images. 5,000 of the 60,000 training images are held back for validation. After training the model, we group the 60,000 `activation values` of the hidden node for the (original) set of training images by the 10 predicted classes and visualize these sets of values using a `boxplot`. The overlap between the range of values in the "boxes" is expected to be minimal.

Build network

This network consists of a sequence of two `Dense` layers, which are densely-connected (also called "fully-connected") neural layers.

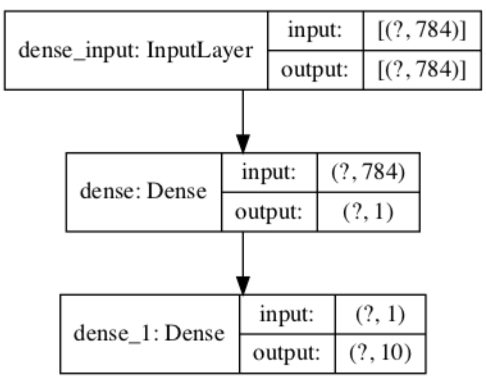
- The first `Dense` layer, the hidden layer, consists of just `one node`.
- The second (and last) layer is a 10-way "softmax" layer, which means it will return an array of 10 probability scores (summing to 1). Each score will be the probability that the current digit image belongs to one of our 10 digit classes.

In [9]:
```
# Define model using Sequential class
# reLu activation
model_1 = models.Sequential()
model_1.add(layers.Dense(1, activation='relu', input_shape=(28 * 28,)))
model_1.add(layers.Dense(10, activation='softmax'))
```

```
WARNING:tensorflow:From /anaconda3/lib/python3.6/site-packages/tensorflow_core/python/ops/resource_variable_ops.py:16
30: calling BaseResourceVariable.__init__ (from tensorflow.python.ops.resource_variable_ops) with constraint is depre
cated and will be removed in a future version.
Instructions for updating:
If using Keras pass *_constraint arguments to layers.
```

In [41]:
```
# Plot a graph of the model
keras.utils.plot_model(model_1, "figures/mnist_model_1_1hnode.png", show_shapes=True)
```

Out[41]:



In [12]:
```
# Model summary
model_1.summary()
```

```
Model: "sequential"
_________________________________________________________________
Layer (type)                 Output Shape              Param #   
=================================================================
dense (Dense)                (None, 1)                 785       
_________________________________________________________________
dense_1 (Dense)              (None, 10)                20        
=================================================================
Total params: 805
Trainable params: 805
Non-trainable params: 0
_________________________________________________________________
```

Compile model

To make our network ready for training, we need to pick three more things, as part of "compilation" step:

- A loss function: the is how the network will be able to measure how good a job it is doing on its training data, and thus how it will be able to steer itself in the right direction.
- An optimizer: this is the mechanism through which the network will update itself based on the data it sees and its loss function.
- Metrics to monitor during training and testing. Here we will only care about accuracy (the fraction of the images that were correctly classified).

In [13]:
```
# Compile model
model_1.compile(loss='sparse_categorical_crossentropy', optimizer='rmsprop', metrics=['accuracy'])
```

Train model

We are now ready to train our network, which in Keras is done via a call to the `fit` method of the network: we "fit" the model to its training data. We train the model for 30 epochs with batch size 32 (the default).

In [14]:
```
# If loading previous model:
# model = model_1()
# model = load_model_1('results/keras_mnist_model_2.h5')

# Train the model
start_time = time.time()
history = model_1.fit(train_images, train_labels, validation_data=(val_images, val_labels),
                      epochs=30)
# fit(train_images, train_labels, epochs=30,
#                       validation_data=(val_images, val_labels))
elapsed_time = time.time() - start_time
print('---------------------------')
print('Training time in seconds: ', round(elapsed_time,2))
print('---------------------------')

# Saving models locally after fitting
save_dir = "results/"
model_name = 'mnist_model_1.h5'
model_path = os.path.join(save_dir, model_name)
model_1.save(model_path)
print('Saved trained model at %s ' % model_path)
print('---------------------------')
```

```
Train on 55000 samples, validate on 5000 samples
Epoch 1/30
55000/55000 [==============================] - 3s 62us/sample - loss: 1.9612 - accuracy: 0.2236 - val_loss: 1.8292 -
val_accuracy: 0.2822
Epoch 2/30
55000/55000 [==============================] - 3s 61us/sample - loss: 1.8108 - accuracy: 0.2961 - val_loss: 1.7554 -
val_accuracy: 0.3192
Epoch 3/30
55000/55000 [==============================] - 3s 52us/sample - loss: 1.7606 - accuracy: 0.3187 - val_loss: 1.7152 -
val_accuracy: 0.3384
Epoch 4/30
55000/55000 [==============================] - 4s 78us/sample - loss: 1.7309 - accuracy: 0.3283 - val_loss: 1.6901 -
val_accuracy: 0.3436
Epoch 5/30
55000/55000 [==============================] - 3s 49us/sample - loss: 1.7117 - accuracy: 0.3363 - val_loss: 1.6732 -
val_accuracy: 0.3432
Epoch 6/30
55000/55000 [==============================] - 3s 47us/sample - loss: 1.6968 - accuracy: 0.3423 - val_loss: 1.6629 -
val_accuracy: 0.3530
Epoch 7/30
55000/55000 [==============================] - 3s 48us/sample - loss: 1.6830 - accuracy: 0.3501 - val_loss: 1.6432 -
val_accuracy: 0.3530
Epoch 8/30
55000/55000 [==============================] - 2s 44us/sample - loss: 1.6656 - accuracy: 0.3602 - val_loss: 1.6242 -
val_accuracy: 0.3652
Epoch 9/30
55000/55000 [==============================] - 2s 45us/sample - loss: 1.6486 - accuracy: 0.3654 - val_loss: 1.6085 -
val_accuracy: 0.3686
Epoch 10/30
55000/55000 [==============================] - 3s 46us/sample - loss: 1.6360 - accuracy: 0.3662 - val_loss: 1.5978 -
val_accuracy: 0.3692
Epoch 11/30
55000/55000 [==============================] - 3s 49us/sample - loss: 1.6260 - accuracy: 0.3663 - val_loss: 1.5875 -
val_accuracy: 0.3762
Epoch 12/30
55000/55000 [==============================] - 3s 51us/sample - loss: 1.6178 - accuracy: 0.3691 - val_loss: 1.5802 -
val_accuracy: 0.3760
Epoch 13/30
55000/55000 [==============================] - 3s 58us/sample - loss: 1.6121 - accuracy: 0.3744 - val_loss: 1.5804 -
val_accuracy: 0.3806
Epoch 14/30
55000/55000 [==============================] - 3s 50us/sample - loss: 1.6069 - accuracy: 0.3827 - val_loss: 1.5749 -
val_accuracy: 0.3824
Epoch 15/30
55000/55000 [==============================] - 3s 48us/sample - loss: 1.6030 - accuracy: 0.3900 - val_loss: 1.5722 -
val_accuracy: 0.4004
Epoch 16/30
55000/55000 [==============================] - 3s 49us/sample - loss: 1.5999 - accuracy: 0.3950 - val_loss: 1.5642 -
val_accuracy: 0.4102
Epoch 17/30
55000/55000 [==============================] - 3s 51us/sample - loss: 1.5970 - accuracy: 0.3982 - val_loss: 1.5705 -
val_accuracy: 0.3998
Epoch 18/30
55000/55000 [==============================] - 3s 49us/sample - loss: 1.5947 - accuracy: 0.4011 - val_loss: 1.5629 -
val_accuracy: 0.4140
Epoch 19/30
55000/55000 [==============================] - 3s 47us/sample - loss: 1.5931 - accuracy: 0.4035 - val_loss: 1.5632 -
val_accuracy: 0.4154
Epoch 20/30
55000/55000 [==============================] - 3s 52us/sample - loss: 1.5915 - accuracy: 0.4073 - val_loss: 1.5570 -
val_accuracy: 0.4216
Epoch 21/30
55000/55000 [==============================] - 3s 49us/sample - loss: 1.5891 - accuracy: 0.4102 - val_loss: 1.5586 -
val_accuracy: 0.4266
Epoch 22/30
55000/55000 [==============================] - 3s 52us/sample - loss: 1.5865 - accuracy: 0.4120 - val_loss: 1.5511 -
val_accuracy: 0.4346
Epoch 23/30
55000/55000 [==============================] - 3s 47us/sample - loss: 1.5827 - accuracy: 0.4186 - val_loss: 1.5506 -
val_accuracy: 0.4196
Epoch 24/30
55000/55000 [==============================] - 3s 48us/sample - loss: 1.5766 - accuracy: 0.4190 - val_loss: 1.5407 -
val_accuracy: 0.4374
Epoch 25/30
55000/55000 [==============================] - 2s 44us/sample - loss: 1.5703 - accuracy: 0.4191 - val_loss: 1.5343 -
val_accuracy: 0.4296
Epoch 26/30
55000/55000 [==============================] - 2s 45us/sample - loss: 1.5645 - accuracy: 0.4212 - val_loss: 1.5361 -
val_accuracy: 0.4132
Epoch 27/30
55000/55000 [==============================] - 3s 48us/sample - loss: 1.5606 - accuracy: 0.4203 - val_loss: 1.5311 -
val_accuracy: 0.4232
Epoch 28/30
55000/55000 [==============================] - 3s 52us/sample - loss: 1.5573 - accuracy: 0.4271 - val_loss: 1.5269 -
val_accuracy: 0.4294
```

Test model

Evaluate the model on the test dataset.

In [15]:
```
test_loss, test_acc = model_1.evaluate(test_images, test_labels)
```

```
10000/10000 [==============================] - 0s 37us/sample - loss: 1.5850 - accuracy: 0.4333
```

In [16]:
```
print(f'test accuracy: {test_acc}, test loss: {test_loss}')
```

```
test accuracy: 0.4332999885082245, test loss: 1.5849745712280274
```

Plot model performance

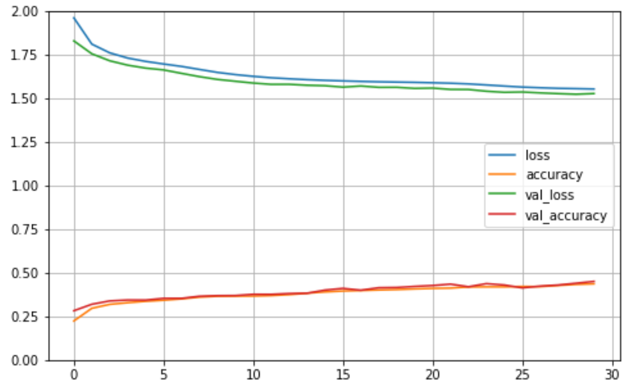
Assess accuracy and loss

In [17]:
```
history_dict = history.history
history_dict.keys()
```

Out[17]:
```
dict_keys(['loss', 'accuracy', 'val_loss', 'val_accuracy'])
```

Figure 1: Learning curves for Model 1

In [40]:
```
# Alternate code for accuracy and loss
# Plot learning curves for model
pd.DataFrame(history.history).plot(figsize=(8, 5))
plt.grid(True)
plt.gca().set_ylim(0, 2) # set the vertical range to [0-1]
plt.savefig("figures/learning_curves_model_1", tight_layout=False)
plt.show()
```



Confusion matrix comparison

Evaluate prediction errors with Model 1

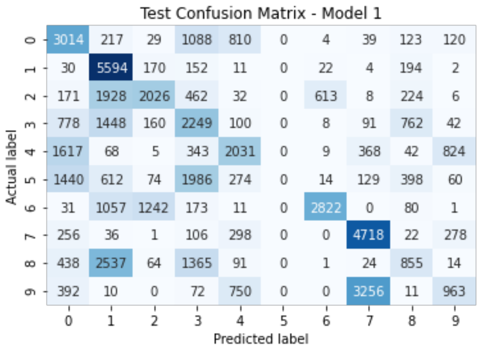
In [20]:
```
# Assign predicted classes
pred_classes = model_1.predict_classes(train_images)
```

In [21]:
```
# Create confusion matrix with values
conf_mx = confusion_matrix(train_labels,pred_classes)
conf_mx
```

Out[21]:
```
array([[3014,  217,   29, 1088,  810,    0,    4,   39,  123,  120],
       [  30, 5594,  170,  152,   11,    0,   22,    4,  194,    2],
       [ 171, 1928, 2026,  462,   32,    0,  613,    8,  224,    6],
       [ 778, 1448,  160, 2249,  100,    0,    8,   91,  762,   42],
       [1617,   68,    5,  343, 2031,    0,    9,  368,   42,  824],
       [1440,  612,   74, 1986,  274,    0,   14,  129,  398,   60],
       [  31, 1057, 1242,  173,   11,    0, 2822,    0,   80,    1],
       [ 256,   36,    1,  106,  298,    0,    0, 4718,   22,  278],
       [ 438, 2537,   64, 1365,   91,    0,    1,   24,  855,   14],
       [ 392,   10,    0,   72,  750,    0,    0, 3256,   11,  963]])
```

Figure 2: Confusion matrix for Model 1

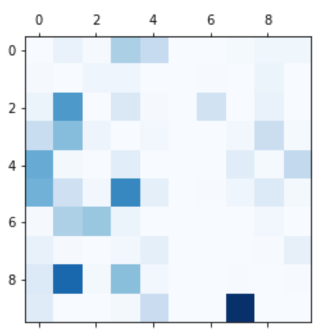
In [39]:
```
# Plot with values transposed to show actual versus predicted values
# m1_cm = confusion_matrix(train_labels, pred_classes)
m1_cm_plt=sns.heatmap(conf_mx, square=False, annot=True, fmt='d', cbar=False, cmap="Blues")
plt.xlabel('Predicted label')
plt.ylabel('Actual label')
plt.title("Test Confusion Matrix - Model 1")
plt.savefig("figures/confusion_matrix_errors_plot_model_1", tight_layout=False)
plt.show();
```



In [23]:
```
# Plot normalized confusion matrix
row_sums = conf_mx.sum(axis=1, keepdims=True)
norm_conf_mx = conf_mx / row_sums
```

Figure 3: Normalized confusion matrix for Model 1

In [38]:
```
np.fill_diagonal(norm_conf_mx, 0)
plt.matshow(norm_conf_mx, cmap="Blues")
plt.savefig("figures/confusion_matrix_errors_plot_norm_model_1", tight_layout=False)
plt.show()
```



Error analysis

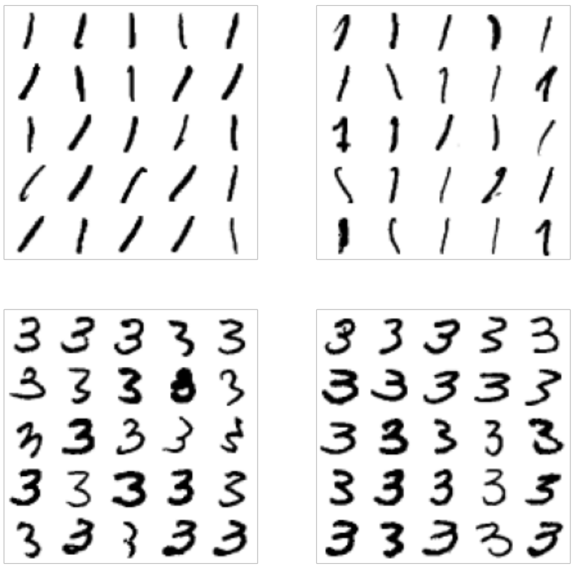
No 5s were predicted, which is an issue considering the general balance of 0-9 classes in the data.

In [25]:
```
# Define function for plotting example digits
def plot_digits(instances, images_per_row=10, **options):
    size = 28
    images_per_row = min(len(instances), images_per_row)
    images = [instance.reshape(size,size) for instance in instances]
    n_rows = (len(instances) - 1) // images_per_row + 1
    row_images = []
    n_empty = n_rows * images_per_row - len(instances)
    images.append(np.zeros((size, size * n_empty)))
    for row in range(n_rows):
        rimages = images[row * images_per_row : (row + 1) * images_per_row]
        row_images.append(np.concatenate(rimages, axis=1))
    image = np.concatenate(row_images, axis=0)
    plt.imshow(image, cmap = 'binary', **options)
    plt.axis("off")
```

In [26]:
```
# Try some examples of mis-classes digits
# Upper left: 1s classified correctly as 1s
# Upper right: 1s classified as 3s
# Lower left: 3s classified as 1s
# Lower right: 3s classified correctly as 3sx

cl_a, cl_b = 1, 3
X_aa = train_images[(train_labels == cl_a) & (pred_classes == cl_a)]
X_ab = train_images[(train_labels == cl_a) & (pred_classes == cl_b)]
X_ba = train_images[(train_labels == cl_b) & (pred_classes == cl_a)]
X_bb = train_images[(train_labels == cl_b) & (pred_classes == cl_b)]

plt.figure(figsize=(8,8))
plt.subplot(221); plot_digits(X_aa[:25], images_per_row=5)
plt.subplot(222); plot_digits(X_ab[:25], images_per_row=5)
plt.subplot(223); plot_digits(X_ba[:25], images_per_row=5)
plt.subplot(224); plot_digits(X_bb[:25], images_per_row=5)
plt.savefig("error_analysis_digits_plot_EXP1_valid")
plt.show()
```



Get the activation values of the hidden nodes

To get the activation values of the hidden nodes, we need to create a new model, `activation_model` , that takes the same input as our current model but outputs the activation value of the hidden layer, i.e. of the hidden node. Then use the `predict` function to get the activation values.

In [27]:
```
# Extract the outputs of the 2 layers
layer_outputs = [layer.output for layer in model_1.layers]

# Create a model that will return these outputs, given the model input
activation_model_1 = models.Model(inputs=model_1.input, outputs=layer_outputs)

# Print a description of the layers
print(f"There are {len(layer_outputs)} layers")
layer_outputs
```

```
There are 2 layers
```

Out[27]:
```
[<tf.Tensor 'dense/Relu:0' shape=(None, 1) dtype=float32>,
 <tf.Tensor 'dense_1/Softmax:0' shape=(None, 10) dtype=float32>]
```

In [28]:
```
# Get the output of the hidden node for each of the 55000 training images
activations = activation_model_1.predict(train_images)
hidden_layer_activation = activations[0]
hidden_layer_activation.shape   #  hidden node has one activation value per training image
```

Out[28]:
```
(55000, 1)
```

In [29]:
```
print(f"The maximum activation value of the hidden node is {hidden_layer_activation.max()}")
```

```
The maximum activation value of the hidden node is 65.29608154296875
```

In [30]:
```
# Output layer stats
np.set_printoptions(suppress = True)  # display probabilities as decimals and NOT in scientific notation
ouput_layer_activation = activations[1]
print(f"The output node has shape {ouput_layer_activation.shape}")
print(f"The output for the first image are {ouput_layer_activation[0].round(4)}")
print(f"The sum of the probabilities is (approximately) {ouput_layer_activation[0].sum()}")
```

```
The output node has shape (55000, 10)
The output for the first image are [0.0398 0.0001 0.     0.0031 0.0935 0.0083 0.     0.4896 0.0014 0.3642]
The sum of the probabilities is (approximately) 1.0
```

Activation values boxplot

We combine the activation values of the one hidden node with the corresponding predicted classes into a DataFrame. We use both `matplotlib` and `seaborn` to create boxplots from the DataFrame.

In [31]:
```
# Create DataFrame of activation values and corresponding predicted class
boxplot_df = pd.DataFrame({'act_value':hidden_layer_activation.reshape(55000),
                           'pred_class':pred_classes})
boxplot_df.head()
```

Out[31]:

	act_value	pred_class
0	-0.000000	7
1	3.236026	3
2	0.152618	7
3	5.412035	1
4	5.979540	1

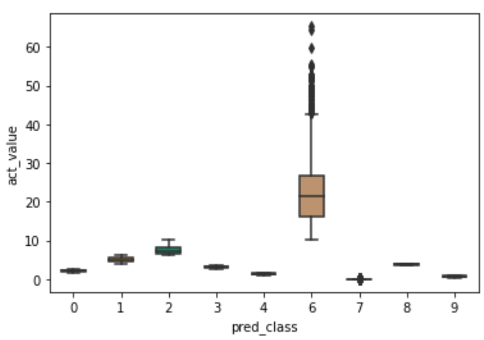
In [32]:
```
# Value counts for boxplot
# Note missing class - 5
boxplot_df['pred_class'].value_counts()
```

Out[32]:
```
1    13507
7     8637
0     8167
3     7996
4     4408
2     3771
6     3493
8     2711
9     2310
Name: pred_class, dtype: int64
```

Figure 4: Activation values boxplot

In [37]:
```
# Activation value boxplot
bplot = sns.boxplot(y='act_value', x='pred_class',
                 data=boxplot_df,
                 width=0.5,
                 palette="colorblind")
# bplot.savefig("act_values_model_1.png")

figure = bplot.get_figure()
figure.savefig("figures/act_values_model_1.png", dpi=400)
# ax.set(ylim=(10, 40))
```



EXPERIMENT 2:

This network will consist of 784 input nodes, a hidden layer with `2 nodes` and 10 output nodes (corresponding to the 10 digits). We use `mnist.load_data()` to get the 70,000 images divided into a set of 60,000 training images and 10,000 test images. We hold back 5,000 of the 60,000 training images for validation.

For each of the 60,000 training images, the output of the two hidden nodes are plotted using a scatterplot. We color code the points according to which of the 10 classes the the output of the two nodes predicts. Ideally, just like in EXPERIMENT 1, the color clusters should have very little overlap.

Build the network

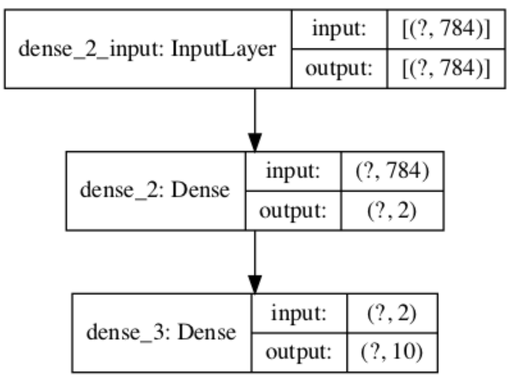
Here our network consists of a sequence of two `Dense` layers, which are densely-connected (also called "fully-connected") neural layers.

- The first `Dense` layer, the hidden layer, consists of `2 nodes` .
- The second (and last) layer is a 10-way "softmax" layer, which means it will return an array of 10 probability scores (summing to 1). Each score will be the probability that the current digit image belongs to one of our 10 digit classes.

In [23]:
```
# Define model
model_2 = models.Sequential()
model_2.add(layers.Dense(2, activation='relu', input_shape=(28 * 28,)))
model_2.add(layers.Dense(10, activation='softmax'))
```

In [24]:
```
# Plot a graph of the model
keras.utils.plot_model(model_2, "figures/mnist_model_2_1hnode.png", show_shapes=True)
```

Out[24]:



In [25]:
```
# Model summary
model_2.summary()
```

```
Model: "sequential_1"
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
dense_2 (Dense)              (None, 2)                 1570
_________________________________________________________________
dense_3 (Dense)              (None, 10)                30
=================================================================
Total params: 1,600
Trainable params: 1,600
Non-trainable params: 0
_________________________________________________________________
```

Compile model

To make our network ready for training, we need to pick three more things, as part of "compilation" step:

- A loss function: the is how the network will be able to measure how good a job it is doing on its training data, and thus how it will be able to steer itself in the right direction.
- An optimizer: this is the mechanism through which the network will update itself based on the data it sees and its loss function.
- Metrics to monitor during training and testing. Here we will only care about accuracy (the fraction of the images that were correctly classified).

In [26]:
```
# Compile the model
model_2.compile(loss='sparse_categorical_crossentropy', optimizer='rmsprop', metrics=['accuracy'])
```

Train model

We are now ready to train our network, which in Keras is done via a call to the `fit` method of the network: we "fit" the model to its training data. We train the model for 30 epochs with batch size 32 (the default).

In [27]:
```
# If loading previous model:
# model = model_2()
# model = load_model_2('results/mnist_model_2.h5')

# Train the model
start_time = time.time()
history = model_2.fit(train_images, train_labels, validation_data=(val_images, val_labels),
                      epochs=30)
# fit(train_images, train_labels, epochs=30,
#                      validation_data=(val_images, val_labels))
elapsed_time = time.time() - start_time
print('---------------------------')
print('Training time in seconds: ', round(elapsed_time,2))
print('---------------------------')

# Saving models locally after fitting
save_dir = "results/"
model_name = 'mnist_model_2.h5'
model_path = os.path.join(save_dir, model_name)
model_2.save(model_path)
print('Saved trained model at %s ' % model_path)
print('---------------------------')
```

```
Train on 55000 samples, validate on 5000 samples
Epoch 1/30
55000/55000 [==============================] - 4s 66us/sample - loss: 1.5698 - accuracy: 0.4084 - val_loss: 1.2846 -
val_accuracy: 0.5170
Epoch 2/30
55000/55000 [==============================] - 3s 56us/sample - loss: 1.2181 - accuracy: 0.5407 - val_loss: 1.1320 -
val_accuracy: 0.5918
Epoch 3/30
55000/55000 [==============================] - 3s 58us/sample - loss: 1.1265 - accuracy: 0.5863 - val_loss: 1.0766 -
val_accuracy: 0.6232
Epoch 4/30
55000/55000 [==============================] - 3s 59us/sample - loss: 1.0876 - accuracy: 0.6127 - val_loss: 1.0465 -
val_accuracy: 0.6404
Epoch 5/30
55000/55000 [==============================] - 3s 60us/sample - loss: 1.0644 - accuracy: 0.6305 - val_loss: 1.0319 -
val_accuracy: 0.6584
Epoch 6/30
55000/55000 [==============================] - 3s 56us/sample - loss: 1.0484 - accuracy: 0.6419 - val_loss: 1.0135 -
val_accuracy: 0.6584
Epoch 7/30
55000/55000 [==============================] - 3s 54us/sample - loss: 1.0367 - accuracy: 0.6482 - val_loss: 1.0101 -
val_accuracy: 0.6638
Epoch 8/30
55000/55000 [==============================] - 3s 49us/sample - loss: 1.0290 - accuracy: 0.6512 - val_loss: 0.9983 -
val_accuracy: 0.6654
Epoch 9/30
55000/55000 [==============================] - 3s 51us/sample - loss: 1.0230 - accuracy: 0.6544 - val_loss: 0.9906 -
val_accuracy: 0.6682
Epoch 10/30
55000/55000 [==============================] - 3s 50us/sample - loss: 1.0178 - accuracy: 0.6575 - val_loss: 0.9982 -
val_accuracy: 0.6736
Epoch 11/30
55000/55000 [==============================] - 3s 53us/sample - loss: 1.0139 - accuracy: 0.6578 - val_loss: 0.9883 -
val_accuracy: 0.6714
Epoch 12/30
55000/55000 [==============================] - 3s 52us/sample - loss: 1.0102 - accuracy: 0.6602 - val_loss: 0.9903 -
val_accuracy: 0.6664
Epoch 13/30
55000/55000 [==============================] - 3s 63us/sample - loss: 1.0068 - accuracy: 0.6604 - val_loss: 0.9879 -
val_accuracy: 0.6690
Epoch 14/30
55000/55000 [==============================] - 3s 60us/sample - loss: 1.0035 - accuracy: 0.6616 - val_loss: 0.9849 -
val_accuracy: 0.6762
Epoch 15/30
55000/55000 [==============================] - 3s 59us/sample - loss: 1.0003 - accuracy: 0.6640 - val_loss: 0.9807 -
val_accuracy: 0.6764
Epoch 16/30
55000/55000 [==============================] - 3s 57us/sample - loss: 0.9967 - accuracy: 0.6640 - val_loss: 0.9771 -
val_accuracy: 0.6724
Epoch 17/30
55000/55000 [==============================] - 3s 57us/sample - loss: 0.9940 - accuracy: 0.6643 - val_loss: 0.9737 -
val_accuracy: 0.6774
Epoch 18/30
55000/55000 [==============================] - 3s 57us/sample - loss: 0.9896 - accuracy: 0.6667 - val_loss: 0.9727 -
val_accuracy: 0.6744
Epoch 19/30
55000/55000 [==============================] - 3s 60us/sample - loss: 0.9850 - accuracy: 0.6699 - val_loss: 0.9705 -
val_accuracy: 0.6762
Epoch 20/30
55000/55000 [==============================] - 3s 58us/sample - loss: 0.9805 - accuracy: 0.6699 - val_loss: 0.9586 -
val_accuracy: 0.6834
Epoch 21/30
55000/55000 [==============================] - 3s 60us/sample - loss: 0.9760 - accuracy: 0.6730 - val_loss: 0.9695 -
val_accuracy: 0.6790
Epoch 22/30
55000/55000 [==============================] - 3s 58us/sample - loss: 0.9697 - accuracy: 0.6762 - val_loss: 0.9625 -
val_accuracy: 0.6834
Epoch 23/30
55000/55000 [==============================] - 3s 58us/sample - loss: 0.9646 - accuracy: 0.6793 - val_loss: 0.9584 -
val_accuracy: 0.6804
Epoch 24/30
55000/55000 [==============================] - 3s 58us/sample - loss: 0.9599 - accuracy: 0.6823 - val_loss: 0.9447 -
val_accuracy: 0.6922
Epoch 25/30
55000/55000 [==============================] - 3s 58us/sample - loss: 0.9543 - accuracy: 0.6860 - val_loss: 0.9388 -
val_accuracy: 0.6984
Epoch 26/30
55000/55000 [==============================] - 3s 59us/sample - loss: 0.9493 - accuracy: 0.6892 - val_loss: 0.9426 -
val_accuracy: 0.7006
Epoch 27/30
55000/55000 [==============================] - 3s 60us/sample - loss: 0.9450 - accuracy: 0.6895 - val_loss: 0.9361 -
val_accuracy: 0.7064
Epoch 28/30
55000/55000 [==============================] - 3s 59us/sample - loss: 0.9409 - accuracy: 0.6946 - val_loss: 0.9519 -
val_accuracy: 0.6958
```

Test model

Evaluate the model on the test dataset.

In [28]:
```
test_loss, test_acc = model_2.evaluate(test_images, test_labels)
```

```
10000/10000 [==============================] - 0s 35us/sample - loss: 0.9423 - accuracy: 0.7006
```

In [29]:
```
print(f'test accuracy: {test_acc}, test loss: {test_loss}')
```

```
test accuracy: 0.7006000280380249, test loss: 0.9423021728515625
```

Plot model performance

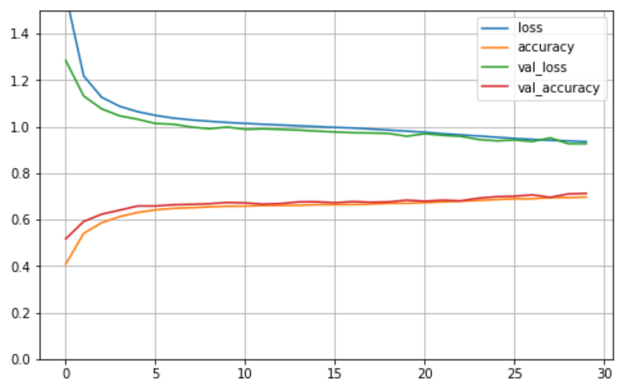
Accuracy and loss

In [30]:
```
history_dict = history.history
history_dict.keys()
```

Out[30]:
```
dict_keys(['loss', 'accuracy', 'val_loss', 'val_accuracy'])
```

Figure 5: Learning curves for Model 2

In [31]:
```
# Alternate code for accuracy and loss
# Plot learning curves for model
pd.DataFrame(history.history).plot(figsize=(8, 5))
plt.grid(True)
plt.gca().set_ylim(0, 1.5) # set the vertical range to [0-1]
plt.savefig("figures/learning_curves_model_2", tight_layout=False)
plt.show()
```



Create confusion matrix with values

Confusion matrix is used to assess errors in prediction

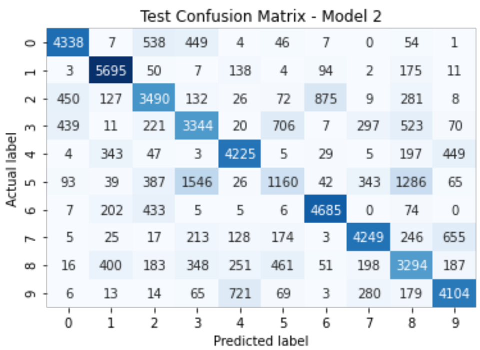
In [32]:
```
# Assign predicted classes
pred_classes = model_2.predict_classes(train_images)
```

In [33]:
```
# Create confusion matrix with values
conf_mx = confusion_matrix(train_labels,pred_classes)
conf_mx
```

Out[33]:
```
array([[4338,    7,  538,  449,    4,   46,    7,    0,   54,    1],
       [   3, 5695,   50,    7,  138,    4,   94,    2,  175,   11],
       [ 450,  127, 3490,  132,   26,   72,  875,    9,  281,    8],
       [ 439,   11,  221, 3344,   20,  706,    7,  297,  523,   70],
       [   4,  343,   47,    3, 4225,    5,   29,    5,  197,  449],
       [  93,   39,  387, 1546,   26, 1160,   42,  343, 1286,   65],
       [   7,  202,  433,    5,    5,    6, 4685,    0,   74,    0],
       [   5,   25,   17,  213,  128,  174,    3, 4249,  246,  655],
       [  16,  400,  183,  348,  251,  461,   51,  198, 3294,  187],
       [   6,   13,   14,   65,  721,   69,    3,  280,  179, 4104]])
```

Figure 6: Confusion matrix for Model 2

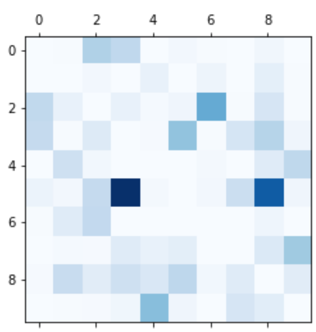
In [34]:
```
# Plot with values transposed to show actual versus predicted values
# m1_cm = confusion_matrix(train_labels, pred_classes)
m2_cm_plt=sns.heatmap(conf_mx, square=False, annot=True, fmt='d', cbar=False, cmap="Blues")
plt.xlabel('Predicted label')
plt.ylabel('Actual label')
plt.title("Test Confusion Matrix - Model 2")
plt.savefig("figures/confusion_matrix_errors_plot_model_2", tight_layout=False)
plt.show();
```



In [35]:
```
# Plot normalized confusion matrix
row_sums = conf_mx.sum(axis=1, keepdims=True)
norm_conf_mx = conf_mx / row_sums
```

Figure 7: Normalized confusion matrix for Model 2

In [36]:
```
np.fill_diagonal(norm_conf_mx, 0)
plt.matshow(norm_conf_mx, cmap="Blues")
plt.savefig("figures/confusion_matrix_errors_plot__norm_model_1", tight_layout=False)
plt.show()
```



Error analysis

In [37]:
```
# Define function for plotting example digits
def plot_digits(instances, images_per_row=10, **options):
    size = 28
    images_per_row = min(len(instances), images_per_row)
    images = [instance.reshape(size,size) for instance in instances]
    n_rows = (len(instances) - 1) // images_per_row + 1
    row_images = []
    n_empty = n_rows * images_per_row - len(instances)
    images.append(np.zeros((size, size * n_empty)))
    for row in range(n_rows):
        rimages = images[row * images_per_row : (row + 1) * images_per_row]
        row_images.append(np.concatenate(rimages, axis=1))
    image = np.concatenate(row_images, axis=0)
    plt.imshow(image, cmap = 'binary', **options)
    plt.axis("off")
```

In [38]:
```
# Try some examples of mis-classes digits
# Upper left: 5s classified correctly as 5s
# Upper right: 5s classified as 8s
# Lower left: 8s classified as 5s
# Lower right: 8s classified correctly as 8s

cl_a, cl_b = 5, 8
X_aa = train_images[(train_labels == cl_a) & (pred_classes == cl_a)]
X_ab = train_images[(train_labels == cl_a) & (pred_classes == cl_b)]
X_ba = train_images[(train_labels == cl_b) & (pred_classes == cl_a)]
X_bb = train_images[(train_labels == cl_b) & (pred_classes == cl_b)]

plt.figure(figsize=(8,8))
plt.subplot(221); plot_digits(X_aa[:25], images_per_row=5)
plt.subplot(222); plot_digits(X_ab[:25], images_per_row=5)
plt.subplot(223); plot_digits(X_ba[:25], images_per_row=5)
plt.subplot(224); plot_digits(X_bb[:25], images_per_row=5)
plt.savefig("error_analysis_digits_plot_EXP1_valid")
plt.show()
```

Get the activation values of the hidden nodes

To get the activation values of the hidden nodes, we need to create a new model, `activation_model`, that takes the same input as our current model but outputs the activation value of the hidden layer, i.e. of the hidden node. Then use the `predict` function to get the activation values.

In [39]:
```
# Extract the outputs of the 2 layers
layer_outputs = [layer.output for layer in model_2.layers]

# Create a model that will return these outputs, given the model input
activation_model_2 = models.Model(inputs=model_2.input, outputs=layer_outputs)

print(f"There are {len(layer_outputs)} layers")
layer_outputs # description of the layers
```

```
There are 2 layers
```

Out[39]:
```
[<tf.Tensor 'dense_2/Relu:0' shape=(None, 2) dtype=float32>,
 <tf.Tensor 'dense_3/Softmax:0' shape=(None, 10) dtype=float32>]
```

In [40]:
```
# Get the output of the hidden node for each of the 55000 training images
activations = activation_model_2.predict(train_images)
hidden_layer_activation = activations[0]
hidden_layer_activation.shape    #  2 hidden node each has one activation value per training image
```

Out[40]:
```
(55000, 2)
```

In [41]:
```
hidden_node1_activation = hidden_layer_activation[:,0] # get activation values of the first hidden node
hidden_node2_activation = hidden_layer_activation[:,1] # get activation values of the second hidden node

print(f"The maximum activation value of the first hidden node is {hidden_node1_activation.max()}")
print(f"The maximum activation value of the second hidden node is {hidden_node2_activation.max()}")
```

```
The maximum activation value of the first hidden node is 39.530494689941406
The maximum activation value of the second hidden node is 33.32828903198242
```

In [42]:
```
# Output layer stats
np.set_printoptions(suppress = True)  # display probabilities as decimals and NOT in scientific notation
ouput_layer_activation = activations[1]
print(f"The output node has shape {ouput_layer_activation.shape}")
print(f"The output for the first image are {ouput_layer_activation[0].round(4)}")
print(f"The sum of the probabilities is (approximately) {ouput_layer_activation[0].sum()}")
```

```
The output node has shape (55000, 10)
The output for the first image are [0.     0.     0.     0.0242 0.     0.0106 0.     0.9463 0.0002 0.0186]
The sum of the probabilities is (approximately) 1.0
```

Activation values scatterplots

We combine the activation values of the two hidden nodes together with the corresponding predicted classes into a DataFrame. We use both `matplotlib` and `seaborn` to create boxplots from the DataFrame.

In [43]:
```
scatterPlot_df =  pd.DataFrame({'act_value_h1':hidden_node1_activation,
                                'act_value_h2':hidden_node2_activation,
                                'pred_class':pred_classes})
scatterPlot_df.head()
```

Out[43]:

	act_value_h1	act_value_h2	pred_class
0	15.107007	12.968557	7
1	7.321884	3.538808	3
2	5.511035	9.155907	9
3	0.206997	-0.000000	6
4	-0.000000	1.498670	1

Figure 8: Activation values scatterplot for Model 2

In [44]:
```
# plt.legend(loc='upper left', prop={'size':6}, bbox_to_anchor=(1,1),ncol=1)
plt.scatter(scatterPlot_df.act_value_h1,
            scatterPlot_df.act_value_h2,
            c=scatterPlot_df.pred_class,
            label=scatterPlot_df.pred_class)

plt.savefig("figures/act_values_model_2", tight_layout=False)

plt.show()
```

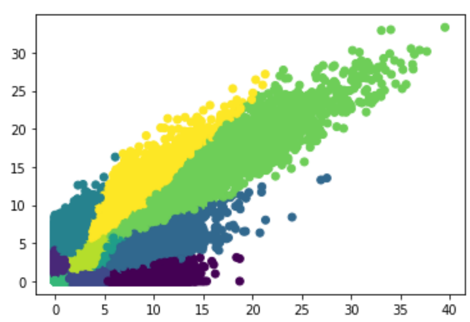


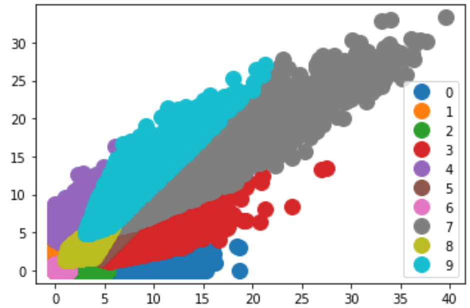
Figure 9: Grouped activation values for Model 2

In [45]:
```
# Scatterplot grouped by predicted class
groups = scatterPlot_df.groupby('pred_class')

# Plot
fig, ax = plt.subplots()
ax.margins(0.05) # Optional, just adds 5% padding to the autoscaling
for name, group in groups:
    ax.plot(group.act_value_h1, group.act_value_h2, marker='o', linestyle='', ms=12, label=name)
ax.legend()

plt.savefig("figures/act_values_grouped_model_2", tight_layout=False)

plt.show()
```



EXPERIMENT 3:

We want to select the *best* `DNN` model subject to certain restrictions on `hyperparameters` :

- The number of hidden layers will be one (network depth).
- The number of nodes will be XXX (network width).

We will use `sklearn.grid_search.GridSearchCV` to find the `best` number of neurons for the hidden layer.

As before we will need 784 input nodes and 10 output nodes (corresponding to the 10 digits). We use `mnist.load_data()` to get the 70,000 images divided into a set of 60,000 training images and 10,000 test images. We hold back 5,000 of the 60,000 training images for validation.

In [69]:
```
# Define a function to create a DNN with a given number of hidden layers and a fixed given number
# of nodes per hidden layer
def build_model(n_hidden=1, n_neurons=2, learning_rate=0.001, input_shape=(28 * 28,)):
    model = models.Sequential()
    model.add(layers.InputLayer(input_shape=input_shape))
    for layer in range(n_hidden):
        model.add(keras.layers.Dense(n_neurons, activation="relu"))
    model.add(layers.Dense(10, activation='softmax'))
    optimizer = keras.optimizers.RMSprop(lr=learning_rate)
    model.compile(loss='sparse_categorical_crossentropy', optimizer=optimizer,  metrics=['accuracy'])
    return model
```

In [ ]:
```
# Define a second model that takes a tuple `(n1,n2...)`, where the number of coordinates is the
# number of `hidden layers` with `n1` nodes in the first hidden layer, `n2` nodes in the second, etc.
def build_model2(n_neurons=(2,3), learning_rate=0.001, input_shape=(28 * 28,)):
    model = models.Sequential()
    model.add(layers.InputLayer(input_shape=input_shape))
    for layer in range(len(n_neurons)):
        model.add(keras.layers.Dense(n_neurons[layer], activation="relu"))
    model.add(layers.Dense(10, activation='softmax'))
    optimizer = keras.optimizers.RMSprop(lr=learning_rate)
    model.compile(loss='sparse_categorical_crossentropy', optimizer=optimizer,  metrics=['accuracy'])
    return model
```

In [70]:
```
# Create a `KerasClassifier` object, the class is an implementation of the scikit-learn classifier API for Keras.
# It is actually a thin wrapper around the model that is built using our `build_model` function.
keras_clf = KerasClassifier(build_model)
```

Hyperparameter tuning with grid search: range of neurons

Testing a range of neurons: 100, 900, 50 (100 to 900 in increments of 50)

```
param_grid = {'n_neurons': range(100,900,50)}
param_grid
```

In [75]:
```
param_grid = {'n_neurons': range(100,900,50)}
param_grid
```

Out[75]:
```
{'n_neurons': range(100, 900, 50)}
```

In [76]:
```
# Run grid search
grid_cv = GridSearchCV(estimator=keras_clf, param_grid=param_grid,cv=3,verbose = 2)
grid_cv.fit(train_images, train_labels, epochs=30,
                  validation_data=(val_images, val_labels),
                  callbacks=[keras.callbacks.EarlyStopping(patience=2)])
```

```
Fitting 3 folds for each of 16 candidates, totalling 48 fits
[CV] n_neurons=100 ..................................................

[Parallel(n_jobs=1)]: Using backend SequentialBackend with 1 concurrent workers.

Train on 36666 samples, validate on 5000 samples
Epoch 1/30
36666/36666 [==============================] - 6s 154us/sample - loss: 0.3270 - accuracy: 0.9073 - val_loss: 0.1802 -
val_accuracy: 0.9482
Epoch 2/30
36666/36666 [==============================] - 6s 155us/sample - loss: 0.1643 - accuracy: 0.9518 - val_loss: 0.1352 -
val_accuracy: 0.9592
Epoch 3/30
36666/36666 [==============================] - 5s 144us/sample - loss: 0.1191 - accuracy: 0.9646 - val_loss: 0.1180 -
val_accuracy: 0.9652
Epoch 4/30
36666/36666 [==============================] - 5s 144us/sample - loss: 0.0961 - accuracy: 0.9724 - val_loss: 0.1064 -
val_accuracy: 0.9696
Epoch 5/30
36666/36666 [==============================] - 5s 142us/sample - loss: 0.0808 - accuracy: 0.9764 - val_loss: 0.1057 -
val_accuracy: 0.9712
Epoch 6/30
36666/36666 [==============================] - 5s 144us/sample - loss: 0.0685 - accuracy: 0.9801 - val_loss: 0.1117 -
val_accuracy: 0.9660
Epoch 7/30
36666/36666 [==============================] - 5s 143us/sample - loss: 0.0592 - accuracy: 0.9828 - val_loss: 0.0997 -
val_accuracy: 0.9712
Epoch 8/30
36666/36666 [==============================] - 5s 142us/sample - loss: 0.0513 - accuracy: 0.9850 - val_loss: 0.1105 -
val_accuracy: 0.9688
Epoch 9/30
36666/36666 [==============================] - 5s 143us/sample - loss: 0.0459 - accuracy: 0.9872 - val_loss: 0.1070 -
val_accuracy: 0.9702
18334/18334 [==============================] - 1s 41us/sample - loss: 0.1287 - accuracy: 0.9691
[CV] ................................... n_neurons=100, total=  49.3s
[CV] n_neurons=100 ..................................................

[Parallel(n_jobs=1)]: Done   1 out of   1 | elapsed:   49.3s remaining:    0.0s
```

```
Train on 36667 samples, validate on 5000 samples
Epoch 1/30
36667/36667 [==============================] - 5s 149us/sample - loss: 0.3250 - accuracy: 0.9089 - val_loss: 0.1825 -
val_accuracy: 0.9482
Epoch 2/30
36667/36667 [==============================] - 5s 138us/sample - loss: 0.1620 - accuracy: 0.9530 - val_loss: 0.1378 -
val_accuracy: 0.9618
Epoch 3/30
36667/36667 [==============================] - 5s 139us/sample - loss: 0.1196 - accuracy: 0.9651 - val_loss: 0.1174 -
val_accuracy: 0.9680
Epoch 4/30
36667/36667 [==============================] - 5s 139us/sample - loss: 0.0957 - accuracy: 0.9720 - val_loss: 0.1143 -
val_accuracy: 0.9654
Epoch 5/30
36667/36667 [==============================] - 5s 142us/sample - loss: 0.0795 - accuracy: 0.9763 - val_loss: 0.1055 -
val_accuracy: 0.9704
Epoch 6/30
36667/36667 [==============================] - 5s 145us/sample - loss: 0.0673 - accuracy: 0.9805 - val_loss: 0.1056 -
val_accuracy: 0.9698
Epoch 7/30
36667/36667 [==============================] - 5s 144us/sample - loss: 0.0570 - accuracy: 0.9838 - val_loss: 0.1111 -
val_accuracy: 0.9682
18333/18333 [==============================] - 1s 42us/sample - loss: 0.1324 - accuracy: 0.9663
[CV] .................................... n_neurons=100, total=  37.8s
[CV] n_neurons=100 .................................................
Train on 36667 samples, validate on 5000 samples
Epoch 1/30
36667/36667 [==============================] - 6s 154us/sample - loss: 0.3357 - accuracy: 0.9046 - val_loss: 0.1810 -
val_accuracy: 0.9494
Epoch 2/30
36667/36667 [==============================] - 5s 142us/sample - loss: 0.1669 - accuracy: 0.9506 - val_loss: 0.1418 -
val_accuracy: 0.9592
Epoch 3/30
36667/36667 [==============================] - 5s 145us/sample - loss: 0.1202 - accuracy: 0.9649 - val_loss: 0.1145 -
val_accuracy: 0.9666
Epoch 4/30
36667/36667 [==============================] - 5s 145us/sample - loss: 0.0944 - accuracy: 0.9731 - val_loss: 0.1091 -
val_accuracy: 0.9682
Epoch 5/30
36667/36667 [==============================] - 5s 142us/sample - loss: 0.0773 - accuracy: 0.9779 - val_loss: 0.0973 -
val_accuracy: 0.9728
Epoch 6/30
36667/36667 [==============================] - 5s 143us/sample - loss: 0.0652 - accuracy: 0.9817 - val_loss: 0.0987 -
val_accuracy: 0.9718
Epoch 7/30
36667/36667 [==============================] - 5s 142us/sample - loss: 0.0550 - accuracy: 0.9848 - val_loss: 0.1049 -
val_accuracy: 0.9718
18333/18333 [==============================] - 1s 43us/sample - loss: 0.1320 - accuracy: 0.9668
[CV] .................................... n_neurons=100, total=  38.6s
[CV] n_neurons=150 .................................................
Train on 36666 samples, validate on 5000 samples
Epoch 1/30
36666/36666 [==============================] - 6s 170us/sample - loss: 0.2979 - accuracy: 0.9146 - val_loss: 0.1721 -
val_accuracy: 0.9528
Epoch 2/30
36666/36666 [==============================] - 6s 159us/sample - loss: 0.1447 - accuracy: 0.9576 - val_loss: 0.1340 -
val_accuracy: 0.9634
Epoch 3/30
36666/36666 [==============================] - 6s 158us/sample - loss: 0.1036 - accuracy: 0.9697 - val_loss: 0.1088 -
val_accuracy: 0.9686
Epoch 4/30
36666/36666 [==============================] - 6s 167us/sample - loss: 0.0803 - accuracy: 0.9759 - val_loss: 0.1093 -
val_accuracy: 0.9724
Epoch 5/30
36666/36666 [==============================] - 6s 160us/sample - loss: 0.0659 - accuracy: 0.9807 - val_loss: 0.0990 -
val_accuracy: 0.9740
Epoch 6/30
36666/36666 [==============================] - 6s 166us/sample - loss: 0.0535 - accuracy: 0.9846 - val_loss: 0.0998 -
val_accuracy: 0.9752
Epoch 7/30
36666/36666 [==============================] - 6s 161us/sample - loss: 0.0454 - accuracy: 0.9879 - val_loss: 0.0993 -
val_accuracy: 0.9748
18334/18334 [==============================] - 1s 43us/sample - loss: 0.1174 - accuracy: 0.9700
[CV] .................................... n_neurons=150, total=  43.2s
[CV] n_neurons=150 .................................................
Train on 36667 samples, validate on 5000 samples
Epoch 1/30
36667/36667 [==============================] - 6s 175us/sample - loss: 0.3008 - accuracy: 0.9128 - val_loss: 0.1641 -
val_accuracy: 0.9544
Epoch 2/30
36667/36667 [==============================] - 6s 162us/sample - loss: 0.1430 - accuracy: 0.9585 - val_loss: 0.1228 -
val_accuracy: 0.9644
Epoch 3/30
36667/36667 [==============================] - 6s 161us/sample - loss: 0.1031 - accuracy: 0.9692 - val_loss: 0.1090 -
val_accuracy: 0.9676
```

```
[Parallel(n_jobs=1)]: Done  48 out of  48 | elapsed: 105.2min finished

Train on 55000 samples, validate on 5000 samples
Epoch 1/30
55000/55000 [==============================] - 23s 417us/sample - loss: 0.2031 - accuracy: 0.9398 - val_loss: 0.1027
- val_accuracy: 0.9712
Epoch 2/30
55000/55000 [==============================] - 23s 417us/sample - loss: 0.0901 - accuracy: 0.9744 - val_loss: 0.0983
- val_accuracy: 0.9702
Epoch 3/30
55000/55000 [==============================] - 24s 433us/sample - loss: 0.0633 - accuracy: 0.9824 - val_loss: 0.0915
- val_accuracy: 0.9772
Epoch 4/30
55000/55000 [==============================] - 24s 429us/sample - loss: 0.0499 - accuracy: 0.9862 - val_loss: 0.0826
- val_accuracy: 0.9774
Epoch 5/30
55000/55000 [==============================] - 22s 399us/sample - loss: 0.0395 - accuracy: 0.9889 - val_loss: 0.0891
- val_accuracy: 0.9778
Epoch 6/30
55000/55000 [==============================] - 27s 498us/sample - loss: 0.0304 - accuracy: 0.9922 - val_loss: 0.0909
- val_accuracy: 0.9778
```

Out[76]:
```
GridSearchCV(cv=3, error_score=nan,
             estimator=<tensorflow.python.keras.wrappers.scikit_learn.KerasClassifier object at 0x1a40c165f8>,
             iid='deprecated', n_jobs=None,
             param_grid={'n_neurons': range(100, 900, 50)},
             pre_dispatch='2*n_jobs', refit=True, return_train_score=False,
             scoring=None, verbose=2)
```

In [77]:
```
# Evaluate the number of neurons that produced the best estimator
grid_cv.best_params_
```

Out[77]:
```
{'n_neurons': 700}
```

In [78]:
```
# Results to compare the performance of the models
results = grid_cv.cv_results_
results
```

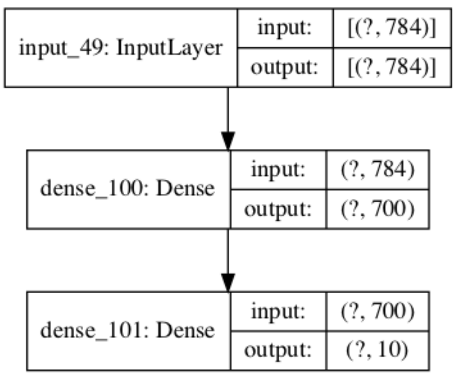
Out[78]:
```
{'mean_fit_time': array([ 41.12912011, 630.86422348,  77.88898277,  59.92755167,
         57.86564732,  61.81225936,  74.93807228,  96.85294294,
         72.20501359, 104.03751699, 270.95088029,  93.30652022,
        109.73006535, 112.4122289 , 115.95351354, 103.50072034]),
 'std_fit_time': array([  5.27420829, 832.59516307,  31.84032017,  12.41713136,
          8.92254138,  12.39513237,  10.10858184,  16.88657514,
          3.16720018,   7.74588704, 274.00334358,  25.66947123,
         25.42976736,  49.70870004,  26.68701655,  30.01017496]),
 'mean_score_time': array([0.77198696, 0.76900085, 1.05369322, 1.03885627, 1.02395908,
        1.09872262, 1.092134  , 1.30000401, 1.45409743, 1.26738   ,
        1.45491107, 1.57188773, 1.40401681, 1.68524734, 1.71294999,
        1.89409161]),
 'std_score_time': array([0.01318589, 0.05691487, 0.19421153, 0.0942548 , 0.01745575,
        0.03468624, 0.02306103, 0.0988736 , 0.27488174, 0.09061485,
        0.13951719, 0.01138051, 0.02879567, 0.37277947, 0.0459696 ,
        0.15512186]),
 'param_n_neurons': masked_array(data=[100, 150, 200, 250, 300, 350, 400, 450, 500, 550, 600,
                    650, 700, 750, 800, 850],
              mask=[False, False, False, False, False, False, False, False,
                    False, False, False, False, False, False, False, False],
        fill_value='?',
             dtype=object),
 'params': [{'n_neurons': 100},
  {'n_neurons': 150},
  {'n_neurons': 200},
  {'n_neurons': 250},
  {'n_neurons': 300},
  {'n_neurons': 350},
  {'n_neurons': 400},
  {'n_neurons': 450},
  {'n_neurons': 500},
  {'n_neurons': 550},
  {'n_neurons': 600},
  {'n_neurons': 650},
  {'n_neurons': 700},
  {'n_neurons': 750},
  {'n_neurons': 800},
  {'n_neurons': 850}],
 'split0_test_score': array([0.96907383, 0.9700011 , 0.97131014, 0.97169197, 0.97060108,
        0.97267371, 0.97414637, 0.97212827, 0.97212827, 0.97463727,
        0.97218281, 0.97398275, 0.97600085, 0.97229195, 0.97294647,
        0.97016472]),
 'split1_test_score': array([0.96634483, 0.97239947, 0.9722904 , 0.97501773, 0.97430861,
        0.97272676, 0.97501773, 0.97539955, 0.97370863, 0.97212678,
        0.97021765, 0.974145  , 0.97201765, 0.97556317, 0.97185403,
        0.97349042]),
 'split2_test_score': array([0.9667812 , 0.97218132, 0.97327226, 0.97185403, 0.96885395,
        0.96879941, 0.97447228, 0.97327226, 0.97239947, 0.97376317,
        0.97490865, 0.97425407, 0.97703594, 0.97670865, 0.97152674,
        0.97130859]),
 'mean_test_score': array([0.96739995, 0.9715273 , 0.97229093, 0.97285457, 0.97125455,
        0.97139996, 0.97454546, 0.97360003, 0.97274546, 0.97350907,
        0.97243637, 0.97412727, 0.97501814, 0.97485459, 0.97210908,
        0.97165457]),
 'std_test_score': array([0.00119694, 0.00108285, 0.00080103, 0.00153101, 0.00227429,
        0.00183899, 0.00035947, 0.00135546, 0.00069001, 0.00104053,
        0.00192347, 0.00011147, 0.00216335, 0.00187143, 0.00060701,
        0.00137958]),
 'rank_test_score': array([16, 13, 10,  7, 15, 14,  3,  5,  8,  6,  9,  4,  1,  2, 11, 12],
       dtype=int32)}
```

In [79]:
```
results['params'], results['rank_test_score']
```

Out[79]:
```
([{'n_neurons': 100},
  {'n_neurons': 150},
  {'n_neurons': 200},
  {'n_neurons': 250},
  {'n_neurons': 300},
  {'n_neurons': 350},
  {'n_neurons': 400},
  {'n_neurons': 450},
  {'n_neurons': 500},
  {'n_neurons': 550},
  {'n_neurons': 600},
  {'n_neurons': 650},
  {'n_neurons': 700},
  {'n_neurons': 750},
  {'n_neurons': 800},
  {'n_neurons': 850}],
 array([16, 13, 10,  7, 15, 14,  3,  5,  8,  6,  9,  4,  1,  2, 11, 12],
       dtype=int32))
```

In [80]:
```
# Assign the best model and plot
best_model = grid_cv.best_estimator_.model
keras.utils.plot_model(best_model, "figures/mnist_model_3_1hnode.png", show_shapes=True)
```

Out[80]:



In [81]:
```
# Evaluate best model with test data
best_model.evaluate(test_images, test_labels)
```

```
10000/10000 [==============================] - 1s 123us/sample - loss: 0.0901 - accuracy: 0.9796
```

Out[81]:
```
[0.0901317909641477, 0.9796]
```

In [82]:
```
# Save model for re-use
save_dir = "results/"
model_name = 'exp_3_best_model.h5'
model_path = os.path.join(save_dir, model_name)
best_model.save(model_path)
print('Saved trained model at %s ' % model_path)
print('--------------------------')
# best_model.save("mnist_model_best.h5")
```

```
Saved trained model at results/exp_3_best_model.h5
--------------------------
```

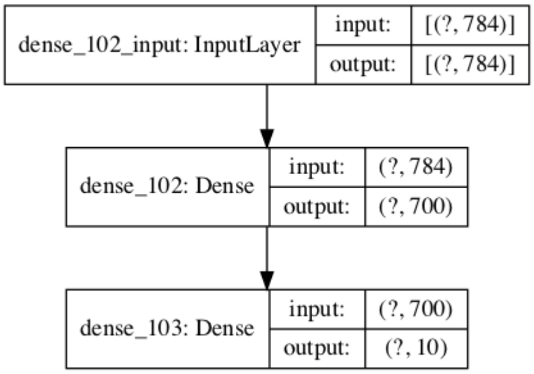
Build the network

- Sequence of two `Dense` layers, which are densely-connected (also called "fully-connected") neural layers.
- The first `Dense` layer, the hidden layer, consists of `700 nodes` with reLu activation, identified via grid search.
- The second (and last) layer is a 10-way "softmax" layer, which means it will return an array of 10 probability scores (summing to 1). Each score will be the probability that the current digit image belongs to one of our 10 digit classes.

In [83]:
```
model_3 = models.Sequential()
model_3.add(layers.Dense(700, activation='relu', input_shape=(28 * 28,)))
model_3.add(layers.Dense(10, activation='softmax'))
```

In [84]:
```
keras.utils.plot_model(model_3, "figures/mnist_model_3_1hnode.png", show_shapes=True)
```

Out[84]:



Compile model

To make our network ready for training, we need to pick three more things, as part of "compilation" step:

- A loss function: the is how the network will be able to measure how good a job it is doing on its training data, and thus how it will be able to steer itself in the right direction.
- An optimizer: this is the mechanism through which the network will update itself based on the data it sees and its loss function.
- Metrics to monitor during training and testing. Here we will only care about accuracy (the fraction of the images that were correctly classified).

In [85]:
```
# Compile the model
model_3.compile(loss='sparse_categorical_crossentropy', optimizer='rmsprop', metrics=['accuracy'])
```

Train model

We are now ready to train our network, which in Keras is done via a call to the `fit` method of the network: we "fit" the model to its training data. We train the model for 30 epochs with batch size 32 (the default).

In [86]:
```
# If loading previous model:
# # model = model_2()
# model_3 = load_model_3('results/exp_3_best_model.h5')

# Train the model
start_time = time.time()
history = model_3.fit(train_images, train_labels, epochs=30,
                    validation_data=(val_images, val_labels),
                    callbacks=[keras.callbacks.EarlyStopping(patience=10)])
elapsed_time = time.time() - start_time
print('--------------------------')
print('Training time in seconds: ', round(elapsed_time,2))
print('--------------------------')

# Saving models locally after fitting
save_dir = "results/"
model_name = 'mnist_model_3.h5'
model_path = os.path.join(save_dir, model_name)
model_3.save(model_path)
print('Saved trained model at %s ' % model_path)
print('--------------------------')
```

```
Train on 55000 samples, validate on 5000 samples
Epoch 1/30
55000/55000 [==============================] - 27s 493us/sample - loss: 0.2042 - accuracy: 0.9398 - val_loss: 0.1131
- val_accuracy: 0.9672
Epoch 2/30
55000/55000 [==============================] - 26s 464us/sample - loss: 0.0933 - accuracy: 0.9730 - val_loss: 0.0821
- val_accuracy: 0.9770
Epoch 3/30
55000/55000 [==============================] - 24s 439us/sample - loss: 0.0653 - accuracy: 0.9819 - val_loss: 0.0862
- val_accuracy: 0.9790
Epoch 4/30
55000/55000 [==============================] - 22s 408us/sample - loss: 0.0515 - accuracy: 0.9859 - val_loss: 0.0793
- val_accuracy: 0.9796
Epoch 5/30
55000/55000 [==============================] - 23s 412us/sample - loss: 0.0401 - accuracy: 0.9893 - val_loss: 0.0875
- val_accuracy: 0.9800
Epoch 6/30
55000/55000 [==============================] - 22s 407us/sample - loss: 0.0313 - accuracy: 0.9913 - val_loss: 0.0828
- val_accuracy: 0.9818
Epoch 7/30
55000/55000 [==============================] - 23s 418us/sample - loss: 0.0261 - accuracy: 0.9932 - val_loss: 0.0978
- val_accuracy: 0.9804
Epoch 8/30
55000/55000 [==============================] - 23s 413us/sample - loss: 0.0203 - accuracy: 0.9945 - val_loss: 0.1041
- val_accuracy: 0.9812
Epoch 9/30
55000/55000 [==============================] - 23s 413us/sample - loss: 0.0168 - accuracy: 0.9955 - val_loss: 0.0951
- val_accuracy: 0.9826
Epoch 10/30
55000/55000 [==============================] - 23s 420us/sample - loss: 0.0128 - accuracy: 0.9966 - val_loss: 0.1119
- val_accuracy: 0.9800
Epoch 11/30
55000/55000 [==============================] - 23s 414us/sample - loss: 0.0111 - accuracy: 0.9968 - val_loss: 0.1049
- val_accuracy: 0.9818
Epoch 12/30
55000/55000 [==============================] - 23s 417us/sample - loss: 0.0088 - accuracy: 0.9980 - val_loss: 0.1186
- val_accuracy: 0.9808
Epoch 13/30
55000/55000 [==============================] - 24s 437us/sample - loss: 0.0075 - accuracy: 0.9979 - val_loss: 0.1059
- val_accuracy: 0.9814
Epoch 14/30
55000/55000 [==============================] - 23s 419us/sample - loss: 0.0062 - accuracy: 0.9984 - val_loss: 0.1321
- val_accuracy: 0.9802
--------------------------
Training time in seconds:  330.92
--------------------------
Saved trained model at results/mnist_model_3.h5
--------------------------
```

Test model

Evaluate the model on the test dataset.

In [87]:
```
test_loss, test_acc = model_3.evaluate(test_images, test_labels)
```

```
10000/10000 [==============================] - 1s 112us/sample - loss: 0.1188 - accuracy: 0.9820
```

In [88]:
```
print(f'test accuracy: {test_acc}, test loss: {test_loss}')
```

```
test accuracy: 0.9819999933242798, test loss: 0.11882708966166242
```

Plot model performance

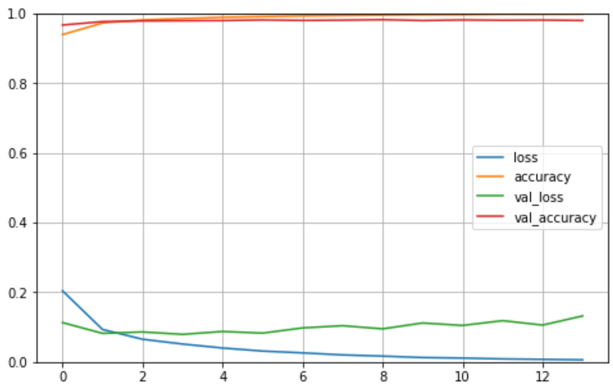
Accuracy and loss

In [90]:
```
history_dict = history.history
history_dict.keys()
```

Out[90]:
```
dict_keys(['loss', 'accuracy', 'val_loss', 'val_accuracy'])
```

Figure 10: Learning curves for Model 3

In [91]:
```
# Alternate code for accuracy and loss
# Plot learning curves for model
pd.DataFrame(history.history).plot(figsize=(8, 5))
plt.grid(True)
plt.gca().set_ylim(0, 1) # set the vertical range to [0-1]
plt.savefig("figures/learning_curves_model_3", tight_layout=False)
plt.show()
```



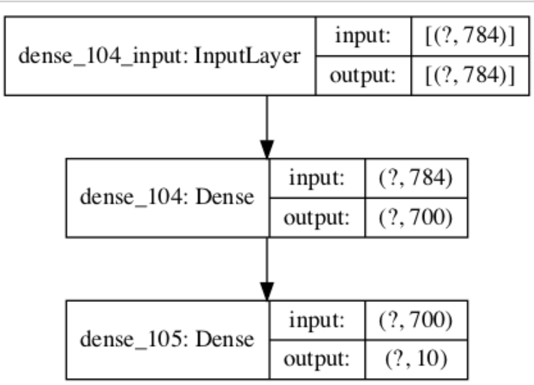
Hyperparameter tuning and comparison

- Sequence of two `Dense` layers, which are densely-connected (also called "fully-connected") neural layers.
- The first `Dense` layer, the hidden layer, consists of `700 nodes` (identified via grid search) with sigmoid activation.
- The second (and last) layer is a 10-way "softmax" layer, which means it will return an array of 10 probability scores (summing to 1). Each score will be the probability that the current digit image belongs to one of our 10 digit classes.

In [92]:
```
model_3_a = models.Sequential()
model_3_a.add(layers.Dense(700, activation='sigmoid', input_shape=(28 * 28,)))
model_3_a.add(layers.Dense(10, activation='softmax'))
```

In [93]:
```
keras.utils.plot_model(model_3_a, show_shapes=True)
```

Out[93]:



Compile model

To make our network ready for training, we need to pick three more things, as part of "compilation" step:

- A loss function: the is how the network will be able to measure how good a job it is doing on its training data, and thus how it will be able to steer itself in the right direction.
- An optimizer: this is the mechanism through which the network will update itself based on the data it sees and its loss function. In this case, Adam is used as a comparison to rmsprop in the previous model, to see if adding momentum leads to performance improvement.
- Metrics to monitor during training and testing. Here we will only care about accuracy (the fraction of the images that were correctly classified).

In [94]:
```
# Compile the model
model_3_a.compile(loss='sparse_categorical_crossentropy', optimizer='Adam', metrics=['accuracy'])
```

Train model

We are now ready to train our network, which in Keras is done via a call to the `fit` method of the network: we "fit" the model to its training data. We train the model for 30 epochs with batch size 32 (the default).

In [95]:
```
# If loading previous model:
# model = model_2()
# model = load_model_3('results/keras_mnist_model_3.h5')

# Train the model
start_time = time.time()
history = model_3_a.fit(train_images, train_labels, epochs=30,
                    validation_data=(val_images, val_labels),
                    callbacks=[keras.callbacks.EarlyStopping(patience=10)])
elapsed_time = time.time() - start_time
print('--------------------------')
print('Training time in seconds: ', round(elapsed_time,2))
print('--------------------------')

# Saving models locally after fitting
save_dir = "results/"
model_name = 'mnist_model_3_a.h5'
model_path = os.path.join(save_dir, model_name)
model_3_a.save(model_path)
print('Saved trained model at %s ' % model_path)
print('--------------------------')
```

```
Train on 55000 samples, validate on 5000 samples
Epoch 1/30
55000/55000 [==============================] - 24s 440us/sample - loss: 0.3475 - accuracy: 0.9010 - val_loss: 0.2081
- val_accuracy: 0.9406
Epoch 2/30
55000/55000 [==============================] - 22s 408us/sample - loss: 0.1784 - accuracy: 0.9468 - val_loss: 0.1369
- val_accuracy: 0.9642
Epoch 3/30
55000/55000 [==============================] - 22s 408us/sample - loss: 0.1156 - accuracy: 0.9656 - val_loss: 0.0982
- val_accuracy: 0.9720
Epoch 4/30
55000/55000 [==============================] - 23s 418us/sample - loss: 0.0810 - accuracy: 0.9757 - val_loss: 0.0823
- val_accuracy: 0.9760
Epoch 5/30
55000/55000 [==============================] - 23s 412us/sample - loss: 0.0577 - accuracy: 0.9824 - val_loss: 0.0748
- val_accuracy: 0.9766
Epoch 6/30
55000/55000 [==============================] - 23s 421us/sample - loss: 0.0422 - accuracy: 0.9870 - val_loss: 0.0682
- val_accuracy: 0.9804
Epoch 7/30
55000/55000 [==============================] - 23s 419us/sample - loss: 0.0317 - accuracy: 0.9906 - val_loss: 0.0603
- val_accuracy: 0.9814
Epoch 8/30
55000/55000 [==============================] - 23s 416us/sample - loss: 0.0223 - accuracy: 0.9937 - val_loss: 0.0669
- val_accuracy: 0.9784
Epoch 9/30
55000/55000 [==============================] - 23s 425us/sample - loss: 0.0173 - accuracy: 0.9953 - val_loss: 0.0597
- val_accuracy: 0.9818
Epoch 10/30
55000/55000 [==============================] - 1828s 33ms/sample - loss: 0.0122 - accuracy: 0.9973 - val_loss: 0.0697
- val_accuracy: 0.9812
Epoch 11/30
55000/55000 [==============================] - 27s 498us/sample - loss: 0.0093 - accuracy: 0.9976 - val_loss: 0.0548
- val_accuracy: 0.9852
Epoch 12/30
55000/55000 [==============================] - 133s 2ms/sample - loss: 0.0070 - accuracy: 0.9985 - val_loss: 0.0642 -
val_accuracy: 0.9826
Epoch 13/30
55000/55000 [==============================] - 26s 471us/sample - loss: 0.0060 - accuracy: 0.9985 - val_loss: 0.0797
- val_accuracy: 0.9792
Epoch 14/30
55000/55000 [==============================] - 27s 483us/sample - loss: 0.0045 - accuracy: 0.9991 - val_loss: 0.0652
- val_accuracy: 0.9824
Epoch 15/30
55000/55000 [==============================] - 24s 438us/sample - loss: 0.0037 - accuracy: 0.9991 - val_loss: 0.0671
- val_accuracy: 0.9844
Epoch 16/30
55000/55000 [==============================] - 24s 441us/sample - loss: 0.0035 - accuracy: 0.9991 - val_loss: 0.0708
- val_accuracy: 0.9830
Epoch 17/30
55000/55000 [==============================] - 24s 444us/sample - loss: 0.0015 - accuracy: 0.9998 - val_loss: 0.0708
- val_accuracy: 0.9836
Epoch 18/30
55000/55000 [==============================] - 24s 438us/sample - loss: 0.0029 - accuracy: 0.9993 - val_loss: 0.0755
- val_accuracy: 0.9834
Epoch 19/30
55000/55000 [==============================] - 25s 454us/sample - loss: 0.0015 - accuracy: 0.9997 - val_loss: 0.0643
- val_accuracy: 0.9842
Epoch 20/30
55000/55000 [==============================] - 24s 442us/sample - loss: 0.0018 - accuracy: 0.9995 - val_loss: 0.0991
- val_accuracy: 0.9790
Epoch 21/30
55000/55000 [==============================] - 24s 440us/sample - loss: 0.0027 - accuracy: 0.9991 - val_loss: 0.0690
- val_accuracy: 0.9854
--------------------------
Training time in seconds:  2421.43
--------------------------
Saved trained model at results/mnist_model_3_a.h5
--------------------------
```

Test model

Evaluate the model on the test dataset.

In [96]:
```
test_loss, test_acc = model_3_a.evaluate(test_images, test_labels)
```

```
10000/10000 [==============================] - 1s 126us/sample - loss: 0.0776 - accuracy: 0.9818
```

In [97]:
```
print(f'test accuracy: {test_acc}, test loss: {test_loss}')
```

```
test accuracy: 0.9818000197410583, test loss: 0.07755896054905373
```

Plot model performance

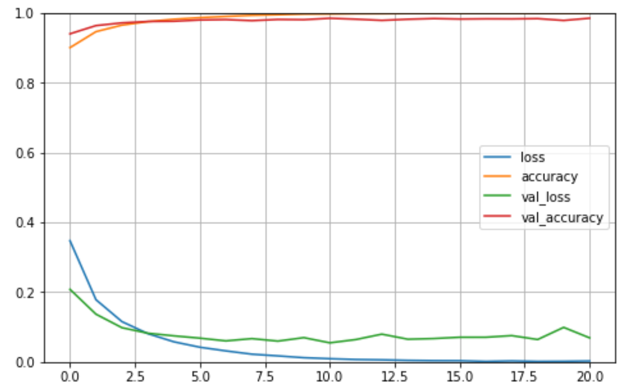
Accuracy and loss

In [98]:
```
history_dict = history.history
history_dict.keys()
```

Out[98]:
```
dict_keys(['loss', 'accuracy', 'val_loss', 'val_accuracy'])
```

Figure 11: Learning curves for Model 3a

In [99]:
```
# Alternate code for accuracy and loss
# Plot learning curves for model
pd.DataFrame(history.history).plot(figsize=(8, 5))
plt.grid(True)
plt.gca().set_ylim(0, 1) # set the vertical range to [0-1]
plt.savefig("figures/learning_curves_model_3_a", tight_layout=False)
plt.show()
```



Confusion matrix

Create a confusion matrix to assess prediction errors. Compare Model 3 (reLu activation and rmsprop optimizer) with Model 3a (sigmoid activation and Adam optimizer).

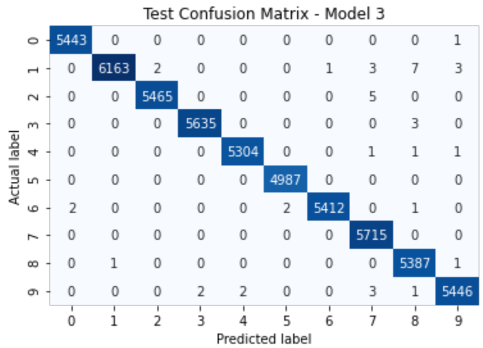
In [100]:
```
# Assign predicted classes
pred_classes = model_3.predict_classes(train_images)
```

In [101]:
```
# Create confusion matrix with values
conf_mx = confusion_matrix(train_labels,pred_classes)
conf_mx
```

Out[101]:
```
array([[5443,    0,    0,    0,    0,    0,    0,    0,    0,    1],
       [   0, 6163,    2,    0,    0,    0,    1,    3,    7,    3],
       [   0,    0, 5465,    0,    0,    0,    0,    5,    0,    0],
       [   0,    0,    0, 5635,    0,    0,    0,    0,    3,    0],
       [   0,    0,    0,    0, 5304,    0,    0,    1,    1,    1],
       [   0,    0,    0,    0,    0, 4987,    0,    0,    0,    0],
       [   2,    0,    0,    0,    0,    2, 5412,    0,    1,    0],
       [   0,    0,    0,    0,    0,    0,    0, 5715,    0,    0],
       [   0,    1,    0,    0,    0,    0,    0,    0, 5387,    1],
       [   0,    0,    0,    2,    2,    0,    0,    3,    1, 5446]])
```

Figure 11: Confusion matrix for Model 3

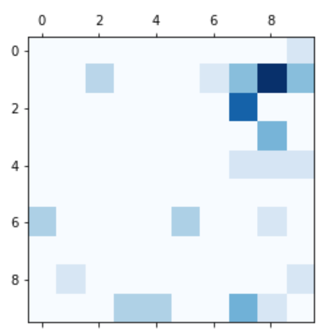
In [102]:
```
# Plot with values transposed to show actual versus predicted values
# m1_cm = confusion_matrix(train_labels, pred_classes)
m3_cm_plt=sns.heatmap(conf_mx, square=False, annot=True, fmt='d', cbar=False, cmap="Blues")
plt.xlabel('Predicted label')
plt.ylabel('Actual label')
plt.title("Test Confusion Matrix - Model 3")
plt.savefig("figures/confusion_matrix_errors_plot_model_3", tight_layout=False)
plt.show();
```



In [103]:
```
# Plot normalized confusion matrix
row_sums = conf_mx.sum(axis=1, keepdims=True)
norm_conf_mx = conf_mx / row_sums
```

Figure 12: Normalized confusion matrix for Model 3

In [104]:
```
np.fill_diagonal(norm_conf_mx, 0)
plt.matshow(norm_conf_mx, cmap="Blues")
plt.savefig("figures/confusion_matrix_errors_plot_norm_model_3", tight_layout=False)
plt.show()
```



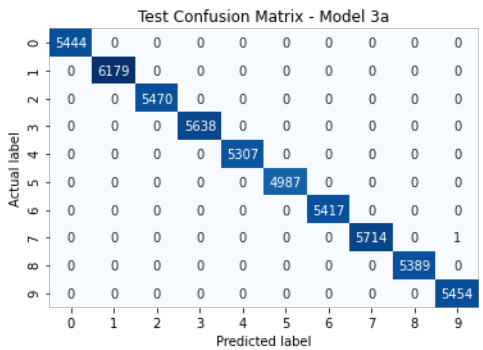
In [106]:
```
# Assign predicted classes
pred_classes = model_3_a.predict_classes(train_images)
```

In [107]:
```
# Create confusion matrix with values
conf_mx = confusion_matrix(train_labels,pred_classes)
conf_mx
```

Out[107]:
```
array([[5444,    0,    0,    0,    0,    0,    0,    0,    0,    0],
       [   0, 6179,    0,    0,    0,    0,    0,    0,    0,    0],
       [   0,    0, 5470,    0,    0,    0,    0,    0,    0,    0],
       [   0,    0,    0, 5638,    0,    0,    0,    0,    0,    0],
       [   0,    0,    0,    0, 5307,    0,    0,    0,    0,    0],
       [   0,    0,    0,    0,    0, 4987,    0,    0,    0,    0],
       [   0,    0,    0,    0,    0,    0, 5417,    0,    0,    0],
       [   0,    0,    0,    0,    0,    0,    0, 5714,    0,    1],
       [   0,    0,    0,    0,    0,    0,    0,    0, 5389,    0],
       [   0,    0,    0,    0,    0,    0,    0,    0,    0, 5454]])
```

Figure 13: Confusion matrix for Model 3a

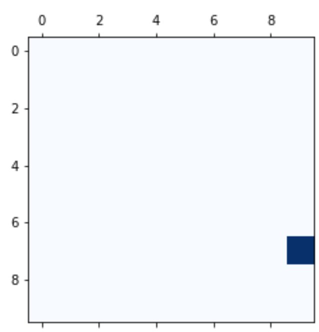
In [108]:
```
# Plot with values transposed to show actual versus predicted values
# m1_cm = confusion_matrix(train_labels, pred_classes)
m3_cm_plt=sns.heatmap(conf_mx, square=False, annot=True, fmt='d', cbar=False, cmap="Blues")
plt.xlabel('Predicted label')
plt.ylabel('Actual label')
plt.title("Test Confusion Matrix - Model 3a")
plt.savefig("figures/confusion_matrix_errors_plot_model_3a", tight_layout=False)
plt.show();
```



In [109]:
```
# Plot normalized confusion matrix
row_sums = conf_mx.sum(axis=1, keepdims=True)
norm_conf_mx = conf_mx / row_sums
```

Figure 14: Normalized confusion matrix for Model 3a

In [110]:
```
np.fill_diagonal(norm_conf_mx, 0)
plt.matshow(norm_conf_mx, cmap="Blues")
plt.savefig("figures/confusion_matrix_errors_plot_norm_model_3a", tight_layout=False)
plt.show()
```



Error analysis

In [46]:
```
# Define function for plotting example digits
def plot_digits(instances, images_per_row=10, **options):
    size = 28
    images_per_row = min(len(instances), images_per_row)
    images = [instance.reshape(size,size) for instance in instances]
    n_rows = (len(instances) - 1) // images_per_row + 1
    row_images = []
    n_empty = n_rows * images_per_row - len(instances)
    images.append(np.zeros((size, size * n_empty)))
    for row in range(n_rows):
        rimages = images[row * images_per_row : (row + 1) * images_per_row]
        row_images.append(np.concatenate(rimages, axis=1))
    image = np.concatenate(row_images, axis=0)
    plt.imshow(image, cmap = 'binary', **options)
    plt.axis("off")
```

In [47]:
```
# Try some examples of mis-classes digits
# Upper left: 5s classified correctly as 5s
# Upper right: 5s classified as 8s
# Lower left: 8s classified as 5s
# Lower right: 8s classified correctly as 8s

cl_a, cl_b = 5, 8
X_aa = train_images[(train_labels == cl_a) & (pred_classes == cl_a)]
X_ab = train_images[(train_labels == cl_a) & (pred_classes == cl_b)]
X_ba = train_images[(train_labels == cl_b) & (pred_classes == cl_a)]
X_bb = train_images[(train_labels == cl_b) & (pred_classes == cl_b)]

plt.figure(figsize=(8,8))
plt.subplot(221); plot_digits(X_aa[:25], images_per_row=5)
plt.subplot(222); plot_digits(X_ab[:25], images_per_row=5)
plt.subplot(223); plot_digits(X_ba[:25], images_per_row=5)
plt.subplot(224); plot_digits(X_bb[:25], images_per_row=5)
plt.savefig("results/error_analysis_digits_plot_EXP3_valid")
plt.show()
```

Experiment 4

We use PCA decomposition to reduce the number of dimensions of our training set of 28x28 dimensional MNIST images. By setting `n_components=0.95` we get the 154 principal components that contain 95% of the variance (information) in the training images. We transform the training images to reduce its dimensionality from 784 to 154. We also reduce the number of dimensions of 'best' model from Experiment 3 to 154 inputs nodes and train it on the new lower dimensional data.

In [113]:
```
# Reduce dimensions to those containing 95% of the variance in the training images
pca = PCA(n_components=0.95)
train_images_red = pca.fit_transform(train_images)
val_images_red = pca.transform(val_images)
test_images_red = pca.transform(test_images)
```

In [114]:
```
# Evaluate shape of data with reduced dimensions
test_images_red.shape, train_images_red.shape, val_images_red.shape
```

Out[114]:
```
((10000, 154), (55000, 154), (5000, 154))
```

Build "best" network before dimension reduction

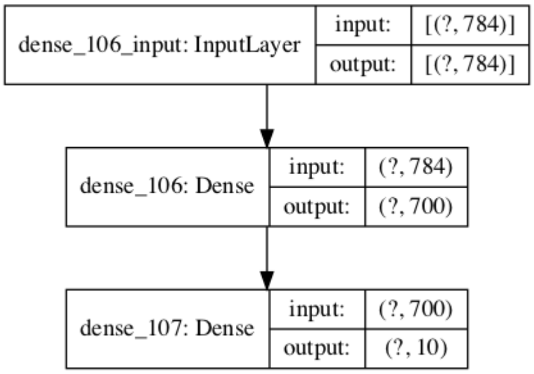
- Using the "best" model from Experiment 3, our baseline network consists of a sequence of two Dense layers, which are densely-connected (also called "fully-connected") neural layers.
- The first Dense layer, the hidden layer, consists of 600 nodes.
- The second (and last) layer is a 10-way "softmax" layer, which means it will return an array of 10 probability scores (summing to 1). Each score will be the probability that the current digit image belongs to one of our 10 digit classes.

In [116]:
```
# # If loading previous model:
# model = model_3()
# model_4 = load_model_3('results/mnist_model_3.h5')

model_4 = models.Sequential()
model_4.add(layers.Dense(700, activation='sigmoid', input_shape=(28 * 28,)))
model_4.add(layers.Dense(10, activation='softmax'))
```

In [117]:
```
# Plot model
keras.utils.plot_model(model_4, "figures/mnist_model_4.png", show_shapes=True) # plot a graph of the model
```

Out[117]:



In [118]:
```
model_4.summary()
```

```
Model: "sequential_54"
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
dense_106 (Dense)            (None, 700)               549500
_________________________________________________________________
dense_107 (Dense)            (None, 10)                7010
=================================================================
Total params: 556,510
Trainable params: 556,510
Non-trainable params: 0
_________________________________________________________________
```

Compile model

To make our network ready for training, we need to pick three more things, as part of "compilation" step:

- A loss function: the is how the network will be able to measure how good a job it is doing on its training data, and thus how it will be able to steer itself in the right direction.
- An optimizer: this is the mechanism through which the network will update itself based on the data it sees and its loss function.
- Metrics to monitor during training and testing. Here we will only care about accuracy (the fraction of the images that were correctly classified).

In [119]:
```
# Compile the model
model_4.compile(loss='sparse_categorical_crossentropy', optimizer='Adam', metrics=['accuracy'])
```

Train model

We are now ready to train our network, which in Keras is done via a call to the `fit` method of the network: we "fit" the model to its training data. We train the model for 30 epochs with batch size 32 (the default).

In [120]:
```
# If loading previous model:
# model = model_2()
# model = load_model_3('results/keras_mnist_model_3.h5')

# Train the model
start_time = time.time()
history = model_4.fit(train_images, train_labels, epochs=30,
                    validation_data=(val_images, val_labels),
                    callbacks=[keras.callbacks.EarlyStopping(patience=10)])
elapsed_time = time.time() - start_time
print('--------------------------')
print('Training time in seconds: ', round(elapsed_time,2))
print('--------------------------')

# Saving models locally after fitting
save_dir = "results/"
model_name = 'mnist_model_4.h5'
model_path = os.path.join(save_dir, model_name)
model_4.save(model_path)
print('Saved trained model at %s ' % model_path)
print('--------------------------')
```

```
Train on 55000 samples, validate on 5000 samples
Epoch 1/30
55000/55000 [==============================] - 26s 479us/sample - loss: 0.3431 - accuracy: 0.9017 - val_loss: 0.2053
- val_accuracy: 0.9390
Epoch 2/30
55000/55000 [==============================] - 23s 422us/sample - loss: 0.1796 - accuracy: 0.9465 - val_loss: 0.1476
- val_accuracy: 0.9572
Epoch 3/30
55000/55000 [==============================] - 24s 438us/sample - loss: 0.1166 - accuracy: 0.9651 - val_loss: 0.1052
- val_accuracy: 0.9690
Epoch 4/30
55000/55000 [==============================] - 24s 443us/sample - loss: 0.0812 - accuracy: 0.9755 - val_loss: 0.0874
- val_accuracy: 0.9732
Epoch 5/30
55000/55000 [==============================] - 24s 429us/sample - loss: 0.0585 - accuracy: 0.9822 - val_loss: 0.0737
- val_accuracy: 0.9766
Epoch 6/30
55000/55000 [==============================] - 23s 417us/sample - loss: 0.0432 - accuracy: 0.9867 - val_loss: 0.0624
- val_accuracy: 0.9816
Epoch 7/30
55000/55000 [==============================] - 23s 422us/sample - loss: 0.0321 - accuracy: 0.9903 - val_loss: 0.0695
- val_accuracy: 0.9808
Epoch 8/30
55000/55000 [==============================] - 24s 431us/sample - loss: 0.0233 - accuracy: 0.9934 - val_loss: 0.0622
- val_accuracy: 0.9800
Epoch 9/30
55000/55000 [==============================] - 24s 428us/sample - loss: 0.0162 - accuracy: 0.9957 - val_loss: 0.0660
- val_accuracy: 0.9804
Epoch 10/30
55000/55000 [==============================] - 24s 436us/sample - loss: 0.0125 - accuracy: 0.9969 - val_loss: 0.0631
- val_accuracy: 0.9814
Epoch 11/30
55000/55000 [==============================] - 24s 429us/sample - loss: 0.0097 - accuracy: 0.9977 - val_loss: 0.0616
- val_accuracy: 0.9834
Epoch 12/30
55000/55000 [==============================] - 25s 459us/sample - loss: 0.0074 - accuracy: 0.9981 - val_loss: 0.0610
- val_accuracy: 0.9828
Epoch 13/30
55000/55000 [==============================] - 24s 445us/sample - loss: 0.0052 - accuracy: 0.9988 - val_loss: 0.0759
- val_accuracy: 0.9788
Epoch 14/30
55000/55000 [==============================] - 24s 435us/sample - loss: 0.0050 - accuracy: 0.9988 - val_loss: 0.0726
- val_accuracy: 0.9806
Epoch 15/30
55000/55000 [==============================] - 26s 464us/sample - loss: 0.0046 - accuracy: 0.9988 - val_loss: 0.0642
- val_accuracy: 0.9844
Epoch 16/30
55000/55000 [==============================] - 24s 443us/sample - loss: 0.0019 - accuracy: 0.9997 - val_loss: 0.0667
- val_accuracy: 0.9834
Epoch 17/30
55000/55000 [==============================] - 24s 439us/sample - loss: 0.0040 - accuracy: 0.9989 - val_loss: 0.0699
- val_accuracy: 0.9810
Epoch 18/30
55000/55000 [==============================] - 25s 449us/sample - loss: 0.0016 - accuracy: 0.9997 - val_loss: 0.0686
- val_accuracy: 0.9842
Epoch 19/30
55000/55000 [==============================] - 24s 440us/sample - loss: 0.0027 - accuracy: 0.9993 - val_loss: 0.0832
- val_accuracy: 0.9810
Epoch 20/30
55000/55000 [==============================] - 24s 439us/sample - loss: 0.0023 - accuracy: 0.9993 - val_loss: 0.0796
- val_accuracy: 0.9816
Epoch 21/30
55000/55000 [==============================] - 25s 450us/sample - loss: 0.0011 - accuracy: 0.9999 - val_loss: 0.0714
- val_accuracy: 0.9842
Epoch 22/30
55000/55000 [==============================] - 24s 442us/sample - loss: 0.0030 - accuracy: 0.9991 - val_loss: 0.0741
- val_accuracy: 0.9834
--------------------------
Training time in seconds:  534.9
--------------------------
Saved trained model at results/mnist_model_4.h5
--------------------------
```

Test model

Evaluate the model on the test dataset.

In [121]:
```
test_loss, test_acc = model_4.evaluate(test_images, test_labels)
```

```
10000/10000 [==============================] - 1s 118us/sample - loss: 0.0917 - accuracy: 0.9805
```

In [122]:
```
print(f'test accuracy: {test_acc}, test loss: {test_loss}')
```

```
test accuracy: 0.9804999828338623, test loss: 0.09173274229887722
```

Building, training and testing the model on the new data

We modify the model to take input with dimension (154,)

In [125]:
```
# check started code for input dimensions

model_4_pca = models.Sequential()
model_4_pca.add(layers.Dense(700, activation='sigmoid', input_shape=(154,)))
model_4_pca.add(layers.Dense(10, activation='softmax'))
```

In [126]:
```
# For use with non-categorical labels
model_4_pca.compile(optimizer='Adam',
                loss='sparse_categorical_crossentropy',
                metrics=['accuracy'])
```

In [127]:
```
# Train the model with the lower-dimensional data
start_time = time.time()
history = model_4_pca.fit(train_images_red, train_labels, epochs=30,
                      validation_data=(val_images_red, val_labels),
                      callbacks=[keras.callbacks.EarlyStopping(patience=10)])
elapsed_time = time.time() - start_time
print('---------------------------')
print('Training time in seconds: ', round(elapsed_time,2))
print('---------------------------')

# Saving models locally after fitting
save_dir = "results/"
model_name = 'mnist_model_4_pca.h5'
model_path = os.path.join(save_dir, model_name)
model_4.save(model_path)
print('Saved trained model at %s ' % model_path)
print('---------------------------')
```

```
Train on 55000 samples, validate on 5000 samples
Epoch 1/30
55000/55000 [==============================] - 12s 224us/sample - loss: 0.4356 - accuracy: 0.8837 - val_loss: 0.2793
- val_accuracy: 0.9218
Epoch 2/30
55000/55000 [==============================] - 11s 205us/sample - loss: 0.2906 - accuracy: 0.9158 - val_loss: 0.2676
- val_accuracy: 0.9244
Epoch 3/30
55000/55000 [==============================] - 11s 198us/sample - loss: 0.2646 - accuracy: 0.9233 - val_loss: 0.2308
- val_accuracy: 0.9332
Epoch 4/30
55000/55000 [==============================] - 11s 202us/sample - loss: 0.2295 - accuracy: 0.9332 - val_loss: 0.2043
- val_accuracy: 0.9424
Epoch 5/30
55000/55000 [==============================] - 11s 200us/sample - loss: 0.1905 - accuracy: 0.9447 - val_loss: 0.1739
- val_accuracy: 0.9498
Epoch 6/30
55000/55000 [==============================] - 11s 201us/sample - loss: 0.1550 - accuracy: 0.9550 - val_loss: 0.1447
- val_accuracy: 0.9588
Epoch 7/30
55000/55000 [==============================] - 12s 209us/sample - loss: 0.1251 - accuracy: 0.9637 - val_loss: 0.1307
- val_accuracy: 0.9600
Epoch 8/30
55000/55000 [==============================] - 11s 204us/sample - loss: 0.1024 - accuracy: 0.9703 - val_loss: 0.1041
- val_accuracy: 0.9698
Epoch 9/30
55000/55000 [==============================] - 11s 203us/sample - loss: 0.0820 - accuracy: 0.9765 - val_loss: 0.0935
- val_accuracy: 0.9728
Epoch 10/30
55000/55000 [==============================] - 11s 204us/sample - loss: 0.0659 - accuracy: 0.9813 - val_loss: 0.0790
- val_accuracy: 0.9740
Epoch 11/30
55000/55000 [==============================] - 11s 204us/sample - loss: 0.0537 - accuracy: 0.9844 - val_loss: 0.0784
- val_accuracy: 0.9746
Epoch 12/30
55000/55000 [==============================] - 11s 204us/sample - loss: 0.0425 - accuracy: 0.9881 - val_loss: 0.0770
- val_accuracy: 0.9756
Epoch 13/30
55000/55000 [==============================] - 12s 212us/sample - loss: 0.0348 - accuracy: 0.9900 - val_loss: 0.0744
- val_accuracy: 0.9782
Epoch 14/30
55000/55000 [==============================] - 11s 206us/sample - loss: 0.0257 - accuracy: 0.9936 - val_loss: 0.0587
- val_accuracy: 0.9822
Epoch 15/30
55000/55000 [==============================] - 11s 209us/sample - loss: 0.0198 - accuracy: 0.9955 - val_loss: 0.0601
- val_accuracy: 0.9828
Epoch 16/30
55000/55000 [==============================] - 11s 205us/sample - loss: 0.0159 - accuracy: 0.9967 - val_loss: 0.0607
- val_accuracy: 0.9818
Epoch 17/30
55000/55000 [==============================] - 11s 206us/sample - loss: 0.0120 - accuracy: 0.9974 - val_loss: 0.0615
- val_accuracy: 0.9814
Epoch 18/30
55000/55000 [==============================] - 12s 210us/sample - loss: 0.0091 - accuracy: 0.9983 - val_loss: 0.0625
- val_accuracy: 0.9814
Epoch 19/30
55000/55000 [==============================] - 12s 210us/sample - loss: 0.0071 - accuracy: 0.9989 - val_loss: 0.0700
- val_accuracy: 0.9820
Epoch 20/30
55000/55000 [==============================] - 11s 209us/sample - loss: 0.0055 - accuracy: 0.9992 - val_loss: 0.0617
- val_accuracy: 0.9828
Epoch 21/30
55000/55000 [==============================] - 11s 207us/sample - loss: 0.0036 - accuracy: 0.9996 - val_loss: 0.0606
- val_accuracy: 0.9828
Epoch 22/30
55000/55000 [==============================] - 11s 208us/sample - loss: 0.0039 - accuracy: 0.9993 - val_loss: 0.0624
- val_accuracy: 0.9814
Epoch 23/30
55000/55000 [==============================] - 11s 209us/sample - loss: 0.0028 - accuracy: 0.9995 - val_loss: 0.0718
- val_accuracy: 0.9818
Epoch 24/30
55000/55000 [==============================] - 12s 215us/sample - loss: 0.0021 - accuracy: 0.9997 - val_loss: 0.0687
- val_accuracy: 0.9802
--------------------------
Training time in seconds:  275.36
--------------------------
Saved trained model at results/mnist_model_4_pca.h5
--------------------------
```

In [128]:
```
hist_dict = history.history
hist_dict.keys()
```

Out[128]:
```
dict_keys(['loss', 'accuracy', 'val_loss', 'val_accuracy'])
```

In [129]:
```
print(f'''acc: {hist_dict['accuracy'][-1]:.4}, val acc: {hist_dict['val_accuracy'][-1]:.4},
loss: {hist_dict['loss'][-1]:.4}, val loss: {hist_dict['val_loss'][-1]:.4}''')
```

```
acc: 0.9997, val acc: 0.9802,
loss: 0.002079, val loss: 0.06871
```

Test model

Evaluate the model on the test dataset.

In [131]:
```
test_loss, test_acc = model_4_pca.evaluate(test_images_red, test_labels)
```

```
10000/10000 [==============================] - 2s 157us/sample - loss: 0.0798 - accuracy: 0.9789
```

In [132]:
```
# Results
print(f'test acc: {test_acc}, test loss: {test_loss}')
```

```
test acc: 0.9789000153541565, test loss: 0.07981216991242036
```

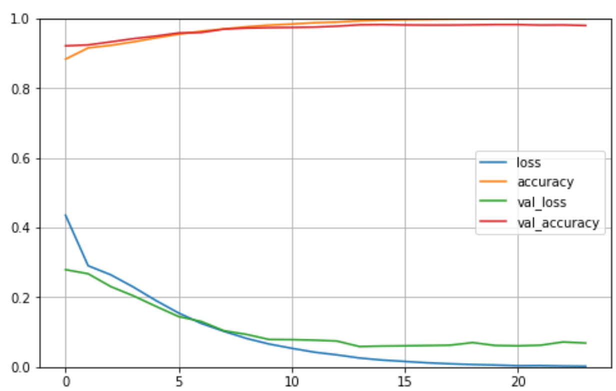
Comparison of the two models

Reducing the dimensions from 784 to 154 had no negative impact on the performance of our 'best' model. Training time was greatly reduced, which could be a factor for higher-dimensionality data.

Plot performance of model with dimension reduction applied

Figure 15: Learning curves for Model 4 with PCA

In [133]:
```
# Alternate code for accuracy and loss
# Plot learning curves for model
pd.DataFrame(history.history).plot(figsize=(8, 5))
plt.grid(True)
plt.gca().set_ylim(0, 1) # set the vertical range to [0-1]
plt.savefig("figures/learning_curves_model_4_pca", tight_layout=False)
plt.show()
```



EXPERIMENT 5

We use a Random Forest classifier to get the relative importance of the 784 features (pixels) of the 28x28 dimensional images in training set of MNIST images and select the top 70 features (pixels). We train our 'best' dense neural network using these 70 features and compare its performance to the the DNN models from EXPERIMENTS 3 and 4.

Reducing dimensionality of the data with Random Forests

We create a Random Forest Classifier (with the default 100 trees) and use it to find the relative importance of the 784 features (pixels) in the training set. We produce a heat map to visual the relative importance of the features (using code from Hands On Machine Learning by A. Geron). Finally, we select the 70 most important feature (pixels) from the training, validation and test images to test our 'best' model on.

In [49]:
```
# Define and train model
rnd_clf = RandomForestClassifier(n_estimators=100, random_state=42)
rnd_clf.fit(train_images,train_labels)
```

Out[49]:
```
RandomForestClassifier(bootstrap=True, ccp_alpha=0.0, class_weight=None,
                       criterion='gini', max_depth=None, max_features='auto',
                       max_leaf_nodes=None, max_samples=None,
                       min_impurity_decrease=0.0, min_impurity_split=None,
                       min_samples_leaf=1, min_samples_split=2,
                       min_weight_fraction_leaf=0.0, n_estimators=100,
                       n_jobs=None, oob_score=False, random_state=42, verbose=0,
                       warm_start=False)
```

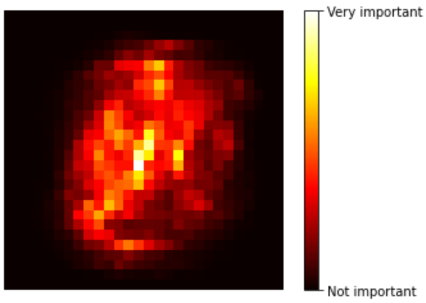
In [50]:
```
# https://github.com/ageron/handson-ml2/blob/master/07_ensemble_learning_and_random_forests.ipynb
import matplotlib as mpl
# import matplotlib.pyplot as plt

def plot_digit(data):
    image = data.reshape(28, 28)
    plt.imshow(image, cmap = mpl.cm.hot,
               interpolation="nearest")
    plt.axis("off")

plot_digit(rnd_clf.feature_importances_)

cbar = plt.colorbar(ticks=[rnd_clf.feature_importances_.min(), rnd_clf.feature_importances_.max()])
cbar.ax.set_yticklabels(['Not important', 'Very important'])

plt.savefig("results/feature_importance_plot_rf")
plt.show()
```



In [136]:
```
# Get the indices of the 70 most "important" features
# https://stackoverflow.com/questions/6910641/how-do-i-get-indices-of-n-maximum-values-in-a-numpy-array
n = 70
imp_arr = rnd_clf.feature_importances_
idx = (-imp_arr).argsort()[:n]          # get the indices of the 70 "most important" features/pixels
len(idx)
```

Out[136]:
```
70
```

In [137]:
```
# Create training, validation and test images using just the 70 pixel locations obtained above
train_images_sm = train_images[:,idx]
val_images_sm = val_images[:,idx]
test_images_sm = test_images[:,idx]
train_images_sm.shape, val_images.shape, test_images_sm.shape # the reduced images have dimension 70
```

Out[137]:
```
((55000, 70), (5000, 784), (10000, 70))
```

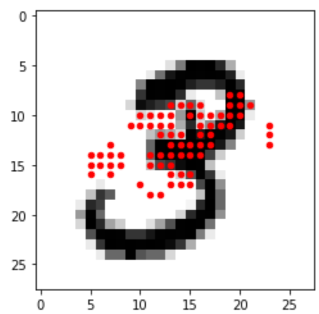
Visualize the 70 pixels

We convert the array of indexes to ordered pairs and plot them as red circles on the second training image. These are the feature we are training our neural network on.

In [138]:
```
# to convert an index n, 0<= n < 784
def pair(n,size):
    x = n//size
    y = n%size
    return x,y
```

In [139]:
```
# Visualize features
plt.imshow(train_images[1].reshape(28,28),cmap='binary')
x, y = np.array([pair(k,28) for k in idx]).T
plt.scatter(x,y,color='red',s=20)
```

Out[139]: <matplotlib.collections.PathCollection at 0x1a4f3d79e8>



Build 'best' network (before reducing dimensions)

Recall from `Experiment 3` that our network consists of a sequence of two `Dense` layers, which are densely-connected (also called "fully-connected") neural layers.

The first `Dense` layer, the hidden layer, consists of `700 nodes` .

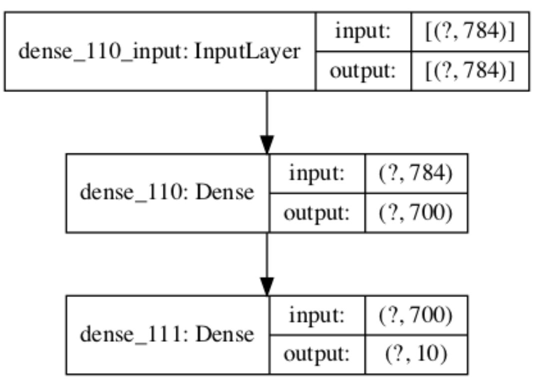
The second (and last) layer is a 10-way "softmax" layer, which means it will return an array of 10 probability scores (summing to 1). Each score will be the probability that the current digit image belongs to one of our 10 digit classes.

In [140]:
```
# If loading previous model:
# model = model_5()
# model_5 = load_model_5('results/mnist_model_4.h5')

model_5 = models.Sequential()
model_5.add(layers.Dense(700, activation='sigmoid', input_shape=(28 * 28,)))
model_5.add(layers.Dense(10, activation='softmax'))
```

In [142]:
```
# Plot model
keras.utils.plot_model(model_5, "figures/mnist_model_5hnode.png", show_shapes=True) # plot a graph of the model
```

Out[142]:



In [143]:
```
model_5.summary()
```

```
Model: "sequential_56"
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
dense_110 (Dense)            (None, 700)               549500
_________________________________________________________________
dense_111 (Dense)            (None, 10)                7010
=================================================================
Total params: 556,510
Trainable params: 556,510
Non-trainable params: 0
_________________________________________________________________
```

Compile model

To make our network ready for training, we need to pick three more things, as part of "compilation" step:

- A loss function: the is how the network will be able to measure how good a job it is doing on its training data, and thus how it will be able to steer itself in the right direction.
- An optimizer: this is the mechanism through which the network will update itself based on the data it sees and its loss function.
- Metrics to monitor during training and testing. Here we will only care about accuracy (the fraction of the images that were correctly classified).

In [144]:
```
# For use with non-categorical labels
model_5.compile(optimizer='Adam',
                loss='sparse_categorical_crossentropy',
                metrics=['accuracy'])
```

Train model

We are now ready to train our network, which in Keras is done via a call to the `fit` method of the network: we "fit" the model to its training data. We train the model for 30 epochs with batch size 32 (the default).

In [145]:
```
# If loading previous model:
# model = model_2()
# model = load_model_3('results/keras_mnist_model_3.h5')

# Train the model
start_time = time.time()
history = model_5.fit(train_images, train_labels, epochs=30,
                    validation_data=(val_images, val_labels),
                    callbacks=[keras.callbacks.EarlyStopping(patience=10)])
elapsed_time = time.time() - start_time
print('---------------------------')
print('Training time in seconds: ', round(elapsed_time,2))
print('---------------------------')

# Saving models locally after fitting
save_dir = "results/"
model_name = 'mnist_model_5.h5'
model_path = os.path.join(save_dir, model_name)
model_5.save(model_path)
print('Saved trained model at %s ' % model_path)
print('---------------------------')
```

```
Train on 55000 samples, validate on 5000 samples
Epoch 1/30
55000/55000 [==============================] - 26s 469us/sample - loss: 0.3430 - accuracy: 0.9010 - val_loss: 0.1968
- val_accuracy: 0.9426
Epoch 2/30
55000/55000 [==============================] - 23s 416us/sample - loss: 0.1789 - accuracy: 0.9469 - val_loss: 0.1325
- val_accuracy: 0.9624
Epoch 3/30
55000/55000 [==============================] - 23s 418us/sample - loss: 0.1172 - accuracy: 0.9645 - val_loss: 0.0990
- val_accuracy: 0.9710
Epoch 4/30
55000/55000 [==============================] - 23s 424us/sample - loss: 0.0822 - accuracy: 0.9755 - val_loss: 0.0825
- val_accuracy: 0.9746
Epoch 5/30
55000/55000 [==============================] - 23s 423us/sample - loss: 0.0589 - accuracy: 0.9821 - val_loss: 0.0802
- val_accuracy: 0.9748
Epoch 6/30
55000/55000 [==============================] - 24s 427us/sample - loss: 0.0429 - accuracy: 0.9866 - val_loss: 0.0700
- val_accuracy: 0.9794
Epoch 7/30
55000/55000 [==============================] - 23s 426us/sample - loss: 0.0317 - accuracy: 0.9906 - val_loss: 0.0684
- val_accuracy: 0.9808
Epoch 8/30
55000/55000 [==============================] - 23s 422us/sample - loss: 0.0227 - accuracy: 0.9938 - val_loss: 0.0631
- val_accuracy: 0.9810
Epoch 9/30
55000/55000 [==============================] - 24s 437us/sample - loss: 0.0174 - accuracy: 0.9951 - val_loss: 0.0640
- val_accuracy: 0.9826
Epoch 10/30
55000/55000 [==============================] - 263s 5ms/sample - loss: 0.0130 - accuracy: 0.9964 - val_loss: 0.0691 -
val_accuracy: 0.9792
Epoch 11/30
55000/55000 [==============================] - 25s 460us/sample - loss: 0.0094 - accuracy: 0.9975 - val_loss: 0.0643
- val_accuracy: 0.9824
Epoch 12/30
55000/55000 [==============================] - 24s 440us/sample - loss: 0.0080 - accuracy: 0.9980 - val_loss: 0.0803
- val_accuracy: 0.9782
Epoch 13/30
55000/55000 [==============================] - 24s 438us/sample - loss: 0.0052 - accuracy: 0.9989 - val_loss: 0.0727
- val_accuracy: 0.9802
Epoch 14/30
55000/55000 [==============================] - 24s 440us/sample - loss: 0.0050 - accuracy: 0.9986 - val_loss: 0.0628
- val_accuracy: 0.9850
Epoch 15/30
55000/55000 [==============================] - 24s 440us/sample - loss: 0.0036 - accuracy: 0.9992 - val_loss: 0.0657
- val_accuracy: 0.9830
Epoch 16/30
55000/55000 [==============================] - 24s 437us/sample - loss: 0.0028 - accuracy: 0.9995 - val_loss: 0.0706
- val_accuracy: 0.9848
Epoch 17/30
55000/55000 [==============================] - 24s 444us/sample - loss: 0.0031 - accuracy: 0.9992 - val_loss: 0.0833
- val_accuracy: 0.9814
Epoch 18/30
55000/55000 [==============================] - 24s 437us/sample - loss: 0.0024 - accuracy: 0.9994 - val_loss: 0.0737
- val_accuracy: 0.9822
Epoch 19/30
55000/55000 [==============================] - 24s 439us/sample - loss: 0.0022 - accuracy: 0.9994 - val_loss: 0.0711
- val_accuracy: 0.9826
Epoch 20/30
55000/55000 [==============================] - 114s 2ms/sample - loss: 0.0022 - accuracy: 0.9995 - val_loss: 0.0829 -
val_accuracy: 0.9832
Epoch 21/30
55000/55000 [==============================] - 25s 457us/sample - loss: 9.7210e-04 - accuracy: 0.9998 - val_loss: 0.0
702 - val_accuracy: 0.9842
Epoch 22/30
55000/55000 [==============================] - 27s 482us/sample - loss: 0.0030 - accuracy: 0.9991 - val_loss: 0.0712
- val_accuracy: 0.9832
Epoch 23/30
55000/55000 [==============================] - 26s 464us/sample - loss: 3.6390e-04 - accuracy: 1.0000 - val_loss: 0.0
710 - val_accuracy: 0.9844
Epoch 24/30
55000/55000 [==============================] - 25s 447us/sample - loss: 0.0019 - accuracy: 0.9993 - val_loss: 0.1009
- val_accuracy: 0.9784
--------------------------
Training time in seconds:  912.39
--------------------------
Saved trained model at results/mnist_model_5.h5
--------------------------
```

In [146]:
```
hist_dict = history.history
hist_dict.keys()
```

Out[146]:
```
dict_keys(['loss', 'accuracy', 'val_loss', 'val_accuracy'])
```

In [147]:
```
print(f'''acc: {hist_dict['accuracy'][-1]:.4}, val acc: {hist_dict['val_accuracy'][-1]:.4},
loss: {hist_dict['loss'][-1]:.4}, val loss: {hist_dict['val_loss'][-1]:.4}''')
```

```
acc: 0.9993, val acc: 0.9784,
loss: 0.00188, val loss: 0.1009
```

Test model

Evaluate the model on the test dataset.

In [148]:
```
test_loss, test_acc = model_5.evaluate(test_images, test_labels)
```

```
10000/10000 [==============================] - 1s 123us/sample - loss: 0.1063 - accuracy: 0.9769
```

In [149]:
```
print(f'test accuracy: {test_acc}, test loss: {test_loss}')
```

```
test accuracy: 0.9768999814987183, test loss: 0.10626666153988776
```

Build, train and test the model on the new data

We modify the model to take input with dimension (70,)

In [150]:
```
model_5_rf = models.Sequential()
model_5_rf.add(layers.Dense(700, activation='relu', input_shape=(70,)))
model_5_rf.add(layers.Dense(10, activation='softmax'))
```

In [151]:
```
# Compile the model
model_5_rf.compile(optimizer='Adam',
                loss='sparse_categorical_crossentropy',
                metrics=['accuracy'])
```

In [152]:
```
# Train the model
start_time = time.time()
history = model_5_rf.fit(train_images_sm, train_labels, epochs=30,
                    validation_data=(val_images_sm, val_labels),
                    callbacks=[keras.callbacks.EarlyStopping(patience=10)])
elapsed_time = time.time() - start_time
print('--------------------------')
print('Training time in seconds: ', round(elapsed_time,2))
print('--------------------------')

# Saving models locally after fitting
save_dir = "results/"
model_name = 'mnist_model_5_rf.h5'
model_path = os.path.join(save_dir, model_name)
model_5_rf.save(model_path)
print('Saved trained model at %s ' % model_path)
print('--------------------------')
```

```
Train on 55000 samples, validate on 5000 samples
Epoch 1/30
55000/55000 [==============================] - 10s 179us/sample - loss: 0.4589 - accuracy: 0.8613 - val_loss: 0.3001
- val_accuracy: 0.9124
Epoch 2/30
55000/55000 [==============================] - 8s 153us/sample - loss: 0.2663 - accuracy: 0.9190 - val_loss: 0.2508 -
val_accuracy: 0.9254
Epoch 3/30
55000/55000 [==============================] - 8s 142us/sample - loss: 0.2143 - accuracy: 0.9327 - val_loss: 0.2180 -
val_accuracy: 0.9338
Epoch 4/30
55000/55000 [==============================] - 8s 148us/sample - loss: 0.1854 - accuracy: 0.9417 - val_loss: 0.1835 -
val_accuracy: 0.9446
Epoch 5/30
55000/55000 [==============================] - 8s 149us/sample - loss: 0.1639 - accuracy: 0.9484 - val_loss: 0.1823 -
val_accuracy: 0.9448
Epoch 6/30
55000/55000 [==============================] - 8s 151us/sample - loss: 0.1493 - accuracy: 0.9529 - val_loss: 0.1771 -
val_accuracy: 0.9478
Epoch 7/30
55000/55000 [==============================] - 8s 147us/sample - loss: 0.1341 - accuracy: 0.9578 - val_loss: 0.1725 -
val_accuracy: 0.9494
Epoch 8/30
55000/55000 [==============================] - 8s 147us/sample - loss: 0.1241 - accuracy: 0.9599 - val_loss: 0.1641 -
val_accuracy: 0.9512
Epoch 9/30
55000/55000 [==============================] - 8s 147us/sample - loss: 0.1135 - accuracy: 0.9637 - val_loss: 0.1736 -
val_accuracy: 0.9486
Epoch 10/30
55000/55000 [==============================] - 9s 158us/sample - loss: 0.1049 - accuracy: 0.9664 - val_loss: 0.1806 -
val_accuracy: 0.9456
Epoch 11/30
55000/55000 [==============================] - 8s 148us/sample - loss: 0.0967 - accuracy: 0.9679 - val_loss: 0.1717 -
val_accuracy: 0.9508
Epoch 12/30
55000/55000 [==============================] - 8s 148us/sample - loss: 0.0887 - accuracy: 0.9715 - val_loss: 0.1669 -
val_accuracy: 0.9558
Epoch 13/30
55000/55000 [==============================] - 9s 163us/sample - loss: 0.0835 - accuracy: 0.9730 - val_loss: 0.1673 -
val_accuracy: 0.9528
Epoch 14/30
55000/55000 [==============================] - 8s 152us/sample - loss: 0.0767 - accuracy: 0.9749 - val_loss: 0.1787 -
val_accuracy: 0.9498
Epoch 15/30
55000/55000 [==============================] - 9s 159us/sample - loss: 0.0722 - accuracy: 0.9764 - val_loss: 0.1916 -
val_accuracy: 0.9472
Epoch 16/30
55000/55000 [==============================] - 8s 150us/sample - loss: 0.0662 - accuracy: 0.9780 - val_loss: 0.1772 -
val_accuracy: 0.9504
Epoch 17/30
55000/55000 [==============================] - 8s 154us/sample - loss: 0.0619 - accuracy: 0.9799 - val_loss: 0.1758 -
val_accuracy: 0.9526
Epoch 18/30
55000/55000 [==============================] - 8s 151us/sample - loss: 0.0560 - accuracy: 0.9820 - val_loss: 0.1806 -
val_accuracy: 0.9542
--------------------------
Training time in seconds:  153.65
--------------------------
Saved trained model at results/mnist_model_5_rf.h5
--------------------------
```

In [153]:
```
hist_dict = history.history
hist_dict.keys()
```

Out[153]:
```
dict_keys(['loss', 'accuracy', 'val_loss', 'val_accuracy'])
```

In [154]:
```
print(f'''acc: {hist_dict['accuracy'][-1]:.4}, val acc: {hist_dict['val_accuracy'][-1]:.4},
loss: {hist_dict['loss'][-1]:.4}, val loss: {hist_dict['val_loss'][-1]:.4}''')
```

```
acc: 0.982, val acc: 0.9542,
loss: 0.05598, val loss: 0.1806
```

Test model

Evaluate the model on the test dataset.

In [156]:
```
test_loss, test_acc = model_5_rf.evaluate(test_images_sm, test_labels)
```

```
10000/10000 [==============================] - 1s 89us/sample - loss: 0.1956 - accuracy: 0.9482
```

In [157]:
```
print(f'test accuracy: {test_acc}, test loss: {test_loss}')
```

```
test accuracy: 0.948199987411499, test loss: 0.19557065222747624
```

Comparison of the three models: "best", PCA, and Random Forest

The performance of the this newest model is surprisingly good given the dimensions were reduced from 784 to 70. Future experiments could reduce the dimensions from 784 to 154 so we can compare the effects of the `PCA` and `Random Forest Classifier` approaches to dimension reduction on the performance of our `best` model. As we noted in our analysis in `EXPERIMENT 3`, using `PCA` to reduce the number of dimensions from 784 to 154 had no negative impact on the performance of our 'best' model.

Figure 16: Learning curves for Model 5 with reduced dimensions (via Random Forest)

In [158]:
```
# Alternate code for accuracy and loss
# Plot learning curves for model
pd.DataFrame(history.history).plot(figsize=(8, 5))
plt.grid(True)
plt.gca().set_ylim(0, 1) # set the vertical range to [0-1]
plt.savefig("learning_curves_model_5_rf", tight_layout=False)
plt.show()
```