# Problem 1

**a.) Same**. $n(n+1) = n^2 + n$. $n^2 + n$ and $2000n^2$ have the same order of growth.

**b.) Lower**. $n^2$ is a lower order of growth than $n^3$ regardless of coefficient

**c.) Same**. All logarithms have the same order of growth.

**d.)Higher**. $log_2^2(n) = (log_2(n))^2$ and $log_2(n^2) = 2(log_2(n))$

**e.) Same**. $2^{n-1} = \frac{1}{2}(2^n)$ and $\frac{1}{2}(2^n)$ has the same order of magnitude as $2^n$

**f.) Lower**. $(n-1)!$ is one order of growth lower then $n!$

# Problem 2

**a.)** $C_{worst}(n) \in \theta(n)$.

$C_{worst}(n) = n$ Since there are n number of elements in the array. If the key is either the last element or not in the array, the order of execution is n.

For $C_{worst}(n) \in O(n)$. $\exists c$ such that $cn \leq n$. Let $c = 1$ $cn \leq n \rightarrow$1n $\leq n \rightarrow$n $\leq n$

$\therefore$C$_{worst} \in O(n)$

For $C_{worst}(n) \in \Omega(n)$. $\exists c$ such that $cn \geq n$. Let $c = 1$ $cn \geq n \rightarrow$1n $\geq n \rightarrow$n $\geq n$

$\therefore$C$_{worst} \in \Omega(n)$

Since $C_{worst} \in O(n)$ and $C_{worst} \in \Omega(n), C_{worse} \in \Theta(n)$

**b.)** $C_{best}(n) \in \theta(1)$.

$C_{best}(n) = 1$ Since the best case is the first element of the array.

By definition $C_{best} \in \Omega(1)$ and $C_{best} \in O(1)$

And if $C_{best} \in \Omega(1)$ and $C_{best} \in O(1)$ then $C_{best} \in \theta(1)$

**c.)** $C_{avg}(n) \in \theta(n)$

$C_{avg}(n) = \frac{p(n+1))}{2} + n(1-p)$

For $C_{avg}(n) \in O(n), c(\frac{p(n+1))}{2} + n(1-p)) \leq \frac{p(n+1))}{2} + n(1-p)$

Let $0 < c < 1$ so $c(\frac{p(n+1))}{2} + n(1-p)) \leq \frac{p(n+1))}{2} + n(1-p)$

$\therefore C_{avg} \in O(n)$

For $C_{avg}(n) \in \Omega(n), c(\frac{p(n+1))}{2} + n(1-p)) \geq \frac{p(n+1))}{2} + n(1-p)$

Let $c > 1$ so $c(\frac{p(n+1))}{2} + n(1-p)) \geq \frac{p(n+1))}{2} + n(1-p)$

$\therefore C_{avg} \in \Omega(n)$

And if $C_{avg} \in O(n)$ and $C_{avg} \in \Omega(n)$ then $C_{avg} in \Theta(n)$

# Problem 3

$5lg(n+100)^{100}$ →$\ln^2 n$ →$\sqrt[3]{n}$ →.001$n^4 + 3n^3 + 1$ →$3^n$ →$2^{2n}$ →(n-2)!

# Problem 4

**a.)** If $f(n) = n$ and $g(n) = n + sin(n)$ then $g(f(n)) = f(n) + sin(f(n))$ which means that $g(n) \leq f(n)$ $\therefore$g(n) $\in O(f(n))$

**b.)** If $f(n) = n$ and $g(n) = n|sin(n)|$ then $g(f(n)) = f(n)|sin(f(n))|$ which means that $g(n) \leq f(n)$ $\therefore$g(n) $\in O(f(n))$