

PHYS 3038 Optics

L22 Fourier Optics

Reading Material: Ch11



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2015, the Year of Light

11.3.3 Fourier Methods in Diffraction Theory

$$H(k_x, k_y, z) = e^{ik_z z} = e^{i\sqrt{k^2 - k_x^2 - k_y^2} z}$$

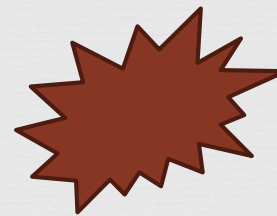


$$f(x, y, z) = \iint F(k_x, k_y) H(k_x, k_y, z) e^{i[k_x x + k_y y]} dk_x dk_y$$

$$f_0(x, y) = f(x, y, z = 0) = \iint F(k_x, k_y) e^{i[k_x x + k_y y]} dk_x dk_y$$

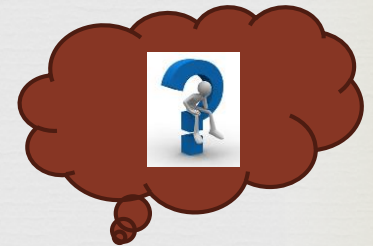
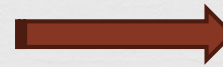
$$F(k_x, k_y) = \mathcal{F}\{f_0(x, y)\}$$

$$F(k_x, k_y) H(k_x, k_y, z) = \mathcal{F}\{f(x, y, z)\}$$



$f_0(x, y)$

$H(k_x, k_y, z)$



$f(x, y, z)$

$$\mathcal{F}^{-1}\{\mathcal{F}\{f_0(x, y)\} H(k_x, k_y, z)\}$$

Fraunhofer Diffraction

10.2.4 2D Aperture



$$E = \iint \frac{\epsilon_A}{r} e^{i(kr - \omega t)} dS \cong \frac{\epsilon_A}{R} e^{i(kR - \omega t)} \iint e^{-ik(Xx + Yy)/R} dS$$

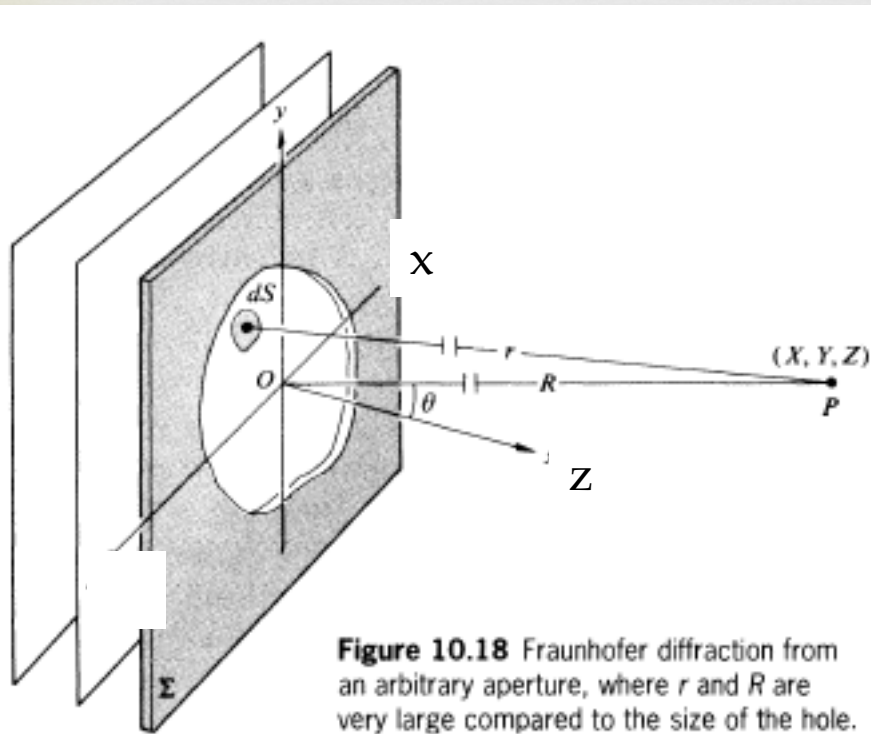


Figure 10.18 Fraunhofer diffraction from an arbitrary aperture, where r and R are very large compared to the size of the hole.

$$\begin{aligned} &\cong \frac{\epsilon_A}{R} e^{i(kR - \omega t)} \iint e^{-i(\frac{kX}{R}x + \frac{kY}{R}y)} dS \\ &\cong \frac{\epsilon_A}{R} e^{i(kR - \omega t)} \iint e^{-i(k_X x + k_Y y)} dS \\ &\cong \frac{\epsilon_A}{R} e^{i(kR - \omega t)} \mathcal{F}\{A(x, y)\} \end{aligned}$$

$$k_X = \frac{kX}{R} = k \cos \beta \qquad k_Y = \frac{kY}{R} = k \cos \gamma$$

The Single Slit



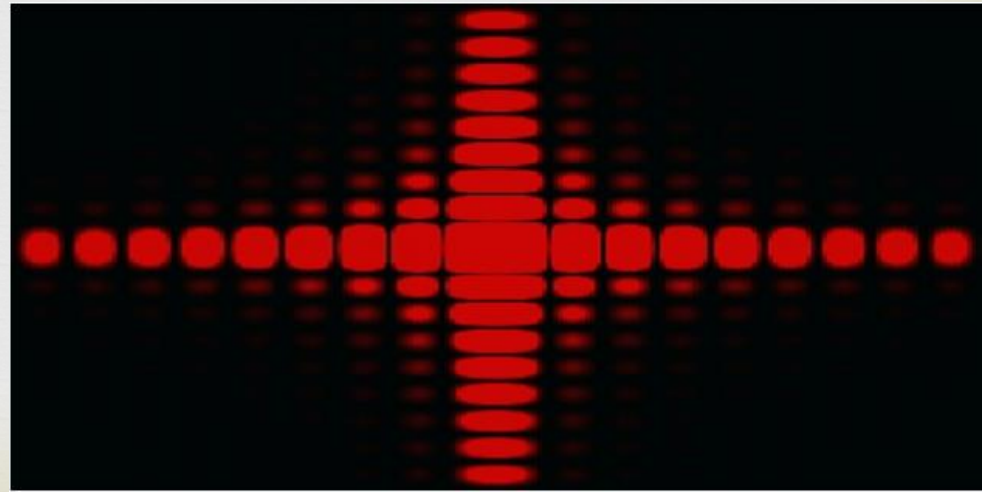
$$A(x) = \begin{cases} A_0 & |x| \leq a/2 \\ 0 & |x| > a/2 \end{cases}$$

$$k_x = \frac{kX}{R} = k \cos \beta$$

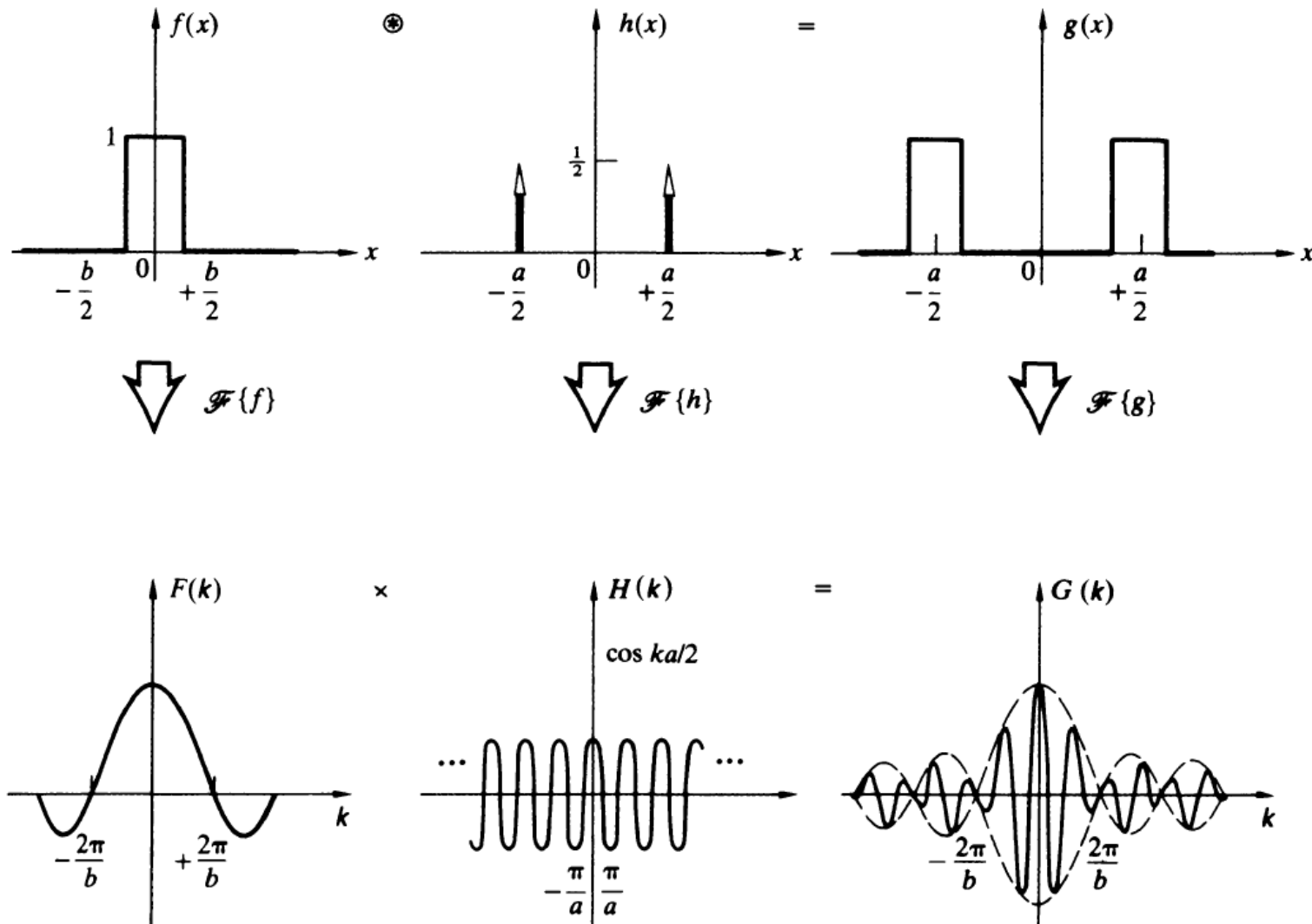
$$E(k_x) = \mathcal{F}\{A(x)\} = A_0 a \operatorname{sinc} k_x a/2$$

2D:

$$\begin{aligned} E(k_x, k_y) &= \mathcal{F}\{A(x, y)\} \\ &= A_0 ab \operatorname{sinc}\left(\frac{k_x a}{2}\right) \operatorname{sinc}\left(\frac{k_y b}{2}\right) \end{aligned}$$



Young's Double Slit



Three Slits

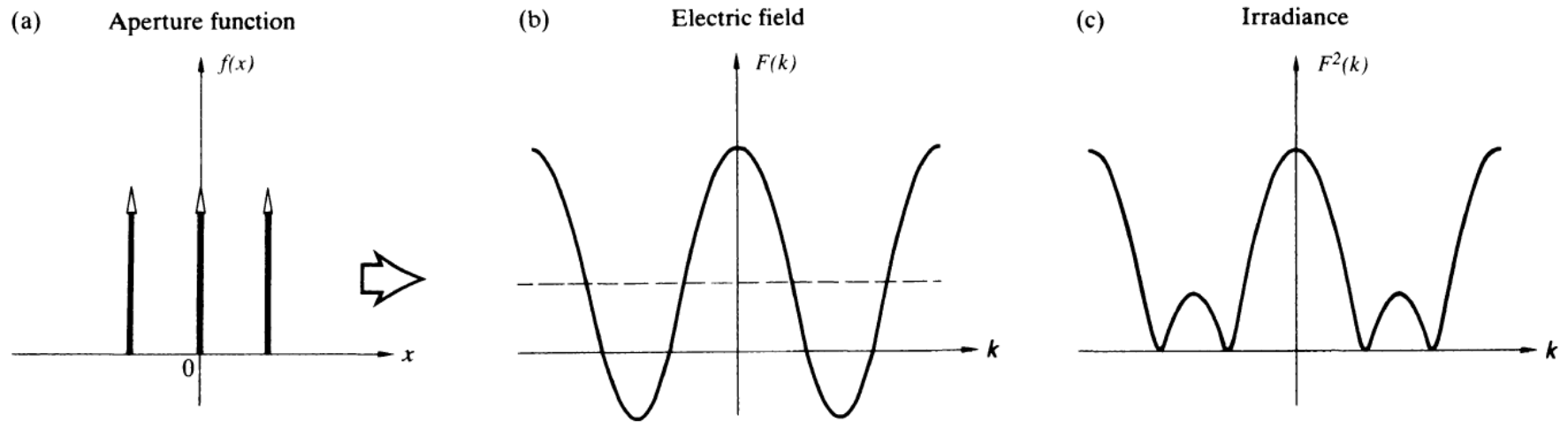


Figure 11.32 The Fourier transform of three equal δ -functions representing three slits.

11.3.5 Transfer Functions

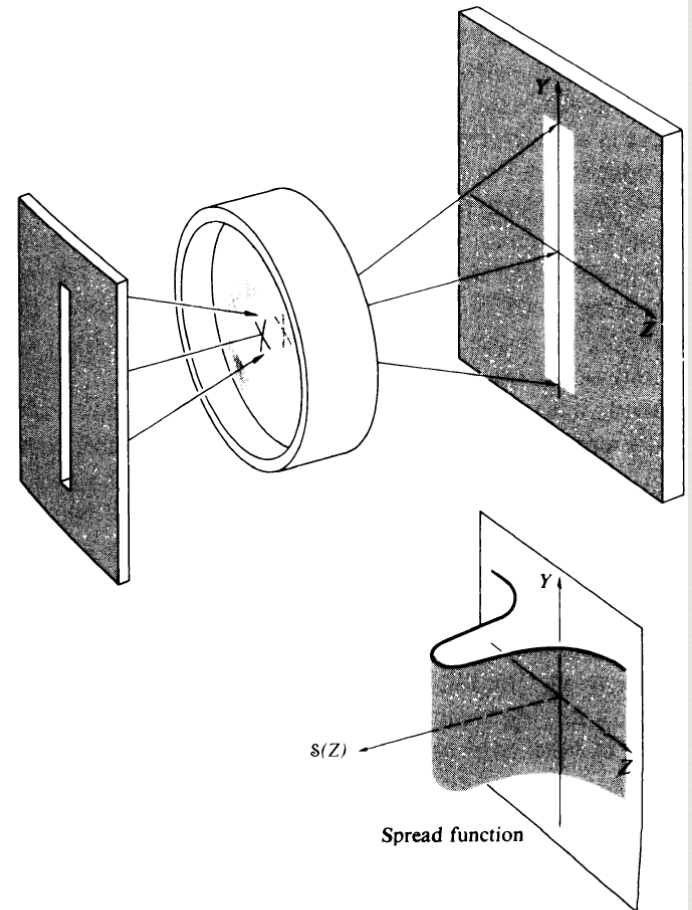
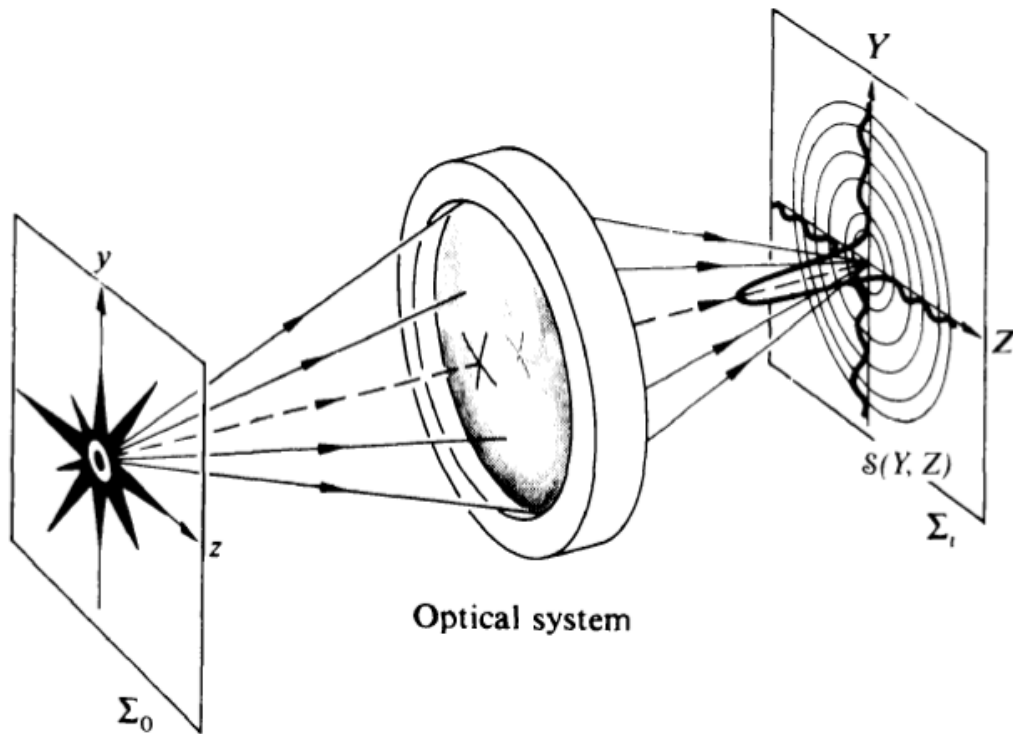


Figure 11.43 The line-spread function.

Final Review



- ❧ Ch2. Wave motion (L2)
- ❧ Ch3. EM theory (L3-L4)
- ❧ Ch5. Geometrical optics (L5-L6)
- ❧ Ch6. Geometrical optics (L7-L8)
- ❧ Ch4. Light propagation (L9)
- ❧ Ch7. Superposition (L10)
- ❧ Ch8. Polarization (L11-L12)
- ❧ Ch9. Interference (L13-L16)
- ❧ Ch10. Diffraction (L17-L20)
- ❧ Ch11. Fourier Optics (L21-L22)

Ch2 Wave Motion



- ❧ Concept of wave
- ❧ Wave equation
- ❧ Superposition principle
- ❧ Complex representation

Ch3 EM Theory



- ⌘ Maxwell equations (in vacuum/medium)
- ⌘ EM Wave (connections between wavelength, frequency, K number/vector, phase velocity...)
- ⌘ Enrgy & Momentum: Time-averaged energy, intensity (irradiance), power
- ⌘ Photon
- ⌘ Light in bulk (dielectric) matter

Ch5-6 Geometry Optics



- ❧ Ray optics
- ❧ Thin Lens & Lens combination
- ❧ Imaging with lens: How, AS, FS, Entrance & Exit pupils...
- ❧ Mirrors: Plane, Aspherical, Spherical
- ❧ Prisms, Fibers, Optical systems
- ❧ Thick lens & lens systems (focal planes, principle planes)
- ❧ Analytical ray tracing: ray vector and matrix
- ❧ Aberrations

Ch4 Light Propagation



- ⌘ Light in bulk (dielectric) matter
- ⌘ Dispersion
- ⌘ Reflection & Refraction

Ch7 Superposition



- ✧ Phasor addition
- ✧ Standing waves & beats
- ✧ Group velocity & phase velocity

Ch8 Polarization



⌘ Polarizations: linear, circular, ... (graphic +
Mathematica)

⌘ Polarizers

⌘ Retarders (wave plates)

Midterm Exam

⌘ Jones vectors & Matrix operations

Ch9 Interference



- ❧ Math & Physics
- ❧ Temporal & spatial coherence
- ❧ Wavefront-splitting interferometers
 - ❧ Young's experiment
 - ❧ ...
- ❧ Amplitude-splitting interferometers
 - ❧ Dielectric films
 - ❧ Haidinger & Newton fringes
 - ❧ Michelson interferometer
 - ❧ Mach-Zehnder Interferometer
 - ❧ Sagnac Interferometer
 - ❧ Fabry-Perot Interferometer
- ❧ Applications: coating

Ch10 Diffraction



- ❧ Fraunhofer diffraction
 - ❧ Single-slit
 - ❧ Double-slit
 - ❧ Many slits
 - ❧ 2D Apertures
- ❧ Diffraction limited resolution
- ❧ Beam propagation & diffraction
- ❧ (Quaso) Bessel beam generation
- ❧ Diffraction gratings (grating equation) & grating spectroscopy
- ❧ Fresnel Diffraction
 - ❧ Obliquiy
 - ❧ Fresnel zones
 - ❧ Circular Apertures & Obstacles
 - ❧ Fresnel zone plates

Ch11 Fourier Optics



- ❧ Fourier transform (1D & 2D)
 - ❧ FT and IFT of the standard waveforms
 - ❧ Displacements & Phase shifts
- ❧ Lens as a FT
- ❧ Free-space transform function
- ❧ Point spread function (concept)
- ❧ Fourier method for Fraunhofer diffraction