PHYS 3033 Assignment 2

Problem 1.

a) Ten equal charges, q, are situated at the corners of a regular 10-sided polygon. What is the net force on a test charge Q at the center?

b) Suppose *one* of the 10 q's is removed. What is the force on Q? Explain your reasoning carefully.

c) Now 11 equal charges, q, are placed at the corners of a regular 11-sided polygon. What is the force on a test charge Q at the center?

d) If one of the 11 q's is removed, what is the force on Q? Explain your reasoning.

Solution 1.

(a) Zero (due to cancellations of pairs of opposite charges, or due to symmetry).

(b) $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{qQ}{r^2}$ where r is the distance from the center to the charges. **F** points *toward* the missing q.

Explanation: by superposition, this is equivalent to (a), with an extra $-q$ at the position of the missing charge. Since the force of all ten is zero, the net force is that of $-q$ only.

(c) Zero (due to symmetry).

(d) $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{qQ}{r^2}$ where r is the distance from the center to the charges. **F** points *toward* the missing q. Same reason as (b).

Problem 2.

One of these is an impossible electrostatic field. Which one?

(a) $\mathbf{E} = k[3xy\hat{\mathbf{x}} + 5yz\hat{\mathbf{y}} + 7xz\hat{\mathbf{z}}]$;

(b) $\mathbf{E} = k[2y^3\hat{\mathbf{x}} + (6xy^2 + z^5)\hat{\mathbf{y}} + 5yz^4\hat{\mathbf{z}}]$.

Here k is a constant with the appropriate units.

Solution 2.

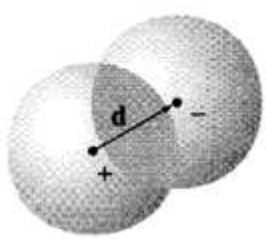
.(a) $\nabla\times\mathbf{E}=k\begin{vmatrix}\hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ 3xy & 5yz & 7xz\end{vmatrix}=k\left[\hat{\mathbf{x}}(0-5y)+\hat{\mathbf{y}}(0-7z)+\hat{\mathbf{z}}(0-3x)\right]\neq\mathbf{0}$, so it is an *impossible* electrostatic field.

(b) $\nabla\times\mathbf{E}=k\begin{vmatrix}\hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ 2y^3 & 6xy^2+z^5 & 5yz^4\end{vmatrix}=k\left[\hat{\mathbf{x}}\left(5z^4-5z^4\right)+\hat{\mathbf{y}}(0-0)+\hat{\mathbf{z}}\left(6y^2-6y^2\right)\right]=\mathbf{0}$, so it is a *possible* electrostatic field.

Problem 3.

(a) Show that the electric field *inside* a sphere with uniform charge density ρ is given by $\mathbf{E}=\dfrac{\rho\mathbf{r}}{3\varepsilon_0}$, where **r** is the vector pointing from the center of the sphere to the observation point.

(b) Two spheres, each of radius R and carrying uniform charge densities $+\rho$ and $-\rho$, respectively, are placed so that they partially overlap. Call the vector from the positive center to the negative center **d**. Show that the field in the region of overlap is constant and find its value.



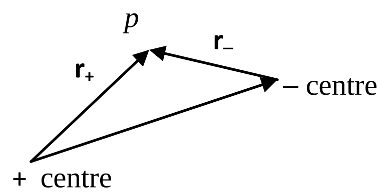
Solution 3.

(a) By Gauss's law, with a Gaussian surface centered at the origin with radius r, we have

$$E\times 4\pi r^2=\frac{Q_{\text{int}}}{\varepsilon_0}\ , \quad \text{where } Q_{\text{int}}=\frac{4\pi r^3}{3}\rho$$

So $\mathbf{E}=\dfrac{\rho\mathbf{r}}{3\varepsilon_0}$

(b)



Let p be our observation point which is inside the overlapping region· Then

$$\mathbf{E}_+ = \frac{\rho \mathbf{r}_+}{3\varepsilon_0}$$

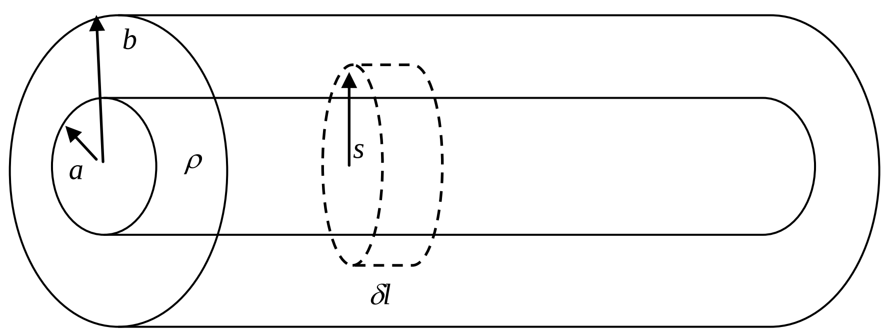
$$\mathbf{E}_- = -\frac{\rho \mathbf{r}_-}{3\varepsilon_0}$$

$$\mathbf{E} = \mathbf{E}_+ + \mathbf{E}_- = \frac{\rho}{3\varepsilon_0}\left(\mathbf{r}_+ - \mathbf{r}_-\right) = \frac{\rho \mathbf{d}}{3\varepsilon_0} \text{ (from the diagram).}$$

Problem 4.

A long coaxial cable carries a uniform *volume* charge density ρ on the inner cylinder (radius a), and a uniform *surface* charge density on the outer cylindrical shell (radius b). This surface charge is negative and of just the right magnitude so that the cable as a whole is electrically neutral. Find the electric field in each of the three regions: (i) inside the inner cylinder ($s < a$), (ii) between the cylinders ($a < s < b$), (iii) outside the cable ($s > b$). Plot $|\mathbf{E}|$ as a function of s.

Solution 5.



Consider the cross section of the cable with a tiny thickness δl

The dashed line represents our Gaussian surface

$$\oint \mathbf{E}\cdot d\mathbf{a} = E\cdot 2\pi s\cdot\delta l = \frac{1}{\varepsilon_0}Q_{enc} = \frac{1}{\varepsilon_0}\rho\left(\pi s^2\delta l\right) \quad ,s<a$$

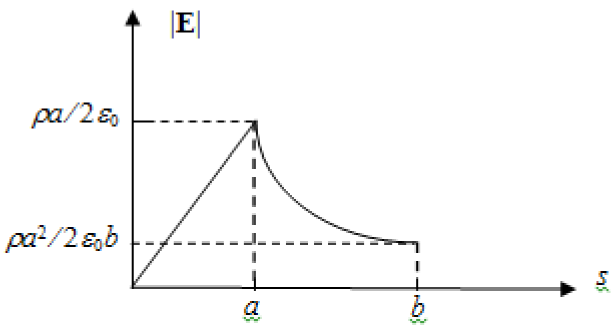
$$\Rightarrow \mathbf{E} = \frac{\rho s}{2\varepsilon_0}\hat{\mathbf{s}} \quad \text{inside the cylinder}$$

$$\oint \mathbf{E}\cdot d\mathbf{a} = E\cdot 2\pi s\cdot\delta l = \frac{1}{\varepsilon_0}Q_{enc} = \frac{1}{\varepsilon_0}\rho\left(\pi a^2\delta l\right) \quad ,a<s<b$$

$$\Rightarrow \mathbf{E} = \frac{\rho a^2}{2\varepsilon_0 s}\hat{\mathbf{s}} \quad \text{between the cylinders}$$

$$\oint \mathbf{E}\cdot d\mathbf{a} = E\cdot 2\pi s\cdot\delta l = \frac{1}{\varepsilon_0}Q_{enc} = 0 \quad ,b<s$$

$$\Rightarrow \mathbf{E} = 0 \quad \text{outside the cable}$$



Problem 5.

a) If the electric field in some region is given (in spherical coordinates) by the expression

$$\mathbf{E} = \frac{A\hat{\mathbf{r}} + Br\sin q\cos f\ \hat{f}}{r^2}$$

where A and B are constants, what is the charge density?

b) The electric field in a certain region is given by (in spherical coordinates)

$$\mathbf{E} = \frac{A\hat{\mathbf{r}} + B\sin\theta\ \hat{\phi}}{r}$$

Determine whether it is an electrostatic field.

Solution 5.

a)

$$\nabla\cdot\mathbf{E} = A\nabla\cdot\left(\frac{\hat{\mathbf{r}}}{r^2}\right) + B\nabla\cdot\left(\frac{r\sin\theta\cos\phi\ \hat{\phi}}{r^2}\right)$$

$$= A\cdot 4\pi\delta^{(3)}(\mathbf{r}) + \frac{B}{r\sin\theta}\frac{\partial}{\partial\phi}\frac{\sin\theta\cos\phi}{r}$$

$$= A\cdot 4\pi\delta^{(3)}(\mathbf{r}) + \frac{B}{r^2}\frac{\partial}{\partial\phi}\cos\phi$$

$$= A\cdot 4\pi\delta^{(3)}(\mathbf{r}) - \frac{B\sin\phi}{r^2}$$

$$\rho = \varepsilon_0\nabla\cdot\mathbf{E} = \varepsilon_0\left(A\cdot 4\pi\delta^{(3)}(\mathbf{r}) - \frac{B\sin\phi}{r^2}\right)$$

b)

$$\nabla\times\mathbf{E} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(E_\phi\sin\theta) - \frac{\partial E_\theta}{\partial\phi}\right]\hat{\mathbf{r}} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial E_r}{\partial\phi} - \frac{\partial}{\partial r}(rE_\phi)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(rE_\theta) - \frac{\partial E_r}{\partial\theta}\right]\hat{\boldsymbol{\phi}}$$

Here $E_r = \frac{A}{r}$, $E_\theta = 0$, $E_\phi = \frac{B\sin\theta}{r}$

$$\nabla\times\boldsymbol{E} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\left(\frac{B\sin\theta}{r}\sin\theta\right)\right]\hat{\boldsymbol{r}} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\phi}\frac{A}{r} - \frac{\partial}{\partial r}\left(r\frac{B\sin\theta}{r}\right)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[-\frac{\partial}{\partial\theta}\frac{A}{r}\right]\hat{\boldsymbol{\phi}}$$

$$= \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\left(\frac{B\sin^2\theta}{r}\right)\right]\hat{\boldsymbol{r}}$$

$$= \frac{1}{r^2\sin\theta}(2B\ \sin\theta\cos\theta)\hat{\boldsymbol{r}}$$

$$= \frac{1}{r^2}(2B\cos\theta)\hat{\boldsymbol{r}}$$

$$\neq 0$$

Hence it is not an electrostatic field.