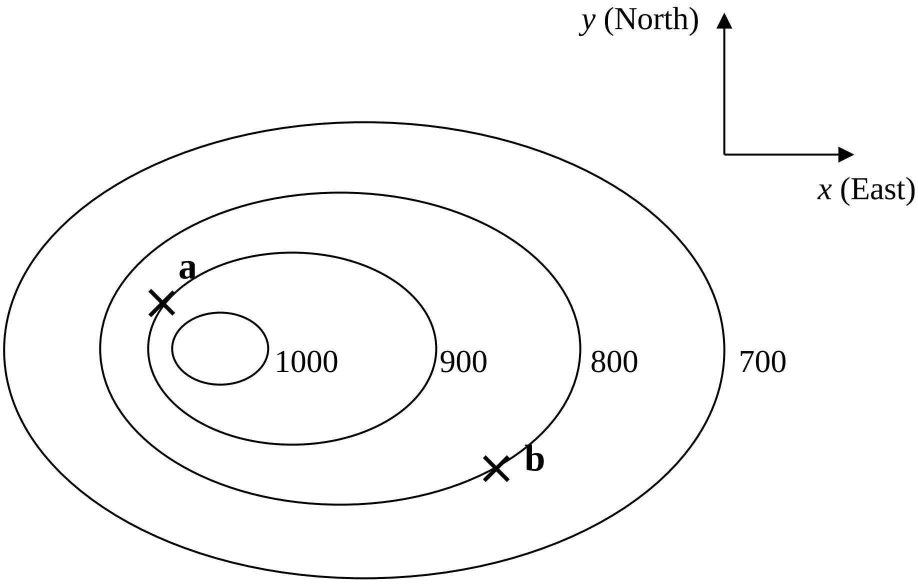
PHYS 3033/3053 Assignment 1

Due: 11 Sep 2015 at begin of lecture at 3:00 pm

1. (a) The figure below is the contour plot of the height (H) of a mountain on the horizontal (xy) plane. Sketch the gradients ∇H at point **a** and point **b**. Pay attention to both the correct directions and relative magnitudes of the gradients.



   (b) Suppose the height in (a) is measured in m, and the contour plot is drawn with a horizontal scale of 1cm ←→ 1km (when printed on A4 paper).

   Estimate $|\nabla H|$ at point **a**.

   (c) If given that $|\nabla H| = 0.05$ at point **b**, estimate ΔH when one starts at **b** and moves 100 m in the north direction.

2. Evaluate the closed line integral of the vector field $\mathbf{F} = y^2\hat{\mathbf{x}} + xy\hat{\mathbf{y}}$ along the edge of the rectangle $-1 \le x \le 1,\ \ 0 \le y \le 1$ using the Stokes' theorem. The direction of the integration is counter-clockwise.

3. Consider the vector field $\mathbf{F} = -2y\hat{\mathbf{x}}$. Compute the line integral along the upper half of the unit circle in the counter-clockwise direction by
   (a) directly carrying out the integral, and
   (b) using the Stokes' theorem.

4. Consider the vector field $\mathbf{F} = zx\hat{\mathbf{x}} + zy\hat{\mathbf{y}} - z^2\hat{\mathbf{z}}$.

   (a) Express the same vector field in spherical coordinates.

   (b) Using the form of divergence in spherical coordinates, show that $\nabla \cdot \mathbf{F} = 0$.

5. (a) Show that $\nabla\left(\frac{1}{\mathscr{r}}\right) = -\frac{\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}$. Here $\boldsymbol{\mathscr{r}} = \mathbf{r} - \mathbf{r}'$ and the gradient is taken with respect to $\mathbf{r}$, with $\mathbf{r}'$ being considered a constant vector.

   (b) Given that $\nabla \cdot \left(\frac{\hat{\mathbf{r}}}{r^2}\right) = 4\pi\delta^3(\mathbf{r})$, show that $\nabla^2 \frac{1}{\mathscr{r}} = -4\pi\delta^3(\boldsymbol{\mathscr{r}})$.