Optimizing Purchase Installments with a Brownian Motion Model

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We seek to solve for the optimal number of purchase installments, the number of monthly interest-free payments that one should take when making a purchase. While intuition says "split as much as possible" knowing that the longer you hold your money the more interest you can make off it in investments, logic also says that holding on to a debt in perpetuity can cause future issues as finances deviate.

We seek to model this risk of financial deviation. Firstly, we make the following (potentially bold) assumptions:

- All purchases through the Nu+ extension are pure expenses (not investments)
- People's risk rise porportionally with wealth
- Wealth of a client has a similar stochastic motion as the markets.

We first denote our client's wealth with $W(t)$, which note is a random function of time. Additionally, our client has a "Danger Zone" value $DZ(t)$, that as assumed above, runs porportionally to wealth. Our objective is to minimize the probability that our client's wealth gets below DZ.

We will (and perhaps this is the most naive step), assume that our client's wealth will exhibit a similar stochastic motion as the markets, that is a Brownian Motion (Geometric Brownian Motion). We do not add consiteration for sudden financial movements. While this is pretty outrageous, it isn't completely rediculous. Retirement savings, personal investments, job outlook and pay, all can be related to market movements, so we could expect micro-values to fluctuate similarly.

Note, giving positive or negative saving habits (living above or below your means) $W(t)$ will trend upwards or downwards. We will assume that while $W(t)$ may drift, $W(t) - DZ(t)$ will not, and actually can be modeled with a Brownian motion with parameter σ.

Now consider our client having c liquid cash (note time independet since happening instantaneously for our optimization), and about to purchase a product for p price (split over i installments). We then hope that $B_t(\sigma) \sim W(t) - DZ(t) > c - \frac{p}{i}$.

(Notice, by the symmetry of Brownian Motion, the distribution of the max is equal to the distribution of the min).

We know from stochastic theory, letting

$$M_t := max_{s \leq t} B(s)$$

that

$$P(M_t \geq a) = 2P(B_t \geq a)$$

We also know that:

$$P(B_t \geq a) = P(N(0, t\sigma) \geq a)$$

which allows us to solve for the probabilty that a purchase price, p, with i installments could cause bankrupcy is

$$= P(M_t \geq c - p/i) = 2P(N(t\sigma) \geq c - p/i)) = p(i)$$

where we let $p(i)$ be our probability of bankrupcy for a certain number of installments, of which we hope to maximize for our choice of i.

We can solve this numerically without too much effort. We also can solve this with respect to our time parameter t, or for our period of time where $t = i$ (where we stop worrying about minimizing bankrupcy after the purchase is completed). The fact that all potential installment levels are being compared while the user is in the same Danger-Zone financial bracket allows the resultant solution to actually be a pretty good proxy for the best number of installments.