PHYS20161 Final assignment: Doppler Spectroscopy

November 24, 2020

Doppler spectroscopy is an indirect method for detecting extrasolar planets in astronomy. As the star and planet orbit around a common centre-of-mass, the variation in the star's velocity can be measured by inspecting the variation in Doppler shift. From this, the planet's mass can be obtained.

In this assignment you are tasked with fitting the observed variation in wavelength of the light emitted from the star over time to find the mass of the exoplanet and, if possible, an uncertainty on these values. Your work should produce some graphics to support your analysis.

1 Theory

A planet in orbit around a star, of mass M_s, abides by Kepler's third law,

$$r^3 = \frac{GM_s}{4\pi^2}P^2, \tag{1}$$

which relates the distance from the star, r, (assuming circular orbits) to the period of its motion, P, where G is Newton's gravitational constant. The force of the planet on the star causes it also to orbit, albeit along a much smaller path, a diagram depicting this is shown in figure 1. If the plane of the orbit is such that the entirety of the star's motion is in the observer's line-of-sight, the movement of the star can be seen in as a varying Doppler shift in the emitted light. For classical speeds, assuming a stationary observer, the relationship between emitted (λ_0) and observed ($\lambda(t)$) wavelength is

$$\lambda(t) = \frac{c + v_s(t)}{c}\lambda_0, \tag{2}$$

where c is the speed of light and $v_s(t)$ is the star's velocity along the line-of-sight. $v_s(t)$ varies with time sinusoidally; i.e.,

$$v_s(t) = v_0 \sin(\omega t + \phi), \tag{3}$$

where v_0 is the magnitude of the star's velocity, $\omega = \frac{2\pi}{P}$, where P is the same period as in equation 1, and ϕ is an initial phase of the motion.

Once the period is known we can determine the distance between the bodies, r, from which we can find the planet's velocity,

$$v_p = \sqrt{\frac{GM_s}{r}}. \tag{4}$$

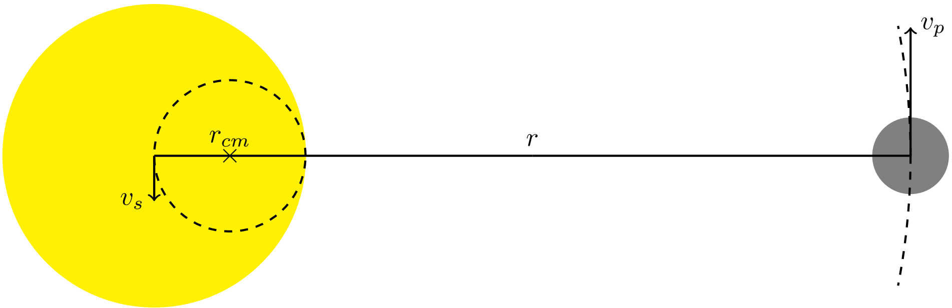

Figure 1: Schematic of star-planet system orbiting around common centre-of-mass whose normal is directed out of the page. The star, represented by a large yellow circle, orbits around the circular path shown with velocity v_s. The planet, represented by a small grey circle, follows a much longer circular path, of which only an arc is shown, with velocity v_p.

Figure 2: Schematic of orbit whose normal is at an angle θ_i from the line-of-sight of the observer.

Then, by taking moments, the planet's mass can be found,

$$m_p = \frac{M_s v_0}{v_p}. \tag{5}$$

If the plane of the orbit is subtended from the line-of-sight of the observer, then the observed velocity is reduced due to the geometry. This inclination can be taken into account in eq. 2 with a factor of $\sin\theta_i$ multiplying $v_s(t)$. We illustrate this for clarity in figure 2.

2 Project description

An experiment has taken place that observed the variation in light emitted from a star over a number of years. The data were collected by two telescopes that were in operation at different times of year. The data has had relative motion of the Earth removed. You can find the data in files `doppler_data_1.csv` and `doppler_data_2.csv`. Unfortunately there were some faults that led to not all the points being recorded properly which you must filter.

You are tasked with obtaining the measured velocities from this data, then fitting to equation 3 by varying v_0 and ω. From this, find the mass of the planet, m_p, causing this wobble quoting uncertainties on this measurement.

Do not attempt to turn the fit into a linear problem and do not try and fit the parameters separately. This will significantly overcomplicates the problem and will almost certainly return the wrong result.

The data taking commenced when the star was closest to Earth. From spectral analysis we know the emitted wavelength, λ_0, is 656.281 nm, the H-alpha Balmer emission line. Previous studies have found this star to have a mass of 2.78 solar masses and suggest $v_0 \approx 50$ m/s and $\omega \approx 3 \times 10^{-8}$ rad/s and the orbit to be along the line-of-sight from Earth.

Your programme should:

- Read in, validate, and combine both data files.
- Perform a minimised χ^2 fit by simultaneously varying ω and v_0.
- Calculate both v_0 and ω to four significant figures in m/s and rad/s respectively[1].
- Calculate m_p and r to four significant figures in Jovian masses and AU respectively[1].
- Calculate $\chi^2_{\text{red.}}$ to three decimal places[1].
- Produce a useful plot of your result.
- Ideally, you should also find the uncertainties on ω, v_0, m_p and r to the appropriate precision[1].

With regards to style, in addition to what was asked for in the previous assignment, we expect your code:

- To have a useful file check that halts the code if there is an issue.
- Read in the data files using inbuilt functions. Do not ask the user to input the file names.
- Use inbuilt functions to perform the minimisation.
- Be versatile and applicable to similar data files with similar validation issues.
- To make plots by attaching axes attributes to figure objects.
- Save any plots as a `.png` file.
- To achieve a linter score of at least 9.90/10.00 (maximum penalty of 10 marks)[2].

Additional marks are available for extra features. You do not need to include them all to get full marks for this aspect. Can you display extra information in these plots? Can you format these plots nicely? Can it be applied to systems with different line-of-sight angles, θ_i? Could it be applied to different files with *different validation issues*? Can you make the initial guess on v_0 and ω general? Could it work without knowing the initial value for ϕ?

More detail on how the mark is split can be found in the illustrative rubric on BlackBoard.

[1] When comparing to colleagues you might see minor discrepancies in the last significant figure. These are fine and accepted as correct when marking.

[2] E.g. 8.33/10.00 corresponds to a deduction of 1.57 marks and -2.40/10.00 corresponds to a deduction of 10.00 marks.