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Diffusion Effects in Drift Mobility Measurements in Semiconductors*

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Drift mobility measurements in semiconductors using small steady dc fields are described. It is necessary in such cases to take diffusion as well as drift into account in calculating the mobility from the transit time, and a calculation is made which considers both effects. If the diffusion corrections are made, one may eliminate the pulsed sweep fields which are customarily used in these experiments. Measurements of transit time *vs* sweep field show that the diffusion corrections are in agreement with experiment.

I. INTRODUCTION

THE direct measurement of drift mobility of minority carriers in semiconductors was first accomplished by Haynes and Shockley[1] who observed the drift time of minority carriers in a known electric field between an emitter and a collector electrode. Further refinements to the experimental technique were made by Prince who measured the drift mobility of minority carriers in germanium and silicon over a wide temperature range.[2,3] The experimental technique has found wide application and has come to be a more or less standard measurement technique in investigations of semiconductors. In this experiment carrier injection is initiated at an emitter electrode and the injected carriers drift toward a collector under the influence of a sweeping field E_0. If the distance between emitter and collector is d, then the transit time t_0 which elapses before the arrival of the carriers is detected at the collector may be written (in the absence of diffusion) as

$$t_0 = d/(\mu_0 E_0). \tag{1}$$

Unfortunately, it has not been possible up to the present time to obtain accurate measurements of drift mobility using the simplest form of the experiment where dc sweep voltage is used and square waves or pulses applied only to the emitter electrode. There are two difficulties encountered. First, if the dc sweep field is too high, heating effects destroy the accuracy of the measurements; and second, if the sweep field is reduced to the point where the heating effects become negligible, the effect of diffusion is so great that simple drift time calculations do not yield the correct mobility.

This latter difficulty arises because Eq. (1) is obtained by considering only drift under the influence of the field. The quantity μ_0 defined by Eq. (1) approaches the drift mobility μ only for large values of E_0. If E_0 is small, the arrival of carriers at the collector is spread out in time due to diffusion, and the effect of this diffusion must be considered to arrive at an expression for the drift mobility μ.

Haynes and Shockley recognized these difficulties and circumvented them by using very high values of sweep field and pulsing the sweep voltage to avoid heating effects.[1] This, however, complicates the circuitry and requires synchronization between emitter and sweep circuits. Furthermore, it renders potentiometric measurement of the voltage between emitter and collector impossible, and Haynes and Shockley were forced to substitute a high-impedance oscilloscopic method of measuring the potentials which were involved.

In the present work, however, the simple dc sweep circuit was retained, and measurements were made at very low sweep fields. Diffusion effects have been taken into account, and by using the results of the analysis it has been found possible to make quite accurate determinations of drift mobility using very simple experimental equipment and applying a simple correction formula to account for carrier diffusion.

II. ANALYSIS OF DIFFUSION EFFECTS

If p is the concentration of *injected* minority carriers, D the diffusion constant, μ the drift mobility, $\mathbf{E}$ the electric field, and τ the effective lifetime for these carriers, then the continuity equation for the added carriers may be written[4]

$$\nabla\cdot(D\nabla p) - \mu\mathbf{E}\cdot\nabla p - \frac{p}{\tau} = \frac{\partial p}{\partial t}. \tag{2}$$

It will be assumed that the geometry of the experiment is one dimensional, as shown in Fig. 1, that the sweeping field is applied such as to always sweep carriers in the $+x$-direction and that this field has a constant magnitude E_0. These conditions will be fulfilled if the sample is of uniform resistivity and cross section and if the cross-sectional dimension $h \ll d$, the

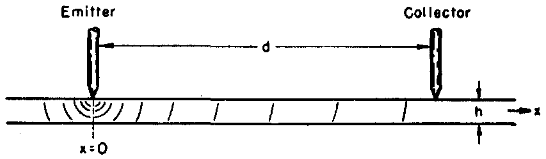

FIG. 1. Schematic representation of the Haynes-Shockley experiment.

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[1] J. R. Haynes and W. Shockley, Phys. Rev. 81, 835 (1951).
[2] M. B. Prince, Phys. Rev. 92, 681 (1953).
[3] M. B. Prince, Phys. Rev. 93, 1204 (1954).
[4] W. van Roosbroeck, Phys. Rev. 91, 282 (1953).

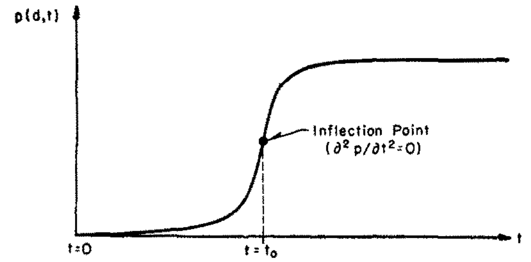

FIG. 2. Typical form of the arrival curve for the Haynes-Shockely experiment showing transit time defined by Eq. (8).

distance between emitter and collector. In addition, it will be assumed that the ends of the sample are sufficiently far from the electrodes that end effects may be neglected and the sample treated as an infinite rod in the $+$ and $-x$-directions. Under these conditions, Eq. (2) may be written in the form

$$D\frac{\partial^2 p}{\partial x^2}-\mu E_0\frac{\partial p}{\partial x}-\frac{p}{\tau}=\frac{\partial p}{\partial t}. \tag{3}$$

Consider now the following situation: a number N minority carriers are injected instantaneously at $t=0$ at the origin. These diffuse and drift according to Eq. (3), and a function which is a solution of Eq. (3) which satisfies these boundary conditions is found to be

$$p_0(x,t)=\frac{N}{(4\pi Dt)^{\frac{1}{2}}}\exp\left[-\frac{(x-\mu E_0 t)^2}{4Dt}-\frac{t}{\tau}\right]. \tag{4}$$

If instead a number of carriers dN are injected in an interval dt about $t=0$ and this rate of injection maintained for a time t, the resulting concentration as a function of x at time t will be given as a superposition of solutions of the form Eq. (4) summed over the injection time interval. This procedure is formally the same as the integration of a Green's function over a spatial source distribution; the boundary conditions are incorporated in Eq. (4) from the outset. The solution is found to be

$$\begin{aligned}p(x,t)&=\int_0^t p_0(x,t')dt'\\&=N\int_0^t\frac{1}{(4\pi Dt')^{\frac{1}{2}}}\exp\left[-\frac{(x-\mu E_0 t')^2}{4Dt'}-\frac{t'}{\tau}\right]dt'. \end{aligned}\tag{5}$$

In this equation N represents the number of carriers injected per second. Setting $x=d$ in Eq. (5), expanding the quadratic term in the exponent, and simplifying, using Einstein's relation[5] to relate D and μ one obtains finally for the injected carrier concentration at the collector as a function of time

$$p(d,t)=N_0\int_0^t\frac{1}{(4\pi Dt')^{\frac{1}{2}}}\exp\left[-\frac{d^2}{4Dt'}-\alpha t'\right]dt' \tag{6}$$

[5] Transistor Teachers Summer School, BTL, Phys. Rev. **88**, 1368 (1952).

where

$$\alpha=\frac{\mu e_0 E_0^2}{4kT}+\frac{1}{\tau} \tag{7}$$

and where $N_0(=N\exp[e_0E_0d/2kT])$ is a factor constant with respect to time. This function Eq. (6) is the "arrival signal" of the Haynes-Shockley experiment. It has, qualitatively, the properties observed in practice as shown in Fig. 2.

Most experimenters take as the transit time the time interval required for the amplitude of the arrival function to rise to half of its maximum value. It has been found that a somewhat different definition of transit time facilitates these calculations and in addition is well suited to measurement experimentally. Therefore, an analytical definition of the transit time will be adopted and used for the purposes of calculation and experiment. The transit time is defined as the interval between onset of injection ($t=0$) and the time when the inflection point of the arrival curve occurs. Thus the transit time is defined by

$$\left(\frac{\partial^2 p}{\partial t^2}\right)_{t=t_0}=0. \tag{8}$$

This definition allows one to arrive at exact analytic expressions for transit time and drift mobility, since when Eq. (6) is differentiated with respect to t, the integral sign vanishes. When the second derivative is taken and set equal to zero, furthermore, the exponential factor also drops out. Performing these operations and solving for t_0, it is found that

$$t_0=\frac{1}{4\alpha}\left[\left(1+\frac{4\alpha d^2}{D}\right)^{\frac{1}{2}}-1\right]. \tag{9}$$

For $d=0.5$ cm, $D=45$ cm²/sec, $\tau=200\,\mu$sec, $E_0=0$, one finds $t_0=475\,\mu$sec. A diffusion contribution of this magnitude cannot be disregarded when making accurate measurements at low values of E_0. It can easily be shown that Eq. (9) reduces to Eq. (1) for large values of E_0.

Substituting the value of α from Eq. (7) into Eq. (9), expressing D in terms of μ with the Einstein relation, and solving for μ, one obtains

$$\mu=\mu_0((1+x^2)^{\frac{1}{2}}-x) \tag{10}$$

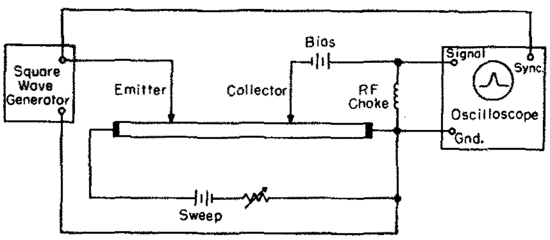

FIG. 3. Block diagram of the circuitry involved in the simplified Haynes-Shockley experiment.

where

$$x=\frac{2kT}{e_0E_0d}\left(\frac{t_0}{\tau}+\frac{1}{2}\right) \tag{11}$$

and where μ_0 is the drift mobility computed from the transit time by Eq. (1) neglecting diffusion. It is clear that the actual drift mobility μ differs from μ_0 by a factor involving E_0 and τ which may be large if E_0 or τ is small. One must know τ in order to compute the correct value of μ, but this is a parameter which is easily measured independently.[6-8] It is in principle possible to measure τ and μ both from the Haynes-Shockley experiment by making transit time measurements for different values of E_0, but the form of the equations which must be employed is such that the measurements of τ obtained in this manner are not very precise. In actual experimental practice the value of x may be as large as 0.15, and experimental values of μ usually differ from μ_0 by 5% to 8%.

III. EXPERIMENTAL RESULTS

Experimentally, the transit time is found to vary with field in agreement with the predictions of Eq. (9). When making measurements of drift mobility the effect of diffusion is to increase the value of μ_0 computed from the transit time by Eq. (1) as E_0 decreases. In the limit of zero field, of course, Eq. (1) predicts that a finite transit time corresponds to an infinite value of μ_0. The value of μ computed from Eq. (10), of course, should not vary with the field.

The apparatus used to take experimental data is quite simple, and is shown in Fig. 3. The carriers are injected by a square wave applied to the emitter, the time interval being measured by the oscilloscope sweep, which is calibrated against a crystal-controlled marker system. The signal voltage is taken across the *RF* choke, and represents the time derivative of the collector current and thus the time derivative of the arrival curve. The maximum of the time derivative, and thus of the signal pip, represents the condition that the second derivative of the arrival curve be zero, and the time measured from the initiation of the sweep to the maximum of the signal is the transit time defined by Eq. (8). A similar observation is made on the departure curve, where, of course, the time interval to the minimum of the observed signal is measured. It can be shown that the results of the diffusion calculation made above on the arrival of carriers apply to the departure of carriers as well. A minimal value of emitter current is used for injection; arrival and departure intervals are equal within a few percent. The field is measured by measuring the voltage between the electrodes with a potentiometer, the bias voltages having been removed. The distance between probes is measured accurately with a measuring microscope.

[6] J. R. Haynes and J. A. Hornbeck, Phys. Rev. **90**, 152 (1953).
[7] Navon, Bray, and Fan, Proc. Inst. Radio Engrs. **40**, 1342 (1952).
[8] J. P. McKelvey and R. L. Longini, J. Appl. Phys. **25**, 634 (1954).

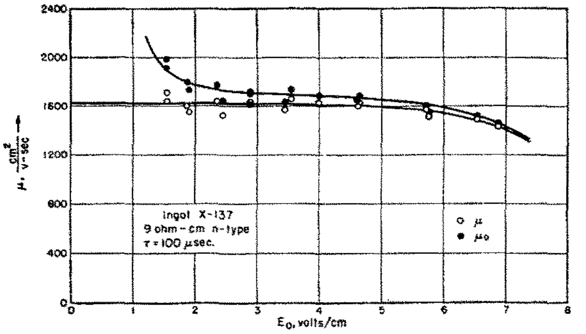

FIG. 4. The drift mobility μ from Eq. (10) and the quantity μ_0 defined by Eq. (1) plotted *vs* the sweep field E_0 for a typical specimen of *n*-type germanium. Note the field dependence of μ_0 and the convergence of μ and μ_0 for larger values of E_0.

Figure 4 shows a typical set of measurements of mobility *vs* field. Note that at low fields the value μ_0 calculated from Eq. (1) rises sharply while the corrected mobility μ remains constant with field. The drop of mobility at higher values of fields is attributed to heating of the sample by the sweep current, since approximate calculations indicate that the power input in such cases is large enough to cause considerable rise in temperature. This causes a drop in mobility due to the change in temperature and in some cases an additional drop due to the fact that above room temperature the measured pulse mobility may depart from the true carrier mobility.[9] In the example shown here the measured mobility is 1613 ± 38 cm^2 volt^{-1} sec^{-1} (an experimental uncertainty of $\pm2.4\%$) but had diffusion effects been ignored, the measured average value would have been 7.6% higher. It is believed that careful measurements made under the experimental conditions described in the foregoing and using the corrections derived herein will yield values of drift mobility accurate to within about $\pm2\%$ on germanium samples.

It is evident from the analysis of the previous section that if diffusion effects are to be avoided rather than corrected for, the field must be kept reasonably large, which of course usually necessitates pulsing to avoid heating. If the diffusion effects are to affect the mobility μ_0 calculated from Eq. (1) by less than 2%, then the parameter x in Eq. (10) must be smaller than about 0.02. This condition can easily be shown to require in turn that the voltage V_0 between the emitter and collector probes satisfy the relation

$$V_0>2.6\left(\frac{t_0}{\tau}+\frac{1}{2}\right) \tag{12}$$

where V_0 is measured in volts and $T=300$°K.

IV. ACKNOWLEDGMENTS

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[9] M. B. Prince, Phys. Rev. **91**, 271 (1953).