**SF2822 Applied Nonlinear Optimization, 2017/2018**
**Project assignment 1A**
**Due Wednesday April 25 2018 23.59**

Discussion between the groups is encouraged, but each group must individually solve the assignments. It is *not* allowed to use solutions made by others in any form. Please see the course web page for more detailed information on the rules for the assignments.
Instructions on how to present the project assignments can be found at the course web page.
The exercises are divided into basic exercises and advanced exercises. Sufficient treatment of the basic exercises gives a passing grade. Inclusion of the advanced exercises is necessary for the higher grades (typically A-C). A member of a group who has not worked on the advanced exercises says so in the self assessment form.
Supplementary instructions:

- The report should have a leading title page where the project name and the group members' names, personal number and e-mail addresses are clearly stated.
- The report should be written using a suitable word processor.
- The contents should be such that another student in the course, who is not familiar with the project, should be able to read the report and easily understand:
    1. What is the problem? What it the problem background? This does *not* mean a copy of the project description, but rather a suitable summary of necessary information needed in order to understand the problem statement.
    2. How has the group chosen to formulate the problem mathematically? What assumptions have been made? If these assumptions affect the solution, this should be noted.
    3. What is the meaning of constraints, variables and objective function in the mathematical formulation?
    4. What is the solution of the formulated optimization problem? If suitable, refer the mathematical solution to the terminology of the (non-mathematical) problem formulation. (There could be more than one optimization problem.)
- Most project descriptions contain a number of questions to be answered in the report. The report *must* contain the answers to these questions. They should, however, in a natural way be part of the content of the report and not be given in a "list of answers". The purpose of the questions is to suggest suitable issues to consider in the part of the report where the results are interpreted and analyzed. Additional interpretations are encouraged as well as generalizations and other ways of modeling the problem.
- A suggested outline of the report is as follows:
    1. Possibly a short abstract.
    2. Problem description and background information.
    3. Mathematical formulation.
    4. Results and analysis (interpretation of results).
    5. A concluding section with summary and conclusions.

Deviations from the outline can of course be done.

- GAMS code should not be part of the report, and should not be referred to in the report.
- Each group should upload the following documents via the Canvas page of the course no later than by the deadline of the assignment:
  - The report as a pdf file.
  - GAMS files.

  Please upload your documents as individual pdf and gms files, and not as zip files.
- Each student should fill out a paper copy of the self assessment form and hand in at the beginning of the presentation lecture.

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The optimal power flow problem, OPF, is a very useful mathematical optimization tool in the field of electric power systems operation and planning. The objective function of OPF can be e.g. minimizing transmission losses, minimizing the power production cost or maximizing system security margins. The constraints of this problem are e.g. bounds on power generation, bounds voltage magnitudes and phase angles and physical laws of power transmission.
Generally in power systems analysis a per-unit system is used in order to limit numerical errors. This implies that all system quantities are given as fractions of some base unit quantity. All system values in this project are given as per-unit values.
Assume that we have a small-scale power transmission system with four nodes $\{1, 2, 3, 4\}$ connected by the links $(1, 2), (1, 3), (2, 4), (3, 4)$. Assume further that there are two generator, G1 and G2, located at node 1 and four generators, G3-G6, at node 2. The operational cost and maximum capacity of these generators, in power units (pu), are given in the following table:

| Generator | Variable cost [SEK/pu/h] | Maximum capacity [pu] |
|---|---|---|
| G1 | 100 | 0.4 |
| G2 | 200 | 0.4 |
| G3 | 100 | 0.5 |
| G4 | 200 | 0.6 |
| G5 | 300 | 0.7 |
| G6 | 400 | 0.8 |

We assume a base of 100 MVA, which means that the maximum capacities of the generators are $0.4 \times 100 = 40$ MW, $0.5 \times 100 = 50$ MW, *etc.*
In a power transmission system one has to consider both active and reactive power. When it comes to power generation, a generator can generate a non-negative amount of active power up to its capacity. For the reactive power we assume that the generator can either generate or absorb some amount of reactive power, this amount can be in the range of minus the capacity and plus the capacity of the generator. Moreover, we assume that nothing else in the system can absorb reactive power except the generators.
In each node $i$ there is a demand of active power given in the following table:

| Node | Demand active power [pu] |
|---|---|
| 1 | 0.78 |
| 2 | 0.50 |
| 3 | 0.14 |
| 4 | 0.89 |

The physical laws of power transmission in a link is given by

$$P_{km} = U_k^2 g_{km} - U_k U_m g_{km} \cos(\theta_k - \theta_m) - U_k U_m b_{km} \sin(\theta_k - \theta_m)$$

for the active power transmission through link $(k, m)$ and

$$Q_{km} = -U_k^2 b_{km} + U_k U_m b_{km} \cos(\theta_k - \theta_m) - U_k U_m g_{km} \sin(\theta_k - \theta_m)$$

for the reactive power transmission through link $(k, m)$. The voltage amplitude at node $k$ is denoted by $U_k$, and these variables should be in the range of 0.9 and 1.1 pu. The voltage phase angles $\theta_k$ should be in the range of $-\pi$ and $\pi$. The parameters $b_{km}$ and $g_{km}$ are known for each link $(k, m)$ in the system according to the following table:

| Link | $g_{km}$ | $b_{km}$ |
|---|---|---|
| (1,2) | 2.65 | -20.67 |
| (1,3) | 3.04 | -23.53 |
| (2,4) | 1.46 | -11.37 |
| (3,4) | 3.25 | -25.29 |

The parameter values are given for the link irrespective of the direction, so that $g_{km} = g_{mk}$ and $b_{km} = b_{mk}$. There should be a flow-balance of both active and reactive power flow at each node. Note also that there are losses, both real and reactive, in a transmission line so generally $P_{km} \neq -P_{mk}$ and $Q_{km} \neq -Q_{mk}$.

## Basic exercises

**1.** Formulate the above power flow problem as an optimization problem that minimizes the power production cost per hour of the system.
*Remark 1*: Note that only the active power generated should be included in the objective function.
*Remark 2*: Note that the transmission line flow is not equal in both directions since there are losses in the transmission lines. Also note that the active and reactive flow in the same link can be in different directions.

**2.** Create a model in GAMS and solve the problem.

**3.** Is the optimization problem you formulated a convex problem or not? Explain.

**4.** If it was possible to increase the capacity of one generator with 0.1 pu, which generator(s) would be the most beneficial to choose. (Answer the question without running the program again)

# Advanced exercises

**5.** There is a useful approximation of the OPF in which one disregards the reactive power all together and in addition one assumes that

- $U_k = 1$ for all nodes.
- The phase angle difference, $\theta_k - \theta_m$, is small enough such that $\sin(\theta_k - \theta_m) \approx \theta_k - \theta_m$ and $\cos(\theta_k - \theta_m) \approx 1$ for all links.

Based on your result in the basic exercises is this approximation reasonable?

**6.** Answer questions 1–3 again for this approximation of OPF.

**7.** What is the interpretation of the dual variables associated with the power flow balance constraint at each node? Compare the approximate and the nonlinear models.

**8.** Investigate the effect of imposing bounds on the active power transmission in the links.

**9.** Can you relate the approximate problem to the nonlinear one and obtain bounds on the optimal value of the nonlinear problem?

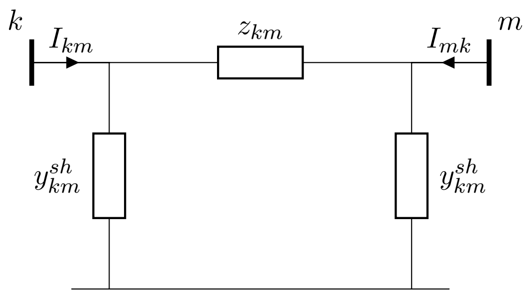
*Good luck!*

# Acknowledgement

This project assignment has been created by Tove Odland (Optimization and Systems Theory, Department of Mathematics, KTH) and Egill Tómasson (Electric Power and Energy Systems, School of Electrical Engineering, KTH).

# Appendix A

In this appendix, the power flow equations are derived. This is for the interested reader, they are not needed for the project assignment.
Figure 1 shows a $\pi$-model of an AC transmission line between nodes $k$ and $m$. The current




Figur 1: $\pi$-model of a transmission line.

$I_{km}$, going from node $k$ is composed of two parts; the part flowing through the capacitive shunt $y_{km}^{sh} \approx jb_{km}^{sh}$ to ground and the part flowing through the impedance $z_{km}$. This current is given by

$$I_{km} = y_{km}(E_k - E_m) + jb_{km}^{sh} E_k$$

where $E_k$ and $E_m$ are the complex voltages of the two nodes and $y_{km}$ is the admittance given by

$$\begin{aligned} y_{km} &= \frac{1}{z_{km}} = \frac{1}{r_{km} + jx_{km}} \\ &= g_{km} + jb_{km} \end{aligned}$$

where

$$\begin{aligned} g_{km} &= \frac{r_{km}}{r_{km}^2 + x_{km}^2} \\ b_{km} &= \frac{-x_{km}}{r_{km}^2 + x_{km}^2} \ . \end{aligned}$$

The complex power $S_{km}$, flowing from node $k$, is given by

$$\begin{aligned} S_{km} &= E_k I_{km}^* \\ &= y_{km}^* U_k e^{j\theta_k}(U_k e^{-j\theta_k} - U_m e^{-j\theta_m}) - jb_{km}^{sh} U_k^2 \\ &= P_{km} + jQ_{km} \end{aligned}$$

where $U_k$ denotes the amplitude of the voltage at node $k$ and $\theta_k$ denotes the voltage phase angle at node $k$. The complex power can be separated into real power flow $P_{km}$ and reactive power flow $Q_{km}$ as follows

$$\begin{aligned} P_{km} &= U_k^2 g_{km} - U_k U_m g_{km} \cos(\theta_k - \theta_m) - U_k U_m b_{km} \sin(\theta_k - \theta_m), \\ Q_{km} &= -U_k^2 (b_{km} + b_{km}^{sh}) + U_k U_m b_{km} \cos(\theta_k - \theta_m) - U_k U_m g_{km} \sin(\theta_k - \theta_m). \end{aligned}$$

For relatively short transmission lines ($< 80\,\mathrm{km}$), the capacitive shunt $y_{km}^{sh}$ can be neglected which gives the power flow equations of this project assignment.

# Appendix B

Reactive power is a difficult concept. For the interested reader, the following comment has been given by Egil Tómasson:

*The transmission line is represented by an impedance which is a circuit element composed of a "resistance" and a "reactance". There will be losses in both of those elements. The resistance losses can be thought of as "heat loss" but the reactance losses are more imaginary. Both $q_{ij}$ and $q_{ji}$ can therefore be positive, since they are both contributing to those reactive losses.*

*Regarding reactive power in general, it is a hard concept to grasp. Attached is a figure that explains the concept in terms of beer and foam, perhaps not a perfect explanation but an attempt:*

*The beer is what you want (real power) but nevertheless you have to take care of the foam (reactive power). If you are not careful, for example pour the beer too quickly, you will have your foam out of control and your precious beer will spill. The same is true for reactive power, it is somehow a by-product that you have to take care of or else you will have a real mess on your hands; voltage out of control which eventually results in a blackout.*