EL2520 – Control Theory and Practice

Classical Loop-Shaping

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Abstract

In this report, we consider the basic classical loop-shaping procedure for control design of a single input single output system.

Basics

A system is modeled by the transfer function (given in [1])

$$G(s) = \frac{3(-s+1)}{(5s+1)(10s+1)} \qquad (1)$$

We will design a lead-lag compensator F such that the closed loop system in Figure 1 fulfills the following specification:

- Crossover frequency $\omega_c = 0.4\,rad/s$.
- Phase margin $\varphi_m = 30^{\circ}$.
- No stationary error for a step response.

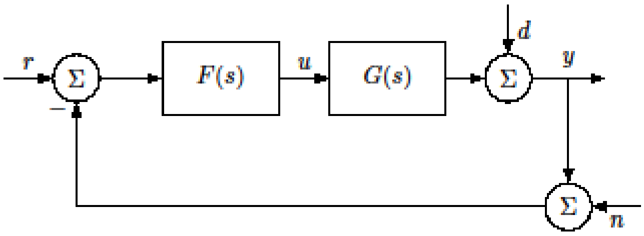

Figure 1: Closed loop block diagram, where F–controller, G–system, r–reference signal, u–control signal,

We follow the procedure from [2] to determine the parameters K, β, τ_I, τ_D, and γ in the lead-lag compensator

$$F(s) = K\frac{(\tau_D s+1)(\tau_I s+1)}{(\beta\tau_D s+1)(\tau_I s+\gamma)} \qquad (2)$$

The system's phase at $\omega_c = 0.4$ is, $arg\big(G(i\omega c)\big) = 18.8°$, is determined with Matlab. The Bode diagram of the system is showed in Figure 2.

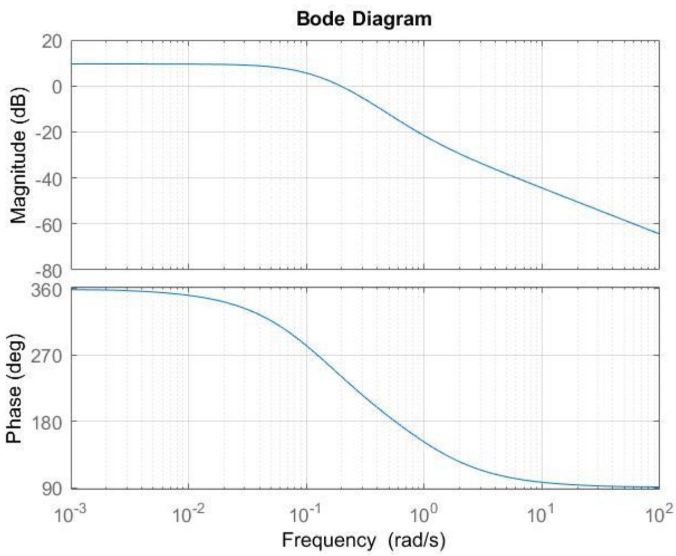

Figure 2: Bode diagram for system G(s) in (1).

Thus, the necessary phase shift is

$$30° - 18.8 + 6° = 17.2°$$

where an extra 6° has been added to account for the lag-part. The first parameter can now be selected from the course book. If we approximate the phase advancement to be around 20° as $\beta = 1/2 = 0.5$. With this data, we can calculate the corresponding φ_{max} and τ_D.

$$\tau_D = \frac{1}{\omega_c\sqrt{\beta}} = 3.54$$

Then we shall get the lead controller:

$$F_{lead} = \frac{3.54s + 1}{1.77s + 1}$$

Thus, at the desired crossover frequency ω_c, we can calculate $|F_{lead}|$.

$$|F_{lead}| = \left|\frac{3.54 \times 0.4i + 1}{1.77 \times 0.4i + 1}\right| = 1.41$$

To get the corresponding value of K we use the following equation since $K|G(i\omega_c)||F_{lead}(i\omega_c)| = 1$.

$$K = \frac{1}{|G(i\omega_c)||F_{lead}(i\omega_c)|} \Rightarrow K = \frac{1}{0.35 \times 1.41} = 2.03$$

With these data, we can continue to the designing of the lag controller.

$$F_{lag} = \frac{\tau_I s + 1}{\tau_I s + \gamma}$$

Here we want to make the static error vanish, so we shall let $\gamma = 0$. To use the compensate phase angle 6°, we can choose $\frac{1}{\tau_I} = 0.1\omega_c$ and thus τ_I shall be the following.

$$\tau_I = \frac{1}{0.1 \times \omega_c} = 25$$

So, we will get the following parameters.

Table 1: Parameters for the lead-lag compensator

K	β	τ_I	τ_D	Γ
2.03	0.5	25	3.54	0

The final controller is given by eq. (2) with the parameters in Table 1.

$$F(s) = 2.03\frac{(3.54s + 1)(25s + 1)}{(1.77s + 1)25s}$$

The rise time and overshoot are determined form the step response in Figure 3, and given in table 2. We can also get the values with the Matlab function $stepinfo$.

To make a comparison, we shall also compute the corresponding values for the system without the controller. These are given in the Table 3.

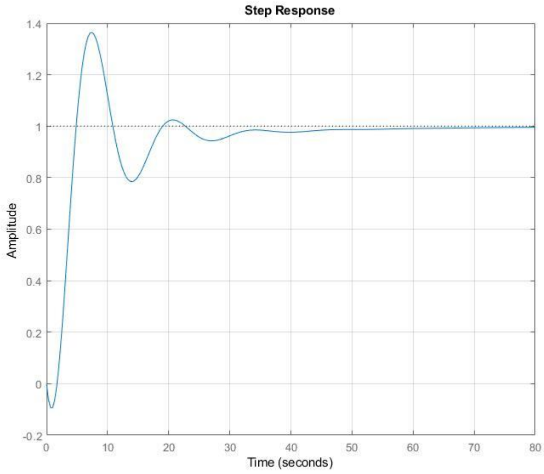

Figure 3: Step response for the closed loop system in fig. 1, with the lead-lag compensator.

Table 2: Characteristics of the closed loop system.

Bandwidth	M_T	T_r [s]	M [%]
0.7844	1.936	2.389	36.385

If we let the phase margin increases to $50°$ without change the crossover frequency we can perform the same procedure with the same method. We shall get the following results:

The phase needs to be advanced is around $50° - 18.8° + 6° = 37.2°$. So $\beta = \frac{1}{5.8} = 0.17$. $\tau_D = \frac{1}{\omega_c\sqrt{\beta}} = 6.06$. $|F_{lead}(i\omega_c)| = 2.42 \Rightarrow K = \frac{1}{|F_{lead}(i\omega_c)||G(i\omega_c)|} = 1.18$. The lag part is the same as the previous one.

Table 3: Parameters for the lead-lag compensator

K	β	τ_I	τ_D	Γ
1.176	0.017	25	6.06	0

So, the lead-lag controller shall be the following:

$$F(s) = 1.176\frac{(6.06s + 1)(25s + 1)}{(1.03s + 1)25s}$$

The characteristics of this closed loop system is given in the following.

Table 4: Characteristics of the closed loop system.

Bandwidth	M_T	T_r [s]	M [%]
1.0044	1.145	2.455	2.805

Disturbance attenuation

In this part of the report we want to construct a controller which both tracks the reference signal and attenuates disturbances. We consider the following system that has been showed in the block diagram.

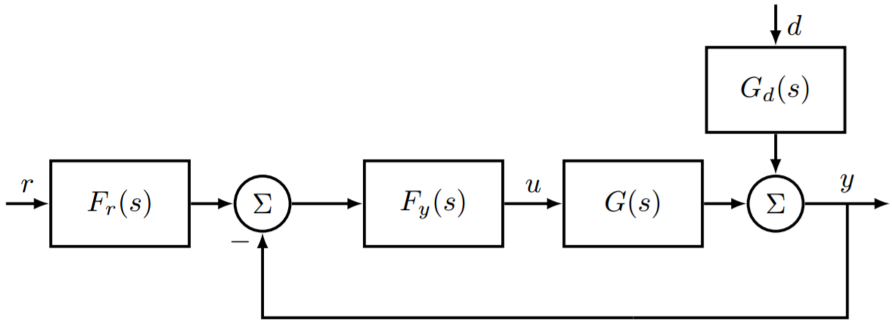

Figure 4: Closed loop block diagram with disturbances, F_r-prefilter, F_y-feedback controller, G-system, G_d-disturbance dynamics, r-reference signal, u-control signal, d-disturbance signal, y-measurement signal

We want to design the two controllers F_r and F_y so we can archive the following specifications:

- Rise time of a step of the reference T_r less than $0.2s$.
- Step responds of a step of the reference has overshoot less than 10%.
- The step responds of a step of disturbance $|y(t)| \leq 1 \ \forall t$ and $|y(t)| \leq 0.1 \ \forall t > 0.5s$.
- The step responds of a step of disturbance has $|u(t)| \leq 1 \ \forall t$.

The transfer function is given to:

$$G(s) = \frac{20}{(s+1)\left(\left(\frac{s}{20}\right)^2 + \frac{s}{20} + 1\right)} \ and \ G_d(s) = \frac{10}{s+1}$$

According to the block-diagram we can find the following transfer function from disturbance d to output y.

$$T_d d = \frac{G_d}{1 + F_y G} d = y$$

The disturbance d needs to be attenuated at least when $|G_d(j\omega)| > 1$ which corresponds to $\frac{100}{\omega^2+1} > 1 \Rightarrow \omega^2 < 99 \Rightarrow \omega < 9.95 = \omega_c$. Thus, we start with letting $F_y = G^{-1}\omega_c/s$. However, this controller is not proper because the order of its numerator is 2 higher than its denominator. So, two additional poles are needed. Because we want the controller F_y to approximate $F_y = G^{-1}\omega_c/s$ for $\omega \in [0,9.95]$ so the added poles should be far away from these values and a proportional constant shall be added to compensate the poles. So, we add a double pole at $-10\omega_c$. To compensate this, the proportional constant shall be $10\omega_c \times 10\omega_c = 100\omega_c^2$. The resulting controller has the following form:

$$F_y = G^{-1}\frac{\omega_c}{s}\frac{100\omega_c^2}{(s+10\omega_c)^2}$$

With this controller, the following results are achieved.

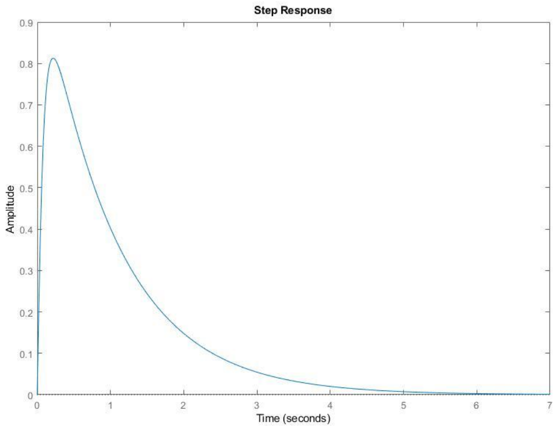

Figure 5: Step response of disturbance

However, this controller is not fast enough since the loop gain has slope of -1 at all frequencies. We would love to have an integral action on the controller. A start point of this kind of controller has the form:

$$F_y = \frac{s + \omega_I}{s} G^{-1} G_d$$

Here ω_I denote the frequency region of effective integration. This controller is not proper neither and the same poles from previous question can be used here. ω_I can be chosen to be $\omega_c/2$ to make the system faster meanwhile reduce oscillation. So, the controller shall be:

$$F_y = G^{-1} G_d \frac{s + \frac{\omega_c}{2}}{s} \frac{100\omega_c^2}{(s + 10\omega_c)^2}$$

The resulting step response is plotted below.

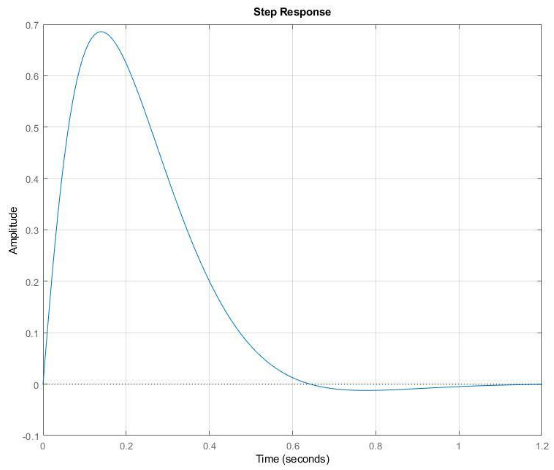

Figure 6: Step response of disturbance

By looking at the graph, we see that the disturbance is attenuated. By zoom in, we can see that we have: $|y(t)| \leq 1 \;\; \forall t$ and $|y(t)| \leq 0.1 \;\; \forall t > 0.5s$.

Now we shall try to fulfill the specification on the reference signal. Without any change, we receive the following step response. Its rise time is $0.1062s$ and its overshoot is 19.08%. We see that the rise time specification is fulfilled however the overshoot is too large.

This can be compensated by a lead controller. Beside the overshoot, the control signal needs to have a norm smaller than 1. From the block diagram, the transfer function of u can be derived: $u = F_y F_r S r - F_y G_d S d$.

Since there is no specification for the phase margin, we can set it to around $10°$ at a crossover frequency larger than the actual one. After some experimental test, we choose to increase the crossover frequency by $4\ rad/s$, so $N = 1.5$, $b = 11.39$. Thus,

$$F_{lead} = N\frac{s+b}{s+bN} = \frac{1.5s + 11.39}{s + 17.09}$$

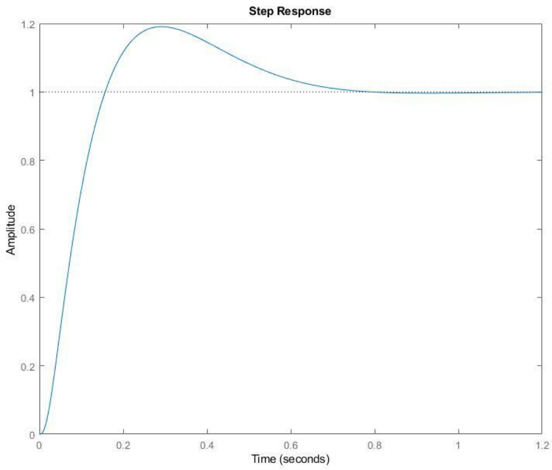

Figure 7: Step response of the reference

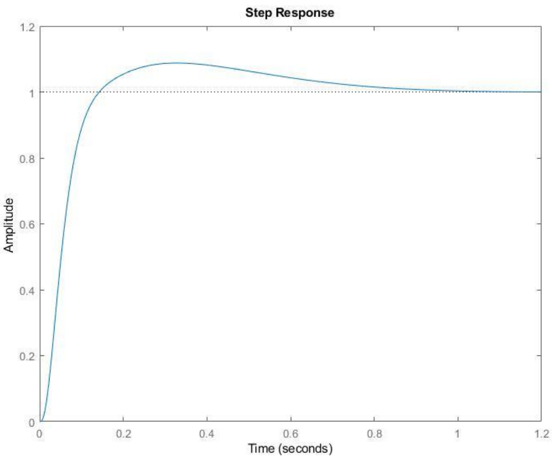

Figure 8: Step response of the reference with lead controller

This controller is good for the reference signal tracking but the control signal does not fulfill the specification $|u(t)| < 1\ \forall t$. So, a prefilter is added to the reference and the lead controller is changed to reduce the margin of u. After several tests, we choose to have the following lead controller and prefilter.

$$F_{lead} = \frac{1.817s + 14.63}{s + 13.95}, \qquad F_r = \frac{1}{1 + 0.1s}$$

The step responds of the system are showed below.

The achieved rise time is $0.1938s < 0.2s$. The overshoot is $6.6538\% < 10\%$. With ZOOM function, it is possible to see that $|y(t)| < 0.1$ for $t > 0.5s$. The graphs below show clearly that $|u(t)| < 1\ \forall t$.

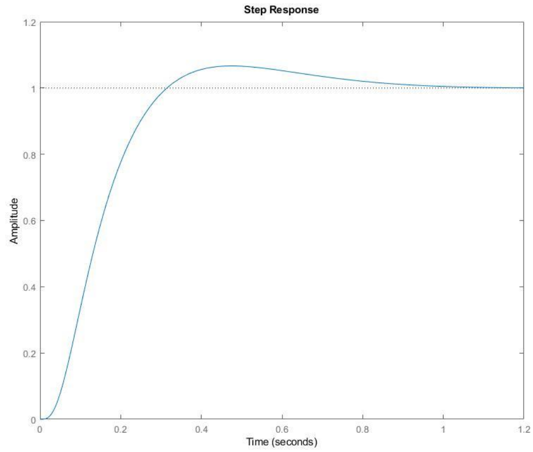

Figure 9: Step response of the reference to the output signal

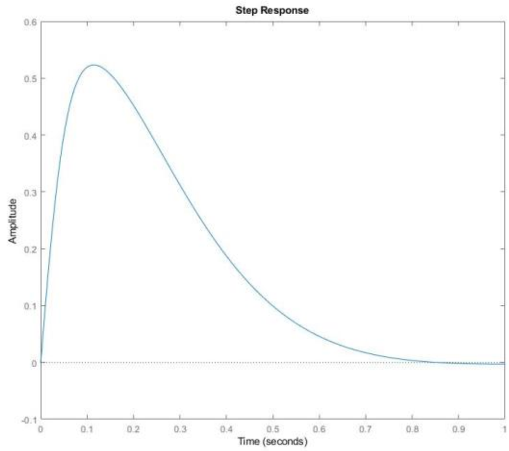

Figure 10: Step response of the disturbance to the output signal

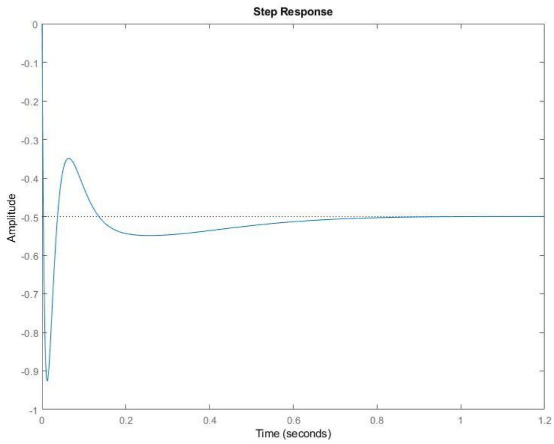

Figure 11: Step response of the reference to the control signal

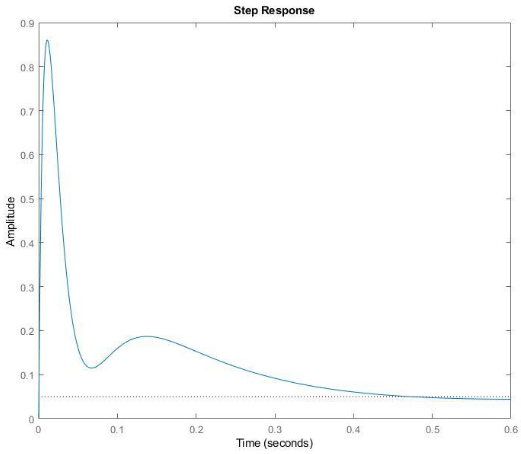

Figure 12: Step response of the disturbance to the control signal

Conclusions

In this report we used the classical loop shaping method to solve several control problems.

References

[1] EL2520 Control Theory and Practice Advanced Course, Computer Exercise: Classical Loop-Shaping, 2014.
[2] T. Glad and L. Ljung, Reglerteknik, Grundläggande teori, Studentlitteratur, 2006.