DD2352 - Lab 2: NP-completeness reductions

1. A solution to the positive instance of the casting problem shall be the following:

This solution satisfies the requirement.

1. If we have a solution, then it is easy to check the constraints. We can iterate through the constraints of type 1 to check if each role is played by a player that can do it. The time complexity here is simply . Then we can iterate through the constraints of type 2 to check if two roles are played by the same player or if player 1 and player 2 are in the same scene. This can be done in linear time complexity by creating a table of Boolean values that symbolize if a player is in the scene. The time complexity of this iteration is then . The total time complexity is then which is polynomial time. So, the question is a NP-question.
2. One additional player is needed to make this instance positive. With one additional player, the following solution is a positive solution to the problem:
3. Two scenes, two roles and two players. The two guaranteed players play two respective roles. Each scene only includes one role. This is the smallest possible production.
4. There are only two groups and players from same group shall not play together. So, each scene can only include two players at most. Thus, if there are scenes then at most players are needed.
5. It will be identical. This is because that instance A means that the roles 4, 7 and 12 must be three different players which cannot include both player 1 and player 2. This is exactly what the instance B tells. So, they are the same.