Algorithms and Complexity

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Mästarprov1: Algorithms

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1. On default, the Dijkstra algorithm can calculate the shortest path in a directed graph by letting each node memorize the shortest length travelled when reaching the node and where did the edge come from as we iterate through all of the nodes. Our problem is to find two edge-distinct shortest paths. This can be done by use the same Dijkstra algorithm and let each node remembering two shortest lengths that comes from two different previous nodes. When the algorithm is finished, check the end node if it contains two lengths which is equal. If there are two lengths stored that are equal then there are two edge-distinct shortest paths, otherwise not.

The algorithm is basically the same as the Dijkstra algorithm. It has double as many operations and double as much memory usage as Dijkstra algorithm. So, the complexity for this algorithm is . The correctness of this algorithm can be argued in the following way. First since the Dijkstra algorithm can calculate the shortest path in the graph, our algorithm should give the correct shortest paths as well. If the two shortest paths do not have any common nodes then they are clearly edge-distinct. If they have any common node, then they must come to the common node from different previous nodes so the edge must be different. Then we have proven that the algorithm determines if there exist two edge-distinct shortest paths.

1. The greedy algorithm can be described as the following procedure. Beginning from the painting , put a guard at the position Then the area between and can be guarded. Additional paintings can be guarded as well if their position satisfies . Then we can go to the next valuable painting that is not guarded, , and put a guard at and repeat this procedure until all valuable painting are guarded. The complexity of this algorithm is since it will iterate all valuable painting once.
2. Let be the biggest value that can be guarded in the first positions with guards. The maximum of is the number of paintings which is . The maximum of is the number of guards which is . So is a matrix with size .