To achieve the maximum payload ratio for the rocket we choose to use the gravity turn for turning our rocket from a vertical state to a horizontal orbit. This leads to that we shall use the following equations to describe our rocket’s dynamic.

In this equations we have following parameters. The thrust is , the air drag to the rocket is , the mass of the rocket is , the radius of the earth is , the horizontal position of rocket in a rotating coordinate is , the vertical position of rocket is , the vertical altitude of the rocket , the flight path angle between the rocket and the horizontal line is , the velocity of rocket is and and are the specific impulse of the rocket and gravitation constant of earth at sea level.

By using this equation system, we can describe location, velocity and mass of a rocket at time with a set of initial values. To make the solution more accurate we shall use the following expressions to describe some of the parameters above.

The thrust is a function of the external pressure which depends on the altitude . The relationship is then:

We assume the exhaust speed and the burn rate is constant so the thrust of our rocket is linearly increasing with decreasing external pressure. We assume that our rocket is going to be launched within an international standard atmosphere (ISA). This model tells us how to calculate the atmosphere pressure. From our source we can find at sea level. (<http://www.spacex.com/sites/spacex/files/falcon_9_users_guide_rev_2.0.pdf>)

To calculate the air drag acting on our rocket we use the equation of air drag.

The density is a function of altitude. We assume the cross section area of the rocket to be the area we can calculate with the help of the diameter of the rocket. Because that the rocket has a circle cross section area so we can use the value . (<https://spaceflightsystems.grc.nasa.gov/education/rocket/termvr.html>) Then we can with those information calculate the air drag acting on the rocket.

Since we are using Falcon 9 which is a two stage rocket with more than enough power to launch our satellite, we can assume that the first stage is going to send our satellite to an orbit where the air drag can be ignored and then the second stage is going to launch our satellite to our goal altitude. So we can assume the air drag is and the thrust is constant at the second stage of the rocket.

There is a problem with our assumption. With a too small initial velocity the change of the flight path angle will be very big since . This will give a very unrealistic result. So we choose to let the rocket fly straight up for a very short time and then let it turn with the help of the gravity.

So summarize these assumptions we can have following result. We first fly for XXXX seconds and then gives the rocket an initial angle GGGG. The second stage shall in result has an initial velocity VVVV and initial angle FFFF. Totally with this initial value we can launch our rocket to the altitude HHHH.