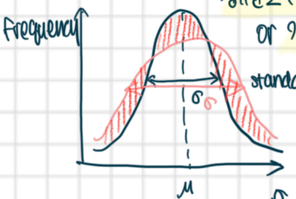
Linear Algebra - cosera / UCL

〈MODULE1: introduction〉

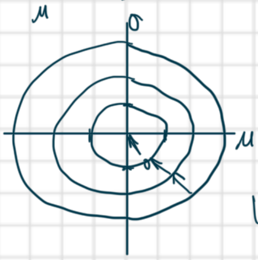
정규분포 (Normal Distribution)
or 가우시안 분포 (Gaussian Distribution)



standard deviation (표준편차)

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{ \frac{-(x-\mu)^2}{2\sigma^2} \right\}$$

parameter space
매개변수 공간



$\begin{bmatrix}\mu\\ \sigma\end{bmatrix}$ $\begin{bmatrix}\mu'\\ \sigma'\end{bmatrix}$ $\begin{bmatrix}155\\ 12.09\ldots\end{bmatrix}$

$12 - 9 = 3$, $\mu = 3$, $\sigma = 3$

$0.03 = \frac{1}{\sigma\sqrt{2\pi}}$ $\quad \sigma = \frac{1}{0.03\times\sqrt{2\pi}}$ $= 1$

SSR 잔차제곱합 Sum of Squared Residuals

$$= \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

실제데이터값, 모델값

$(y_i - \hat{y}_i)$ = 잔차 (residuals)

similar

1. 2, 4 ($\frac{~~1550~~}{f}$)
2. 1 (145, 12)
3. 2
4. 2, 5 increase μ, decrease σ
5. 179, 7.7
6. 1, 4

〈MODULE2: Vectors and Objects〉

- modulus 절대값
- magnitude 크기
- dot product 내적
- scala projection 스칼라투영
- vector projection 벡터투영
- basis vector 기저벡터 (좌표계)
- linear independence 선형독립
- linear combination 선형결합
- hypotenuse 빗변
- perpendicular 직각

$|r| = \sqrt{a^2+b^2}$
$r = a\hat{i} + b\hat{j} = \begin{bmatrix}a\\ b\end{bmatrix}$
단위벡터
$r = \begin{bmatrix}a\\ b\end{bmatrix}$, $|r| = \sqrt{a^2+b^2}$

$s = \begin{bmatrix}-1\\ 2\end{bmatrix} = \begin{bmatrix}s_i\\ s_j\end{bmatrix}$ $\quad r = \begin{bmatrix}3\\ 2\end{bmatrix} = \begin{bmatrix}r_i\\ r_j\end{bmatrix}$

$s\cdot r = r\cdot s = r_i s_i + r_j s_j$
$= -3 + 4 = 1$

commutative
distributive $\rightarrow r\cdot(s+t) = r\cdot s + r\cdot t$
associative $\rightarrow r\cdot(as) = a(r\cdot s)$

cosine rule

$c^2 = a^2 + b^2 - 2ab\cos\theta$
$|r-s|^2 = |r|^2 + |s|^2 - 2|r||s|\cos\theta$
$(r-s)\cdot(r-s) = |r|^2 + |s|^2 - 2s\cdot r$
$\boxed{r\cdot s = |r||s|\cos\theta}$

$\theta = 90°$: $\cos 90 = 0$, $r\cdot s = 0$
$\cos 0 = 1 = r\cdot s = |r||s|$
$\cos 180 = -1$, $r\cdot s = -|r||s|$

Projection

$\cos\theta = \frac{adj}{hyp} = \frac{adj}{|s|}$

$r\cdot s = |r||s|\cos\theta$ (adj projection)

$\frac{r\cdot s}{|r|} = |s|\cos\theta$ = scalar projection.

$\sqrt{1^2+3^2+4^2+2^2}$
$1\ 9\ 16\ 4$, 30
$-5+6-2$
-1

1. $\sqrt{30}$
2. -1
3. 2
4. ...
5. $|a||b| \leq |a|+|b|$
6. we the size the vec

$\frac{30}{-30}$ 6 -4 0

$\frac{3}{5}$

$r = 3\hat{e}_1 + 4\hat{e}_2 = \begin{bmatrix}3\\ 4\end{bmatrix}$

$b_2 = \begin{bmatrix}-2\\ 4\end{bmatrix}$, $r_b = \begin{bmatrix}2\\ \frac{1}{2}\end{bmatrix}$, $b_1 = \begin{bmatrix}2\\ 1\end{bmatrix}$, $\hat{e}_2 = \begin{bmatrix}0\\ 1\end{bmatrix}$, $\hat{e}_1 = \begin{bmatrix}1\\ 0\end{bmatrix}$

$\frac{r_e\cdot b_1}{|b_1|^2} = \frac{3\times 2 + 4\times 1}{2^2+1^2} = \frac{10}{5} = 2$

$\cos\theta = \frac{b_1\cdot b_2}{|b_1||b_2|}$

$\frac{r_e\cdot b_1}{|b_1|^2}\times b_1 = 2\begin{bmatrix}2\\ 1\end{bmatrix} = \begin{bmatrix}4\\ 2\end{bmatrix}$

$\begin{bmatrix}4\\ 2\end{bmatrix} + \begin{bmatrix}-1\\ 2\end{bmatrix} = \begin{bmatrix}3\\ 4\end{bmatrix}$

$b_1\cdot b_2 = 2\times -2 + 1\times 4 = 0$ $\quad \frac{r_e\cdot b_2}{|b_2|^2} = \frac{3\times -2 + 4\times 4}{2^2+4^2} = \frac{-6+16}{20} = \frac{1}{2}$

$\frac{r_e\cdot b_2}{|b_2|^2}\times b_2 = \frac{1}{2}\begin{bmatrix}-2\\ 4\end{bmatrix} = \begin{bmatrix}-1\\ 2\end{bmatrix}$

1. $v = \begin{bmatrix}5\\ -1\end{bmatrix}$ $b_1 = \begin{bmatrix}1\\ 1\end{bmatrix}$ $b_2 = \begin{bmatrix}1\\ -1\end{bmatrix}$, $v_b = ?$

$\frac{v_b\cdot b_1}{|b_1|^2} = \frac{5-1}{2} = 2$

$\frac{v_b\cdot b_2}{|b_2|^2} = \frac{5+1}{2} = 3$

2. $v = \begin{bmatrix}10\\ -5\end{bmatrix}$ $b_1 = \begin{bmatrix}3\\ 4\end{bmatrix}$ $b_2 = \begin{bmatrix}4\\ -3\end{bmatrix}$, $v_b = ?$

$\frac{v_b\cdot b_1}{|b_1|^2} = \frac{30-20}{25} = \frac{2}{5}$

$\frac{v_b\cdot b_2}{|b_2|^2} = \frac{40+15}{25} = \frac{11}{5}$

3. $v = \begin{bmatrix}2\\ 2\end{bmatrix}$ $b_1 = \begin{bmatrix}-3\\ 1\end{bmatrix}$ $b_2 = \begin{bmatrix}1\\ 3\end{bmatrix}$

$v_b = \begin{bmatrix}\frac{-6+2=-4}{10} = -\frac{2}{5}\\ \frac{2+6=8}{10} = \frac{4}{5}\end{bmatrix}$

4. $v = \begin{bmatrix}1\\ 1\\ 1\end{bmatrix}$ $b_1 = \begin{bmatrix}2\\ 1\\ 0\end{bmatrix}$ $b_2 = \begin{bmatrix}1\\ -2\\ -1\end{bmatrix}$ $b_3 = \begin{bmatrix}-1\\ 2\\ -5\end{bmatrix}$

$v_b = \begin{bmatrix}\frac{3}{5}\\ \frac{1-2-1}{6} = -\frac{1}{3}\\ \frac{-1+2-5}{30} = \frac{-4}{30} = \frac{-2}{15}\end{bmatrix}$

5. $v = \begin{bmatrix}1\\ 1\\ 2\\ 3\end{bmatrix}$ $b_1 = \begin{bmatrix}1\\ 0\\ 0\\ 0\end{bmatrix}$ $b_2 = \begin{bmatrix}0\\ 2\\ -1\\ 0\end{bmatrix}$ $b_3 = \begin{bmatrix}0\\ 1\\ 2\\ 0\end{bmatrix}$ $b_4 = \begin{bmatrix}0\\ 0\\ 0\\ 3\end{bmatrix}$

$v_b = \begin{bmatrix}1\\ \frac{0}{5}\\ \frac{5}{5} = 1\\ \frac{9}{9} = 1\end{bmatrix}$

$\frac{4-6+16}{1^2+2^2+3^2} = \frac{14}{14} = 1$

$\frac{8-3}{4+1} = \frac{5}{5} = 1$

$2+18+40$
$9+36+25$

Basis is a set of n vectors that:
- (i) are not linear combinations of each other (linearly independent)
- (ii) span the space
- The space is then n-dimensional.

$b_3 \neq a_1 b_1 + a_2 b_2$

linearly independent 선형독립

벡터가 배수 $\nearrow b=3a$ 면 선형종속

1. yes
2. yes
3. $-7, -3$
4. yes
5. NO
6. not lin, not span 3

1
2 $-\frac{1}{5}$
3 $\begin{pmatrix}1\\ 1\end{pmatrix}$
4 NO
5 $\begin{pmatrix}\frac{1}{2}\end{pmatrix}$
6

$\begin{bmatrix}1\\ 2\end{bmatrix}$ $\begin{bmatrix}1\\ 1\end{bmatrix}$

size?

$c^2 = a^2 + b^2 - 2ab\cos\theta$

$\frac{6-4}{\sqrt{3^2+4^2}} = \frac{2}{5}$

<MODULE 3 : Objects that operate on vectors>

$2a+3b=8$
$10a+1b=13$
$\begin{pmatrix}2&3\\10&1\end{pmatrix}\begin{pmatrix}a\\b\end{pmatrix}=\begin{pmatrix}8\\13\end{pmatrix}$

$\begin{pmatrix}35-36\\60+48\end{pmatrix}$ $\begin{matrix}-1\\108\end{matrix}$

$\begin{bmatrix}1&0\\0&1\end{bmatrix}$ $\begin{bmatrix}-1&0\\0&1\end{bmatrix}$ $\begin{bmatrix}0&-1\\1&0\end{bmatrix}$

☆ $\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}$

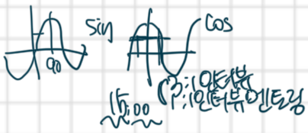
Identity Matrix : 단위행렬 I

$A_1A_2 \neq A_2A_1$

$\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}1&1\\0&1\end{pmatrix}$ $\begin{pmatrix}0&1\\1&1\end{pmatrix}$

1. $\begin{pmatrix}\frac{3}{2}&2&-\frac{1}{2}\\ \frac{3}{2}\end{pmatrix}$ 2. $\begin{pmatrix}\frac{1}{2}&-1\\0&\frac{3}{4}\end{pmatrix}\begin{pmatrix}-2\\4\end{pmatrix}=\begin{pmatrix}-1-4\\3\end{pmatrix}=\begin{pmatrix}-5\\3\end{pmatrix}$

5. $\begin{pmatrix}1&0\\0&8\end{pmatrix}\begin{pmatrix}1&0\\\frac{1}{2}&1\end{pmatrix}=\begin{pmatrix}1&0\\4&8\end{pmatrix}$ $\begin{bmatrix}-\frac{1}{2}\\ \frac{1}{2}\end{bmatrix}$



A^{-1}: inverse

$B=A^{-1}$
$AA^{-1}=I$
$AB=I$

$\begin{pmatrix}1&1&3\\1&2&4\\1&1&2\end{pmatrix}\begin{pmatrix}b_{11}&b_{12}&b_{13}\\b_{21}&b_{22}&b_{23}\\b_{31}&b_{32}&b_{33}\end{pmatrix}=I=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$

$\begin{pmatrix}b_{11}\\b_{21}\\b_{31}\end{pmatrix}=\begin{pmatrix}1\\0\\0\end{pmatrix}$

$\begin{pmatrix}1&1&3\\0&1&1\\0&0&-1\end{pmatrix}$ $\begin{pmatrix}1&0&0\\-1&1&0\\-1&0&-1\end{pmatrix}$

$3a+4b+c=7$
$b+c=-\frac{1}{2}$

$\begin{pmatrix}1&1&0\\0&1&0\\0&0&1\end{pmatrix}$

$\begin{matrix}-2&0&3\\-2&1&1\\1&0&-1\end{matrix}$

지루한 과정
$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$ $\begin{pmatrix}0&-1&2\\-2&1&1\\1&0&-1\end{pmatrix}$ $A^{-1}=B$ inverse matrix

1. $\boxed{\begin{pmatrix}1&1&1\\3&2&1\\2&1&2\end{pmatrix}\begin{pmatrix}a\\b\\c\end{pmatrix}=\begin{pmatrix}15\\28\\23\end{pmatrix}}$

$\begin{matrix}-3&-\frac{9}{2}&-\frac{3}{2}&-\frac{27}{4}\\3&4&1&\cancel{\frac{28}{4}}\end{matrix}$
$\begin{matrix}0&-\frac{1}{2}&+\frac{1}{2}&+\frac{1}{4}\end{matrix}$

3. $\begin{pmatrix}1&\frac{3}{2}&\frac{1}{2}\\0&1&1\\0&0&1\end{pmatrix}\begin{pmatrix}a\\b\\c\end{pmatrix}=\begin{pmatrix}\frac{9}{4}\\-\frac{1}{2}\\0\end{pmatrix}$

2. $③'' = (② - 3①)(-2)$

$\begin{pmatrix}2&3&1\\0&1&1\\2&8&13\end{pmatrix}$ $\begin{pmatrix}\frac{9}{2}\\-\frac{1}{2}\\0\end{pmatrix}$

4. $a=3$, $b=-\frac{1}{2}$, $c=0$

6. [3, 7, 5]

$\begin{pmatrix}1&1&1\\3&2&1\\2&1&2\end{pmatrix}$ $\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$

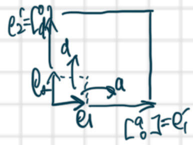
$\begin{pmatrix}1&1&1\\-2&1&0\\1&0&1\end{pmatrix}$ $\begin{pmatrix}1&0&0\\-1&1&0\\-1&0&1\end{pmatrix}$

$\begin{matrix}1&1&1\\0&1&-2\\1&0&1\end{matrix}$ $\begin{pmatrix}1&0&0\\1&1&-2\\-1&0&1\end{pmatrix}$

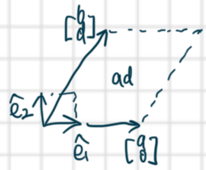
$\begin{pmatrix}1&0&3\\0&1&-2\\1&0&1\end{pmatrix}$ $\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$ $\begin{pmatrix}0&-1&2\\1&1&-2\\-1&0&1\end{pmatrix}$

$\begin{pmatrix}1&0&3\\0&1&-2\\0&0&2\end{pmatrix}$ $\begin{pmatrix}1&0&3\\0&1&0\\0&0&1\end{pmatrix}$ $\begin{pmatrix}0&-1&2\\1&1&-2\\1&-1&1\end{pmatrix}$ $\begin{pmatrix}0&-1&2\\2&0&-1\\\frac{1}{2}&-\frac{1}{2}&\frac{1}{2}\end{pmatrix}$

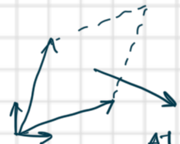
determinant : 행렬식



$\begin{pmatrix}a&0\\0&d\end{pmatrix}$



$\begin{pmatrix}a&b\\0&d\end{pmatrix}$



$\begin{pmatrix}a&b\\c&d\end{pmatrix}$ Area=ad−bc

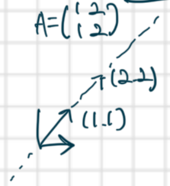
$\boxed{\frac{1}{ad-bc}}\begin{pmatrix}a&b\\c&d\end{pmatrix}\boxed{\begin{pmatrix}d&-b\\-c&a\end{pmatrix}}=\begin{pmatrix}\cancel{ad-bc}&0\\0&\cancel{ad-bc}\end{pmatrix}$

A A^{-1}

$\begin{pmatrix}1&2&3&4\\5&6&7&8\end{pmatrix}$

$[5.6.7.8]-5\times[1.2.3.4]$
$5.10.15.20$
$[0.-4.-8.-12]$

$|A|=0$ ☆
$A=\begin{pmatrix}1&2\\1&2\end{pmatrix}$



) not independent

$\begin{pmatrix}1&1&3\\1&2&4\\2&3&7\end{pmatrix}\begin{pmatrix}a\\b\\c\end{pmatrix}=\begin{pmatrix}12\\17\\29\end{pmatrix}$

row③ = row① + row②
col③ = col① × 2 + col②

가우스소거하다가 하나의 row가 (col도) 사라지면 → 선형독립X.

$-\frac{9}{2}+\frac{4}{2}+\frac{1}{2}$

$\boxed{\begin{pmatrix}-\frac{3}{2}&\frac{1}{2}&\frac{1}{2}\\2&0&-1\\\frac{1}{2}&-\frac{1}{2}&\frac{1}{2}\end{pmatrix}}$ ✓

$\begin{pmatrix}1&1&1\\3&2&1\\2&1&2\end{pmatrix}$

$\begin{matrix}-\frac{3}{2}&-1&2\\-\frac{3}{2}&\frac{1}{2}&\frac{1}{2}\\\frac{3}{2}&+\frac{3}{2}&-\frac{3}{2}\end{matrix}$

$\begin{pmatrix}-\frac{3}{2}&\frac{1}{2}&\frac{1}{2}\\2&0&-1\\\frac{1}{2}&-\frac{1}{2}&\frac{1}{2}\end{pmatrix}$

<MODULE 4: Matrics make linear mappings>

A B AB

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & & & \\ \vdots & & & \\ a_{n1} & & & a_{nm} \end{pmatrix}\begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1m} \\ b_{21} & & & \vdots \\ \vdots & & \ddots & \vdots \\ b_{n1} & & \cdots & b_{nm} \end{pmatrix} =$$

row col

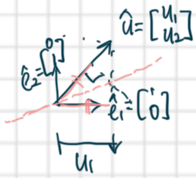
$(ab)_{23} = a_{21}b_{13} + a_{22}b_{23} + \cdots + a_{2m}b_{n3}$

$(ab)_{ik} = \sum_j a_{ij} b_{jk}$

$\boxed{AB = C \quad C_{ik} = a_{ij} b_{jk}}$

$2\begin{pmatrix}\cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot\end{pmatrix}_3 \begin{pmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{pmatrix}^4 = 2\begin{pmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{pmatrix}^4$

$U \cdot V$: $\begin{pmatrix} u_i \end{pmatrix} \cdot \begin{pmatrix} v_i \end{pmatrix} = u_i v_i = [u_1 u_2 \cdots u_n]\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$



1. $\cancel{B} = 4+1 = 5$
2. $\begin{pmatrix} 4 & 3 & 5 \\ 5 & 4 & 1 \end{pmatrix}$
3. $\underbrace{2+12}_{22}+\underbrace{10+6}_{8} = 30$

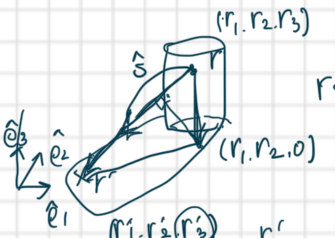
4. $\begin{pmatrix} 2 & 4 & 5 & 6 \\ 6 & 12 & 15 & 18 \end{pmatrix}$

5. $\begin{pmatrix} 2 & 2 & 8 & -4 \\ & & & \end{pmatrix}$

6. $D = 5 \times 4$

7. $\begin{pmatrix} 1 & 2 & 3 \\ 4 & & \end{pmatrix}$

8 3개



$r' = r + \lambda \hat{s}$

$r_3' = 0 \quad r' \cdot \hat{e}_3 = 0$

$r\hat{e}_3 + \lambda s_3 = 0 \quad s_3 = \hat{s} \cdot \hat{e}_3$

$(r_1, r_2, r_3) \quad r_i' \quad \lambda = -\frac{r\hat{e}_3}{s_3} \quad r' = Ar$

① $r' = r - \hat{s}(r \cdot \hat{e}_3)/s_3$ $\quad r' = r - \frac{r\hat{e}_3}{s_3} \times \hat{s}$

$r = Ar$

$\frac{r'}{r} = A$

$r = 3D$, $r' = 2D$, $A = 2\times 3$

$r_i' = r_i - s_i \frac{r_j \hat{e}_{3j}}{s_3} \to r_3$

$r_i' = \left(I_{ij} - s_i \frac{I_{3j}}{s_3}\right) r_j$

② $r_i' = r_i - \frac{s_i r_3}{s_3}$ / $r_i' = (I_{ij} - s_i \frac{I_{3j}}{s_3}) r_j$

③ $\begin{pmatrix} 1 & 0 & -s_1/s_3 \\ 0 & 1 & -s_2/s_3 \end{pmatrix}$

3D→2D 2차원으로!

$G(x', y') = (x - s_1\frac{z}{s_3}, y - s_2\frac{z}{s_3})$

1 … 0 0

$\begin{pmatrix} 1 & 0 & -s_1/s_3 \\ 0 & 1 & -s_2/s_3 \end{pmatrix}$

④ $(0, 0, 0)$

$A = \begin{pmatrix} 1 & 0 & -\frac{s_1}{s_3} \\ 0 & 1 & -\frac{s_2}{s_3} \end{pmatrix}$

$\hat{s} = \begin{pmatrix} 4/13 \\ -3/13 \\ -12/13 \end{pmatrix} \quad r = \begin{pmatrix} 6 \\ 2 \\ 3 \end{pmatrix} \quad r' \quad -\frac{4/13}{12/13} \quad r' = Ar$

⑤ $(7, 1.25)$

⑥ $\begin{pmatrix} 8 & 0 & -3 & 11 \\ 1.75 & -4.75 & 1 & -5 \end{pmatrix}$

$(x', y') = \begin{pmatrix} 1 & 0 & 1/3 \\ 0 & 1 & -1/4 \end{pmatrix}\begin{pmatrix} 6 \\ 2 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 6+1 \\ 2-\frac{3}{4} \end{pmatrix} \begin{pmatrix} 7 \\ 5/4 \end{pmatrix}$

$R' = AR \quad r' = Ar$

$r_i' = A_{ij} r_j \quad R_{ia}' = A_{ij} R_{ja} \quad \hat{s} = \begin{pmatrix} 4/13 \\ -3/13 \\ -12/13 \end{pmatrix}$

$\begin{pmatrix} r_1' & s_1' & t_1' & u_1' \\ r_2' & s_2' & t_2' & u_2' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1/3 \\ 0 & 1 & -\frac{1}{4} \end{pmatrix}\begin{pmatrix} 5 & -1 & -3 & 7 \\ 4 & 4 & 1 & -2 \\ 9 & 3 & 0 & 12 \end{pmatrix}$

R' $\quad 2\times 4 \quad \cancel{2}\times 3 \quad 3 \times 4$

$= \begin{pmatrix} 5+3 & -1+1 & -3 & 7+4 \\ 4-\frac{9}{4} & 4-\frac{3}{4} & 1 & -2-3 \end{pmatrix}$

8 0 −3 11

$\frac{16}{4} \quad \frac{7}{4}$

1.75 −4.75 1 −5

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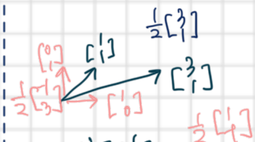
orthogonal 정리



나열때 서로 dot product를 통해 구할수 OK

= Matrices changing.

변환행렬 정의. A의 기준벡터 (3,1) (1,1)

$\begin{bmatrix} 3 \\ 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\to \underbrace{\begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}}_{\text{Bear's basis in my coor}} \underbrace{\begin{bmatrix} 3/2 \\ 1/2 \end{bmatrix}}_{\text{bear's}} = \underbrace{\begin{bmatrix} 5 \\ 2 \end{bmatrix}}_{\text{my vector}}$

B

→ 변환행렬 $\begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix}$

→ 변환행렬의 역행렬

$\underbrace{\frac{1}{2}\begin{pmatrix} 1 & -1 \\ -1 & 3 \end{pmatrix}}$ $\underline{B \to A}$

B'

$\underbrace{\frac{1}{2}\begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix}}_{\text{my basis in Bear's world}}\begin{bmatrix} 5 \\ 2 \end{bmatrix} = \underbrace{\begin{bmatrix} 3/2 \\ 1/2 \end{bmatrix}}_{\text{Bear's vector}}$

→ B축의 행렬을 A로 바꿀때 $\frac{1}{2}\begin{pmatrix} 1 & -1 \\ -1 & 3 \end{pmatrix}$을 쓰면됨.

$B^{-1} R^{45} B$ 새로운 기준계에서 회전행렬.

$\underbrace{\frac{1}{2}\begin{pmatrix} 1 & -1 \\ -1 & 3 \end{pmatrix}}_{B^{-1}} \underbrace{\underbrace{\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}}_{R^{45}} \underbrace{\begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix}}_{B}\begin{bmatrix} x \\ y \end{bmatrix}}_{\text{vector in my frame}} = R_B$

vector in bear's frame

$\frac{1}{\sqrt{2}}\begin{pmatrix} 3 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} 2 & -2 \\ 2 & 2 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$

$r' = \begin{pmatrix} 1 & 0 & +\frac{1}{3} \\ 0 & 1 & -\frac{1}{4} \end{pmatrix}\begin{pmatrix} 6 \\ 2 \\ 3 \end{pmatrix}$

$\begin{pmatrix} 6+1 \\ 2-\frac{3}{4} \end{pmatrix} \begin{matrix} 7 \\ \frac{5}{4} \end{matrix}$

Transpose $A^T_{ij} = A_{ji}$

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}^T = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$$

$A^TA = I$ $\quad \underline{A^T = A^{-1}}$

$$\overset{A^T}{\begin{pmatrix} (& a_1 &) \\ (& a_2 &) \\ & \vdots & \\ (& a_n &) \end{pmatrix}} \overset{A \quad n\times n}{\begin{pmatrix} \begin{pmatrix} a_1 \end{pmatrix} \begin{pmatrix} a_2 \end{pmatrix} \cdots \begin{pmatrix} a_n \end{pmatrix} \end{pmatrix}} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 1 & 0 & 0 & \cdots \\ & & & \ddots \end{pmatrix}$$

직교행렬!

$a_i \cdot a_j = 0$ "직교" $i \neq j$

$a_i \cdot a_j = 1$ "자기" $\xrightarrow{i=j}$ 답 ①.

Gram-Schmidt process.

$V = \{v_1, v_2 \ldots v_n\}$ ← 개별 vector

v_1 $\quad e_1 = \frac{v_1}{|v_1|}$ 정규화 (normalized)

정규화 하는 이유
i) 일관성: 모든 벡터 동일 크기. 비교 easy
ii) 계산 용이: 단위벡터 = 수학적 계산 easy

projection 성분 단위벡터

$v_2 = (v_2 \cdot e_1)\frac{e_1}{|e_1|} + u_2$ — e_1과 직교하는 성분들으로 u_2

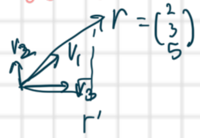
$u_2 = v_2 - (v_2 \cdot e_1)e_1$ $\quad \boxed{\frac{u_2}{|u_2|} = e_2}$

$u_3 = v_3 - (v_3 \cdot e_1)e_1 - (v_3 \cdot e_2)e_2$ $\quad \frac{u_3}{|u_3|} = e_3$

e_1 성분 $\quad e_2$ 성분

내적: v_3가 e_1 방향으로 얼마나 뻗어있는지

$$\underset{v_1}{\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}} \underset{v_2}{\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}} \underset{v_3}{\begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}}$$



$e_1 = \frac{v_1}{|v_1|} = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$$v_2 = v_2 - (v_2 \cdot e_1)e_1 = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} - \left[\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \cdot \frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\right] \frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$
$$= \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} - 3\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

$e_2 = \frac{v_2}{|v_2|} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$

$$v_3 = v_3 - (v_3 \cdot e_1)e_1 - (v_3 \cdot e_2)e_2$$
$$= \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} - \left[\begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} \cdot \frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\right]\frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} - \left[\begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} \cdot \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}\right]\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$
$$= \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} - \frac{1}{3} \cdot 3\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} - \frac{1}{\cancel{2}} \cdot \cancel{2}\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$$

$e_3 = \frac{v_3}{|v_3|} = \frac{1}{\sqrt{6}}\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$

$$E = \begin{pmatrix} (e_1) & (e_2) & (e_3) \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} & \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} & \frac{1}{\sqrt{6}}\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} \end{pmatrix}$$

$$T_E = \begin{array}{c} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \\ e_1 \; e_2 \; e_3 \end{array}$$

Reflecting in a plane $\quad E^T = E^{-1}$

$$\begin{tikzcd} r \arrow[r, "hard"] \arrow[d, "E^{-1}"'] & r' \\ r_E \arrow[r, "T_E"'] & r'_E \arrow[u, "E"'] \end{tikzcd}$$

$$E \, \underline{T_E E^{-1}} r = r'$$

$$T_E \underset{E^T}{\overset{\shortparallel}{E^{-1}}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{3}}(1 & 1 & 1) \\ \frac{1}{\sqrt{2}}(1 & -1 & 0) \\ \frac{1}{\sqrt{6}}(1 & 1 & -2) \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{3}}(1, 1, 1) \\ \frac{1}{\sqrt{2}}(1, -1, 0) \\ \frac{1}{\sqrt{6}}(-1, -1, 2) \end{pmatrix}$$

$$E \times \underset{\sim\sim}{T_E E^T} = \begin{pmatrix} \frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} & \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} & \frac{1}{\sqrt{6}}\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{3}}(1, 1, 1) \\ \frac{1}{\sqrt{2}}(1, -1, 0) \\ \frac{1}{\sqrt{6}}(-1, -1, 2) \end{pmatrix} = \begin{pmatrix} \frac{1}{3}+\frac{1}{2}-\frac{1}{6} & \frac{1}{3}-\frac{1}{2}-\frac{1}{6} & \frac{1}{3}+0-\frac{2}{6} \\ \frac{1}{3}-\frac{1}{2}-\frac{1}{6} & & \\ & & \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 2 & -1 & 2 \\ -1 & 2 & 2 \\ 2 & 2 & -1 \end{pmatrix} = T$$

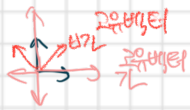
$$\underline{\underline{r' = T \times r}} = T\begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix} = \underline{\underline{\frac{1}{3}\begin{pmatrix} 11 \\ 14 \\ 5 \end{pmatrix}}}$$

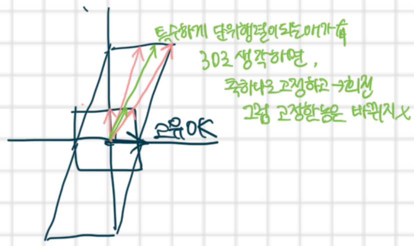
$T = E T_E E^{-1}$

$=$

<MODULE 5: Eigenvalues & Eigenvectors>
고유값 고유벡터

1. $\begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}$
2. $\begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}$
3. $\begin{pmatrix}1\\0\end{pmatrix}$
4. none of the above
5. $\begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}$
6. $\begin{pmatrix}1\\0\end{pmatrix}$





$Ax = \lambda x$ (λ: number)

변환행렬 A가 고유벡터 x를 가질경우, 변환후에도 같은 방향임. 고유값 λ을 찾는다.

$(A-\lambda I)x = 0$ (λI: matrix로 계산하려고. $x \neq 0$)

행렬식 $\det(A-\lambda I)=0$ $A=\begin{pmatrix}a&b\\c&d\end{pmatrix}$

$\det\left(\begin{pmatrix}a&b\\c&d\end{pmatrix}-\begin{pmatrix}\lambda&0\\0&\lambda\end{pmatrix}\right)=0$ $\begin{pmatrix}a-\lambda&b\\c&d-\lambda\end{pmatrix}$

$\lambda^2-(a+d)\lambda+ad-bc=0$ $(a-\lambda)(d-\lambda)-bc=0$

$ad-(a+d)\lambda+\lambda^2-bc=0$

ex) $A=\begin{pmatrix}1&0\\0&2\end{pmatrix}$

$\det\begin{pmatrix}1-\lambda&0\\0&2-\lambda\end{pmatrix}=(1-\lambda)(2-\lambda)=0$ $\lambda=1,2$

$(A-\lambda I)x=0$ $\lambda^2-3\lambda+2$

i) $\lambda=1$: $\begin{pmatrix}1-1&0\\0&2-1\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}=\begin{pmatrix}0&0\\0&1\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}=\begin{pmatrix}0\\x_2\end{pmatrix}=0$

ii) $\lambda=2$: $\begin{pmatrix}1-2&0\\0&2-2\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}=\begin{pmatrix}-1&0\\0&0\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}=\begin{pmatrix}-x_1\\0\end{pmatrix}=0$

i) $\lambda=1$: $x=\begin{pmatrix}t\\0\end{pmatrix}$

ii) $\lambda=2$: $x=\begin{pmatrix}0\\t\end{pmatrix}$

ex2) $A\begin{pmatrix}0&-1\\1&0\end{pmatrix}$ → 90° ccw 회전)

$\det\begin{pmatrix}-\lambda&-1\\1&-\lambda\end{pmatrix}=\lambda^2+1=0$ → 고유벡터 X

① $\lambda^2-3\lambda+2=0$

② $\begin{pmatrix}0\\3\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}0\\2\end{pmatrix}$

③ $(3-\lambda)(5-\lambda)$ $\begin{pmatrix}3&4\\0&5\end{pmatrix}$ $\lambda^2-8\lambda+15=0$

$\lambda=3$: $\begin{pmatrix}3-3&4\\0&5-3\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}=\begin{pmatrix}0&4\\0&2\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}=\begin{pmatrix}4x_2\\2x_2\end{pmatrix}$

$\lambda=5$: $\begin{pmatrix}3-5&4\\0&5-5\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}=\begin{pmatrix}-2&4\\0&0\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}=\begin{pmatrix}-2x_1+4x_2\\0\end{pmatrix}$

④ $\begin{pmatrix}-1\\-1/2\end{pmatrix}\begin{pmatrix}2\\1\end{pmatrix}\begin{pmatrix}3\\0\end{pmatrix}$

⑤ $\begin{pmatrix}1-\lambda&0\\-1&4-\lambda\end{pmatrix}$ $\lambda^2-5\lambda+4=0$

$\lambda=1$: $\begin{pmatrix}0&0\\-1&3\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}=\begin{pmatrix}0\\-x_1+3x_2\end{pmatrix}$

$\lambda=4$: $\begin{pmatrix}-3&0\\-1&0\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}=\begin{pmatrix}-3x_1\\-x_1\end{pmatrix}$

⑥ $\begin{pmatrix}3\\1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}$

(0,0) never consider as eigenvector.

⑦ $\begin{pmatrix}-3-\lambda&8\\2&3-\lambda\end{pmatrix}$ λ^2-9+16 $\lambda=\pm5$

$\lambda=5$: $\begin{pmatrix}-8&8\\2&-2\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}=\begin{pmatrix}-8x_1+8x_2\\2x_1-2x_2\end{pmatrix}$

$\lambda=-5$: $\begin{pmatrix}2&8\\2&8\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}=\begin{pmatrix}2x_1+8x_2\\2x_1+8x_2\end{pmatrix}$

⑧ $\begin{pmatrix}-1\\1\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}\begin{pmatrix}4\\-1\end{pmatrix}$

⑨ $\begin{pmatrix}5-\lambda&4\\-4&-3-\lambda\end{pmatrix}$ $\begin{pmatrix}4&4\\-4&-4\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}=\begin{pmatrix}4x_1+4x_2\\-4x_1-4x_2\end{pmatrix}$

$(\lambda-5)(\lambda+3)$

$\lambda^2-2\lambda-15+16$ $\lambda_1=\lambda_2$

$\lambda^2-2\lambda+1=0$ $\lambda=1$.

⑩ $\begin{pmatrix}-2-\lambda&-3\\1&1-\lambda\end{pmatrix}$

$(\lambda+2)(\lambda-1)$ $\lambda^2+\lambda-2+3$

$\lambda^2+\lambda+1=0$ No

대각화 (diagonalisation)

ex) $T=\begin{pmatrix}0.9&0.8\\-1&0.35\end{pmatrix}$ $v_0=\begin{pmatrix}0.5\\1\end{pmatrix}$

$v_1=Tv_0$

$v_2=Tv_1$

⋮

$v_n=T^n \times v_0$

$T^n=\begin{pmatrix}a^n&0&0\\0&b^n&0\\0&0&c^n\end{pmatrix}$ 대각행렬.

but, T처럼 대각행렬이 아닐경우? T를 대각화 할수있는 새로운 기저 (eigen basis) 고유기저

$C=\begin{pmatrix}x_1&x_2&x_3\\ \vdots&\vdots&\vdots\end{pmatrix}$ $D=\begin{pmatrix}\lambda_1&0&0\\0&\lambda_2&0\\0&0&\lambda_3\end{pmatrix}$

$T=CDC^{-1}$

$T^2=CDC^{-1}CDC^{-1}=CD^2C^{-1}$

☆ $T^n=CD^nC^{-1}$

$\begin{pmatrix}1&1\\0&2\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}$

$$\begin{array}{ccc} r & \xrightarrow{\text{hard}} & r' \\ E^{-1}\downarrow & & \uparrow E \\ r_E & \xrightarrow[T_E]{} & r'_E \end{array}$$

랑 비슷한 개념

$r'=\underbrace{E T_E E^{-1}}_{T}r$

ex) $T=\begin{pmatrix}1-\lambda&1\\0&2-\lambda\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}=\begin{pmatrix}2\\2\end{pmatrix}$

$\lambda=1,2$ $\lambda=1$ $\begin{pmatrix}0&1\\0&1\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}=\begin{pmatrix}x_2\\x_2\end{pmatrix}=0$ $\boxed{v=\begin{pmatrix}1\\0\end{pmatrix}}$

→ $x_2=0$, 따라서 x_1은 자유변수 $x_1\begin{pmatrix}1\\0\end{pmatrix}$ 고유vector

$\lambda=2$ $\begin{pmatrix}-1&1\\0&0\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}=\begin{pmatrix}-x_1+x_2\\0\end{pmatrix}$ $\boxed{v=\begin{pmatrix}1\\1\end{pmatrix}}$

$-x_1+x_2=0 \Rightarrow x_1=x_2$

$x_1\begin{pmatrix}1\\1\end{pmatrix}$

$\begin{pmatrix}1&1\\0&2\end{pmatrix}\begin{pmatrix}-1\\1\end{pmatrix}=\begin{pmatrix}0\\2\end{pmatrix}$ $T^2=\begin{pmatrix}1&1\\0&2\end{pmatrix}\begin{pmatrix}1&1\\0&2\end{pmatrix}=\begin{pmatrix}1&3\\0&4\end{pmatrix}$

$\begin{pmatrix}1&1\\0&2\end{pmatrix}\begin{pmatrix}0\\2\end{pmatrix}=\begin{pmatrix}2\\4\end{pmatrix}$

$\begin{pmatrix}1&3\\0&4\end{pmatrix}\begin{pmatrix}-1\\1\end{pmatrix}=\begin{pmatrix}2\\4\end{pmatrix}$

고유벡터 기반 구성

$C=\boxed{\begin{pmatrix}1&1\\0&1\end{pmatrix}}$ $C^{-1}=\begin{pmatrix}1&-1\\0&1\end{pmatrix}$ $T=\begin{pmatrix}1&0\\0&2\end{pmatrix}$

$T^2=CD^2C^{-1}=\begin{pmatrix}1&1\\0&1\end{pmatrix}\begin{pmatrix}1&0\\0&2\end{pmatrix}^2\begin{pmatrix}1&-1\\0&1\end{pmatrix}$

$=\begin{pmatrix}1&1\\0&1\end{pmatrix}\begin{pmatrix}1&0\\0&4\end{pmatrix}\begin{pmatrix}1&-1\\0&1\end{pmatrix}=\begin{pmatrix}1&1\\0&1\end{pmatrix}\begin{pmatrix}1&-1\\0&4\end{pmatrix}$

$=\begin{pmatrix}1&3\\0&4\end{pmatrix}\begin{pmatrix}-1\\1\end{pmatrix}=\begin{pmatrix}2\\4\end{pmatrix}$

① $T=\begin{pmatrix}6&-1\\2&3\end{pmatrix}$ $C=\begin{pmatrix}1&1\\1&2\end{pmatrix}$

$D=C^{-1}TC$ $C^{-1}=\begin{pmatrix}2&-1\\-1&1\end{pmatrix}$

$=\begin{pmatrix}2&-1\\-1&1\end{pmatrix}\begin{pmatrix}6&-1\\2&3\end{pmatrix}\begin{pmatrix}1&1\\1&2\end{pmatrix}$

$=\begin{pmatrix}10&-5\\-4&4\end{pmatrix}\begin{pmatrix}1&1\\1&2\end{pmatrix}=\boxed{\begin{pmatrix}5&0\\0&4\end{pmatrix}}$

② $T=\begin{pmatrix}2&7\\0&-1\end{pmatrix}$ $C=\begin{pmatrix}7&1\\-3&0\end{pmatrix}$ $C^{-1}=\frac{1}{3}\begin{pmatrix}0&-1\\3&7\end{pmatrix}$

$=\frac{1}{3}\begin{pmatrix}0&-1\\3&7\end{pmatrix}\begin{pmatrix}2&7\\0&-1\end{pmatrix}\begin{pmatrix}7&1\\-3&0\end{pmatrix}$

$=\frac{1}{3}\begin{pmatrix}0&1\\6&14\end{pmatrix}\begin{pmatrix}7&1\\-3&0\end{pmatrix}=\frac{1}{3}\begin{pmatrix}-3&0\\0&6\end{pmatrix}=\begin{pmatrix}-1&0\\0&2\end{pmatrix}$

③ $D=C^{-1}TC$

$T=\begin{pmatrix}1&0\\2&-1\end{pmatrix}$ $C=\begin{pmatrix}1&0\\1&1\end{pmatrix}$ $C^{-1}=\begin{pmatrix}1&0\\-1&1\end{pmatrix}$

$D=\begin{pmatrix}1&0\\-1&1\end{pmatrix}\begin{pmatrix}1&0\\2&-1\end{pmatrix}\begin{pmatrix}1&0\\1&1\end{pmatrix}$

$=\begin{pmatrix}1&0\\1&-1\end{pmatrix}\begin{pmatrix}1&0\\1&1\end{pmatrix}=\begin{pmatrix}1&0\\0&-1\end{pmatrix}$

④ $D=\begin{pmatrix}a&0\\0&a\end{pmatrix}$ $C=\begin{pmatrix}1&2\\0&1\end{pmatrix}$ $C^{-1}=\begin{pmatrix}1&-2\\0&1\end{pmatrix}$

$a\,CC^{-1}I$

$T=a\cancel{\begin{pmatrix}1&2\\0&1\end{pmatrix}\begin{pmatrix}1&-2\\0&1\end{pmatrix}}^{I}=\begin{pmatrix}a&0\\0&a\end{pmatrix}$

⑤ $T=\begin{pmatrix}6&-1\\2&3\end{pmatrix}=\underset{C}{\begin{pmatrix}1&1\\1&2\end{pmatrix}}\underset{D}{\begin{pmatrix}5&0\\0&4\end{pmatrix}}\underset{C^{-1}}{\begin{pmatrix}2&-1\\-1&1\end{pmatrix}}$ T^3

$T^3=\begin{pmatrix}1&1\\1&2\end{pmatrix}\begin{pmatrix}125&0\\0&64\end{pmatrix}\begin{pmatrix}2&-1\\-1&1\end{pmatrix}$

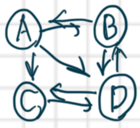
$=\begin{pmatrix}125&64\\125&128\end{pmatrix}\begin{pmatrix}2&-1\\-1&1\end{pmatrix}=\begin{pmatrix}186&-61\\122&3\end{pmatrix}$

⑥ $T=\begin{pmatrix}2&7\\0&-1\end{pmatrix}=\begin{pmatrix}7&1\\-3&0\end{pmatrix}\begin{pmatrix}-1&0\\0&8\end{pmatrix}\begin{pmatrix}0&-1/3\\1&7/3\end{pmatrix}$ T^3

$=\begin{pmatrix}-7&8\\3&0\end{pmatrix}\begin{pmatrix}0&-1/3\\1&7/3\end{pmatrix}=\begin{pmatrix}8&21\\0&-1\end{pmatrix}$

⑦ $T=\begin{pmatrix}1&0\\2&-1\end{pmatrix}=\begin{pmatrix}1&0\\1&1\end{pmatrix}\begin{pmatrix}1&0\\0&-1\end{pmatrix}\begin{pmatrix}1&0\\-1&1\end{pmatrix}$

$=\begin{pmatrix}1&0\\1&-1\end{pmatrix}\begin{pmatrix}1&0\\-1&1\end{pmatrix}=\begin{pmatrix}1&0\\2&-1\end{pmatrix}$



PageRank

$L_A=(0.\ 1/3.\ 1/3.\ 1/3)$

$L_B=(1/2.\ 0.\ 0.\ 1/2)$

$L_C=(0.\ 0.\ 0.\ 1)$

$L_D=(0.\ 1/2.\ 1/2.\ 0)$

얘 세로? 일반적이래...

$\Rightarrow L=\begin{array}{c|cccc} & A & & & \\ A & 0 & 1/2 & 0 & 0 \\ B & 1/3 & 0 & 0 & 1/2 \\ & 1/3 & 0 & 0 & 1/2 \\ & 1/3 & 1/2 & 1 & 0 \end{array}$

$r_A=\sum_{j=1}^{n} L_{Aj}r_j$ $r=\begin{pmatrix}1/4\\1/4\\1/4\\1/4\end{pmatrix}$

$r=Lr$ $r^{i+1}=Lr^i$ d: damping factor 감쇠계수

$r^{i+1}=d(Lr^i)+\frac{1-d}{n}$

① $\begin{pmatrix}-3\\-3\\-1\end{pmatrix}\begin{pmatrix}-2/\sqrt{9}\\-2/\sqrt{9}\\1/\sqrt{9}\end{pmatrix}\begin{pmatrix}1/2\\-1/2\\1\end{pmatrix}$

② Because of the loop — Other eigen —

③ The other eigen —
There is now a probability —

④ There isn't a uni
There are loops
There are two eigen value 1.

⑤ None of the —

⑥ $\lambda^2-2\lambda+\frac{1}{4}$

⑦ $1\pm\frac{\sqrt{3}}{2}$

⑧ $-1\pm\sqrt{3}$

⑨ $\begin{pmatrix}1+\frac{\sqrt{3}}{2}&0\\0&1-\frac{\sqrt{3}}{2}\end{pmatrix}$

⑩ $\begin{pmatrix}11/4&-2\\-1&3/4\end{pmatrix}$

$D^2=C^{-1}AC$

$A^2\begin{pmatrix}3/2&-2\\-\frac{1}{2}&\frac{1}{2}\end{pmatrix}\begin{pmatrix}3/2&-2\\-\frac{1}{2}&1/2\end{pmatrix}$

$\frac{1}{4}\begin{pmatrix}3&-2\\-1&1\end{pmatrix}\begin{pmatrix}3&-2\\-1&1\end{pmatrix}$

$\frac{1}{4}\begin{pmatrix}11&-8\\-4&3\end{pmatrix}$