SDM366 Sp24 **Homework # 3**
Assigned March 9, 2024 Due: March 16, 2024

- Please submit your solution through Blackboard system
- To receive credits, please write down all the necessary steps leading to final answer.

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1. **Sensor Networks:** Consider a sensor network with 5 nodes as shown in Fig. 1. At each discrete time step k, each sensor i, $i = 1, \ldots, 5$, sends its value $x_i(k)$ to other sensors that are directly connected with sensor i. For example, sensor 3 sends its value $x_3(k)$ to sensor 2, while it receives data from sensor 1 and sensor 4. Find the discrete-time state space model for the dynamics of the sensor values under the following two cases

   (a) For each sensor i, the new value at time $k+1$, $x_i(k+1)$, is the maximum among the data it receives from the neighbors and its own value at time k.

   (b) For each sensor i, the new value at time $k+1$, $x_i(k+1)$, is the average of the data it receives from the neighbors and its own value at time k.

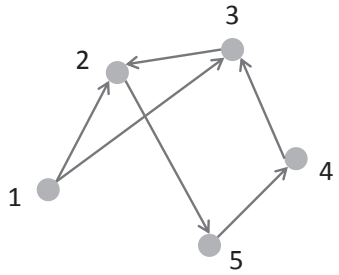

Figure 1: A small sensor network

2. **Power Electronics:** Consider the buck converter as show in Fig. 2 with ideal switches. Let the inductor current and capacitor voltage be the system states: $x_1(t) = v_c(t)$ and $x_2(t) = i_L(t)$. Let the output of the system be $y(t) = v_o(t)$. Find a linear state space model for each switching position as shown in Fig. 2.

3. **Permanent Magnet DC Motor Model:** Please read the supplemental material associated with HW3 (Section 8.2 and 8.3). Derive the state-space model for the motor + load system shown in Figure 8.5 or equivalently Fig. 8.6 with the armature voltage V_a as control input and rotor position θ_m as the output. Note that this is a

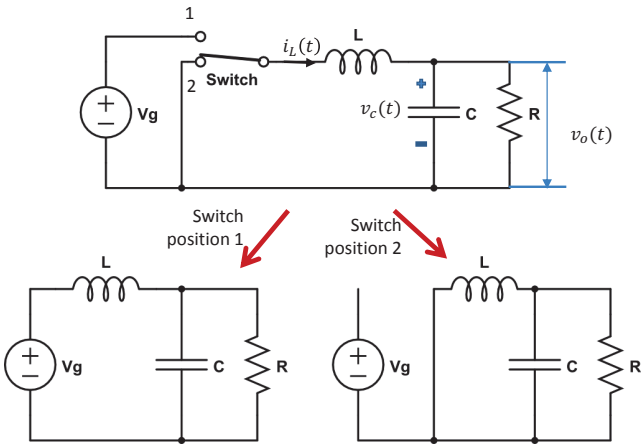

Figure 2: Buck converter

linear system with the load torque as disturbance. You should obtain a model of the following form:

$$\begin{cases} \dot{x} = Ax + Bu + Fd \\ y = Cx + Du \end{cases}$$

where the disturbance $d = \tau_l$ is the load torque.

4. **Linearized Pendulum Model:** Consider a pendulum as shown in Fig. 3 with linear friction at the pivot. Assume the rod is rigid with zero mass and its length is l. The equations of motion is given by

$$ml^2\ddot{\theta} = -mgl\sin\theta - bl^2\dot{\theta}$$

where b is the friction coefficient.

   (a) Find an equivalent continuous time state-space model. (choose your state vector as $x = [\theta, \dot{\theta}]^T$, and your output as $y = \theta$. There is no control input in this case)

   (b) Compute the linearized model around $\hat{x} = [0, 0]^T$.

   (c) Find the discrete time linear state space model with sampling period $\Delta t = 2$ (sec).

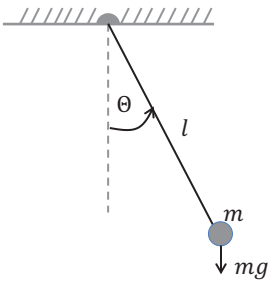

Figure 3: Pendulum Example