ECE 1505 Convex Optimization
Winter 2020

Homework Set #1

1. Consider the function $f(x) = -\sum_{i=1}^{m} \log(b_i - a_i^T x)$, where $x \in \mathbf{R}^n$, $b_i \in \mathbf{R}$ and $a_i \in \mathbf{R}^n$. Compute ∇f and $\nabla^2 f$. Write down the first three terms of the Taylor series expansion of $f(x)$ around some x_0.

2. Problem 2.5 of Boyd and Vandenberghe

3. Problem 2.14(a) of Boyd and Vandenberghe

4. Problem 3.14 of Boyd and Vandenberghe

5. Problem 3.16(a-c) of Boyd and Vandenberghe

6. Problem 3.32(a) of Boyd and Vandenberghe

7. Consider the function $f(x, y) = x^2 + y^2 + \beta xy + x + 2y$. Find (x^*, y^*) for which $\nabla f = 0$. Express your answer as a function of β. For which values of β is the (x^*, y^*) a global minimum of $f(x, y)$?

8. A function $f(x)$ is strongly convex with a positive factor m if $\nabla^2 f(x) \succeq mI$, for all x, where I denotes the identity matrix. Another equivalent definition of a m-strongly convex function, with respect to ℓ_2-norm $\| \, . \, \|_2$, is given by $f(y) \geq f(x) + \nabla f(x)^T (y - x) + \frac{m}{2} \|y - x\|_2^2$ for all x, y.
   (a) Assume $f(x)$ is a strongly convex function with factor m. Is $f(x)$ also a strictly convex function?
   (b) Assume $g(x)$ is a strictly convex function. Is $g(x)$ also a strongly convex function? Find the largest factor of strong convexity.
   Hint: You may assume that the eigen values of the hessian matrix, represented by $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_n$, are given and known. You may describe the largest strong convexity factor in terms of the eigen values of the hessian matrix.

9. In this problem, we are given a set of data points (x_i, y_i), $i = 1 \cdots 100$. We wish to fit a quadratic model, $y_i = ax_i^2 + bx_i + c + n_i$, to the data. Here, (a, b, c) are the parameters to be determined and n_i is the unknown observation noise. The (x_i, y_i) points are contained in a file `hw1data.mat` available on the course webpage. You may load the data to `MATLAB` using the command `load hw1data` and view them using `scatter(x,y,'+')`. Please use the same data set and find the maximum likelihood estimate of (a, b, c) assuming n_i's are i.i.d., and

   (a) $n_i \sim \mathcal{N}(0, 1)$;
   (b) n_i is always positive and $n_i \sim e^{-z}$ for $z \geq 0$.

Please plot the data and the models on the same `MATLAB` figure and submit the figure as a part of your solution.

(`MATLAB` has built-in functions to solve many optimization problems. For example, `linprog` solves a linear programming problem, `quadprog` solves a quadratic programming problem. You may use `help linprog` to get more details. Hint: part (a) has an analytic solution.)