1. Q 4.11(a,c,e) textbook
2. Q 4.16 textbook
3. Q 4.21(a) textbook
4. Q 4.25 textbook

$$\varepsilon_i = \{P_i u + q_i : \|u\|_2 \leq 1\}, i = 1, .., K+L, P_i \in S^n$$

Find a feasible hyperplane strictly separating $\varepsilon_1, .., \varepsilon_K$ from $\varepsilon_{K+1}, .., \varepsilon_{K+L}$.

$$a^T x + b > 0, x \in \bigcup_{i=1}^{K} \varepsilon_i$$

$$a^T x + b < 0, x \in \bigcup_{i=K+1}^{K+L} \varepsilon_i$$

let $\epsilon > 0$, a constant for strict separation

relax inequalities to:

$$a^T x + b \leq -\epsilon, x \in \bigcup_{i=1}^{K} \varepsilon_i$$

$$a^T x + b \geq \epsilon, x \in \bigcup_{i=K+1}^{K+L} \varepsilon_i$$

$$a^T(P_i u + q_i) + b \leq -\epsilon, \|u\|_2 \leq 1, i = [1, K]$$

$$a^T(P_i u + q_i) + b \geq \epsilon, \|u\|_2 \leq 1, i = [K+1, K+L]$$

$$\sup_{\|u\|_2 \leq 1} a^T P_i u + a^T q_i + b \leq -\epsilon, i = [1, K]$$

$$\sup_{\|u\|_2 \leq 1} a^T P_i u = \frac{a^T P_i (a^T P_i)^T}{\|a^T P_i\|_2} = \|a^T P_i\|_2$$

$$\|a^T P_i\|_2 + a^T q_i + b \leq -\epsilon, i = [1, K]$$

$$\|a^T P_i\|_2 \leq -a^T q_i - b - \epsilon, i = [1, K]$$

$$\inf_{\|u\|_2 \leq 1} a^T P_i u + a^T q_i + b \geq \epsilon, i = [K+1, K+L]$$

$$\inf_{\|u\|_2 \leq 1} a^T P_i u = \frac{a^T P_i (-a^T P_i)^T}{\|a^T P_i\|_2} = -\|a^T P_i\|_2$$

$$-\|a^T P_i\|_2 + a^T q_i + b \geq \epsilon, i = [K+1, K+L]$$

$$\|a^T P_i\|_2 \leq a^T q_i + b - \epsilon, i = [K+1, K+L]$$

Second order cone formulation:

$$\begin{aligned}
&\min_{a,b} 0 \\
&\|a^T P_i\|_2 \leq -a^T q_i - b - \epsilon, i = [1, K] \\
&\|a^T P_i\|_2 \leq a^T q_i + b - \epsilon, i = [K+1, K+L] \\
&\text{where } \epsilon > 0
\end{aligned}$$

5. Q 4.30 textbook
6. Q 4.43(a-b) textbook