1. dual norm: $\|z\|_*:=\sup_x\{z^T x : \|x\|_p\leq 1\}$
   dual norm of $L_p$-norm is $L_q$-norm where $1/p+1/q=1 \implies p=\frac{q}{q-1}$
2. SVD: $A=U\Sigma V^T$
3. derivative: $f:\mathbb{R}\to\mathbb{R}, g:\mathbb{R}\to\mathbb{R}, h(x)=f(g(x))\ \nabla h(x)=g'(f(x))\nabla f(x)$
4. $A$ real symmetric, $(\forall v)v^T Av \geq 0 \implies A$ PSD $\iff A\geq 0$ $A$ real symmetric, $(\forall v)v^T Av > 0 \implies A$ PD $\iff A>0$
5. $S^n \equiv$ set of all symmtric matrices
6. $M_+^n \equiv$ set of all positive definite matrices (PD)
7. $S_+^n \equiv$ set of all symmtric positive semidefinite matrices (SPSD)
8. $S_+ +^n \equiv$ set of all symmtric positive definite matrices (SPD)
9. square root of PSD: take square root of eigenvalues
10. affine property: $(\forall\theta\ in\mathbb{R})\theta x_1+(1-\theta)x_2\in C$
11. affine combination: coefficients add to 1
12. convex property: constrain $\theta\in[0,1]$
13. convex combination: coefficients add to 1 and all non-negative
14. converse seprating hyperplane theorem: $C, D$ convex, $C$ open, $(\exists f)f$ affine, non-positive on $C$, non-negative on $D \implies C, D$ disjoint
15. separating hyperplane theorem: $C, D$ non-empty, $C\bigcap D=\emptyset \implies (\exists a\neq 0)a^T x\leq b$
16. strict separating between convex sets: $((\forall x\in C)a^T x<b)\wedge((\forall x\in D)a^T x>b) \implies (\exists x_0)a^T x=b$ is a strict separating hyperplane
17. strict separating hyperplane between point and and closed convex set: $C$ closed $\wedge x_0\notin C \implies ((\exists a\neq 0, b)a^T x_0<b\wedge(\forall x\in C)a^T x>b)$
18. operator norm: $\|X\|_{a,b}=sup\{\|Xu\|_a\colon \|u\|_b\leq 1\}, X\in\mathbb{R}^{m\times n}$
    item dual norm properties:
    - $K^*$ closed and convex
    - $K_1\subseteq K_2 \implies k_2^*\subseteq K_1^*$
    - $K$ has non-empty interior $\implies K^*$ pointed
    - $cl(K)$ pointed $\implies K^*$ has non-tempty interior
    - $K^{**}=cl(convhull(K))$
    - $K$ convex and closed $\implies K=K^{**}$

    see exercise 2.31

19. dual generalized inequality: $K$ is convex and proper cone $\implies K^*$ is proper $\implies \preceq_{K^*}$
20. minimum: $x$ is a minimum element of $S \iff (\forall\lambda : \lambda \succeq_{K^*} 0)(\lambda \in K^*) \wedge x = \min_{z\in S} \lambda^T z$
21. minimum: $(\forall\lambda \succ 0)\{z : \lambda^T(z - x) = 0\}$ is a strict supporting hyperplane (only intersects S at point x)
22. generalized inequaliy ($\preceq_K$), pg44:
    - preserved addition
    - transitive
    - preserved non-negative scaling
    - reflexive
    - antisymmetric
    - preserved under limits: $x_i \preceq_K y_i, \forall i, i \to \infty \implies x \preceq_K y, x_i \to x, y_i \to y$
23. generalized strict inequality:
    - relaxation to instrict inequality always possible
    - preservation under inequality addition: $a_K b, c_k d \implies c + a \prec_K b + d$
    - positive scaling: $\alpha A \prec_K \alpha B, \forall\alpha > 0$
    - irreflecive
    - existence of epsilon on both side: $x + \epsilon \prec y + \epsilon$
24. minimum element x in S (it is unique): $x \in S, (\forall y \in S)x \preceq y$
25. minimal element x in S: $x \in S, (\forall y \in S)y \preceq x \implies y = x$
    $(x - K)\bigcap S = \{x\}$
26. maximum element x in S (it is unique): $x \in S, (\forall y \in S)x \succeq y$
27. maximal element x in S: $x \in S, (\forall y \in S)y \succeq x \implies x = y$
    $(x + K)\bigcap S = \{x\}$
28. supporting hyperplane: $x_0 \in Bd(C)(\exists a \neq 0)(\forall x \in C)(a^T x \leq a^T x_0) \implies \{x : a^T x = a^T x_0\}$ is a separating hyperplane
29. separating hyperplane: A,B are convex, A is open, there is a separating hyperplane between A and B $\implies$ A and B are disjoint
30. closed convex set is equal to intersection of many halfspaces
31. operations preserving convex sets:
    - partial sum
    - sum

- coordination projection
- scaling
- translation
- intersection between any convex sets

32. norm cone: $K = \{(x,t) \in \mathbb{R}^{n+1} : \|x\| \leq t\}, x \in \mathbb{R}^n$
33. dual norm cone:
$\|u\|_* = sup\{u^T x : \|x\| \leq 1\}$
interpreted as norm of $u^T$
from dual norm definition:
$z^T x \leq \|x\| \|z\|_*$
can be tightened: given x, $(\exists z) z^T x = \|x\| \|z\|_*$
given z, $(\exists x) z^T x = \|x\| \|z\|_*$
34. convex function, secant line is above f in the evaluated domain
35. convex function, 1st order conditions:
$\nabla f$ exists for $dom(f)$
$f$ is open
$dom(f)$ is convex $\wedge (\forall x, y \in dom(f)) f(y) \geq f(x) + \nabla f(x)^T (y - x) \iff f$ is convex
1st order tayler expansion is a global underestimator of f
can use convex combination of any 2 points in the domain to prove 1st order condition (taylor approx)
36. extended convex function: use $+\infty$ when outside of the original $dom(f)$:
$\tilde{f}(x) = \begin{cases} f(x), & x \in dom(f) \\ \infty, & \text{otherwise} \end{cases}$
useful when doing intersections between multiple domains: any out of bound domain causes intersection to take on extended value of $\infty$
37. indicator function: $I_C(x) = \begin{cases} 0, & x \in C \\ \infty, & \text{otherwise} \end{cases}$
38. useful math properties:
    - $(a + x)^{-1} \approx 1 - x$
    - $\lim_{t \to 0} \frac{f(x + \epsilon t) - f(x)}{t} = \frac{\partial f(x)}{\partial x} \epsilon$