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Zeth Protocol Specification

2

Clearmatics Cryptography R&D

3

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Abstract

5 This document specifies the Zeth protocol with various security fixes and performance
6 improvements from the initial design [RZ19].

7 **Keywords**— Ethereum, Zerocash, Zcash, financial-privacy, zero-knowledge proofs,
8 Zeth, privacy-preserving state transitions

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107 Notation

108 Basic mathematical notation

109	\emptyset	The empty set, i.e. $\emptyset = \{\}$
110	$\#S$	The number of elements in the finite set S (also referred to as “cardinality of the
111		set S ”). By convention, $\#\emptyset = 0$
112	$x \in S$	Represents that x is an element of S . If x is a variable such that $x \in S$, we will
113		say that “ x has type S ”, i.e. the unordered collection of objects S represents all
114		the values that x can take
115	$S \setminus T$	Set difference of sets S and T , i.e. $S \setminus T = \{x \in S : x \notin T\}$ (voiced “the set of
116		elements x in S such that x is not in T ”)
117	$S \subseteq T$	S is a subset of T , i.e. $x \in S \Rightarrow x \in T$
118	$S \subset T$	S is a <i>proper</i> (or “strict”) subset of T , i.e. $x \in S \Rightarrow x \in T \wedge \exists y \in T, y \notin S$
119	$S = T$	$S \subseteq T \wedge T \subseteq S$
120	$S \cup T$	Union of set S and set T , i.e. $\{x : x \in S \vee x \in T\}$
121	$S \cap T$	Intersection of set S with set T , i.e. $\{x : x \in S \wedge x \in T\}$
122	$f: S \rightarrow T$	Function f that maps elements of the non-empty set S , the “domain”, to the
123		non-empty set T , the “codomain”
124	\mathbb{N}	Set of natural numbers. \mathbb{N}^+ represents $\mathbb{N} \setminus \{0\} = \{1, 2, \dots\}$, where $\{n, \dots\}$ rep-
125		resents the application of the successor operator $\text{Succ}(n) = n + 1$, defined by the
126		Peano axioms, infinitely many times
127	\mathbb{Z}	Set of integers, i.e. $\{\dots, -2, -1, 0, 1, 2, \dots\}$, where $\{\dots, n\}$ represents the appli-
128		cation of the predecessor operator $\text{Pred}(n) = n - 1$ infinitely many times
129	\mathbb{Q}, \mathbb{R}	Set of rational, real numbers
130	$[n]$	Set $\{0, \dots, n - 1\}$, where $n \in \mathbb{N}$
131	$\{a, \dots, b\}$	Set of integers from a through b inclusive, where $a \leq b$

132	$(a_0, a_1, \dots, a_{n-1})$	n -tuple, i.e. ordered collection of items of length n . If $n = 1$, we call
133		it a “singleton”, if $n = 2$, we call the tuple a “pair”. Finally, if $n = 3$, we call it
134		a “triple”. We use the terms “tuples” and “lists” interchangeably.
135	$S \times T$	Cartesian product of sets S and T , i.e. set of all ordered pairs $\{(x, y) : x \in S \wedge y \in T\}$
136	S^n	n -fold Cartesian product of S with itself, i.e. $S^n = \{(x_0, \dots, x_{n-1}) : x_i \in S \forall i \in$
137		$[n]\}$, where $n \in \mathbb{N}$
138	Λ	General notation for an alphabet, i.e. a <i>non-empty finite set</i> such that every string
139		(ordered collection of symbols, or letters, all in Λ) has a unique decomposition.
140		The number of symbols in a string is denoted the “length” of the string
141	ε	The empty string. ε is a string over any alphabet.
142	Λ^n	Set of all strings, defined over alphabet Λ , containing n symbols (i.e. “of length
143		n ”)
144	Λ^*	The Kleene star of Λ represents the set of all strings of finite length, defined over
145		alphabet Λ , including the empty string ε , i.e. $\Lambda^* = \bigcup_{n \in \mathbb{N}} \Lambda^n$
146	$\text{length}(x)$	$\text{length} : \Lambda^* \rightarrow \mathbb{N}$ computes the length of a string x defined over Λ , i.e. $\text{length}(x)$
147		returns the number of symbols composing the string x . By convention, $\text{length}(\varepsilon) =$
148		0
149	$x \parallel y$	Infix notation for the concatenation function, $\parallel : \Lambda^* \times \Lambda^* \rightarrow \Lambda^*$. If $\text{length}(x) =$
150		$n, \text{length}(y) = m$ and $(n, m) \in \mathbb{N}^2$, then for $z = x \parallel y$ holds $\text{length}(z) = n + m$
151	$\text{trunc}_x(k)$	$\text{trunc} : \Lambda^* \rightarrow \Lambda^k$ is the truncation function that returns the sequence formed
152		from the first k elements of x , where $x \in \Lambda^*$. If $k > \text{length}(x)$, then $\text{trunc}_x(k) = x$
153	$x[a:b]$	$[\cdot] : \Lambda^n \times \mathbb{N} \times \mathbb{N} \rightarrow \Lambda^{\leq b-a}$ is the slice function that, if $b \geq a$, returns the string
154		starting at index $\min(n, a)$ of x and finishing at index $\min(n, b)$. The function
155		additionally interprets $x[:b]$ as $x[0:b]$ and $x[a:]$ as $x[a:n]$
156	$\text{pad}_n(x)$	$\text{pad} : \Lambda^{\leq n} \rightarrow \Lambda^n$ is the padding function which pads x by 0’s to reach a size of
157		n . The padding depends on the variable type and endianness.
158	$\text{append}(l, x)$	$\text{append} : D^n \times D \rightarrow D^{n+1}$ is the algorithm that appends x to the list of n
159		element(s) l , if all x and l share the same data type D
160	\mathbb{B}	Alphabet of binary symbols, i.e. $\{0, 1\}$
161	$\langle \mathbf{g}_1, \dots, \mathbf{g}_l \rangle$	Cyclic group generated by $\{\mathbf{g}_1, \dots, \mathbf{g}_l\}$
162	$(q, \mathbb{G}, \mathbf{g}, \otimes)$	Description of the cyclic group $\mathbb{G} = \langle \mathbf{g} \rangle$ of order q , with operation \otimes
163	\mathbb{G}_{CUR}	Safe subgroup of the cyclic group induced by the set of points on the elliptic curve
164		Curve (i.e. elliptic curve subgroup suited for cryptographic use, in which hardness
165		assumptions hold)

166	$\mathbb{Z}/r\mathbb{Z}$	Quotient group defined as the set of equivalence classes modulo r . $\mathbb{Z}/r\mathbb{Z}$, also written \mathbb{Z}_r , is an additive group. If $r = p$ a prime number, then $\mathbb{Z}_p = \{0, \dots, p-1\} = \mathbb{Z}/p\mathbb{Z}$ is a finite field of elements modulo prime p , also denoted \mathbb{F}_p , where $0_{\mathbb{F}_p}$ and $1_{\mathbb{F}_p}$ respectively represent the additive and multiplicative identity
170	\mathbb{F}_q	Finite field of cardinality $q = p^m$, where p is prime, and $m \in \mathbb{N}$
171	$\llbracket x \rrbracket$	Represents the encoding of the scalar x in a group \mathbb{G} described as $(p, \mathbb{G}, \langle \mathbf{g} \rangle, \otimes)$, i.e. $\llbracket x \rrbracket = x \cdot \llbracket 1 \rrbracket = \mathbf{g} \otimes \dots \otimes \mathbf{g}$ (x times). Thus, by convention, $\llbracket 1 \rrbracket = \mathbf{g}$
173	\bullet	Represents an inline operator for bilinear pairing. That is for a bilinear pairing from $\mathbb{G}_1 \times \mathbb{G}_2$ to \mathbb{G}_T and elements $\llbracket a \rrbracket_1, \llbracket b \rrbracket_2$ we write $\llbracket ab \rrbracket_t = \llbracket a \rrbracket_1 \bullet \llbracket b \rrbracket_2$
175	$\lceil x \rceil$	Round $x \in \mathbb{R}$ to the next integer
176	$\lfloor x \rfloor$	Round $x \in \mathbb{R}$ to the previous integer
177	$\log_b(x)$	Logarithm with respect to base b , i.e. $x = b^y, \log_b(x) = y$
178	Algorithmic notation	
179	$x \leftarrow \$ \mathcal{X}$	Element chosen uniformly at random from set \mathcal{X}
180	$x \leftarrow y$	The value y is assigned to the variable x (i.e. “ x receives the value y ”)
181	PPT	Probabilistic polynomial time. A polynomial time algorithm A is one for which there exists a polynomial f such that the running time of A on input $x \in \{0, 1\}^*$ is $f(x)$. A probabilistic algorithm has the ability to “flip” random coins and use the result of these coin tosses in its computation
185	NUPPT	Non-uniform probabilistic polynomial time
186	$\mathcal{O}(f)$	Big-O notation
187	il, kl, nl, rl, ol	The input il, key kl, nonce nl, randomness rl and output ol length
188	Cryptography notation	
189	$\mathcal{O}^X(n)$	Public oracle for algorithm X which can be accessed at most n times; \mathcal{O}^X is an unrestricted oracle for algorithm X
191	λ	Security parameter ($\lambda \in \mathbb{N}$)
192	negl	Negligible function. In this document, negligible will usually mean $\mathcal{O}(2^{-\lambda})$
193	poly	Polynomial function
194	\mathcal{A}	Adversary algorithm

195	$\text{Adv}_{F,\mathcal{A}}^{\text{prop}}(\lambda)$	Advantage of the adversary \mathcal{A} with regard to the attack game prop on F	
196		(e.g. F can be a function, a family of functions or a group on which a given	
197		property represented by the game prop is supposed to hold)	
198	$\text{prop}^{\mathcal{A}}$	Adversary \mathcal{A} running a security game prop	
199	Zeth notation		
200	π	Output of the proving algorithm of a zk-SNARK scheme. π is also informally	
201		referred to as a “zk-SNARK proof”, “zk-proof”, or simply “proof”	
202	$\mathcal{P}_{\mathcal{Z}}$	Standard notation for a Zeth user	
203	$\widetilde{\text{Mixer}}$	The mixer smart-contract instance	
204	EncSch	In-band encryption scheme used to share Zeth notes	
205	Ethereum notation		
206	Account	Standard notation for an Ethereum account object	
207	$\widetilde{\text{Cntrct}}$	Standard notation for an Ethereum smart-contract instance	
208	$\mathcal{P}_{\mathcal{E}}$	Standard notation for an Ethereum user	
209	ς	Mapping representing the Ethereum state (i.e. “World state”)	
210	$\varsigma[a]$	Account object stored at address a in ς if it exists, \perp is returned otherwise	
211	Constants		
212	ADDRLEN	The bit-length of an Ethereum address	160 <i>bits</i>
213	BLAKE2sCLEN	Output size of Blake2s compression function [ANWOW13]	256 <i>bits</i>
214	FIELD_{BLS}CAP	Field capacity of $\mathbb{F}_{\mathbf{r}_{\text{BLS}}}$.	$\lfloor \log_2 \mathbf{r}_{\text{BLS}} \rfloor = 252$ bits
215	FIELD_{BLS}LEN	Bit-length of a field element $x \in \mathbb{F}_{\mathbf{r}_{\text{BLS}}}$	$\lceil \log_2 \mathbf{r}_{\text{BLS}} \rceil = 253$ bits
216	FIELD_{BN}CAP	Field capacity of $\mathbb{F}_{\mathbf{r}_{\text{BN}}}$.	$\lfloor \log_2 \mathbf{r}_{\text{BN}} \rfloor = 253$ bits
217	FIELD_{BN}LEN	Bit-length of a field element $x \in \mathbb{F}_{\mathbf{r}_{\text{BN}}}$	$\lceil \log_2 \mathbf{r}_{\text{BN}} \rceil = 254$ bits
218	BYTELEN	Bit-length of a byte	8 <i>bits</i>
219	ENCZETHNOTELEN	Size of an encrypted note (see Section 3.5.3)	$\text{CTBYTELEN} * \text{BYTELEN}$ <i>bits</i>
220	ETHWORDLEN	Width of a storage cell on the Ethereum Virtual Machine stack, i.e. size of	
221		a word on the EVM	256 <i>bits</i>

222	FIELD CAP	Field capacity of $\mathbb{F}_{r_{\text{CUR}}}$, defined as the maximum bit length l such that all	
223		numbers x encoded on l bits are elements of $\mathbb{F}_{r_{\text{CUR}}}$. In other words, $\text{FIELD CAP} =$	
224		$\max_{x \in \mathbb{F}_{r_{\text{CUR}}}} \{\lceil \log_2 x \rceil\}$ s.t. $\sum_{i \in [\text{FIELD CAP}]} 2^i \in \mathbb{F}_{r_{\text{CUR}}}$	
225	FIELD LEN	Bit-length of elements in field element $x \in \mathbb{F}_{r_{\text{CUR}}}$	$\lceil \log_2 r_{\text{CUR}} \rceil$ bits
226	JSIN, JSOUT, JS MAX	The number of inputs and outputs of a joinsplit and $\text{JS MAX} = \max \{\text{JSIN}, \text{JSOUT}\}$	
227	KEK256 DLEN	Message digest size of Keccak256 [GJMG11]	256 bits
228	MKDEPTH	The depth of the Merkle tree used to store commitments	
229	p_{BLS}	Characteristic of the prime (base) finite field over which curve BLS12-377 is de-	
230		defined, $p_{\text{BLS}} = 2586644260129690940106527336948935335363935127549146605398$	
231		$84262666720468348340822774968888139573360124440321458177$ [BCG ⁺ 20]	
232	p_{BN}	Characteristic of the prime (base) finite field over which curve BN-254 is defined,	
233		$p_{\text{BN}} = 218882428718392752224640574525727508869631115729782366268903789$	
234		4645226208583 [Rk19]	
235	p_{SECP}	Characteristic of the prime (base) finite field over which curve secp256k1 is de-	
236		defined, $p_{\text{SECP}} = 115792089237316195423570985008687907853269984665640564039$	
237		457584007908834671663 [wik]	
238	r_{BLS}	Characteristic of the scalar field of BLS12-377, $r_{\text{BLS}} = 84444617494283704242488$	
239		$24938781546531375899335154063827935233455917409239041$ [BCG ⁺ 20]	
240	r_{BN}	Characteristic of the scalar field of BN-254, $r_{\text{BN}} = 2188824287183927522246405$	
241		$745257275088548364400416034343698204186575808495617$ [Rk19]	
242	r_{CUR}	Characteristic of the scalar field of some chosen curve Curve	
243	r_{SECP}	Characteristic of the scalar field of secp256k1, $r_{\text{SECP}} = 1157920892373161954235$	
244		$70985008687907852837564279074904382605163141518161494337$ [wik]	
245	SHA256 BLEN	Block size of SHA256 [oST15, Figure 1]	512 bits
246	SHA256 DLEN	Message digest size of SHA256 [oST15, Figure 1]	256 bits
247	SHA256 MLEN	Message size of SHA256 [oST15, Figure 1]	$< 2^{64}$ bits
248	DGAS	The default/intrinsic cost of an Ethereum transaction	21000 gas
249	ZVALUE LEN	Size in bits of the transferable maximal value	64 bits

Change log

- **Version:** 0.0, **Date:** 04/12/2019, **Contributor:** Antoine Rondelet, **Description:** Creation of the document. Established initial table of content and started to populate sections with bullet lists to develop in further versions of the document.
- **Version:** 0.1, **Date:** 20/12/2019, **Contributor:** Antoine Rondelet, **Description:** Refactored the structure of the document. Finalized the table of content, wrote sections on notations and preliminaries, and introduced the content related to the malleability fix.
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 - **Date:** 04/03/2020, **Contributor:** Raphael Toledo, **Description:** Wrote section on the packing policy and corresponding attack.
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 - **Date:** 24/03/2020, **Contributor:** Raphael Toledo, **Description:** Changed the PRF and commitment instantiation with Blake2s compression function.
 - **Date:** 17/04/2020, **Contributor:** Giuseppe Giffone, **Description:** Added DHAES encryption scheme.
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281 added examples, figures and merged sections 5 and 6 of Chapter 1 for clarity, added
282 missing references.

283 • **Version:** 1.1, **Date:** 06/11/2020, **Contributor:** Duncan Tebbs, **Description:**
284 Specification in terms of a generic curve, with constants provided for BN-254 and
285 BLS12-377. Some clarification and grammar fixes.

286 • **Version:** 1.2, **Date:** 27/01/2021, **Contributor:** Antoine Rondelet, Michal Zajac,
287 **Description:** Fix erroneous prime fields used in data types, added elliptic curve
288 group notations, fixed erroneous Groth16 formulas and notation inconsistencies
289 between group and field elements.

Chapter 1

Preliminaries

Zeth is a protocol which enables private transactions on **Ethereum** [Woo19]. It is a modification of the Decentralized Anonymous Payment (DAP) system **ZeroCash** [BSCG⁺14]. The design described in [RZ19] presents the mechanisms by which **ZeroCash** can be used on **Ethereum**, and argues that the information leakages of the solution are well defined and controlled. This document, however, serves as a specification of the protocol and provides security fixes and optimizations from the first proof of concept release of the protocol [Cle19].

This document assumes familiarity with blockchain and **Ethereum** in particular. It does not, in any way, aim at replacing the Ethereum yellow paper [Woo19]. The reader is strongly advised to read about **Ethereum** before delving into this specification document.

The key words **MUST**, **MUST NOT**, **SHOULD**, **SHOULD NOT**, **MAY**, and **RECOMMENDED** in this document are to be interpreted as described in [Bra97] when they appear in **ALL CAPS**. These words may also appear in this document in lower case as plain English words, absent their normative meanings.

1.1 Data structures and representation

1.1.1 Structured data

When describing the operations to be performed and the data to be manipulated as part of the protocol, we commonly employ tuples of related data where each element of the tuple has some associated semantic meaning and which must often satisfy some conditions. In this section, we develop a framework to reason about such *structured* data, where a single datum may consist of one or more logical parts (called *fields*). The framework is built on top of simple mathematical concepts such as sets, and mappings between them, ensuring that we can always reason about structured data in a rigorous way. We also define notation to aid the specification of structured data, and to refer to specific components of a datum. This will be used extensively in the specification of the protocol.

318 As a simple motivating example, consider a protocol that processes data relating to
 319 individual people. This fictional system may send and receive data such as *name*, *age*
 320 and *address* for a single person, grouping this data into a logical unit. Further, each
 321 piece of data must satisfy specific conditions (*name* must be a series of characters from
 322 some alphabet, *age* must be a positive integer, etc.) We shall make use of this example
 323 several times during the formulation below.

324 In what follows, let $\text{STR} = \{a, b, \dots, y, z\}^*$ (the Kleene star of the *Roman alphabet*).
 325 In our formulation, field names f_i will be elements in this set.

326 **Remark 1.1.1.** Note that a similar formulation could be made using an arbitrary set,
 327 such as the same alphabet augmented with specific symbols, or the alphabet of a different
 328 language. Our choice of STR here is for simplicity.

329 We begin by defining a data type as a set of values called “fields”, each with a “name”
 330 from STR . Abstract sets are used to constrain the values of each field.

Definition 1.1.2 (Structured Data Type). Let f_0, \dots, f_{n-1} be n distinct elements of STR and let V_0, \dots, V_{n-1} be sets, for some $n \in \mathbb{N}$. We define the *structured data type* \mathbf{T} with fields $\{(f_i, V_i)\}_{i \in [n]}$ to be a set of values:

$$\mathbf{T} = V_0 \times \dots \times V_{n-1}$$

with associated post-fix “dot” operators $.f_i : \mathbf{T} \rightarrow V_i$ for $i = 0, \dots, n-1$, acting on values $\mathbf{x} \in \mathbf{T}$ to extract the individual elements:

$$\mathbf{x}.f_i = v_i, \text{ where } \mathbf{x} = (v_0, \dots, v_{n-1}) \in \mathbf{T}$$

331 Here, we say that the i -th field has *field name* f_i , with *value set* V_i . Each “dot”
 332 operator $.f_i$ *extracts* the i -th component, or the *value with field name* f_i .

Example 1.1.3. Consider our example protocol that processes information about people. A potentially useful structured data type **Person** may be defined with fields:

$$\{(name, \text{STR}), (age, \mathbb{N}), (height, \mathbb{R}^+)\}$$

333 Values \mathbf{p} in **Person** are simply tuples in $\text{STR} \times \mathbb{N} \times \mathbb{R}^+$, with semantic meaning (name,
 334 age, height) assigned to each component of \mathbf{p} .

Examples of valid elements in **Person** include $\mathbf{a} = (alice, 28, 1.65)$ and $\mathbf{b} = (bob, 31, 1.74)$, where the following equalities hold:

$$\begin{aligned} \mathbf{a}.name &= alice, \\ \mathbf{b}.age &= 31, \\ \mathbf{b}.height &= 1.74; \end{aligned}$$

335 For clarity, structured data types may be specified using tables of names, descriptions
 336 and value sets, rather than sets of the form $\{(f_i, V_i)\}_{i \in [n]}$. Similarly, it is frequently
 337 convenient to include the *field names* alongside values when specifying structured data
 338 values.

339 **Example 1.1.4.** Person from Example 1.1.3 might be described in table-form as follows:

Field	Description	Data type
<i>name</i>	Name of the person	STR
<i>age</i>	Age in years	\mathbb{N}
<i>height</i>	Height in meters	\mathbb{R}^+

Example 1.1.5. The values **a** and **b** in Example 1.1.3 might be written as follows:

$$\begin{aligned}\mathbf{a} &= \{name : \textit{alice}, age : 28, height : 1.65\} \\ \mathbf{b} &= \{name : \textit{bob}, age : 31, height : 1.74\}\end{aligned}$$

340 **Remark 1.1.6** (“dot” operators in assignment). The “dot” operators may be used in
341 algorithm descriptions to indicate *assignment to a specific component*. For example
342 $\mathbf{a}.age \leftarrow 29$ means that the value of the *age* field of **a** is replaced by the value 29.

Formally, for a structured data type **T** with fields $\{(f_i, V_i)\}_{i \in [n]}$ where $\mathbf{x} = (v_0, \dots, v_{n-1}) \in \mathbf{T}$ and $v'_i \in V_i$:

$$\mathbf{x}.f_i \leftarrow v'_i$$

is equivalent to:

$$\mathbf{x} \leftarrow (v_0, \dots, v_{i-1}, v'_i, v_{i+1}, \dots, v_{n-1})$$

343 We define one further operator and related assignment notation, convenient in cases
344 where $V_i = X^m$ for sets X and $m \in \mathbb{N}$.

Definition 1.1.7 (Square bracket operator). For $m \in \mathbb{N}$ and set X , define the operator $[] : X^m \times [m] \rightarrow X$ as:

$$\mathbf{x}[i] = x_i \text{ where } \mathbf{x} = (x_0, \dots, x_m)$$

For the set X^* , the operator takes the form $[] : X^* \times \mathbb{N} \rightarrow X$, defined as:

$$\mathbf{x}[i] = \begin{cases} x_i & \text{if } \text{length}(\mathbf{x}) > i \text{ where } \mathbf{x} = (x_0, \dots) \\ \perp & \text{otherwise} \end{cases}$$

Remark 1.1.8 (Square bracket operators in assignment). Similarly to Remark 1.1.6, we develop assignment notation for the square bracket operator $[]$. Let $\mathbf{x} = (x_0, \dots, x_{m-1})$ be a member of X^m , and x'_i be some element in X . The statement:

$$\mathbf{x}[i] \leftarrow x'_i$$

is equivalent to:

$$\mathbf{x} \leftarrow (x_0, \dots, x_{i-1}, x'_i, x_{i+1}, \dots, x_{m-1})$$

345 Informally, this can be interpreted as replacing the i -th component of \mathbf{x} with x'_i .

346 **Remark 1.1.9** (Deep structures and chained “dot” operators). Consider the case of
 347 structured data \mathbf{T} with fields $\{(f_i, V_i)\}_{i \in [n]}$ for $n \in \mathbb{N}$. Let \mathbf{T}' be another structured data
 348 type with fields $\{(f'_i, V'_i)\}_{i \in [n']}$ for $n' \in \mathbb{N}$, and assume that $V_j = \mathbf{T}'$ for some $j \in [n]$.
 349 Informally, the values of the j -th field of elements of \mathbf{T} are themselves structured data
 350 of type \mathbf{T}' .

351 In this case, “dot” operators may be *chained*, so that $\mathbf{x}.f_j.f'_k$ refers to the k -th field
 352 v'_k of the j -th field v_j of $\mathbf{x} \in \mathbf{T}$.

353 **Example 1.1.10.** Define a structured data type **Address** with fields $(country, \text{STR}), (zipcode, \text{STR})$.
 354 We redefine the structured data type **Person** from Example 1.1.3, with an extra field
 355 *address* of type **Address**. That is, **Person** is the structured data type with fields:

Field	Description	Data type
<i>name</i>	Name of the person	STR
<i>age</i>	Age in years	\mathbb{N}
<i>height</i>	Height in meters	\mathbb{R}^+
<i>address</i>	Address of the person	Address

An example element \mathbf{a} in **Person** is:

$$\mathbf{a} = \{ \begin{array}{l} name : \textit{alice}, \\ age : 28, \\ height : 1.65, \\ address : (country : UK, zipcode : SW1A) \end{array} \}$$

In this case, the following equalities using the dot and square bracket operators all hold:

$$\begin{aligned} \mathbf{a}.name &= \textit{alice} \\ \mathbf{a}.height &= 1.65 \\ \mathbf{a}.address.country &= UK \\ \mathbf{a}.address.zipcode &= SW1A \\ \mathbf{a}.address.country[1] &= K \end{aligned}$$

356 1.1.2 Representations

357 The binary alphabet $\{0, 1\}$, denoted \mathbb{B} , is used to represent the presence or absence of an
 358 electrical signal in a computer. In fact, every piece of information in a computer is rep-
 359 resented as a string of bits. We assume the existence of an efficient binary representation
 360 for some set of primitive datatypes (such as the natural numbers \mathbb{N} , or alphanumeric

characters). Structured data types built up from primitive types (as described above) can then recursively be assigned similarly efficient representations. This is used to define the following functions to *encode* data to its bit-string representation, and to *decode* such bit-strings back to elements of the original type.

Definition 1.1.11. For a set X of values which are to be represented as bit strings, we define functions:

$$\begin{aligned}\text{encode}_X &: X \rightarrow \mathbb{B}^* \\ \text{decode}_X &: \mathbb{B}^* \rightarrow X \cup \perp\end{aligned}$$

satisfying

$$\text{decode}_X(\text{encode}_X(x)) = x \quad \forall x \in X$$

to be the functions which encode (resp. decode) elements of X into (resp. from) the bit-string representations chosen above. We note that decode_X may return \perp in the case that the input bit-string is malformed.

Without ambiguity, we overload the functions **encode** and **decode** to mean encode_X and decode_X where the set X is clear from context.

In the following sections, we assume that elements of \mathbb{N} are encoded as big-endian binary numbers in the natural way. We denote by \mathbb{N}_b the set of natural numbers that can be uniquely encoded in this way using b bits (possibly with padding). In other words,

$$\mathbb{N}_b = \left\{ x \in \mathbb{N} \text{ s.t. } \text{encode}_{\mathbb{N}}(x) \in \mathbb{B}^b \right\}$$

1.2 Ethereum

In a nutshell, **Ethereum** is a distributed deterministic state machine, consisting of a globally accessible singleton state (“the World state”) and a virtual machine that applies changes to that state [AG18]. State transitions in the state machine are represented by transactions on the system. As such, each transaction represents a change in the global state represented as a Merkle Patricia Tree [wc] whose nodes are objects called “accounts” (Section 1.2.1). The Ethereum Virtual Machine (EVM) allows state transitions to be specified by creating a type of accounts which are associated with a piece of code (smart-contracts). The code of such accounts, and so, the corresponding state transitions, can be executed to transition to another state in the automata, by creating a transaction that calls the given piece of code (Section 1.2.2).

To prevent unbounded state transitions in the state machine, each instruction executed by the EVM is associated with a cost in **Wei**, referred to as “the gas necessary to run the operation”. The “gas cost” of a transaction needs to be paid by the transaction originator (deduced from their account balance), and is awarded to the miner (added to their account balance) who successfully mines the block containing the transaction. In addition to the cost of every instruction executed as part of a state transition, every

transaction has an intrinsic cost of `DGAS` gas [Woo19, Appendix G]. Bounding modifications to the `Ethereum` state by the amount of `Wei` held in the transaction originator’s account allows the system to avoid the Halting problem¹ and protects against a range of Denial of Service (DOS) attacks.

1.2.1 Ethereum account

An `Ethereum` account [Woo19, Section 4.1] is an object containing 4 attributes, as represented Table 1.1. We distinguish two types of accounts:

- “Externally Owned Accounts” (EOA), that are created by derivation of an ECDSA secret key; and
- Smart-contract accounts, that are derived from EVM code specifying a state transition on the state machine.

Each account object is accessible in the Merkle Patricia Tree representing the “World state” by a unique `ADDRLEN`-bit long identifier called the address. In the context of EOA, the address is obtained by generating a new ECDSA [JMV01] key pair (sk, vk) over curve `secp256k1` [Qu99] and taking the rightmost `ADDRLEN` bits of the `Keccak256` hash of the verification key vk .

Field	Description	Data type
<i>nce</i>	The nonce of an account is a scalar value representing the number of transactions that have originated from the account, starting at 0.	$\mathbb{N}_{\text{ETHWORDLEN}}$
<i>bal</i>	The balance of an account is a scalar value representing the amount of <code>Wei</code> in the account.	$\mathbb{N}_{\text{ETHWORDLEN}}$
<i>sRoot</i>	The storage root is the <code>Keccak256</code> hash representing the storage of the account.	$\mathbb{B}^{\text{KEK256DLEN}}$
<i>codeh</i>	The code hash is the hash of the EVM code governing the account. If this field is the <code>Keccak256</code> hash of the empty string, then the account is said to be an “Externally owned Account” (EOA), and is controlled by the corresponding ECDSA private key. If, however, this field is not the <code>Keccak256</code> hash of the empty string, the account represents a smart contract whose interactions are governed by its EVM code.	$\mathbb{B}^{\text{KEK256DLEN}}$

Table 1.1: Ethereum Account structure

¹https://en.wikipedia.org/wiki/Halting_problem

Note

In the rest of this document, we will refer to an *Ethereum user* $\mathcal{U}_{\mathcal{E}}$ as a person, modeled as an object, holding *one*^a secret key, sk (object attribute), associated with an existing EOA in the “World state”. We denote by $\mathcal{U}_{\mathcal{E}}.Addr$ the **Ethereum** address of $\mathcal{U}_{\mathcal{E}}$ derived from $\mathcal{U}_{\mathcal{E}}.sk$, and which allows $\mathcal{U}_{\mathcal{E}}$ to access the state of their account $\varsigma[\mathcal{U}_{\mathcal{E}}.Addr]$.

We denote by $\widetilde{\mathbf{SmartC}}$ a smart-contract instance/object (i.e. deployed smart-contract with an address, Section 1.2.2), and denote by $\widetilde{\mathbf{SmartC}}.Addr$ its address.

^aThe same physical person may correspond to multiple “Ethereum users” and thus control multiple accounts in the Merkle Patricia Tree.

1.2.2 Ethereum transaction

We now briefly mention what **Ethereum** transactions [Woo19, Section 4.2] are, and how they are created, signed and validated. Once more, the reader is highly encouraged to refer to [Woo19] for a detailed presentation. Informally, a transaction object (tx) is a signed message originating from an **Ethereum** user $\mathcal{U}_{\mathcal{E}}$ (the *transaction originator*, or simply *sender*) that represents a state transition on the distributed state machine (i.e. a change in the “World state” ς).

Raw transaction

In the following, we define a raw transaction as an unsigned transaction (Table 1.2).

Field	Description	Data type
nce	Transaction nonce	$\mathbb{N}_{\text{ETHWORDLEN}}$
$gasP$	gasPrice	$\mathbb{N}_{\text{ETHWORDLEN}}$
$gasL$	gasLimit	$\mathbb{N}_{\text{ETHWORDLEN}}$
to	Recipient’s address	$\mathbb{B}^{\text{ADDRLEN}}$
val	Value of the transaction in Wei	$\mathbb{N}_{\text{ETHWORDLEN}}$
$init / data$	Contract Creation data $init$ Message call data $data$	\mathbb{B}^*

Table 1.2: Structure of a *raw transaction data type* TxRawDType

Finalizing raw transactions

A raw transaction needs to be finalized to be accepted. In the context of this document, “finalizing a raw transaction” will be a synonym of “signing a raw transaction”. The transaction structure is represented in Table 1.3.

Field	Description	Data type
tx_{raw}	Raw transaction object	TxRawDType
v	Field v of ECDSA signature used for public key recovery	$\mathbb{B}^{\text{BYTELEN}}$
r	Field r of ECDSA signature [Por13]	$\mathbb{F}_{\text{rSECP}}$
s	Field s of ECDSA signature [Por13]	$\mathbb{F}_{\text{rSECP}}$

Table 1.3: Structure of a (finalized) *transaction data type* TxDType

We define the transaction generation function, cf. Fig. 1.1, as the function taking the sender’s ECDSA signing key and the components of a raw transaction as arguments, and returning a signed (or finalized) transaction (tx_{final} or tx for short).

$$\begin{aligned}
tx_{final} &= \text{TxGen}(sk_{\text{ECDSA}}, nce_{in}, gasP_{in}, gasL_{in}, to_{in}, val_{in}, init_{in}, data_{in}) \\
tx_{final} &= \{ \\
&\quad \left. \begin{array}{ll} nce & : nce_{in}, \\ gasP & : gasP_{in}, \\ gasL & : gasL_{in}, \\ to & : to_{in}, \\ val & : val_{in}, \\ init/data & : init_{in}/data_{in}, \end{array} \right\} tx_{raw} \\
&\quad \left. \begin{array}{ll} v : \sigma_{\text{ECDSA}}.v, \\ r : \sigma_{\text{ECDSA}}.r, \\ s : \sigma_{\text{ECDSA}}.s \end{array} \right\} \sigma_{\text{ECDSA}} \\
&\}
\end{aligned}$$

417 To sign a transaction, the sender first computes the hash of the raw transaction using
418 Keccak256, cf. Eq. (1.1), and then uses their ECDSA signing key, sk_{ECDSA} , to sign the
419 obtained digest. cf. Eq. (1.2). The signature is then appended to the raw transaction to
420 obtain a finalized transaction, cf. Fig. 1.1.

$$digest_{\text{ECDSA}} = \text{Keccak256}(nce_{in}, gasP_{in}, gasL_{in}, to_{in}, val_{in}, init_{in}/data_{in}) \quad (1.1)$$

$$\sigma_{\text{ECDSA}} = \text{SigSch}_{\text{ECDSA}}.\text{Sig}(sk_{\text{ECDSA}}, digest_{\text{ECDSA}}) (= (v, r, s)) \quad (1.2)$$

```

TxGen( $sk_{\text{ECDSA}}, nce_{in}, gasP_{in}, gasL_{in}, to_{in}, val_{in}, init_{in}, data_{in}$ )
1 : if  $to_{in} = \emptyset$  do
2 :    $tx_{raw} \leftarrow \{nce : nce_{in}, gasP : gasP_{in}, gasL : gasL_{in}, to : to_{in}, val : val_{in}, init : init_{in}\};$ 
3 : else
4 :    $tx_{raw} \leftarrow \{nce : nce_{in}, gasP : gasP_{in}, gasL : gasL_{in}, to : to_{in}, val : val_{in}, data : data_{in}\};$ 
5 : endif
6 :  $\sigma_{\text{ECDSA}} \leftarrow \text{SigSch}_{\text{ECDSA}}.\text{Sig}(sk_{\text{ECDSA}}, \text{Keccak256}(tx_{raw}));$ 
7 :  $tx_{final} \leftarrow \{tx_{raw}, v : \sigma_{\text{ECDSA}}.v, r : \sigma_{\text{ECDSA}}.r, s : \sigma_{\text{ECDSA}}.s\};$ 
8 : return  $tx_{final};$ 

```

Figure 1.1: Transaction generation function TxGen

Remark 1.2.1. As one can see, there is no “from” attribute in a transaction. The sender’s Ethereum address can be recovered from the ECDSA signature. This method is defined in the Ethereum yellow paper as a “sender function” S [Woo19, Appendix F] which maps each transaction to its sender.

Types of transactions

While only two types of transactions are described in [Woo19, Section 4.2]; namely those which result in message calls and those which result in the creation of new accounts with associated code, we will instead differentiate the types of transactions based on their purpose. The reader is encouraged to read [Woo19] for a formal discussion.

Informally, a transaction can be used to achieve three things: transferring Wei from an EOA to another EOA, creating a new account with associated code (i.e. “deploying a smart-contract”), and calling a function of a smart-contract. We will detail here the differences between these usages.

Creating a contract The $tx.to$ address is set to \emptyset in the transaction. The contract creation data ($tx.init$) includes the new contract’s code. The contract address is computed as the rightmost ADDRLEN bits of the Keccak256 hash of the RLP encoding [wc19] of the transaction originator’s address and account nonce [Woo19, Section 6].

Calling a contract function The $tx.to$ address is set to the address of the contract. The message call data byte array ($tx.data$) is set to the contract’s function address (or “Function Selector” [abi]) which are the first 4 bytes of the Keccak256 hash of the function signature, and the function input arguments (ETHWORDLEN bits per input) [Woo19, Section 8].

Transferring Wei from an EOA to another EOA This corresponds to a “plain transaction” spending Wei from an address to send them to another. In that case the $tx.to$ address corresponds to the recipient’s address while the transaction data is left empty.

Note

In order to keep notations simple, we assume, in the rest of the document, that smart-contract functions are uniquely determined by their name. As such, we denote by $\text{FS}(\cdot): \mathbb{B}^* \rightarrow \mathbb{B}^{4 \cdot \text{BYTELEN}}$ the function that takes a function name as input and returns its function selector.

Transaction validity

Importantly, not all finalized transactions constitute valid state transitions on the state machine [Woo19, Section 6]. We denote by `EthVerifyTx` the function that takes an `Ethereum` transaction object tx as input and return `true` (resp. `false`) if tx is valid (resp. invalid). To be deemed valid, a transaction **MUST** satisfy *all* the following conditions:

1. The transaction is correctly RLP encoded, with no additional trailing bytes;
2. the transaction signature (v, r, s) is valid;
3. the transaction nonce $(tx.nce)$ is valid, i.e. it is equal to the account nonce of the transaction originator;
4. the gas limit is no smaller than the gas used by the transaction;
5. the transactor has enough funds on his account balance to cover at least the cost $tx.val + tx.gasP \cdot tx.gasL$.

Lifecycle of a transaction, and miners' incentives

After the creation of an `Ethereum` transaction tx by a user from an `Ethereum` client (machine running a piece of software that enables to be connected to the `Ethereum` network), the transaction is broadcasted to the network and received by a set of peers/nodes.

The transaction is then stored in each node's transaction pool, which is a data structure containing all transactions that should be validated (pending transactions) by the node and mined. To maximize miners' returns, the transaction pools are ordered according to the gas price of the transactions. As such, transactions with the highest $tx.gasP$ are subject to be validated and included into a block first. Once tx is selected from the transaction pool, it is validated (fed into `EthVerifyTx`), executed, and included into a block (i.e. "mined"). The block is then broadcasted to all the nodes of the network and is used as the predecessor for the next block to be mined on the network (i.e. "it is added to the chain").

1.2.3 Ethereum events and Bloom filters

The EVM contains the set of "LOGX" instructions enabling smart-contract functions to "emit events" (i.e. log data) when they are executed²

²see <https://ethgastable.info/>

As such, when a block is generated by a miner or verified by the rest of the network, the address of any logging contract, and all the indexed fields from the logs generated by executing those transactions are added to a Bloom filter [Blo70], which is included in the block header [Woo19, Section 4.3]. Importantly, the actual logs *are not included in the block data* in order to save space. As such, when an application wants to find (“consume”) all the log entries from a given contract, or with specific indexed fields (or both), the node can quickly scan over the header of each block, checking the Bloom filter to see if it may contain relevant logs. If it does, *the node re-executes the transactions from that block, regenerating the logs, and returning the relevant ones to the application* [Joh16].

Note

The ability for a smart-contract function to “emit” some pieces of data when executed, and for an application to “consume” such pieces of data, is used in Zeth in order to construct a *confidential receiver-anonymous channel* [KMO⁺13].

1.3 zk-SNARKs

In this section we introduce notions necessary to understand zero-knowledge proofs, define properties crucial for them, and introduce zk-SNARKs. We refer the reader to Section 3.6 in which we describe the zk-SNARK scheme used in Zeth.

1.3.1 Preliminary definitions

NP class of languages. Since the considered proof systems are designed to work with languages in NP we begin with defining this class. Intuitively, a language \mathbf{L} belongs to NP if for each element $prim$ from the language there is a short witness aux that allows to efficiently³ verify that in fact $prim \in \mathbf{L}$.

Definition 1.3.1 (NP class of languages, cf. [Gol01]). We say that a language \mathbf{L} belongs to a class NP if there exist a polynomial p and a Turing machine M such that for every primary input $prim \in \{0, 1\}^*$, $prim \in \mathbf{L}$ iff there exists an auxiliary input aux such that M accepts the pair $(prim, aux)$ in time at most $p(\text{length}(prim))$.

The set of all pairs $(prim, aux)$ acceptable by M constitutes an NP relation \mathbf{R} corresponding to the language \mathbf{L} .

Non-interactive zero knowledge. A non-interactive zero-knowledge proof system NIZK for an NP language \mathbf{L} is a tuple of four algorithms $\text{NIZK} = (\text{KGen}, P, V, \text{Sim})$. NIZK for a language \mathbf{L} and instance $prim \in \mathbf{L}$ allows a party, called prover and denoted by P , to convince another party, called verifier and denoted by V , that $prim \in \mathbf{L}$ and nothing else.

Without loss of generality, we focus on zk-proof systems that are universal, that is, are able to work with any given NP relation \mathbf{R} . To that end, we define a *relation*

³Informally we say that an algorithm is efficient if it runs in time polynomial in the size of its inputs.

generator \mathcal{R} that on input 1^λ (i.e. the security parameter represented in unary) outputs an NP relation \mathbf{R} . We assume that the security parameter λ can be easily deduced from \mathbf{R} .

We require from a NIZK to have three substantial properties, cf. [Gro06]:

Completeness that assures that an honest prover, who proves that $\text{prim} \in \mathbf{L}$ succeeds, i.e. gets his proof accepted by the verifier \mathbf{V} . Formally we require that for any λ , $\mathbf{R} \leftarrow \mathcal{R}(1^\lambda)$, $(\text{prim}, \text{aux}) \in \mathbf{R}$

$$\Pr \left[\mathbf{V}(\mathbf{R}, \text{crs}, \text{prim}, \mathbf{P}(\mathbf{R}, \text{crs}, \text{prim}, \text{aux})) \mid \begin{array}{l} \mathbf{R} \leftarrow \mathcal{R}(1^\lambda); \\ (\text{crs}, \text{td}) \leftarrow \text{KGen}(\mathbf{R}, 1^\lambda) \end{array} \right] = 1 .$$

Computational soundness which states that in case $\text{prim} \notin \mathbf{L}$ the verifier accepts the proof for prim with negligible probability only. Formally we require that for any $\mathbf{R} \leftarrow \mathcal{R}(1^\lambda)$ and PPT adversary \mathcal{A}

$$\Pr \left[\mathbf{V}(\mathbf{R}, \text{crs}, \text{prim}, \pi) \mid \begin{array}{l} \mathbf{R} \leftarrow \mathcal{R}(1^\lambda); \\ (\text{crs}, \text{td}) \leftarrow \text{KGen}(\mathbf{R}, 1^\lambda); \\ (\text{prim}, \pi) \leftarrow \mathcal{A}(\mathbf{R}, \text{crs}); \\ \text{prim} \notin \mathbf{L} \end{array} \right] \leq \text{negl}(\lambda).$$

Zero knowledge assures that the verifier learns from a proof nothing except the veracity of the proven statement. More precisely we require that there exist a PPT algorithm Sim and negligible function $\eta(\lambda)$ such that for every adversary \mathcal{A} and security parameter λ

$$\left| \Pr \left[\mathcal{A}(\mathbf{R}, \text{crs}, \pi) = 1 \mid \begin{array}{l} \mathbf{R} \leftarrow \mathcal{R}(1^\lambda); \\ (\text{crs}, \text{td}) \leftarrow \text{KGen}(\mathbf{R}, 1^\lambda); \\ (\text{prim}, \text{aux}) \leftarrow \mathcal{A}(\mathbf{R}, \text{crs}); \\ (\text{prim}, \text{aux}) \in \mathbf{R}; \\ \pi \leftarrow \text{Sim}(\mathbf{R}, \text{crs}, \text{td}, \text{prim}) \end{array} \right] - \Pr \left[\mathcal{A}(\mathbf{R}, \text{crs}, \pi) = 1 \mid \begin{array}{l} \mathbf{R} \leftarrow \mathcal{R}(1^\lambda); \\ (\text{crs}, \text{td}) \leftarrow \text{KGen}(\mathbf{R}, 1^\lambda); \\ (\text{prim}, \text{aux}) \leftarrow \mathcal{A}(\mathbf{R}, \text{crs}); \\ (\text{prim}, \text{aux}) \in \mathbf{R}; \\ \pi \leftarrow \mathbf{P}(\mathbf{R}, \text{crs}, \text{prim}, \text{aux}) \end{array} \right] \right| \leq \eta(\lambda).$$

We say that NIZK is *perfectly* zero-knowledge if $\eta = 0$.

We note that the existence of the simulator which by using the trapdoor is able to make a proof for a false statement (i.e. for $\text{prim} \notin \mathbf{L}$) makes the whole zk-proof system

516 vulnerable to adversaries that also know the trapdoor. More precisely, an adversary
 517 who knows a trapdoor td can break the soundness property. This vulnerability comes
 518 with each CRS-based NIZK (for languages in NP). Thus in the real-life deployment of a
 519 CRS-based NIZK it has to be enforced that nobody learns the trapdoor.

520 A zk-SNARK scheme, denoted ZkSnarkSch , is a special type of NIZK which is equipped
 521 with two more properties. First, zk-SNARKs are arguments *of knowledge*, as such they
 522 have to follow a stronger definition of soundness, called *knowledge soundness*.

Knowledge soundness assures that if a prover provided a proof π for a statement
 $prim$ acceptable to a verifier, then she knows the corresponding auxiliary input aux .
 More precisely, we require that for each $\mathbf{R} \leftarrow \mathcal{R}(1^\lambda)$, and malicious PPT prover \mathcal{A}
 there exists a machine $\text{Ext}_{\mathcal{A}}$, called extractor, that given access to randomness r
 used by \mathcal{A} and its inputs, *extracts* the auxiliary input aux from \mathcal{A} ; that is:

$$\Pr \left[\begin{array}{c} \neg(\mathbf{R}(prim, aux)) \wedge \\ \mathbf{V}(\mathbf{R}, crs, prim, \pi) \end{array} \middle| \begin{array}{c} \mathbf{R} \leftarrow \mathcal{R}(1^\lambda); \\ (crs, td) \leftarrow \text{KGen}(\mathbf{R}, 1^\lambda); \\ (prim, \pi) \leftarrow \mathcal{A}(\mathbf{R}, crs; r); \\ aux \leftarrow \text{Ext}_{\mathcal{A}}(\mathbf{R}, crs; r) \end{array} \right] \leq \text{negl}(\lambda).$$

523 Second, zk-SNARKs are *succinct*, and so we require that proofs produced by ZkSnarkSch.P
 524 are short, i.e. sublinear to the size of the primary and auxiliary inputs. Importantly, in
 525 many modern zk-SNARKs, like [Gro16, MBKM19, Gab19, GWC19, CHM⁺20] the proof
 526 size is constant regardless the size of the input.

527 1.3.2 Computation representation – arithmetization

528 In **Zeth** the sender shows that the transaction is correct by arguing (in zero knowledge,
 529 i.e. hiding private inputs) about correctness of evaluation of some predefined predicate.
 530 This predicate ensures that the soundness of the blockchain system is not violated, i.e. the
 531 zk-proof is used to prove that a transaction follows the “rules of the system” without
 532 disclosing its attributes. The proof system that **Zeth** uses operates on an algebraic
 533 representation of the “predicate to prove”. Informally, representing the computation as
 534 a set of algebraic constraints is called *arithmetization*. One of such representations is
 535 Quadratic Arithmetic Programs (QAP) [GGPR13], which, following [Gro16], is used in
 536 **Zeth**. This representation is considered one of the most efficient for general arithmetic
 537 circuits.

538 **Remark 1.3.2.** Preprocessing SNARKs such as [Gro16] rely on common reference
 539 strings with a specific structure. As such, we may use crs and srs (*structured refer-*
 540 *ence string*) interchangeably in the rest of this document.

541 **QAP (R1CS).** Let C be an arithmetic circuit of fan-in 2 over \mathbb{F}_p . The number of
 542 multiplication gates in C is denoted by $constNo$. Likewise, the number of all wires in C
 543 is denoted by $inpNo$.

Before we formally introduce the QAP relation \mathbf{R}_{QAP} we provide some intuitions behind it. First, we observe that the circuit \mathbf{C} can be represented by three matrices $\vec{A}, \vec{B}, \vec{C}$ all in $\mathbb{F}_p^{\text{constNo} \times \text{inpNo} + 1}$ such that the i -th row in matrix \vec{A} (and \vec{B}) denotes left (and right) input to the i -th multiplication gate, which is also the k -th input to the circuit. That is for a circuit evaluation $z \in \mathbb{F}_p^{\text{inpNo} + 1}$ the left input for the i -th gate is $\sum_{j=0}^{\text{inpNo}} A_{ij} z_j$ and the right input is $\sum_{j=0}^{\text{inpNo}} B_{ij} z_j$. Furthermore, entry \vec{C}_{ik} contains the output of i -th multiplication gate that is k -th input to the circuit.

Second, for the sake of efficiency we represent each matrix as a sequence of polynomials. Each matrix's column is represented by a polynomial in $\mathbb{F}_p[X]$ such that the column's i -th input equals polynomial's evaluation at ω^i – the i -th primitive root of unity modulo p . More precisely, we define polynomials:

- $u_j(X)$, for $j \in \{0, \dots, \text{inpNo}\}$, such that $u_j(\omega^i) = \vec{A}_{ij}$;
- $v_j(X)$, for $j \in \{0, \dots, \text{inpNo}\}$, such that $v_j(\omega^i) = \vec{B}_{ij}$;
- $w_j(X)$, for $j \in \{0, \dots, \text{inpNo}\}$, such that $w_j(\omega^i) = \vec{C}_{ij}$.

We consider inputs from 1 to inpNoPrim public (primary input), for some $\text{inpNoPrim} \leq \text{inpNo}$. The rest of the inputs is considered private (auxiliary input). The QAP relation \mathbf{R}_{QAP} is defined as follows:

$$\mathbf{R}_{\text{QAP}} = \left\{ (prim, aux) \left| \begin{array}{l} a_0 = 1; prim = (a_1, \dots, a_{\text{inpNoPrim}}) \in \mathbb{F}_p^{\text{inpNoPrim}}; \\ aux = (a_{\text{inpNoPrim}+1}, \dots, a_{\text{inpNo}}) \in \mathbb{F}_p^{\text{inpNo} - \text{inpNoPrim}}; \\ \sum_{j=0}^{\text{inpNo}} a_j u_j(X) \cdot \sum_{j=0}^{\text{inpNo}} a_j v_j(X) = \sum_{j=0}^{\text{inpNo}} a_j w_j(X) \end{array} \right. \right\}.$$

Note

Importantly, we note that efficient computation on standard hardware may not necessarily lead to an efficient QAP representation. As such, a function can be very efficient to evaluate on a standard computer, but very slow to evaluate in QAP form.

558

1.4 Decentralized Anonymous Payment schemes (DAP)

Zeth [RZ19] is a Decentralized Anonymous Payment scheme (DAP) [BSCG⁺14, Section 3] defined on top of an **Ethereum** ledger L . A DAP is a tuple of polynomial-time algorithms $\text{DAP} = (\text{Setup}, \text{GenAddr}, \text{SendTx}, \text{VerifyTx}, \text{Receive})$ that manipulate (*create*, *spend*) data objects called *Notes*. These objects are bound to a given owner and have a value v attribute (see Section 2.1).

System Setup The algorithm **Setup** takes the security parameter λ as input and generates the public parameters pp . The algorithm **Setup** is executed by a trusted

566

567 party. The resulting public parameters pp are published and made available to all
 568 parties.

569 **Creating Zeth addresses** The algorithm `GenAddr` takes as input the public parame-
 570 ters pp and generates a new DAP address object $Addr = \{pub : Addr_{pk}, priv : Addr_{sk}\}$. More precisely, $Addr_{pk}$ is an object referred to as the “payment ad-
 571 dress” (Table 1.4), and $Addr_{sk}$ is an object referred to as the “private address”
 572 (Table 1.5) [ZCa19].
 573

574 **Transfer notes** The algorithm `SendTx` is used to transfer some public input vin as
 575 well as the value of a set of input (“old”) $Notes$ into a set of output (“new”) $Notes$
 576 as well as some public output value $vout$. The inputs $Notes$ are marked as
 577 “consumed” (alternatively, we say that the input $Notes$ are “spent”). `SendTx` takes
 578 as inputs the public parameters pp , the input value and the input (“old”) $Notes$
 579 to be transferred, as well as the Merkle root and the Merkle authentications paths
 580 of the commitments to the input $Notes$, the “spending keys” related to the input
 581 $Notes$, the output value to create and the “payment addresses” for the output
 582 (“new”) $Notes$. If the joinsplit equation is satisfied, the algorithm returns the new
 583 $Notes$ and the corresponding **Ethereum** transaction tx , else it returns \perp .

584 **Verifying transactions** The algorithm `VerifyTx` checks the validity of a transaction.
 585 It takes as inputs the public parameters pp , a transaction and the current ledger
 586 L and outputs a bit equal 1 iff the transaction is valid, 0 otherwise.

587 **Receiving notes** The algorithm `Receive` scans the ledger L and retrieves unspent $Notes$
 588 paid to a particular user address. It takes as input the recipient address key pair
 589 $\{pub : Addr_{pk}, priv : Addr_{sk}\}$ and the current ledger L and outputs the set of
 590 (unspent) received $Notes$.

Note

In the rest of this document, we will refer to a *Zeth user* $\mathcal{U}_{\mathcal{Z}}$ as a person, modeled as an object, holding one **Zeth** address (object attribute), and thus holding a *private address*, $Addr_{sk}$. We denote by $\mathcal{U}_{\mathcal{Z}}.Addr$ the **Zeth** address of $\mathcal{U}_{\mathcal{Z}}$ derived from $Addr_{sk}$, and which allows $\mathcal{U}_{\mathcal{Z}}$ to be the recipient of payments via **Zeth**, and to send funds via **Zeth**. Importantly, *not all Ethereum users are Zeth users, and vice-versa!*

591

Field	Description
apk	The <i>paying key</i>
$pkenc$	The <i>transmission key</i>

Table 1.4: “Payment address”, $Addr_{pk}$, of a DAP address

Field	Description
<i>ask</i>	The <i>spending key</i>
<i>skenc</i>	The <i>receiving key</i>

Table 1.5: “Private address”, $Addr_{sk}$, of a DAP address

592 **Zeth** leverages zk-SNARKs (Section 1.3) and the possibility to deploy smart-contracts
593 to specify privacy-preserving state transitions altering the **Ethereum** state ς (Section 1.2).
594 As such, **Zeth** defines a smart-contract, $\widetilde{\mathbf{Mixer}}$, that keeps track of the set of *ZethNotes*
595 (Section 2.1) in a committed form, stored in a Merkle tree; and which verifies the va-
596 lidity of the state transitions generated by the **Zeth** users. As such a **Zeth** DAP is
597 entirely determined by $\widetilde{\mathbf{Mixer}}$, the instance of the mixer smart-contract deployed on the
598 **Ethereum** ledger. State transitions are executed on-chain by calling the **Mix** function of
599 $\widetilde{\mathbf{Mixer}}$, which implements the algorithm **VerifyTx** of DAP, and which modifies ς iff the
600 transaction is deemed valid.

Note

We denote by Mix_{in} the inputs taken by the **Mix** function defined on $\widetilde{\mathbf{Mixer}}$. Let $zdata$ be the value of the *data* field of an **Ethereum** transaction such that:

$$zdata = FS(\mathbf{Mix}) \parallel Mix_{in}$$

Then, we define $tx_{\mathbf{Mix}}$ as being the **Ethereum** transaction object returned by **SendTx** such that:

$$tx_{\mathbf{Mix}}.to = \widetilde{\mathbf{Mixer}}.Addr \wedge tx_{\mathbf{Mix}}.data = zdata$$

Importantly, when it is clear from context, we will omit the function selector from the definition of $zdata$, and only assume that $zdata = Mix_{in}$.

601

602 1.5 Definitions

603 1.5.1 Negligible function

604 **Definition 1.5.1** (Negligible function, [KL14, Definition 3.4]). A function f from \mathbb{N} to
605 \mathbb{R}^+ (positive real numbers) is negligible if for every positive polynomial p there exists N
606 such that for all integers $n > N$ it holds that $f(n) < \frac{1}{p(n)}$.

1.5.2 Basic algebra notions

Definition 1.5.2 (Group, see [Bou03, Section I.4]). A group is given by a tuple (\mathbb{G}, \otimes) , where \mathbb{G} is a set and \otimes is a binary operation in \mathbb{G} , i.e. $\otimes : \mathbb{G} \times \mathbb{G} \rightarrow \mathbb{G}$, with the following properties:

- $(\mathbf{g} \otimes \mathbf{h}) \otimes \mathbf{k} = \mathbf{g} \otimes (\mathbf{h} \otimes \mathbf{k})$ (associativity)
- There exists an element $\epsilon \in \mathbb{G}$ s.t. for each $\mathbf{g} \in \mathbb{G}$, $\mathbf{g} \otimes \epsilon = \epsilon \otimes \mathbf{g} = \mathbf{g}$ (identity element).
- For each $\mathbf{g} \in \mathbb{G}$ there exist $\mathbf{h} \in \mathbb{G}$ s.t. $\mathbf{g} \otimes \mathbf{h} = \mathbf{h} \otimes \mathbf{g} = \epsilon$ (inverse element).

For simplicity, we may also use the additive notation for groups: \otimes is denoted as $+$, the identity element as \mathbf{o} and the inverse element of \mathbf{g} as $-\mathbf{g}$. Given $\mathbf{g} \in \mathbb{G}$ and $x \in \mathbb{Z}$, we have that:

$$x \cdot \mathbf{g} = \begin{cases} \mathbf{o} & \text{if } x = 0 \\ \mathbf{g} + \dots + \mathbf{g}, (x \text{ times}) & \text{if } x > 0. \\ -\mathbf{g} + \dots + (-\mathbf{g}), (x \text{ times}) & \text{if } x < 0 \end{cases}$$

Definition 1.5.3 (Finite Cyclic Group, adapted from [KL14, Sections 7.1.3, 7.3.2]). A finite cyclic group is given by a tuple $(q, \mathbb{G}, \mathbf{g}, \otimes)$, called the *group description*, where \mathbb{G} represents the set of group elements, \mathbf{g} is a generator and q is the order. The generator \mathbf{g} generates the group; namely, each $\mathbf{h} \in \mathbb{G}$ can be expressed by the generator as $\mathbf{h} = \mathbf{g} \otimes \dots \otimes \mathbf{g}$. Given a scalar x , we denote by $\llbracket x \rrbracket$ the *encoding* of x in \mathbb{G} : i.e. $\llbracket x \rrbracket = \mathbf{g} \otimes \dots \otimes \mathbf{g}$ (x times). As consequence, $\llbracket 1 \rrbracket = \mathbf{g}$.

For theoretical purposes, we introduce the **SetupG** algorithm that for a given security parameter λ outputs a cyclic group, formally:

Definition 1.5.4 (Group Setup Algorithm, taken from [KL14, Sections 7.1.3, 7.3.2]). A group setup algorithm **SetupG** is a PPT algorithm which takes as input a security parameter 1^λ and outputs a group description $(q, \mathbb{G}, \mathbf{g}, \otimes)$, where the binary representation of q is given by λ bits and each group element can be represented by $gLen(\lambda)$ bits. Note that $gLen$ is $\text{poly}(\lambda)$.⁴

1.5.3 Security assumptions

Definition 1.5.5 (Discrete Log Problem(DLog), cf. [BS07]). Let \mathbb{G} denote a group (Section 1.5.2) whose order p is prime and written over λ bits. We let $\log_{\mathbf{g}}(h)$ denote the discrete logarithm of h in the basis \mathbf{g} . We assume \mathbb{G} , p are fixed and known to all parties. We denote the advantage of a PPT adversary \mathcal{A} in attacking the discrete logarithm problem as

$$\text{Adv}_{\mathbb{G}, \mathcal{A}}^{\text{dlog}} = \Pr[\mathbf{g} \leftarrow \mathbb{G}^*, x \leftarrow \mathbb{F}_p, x' \leftarrow \mathcal{A}(\llbracket 1 \rrbracket, \llbracket x \rrbracket) : \llbracket x' \rrbracket = \llbracket x \rrbracket]$$

⁴For simplicity, we may denote $gLen(\lambda)$ as $gLen$.

629 We say that the DLog is hard in \mathbb{G} if and only if $\text{Adv}_{\mathbb{G}, \mathcal{A}}^{\text{dlog}}(\lambda)$ is negligible for any PPT
 630 adversary \mathcal{A} .

Definition 1.5.6 (One More Discrete Log Problem (om-DLog), cf. [PV05]). Let \mathbb{G} denote a group whose order p is prime and written over λ bits. We let $\log_{\mathbf{g}}(h)$ denote the discrete logarithm of h in the basis \mathbf{g} . A PPT adversary \mathcal{A} solving the om-DLog is given $q + 1$ random group elements as well as limited access to a discrete logarithm oracle $\text{O}^{\text{DLog}_{\mathbf{g}}}(q)$. \mathcal{A} is allowed to query this oracle at most q times, thus obtaining the discrete logarithm of q group elements of his choice with respect to a fixed base \mathbf{g} . Eventually, \mathcal{A} must output the $q + 1$ discrete logarithms. We denote the advantage of a PPT adversary \mathcal{A} in attacking the one more discrete logarithm problem as

$$\text{Adv}_{\mathbb{G}, \mathcal{A}}^{\text{om-dlog}}(\lambda) = \Pr \left[\begin{array}{l} \mathbf{g} \leftarrow \mathbb{G}^*, \{ \llbracket r_i \rrbracket \}_{i \in [q+1]} \leftarrow \mathbb{G}^{q+1}, \\ \{ r'_i \}_{i \in [q+1]} \leftarrow \mathcal{A}^{\text{O}^{\text{DLog}_{\mathbf{g}}}(q)}(\llbracket 1 \rrbracket, \{ \llbracket r_i \rrbracket \}_{i \in [q+1]}) : \\ \forall i \in [q+1], r'_i = \log_{\mathbf{g}}(\llbracket r_i \rrbracket) \end{array} \right]$$

631 We say that the om-DLog is hard in \mathbb{G} if and only if $\text{Adv}_{\mathbb{G}, \mathcal{A}}^{\text{om-dlog}}(\lambda)$ is negligible for any
 632 PPT adversary \mathcal{A} .

633 1.5.4 Symmetric encryption

634 **Definition 1.5.7** (Symmetric Encryption, [KL14, Definition 3.8]). A symmetric encryption
 635 scheme Sym is given by a tuple of PPT algorithms $(\text{KGen}, \text{Enc}, \text{Dec})$ where:

- 636 • KGen , the key generation algorithm, takes a security parameter 1^λ and outputs a
 637 secret key ek ; we assume, without loss of generality, that $kLen(\lambda) = \text{length}(ek) \geq \lambda$.
 638 Note that $kLen(\lambda)$ is a polynomial function in λ .⁵
- 639 • Enc , the encryption algorithm, takes a key ek , a plaintext $m \in \{0, 1\}^*$ and returns
 640 a ciphertext ct .
- 641 • Dec , the decryption algorithm, takes a key ek and a ciphertext ct , and returns a
 642 message m . We assume, without loss of generality, that Dec is deterministic.

643 For every security parameter λ , key ek output by $\text{KGen}(1^\lambda)$, and message $m \in \{0, 1\}^*$,
 644 it holds that $\text{Dec}(ek, \text{Enc}(ek, m)) = m$ (*correctness property*).

645 Let $(\text{KGen}, \text{Enc}, \text{Dec})$ be a symmetric encryption scheme. If there exists a polynomial
 646 l such that, for all $\lambda > 0$ and key ek output by $\text{KGen}(1^\lambda)$, $\text{Enc}(ek, \cdot)$ is only defined for
 647 messages $m \in \{0, 1\}^{l(\lambda)}$, then we say that $(\text{KGen}, \text{Enc}, \text{Dec})$ is a *fixed-length symmetric*
 648 *encryption scheme* with *length parameter* $l(\lambda)$. A security notion for Sym follows:

Definition 1.5.8 (IND-CPA). Let Sym be a symmetric encryption scheme and let \mathcal{A} be an adversary. Consider the IND-CPA game described in Figure 1.2. We define the IND-CPA advantage of \mathcal{A} as follows:

$$\text{Adv}_{\text{Sym}, \mathcal{A}}^{\text{ind-cpa}}(\lambda) = |2 \cdot \Pr[\text{IND-CPA}(\lambda) = 1] - 1|.$$

⁵For simplicity, we may denote $kLen(\lambda)$ as $kLen$.

```

IND-CPA( $\lambda$ )
 $ek \leftarrow \text{KGen}(1^\lambda)$ 
 $(m_0, m_1, \text{state}) \leftarrow \mathcal{A}^{\text{O}^{\text{Enc}_{ek}}} \text{ with } \text{length}(m_0) = \text{length}(m_1)$ 
 $b \leftarrow \$\{0, 1\}$ 
 $ct \leftarrow \text{Enc}(ek, m_b)$ 
 $\tilde{b} \leftarrow \mathcal{A}^{\text{O}^{\text{Enc}_{ek}}}(ct, \text{state})$ 
return  $\tilde{b} = b$ 

```

Figure 1.2: IND-CPA game for Sym.

649 Sym is said to be IND-CPA secure if, for every PPT adversary \mathcal{A} , the advantage $\text{Adv}_{\text{Sym}, \mathcal{A}}^{\text{ind-cpa}}(\lambda)$
650 is a negligible function.

651 1.5.5 Asymmetric encryption

652 **Definition 1.5.9** (Asymmetric encryption, [KL14, Definition 10.1]). An *asymmetric*
653 *encryption scheme* Asym is given by a tuple of PPT algorithms $(\text{KGen}, \text{Enc}, \text{Dec})$ where:

- 654 • KGen , the key generation algorithm, takes a security parameter 1^λ and returns a
655 pair of keys (sk, pk) . We refer to the first of these as the *private key* and the second
656 as the *public key*. We assume for convenience that pk and sk each have length at
657 least λ , and that λ can be determined from pk, sk ;
- 658 • Enc , the encryption algorithm, takes a public key pk , a plaintext m , from some
659 underlying plaintext space (that may depend on pk) and returns a ciphertext ct ;
- 660 • Dec , the decryption algorithm, takes a private key sk and a ciphertext ct , and
661 returns a message m or a special symbol \perp denoting decryption failure. We assume,
662 without loss of generality, that Dec is deterministic.

663 We require that for all (sk, pk) returned by KGen , and every message m in the appropriate
664 underlying plaintext space, it holds that $\text{Dec}(sk, \text{Enc}(pk, m)) = m$ (*correctness property*).

665 Secure communication usually requires ciphertext indistinguishability (e.g. IND-CCA2
666 [ABR99, Definition 8]). In **Zeth**, however, the key privacy property IK-CCA [BBDP01]
667 is also required – it ensures indistinguishability of the key under which an encryption is
668 performed.

Definition 1.5.10 (IK-CCA). Let $\text{Asym} = (\text{KGen}, \text{Enc}, \text{Dec})$ be an asymmetric encryption scheme and let \mathcal{A} be an adversary. Given the IK-CCA game described in Figure 1.3, with the condition that \mathcal{A} cannot query $\text{O}^{\text{Dec}_{sk_0}}$ or $\text{O}^{\text{Dec}_{sk_1}}$ on the challenge ciphertext

IK-CCA(λ)

$(sk_0, pk_0), (sk_1, pk_1) \leftarrow \text{KGen}(1^\lambda)$

$(m, state) \leftarrow \mathcal{A}^{\mathcal{O}^{\text{Dec}_{sk_0}}, \mathcal{O}^{\text{Dec}_{sk_1}}}(pk_0, pk_1)$

$b \leftarrow \$\{0, 1\}$

$ct \leftarrow \text{Enc}(pk_b, m)$

$\tilde{b} \leftarrow \mathcal{A}^{\mathcal{O}^{\text{Dec}_{sk_0}}, \mathcal{O}^{\text{Dec}_{sk_1}}}(ct, state)$

return $\tilde{b} = b$

Figure 1.3: IK-CCA game.

ct^6 , we define the IK-CCA advantage of \mathcal{A} as follows:

$$\text{Adv}_{\text{Asym}, \mathcal{A}}^{\text{ik-cca}}(\lambda) = |2 \cdot \Pr[\text{IK-CCA}(\lambda) = 1] - 1|$$

We say that Asym is IK-CCA secure if for every PPT adversary \mathcal{A} the advantage $\text{Adv}_{\text{Asym}, \mathcal{A}}^{\text{ik-cca}}(\lambda)$ is a negligible function.

1.5.6 Block cipher-based compression functions

Definition 1.5.11. Let $kl, il > 1$. A *block cipher* is a map $E: \{0, 1\}^{kl} \times \{0, 1\}^{il} \rightarrow \{0, 1\}^{il}$ where, for each key $k \in \{0, 1\}^{kl}$, the function $E_k(\cdot) = E(k, \cdot)$ is a permutation on $\{0, 1\}^{il}$. If E is a block cipher then E^{-1} is its inverse, that on input (k, y) returns m such that $E_k(m) = y$.

Let $\mathcal{BK}(kl, il)$ be the set of all block ciphers $E: \{0, 1\}^{kl} \times \{0, 1\}^{il} \rightarrow \{0, 1\}^{il}$. In order to analyse the security properties of block cipher-based cryptographic constructions it is common to use a security model denoted *the ideal cipher model (ICM)*. Informally speaking, in ICM attackers are allowed to query an oracle simulating a random block cipher, but have no information about the oracle's internal structure. We formalize this notion in the following definition:

Definition 1.5.12 (Ideal Cipher Model [HKT11]). The Ideal Cipher Model (ICM), is a security model where all parties are granted access to an ideal cipher $E: \{0, 1\}^{kl} \times \{0, 1\}^{il} \rightarrow \{0, 1\}^{il}$, a random primitive such that $E(k, \cdot)$ for $k \in \{0, 1\}^{kl}$ are 2^{kl} independent random permutations.

For fixed kl and il , each party is given access to the oracles \mathcal{O}^E and $\mathcal{O}^{E^{-1}}$, simulating E and E^{-1} , which can be queried for encryption and decryption a polynomial number of times. The encryption oracle takes as input a key, $k \in \{0, 1\}^{kl}$, and a preimage, $m \in \{0, 1\}^{il}$, and returns a tuple comprising the image, $y \in \{0, 1\}^{il}$, along with the inputs, k and m . If (k, m) is queried for the first time, the image y is taken uniformly

⁶*state* is some state information that the adversary outputs after the choice of the message to encrypt. It can be some preprocessed information that can be helpful to win the game

$O^E(k, m)$	$O^{E^{-1}}(k, y)$
if $(k, m, \cdot) \notin \text{Table}_O$ $y \leftarrow \$ \{0, 1\}^{\text{il}}$ $\text{Table}_O.\text{append}(k, m, y)$	if $(k, \cdot, y) \notin \text{Table}_O$ $m \leftarrow \$ \{0, 1\}^{\text{il}}$ $\text{Table}_O.\text{append}(k, m, y)$
else $y \leftarrow \text{Table}_O(k, m)$	else $m \leftarrow \text{Table}_O(k, y)$
return (k, m, y)	return (k, m, y)

Figure 1.4: Oracles of an ideal block cipher, with Table_O being a table of tuples (key, preimage, image) of queries already answered by the oracle.

691 at random from $\{0, 1\}^{\text{il}}$ and added to the oracle's table. Otherwise, the oracle returns
692 y associated with query (k, m) in its table. The decryption oracle is defined similarly
693 with the image and key defined as inputs and the preimage chosen randomly, for details
694 see Fig. 1.4.

Definition 1.5.13 (Block cipher-based compression function [BRS02]). A *block cipher-based compression function* is a map f such that

$$f: \mathcal{BK}(\text{kl}, \text{il}) \times \{0, 1\}^a \times \{0, 1\}^b \rightarrow \{0, 1\}^c$$

695 where $\text{kl}, \text{il}, a, b, c > 1$ and $a + b > c$. The function f , given $m \in \{0, 1\}^a \times \{0, 1\}^b$,
696 computes $f(E, m)$ using an E -oracle.

697 **Remark 1.5.14.** We use f_E to denote a block cipher-based compression function f
698 restricted to a given block cipher E , i.e. $f_E: \{0, 1\}^a \times \{0, 1\}^b \rightarrow \{0, 1\}^c$ and $f_E = f(E, \cdot)$,
699 for a, b, c as given in the definition above.

Let f be a compression function based on a block cipher. Fix a constant $h_0 \in \{0, 1\}^c$ and an adversary \mathcal{A} . We define the advantage in finding a collision in f as

$$\text{Adv}_{f, \mathcal{A}}^{\text{coll}} = \Pr \left[E \leftarrow \$ \mathcal{BK}(\text{kl}, \text{il}); ((k, m), (k', m')) \leftarrow \mathcal{A}^{O^E, O^{E^{-1}}} (f_E, h_0) : \right. \\ \left. ((k, m) \neq (k', m') \wedge f_E(k, m) = f_E(k', m')) \vee f_E(k, m) = h_0 \right].$$

700 The previous definition gives credit for finding an (k, m) such that $f_E(k, m) = h_0$ for
701 a fixed $h_0 \in \{0, 1\}^c$.

702 1.5.7 Hash functions

703 **Definition 1.5.15** (Hash function, [KL14, Definition 4.9]). A hash function \mathcal{H} is a pair
704 of algorithms (Setup, H) fulfilling the following properties:

- 705 • **Setup** is a PPT algorithm which takes as input a security parameter 1^λ and outputs
706 a key hk . We assume that 1^λ is included in hk .

707 • H is (deterministic) polynomial-time algorithm that takes as input a key hk and
 708 any string $x \in \{0, 1\}^*$, and outputs a string $H(hk, x) = H_{hk}(x) \in \{0, 1\}^{hLen}$, where
 709 $hLen$ is a polynomial in λ .⁷

710 If for every λ and hk , H_{hk} is defined only over inputs of length $hInpLen(\lambda)$ and $hInpLen(\lambda) >$
 711 $hLen(\lambda)$, then we say that \mathcal{H} is a *fixed-length hash function* with length parameter
 712 $hInpLen$. Note that $hInpLen(\lambda)$ is a polynomial in λ .

713 Informally, for a given function f we say that (x, y) is a *collision* if $f(x) = f(y)$ and
 714 $x \neq y$. In the following, we formalize this notion for a hash function \mathcal{H} .

Definition 1.5.16 (Collision Resistance [KL14, Definitions 4.10]). A hash function $\mathcal{H} = (\text{Setup}, H)$ is collision resistant if for all PPT adversaries \mathcal{A} there exists a negligible function $\text{negl}(\lambda)$ such that:

$$\text{Adv}_{\mathcal{H}, \mathcal{A}}^{\text{cr}}(\lambda) = \Pr \left[hk \leftarrow \text{Setup}(1^\lambda), (x, y) \leftarrow \mathcal{A}(hk) : x \neq y \wedge H_{hk}(x) = H_{hk}(y) \right] \leq \text{negl}(\lambda).$$

715 HDHI and HDHI2 assumptions

716 The Hash Diffie-Hellmann Independence (HDHI) assumption states that, given H in \mathcal{H}
 717 and a group description $(p, \mathbb{G}, \mathbf{g}, \otimes)$, for $\llbracket u \rrbracket$ and $\llbracket v \rrbracket$, with u, v sampled at random, it
 718 is hard for an attacker to distinguish $H(\llbracket u \rrbracket \parallel \llbracket uv \rrbracket)$ from a random string of the same
 719 size.⁸ This is formalized in Definition 1.5.17, where an attacker can also access an oracle
 720 $\mathcal{O}^{\text{HDHI}_v}$ that on input $\mathbf{r} \in \mathbb{G}$ returns $H(\mathbf{r} \parallel v \cdot \mathbf{r})$ (queries on $\llbracket u \rrbracket$ are forbidden).⁹ In other
 721 words, the HDHI assumption measures the sense in which H is “independent” of the
 722 underlying Diffie-Hellman problem.

Definition 1.5.17 (HDHI, [ABR99, Definition 7]). Let \mathcal{H} be a hash function, SetupG be a group generation algorithm and \mathcal{A} be an adversary. Consider the HDHI game described in Figure 1.5. We define the advantage of \mathcal{A} in violating the HDHI assumption as:

$$\text{Adv}_{\mathcal{H}, \text{SetupG}, \mathcal{A}}^{\text{hdhi}}(\lambda) = |2 \cdot \Pr[\text{HDHI}(\lambda) = 1] - 1|.$$

723 Note that the above definition corresponds to [ABR99, Section 3.2.1, Definition 3].
 724 In the following, we introduce a similar notion denoted as HDHI2 (this is an adapta-
 725 tion of the ODH2 notion in [ABN10, Section 6]) which will be useful in the IK-CCA
 726 proof Section 3.5.4.

Definition 1.5.18 (HDHI2). Let \mathcal{H} be a hash function, SetupG a group generation algorithm and let \mathcal{A} be an adversary. Consider the HDHI2 game described in Figure 1.6. We define the advantage of \mathcal{A} in violating the HDHI2 assumption as:

$$\text{Adv}_{\mathcal{H}, \text{SetupG}, \mathcal{A}}^{\text{hdhi2}}(\lambda) = |2 \cdot \Pr[\text{HDHI2}(\lambda) = 1] - 1|.$$

⁷For simplicity, we may denote $hLen(\lambda)$ as $hLen$.

⁸Note that H takes as inputs bit strings, so technically we should make use of an encoding function from \mathbb{G} to $\{0, 1\}^{gLen}$ but we may omit this step through the document to improve readability.

⁹In [ABR99, Section 3.2.1] this notion is denoted as adaptive HDH independence assumption. Since we only introduce the adaptive version we denote it as HDHI.

$\text{HDHI}(\lambda)$
 $hk \leftarrow \mathcal{H}.\text{Setup}(1^\lambda)$
 $(q, \mathbb{G}, \mathfrak{g}, \otimes) \leftarrow \text{SetupG}(1^\lambda)$
 $u, v \leftarrow \$[q]$
 $w_0 \leftarrow \mathcal{H}.H_{hk}(\llbracket u \rrbracket \parallel \llbracket uv \rrbracket)$
 $w_1 \leftarrow \$\{0, 1\}^{hLen}$
 $b \leftarrow \$\{0, 1\}$
 $\tilde{b} \leftarrow \mathcal{A}^{\text{O}^{\text{HDHI}_v}}(\llbracket u \rrbracket, \llbracket v \rrbracket, w_b)$
return $\tilde{b} = b$

Figure 1.5: HDHI game.

$\text{HDHI2}(\lambda)$
 $hk \leftarrow \mathcal{H}.\text{Setup}(1^\lambda)$
 $(q, \mathbb{G}, \mathfrak{g}, \otimes) \leftarrow \text{SetupG}(1^\lambda)$
 $u, v_0, v_1 \leftarrow \$[q]$
 $w_{0,0} \leftarrow \mathcal{H}.H_{hk}(\llbracket u \rrbracket \parallel \llbracket uv_0 \rrbracket), w_{0,1} \leftarrow \mathcal{H}.H_{hk}(\llbracket u \rrbracket \parallel \llbracket uv_1 \rrbracket)$
 $w_{1,0} \leftarrow \$\{0, 1\}^{hLen}, w_{1,1} \leftarrow \$\{0, 1\}^{hLen}$
 $b \leftarrow \$\{0, 1\}$
 $\tilde{b} \leftarrow \mathcal{A}^{\text{O}^{\text{HDHI}_{v_0}}, \text{O}^{\text{HDHI}_{v_1}}}(\llbracket u \rrbracket, \llbracket v_0 \rrbracket, \llbracket v_1 \rrbracket, w_{b,0}, w_{b,1})$
return $\tilde{b} = b$

Figure 1.6: HDHI2 game.

Lemma 1.5.1. *Let \mathcal{A} be an adversary with advantage $\text{Adv}_{\mathcal{H}, \text{SetupG}, \mathcal{A}}^{\text{hdhi2}}$ in solving the HDHI2 problem. Then there exists an adversary \mathcal{B} such that*

$$\text{Adv}_{\mathcal{H}, \text{SetupG}, \mathcal{A}}^{\text{hdhi2}}(\lambda) \leq 2 \cdot \text{Adv}_{\mathcal{H}, \text{SetupG}, \mathcal{B}}^{\text{hdhi}}(\lambda).$$

Proof. We reuse the proof described in [ABN10, Lemma 6.1] by applying minor modifications. In fact, HDHI and HDHI2 are, respectively, slightly different from ODH and ODH2 notions: in the related security games, if $b = 0$ the challenges are constructed as $H(\llbracket u \rrbracket \parallel \llbracket uv \rrbracket)$ and $\{H(\llbracket u \rrbracket \parallel \llbracket uv_0 \rrbracket), H(\llbracket u \rrbracket \parallel \llbracket uv_1 \rrbracket)\}$ instead of $H(\llbracket uv \rrbracket)$ and $\{H(\llbracket uv_0 \rrbracket), H(\llbracket uv_1 \rrbracket)\}$. By accordingly changing the instances of H in the games G_0, G_1, G_2 of [ABN10, Lemma 6.1] our lemma follows. \square

1.5.8 Pseudo Random Functions

Informally, a pseudorandom function family $\mathcal{PRF} = \{\text{PRF}_k : D \rightarrow C\}_{k \in \mathcal{K}}$ is a collection of functions such that for a randomly chosen $k \in \mathcal{K}$, the function PRF_k is indistinguishable from a random function that maps D to C .

Definition 1.5.19 (PRF Family [KL14, Definition 3.23]). Let $\mathcal{F} : \{0, 1\}^* \times \{0, 1\}^* \rightarrow \{0, 1\}^*$ be an efficient, length-preserving, keyed function. We say \mathcal{F} is a pseudo random function if for all probabilistic polynomial-time distinguishers Dist , there exists a negligible function negl such that:

$$\text{Adv}_{\mathcal{F}, \text{Dist}}^{\text{prf}}(\lambda) = \left| \Pr[\text{Dist}^{\mathcal{F}_k(\cdot)}(1^\lambda) = 1] - \Pr[\text{Dist}^{f_\lambda(\cdot)}(1^\lambda) = 1] \right| \leq \text{negl}(\lambda),$$

where $k \leftarrow \$\mathcal{K} = \{0, 1\}^\lambda$ is chosen uniformly at random and f_λ is chosen uniformly at random from the set of functions mapping λ -bit strings to λ -bit strings.

1.5.9 Commitment scheme

Definition 1.5.20 (Non-interactive commitment scheme [BCC⁺15, Section 2.1]). A non-interactive commitment scheme ComSch is defined by the following algorithms:

- 742 • **Setup**, is a PPT algorithm that takes a security parameter 1^λ and outputs public
743 parameters pp .
- 744 • **Com**, is a polynomial-time algorithm that takes a message $m \in \mathbb{B}^l$, a random coin
745 $r \in \mathbb{B}^n$ and outputs a commitment $cm \in \mathbb{B}^l$.

746 We assume that pp is implicitly passed to **Com**.

Definition 1.5.21 (Computational hiding). We say that a commitment scheme is computationally hiding if for all PPT adversary \mathcal{A} , the advantage:

$$\left| \Pr \left[pp \leftarrow \text{Setup}(1^\lambda), (m_0, m_1) \leftarrow \mathcal{A}(pp), b \leftarrow \{0, 1\}, \right. \right. \\ \left. \left. r \leftarrow \mathbb{B}^n, cm \leftarrow \text{Com}(m_b; r), \tilde{b} \leftarrow \mathcal{A}(cm), b = \tilde{b} \right] - \frac{1}{2} \right|$$

747 is at most negligible in λ .

Definition 1.5.22 (Computational binding). We say that a commitment scheme is computationally binding if for all PPT adversary \mathcal{A} , the advantage:

$$\Pr \left[pp \leftarrow \text{Setup}(1^\lambda), (m_0, r_0, m_1, r_1) \leftarrow \mathcal{A}(pp) \right. \\ \left. m_0 \neq m_1 \wedge \text{Com}(m_0; r_0) = \text{Com}(m_1; r_1) \right]$$

748 is at most negligible in λ .

749 Note that the previous definitions can be made *statistical* if we consider unbounded
750 attackers \mathcal{A} .

751 1.5.10 Digital Signature

752 **Definition 1.5.23** (Digital signature [KL14, Definition 12.1]). A digital signature scheme
753 **SigSch** is defined by the tuple of functions **SigSch** = (**KGen**, **Sig**, **Vf**),

- 754 • $(sk, vk) \leftarrow \text{KGen}(1^\lambda)$. Key Generation randomized algorithm takes as input the
755 security parameter 1^λ and returns a signing key sk and verifying key vk .
- 756 • $\sigma \leftarrow \text{Sig}(sk, m)$. Given a signing key sk and a message m , the **Sig** algorithm
757 computes and outputs a signature σ .
- 758 • $\{0, 1\} \leftarrow \text{Vf}(vk, m, \sigma)$. Given a verification key vk , a message m and a signature
759 σ , the **Vf** algorithm returns 1 if σ is a valid signature else 0.

760 A signature scheme must satisfy the *correctness property* (i.e $\text{Vf}(vk, m, \text{Sig}(sk, m)) =$
761 **true**, where $(sk, vk) \leftarrow \text{KGen}(1^\lambda)$) and be *unforgeable* (i.e. it is intractable to produce a
762 signature, without knowing the signing key sk , on a message that has not been signed
763 yet). In addition to these properties, certain digital signature schemes have an additional
764 property called *one-timeness*, also defined below.

UF-CMA($1^\lambda, t, q$)

```

1 :   $(sk, vk) \leftarrow \text{KGen}(1^\lambda)$ 
2 :   $state \leftarrow \mathcal{A}^{\text{OSig}_{sk}}(vk, \cdot)$ 
3 :  //  $state = \{(m_i, \sigma_i)\}_{i \in [q]}$  where  $m_i$  denotes
4 :  // the  $i$ th query made to  $\text{OSig}_{sk}$  and
5 :  //  $\sigma_i$  denotes the  $i$ th oracle answers
6 :   $(m^*, \sigma^*) \leftarrow \mathcal{A}(state)$ 
7 :  return  $\text{Vf}(vk, m^*, \sigma^*) = 1$ 
8 :   $\wedge m^* \notin \{m_i\}_{i \in [q]}$ 

```

Figure 1.7: UF-CMA game

SUF-CMA($1^\lambda, t, q$)

```

1 :   $(sk, vk) \leftarrow \text{KGen}(1^\lambda)$ 
2 :   $state \leftarrow \mathcal{A}^{\text{OSig}_{sk}}(vk, \cdot)$ 
3 :  //  $state = \{(m_i, \sigma_i)\}_{i \in [q]}$  where  $m_i$  denotes
4 :  // the  $i$ th query made to  $\text{OSig}_{sk}$  and
5 :  //  $\sigma_i$  denotes the  $i$ th oracle answers
6 :   $(m^*, \sigma^*) \leftarrow \mathcal{A}(state)$ 
7 :  return  $\text{Vf}(vk, m^*, \sigma^*) = 1$ 
8 :   $\wedge (m^*, \sigma^*) \notin \{(m_i, \sigma_i)\}_{i \in [q]}$ 

```

Figure 1.8: SUF-CMA game

Definition 1.5.24 (Unforgeability (UF-CMA) [KL14, Definition 12.2]). A digital signature scheme SigSch is UF-CMA if for any PPT adversary \mathcal{A} , the probability that \mathcal{A} wins the UF-CMA game, depicted in Fig. 1.7, is negligible.

Definition 1.5.25 (Strong Unforgeability (SUF-CMA)). A digital signature scheme SigSch is SUF-CMA if the probability that any PPT adversary \mathcal{A} wins the SUF-CMA game, depicted in Fig. 1.8, is negligible.

Definition 1.5.26 (One-Time (OT) Signature [KL14, Definition 12.6]). A *one-time* signature scheme is a digital signature scheme that uses each key-pair at most once.

Remark 1.5.27. It is worth noting that users may use one-time signing keys to sign multiple messages. In this case no security claims can be made.

1.5.11 Message Authentication Code

A message authentication code is a scheme that enables users to tag data for the purpose of authenticity and integrity. Formally:

Definition 1.5.28 (Message Authentication Code, [KL14, Definition 4.1]). A message authentication code MAC is given by a tuple of PPT algorithms $(\text{KGen}, \text{Tag}, \text{Vf})$ where:

- **KGen**, the key generation algorithm, takes a security parameter 1^λ , and returns a key $mk \in \{0, 1\}^{mLen(\lambda)}$.¹⁰
- **Tag**, the tag generation algorithm, takes a key mk and a message $y \in \{0, 1\}^*$ and returns a string $\tau \in \{0, 1\}^*$, called *tag*.
- **Vf**, the tag verification algorithm, takes a key mk , a message $y \in \{0, 1\}^*$ and a tag $\tau \in \{0, 1\}^*$. It returns a value in $\{0, 1\}$ where: 0 denotes that the message was rejected (i.e. deemed unauthentic) and 1 denotes that the message was accepted (i.e. deemed authentic).

¹⁰For simplicity, we may denote $mLen(\lambda)$ as $mLen$.

SUF-CMA (λ)
 $mk \leftarrow \text{KGen}(1^\lambda)$
 $(\bar{y}, \bar{\tau}) \leftarrow \mathcal{A}^{\text{O}^{\text{Tag}_{mk}}, \text{O}^{\text{Vf}_{mk}}}$
return $\text{Vf}(mk, \bar{y}, \bar{\tau}) = 1$

Figure 1.9: SUF-CMA game.

788 We require that for all $mk \in \{0, 1\}^\lambda$ and $y \in \{0, 1\}^*$ we have $\text{Vf}(mk, y, \text{Tag}(mk, y)) = 1$.
 789 If $\text{Tag}(mk, \cdot)$ is defined only over messages of length $l(\lambda)$ and $\text{Vf}(mk, y, \tau)$ outputs 0 for
 790 every y that is not of length $l(\lambda)$, then we say that $(\text{KGen}, \text{Tag}, \text{Vf})$ is a *fixed-length MAC*
 791 with length parameter $l(\lambda)$.

792 A security notion for MAC follows:

793 **Definition 1.5.29** (SUF-CMA, [ABR99, Section 3.2.3]). Let $\text{MAC} = (\text{KGen}, \text{Tag}, \text{Vf})$ be
 794 a message authentication scheme and let \mathcal{A} be an adversary. Consider the SUF-CMA
 795 game described in Figure 1.9, with the condition that $\text{Tag}(mk, \bar{y}) \neq \bar{\tau}$. We say that an
 796 adversary \mathcal{A} has *forged* a tag when it outputs a pair $(\bar{y}, \bar{\tau})$ such that $\text{Vf}_k(\bar{y}, \bar{\tau}) = 1$, where
 797 $(\bar{y}, \bar{\tau})$ was not previously obtained via a query to the tag oracle.

We define the SUF-CMA advantage of \mathcal{A} as follows:

$$\text{Adv}_{\text{MAC}, \mathcal{A}}^{\text{suf-cma}}(\lambda) = \Pr[\text{SUF-CMA}(\lambda) = 1]$$

798 We say that MAC is SUF-CMA secure if for every PPT adversary \mathcal{A} the advantage
 799 $\text{Adv}_{\text{MAC}, \mathcal{A}}^{\text{suf-cma}}(\lambda)$ is a negligible function.

Chapter 2

Zeth protocol

In this section, we detail the **Zeth** protocol and provide a set of requirements that need to be respected to guarantee the security of the protocol.

2.1 Zeth Data Types

We begin by describing, and giving intuition about, the data types (see Section 1.1) used in **Zeth**. We follow some design rationale from **ZeroCash** [BSCG⁺14], and **Zcash** [ZCa19] in order to prevent the transaction malleability attack, and the Faerie Gold attack [ZCa19, Section 8.4]. We refer the reader to Appendix A for more details.

In what follows **Curve** represents a curve with scalar field $\mathbb{F}_{\mathbf{r}_{\text{CUR}}}$, satisfying the requirements of Section 3.6. The specification is described in terms of this generic curve, with examples and notes relating to specific instances of interest (namely BN-254 and BLS12-377, see Chapter 3).

ZethNoteDType Represents a note in **Zeth**. This data type consists of the note owner’s public address apk , identifier ρ , randomness r and value v .

Field	Description	Data type
apk	Note owner’s paying key	$\mathbb{B}^{\text{PRFADDRROUTLEN}}$
r	Note randomness	$\mathbb{B}^{\text{RTRAPLEN}}$
v	Note value	$\mathbb{B}^{\text{ZVALUELEN}}$
ρ	Note identifier	$\mathbb{B}^{\text{PRFRHOOUTLEN}}$

Table 2.1: **ZethNoteDType** data type

JSInputDType Denotes a joinsplit input. It comprises the opening of a commitment cm which is in the set of leaves in the Merkle tree of **Mixer** (i.e. a *ZethNote*), its address $mkaddr$ and authentication path $mkpath$ on the contract’s Merkle tree as well as the spending key ask of the note holder and the note nullifier nf .

Field	Description	Data type
<i>mkpath</i>	Merkle authentication path to the commitment corresponding to the <i>ZethNote</i> to spend	$(\mathbb{F}_{\mathbf{r}_{\text{CUR}}})^{\text{MKDEPTH}}$
<i>mkaddr</i>	Commitment address in the Merkle tree	$\mathbb{B}^{\text{MKDEPTH}}$
<i>znote</i>	Zeth note object	ZethNoteDType
<i>cm</i>	Zeth note commitment	$\mathbb{F}_{\mathbf{r}_{\text{CUR}}}$
<i>ask</i>	Note owner's spending key	$\mathbb{B}^{\text{ASKLEN}}$
<i>nf</i>	Note nullifier	$\mathbb{B}^{\text{PRNFOUTLEN}}$

Table 2.2: JSInputDType data type

819 **PrimInputDType** Represents the primary inputs used to generate the zk-SNARK proof
820 π . *prim* is a tuple defined as the current Merkle root *mkroot* of the Merkle tree
821 maintained by **Mixer**, the input notes nullifiers *nfs* = $(nf_0, \dots, nf_{\text{JSIN}-1})$, the
822 output notes commitments *cms* = $(cm_0, \dots, cm_{\text{JSOUT}-1})$, the signature hash *hsig*,
823 the message authentication tags *htags* = $(h_0, \dots, h_{\text{JSIN}-1})$ and the residual bits
824 field *rsd*, which aggregates the former's fields bits which could not be contained in
825 a field element.

Field	Description	Data type
<i>mkroot</i>	Merkle root of the Merkle tree	$\mathbb{F}_{\mathbf{r}_{\text{CUR}}}$
<i>nfs</i>	Indexed set of nullifiers of the “old” notes to spend (see Section 3.3.1 for definition of NFFLEN)	$((\mathbb{F}_{\mathbf{r}_{\text{CUR}}})^{\text{NFFLEN}})^{\text{JSIN}}$
<i>cms</i>	Indexed set of commitments to the newly created notes	$(\mathbb{F}_{\mathbf{r}_{\text{CUR}}})^{\text{JSOUT}}$
<i>hsig</i>	Signature hash (non-malleability, see Appendix A and Section 3.3.1 for definition of HSIGFLEN))	$(\mathbb{F}_{\mathbf{r}_{\text{CUR}}})^{\text{HSIGFLEN}}$
<i>htags</i>	Indexed set of message authentication tags (non-malleability, see Appendix A and Section 3.3.1 for definition of HFLEN))	$((\mathbb{F}_{\mathbf{r}_{\text{CUR}}})^{\text{HFLEN}})^{\text{JSIN}}$
<i>rsd</i>	Residual bits corresponding to unpacked bits of former fields (see Section 3.3.1 for definition of RSDFLEN)	$(\mathbb{F}_{\mathbf{r}_{\text{CUR}}})^{\text{RSDFLEN}}$

Table 2.3: PrimInputDType data type

826 **AuxInputDType** Represents the auxiliary inputs used to generate the zk-SNARK proof
827 π . *aux* is a tuple defined as joinsplit inputs (i.e. “old outputs to be spent”), the new
828 *ZethNotes*, the joinsplit’s randomness ϕ as well the public values *vin* and *vout*, the
829 signature hash *hsig* and the message authentication tags *htags* = $(h_0, \dots, h_{\text{JSIN}-1})$.

Field	Description	Data type
<i>jsins</i>	Indexed set of JSIN joinsplit inputs	JSInputDType ^{JSIN}
<i>znotes</i>	Indexed set of JSOUT newly created notes	ZethNoteDType ^{JSOUT}
ϕ	The joinsplit randomness (non-malleability, see Appendix A)	$\mathbb{B}^{\text{PHILEN}}$
<i>vin</i>	Public input value to the joinsplit	$\mathbb{B}^{\text{ZVALUELEN}}$
<i>vout</i>	Public output value to the joinsplit	$\mathbb{B}^{\text{ZVALUELEN}}$
<i>hsig</i>	Signature hash (non-malleability, see Appendix A)	$\mathbb{B}^{\text{CRHHSIGOUTLEN}}$
<i>htags</i>	Indexed set of message authentication tags (non-malleability, see Appendix A)	$(\mathbb{B}^{\text{PRFPKOUTLEN}})^{\text{JSIN}}$

Table 2.4: **AuxInputDType** data type

830 **MixInputDType** Represents the set of inputs to the Mix function of **Mixer**. The input of
831 the Mix function is a tuple defined as the primary inputs *prim*, the zk-proof π , the
832 ciphertexts of the newly created notes *ciphers* = $(ct_0, \dots, ct_{\text{JSOUT}-1})$, a one-time
833 signature σ and the associated verification key *vk*.

Field	Description	Data type
<i>primIn</i>	Primary input object associated with the zk-proof π	PrimInputDType
<i>proof</i>	The zk-SNARK associated to the Zeth statement (see Section 2.2)	ZKPDType (see Section 3.6)
<i>otssig</i>	The one-time signature used to prevent transaction malleability (see Appendix A)	SigOtsDType (see Section 3.4.2)
<i>otsvk</i>	The verification key associated with the signature <i>otssig</i> used to prevent transaction malleability (see Appendix A)	VKOtsDType (see Section 3.4.2)
<i>ciphers</i>	Indexed set of ciphertexts of the newly generated notes	$(\mathbb{B}^{\text{ENCZETHNOTELEN}})^{\text{JSOUT}}$ (see Section 3.5)

Table 2.5: **MixInputDType** data type

834 **MixEventDType** Represents the data emitted as an **Ethereum** event (Section 1.2.3) dur-
 835 ing a successful execution of the Mix function of **Mixer**. Clients are required to
 836 read this data and use it to update their representation of **Mixer**'s state.

Field	Description	Data type
<i>mkroot</i>	New root of Merkle tree of commitments	$\mathbb{F}_{\mathbf{r}_{\text{CUR}}}$
<i>nfs</i>	Nullifiers for input notes consumed	$(\mathbb{B}^{\text{PRFNFOUTLEN}})^{\text{JSIN}}$
<i>cms</i>	Commitments to the output notes	$(\mathbb{F}_{\mathbf{r}_{\text{CUR}}})^{\text{JSOUT}}$
<i>ciphers</i>	Ciphertexts for the output notes	$(\mathbb{B}^{\text{ENCZETHNOTELEN}})^{\text{JSOUT}}$

Table 2.6: **MixEventDType** data type

837 2.2 Zeth statement

838 As explained in [RZ19], the Mix function of **Mixer** verifies the validity of π on the
 839 given primary inputs in order to determine whether the state transition is valid. As
 840 such, **Mixer** verifies whether for π , and primary input *prim*, there exists an auxiliary
 841 input *aux*, such that the tuple $(\text{prim}, \text{aux})$ satisfies the NP-relation \mathbf{R}^z , consisting of the
 842 following constraints:

- 843 • For each $i \in [\text{JSIN}]$:
 - 844 1. $\text{aux.jsins}[i].\text{znote.apk} = \text{PRF}_{\text{aux.jsins}[i].\text{ask}}^{\text{addr}}(0)$
 - 845 2. $\text{aux.jsins}[i].\text{cm} = \text{ComSch.Com}(\text{aux.jsins}[i].\text{znote.apk}, \text{aux.jsins}[i].\text{znote}.\rho, \text{aux.jsins}[i].\text{znote}.v;$
 846 $\text{aux.jsins}[i].\text{znote}.r)$
 - 847 3. $\text{aux.jsins}[i].\text{nf} = \text{PRF}_{\text{aux.jsins}[i].\text{ask}}^{\text{nf}}(\text{aux.jsins}[i].\text{znote}.\rho)$
 - 848 4. $\text{aux.htags}[i] = \text{PRF}_{\text{aux.jsins}[i].\text{ask}}^{\text{pk}}(i, \text{aux.hsigs})$ (non-malleability, see Appendix A)
 - 849 5. $(\text{aux.jsins}[i].\text{znote}.v) \cdot (1 - e) = 0$ is satisfied for the boolean value e set such
 850 that if $\text{aux.jsins}[i].\text{znote}.v > 0$ then $e = 1$.
 - 851 6. The Merkle root mkroot' obtained after checking the Merkle authentica-
 852 tion path $\text{aux.jsins}[i].\text{mkpath}$ of commitment $\text{aux.jsins}[i].\text{cm}$, with MKHASH,
 853 equals to prim.mkroot if $e = 1$.
 - 854 7. $\text{prim.nfs}[i]$
 855 $= \{\text{Pack}_{\mathbb{F}_{\mathbf{r}_{\text{CUR}}}}(\text{aux.jsins}[i].\text{nf}[k \cdot \text{FIELD CAP}:(k+1) \cdot \text{FIELD CAP}])\}_{k \in [\lfloor \text{PRFNFOUTLEN}/\text{FIELD CAP} \rfloor]}$
 856 (see Section 3.3.1 for definition of Pack)
 - 857 8. $\text{prim.htags}[i]$
 858 $= \{\text{Pack}_{\mathbb{F}_{\mathbf{r}_{\text{CUR}}}}(\text{aux.htags}[i][k \cdot \text{FIELD CAP}:(k+1) \cdot \text{FIELD CAP}])\}_{k \in [\lfloor \text{PRFPKOUTLEN}/\text{FIELD CAP} \rfloor]}$
 859 (see Section 3.3.1 for definition of Pack)

- 860 • For each $j \in [\text{JSOUT}]$:
 - 861 1. $\text{aux.znotes}[j].\rho = \text{PRF}_{\text{aux}.\phi}^{\text{rho}}(j, \text{aux.hsig})$ (non-malleability, see Appendix A)
 - 862 2. $\text{prim.cms}[j] = \text{ComSch.Com}(\text{aux.znotes}[j].\text{apk}, \text{aux.znotes}[j].\rho, \text{aux.znotes}[j].v;$
 - 863 $\text{aux.znotes}[j].r)$
- 864 • $\text{prim.hsig} = \{\text{Pack}_{\mathbb{F}_{\text{rCUR}}}(\text{aux.hsig}[k \cdot \text{FIELD CAP}:(k+1) \cdot \text{FIELD CAP}])\}_{k \in [\lfloor \text{CRHHSIGOUTLEN}/\text{FIELD CAP} \rfloor]}$
- 865 (see Section 3.3.1 for definition of Pack)
- 866 • $\text{prim.rsd} = \text{Pack}_{\text{rsd}}(\{\text{aux.jsins}[i].\text{nf}\}_{i \in [\text{JSIN}]}, \text{aux.vin}, \text{aux.vout}, \text{aux.hsig}, \{\text{aux.htags}[i]\}_{i \in [\text{JSIN}]})$
- 867 (see Section 3.3.1 for definition of Pack_{rsd})
- Check that the “joinsplit is balanced”, i.e. check that the joinsplit equation holds:¹

$$\begin{aligned}
 & \text{Pack}_{\mathbb{F}_{\text{rCUR}}}(\text{aux.vin}) + \sum_{i \in [\text{JSIN}]} \text{Pack}_{\mathbb{F}_{\text{rCUR}}}(\text{aux.jsins}[i].\text{note}.v) \\
 &= \sum_{j \in [\text{JSOUT}]} \text{Pack}_{\mathbb{F}_{\text{rCUR}}}(\text{aux.znotes}[j].v) + \text{Pack}_{\mathbb{F}_{\text{rCUR}}}(\text{aux.vout})
 \end{aligned}$$

868 2.3 Generating the inputs of the Mix function (Mix_{in})

869 In the following section, we assume that the system is initialized. In other words, we
 870 assume that a ledger L is available (i.e. an **Ethereum** network is operated by a set of
 871 miners), the **Mixer** contract is deployed on L . Likewise, we assume that the public
 872 parameters $pp_{\text{ZkSnarkSch}} \leftarrow \text{ZkSnarkSch.KGen}(1^\lambda, \mathbf{R}^z)$ are available to **Mixer** and to all
 873 parties willing to call the Mix function of **Mixer**. Furthermore, we assume that there
 874 exists a set of **Ethereum** and **Zeth** users, and that the *payment address* of each **Zeth** user
 875 is easily discoverable. In the rest of this section, the set of *payment addresses* discovered
 876 by a zeth user \mathcal{U}_Z is represented as a list attribute $\mathcal{U}_Z.\text{keystore}$ indexed by usernames.

877 In order for \mathcal{U}_Z to transact via **Zeth**, \mathcal{U}_Z needs to create an object Mix_{in} of type
 878 **MixInputDType** to pass to the Mix function of **Mixer**:

- 879 1. Create an object *prim* of type **PrimInputDType** to represent the primary input,
- 880 and an object *aux* of type **AuxInputDType** to represent the auxiliary input, where:
 - 881 (a) $\text{prim.mkroot} \in \text{Roots}$, where *Roots* is the set of *all* Merkle roots corresponding
 - 882 to one of the state of the Merkle tree on **Mixer** containing *all* the commit-
 - 883 ments to the input notes, in aux.jsins , in its set of leaves.
 - 884 (b) $\text{aux.znotes}[j].r \leftarrow \$\mathbb{B}^{\text{RTRAPLEN}}, \forall j \in [\text{JSOUT}]$, and $\text{aux}.\phi \leftarrow \$\mathbb{B}^{\text{PHILEN}}$

¹where $\text{Pack}_{\mathbb{F}_{\text{rCUR}}}(x)$ outputs the numerical value of x in \mathbb{F}_{rCUR} . We rely on the fact that $\text{ZVALUELEN} < \text{FIELD CAP}$ to perform this sum.

- 885 (c) The public values $(aux.vin, aux.vout) \in (\mathbb{B}^{ZVALUELEN})^2$, $aux.znotes[j].v$ and
 886 $aux.znotes[j].apk \forall j \in [JSOUT]$ are all set by the sender, \mathcal{U}_Z , as desired as
 887 long as they satisfy the joinsplit equation.
- (d) All attributes of the *prim* and *aux* objects should be derived as specified in the statement (see Section 2.2), alongside a signature hash ($aux.hsigs$) that is generated as the hash of the nullifiers and a one-time signing verification key (non-malleability, see Appendix A), using the desired signature scheme $\text{SigSch}_{\text{OT-SIG}}$ (see Section 3.4):

$$(sk_{\text{OT-SIG}}, vk_{\text{OT-SIG}}) = \text{SigSch}_{\text{OT-SIG}}.\text{KGen}(1^\lambda) \quad (2.1)$$

$$aux.hsigs = \text{CRH}^{\text{hsig}}(\{aux.jsins[i].nf\}_{i \in [JSIN]}, vk_{\text{OT-SIG}}) \quad (2.2)$$

- 888 (e) $\text{Mix}_{in}.primIn \leftarrow prim$

Note

If one of the attributes of *prim* and *aux* is not correctly generated, then the proof of computational integrity generated in the next step will be rejected on **Mixer**, and the state of **Mixer** will not be modified.

- 889
- 890 2. Generate a zk-SNARK proof π to prove, in zero-knowledge, that the relation \mathbf{R}^z
 891 (Section 2.2) holds on the primary and auxiliary inputs, using the desired zk-
 892 SNARK scheme ZkSnarkSch (see Section 3.6):
- 893 (a) $\pi \leftarrow \text{ZkSnarkSch}.P(pp_{\text{ZkSnarkSch}}, prim, aux)$
- 894 (b) $\text{Mix}_{in}.proof \leftarrow \pi$
- 895 3. Encrypt all the *aux.znotes* using the recipient's *payment address*, using the en-
 896 cryptation scheme EncSch (see Section 3.5).
- (a) For all $j \in [JSOUT]$, do:

$$ct_j \leftarrow \text{EncSch}.Enc(aux.znotes[j], \mathcal{U}_Z.keystore[recipient_j].pub.pkenc)$$

- 897 (b) $\text{Mix}_{in}.ciphers \leftarrow \{ct_j\}_{j \in [JSOUT]}$

- 898 4. Generate a signature $\sigma_{\text{OT-SIG}}$ on the inputs of the *Mix* function, in order to prevent
 899 any malleability attacks (c.f. Appendix A), using the desired signature scheme
 900 $\text{SigSch}_{\text{OT-SIG}}$ (see Section 3.4):

- (a) Using the one-time signature keypair generated in Eq. (2.1), do:

$$\begin{aligned} dataToBeSigned &= \mathcal{S}_E.Addr \parallel \text{Mix}_{in}.primIn \parallel \text{Mix}_{in}.\pi \parallel \text{Mix}_{in}.ciphers \\ \sigma_{\text{OT-SIG}} &= \text{SigSch}_{\text{OT-SIG}}.\text{Sig}(sk_{\text{OT-SIG}}, \text{CRH}^{\text{ots}}(dataToBeSigned)) \end{aligned}$$

- 901 (b) $\text{Mix}_{in}.otssig \leftarrow \sigma_{\text{OT-SIG}}$

902 (c) $\text{Mix}_{in}.otsvk \leftarrow vk_{\text{OT-SIG}}$

903 Here, $\mathcal{S}_{\mathcal{E}}.Addr$ represents the address of the **Ethereum** user $\mathcal{S}_{\mathcal{E}}$ who must sign the
 904 transaction (see Section 2.4). In general, this is likely to be owned by the holder
 905 $\mathcal{U}_{\mathcal{Z}}$ of the **Zeth** notes to be spent, but this is not a requirement.

906 2.4 Creating an Ethereum transaction tx_{Mix} to call $\widetilde{\text{Mixer}}$

After generating a Mix_{in} object, $\mathcal{U}_{\mathcal{Z}}$ can generate an object tx_{raw} of type **TxRawDType**, such that:

$$tx_{raw}.to = \widetilde{\text{Mixer}}.Addr \wedge tx_{raw}.data = zdata$$

907 Then, an **Ethereum** user $\mathcal{S}_{\mathcal{E}}$ can ECDSA sign tx_{raw} , under $\mathcal{S}_{\mathcal{E}}.sk$ in order to transform
 908 this object of type **TxRawDType** into an finalized transaction, i.e. an object tx_{Mix} of type
 909 **TxDType**.

910 Finally, the transaction tx_{Mix} is broadcasted on the **Ethereum** network and eventually
 911 gets mined.

Note

Here, the **Ethereum** user $\mathcal{S}_{\mathcal{E}}$ who sends the final transaction, and the **Zeth** user $\mathcal{U}_{\mathcal{Z}}$ may represent the same person or entity, but this is not necessarily the case. It is perfectly feasible (and in some cases may be desirable) for a **Zeth** user $\mathcal{U}_{\mathcal{Z}}$ to create a **Zeth** transaction which is later signed by a distinct party $\mathcal{S}_{\mathcal{E}}$. In particular, the only identifying information that appears in plaintext on the ledger will be that of $\mathcal{S}_{\mathcal{E}}$.

912

913 2.5 Processing tx_{Mix}

914 When a tx_{Mix} is mined (hence assuming that $\text{EthVerifyTx}(tx_{\text{Mix}})$ returns true), the state
 915 transition specified by the **Mix** function of $\widetilde{\text{Mixer}}$ is executed.

916 To preserve the soundness of **Zeth**, and make sure that no $\mathcal{U}_{\mathcal{Z}}$ is able to create
 917 value by double spending *ZethNotes*, various checks need to be satisfied. The function
 918 **ZethVerifyTx** is defined as the function that returns true if all the checks are satisfied,
 919 and false otherwise.

920 If **ZethVerifyTx**(tx_{Mix}) returns true, then **Mix** modifies the “World state” ς to account
 921 for the spent *ZethNotes* and the newly generated ones. However, if **ZethVerifyTx**(tx_{Mix})
 922 returns false, then the state transition ends.

Note

Even if $\text{ZethVerifyTx}(tx_{\text{Mix}})$ returns false, ς is modified since the **Ethereum** balances of the transaction originator is decremented by the sum of **DGAS** and the gas consumed by the **ZethVerifyTx** function, and the balance of the **Ethereum** account of the miner gets incremented by the same amount.

923

924 Thus, Mix proceeds as follows:

1. Check that all the values of the primary inputs' ($\text{Mix}_{in}.primIn$) entries are elements of the scalar field over which the zk-proof is generated:

$$\text{Mix}_{in}.primIn \in \mathbb{F}_{\text{rCUR}}^*$$

2. Unpack the nullifiers, signature hash and public values (see Section 3.3.1 for the definitions of the **Unpack** functions):

$$\begin{aligned} nf_i &= \text{Unpack}_{nf}(\text{Mix}_{in}.primIn.nfs[i], \text{Mix}_{in}.primIn.rsd) \quad \forall i \in [\text{JSIN}] \\ vin &= \text{decode}_{\mathbb{N}}(\text{Unpack}_{vin}(), \text{Mix}_{in}.primIn.rsd) \\ vout &= \text{decode}_{\mathbb{N}}(\text{Unpack}_{vout}(), \text{Mix}_{in}.primIn.rsd) \\ hsig &= \text{Unpack}_{hsig}(\text{Mix}_{in}.primIn.hsig, \text{Mix}_{in}.primIn.rsd) \end{aligned}$$

925

3. Check the validity of the tx_{Mix} object (**ZethVerifyTx**):

- Check that $\text{Mix}_{in}.primIn.hsig$ is correctly computed, i.e. check that the following equation holds (to prevent transaction malleability, see Appendix A):

$$hsig = \text{CRH}^{\text{hsig}}(\text{Mix}_{in}.primIn.nfs, \text{Mix}_{in}.otsvk)$$

- Check that π is a valid zk-SNARK proof for $\text{Mix}_{in}.primIn$, i.e. check that:

$$\text{ZkSnarkSch.V}(pp_{\text{ZkSnarkSch}}, \pi, \text{Mix}_{in}.primIn) = \text{true}$$

- Check that none of the nullifiers in $\text{Mix}_{in}.primIn.nfs$ have already been used, i.e. check that:

$$nf_i \notin \text{Nulls}, \forall i \in [\text{JSIN}]$$

926

where Nulls is the set of all nullifiers that are “declared” on $\widetilde{\text{Mixer}}$.

- Check that $\text{Mix}_{in}.otssig$ is a valid signature of the **Ethereum** sender’s address Addr (see Section 2.4) and the attributes of Mix_{in} , to prevent transaction malleability (see Appendix A), i.e. check that:

$$\begin{aligned} \text{SigSch}_{\text{OT-SIG}}.Vf(\text{Mix}_{in}.otsvk, m, \text{Mix}_{in}.otssig) &= \text{true} \\ \text{where } m &= \text{CRH}^{\text{ots}}(\text{Addr} \parallel \text{Mix}_{in}.primIn \parallel \text{Mix}_{in}.\pi \parallel \text{Mix}_{in}.ciphers) \end{aligned}$$

- Check that $\widetilde{\text{Mix}_{in}.primIn.mkroot}$ corresponds to a valid state of the Merkle tree held on **Mixer**, i.e. check that:

$$\text{Mix}_{in}.primIn.mkroot \in \text{Roots}'$$

927 where Roots' is the set of all Merkle roots corresponding to one of the states
928 of the Merkle tree.

- Check that vin corresponds to the value val of the transaction object, i.e. check that:

$$vin = tx_{\text{Mix}}.val$$

929 4. If all checks above pass, i.e. if $\text{ZethVerifyTx}(tx_{\text{Mix}})$ returns **true**, then the following
930 additional modifications are made in ς :

- 931 • Add the commitments $\text{Mix}_{in}.primIn.cms$ to the Merkle tree held on $\widetilde{\text{Mixer}}$.
- 932 • $\text{Roots}' \leftarrow \text{Roots}' \cup \{mkroot'\}$, where $mkroot'$ is the Merkle root of the Merkle
933 tree after insertion of the commitments $\text{Mix}_{in}.primIn.cms$ in the Merkle tree.
- 934 • $\text{Nulls} \leftarrow \text{Nulls} \cup \{nf_i\}_{i \in [\text{JSIN}]}$, i.e. the nullifiers nfs become “declared”.
- 935 • Modify the **Ethereum** balances according to the public values:
936 – $\varsigma[\mathcal{SE}.Addr].bal = \varsigma[\mathcal{SE}.Addr].bal - vin$
937 – $\varsigma[\mathcal{SE}.Addr].bal = \varsigma[\mathcal{SE}.Addr].bal + vout$
938 – $\widetilde{\text{Mixer}}.bal = \widetilde{\text{Mixer}}.bal + vin$
939 – $\widetilde{\text{Mixer}}.bal = \widetilde{\text{Mixer}}.bal - vout$
- 940 • Emit an event (Section 1.2.3) $evMixOut$ of type **MixEventDType**, contain-
941 ing the new root $mkroot'$ of the Merkle tree of commitments, the nullifiers
942 $\{nf_i\}_{i \in [\text{JSIN}]}$, commitments to the newly created *ZethNotes* $\text{Mix}_{in}.primIn.cms$,
943 and the corresponding ciphertexts $\text{Mix}_{in}.primIn.ciphers$.

944 2.6 Receiving *ZethNotes*

945 In order to confirm the reception of *ZethNotes*, \mathcal{R}_Z must listen to the events (Sec-
946 tion 1.2.3) of type **MixEventDType** emitted by the processing of tx_{Mix} , and try to decrypt
947 the ciphertexts using $\mathcal{R}_Z.priv.skenc$ to see if he is the recipient of a **Zeth** payment. If
948 the decryption is successful (\mathcal{R}_Z is the recipient of a payment), \mathcal{R}_Z must verify that the
949 *ZethNote* recovered is the opening of a commitment in the Merkle tree of $\widetilde{\text{Mixer}}$. If not,
950 \mathcal{R}_Z rejects the (invalid) payment.

951 We describe below the steps that \mathcal{R}_Z needs to carry out for all events $evMixOut \in$
952 **MixEventDType** emitted by $\widetilde{\text{Mixer}}$, in order to receive payments:

- 953 1. Compute the new root $mkroot'$ of the Merkle tree of commitments, after adding the
954 new values $ev\widetilde{MixOut}.cms$. If this value does not match the new root $evMixOut.mkroot$
955 emitted by **Mixer**, abort.

2. Try to decrypt the ciphertexts:

$$zn_j = \text{EncSch.Dec}(\mathcal{R}_Z.\text{priv.skenc}, \text{evMixOut.ciphers}[j])$$

956 3. For each successful decryption, let j be the index of the decrypted ciphertext:

- 957 (a) Check whether the recovered plaintext zn_j is a well-formed *ZethNote*. Abort
958 if it is not well-formed.
- (b) Check that the recovered *ZethNote* zn_j is the opening of the corresponding
commitment $\text{evMixOut.cms}[j]$:

$$\text{evMixOut.cms}[j] = \text{ComSch.Com}(zn_j.\text{apk}, zn_j.\rho, zn_j.v; zn_j.r)$$

959 Abort if the note is not a valid opening.

- 960 (c) Additionally, if sender \mathcal{S}_Z , and recipient \mathcal{R}_Z had a contractual agreement,
961 then \mathcal{R}_Z needs to check that the terms of this agreement are fulfilled by all
962 the recovered *ZethNotes*, abort otherwise.

963 Note that Steps 1 and 3b are required to ensure that data decrypted by \mathcal{R}_Z ex-
964 actly matches the data committed to in **Mixer**. In particular, Step 1 requires \mathcal{R}_Z
965 to maintain or have access to some representation of the Merkle tree of commitments.
966 See Section 4.1.2 for further details.

967 2.7 Security requirements for the primitives

968 We list below the security requirements to instantiate the primitives of the **Zeth** protocol.

- 969 • CRH^{hsig} and CRH^{ots} MUST be collision resistant functions (see Definition 1.5.16).
- 970 • PRF^{addr} , PRF^{nf} , PRF^{rho} and PRF^{pk} MUST be PRF when keyed by ask and ϕ , and be
971 collision resistant (see Definition 1.5.16, and Section 1.5.8).
- 972 • $\text{SigSch}_{\text{OT-SIG}}$ MUST be UF-CMA (see Definition 1.5.24 and Appendix A.2.3).
- 973 • ComSch MUST be computationally hiding and binding (see Section 1.5.9).
- 974 • MKHASH MUST be collision resistant with $h_0 = 0_{\mathbb{F}_{\text{rCUR}}}$ (see Section 1.5.6).²
- 975 • EncSch MUST be IND-CCA2 and IK-CCA (see, respectively, [ABR99, Definition 8]
976 and Definition 1.5.10).
- 977 • $\text{Unpack}(\text{Pack}(X)) = X$ and $\text{Unpack}(\text{Pack}_{\text{rsd}}(X)) = X$ MUST hold.
- 978 • $\text{decode}(\text{encode}(X)) = X$ MUST hold.

²This security requirement is equivalent to the one in [ZCa19, Section 5.4.1.3] where finding a preimage of $0^{\text{SHA256DLEN}}$ must be hard.

979 2.7.1 Additional notes

980 Defining $hsig$

The signature hash $hsig$ is a variable used to bind the signature keys to the primary inputs. We use the same definition of $hsig$ as **Zcash** to prevent the Faerie Gold attack and thus

$$hsig = \text{CRH}^{\text{hsig}}(nfs, vk).$$

981 As a private transaction is uniquely determined by its nullifiers $nfs = (nf_0, \dots, nf_{\text{JSIN}-1})$,
 982 and because of the collision resistance of CRH^{hsig} , a transaction is uniquely determined
 983 by $hsig$ (with overwhelming probability). We did not use the *randomSeed* defined in
 984 **Zcash** however, since this is only necessary to achieve uniqueness of $hsig$ for transactions
 985 *in transit* (i.e. not mined yet) [Hop16]. The uniqueness of $hsig$ is a requirement to
 986 prevent the Fairy Gold attack.

987 Security Requirement.

- 988 • The variable $hsig$ **MUST** be derived from the nullifiers $\{nf_i\}_{i \in [\text{JSIN}]}$ and the signing
 989 key vk using a collision resistant function. Doing so, makes sure that $hsig$ is unique
 990 for each tx_{Mix} with overwhelming probability.

991 Defining ρ

We define ρ like in **Zcash** in order to prevent the Faerie Gold attack. A malicious sender could reuse the same ρ for a given recipient, hence correctly generating a *ZethNote* which could become unspendable by the recipient. Making ρ the output of a collision resistant PRF with random variable ϕ as key and with tx_{Mix} 's $hsig$ as input ensures, with overwhelming probability, the uniqueness of ρ and prevents this attack. Thus,

$$\rho_j = \text{PRF}_{\phi}^{\text{rho}}(j, hsig).$$

992 Message authentication tags h_i

The message authentication tags are used to bind the signature hash to the input notes spending keys, to show ownership of the spent notes. Each tag derived from a note owner's spending key and the signature hash **MUST** be unique for each note with overwhelming probability. We define

$$h_i = \text{PRF}_{ask_i}^{\text{pk}}(i, hsig).$$

Chapter 3

Instantiation of the cryptographic primitives

In this chapter, we start by instantiating the cryptographic building blocks used in previous sections to describe the **Zeth** DAP design. Finally, we proceed by providing security proofs justifying that our instantiation complies with the security requirements listed in previous sections.

Note that, in several cases, it is necessary to specify details in terms of concrete properties of the curve **Curve** and associated scalar field \mathbb{F}_{CUR} . In these cases, we focus on two curves of interest: **BN-254** and **BLS12-377**. We note, however, that other suitable curves could be used.

BN-254 [Rk19] has several properties that make it implementation-friendly. Elements of both the base field and scalar field can be represented in **ETHWORDLEN** bits (the native word size of the EVM), allowing efficient encoding and manipulation of such elements. Moreover, a subset of operations on **BN-254** are supported by the EVM through precompiled contracts. These precompiled contracts enable verification of signatures (Section 3.4) and zero-knowledge proofs (Section 3.6), required by this protocol, with minimal gas overhead.

BLS12-377 [BCG⁺20], like **BN-254**, has the advantage that scalar field elements can be represented within **ETHWORDLEN**-bit words (although the same is not true of base field elements). However, the **EVM** provides no native support for **BLS12-377**, which increases the complexity of the **Mixer** implementation (see Section 2.5 for details of the operations to be performed). An advantage that **BLS12-377** does provide, is that it is the “inner” curve of a one-layer chain (as described in [BCG⁺20, HG20]). Therefore zero-knowledge proofs using **BLS12-377** can be efficiently verified by statements in other zero-knowledge proofs using an appropriate “outer” pairing. Support for **BLS12-377** in **Zeth** therefore admits several applications (not explicitly covered by this document), such as aggregation of proofs over multiple **Zeth** transactions (e.g. [Ron20]).

Further details related to implementation and optimization are given in Chapter 4.

3.1 Instantiating the PRFs, ComSch and CRHs

The functions CRH^{hsig} and CRH^{ots} are instantiated with SHA256 [oST15] which we assume to be collision resistant. Furthermore, ComSch , $\text{PRF}^{\text{pk}}(x)$, $\text{PRF}^{\text{rho}}(x)$, $\text{PRF}^{\text{addr}}(x)$, and $\text{PRF}^{\text{nf}}(x)$ are all instantiated with Blake2’s hash function optimized for 32-bit platforms, Blake2s, which we prove in the Weakly Ideal Cipher Model [LMN16] to be from a family of PRF and collision resistant functions. The Weakly Ideal Cipher model assumes that Blake2’s underlying block cipher is ideal and has no structural weaknesses (see Appendix D.2). In addition to that, and to ensure that the functions $\text{PRF}^{\text{pk}}(x)$, $\text{PRF}^{\text{rho}}(x)$, $\text{PRF}^{\text{addr}}(x)$, and $\text{PRF}^{\text{nf}}(x)$ compute images lying in different domains, we use different message prefixes (or “domain separators”) for the PRFs inputs. This approach ensures that the apk_i ’s, nf_i ’s, ρ_i ’s, and h_i ’s have independent distributions from a PPT adversary point of view.

Note

It is important to note that, for this approach to be secure, the hash function used needs to be secure against *chosen-prefix collision attacks* [Ste15].

Furthermore, we take:

- $\text{RTRAPLEN}, \text{ASKLEN}, \text{PHILEN} = \text{BLAKE2sCLEN}$

3.1.1 Blake2 primitive

Blake [AHMP08] is a hash family that was presented as a candidate at the SHA3 competition. Blake2 is the next iteration of the family which has been further optimized to achieve higher throughput thanks to some optimizations and by being less conservative on its security¹. Blake and Blake2 are based on the ChaCha stream cipher [Ber08a] composed with the HAIFA framework [BD07]. ChaCha defined over 20 rounds, as used in Blake2, is deemed secure and a PRF based on today’s cryptanalysis [Pro14, CM16]. Blake2 is specified in RFC-7693 [MJS15] and licensed under CC0. Blake2s is an instantiation of Blake2 optimized for 32-bit platforms. As such, to reason about the security of Blake2s we prove the security of Blake2.

Blake security Blake security has been heavily scrutinized through the SHA3 competition [VNP10, MQZ10, AMP10, AAM12, AMPŠ12, ALM12, HMRS12]. Blake2 has also been thoroughly cryptanalyzed independently [GKN⁺14, Hao14, EFK15, NA19]. For n -bit long digests/outputs, the hash and compression functions present $n/2$ -bit of collision resistance and n -bit of preimage resistance, immunity to length extension, and indistinguishability from a random oracle [ANWOW13]. They have furthermore been demonstrated secure in the Weakly Ideal Cipher Model [LMN16] (WICM, see Appendix D.1.1). More

¹The authors increased the number of rounds of Blake for the SHA3 competition to be more conservative on security. They however showed afterwards that this change was not “meaningfully more secure” and thus reverted it for Blake2 (see [ANWOW13, Section 2.1]).

1054 particularly, Luykx et al. show that Blake2 is indifferentiable from a random oracle in
 1055 this model and is a PRF. We use this result in Appendix D.2 to show the collision
 1056 resistance of Blake2. We also prove that, given that Blake2 is collision resistant and a
 1057 PRF, Blake2($r||x$) is a computationally binding and computationally hiding commitment
 1058 scheme for input x and randomness r .

Note

We assume that the encryption scheme used in the Blake2 underlying compression function – which is derived from ChaCha20 – has no exploitable structural behaviour. More precisely, that this encryption scheme behaves like a weak ideal cipher. We provide proofs in this model.

1059

1060 3.1.2 Commitment scheme

We define our commitment scheme as follows,

$$\begin{aligned} \text{ComSch.Setup} &: \{1^\lambda \text{ s.t. } \lambda \in \mathbb{N}\} \rightarrow \mathbb{B}^* \\ \text{ComSch.Com} &: (\mathbb{B}^{\text{PRFADDRROUTLEN}} \times \mathbb{B}^{\text{PRFRHOOUTLEN}} \times \mathbb{B}^{\text{ZVALUELEN}}) \times \mathbb{B}^{\text{RTRAPLEN}} \rightarrow \mathbb{F}_{\mathbf{r}_{\text{CUR}}} \end{aligned}$$

We instantiate the commitment scheme with Blake2s as follows,

$$\begin{aligned} pp &= \text{ComSch.Setup}(1^\lambda) \text{ (corresponds to Blake2s's constant PB and } \mathbf{r}_{\text{CUR}}) \\ cm &= \text{ComSch.Com}(m = (apk, \rho, v); r) \\ &= \text{decode}_{\mathbb{N}}(\text{Blake2s}(r||apk||\rho||v)) \pmod{\mathbf{r}_{\text{CUR}}} \end{aligned}$$

1061 **Remark 3.1.1.** We set the commitment digest length in the parameter block PB [MJS15].

1062 Security proof

1063 The commitment scheme defined above is computationally hiding and binding in the
 1064 WICM, see Appendix D.2.4. However, because of the modulo \mathbf{r}_{CUR} operation, the scheme
 1065 is only $(\text{FIELDLEN}/2)$ -bit binding.

1066 3.1.3 PRFs

We show in this section how we instantiate the PRFs with Blake primitives. As a reminder the PRFs are defined as follows,

$$\begin{aligned} \text{PRF}^{\text{addr}} &: \mathbb{B}^{\text{ASKLEN}} \times \{0\} \rightarrow \mathbb{B}^{\text{PRFADDRROUTLEN}} \\ \text{PRF}^{\text{pk}} &: (\mathbb{B}^{\text{ASKLEN}} \times [\text{JSIN}]) \times \mathbb{B}^{\text{CRHHSIGOUTLEN}} \rightarrow \mathbb{B}^{\text{PRFPKOUTLEN}} \\ \text{PRF}^{\text{nf}} &: \mathbb{B}^{\text{ASKLEN}} \times \mathbb{B}^{\text{PRFRHOOUTLEN}} \rightarrow \mathbb{B}^{\text{PRFNFOUTLEN}} \\ \text{PRF}^{\text{rho}} &: (\mathbb{B}^{\text{PHILEN}} \times [\text{JSOUT}]) \times \mathbb{B}^{\text{CRHHSIGOUTLEN}} \rightarrow \mathbb{B}^{\text{PRFRHOOUTLEN}} \end{aligned}$$

As we instantiate the PRFs with Blake2s we have,

$$\text{PRFADDROUTLEN}, \text{PRFNFOUTLEN}, \text{PRFPKOUTLEN}, \text{PRFRHOOUTLEN} = \text{BLAKE2sCLEN}$$

To ensure that the PRFs have independent distributions, we first introduce tagging functions tag^x which truncate and prepend with a distinct tag the PRFs key. We have,

$$\begin{aligned} \text{tag}^{\text{addr}} &: \mathbb{B}^{\text{ASKLEN}} \rightarrow \mathbb{B}^{\text{BLAKE2sCLEN}} \\ \text{tag}^{\text{pk}} &: \mathbb{B}^{\text{ASKLEN}} \times [\text{JSIN}] \rightarrow \mathbb{B}^{\text{BLAKE2sCLEN}} \\ \text{tag}^{\text{nf}} &: \mathbb{B}^{\text{ASKLEN}} \rightarrow \mathbb{B}^{\text{BLAKE2sCLEN}} \\ \text{tag}^{\text{rho}} &: \mathbb{B}^{\text{PHILEN}} \times [\text{JSOUT}] \rightarrow \mathbb{B}^{\text{BLAKE2sCLEN}} \end{aligned}$$

The tagging functions are instantiated as follows,

$$\begin{aligned} \text{tag}^{\text{addr}}(\text{aux.jsins}[i].ask) &= \text{tag}_{ask}^{\text{addr}} \\ &= (1) \parallel (1)^{\lceil \frac{\text{JSMAX}}{2} \rceil} \parallel (0, 0) \parallel \text{trunc}_{\text{BLAKE2sCLEN}-3-\lceil \frac{\text{JSMAX}}{2} \rceil}(\text{aux.jsins}[i].ask) \\ \text{tag}^{\text{nf}}(\text{aux.jsins}[i].ask) &= \text{tag}_{ask}^{\text{nf}} \\ &= (1) \parallel (1)^{\lceil \frac{\text{JSMAX}}{2} \rceil} \parallel (1, 0) \parallel \text{trunc}_{\text{BLAKE2sCLEN}-3-\lceil \frac{\text{JSMAX}}{2} \rceil}(\text{aux.jsins}[i].ask) \\ \text{tag}^{\text{pk}}(\text{aux.jsins}[i].ask, i) &= \text{tag}_{ask, i}^{\text{pk}} \\ &= (0) \parallel \text{pad}_{\lceil \frac{\text{JSMAX}}{2} \rceil}(\text{encode}_{\mathbb{N}}(i)) \parallel (0, 0) \parallel \text{trunc}_{\text{BLAKE2sCLEN}-3-\lceil \frac{\text{JSMAX}}{2} \rceil}(\text{aux.jsins}[i].ask) \\ \text{tag}^{\text{rho}}(\text{aux}. \phi, j) &= \text{tag}_{ask, j}^{\text{rho}} \\ &= (0) \parallel \text{pad}_{\lceil \frac{\text{JSMAX}}{2} \rceil}(\text{encode}_{\mathbb{N}}(j)) \parallel (1, 0) \parallel \text{trunc}_{\text{BLAKE2sCLEN}-3-\lceil \frac{\text{JSMAX}}{2} \rceil}(\text{aux}. \phi) \end{aligned}$$

1067 where $\text{pad}_{\lceil \frac{\text{JSMAX}}{2} \rceil}(\text{encode}_{\mathbb{N}}(i))$ is the function that pads the binary representation of i by
 1068 adding 0's before the most significant bit (e.g. assuming big endian encoding, $\text{pad}_2(\text{encode}_{\mathbb{N}}(1)) =$
 1069 01).

We now present how the PRFs are instantiated,

$$\begin{aligned} \text{PRF}_{\text{aux.jsins}[i].ask}^{\text{addr}}(0) &= \text{aux.jsins}[i].znote.apk \\ &= \text{Blake2s}(\text{tag}^{\text{addr}}(\text{aux.jsins}[i].ask) \parallel \text{pad}_{\text{BLAKE2sCLEN}}(0)) \\ \text{PRF}_{\text{aux.jsins}[i].ask}^{\text{nf}}(\text{aux.jsins}[i].\rho) &= \text{prim.nfs}[i] \\ &= \text{Blake2s}(\text{tag}^{\text{nf}}(\text{aux.jsins}[i].ask) \parallel \text{aux.jsins}[i].znote.\rho) \\ \text{PRF}_{\text{aux.jsins}[i].ask}^{\text{pk}}(i, \text{prim.hsig}) &= \text{prim.htags}[i] \\ &= \text{Blake2s}(\text{tag}^{\text{pk}}(\text{aux.jsins}[i].ask, i) \parallel \text{prim.hsig}) \\ \text{PRF}_{\text{aux}. \phi}^{\text{rho}}(j, \text{prim.hsig}) &= \text{aux.znotes}[j].\rho \\ &= \text{Blake2s}(\text{tag}^{\text{rho}}(\text{aux}. \phi, j) \parallel \text{prim.hsig}) \end{aligned}$$

1070 **Remark 3.1.2.** We set the PRFs' output length in the Blake2s's parameter block PB.

Security proof

The functions defined above are collision resistant and PRFs in the WICM, see Appendix D.2. Because of the tagging functions, the security parameter of the PRFs becomes $\lambda = \text{BLAKE2sCLEN}/2 - \text{JSMAX}/4 - 3/2$.

3.1.4 Collision resistant hashes

We instantiate in this section the collision resistant hash functions CRH^{hsig} and CRH^{ots} with SHA256. As a consequence, we have,

$$\text{CRHHSIGOUTLEN} = \text{CRHOTSOUTLEN} = \text{SHA256DLEN}$$

SHA256 Security SHA-256 (Secure Hash Algorithm 256) is a hash function designed by the National Security Agency (NSA) in 2001. It is based on the Merkle–Damgård structure, the Davies–Meyer compression function construct [BRS02, Function f_5 in Figure 3] and the classified SHACAL-2 block cipher.

Collision attacks have been thoroughly studied by the research community [SS08, MNS11]. The best attacks at this day, are second-order differential attack by Lamberger et al. [LM11] on the SHA-256 compression function reduced to 46 out of 64 rounds.

Many researchers [IS09, AGM⁺09] have also studied preimage attacks on SHA-256 with reduced rounds. Guo et al. [GLRW10] in particular were among the first to use the meet in the middle strategy [AS09] and achieved more efficient ones on 42-step SHA-256. Khovratovich et al. in 2012 [KRS12] have so far presented the best preimage attacks, on 45-round and 52-round SHA-256 as well as a 52-round attack on the SHA-256 compression function.

Li et al. have published in 2012 [LIS12] a noteworthy paper on converting meet in the middle preimage attack into pseudo collision attack. Using preimage attacks by bicliques, they found pseudo collisions attacks on 52 steps of SHA-256.

Claim 1. SHA256 is 128-bit collision resistant.

3.2 Instantiating MKHASH

In this section we describe the instantiation of MKHASH with a compression function based on MIMC [AGR⁺16]. We firstly show how the compression function is constructed, and prove that this instantiation complies with the security requirements mentioned in Section 2.7

3.2.1 MIMC Encryption

MIMC is a block cipher with a simple design, consisting of a number of rounds (denoted *rounds*). During the i -th round, the message m is mixed with the encryption key k and a randomly chosen constant $c[i]$, and a permutation function is applied to generate a new value of m . The permutation function consists of exponentiation with a carefully chosen

1103 exponent e (see Section 3.2.1). Note that *rounds* depends on the desired security level
 1104 λ . We denote the encryption function by **MIMC-Encrypt** and illustrate it in Fig. 3.1.

```

MIMC-Encrypt( $k, m, c, e, rounds$ )
1 : foreach  $i \in [rounds]$  :
2 :    $m \leftarrow (k \text{ OP } c[i] \text{ OP } m)^e$ 
3 : return ( $m \text{ OP } k$ )

```

Figure 3.1: MIMC Encryption function.

MIMC-Encrypt can be defined on both binary and prime fields. As such, the **OP** operation corresponds to either \oplus or $+$ (mod p) [AGR⁺16, GRR⁺16]. For general prime p , we denote by **MIMC_p** (resp. **MIMC_{2ⁿ}**) the **MIMC-Encrypt** function defined over \mathbb{F}_p (resp. \mathbb{F}_{2^n}). Since block ciphers are usually defined over the product space of keys and messages, we consider the variables c , *rounds* and e as fixed. The function signatures thus become,

$$\begin{aligned} \text{MIMC}_p &: \mathbb{F}_p \times \mathbb{F}_p \rightarrow \mathbb{F}_p \\ \text{MIMC}_{2^n} &: \mathbb{F}_{2^n} \times \mathbb{F}_{2^n} \rightarrow \mathbb{F}_{2^n} \end{aligned}$$

1105 From now on, we only consider **MIMC** defined over prime fields. In particular, the
 1106 field $\mathbb{F}_{\mathbf{r}_{\text{CUR}}}$ with elements written over λ bits, over which **ZkSnarkSch** operates.

1107 Security parameters and analysis

1108 To ensure that the exponentiation leads to a permutation in $\mathbb{F}_{\mathbf{r}_{\text{CUR}}}$, e of the form $e = 2^t - 1$
 1109 such that $\gcd(e, \mathbf{r}_{\text{CUR}} - 1) = 1$. To achieve a security of λ , we require $rounds = \left\lceil \frac{\log_2 \mathbf{r}_{\text{CUR}}}{\log_2 e} \right\rceil$.²

1110 We refer to the **MIMC** paper [AGR⁺16, Section 4.2 and 5.1] for more details on the
 1111 security analysis and attacks on the scheme. Note that **MIMC_{r_{CUR}}** does not suffer from
 1112 *inversion subfield attacks* as there are no proper subfields of $\mathbb{F}_{\mathbf{r}_{\text{CUR}}}$.

1113 3.2.2 MIMC-based compression function

1114 There exist two main techniques to construct a hash function from a block-cipher (or
 1115 permutation): sponge functions [BDPVA07] and iterated compression functions [BRS02].

1116 A Merkle tree is a binary tree of values of fixed size, where the values in each “layer”
 1117 are generated by hashing pairs of values from the previous “layer”. That is, we require a
 1118 compression function **MKHASH**, which we construct via the Miyaguchi-Preneel scheme.
 1119 (Miyaguchi-Preneel is more secure [BRS02, f_5 function] than the more flexible Davies-
 1120 Meyer construct [GFBR06, Section 3], but this flexibility is not required in our case).

²We do not consider exponents of type $2^t + 1$ since the polynomial representing **MIMC-Encrypt** would be sparse and as such, the number of rounds becomes constant (see [AGR⁺16, Section 5.3]).

1121 Miyaguchi-Preneel compression construct

1122 Miyaguchi-Preneel (MP) [BRS02, f_3 function] is a general scheme for constructing com-
 1123 pression functions from block ciphers (see Section 1.5.6). Given a block cipher E , the
 1124 corresponding compression function by f_E^{MP} is given in Fig. 3.2. The original construc-
 1125 tion is defined over binary fields, however **Zeth** operates over prime fields. Hence, in the
 1126 general discussion here we replace the bitwise addition operator \oplus by modular addition
 1127 in $\mathbb{F}_{r_{\text{CUR}}}$ (see [Har19]).

1128 We denote by **MIMC-MP** the compression function defined by the application of
 1129 the Miyaguchi-Preneel construct over **MIMC**. Similarly, for general prime p we denote
 1130 by **MIMC-MP_p** (see Fig. 3.3) the compression function defined by application of the
 1131 Miyaguchi-Preneel construct over **MIMC_p**.

$f_E^{\text{MP}}(k, m)$
1 : $res \leftarrow E_k(m)$ 2 : return $(res + m + k) \pmod{r_{\text{CUR}}}$

Figure 3.2: MP construct in $\mathbb{F}_{r_{\text{CUR}}}$.

MIMC-MP_{r_{CUR}} (k, m)
1 : $res \leftarrow \text{MIMC}_{r_{\text{CUR}}}(k, m)$ 2 : return $(res + k + m) \pmod{r_{\text{CUR}}}$

Figure 3.3: **MIMC-MP_{r_{CUR}}** construction.

1132 3.2.3 An efficient instantiation of **MIMC** primitives

1133 To select appropriate instances of **MIMC_{r_{CUR}}** and **MIMC-MP_{r_{CUR}}**, we consider the cost
 1134 (in terms of gas consumption and prover efficiency). For given e and $rounds$, the final
 1135 definition of **MIMC-MP_{r_{CUR}}** is given in Fig. 3.4 and Fig. 3.5.

MIMC_{r_{CUR}} (k, m)	InitRoundConstants ()
1 : $c \leftarrow \text{InitRoundConstants}()$ 2 : foreach $i \in [rounds]$: 3 : $m \leftarrow (k + c[i] + m)^e \pmod{r_{\text{CUR}}}$ 4 : return $(m + k) \pmod{r_{\text{CUR}}}$	$iv \leftarrow \text{Keccak256}(\text{"clearmatics_mt_seed"})$ $c[0] \leftarrow 0$ $c[1] \leftarrow \text{Keccak256}(iv)$ foreach $i \in \{2, \dots, rounds\}$: $c[i] \leftarrow \text{Keccak256}(c[i-1])$ return $c = (c[0], \dots, c[rounds-1])$

Figure 3.4: **MIMC_{r_{CUR}}** full construction

MIMC-MP_{r_{CUR}} (k, m)
return MIMC_{r_{CUR}} (k, m) + $m + k \pmod{r_{\text{CUR}}}$

Figure 3.5: **MIMC-MP_{r_{CUR}}** full construction

1136 **Remark 3.2.1.** Note that Keccak256 is the 256-bit digest instance of the Keccak family
 1137 that won the NIST SHA-3 competition [GJMG11]. It is supported by the EVM via an
 1138 opcode (see [W⁺, Appendix G]), making it convenient for use in smart contracts.

1139 **Remark 3.2.2.** To increase the security of the MKHASH, different round constants for
 1140 each level of the Merkle tree could be used.

We define MKHASH to be MIMC-MP over $\mathbb{F}_{\mathbf{r}_{\text{CUR}}}$. Thereby, for input values m_0 and m_1 , $\text{MKHASH} : \mathbb{F}_{\mathbf{r}_{\text{CUR}}} \times \mathbb{F}_{\mathbf{r}_{\text{CUR}}} \rightarrow \mathbb{F}_{\mathbf{r}_{\text{CUR}}}$ is defined by

$$\text{MKHASH}(m_0, m_1) = \text{MIMC-MP}_{\mathbf{r}_{\text{CUR}}}(m_0, m_1) \quad (3.1)$$

1141 For specific values of \mathbf{r}_{CUR} (such as \mathbf{r}_{BN} for BN-254 or \mathbf{r}_{BLS} for BLS12-377), it remains
 1142 to choose concrete values of e and *rounds*.

1143 Note that small exponents e result in fewer constraints in the arithmetic circuit
 1144 (see Section 2.2), while larger exponents can reduce the cost of Merkle tree operations
 1145 on the contract (see Section 2.5). This is due to two factors, namely that exponentiation
 1146 is cheaper to execute on a contract than in an arithmetic circuit, and that the number of
 1147 rounds decreases with higher e . For instance, choosing $e = 7$ results in 365 constraints
 1148 and $\approx 20k$ gas while $e = 31$ corresponds to 417 constraints (+15%) and $\approx 17k$ (−10%)
 1149 in gas consumption. Repeating the same process for different exponents, we observe
 1150 roughly the same order of magnitude gain on the gas consumption and loss on the
 1151 number of constraints.

The number of constraints of MIMC-MP for several exponents e is given by the formula

$$\text{constraints} = \text{rounds} \cdot \text{mults} + 1$$

1152 where $\text{rounds} = \lceil \frac{\log_2 \mathbf{r}_{\text{CUR}}}{\log_2 e} \rceil$, *mults* is the number of multiplications required for exponenti-
 1153 ation and the additional constraint (corresponding to +1 in the above formula) is a result
 1154 of the final message and key addition. Note that for $e = 2^t - 1$ we have $\text{mults} = 2 \cdot t - 2$
 1155 using the *square-and-multiply* algorithm [MVOV96].

1156 For several concrete values of e , the number of *rounds* required to attain the desired
 1157 security level, along with the number of constraints, are shown in Table 3.1.

1158 In conclusion, for the case of BN-254 we set $e = 7$ and *rounds* = 91, targetting a 254-
 1159 bit security level. Similarly, for BLS12-377 we set $e = 31$ and *rounds* = 51, targetting
 1160 a 253-bit security level. These values are chosen such that they satisfy the requirement
 1161 that $\text{gcd}(e, \mathbf{r}_{\text{CUR}} - 1) = 1$ and give a good balance between the number of constraints in
 1162 the arithmetic circuit and the gas cost of hashing on the contract.

1163 3.2.4 Security requirements satisfaction

1164 After presenting the state of the art of MiMC cryptanalysis, we present the security
 1165 proof of MIMC-MP collision resistance.

1166 Cryptanalysis of MIMC block cipher and primitives

1167 MIMC's security is increasingly being analysed since the primitive has gained traction
 1168 in zero-knowledge and cryptocurrency communities for its succinct algebraic constraint
 1169 representation. As of today, we do not know of any attacks breaking MIMC on prime
 1170 fields on full rounds.

e	BN-254		BLS12-377	
	<i>rounds</i>	<i>constraints</i>	<i>rounds</i>	<i>constraints</i>
7	91	365		
31	52	417	51	409
127	37	445	37	445
511	29	465		
2047	24	481	23	461
8191	20	481	20	481
32676	17	477		
131071	15	481	15	481
524287	14	505	14	505
2097151	13	521		

Table 3.1: Arithmetic constraints required to represent MIMC-MP as an R1CS program, for different exponents e and curves. Missing entries where $\gcd(e, r_{\text{CUR}} - 1) \neq 1$

The first attack on MIMC was an interpolation attack [LP19] which targets a reduced-round version for a scenario in which the attacker has only limited memory. An attack on Feistel-based MIMC [Bon19] was discovered shortly after, by using generic properties of the used Feistel construction (instead of exploiting properties of the primitive itself). Additionally, [ACG⁺19] proposes an attack based on Gröbner basis. The authors state that by introducing a new intermediate variable in each round, the resulting multivariate system of equations is a Gröbner basis. As such, the first step of a Gröbner basis attack can be obtained for free. However, the following steps of the attack are so computationally demanding that the attack becomes infeasible in practice. A recent work [EGL⁺20] targets MIMC on binary fields, and achieves a full-round break of the scheme. While, the attack presented does not apply to prime fields, the authors note that it “can be generalized to include ciphers over \mathbb{F}_p ”, and that only the lack of efficient distinguishers over prime fields precludes this. Another attack from Beyne et al [BCD⁺20] uses a low complexity distinguisher against full MIMC permutation leading to a practical collision attack on reduced round sponge-based MIMC hash defined with security of 128 bits.

Security proof of MIMC-MP collision resistance

We now prove that this compression scheme satisfies all the security requirements listed in Section 2.7. To do so, we first assume that the round constants are pseudo-random, i.e. that Keccak256 is a PRF.

Lemma 3.2.1. *Keccak256 is a PRF with $\lambda = 128$.*

The security of MIMC-MP derives from a more general result, i.e. from modelling MIMC as an ideal cipher (see Definition 1.5.12). More specifically, we show a security

1193 result for the MP construction on $\mathbb{F}_{\mathbf{r}_{\text{CUR}}}$ by proving that, in the Ideal Cipher Model,
 1194 the collision resistance advantage of any adversary is bounded by $\frac{q(q+1)}{\mathbf{r}_{\text{CUR}}}$, where q is the
 1195 number of different queries that the attacker makes to the oracle. This means that,
 1196 assuming a maximum q number of possible encryption/decryption queries, parameter
 1197 \mathbf{r}_{CUR} can be chosen to make the advantage small as needed and $\mathbf{f}_{\text{E}}^{\text{MP}}$ considered collision
 1198 resistant. Similar result applies to the 2^n case.

1199 The instance of MIMC we use is modelled as an ideal cipher defined on field elements,
 1200 for this reason we consider a variant of the ICM model where the keys, inputs and outputs
 1201 are field elements in $\mathbb{F}_{\mathbf{r}_{\text{CUR}}}$ and the block cipher scheme, with key k , correspond to a family
 1202 of \mathbf{r}_{CUR} independent random permutations $f_k : \mathbb{F}_{\mathbf{r}_{\text{CUR}}} \times \mathbb{F}_{\mathbf{r}_{\text{CUR}}} \rightarrow \mathbb{F}_{\mathbf{r}_{\text{CUR}}}$.

1203 In the proof, without loss of generality, we assume the following conventions for an
 1204 adversary \mathcal{A} :

- 1205 • the adversary asks distinct queries: i.e. if \mathcal{A} asks a query $\text{O}^{\text{E}}(k, m)$ and this returns
 1206 y , then \mathcal{A} does not ask a subsequent query of $\text{O}^{\text{E}}(k, m)$ or $\text{O}^{\text{E}^{-1}}(k, y)$, and inversely;
- 1207 • the adversary necessarily obtained the candidate collision from the oracle. This
 1208 property follows suite from modelling MIMC as an ideal cipher.

1209 **Lemma 3.2.2.** *Let $\mathbf{f}_{\text{E}}^{\text{MP}}$ be the MP compression function built on an ideal block-cipher*
 1210 *E on $\mathbb{F}_{\mathbf{r}_{\text{CUR}}}$, the probability for an adversary \mathcal{A} to find a collision is not greater than*
 1211 *$q(q+1)/\mathbf{r}_{\text{CUR}}$ where q is a (positive) number of distinct oracle queries.*

1212 The following proof has been adapted from [BRS02, Lemma 3.3]³.

1213 *Proof.* Fix $h_0 \in \mathbb{F}_{\mathbf{r}_{\text{CUR}}}$. Let \mathcal{A} be an adversary attacking the compression function $\mathbf{f}_{\text{E}}^{\text{MP}}$.
 1214 Assume that \mathcal{A} asks the oracles O^{E} and $\text{O}^{\text{E}^{-1}}$ a total of *distinct* q queries. Let us
 1215 denote the result of the q queries and output of the attacker (candidate collision) as
 1216 $((k_1, m_1, y_1), \dots, (k_q, m_q, y_q), \text{out})$. If \mathcal{A} is successful it means that it outputs (k, m) ,
 1217 (k', m') such that either $(k, m) \neq (k', m')$ and $\mathbf{f}_{\text{E}}^{\text{MP}}(k, m) = \mathbf{f}_{\text{E}}^{\text{MP}}(k', m')$ or $\mathbf{f}_{\text{E}}^{\text{MP}}(k, m) =$
 1218 h_0 . By the definition of $\mathbf{f}_{\text{E}}^{\text{MP}}$, we have that $\text{E}_k(m) + m + k = \text{E}_{k'}(m') + m' + k'$ for
 1219 the first case, or $\text{E}_k(m) + m + k = h_0$ for the second. So either there are distinct
 1220 $r, s \in [1, \dots, q]$ such that $(k_r, m_r, y_r) = (k, m, \text{E}_k(m))$ and $(k_s, m_s, y_s) = (k', m', \text{E}_{k'}(m'))$
 1221 and $\text{E}_{k_r}(m_r) + m_r + k_r = \text{E}_{k_s}(m_s) + m_s + k_s$ or else there is an $r \in [1, \dots, q]$ s.t. $(k_r, m_r, y_r) =$
 1222 (k, m, h_0) and $\text{E}_{k_r}(m_r) + m_r + k_r = h_0$. We show that this event is unlikely.

1223 In fact, for each $i \in [1, \dots, q]$, let C_i be the event that either $y_i + m_i + k_i = h_0$ or
 1224 does exist $j \in [1, \dots, i-1]$ s.t. $y_i + m_i + k_i = y_j + m_j + k_j$. When carrying out the
 1225 simulation y_i or m_i was randomly selected from a set of at least $\mathbf{r}_{\text{CUR}} - (i-1)$ elements,
 1226 so $\Pr[C_i] \leq i/(\mathbf{r}_{\text{CUR}} - i)$. This means that for the collision advantage of \mathcal{A} , $\text{Adv}_{\mathbf{f}_{\text{E}}^{\text{MP}}, \mathcal{A}}^{\text{coll}}$ it
 1227 holds that $\text{Adv}_{\mathbf{f}_{\text{E}}^{\text{MP}}, \mathcal{A}}^{\text{coll}} \leq \Pr[C_1 \vee \dots \vee C_q] \leq \sum_{i=1}^q \Pr[C_i]$. For $q \leq \frac{\mathbf{r}_{\text{CUR}}}{2}$ this probability is

³It states the collision resistance of a set of compression functions f_1, \dots, f_{12} , denoted as *group-1 compression functions* and showed in [BRS02, Figure 3]. As mentioned above, Miyaguchi-Preneel corresponds to f_3 of that group. Since the proof of [BRS02, Lemma 3.3] shows collision resistance of f_1 , we slightly modified it to work for f_3 .

1228 bounded by $l \cdot \frac{q(q+1)}{r_{\text{CUR}}}$. However, we allow only a polynomial number of queries, thus for
 1229 $q = \text{poly}(\lambda)$ this probability becomes $\frac{\text{poly}(\lambda)}{r_{\text{CUR}}}$, where $r_{\text{CUR}} \approx 2^\lambda$. \square

Note

Lemma 3.2.2 is applicable to our case by the strong assumption of $\text{MIMC}_{r_{\text{CUR}}}$ being an ideal cipher. In other words, the proof does not take into account any structural weakness or knowledge that an attacker is aware of. Any such additional information could make Lemma 3.2.2 invalid, and consequently could be used to break the collision resistance.

1230

1231 **Remark 3.2.3.** Note that from Lemma 3.2.2 follows that the collision resistance security
 1232 of the Zeth Merkle tree is $\log_2(r_{\text{CUR}}/2)$ (around 127 bits for $r_{\text{CUR}} = r_{\text{BN}}$ or r_{BLS}).

Note

MIMC has *not* received as much cryptanalytic scrutiny as other “older” and more established hash functions. This is important to note since, for these type of primitives which are not provably secure, the amount of attacks received by a scheme is a great indicator of its security and robustness. A natural alternative to MIMC here consists in using Pedersen hash which is provably collision resistant under the discrete-logarithm assumption.

1233

1234 3.3 Zeth statement after primitive instantiation

1235 After instantiating the various primitives and providing security proofs to justify that
 1236 they comply with the security requirements listed in previous sections, \mathbf{R}^Z now becomes:

1237 • For each $i \in [\text{JSIN}]$:

- 1238 1. $\text{aux.jsins}[i].\text{znote.apk} = \text{Blake2s}(\text{tag}_{\text{ask}}^{\text{addr}} \parallel \text{pad}_{\text{BLAKE2sCLEN}}(0))$
 1239 with $\text{tag}_{\text{ask}}^{\text{addr}}$ defined in Section 3.1.3
- 1240 2. $\text{aux.jsins}[i].\text{nf} = \text{Blake2s}(\text{tag}_{\text{ask}}^{\text{nf}} \parallel \text{aux.jsins}[i].\text{znote}.\rho)$
 1241 with $\text{tag}_{\text{ask}}^{\text{nf}}$ defined in Section 3.1.3
- 1242 3. $\text{aux.jsins}[i].\text{cm} = \text{Blake2s}(\text{aux.jsins}[i].\text{znote}.r \parallel m)$
 1243 with $m = \text{aux.jsins}[i].\text{znote.apk} \parallel \text{aux.jsins}[i].\text{znote}.\rho \parallel \text{aux.jsins}[i].\text{znote}.v$
- 1244 4. $\text{aux.htags}[i] = \text{Blake2s}(\text{tag}_{\text{ask},i}^{\text{pk}} \parallel \text{prim.hsigs})$ (malleability fix, see Appendix A)
 1245 with $\text{tag}_{\text{ask},i}^{\text{pk}}$ defined in Section 3.1.3
- 1246 5. $(\text{aux.jsins}[i].\text{znote}.v) \cdot (1 - e) = 0$ is satisfied for the boolean value e set such
 1247 that if $\text{aux.jsins}[i].\text{znote}.v > 0$ then $e = 1$.
- 1248 6. The Merkle root mkroot' used to check the Merkle authentication path $\text{aux.jsins}[i].\text{mkpath}$
 1249 of commitment $\text{aux.jsins}[i].\text{cm}$, with $\text{MIMC-MP}_{r_{\text{CUR}}}$, equals prim.mkroot if
 1250 $e = 1$.

- 1251 7. $\text{prim.nfs}[i]$
1252 $= \left\{ \text{Pack}_{\mathbb{F}_{\text{rCUR}}}(\text{aux.jsins}[i].\text{nf}[k \cdot \text{FIELD CAP}:(k+1) \cdot \text{FIELD CAP}]) \right\}_{k \in [\lfloor \text{PRFNFOUTLEN}/\text{FIELD CAP} \rfloor]}$
- 1253 8. $\text{prim.htags}[i]$
1254 $= \left\{ \text{Pack}_{\mathbb{F}_{\text{rCUR}}}(\text{aux.htags}[i][k \cdot \text{FIELD CAP}:(k+1) \cdot \text{FIELD CAP}]) \right\}_{k \in [\lfloor \text{PRFPKOUTLEN}/\text{FIELD CAP} \rfloor]}$
- 1255 • For each $j \in [\text{JSOUT}]$:
- 1256 1. $\text{aux.znotes}[j].\rho = \text{Blake2s}(\text{tag}_{\text{ask},j}^\rho \parallel \text{prim.hsig})$ (malleability fix, see Appendix A)
1257 with $\text{tag}_{\text{ask},j}^\rho$ defined in Section 3.1.3
- 1258 2. $\text{prim.cms}[j] = \text{Blake2s}(\text{aux.znotes}[j].r \parallel m)$
1259 with $m = \text{aux.znotes}[j].\text{apk} \parallel \text{aux.znotes}[j].\rho \parallel \text{aux.znotes}[j].v$
- 1260 • $\text{prim.hsig} = \left\{ \text{Pack}_{\mathbb{F}_{\text{rCUR}}}(\text{aux.hsig}[k \cdot \text{FIELD CAP}:(k+1) \cdot \text{FIELD CAP}]) \right\}_{k \in [\lfloor \text{CRHHSIGOUTLEN}/\text{FIELD CAP} \rfloor]}$
- 1261 • $\text{prim.rsd} = \text{Pack}_{\text{rsd}}(\{\text{aux.jsins}[i].\text{nf}\}_{i \in [\text{JSIN}]}, \text{aux.vin}, \text{aux.vout}, \text{aux.hsig}, \{\text{aux.htags}[i]\}_{i \in [\text{JSIN}]})$
- Check that the “joinsplit is balanced”, i.e. check that the joinsplit equation holds:

$$\begin{aligned} & \text{Pack}_{\mathbb{F}_{\text{rCUR}}}(\text{aux.vin}) + \sum_{i \in [\text{JSIN}]} \text{Pack}_{\mathbb{F}_{\text{rCUR}}}(\text{aux.jsins}[i].\text{note}.v) \\ &= \sum_{j \in [\text{JSOUT}]} \text{Pack}_{\mathbb{F}_{\text{rCUR}}}(\text{aux.znotes}[j].v) + \text{Pack}_{\mathbb{F}_{\text{rCUR}}}(\text{aux.vout}) \end{aligned}$$

1262 **Remark 3.3.1.** For higher security, we could use Blake2b with 32-byte output instead
1263 of SHA256. In fact, since a precompiled contract computing the Blake2 compression
1264 function [MJS15] has been added to the Istanbul release of Ethereum (EIP 152 [THH15]),
1265 it could be possible to write a small wrapper on the smart contracts, in order to hash
1266 with Blake2b with any parameter.

1267 3.3.1 Instantiating the packing functions

1268 As we consider SNARKs based on arithmetic circuits defined over a prime field, all
1269 variables in the constraint system are interpreted as field elements. Nevertheless, as
1270 illustrated in Section 2.2, part of the statement consists of functions whose co-domains
1271 are sets of binary strings (which may be longer than the bit representation of elements of
1272 the finite field). While a bit (i.e. $\{0, 1\}$) is an element of \mathbb{F}_p (p prime), it is important to
1273 minimize the number of gates in the arithmetic circuit (for proof generation efficiency),
1274 and to minimize the number of input wires (to improve verification time). This can be
1275 done by representing fragments of binary strings as the base 2 decomposition of field
1276 elements, thereby “packing” binary strings into multiple elements. Converting binary
1277 strings into field elements requires the addition of some arithmetic gates (extending
1278 the statement to be proven), but reduces the number of primary inputs (reducing the
1279 complexity of the SNARK verification carried out on-chain). The cost of Groth16 zk-
1280 SNARK [Gro16] proof verification is linear in the number of primary inputs, since each

input acts as a scalar in a costly scalar multiplication of a curve point in \mathbb{G}_1 . Hence, while packing slightly increases the prover cost – by adding constraints to the circuit – it simplifies the verifier’s work.

In this section, we detail the method by which we encode (resp. decode) a set of binary strings to (resp. from) sets of field elements. In the rest of this section, the notion of *packing policy* refers to the set of *packing* and *unpacking* functions.

The set of primary inputs is composed of the input nullifiers, the output commitments, the public values (see [RZ19, Section 3.4.3]) along with the signature hash and the authentication tags for security (malleability fix, see Appendix A). The complete description of the public inputs is represented in Eq. (3.2).

$$(\{prim.nf_i\}_{i \in [JSIN]}, \{prim.cms[j]\}_{j \in [JSOUT]}, vin, vout, hsig, \{prim.htags[i]\}_{i \in [JSIN]}) \quad (3.2)$$

The primary inputs that consist of binary strings are: the nullifiers *nfs*, the public values *vin* and *vout*, the signature hash *hsig* and the authentication tags *htags*.

For a binary string x , let $\alpha_x = \lceil \text{length}(x) / \text{FIELD CAP} \rceil$ be the number of field elements required to completely encode x and let $\beta_x = \lfloor \text{length}(x) / \text{FIELD CAP} \rfloor$ be the number of field elements whose capacity is fully used. Let $\gamma_x = \text{length}(x) \pmod{\text{FIELD CAP}}$ be the number of “residual” bits remaining after fully using β_x field elements.

Example 3.3.2. Consider binary strings $A \in \{0, 1\}^7$ of length 7, to be encoded over the field \mathbb{F}_{41} . This field has a capacity of 5 bits, and therefore $\alpha_A = 2$, $\beta_A = 1$, and $\gamma_A = 2$. That is, A can be represented as 2 field elements, or as 1 field element with 2 “residual” bits.

Consider $A = (1111011)$. Fig. 3.6 illustrates how A can be packed as field elements. Note that the 2 residual bits are taken from the “beginning” of the bit string, that is, the highest order bits.

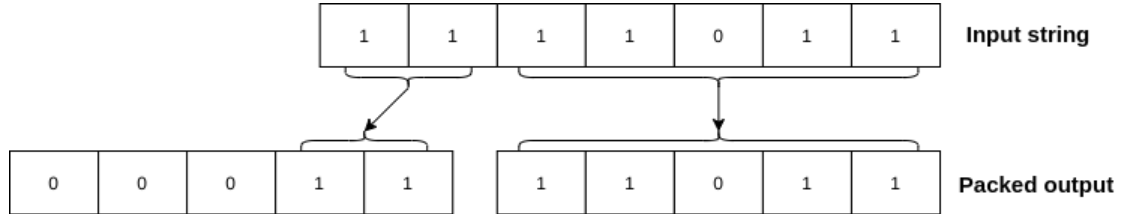


Figure 3.6: Packing of string A (see Example 3.3.2)

We now consider strategies to pack all primary inputs that are binary strings. A naive approach is to encode each binary string x as α_x field elements. In general, this results in significant waste (and consequently more field elements than necessary), especially when the number of residual bits is small compared to **FIELD CAP** (see Fig. 3.7). An alternative strategy could be to concatenate all binary strings into a single string y and pack this string into α_y field elements. While this approach minimizes the set of unused bits, each unpack operation would require different shift and mask operations over 2 or

1307 3 field elements. This significantly increases the complexity of the unpacking operation
 1308 that must be performed on-chain, resulting in a higher gas cost (due to extra logic) or
 1309 more contract code (if each unpack operation is hard-coded).

The **Zeth** protocol requires that each binary string variable x is packed into β_x field elements, and the residual bits from all binary strings, along with the public values vin and $vout$, are aggregated into a variable rsd . Let **RSDBLEN** be the total number of residual bits, and **RSDFLEN** be the number of field elements required to represent rsd . We assume that **ZVALUELEN** < **FIELD CAP**, and define the notation $\gamma_v = \text{ZVALUELEN}$ for the bit lengths of public values vin and $vout$. Thus **RSDBLEN** is given by

$$\text{RSDBLEN} = \gamma_{hsig} + 2 \cdot \gamma_v + \text{JSIN} \cdot (\gamma_{nf} + \gamma_h)$$

and the lengths, in field elements, of each of the corresponding public inputs are

$$\begin{aligned} \text{NFFLEN} &= \beta_{nf} \\ \text{HSIGFLEN} &= \beta_{hsig} \\ \text{HFLEN} &= \beta_h \\ \text{RSDFLEN} &= \lceil \text{RSDBLEN} / \text{FIELD CAP} \rceil \end{aligned}$$

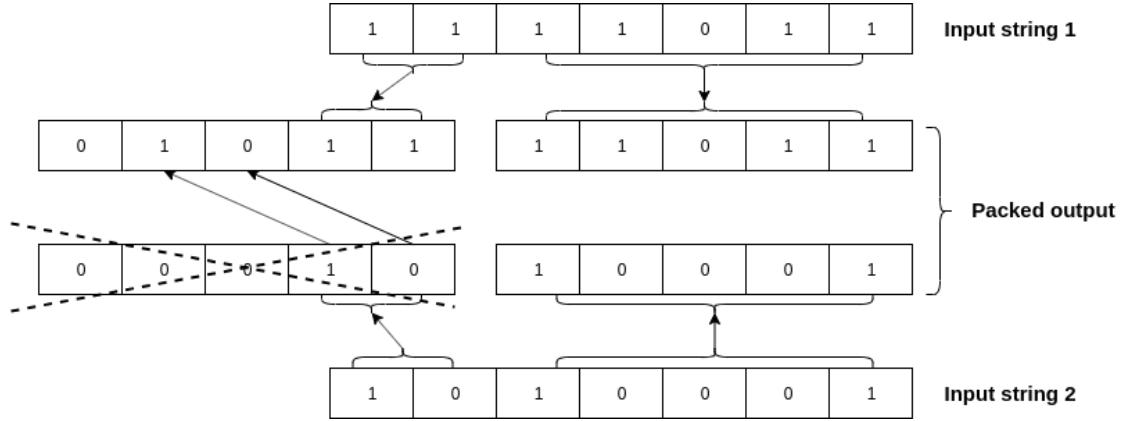


Figure 3.7: Packing of multiple strings. Observe that, by carefully arranging the bits of the input strings, it is possible to output fewer field elements

The residual bits rsd are formatted as follows:

$$\widetilde{hsig} \parallel \widetilde{nfs} \parallel \widetilde{htags} \parallel vin \parallel vout$$

1310 where $\widetilde{hsig}, \widetilde{nfs}, \widetilde{htags}$ are, respectively, the $\gamma_{hsig}, \gamma_{nf}, \gamma_h$ bits.

1311 Note that the public values are packed into the “last”, or lowest order, $2 \cdot \gamma_v$ bits of
 1312 the resulting field element(s). In this way, their unpack functions are independent of the
 1313 values **JSIN** and **JSOUT** and of the number of residual bits required for each bit string
 1314 (and consequently, independent of the finite field used).

 $\text{Pack}_{\mathbb{F}_{\text{rCUR}}}(x)$

```

out  $\leftarrow 0_{\mathbb{F}_{\text{rCUR}}}$ ;
for  $i \in [\text{length}(x)]$  do :
  if  $x[i] = 1$  do :
    out  $\leftarrow out +_{\mathbb{F}_{\text{rCUR}}} 2^{\text{length}(x)-1-i}$ 
return out;

```

Figure 3.8: Algorithm to pack bits into a field element.

 $\text{Pack}_{\text{rsd}}(nfs, vin, vout, hsig, htags)$

```

out  $\leftarrow []$ ;  $r \leftarrow \epsilon$ ;
 $r \leftarrow vout$ ;
 $r \leftarrow vin || r$ ;
for  $i \in [\text{JSIN}]$  do :
   $r \leftarrow htags[i][\beta_{htags[i]} \cdot \text{FIELD CAP} : ] || r$ ;
for  $i \in [\text{JSIN}]$  do :
   $r \leftarrow nfs[i][\beta_{nfs[i]} \cdot \text{FIELD CAP} : ] || r$ ;
 $r \leftarrow hsig[\beta_{hsig} \cdot \text{FIELD CAP} : ] || r$ ;
for  $i \in [\lceil \text{length}(r) / \text{FIELD CAP} \rceil]$  do :
  out[i]  $\leftarrow \text{Pack}_{\mathbb{F}_{\text{rBN}}}(r[i \cdot \text{FIELD CAP} : (i+1) \cdot \text{FIELD CAP}])$ ;
return out;

```

Figure 3.9: Algorithm to pack residual bits.

To format the unpacked primary inputs into field elements, we define the following functions. Given a bit string of length less than FIELD CAP , the algorithm Pack (see Fig. 3.8) returns a field element. Given the nullifiers, public values and authentication tags, the algorithm Pack_{rsd} (see Fig. 3.9) outputs the residual bits. Given a set of packed field elements and the residual bits, the algorithm Unpack returns the variables reassembled as binary strings. In particular, we have that $\text{Unpack}_{nf}(\text{prim}.nfs, \text{rsd}) = \{aux.jsins[i].nf\}_{i \in [\text{JSIN}]}$.

$\text{Pack} : \mathbb{B}^{\leq \text{FIELD CAP}} \rightarrow \mathbb{F}_{\text{rCUR}}$

$\text{Pack}_{\text{rsd}} : (\mathbb{B}^{\text{PRNFOUTLEN}})^{\text{JSIN}} \times (\mathbb{B}^{\text{ZVALUELEN}})^2 \times \mathbb{B}^{\text{CRHHSIGOUTLEN}} \times (\mathbb{B}^{\text{PRFPKOUTLEN}})^{\text{JSIN}} \rightarrow (\mathbb{F}_{\text{rCUR}})^{\text{RSDFLEN}}$

$\text{Unpack} : \mathbb{F}_{\text{rCUR}}^* \times (\mathbb{F}_{\text{rCUR}})^{\text{RSDFLEN}} \rightarrow \mathbb{B}^*$

The Unpack functions for nullifiers, public values and signature hash are represented as follows.

$\text{Unpack}_{hsig} : (\mathbb{F}_{\text{rCUR}})^{\text{HSIGFLEN}} \times (\mathbb{F}_{\text{rCUR}})^{\text{RSDFLEN}} \rightarrow \mathbb{B}^{\text{CRHHSIGOUTLEN}}$

$\text{Unpack}_{nf} : (\mathbb{F}_{\text{rCUR}})^{\text{NFFLEN}} \times (\mathbb{F}_{\text{rCUR}})^{\text{RSDFLEN}} \rightarrow \mathbb{B}^{\text{PRNFOUTLEN}}$

$\text{Unpack}_{vin} : \mathbb{F}_{\text{rCUR}}^0 \times (\mathbb{F}_{\text{rCUR}})^{\text{RSDFLEN}} \rightarrow \mathbb{B}^{\text{ZVALUELEN}}$

$\text{Unpack}_{vout} : \mathbb{F}_{\text{rCUR}}^0 \times (\mathbb{F}_{\text{rCUR}})^{\text{RSDFLEN}} \rightarrow \mathbb{B}^{\text{ZVALUELEN}}$

1315 Packing Policy Security

1316 **Proposition 3.3.1** (Packing security). *For a binary string x , it holds that $\text{Unpack}(\text{Pack}(x)) =$*
 1317 *x and $\text{Unpack}(\text{Pack}_{\text{rsd}}(x)) = x$.*

1318 Packing Policy Example

In the case where $\text{JSIN} = \text{JSOUT} = 2$, the BN-254 is being used (in which field elements hold $\text{FIELD CAP}_{\text{BN}}$ bits) and all PRFs and CRH^{hsig} output bit-strings of length 256, the unpacked primary inputs are 2167-bit long. The packing parameters are therefore:

$$\begin{aligned}\text{RSDBLEN} &= 5 \times 3 + 64 + 64 = 143 \\ \text{NFFLEN} &= \text{HSIGFLEN} = \text{HFLEN} = \text{RSDFLLEN} = 1\end{aligned}$$

The packed primary inputs are 2277 bits long, corresponding to a small space overhead of $\approx 5\%$ unused bits. Moreover, the 143-bit residual bits can be packed into a single field element. As such, the primary inputs are encoded as 9 field elements. Finally, the residual bits are formatted as follows,

$$\underbrace{\text{padding}}_{113 \text{ bits}} \parallel \underbrace{\text{hsig}}_{3 \text{ bits}} \parallel \underbrace{\text{nf}_1}_{3 \text{ bits}} \parallel \underbrace{\text{nf}_0}_{3 \text{ bits}} \parallel \underbrace{h_1}_{3 \text{ bits}} \parallel \underbrace{h_0}_{3 \text{ bits}} \parallel \underbrace{\text{vin}}_{64 \text{ bits}} \parallel \underbrace{\text{vout}}_{64 \text{ bits}}$$

For the analogous case using BLS12-377 (in which field elements hold $\text{FIELD CAP}_{\text{BLS}}$ bits), the packing parameters are:

$$\begin{aligned}\text{RSDBLEN} &= 5 \times 4 + 64 + 64 = 148 \\ \text{NFFLEN} &= \text{HSIGFLEN} = \text{HFLEN} = \text{RSDFLLEN} = 1\end{aligned}$$

The residual bits can be packed into a single field element of the form

$$\underbrace{\text{padding}}_{108 \text{ bits}} \parallel \underbrace{\text{hsig}}_{4 \text{ bits}} \parallel \underbrace{\text{nf}_0}_{4 \text{ bits}} \parallel \underbrace{\text{nf}_1}_{4 \text{ bits}} \parallel \underbrace{h_0}_{4 \text{ bits}} \parallel \underbrace{h_1}_{4 \text{ bits}} \parallel \underbrace{\text{vin}}_{64 \text{ bits}} \parallel \underbrace{\text{vout}}_{64 \text{ bits}}$$

1319 and the primary inputs are again encoded as 9 field elements.

1320 3.4 Instantiate SigSch_{OT-SIG}

1321 **Zeth** uses the one-time Schnorr-based signature scheme introduced by Bellare and Shoup [BS07]
1322 for its long proven security, simplicity, speed and size. Its security relies on the one-more
1323 discrete log problem (see Definition 1.5.6) and the collision resistance of the underlying
1324 hash function **CRH** (see Definition 1.5.16) that we instantiate with **SHA256**.

1325 Note that no signature operations or data are used in the arithmetic circuit describing
1326 the **Zeth** statement. Hence the curve used for the signature scheme can be chosen
1327 independently of **Curve** (the scalar field of which is used for the arithmetic circuit, and
1328 consequently for commitments and bit string encodings described in Section 3.1 and
1329 Section 3.2). BN-254 is used since it is supported by the EVM, in the form of precompiled
1330 contracts. This allows a gas-efficient implementation in the **Mixer** contract.

1331 This one-time signature scheme (see Definition 1.5.26) is defined by the two-tier
1332 signature scheme over a cyclic group $(p, \mathbb{G}, \langle \mathbf{g} \rangle, \otimes)$. In the two-tier signature scheme, the
1333 hash function **CRH** only needs to be collision resistant (the random oracle model is not

1334 used). Similarly, the variable hk represents the key of the hash function (a particular
1335 instance).

1336 To turn this two-tier signature scheme into a one-time signature scheme, one simply
1337 has to define the one-time signature key generation **KGen** as the combination of both
1338 primary and secondary key generations of the two-tier (see [BS07, Section 6]). The
1339 one-time signing key (respectively verification key) of the one time signature scheme is
1340 defined as both the primary and secondary signing key (respectively verification key) of
1341 the two-tier scheme, Fig. 3.10

$\text{KGen}(1^\lambda) :$ $hk \leftarrow \$ \mathbb{B}^{kl}$ $\mathbf{g} \leftarrow \$ \mathbb{G}^*$ $x \leftarrow \$ \mathbb{F}_p$ $ppk = (hk, \mathbf{g}, \llbracket x \rrbracket)$ $psk = (hk, \mathbf{g}, x)$ $y \leftarrow \$ \mathbb{F}_p$ $spk = \llbracket y \rrbracket$ $ssk = (y, \llbracket y \rrbracket)$ $pk = (ppk, spk)$ $sk = (psk, ssk)$	$\text{Sig}(sk, m) :$ $hk, \mathbf{g}, x = sk.psk$ $y, \llbracket y \rrbracket = sk.ssk$ $c = \text{CRH}(hk, \llbracket y \rrbracket m)$ $\sigma = y \bmod p$ $\sigma += c \cdot x \bmod p$ return σ	$\text{Vf}(pk, m, \sigma) :$ $hk, \mathbf{g}, \llbracket x \rrbracket = pk.ppk$ $\llbracket y \rrbracket = pk.spk$ $c = \text{CRH}(hk, \llbracket y \rrbracket m)$ if $\sigma = \llbracket y \rrbracket \otimes c \cdot \llbracket x \rrbracket$ then return 1 else return 0 endif
--	---	---

Figure 3.10: One-time signature scheme from two tier Schnorr based signature scheme by Bellare and Shoup [BS07]

1342 3.4.1 Security requirements satisfaction

1343 We now prove that this signature scheme satisfies all the security requirements listed
1344 in Section 2.7.

1345 **Theorem 3.4.1.** *The One-Time Schnorr signature is strongly unforgeable under chosen-*
1346 *message attacks (SUF-CMA) assuming that the om-DLog problem is hard in \mathbb{G} and that*
1347 *the hash function CRH is collision resistant.*

1348 *Proof.* See [BS07, Theorems 5.1, 5.2 and 6.1]. □

1349 3.4.2 Data types

1350 We now describe the data types and operations associated with this signature scheme.

1351 **VK0tsDType** Denotes the verification key associated with the one-time signature scheme.

Field	Description	Data type
ppk	Encoding of the scalar x in the group	\mathbb{G}_{BN}
spk	Encoding of the scalar y in the group	\mathbb{G}_{BN}

Table 3.2: VK0tsDType data type

1352 **SK0tsDType** Denotes the signing key associated with the one-time signature scheme.

Field	Description	Data type
psk	Scalar element x	\mathbb{F}_{rBN}
ssk	Scalar element y and its encoding in the group	$\mathbb{F}_{\text{rBN}} \times \mathbb{G}_{\text{BN}}$

Table 3.3: SK0tsDType data type

1353 **Sig0tsDType** Denotes the signature data type associated with the one-time signature
 1354 scheme. **Sig0tsDType** is an alias for \mathbb{F}_{rBN} .

1355 3.5 Instantiate EncSch

1356 In this section we describe the instantiation of **EncSch** primitive introduced in Section 2.3.
 1357 First, we present a general asymmetric encryption scheme called **DHAES** (Diffie-Hellman
 1358 Asymmetric Encryption Scheme [ABR99]), which satisfies all the required security prop-
 1359 erties for the in-band encryption scheme **EncSch** (see Section 1.5.3). Then, we give details
 1360 of the concrete algorithms used for the implementation.

1361 3.5.1 DHAES encryption scheme

1362 Given a symmetric encryption scheme **Sym**, a group defined by **SetupG**, a family of hash
 1363 function \mathcal{H}^4 and a message authentication scheme **MAC** as defined in Section 1.5, we
 1364 define a **DHAES** scheme as the following public-key encryption scheme:

- 1365 • **Setup**, setup algorithm, takes as input a security parameter 1^λ . It runs $\mathcal{H}.\text{Setup}$,
 1366 **SetupG** and returns public parameters $pp = (hk, (q, \mathbb{G}, \mathbf{g}, +))$.
- 1367 • **KGen**, key generation algorithm, takes as input public parameters pp . It samples
 1368 at random $v \leftarrow_{\$} [q]$ and returns a keypair $(sk, pk) = (v, \llbracket v \rrbracket)$.

⁴Here, we only consider fixed-length hash functions with $h\text{InpLen}(\lambda) = 2g\text{Len}$ and $h\text{Len}(\lambda) = k\text{Len}(\lambda) + m\text{Len}(\lambda)$ (see Section 1.5).

- 1369 • **Enc**, encryption algorithm, takes as input public parameters pp , a message m and
1370 a public key pk . It runs **KGen** that returns an ephemeral keypair $(esk, epk) =$
1371 $(u, \llbracket u \rrbracket)$. Then, it computes a shared secret $ss = H_{hk}(epk \parallel esk \cdot pk) = H_{hk}(epk \parallel sk \cdot$
1372 $epk)$, parsed as $ek \parallel mk$ ⁵. It computes $ct_{\text{Sym}} = \text{Sym.Enc}(ek, m)$ and $\tau = \text{MAC.Tag}(mk, ct_{\text{Sym}})$
1373 and finally outputs the ciphertext $epk \parallel ct_{\text{Sym}} \parallel \tau$.
- 1374 • **Dec**, decryption algorithm, takes as input public parameters pp , a private key sk
1375 a ciphertext $epk \parallel ct_{\text{Sym}} \parallel \tau$. It computes $ss = H_{hk}(epk \parallel sk \cdot epk)$ and parses it, as
1376 above, as $ek \parallel mk$. If MAC verification passes, i.e. $\text{MAC.Vf}(mk, \tau) = 1$, the algorithm
1377 returns $\text{Sym.Dec}(ek, ct_{\text{Sym}})$ and \perp otherwise.

1378 The DHAES definition given above is an asymptotic adaptation of [ABR99, Section
1379 1.3].

1380 Inclusion of ephemeral key in hash input

1381 Given an ephemeral keypair $(u_0, \llbracket u_0 \rrbracket)$, If the group $\langle \mathbf{g} \rangle$, generated by **SetupG**, has com-
1382 posite order, then $\llbracket u_0 \rrbracket$ is required to be part of the hash input because $\llbracket u_0 v \rrbracket$ and $\llbracket v \rrbracket$
1383 together may not uniquely determine $\llbracket u_0 \rrbracket$. Equivalently, there may exist two values
1384 u_0 and u_1 such that $u_0 \neq u_1$ and $\llbracket u_0 v \rrbracket = \llbracket u_1 v \rrbracket$. As a result, both u_0 and u_1 can be
1385 used to produce two different *valid* ciphertexts of the same plaintext m , under different
1386 ephemeral keys $(\llbracket u_0 \rrbracket, \llbracket u_1 \rrbracket)$. It is easy to show this, for example, in the multiplicative
1387 group $\mathbb{Z}_p \setminus \{0\}$, where p is a prime (see [ABR99, Section 3.1]). A scheme having such
1388 malleability property clearly cannot be proven IND-CCA2 secure: an attacker could eas-
1389 ily win the related security game by altering the challenged ciphertext and query the
1390 decryption oracle that would not recognize that as a not allowed query. If the group
1391 has prime order this problem does not arise so only $\llbracket u_0 v \rrbracket$ is required as input of the **H**
1392 function [ABR01, Section 3].

1393 3.5.2 A DHAES instance

1394 Curve25519

1395 For a cyclic group we propose the use of a subgroup of **Curve25519** described in [Ber06]
1396 and in [LHT16]. **Curve25519** is a Montgomery elliptic curve [Mon87] defined by the
1397 equation $y^2 = x^3 + 486662x^2 + x$ and coordinates on \mathbb{F}_p , where p is the prime number
1398 $2^{255} - 19$. It has a prime order subgroup of order $2^{252} + 27742317777372353535851937$
1399 790883648493 and cofactor 8. **Curve25519** comes with an efficient scalar multiplication
1400 denoted as **X25519**⁶. In a Diffie-Hellman-based scheme it allows to have 32-byte long
1401 public and private keys (given a point $P = (x, y)$ only the x coordinate is actually used)
1402 and the 32-byte sequence representing 9 is specified as base point.

⁵Note that ek and mk must have the same length.

⁶**X25519** is actually introduced in [LHT16] in order to avoid notation issues due to the use **Curve25519** to indicate both curve and scalar multiplication as done in [Ber06]

Efficiency and security of Curve25519

High-speed and timing-attack resistant implementations of X25519 are available and its security level is conjectured to be 128 bits [Ber06, Section 1]. However, combined attacks can lead to 124 bits of security (see [BL, Section “Twist Security”]). By design, Curve25519 is resistant to state-of-the-art attacks and satisfies all security criteria and principles listed in *Safecurves* [BL]⁷.

Interestingly, Curve25519 does not require *public key validation*⁸, while we know that, on other curves, active attacks – consisting of sending malformed public keys – could be carried out by adversaries, to violate the confidentiality of private keys, e.g. [ABM⁺03]. However, Curve25519 specification mandates the *clamping* of private keys: that is, after the random sampling of 32 bytes, the user clears bits 0, 1 and 2 of the first byte, clears bit 7 and sets bit 6 of the last byte. The resulting 32 bytes are then used as private key. This particular structure for private keys prevents various types of attacks (see [Ber06, Section 3] for more details).

Note

Note that the *clamping* procedure is vital to ensure the security guarantees of the Curve25519 specification, and implementations **MUST** perform this exactly as described.

Chacha20

ChaCha20 is an ARX-based⁹ stream cipher introduced in [Ber08a]. It is an improved version of Salsa20 [Ber08b] that won the *eSTREAM* challenge [est]. Compared with Salsa20, it has been designed to improve diffusion per round, conjecturally increasing resistance to cryptanalysis, while preserving time efficiency per round. It is considerably faster than AES in software-only implementations and can be easily implemented to be timing-attacks resistant. Several versions of the cipher can be used. The original paper presents ChaCha20 with a 128-bit key and 64-bit nonce/block count. However, the length of the key, nonce and block count – which indicates how many chunks can be processed by using the same key and nonce – can be modified depending on the application. In [LN18][Section 2.3], for instance, the key is a 256-bit string, the nonce is a string of 96 bits and the block count is encoded on a 32-bit word. This configuration allows to process around 2^{32} blocks, corresponding to roughly 256 GB of data. We propose to use the same parameters in Zeth.

$$\text{ChaCha20} : \mathbb{B}^{256} \times \mathbb{B}^{32} \times \mathbb{B}^{96} \times \mathbb{B}^* \rightarrow \mathbb{B}^*$$

⁷In this work, the authors take into account both Elliptic Curve Discrete Logarithm Problem (ECDLP) and Elliptic Curve Cryptosystems (ECC) security, that allows to have an overall evaluation of the security guarantees.

⁸Informally, it is a set of security checks that a user performs before using a not trusted public key (e.g. see [BCK⁺18])

⁹Addition-Rotation-XOR

1432 Security of Chacha

1433 Recent cryptanalysis results for ChaCha are available in [AFK⁺08, Ish12, SZFW12,
1434 Mai16, CM16, CM17]: all of them make use of advanced cryptanalysis techniques able
1435 to perform key-recovery attacks only on reduced versions (6 and 7 rounds) of ChaCha.

Note

Importantly, the security properties of ChaCha rely on the fact that, for a given key, all blocks are processed with distinct values in the state words 12 to 15 (storing the counter and the nonce) [LN18, Section 2.3].

1436

1437 Poly1305

1438 Poly1305 [Ber05] is a high-speed message authentication code, easy to implement and
1439 make side-channel attack resistant. It takes a 32-byte one-time key mk and a message m
1440 and produces a 16-byte tag τ that authenticates the message. mk must be unpredictable
1441 and it is represented as a couple (r, s) , where both components are given as a sequence
1442 of 16 bytes each. It can be generated by using pseudorandom algorithms: in [Ber05,
1443 Section 2], for example, AES and a nonce are used to generate s . The second part of
1444 the key, r , is expected to have a given form [Ber05, Section 2], and must be “clamped”
1445 as follows: top four bits of $r[3]$, $r[7]$, $r[11]$, $r[15]$ and bottom two bits of $r[4]$, $r[8]$, $r[12]$
1446 are cleared (see also Section 3.5.3).

Note

Similarly to Curve25519, the *clamping* procedure here is essential to the security of the Poly1305 scheme. Implementations MUST ensure that this is performed correctly in order for all security guarantees to hold.

1447

1448 We refer to [LN18, Section 2.5, Section 3] for Tag and Vf implementations of Poly1305.

$$\begin{aligned}\text{Poly1305.Tag} &: \mathbb{B}_Y^{32} \times \mathbb{B}_Y^* \rightarrow \mathbb{B}_Y^{16} \\ \text{Poly1305.Vf} &: \mathbb{B}_Y^{32} \times \mathbb{B}_Y^{16} \times \mathbb{B}_Y^* \rightarrow \mathbb{B}\end{aligned}$$

1449 Security of Poly1305

1450 Citing Poly1305 [LN18, Section 4], “the Poly1305 authenticator is designed to ensure that
1451 forged messages are rejected with a probability of $1 - (n/(2^{102}))$ for a $16n$ -byte message,
1452 even after sending 2^{64} legitimate messages, so it is SUF-CMA (strong unforgeability
1453 against chosen-message attacks)”.

1454 Blake2b-512

1455 Since we need a total of 64 bytes for the key material (32 for ChaCha20 and 32 for
 1456 Poly1305) Blake2b512 can be used. ZCash protocol [ZCa19, Section 5.4.3], instead, makes
 1457 use of Blake2b256 since a DHAES variant, denoted as ChaCha20-Poly1305, is adopted
 1458 (see [LN18, Section 2.8]).

$$\text{Blake2b512} : \mathbb{B}^* \rightarrow \mathbb{B}_{\mathbb{Y}}^{32}$$

1459 3.5.3 EncSch instantiation

In the following we instantiate EncSch as a DHAES scheme, detailing the KGen, Enc and Dec components. First, we introduce some required constant values:

$$\begin{aligned} \text{ESKBYTELEN} &= 32 \\ \text{EPKBYTELEN} &= 32 \\ \text{NOTEBYTELEN} &= (\text{PRFADDRROUTLEN} + \text{RTRAPLEN} + \text{ZVALUELEN} + \text{PRFRHOOUTLEN})/\text{BYTELEN} \\ \text{SYMKEYBYTELEN} &= 32 \\ \text{MACKEYBYTELEN} &= 32 \\ \text{KDFDIGESTBYTELEN} &= \text{SYMKEYBYTELEN} + \text{MACKEYBYTELEN} \\ \text{CTBYTELEN} &= \text{EPKBYTELEN} + \text{NOTEBYTELEN} + \text{TAGBYTELEN} \\ \text{TAGBYTELEN} &= 16 \\ \text{CHACHANONCEVALUE} &= 0^{32} \\ \text{CHACHABLOCKCOUNTERVALUE} &= 0^{96} \end{aligned}$$

1460 EncSch.KGen

1461 The keypair (sk, pk) generation is defined as:

- 1462 • Randomly sample a sequence of ESKBYTELEN bytes and assign to sk .
- Clamp sk as follows:

$$\begin{aligned} sk[0] &\leftarrow sk[0] \& 0xF8 \\ sk[31] &\leftarrow sk[31] \& 0x7F \\ sk[31] &\leftarrow sk[31] | 0x40 \end{aligned}$$

1463 where $|$ and $\&$ denotes, respectively, OR and AND binary operators between bit
 1464 strings of same the length.¹⁰

- 1465 • Compute $pk = \text{X25519}(sk, 0x09)$.
- 1466 • Return $(sk, pk) \in \mathbb{B}_{\mathbb{Y}}^{\text{ESKBYTELEN}} \times \mathbb{B}_{\mathbb{Y}}^{\text{EPKBYTELEN}}$

¹⁰E.g Given two bytes 0x15 and 0x03 then $0x15|0x03 = 0x17$ and $0x15\&0x03 = 0x01$.

1467 EncSch.Enc

1468 The encryption, on inputs $(pk, m) \in \mathbb{B}_{\mathbb{Y}}^{\text{EPKBYTELEN}} \times \mathbb{B}_{\mathbb{Y}}^{\text{NOTEBYTELEN}}$, is defined as follows:

1469 1. Generate an ephemeral Curve25519 keypair $(esk, epk) \in \mathbb{B}_{\mathbb{Y}}^{\text{ESKBYTELEN}} \times \mathbb{B}_{\mathbb{Y}}^{\text{EPKBYTELEN}}$
 1470 (as above).

2. Compute the shared secret¹¹ $ss \in \mathbb{B}_{\mathbb{Y}}^{\text{EPKBYTELEN}}$:

$$ss = \text{X25519}(esk, pk) \in \mathbb{B}_{\mathbb{Y}}^{\text{EPKBYTELEN}}$$

3. Generate a session key:

$$\text{Blake2b512}(\text{encTag} \| epk \| ss) \in \mathbb{B}_{\mathbb{Y}}^{\text{KDFDIGESTBYTELEN}}$$

where $\text{encTag} = 0x5A \| 0x65 \| 0x74 \| 0x68 \| 0x45 \| 0x6E \| 0x63$, that is the UTF-8 encoding of “ZethEnc” string (used for domain separation purposes). The result, then, is parsed as follows:

$$ek = \text{Blake2b512}(\text{encTag} \| epk \| ss)[\text{SYMKEYBYTELEN} - 1]$$

$$mk = \text{Blake2b512}(\text{encTag} \| epk \| ss)[\text{SYMKEYBYTELEN} : \text{SYMKEYBYTELEN} + \text{MACKEYBYTELEN} - 1].$$

4. Encrypt the confidential data:

$$ct_{\text{sym}} = \text{ChaCha20}(ek, \text{CHACHABLOCKCOUNTERVALUE}, \text{CHACHANONCEVALUE}, m) \in \mathbb{B}^{\text{NOTEBYTELEN} * \text{BYTELEN}}$$

1471 **Remark 3.5.1.** Formally speaking we should have written $ct_{\text{sym}} \in \mathbb{B}^n$, where
 1472 n is the length of binary representation of the encrypted message m . In Zeth
 1473 however, the only data encrypted are the notes. As such, the size of the plaintexts
 1474 is $\text{NOTEBYTELEN} * \text{BYTELEN}$ bits.

1475 **Remark 3.5.2.** In the following, we omit the explicit conversion from \mathbb{B}^n to
 1476 $\mathbb{B}_{\mathbb{Y}}^{\lceil n/\text{BYTELEN} \rceil}$ when passing the output of ChaCha20 to the Poly1305 algorithms.

5. Randomly generate $(r, s) \in \mathbb{B}_{\mathbb{Y}}^{\text{MACKEYBYTELEN}/2} \times \mathbb{B}_{\mathbb{Y}}^{\text{MACKEYBYTELEN}/2}$ and clamp it:

$$\begin{aligned} r[3] &\leftarrow r[3] \ \& \ 0x0F \\ r[7] &\leftarrow r[7] \ \& \ 0x0F \\ r[11] &\leftarrow r[11] \ \& \ 0x0F \\ r[15] &\leftarrow r[15] \ \& \ 0x0F \\ r[4] &\leftarrow r[4] \ \& \ 0xFC \\ r[8] &\leftarrow r[8] \ \& \ 0xFC \\ r[12] &\leftarrow r[12] \ \& \ 0xFC \end{aligned}$$

¹¹We assume here that esk has been clamped as discussed in Section 3.5.2

6. Generate the related tag:

$$\tau = \text{Poly1305.Tag}(mk, ct_{\text{Sym}}) \in \mathbb{B}_{\mathbb{Y}}^{\text{TAGBYTELEN}}.$$

7. Create the asymmetric ciphertext as:

$$ct = epk \parallel ct_{\text{Sym}} \parallel \tau \in \mathbb{B}_{\mathbb{Y}}^{\text{CTBYTELEN}}.$$

1477 8. Return ct . As consequence $\text{ENCZETHNOTELEN} = \text{CTBYTELEN} * \text{BYTELEN}$ bits.

1478 **EncSch.Dec**

1479 The decryption, on inputs $(sk, ct) \in \mathbb{B}_{\mathbb{Y}}^{\text{ESKBYTELEN}} \times \mathbb{B}_{\mathbb{Y}}^{\text{CTBYTELEN}}$, is defined as follows:

1. Parse the ciphertext ct as:

$$\begin{aligned} epk &\leftarrow ct[: \text{EPKBYTELEN} - 1] \\ ct_{\text{Sym}} &\leftarrow ct[\text{EPKBYTELEN} : \text{EPKBYTELEN} + \text{NOTEBYTELEN} - 1] \\ \tau &\leftarrow ct[\text{EPKBYTELEN} + \text{NOTEBYTELEN} : \text{EPKBYTELEN} + \text{NOTEBYTELEN} + \text{TAGBYTELEN} - 1] \end{aligned}$$

2. Recover the shared secret

$$ss = \text{X25519}(sk, epk).$$

3. Compute the $ek \parallel mk$

$$\begin{aligned} ek &= \text{Blake2b512}(\text{encTag} \parallel epk \parallel ss)[: \text{SYMKEYBYTELEN} - 1] \\ mk &= \text{Blake2b512}(\text{encTag} \parallel epk \parallel ss)[\text{SYMKEYBYTELEN} : \text{SYMKEYBYTELEN} + \text{MACKEYBYTELEN} - 1]. \end{aligned}$$

4. Verify that the ciphertext has not been forged:

$$\text{Poly1305.Vf}(mk, \tau, ct_{\text{Sym}})$$

5. (If the MAC verifies) decrypt:

$$m = \text{ChaCha20.Dec}(ek, \text{CHACHABLOCKCOUNTERVALUE}, \text{CHACHANONCEVALUE}, ct_{\text{Sym}})$$

1480 6. Return m .

3.5.4 Security requirements satisfaction

DHAES has already been proved to be IND-CCA2 secure (see [ABR99, Section 3.5, Theorem 6])¹² and to the best of our knowledge there is no paper showing IK-CCA security. The only proof we have found is related to DHIES scheme [ABN10], that is a prime order group version of DHAES. In the following, we provide a proof for IK-CCA security of DHAES by adapting that proof to our case.

Theorem 3.5.1 (IK-CCA of DHAES). *Let DHAES be the asymmetric encryption scheme as defined above. Let \mathcal{A} be an adversary for the IK-CCA game, then there exists a HDHI adversary \mathcal{B} of $(\mathcal{H}, \text{SetupG})$ and a SUF-CMA adversary \mathcal{C} of MAC such that*

$$\text{Adv}_{\text{DHAES}, \mathcal{A}}^{\text{ik-cca}}(\lambda) \leq 2 \cdot \text{Adv}_{\mathcal{H}, \text{SetupG}, \mathcal{B}}^{\text{hdhi}}(\lambda) + \text{Adv}_{\text{MAC}, \mathcal{C}}^{\text{suf-cma}}(\lambda).$$

The adversaries \mathcal{B} and \mathcal{C} have the same running time as \mathcal{A} ¹³.

Informal proof. As already mentioned, DHAES is similar to DHIES scheme, except for the underlying group and the way the symmetric keys are constructed. As consequence, IK-CCA property for DHAES can be shown similarly to the approach in [ABN10, Theorem 6.2]. More precisely, they show that one can construct from an attacker \mathcal{A} for the IK-CCA game two attackers \mathcal{B} and \mathcal{C} for the ODH and SUF-CMA games. Actually, they make use of a $\bar{\mathcal{B}}$ attacker for the ODH2 game [ABN10, Figure 20] and then apply [ABN10, Lemma 6.1] to obtain an attacker \mathcal{B} ¹⁴ in the ODH game. We adopt a similar strategy, working with HDHI, HDHI2 and Lemma 1.5.1.

Let \mathcal{A} be an attacker for the IK-CCA game, and let $\bar{\mathcal{B}}$ be an attacker for the HDHI2 game described in Fig. 3.11. We show that,

$$\text{Adv}_{\mathcal{H}, \text{SetupG}, \bar{\mathcal{B}}}^{\text{hdhi2}}(\lambda) = |\Pr[\text{IK-CCA}^{\mathcal{A}}(\lambda) = 1] + \Pr[\text{G}_0^{\mathcal{A}}(\lambda) = 1] - 1|$$

where G_0 is the security game described in Fig. 3.12.

Given an HDHI2 challenge $(\llbracket u \rrbracket, \llbracket v_0 \rrbracket, \llbracket v_1 \rrbracket, w_{b_2,0}, w_{b_2,1})$, an adversary $\bar{\mathcal{B}}$ samples $b \leftarrow \{0, 1\}$ and runs \mathcal{A} on $\llbracket v_0 \rrbracket, \llbracket v_1 \rrbracket$ (note that b_2 is the random bit chosen by the $\bar{\mathcal{B}}$ challenger in the HDHI2 game). $\bar{\mathcal{B}}$ constructs oracles $\text{O}^{\text{Dec}_{sk_i}}$ where the queries $(\tau \| ct_{\text{Sym}} \| \tau)$ are processed as follows: if $\tau \neq \llbracket u \rrbracket$, then $\bar{\mathcal{B}}$ queries related HDHI2 oracle to obtain $ek \| mk \leftarrow \text{O}^{\text{HDHI}_{v_i}}(\tau)$ (see Fig. 3.11). If $\tau = \llbracket u \rrbracket$, $w_{b_2,i}$ is parsed as $ek \| mk$. In both cases, it checks that $\text{MAC.Vf}(mk, ct_{\text{Sym}}, \tau) = 1$ and, if so, returns $m \leftarrow \text{Sym.Dec}(ek, ct_{\text{Sym}})$. We note that \mathcal{A} cannot query the challenged ciphertext. $\bar{\mathcal{B}}$ returns 0 if and only if $b = \tilde{b}$. It easy to see that if b_2 is equal to 0, then all symmetric encryption and MAC keys used for the challenge ciphertext $(\tau^* \| ct_{\text{Sym}}^* \| \tau^*)$ and decryption responses are exactly as in a DHAES game.

¹²Specifically, if Sym is IND-CPA secure, it holds that H is HDHI secure and MAC is SUF-CMA secure.

¹³In order to give an asymptotic version of the theorem, the number of queries q has been substituted by the fact of considering PPT adversaries.

¹⁴Note that in [ABN10] the IK-CCA game is a particular case of the AI-CCA game that requires two input messages in the LR query. In order to reason only about the key-privacy, the two messages m_0 and m_1 are constrained to be equal.

Adversary $\bar{\mathcal{B}}(\llbracket u \rrbracket, \llbracket v_0 \rrbracket, \llbracket v_1 \rrbracket, w_{b_2,0}, w_{b_2,1})$	$\bar{\mathcal{B}}$ simulation of $\mathcal{O}^{\text{Dec}_{sk_i}}(\tau \parallel ct_{\text{Sym}} \parallel \tau)$
$b \leftarrow \$\{0, 1\}$	if $\tau \neq \llbracket u \rrbracket$
$(m, state) \leftarrow \mathcal{A}^{\mathcal{O}^{\text{Dec}_{sk_0}}, \mathcal{O}^{\text{Dec}_{sk_1}}}(\llbracket v_0 \rrbracket, \llbracket v_1 \rrbracket)$	$ek \parallel mk \leftarrow \mathcal{O}^{\text{HDH}_{v_i}}(\tau)$
$ek \parallel mk \leftarrow w_{b_2,b}$	else
$\tau^* \leftarrow u$	$ek \parallel mk \leftarrow w_{b_2,i}$
$ct_{\text{Sym}}^* \leftarrow \text{Sym.Enc}(ek, m)$	fi
$\tau^* \leftarrow \text{MAC.Tag}(mk, ct_{\text{Sym}}^*)$	if $\text{MAC.Vf}(mk, ct_{\text{Sym}}, \tau) = 1$
$\tilde{b} \leftarrow \mathcal{A}^{\mathcal{O}^{\text{Dec}_{sk_0}}, \mathcal{O}^{\text{Dec}_{sk_1}}}(\tau^* \parallel ct_{\text{Sym}}^* \parallel \tau^*, state)$	return $\text{Sym.Dec}(ek, ct_{\text{Sym}})$
return $\tilde{b} = b$	else
	return \perp
	fi

Figure 3.11: Description of the adversary $\bar{\mathcal{B}}$ for HDH12, simulating DHAES game for \mathcal{A} .

If $b_2 = 1$, then $w_{1,0}$ and $w_{1,1}$ are random strings and the challenge ciphertext and decryption responses are given as in the \mathbf{G}_0 game described in Fig. 3.12. So we get,

$$\Pr[\text{HDH12}^{\bar{\mathcal{B}}}(\lambda) = 1] = \frac{1}{2} \cdot \Pr[\text{IK-CCA}^{\mathcal{A}}(\lambda) = 1] + \frac{1}{2} \cdot \Pr[\mathbf{G}_0^{\mathcal{A}}(\lambda) = 1].$$

1507 And from the definition of HDH12 advantage we have

$$\text{Adv}_{\mathcal{H}, \text{Setup}_{\mathbf{G}}, \bar{\mathcal{B}}}^{\text{hdhi2}}(\lambda) = |\Pr[\text{IK-CCA}^{\mathcal{A}}(\lambda) = 1] + \Pr[\mathbf{G}_0^{\mathcal{A}}(\lambda) = 1] - 1|.$$

1508 At this point, we can conclude as in [ABN10, Theorem 6.2], with the only difference
 1509 of applying Lemma 1.5.1 instead of [ABN10, Lemma 6.1] and by defining a game \mathbf{G}_1
 1510 that is *identical until bad*¹⁵ \mathbf{G}_0 defined in Fig. 3.12. \square

1511 3.5.5 Final notes and observations

1512 In this section we list some notes regarding the approach taken in **Zcash** (see [ZCa19,
 1513 Section 8.7]), and other observations:

- 1514 • *Key derivation parameters:* in DHAES construction, the only required input vari-
 1515 ables are the shared secret ss and epk . In the Sprout release of **Zcash**, additional
 1516 parameters were added (i.e. h_{sig} , pk_{enc} and a counter i) (see [ZCa19, 5.4.4.2]):
 1517 they state that h_{sig} was used in order to get a different randomness extractor for
 1518 each joinsplit transfer in order to limit the degradation of the security and weaken
 1519 assumption on the hash. The authors believed, about the use of long-standing
 1520 public key pk_{enc} , that it might be necessary for IND-CCA2 security and for post-
 1521 quantum privacy (in the case where the quantum attacker does not have the public

¹⁵Games \mathbf{G}_i and \mathbf{G}_j are said to be *identical until bad* if they differ only in statements that follow the setting of the **bad** variable to *True*. **bad** is initialized with *False*

$G_0(\lambda)$	Oracle $O^{\overline{\text{Dec}}_{sk_i}}(\tau \ ct_{\text{Sym}} \ \tau)$
$(q, \mathbb{G}, g, +) \leftarrow \text{SetupG}(1^\lambda)$	if $\tau = \tau^*$
$(sk_0, pk_0), (sk_1, pk_1) \leftarrow \$ \text{KGen}(1^\lambda)$	$m \leftarrow \perp$
$\tau^* \leftarrow \$ \mathbb{G}$	if $\text{MAC.Vf}(mk^*, ct_{\text{Sym}}, \tau) = 1$
$ek^* \leftarrow \$ \{0, 1\}^{kLen}$	$\text{bad} \leftarrow \text{true}$
$mk^* \leftarrow \$ \{0, 1\}^{mLen}$	$m \leftarrow \text{Sym.Dec}(ek^*, ct_{\text{Sym}})$
$(m, state) \leftarrow \mathcal{A}^{O^{\overline{\text{Dec}}_{sk_0}}, O^{\overline{\text{Dec}}_{sk_1}}}(pk_0, pk_1)$	fi
$b \leftarrow \$ \{0, 1\}$	else
$ct_{\text{Sym}}^* \leftarrow \text{Sym.Enc}(ek^*, m)$	$m \leftarrow \text{Dec}(sk_i, \tau \ ct_{\text{Sym}} \ \tau)$
$\tau^* \leftarrow \text{MAC.Tag}(mk^*, ct_{\text{Sym}}^*)$	fi
$\tilde{b} \leftarrow \mathcal{A}^{O^{\overline{\text{Dec}}_{sk_0}}, O^{\overline{\text{Dec}}_{sk_1}}}(\tau^* \ ct_{\text{Sym}}^* \ \tau^*, state)$	return m
return $\tilde{b} = b$	

Figure 3.12: G_0 game and related decryption oracles for Theorem 3.5.1.

- key) [zcaa]. None of these additional components are used any longer starting from the Sapling release (see [ZCa19, 5.4.4.4]). To the best of our knowledge there is no formal reason to use the note counter i as an input to the KDF: an explanation could be to avoid the same session key being reused for multiple notes, but this should not be a problem since a different nonce or block counter is used for the symmetric cipher (actually this is already mandated in the case where epk is reused, as described below).
- *Reuse of ephemeral keys epk :* **Zcash** reuses the same ephemeral keys epk (and different nonces) for two ciphertexts in a joinsplit description, claiming that this does not affect the security of the scheme as soon as the HDHI assumption of the DHAES security proof is adapted. Note that the proof they refer to is related to the IND-CCA2 notion.
 - Note that in **Zcash** Sprout and Sapling, being able to break the Elliptic Curve Diffie-Hellman Problem on Curve25519 or Jubjub would not help to decrypt the transmitted notes ciphertext unless the receiver pk_{enc} is known or guessed. On the other hand, having pk_{enc} into the hash (as used in Sprout) may violate in principle the key-privacy of the encryption scheme. For these reasons, we underline that the protocol should enforce a mechanism that does not reveal users public keys to increase the security.
 - In [ABN10], the concept of *robustness* for an asymmetric encryption scheme is introduced: it formalizes the infeasibility of producing a ciphertext valid under two different public encryption keys. We note that this is particularly useful for **Zeth** since only the intended receiver will be able to decrypt the encrypted note. In fact, the definition is more general since it also covers the case in which a decryption

1546 is successful but returns an incorrect plaintext. This prevents situations where
1547 a user, scanning the **Mixer** logs for incoming transactions, gets a false positive
1548 decryption and stores garbage notes.

Note

We note however, that the “false-positive” situation above can be prevented by relying on a weaker notion of robustness called *collision-freeness* [Moh10]. In fact, as described in Section 2.6, the procedure to receive a *ZethNote* requires to decrypt the ciphertext emitted by the **Mixer**, and then to verify that the recovered plaintext is the opening of a commitment in the Merkle tree. As such, since the *collision-freeness* of the encryption ensures that plaintexts recovered under different keys are different (i.e. “do not produce a collision”), then we know that plaintexts recovered by parties who are not the intended recipient will fail the “commitment opening verification”, leading the payment to be rejected, and solving the aforementioned false-positive issue.

1549

1550 In [ABN10, Section 6], the authors prove that DHIES can be made strongly robust.
1551 The proof can be easily adapted to work with DHAES.

- 1552 • *No public key validation for X25519*: cryptographers have been discussing the ab-
1553 sence of any mandated public key validation or checks on the result of X25519.
1554 For example, in [LHT16, Section 6.1], an optional zero check is introduced in order
1555 to assure that the result of X25519 is not 0: this avoids a situation in which one
1556 of the two parties can force the result of the key-exchange by using a small order
1557 point as public key. This property is generally defined as *contributory behaviour*,
1558 that is, none of the parties is able to force the output of a key exchange. However,
1559 protocols do not have all the same security requirements and adding default checks
1560 in the Curve25519 specifications would be superfluous in most cases and would add
1561 complexities that Bernstein has deliberately chosen to avoid (*simple implementa-*
1562 *tion principle*). More importantly, Diffie-Hellman does not require *contributory*
1563 *behaviour* property [Per17]: modern view is that the only requirements are key
1564 indistinguishability and, in case of an active attacker, that the output of the key
1565 exchange should not produce a low-entropy function of the honest party’s private
1566 key (e.g. small-subgroup and invalid-curve attacks). Since these two properties are
1567 considered satisfied by Curve25519, there is no need to add extra checks to the
1568 Curve25519 specification. We conclude by observing that in the Sprout release, the
1569 Zcash protocol does not specify any point validation and makes use only of the
1570 private key clamping to keep Diffie-Hellman key exchange secure.

3.6 ZkSnarkSch instantiation

Groth’s proof system **Groth16** [Gro16] is the most efficient known zk-SNARK (in terms of the proof size and proof and verification cost) for QAPs, and thus one of the most efficient NIZK for proving statements on arithmetic circuits (consisting of addition and multiplication gates over a finite field \mathbb{F}). Below we present **Groth16**’s key generation, prover, verifier, and simulator algorithms, adjusted as described in [BGM17] to further reduce the size of *srs* and proofs, and to make the **KGen** algorithm more amenable to implementation as a multi-party computation.

In what follows, let the number *constNo* of constraints in the relation **R** be fixed. Without loss of generality we consider *constNo* to be an *upper bound* on the number of constraints in the **R** parameter, and assume that there exists some *constNo*-th root of unity $\omega \in \mathbb{F}_{\text{rCUR}}$. Define $\ell_i(X)$ to be the *i*-th Lagrange polynomial of degree $(\text{constNo} - 1)$ over the set $\{\omega^i\}_{i \in [\text{constNo}]}$, and let $\ell(X)$ be the unique non-zero polynomial of degree *constNo* that satisfies $\ell(\omega^i) = 0$ for all $i \in [\text{constNo}]$.

We note that the requirement that there exists a *constNo*-th root of unity ω imposes a restriction on the maximum number of constraints in **R** that the scheme can support. In the particular case of $\omega \in \mathbb{F}_{\text{rBN}}$, the restriction becomes $\text{constNo} \leq 2^{28}$. For \mathbb{F}_{rBLS} this becomes $\text{constNo} \leq 2^{47}$.

Furthermore, we denote by $\text{inp} \in \mathbb{F}^{\text{inpNo}+1}$ the tuple of variables (i.e. “circuit wires”) in the algebraic representation of the relation **R**, such that:

- $\text{inp}_0 = 1_{\mathbb{F}}$ (the multiplicative identity in \mathbb{F}),
- $(\text{inp}_1, \dots, \text{inp}_{\text{inpNoPrim}})$ represent variables in the statement,
- $(\text{inp}_{\text{inpNoPrim}+1}, \dots, \text{inp}_{\text{inpNo}})$ represent variables in the witness (so-called “auxiliary input”).

KGen(**R**, 1^λ):

- i. Pick trapdoor $td = (\tau, \alpha, \beta, \delta) \leftarrow \$(\mathbb{Z}_p^* \setminus \{\omega^{i-1}\}_{i=1}^{\text{constNo}}) \times (\mathbb{Z}_p^*)^3$;
- ii. For $j \in \{0, \dots, \text{inpNo}\}$, let

$$u_j(\tau) = \sum_{i=1}^{\text{constNo}} U_{ij} \ell_i(\tau),$$

$$v_j(\tau) = \sum_{i=1}^{\text{constNo}} V_{ij} \ell_i(\tau),$$

$$w_j(\tau) = \sum_{i=1}^{\text{constNo}} W_{ij} \ell_i(\tau);$$

iii. Set

$$\begin{aligned} srs_P &\leftarrow \left(\llbracket \alpha \rrbracket_1, \llbracket \beta \rrbracket, \llbracket \delta \rrbracket, \{ \llbracket u_j(\tau) \rrbracket_1 \}_{j=0}^{inpNo}, \{ \llbracket v_j(\tau) \rrbracket \}_{j=0}^{inpNo}, \right. \\ &\quad \left. \{ \llbracket (u_j(\tau)\beta + v_j(\tau)\alpha + w_j(\tau))/\delta \rrbracket_1 \}_{j=inpNoPrim+1}^{inpNo}, \right. \\ &\quad \left. \{ \llbracket \tau^i \ell(\tau)/\delta \rrbracket_1 \}_{i=0}^{constNo-2} \right) \\ srs_V &\leftarrow \left(\llbracket \alpha \rrbracket_1, \llbracket \beta \rrbracket_2, \llbracket \delta \rrbracket_2, \{ \llbracket \beta u_j(\tau) + \alpha v_j(\tau) + w_j \rrbracket_1 \}_{j=0}^{inpNoPrim} \right) \\ srs &\leftarrow (srs_P, srs_V) \end{aligned}$$

1597 **return** srs, td

1598 $P(\mathbf{R}, srs_P, prim = (inp_j)_{j=1}^{inpNoPrim}, aux = (inp_j)_{j=inpNoPrim+1}^{inpNo})$:

i. Define

$$a^\dagger(X) = \sum_{j=0}^{inpNo} inp_j u_j(X), \quad b^\dagger(X) = \sum_{j=0}^{inpNo} inp_j v_j(X), \quad c^\dagger(X) = \sum_{j=0}^{inpNo} inp_j w_j(X);$$

1599 ii. Define the polynomial $h(X) = (a^\dagger(X)b^\dagger(X) - c^\dagger(X))/\ell(X)$ and compute the
1600 coefficients $\{h_i\}_{i=0}^{constNo-2}$ of h , such that $h(X) = \sum_{i=0}^{constNo-2} h_i X^i$.

1601 iii. $r_a \leftarrow \$\mathbb{Z}_p$;

1602 iv. $r_b \leftarrow \$\mathbb{Z}_p$;

v. Compute proof elements:

$$\begin{aligned} \mathbf{a} &\leftarrow \sum_{j=0}^{inpNo} inp_j \llbracket u_j(\tau) \rrbracket_1 + \llbracket \alpha \rrbracket_1 + r_a \llbracket \delta \rrbracket_1 \\ \mathbf{b} &\leftarrow \sum_{j=0}^{inpNo} inp_j \llbracket v_j(\tau) \rrbracket_2 + \llbracket \beta \rrbracket_2 + r_b \llbracket \delta \rrbracket_2 \\ \mathbf{c} &\leftarrow \sum_{j=inpNoPrim+1}^{inpNo} inp_j \left\llbracket \frac{u_j(\tau)\beta + v_j(\tau)\alpha + w_j(\tau)}{\delta} \right\rrbracket_1 + \sum_{i=0}^{constNo-2} h_i \llbracket \tau^i \ell(\tau)/\delta \rrbracket_1 + \\ &\quad r_b \mathbf{a} + r_a \left(\sum_{j=0}^{inpNo} inp_j \llbracket v_j(\tau) \rrbracket_1 + \llbracket \beta \rrbracket_1 + r_b \llbracket \delta \rrbracket_1 \right) - r_a r_b \llbracket \delta \rrbracket_1 \end{aligned}$$

1603 **return** $\pi \leftarrow (\mathbf{a}, \mathbf{b}, \mathbf{c})$;

1604 $V(\mathbf{R}, srs_V, prim = (inp_j)_{j=1}^{inpNoPrim}, \pi)$:

i. Check that:

$$\begin{aligned} \mathbf{a} \bullet \mathbf{b} &= \mathbf{c} \bullet \llbracket \delta \rrbracket_2 \\ &\quad + \left(\sum_{j=0}^{inpNoPrim} inp_j \llbracket u_j(\tau)\beta + v_j(\tau)\alpha + w_j(\tau) \rrbracket_1 \right) \bullet \llbracket 1 \rrbracket_2 \\ &\quad + \llbracket \alpha \rrbracket_1 \bullet \llbracket \beta \rrbracket_2 \end{aligned}$$

1605 Note that $\llbracket \alpha \rrbracket_1$ and $\llbracket \beta \rrbracket_2$ are stored individually and used by the prover to re-
 1606 compute $\llbracket \alpha\beta \rrbracket_T$ seemingly redundantly. This is required in order to leverage the
 1607 pairing check functionality built in to **Ethereum**, which accepts a sequence of tuples
 1608 in $\mathbb{G}_1 \times \mathbb{G}_2$ and returns **true** if and only if the product of the resulting pairings
 1609 equals $\llbracket 1 \rrbracket_T$.

1610 **Sim**(**R**, *srs*, *td*, *prim*):

- 1611 i. Sample $a \leftarrow \$\mathbb{Z}_p$; $b \leftarrow \$\mathbb{Z}_p$;
 ii. Compute proof elements:

$$\begin{aligned} \mathbf{a} &\leftarrow \llbracket a \rrbracket_1 \\ \mathbf{b} &\leftarrow \llbracket b \rrbracket_2 \\ \mathbf{c} &\leftarrow \frac{1}{\delta} \cdot \left(ab - \alpha\beta - \sum_{j=0}^{inpNoPrim} inp_j(u_j(\tau)\beta + v_j(\tau)\alpha + w_j(\tau)) \right) \llbracket 1 \rrbracket_1 \end{aligned}$$

1612 **return** $\pi \leftarrow (\mathbf{a}, \mathbf{b}, \mathbf{c})$;

Chapter 4

Implementation considerations and optimizations

4.1 Client security considerations

In this section we consider some details of client *wallet* software that manages user's private and public keys, **Zeth** notes, and interacts with the **Mixer** contract.

Due to the processing and storage requirements involved, we consider it impractical for all **Zeth** client implementations to assume that a dedicated **Ethereum** node (miner node or archive node) is run on the same host as the wallet. Therefore, in order to interact with the **Ethereum** network, wallet software must communicate with external **Ethereum** P2P nodes via their RPC channel, and must assume that these nodes are completely outside the wallet's control. *From a security standpoint, connected **Ethereum** nodes should therefore be considered untrusted, and in particular the details of all RPC calls and responses should be considered publicly visible.* Note that even if the connected **Ethereum** node itself is not malicious, 3rd parties able to see network traffic may also be able to gain an insight into the RPC communication of a specific **Zeth** client.

Note

Note that there are several possible models besides the fully untrusted **Ethereum** node. Organizations or individuals could host one or more “trusted” **Ethereum** nodes, which clients can securely connect to (if they trust the host). This centralization would represent a security trade-off. From the point of view of clients it would create a single point of trust, and for potential malicious observers or attackers it would represent a valuable target.

In what follows we focus on preventing data leaks through network traffic. We do not consider adversaries with physical access to the machine running the wallet (see Appendix C).

Note

Importantly, we focus here on information leakages intrinsic to network communication patterns of the **Zeth** protocol. However, in order to protect against sophisticated adversaries, it is necessary to use network-level anonymity solutions to protect the source of messages emitted on the network. While this is outside of the scope of the **Zeth** protocol, we highly encourage implementers to establish threat models and consider using technologies like *mixnets* to protect against network analysis (see e.g. [PHE⁺17, DG09]).

1633

1634 4.1.1 Syncing and waiting

1635 **Zeth** clients must periodically synchronize with the latest state of the blockchain. This
1636 is necessary to keep track of the data held by the **Mixer** contract, and to detect notes
1637 received by the user of the wallet, storing them for future transactions.

1638 Clients should synchronize with **Ethereum** nodes in such a way that information is
1639 not leaked. As such:

- 1640 1. Clients **MUST** use consensus evidence and block headers to verify all data they
1641 receive from **Ethereum** nodes.
- 1642 2. Clients **MUST** locally store all parts of the **Mixer** state they require in order to
1643 function.
- 1644 3. Clients **MUST** obtain all such information by “synchronizing” with the **Ethereum**
1645 blockchain and parsing relevant events emitted by **Mixer**. Clients **MUST NOT** query
1646 the **Mixer** state via RPC.
- 1647 4. Clients **SHOULD** take steps to avoid being identified while synchronizing (see Ap-
1648 pendix C.2. For example, clients **SHOULD** vary the set of **Ethereum** nodes that they
1649 connect to, and **SHOULD NOT** always sync from the block following the last one that
1650 they processed.
- 1651 5. Clients **SHOULD NOT** re-request blocks or transaction receipts that are of particular
1652 interest to them. They **SHOULD** process all events, emitted by **Mixer**, in the same
1653 way.
- 1654 6. Clients **SHOULD NOT** make any RPC calls or change their externally visible behaviour
1655 in response to blocks or transaction receipts that are of interest to them.

1656 Use of contract queries

1657 We suggest that clients **SHOULD NOT** directly query the contract state, for the reasons
1658 discussed in Appendix C.2 and Appendix C.3 (and consequently, Section 4.2 suggests
1659 that the **Mixer** contract should, as far as possible, not expose public methods). The

1660 Zeth protocol prohibits direct queries of the state of $\widetilde{\text{Mixer}}$ (via *public* smart-contract
1661 functions) because they introduce a risk that client implementations will leak information
1662 by using them.

1663 If implementers choose to add public methods to the $\widetilde{\text{Mixer}}$ contract (for application-
1664 specific reasons), they should consider carefully the security issues raised in Appendix C.
1665 This specification assumes that `Mix` is the only public method of the $\widetilde{\text{Mixer}}$ contract.

1666 4.1.2 Note management

1667 `Mix` calls on the $\widetilde{\text{Mixer}}$ contract emit log events containing new commitment values,
1668 nullifiers, the new Merkle root and the secret data for new notes (encrypted using a key
1669 derived from the recipients public key). As clients synchronize with the latest state of
1670 the blockchain, they **MUST** read these events and correctly process the data they contain.

- 1671 1. Clients **MUST** process the `MixEventDType` event for every `Mix` transaction, in the
1672 order in which they appear in the blockchain.
- 1673 2. Clients implementing spending functionality **MUST** use the commitment values in
1674 events to track the state of the Merkle tree. The Merkle tree state will be used
1675 to generate Merkle paths for future transactions, and **MUST** be made available to
1676 the client without the need to query the contract. (Note that not all commitments
1677 must necessarily be persisted – see Section 4.3).
- 1678 3. Clients that can receive notes **MUST** attempt to decrypt the ciphertexts for every
1679 transaction (see Item 2 in Section 2.6).
- 1680 4. Clients **MUST NOT** perform any network-related action, including closing the RPC
1681 connection, dependent on successful/unsuccessful decryption of ciphertexts (see
1682 Appendix C.3).
- 1683 5. Clients that can receive notes **MUST** attempt to parse any successfully decrypted
1684 plaintext (that is, ensure it is well-formed as in Item 3a in Section 2.6).
- 1685 6. Clients **MUST NOT** perform any network-related action, including closing the con-
1686 nection, dependent on successful / unsuccessful parsing (see Appendix C.4).
- 1687 7. Clients that can receive notes **MUST** verify that successfully parsed plaintext data
1688 is the opening of the corresponding commitment in the transaction (see Item 3b
1689 in Section 2.6).
- 1690 8. Clients **MUST NOT** perform any network-related action, including closing the con-
1691 nection, dependent on whether the parsed note data is the opening of the corre-
1692 sponding commitment (see Appendix C.4).
- 1693 9. Clients **MUST** confirm that, after adding the new commitments, the local repre-
1694 sentation of the Merkle tree of commitments has a root consistent with the event
1695 data.

- 1696 10. Clients **SHOULD** keep a *local* record of the data related to valid decrypted notes.
1697 This will be required in order to spend the notes in a future transaction.
- 1698 11. Clients implementing spending functionality **SHOULD** process all nullifiers in Mix
1699 transaction events, checking for any corresponding notes previously recorded. Any
1700 such note should be marked as spent in the client's local record.

1701 4.1.3 Prepare arguments for Mix transaction

1702 Clients **MUST NOT** query **Ethereum** nodes while generating any arguments to a Mix call.
1703 In particular, Merkle paths **MUST** be calculated using the client's local representation of
1704 the Merkle tree of commitments that was constructed by parsing events.

1705 Where the zero-knowledge proof is generated by some external process, clients **MUST**
1706 put in place sufficient security schemes to ensure that:

- 1707 • they are communicating with an authentic proof generation process (not a man-
1708 in-the-middle), and
- 1709 • data sent to and from the proving process cannot be observed in transit and tam-
1710 pered with by a third party, and
- 1711 • the proof has been generated for the correct instance-witness pair¹

1712 Without these safe-guards, the operation of the system and the secret data required
1713 to spend the input notes may be compromised. See Appendix C.6.

1714 4.1.4 Wallet backup and recovery

1715 Given the restrictions placed on clients and their interaction with the **Mixer** contract,
1716 it follows that clients must store all data required to spend notes owned by their users'
1717 addresses, and to verify the validity of incoming notes. If this local data is lost, it must
1718 be reconstructed before client operations can resume.

1719 **Zeth** private keys (see Table 1.5) can be used to fully restore client state. In this
1720 case, clients **MUST** retrieve all events from the beginning of the **Mixer** contract's his-
1721 tory, decrypting notes and tracking nullifiers, as described in the previous sections, to
1722 reconstruct the set of unspent notes that they own.

1723 Without a backup of the private keys it is not possible to restore wallet state. As
1724 such, private keys are the minimal set of data that must be securely stored and backed
1725 up, and clients **SHOULD** provide support for this mode of recovery. However, to avoid the
1726 need to scan all events emitted by **Mixer** (a very expensive operation) implementations
1727 **SHOULD** also support back ups of further state data (such as the representation of the
1728 Merkle tree of note commitments, the set of unspent notes, etc) to allow more efficient
1729 modes of recovery.

¹Although given an acceptable zk-proof π for an instance *prim* it is infeasible to check which witness has been used – which comes directly from the zero-knowledge property – we need to assure security measures that prevents any third party from mauling and tampering with the proof generation process.

1730 4.2 Contract security considerations

1731 Section 4.1 mentions several considerations for client implementations, concerning how
1732 they interact with the contract. These must be taken into account when authoring the
1733 contract code, to ensure that clients can securely retrieve the information they need to
1734 operate without encouraging them to perform insecure operations.

- 1735 1. **Mixer** MUST validate inputs, the contract needs to ensure that the primary inputs
1736 are elements of the scalar field $\mathbb{F}_{r_{\text{CUR}}}$ (that is, they are in the range $\{0, \dots, r_{\text{CUR}} - 1\}$).
- 1737 2. **Mixer** MUST output events for valid Mix calls, including:
 - 1738 (a) commitment for each new note;
 - 1739 (b) nullifier for each spent note;
 - 1740 (c) value of new Merkle root of commitments;
 - 1741 (d) ciphertexts for each new note;
 - 1742 (e) implementation-specific data (such as the one-time sender public key specified
1743 in Section 3.5, required to decrypt the ciphertexts).
- 1744 3. The Mix function MUST be *payable*², to support non-zero *vin*.
- 1745 4. **Mixer** MUST NOT expose any public methods except for Mix.

1746 4.3 Efficiency and scalability

1747 4.3.1 Importance of performance

1748 Poor performance and scalability has several impacts on the viability of the system.

1749 Efficiency and performance are arguably most important for the **Mixer** contract,
1750 where gas usage directly affects the monetary cost of using **Zeth** to transfer value. That
1751 is, high gas costs could make transactions very expensive, and therefore not practical for
1752 many use-cases, undermining the utility and viability.

1753 High storage or compute requirements on the client would severely restrict the set
1754 of devices on which **Zeth** client software can run, and long delays when sending or
1755 receiving transactions can adversely affect the user-experience, discouraging some users
1756 and undermining the privacy promises of the system.

1757 Although the proof-of-concept implementation of **Zeth** is not intended to be used in a
1758 production environment, one of its aims is to demonstrate the practicality of the protocol
1759 in terms of transaction costs. Therefore, some of the techniques described here have been
1760 included in the proof-of-concept implementation, while in some cases implementers of
1761 production software may wish to make different trade-offs.

²see <https://solidity.readthedocs.io/en/v0.6.2/types.html?highlight=payable#function-types>

4.3.2 Cost centers

One important factor, primarily affecting client performance, is the cost of zero-knowledge proof generation. This is directly related to the number of constraints used to represent the statement in Section 2.2, which in turn depends on the specific cryptographic primitives used (see Chapter 3).

Note that cryptographic primitives which are “snark-friendly” (i.e. can be implemented using fewer gates in an arithmetic circuit) may not necessarily run efficiently on the EVM or on standard hardware. As such, trade-offs must be made between proof generation cost and the gas costs of state transitions. An example of this is the hash function used in the Merkle tree of commitments. This is not only used in the statement of Section 2.2 (to verify Merkle proofs, see Section 2.2), but also on the client (to create Merkle proofs, see Section 2.3) and in the **Mixer** contract (to compute the Merkle root, see Section 2.5).

Aside from the specific hash function used, implementers have some freedom in the data structures and algorithms used to maintain the Merkle tree and generate proofs. Because of this freedom, and the importance of the chosen algorithms on performance across all components of the system, the majority of this section focuses on the details of the Merkle tree.

As described in Chapter 2, **Zeth** notes are maintained and secured by the Merkle tree, whose depth `MKDEPTH` must be fixed when the contract is deployed. Therefore, `MKDEPTH` determines the maximum number of notes (2^{MKDEPTH}) that may be created over the lifetime of the deployment. To ensure the utility of **Zeth**, `MKDEPTH` must be sufficiently large, and therefore the following includes a discussion of *scalability* with respect to `MKDEPTH`.

Also, due to the fact that `MKDEPTH` is fixed, we assume that Merkle proofs are computed as `MKDEPTH`-tuples, no matter how many leaves have been populated. Unpopulated leaves are assumed to take some default value (usually a string of zero bits).

4.3.3 Client performance

Commitment Merkle tree

The simplest possible implementation, which stores only the data items at the leaves of the tree, requires $2^{\text{MKDEPTH}} - 1$ hash invocations to compute the Merkle root or to generate a Merkle proof. The cost of this is too high to be practical for non-trivial values of `MKDEPTH`.

An immediate improvement in compute costs can be achieved by simply storing all nodes (or all nodes whose value is not the default value) and updating only those necessary as new commitments are added. When adding `JSOUT` consecutive leaves to the tree, after $\mathcal{O}(\log_2(\text{JSOUT}))$ layers (requiring $\mathcal{O}(\text{JSOUT})$ hashes) we reach the common ancestor of all new leaves and can update the Merkle tree by proceeding along a single branch (of approximately `MKDEPTH` - $\log_2(\text{JSOUT})$ layers). Thus, the cost of updating the Merkle tree for a single transaction has a fixed bound which is linear in `JSOUT` and

1801 **MKDEPTH.** However, this doubles the storage cost of the tree since non-leaf nodes must
 1802 also be persisted.

1803 In the case of the client, the Merkle tree will only be used to generate proofs for notes
 1804 owned by the user of the client. Thereby **Zeth** clients need only store nodes of the Merkle
 1805 tree that are required for this purpose, and may discard all others. In particular, any
 1806 full sub-tree need only contain nodes that are part of Merkle paths associated with the
 1807 user’s notes. Implementations that discard unnecessary nodes can achieve vast savings
 1808 in storage space.

1809 **Zero-knowledge proof generation**

1810 As well as keeping the number of constraints as low as possible, it is important to ensure
 1811 that the prover implementation is optimal and thereby that proving times are as short as
 1812 possible. Proof generation should also exploit any available parallelism, to help reduce
 1813 the time taken. This may require specific programming languages or frameworks to be
 1814 used, necessitating that proof generation be performed by some external process (as is
 1815 the case in the proof-of-concept implementation).

1816 The proof generation process can also be very memory intensive (in part due to the
 1817 FFT calculations required), and so ensuring that enough RAM is present in the system
 1818 is important to avoid long proof times.

1819 See Appendix C.6 for a discussion of related security concerns.

1820 **4.3.4 Zero-knowledge proof verification (on-chain)**

1821 Verification of the joinsplit statement via a zero-knowledge proof represents a significant
 1822 computation, which must be carried out on-chain (by the **Mixer** contract) for each valid
 1823 **Zeth** transaction. As described in Section 3.6, this verification cost increases linearly
 1824 with the number of primary inputs to the statement – a scalar multiplication of a group
 1825 element and a group addition operation must be performed for each primary input. A
 1826 technique presented in [GGPR13, Section 4.5.1] can be applied to reduce this linear cost.

Given a relation \mathbf{R} , the corresponding language \mathbf{L} , and a collision resistant hash
 function $H : \mathbf{L} \rightarrow \mathbb{F}_{\mathbf{r}_{\text{CUR}}}$, let

$$\mathbf{R}' = \{(prim', aux') \mid prim' = H(prim), aux' = (prim, aux), \text{ for } (prim, aux) \in \mathbf{R}\}$$

1827 be a new relation, with corresponding language $\mathbf{L}' \subset \mathbb{F}_{\mathbf{r}_{\text{CUR}}}$. To (probabilistically) verify
 1828 that $prim \in \mathbf{L}$, a verifier can compute $H(prim)$ and check that $H(prim) \in \mathbf{L}'$. (By
 1829 construction, if $H(prim) \in \mathbf{L}'$, there exists $(prim_0, aux) \in \mathbf{R}$, i.e. $prim_0 \in \mathbf{L}$ with
 1830 $H(prim_0) = H(prim)$. By the collision-resistance of H we have $prim_0 = prim$ with
 1831 overwhelming probability.)

1832 Informally, the original circuit is transformed as follows:

- 1833 • all *primary* inputs $prim$ become *auxiliary* inputs,
- 1834 • a single primary input h is added, and

- the statement is extended such that h is the digest of the original primary inputs.

This slightly increases the complexity of the statement to be proven, adding to the cost of generating proofs π' for the augmented statement, but minimizes the linear component of the verification cost (since the verifier must now only process a single primary input). Note that this technique does not require any change to the initial statement itself (in this case the joinsplit statement described in Section 2.2), or the data upon which it operates. The **Mixer** contract must perform this hash step before the zk-SNARK verification, although we note that the parameters are also unchanged.

In the proof-of-concept implementation of Zeth, this technique is employed using a snark-friendly hash function constructed as follows.

The Merkle-Damgård construction (see [MVOV96, Chapter 9]) can be applied to a collision-resistant compression function to yield a collision-resistant hash function, accepting an arbitrary length input. We apply this to the compression function described in Section 3.2, which is chosen to be collision resistant over domain $\mathbb{F}_{r_{\text{CUR}}}$, and efficiently implementable as arithmetic constraints. Thereby, the resulting hash function, in common with the underlying compression function, can also be efficiently implemented to hash lists of elements in $\mathbb{F}_{r_{\text{CUR}}}$ (and this is exactly the form of the original primary inputs).

4.3.5 Merkle tree updates (on-chain)

For most components of the contract, the set of operations to be performed is strictly defined and the set of possible algorithmic optimizations that can be made is limited. In these cases, it is important to ensure that code is benchmarked and optimized to a reasonable degree, to minimize gas costs. We note that apart from the number and type of compute instructions executed, store and lookup operations have a significant impact on the gas used. In particular, storing new values is more expensive than overwriting existing values, and a gas rebate is made when contracts release stored values. See [Woo19, Appendix H.1] for further details.

The primary component in which algorithmic optimizations can be made is the Merkle tree of note commitments. The **Mixer** contract must compute (and store) the new Merkle root after adding the JSOUT new commitments as leaves.

As in Section 4.3.3, the simplest possible implementation which stores only the data items at the leaves of the tree, requires the full root to be recomputed, involving $2^{\text{MKDEPTH}} - 1$ hash invocations. This quickly becomes impractical for non-trivial values of MKDEPTH.

The first-pass optimization (also described in Section 4.3.3) can be used to ensure that the cost of updating the Merkle tree (number of hash computations, stores and loads) is bounded by a constant that is linear in the Merkle tree depth. This is the strategy used in the proof-of-concept implementation of **Mixer**.

It may be possible to gain further improvements in gas costs by discarding nodes from the Merkle tree that are not required. Unlike clients, **Mixer** is only required to compute the new Merkle root, and does not need to create or validate Merkle proofs (as these are checked as part of the zero-knowledge proof). Consequently, *all* nodes in a

sub-tree can be discarded when the sub-tree is full, and the optimization is much simpler to implement than on the client.

Another possible strategy for decreasing the gas costs associated with Merkle trees is *Merkle Shrubs*, described in [Lab19, Section 2.2]. Under this scheme, the contract maintains a “frontier” of roots of sub-trees and Merkle proofs provided by clients (as auxiliary inputs to the \mathbf{R}^z circuit) contain a path from the leaf to one of the nodes in the frontier. The gas savings in this scheme are due to the fact that, for new commitments, the contract need only recompute the value of nodes from the leaf to the “frontier” (not all the way to the root of the tree). However this comes at the cost of complexity in the arithmetic circuit, which must verify a Merkle path to one of several frontier nodes.

When choosing cryptographic primitives to be used on the EVM (and considering the trade-off with other platforms, described in Section 4.3.1) it may be valuable to note that the EVM supports so-called “pre-compiled contracts”. These behave like built-in functions providing very gas-efficient access to certain algorithms, such as Keccak. However, pre-compiled contracts exist only for a limited set of algorithms. Others must be implemented using EVM instructions.

4.3.6 Optimizing Blake2’s circuit.

After presenting Blake2s circuit and its components working on little endian variables, we show a few optimizations.

Helper circuits

We first define the following helper circuits needed in the Blake2s routine, operating on w -bit long words.

XOR circuits The following XOR circuits on w -bit long variables have been implemented, we assume the inputs are boolean (this is not checked in these circuits),

- “Classic” XOR circuit, which xors 2 variables,
 $a \oplus b = c$;
- XOR with constant, which xors two variables and a constant,
 $a \oplus b \oplus c = d$, with c constant;
- XOR with rotation, which xors two variables and rotates the result.
 $a \oplus b \ggg r = c$, with r constant, and \ggg the rightward rotation [MJS15, Section 2.3]; i.e. for and constant $r < w$ we have $a_i \oplus b_i = c_{i+r \pmod{w}}$, for $i = 0, \dots, w$,

Each of these circuits presents w constraints. Assuming that the inputs are boolean, the output is automatically boolean. To ascertain that both inputs are boolean (a and b), we would need $2 \cdot w$ more gates per circuit.³

³Making sure that no gates are duplicated in the circuit is very important to keep the proving time as small as possible. One challenge of writing RICS programs is to make sure that the statement is correctly represented, without redundancy, in order to keep the constraint system as small as possible.

1909 **Modular addition** We present here two circuits to verify modular arithmetic.

1910 **Double modular addition: $a + b = c \pmod{2^w}$.** This circuit checks that the
 1911 sum of two w -bit long variables in little endian format modulo 2^w is equal to a w -bit
 1912 long variable. More precisely, it checks the equality of the modular addition of $a + b$
 1913 $\pmod{2^w}$ and c and the booleanness of the later. We assume the inputs are boolean (this
 1914 is not checked in this circuit).

1915 As the addition of two w -bit long integers results in at most an $(w + 1)$ -bit integer,
 1916 we consider c to be $(w + 1)$ -bit long. We do not care about the last bit value, c_w , but
 1917 have to ensure its booleanness.

1918 The circuit presents the following $w + 2$ constraints, for a and b of size w , where
 1919 $w = 32$ in practice, and variable c of size $w + 1$, that:

$$\sum_{i=0}^{w-1} (a_i + b_i) \cdot 2^i = \sum_{j=0}^w c_j \cdot 2^j \quad (4.1)$$

$$\forall j \in \{0, \dots, w\}, (c_j - 0) \cdot (c_j - 1) = 0 \quad (4.2)$$

1920 **Triple modular addition: $a + b + c = d \pmod{2^w}$.** This circuit checks the
 1921 equality of a w -bit long variable d with the sum of three w -bit long variables in little
 1922 endian format modulo 2^w . More precisely, it checks the equality of the modular addition
 1923 of $a + b + c \pmod{2^w}$ and d and the booleanness of the latter. We assume the inputs are
 1924 boolean (this is not checked in this circuit).

1925 As the addition of three w -bit long integers results in at most an $(w + 2)$ -bit integer,
 1926 we consider d to be $(w + 2)$ -bit long. We do not care about the values of the last two
 1927 bits (d_w and d_{w+1}), but have to ensure their booleanness.

The circuit presents the following $w + 3$ constraints, for a , b and c of size w , where
 $w = 32$ in practice, and variable d of size $w + 2$, that:

$$\sum_{i=0}^{w-1} (a_i + b_i + c_i) \cdot 2^i = \sum_{j=0}^{w+1} d_j \cdot 2^j \quad (4.3)$$

$$\forall j \in \{0, \dots, w + 1\}, (d_j - 0) \cdot (d_j - 1) = 0 \quad (4.4)$$

1928 Blake2s routine circuit

1929 We define in this section the circuit of the Blake2 routine (see [MJS15, Section 3.1]
 1930 and Fig. 4.1) known as “G function” [ANWOW13, Section 2.4]. G is based on ChaCha
 1931 encryption [Ber08a]. It works on w -bit long words, and presents $8 \cdot w + 10$ constraints.
 1932 The function mixes a state (a , b , c and d) with the inputs (x and y) and returns the
 1933 updated state.

1934 This circuit does not check the booleanness of the inputs or state. However, given that
 1935 the state is boolean, the output is automatically boolean due to the use of the modular
 1936 addition circuits.

$G(a, b, c, d; x, y) \mapsto (a_2, b_2, c_2, d_2)$	$\text{getSigma}()$
1 : $a_1 \leftarrow a + b + x \pmod{2^w}$	1 : $\Sigma \in (\mathbb{N}^{16})^{10}$
2 : $d_1 \leftarrow d \oplus a_1 \ggg r_1$	2 : $\Sigma[0] \leftarrow (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15)$
3 : $c_1 \leftarrow c + d_1 \pmod{2^w}$	3 : $\Sigma[1] \leftarrow (14, 10, 4, 8, 9, 15, 13, 6, 1, 12, 0, 2, 11, 7, 5, 3)$
4 : $b_1 \leftarrow b \oplus c_1 \ggg r_2$	4 : $\Sigma[2] \leftarrow (11, 8, 12, 0, 5, 2, 15, 13, 10, 14, 3, 6, 7, 1, 9, 4)$
5 : $a_2 \leftarrow a_1 + b_1 + y \pmod{2^w}$	5 : $\Sigma[3] \leftarrow (7, 9, 3, 1, 13, 12, 11, 14, 2, 6, 5, 10, 4, 0, 15, 8)$
6 : $d_2 \leftarrow d_1 \oplus a_2 \ggg r_3$	6 : $\Sigma[4] \leftarrow (9, 0, 5, 7, 2, 4, 10, 15, 14, 1, 11, 12, 6, 8, 3, 13)$
7 : $c_2 \leftarrow c_1 + d_2 \pmod{2^w}$	7 : $\Sigma[5] \leftarrow (2, 12, 6, 10, 0, 11, 8, 3, 4, 13, 7, 5, 15, 14, 1, 9)$
8 : $b_2 \leftarrow b_1 \oplus c_2 \ggg r_4$	8 : $\Sigma[6] \leftarrow (12, 5, 1, 15, 14, 13, 4, 10, 0, 7, 6, 3, 9, 2, 8, 11)$
9 : return a_2, b_2, c_2, d_2	9 : $\Sigma[7] \leftarrow (13, 11, 7, 14, 12, 1, 3, 9, 5, 0, 15, 4, 8, 6, 2, 10)$
	10 : $\Sigma[8] \leftarrow (6, 15, 14, 9, 11, 3, 0, 8, 12, 2, 13, 7, 1, 4, 10, 5)$
	11 : $\Sigma[9] \leftarrow (10, 2, 8, 4, 7, 6, 1, 5, 15, 11, 9, 14, 3, 12, 13, 0)$
	12 : return Σ

Figure 4.1: G primitive [MJS15, Section 3.1]

Figure 4.2: Blake2 permutation table [MJS15, Section 2.7]

1937 For Blake2s, we have $w = 32$, $r_1 = 16$, $r_2 = 12$, $r_3 = 3$ and $r_4 = 7$.

1938 Blake2s compression function circuit

The compression function is defined as follows, for more details see Fig. 4.3,

$$\text{Blake2sC} : \mathbb{B}^n \times \mathbb{B}^{2n} \times \mathbb{B}^{n/4} \times \mathbb{B}^{n/4} \rightarrow \mathbb{B}^n.$$

1939 Blake2C takes as input a state $h \in \mathbb{B}^n$ which is used as chaining value when hashing,
1940 a message to compress $x \in \mathbb{B}^{2n}$, a message length written in binary $t \in \mathbb{B}^{n/4}$ which is
1941 incremented when hashing and a binary flag $f \in \mathbb{B}^{n/4}$ to tell whether the current block
1942 is the last to be compressed to prevent length extension attacks.

1943 Blake2C uses the G function iteratively over **rounds** number of rounds on a state
1944 and message. The constant initialization vector IV and the permutation table Σ are
1945 hard-coded. Blake2sC works in little endian (see [MJS15, Section 2.4]) on n -bit long
1946 variables ($n = 256$), w -bit long words ($w = 32$), and the rotation constants specified
1947 in Section 4.3.6 (see [MJS15, Section 2.1]). We have the following constants (see speci-
1948 fications [ANWOW13] and [MJS15, Section 2.2]),

- 1949 • IV is the $(8 \cdot w)$ -bit long initialization vector; it corresponds to the first w bits of the
1950 fractional parts of the square roots of the first eight prime numbers $(2, 3, 5, 7, \dots)$
1951 (see [MJS15, Section 2.6]);
- 1952 • Σ are the $10 \cdot 16$ permutation constants of Blake2 (see Fig. 4.2 and [MJS15, Section
1953 2.7]);
- 1954 • **rounds**, the number of rounds: 10 for Blake2sC, 12 for Blake2bC.

1955 We have the following variables (see specifications [ANWOW13] and [MJS15, Section
1956 2.2]),

- 1957 • H is the $(8 \cdot w)$ -bit long initial state while v is the $(16 \cdot w)$ -bit long final state;
- 1958 • $T[i]$ are two w -bit long counters encoding the block length;
- 1959 • $F[i]$ are two w -bit long finalization flags. We set the first one $F[0]$ to $2^w - 1$ to state
1960 when the input block is the last one to be hashed. The second, $F[1] = 0$ is only set
1961 for tree hashing mode (which is not our case) and is therefore unused.

1962 We introduce the following functions to write Blake2C (see specifications [ANWOW13]
1963 and [MJS15, Section 2.6]):

- 1964 • The function **prime** takes a positive integer i as input and outputs the i -th prime
1965 number;
- 1966 • The function **dec** takes a real number x as input outputs its positive decimal part.

1967 This circuit presents $((64 \cdot \mathbf{rounds} + 8) \cdot w + 8 \cdot \mathbf{rounds} + 10)$ constraints. For Blake2sC,
1968 as $w = 32$ and $\mathbf{rounds} = 10$, we have 21536 constraints.

1969 We do not check the input block booleaness in this circuit. Given that the initial
1970 state is boolean, the output is automatically boolean. This can be proved iteratively by
1971 the booleaness of G primitive's output.

1972 **Security requirement.** The inputs to Blake2sC MUST be boolean.

1973 Blake2s hash function

The hash function is defined as follows, for more details see Fig. 4.3,

$$\text{Blake2s} : \mathbb{B}^{\leq 2n} \times \mathbb{B}^* \rightarrow \mathbb{B}^n$$

1974 Blake2 takes as input a hash key $k \in \mathbb{B}^n$ and the message to hash $x \in \mathbb{B}^{2n}$. Blake2
1975 uses the Blake2C function iteratively over each $2n$ -bit long chunk of the padded message.
1976 If the key is non null, it is used as the first block to be hashed. The constant initialization
1977 vector **IV** and part of the parameter block **PB** are hard-coded. We have the following
1978 constants (see specifications [ANWOW13] and [MJS15, Section 2.2]),

- 1979 • **IV** is the $(8 \cdot w)$ -bit long Initialization Vector; it corresponds to the first w bits of the
1980 fractional parts of the square roots of the first eight prime numbers $(2, 3, 5, 7, \dots)$
1981 (see [MJS15, Section 2.6]).

1982 We have the following variables (see specifications [ANWOW13] and [MJS15, Section
1983 2.2]),

Blake2C(h, m, t, f)

```

1 :  $\mathbf{T}, \mathbf{F}, \mathbf{H}, \mathbf{IV}, v \in (\mathbb{B}^w)^2 \times (\mathbb{B}^w)^2 \times (\mathbb{B}^w)^8 \times (\mathbb{B}^w)^8 \times (\mathbb{B}^w)^{16}$ 
2 :  $\{\mathbf{IV}[i]\}_{i \in [8]} \leftarrow \left\{ \left\lfloor 2^w \cdot \text{dec}(\sqrt{\text{prime}(i+1)}) \right\rfloor \right\}_{i \in [8]}$ 
3 :  $\Sigma \leftarrow \text{getSigma}()$ 
4 :  $\{\mathbf{H}[i]\}_{i \in [8]} \leftarrow \{h[i \cdot w : (i+1) \cdot w]\}_{i \in [8]}$ 
5 :  $\{m[i]\}_{i \in [8]} \leftarrow \{x[i \cdot w : (i+1) \cdot w]\}_{i \in [8]}$ 
6 :  $\mathbf{T}[0], \mathbf{T}[1] \leftarrow t[w:2w], t[0:w]$ 
7 :  $\mathbf{F}[0], \mathbf{F}[1] \leftarrow f[w:2w], f[0:w]$ 
8 :  $\{v[i]\}_{i \in [8]} \leftarrow \{\mathbf{H}[i]\}_{i \in [8]}$ 
9 :  $\{v[i+8]\}_{i \in [8]} \leftarrow \{\mathbf{IV}[i]\}_{i \in [8]}$ 
10 :  $v[12], v[13] \leftarrow v[12] \oplus \mathbf{T}[0], v[13] \oplus \mathbf{T}[1]$ 
11 :  $v[14], v[15] \leftarrow v[14] \oplus \mathbf{F}[0], v[15] \oplus \mathbf{F}[1]$ 
12 : foreach  $r \in [\text{rounds}]$  do
13 :    $\tau \leftarrow \Sigma[r \pmod{15}]$ 
14 :    $v[0], v[4], v[8], v[12] \leftarrow \mathbf{G}(v[0], v[4], v[8], v[12], m[\tau[0]], m[\tau[1]])$ 
15 :    $v[1], v[5], v[9], v[13] \leftarrow \mathbf{G}(v[1], v[5], v[9], v[13], m[\tau[2]], m[\tau[3]])$ 
16 :    $v[2], v[6], v[10], v[14] \leftarrow \mathbf{G}(v[2], v[6], v[10], v[14], m[\tau[4]], m[\tau[5]])$ 
17 :    $v[3], v[7], v[11], v[15] \leftarrow \mathbf{G}(v[3], v[7], v[11], v[15], m[\tau[6]], m[\tau[7]])$ 
18 :    $v[0], v[5], v[10], v[15] \leftarrow \mathbf{G}(v[0], v[5], v[10], v[15], m[\tau[8]], m[\tau[9]])$ 
19 :    $v[1], v[6], v[11], v[12] \leftarrow \mathbf{G}(v[1], v[6], v[11], v[12], m[\tau[10]], m[\tau[11]])$ 
20 :    $v[2], v[7], v[8], v[13] \leftarrow \mathbf{G}(v[2], v[7], v[8], v[13], m[\tau[12]], m[\tau[13]])$ 
21 :    $v[3], v[4], v[9], v[14] \leftarrow \mathbf{G}(v[3], v[4], v[9], v[14], m[\tau[14]], m[\tau[15]])$ 
22 : return  $\|_{i=0}^8 \mathbf{H}[i] \oplus v[i] \oplus v[i+8]$ 

```

Figure 4.3: Blake2 compression function [MJS15, Section 3.2]. Set n , w and \mathbf{G} 's constants to obtain Blake2sC.

- 1984 • PB is the $(16 \cdot w)$ -bit long parameter block used to initialize the state (see [MJS15,
1985 Section 2.5]). In big endian encoding, the first byte corresponds to the digest
1986 length (fixed to 32 bytes), the second byte to the key length, the third and fourth
1987 bytes correspond to the use of the serial mode;
- 1988 • $\mathbf{H} \in \mathbb{B}^{\text{BLAKE2sCLEN}}$, the chaining value.

1989 We do not check the input block booleaness in this circuit. Given that the initial
1990 state is boolean, the output is automatically boolean. This can be proved iteratively by
1991 the booleaness of Blake2C primitive's output.

1992 **Security requirement** To ensure the correct use of Blake2s, Blake2s's inputs MUST be
1993 boolean.

Blake2(k, x)

```

1 :  H, IV, PB  $\in \mathbb{B}^{8w} \times \mathbb{B}^{8w} \times \mathbb{B}^{8w}$ 
2 :  PB  $\leftarrow \text{pad}_{8 \cdot w}(\text{encode}_{\mathbb{N}}(0x0101)) \parallel \text{pad}_w(\text{encode}_{\mathbb{N}}(\lceil \text{length}(k)/\text{BYTELEN} \rceil)) \parallel \text{encode}_{\mathbb{N}}(0x20)$ 
3 :  IV  $\leftarrow \parallel_{i=0}^8 \left[ 2^w \cdot \text{dec}(\sqrt{\text{prime}(i+1)}) \right]$ 
4 :  H  $\leftarrow \text{PB} \oplus \text{IV}$ 
5 :  y  $\leftarrow x$ 
6 :  if length( $k$ )  $\neq 0$  do
7 :    y  $\leftarrow \text{pad}_{2n}(k) \parallel y$ 
8 :    z  $\leftarrow \text{pad}_{2n \cdot \lceil \text{length}(y)/2n \rceil}(y)$ 
9 :    for  $i \in [\lceil \text{length}(z)/2n \rceil]$  do
10 :      if  $i = \lceil \text{length}(z)/2n \rceil - 1$  do
11 :        H  $\leftarrow \text{Blake2C}(H, z[i \cdot 2n:(i+1) \cdot 2n], \text{pad}_{2w}(\text{encode}_{\mathbb{N}}(\lceil \text{length}(y)/\text{BYTELEN} \rceil)), \text{pad}_{2w}(\text{encode}_{\mathbb{N}}(2^w - 1)))$ 
12 :      else
13 :        H  $\leftarrow \text{Blake2C}(H, z[i \cdot 2n:(i+1) \cdot 2n], \text{pad}_{2w}(\text{encode}_{\mathbb{N}}((i+1) \cdot 2n/\text{BYTELEN})), \text{pad}_{2w}(0))$ 
14 :    return H

```

Figure 4.4: Blake2 hash function [MJS15, Section 3.3]. Set $n = 16w$ and G's constants accordingly to obtain Blake2s.

1994 Optimizing the circuits

1995 The above helper circuits form the building blocks of the Blake2s compression function.
 1996 We show here two exclusive methods to optimize these circuits.

1997 Optimizing the Modular additions

Double modular addition: $a + b = c \pmod{2^w}$. We present here an optimization on the circuit to save one constraint by merging the modular constraint with a boolean constraint. The optimized circuit presents the following constraints:

$$\left(\sum_{i=0}^{w-1} (a_i + b_i - c_i) \cdot 2^i \right) \cdot \left(\sum_{i=0}^{w-1} (a_i + b_i - c_i) \cdot 2^i - 2^w \right) = 0 \quad (4.5)$$

$$\forall j \in \{0, \dots, w-1\}, (c_j - 0) \cdot (c_j - 1) = 0 \quad (4.6)$$

1998 with $\sum_{i=0}^{w-1} x_i \cdot 2^i$ a binary encoding of x (x_i is the i^{th} bit of x).

1999 These equations describe $w + 1$ constraints to prove the bit equality $a + b = c$ (note
 2000 that an additional $2 \cdot w$ constraints would be required to prove the booleanness of input
 2001 variables a and b). We now explain how we obtained them.

2002 *Proof.* The most straightforward way to prove that $a + b = c \pmod{2^w}$ and c booleanness
 2003 is with the set of constraints illustrated in Eq. (4.1) and in Eq. (4.2).

As we perform arithmetic modulo 2^w , we do not care about the value of c_w but would like to ensure its booleanness. As one may notice, the summing constraint Eq. (4.1) is an equality of two linear combinations with no multiplication by a variable. Hence, we can combine it with the boolean constraint of c_w to remove any reference to c_w and still have a bilinear gate. To do so, we first rewrite Eq. (4.1) as an equality check over $c_w \cdot 2^w$ and multiply Eq. (4.2) for $j = n$ by $2^{2 \cdot w}$.

$$\sum_{i=0}^{w-1} (a_i + b_i - c_i) \cdot 2^i = c_w \cdot 2^w \quad (4.7)$$

$$2^w \cdot (c_w - 0) \cdot 2^w \cdot (c_w - 1) = 0 \quad (4.8)$$

We finally replace $c_w \cdot 2^w$ in Eq. (4.8) by the value from Eq. (4.7).

$$\begin{aligned} 0 &= 2^w \cdot (c_w - 0) \cdot 2^w \cdot (c_w - 1) = 2^w \cdot c_w \cdot (2^w \cdot c_w - 2^w) \\ &= \left(\sum_{i=0}^{w-1} (a_i + b_i - c_i) \cdot 2^i \right) \cdot \left(\left(\sum_{i=0}^{w-1} (a_i + b_i - c_i) \cdot 2^i \right) - 2^w \right) \end{aligned}$$

2004 This results in Eq. (4.5) and Eq. (4.6). All references to c_w have disappeared and, with
2005 a single multiplication by a variable, we still have bilinear gates. \square

2006 **Triple modular addition: $a + b + c = d \pmod{2^w}$.** To optimize, we use the
2007 above circuit twice. We define a temporary variable d' such that $a + b = d' \pmod{2^w}$.
2008 As such, we have $c + d' = d \pmod{2^w}$. As d' is the addition of two w -bit long variables,
2009 it is $(w+1)$ -bit long. However as we evaluate the sum modulo 2^w , we discard the last bit
2010 of d' . We proceed similarly for d . To ensure that d is boolean, we check the booleanness
2011 of the $w+1$ bits of d as well as the booleanness of the last bit of d' (to account for d 's
2012 $w+2^{th}$ bit in the original expression $(a + b + c = d \pmod{2^w})$).

We thus obtain the following circuit with $w+2$ constraints,

$$\begin{aligned} \left(\sum_{i=0}^{w-1} (a_i + b_i - d'_i) \cdot 2^i \right) \cdot \left(\sum_{i=0}^{w-1} (a_i + b_i - d'_i) \cdot 2^i - 2^w \right) &= 0 \\ \left(\sum_{i=0}^{w-1} (c_i + d'_i - d_i) \cdot 2^i \right) \cdot \left(\sum_{i=0}^{w-1} (c_i + d'_i - d_i) \cdot 2^i - 2^w \right) &= 0 \\ \forall j \in \{0, \dots, w-1\}, (d_j - 0) \cdot (d_j - 1) &= 0 \end{aligned}$$

2013 These optimizations lead to a gain of 320 constraints ($= 4 \cdot 8 \cdot \text{rounds}$).

2014 **Optimizing Blake2s routine's circuit** As seen in Fig. 4.1, our routine presents 2
2015 double and 2 triple modular additions. Each of these circuits comprises at least one
2016 modular constraint which pack several w -bit long variables. The circuit is however
2017 processed in \mathbb{F}_{CUR} , that is to say most integers can be written over **FIELD** bits. We

can thus batch the modular constraints. As the G primitive performs 2 double modular and 2 triple modular, we have in total 6 modular checks per iteration. We can batch up to $\text{FIELD CAP}/w$ constraints together. For $w = 32$ and $\text{FIELD CAP} \geq 224$ (which holds for BN-254 and BLS12-377), we can encode up to 7 words per field element, that is to say we can include all the modular constraints into a single one.

This optimization leads to a gain of 274 constraints ($= 4 \cdot 8 \cdot 10 - \lceil \frac{4 \cdot 8 \cdot 10}{7} \rceil$).

Optimization conclusion Using the more efficient optimization on the modular additions, the Blake2s compression function comprises 21216 constraints.

Increasing the PRF security with Blake

As Blake2 comprises a personalization tag in its parameter block PB , one could ensure the independence of the PRFs by writing different tags for each of them (we would be able to consider up to 2^{30} inputs and outputs). We did not choose to write this enhancement in the instantiation to keep a general tagging method in case of a change of hash function.

4.4 Encryption of the notes

In this section we give some remarks concerning the implementation of the Zeth encryption scheme, described in Section 3.5. As noted, there are several details in the specification of the underlying primitives which can impact security if not carefully implemented. The following list is by no means exhaustive but includes several details noted during development of the proof-of-concept system.

- Private keys for Curve25519 **MUST** be randomly generated as 32 bytes where the first byte is a multiple of 8, and the last byte takes a value between 64 and 127 (inclusive). Further details are given in [Ber06], including an example algorithm for generation. Implementations **MUST** take care to ensure that their code, or any external libraries they rely upon, correctly perform this step.
- A similar observation holds for Poly1305 in which the r component of the MAC key (r, s) **MUST** be *clamped* in a specific way (see Section 3.5.3). This step is also essential and **MUST** be performed.
- In the implementation of the ChaCha stream cipher, correct use of the *key*, *counter* and *nonce* **MUST** be ensured in order to adhere to the standard and guarantee the appropriate security properties.

During the proof-of-concept implementation it was not obvious that the cryptography library⁴ adhered to the specifications with respect to the above points. In particular, it was not clear whether key clamping was performed at generation time and/or when

⁴<https://cryptography.io/en/latest/>

2051 performing operations. Moreover, the interface to the **ChaCha** cipher accepted a differ-
2052 ent set of input parameters (namely *key* and *nonce* with no *counter*). This left some
2053 ambiguity about the responsibility for clamping, and whether the **ChaCha** block data
2054 would be updated as described in the specification. Details of how this was resolved are
2055 given in the proof-of-concept encryption code, which may prove a useful reference for
2056 implementers⁵.

⁵see <https://github.com/clearmatics/zeth/blob/v0.4/client/zeth/encryption.py>

Appendix A

Transaction non malleability

The transaction malleability problem for a DAP (Section 1.4) is characterized by a game TR-NM involving a polynomial-time adversary \mathcal{A} as described below.

Definition A.0.1. Let DAP be a (candidate) Decentralized Anonymous Payment scheme.

$$\text{DAP} = (\text{Setup}, \text{GenAddr}, \text{SendTx}, \text{VerifyTx}, \text{Receive})$$

We say that DAP is TR-NM secure if, for every $\text{poly}(\lambda)$ -time adversary \mathcal{A}

$$\text{Adv}_{\text{DAP}, \mathcal{A}}^{\text{tr-nm}}(\lambda) < \text{negl}(\lambda),$$

where $\text{Adv}_{\text{DAP}, \mathcal{A}}^{\text{tr-nm}}(\lambda) = \Pr[\text{TR-NM}(\text{DAP}, \mathcal{A}, \lambda) = 1]$ is \mathcal{A} 's advantage in the TR-NM experiment.

Below, we adapt [BSCG⁺14, Appendix C.2] to our specific DAP—Zeth.

We start by describing the TR-NM experiment. Given a (candidate) Zeth DAP, adversary \mathcal{A} , and security parameter λ , the (probabilistic) game $\text{TR-NM}(\text{DAP}, \mathcal{A}, \lambda)$ consists of an interaction between \mathcal{A} and a challenger \mathcal{C} , terminating with a binary output by \mathcal{C} .

At the beginning of the game, \mathcal{C} samples $pp \leftarrow \text{Setup}(\lambda)$ and sends pp to \mathcal{A} . Next, \mathcal{C} initializes a DAP oracle O^{DAP} with pp and allows \mathcal{A} to issue queries to it [RZ19, Appendix B].

At the end of the experiment, \mathcal{A} sends to \mathcal{C} a **Mixer** contract call transaction tx_{Mix}^* , and \mathcal{C} outputs 1 iff the following conditions hold. Letting T be the set of transactions generated by O^{DAP} in response to **SendTx** queries, there exists $tx_{\text{Mix}} \in T$ such that:

1. tx_{Mix} was not inserted in L by \mathcal{A} ;
2. $tx_{\text{Mix}}^*.data \neq tx_{\text{Mix}}.data$;
3. $\text{VerifyTx}(pp, tx_{\text{Mix}}^*, L') = 1$ where L' is the portion of the ledger L preceding tx_{Mix} ;
4. a serial number revealed in tx_{Mix}^* is also revealed in tx_{Mix} .

A.1 Transaction malleability attack on Zeth

In this section we present the threat related to the transaction malleability attack on Zeth and expose the solutions by ZeroCash [BSCG⁺14] and Zcash [ZCa19] that we adapted.

First, we start by assuming that none of the checks related to transaction malleability attack have been added in the protocol Chapter 2. As such, we assume that *hsig* and *htags* are not attributes of `PrimInputDType`, ϕ is not an attribute of `AuxInputDType`, and *otssig* and *otsvk* are not attributes of the `MixInputDType` data type anymore. As a consequence, all checks related to these attributes are removed from the protocol. Moreover, if *zn* is an object of type `ZethNoteDType`, then *zn*. ρ is chosen at random. Finally, the NP-relation used in Zeth, now denoted \mathbf{R}^{mal} , becomes the following:

- For each $i \in [\text{JSIN}]$:

1. $\text{aux.jsins}[i].\text{znote.apk} = \text{Blake2s}(\text{tag}_{\text{ask}}^{\text{addr}} \parallel \text{pad}_{\text{BLAKE2sCLEN}}(0))$
with $\text{tag}_{\text{ask}}^{\text{addr}}$ defined in Section 3.1.3
2. $\text{aux.jsins}[i].\text{nf} = \text{Blake2s}(\text{tag}_{\text{ask}}^{\text{nf}} \parallel \text{aux.jsins}[i].\text{znote}.\rho)$
with $\text{tag}_{\text{ask}}^{\text{nf}}$ defined in Section 3.1.3
3. $\text{aux.jsins}[i].\text{cm} = \text{Blake2s}(\text{aux.jsins}[i].\text{znote}.r \parallel m)$
with $m = \text{aux.jsins}[i].\text{znote.apk} \parallel \text{aux.jsins}[i].\text{znote}.\rho \parallel \text{aux.jsins}[i].\text{znote}.v$
4. $(\text{aux.jsins}[i].\text{znote}.v) \cdot (1 - e) = 0$ is satisfied for the boolean value e set such that if $\text{aux.jsins}[i].\text{znote}.v > 0$ then $e = 1$.
5. The Merkle root mkroot' obtained after checking the Merkle authentication path $\text{aux.jsins}[i].\text{mkpath}$ of commitment $\text{aux.jsins}[i].\text{cm}$, with MKHASH, is equal to prim.mkroot if $e = 1$.
6. $\text{prim.nfs}[i]$
 $= \{ \text{Pack}_{\mathbb{F}_{\text{rCUR}}}(\text{aux.jsins}[i].\text{nf}[k \cdot \text{FIELD CAP}:(k+1) \cdot \text{FIELD CAP}]) \}_{k \in [\lfloor \text{PRNFOUTLEN}/\text{FIELD CAP} \rfloor]}$

- For each $j \in [\text{JSOUT}]$:

1. $\text{prim.cms}[j] = \text{Blake2s}(\text{aux.znotes}[j].r \parallel m)$
with $m = \text{aux.znotes}[j].\text{apk} \parallel \text{aux.znotes}[j].\rho \parallel \text{aux.znotes}[j].v$

- $\text{prim.rsd} = \text{Pack}_{\text{rsd}}(\{ \text{aux.jsins}[i].\text{nf} \}_{i \in [\text{JSIN}]}, \text{aux.vin}, \text{aux.vout})$
- Check that the “joinsplit is balanced”, i.e. check that the joinsplit equation holds:

$$\begin{aligned} & \text{Pack}_{\mathbb{F}_{\text{rCUR}}}(\text{aux.vin}) + \sum_{i \in [\text{JSIN}]} \text{Pack}_{\mathbb{F}_{\text{rCUR}}}(\text{aux.jsins}[i].\text{znote}.v) \\ &= \sum_{j \in [\text{JSOUT}]} \text{Pack}_{\mathbb{F}_{\text{rCUR}}}(\text{aux.znotes}[j].v) + \text{Pack}_{\mathbb{F}_{\text{rCUR}}}(\text{aux.vout}) \end{aligned}$$

2107 A.1.1 The attack

2108 In order to win the game TR-NM on the weak **Zeth** DAP above, an adversary \mathcal{A} inter-
 2109 cepts a target transaction tx_{Mix} by passively listening to the network (remember that
 2110 transactions are broadcasted to the **Ethereum** network in order to be mined, see Sec-
 2111 tion 1.2.2), extracts the zk-proof and primary inputs from $tx_{\text{Mix}}.data$ and uses these
 2112 extracted pieces of information in order to create a malicious transaction tx_{Mix}' , where
 2113 the ciphertexts are replaced by arbitrary data. The adversary can then broadcast tx_{Mix}'
 2114 to the network in order for it to be mined. If the malicious transaction gets mined before
 2115 the legitimate one, the input notes become spent and the ciphertexts are undecryptable
 2116 making the new notes unredeemable (by any **Zeth** user!), since all attempts to decrypt
 2117 the ciphertexts will fail (see Section 2.6).

```

TxMalGen( $sk'_{\text{ECDSA}}, nce_{in}, tx_{\text{Mix}}$ )
1 :  $p \leftarrow tx_{\text{Mix}}.gasP + 1$ 
2 :  $l \leftarrow tx_{\text{Mix}}.gasL + 1$ 
3 :  $zdata' \leftarrow tx_{\text{Mix}}.data$ 
4 :  $zdata'.ciphers \leftarrow \mathbb{B}^*$ 
5 :  $tx_{raw} \leftarrow \{nce : nce_{in}, gasP : p, gasL : l, to : tx_{\text{Mix}}.to, val : tx_{\text{Mix}}.val, data : zdata'\};$ 
6 :  $\sigma_{\text{ECDSA}} \leftarrow \text{SigSch}_{\text{ECDSA}}.\text{Sig}(sk'_{\text{ECDSA}}, \text{Keccak256}(tx_{raw}));$ 
7 :  $tx_{final} \leftarrow \{tx_{raw}, v : \sigma_{\text{ECDSA}}.v', r : \sigma_{\text{ECDSA}}.r', s : \sigma_{\text{ECDSA}}.s'\};$ 
8 : return  $tx_{final}$ ;

```

Figure A.1: Transaction malleability attack function TxMalGen

2118 As shown on Fig. A.1, during the attack, the adversary extracts the proof and pri-
 2119 mary inputs from the honest transaction, and replaces the ciphertexts by some arbitrary
 2120 information. The attacker then formats this data into a transaction that calls the Mix
 2121 function of **Mixer**, and submits it to the network. While the data fields ($tx_{\text{Mix}}.data$
 2122 and $tx_{\text{Mix}}'.data$) are different, the nullifiers revealed by both transactions are the same
 2123 (i.e. $tx_{\text{Mix}}.data.proof = tx_{\text{Mix}}'.data.proof$, and $tx_{\text{Mix}}.data.prim = tx_{\text{Mix}}'.data.prim$).
 2124 As a consequence, if the adversary makes sure that tx_{Mix}' satisfies all the checks of
 2125 **EthVerifyTx** (Section 1.2.2), he can ensure that **ZethVerifyTx**(tx_{Mix}') will return the same
 2126 value as **ZethVerifyTx**(tx_{Mix}). Furthermore, if $tx_{\text{Mix}}'.gasP > tx_{\text{Mix}}.gasP$, then the adver-
 2127 sary maximizes his chances of having his transaction mined first (Section 1.2.2), and so
 2128 maximizes the chances for the malleability attack to be successful; leading to lost funds
 2129 on **Mixer**.

2130 **Remark A.1.1.** Note that, although not directly contained within the *data* field of a
 2131 **Mixer** call transaction, the **Ethereum** address $\mathcal{S}_{\mathcal{E}}.Addr$ of the transaction sender is also
 2132 used by the **Mixer** call (this is either the calling contract's address, or the transaction
 2133 signer's address recovered as described in Remark 1.2.1). In particular, the balance
 2134 of this **Ethereum** address is incremented by the value *vout* by successful Mix calls. If

we again assume the absence of the malleability checks, an attacker could re-sign any **Mixer** call transaction with a key under his control, rebroadcast it as described above, and (with some reasonable probability) become the recipient of any public output value $vout$.

Remark A.1.2. We note that the attack described above cannot be prevented by merely substituting a malleable Groth16 zk-SNARK by a simulation-extractable one like e.g. [GM17]. This comes since the attack does not utilise malleability of the proof system, but malleability of data that are broadcasted along with the zk-proof.

A.2 Solutions to address the transaction malleability attack

A.2.1 ZeroCash solution

The idea of the solution presented in [BSCG⁺14] is to use a one-time SUF-CMA digital signature and bind its verification key with the zk-proof primary inputs to prevent an adversary from corrupting part of a transaction's data.

Specifically, to transact via **Zeth**, the user first samples a key pair (sk, vk) for a one-time signature scheme. He then computes the hash $hsig = CRH(vk)$, where CRH is a collision resistant hash function, see [BSCG⁺14], and derives a value $h_i = PRF_{ask_i}^{pk}(hsig)$, for each input note (i.e. $i \in [JSIN]$), which acts as a MAC binding $hsig$ to the address spending key of a note (ask_i) .

The user then generates the zk-proof with the additional statement that the values $\{h_i\}_{i \in [JSIN]}$ are computed correctly. He finally uses sk to sign every value associated with the operation, thus obtaining a signature, which is included, along with the signature verification key vk , in the transaction. To verify a transaction on the DAP, it is necessary to verify that

- the primary inputs are correctly formatted,
- the Merkle root corresponds to one of the previous states of the Merkle tree,
- the nullifiers have not been declared in a previous transaction,
- the $hsig$ is correctly computed from vk , and
- both the zk-proof and the one-time signature verifications pass successfully.

Now, an adversary trying to carry out the aforementioned attack has to either change the ciphertexts or the encryption key. Nevertheless, doing so should lead to the one-time signature verification to fail or should yield an attack that breaks the UF-CMA property of the one-time signature (as this corresponds to creating a forgery on a different message, not changing the signature). Thereby, the adversary also has to modify the signature, however he does not know the one-time signing key used by the creator of the targeted transaction. As such, the adversary needs to use another signing key pair, however

2171 this leads to the check verifying that *hsig* is correctly computed to fail. If the adversary
 2172 attempts to change *hsig*, the zk-proof verification fails as the NP-statement has changed.
 2173 Hence, any attempt to carry out a malleability attack results in the violation of at least
 2174 one check in the verification of the transaction on the DAP. The solution presented
 2175 effectively solves the transaction-malleability attack initially described.

2176 **Remark A.2.1.** The one-timeness property of the signature scheme was required in
 2177 **ZeroCash** to retain anonymity. It also makes analysing non-adaptive adversary sufficient.
 2178 As **Ethereum** transaction senders need to pay the gas cost associated with their trans-
 2179 actions, the senders are not anonymous. This said, making sure that **Zeth** is designed
 2180 with anonymity in mind is worth the effort in order to minimize information leakages
 2181 and be ready if/when **Ethereum** incorporates protocol changes that enable anonymous
 2182 transactions.

2183 A.2.2 Zcash’s solution

2184 In addition to the changes aforementioned, **Zcash**’s solution [ZCa19] also consists of:

- Redefining the variable *hsig* as,

$$hsig = \text{CRH}(\text{randomSeed}, \{nf_i\}_{i \in [\text{JSIN}]}, vk)$$

2185 for some random seed *randomSeed*.

- Defining a new random variable ϕ and using it with *hsig*, as key and input of a
 2186 PRF respectively, to compute the identifier of each output notes ρ_j ($j \in [\text{JSOUT}]$)
 2187 and ensure their uniqueness (with overwhelming probability).
 2188

2189 These changes were made to prevent the Faerie Gold attack [ZCa19, Section 8.4], as well
 2190 as to prevent linkability: if *hsig* were repeated in two transactions, the circuit would
 2191 leak, via $\{h_i\}_{i \in [\text{JSIN}]}$, the fact that the input notes in both transactions were spent with
 2192 the same *ask_i* (if that were the case).

2193 More particularly, using the input notes’ nullifiers to derive *hsig* ensures that *hsig* is
 2194 unique with overwhelming probability for all *accepted* transaction. Furthermore, us-
 2195 ing *randomSeed* ensures the uniqueness of *hsig* for transactions *in transit* (as before
 2196 validation there may be several in transit transactions with the same set of nullifiers).

2197 A.2.3 Solution on Ethereum

2198 As described in the **Ethereum** yellow paper [Woo19, Appendix F], **Ethereum** transactions
 2199 are ECDSA signed. Further, as described in Section 2.3, the one-time signature used to
 2200 sign the Mix data also signs the **Ethereum** address used to sign the transaction. As such,
 2201 any modification to the transaction object will result in a new transaction hash, and
 2202 any attempt to sign the transaction with a different ECDSA key will be rejected by the
 2203 **Mixer** contract (see Section 2.5). We thereby conclude that the one-time signature used

2204 to sign the transaction data does not need to be **SUF-CMA**, but *only needs to achieve*
 2205 **UF-CMA**.

2206 Specifically, carrying out any change on the one-time signature will change the
 2207 **Ethereum** transaction data and result in a failure to verify the **ECDSA** signature of
 2208 the **Ethereum** transaction. To obtain a new valid signature on this transaction, the
 2209 adversary needs to break the **UF-CMA** property of the **ECDSA** signature scheme or use
 2210 another **ECDSA** keypair to sign the transaction. In the last case, the one-time signature
 2211 will no longer be valid.

2212 Note that including the **Ethereum** transaction sender in the data to be signed by the
 2213 one-time signature scheme also addresses the possible attack described in Remark A.1.1.
 2214 An attacker trying to resign the same **Ethereum** transaction with a different key will
 2215 cause **Mixer** to reject the transaction when the one-time signature is checked.

Remark A.2.2. We note that the transaction malleability issue can also be addressed
 in another way. In fact, one could use the **ECDSA** signatures on **Ethereum** transactions
 to fix all inputs and ciphertexts, and then tie the sender of the **Ethereum** transaction to
 the zk-snark by putting the sender address $\mathcal{S}_{\mathcal{E}}.Addr$ in $hsig$. In other words, it is also
 possible to define $hsig$ as:

$$hsig = \text{CRH}(\{nf_i\}_{i \in [\text{JSIN}]}, \mathcal{S}_{\mathcal{E}}.Addr)$$

2216 As such, if an attacker extracts the ciphertexts of a tx_{Mix} transaction in order to craft
 2217 another malicious transaction tx_{Mix}' , the key-pair used to sign tx_{Mix}' differs from the one
 2218 used to sign tx_{Mix} , which changes the transaction sender address recovered on **Mixer**. As
 2219 a consequence, the check on $hsig$ would fail on the **Mixer**, invalidating the transaction,
 2220 and preventing the attack.

2221 While such a solution would avoid the need to generate one-time signing keys and
 2222 could avoid a signature check in the **Mixer**, it would also require every **Zeth** user to
 2223 have an **Ethereum** account. Doing so, would be a major hindrance toward the design of
 2224 mechanisms aiming to provide anonymity to **Zeth** transactions initiators. In fact, the
 2225 addressing scheme used in **Zeth** along with the solution to the malleability introduced in
 2226 **Zcash** makes it possible to generate raw **Zeth** transactions without having an **Ethereum**
 2227 account. These raw transactions could then be broadcasted – to a set of **Ethereum** user
 2228 nodes – on an anonymous p2p network, before being finalized and submitted to the
 2229 **Ethereum** network by **Ethereum** users who would be rewarded according to an incentive
 2230 structure. While such a protocol is outside of the scope of this document, it shows that
 2231 defining $hsig$ using the senders address alters the flexibility of **Zeth**; hence this solution
 2232 has not been favoured.

Appendix B

Double spend attack on equivalent class

The primary inputs of our zk-SNARK are elements of $\mathbb{F}_{r_{\text{CUR}}}$ and they can be written over FIELDLEN bits. Note that the projection of $\mathbb{B}^{\text{FIELDLEN}}$ onto $\mathbb{F}_{r_{\text{CUR}}}$ formed by interpreting elements in $\mathbb{B}^{\text{FIELDLEN}}$ as FIELDLEN -bit numbers and reducing modulo r_{CUR} , is surjective.

When we pass the primary inputs to the **Mixer** contract, they are interpreted as elements of $\mathbb{B}^{\text{ETHWORDLEN}}$, and $\mathbb{B}^{\text{FIELDLEN}} \subset \mathbb{B}^{\text{ETHWORDLEN}}$. As previously noted, this means that there exist pairs of elements in $\mathbb{B}^{\text{ETHWORDLEN}}$ with the same projection in $\mathbb{F}_{r_{\text{CUR}}}$. An adversary could make use of this to perform a double spend attack.

Indeed, to check that a note is not double spent, the contract stores the nullifiers of spent notes (as elements of $\mathbb{B}^{\text{ETHWORDLEN}}$) and verifies that the nullifier of the note to be spent is not stored. The adversary could thus modify the nullifier to a different value with the same projection. As the SNARK verification operates in $\mathbb{F}_{r_{\text{CUR}}}$, the proof would still be valid. However, the value stored for this nullifier would be different from the adversarial one. Hence, the nullifier would be validated, the transaction would succeed and the note would be double spent. In practice, the adversary can perform the attack by simply adding r_{CUR} to one of the elements representing the nullifier.

To prevent this attack, the contract checks that all primary inputs are elements of $\mathbb{F}_{r_{\text{CUR}}}$, that is to say that they are smaller than r_{CUR} . As one may see, the attack described above is not due to the packing of hash digests into field elements but to the contract storage of field elements as **Ethereum** words.

2255 Appendix C

2256 Side-channel attacks and 2257 information leaks

2258 The following subsections describe several side-channel attacks and possible weaknesses
2259 that implementers should be aware of and attempt to mitigate.

2260 We consider cases in which the attacker is able to observe the RPC communications
2261 between **Zeth** client software, and Ethereum P2P nodes. This situation may occur if an
2262 observer is able to monitor the network traffic between the Ethereum node and the **Zeth**
2263 client software, or if the Ethereum node itself is compromised.

Note

In this discussion, we do not consider adversaries with physical access to the machine running the client software. Such adversaries could make precise measurements of timing, power-consumption or other physical quantities that could reveal fine-grained details of the operations being carried out by the software, or the data it is operating on. Protecting against attacks of this kind often involves implementation techniques such as: avoiding branches based on private data, being careful with memory access patterns, and making all operations constant time, to only name a few. We leave consideration of these attacks and prevention methods for a future discussion.

2264

2265 C.1 Counterfeit data

2266 Malicious Ethereum nodes or attackers able to compromise the network have the oppor-
2267 tunity to send invalid data to RPC clients. This could be used to inject invalid data
2268 into the client's record of state, which could prevent it from generating valid **Mix** calls
2269 or allow it to be identified in the future. In general, data from any remote host should
2270 be treated as malicious, unless accompanied by evidence that convinces the client of its
2271 authenticity.

2272 In the case of Ethereum event logs (the main source of data used to track the on-
2273 chain state – see Section 4.1.1 for details), clients **MUST** leverage the consensus evidence
2274 and block headers to verify that log data is genuine and has been committed to the
2275 blockchain. See Section 1.2.3 for further information about how such data is secured.

2276 C.2 Data leaked during synchronization

2277 In order to receive private payments and keep their local data up-to-date, **Zeth** client
2278 software **MUST** scan the blockchain and process *all* the event data emitted by **Mixer**
2279 during Mix calls (as described in Section 4.1.1). There are several issues to consider
2280 when determining exactly how and when this “synchronization” takes place.

2281 Client implementations that only connect to the RPC endpoint in response to user
2282 input, or in preparation for performing a Mix call, may leak information. Observers may
2283 deduce that such client are likely to be the recipient of a recent or upcoming transaction,
2284 or that they may be about to perform a Mix call.

2285 Similarly, payment provider software that only listens for events when awaiting a
2286 transaction, and remains disconnected otherwise, may reveal that it is the recipient of
2287 an upcoming transaction, and possibly *which* transaction or block it was paid by (based
2288 on when it stops listening).

2289 Further, consider wallet software that performs RPC operations to explicitly wait
2290 for the Ethereum transaction corresponding to a specific Mix call. This would most
2291 likely be for transactions emitted by the **Zeth** client, in order to inform the user and
2292 update the wallet state once the payment is complete (but could possibly happen on
2293 the receiver side, if he somehow knows the ID of the transaction of interest – e.g. via
2294 off-chain communication with the sender). If such a *wait* procedure is implemented by
2295 querying the status of a specific transaction by its ID, or by listening for blocks *until*
2296 the transaction of interest is received, the connected Ethereum node may infer that this
2297 client is interested in the transaction, and likely to be the sender or recipient.

2298 Consider a client which periodically connects to some Ethereum node and requests
2299 all relevant data from the last block it saw, up to the latest block available. Each client
2300 will have information up to some block n (where n varies per client), and n is known to
2301 the Ethereum node that served the client. The client could then potentially be identified
2302 by n (even if it hides its IP for each connection) since a client that connects and queries
2303 **Zeth** transactions from block $n + 1$ reveals that it is one of the clients who synced up to
2304 block n when it last connected.

2305 Note that, if the client always broadcasts the Mix transaction via this same Ethereum
2306 node, then the Ethereum node may already deduce that the client is the sender. However,
2307 implementations may wish to use techniques (such as sending transactions from other
2308 nodes or hiding their IP address in other way) to obfuscate any relationship between
2309 transactions and the clients that originated them.

C.3 Queries on successful decryption

The event data emitted by $\widetilde{\text{Mixer}}$ contains the note data for new commitments, encrypted using a key derived from the recipients' public key. As described in Section 2.6, clients scan the blockchain for these events and attempt to decrypt the ciphertext using their secret decryption keys. If they are successful, they are the recipient of the note and can try to parse the plaintext to extract the secret note data.

When decryption is successful and the note data has been extracted from the plaintext (we discuss parsing failure in Appendix C.4), clients **MUST** check that this note data does indeed open the commitment for the note.

A naive implementation of this check could query the state of $\widetilde{\text{Mixer}}$ via RPC to check the relevant entry in the set of commitments. However, this would reveal to an observer that the client had successfully decrypted and parsed the corresponding ciphertext, and was therefore the recipient of that note.

For this reason, the protocol specifies that $\widetilde{\text{Mixer}}$ **MUST** emit events informing clients of new commitment values and locations in the Merkle tree. Clients **MUST** consume *all* such data to maintain their view of contract state (as described in Appendix C.2). Further, clients **MUST** attempt to decrypt *all* ciphertexts and, for successful decryptions, **MUST** verify that the plaintext opens the note's commitment. This avoids the need for any extra RPC queries that would reveal which ciphertexts were successfully decrypted.

Note

Emitting events containing all data necessary to carry out the local checks implemented in the wallet is a way to enforce that all wallets behave exactly the same to the eyes of network (passive) adversaries (regardless whether the user is the recipient of a note or not).

C.4 Invalid ciphertext

The attack described in [TBP20, Section 4.2.1] illustrates the importance of correctly handling invalid data in client software. A so-called “REJECT Attack” is described whereby an attacker creates a Mix call with specially crafted ciphertext. The ciphertext can be successfully decrypted by the correct recipient – that is, the plaintext note is encrypted with an encryption key derived from the recipients public key – but the corresponding plaintext is invalid and cannot be parsed correctly by the recipient.

Note

Note that the above is possible because the plaintext is neither verified by the circuit encoding \mathbf{R}^Z , nor by the contract (which is unable to decrypt it). Hence, $\widetilde{\text{Zeth}}$ allows such transactions with malicious ciphertexts to be accepted by the $\widetilde{\text{Mixer}}$ contract, and clients must handle this case with care.

2338 In the case described in [TBP20], there is no distinction between “client” or “wallet”
2339 software, and the underlying P2P nodes. Before a fix was applied (see [zcab]), nodes
2340 explicitly rejected transactions of the above form, proving to their peers that they were
2341 able to decrypt the ciphertext and were therefore the intended recipient.

2342 In **Zeth**, P2P nodes and wallet software are separated, so there will be no explicit
2343 rejection of the transaction. However, careless error handling (such as exceptions which
2344 causes the RPC connection to be closed) could potentially be detected by the connected
2345 Ethereum node. As in the “REJECT Attack”, this reveals that the connected RPC
2346 client is the intended recipient of a transaction, and the owner of the corresponding
2347 encryption key.

2348 C.5 Using (and retrieving) nullifiers

2349 Any non-trivial wallet implementation will need to track which of the user’s **Zeth** notes
2350 have been spent, and which are still available. Naturally, the wallet software could mark
2351 the notes as it broadcasts transactions that spend them. However, this approach is
2352 subject to several problems.

2353 Firstly, for each note spent, the client software must record the ID of the spending
2354 transaction, in order to track it and confirm that it is accepted into a block. Once each
2355 spending transaction is accepted the client can finally mark the appropriate **Zeth** notes
2356 as “spent”. This requires significant complexity in order to asynchronously mark the
2357 notes, and to deal with the issues described in Appendix C.2.

2358 Secondly, this approach does not support multiple wallets using the same key, or
2359 wallets being restored from **Zeth** addresses. A user that wishes to rebuild his wallet
2360 (see the discussion in Section 4.1.4), or check for any spending activity by other wallets,
2361 would not be able to do so by simply scanning the blockchain.

2362 By using the nullifiers passed to **Mix** calls, clients can determine the availability of
2363 notes in a more robust way. That is, to determine whether a note is spent or available,
2364 the client can compute the nullifier and check whether that nullifier has been seen by
2365 the **Mixer** contract.

2366 In a similar way to Appendix C.3, queries to **Mixer** for specific nullifiers reveals
2367 to observers that the client was the sender of any previous or future transaction that
2368 generates such a nullifier. To mitigate this, **Mixer** MUST include nullifier values in the
2369 event data it emits, and clients SHOULD use this to track which of their notes are spent.
2370 This MUST happen as part of the regular sync operation, so that no extra RPC traffic is
2371 generated and observers cannot distinguish between clients that do and do not recognize
2372 any given nullifier. Note that this approach also supports tracking spent notes from
2373 multiple wallets, and rebuilding wallets by re-syncing the blockchain.

C.6 Proof generation

Generation of the zero-knowledge proofs, required for valid Mix calls, is a very computationally intensive process. The proof generation itself does not require any communication with external parties, and so may not directly leak information about the client, but implementers should consider some indirect ways in which information may be leaked.

Implementers may also wish to consider the possible indirect impact of proof generation on the RPC channel. For example, a client that “waits” for proof generation without servicing the RPC connection may fail to respond to (or take significantly longer to respond to) new log events. The connected Ethereum node might then deduce that its peer is generating a proof and therefore likely to be the sender of an upcoming transaction.

Note

As stated in the introduction to this chapter, this discussion does not consider general timing attacks. We mention this extreme case of a client that completely stalls during proof generation only to illustrate how a poor implementation may leak information to its RPC peer.

In the case where proof generation is carried out on some external host, or by an external process on the same host, there may be a risk of network traffic or other IPC traffic being observed. If an observer can detect that a given client is communicating with a prover process, it can reliably deduce that the client will be the sender of an upcoming transaction.

An observer able to see the content of the communication between the wallet and prover process will also gain knowledge of the auxiliary inputs to the proof (including the data required to spend the input notes and secret attributes of the output notes). It is therefore important to secure any such connection, protect any prover process from being maliciously modified or observed, and to ensure that wallets only communicate with trusted processes.

C.7 Simple mixer calls

The public parameters to a Mix call can reveal information about the nature of a transaction, even though they do not reveal recipient details or note amounts. For example, a Mix call in which $\text{Mix}_{in}.primIn.vout = 0$ and $\text{Mix}_{in}.primIn.vin \neq 0$ may indicate a simple “deposit” of funds into the mixer. Similarly, if both $\text{Mix}_{in}.primIn.vout$ and $\text{Mix}_{in}.primIn.vin$ are zero, the transaction must be spending only notes already within **Mixer**, into new notes. Finally, if $\text{Mix}_{in}.primIn.vin = 0$ and $\text{Mix}_{in}.primIn.vout \neq 0$, the sender may be performing a simple “withdrawal” of funds from some existing notes.

A Mix call can combine all of the above logical operations in a single transaction. That is, it can deposit value into the mixer, spend existing notes, create new notes, and withdraw value from **Mixer** at the same time. Combining logical operations in this way

2407 makes it much more difficult for an observer to attribute a specific purpose to the Mix
2408 call.

2409 Clients can also perform Mix calls in which $vin = vout = 0$ and 0-valued notes are
2410 created from other 0-valued notes. Such “dummy” self-payments can further obfuscate
2411 the activity of a wallet, by adding “noise” to the system. Note, however, that the gas
2412 cost for such transactions must still be paid.

2413 Wallet implementations **SHOULD** encourage the use of these complex calls where pos-
2414 sible, either via the user interface or by automatically adding complexity to transactions,
2415 and **SHOULD** support features such as adding “noise”¹ if the user wishes to pay for extra
2416 protection of this kind.

2417 C.7.1 Small anonymity sets

2418 Until there is a large number of commitments and users of the mixer, it may be easy
2419 for an observer to infer some of the private data that is intended to be hidden by mixer
2420 calls.

2421 In the simple case, if there are very few commitments in the $\widetilde{\text{Mixer}}$ ’s Merkle tree, an
2422 attacker has a small list of candidate commitments that are being spent by subsequent
2423 Mix calls. Similarly, if the number of distinct Ethereum addresses that have been used
2424 to call $\widetilde{\text{Mixer}}$ is very small, observers can trace the original source of funds subsequently
2425 withdrawn to a small set of original depositors.

2426 Client software may wish to track metrics about the $\widetilde{\text{Mixer}}$ state, and either prevent
2427 certain actions or design the user interface to discourage users² from creating trans-
2428 actions whose features can be identified with high probability. We provide below a
2429 non-exhaustive list of metrics of interest:

- 2430 • **Number of commitments.** If there is a low absolute number of commitments,
2431 clearly any non-zero output must spend one of these (although we note that only
2432 $vout$ can be publicly known to be non-zero).
- 2433 • **Number of unspent commitments.** If $\#Comms - \#Nulls$ is small and a new
2434 commitment is created and then spent, observers can deduce that there is a high
2435 chance that the spend operation targeted the new commitment.
- 2436 • **Number of Ethereum addresses.** While very few distinct addresses (or groups
2437 of addresses that are not associated) have used the contract, observers can de-
2438 duce that subsequent Mix calls are likely to spend commitments created by clients
2439 associated with one of this small set of addresses.

2440 The set of Ethereum addresses that have interacted with the contract can leak data
2441 in other ways. An Ethereum address that withdraws value from the contract, but has not
2442 previously been used to make a Mix call (or a Mix call that deposits value into $\widetilde{\text{Mixer}}$),

¹By randomly scheduling dummy payments, for instance

²By, for example, displaying warning messages and/or asking the user for confirmation

2443 must have been the recipient of zeth notes created by a previous depositor. The details
2444 may not be directly available to an observer, but this is another example of information
2445 which could be combined with other leaked data to infer connections between entities
2446 and transactions.

2447 Appendix D

2448 Security proofs of Blake2

2449 This appendix proves the collision resistance, PRF-ness, binding and hiding properties
2450 of the Blake2 hash function in the Weakly Ideal Cipher model (WICM, see [LMN16]).
2451 The proofs use definitions and results of Luykx et al. [LMN16], regarding the indifferen-
2452 tiability of Blake2 and a random oracle in the Weakly Ideal Cipher Model (WICM). In
2453 the following, we assume that the optimization of Blake2 for 8- to 32-bit platforms is as
2454 secure as Blake2 as described in [LMN16].

2455 D.1 Security model of Blake2

2456 The security analysis treats Blake2 as hash function built on top of a block-cipher-based
2457 compression function in the WICM (which derives from the Ideal Cipher Model). In
2458 this section, we present the WICM and prove that Blake2 is a collision resistant PRF,
2459 and thus a commitment scheme.

2460 D.1.1 Weakly Ideal Cipher Model

2461 The research community believes that Blake’s underlying block cipher has no known
2462 weaknesses and could reasonably be modeled as an ideal cipher [LMN16, Section 2.1].
2463 However, Blake2 admits weak keys with a specific structure [LMN16, Section 2.1]. Blake2
2464 is therefore more appropriately analysed in the WICM, which is an extension of the Ideal
2465 Cipher Model that represents a block cipher as a set of independent random permuta-
2466 tions [HKT11]. The WICM may also be viewed as a specialization for Blake2 of the Weak
2467 Cipher Model [MP15], which aims to be realistic by modeling particular characteristics,
2468 invariants or properties a block cipher may have.

2469 A number of definitions in what follows are quoted directly from Luykx et al. [LMN16].

The Weakly Ideal Cipher Model. Let \mathcal{W} and \mathcal{S} be the following partition of $\mathbb{B}^{2 \cdot \text{ol}}$
into weak and strong sets, where w is the word length ($16 \cdot w = 2 \cdot \text{ol}$):

$$\mathcal{W} = \left\{ \text{aaaabbbbccccdddd} \in \mathbb{B}^{2 \cdot \text{ol}} \mid a, b, c, d \in \mathbb{B}^w \right\}$$

$$\mathcal{S} = \mathbb{B}^{2 \cdot \text{ol}} \setminus \mathcal{W}$$

Let $\mathcal{BLC}(2 \cdot \text{ol}, 2 \cdot \text{ol})$ denote the set of all block ciphers $E : \mathbb{B}^{2 \cdot \text{ol}} \times \mathbb{B}^{2 \cdot \text{ol}} \rightarrow \mathbb{B}^{2 \cdot \text{ol}}$. Define $\mathcal{BLC}^*(2 \cdot \text{ol}, 2 \cdot \text{ol})$ as the set of all block ciphers $E \in \mathcal{BLC}(2 \cdot \text{ol}, 2 \cdot \text{ol})$ with the additional restriction that $E(k_w, \cdot)$ is \mathcal{W} - and \mathcal{S} -subspace invariant for all keys $k_w \in \mathcal{K}_{\text{weak}}$. That is, inputs in \mathcal{W} map to \mathcal{W} , and likewise for \mathcal{S} . Here, $\mathcal{K}_{\text{weak}}$ is the set of weak keys, defined as

$$\mathcal{K}_{\text{weak}} = \left\{ k = \text{kkkkkkkkkkkkkkkkkk} \in \mathbb{B}^{2 \cdot \text{ol}} \mid k \in \mathbb{B}^w \right\}.$$

2470 A random $E \leftarrow \$\mathcal{BLC}^*(2 \cdot \text{ol}, 2 \cdot \text{ol})$ can now be modeled as follows:

- 2471 • on input of $(k, x) \in \mathcal{K}_{\text{weak}} \times \mathcal{W}$, E generates its response y randomly from \mathcal{W} up
2472 to repetition;
- 2473 • on input of $(k, x) \in \mathcal{K}_{\text{weak}} \times \mathcal{S}$, E generates its response y randomly from \mathcal{S} up to
2474 repetition.

2475 For key values $k \in \mathbb{B}^{2 \cdot \text{ol}} \setminus \mathcal{K}_{\text{weak}}$, E behaves like an ideal cipher: it either outputs a
2476 new random value or if the key-message-image tuple has already been queried the tuple's
2477 image. The case of inverse queries is analogous.

Blake2C is defined over the following domains and codomain:

$$\text{Blake2C} : \mathcal{BLC}^*(2 \cdot \text{ol}, 2 \cdot \text{ol}) \times \mathbb{B}^{\text{ol}} \times \mathbb{B}^{2 \cdot \text{ol}} \times \mathbb{B}^{\text{ol}/4} \times \mathbb{B}^{\text{ol}/4} \rightarrow \mathbb{B}^{\text{ol}}$$

2478 We write $\text{Blake2C}_E(h, m, t, f)$ for the output of the Blake2 compression function, defined
2479 over encryption scheme E on inputs h , m , t and f . The compression function, in par-
2480 ticular, computes the state $x = (h \parallel \text{pad}_{\text{ol}/2}(0) \parallel t \parallel f) \oplus (\text{pad}_{\text{ol}}(0) \parallel \text{IV})$ for some IV . It then
2481 encrypts x under m (where m is treated as a key for the encryption) and splits $E(m, x)$
2482 in two same size variables, the left part l_E and right part r_E . It finally outputs $l_E \oplus r_E \oplus h$.

2483
2484 **Zeth** uses the Blake2 compression function with a fixed encryption scheme E^* based on
2485 ChaCha stream cipher [Ber08a]. Thus, we write $\text{Blake2C}(h, m, t, f) = \text{Blake2C}_{E^*}(h, m, t, f)$.

2486 **Indifferentiability.** One way to measure the extent to which a certain cryptographic
2487 function behaves like a random function is via the indistinguishability framework where
2488 a distinguisher is given oracle access to either the cryptographic function or the random
2489 function with the goal of determining which one it has access to.

Definition D.1.1. Let \mathcal{C} be a construction with oracle access to an ideal primitive \mathcal{P} . Let \mathcal{R} be an ideal primitive with the same domain and codomain as \mathcal{C} . Let Sim be a simulator with the same domain and codomain as \mathcal{P} with oracle access to \mathcal{R} , and let Dist be a PPT distinguisher. The indifferentiability advantage of Dist is defined as:

$$\text{Indiff}_{\mathcal{C}^{\mathcal{P}}, \text{Sim}}(\text{Dist}) = \left| \Pr \left[\text{Dist}^{\mathcal{C}^{\mathcal{P}}, \mathcal{P}} = 1 \right] - \Pr \left[\text{Dist}^{\mathcal{R}, \text{Sim}^{\mathcal{R}}} = 1 \right] \right|$$

2490 The distinguisher Dist can query both its left oracle (either \mathcal{C} or \mathcal{R}) and its right
 2491 oracle (either \mathcal{P} or Sim). We refer to $\mathcal{C}^{\mathcal{P}}$, \mathcal{P} as the real world, and to \mathcal{R} , $\text{Sim}^{\mathcal{R}}$ as the
 2492 simulated world; the distinguisher Dist converses with either of these worlds and its goal
 2493 is to tell both worlds apart.

Theorem D.1.1 (Indifferentiability of Blake2 [LMN16]). *Let an encryption scheme $E \leftarrow \$ \mathcal{BLC}^*(2 \cdot \text{ol}, 2 \cdot \text{ol})$ be a weakly ideal cipher, and consider the hash function Blake2_E that internally uses E . There exists a simulator Sim such that for any distinguisher Dist with total complexity q , we have:*

$$\text{Indiff}_{\text{Blake2}_E, \text{Sim}}(\text{Dist}) \leq \frac{\binom{q}{2}}{2^{2\text{ol}}} + \frac{2\binom{q}{2}}{2^{\text{ol}}} + \frac{q}{2^{\text{ol}/2}}$$

2494 where Sim makes at most $O(q^3)$ queries to a random function \mathcal{R} .

2495 *Proof.* See [LMN16, Corollary 1]. □

2496 For asymptotic security, we assume the distinguisher to be PPT and that the number
 2497 of queries made is polynomial $q \leq \text{poly}(\text{ol})$.

2498 **Additional remarks.** Luykx et al. [LMN16] remark that, by resorting to the WICM,
 2499 they do not make stronger assumptions than those used in previous results (ICM), and,
 2500 despite the fact that they give distinguishers more power (by weakening the cipher),
 2501 they are able to get similar results.

2502 D.2 Security proofs

2503 D.2.1 Blake2 is a PRF

2504 Luykx et al. already prove the PRFness of Blake2 *keyed* hash function in the multi-key
 2505 setting.

Definition D.2.1 (PRF in multi-key setting [ML15]). Let $\mu \geq 1$ and $k \leftarrow \$ \mathcal{K}^\mu$. Let \mathcal{C} be a keyed construction with key space \mathcal{K} and with oracle access to an ideal primitive \mathcal{P} . Let $\mathcal{R}_1, \dots, \mathcal{R}_\mu$ be random functions with the same domains and ranges as $\mathcal{C}_{k_1}, \dots, \mathcal{C}_{k_\mu}$. Let D be a distinguisher. The PRF distinguishing advantage of D is defined as,

$$\text{PRF}_{\mathcal{C}^{\mathcal{P}}}(\text{D}) = \left| \Pr[\text{Dist}_{\mathcal{C}_{k_1}^{\mathcal{P}}, \dots, \mathcal{C}_{k_\mu}^{\mathcal{P}}, \mathcal{P}} = 1] - \Pr[\text{Dist}_{\mathcal{R}_1, \dots, \mathcal{R}_\mu, \mathcal{P}} = 1] \right|$$

Blake2 supports keyed hashing by simply prepending the key to the message:

$$\text{Blake2}_{E,k}(m) = \text{Blake2}_E(k \| 0^{2\text{ol}-\text{kl}} \| m)$$

2506 where $\text{kl} \leq 2\text{ol}$ denotes the key size. In other words, the key gets processed as other data
 2507 and the HAIFA counter and flags are designated to the key in a similar fashion as if they
 2508 were for normal data blocks.

Theorem D.2.1 (PRF-security of Blake2 keyed mode [LMN16]). *Let $\mu \geq 1$ and let $k \leftarrow \$ (\mathbb{B}^{\text{kl}})^\mu$. Let an encryption scheme $E \leftarrow \$ \mathcal{BLC}^*(2 \cdot \text{ol}, 2 \cdot \text{ol})$ be a weakly ideal cipher, and consider the keyed hash function $\text{Blake2}_{E,k}$ that internally uses Blake2C_E that internally uses E . For any distinguisher Dist with total complexity q :*

$$\text{PRF}_{\text{Blake2}_{E,k}}(\text{Dist}) \leq \frac{\binom{q}{2}}{2^{2\text{ol}}} + \frac{2\binom{q}{2}}{2^{\text{ol}}} + \frac{q}{2^{\text{ol}/2}} + \frac{\mu q}{2^{\text{kl}}} + \frac{\binom{\mu}{2}}{2^{\text{kl}}}$$

2509 *Proof.* See [LMN16, Corollary 3]. □

2510 **Remark D.2.2.** We can note that in the case of keyed hashing, the key is padded only
 2511 to be processed in a single block to differentiate the key from the message. The security
 2512 proof of Theorem D.2.1 does not rely on this padding and as such also works with no
 2513 padding.

Theorem D.2.2 (PRF-security of Blake2 with a single key [LMN16]). *Let $k \leftarrow \$ \mathbb{B}^{\text{kl}}$. Let an encryption scheme $E \leftarrow \$ \mathcal{BLC}^*(2 \cdot \text{ol}, 2 \cdot \text{ol})$ be a weakly ideal cipher, and consider the keyed hash function $\text{Blake2}_E(k, \cdot) = \text{Blake2}_E(k \| \cdot)$ that internally uses Blake2C_E that internally uses E . For any distinguisher Dist with total complexity q :*

$$\text{PRF}_{\text{Blake2}_E}(\text{Dist}) \leq \frac{\binom{q}{2}}{2^{2\text{ol}}} + \frac{2\binom{q}{2}}{2^{\text{ol}}} + \frac{q}{2^{\text{ol}/2}} + \frac{q}{2^{\text{kl}}}$$

2514 *Proof.* See Remark D.2.2 and Theorem D.2.1 with $\mu = 1$. □

2515 **Remark D.2.3.** Since we analyse the security of Blake2 asymptotically, we assume that
 2516 for a security parameter λ holds $\text{ol} = \mathcal{O}(\lambda)$, $\text{kl} = \mathcal{O}(\lambda)$, and $q = \text{poly}(\lambda)$.

2517 D.2.2 Proof of Blake2 collision resistance

2518 We want to prove here the collision resistance of Blake2. To do so, we are going to
 2519 prove by contradiction that if Blake2 is not collision resistant, it is not indifferentiable
 2520 according to Definition D.1.1.

2521 **Theorem D.2.3.** *Blake2 is collision resistant.*

2522 *Informal proof.* Let us assume that there exists a PPT adversary \mathcal{B} which breaks the
 2523 collision resistance of Blake2. We build an adversary \mathcal{A} that uses this adversary to
 2524 differentiate between the real and simulated worlds. More particularly, \mathcal{A} gets left and
 2525 right oracles (see [LMN16, Figure 3]), which are either an oracle for a hash function and
 2526 for a weakly ideal block cipher or a random oracle and an encryption simulator with
 2527 oracle access to the random oracle.

2528 On each \mathcal{B} 's query m_i , $i \in \{1, \dots, q\}$, \mathcal{A} passes them to his left oracle and returns
 2529 the answer h_i to \mathcal{B} . Eventually, if \mathcal{B} finds a collision, that is a pair (m_i, m_j) such that
 2530 $m_i \neq m_j$ and $h_i = h_j$, \mathcal{A} guesses that his oracles were real; else \mathcal{A} returns a random

2531 guess. Otherwise \mathcal{A} guesses his oracles were simulated – if the left oracle was a random
 2532 oracle, the probability of finding a collision would be negligible for $q \leq \text{poly}(\lambda)^1$.
 2533 On the other hand, \mathcal{B} finds a collision with non-negligible probability if the oracles
 2534 were real. Hence, \mathcal{A} wins the indistinguishability game with non-negligible advantage,
 2535 which is a contradiction. \square

2536 D.2.3 Blake2 as a commitment scheme

2537 We prove here that Blake2 is a commitment scheme, i.e. is binding and hiding. To do so
 2538 we rely on the previous results that Blake2 is collision resistant and a PRF.

2539 **Theorem D.2.4.** *Let $E \leftarrow \$\mathcal{BLC}(2\text{ol}, 2\text{ol})$ and for a message $x \in \mathbb{B}^*$ and randomness*
 2540 *$r \in \mathbb{B}^l$ commitment to x using r be $\text{ComSch.Com}(x; r) = \text{Blake2}_E(r \| x)$. Then ComSch*
 2541 *is hiding and binding.*

2542 *Informal proof. Hiding.* A commitment scheme ComSch is computationally hiding if,
 2543 knowing two potential openings, a PPT adversary cannot distinguish which was com-
 2544 mitted. Let us assume that there exists a PPT adversary \mathcal{B} which breaks the hiding
 2545 property of Blake2 with a non-negligible advantage η . We build an adversary \mathcal{A} that
 2546 uses \mathcal{B} to break the PRF property of Blake2 with advantage $\eta/2$.

2547 First, the PRF game is initiated, that is, the challenger chooses a random encryption
 2548 scheme E and key $k \in \mathbb{B}^l$ and instantiates two oracles $O^{\text{Blake2}_k} = \text{Blake2}_E(k, \cdot)$ and O^R
 2549 a random function. The challenger picks an oracle randomly and gives \mathcal{A} access to it.
 2550 \mathcal{B} sends q oracle queries m_1, \dots, m_q to \mathcal{A} (adaptively) who extends them with random
 2551 r_1, \dots, r_q and sends $r_i \| m_i$ to his left oracle. Given the answer from the oracle, \mathcal{A} returns
 2552 them to \mathcal{B} . Eventually, \mathcal{B} then outputs two challenge messages $(\tilde{m}_0, \tilde{m}_1)$ and sends them
 2553 to \mathcal{A} who randomly selects message \tilde{m}_b , extends it with r and sends $r \| \tilde{m}_b$ to his left oracle.
 2554 The oracle answers with y_b which is also sent to \mathcal{B} . Finally, \mathcal{B} returns the decision bit \tilde{b}
 2555 to \mathcal{A} . If $b = \tilde{b}$, \mathcal{A} answers to the challenger that the oracle was instantiating the PRF.
 2556 Otherwise, \mathcal{A} answers with a random guess. The advantage of \mathcal{A} equals advantage of \mathcal{B}
 2557 if it interacts with a real hash function. The advantage of \mathcal{A} equals half the advantage
 2558 of \mathcal{B} when interacting with a random oracle and simulator.

2559 *Binding.* A commitment scheme ComSch is said to be computationally binding if
 2560 it is infeasible to find x, x' and r, r' such that $x \neq x'$ and $\text{Com}(x; r) = \text{Com}(x'; r')$.
 2561 This is implied by collision resistance of Blake2. Thus if \mathcal{B} is an algorithm that breaks
 2562 the biding property with advantage η , there is another algorithm \mathcal{A} that breaks Blake2
 2563 collision resistance with the same advantage. \square

¹The probability would be $\frac{q^2}{2^{\text{ol}}}$ which is negligible for a polynomial number of queries q . This is the sum of the probabilities of finding a collision when doing the i^{th} query. Indeed, let us suppose the adversary has done $i - 1$, $i > 2$, queries without finding a collision, i.e. he knows $i - 1$ distinct tuples of input-output. When receiving the i^{th} value, the adversary has thus $i - 1$ chance to find a collision. The probability for the new output to be equal to any of the previous outputs is thus $(i - 1) \cdot \frac{1}{2^{\text{ol}}}$ (as we are in the random oracle model). Summing this probability over all queries, we find the probability of finding a collision after doing q queries.

Assuming that Blake2s is as secure as Blake2, a commitment scheme based on a Blake2s, i.e. $\text{Com}(x; r) = \text{Blake2s}_E(r \| x)$ is hiding and binding.

D.2.4 Proof of commitment scheme security

To prove the binding and hiding property of ComSch (see Section 3.1.2), we introduce the following commitment scheme ComSch^* ,

$$\begin{aligned} \text{ComSch}^*.\text{Setup} : \{1^\lambda \text{ s.t. } \lambda \in \mathbb{N}\} &\rightarrow \mathbb{B}^* \\ \text{ComSch}^*.\text{Com} : \mathcal{BK}^*(2 \cdot \text{BLAKE2sCLEN}, 2 \cdot \text{BLAKE2sCLEN}) &\times \mathbb{B}^{2 \cdot \text{BLAKE2sCLEN}} \\ &\times (\mathbb{B}^{\text{PRFADDRROUTLEN}} \times \mathbb{B}^{\text{PRFRHOOUTLEN}} \times \mathbb{B}^{\text{ZVALUELEN}}) \times \mathbb{B}^{\text{RTRAPLEN}} \rightarrow \mathbb{B}^{\text{BLAKE2sCLEN}} \end{aligned}$$

The commitment scheme is defined as follows,

$$\begin{aligned} \text{ComSch}^*.\text{Setup}(1^\lambda) &= pp^* = \epsilon \\ \text{ComSch}^*.\text{Com}(m = (apk, \rho, v); r) &= cm \\ &= \text{Blake2E}^*(r \| apk \| \rho \| v) \end{aligned}$$

Given a commitment scheme ComSch^* , the bijective function $\text{decode}_{\mathbb{N}}(\cdot)$ and $p_\lambda \in \mathbb{N}$, a prime which can be represented using λ bits, we define the commitment scheme ComSch' as follows:

$$\begin{aligned} \text{ComSch}'.\text{Setup}(1^\lambda) &= (\text{ComSch}^*.\text{Setup}(1^\lambda), p_\lambda) \\ \text{ComSch}'.\text{Com}(m; r) &= \text{decode}_{\mathbb{N}}(\text{ComSch}^*.\text{Com}(m; r)) \pmod{p_\lambda} \text{ for } m = (apk \| \rho \| v) \end{aligned}$$

Note that ComSch (see Section 3.1.2) is a particular instantiation of ComSch' where E^* is set as ChaCha encryption scheme [Ber08a], k^* is a random key, and p_λ is r_{CUR} .

Theorem D.2.5 (Hiding). *If ComSch^* is hiding then ComSch' is hiding.*

Proof. We prove the theorem by contradiction i.e. we assume that there exists an adversary \mathcal{B} that breaks ComSch' 's hiding property and construct an adversary \mathcal{A} that uses \mathcal{B} to break ComSch^* 's hiding property with non-negligible probability.

Let \mathcal{C} be a challenger that sets up the hiding game for ComSch^* and \mathcal{A} . The adversary \mathcal{A} , given public parameters pp^* of ComSch^* and access to an oracle that runs the Com algorithm of ComSch^* scheme, simulates a hiding game for ComSch' for \mathcal{B} . The adversary \mathcal{A} starts by setting public parameters pp' for ComSch' using public parameters pp^* given by \mathcal{C} . Parameters pp' are passed to \mathcal{B} who outputs a pair of messages m_0, m_1 . The adversary \mathcal{A} forwards them to the challenger who samples a bit b at random and generates $cm^* = \text{ComSch}^*.\text{Com}(m_b; r)$ for some randomness r . The result is returned to \mathcal{A} (see Definition 1.5.21). Then \mathcal{A} passes $cm = \text{decode}_{\mathbb{N}}(cm^*) \pmod{p_\lambda}$ to \mathcal{B} who returns his guess b' . The adversary \mathcal{A} returns the same b' to the challenger.

By construction, it is clear that \mathcal{A} wins the hiding game with the same probability that \mathcal{B} wins the simulated hiding game. Since \mathcal{B} 's advantage is non-negligible, this means that \mathcal{A} wins the ComSch^* hiding game with non-negligible probability as well. \square

2585 **Theorem D.2.6** (Binding). *Let ComSch^* be a computationally binding commitment*
 2586 *scheme and $\text{ComSch}^*.\text{Com}$ indifferentiable from a random oracle. Then ComSch' is also*
 2587 *computationally binding if $l = \lceil 2^\lambda/p_\lambda \rceil$ is at most $\text{poly}(\lambda)$.*

2588 *Proof.* Assume that \mathcal{A} asks the ComSch' commit and open oracles a total of q_λ distinct
 2589 queries. Let us denote the result of the q_λ queries and output of the attacker (the
 2590 candidate collision) as $((m_1, r_1, y_1), \dots, (m_{q_\lambda}, r_{q_\lambda}, y_{q_\lambda}), \text{out})$. If \mathcal{A} is successful it means
 2591 that it outputs (m, r) , (m', r') such that $(m, r) \neq (m', r')$ and $\text{ComSch}'.\text{Com}(m; r) =$
 2592 $\text{ComSch}'.\text{Com}(m'; r')$.

By the definition of ComSch' , we have that,

$$\text{ComSch}'.\text{Com}(m; r) = \text{decode}_{\mathbb{N}}(\text{ComSch}^*.\text{Com}(m; r)) \pmod{p_\lambda}$$

Hence, we have a collision in ComSch' if there exists $k \in [l]$, l being the ratio of the
 codomains of $\text{ComSch}^*.\text{Com}$ and $\text{ComSch}'.\text{Com}$, such that,

$$|\text{decode}_{\mathbb{N}}(\text{ComSch}^*.\text{Com}(m; r)) - \text{decode}_{\mathbb{N}}(\text{ComSch}^*.\text{Com}(m'; r'))| = k \cdot p_\lambda.$$

2593 We show that this event is unlikely.

2594 In fact, for each $i \in [q_\lambda]$, let C_i be the event that the adversary wins at the i -th
 2595 query. That is, the last commitment y_i is a ComSch' collision with one of the previous
 2596 y_j . More precisely there exists $j \leq i$ and $k < l$ such that $y_i = y_j + k \cdot p_\lambda$.

2597 Since ComSch^* is a random oracle, y_i is randomly selected from a set of at least p_λ
 2598 elements. As such, we have $\Pr[C_i] \leq i \cdot l/p_\lambda$.

Thus the probability of finding a collision after q_λ queries is $\Pr[C_1 \vee \dots \vee C_{q_\lambda}] \leq$
 $\sum_{i=1}^{q_\lambda} \Pr[C_i] = l/p_\lambda \cdot \sum_{i=1}^{q_\lambda} i$. This probability is bounded by $l \cdot \frac{q_\lambda(q_\lambda+1)}{p_\lambda}$. However,
 we allow only polynomial number of queries. Thus for $q_\lambda = \text{poly}(\lambda)$ this probability
 becomes,

$$\frac{2^\lambda \cdot \text{poly}(\lambda)}{p_\lambda^2},$$

2599 what is negligible for $2^\lambda/p_\lambda \leq \text{poly}(\lambda)$. □

2600 **Remark D.2.4.** Note that in **Zeth**'s commitment scheme, we set $p_\lambda = \mathbf{r}_{\text{CUR}}$ and $2^\lambda =$
 2601 $2^{\text{BLAKE2sCLEN}}$. Thus, for **BN-254** and **BLS12-377** have $l = 6$ and $l = 14$, respectively.
 2602 Therefore, the probability of an attacker breaking the binding property due to reduction
 2603 modulo \mathbf{r}_{CUR} increases approximately by these factors. This is still negligible.

2604 **Corollary.** *Assume that Blake2 is indifferentiable from a random oracle and a PRF,*
 2605 *then ComSch^* is computationally binding and computationally hiding. Furthermore, the*
 2606 *reduction is tight. That is, the advantage of any PPT adversary against the binding*
 2607 *(resp. hiding) property is the same as the advantage of an adversary against collision*
 2608 *resistance and binding (resp. hiding).*

2609

2610 Glossary

- 2611 **joinsplit** Set of JSIN input *ZethNotes*, and JSOUT output *ZethNotes* as well as the
2612 public values *vin* and *vout* used in a tx_{Mix} transaction. 37, 39, 41, 59, 97, 116, 117
- 2613 **joinsplit equation** Equation that checks that the sum of the values of the SendTx
2614 algorithm of DAP is equal to the sum of the values of its outputs. This equations
2615 checks that the joinsplit is “balanced” and thus, that no value is created while
2616 creating new *ZethNotes*. 25, 41, 42, 59, 97, 116

2617 **Acronyms**

2618 **DOS** Denial of Service (Attack). 16, 116

2619 **EOA** Externally Owned Account. 16, 17, 19, 116

2620 **EVM** Ethereum Virtual Machine. 15, 16, 20, 48, 54, 63, 84, 87, 116

2621 **FFT** Fast Fourier Transform. 85, 116

2622 **MAC** Message Authentication Code. 99, 116

2623 **PoC** Proof of Concept. 116

2624 **RAM** Random-access Memory. 85, 116

2625 **RLP** Recursive Length Prefix. 19, 20, 116

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