

BP

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1 Model

The atomic unit of the factor graph is shown in Figure 1.

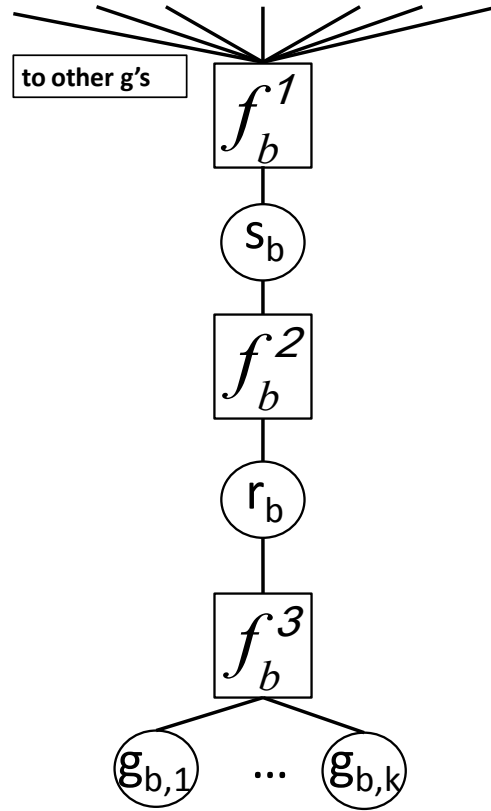


Figure 1: Factor graph

We require efficient computation of the messages.

2 Top-down messages

2.1 $\mu_{f_b^1 \rightarrow s_b}$

The general form of the message is given by

$$\mu_{f_b^1 \rightarrow s_b}(s_b) = \sum_{\vec{g}} f_b^1(s_b, \vec{g}) \prod_{g_{b',k}} \mu_{g_{b',k} \rightarrow f_b^1}(g_{b',k}) \quad (1)$$

We divide the cases into $s_b = 0$, and $s_b = 1$.

$$\mu_{f_b^1 \rightarrow s_b}(s_b = 0) = \sum_{\vec{g}} f_b^1(s_b = 0, \vec{g}) \prod_{g_{b',k}} \mu_{g_{b',k} \rightarrow f_b^1}(g_{b',k}) \quad (2)$$

$$= (1 - \epsilon) \prod_{g_{b',k}} \mu_{g_{b',k} \rightarrow f_b^1}(g_{b',k} = 0) \quad (3)$$

$$\mu_{f_b^1 \rightarrow s_b}(s_b = 1) = \sum_{\vec{g}} f_b^1(s_b = 1, \vec{g}) \prod_{g_{b',k}} \mu_{g_{b',k} \rightarrow f_b^1}(g_{b',k}) \quad (4)$$

$$= \epsilon \times \prod_{g_{b',k}} \mu_{g_{b',k} \rightarrow f_b^1}(g_{b',k} = 0) \quad (5)$$

$$+ \sum_{\vec{g}} \prod_{g_{b',k}} \mu_{g_{b',k} \rightarrow f_b^1}(g_{b',k}) - \prod_{g_{b',k}} \mu_{g_{b',k} \rightarrow f_b^1}(g_{b',k} = 0)$$

$$= \epsilon \times \prod_{g_{b',k}} \mu_{g_{b',k} \rightarrow f_b^1}(g_{b',k} = 0) \quad (6)$$

$$+ 1 - \prod_{g_{b',k}} \mu_{g_{b',k} \rightarrow f_b^1}(g_{b',k} = 0)$$

$$= \prod_{g_{b',k}} \mu_{g_{b',k} \rightarrow f_b^1}(g_{b',k} = 0)(\epsilon - 1) + 1 \quad (7)$$

$$= 1 - (1 - \epsilon) \prod_{g_{b',k}} \mu_{g_{b',k} \rightarrow f_b^1}(g_{b',k} = 0) \quad (8)$$

$$= 1 - \mu_{f_b^1 \rightarrow s_b}(s_b = 0) \quad (9)$$

where we have made use of the property that messages from a node/factor sum to 1

2.2 $\mu_{s_b \rightarrow f_b^2}$

if s_b has not been observed, this message is given by

$$\mu_{s_b \rightarrow f_b^2}(s_b) = \mu_{f_b^1 \rightarrow s_b}(s_b) \quad (10)$$

otherwise, it is a constant message that indicates its observed state.

2.3 $\mu_{f_b^2 \rightarrow r_b}$

$$\mu_{f_b^2 \rightarrow r_b}(r_b) = \sum_{s_b} f_b^2(s_b, r_b) \mu_{s_b \rightarrow f_b^2}(s_b) \quad (11)$$

$$= \sum_{s_b} P(r_b \mid s_b) \mu_{s_b \rightarrow f_b^2}(s_b) \quad (12)$$

s_b only has two values, so this is easy to compute.

2.4 $\mu_{r_b \rightarrow f_b^3}$

$$\mu_{r_b \rightarrow f_b^3}(r_b) = \mu_{f_b^2 \rightarrow r_b}(r_b) \quad (13)$$

2.5 $\mu_{f_b^3 \rightarrow g_{b,k}}$

$$\mu_{f_b^3 \rightarrow g_{b,k}}(g_{b,k}) = \sum_{r_b} \sum_{k' \neq k} f_b^3(r_b, \vec{g}_b) \mu_{r_b \rightarrow f_b^2}(r_b) \prod_{k' \neq k} \mu_{g_{b,k} \rightarrow f_b^3}(g_{b,k'}) \quad (14)$$

$$= \sum_{r_b} \mu_{r_b \rightarrow f_b^2}(r_b) \sum_{k' \neq k} P(\vec{g}_b \mid r_b) \prod_{k' \neq k} \mu_{g_{b,k} \rightarrow f_b^3}(g_{b,k'}) \quad (15)$$

$$= \sum_{r_b} \mu_{r_b \rightarrow f_b^2}(r_b) \sum_{k' \neq k} P(\vec{g}_{b,k} \mid r_b) \prod_{k' \neq k} P(\vec{g}_{b,k'} \mid r_b) \mu_{g_{b,k} \rightarrow f_b^3}(g_{b,k'}) \quad (16)$$

$$= \sum_{r_b} \mu_{r_b \rightarrow f_b^2}(r_b) P(\vec{g}_{b,k} \mid r_b) \prod_{k' \neq k} \sum_{g_{b,k'}} P(\vec{g}_{b,k'} \mid r_b) \mu_{g_{b,k} \rightarrow f_b^3}(g_{b,k'}) \quad (17)$$

The number of values r_b can take should be small (no more than, say 10? 20?), so this should be possible to compute.

2.6 $\mu_{g_{b,k} \rightarrow f_b^1}$

$$\mu_{g_{b,k} \rightarrow f_b^1}(g_{b,k}) = \mu_{f_b^3 \rightarrow g_{b,k}}(g_{b,k}) \prod_{b' \neq b} \mu_{f_{b'}^1 \rightarrow g_{b,k}}(g_{b,k}) \quad (18)$$

3 Bottom-up messages

3.1 $\mu_{f_b^1 \rightarrow g_{b',k}}$

Recall that f_b^1 connects to other bricks. We denote one such brick by b' .

Because f_b^1 only cares about whether $g_{b',k}$ points to b , we reduce the domain of $g_{b',k}$ to 0, 1 which means “doesn’t point to b ” and “points to b ”, respectively. The messages can be “expanded” by $g_{b',k}$ to its original domain when $g_{b',k}$ needs to send a message to f_b^3 .

The general form of this message is

$$\mu_{f_b^1 \rightarrow g_{b',k}}(g_{b,k'}) = \sum_{s_b} \sum_{g \neq g_{b',k}} f_b^1(s_b, \mathbf{g}) \mu_{s_b \rightarrow f_b^1}(s_b) \prod_{g \neq g_{b',k}} \mu_{g \rightarrow f_b^1}(g). \quad (19)$$

We divide the cases into $g_{b',k} = 0$, and $g_{b',k} = 1$

$$\mu_{f_b^1 \rightarrow g_{b',k}}(g_{b',k} = 0) = \mu_{s_b \rightarrow f_b^1}(s_b = 0) \times (1 - \epsilon) \times \prod_{g \neq g_{b',k}} \mu_{g \rightarrow f_b^1}(g = 0) \quad (20)$$

$$\begin{aligned} & + \mu_{s_b \rightarrow f_b^1}(s_b = 1) \times \epsilon \times \prod_{g \neq g_{b',k}} \mu_{g \rightarrow f_b^1}(g = 0) \\ & + \mu_{s_b \rightarrow f_b^1}(s_b = 1) \left(\prod_{g \neq g_{b',k}} \sum_g \mu_{g \rightarrow f_b^1}(g) - \prod_{g \neq g_{b',k}} \mu_{g \rightarrow f_b^1}(g = 0) \right) \\ = & \mu_{s_b \rightarrow f_b^1}(s_b = 0) \times (1 - \epsilon) \times \prod_{g \neq g_{b',k}} \mu_{g \rightarrow f_b^1}(g = 0) \quad (21) \\ & + \epsilon \times \mu_{s_b \rightarrow f_b^1}(s_b = 1) \prod_{g \neq g_{b',k}} \mu_{g \rightarrow f_b^1}(g = 0) \\ & + \mu_{s_b \rightarrow f_b^1}(s_b = 1) \left(1 - \prod_{g \neq g_{b',k}} \mu_{g \rightarrow f_b^1}(g = 0) \right) \end{aligned}$$

$$\begin{aligned} = & \mu_{s_b \rightarrow f_b^1}(s_b = 0) \times (1 - \epsilon) \times \prod_{g \neq g_{b',k}} \mu_{g \rightarrow f_b^1}(g = 0) \\ & + \mu_{s_b \rightarrow f_b^1}(s_b = 1) \left(\prod_{g \neq g_{b',k}} \mu_{g \rightarrow f_b^1}(g = 0) (\epsilon - 1) + 1 \right) \quad (22) \\ = & \mu_{s_b \rightarrow f_b^1}(s_b = 0) \times (1 - \epsilon) \times \frac{\prod_g \mu_{g \rightarrow f_b^1}(g = 0)}{\mu_{g_{b',k} \rightarrow f_b^1}^0(g = 0)} \quad (23) \\ & + \mu_{s_b \rightarrow f_b^1}(s_b = 1) \left(\frac{\prod_g \mu_{g \rightarrow f_b^1}(g = 0)}{\mu_{g_{b',k} \rightarrow f_b^1}^0(g = 0)} (\epsilon - 1) + 1 \right) \end{aligned}$$

$$\begin{aligned} = & \mu_{s_b \rightarrow f_b^1}(s_b = 0) \times \frac{\mu_{s_b \rightarrow f_b^1}(s_b = 0)}{\mu_{g_{b',k} \rightarrow f_b^1}^0(g = 0)} \quad (24) \\ & + \mu_{s_b \rightarrow f_b^1}(s_b = 1) \left(\frac{\mu_{s_b \rightarrow f_b^1}(s_b = 0)}{\mu_{g_{b',k} \rightarrow f_b^1}^0(g = 0)} (\epsilon - 1) + 1 \right) \\ = & \frac{\mu_{s_b \rightarrow f_b^1}(s_b = 0)^2}{\mu_{g_{b',k} \rightarrow f_b^1}^0(g = 0)} \quad (25) \\ & + \frac{\mu_{s_b \rightarrow f_b^1}(s_b = 1) \mu_{s_b \rightarrow f_b^1}(s_b = 0) (\epsilon - 1) + \mu_{g_{b',k} \rightarrow f_b^1}^0(g = 0)}{\mu_{g_{b',k} \rightarrow f_b^1}^0(g = 0)} \end{aligned}$$

$$\begin{aligned} = & \frac{\mu_{s_b \rightarrow f_b^1}(s_b = 0)^2 + \mu_{s_b \rightarrow f_b^1}(s_b = 1) \mu_{s_b \rightarrow f_b^1}(s_b = 0) (\epsilon - 1) + \mu_{g_{b',k} \rightarrow f_b^1}^0(g = 0)}{\mu_{g_{b',k} \rightarrow f_b^1}^0(g = 0)} \quad (26) \end{aligned}$$

where $\mu_{g_{b',k} \rightarrow f_b^1}^0$ means the old message sent by $g_{b',k}$, and we have made use of the property that messages from a node/factor sum to 1. Note that we can compute this message without having to explicitly examine all the other $g'_{b,k}$ s; we re-used the computation from computing $\mu_{s_b \rightarrow f_b^1}(s_b = 0)$.

$$\mu_{f_b^1 \rightarrow g_{b',k}}(g_{b,k'} = 1) = \mu_{s_b \rightarrow f_b^1}(s_b = 1) \prod_{g \neq g_{b',k}} \sum_g \mu_{g \rightarrow f_b^1}(g) \quad (27)$$

$$= \mu_{s_b \rightarrow f_b^1}(s_b = 1) \quad (28)$$

3.2 $\mu_{g_{b,k} \rightarrow f_b^3}$

$$\mu_{g_{b,k} \rightarrow f_b^3}(g_{b,k}) = \prod_{b' \in V(b)} \mu_{f_{b'}^3 \rightarrow g_{b,k}}(g_{b,k}) \quad (29)$$

where $V(b)$ is the after set of brick b . Note that $\mu_{f_{b'}^3 \rightarrow g_{b,k}}(g_{b,k})$ needs to be “expanded” to the proper domain of $g_{b,k}$

3.3 $\mu_{f_b^3 \rightarrow r_b}$

$$\mu_{f_b^3 \rightarrow r_b}(r_b) = \sum_{\vec{g}} f_b^3(\vec{g}, r_b) \prod_k \mu_{g_{b,k} \rightarrow f_b^3}(g_{b,k}) \quad (30)$$

$$= \sum_{\vec{g}} \prod_k P(g_{b,k} \mid r_b) \prod_k \mu_{g_{b,k} \rightarrow f_b^3}(g_{b,k}) \quad (31)$$

$$= \prod_k \sum_{g_{b,k}} P(g_{b,k} \mid r_b) \mu_{g_{b,k} \rightarrow f_b^3}(g_{b,k}) \quad (32)$$

3.4 $\mu_{r_b \rightarrow f_b^2}$

$$\mu_{r_b \rightarrow f_b^2}(r_b) = \mu_{f_b^3 \rightarrow r_b}(r_b) \quad (33)$$

3.5 $\mu_{f_b^2 \rightarrow s_b}$

$$\mu_{f_b^2 \rightarrow s_b}(s_b) = \sum_{r_b} f_b^2(r_b, s_b) \mu_{r_b \rightarrow f_b^2}(r_b) \quad (34)$$

$$= \sum_{r_b} P(r_b \mid s_b) \mu_{r_b \rightarrow f_b^2}(r_b) \quad (35)$$

These messages should be trivial to compute, since the set of values r_b is small.

3.6 $\mu_{s_b \rightarrow f_b^1}$

$$\mu_{s_b \rightarrow f_b^1}(s_b) = \mu_{f_b^2 \rightarrow s_b}(s_b) \quad (36)$$