Jeroen's model

May 19, 2013

1 Notation

- Σ: partially-ordered list of symbols (types).
- Σ_i: ith symbol in partial ordering.
- T: number of symbols in grammar.
- $\mathcal{P}$: pose space.
- $\mathcal{P}(b)$: pose cell of brick b- a set of poses. $\mathcal{P}(b) \in \mathcal{P}$
- $\mathcal{A} = (a_1, \ldots, a_M)$: set of active bricks.
- $\mathcal{A} \subseteq \mathcal{B}$.
- $\mathcal{B} = (b_1, \ldots, b_N)$: set of bricks.
- $t(b)$: type of brick b.
- $\mathcal{R}$: set of production rules of the form $A \to B_1, \ldots, B_k$ with $r = A, B_1, \ldots, B_k \in \Sigma$. $\mathcal{R}$ is acyclic.
- $n(r)$: number of slots/children associated with rule $r \in \mathcal{R}$
- $c(b) = \max\limits_{r \in \mathcal{R}, LHS(r)=t(b)} n(r)$: maximum number of slots/children associated with a brick b
- $p_{r,n}(x_i|x)$, $r \in \mathcal{R}$, $x, x_i \in \mathcal{P}$, $n \in [1...n(r)]$: conditional probability density over pose for B_i given pose for A for this rule and slot.

- $\mathcal{R}(A)$: rules with A in the left-hand-side (LHS).

- p_A: distribution over $\mathcal{R}(A)$.

- $s_b \in \{0, 1\}$: on/off state of brick b.

- $r_b \in \mathcal{R}(t(b))$: rule used by brick b.

- $\mathbf{g_b} \in (\mathcal{B} \cup \perp)^{c(b)}$: array of pointers, where each element can either point to a brick, or null (no child).

- $V(b)$: "before" set of bricks, which we define as $V(b) = \{b' : t(b') < t(b)\}$ where $<$ means "comes before in the partial ordering defined by Σ".

- $W(b)$: "after" set of bricks, which we define as $V(b) = \{b' : t(b') > t(b)\}$ where $>$ means "comes after in the partial ordering defined by Σ".

- $q_b \in \mathcal{P}(b)$: pose for brick b. $q_b = \perp$ is a special "nothing" pose that does not contribute to image evidence.

- $\mathbf{X} = \{\mathbf{s}, \mathbf{r}, \mathbf{g}, \mathbf{q}\}$: collection of hidden variables of model.

- I: the image.

2 Model

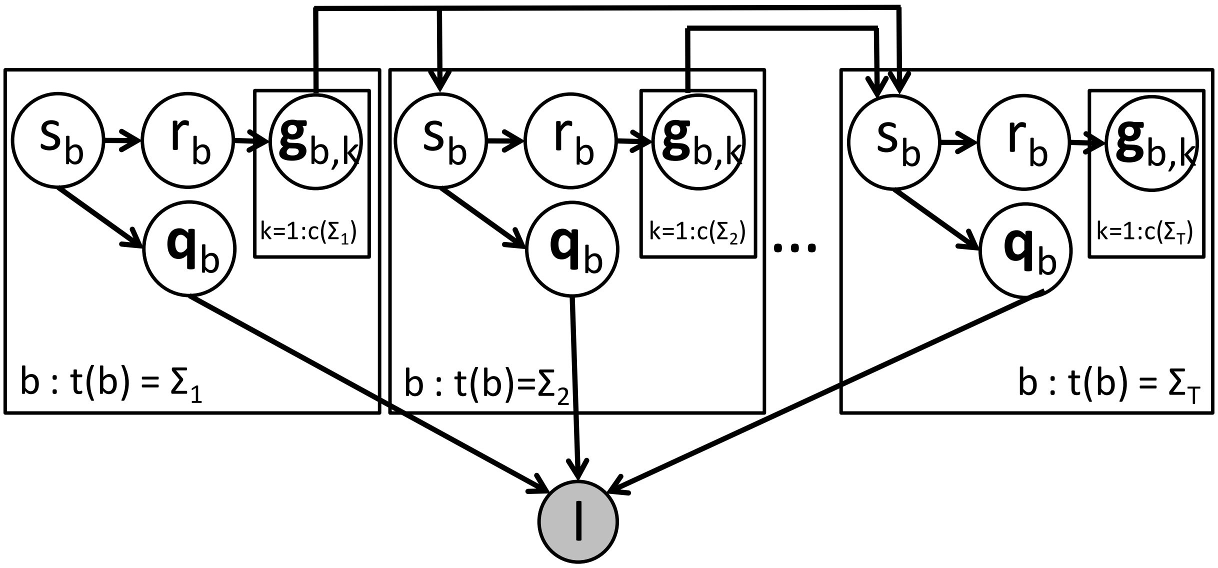

Figure 1: Graphical model

Joint probability is given by:

$$
\begin{aligned}
P(\mathbf{X}, I) \quad &= \quad \Big(\prod_{b\in\mathcal{B}} P(s_b \mid \mathbf{g}_{V(b)})\Big) && (1)\\
&\Big(\prod_{b\in\mathcal{B}} P(r_b \mid s_b) \prod_{k=1}^{c(b)} P(g_{b,k} \mid r_b)\Big) && (2)\\
&\Big(\prod_{b\in\mathcal{B}} P(\mathbf{q_b} \mid s_b)\Big) && (3)\\
&P(I \mid \mathbf{q}) && (4)
\end{aligned}
$$

Before we define localizations, we first define additional notation to ease the presentation.

- $\mathbf{g}_b^{\mathcal{A}} \in (\mathcal{A} \cup \emptyset)^{c(b)}$: array of pointers, where each element can either point to a brick, or blank (child not yet specified). Note that blank is **different** than null (no child) in that blank means "there could be a child in this slow, but we don't know if there is or which one yet" while null (no child) means "there definitely isn't a child in this slot".

For localization, we do not explicitly represent $\mathbf{r}$; instead we express the state configuration in terms of $\mathbf{g}_b^{\mathcal{A}}$ only, which implicitly specifies a set of compatible rules. Rather than selecting specific values for $\mathbf{r}$, we instead marginalize over the set of compatible rules defined by $\mathbf{g}^{\mathcal{A}}$ wherever $\mathbf{r}$ is referred to.

We define localizations in terms of the components $\big(\prod_{b\in\mathcal{B}} P(s_b \mid \mathbf{g}_{V(b)})\big)$, $\big(\prod_{b\in\mathcal{B}} P(r_b \mid s_b)\prod_{k=1}^{c(b)} P(g_{b,k} \mid r_b)\big)$, $\big(\prod_{b\in\mathcal{B}} P(\mathbf{q_b} \mid s_b)\big)$, and $P(I \mid \mathbf{q})$ separately.

$$\Big(\prod_{b\in\mathcal{B}} P(s_b \mid \mathbf{g}_{V(b)})\Big) \quad\Rightarrow\quad \Big(\prod_{a\in\mathcal{A}} P^{\mathcal{A}}(s_a \mid \mathbf{g}^{\mathcal{A}}_{V(b)})\Big) \tag{5}$$

where $\Rightarrow$ is used to mean "localiizes to". Similarily,

$$\Big(\prod_{b\in\mathcal{B}} P(r_b \mid s_b)\prod_{k=1}^{c(b)} P(g_{b,k} \mid r_b)\Big) \quad\Rightarrow\quad \Big(\prod_{a\in\mathcal{A}}\sum_{r_a} P^{\mathcal{A}}(r_a \mid s_a) P^{\mathcal{A}} \prod_{k=1}^{c(b)} P^{\mathcal{A}}(g_{a,k} \mid r_a)\Big) \tag{6}$$

$$P^{\mathcal{A}}(g_{a,k} \mid r_a) \quad = \quad \text{defn here. split empty, not-empty cases} \tag{7}$$

$$\Big(\prod_{b\in\mathcal{B}} P(\mathbf{q_b} \mid s_b)\Big) \Rightarrow \Big(\prod_{a\in\mathcal{A}} P^{\mathcal{A}}(\mathbf{q_a} \mid s_a)\Big) \tag{8}$$

$$P(I \mid \mathbf{q}) \Rightarrow P^{\mathcal{A}}(I \mid \mathbf{q}^{\mathcal{A}}) \tag{9}$$