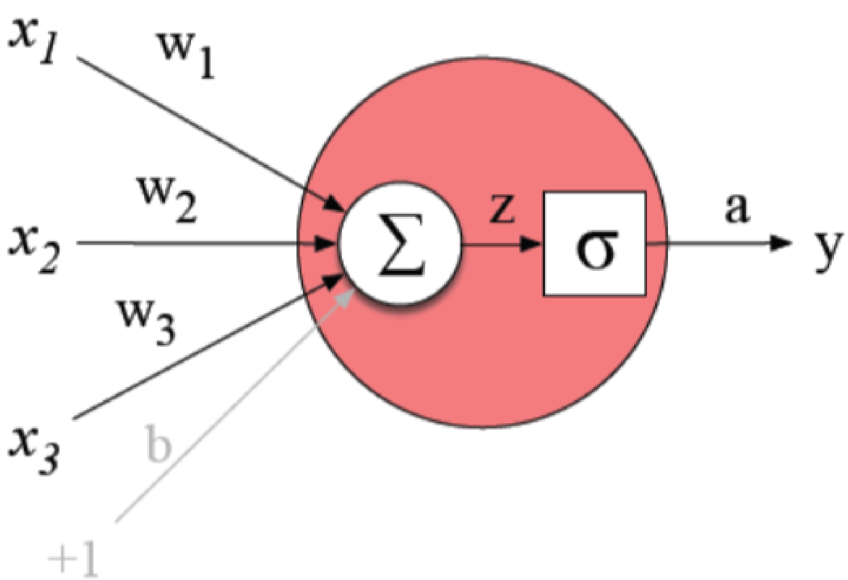
Capitolo 5.2

1. Ret Neurali per nlp

- **Feed forward neural Network**
  - Neural unit:

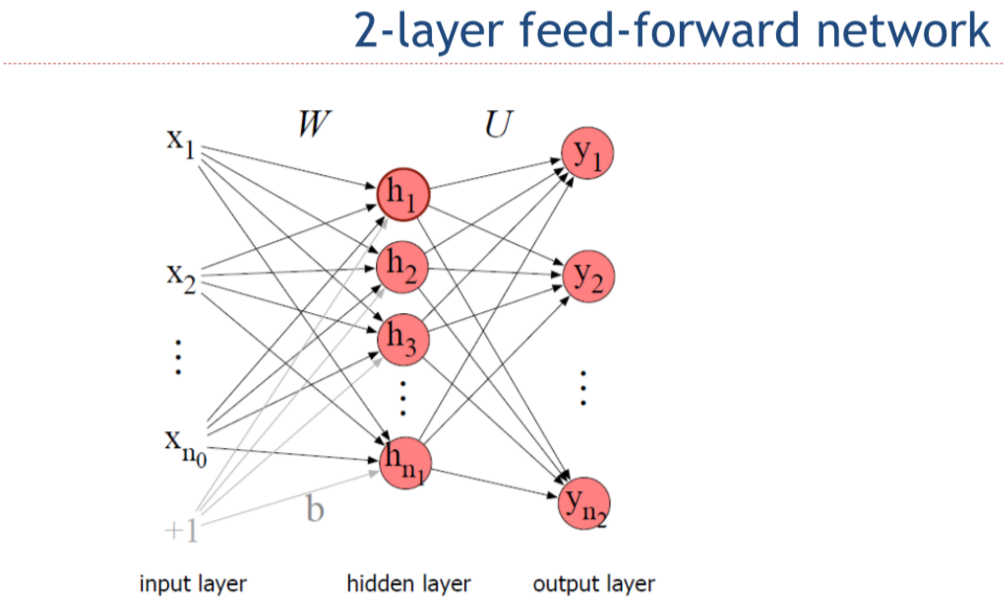


- $y = \sigma(Wx + b)$
  - output of the neural unit;
- $x = (x_1, .., x_n)$ is the input of the neural unit;
- W is the weight matrix of the neural unit;
- b is the bias vector of the neural unit;
- $z = Wx + b$ is the linear combination of the input and the weight matrix plus the bias vector.
- $f(z) = \sigma(z)$ is the activation function of the neural unit, where σ is the sigmoid function.

- other Activation function

  - tanh: $f(z) = tanh(z)$
  - ReLU: $f(z) = max(0, z)$
- the activation function is used to introduce non-linearity in the neural network.
- need non-linearity because the composition of linear functions is a linear function, and otherwise we dont have one W matrix for each layer but only one W matrix for all the layers: $W = W_1W_2...W_n$.
- the union of more neural unit without activation function would be the same of linear regression.

- FFNN is multilayer network with each layer composed of multiple neural units. There is at least a hidden layer, and every layer's output is the input of the next layer.

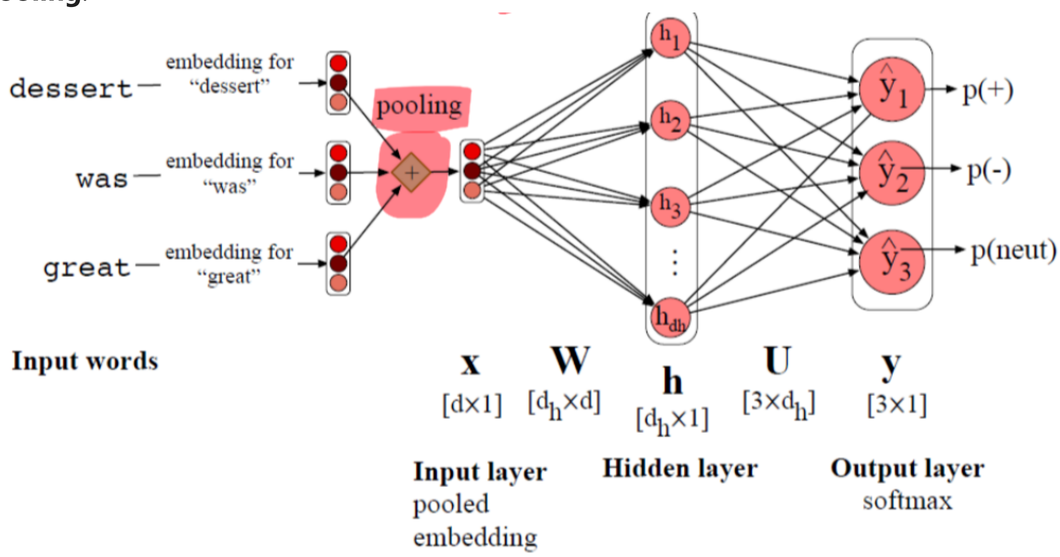


- $h = \sigma(Wx + b)$
- $z = U \times h$
- $y = softmax(z)$
- softmax is used to normalize the output of the network in a probability distribution.
- suppose to have a network with 3 layers:
  - input layer: $x = (x_1, x_2, x_3)$
  - hidden layer: $h = (h_1, h_2, h_3)$
    - $h_1 = \sigma(W_{11}x_1 + W_{12}x_2 + W_{13}x_3 + b_1)$
    - $h_2 = \sigma(W_{21}x_1 + W_{22}x_2 + W_{23}x_3 + b_2)$
    - $h_3 = \sigma(W_{31}x_1 + W_{32}x_2 + W_{33}x_3 + b_3)$
  - output layer: $y = (y_1, y_2, y_3)$
    - $y_i = \sigma(U_{i1}h_1 + U_{i2}h_2 + U_{i3}h_3 + b_i)$
  - $softmax(y) = (\frac{e^{y_1}}{\sum_{i=1}^{3} e^{y_i}}, \frac{e^{y_2}}{\sum_{i=1}^{3} e^{y_i}}, \frac{e^{y_3}}{\sum_{i=1}^{3} e^{y_i}})$
  - the output with softmax generate a probability distribution for each class to classify.
- W are the weight matrix of the layer;
- dimension of W is (n x m), where n is the number of neurons in the current layer and m is the number of neurons in the previous layer;
- b dim is (n x 1);
- U is the weight matrix of the output layer;
- dim of U is (k x n), where k is the number of classes and n is the number of neurons in the previous layer;

- **FNN for NLP (Classification)**:
  - We want to classify a sentence in a class.
  - We need to represent the sentence as a vector.

- in every case of embedding we have a matrix of size (V x d), where V is the size of the vocabulary and d is the dimension of the embedding.
- usually we work with sentences and not with words, so we need to represent the sentence as a vector.
- we need to represent the sentence of word as a single vector.

- **Pooling**:



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- there is various way to rappresent a sequence of vectors as a single vector in order to classify the sequence.
- we can concatenate all the vectors in the sequence and use the concatenation as a single vector.
  - can be not a good idea because the dimension of the vector is too big.
- Pooling is a way to represent a sequence of vectors as a single vector.
- Given a sequence of words $x_1, x_2, \ldots, x_n$.
- typically two type of pooling:
  - 1) **Max Pooling**:
    - $x_{pooled} = max(x_1, x_2, \ldots, x_n)$
    - x_{pooled} is the vector of the max of all the words in the sequence.
  - 2) **Average pooling**
    - $x_{pooled} = \frac{1}{n}\sum_{i=1}^{n} e(x_i)$
    - x_{pooled} is the average of all the words in the sequence.

- coming back to the classification task:

  - given for ex $x_pooled = mean(e(x_1), e(x_2), \ldots, e(x_n))$ as the pooled representation of the sentence.
  - $h = \sigma(Wx_pooled + b)$
  - $z = U \times h$
  - $y = softmax(z)$

- if we want classify a test set of m examples:

  - all input sequence are packed in a matrix X of size (m x d), where m is the number of examples and d is the number of attribute of x_pooled.

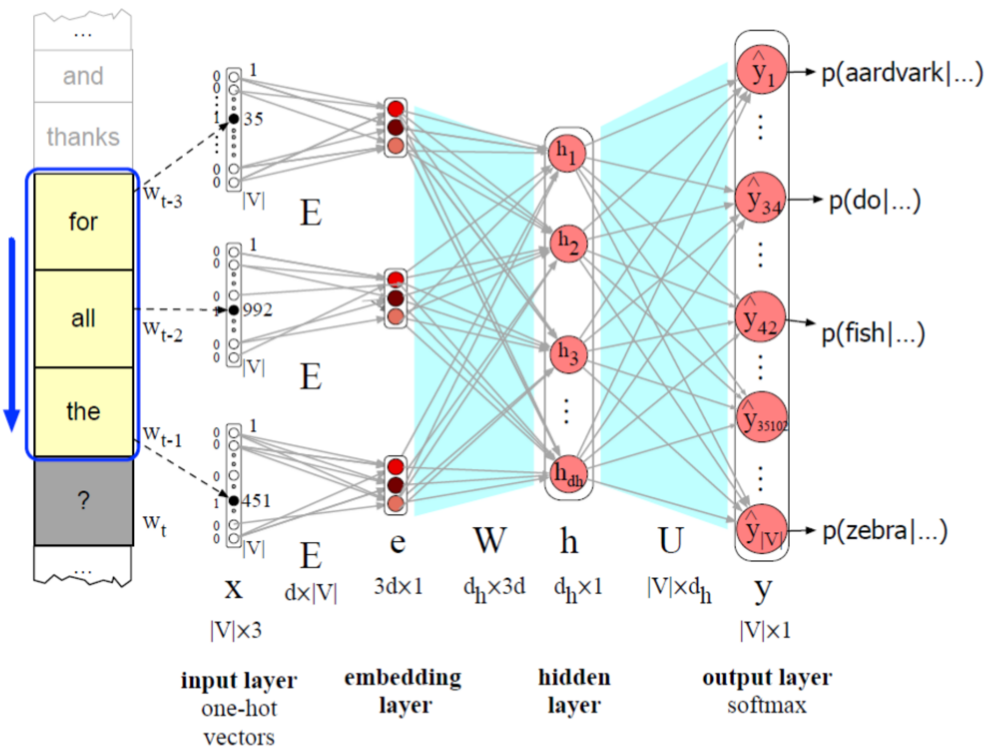
- **Feedforward Neural LM**

- the input at time t of the network is a sequence of words $w_1, w_2, \ldots, w_t$.
- the output at time t of the network is the probability distribution of the next word w_{t+1}.
- $P(w_{t+1}|w_1, w_2, \ldots, w_t)$
- assuming the words are independent:
  - $P(w_{t+1}|w_1, w_2, \ldots, w_t) = \prod_{i=1}^{t} P(w_{i+1}|w_1, w_2, \ldots, w_i) = P(w_2|w_1)P(w_3|w_1$
- so for a sequence of word the probability is:
  - $P(w_1, w_2, \ldots, w_n) = \prod_{t=1}^{n} P(w_t|w_1, w_2, \ldots, w_{t-1})$

- Because NLM rapresent words by their embedding rather than word identity as in n-gram LM

- word identity in n-gram LM is a sparse representation
- sparse representation is not good for generalization because we have a lot of unseen words.
- Using embedding allows NLM to generalize better unseen words.

- **Forward inference**



- F.inf. aka decoding:
  - is the task, given a sequence of words, to predict the next word.
  - each word is a one-hot vector of length V (one for each word in the vocabulary).
  - Embedding matrix E has a column for each word in the vocabulary and each column is the embedding of the word.
  - example of multipliction of one-hot vector with Embedding matrix:
    - $e(w_1) = E \times w_1$
    - $w_1 = [0, 0, 1]$
    - $E = \begin{bmatrix} 0.1 & 0.2 & 0.3 \\ 0.4 & 0.5 & 0.6 \\ 0.7 & 0.8 & 0.9 \end{bmatrix}$

- $e(w_1) = (0 * 0.1 + 0 * 0.4 + 1 * 0.7, 0 * 0.2 + 0 * 0.5 + 1 * 0.8, 0 * 0.3 + 0 * 0$
- multiply one-hot vector with only one nonzero el. and embedding matrix
- this multiplication selects out relevant column of the embedding matrix for word wi.
- Result is the embedding of the word wi.

- **Training Neural nets**:
  - we need loss function to train the network to understand the distance between the prediction and the true label.
  - common loss function for classification task is the cross entropy loss.
  - $L = -\sum_{i=1}^{k} y_i log(\hat{y}_i)$
  - where:
    - y_i is the true label;
      - in case of language model is the next word of source sentence.
    - $\hat{y}_i$ is the predicted label;
      - in case of language model is the probability distribution over the vocabulary of the possible next word.
    - k is the number of classes.
      - in case of language model is the size of the vocabulary.

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- we need to minimize the loss function.
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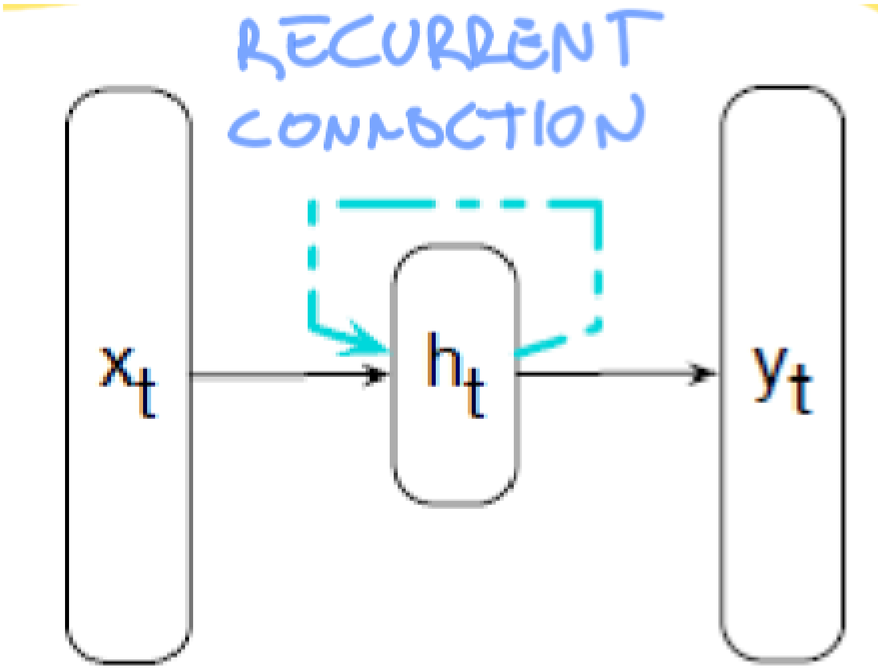
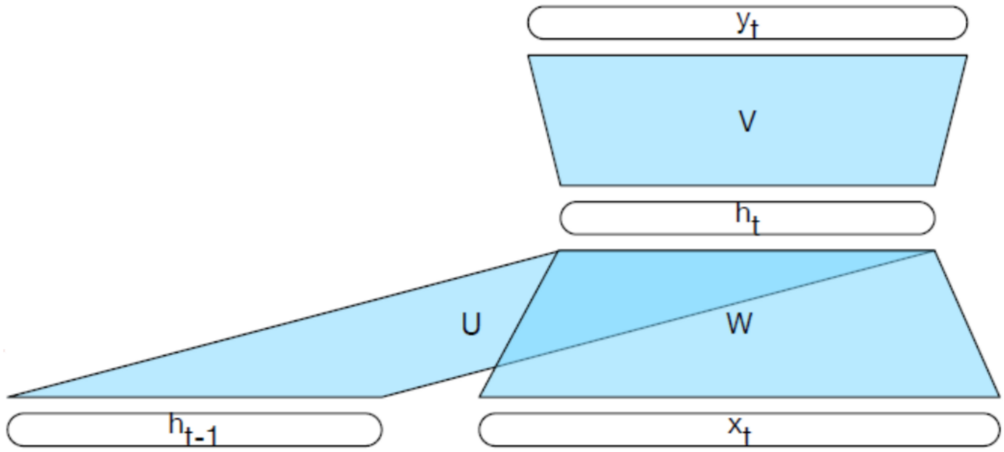
- BACKPROPAGATION:
  - is the algorithm to compute the gradient of the loss function with respect to the parameters of the network.
  - we need to update the parameters of the network to minimize the loss function
  - to do that we use the gradient descent.
  - we need to compute the gradient of the loss function with respect to the parameters of the network: $\nabla \omega L = (\frac{\partial L}{\partial \omega_1}, \frac{\partial L}{\partial \omega_2}, ..., \frac{\partial L}{\partial \omega_n})$
    - ω is the parameter of the network.
    - we use the chain rule to compute the gradient.
    - $\frac{\partial L}{\partial \omega} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial z} \frac{\partial z}{\partial \omega}$
    - $\omega_{new} = \omega_{old} - \eta \frac{\partial L}{\partial \omega}$
    - η is the learning rate.
  - this process is repeated until the loss function is minimized or until the number of epochs of training is reached.

2. RNN



- RNN are a class of neural network that can model sequential data.
- different from feedforward neural network is the recurrent connection.
- recurrent connection allows the network to maintain information about the past.
- is a form of memory.
- memory is stored in a hidden state vector h.
- hidden state is computed as:
  - $h_t = f(Uh_{t-1} + Wx_t)$
  - h_t is the hidden state vector at time t;
  - h_{t-1} is the hidden state vector at time t-1;
  - x_t is the input vector at time t;
  - U is the weight matrix for the recurrent connection;
  - W is the weight matrix for the input;
  - f is the activation function as sigmoid, tanh or relu.
  - the hidden state is in the hidden layer.
  - in LM the hidden state is the context of the sentence.
  - critical aspect: approach that does not impose a fixed length limit on previous context.
    - unlimited context is problematic because the hidden state vector loses information about the beginning of the sequence.
  - in fact h is a summary with loss of information of the past.
  - hidden state vector is updated at each time step;
  - so after a certain number of time steps the h will not contain information about the beginning of the sequence.

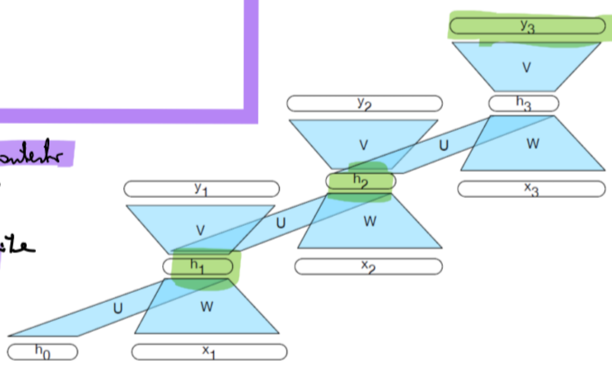
- A schematic representation of the RNN:



- **Inferenza in RNN as LM**

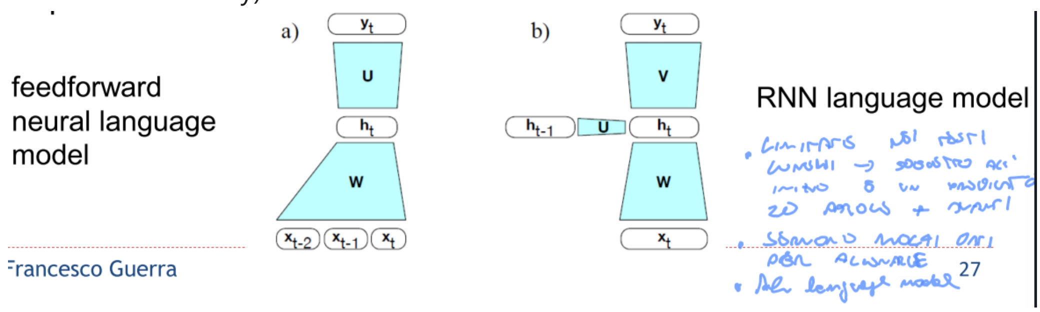
function FORWARDRNN(x, *network*) **returns** output sequence y

$h_0 \leftarrow 0$
for $i \leftarrow 1$ **to** LENGTH(x) **do**
  $h_i \leftarrow g(U\, h_{i-1} + W\, x_i)$
  $y_i \leftarrow f(V\, h_i)$
return y



h_{t-1} serve a tenere conto del contesto permettendolo di propagare lo stato passato nella rete.
h è un riassunto con perdite degli input passati e futuri
Es: in y_3, h_1 è meno rilevante di h_2

- similar to feedforward neural network.
  - the difference is that the input $x_t = [x_1, x_2, \dots, x_t]$ is a sequence of words embedded in a vector space.
    - each word is a one-hot vector of length V (one for each word in the vocabulary).



  - the output y_t is a probability distribution over the vocabulary of the possible next word.
- giving input with dimension d_{in}
- hidden layer with dimension d_h
- output is a vector with dimension d_{out}.
- $h_t = f(Uh_{t-1} + Wx_t)$
  - W is the weight matrix for the input of dimension $d_h \times d_{in}$;

- U is the weight matrix for the recurrent connection of dimension $d_h \times d_h$;
- $y_t = f(Vh_t)$
  - V is the weight matrix for the output of dimension $d_{out} \times d_h$;
  - f is usually a softmax function
  - $y_t = softmax(Vh_t)$

- RNN lenguage model process for each time step:

  - use word matrix E to retrieve the embedding of the current word x_t;
    - $e_t = Ex_t$
  - combining with the previous hidden state h_{t-1} to compute the new hidden state h_t;
    - $h_t = f(Uh_{t-1} + e_t)$
  - generate output layer from the hidden state h_t;
    - $o_t = Vh_t$
  - compute the probability distribution over the vocabulary of the possible next word;
    - $y_t = softmax(o_t)$
  - Training process:
  - this is a self-supervised learning approach.
  - so the training data is unlabeled.
  - the label is the next word in the input sequence, to be compared with the output of the network.
  - compute the loss function and minimize it with backpropagation (through time).

  - using Teacher forcing: the input at time t is the true label at time t-1.

    - $x_t = y_{t-1}$
  - Weight tying:

    - Use the same embedding matrix and output weight matrix($V = E^T$).
    - $e_t = Ex_t$
    - $h_t = f(Uh_{t-1} + We_t)$
    - $y_t = softmax(E^T h_t)$

    - this is useful because:

    - E and V are trained to do the same thing
    - E provides a embedding for each input word
    - V provide an embedding for all the next possible words
    - Using

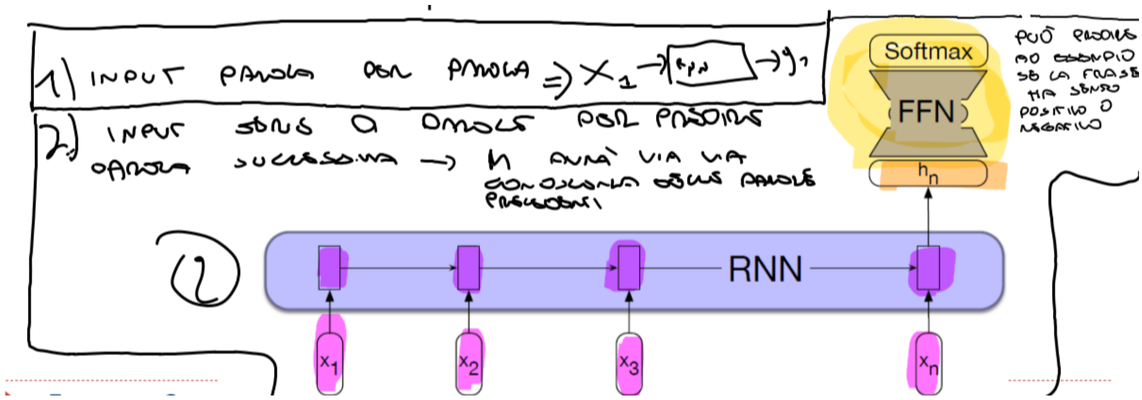
$$V = E^T$$

      we use a single set of embedding weights for both the input and output layers.
    - reduces the number of parameters to train.
    - improves the performance of the model.

- RNN for Sequence classification
  - perform text classification as:

- sentiment analysis;
- spam detection;

- Give a sequence of words, predict a single label.



Rnn's are difficult to train because of vanishing and exploding gradient problem. Lose information about the beginning of the sequence.

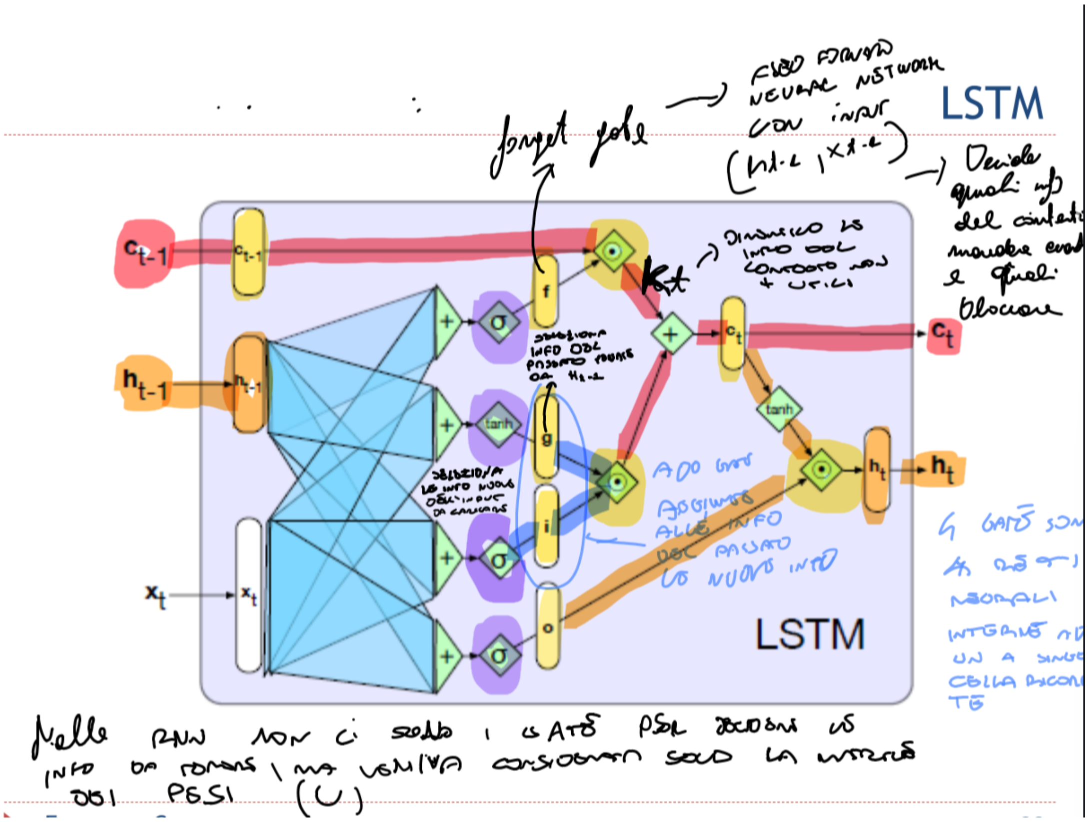
3. LSTM:

why lstm is introduced?

- to solve the problem of vanishing gradient and exploding gradient.
  - Maintain the context over the time.
  - removing information no longer needed from context
  - adding new information likely to be needed for later decision making.
- To manage the context, LSTM use gates.

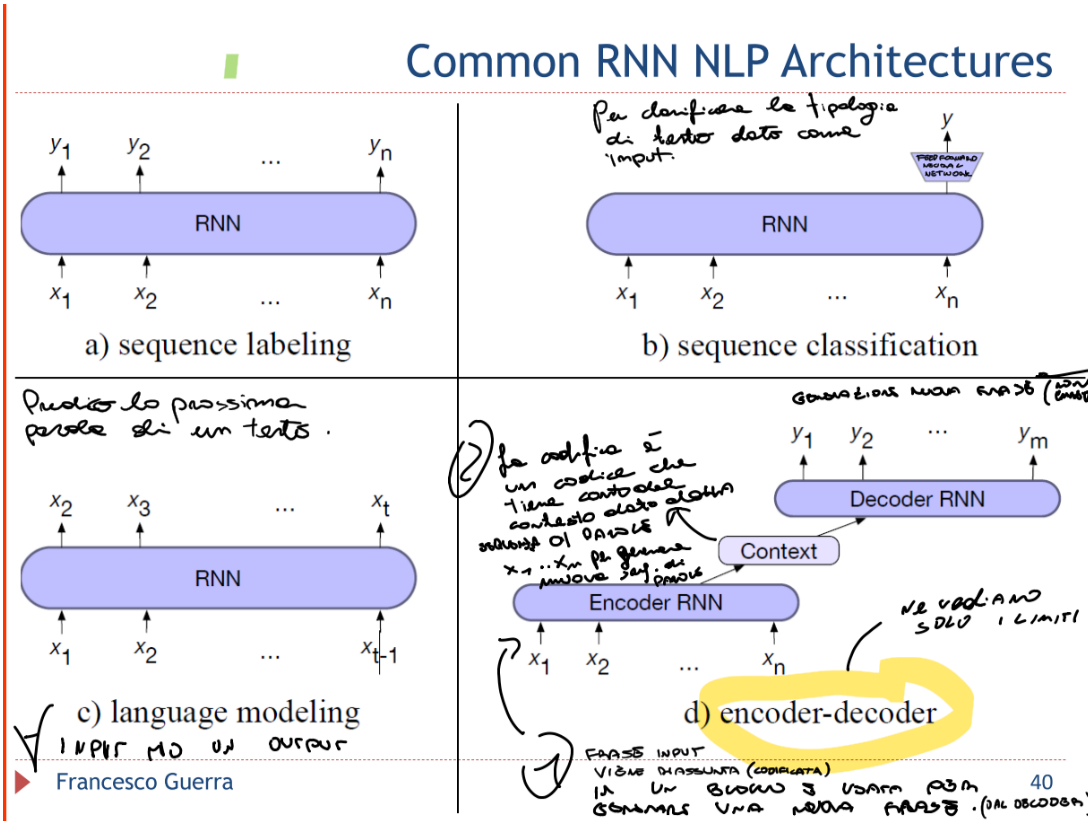
- Gates are neural network layer that control the flow of information.
- Gates are composed of -feedforward layer;
  - sigmoid neural net layer -> push output values between 0 and 1;
  - pointwise multiplication operation
    - combining this operation with sigmoid layer -> have effect similar to a binary mask -> allow or block information flow.

- **Gates**:



  - **forget gate**: compute what information to discard from the context vector.(info no longer needed)
  - $f_t = sigmoid(Uf * h_t + Wf * x_t)$
  - Modidfied context vector: $k_t = c_{t-1} \odot f_t$ (what delete from previous context vector)
  - g_t -> **select candidate state**: select what new information to add from current input and previous hidden state.
    - $g_t = tanh(Ug * h_t + Wg * x_t)$
  - **input gate**:
    - $i_t = sigmoid(Ui * h_t + Wi * x_t)$
  - **add gate**: select what information to add to the context vector from the candidate state.
    - $j_t = g_t \odot i_t$ (add gate)
  - **output gate**: *used to decide what information is required for the current hidden state.*
    - $o_t = sigmoid(Uo * h_t + Wo * x_t)$
  - **new context vector**
    - $c_t = k_t + j_t$
  - **New hidden state**:
    - $h_t = o_t \odot tanh(c_t)$

- **Common RNN NPL Architectures**:

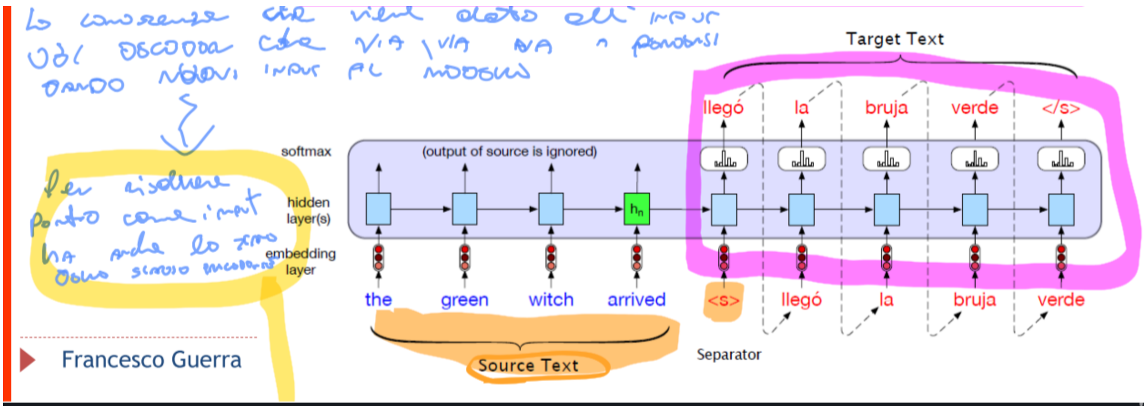


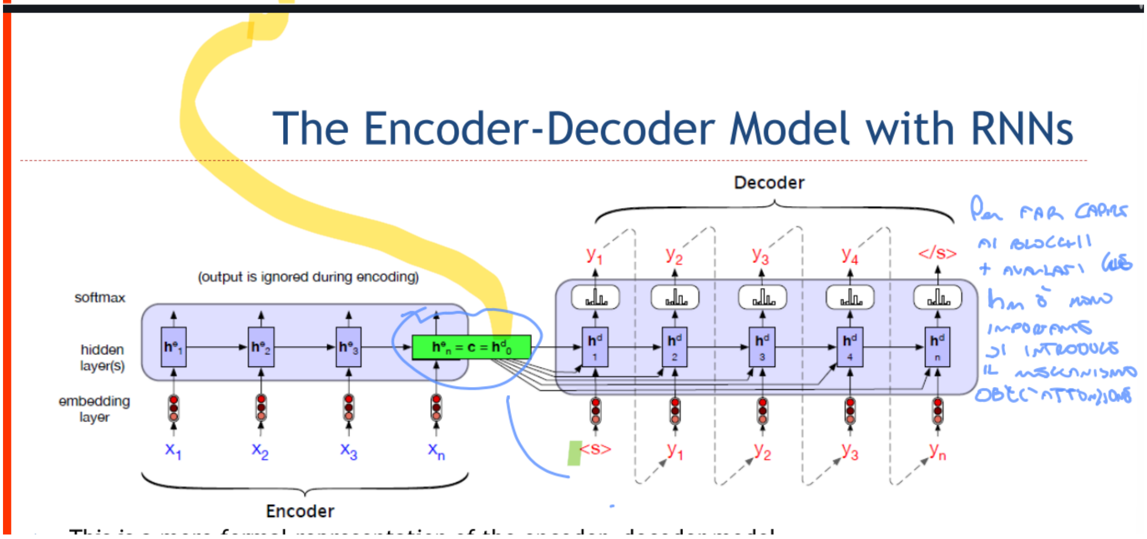
  - a) Sequence labeling (x1 -> y1, x2 -> y2, ...);
  - b) Sequence classification (x1, x2, ... -> y);
  - c) language modeling (x1 -> x2, x2 -> x3, ...);
  - d) encoder-decoder (x1, x2, ... -> y1, y2, ...);
  - $p(y|x) =$
  - $p(y1|x1)p(y2|x1,y1)p(y3|x1,y2,y3)\dots p(y_m|x1,\dots,y_m) = p(y|x)$
  - $= \prod_{t=1}^{m} p(y_t|x,y_1,\dots,y_{t-1})$
  - x is source sentence
  - y is target sentence.

- **Encoder-Decoder model with RNNs**:
  - take input sequence arbitrary length;
  - traslate it into another sequence of arbitrary length.;
  - task can be solved with encoder-decoder architecture:
    - summarization (input: long text, output: short text);
    - machine translation (input: text in one language, output: text in another language);
    - question answering (input: question, output: answer, as gpt3);
  - Encoder (can be LSTMs, CNN, Transformer):
    - 1) take input sequwnce $x = (x_1,\dots,x_n)$; and generate a contextualized representation of the input sequence $h = (h_1,\dots,h_n)$;

- 2) context vector c is derived from the final hidden state of the encoder h_n; -3) c is essence of the input to the decoder.
  - Decoder (can be every model that can generate sequence):
    - 4) Decoder generates from c a arbitrary length of hidden states $h = (h_1, \dots, h_m)$;
    - 5) h is used to generate the output sequence $y = (y_1, \dots, y_m)$;
- RNN lenguage modeling for each timestep t:
  - pass $t-1$ token through embedding layer;
  - Use forward inference to compute sequence of hidden states;
  - Use final hidden state to compute probability distribution over vocabulary;
    - $h_t = g(h_{t-1}, x_t)$
    - $y_t = f(h_t)$
- Thus giving:
  - x: source text;
  - y: target text;
- we get:

$$p(y|x) = p(y1|x)p(y2|x, y1)p(y3|x, y1, y2) \dots p(y_m|x, y_1, \dots, y_{m-1}) = \prod_{t=1}^{m} p(y_t|x, y_1, \dots$$



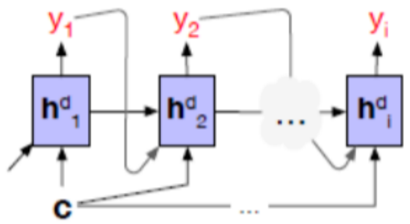


- Rnn's have problem of context influence in long term dependencies from source to target sentence.
- Introduction of attention mechanism to solve this problem.

4. Attention mechanism:

- allowing decoder to look at all the source words at each step of the decoding process.
- I.e. decoder get info from all hidden states of encoder, not just the last one.
  - **idea**:
    - create context vetor c_t as single-fixed length, taking a weighted sum of all the hidden states of the encoder.
    - weight select relevant part of the source sentence, as the decoder generate tokens of the target sentence.
    - encoder hidden state are different for each token in decoder -> context vector is dynamically derived at each step of decoding from the hidden states of the encoder.

$$\mathbf{h}_t^d = g(\hat{y}_{t-1}, \mathbf{h}_{t-1}^d, \mathbf{c})$$

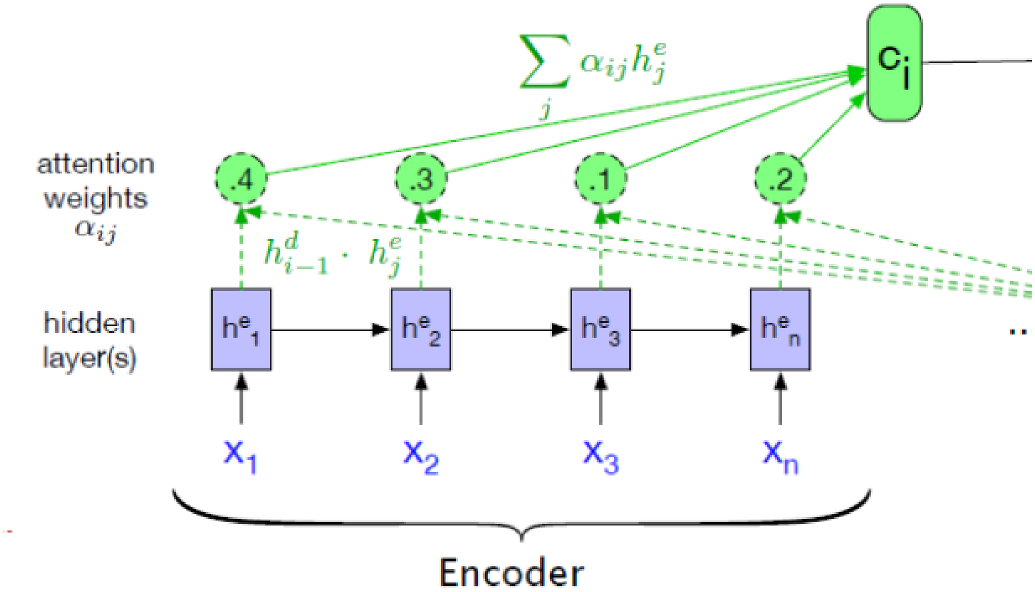


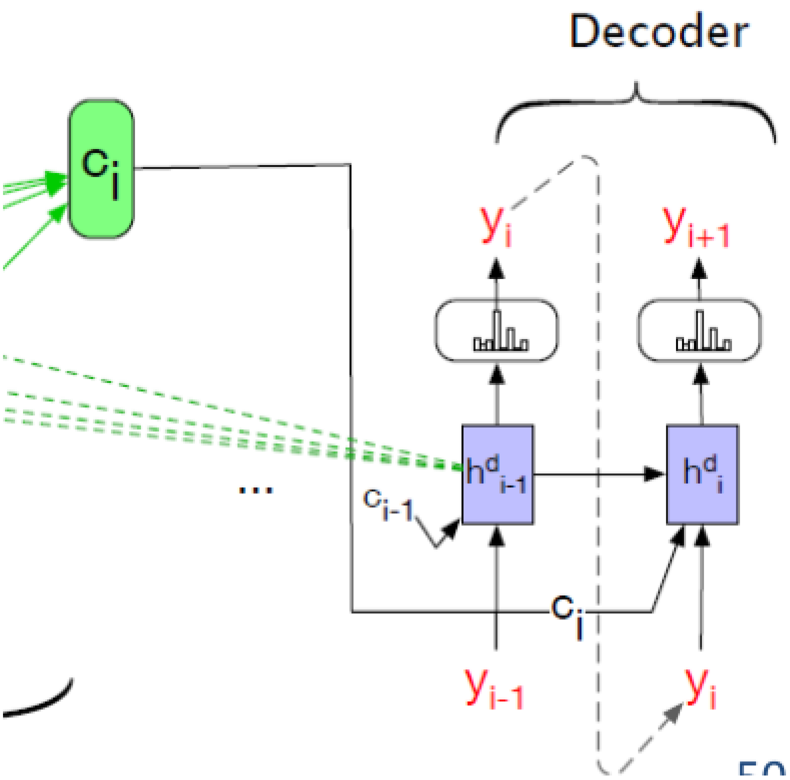
$$\begin{aligned} \mathbf{c} &= \mathbf{h}_n^e \\ \mathbf{h}_0^d &= \mathbf{c} \\ \mathbf{h}_t^d &= g(\hat{y}_{t-1}, \mathbf{h}_{t-1}^d, \mathbf{c}) \\ \mathbf{z}_t &= f(\mathbf{h}_t^d) \\ y_t &= \text{softmax}(\mathbf{z}_t) \end{aligned}$$

- computing C_i consider:
  - how to focus on each encoder hidden state he.
  - how relevant each encoder hidden state is to the current decoder hidden state hd_{i-1}.
    - introduce score to measure the relevance of each encoder hidden state during the decoding process.
- **simplest score function** (dot product attention (degree of similarity)):
  - $score(he_j, hd_{i-1}) = he_j * hd_{i-1}$ -> mesure the similarity between j-th encoder hidden state and i-th decoder hidden state.

- **vector of scores**: describe how relevance each encoder hidden state is to the current decoder hidden state.

- **softmax function**:
  - $\alpha_{ij} = softmax(score(he_j, hd_{i-1})) = \frac{exp(score(he_j, hd_{i-1}))}{\sum_{k=1}^{n} exp(score(he_k, hd_{i-1}))}$
  - give the weights to each encoder hidden state.
- **context vector**:
  - $c_i = \sum_{j=1}^{n} \alpha_{ij} * he_j$

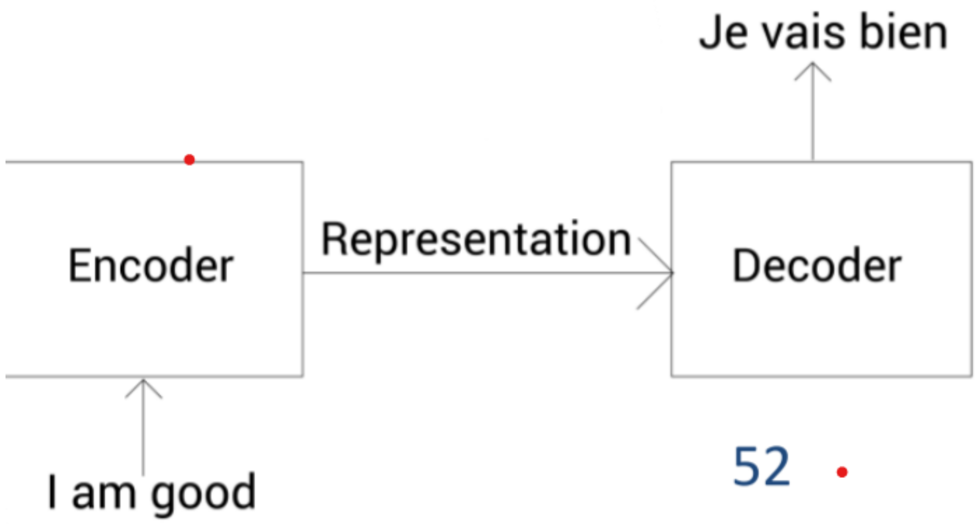




1. **Contextualized word embeddings**

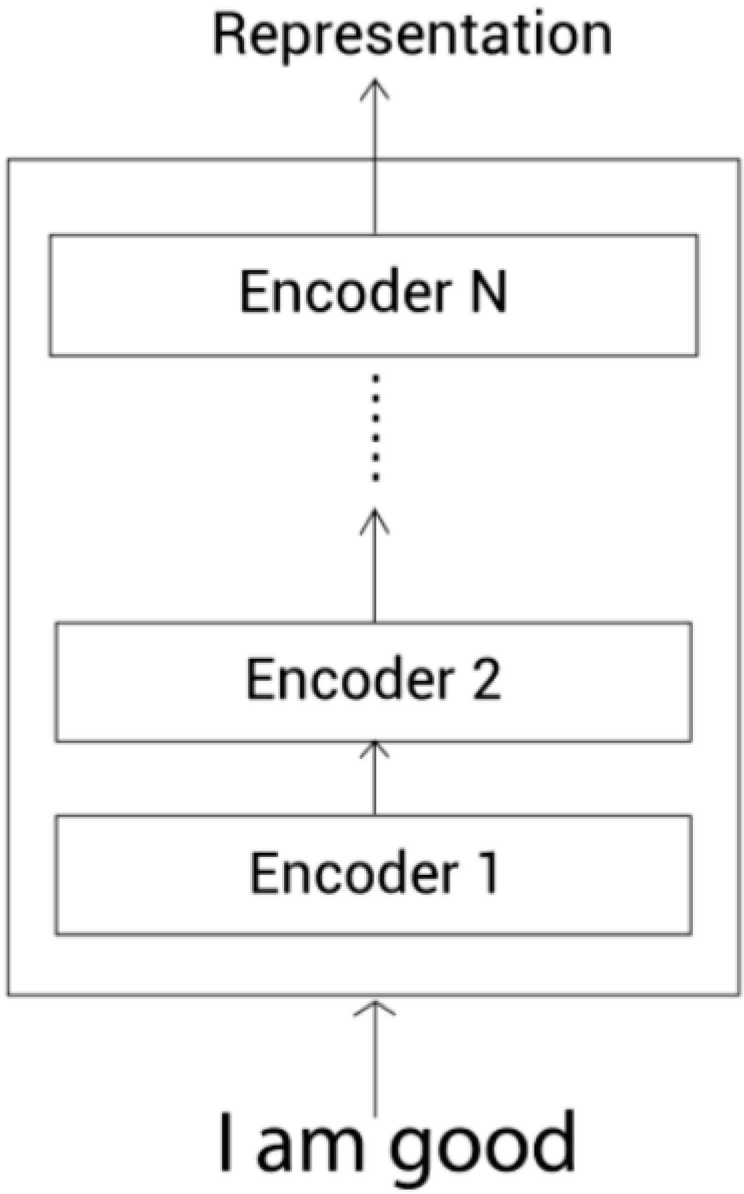
- **Trasformers**:

    - trasformer capture long term dependencies in the input sequence by using attention mechanism.
    - use self-attention to compute a representation of the input sequence.
    - is encoder-decoder architecture.
        - encoder:
            - input: sequence of tokens
            - encoder learns a representation of the input sequence.
            - output: give embedding of each token in the sequence to send to decoder.
        - decoder:
            - input: embedded words from encoder
            - output: target sequence of tokens.

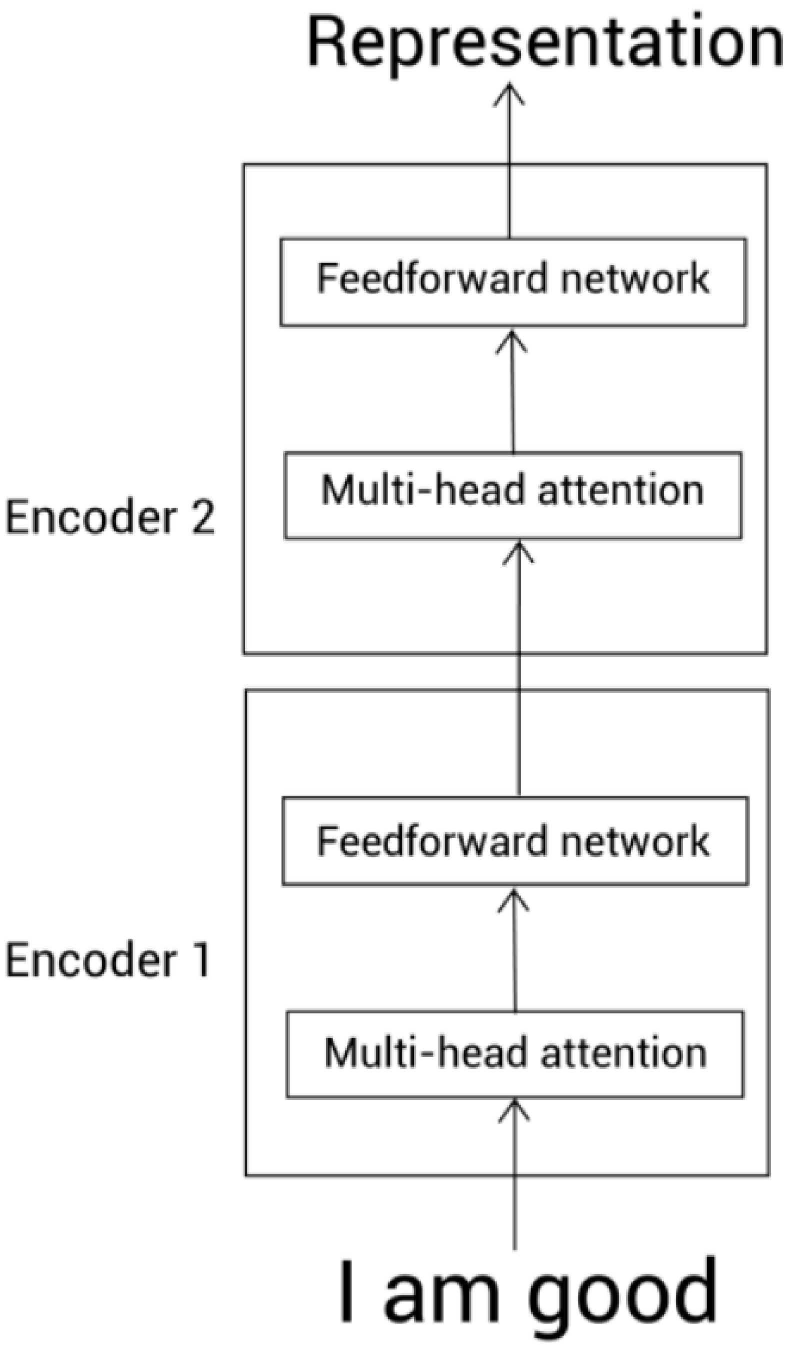


- **Trasformer encoder**:
    - consist on a stack of N encoders.

- output of each encoder is fed to the next encoder as input.

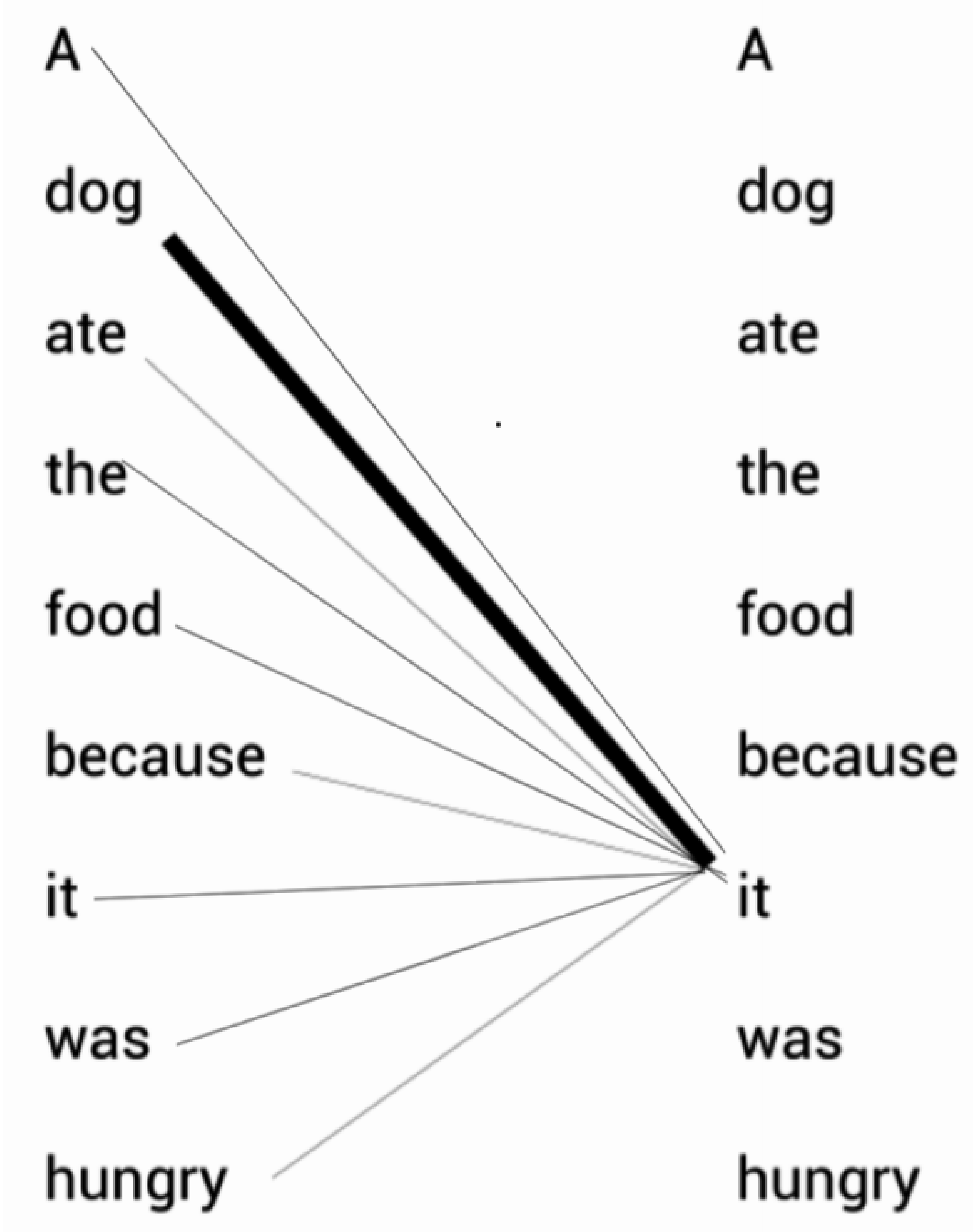


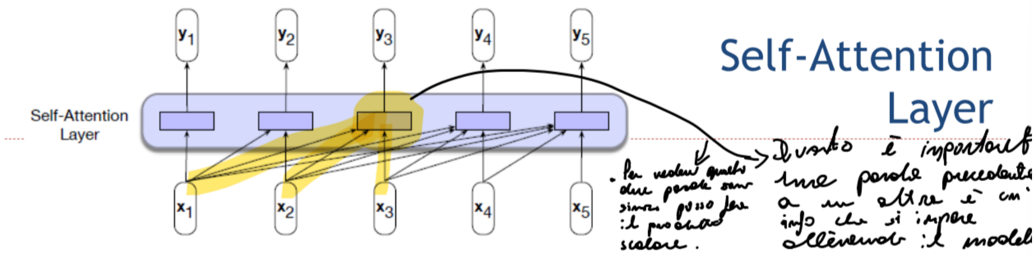
- each encoder is composed of two sublayers:
  - multi-head self-attention layer;
  - feed-forward network.



- **Self-Attention-Layer**:
    - Used to understand the relationship between different words in a sentence by computing a representation of the sentence that takes into account the

relationship between all words.

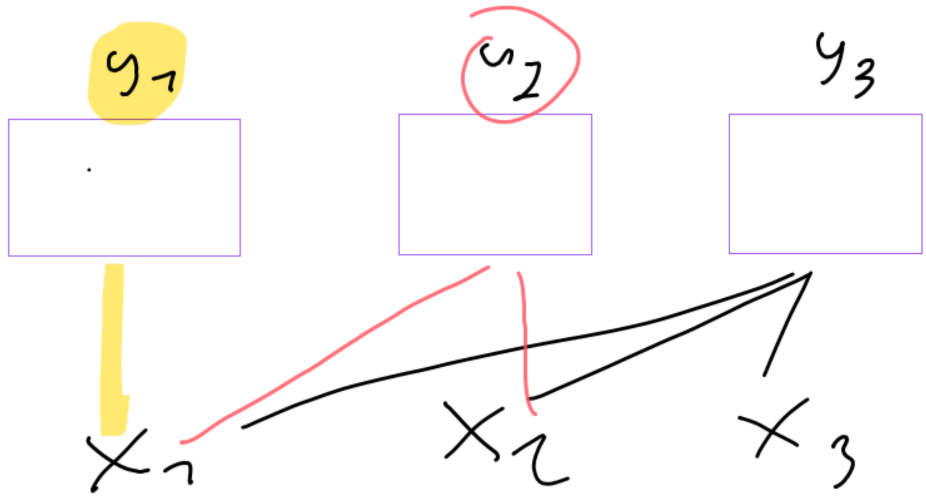




- maps input sequence of tokens $x = (x_1, \ldots, x_n)$ to sequence of vectors $y = (y_1, \ldots, y_n)$ of same length.
- to generate output y_m the model has access to every input token $(x_1, \ldots, x_m)$.
- this ensure to create a LM that can be use for auto-regressive generation i.e. generate one token at a time.

- **Self-Attention-mechanism**

- provide a way to compare a word of interest to other words in the same sentence, to retrieve their relevance in the current context.
- this comparisons are used to compute an output for current input word.



- es: to compute y_3 is necessary to compare x_3 with x_1, x_2 and x_3 itself.
- comparison in a self attention layer with dot-product:
- $score(x_i, x_j) = x_i * x_j$
- greater the dot product, more similar the two words are.
- Score is normalized using softmax function to obtain a probability distribution over all words in the sentence.
- $\alpha ij = softmax(score(x_i, x_j)) = \frac{exp(score(x_i,x_j))}{\sum_{k=1}^{n} exp(score(x_i,x_k))}, \forall j \leq i;$
- the probability distribution is used to compute a weighted sum of the input sequence.
- $y_i = \sum_{j \leq i} \alpha_{ij} * x_j$
- **idea**:
  - compute a representation of the input sequence by computing a weighted sum of the input sequence.
  - each input token is associated with three vectors:
    - Query vector q;
    - Key vector k;
    - Value vector v;
  - Introducing weight matrices W_q, W_k and W_v each of this are used to compute the query, key and value vectors for each input x_i.
  - W_q have dimension $d_{model} * d_q$;
  - W_k have dimension $d_{model} * d_k$;
  - d_{model} is the dimension of the input sequence;
  - d_q and d_k have to match to allow the dot product between query and key vectors;

        - W_v have dimension $d_{model} * d_v$;
        - **query vector**:
            - is the current focus of attention. Is used to compute the similarity between the current input token and the previous input tokens of the sequence.
            - $q_i = W_q * x_i$, the query vector for the i-th input token.
        - **key vector**:
            - is the previous input token respect to the current input, used to compute the score.
            - $k_i = W_k * x_i$, the key vector for the i-th input token.
        - **value vector**:
            - used to compute the weighted sum of the input sequence, aka the output fot the current focus of attention.
            - $v_i = W_v * x_i$, the value vector for the i-th input token.
        - **score function**:
            - $score(x_i, x_j) = \frac{q_i * k_j}{\sqrt{d_k}}$
                - measure the similarity between the actual focus of attention and the previous input token.
            - dot product has been scaled by $\sqrt{d_k}$ to avoid large values of the dot product, this avoid numerical issue during training.

N

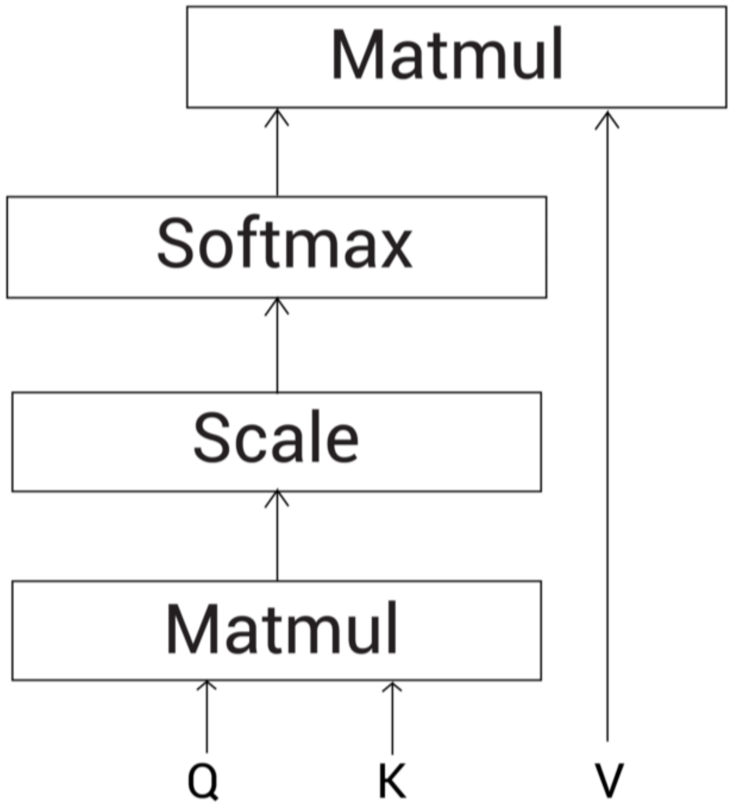
q1•k1	$-\infty$	$-\infty$	$-\infty$	$-\infty$
q2•k1	q2•k2	$-\infty$	$-\infty$	$-\infty$
q3•k1	q3•k2	q3•k3	$-\infty$	$-\infty$
q4•k1	q4•k2	q4•k3	q4•k4	$-\infty$
q5•k1	q5•k2	q5•k3	q5•k4	q5•k5

N

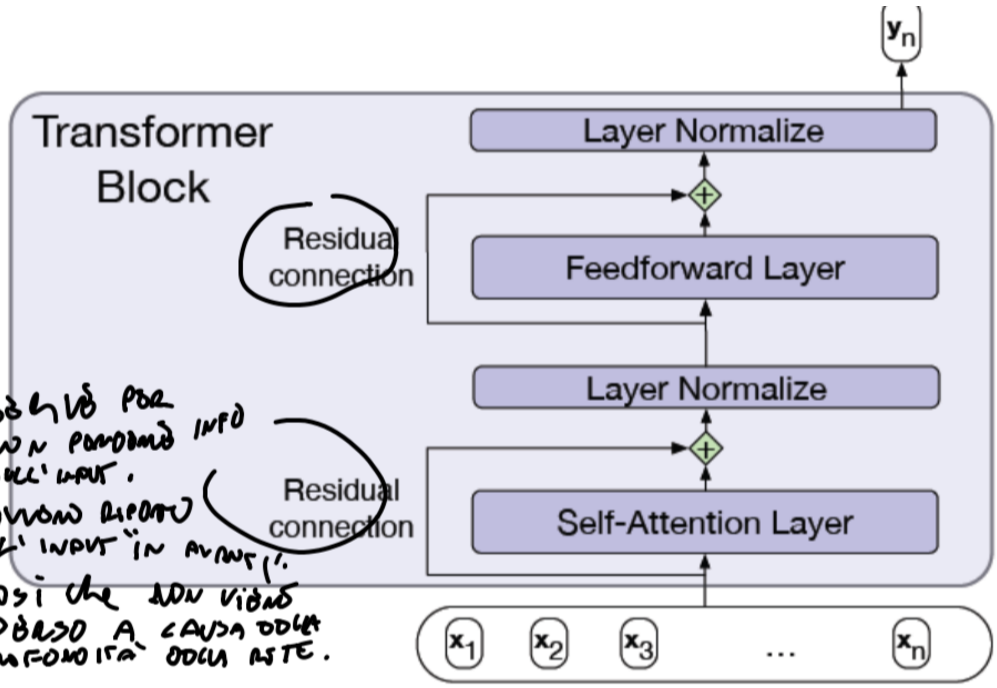
- Attention is quadratic in the length of the input sequence.

- most application have to limit the length of the input sequence to a certain number of tokens.

- **softmax function**:
  - $softmax(score(q,k)) = \frac{exp(score(q,k))}{\sum_{j=1}^{n} exp(score(q,k))}$
  - give the weights to each key vector.
  - **Output at time step i**:
  - $y_i = \sum_{j \leq i} softmax(score(q,k)) * v$
  - is the i-th output of the self attention layer.
  - computed as weighted sum of the value vectors.
  - each y_i is computed independently from the others.
  - this allow to compute the output in parallel.
  - entire process is parallelized by packing input sequence (embeddig) of N tokens into a single matrix $X \in \mathbb{R}^{N \times d_{model}}$.
  - $Q = X * W_q$
  - $K = X * W_k$
  - $V = X * W_v$
  - $SelfAttention(Q,K,V) = softmax(\frac{Q*K^T}{\sqrt{d_k}}) * V$



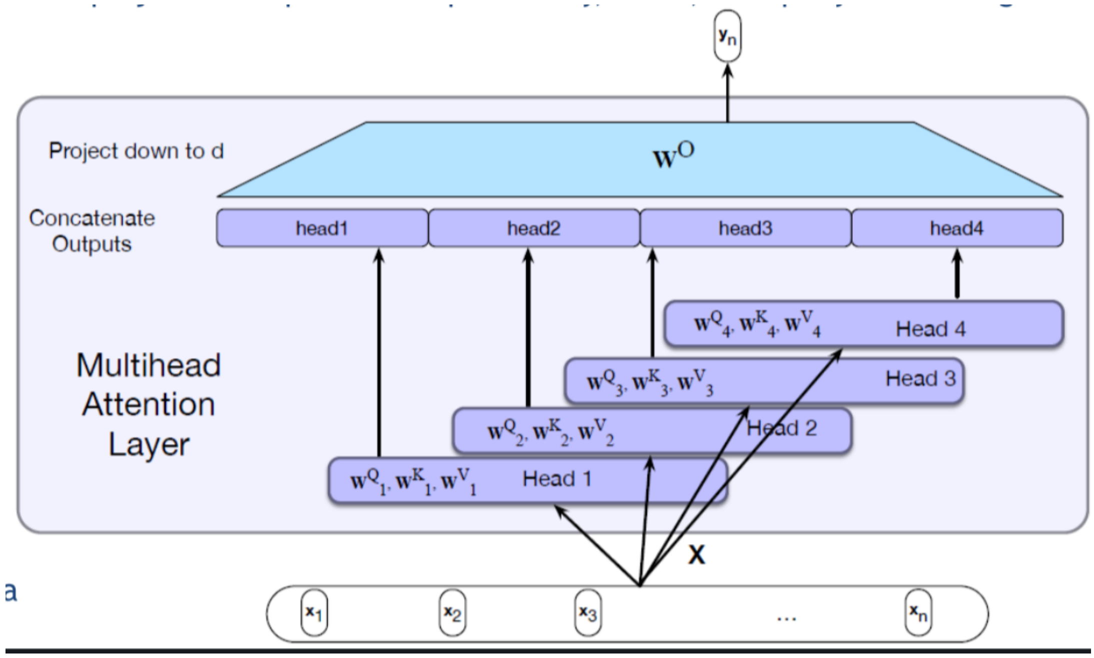
- **Trasformer Bloks**:



Utilizzato per non perdere info sull'input.
Quindi riporto l'input "in avanti".
Così che non viene perso a causa della profondità della rete.

  - Given input sequence $x = (x_1, \dots, x_n)$
  - pass it in self attention layer to obtain a sequence of vector $SelfAttention(x)$
  - to $SelfAttention(x)$ is added the input sequence x with a residual connection.
  - Residual connection is used to avoid to lose input information.
  - after residual connection is applied a normalization layer.
  - $LayerNorm(SelfAttention(x) + x)$
  - $LayerNorm = \gamma * \frac{x-\mu}{\sigma} + \beta$
  - γ and β are learnable parameters.
  - improve the training process because every input sequence will be in the same range of values.
  - having input in the same range of values, the model will be able to learn faster and avoid overfitting
- **multi-head attention**:

- Introduced to capture different relationships between different words in a sentence.
- use multiple self-attention layers in parallel to represent the input sequence in different ways.
- each self-attention layer is called head.
- each head is composed of three weight matrices W_i^Q, W_i^K and W_i^V.
- each head is computed independently and is a diifferent embedding of the input sequence.
- each head is concatenated and multiplied by a weight matrix W^O to obtain the final output of the multi-head attention layer y.



- $MultiHead(Q, K, V) = Concat(head_1, \dots, head_h) * W^O$
- $head_i = Attention(Q * W_i^Q, K * W_i^K, V * W_i^V)$
- $W_i^Q \in \mathbb{R}^{d_{model} \times d_k}$
- $W_i^K \in \mathbb{R}^{d_{model} \times d_k}$
- $W_i^V \in \mathbb{R}^{d_{model} \times d_v}$
- $W^O \in \mathbb{R}^{hd_v \times d_{model}}$
- $d_k = d_v = \frac{d_{model}}{h}$
- h is the number of heads.
- d_{model} is the dimension of the input sequence, aka the embedding dimension for each token.
- d_k and d_v are the dimension of the key and value vectors.

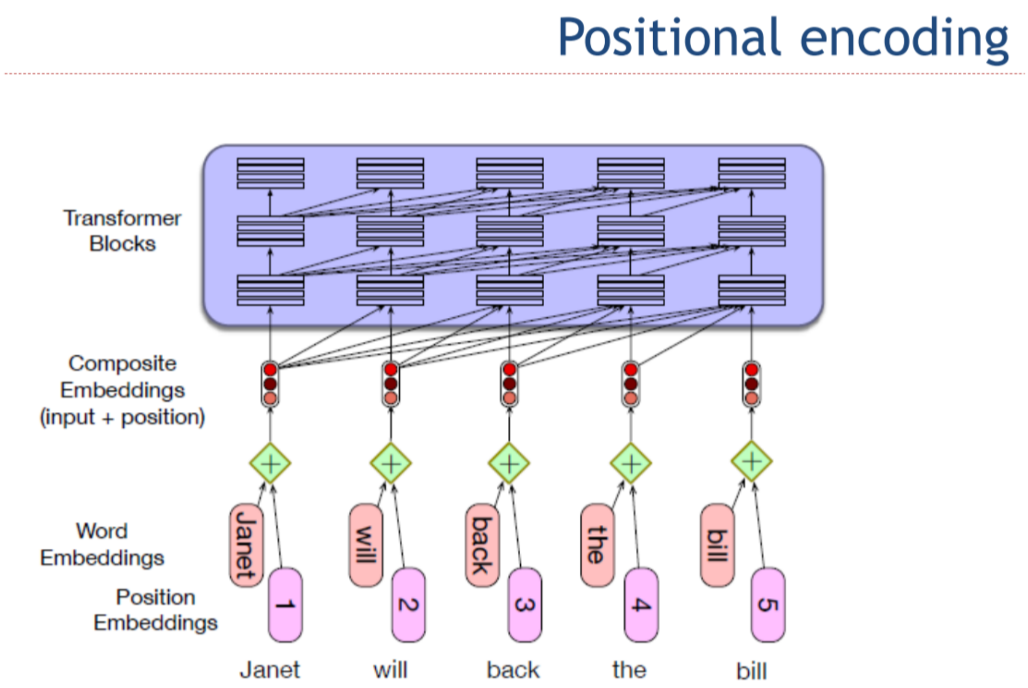
- **Positional Encoding**:
  - since we fed the input sequence to the self-attention layer in parallel, the self-attention layer is not able to capture the order of the input sequence.
  - to capture the order of the input sequence is added a positional encoding to the input sequence.
  - positional encoding is a vector of the same dimension of the input sequence.
  - each element of the positional encoding is a function of the position of the token in the input sequence.

$$X = \underbrace{\begin{bmatrix} 1.769 & 2.22 & 3.4 & 5.8 \\ 7.3 & 9.9 & 8.5 & 7.1 \\ 9.1 & 7.1 & 0.85 & 10.1 \end{bmatrix}}_{X} + \underbrace{\begin{bmatrix} 0 & 1 & 0 & 1 \\ 0.841 & 0.54 & 0.01 & 0.99 \\ 0.909 & -0.416 & 0.02 & 0.99 \end{bmatrix}}_{P}$$

$$= \begin{bmatrix} 1.769 & 3.22 & 3.4 & 6.8 \\ 8.14 & 10.44 & 8.51 & 8.09 \\ 10.0 & 6.68 & 0.87 & 11.09 \end{bmatrix}$$

(non necessario per esame sapere le formule)

- $PE_{(pos,2i)} = sin(pos/10000^{2i/d_{model}})$
- $PE_{(pos,2i+1)} = cos(pos/10000^{2i/d_{model}})$
- d_{model} is the dimension of the input sequence.
- $PE_{(pos,i)}$ is the i-th element of the positional encoding for the token at position pos



- **Basic idea of BERT**: (Bidirectional Encoder Representations from Transformers)
  - It is a context-based model
  - Unlike other embedding models like Word2Vec BERT is able to capture the meaning of a word based on the context in which it is used.
  - ex: Consider the following two sentences:
    - Sentence A: He got bit by Python.
    - Sentence B: Python is my favorite programming language.
    - The meaning of the word 'Python' is different in both sentences

- Context-free embedding model such as word2vec, give the same embedding of the word 'Python'.
- BERT will give different embeddings for the word 'Python' based on the context.