PyTorch practice

Multilayer Perceptron (MLP)

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Agenda

Multilayer perceptron (MLP)

For the purpose of this lecture, we'll stick to the task of image classification.

Let's assume we have a training dataset of N images

$$x_i \in \mathbb{R}^D, i = 1, \ldots, N$$

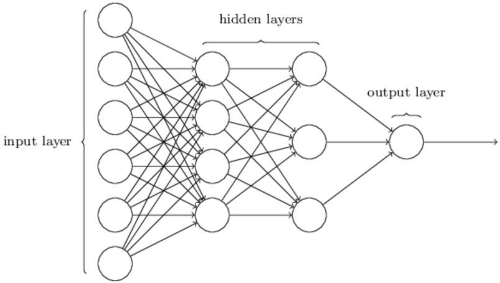
that we want to classify into K distinct classes.

Thus, training set is made by couples:

$$(x_i, y_i), \;\; where\; y_i \in \{1, \ldots, K\}$$

Our goal is to define a function $f : \mathbb{R}^D \mapsto \mathbb{R}^K$ that maps images to class scores.

When we connect an ensemble of neurons in a graph is when the magic happens and we get an actual **Multilayer perceptron (MLP)**.



Neural networks are arranged in **layers**, with one *input layer*, one *output layer* and *N hidden layers* in the middle.

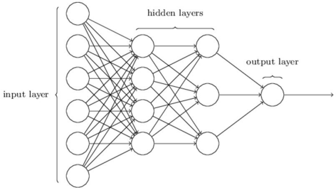
The network depicted here has a total of 47 *learnable parameters. Does this make sense to you?*

The 4-layer network previously depicted can be simply expressed as:

$$out = \phi(\mathbf{W_3}\phi(\mathbf{W_2}\phi(\mathbf{W_1}\mathbf{x}))) \tag{1}$$

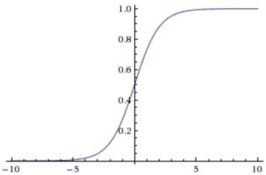
where:

- ϕ is the activation function
- $\mathbf{x} \in \mathbb{R}^6$ is the input
- $\mathbf{W_1} \in \mathbb{R}^{4x6}$ are the weights of first layer
- $\mathbf{W_2} \in \mathbb{R}^{3x4}$ are the weights of second layer
- $\mathbf{W_3} \in \mathbb{R}^{1x3}$ are the weights of third layer



Notice that to ease the notation biases have been incorporated into weight matrices W.

Activation functions are non-linear functions computed on the output of each neuron. There are a number of different activation functions you could use. In practice, the three most widely used functions have been *sigmoid*, *tanh* and *ReLu*. Nonetheless, more complex activation functions exist (*e.g.* [2, 1]).



Sigmoid nonlinearity has form $\sigma(x) = 1/(1 + e^{-x})$. Sigmoid function squashes any real-valued input into range $[0, 1]$. It has seen frequent use historically, yet now it's rarely used. It has two major drawbacks:

- saturates and kill the gradient
- output is not zero centered

Why using non-linear activations at all?

Composition of linear functions is a linear function. Without nonlinearities, neural networks would reduce to 1 layer logistic regression.

Let's say we have the following function (ϕ represent nonlinear activations):

$$f(\boldsymbol{x}) = \phi(\mathbf{W_2}\phi(\mathbf{W_1x}))$$

If we get rid of nonlinearities, this reduces to:

$$f(\boldsymbol{x}) = \mathbf{W_2W_1x} = \boldsymbol{Wx} \quad where \quad \boldsymbol{W} = \boldsymbol{W_2W_1}$$

which is clearly still linear.

Softmax Classifier generalizes Logistic Regression classifier to multi-class classification.

We first introduce the **softmax function**:

$$softmax_j(\mathbf{z}) = \frac{e^{z_j}}{\sum_k e^{z_k}}$$

It takes a vector of arbitrary real-valued scores $\mathbf{z}$ and squashes it to a vector of values between zero and one that sum to one.

e.g.

$$\mathbf{z} = \begin{bmatrix} 1.2 \\ 5.1 \\ 2.7 \end{bmatrix} \quad softmax(\mathbf{z}) = \begin{bmatrix} 0.018 \\ 0.90 \\ 0.08 \end{bmatrix}$$

Softmax Classifier

→ In pytorch è incluso nelle loss. quindi non va fatto.

In the softmax classifier the scores of linear function mapping $f(x_i, W) = Wx_i$ are interpreted as unnormalized log probabilities. To train the classifier we optimize the **cross-entropy loss**:

$$L = -\sum_i y_i^{true} log(y_i^{pred})$$

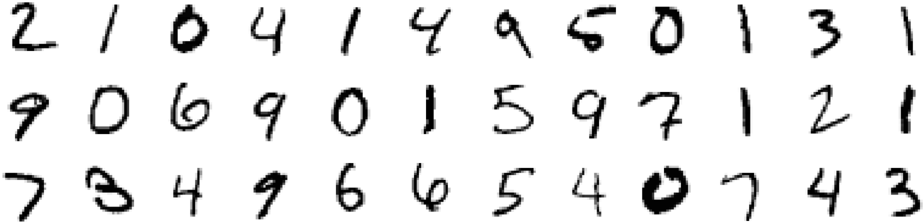
where y_i^{true} is the ground truth output distribution and y_i^{pred} is the predicted output distribution. In practice, y_i^{true} is a *one-hot* vector selecting the output of the right label.

Notice: softmax classifier has the appealing property to produce an easy-to-interpret output, that is the normalized score confidence for each class.

Goal

The **goal** of this practice is to **implement a fully-connected neural network to perform 10-class classification** on the MNIST dataset.

MNIST[3] is a database of handwritten digits consisting of 60K training images and 10K testing images. All digits have been centered in 28x28 grayscale images.



References

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[2] K. He, X. Zhang, S. Ren, and J. Sun.
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[3] Y. LeCun.
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