Ensemble methods: boosting

Machine Learning and Deep Learning

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Rationale

Boosting exploits the wisdom of the crowd:

- building many weak classifiers is easier than devising one complex prediction rule
- can I build a strong classifier out of a crowd of weak classifiers?

Boosting

Provides a general framework featuring:

- weak classifiers: slightly better than random chance
- cascade: each classifier is aware of previous ones errors
- the prediction rule is based on all classifiers' predictions

Boosting

Provides a general framework featuring:

- weak classifiers: slightly better than random chance
  - How?
- cascade: each classifier is aware of previous ones errors
  - How?
- the prediction rule is based on all classifiers' predictions
  - How?

Adaboost: setting

We are given

- a dataset D of examples $\{\vec{x_i}\}_{i=1}^{N}$ $\{y_i\}_{i=1}^{N}$, where $\vec{x_i} \in \mathbb{R}^d$ and $y_i \in \{-1, 1\} \quad \forall i = 1, \ldots, N$
- a predefined number of weak learners K

Adaboost: training algorithm

- initialize a weight w_i for each training example
- repeat for $j = 1, \ldots, K$:
  - from D, sample N examples with replacement and build a temporary dataset D_j. Each example has a probability of being sampled equal to w_i
  - build a weak classifier M_j on D_j
  - compute the error rate ϵ_j of M_j, and the relative *efficiency* α_j
  - update weights for all samples in D_j
    - if correctly classified: decrease the weight
    - if incorrectly classified: increase the weight

Adaboost: a closer look (1)

To build a weak classifier for D_j, repeat until $\epsilon_j < 0.5$

- randomly choose a data dimension
- randomly choose a threshold
- set all points above the threshold (along the chosen axis) to 1 and all points below to -1 (or viceversa)
- compute ϵ_j over D_j

Adaboost: a closer look (2)

To compute the error and the efficiency of a weak classifier M_j

$$\epsilon_j = \sum_{M_j(x_i) \neq y_i} w_i$$

$$\alpha_j = \frac{1}{2} \ln \left(\frac{1 - \epsilon_j}{\epsilon_j} \right)$$

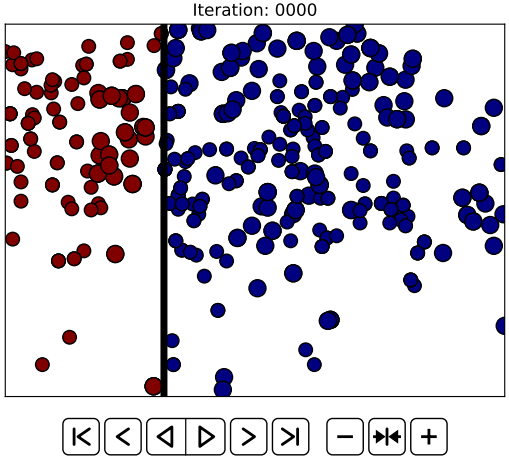
Adaboost: a closer look (3)

To update w_i:

$$w_i^{(j+1)} = \frac{w_i^{(j)}}{Z^{(j)}} \begin{cases} \exp^{-\alpha_j} & \text{if } M_j(x_i) = y_i \\ \exp^{\alpha_j} & \text{if } M_j(x_i) \neq y_i \end{cases}$$

$Z^{(j)}$ is a normalization factor that ensures $\sum_i w_i = 1$

Adaboost: learning

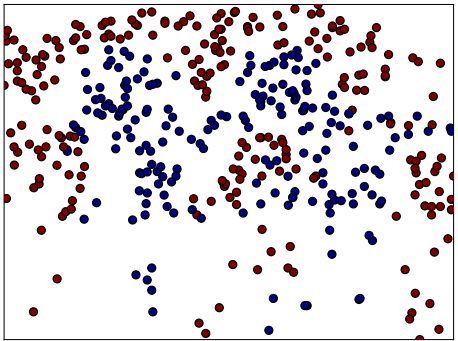


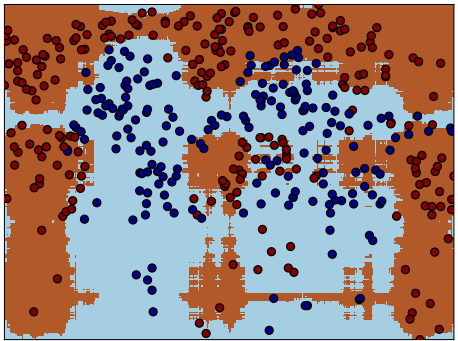
Adaboost: decision rule

At test phase, you can classify an unknown vector $\vec{u}$

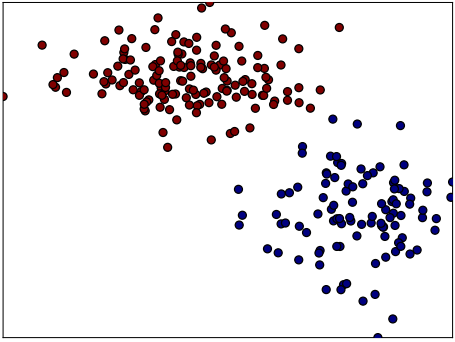
$$\mathcal{H}(\vec{u}) = \text{sign}\Big(\sum_{j=1}^{K} \alpha_j M_j(\vec{u})\Big)$$

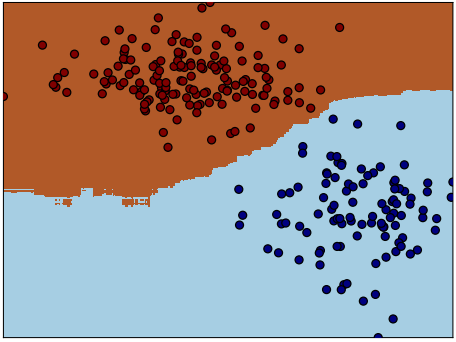
Adaboost: example (1)





Adaboost: example (2)





Adaboost: example (3)