# **Informe RSA**

Becerra Sipiran, Cledy Elizabeth	33,3%
Oviedo Sivincha, Massiel	33,3%
Villanueva Borda, Harold Alejandro	33,3%

### 1. Estructura del main()

int main(){
rsa bloques Elle(1024);

string message = "The idea of an asymmetric public-private key cryptosystem is attributed to Whitfield Diffie and Martin Hellman, who published this concept in 1976. They also introduced digital signatures and attempted to apply number theory. Their formulation used a shared-secret-key created from exponentiation of some number, modulo a prime number. However, they left open the problem of realizing a one-way function, possibly because the difficulty of factoring was not well-studied at the time.[4] Ron Rivest, Adi Shamir, and Leonard Adleman at the Massachusetts Institute of Technology, made several attempts over the course of a year to create a one-way function that was hard to invert. Rivest and Shamir, as computer scientists, proposed many potential functions, while Adleman, as a mathematician, was responsible for finding their weaknesses. They tried many approaches including knapsack-based and permutation polynomials. For a time, they thought what they wanted to achieve was impossible due to contradictory requirements. [5] In April 1977, they spent Passover at the house of a student and drank a good deal of Manischewitz wine before returning to their homes at around midnight. [6] Rivest, unable to sleep, lay on the couch with a math textbook and started thinking about their one-way function. He spent the rest of the night formalizing his idea, and he had much of the paper ready by daybreak. The algorithm is now known as RSA – the initials of their surnames in same order as their paper.[7] Clifford Cocks, an English mathematician working for the British intelligence agency Government Communications Headquarters (GCHQ), described an equivalent system in an internal document in 1973.[8] However, given the relatively expensive computers needed to implement it at the time, it was considered to be mostly a curiosity and, as far as is publicly known, was never deployed. His discovery, however, was not revealed until 1997 due to its top-secret classification. Kid-RSA (KRSA) is a simplified public-key cipher published in 1997, designed for educational purposes. Some people feel that learning Kid-RSA gives insight into RSA and other public-key ciphers, analogous to simplified DES.[9][10][11][12][13] A patent describing the RSA algorithm was granted to MIT on 20 September 1983 U.S. Patent 4,405,829 Cryptographic communications system and method. From DWPIs abstract of the patent The system includes a communications channel coupled to at least one terminal having an encoding device and to at least one terminal having a decoding device. A message-to-be-transferred is enciphered to ciphertext at the encoding terminal by encoding the message as a number M in a predetermined set. That number is then raised to a first predetermined power (associated with the intended receiver) and finally computed. The remainder or residue, C, is... computed when the exponentiated number is divided by the product of two predetermined prime numbers (associated with the intended receiver). A detailed description of the algorithm was published in August 1977, in Scientific Americans Mathematical Games column.[7] This preceded the patents filing date of December 1977. Consequently, the patent had no legal standing outside the United States. Had Cocks's work been publicly known, a patent in the United States would not have been legal either. When the patent was issued, terms of patent were 17 years. The patent was about to expire, on 21 September 2000, when RSA Security released the

algorithm to the public domain, on 6 September 2000.[14] There are a number of attacks against plain RSA as described below. When encrypting with low encryption exponents (e.g., e 3) and small values of the m, (i.e., m <n1e) the result of me is strictly less than the modulus n. In this case, ciphertexts can be decrypted easily by taking the eth root of the ciphertext over the integers. If the same clear text message is sent to e or more recipients in an encrypted way, and the receivers share the same exponent e, but different p, q, and therefore n, then it is easy to decrypt the original clear text message via the Chinese remainder theorem. Johan Håstad noticed that this attack is possible even if the cleartexts are not equal, but the attacker knows a linear relation between them. [22] This attack was later improved by Don Coppersmith (see Coppersmiths attack).[23] Because RSA encryption is a deterministic encryption algorithm (i.e., has no random component) an attacker can successfully launch a chosen plaintext attack against the cryptosystem, by encrypting likely plaintexts under the public key and test if they are equal to the ciphertext. A cryptosystem is called semantically secure if an attacker cannot distinguish two encryptions from each other, even if the attacker knows (or has chosen) the corresponding plaintexts. As described above, RSA without padding is not semantically secure.[24] RSA has the property that the product of two ciphertexts is equal to the encryption of the product of the respective plaintexts. That is m1em2e (m1m2)e (mod n). Because of this multiplicative property a chosen-ciphertext attack is possible. E.g., an attacker who wants to know the decryption of a ciphertext c me (mod n) may ask the holder of the private key d to decrypt an unsuspicious-looking ciphertext c cre (mod n) for some value r chosen by the attacker. Because of the multiplicative property c is the encryption of mr (mod n). Hence, if the attacker is successful with the attack, they will learn mr (mod n) from which they can derive the message m by multiplying mr with the modular inverse of r modulo n.[citation needed] Given the private exponent d one can efficiently factor the modulus n pg. And given factorization of the modulus n pg. one can obtain any private key (d.n) generated against a public key (e,n).[15] To avoid these problems, practical RSA implementations typically embed some form of structured, randomized padding into the value m before encrypting it. This padding ensures that m does not fall into the range of insecure plaintexts, and that a given message, once padded, will encrypt to one of a large number of different possible ciphertexts. Standards such as PKCS1 have been carefully designed to securely pad messages prior to RSA encryption. Because these schemes pad the plaintext m with some number of additional bits, the size of the un-padded message M must be somewhat smaller. RSA padding schemes must be carefully designed so as to prevent sophisticated attacks that may be facilitated by a predictable message structure. Early versions of the PKCS1 standard (up to version 1.5) used a construction that appears to make RSA semantically secure. However, at Crypto 1998, Bleichenbacher showed that this version is vulnerable to a practical adaptive chosen ciphertext attack. Furthermore, at Eurocrypt 2000, Coron et al.[25] showed that for some types of messages, this padding does not provide a high enough level of security. Later versions of the standard include Optimal Asymmetric Encryption Padding (OAEP), which prevents these attacks. As such, OAEP should be used in any new application, and PKCS1 v1.5 padding should be replaced wherever possible. The PKCS1 standard also incorporates processing schemes designed to provide additional security for RSA signatures, e.g. the Probabilistic fSignature Scheme for RSA (RSA-PSS). Secure padding schemes such as RSA-PSS are as essential for the security of message signing as they are for message encryption. Two USA patents on PSS were granted (USPTO 6266771 and USPTO 70360140); however, these patents expired on 24 July 2009 and 25 April 2010, respectively. Use of PSS no longer seems to be encumbered by patents.[original research] Note that using different RSA key-pairs for encryption and signing is potentially more secure.[26]";

```
cout << "\nmessage: " << message << endl;
ZZ n;
ZZ e;
cout << "\nn: "; cin >> n;
cout << "\ne: "; cin >> e;

cout << "\nmensaje cifrado" << endl;
cout << Elle.encrypt(Elle.dividirBloques(message),n,e) << endl;</pre>
```

```
string firma = "Cledy Becerra, 71666666, 11-333-1111"; cout << "\nfirma: \n" << endl; cout << Elle.sign encrypt(firma,n,e);
```

#### /\*string message =

5460282730935308155891";

//string message =

"036713981774726449951121691785546039832472597384630845817817419399289336436327086 6262848388881148254640293072476402932547478903007944586194026374201914857523791389

```
cout << "\nmensaje: \n" << message << endl;
cout << "\nmensaje decifrado: " << endl;
cout << Elle.decrypt(message) << endl;*/
/*ZZ n=</pre>
```

conv<ZZ>("853991922719896096503152601516856098693891821176223242685212919844208112 8606266812965474423514260301370452733540760560658962552656301537875246168230782105

 $2579684275448776212306112922545681109178618798090130834664457261301431717909117641\\4240666481797844358448664672963675776535072548378114898923212457412662294522440252\\7862321175537556593241910726554277889051590319896721594908297650416866781267903004\\4230054587815089927460923550663681303107337679126994703508166888754057212216526773\\3614789006301134608595826246786118625402728175031177671563655976662151785830751312\\8592910977571611257319190630187290454598957622902171");$ 

77.e =

27564";

conv<ZZ>("835557209532937823961444966612144536310933339452294518435136096991532776 2283885135927417629864524571641058079948221737692176994591249324455944905035840775 2924921797079034310359090205882622610610193614608737352128716359413459308157267751 487567975401279393207985294366538400151500759751517041559305331773327601");

string firma =
"013370792928678654627281398596604959279025247936896396384234606374010519952712755
3880389242335140489950994729070620566667513397358600117079279076501143160853099267
0089855600095241556849467581874898147176415592241961980255278976254715788516636912
5974093955115194216885896318925781867239739737195823540084845729975065931512990538
4982950609042953615233430146803237145852544881166617002059490265334080334979593578
44793446467746517171437678010557500556432321556353333642258199267581851043903510834
8606123094867694614303214940971635340718730123916177129401058378745407978509592864
2197697265753154592998833084746036004352484302598941690137816474219242302085339122
6584586519410935060137709999884902563272543231836792533276251231318515286872040095
3929212369614154249979818023143892227214082829168331025547808944724431251785124073
2444922392761730906454202881917678055140942103094107336344693056132712409426444003
6213516235540573042448226494173995242666764649604000765555773914019338675257165506
8332145397932430996507881357625323177857702250273048339487478211592172568543394528
5307548286657766056719992392127917328754581122544879466770546154778594201881939057

```
cout << "\nfirma: \n" << firma << endl;
cout << "\nfirma: " << endl;
cout << Elle.remove_sign(firma,n,e);*/
return 0;</pre>
```

#### 2. Generación de claves

o Generación de números aleatorios

// En esta función accedemos a la arquitectura del computador donde en la variable processInfo guardamos los procesos:

```
void random::fillWithMemoryInfo() {
   DWORD aProcesses[1024], cbNeeded, cProcesses;
   if (EnumProcesses(aProcesses, sizeof(aProcesses), &cbNeeded)) {//obtiene los
   processes en aProcesses
        cProcesses = cbNeeded / sizeof(DWORD);
```

//Calcula cuántos procesos fueron retornados.

```
for (int i = 100; i < cProcesses; i++){
                 HANDLE hProcess = OpenProcess(PROCESS QUERY INFORMATION |
PROCESS VM READ, FALSE, aProcesses[i]);
                 if (NULL != hProcess) {
                       PROCESS MEMORY COUNTERS pmc;
                       if (GetProcessMemoryInfo(hProcess, &pmc, sizeof(pmc))) {
processInfo+=(to string(pmc.PageFaultCount)+to string(pmc.WorkingSetSize)+
to\ string(pmc.QuotaPagedPoolUsage) + to\_string(pmc.QuotaNonPagedPoolUsage) + to\_str
string(pmc.PeakPagefileUsage));
                 CloseHandle(hProcess);
}
//Generamos nuestra semilla
vector<int> random::generateSeed(){
      if((processInfo.size()<contador+15)||processInfo.empty()){
           processInfo.clear();
           fillWithMemoryInfo();
           contador=0;
      vector<int> k;
      for(int i=0, j=contador; i < 5; i++, j+=3)
           int n=stoi(processInfo.substr(j,3));
           while(n > 255) n > = 1;
           while(n < 128) n < < = 1;
           k.push back(n);
      contador+=15;
      return k;
vector<bool> random::RC4(vector<int> semilla){
      vector<int> Or;
      {//permite que S solo pertenezca a este scope
           vector<int> S;
           for(int i=0;i<256;i++) S.push back(i);
            {//permite que k solo exista en este scope, optimiza memoria
                  vector<int> K;
                  for(int i=0,k=0;i<=51;i++)
                       for(int j=0; j<5; j++,k++)
                              K.push back(semilla[j]);
                  for(int i=0, f=0; i<256; i++){
                       f=modint(f+S[i]+K[i], 256);
                       swap(S[i],S[f]);
                  }
           for(int i=0,f=0,k=0;k<8;k++){
                 i=modint(i+1, 256);
                  f=modint(f+S.at(i), 256);
                  swap(S.at(i), S.at(f));
                  Or.push back(S.at(modint(S.at(i) + S.at(f), 256)));//t
```

```
vector<bool> out;
  for(int i=0; i<8; i++){
    bitset<8> aux(Or[i]);
    for(int j=0; j<8; j++)
       out.push back(aux[j]);
  return out;
void izqRotate(vector<bool> &vec, int times){
  //Rota times veces el vector llevando el primero al ultimo y borrando los primeros
  vec.insert(vec.end(),vec.begin(),vec.begin()+times);//Agrega al final del vector
times elementos del inicio del vector
  vec.erase(vec.begin(),vec.begin()+times);//borra los primeros times valores.
}
//Arrays que tienen maximo como número máximo 64 y son al azar
//PC 1 tiene 56 números y PC 2 tiene 48
int
PC 1[]={56,48,40,32,24,16,8,0,57,49,41,33,25,17,9,1,58,50,42,34,26,18,10,2,59,51,4
3,35,62,54,46,38,30,22,14,6,61,53,45,37,29,21,13,5,60,52,44,36,28,20,12,4,27,19,11,
3};
PC 2[]={13,16,10,23,0,4,2,27,14,5,20,9,22,18,11,3,25,7,15,6,26,19,12,1,40,51,30,36,
46,54,29,39,50,44,32,47,43,48,38,55,33,52,45,41,49,35,28,31};
int rotaciones[]={1,1,2,2,2,2,2,2,1,2,2,2,2,2,1,1,1,2,2,2,2};//propia del DES
//PC elección permutada
//Hacemos uso de la generación de claves de DES
//Donde teóricamente se usan 56 bits de la clave
//Teóricamente son 64 bits del PC 1 pero por paridad que no la necesitamos usamos
sólo 56 bits
//Teóricamente se usan 48 bits de la subclave para PC 2
vector<br/>bool> DES(int bits=1024){
  vector<bool> K;//para guardar todas las k
  vector<br/>bool>k=RC4(generateSeed());//64 bools o bits
  int nBits=bits/48+1;
  vector<bool> c; vector<bool> d;
  for(int i=0;i<28;i++) c.push back(k.at(PC 1[i]));//Añade cierta posición de k en c
entre las posiciones adquiridas esta los primeros 28 números de PC 1
  for(int i=28;i<56;i++) d.push_back(k.at(PC_1[i]));//Añade cierta posición de k en d
entre las posiciones adquiridas está de 28 a 56 números de PC 1
  for(int i=0;i \le nBits;i++)
    izqRotate(c,rotaciones[i]);
    izqRotate(d,rotaciones[i]);//Rotara los vectores rotaciones[i] veces
    vector<br/>bool> both(c.begin(),c.end());//Vector duplicamos con los valores de c
    both.insert(both.end(),d.begin(),d.end());//Agregamos todo el vector de d
     for(int i=0;i<48;i++) K.push_back(both.at(PC_2[i]));//Agregamos a k cierta
posición de both entre las posiciones 0-48 de Pc 2
```

```
K.resize(bits);//Modifica el k para quedar con cierto número de bits
          K[0]=1; K[bits-1]=1;//Primer y ultimo valor de k sera 1
          return K;
        }
        //Convierte un vector booleano a ZZ tomándolo como número binario
        ZZ random::generate random(int bits){
          return Random Number(DES(bits));
        }
o Generación de primos
        bool Miller Rabin(ZZ d, ZZ n){
          ZZ a; a=2;
          ZZ x = Power mod(a, d, n);
          if (x == 1 || x == n-1) return true;
          while (d != n-1){
             x = modulo((x * x), n);
             d *= 2;
             if (x == 1) return false;
             if (x == n-1) return true;
          return false;
        bool isPrime(ZZ n){
          ZZ k; k=0;
          if (n \le 1 \text{ or } n == 4) return false;
          if (n \le 3) return true;
          ZZ d = n - 1;
          while (even(d)){
             //d /= 2;
             d >>= 1:
             k++;
          for (int i = 0; i < k+1; i++)
             if (!Miller Rabin(d, n))
               return false;
          return true;
        }
        ZZ gen_prime(ZZ n){//Encuentra el primo más cercano a un número aleatorio
          ZZ a= modulo(n, ZZ(6));
          n-=a+ZZ(5); //para obtener la forma 6n+5
          //Los números primos tienen la forma de 6n+1 y 6n+5, no necesariamente todos
        los números obtenidos con esa
           if(isPrime(n)) return n;//Si es primo retorna ese valor
          else\{//Caso\ contrario\ busca\ los\ siguientes\ números\ de\ la\ forma\ 6n+1\ y\ 6n+5
        hasta encontrar un primo
             while(true){
```

```
n+=2;
if(isPrime(n)) return n;
n+=4;
if(isPrime(n)) return n;
}
}
}
```

o Inversa

```
//evalúa si mcd es 1
bool Existe Inversa(ZZ x, ZZ y){
  if(even(x)&&even(y)) return 0;
  while(x!=0)
    while(even(x)) x=x>>1;
    while(even(y)) y=y>>1;
    ZZ t=valAbs((x-y))>>1;
    if(x>=y) x=t;
    else y=t;
  return y==ZZ(1);
//Determina si es par o impar
bool even(ZZ a){
  ZZ r=(a>>1)<<1;
  if(r<0) r=ZZ(2)+r;
  return r==a;
//Convertir en positivo-Valor Absoluto
ZZ valAbs(ZZ a){
  if (a<0) return (a*-1);
  return a;
}
```

#### o Euclides extendido

//Implementamos para Algoritmo Binario del MCD y lo combinamos con el extendido de euclides para retornar un bool para saber si tiene inversa o no.

```
ZZ inversa(ZZ r1, ZZ r2) {
    ZZ s1=ZZ(1), s2=ZZ(0), b=ZZ(r2);
    if (Existe_Inversa(r1, r2)) {
        while (r2>0) {
            ZZ q=r1/r2;
            ZZ r=r1-q*r2;
            r1=r2;
            r2=r;
            ZZ s=s1-q*s2;
            s1=s2;
            s2=s;
        }
        if(s1<0) return s1+=b;
        return s1;
```

```
else cout<<"No tiene inversa"<<endl;
              return ZZ(0);
            //evalúa si mcd es 1
            bool Existe Inversa(ZZ x, ZZ y){
              if(even(x)&&even(y)) return 0;
              while(x!=0)
                 while(even(x)) x=x>>1;
                 while(even(y)) y=y>>1;
                 ZZ t=valAbs((x-y))>>1;
                 if(x>=y) x=t;
                 else y=t;
              return y==ZZ(1);
            //Determina si es par o impar
            bool even(ZZ a){
              ZZ r=(a>>1)<<1;
              if(r<0) r=ZZ(2)+r;
              return r==a;
            //Convertir en positivo-Valor Absoluto
            ZZ valAbs(ZZ a){
              if (a<0) return (a*-1);
              return a;
            }
3. Formación de Bloques
    o Llenar ceros
            string rsa bloques::completarCeros(string mensaje,ZZ Nr){
              int digit =ZZtoStr(Nr).size()-1;
              int c = modint(mensaje.size(),digit);//las letras que no llegan a completar un
              string cero(digit-c,'0');///la cantidad de 0s que faltan
              return cero+mensaje;
            int modint(int a,int n){
              int q=a/n;
              if(q<0) q--;
              return a-(n*q);
            }
    o ZZ a string
            // Convertimos el número en string lo utilizamos en el cifrado y descifrado con y sin
           firma digital.
            string rsa_bloques::ZZtoStr(ZZ z){
              stringstream ss;
              ss<<z;
```

```
return ss.str();
o Dividir bloques
        string rsa bloques::dividirBloques(string mensaje){
           //Divide en bloques y completa para tener dígitos al igual que el número de dígitos
           int digit=to string(alfabeto.size()).size();//número de dígitos del alfabeto
           string str;
             string cero(digit,'0');//string hecha de 0 para completar
             for(int i=0;i<mensaje.size();i++){
                size_t pos=alfabeto.find(mensaje.at(i));//size t adquiere cualquier tipo de
        variable en los vectores
                int len=to string(pos).size();//la longitud del número
                if(len<digit) str+=cero.substr(0,digit-len);//completa los 0
                str+=to string(pos);
           digit=ZZtoStr(n).size()-1;
           while(modint(str.size(),digit)) str+="22";//Aquí completamos el mensaje con W
        para evitar espacios en blanco
           return str;
```

## 4. Exponenciación modular

```
ZZ Power mod(ZZ base, ZZ exp, ZZ n){
  //Usamos exponenciación modular el documento
  //Exponenciación Modular Binaria
  ZZ salida(1);
  do{
    if(!even(exp))
       salida= modulo(salida * base, n);
    base= modulo(base * base, n);
    \exp >>=1;
  }while(exp!=ZZ(0));
  return salida;
ZZ modulo(ZZ a, ZZ b){
  ZZ q= a/b;
  if(q<0) q--;
  return a-(b * q);
}
```

#### 5. Función de cifrado

```
string rsa_bloques::encrypt(string mensaje, ZZ Nr, ZZ Er){
int digit=ZZtoStr(Nr).size()-1;
string out;
```

```
for(int i=0; i < mensaje.size(); i+=digit){
    ZZ p(conv<ZZ>(mensaje.substr(i, digit).c_str()));
    p=Power_mod(p,Er,Nr);
    string ceros((digit+1-ZZtoStr(p).size()),'0');
    out+=ceros+ZZtoStr(p);
}
return out;
}
```

6. Función de descifrado
 string rsa\_bloques::decrypt(string mensaje) {
 string salida;
 int digitN=ZZtoStr(n).size();
 int digit=digitN-1;
 for(int i=0;i<mensaje.size();i+=digitN) {
 ZZ c(conv<ZZ>(mensaje.substr(i,digitN).c\_str()));
 c=TRC(c);
 string ceros((digit-ZZtoStr(c).size()),'0');
 salida+=ceros+ZZtoStr(c);
 }
 digit=to\_string(alfabeto.size()).size();
 string outLetters;
 for(int i=0;i<salida.size();i+=digit) {
 outLetters+=alfabeto.at(stoi(salida.substr(i,digit)));
 }
 return outLetters;</pre>

#### 7. Firma digital

• Cifrado:

```
string rsa_bloques::sign_encrypt(string mensajeL, ZZ Nr, ZZ Er){
    string aux = encryptD(mensajeL);
    return encrypt(completarCeros(aux, Nr), Nr, Er);
}

string rsa_bloques::encryptD(string mensajeL){
    string mensajeNo = dividirBloques(mensajeL);
    int digit=ZZtoStr(n).size()-1;
    string out;
    for(int i=0;i<mensajeNo.size();i+=digit){
        ZZ c(conv<ZZ>(mensajeNo.substr(i,digit).c_str()));
        c=TRC(c);//se realiza una segunda versión para optimizar con el TRC y mayor velocidad
        string ceros((digit+1-ZZtoStr(c).size()),'0');
```

```
out=out+ceros+ZZtoStr(c);
          return out;
       Descifrado:
        string rsa bloques::remove sign(string mensaje, ZZ Ne, ZZ Ee){
          return decryptE(mensaje, Ne, Ee);
        string rsa bloques::decryptE(string mensaje, ZZ Ne, ZZ Ee){
          string salida;
          int digitN=ZZtoStr(n).size();
          int digit=digitN-1;
          for(int i=0;i<mensaje.size();i+=digitN){
             ZZ c(conv<ZZ>(mensaje.substr(i,digitN).c str()));
             c=TRC(c);
             string ceros((digit-ZZtoStr(c).size()),'0');
             salida+=ceros+ZZtoStr(c);
          string mensajeNo = salida;
          int a= modint(mensajeNo.size(), ZZtoStr(Ne).size());
          mensajeNo = mensajeNo.substr(a);
          string salida2;
          digit=ZZtoStr(Ne).size()-1;
          digitN=digit+1;
          for(int i=0; i < mensajeNo.size(); i+=digitN){
             ZZ c(conv<ZZ>(mensajeNo.substr(i, digitN).c str()));
             c=Power mod(c,Ee,Ne);
             string ceros((digit-ZZtoStr(c).size()),'0');
             salida2+=ceros+ZZtoStr(c);
          digit=to string(alfabeto.size()).size();
          string outLetters;
          for(int i=0;i<salida2.size();i+=digit){
             outLetters+=alfabeto.at(stoi(salida2.substr(i,digit)));
          return outLetters;
ZZ rsa bloques::TRC(ZZ M){
  ZZ q1 = inversa(modulo(q, p), p);
  ZZ a1= Power mod(modulo(M, p), modulo(d, p - 1), p);
  ZZ q2 = inversa(modulo(p, q), q);
  ZZ a2= Power_mod(modulo(M, q), modulo(d, q - 1), q);
  return modulo(modulo(a1 * q * q1, n) + modulo(a2 * p * q2, n), n);
```

8. TRC

```
ZZ Power_mod(ZZ base, ZZ exp, ZZ n) {

//Usamos exponenciación modular el documento

//Exponenciación Modular Binaria

ZZ salida(1);
do {

if(!even(exp))

salida= modulo(salida * base, n);
base= modulo(base * base, n);
exp>>=1;
} while(exp!=ZZ(0));
return salida;
```