1: FD and Keys

Consider a relation R(A,B,C,D,E) with FD's, S=AB → C, CD → E, C →A, C → D, D → B:

Determine all the keys of relation R. Do not list super keys that are not a minimal key.

(solution)
Keys: AB, AD, C
To get the key AB, we can do the following:
From AB → C and C →D, we obtain AB →D.
From AB → C and AB → D, we obtain AB → CD.
From AB → CD and CD → E, we obtain AB → E.
To get the key AD, we can do the following:
From D → B, we can get AD → AB.
From AB, we can obtain the rest of the attributes.
To get the key C, we can do the following:
From C → A and C → B, we obtained C → AB.
From AB, we can obtain the rest of the attributes.

2: FD

Consider a relation R(A, B, C, D, E, F) with the following set of FD's :
S: {A→BC, CD→E, B →D, E→A, CF→B}

(a) Give an example of FD that follows from S and explain your answer.

(Solution)
AB → D, D is in the closure of AB. Because A→ B and B→ D, thus AB → D is a valid FD that follows S.

(b) Give an example of FD that does not follow from S and explain your answer.

(Solution)
B → C, C is not in the closure of B. B doesn't uniquely identify C accordance to S. So, B→C is not valid according to S.

3: BCNF

Consider relation R (A, B, C) with a set of FDs F={AB→C, C→A}. Determine whether R is in BCNF.

(solution)
The keys are AB and BC. R is not in BCNF since left hand side of C→A is not a super key.

4: BCNF

Consider the relation schema R(A, B, C, D, E) with FD's, A →BCDE, C →D, and CE → B . Decompose the relation till it follows BCNF.

(solution)
R is not in BCNF because CE →B and CE is not a super key.
Decompose R: R1= {CEB}, R2={ACDE}
R1 is in BCNF
R2 is not in BCNF, because C → D and C is not a super key
Decompose R2: R21= {C,D} , R22={A,C,E}
R1,R21,R22 are in BCNF.

5: BCNF

Consider a relation R=(A,B,C,D,E) with the following functional dependencies, S= BC → ADE, D → B.

(a) Find all candidate keys.

(solution)
The keys are {B,C} and {C,D}.
{B,C} is a key from BC → ADE.
To get the key {C,D}:
from D→ B we get B, with B and C we have BC → ADE

(b) Identify whether or not R is in BCNF.

(solution)
The relation is not BCNF because D is not a super key which violates BCNF.

6: BCNF

Consider a relation R = (A,B,C,D,E) with the following functional dependencies:
S= {CE → D,D → B,C → A}.

(a) Find all candidate keys.

(solution)
The only key is {C,E}
To get the key CE, we can do the following: From CE → D and D →B, we obtain CE →B. From CE → D and C → A, we obtain CE → AD.

(b) If the relation is not in BCNF, decompose it until it becomes BCNF.

(solution)
Relation R is not in BCNF.
Step 1: Decomposes R into R1=(A,C) and R2=(B,C,D,E).
Resulting R1 is in BCNF. R2 is not.
Step 2: Decompose R2 into, R21=(C,D,E) and R22=(B,D).
Both relations are in BCNF.