The assignment is to be turned in before Midnight (by 11:59pm) on January 28th. You should turn in the solutions to this assignment as a PDF file through Canvas. The solutions should be produced using editing software programs, such as LaTeX or Word, otherwise they will not be graded. The assignment should be done in groups of two students. Each group must submit only one file that contains the full name, OSU email, and ONID of every member of the group.

1: BCNF and 3NF (1.5 points)

Consider a relation R with five attributes A, B, C, D, and E. You are given the following functional dependencies: A $\rightarrow$ B, BC$\rightarrow$E, and ED$\rightarrow$A.

(a) List all keys for R. (0.5 point)

CDE, ACD, BCD

(b) Is R in BCNF? If it is not, decompose it into a collection of BCNF relations. (0.5 point)

(solution)

R is not in BCNF because none of A, BC and ED contain a key. Its BCNF schema can be either of the following:
(A, B) (A, D, E) (C, D, E)
(B, C, E) (A, B) (A, C, D)
(A, D, E) (B, C, E) (B, C, D)

(c) Is R in 3NF? If it is not, convert it into a collection of 3NF relations. (0.5 point)

(solution)

R is in 3NF because B, E and A are all parts of keys.

2: BCNF and 3NF (1.5 points)

Consider the relation schema R with attributes A, B, C, and D and the following functional dependencies: AB$\rightarrow$C, AC$\rightarrow$B, B$\rightarrow$D, BC$\rightarrow$A.

(a) List all keys for R. (0.5 point)

(solution)

AB, AC, BC.

(b) Is R in BCNF? If it is not, decompose it into a collection of BCNF relations. (0.5 point)

(solution)

It is not in BCNF because B is not a key. Its BCNF schema is: (A,B,C) (B,D).

(g) Is R in 3NF? If it is not, convert it into a collection of 3NF relations. (0.5 point)

(solution)

It is not in 3NF. The set of functional dependencies is already a minimal basis. The 3NF schema is: (A,B,C) (B,D).

3: FD Implication & Schema Decomposition (1 point)

(a) Given that X, Y, W, Z are attributes in a relation, using Armstrong's axioms, prove that if we have $X \to Y$ and $YW \to Z$, then $XW \to Z$. (0.25 point)

(solution)

We may add W using argumentation to both sides of $X \to Y$ to get $XW \to YW$. Then, we will use final results by applying the transitivity on $XW \to YW$ and $YW \to Z$.

(b) Given that X, Y, Z are attributes in a relation, using Armstrong's axioms, prove that if we have $X \to Y$ and $X \to Z$, then $X \to YZ$. (0.25 point)

(solution)

Using argumentation, we have $X \to XY$ and $XY \to YZ$. Then, by applying transitivity, we have $X \to YZ$.

(c) Prove that, if relation R has only one simple key, it is in BCNF if and only if it is in 3NF. (0.5 point)

(solution)

Let F (F+) denote the (closure of the) set of functional dependencies satisfied by the schema R which is assumed to be in 3NF. We need to show that for each nontrivial dependency X → A in F+, X is a superkey. To this end, consider such a dependency. If X is not a superkey, the 3NF property guarantees that the attribute A is part of a key. Since the key is simple by assumption, we have that A is a key. This last fact together with the dependency X → A implies that X is a superkey (this follows, from the transitivity axiom) which is a contradiction.

4: Information preservation (1 point)

(a) Suppose you are given a relation R(A,B,C,D) with functional dependencies B→C and D→A. State whether the decomposition of R to S_1(B,C) and S_2(A,D) is lossless or dependency preserving and briefly explain why or why not. (0.5 point)

(solution)

The decomposition into BC and AD does not have lossless join property because the join of S_1 and S_2 is the Cartesian product which is not equal to R. It is, however, a dependency preserving decomposition.

(b) Prove that the 3NF synthesis algorithm produces a lossless-join decomposition of the relation containing all the original attributes. (0.5 point)

(solution)

Let R denote the set of all attributes. N is a minimal cover for the set of all FD's satsified by the schema and K some key for the schema. We will show that the decomposition $\{XA \mid X \rightarrow A \in N\}$ $\{K\}$, where K is any key gives a lossless join 3NF decomposition. First, note that the subschema $\{K\}$ is in 3NF because any FD that holds over it will have its right hand side (attribute) contained in a key (namely, K). To prove the clain, enumerate the set of subschema XA in the decomposition as $R_1, R_2, \ldots R_m$. Let r be an instance and let t_i and a be tuples in r. A formal proof of this will proceed by induction and is based on using the tuple a and the fact that K is a key and functionallly determines the rest of attributes that follows from the FD's $X \rightarrow A$ to "connect" the other tuples and force each tuple in the join to lie in r.