CREDIT DERIVATIVES

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Credit Derivatives Strategy
New York: (1) 212 449-0104
London: (44) 20 7995-3948

Credit Derivatives Handbook 2006 – Vol. 2

A Guide to the Exotics Credit Derivatives Market

Global

Contributors

North America

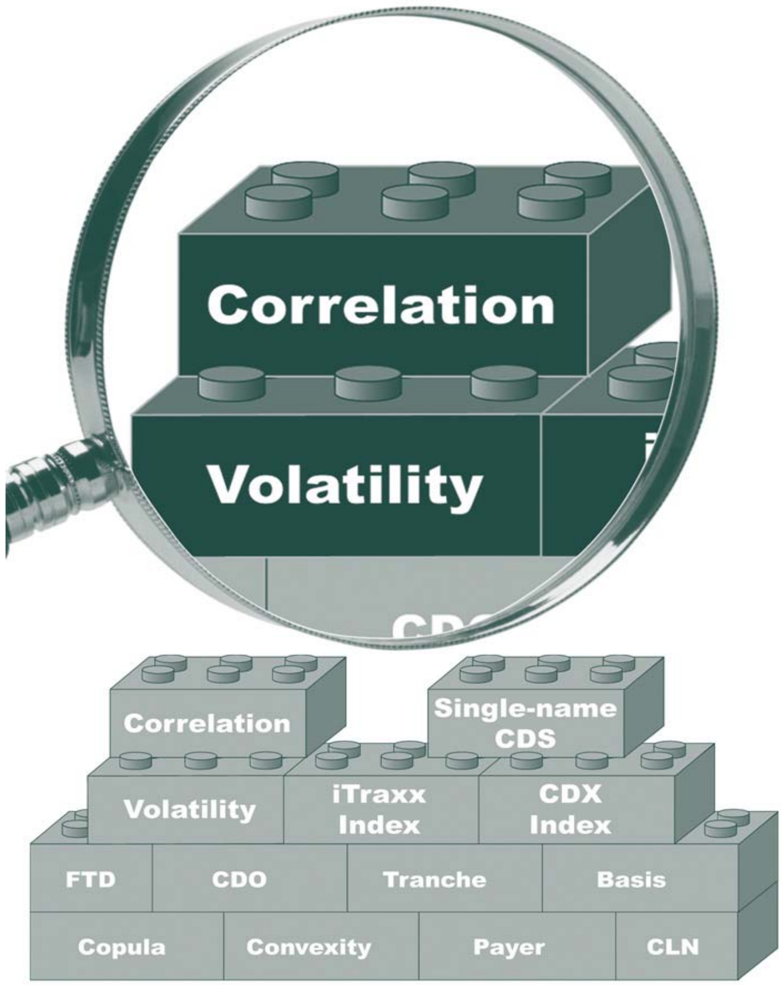
Atish Kakodkar
Strategist, MLPF&S
(1) 212 449-0104
atish_kakodkar@ml.com

Stefano Galiani
Strategist, MLPF&S
(1) 212 449-7416
stefano_galiani@ml.com

Europe

Jón G. Jónsson
Strategist, MLPF&S (UK)
(44) 20 7995-3948
jon_jonsson@ml.com

Alberto Gallo
Strategist, MLPF&S (UK)
(44) 20 7995-8542
alberto_gallo@ml.com



This report has been published in conjunction with "Credit Derivatives Handbook, Volume 1: A Guide to Single-Name and Index CDS Products"

Refer to important disclosures on page 196. Analyst Certification on page 196.

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ADDITIONAL CONTRIBUTORS

Chris Francis (65) 6330-7210
International Credit & Rates,
Merrill Lynch (Singapore)
chris_francis@ml.com

Barnaby Martin (44) 20 7995-0458
Credit Strategist, MLPF&S (UK)
barnaby_martin@ml.com

Lang Gibson (1) 212 449-3942
CDO Analyst, MLPF&S
lang_gibson@ml.com

Alexander Batchvarov (44) 20 7995-8649
Research Analyst, MLPF&S (UK)
alexander_batchvarov@ml.com

Altynay Davletova (44) 20 7995-3968
Research Analyst, MLPF&S (UK)
altynay_davletova@ml.com

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Refer to important disclosures on page 196.

1. Exotic Credit Derivatives: Tried & Tested

Since late 2003, the credit derivatives market transformed itself from a primarily single-name only market to a more complex market of single-name, index, correlation and options. This second volume of the handbook deals primarily with correlation and volatility based products. This includes single-tranche synthetic CDOs (STCDO), first-to-default (FTD) baskets, CDS options and other related products.

Forces Driving the Market

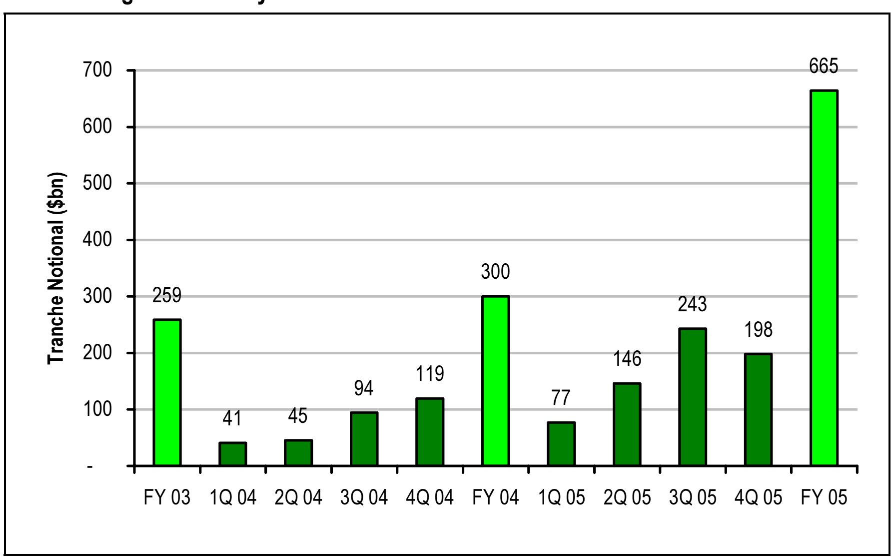
Yield, liquidity and innovation have driven growth

The growth of the exotics credit derivatives market has been driven primarily by three key forces:

- **Yield enhancement**: As spreads have tightened, investors have taken on more leverage via the STCDO market either via the plain vanilla structure or more leveraged synthetic CDO-squared (or CDO^2) structures. Investors with strong credit views have also used FTD structures to enhance yield.
- **Index liquidity**: The wildly successful index (CDX & iTraxx) is the instrument of choice to take on a macro credit view[1]. More importantly, the indices have also served as building blocks for the next generation of products such index tranches or index options.
- **Innovation**. The ever-changing needs of end investors have meant new structures to express spread views with more sophisticated payoffs such as CDS options, cancelable CDS, or constant maturity CDS (CMCDS). This innovation has extended into the tranche market to enable investors to capture value in different parts of the capital structure, e.g. leveraged super senior (LSS) notes.

The STCDO market, in particular, has grown significantly over the last few years. However, the OTC nature of the market makes it difficult to measure the size of the market. Chart 1 represents an estimate of the STCDO volumes (both bespoke and standardized) over the last few years. According to this estimate, a majority of the recent volume relates to standardized index tranches (about 68% in FY05). However, these estimates are not adjusted for the tranche seniority and do not reflect the typically higher notionals for senior relative to junior tranches.

Chart 1: Single Tranche Synthetic CDO Volume Estimates



Source: Creditflux

[1] Indices are described in more detail in Volume 1, Chapter 8.

What Have Investors Done?

Investor Focus

Long-only investors vs. relative value traders

Real money investors such as insurance companies have been focused primarily on yield enhancement. They have taken on exposure to bespoke tranches in order to maximize spread for the given tranche rating (typically AAA or AA rated mezzanine tranches). Unlike standardized tranches, bespoke tranches offer more control via tailor-made portfolios that better suit the investment objectives of the investor.

Real money investors have also been protection sellers of FTD baskets of credits and have used this structure to create leverage on credits where they have a favorable opinion. Others have even sold CDS options on indices and specific credits to enhance yield.

Hedge fund investors on the other hand have been traders of correlation and volatility. They have been the main participants (along with bank prop desks and dealers) in the standardized tranche market based on the CDX and iTraxx indices. They are also the main traders of options on single-names, indices and even tranches (a relatively nascent market).

The Changing Nature of Bespoke Tranches

More leverage in different ways

In order to meet yield bogies in a tighter spread environment, bespoke tranche investors have taken on additional leverage via:

- longer duration,
- lower quality underlying collateral,
- lower attachment points, and
- more structure.

Bespoke tranches now have longer maturities (typical deals are 7y instead of 5y), lower quality underlying credits (more crossover/HY credits), and lower attachment points.

Popularity of longer maturity bespoke tranches has led to more liquidity in 7y & 10y standardized tranches as dealers use them to hedge bespoke tranche risk. As a result, the tranche market has developed a relatively liquid full term structure in 5y, 7y & 10y maturities (and to some extent the 3y).

Mixed Growth in Options

Index options quite active

The options market has been focused primarily on the indices. The new CDX Crossover (XO) index that was recently introduced includes GMAC and FMCC and has become the new “hi vol” index. Options on the XO index have been quite popular since it was introduced.

Single-name option trading on the other hand has been relatively patchy. Investors typically get involved only on the back of some credit-specific news and usually trade in one direction. This makes it less attractive for the dealers to be involved.

Evolution of the Correlation Market

Early Days of Correlation

Long correlation was a popular trade in the early days…

The standardized tranche market developed in response to dealers’ need to hedge their (typically mezzanine) bespoke single tranche exposure. Dealers typically sold protection on standardized mezzanine tranches or bought protection on the remainder of the capital structure to hedge correlation exposure. Importantly, the standardized tranche market led to correlation price discovery and an observable market view of correlation for a particular portfolio. Besides enabling better

Refer to important disclosures on page 196.

pricing of bespoke tranches, it also helped dealers determine correlation reserves. The links between the bespoke and the standardized tranche market were perhaps best evidenced by the strong link between rising bespoke issuance and the rally in standardized mezzanine tranche spreads.

The emergence of the standardized market also paved the way for long correlation trades by hedge funds, which sold equity tranche protection, delta-hedged by buying mezzanine tranche protection. The trades matched dealers' hedging needs, offered a positive carry (or generous up-front) and positive mark-to-market (MTM) on correlated spread changes, but negative MTM from idiosyncratic credit events and jump-to-default risks. Due to dealers' hedging needs, these long correlation trades offered what appeared at the time attractive levels.

Table 1: Long Correlation Trades

Protection Selling	Protection Buying	Market Participants
Index (DH)	Mezzanine or Senior Tranche	Dealers, Hedge Funds
Equity Tranche	Index (DH)	Hedge Funds, Prop Desks
Equity Tranche	Mezzanine or Senior Tranche (DH)	Hedge Funds, Prop Desks

Source: Merrill Lynch. DH=Delta-Hedge.

Coming of Age: The Correlation Shakeout

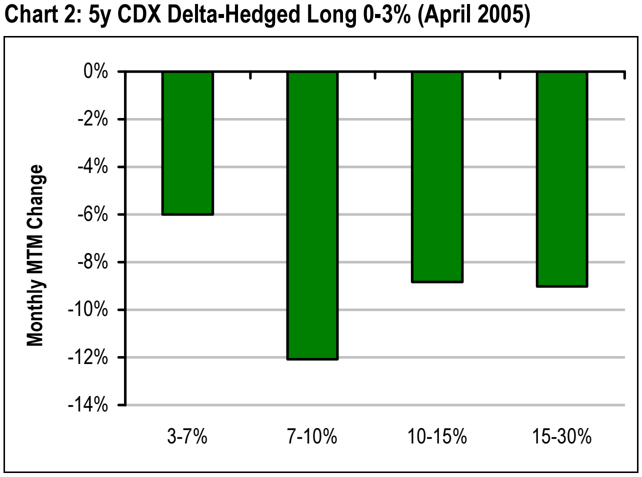
…until the correlation shake-out in mid 2005

The simultaneous downgrade of GM and Ford on 5th May 2005 to non-investment grade by S&P triggered the first real test for the correlation markets. The credit deterioration in early 2005 and eventual downgrade was similar to an idiosyncratic spread movement and caused two key trades:

- Rush to hedge the unhedged bespoke mezzanine risk by dealers, and
- Rush to unwind long correlation trades by prop desks and hedge funds.

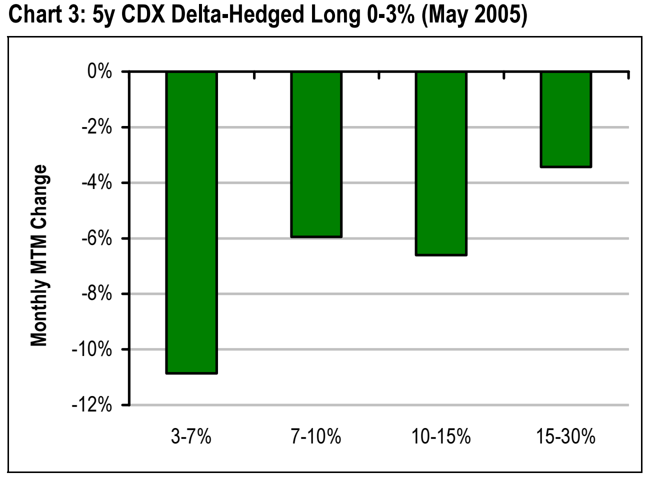
We believe the rush to hedge bespoke was the more dominant trade and resulted in significant losses to some dealers. The net impact was a dramatic tightening of mezzanine spreads and an equally dramatic widening of equity spreads. On the other hand, the underlying indices remained relatively unchanged at the end of May when compared to the beginning of the month. Long correlation trades underperformed significantly in April and May 2005 (Chart 2 and Chart 3). Each of these charts shows the return of the 5y equity tranche delta-hedged with the mezzanine/senior tranches. Monthly returns were as bad as -12%!

Chart 2: 5y CDX Delta-Hedged Long 0-3% (April 2005)



Source: Merrill Lynch.

Chart 3: 5y CDX Delta-Hedged Long 0-3% (May 2005)



Source: Merrill Lynch.

Value moved out of the belly and into the wings

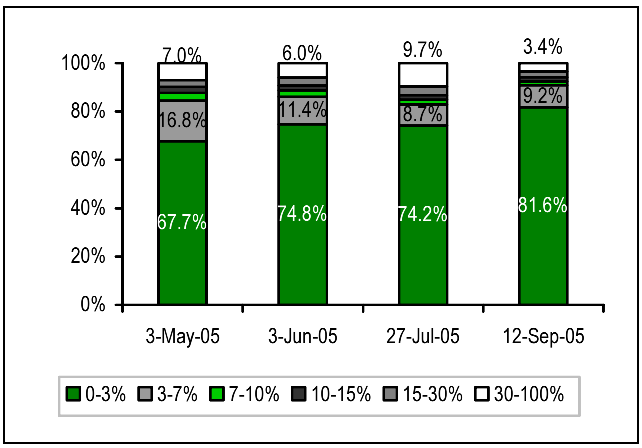
■ Tranche Risk Repricing

The months during and following the correlation shakeout were characterized by a distinct shift of value between different parts of the capital structure. The timeline below highlights this transfer of value:

- In **May**, value moved out of the mezzanine tranches and into the junior tranches as mezzanine spreads tightened and equity spreads widened more than that implied by the movement of the underlying index. While long correlation traders (like hedge funds and traders) lost money, real money investors who had sold protection on mezzanine tranches benefited. Several of them even unwound their positions to lock in their gains.
- In **June** and **July**, value moved out of the equity and mezzanine tranches to the super-senior tranches. This was driven by hedge funds who reestablished long equity positions post the May highs and by dealers, who continued to hedge their bespokes in June and July. By the end of July, the super-senior looked very attractive.
- In **August** and **September**, the most popular trade was the leveraged super-senior (or LSS) which squeezed all the value out of the super-senior and back to the equity tranche. While the mezzanine tranche continued to face spread tightening pressure, increasing likelihood of defaults (Delphi bankruptcy was imminent) once again highlighted the risk of the equity tranche.

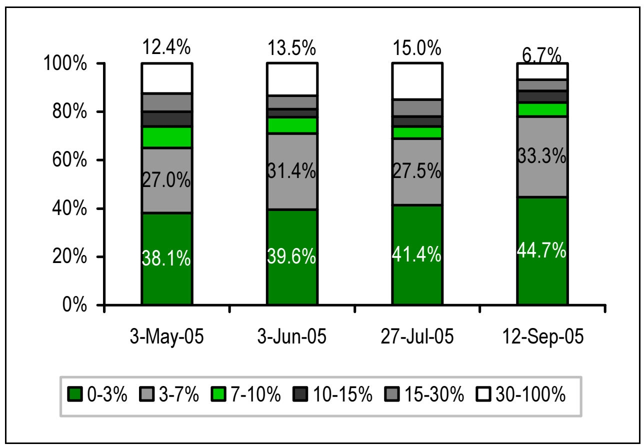
These trends are well illustrated in the following relative loss allocation[2] (RLA) charts, showing the change in percentage of index (portfolio) losses to the different tranches.

Chart 4: 5y CDX IG5 Relative Loss Allocation



Source: Merrill Lynch

Chart 5: 10y CDX IG5 Relative Loss Allocation



Source: Merrill Lynch

Outlook for Growth

Take advantage of value in equity tranches

■ Value in Equity Tranches

Post these correlation gyrations in 2005, the value in the tranche market has settled in the junior part of the capital structure, particularly the equity tranche. The equity tranche should continue to retain more value due to potential for idiosyncratic events such as LBOs and releveraging risk.

We expect real money investors to take advantage of this relative value via principal protected structures. These structures essentially provide the principal "protection" via exposure to a higher-rated senior tranche while receiving a "risky" coupon based on the equity tranche. This type of structure should be popular in 2006

[2] See Volume 2, Chapter 6 for discussion on RLA.

Single-name CDS volatility due to tranche hedging

Increasing popularity of the junior tranches will, however, be accompanied by the need to hedge any "jump" risk. We expect potential blow-up credits, such as LBO candidates or releveragers, to be hedged via single-name CDS. Any hedging would usually occur with (cheaper) front-end protection causing these single-name credit curves to flatten or even invert as evidenced in early 2005 for GM and F.

More Demand for Managed Bespokes

Single tranche still an attractive way to create leverage

We anticipate a steady demand for bespoke single tranches in 2006 as it continues to remain an attractive way to create leverage in a customized fashion. Investors can choose the underlying portfolio, duration, and attachment points to create the appropriate level of credit risk and leverage.

More managed deals to minimize credit risk

As spreads tighten, we expect investors to take on more credit risk to meet their yield bogies via lower attachment points. For example, an attachment point of 7% for CDX IG would need about 15 defaults (assume 40% recovery) before the notional is hit. However, a 3% attachment point would only need about 6 defaults. Given these conditions, we believe investors would be more willing to pay good managers to minimize defaults in bespoke portfolios

Other innovations such as forward tranche exposures can provide further customization. In addition, potential accounting changes could provide a further boost to the bespoke market in 2006.

Modeling Improvements

Marking-to-market exotics to observable parameters

The explosion of customized tranches (both in terms of portfolio composition as well as subordination thresholds other than the standardized ones) has compelled dealers not only to objectively price them but also to identify and measure the additional risks appearing in the bespoke market (such as non observable correlation or composition mismatch with the typically adopted macro hedging with the index).

We believe that a revamp of the current traditional modeling framework (based on the overly blamed Gaussian Copula framework) will lead to a new class of models (possibly based on sophisticated spread dynamics) that will enable dealers to better capture the systematic and idiosyncratic component of correlation products

2. Single-Tranche Synthetic CDO

Synthetic CDO Basics

Synthetic CDOs facilitate credit risk exposure to a portfolio of names where risk is synthetically transferred

A Synthetic CDO is an investment where the underlying collateral is a portfolio of credits as opposed to a just a single bond or loan. Importantly, in a synthetic CDO transaction, the underlying collateral pool is a collection of single-name credit default swaps, which synthetically transfer the credit risk of the names that they reference. **In contrast, traditional cash CDO structures have funded physical portfolios of bonds or loans as the underlying collateral pool, while the CDS assets of a synthetic CDO are typically not funded.**

Table 2: An Example of Tranching Portfolio Credit Risk

Collateral Pool	Characteristics
Collateral Pool	
No of reference entities	100
Notional amount of protection on each credit	€10m
Total Portfolio Size	€1bn
Tranches	***% of Credit Losses***
'Equity' Tranche	0%-5%
'Junior Mezzanine' Tranche	5%-6.5%
'Mezzanine' Tranche	6.5%-9.5%
'Senior' Tranche	9.5%-12.8%
'Super-Senior' Tranche	12.8%-100%

Source: Merrill Lynch

The CDO structure redistributes the credit risk of the underlying portfolio through *tranching*, and cash flow and loss prioritization. For example, all cash collections (interest and principal payments from the underlying portfolio in a cash CDO structure, or CDS premiums and CLN collateral yield in funded or partially funded synthetic structures) are distributed to the CDO notes on a priority basis – from senior to junior, sequentially (in most cases). Shortfalls in the cash flows in cash structures due to losses or credit events crystallized in losses in a synthetic structure are allocated first to the lowest CDO tranche, known as the equity tranche, and then upwards in a reverse priority to the cash flow allocation mentioned above (Chart 6).

The capital structure of a CDO, that is its tranching, is determined by assessing the risks of its asset side and allocating those risks in priority on the liability side, so that the risks of each liability tranche are consistent with a desired rating level, expressed either in expected loss or in probability of default.

Every CDO has an asset side, generating its revenue, and a liability side, whose obligations need to be satisfied. The difference between the two is usually termed the *funding gap* or *excess spread*. One of the reasons that synthetic CDOs have risen in popularity is that they are able to pass on any positive benefits of the CDS-cash basis, thus generating more excess spread for investors.

Default swaps can better match CDO assets and liabilities

Furthermore, the asset and liability sides need to be carefully matched. One of the advantages of using credit default swaps as the underlying collateral is the ability to facilitate better matching (in terms of currency and maturity etc.) of the assets and liabilities of a CDO.

Banks can use CDOs to reduce regulatory capital

Synthetic (and cash) CDO structures are commonly used by banks (balance sheet CDOs) as a means of reducing regulatory capital requirements on their portfolios of risky assets, and so improving the return on equity. In addition, banks can use CDOs to better manage the risks of their balance sheets in terms of exposures, credit lines, diversification etc. Depending on the yield of their portfolios and their funding needs, they may choose to execute either a cash or a synthetic CDO transaction.

An investor can gain exposure to risk of specific losses on the underlying portfolio due to default

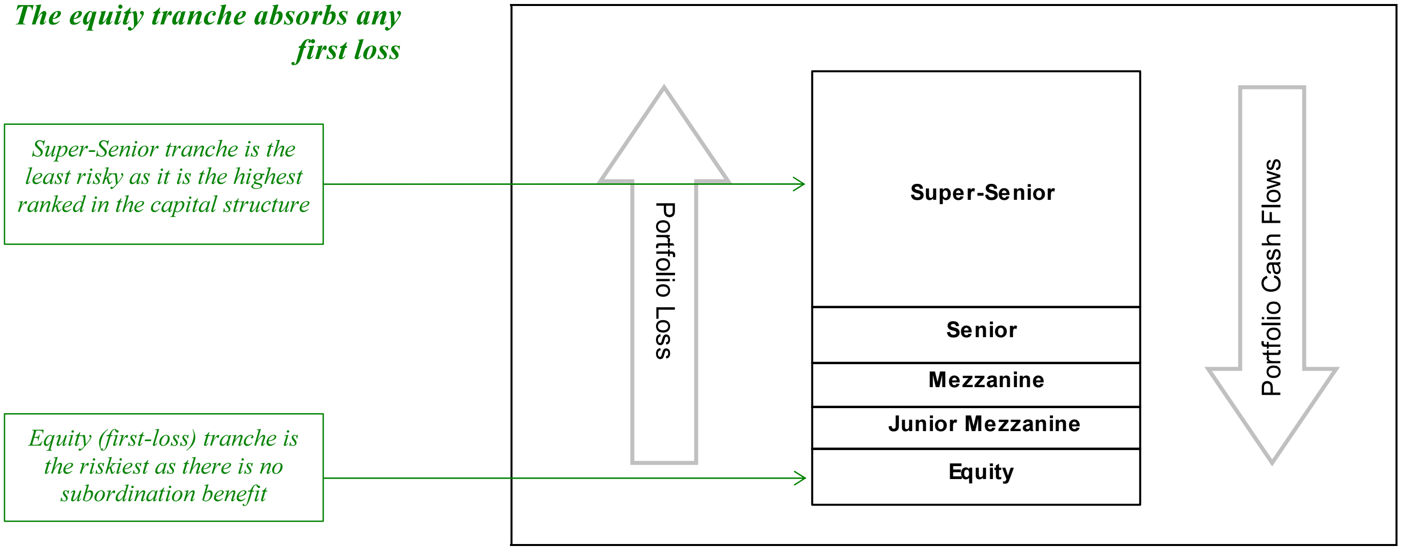
By purchasing the tranches issued from a CDO, an investor can gain exposure in varying degrees to the credit risk of the underlying portfolio. Specifically, the investor can gain exposure to the risk of **specific losses** on the underlying portfolio due to default. That risk is defined by the *attachment point* of the tranche, which defines the point at which losses on the underlying portfolio begin to reduce the notional of the tranche. A lower attachment point implies a riskier tranche than a higher attachment point, other things being equal. The full risk exposure to the investor is defined by the notional amount of the investment, which lies between the lower and the upper attachment points – the boundaries of the trade.

For example, suppose that we have a €1bn portfolio comprising 100 default swaps each transferring €10m notional of credit risk. The first loss or equity tranche might absorb the first 4% of losses (€40mn) on the portfolio due to default. The second loss tranche might absorb losses from 4% to 8% on the portfolio due to default, and so on until 100% of the losses on the portfolio have been tranched.

Refer to important disclosures on page 196.

The equity tranche absorbs any first loss

Chart 6: Prioritization in a CDO Capital Structure



Source: Merrill Lynch

Credit loss allocation would be bottom-up sequentially through the tranches

Table 2 details a hypothetical tranching of a €1bn portfolio into 5 tranches. Losses in the portfolio up until 5% (€50m of losses in total) would result in a write-down of principal of the equity tranche. Further losses in the portfolio in excess of 5% and up until 6.5% (€65m of losses in total) would then result in a write-down of principal in the junior mezzanine tranche. This process of loss allocation would continue bottom-up sequentially through the tranches.

Recovery rates would be key in determining portfolio and tranche loss

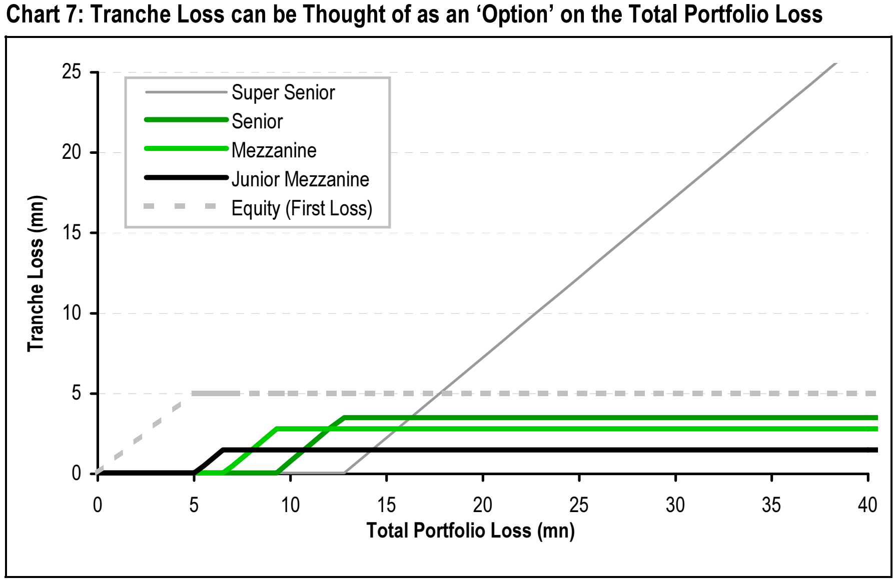
The level of losses in the portfolio (and hence tranche loss) depends on the timing of defaults and the subsequent recovery rate following each default of credits in the underlying portfolio. For instance, if each credit in Table 2 had a 50% recovery rate, each default in the underlying portfolio would result in a €5m loss. Accordingly, it would take 10 defaults to write-down the equity tranche.

Tranche can be viewed as an 'option' on portfolio losses

Tranches across the capital structure of a deal can be viewed as an 'option' on the performance of the underlying portfolio (Chart 7). For instance, the super-senior tranche is effectively "short a put option" on the performance of the underlying portfolio. Alternatively, the mezzanine tranche pay-off can be viewed as a mixture of a call and a put on the pool loss.

The super-senior tranche is effectively 'short a put option' on the portfolio performance

Chart 7: Tranche Loss can be Thought of as an 'Option' on the Total Portfolio Loss



Source: Merrill Lynch. Tranching as in Table 2.

Single Tranche Synthetic CDOs vs. Multi-Tranched Synthetic CDOs

Different tranches attract different investors

In a synthetic CDO, the issued tranches are typically purchased by different investors, according to their different risk and reward preferences, and often there is syndication of the deal. Investors in the equity portion, being the riskiest, are often hedge funds or dedicated structured finance investors. Conversely, investors in the super-senior tranches, which attract very high credit ratings, tend to be insurance companies and monolines.

Single-tranche deals are bilateral contracts where the dealer retains the residual risk

A single-tranche synthetic CDO is crucially different in that just one of the tranches is placed with one investor and the remaining tranches are retained by the broker/dealer. The risks of these tranches are dynamically managed (delta hedged) by the dealer[3]. A single tranche synthetic is non-syndicated, simply being a bilateral contract between the broker/dealer and one investor. This provides the most flexible option for the investor in terms of constructing the underlying portfolio, attachment point of the tranche, and desired tranche rating (which may well be higher than the average rating of the underlying collateral pool). Furthermore, with just two parties to a deal, tranches can be efficiently and speedily placed, providing opportunities to capitalize on arbitrage between the CDS and cash markets. Table 3 compares some common features of single-tranche synthetic CDOs and traditional synthetic CDOs.

Basic Structure of Single-Tranche Deals

Dealer sells protection into the market place on each individual name

The basic structure of a single-tranche synthetic CDO is shown in Chart 8. Credit risk is synthetically transferred via single-name default swaps by the dealer selling protection into the market place on each name. The portfolio is tranched into three simple pieces of equity, mezzanine and senior risk and the investor purchases (via a CLN in this example) the mezzanine tranche referencing losses on the underlying portfolio between 4% and 8% of the total portfolio notional. However, the investor can gain exposure to various tranches of credit risk in both funded and unfunded form:

Unfunded form

- Unfunded Form: the investor directly enters into a *portfolio default swap*, selling portfolio protection. A portfolio default swap is a credit derivative instrument that allows the transfer of exposure to specific losses in the underlying portfolio due to default. The dealer (protection buyer) pays a spread on the remaining notional of the tranche, and the investor (seller of protection) compensates the buyer for any losses in the tranche due to default.

Funded form

- Funded Form: the investor purchases a Credit-Linked Note issued by a Special Purpose Vehicle. In this instance, the dealer buys mezzanine protection from the SPV in return for default payments following losses on the tranche. The SPV issues coupon-paying notes to the investor and the proceeds of the note issuance are used to purchase pre-agreed collateral which i) forms the default payments to the protection buyer and ii) provides an enhanced coupon for the investor[4].

Both the CLN and CDS can be rated

The CLN can pay either a fixed or a floating coupon (depending on seniority) and the currency can be tailored to meet the demands of the investor. **Importantly, both the Credit Linked Note and the Credit Default Swap can be rated**.

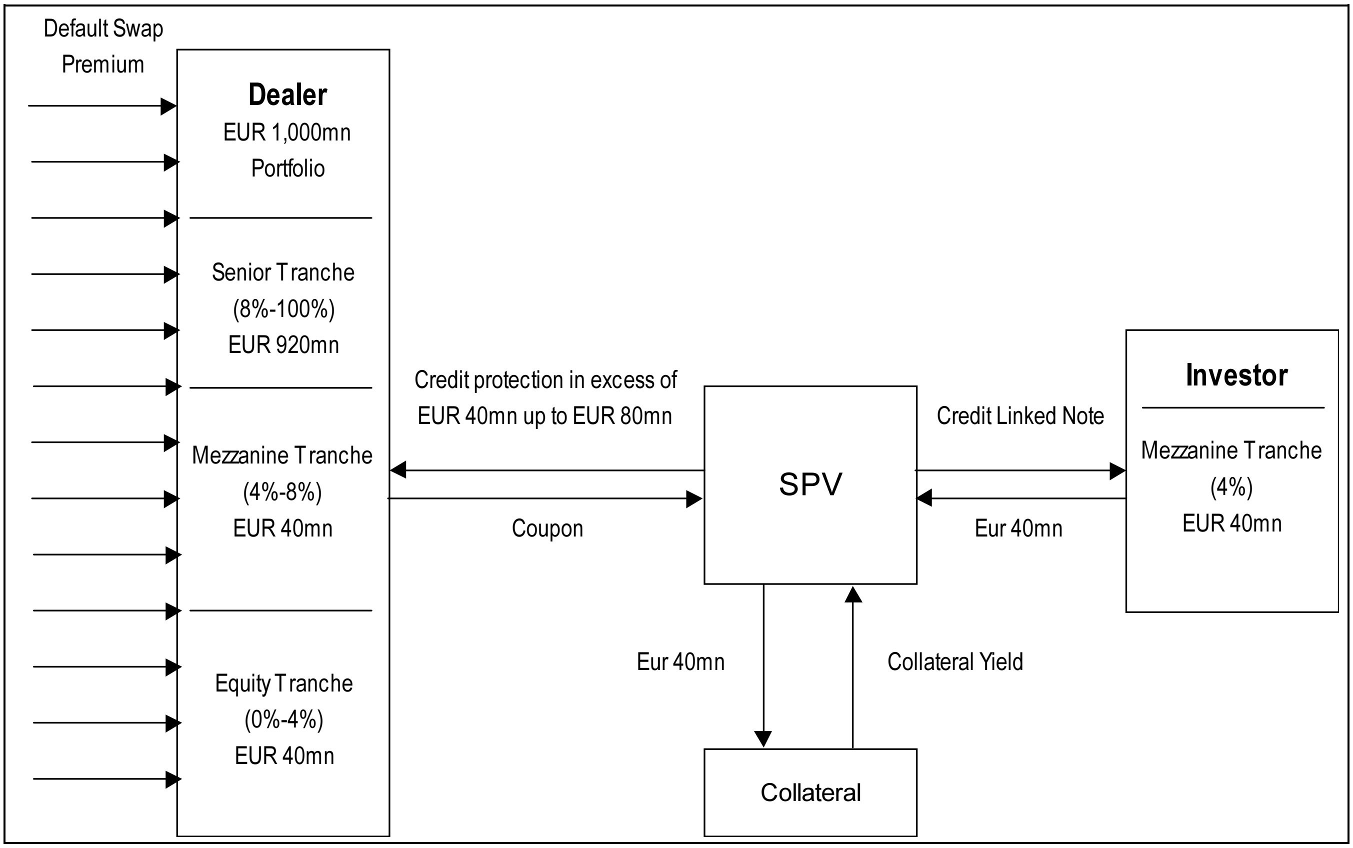
Ratings typically reflect both quantitative and qualitative aspects of the transaction

If rated, the rating process is similar to that used in rating public CDO transactions based on investment grade names executed either in cash or synthetic forms. Ratings will focus on both the quantitative and qualitative aspects of the transaction. Typically this will involve an analysis of the composition of the underlying portfolio, its diversification, any single-name concentrations, the average rating of the underlying portfolio, substitution rights, trading guidelines,

[3] It may be more advantageous for a dealer to place the remaining tranches with other investors at a later date instead of delta hedging them. For example, originating numerous mezzanine deals would result in a concentration of senior and equity risk for the dealer.
[4] Credit Linked Notes are discussed in more detail in Volume 1, Chapter 9

assessment of any CDO manager, legal review of the transaction, default swap documentation etc. We address certain aspects of the rating process further below.

Chart 8: Example Transaction Diagram for a €1bn Portfolio. Investor Purchases €40m CLN tied to Losses on the Mezzanine Tranche



Source: Merrill Lynch

Table 3: Comparison Between Single-Tranche Synthetic CDOs and Traditional Synthetic CDOs

Single-Tranche Synthetic CDO	Multi-Tranche Synthetic CDO
Synthetic risk transference via single-name credit default swaps is used to create the underlying collateral pool.	Synthetic risk transference via single-name credit default swaps is used to create the underlying collateral pool.
Only one tranche is placed with an investor. The remaining tranches are retained by the dealer and the associated risk is actively managed ('delta hedged').	A majority of the tranches are placed with different investors and hedging is more straightforward than in single-tranche deals. Syndication may be involved.
Bilateral contract between protection buyer and protection seller. No reliance or dependence on equity tranche investor or other parties as in traditional CDOs. Client has ultimate flexibility in determining the underlying portfolio, investment size, tranche structure and desired rating.	Different investors will be attracted to different tranches depending on risk/return preference. Less flexibility in determining underlying collateral pool depending on the type of investor.
Investor-driven, single-investor.	Issuer-driven in traditional securitisation or equity investor / portfolio manager driven in case of arbitrage CDO.
Potentially faster execution than full capital structure deals, allowing the investor to capitalize on arbitrage opportunities between the CDS and cash markets, as well as sweet spots in the capital structure.	Typically slower execution as all tranches need to be placed. Deals can collapse if the entire capital structure cannot be placed.

Source: Merrill Lynch

Leverage and Tranche Returns

Mezzanine tranches can often offer enhanced yields versus a direct investment in the underlying portfolio

Single-tranche synthetic CDOs offer a leveraged exposure to the underlying portfolio above the attachment point. Any tranche subordination effectively acts as a 'buffer' against defaults, i.e. there is room for defaults on the underlying portfolio without suffering a loss of notional and return. **For some mezzanine tranches that achieve investment grade ratings, the subordination implies that the Internal Rate of Return (IRR) will likely be much higher compared**

to the IRR on a straight investment in the underlying portfolio (which will have a weighted-average rating similar to the rating of the mezzanine tranche). This can present a compelling investment opportunity.

Below we analyze the dynamics of a leveraged investment in a mezzanine (single) tranche of a synthetic CDO. We need to emphasize, though, that a mezzanine tranche position is actually 'deleveraged' up to the attachment point, and then more leveraged than an investment in the underlying portfolio.

The return on the investment is a function of the level of leverage, number of defaults in the underlying portfolio and the level of recoveries in the case of defaults.

Leveraged Tranche Returns as a Function of Defaults

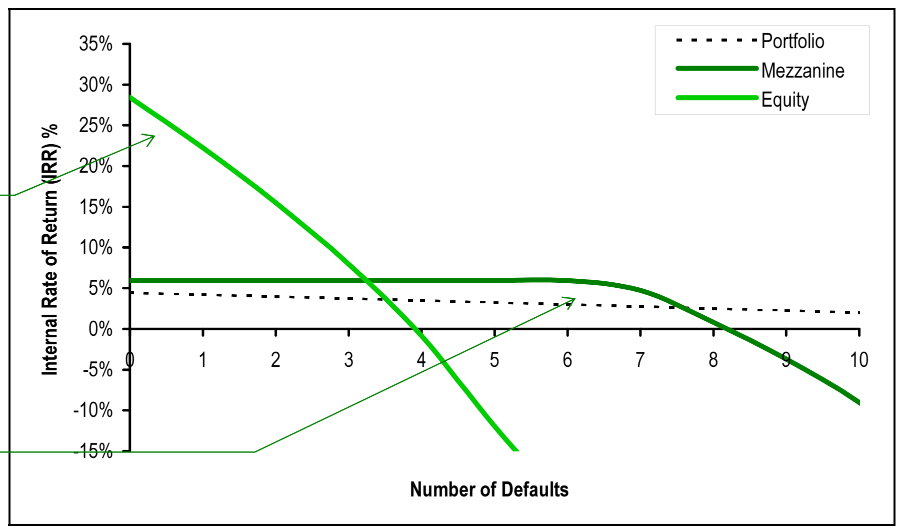
We can describe the leveraged tranche dynamics as a function of defaults in the underlying portfolio. . .

We can describe the leveraged tranche dynamics as a function of the number of defaults under a 40% recovery assumption (Chart 9). We have assumed a portfolio of 100 names with an average spread of 100bps and tranched this portfolio into a first-loss piece of 4% and a mezzanine piece of 4%. For a direct investment in the portfolio ('100% tranche'), each default contributes to a loss in the IRR as there is no subordination. For the mezzanine tranche, subordination means that the IRR is constant until the seventh default, following which the IRR falls much quicker than for the portfolio. **After the attachment point, each default contributes to a greater loss (as a % of initial investment) in the case of the mezzanine tranche than in the case of the whole portfolio.** Naturally, the timing of defaults and recoveries will also have an impact on returns.

Chart 9: IRR Analysis for Tranches vs. IRR for Underlying Portfolio

No subordination in the equity tranche means that the IRR is initially very big but falls very quickly

Mezzanine IRR is constant until subordination is exhausted, then falls quicker than the IRR decline of the whole portfolio



Source: Merrill Lynch
Assuming 5yr swap rate of 3.5%, coupon of Mezzanine tranche of 2.5%. Coupon on Equity tranche of 25%.

Leveraged Tranche Returns as a Function of Recovery Rate

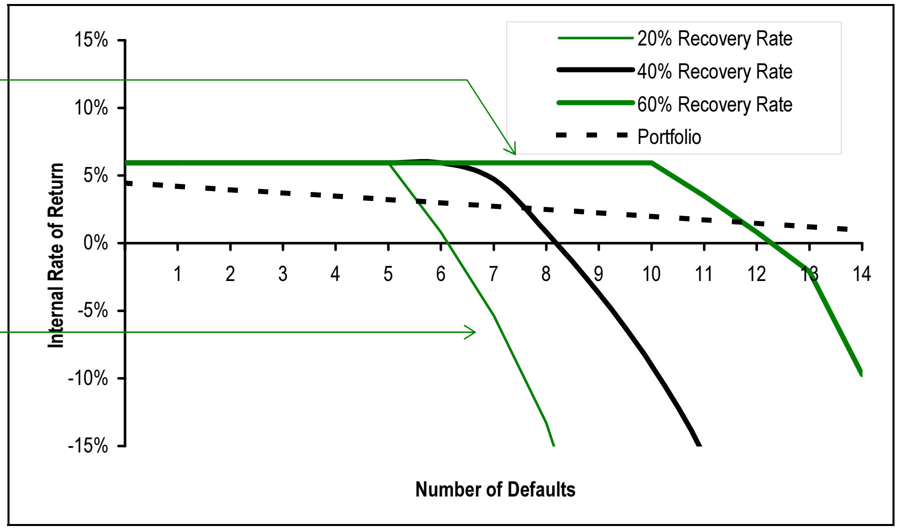
. . .and also as a function of recovery rates

We can also describe the leveraged tranche dynamics of the mezzanine tranche as a function of the recovery rate while keeping all other factors constant (Chart 10). For a greater recovery rate assumption, more defaults in the underlying portfolio are needed before the subordination benefit of the mezzanine tranche is exhausted. Consequently, for higher recovery rate assumptions, the IRR of the tranche remains constant (and higher than the IRR of the underlying portfolio) for longer. The reverse is true for lower recovery rate assumptions.

For higher recovery rates more defaults are needed to exhaust the subordination benefit of the mezzanine tranche. The IRR is constant for longer

Lower recovery rates mean that the subordination benefit of the mezzanine tranche is exhausted quicker.

Chart 10: IRR Analysis for Mezzanine Tranche for Varying Recovery Rates



Source: Merrill Lynch. Coupon on Mezzanine tranche of 2.5%

Defaults and Loss Valuation

Cash Settlement Following a Default

Transactions are typically cash settled upon default

Typically, following the default of a name in the underlying portfolio, the loss amount will be determined by a cash settlement process. This involves polling the market place for the price of the relevant security following the default. While the mechanics of such a process vary from deal to deal and vary between dealers, the following factors play an important role in the overall robustness of determining the loss amount:

- **The number of firms participating in the auction process**. Prices will typically be sourced from major dealers in the specific debt obligation being valued. Usually the minimum required is 5.
- **Method of price discovery**. Valuation could involve bid or mid price calculations. Moreover, the highest price or an average of the prices may be used. Typically, the highest bid is used.
- **Time period for loss to be determined and how long after default this process is begun**. Immediately after a credit event, an obligation's value may not be representative of recovery rates as there may not be enough information available for the market to price the obligation reasonably. Rating agency recovery studies are based on a minimum period of time after default. S&P, for example, requires a minimum of 60 days, while market practice is 72 days.
- **The number of readings involved in determining the loss amount and the frequency between these**. This prevents the final loss amount being artificially low should a poll produce too few prices. Rating agencies require at least two prices.
- **Obligation (s) used for reference pricing.**
- **Calculation agent, usually dealer, and maybe a third-party verifier.**

"Default" and Credit Events

"Default" and "credit event" are not identical terms

Throughout this report we interchangeably use "default" and "credit event". In reality, "default" as captured by rating agency statistics may sometimes be a more severe test than certain credit events. Moody's, for example, notes three categories of default for the purposes of its ratings and historical default statistics:

- Missed or delayed interest or principal payments;
- Bankruptcy or receivership; and
- Distressed exchange either leaving investors with a diminished financial obligation or an exchange for the apparent reason of avoiding default.

Recovery rate statistics reflect "hard default" events

Credit events for corporates are Bankruptcy, Failure to Pay and Restructuring. Rating agency recovery rate statistics reflect the above three events and do not include "soft" restructurings. Expected recovery following a "soft" restructuring credit event, for example, would likely be significantly higher than for liquidation.

Mod-R and Mod-Mod-R would still apply in mixed portfolios

Current market consensus is for **Modified-Modified Restructuring** to be applied to European reference entities and **Modified Restructuring** to be applied to US reference entities. If the underlying portfolio of the single-tranche CDO consists of both European and US reference entities, then the respective restructuring alternatives would still apply to those credits.

Portfolio Construction and Management

Portfolio Construction and Diversification

Initial portfolio construction agreed between the investor and dealer

Prior to the investment decision, the investor and dealer will agree upon the choice of underlying reference entities in the portfolio. The portfolio composition will typically involve decisions based on:

- **Number of Reference Entities**. Single-tranche deals typically reference portfolios of over 100 names.
- **Type of Reference Entities**. They are usually corporate credits, but ABS credits can also be added under some circumstances.
- **Credit Quality of Reference Entities**. Both investment grade and high yield credits can be selected, although typically portfolios are constructed of solely investment grade credits.
- **Subordination**. Credit risk can be transferred into the portfolio at both the senior and subordinated level. In Europe, subordinated contracts tend to be most liquid on banks and insurance companies. Typically, the market assumes a recovery rate of 35% for senior unsecured debt and 20% for subordinated debt but actual recovery rates following default are likely to vary around these averages.
- **Maturity**. Default swaps tend to be most liquid in the 5yr part of the curve with the issued tranches having a similar maturity. There is increasing liquidity in 7y and 10y maturities as investors extend duration for more yield.
- **Domicile of Reference Entity**
- **Notional Amount**

Portfolio diversity avoids the risk of excessive losses relating to a single name, country or industry

When constructing the underlying portfolio, investors should be mindful of the impact of portfolio diversity on deal tranching. **Diversification in a synthetic CDO pool serves to avoid the risk of excessive losses relating to a single name, country or industry category**. All other things equal, higher diversity would mean larger higher-rated tranches. The benefit gained from the diversification of the underlying collateral pool varies though between risk tranches in the deal structure. Most of the benefits of diversification in the pool are actually allocated to the senior tranches, while the mezzanine tranche, depending on its size may lie on the 'inversion point' in the diversification benefits.

Adverse Selection

Wide credits may simply reflect credit deterioration

One of the key determinants to the overall performance of a single-tranche deal is the initial portfolio selection, regardless of any credit substitutions permitted during the transaction. Credits that trade wide for their respective rating category

Refer to important disclosures on page 196.

may well be reflecting negative credit trends, and while cheap for the respective rating, may well result in portfolio losses further down the line. Given that the maturity of single-tranche transactions is typically over 5yrs, the choice of underlying collateral should be based on not only current credit quality but also **expectations about future credit quality**.

Structural models of credit risk are popular in gauging adverse selection

Structural models of credit risk can be used to help assess adverse selection. Market-based indicators of credit risk, such as total debt to total market capitalization and implied equity volatility are popular methods of assessing the market-implied credit quality of an issuer. Moreover, fundamental analysis such as that provided my Merrill Lynch credit analysts can be used to gauge future rating trends of issuers.

Static and Dynamic Transactions

The underlying portfolio is fixed in static transactions . . .

Synthetic CDOs can be both static and dynamic transactions. In static transactions, the underlying portfolio is agreed upfront between the investor and dealer and is fixed throughout the life of the transaction. Should a credit experience a negative trend before maturity of the transaction, there is no scope to substitute this name and prevent potential portfolio loss. In this type of structure, there is also no possibility of trading gains (or losses) from active management of the portfolio.

. . .but dynamic transactions are now becoming popular

Transactions can also be dynamic in nature. In this structure the underlying portfolio may be modified throughout the life of the deal, by either the investor or an elected manager. While the investor gives up some yield on the invested tranche for the right to have substitution (the investor can be thought of as long an 'option' to restructure the portfolio), credits that deteriorate may be substituted with less risky alternatives to prevent credit loss.

Deals may be 'lightly managed', as all single-tranche CDOs tend to be, where a small element of defensive trading is permitted or more fully managed, where the manager has the right to enter into hedging transactions to mitigate the credit risk of earlier trades or buy protection for arbitrage purposes. The trend towards dynamic transactions has been bolstered by the credit volatility and CDO ratings downgrades in recent years. Conversely, proponents of static transactions argue that substitution rights are of limited value in a well-diversified portfolio, as market risk is much more difficult to mitigate. That value also depends on the actual ability to trade the selected credit exposures.

Degrees of Management in Synthetic CDOs

Where active management is permitted, the choices for substitution rights are typically threefold:

Investor manages the portfolio

- **Investor Managed**. The investor has ultimate flexibility on initial composition of the portfolio and the on-going substitutions into the portfolio.

An elected third-party manager maintains the portfolio

- **Third Party Managed**. The investor delegates the management of substitution rights to a third-party asset manager, typically with experience in managing synthetic CDOs. The manager may well invest in a portion of the first-loss piece. The manager is responsible for the credit performance of the collateral portfolio and for ensuring that the transaction meets the diversification, quality and structural guidelines specified by the rating agencies. Part of the credit analysis may focus on the capabilities of the CDO manager and the way the manager is compensated.

  This option may be appealing to new entrants into the credit markets who may not have the capacity to monitor a large portfolio of credits. However, they should keep in mind that in return for managing the collateral portfolio, the manager receives a fee. Such a fee is usually paid on the nominal value of the portfolio rather than on the notional of the investor's tranche.

Dealer manages the portfolio

- **Dealer Managed**. The dealer pays the investor an enhanced coupon to retain the substitution rights (although this option is much less common and was originally used to facilitate dealer hedging).

Credit Substitutions

Substitution for credit deterioration or improvement

Credit substitutions (replacing one credit with another) are allowed in the portfolio subject to certain conditions. Such conditions are set in place to ensure that the initial portfolio criteria remain in place. We note that single-tranche deals do not allow removal without replacement as it leads to deleveraging on a fixed income investment. Substitutions will typically be for reasons of:

Substitutions follow certain guidelines

- *Credit Deterioration*: spreads on the underlying reference entity have widened and the investor believes that a possible default is much more likely. These substitutions may be subject to constraints such as spread widening tests on the outgoing credit, as well as tests on the incoming (substituting) credit similar to the ones mentioned below.
- *Credit Improvement*: Substitutions due to credit improvement may be subject to a minimum spread tightening test as well as constraints meant to protect the credit quality of the portfolio: rating of incoming credit, industry and obligor concentration limits, concentration of certain rating categories, etc.

There may also be restrictions on the number of substitution trades permitted, typically 5-10 p.a. (depending on the trading account). Generally, there will be restrictions on the level of spread of the incoming credits vs. that of the outgoing credits, enforced through spread tests. Rating agencies tend also to stipulate that to maintain ratings, the credit rating for the incoming credit should be better or should not be of any worse credit quality than the outgoing credit.

A trading account may be created with the dealer on day one of the transaction

Where substitutions are allowed, a **trading account** may be created with the dealer on day one of the transaction. In the absence of such an account, all trading losses must be paid for by the investor. The size of the trading account will depend on the investment size, tranche subordination, average spread of the portfolio etc. Trading losses from substitution would be debited from the trading account whilst trading gains from substitution would be added to the trading account. Such an account is for the benefit of the investor and it may choose to withdraw available funds or add funds at any time.

Higher trading accounts would generally mean more trading flexibility

Trading out of 'deteriorating names', generating a loss, will be allowed to the extent that the balance in the trading account is sufficient to cover the loss (where the account cannot cover the loss, trading may still be permitted with the mark-to-market payment settled immediately between the investor and dealer). Higher trading accounts would generally mean more trading flexibility but a lower running spread on the tranche.

The loss or gain from substitution will be a function of two factors:

- **Mark-to-Market on the Outgoing Credit**, a function of its spread change since trade inception and remaining duration[5]. This calculation needs to be adjusted for the characteristics (spread, duration, etc.) of the incoming credit.
- **The Delta of the Credit**. A transaction may or may not adjust the gain/loss for deltas of the incoming and outgoing credits. We talk more about Deltas in Volume 2, Chapter 5.

Delta plays an important part in determining trading gains or losses

Importantly, Delta (which is less than 100%) has the impact of 'dampening' trading losses (but also dampening any profit from trading gains).

In the absence of a trading account, changes in P&L may be reflected in a <u>modification to the running coupon on the tranche.</u> In this case, the NPV of the substitution is converted into a coupon value (the coupon value of a 1bp annuity is needed to determine this).

Subordination can also be adjusted

Furthermore, any trading gains or losses can also be added or subtracted from subordination beneath a mezzanine tranche. This requires converting the trading gain/loss into a mark-to-market change in the tranche by changing the subordination. Such a conversion is done through an adjustment factor.

[5] See Volume 1, Chapter 3 for a discussion on unwinding CDS.

Overall, the trading gain/loss can be reflected in cash (trading account), coupon change or subordination change. In any of these cases a different adjustment factor is applied.

Pricing Drivers

Portfolio credit risk influenced by 4 factors

The loss profiles of portfolio credit derivative products can be complex but are essentially influenced by the following factors:

- The number of credits underlying the portfolio and their granularity,
- The individual default probabilities of each credit,
- The recovery rate assumptions of the underlying credits,
- The **default correlation** assumption between the credits.

Importance of Default Correlation

Default correlation fundamental driver of pricing

Default correlation assumptions estimate the tendency of credits in a portfolio to default together over a given time horizon. While in practice there is a lack of historical data that can be used to reliably extract default correlations, empirical evidence shows that default correlation is linked to credit rating and varies with time. In a negative economic environment, lower rated companies that are generally more leveraged are likely to default together and hence default correlation is expected to be higher. Conversely, in a positive economic environment, default correlation is expected to be lower.

One can estimate default correlation from asset correlation. . .

Proxies for default correlation might include looking at the asset correlations of companies. These models, called 'structural models' consider a firm in default when the value of its assets fall below a certain threshold amount, such as the face-value of its debt. As such, the probability of two firms defaulting is simply the probability that the market value of each firm's assets falls below their corresponding default threshold amounts. Stock market data is usually used to derive asset correlations as equity market information tends to be readily available and have a longer history from which to perform analysis. The asset correlation derived in this manner is deterministically related to the default correlation, i.e. one can be transformed into the other.

. . .or spread correlation. . .

Another approach is spread "jump" models. In these models, credit spread correlation is used to determine the expected spread widening (or mathematically, a jump in the annualized default rate) of the non-defaulted credits in case one credit in the portfolio defaults.

. . .or rating downgrade correlation

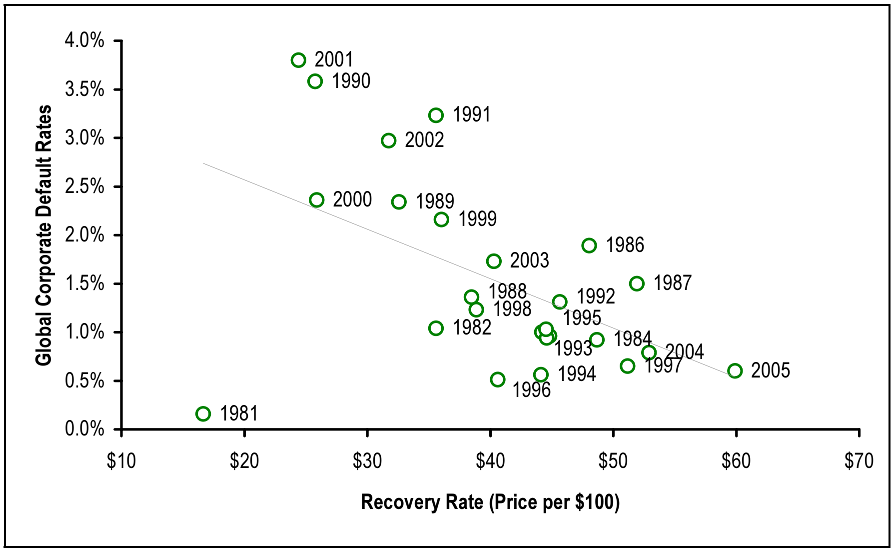
Finally, rating transition studies can also be used to imply default correlation. In such cases, the assumption is that the correlation of two bonds being downgraded more or less at the same time is an indicator that they may default at the same time.

Relationship Between Default Rates and Recovery Rates

Inverse correlation between default rates and recovery rates

A further corollary to the above is that there is a relationship over time between default rates and recovery rates of corporate bond issuers. In particular, there is an inverse correlation between the two: an increase in the default rate (defined as the percentage of issuers defaulting) is generally associated with a decline in the average recovery rate. In other words, default severity will be highest when defaults are at their most severe. Chart 11 plots average yearly recovery rates against associated default rates and highlights the negative correlation between the two.

Chart 11: Inverse Correlation of Default Rates and Recovery Rates



Source: Moody's Default and Recovery Rates of Corporate Bond Issuers, 1920-2005; *January 2006, David Hamilton.*

Default rates and recovery rates tend to be negatively correlated: default severity will be highest when defaults are at their worst

■ Correlation and Portfolio Loss

We can examine how default correlation assumptions can have a significant impact on the **distribution of losses experienced by a portfolio of credits**. It is the loss distribution that encapsulates the default characteristics of a portfolio of credits and ultimately drives pricing of the various tranches. Understanding the shape of the loss distribution and what factors influence it provides intuition as to the pricing and risk of tranched portfolio credit products.

Table 4: Hypothetical Portfolio

Characteristics	Assumptions
Number of Credits	50
Spread for each credit	100bps
Recovery for each credit	50%
Maturity	5y

Source: Merrill Lynch

For the hypothetical portfolio in Table 4, we examine three scenarios: low, medium and high correlation assumption between credits. In all cases we examine the shape of the expected loss distribution from holding this portfolio to maturity.

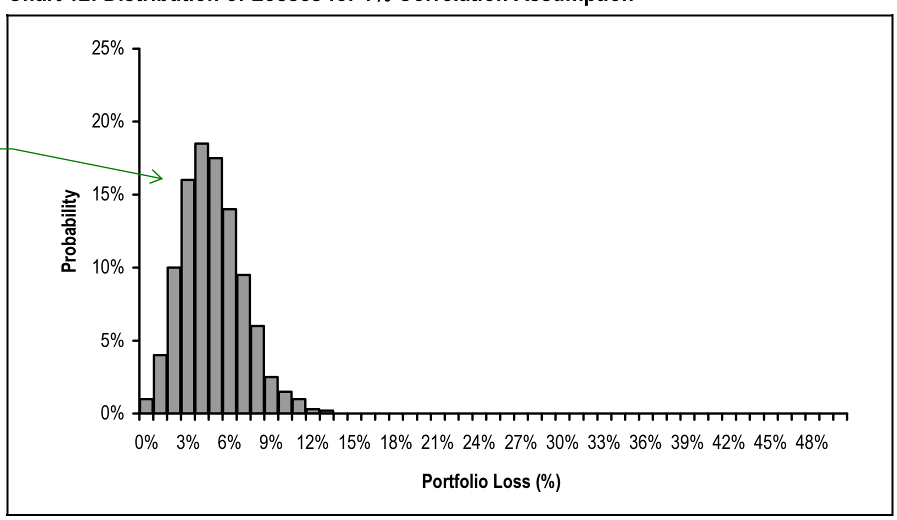
Distribution is 'symmetric' for very low correlation

Case 1: Low Default Correlation Assumption (1%)

With an extremely low default correlation assumption, the distribution is almost 'symmetric'. There is a high probability of experiencing a few losses but **almost no probability of experiencing a very large number of losses**. Furthermore, there is also a **low probability of experiencing zero losses**. Chart 12 plots the loss distribution for the 1% correlation case.

Chart 12: Distribution of Losses for 1% Correlation Assumption

'Symmetric' distribution with low probability of large number of losses but also a low probability of zero losses



Source: Merrill Lynch

Case 2: Medium Correlation Assumption (45%)

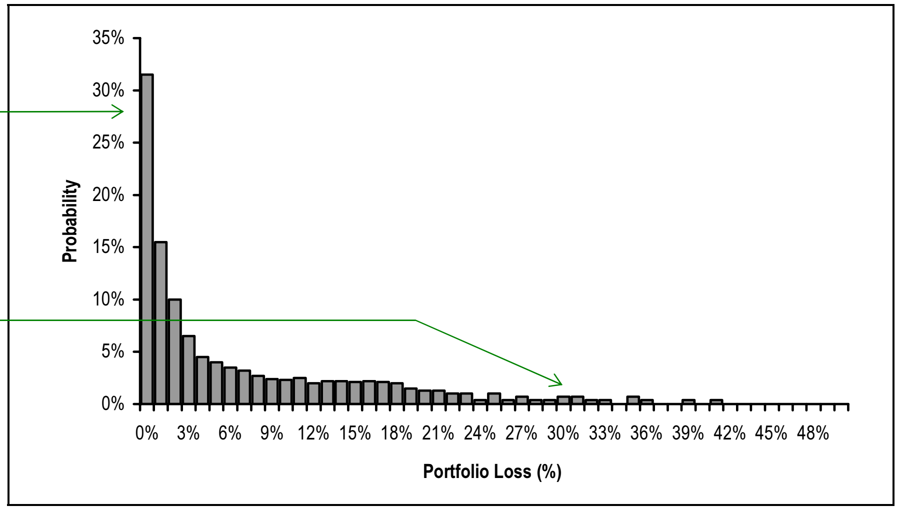
Distribution skewed for increasing correlation

With a medium default correlation assumption, the distribution begins to look more 'skewed'. Compared to Case 1, we now see that there is a **higher probability of experiencing no defaults** but also a **higher probability of experiencing a large number of losses**. As a result, there is now a 'tail' developing in the loss distribution profile, as there is a greater likelihood of assets defaulting together. Chart 13 plots the loss distribution for the 45% correlation case.

Chart 13: Distribution of Losses for 45% Correlation Assumption

Increasing probability of no losses

Increasing probability of a large number of losses



Source: Merrill Lynch

Case 3: High Correlation Assumption (99%)

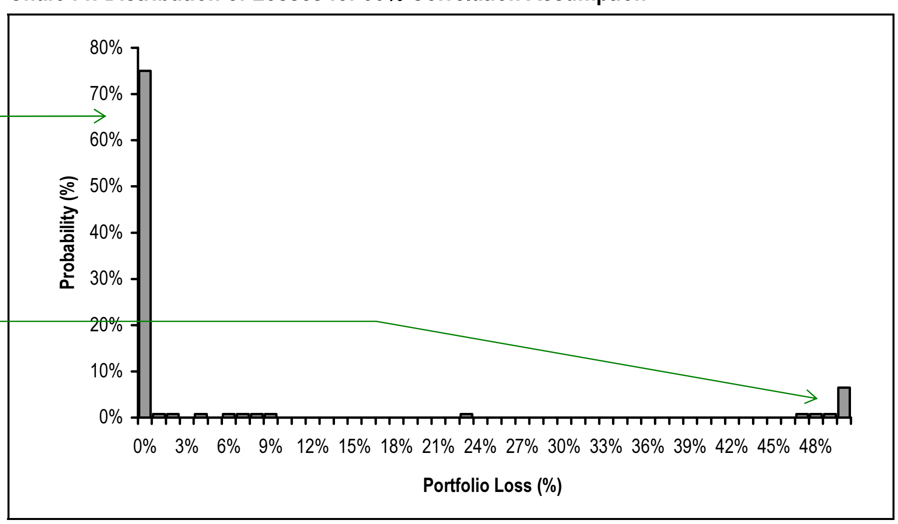
High correlation portfolios behave like a single asset

With an extremely high default correlation assumption, the **portfolio behaves almost like a single credit** and there is no diversification. The loss distribution has virtually disappeared apart from two peaks at 0% and 50% loss. **The assets either all survive, or they all default** with recovery rate equal to our assumption of 50%. Chart 14 plots the loss distribution for the 99% correlation case.

Chart 14: Distribution of Losses for 99% Correlation Assumption

75% probability that nothing will happen and ***losses will be 0%***

Material probability that all credits default with ***50% recovery rate***



Source: Merrill Lynch

Correlation, Recovery, Spread and Tranche Pricing

Tranche Pricing as a Function of Correlation

Correlation drives tranche pricing

We can now analyze how the different outcomes in the above three cases can provide intuition as to the pricing dynamics of the various tranches of a synthetic CDO. To aid the analysis we tranche a 50 name portfolio into a simple structure of equity, mezzanine and senior pieces, all of 4% size, with subordination of 0%, 4% and 8% respectively. Table 5 details transaction specifics. The tranche premium for each of the three tranches as a function of correlation is shown in Chart 15.

Table 5: Hypothetical Portfolio: 3 Tranche Example

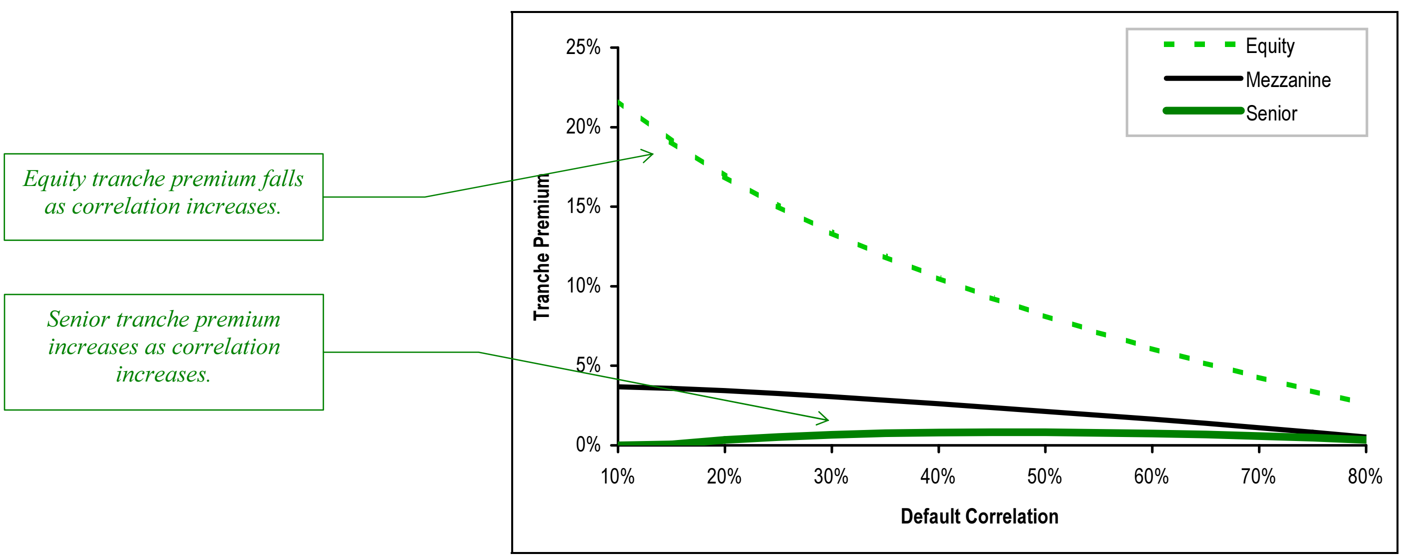
Portfolio and Tranche Characteristics	Assumption
Portfolio Summary	
Number of Credits	50
Notional Size Per Credit	$15m per name
Total Portfolio Size	$750m
Maturity	5 years
Default Swap Premium	Each name trading at 100bp
Recovery Assumption	35% for each name
Tranche Summary	
Equity Tranche	0%-4%
Mezzanine Tranche	4%-8%
Senior Tranche	8%-12%

Source: Merrill Lynch

The equity tranche is the riskiest to own and has the highest premium for all correlation levels

For <u>all levels of correlation</u>, the premium on the equity tranche is the highest, the premium on the senior tranche is the lowest and the premium on the mezzanine tranche lies between the two. This reflects the subordination of the various tranches. The equity tranche, being the first-loss tranche, has no subordination and is inherently the riskiest tranche to own. Conversely, the mezzanine and senior tranches are protected by 4% and 8% subordination respectively and command a lower risk premium than the equity tranche. **It should be noted that tranche premia are very sensitive to the average level of spread in the underlying portfolio**.

Chart 15: Tranche Premia Versus Correlation



Source: Merrill Lynch

Refer to important disclosures on page 196.

Low correlation

At **low** correlation, the assets are virtually independent. Chart 12 shows that the probability of a large number of losses is small. As a result, the probability of losses reaching the senior tranche is low and therefore the spread required to hold this tranche is small (or buying protection on the senior tranche is less valuable). Similarly, Chart 12 also shows that the probability of zero losses is small and hence there is a high probability that the equity tranche will suffer losses. The spread required to hold this tranche is high. The spread of the mezzanine tranche lies between the two.

Medium Correlation

At **medium** correlation, the assets in the portfolio become more likely to default together, and the tail of the portfolio loss distribution is pushed out, pushing more of the risk into the senior tranche. Chart 13 shows that the probability of a large number of losses has increased. As a result, the probability of losses reaching the senior tranche is higher and therefore the spread required to hold this tranche has increased (buying protection on the senior tranche is now more valuable). Chart 13 also shows that the probability of zero losses has increased making the equity tranche less risky to hold. The spread required to hold this tranche has decreased. Again, the spread of the mezzanine tranche lies between the two.

High Correlation

A **high** correlation, the portfolio virtually behaves like one asset, which either defaults or survives. Chart 14 shows that the probability of a large number of losses is high. As a result, the probability of losses reaching the senior tranche is high and therefore the spread required to hold this tranche is high. Similarly, Chart 14 also shows that the probability of zero losses is high making the equity tranche less risky to hold. The spread required to hold this has decreased further (buying protection on the equity tranche is less valuable). Again, the spread of the mezzanine tranche lies between the two.

Mezzanine tranche less sensitive to correlation

Correlation is highly important for stand-alone tranches. However, except in the extremes, the **mezzanine tranche tends to be less sensitive to correlation** than the senior or equity tranches.

An investor's tranche preference summarizes their correlation view

An investor's view on correlation can be summarized as follows:

- The Equity tranche holder (i.e. protection seller) is a buyer of correlation.
- The Senior tranche holder is a seller of correlation.
- The Mezzanine tranche holder can be characterized as more neutral with regards to correlation (although this will depend on transaction specifics).

Tranche Pricing as a Function of Recovery Rate

Tranche pricing also sensitive to recovery rates

We can also look at how the premium of the various tranches changes as recovery rate assumptions are changed, keeping spreads constant (Chart 16). Increasing the recovery rate means that for a fixed credit spread, the annualized probability of default increases[6]. However, if default does occur, the payment received by the protection buyer is lower.

Equity tranche premium rises as recovery rate is increased

The break-even coupon on the equity tranche rises as recovery rate is increased. This at first may seem somewhat counter-intuitive but the reasoning lies in the dominating impact of rising default probability (which is ultimately what buyers of protection on a first-loss piece care most about). Buying protection on the equity tranche is more valuable as recovery rates are increased (for fixed spread) and so the premium of the equity tranche rises.

Senior tranche coupon falls as recovery rate is increased

The break-even coupon on the senior tranche falls as recovery rate is increased. For the senior tranche, the dominating effect is the higher recovery rate. Increasing the recovery rate means that less and less of the losses on the portfolio hit the senior tranche. Buying protection on the senior tranche is less valuable as recovery rates increase, and so the tranche premium falls.

The mezzanine tranche is the least sensitive to recovery rates.

[6] We estimate the annualised probability of default (λ) from the CDS spread S and recovery R: $\lambda \approx S / (1-R)$. Over 5yrs, the default probability is $1 - \exp(-\lambda*5)$

Tranche Pricing as Credit Spreads Shift

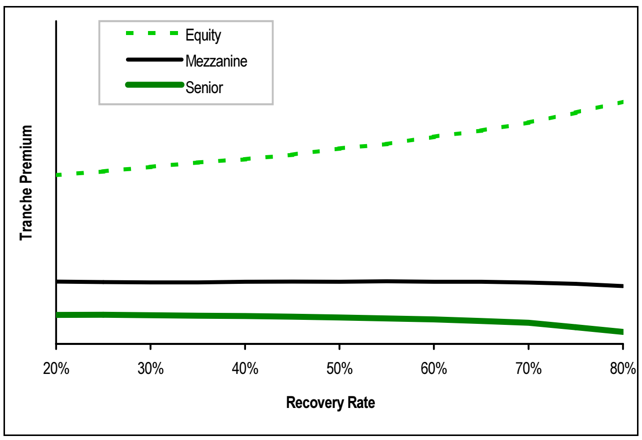
Tranche premium increases for increasing portfolio credit spreads

From our base case of 100bps spread for all credits in the underlying portfolio, we show the impact on tranche premiums as all credits spreads are shifted wider (parallel shifts). Chart 17 shows the premia of the three tranches as credit spreads are increased (keeping correlation and recovery constant). As spreads widen, all break-even coupons increase, and vice versa.

Rising credit spreads imply rising default probability for a fixed recovery rate

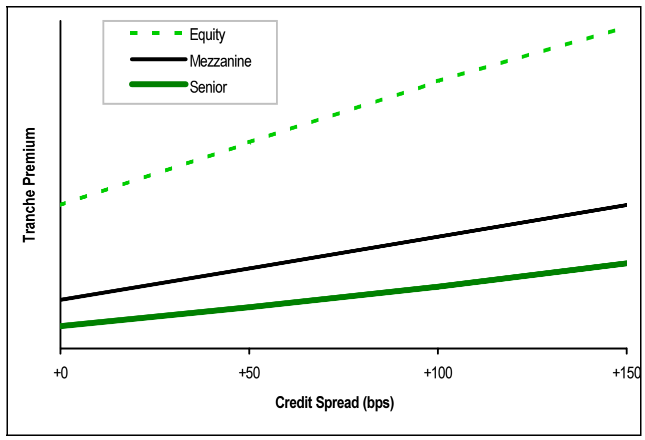
Senior tranche protection only pays out if a large number of individual defaults occur: the likelihood of this occurring increases significantly if credit spread – and hence the probability of default – is increased. Protection on the equity tranche is likely to pay out over the life of the trade, irrespective of whether spreads are increased or not. Increasing credit spreads makes it likely that defaults in the underlying portfolio will happen sooner, but the impact of rising credit spread is more significant on the senior tranche premium.

Chart 16: Tranche Premium Versus Recovery Rate



Source: Merrill Lynch. Credit Spreads for underlying portfolio constant at 100bps.

Chart 17: Tranche Premium Versus Credit Spread Shift



Source: Merrill Lynch. Constant recovery and correlation assumptions. Shifts are from base case of 100bps spread for all credits.

In summary,

- For the equity tranche, premium increases with a decrease in correlation and with an increase in recovery rate.
- For the senior tranche, the converse holds. Premium increases with an increase in correlation and a decrease in recovery rate.
- The mezzanine tranche demonstrates reduced sensitivity to correlation and recovery rate, behaving in a similar fashion to the senior tranches for certain ranges of recovery and correlation and behaving in a similar fashion to the equity tranche for other ranges of recovery and correlation.
- All tranches have higher coupons as the spread on the underlying portfolio increases, and vice versa.

Refer to important disclosures on page 196.

3. First-to-Default (FTD) Baskets

Explaining the Structure

First-to-default basket swap functions like a CDS with a key difference

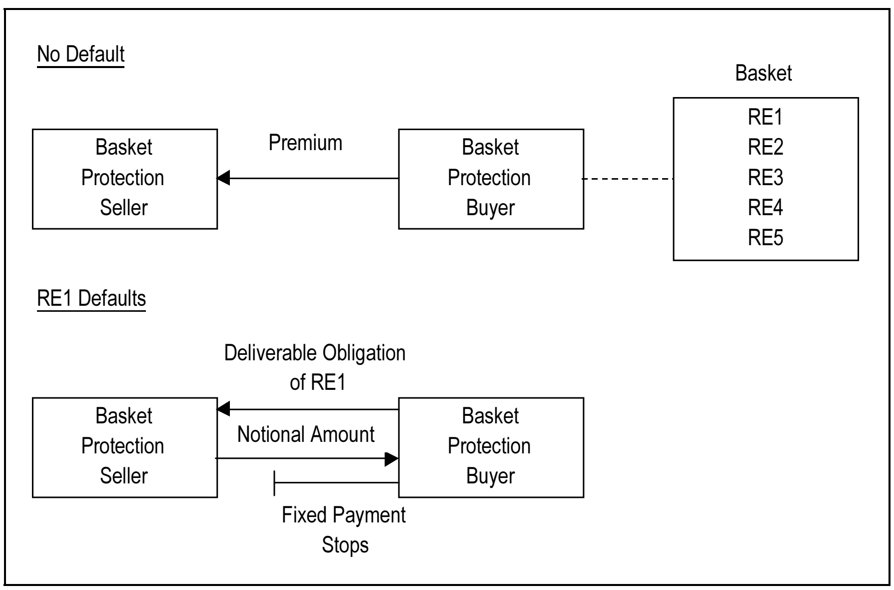
A FTD basket works in a similar manner to a single-name CDS with a crucial difference - the protection seller of a FTD basket provides protection against the first reference entity that experiences a credit event from a basket of more than one reference entity. The protection seller, therefore, assumes the "first-to-default" risk on a basket of credits.

For example, let's assume a basket of five credits with a basket notional amount of €10 million. If any one of these credits experiences a credit event, the basket default swap terminates and the protection seller pays €10million in exchange for the notional amount of a deliverable obligation of the credit that experienced the credit event (see Chart 18).

Sellers are attracted by the leverage

Investors can create a simultaneous exposure to multiple credits by selling protection on a FTD basket. The protection seller is motivated primarily by the leverage obtained by investing in such a structure. In the basket described above, the seller is exposed to the credit risk of five names with a total notional amount of €50 million. The seller receives a premium that is higher than any individual default swap to reflect the higher level of risk. However, in the event of default, the seller's maximum loss is limited to the notional amount for only one of the reference credits, i.e. €10 million. The investor has effectively sold five different default swaps but after the first credit event, the remaining four default swaps are knocked out.

Chart 18: First-to-Default Basket Swap



Note: RE = Reference Entity.
Source: Merrill Lynch

Lower cost method of hedging for buyers

The protection buyer views a basket swap as a lower cost method of hedging multiple credits (or, in effect, providing an equity cushion to this part of its portfolio). However, since the seller is exposed to the notional amount of only one (the first-to-default) credit, the buyer retains the residual risk of multiple defaults. This residual risk represents the imperfect hedge for the buyer. The potential cost of managing the hedge could determine the price the buyer is willing to pay for the basket.

Also available in funded form

FTD baskets can also be offered to investors in the form of credit-linked notes (CLNs). CLNs are created by embedding credit derivatives in new issues from a special purpose vehicle (SPV). A FTD basket CLN has an embedded FTD basket swap and enables an investor to indirectly sell protection while investing in a cash instrument. We discuss CLNs in more detail in Volume 1, Chapter 9.

Basket Pricing

Valuation Inputs

FTD basket pricing more complicated than CDS . . .

As might be expected, pricing a basket is more complicated than pricing a single-name CDS. Any theoretical model of pricing basket swaps would include the following key inputs:

- number of reference entities;
- probability of default of reference entities and protection seller;
- default correlations between reference entities;
- default correlations between reference entities and protection seller;
- maturity of swap;
- expected recovery value of the reference entities.

The basket premium depends not only on the probability of default of each credit in the basket (implied from credit spreads and recovery) but also on the default correlation between these credits.

. . . driven by default correlations

As the seller of a basket, an investor is essentially paid for a single default plus the increased likelihood of the occurrence of default. Given that the reference credits are typically less than perfectly correlated, the credit risk of a basket would, therefore, be greater than a single-name CDS for any of the basket constituents. The seller should be compensated for this risk with a higher yield on the basket than any single-name CDS. The weaker the correlation relationship the greater the degree of additional compensation that should be required.

Basket premium greater than the worst but less than the sum

The following boundary conditions should apply to the basket premium:

1. Basket premium should **exceed the single-name default premium on the weakest credit** in the basket. This compensates the seller for the increased likelihood of default relative to any single reference entity.
2. Basket premium should be **less than the sum of the premiums** available for single-name default swaps for each credit in the basket[7]. This condition should be satisfied because the buyer is not buying protection on all the names in the basket but only on the first one to default.

Unlike a single-name CDS, a FTD basket cannot be replicated in the cash market making it difficult to price this instrument from arbitrage relationships between the cash and the derivative markets. The practical approach to pricing a FTD basket is derived from the dynamic hedging behavior of dealers who buy protection on FTD baskets as described in the following pages.

Dynamic Hedging of the Basket

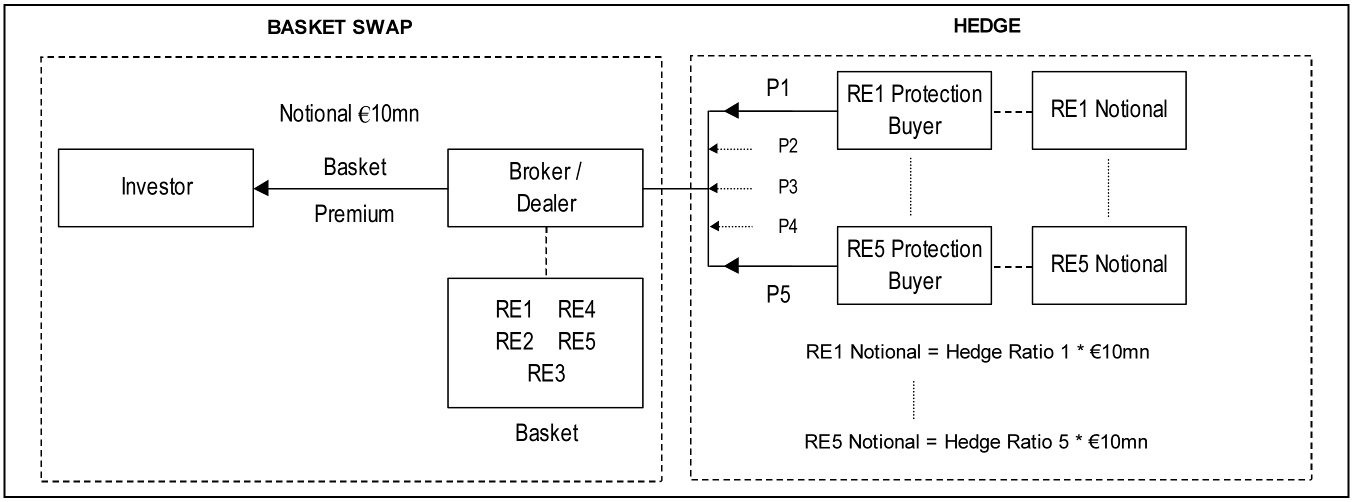
The price of the basket is affected by the cost of managing the hedge

The hedging behavior of a dealer provides some intuition behind the actual basket premium. **A dealer that buys protection on a basket from an investor would normally hedge this transaction by selling default protection on each individual name in the basket**. Chart 19 illustrates the hedge.

The amount of protection sold by the dealer in each name is known as the **delta** or the **hedge ratio** of that name. Among other factors, hedge ratios depend on default correlations and relative premiums of the single-name default swaps of the underlying credits. If single-name default swaps trade at similar levels, all credits would have similar hedge ratios assuming similar recovery rates.

[7] We assume that the reference entities are positively correlated.

Chart 19: Hedging a First-to-Default Basket Swap: Broker/Dealer is the Protection Buyer



Source: Merrill Lynch

The hedge is exposed to an unwind risk – greater the correlation, greater the risk

As the underlying default premiums shift, the deltas will change and the hedges will need to be rebalanced dynamically. The efficiency with which the hedge can be managed is a key factor that determines the basket premium. For small movements in the hedge ratio, the dealer may not be able to sell or buy protection and may instead buy or sell bonds to hedge thus taking on basis risk.

Following a credit event, the dealer will be forced to unwind the hedges on the other credits (assuming non-zero deltas for these credits). The cost of unwinding the hedge would depend on the spread movement for each of the non-defaulted credits. This, in turn, would depend on the correlation between the defaulted and the non-defaulted credits.

The greater this correlation, the greater the expected spread widening for a non-defaulted single-name default swap. This would imply a greater cost of unwinding the hedge. The dealer would therefore maintain a lower delta i.e., sell a lower amount of protection, to minimize losses from the unwind. This would, in turn, provide a lower premium to pay for the basket protection.

On the other hand, a low correlation would imply a lower expected spread change in a non-defaulted credit in the event of default and consequently a lower cost of unwinding that hedge. The hedger could therefore maintain a higher delta to manage the hedge i.e., sell a higher amount of protection. This provides a higher premium to pay for the basket protection.

Hedge is typically negative carry trade for dealer . . .

Negative Carry & Long Gamma Trade

Consider the following basket example:

- Three-credit basket, each 5y single-name default swap trades at 100bps.
- At 50% correlation, model-implied **breakeven basket premium**[8] is 236bps.
- The hedge ratio for each name in the basket is 68.4%.
- The **hedge carry**[9] is therefore 205bps (68.4% x 100 x 3).
- The hedge carry is less than the breakeven basket coupon and thus the dealer has a **negative carry** of 31bps.

[8] The breakeven basket premium is one that makes the expected value of the trade zero on day one.
[9] Hedge carry is defined as the sum of the premiums received from selling single-name default swaps of credits in the basket.

For typical baskets, **the hedge is a negative carry trade for the dealer**, i.e., the breakeven basket premium is greater than the hedge carry. This is due to a positive net expected gain[10] following a credit default.

Basket swaps cannot be fully replicated using only single-name default swaps. In other words, single-name default swaps cannot be used to hedge simultaneously the stochastic process of the spread movement of individual credits and the stochastic process of the actual default of any one of the credits. Dealers typically hedge only the spread process and are thus less than fully hedged. As a result, they pay a negative carry in return for a net expected gain on default. The difference also reflects the fact that the dealer is long gamma as described below.

. . . as well as a long gamma position

Gamma is defined as the rate of change of delta. As the spread of an underlying credit widens, the dealer needs to sell more protection on that credit to rebalance the hedge. Thus the hedge ratio or delta for this credit increases, i.e. gamma is positive. The dealer's hedge is a long gamma trade and dynamic hedging benefits the dealer in the following way:

- If a reference credit widens, the delta increases and the dealer sells more protection increasing the carry on the trade.
- If a reference credit tightens, the delta decreases and the dealer buys more protection thus booking gains and reducing risk.

For typical baskets, a static hedge would have a negative carry that can be recaptured in the process of dynamic hedging. From an arbitrage perspective it is intuitively satisfying to infer that if the dealer is hedging only the spread process of underlying reference credits, the hedge should have a negative carry.

Default Correlation

Basket premiums decline with increasing correlation

Default correlations are key determinants of hedge ratios which determine basket premiums that dealers are willing to pay. The boundary conditions for the basket premium can be restated in terms of the default correlation as follows:

1. If the default correlation among the credits is equal to 0, the basket premium should be equal to the sum of all the single-name default premiums.
2. If the default correlation among credits is equal to 1, the basket premium should be equal to the widest single-name default premium (or the lowest quality credit).

Basket premiums should, therefore, decline with an increase in correlation. A basket of **uncorrelated credits trading at similar spreads produces the largest relative increase in premium** compared to the average single-name default swap premium.

Default, asset & spread correlations

Default correlations impact the likelihood of multiple defaults up to a given time horizon. In practice, there is a lack of historical data that could be used to extract default correlations. Instead, market players use the **asset correlation** to calculate default correlation.

Asset correlations can be extracted from the "ability to pay" process of a portfolio of firms. Such a process is modeled for an individual firm as its market value of assets minus liabilities. Market inputs are equity and debt data. The asset correlation derived in this manner is deterministically related to the default correlation, i.e. one can be transformed into the other.

Another approach is to apply "jump" models. In these models, a **spread correlation** is used to determine the expected spread widening (or mathematically, a jump in the annualized default rate) of the non-defaulted credits in case one credit in the basket defaults.

[10] Net expected gain following a credit default = Expected gain on the defaulted credit *less* the expected loss on unwinding the surviving credits.

Refer to important disclosures on page 196.

Sensitivity of Basket Swaps

Creating a Suitable Basket

Higher yielding FTD baskets provide greater leverage

A basket of credits needs to be carefully chosen to provide the desired level of leverage to the protection seller. A basket that that is based on credits with a low likelihood of multiple defaults (i.e. low correlation) would provide the seller with the highest leverage and the buyer with the most effective hedge. We would also expect such a basket to be relatively high yielding.

High-yield baskets are less attractive for FTD basket protection buyers

It makes more sense to use **investment grade rather than high yield** credits in a basket. Even though the high yield credits in a basket may be uncorrelated, the higher individual probabilities of default associated with each high yield credit could lead to simultaneous multiple defaults. In the event of a single default, the non-defaulted high yield credits may have deteriorated significantly to make purchase of new protection on them extremely expensive. Investment grade credits, on the other hand, would be less likely to experience such credit deterioration. If one member of a higher quality basket defaults, it is quite likely that the others can be rehedged at cost-effective levels.

Sample Basket Analysis

Basket premiums are driven by several factors including default correlations, number of credits in basket as well as the quality of the credits. In order to examine the sensitivity of the basket premium with respect to some of these factors, we use a sample basket with the following characteristics:

- 5 reference entities, 5-year maturity.
- €10mn notional amount.
- Each single-name default swap trades at 100bps.

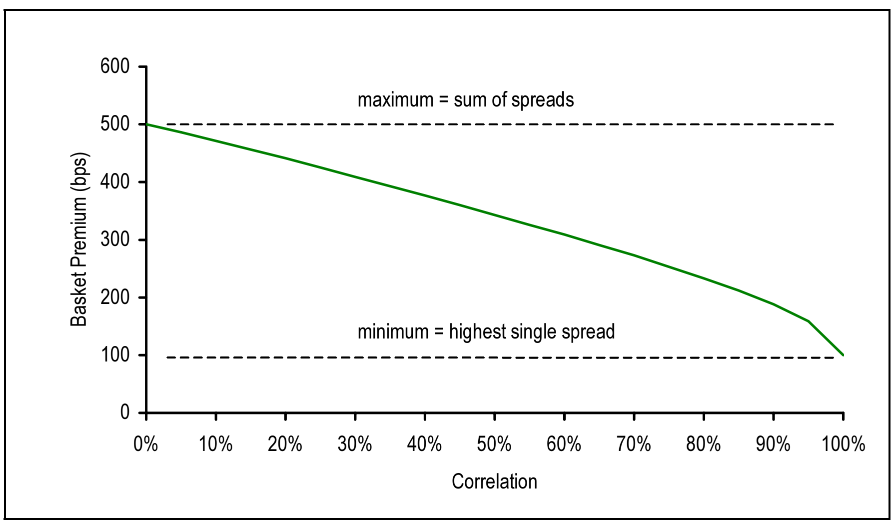
Correlation

Basket premium declines with increase in correlation . . .

As explained in the previous section, correlation drives the risk/reward tradeoff in a basket structure. The greater the correlation, the greater the probability of multiple defaults, i.e., the lower the value of protection to the buyer. Chart 20 highlights the relationship between the basket premium and correlation for our sample basket.

. . . and at an increasing rate

Chart 20: Basket Premium Declines as Correlation Increases



Source: Merrill Lynch

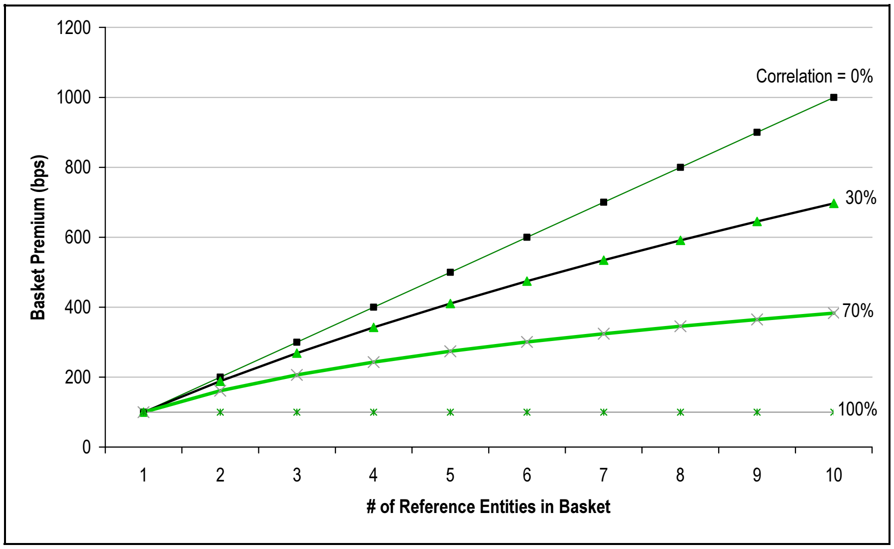
Number of Reference Entities

Additional REs increase the basket premium . . .

Assuming constant correlation, an increase in the number of credits increases the basket premium (Chart 21). As more credits are added to the basket, the risk of the first-to-default event increases and the seller requires a greater level of compensation. However, the rate of increase in the basket premium declines with an increase in the number of reference entities. From a dealer's perspective, balanced baskets with 3-7 reference credits can be hedged most effectively. More credits would imply low deltas (and therefore low hedge notionals) resulting in lower market liquidity to set up the hedges.

. . . but at a diminishing rate

Chart 21: Basket Premium Increases with Number of Reference Entities in Basket

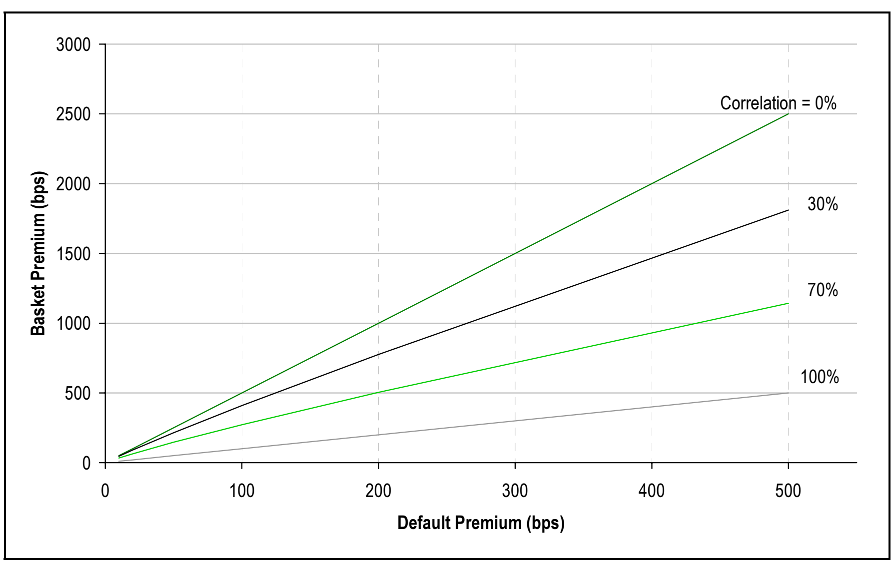


Source: Merrill Lynch

Default Premium

Chart 22 shows us the movement of basket price with a change in default premiums for all the reference entities in the basket. As the premiums increase by equal amounts for all the credits, the risk of first default of the basket increases. If basket entities are uncorrelated, we note that the price of the basket is equal to the sum of the individual default premiums.

Chart 22: Basket Premium Increases with Default Premiums



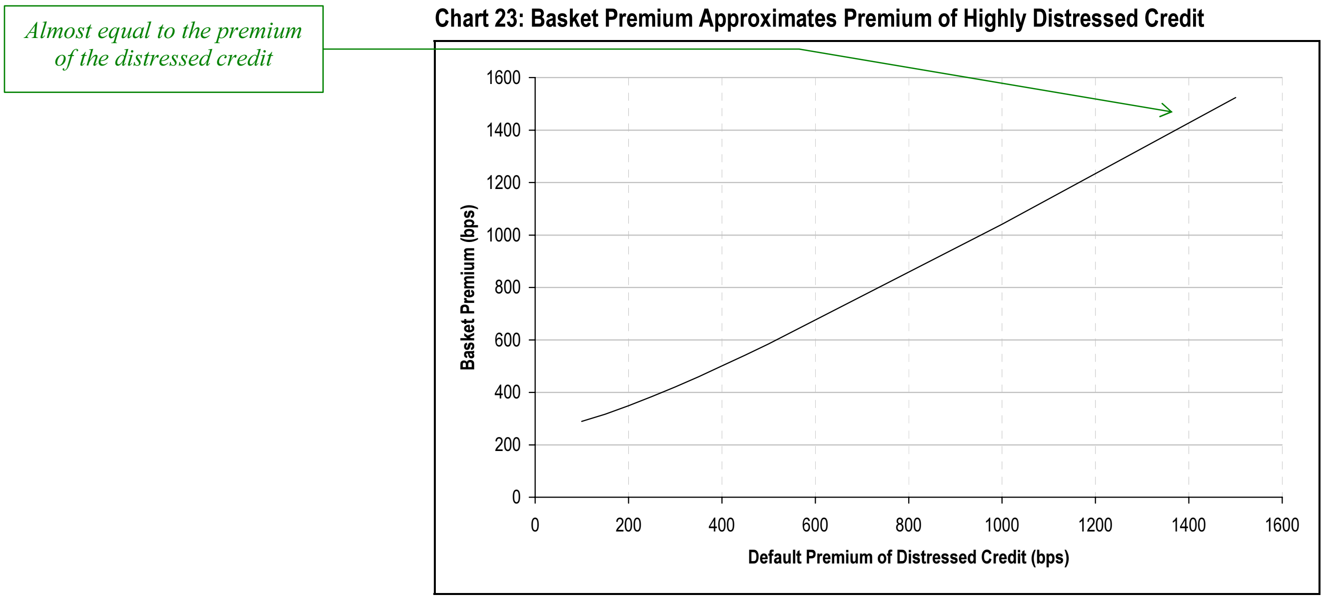
Source: Merrill Lynch

Refer to important disclosures on page 196.

Distressed credits dictate basket pricing

Basket structures make more sense for credits that are trading at similar spreads or those that have similar credit ratings. If one of the credits is extremely weak, then it would dictate the pricing of the basket making the protection on the other credits less valuable. This is demonstrated in Chart 23 for the same basket of five credits. The chart plots the basket premium as the single-name default premium of one of the credits widens while that of the other four remain steady at 100bps.

Chart 23: Basket Premium Approximates Premium of Highly Distressed Credit



Source: Merrill Lynch

Basket Swap Strategies

Investment Motivators

FTD baskets provide unique relative value opportunities to portfolio managers . . .

Basket swaps cannot be replicated in the cash market and provide some unique benefits to investors in terms of relative value and leverage.

- **Improving portfolio yields**: As credit spreads tighten, unleveraged investments in individual credits could fail to meet portfolio yield hurdles and become less attractive. In order to improve yields, portfolio managers can expand their gamut of investment opportunities by investing in lower rated, higher yielding assets. Alternatively, they can **sell protection** on a basket of approved names that meets the yield hurdle even though single-name default swaps on the reference entities may not meet the hurdle on their own.

- **Express a view on correlation**: Investors who believe that a group of credits have a higher correlation than that expressed by a basket swap on the same credits can express this opinion by **selling protection** on the basket. This trade looks more attractive as investors' opinion of correlation increases relative to that expressed in the market price of the basket.

- **Protection from a credit landmine**: Accounting and related uncertainties have increased concerns about unexpected deterioration for a particular credit. Though an investor may make the right sector allocations, sudden and sharp credit deterioration (e.g. Enron) could severely diminish portfolio returns. Investors can mitigate the effect of a credit landmine by **buying protection** on a basket swap. Though this protection would reduce overall portfolio return, investors would be protected in the current uncertain financial environment.

. . . but also have some inherent limitations

Investors also need to be aware of some of the limitations of basket swaps.

- **Liquidity**: Basket swaps are investor-specific and typically negotiated for baskets selected by investors for specified maturities. Investors can usually sell basket protection in maturities that correspond to the liquid single-name default swaps, usually 5 years.
- **Cheapest to deliver (CTD) risk**: Protection sellers take on CTD risk following a credit event. Physical settlement of the basket swap will likely consist of the lowest priced bond ranking pari passu with the reference obligation of the entity that experienced a credit event. This risk, however, is not specific to baskets. Since baskets are special forms of credit default swaps they share similar characteristics including the CTD risk.

Investor Strategies

Basket strategies can capitalize on these views

The potential benefits of basket investments can be translated into clear trading strategies for investors who wish to express particular views. We have outlined a few **examples of strategies with FTDs** in Volume 2, Chapter 7.

We discuss some of the strategies below.

1. **Creating leveraged positions**: As discussed above, as credit spreads tighten investors can sell FTD protection on a basket of approved names to increase portfolio yield rather than moving down the credit curve and investing in high-yield credits. Though the basket may consist of approved credits, the less than perfect correlation between them increases the risk of basket default relative to each individual credit.
2. **Creating a synthetic "senior" position**: Investors can take a long position in a small portfolio of credits and buy first-to-default protection on the portfolio. The net carry from this trade is lower than that from the individual credits but the trade is less risky as a loss will only occur if there are multiple defaults. Investors take the risk that the actual correlation is higher than the expected correlation increasing the likelihood of multiple defaults.
3. **Credit convexity trade**: Investors buy FTD basket protection and dynamically hedge by selling single-name default swaps on underlying credits. The investor is long gamma and has a potentially large upside. Due to the hedge the downside is limited except when actual correlation is greater than expected correlation. This trade typically has a negative carry, is non-directional and does not require price convergence or suffer during price divergence like most long/short strategies. The key risk in this trade is that FTD baskets are illiquid and the best "way out" of this trade is dynamic hedging until maturity. This requires active management and a commitment to follow and participate in the CDS market.
4. **Creating a cheap senior short**: Investor sells FTD basket protection and buys protection on each individual credit. The net position is similar to being short the senior tranche in this portfolio. If this position can be set up at really low rates then the investor has a small negative carry. The trade is then equivalent to buying cheap deeply out-of-the-money portfolio puts that have a big payoff when the entire market blows up and there are multiple defaults.

Replacement Language in FTD

FTDs have two types of replacement languages

A key issue in FTD documentation is the nature of the replacement language following a Succession Event for any of the credits in the basket. Users can select whether Substitution is (a) Applicable or (b) Not Applicable. The provision that is selected most often by investors is Substitution Applicable.

While this section aims to illustrate this issue for investors, we recommend that users take legal advice on any documentation issues.

Substitution Not Applicable

Successors that are Reference Entities are not replaced

If Substitution is Not Applicable, then the FTD contract is divided into the same number as the number of Successors determined by the 2003 definitions[11]. Each new transaction will include one of the Successors along with the other Reference Entities (that did not experience a Succession Event). If one Reference Entity is succeeded by one or more of the other Reference Entities in the FTD basket, the total number of Reference Entities may be reduced. If the protection seller merges with a Reference Entity the FTD contract will terminate.

Example

- Initial FTD basket: A, B, C, D ($10mn notional)
- B and X are Successors to A following a Succession Event
- Result: Two new FTD baskets
    - Basket 1: B, C, D ($5mn notional)
    - Basket 2: X, B, C, D ($5mn notional)

Basket 1 is reduced to a three-name FTD basket as B is one of the Successors to A and would therefore appear twice in the basket according to the rules, i.e. basket 1 should have B, B, C and D. However, the documentation does not allow duplication as a result of Succession and provides that if a Reference Entity is specified twice then it is deemed to be specified only once.

Substitution Applicable

Successors that are Reference Entities are replaced…

If Substitution is applicable following a Succession Event, then a similar process is followed i.e. the FTD contract is divided into the same number as the number of Successors determined by the 2003 definitions. However, there is one key change: if a Reference Entity is succeeded by another Reference Entity in the FTD, then this Surviving Reference Entity is not deemed to be a Successor and a Replacement Reference Entity is selected. The Surviving Reference Entity, however, remains in the basket. This should prevent the decrease in the number of Reference Entities in the FTD except in the case where there is a failure to select Replacement Reference Entities. If the protection seller merges with a Reference Entity, that Reference Entity is replaced by another credit.

The examples below illustrate two cases.

Example 1

- Initial FTD basket: A, B, C, D ($15mn notional)
- B, C, & X are Successors to A following a Succession Event
- B is replaced by Y; C is replaced by Z
- Result: Three new baskets
    - Basket 1: X, B, C, D ($5mn notional)
    - Basket 2: Y, B, C, D ($5mn notional)
    - Basket 3: Z, B, C, D ($5mn notional)

Example 2

- Initial FTD basket: A, B, C, D ($10mn notional)
- B & X are Successors to A following a Succession Event
- B needs to be replaced. However, protection buyer fails to deliver a Reference Entity List or both protection buyer and seller fail to select a Replacement Reference Entity.

[11] See Volume 1, Chapter 7 for in-depth discussion of Succession.

- Result: Two new baskets
  - Basket 1: X, B, C, D ($5mn notional)
  - Basket 2: B, C, D ($5mn notional)

If X is the protection seller itself, then a replacement has to be found according to the rules. If a replacement cannot be found, then the basket is reduced to a three-name basket.

Replacing the Reference Entity

…within 18 business days…

The replacement process of a Reference Entity is a three-step process:

- Within ten business days of the Succession Event, the protection buyer has to deliver a list of at least three Eligible Reference Entities (i.e. Proposed Reference Entities).
- Within five business days of receiving this list, the protection seller has to select one of the above Proposed Reference Entities and notify the protection buyer.
- If the seller fails to make a selection within the above timeframe, the protection buyer can then select a name from the above list within three business days and specify the name to the seller.

Eligible Reference Entity

…with appropriate credits

An Eligible Reference Entity has to satisfy the following conditions:

- It is in the same Moody's or S&P industry group as the entity to be replaced (Surviving Reference Entity).
- It has a bid-side credit spread no more than 110% of the bid-side credit spread of the entity to be replaced at the time the list is provided to the protection seller.
- It is traded in the same geographical credit derivatives market as the entity to be replaced.
- It is not an affiliate of (a) any Reference Entity in the FTD basket, (b) the protection seller or (c) the protection buyer.

Refer to important disclosures on page 196.

4. Standardized Tranches

This chapter assumes readers are familiar with the CDX and iTraxx family of indices described in Volume 1, Chapter 8.

Standardized tranches represent the most liquid part of the single-tranche synthetic CDO market. These tranches are carved out the standardized CDS indices that have been trading since late 2003. The most liquid standardized tranches are based on the following three underlying portfolios:

- CDX IG (125-name, investment grade)
- iTraxx Europe (125-name, investment grade)
- CDX HY (100-name, high yield)

The single-tranche synthetic CDO market is divided primarily into the (a) **bespoke** (or customized) market and (b) the **standardized** index tranche market. The standardized tranche market initially grew out of dealers' need to hedge out exposure to bespoke single-tranche synthetic CDOs which provided investors who sold these tranches with enhanced yield. The development of the standardized tranche market also led to correlation price discovery and an observable market view of correlation for a particular portfolio. Besides enabling better pricing of bespoke tranches, it also helped dealers release correlation reserves. Standardized tranche trading which was primarily a dealer activity has attracted more of the other investors including hedge funds, banks, insurance companies and private clients.

Investment Grade Standardized Tranches

5 liquid tranches for each index in North America and Europe

The most liquid single-tranches are those carved out of the two main investment grade indices: (a) **CDX IG** index in North America and (b) **iTraxx Europe** index in Europe. Each of these indices has five liquid tranches as shown in Table 6. Each tranche is defined by its

(a) **attachment point** which defines the level of subordination of a tranche, and

(b) **detachment (or exhaustion) point** which defines the maximum loss of the underlying portfolio that would result in a full loss of tranche notional.

For example, the 3-7% tranche implies a 3% attachment point and a 7% detachment point. In our discussion of these tranches, we generally refer to the **on-the-run tranches** that are derived from the on-the-run indices. The current on-the-run is the Series 5 for CDX and Series 4 for iTraxx (Table 6)[12].

Table 6: CDX IG and iTraxx Tranches

CDX IG (North America)	5y Mid (bps)	10y Mid (bps)	iTraxx (Europe)	5y Mid (bps)	10y Mid (bps)
0-3% (+500 bps)	35.5%	60 5/8%	0-3% (+500 bps)	27.75%	58%
3-7%	106	635	3-6%	78.5	527.5
7-10%	27	114	6-9%	27	100
10-15%	12	55.5	9-12%	11.5	44.5
15-30%	5	16	12-22%	5.25	24

Underlying 5y CDX: 45 mid; 10y CDX: 67 mid; 5y iTraxx 36 mid; 10y iTraxx 58 mid. All levels are indicative as of 30th January 2006; Assume exchange of deltas at mid; Equity tranche trades Upfront + 500bps
Source: Merrill Lynch

Tranche width is the difference between the detachment and attachment points. The notional amount of the underlying portfolio can be determined by dividing the tranche notional by the tranche width. For example, a $40mn notional of the 3-7% tranche implies an underlying CDX IG notional size of $1bn ($40mn/4%).

[12] Both indices roll every six months around 20th September and 20th March. See Volume 1, Chapter 8.

Since the CDX IG and iTraxx Europe are equally-weighted indices of 125 different credits, the **reference weight** of each credit is 0.8% (1/125) of the notional of the index portfolio. For example, exposure to $30mn notional of the equity (0-3%) tranche implies an exposure to $1bn notional of the underlying index and $8mn exposure to each of the individual index credits.

Liquidity has improved signficantly

The liquidity of the CDX standardized tranches has increased substantially since their inception in October 2003 (CDX was known as iBoxx at the time). The bid/offer on the 7-10% tranche (5y CDX IG index) is around 2bps compared to about 40bps in October 2003 (both cases assume deltas are exchanged at mid). In Europe too we have seen a similar tightening of the bid-offer spreads of the iTraxx tranches. The extremely liquid underlying index market has made it relatively easy to hedge spread risk in tranche exposures while at the same time isolating a specific correlation view. However, as liquidity in the tranche market builds up, correlation trends, in our opinion, continue to be driven primarily by demand/supply technicals leading to effects such as the correlation skew[13].

More demand for 7y and 10y tranches

While the 5y tranche remains the most liquid, since mid 2005 we have seen an increasing demand for 7y and 10y tranches as spreads have tightened and investors continue to seek higher spreads. We see strong liquidity in 5y, 7y and 10y for both CDX IG and iTraxx Europe standardized tranches.

Mechanics

Equity premiums typically paid upfront

The protection buyer of a tranche makes quarterly payments to the protection seller in return for credit protection on losses in the underlying portfolio. The equity tranche (0-3%) trades on the basis of an upfront payment plus a running spread of 500bps per annum. All the other tranches trade on a full running spread basis.

Tranche notional reduced if credit losses exceed attachment point

Premium payments are made until maturity or until the notional amount of the tranche gets fully written down following credit events. The protection seller makes payments to the protection buyer as long as credit losses are more than the attachment point of the tranche and less than the detachment point. When the total credit losses equal the detachment point, the tranche notional is fully written down. For each credit event that affects the tranche, the notional of the tranche is reduced by the loss amount and the premium payments are made on this reduced notional (see case study later in the chapter). In the next section, we highlight the number of losses required for a full notional loss for each tranche.

Tranche contracts based on ISDA defintions

The 2003 ISDA Credit Derivatives Definitions govern all tranche contracts. In North America, the underlying CDX IG portfolio has two credit events: Bankruptcy and Failure to Pay. iTraxx Europe, however, has three credit events: Bankruptcy, Failure to Pay and Modified-Modified-Restructuring (mod-mod-R).

Tranches of both indices are typically cash settled. Following the Delphi (DPH) default in 2005, CDX IG tranches were cash settled in an orderly manner.

The market uses standard confirmations to trade tranches. This lowers basis risk as an investor with a tranche exposure to one dealer can offset it via a trade with another dealer.

[13] See Volume 2, Chapter 9

Spread vs. Subordination

The 0-3% tranche of both indices is the first-loss or the equity tranche. This tranche is the riskiest as there is no benefit of subordination. The riskiness of each tranche decreases with an increase in subordination. This is reflected in the decreasing spreads and increasing ratings quality for each tranche as shown in Table 6.

Table 7: Credit Assessment for Tranched CDX IG Portfolio

	5Y	7Y	10Y
AAA	5.8%	6.9%	8.5%
AA	5.0%	6.0%	7.6%
A	4.4%	5.4%	6.8%
BBB	3.4%	4.3%	5.6%
BB	2.3%	3.0%	4.0%
B	1.5%	2.0%	2.9%
CCC	0.5%	0.8%	1.4%

Source: Merrill Lynch; Estimated using S&P's CDO Evaluator (v3.0) ratings model. Calculations as of 31st January 2006.

CDO ratings reflect more quantitative factors such as the default risk on the individual credits, the correlation of those risks, expected loss following default and the tranche structure of the CDO (attachment point and width). Table 7 highlights credit assessments for different attachment points for the current CDX IG index. These assessments have been estimated by Merrill Lynch using S&P's CDO Evaluator ratings model.

This is quite different from rating a debt instrument where a qualitative analysis forms a key factor in the ratings process. However, the ratings of the individual components of the index will themselves be based on corporate criteria. As a result, we find that CDO tranches can offer attractive relative value compared to other similarly rated assets.

Attractive spread for subordination level

Table 8 highlights the number of defaults required to absorb the full notional exposure for each tranche. A full notional loss of the equity tranche (either CDX or iTraxx) would require at least 6 defaults assuming a recovery of 30%. The AAA rated 7-10% tranche, which is trading at about 27 bps (5y), would require about 13 defaults at a 30% recovery before the tranche notional is hit. Similarly a AAA+ rated 10-15% tranche trades at 12 bps (5y) and would require about 18 defaults (at 30% recovery) before there is a loss on the notional.

Table 8: Number of Defaults for Full Loss of Notional

CDX IG	# of Defaults for Full Loss (by Recovery)				iTraxx Europe	# of Defaults for Full Loss (by Recovery)			
(125 credits)	20%	30%	40%	50%	(125 credits)	20%	30%	40%	50%
0-3%	4.7	5.4	6.3	7.5	0-3%	4.7	5.4	6.3	7.5
3-7%	10.9	12.5	14.6	17.5	3-6%	9.4	10.7	12.5	15.0
7-10%	15.6	17.9	20.8	25.0	6-9%	14.1	16.1	18.8	22.5
10-15%	23.4	26.8	31.3	37.5	9-12%	18.8	21.4	25.0	30.0
15-30%	46.9	53.6	62.5	75.0	12-22%	34.4	39.3	45.8	55.0

Source: Merrill Lynch

Spread vs. Correlation

Long/short tranche credit also implies a view on correlation

Taking a long or short credit position in a tranche also implies taking a view on correlation. Assuming all other factors remain constant we observe that long equity tranches are typically long correlation, i.e. spreads tighten if default correlation increases; long senior tranches are typically short correlation; and long mezzanine tranches are more or less correlation neutral depending on the attachment and detachment points of the tranche[14].

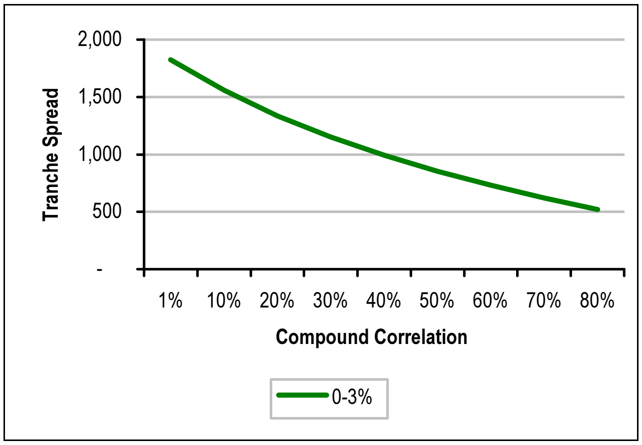
The charts below highlight the spread sensitivity of the CDX and iTraxx tranches for different underlying default correlations. **We assume that all other factors including underlying index spreads and recovery rates remain constant**. If the index moves, the shape of these graphs may change depending on the amount of index widening or tightening.

Implied correlation derived from market spreads

These charts illustrate that even if underlying credit spreads remain constant, tranche premiums change with changes in default correlation. All variables that affect the premium of a standardized tranche are typically known except default correlation. Therefore, for a given market premium, we can derive an **implied correlation** of the tranche via a model. The model that is used as a market standard to derive this implied correlation is called the Gaussian Copula model.

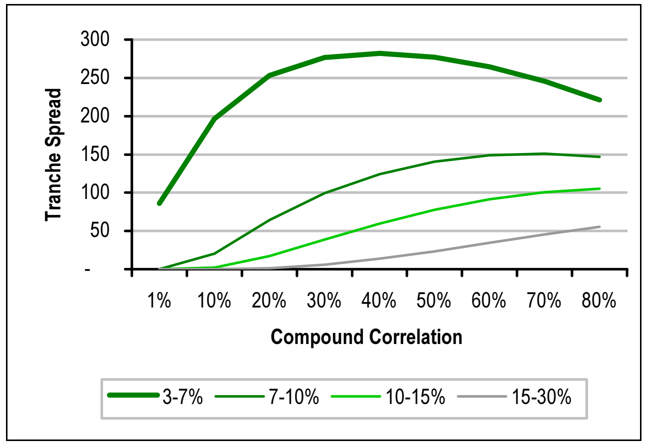
[14] See Volume 2, Chapter 2.

Chart 24: 5y CDX IG Equity Tranche Spread vs. Correlation



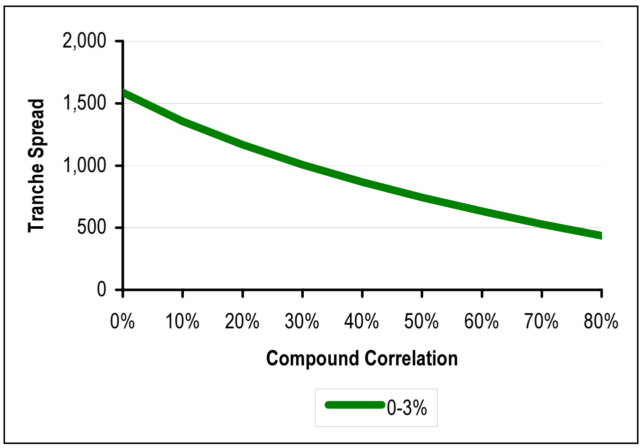
Underlying 5y CDX: 45 mid
Source: Merrill Lynch

Chart 25: 5y CDX IG Mezzanine and Senior Tranche Spreads vs. Correlation



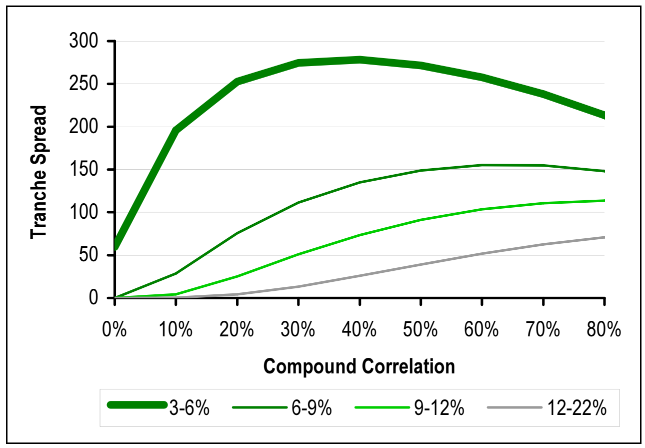
Underlying 5y CDX: 45 mid
Source: Merrill Lynch

Chart 26: 5y iTraxx Europe Equity Tranche Spread vs. Correlation



Underlying 5y iTraxx: 36 mid
Source: Merrill Lynch

Chart 27: 5y iTraxx Europe Mezzanine and Senior Tranche Spreads vs. Correlation



Underlying 5y iTraxx: 36 mid
Source: Merrill Lynch

Spread Sensitivity of Tranches (Delta)

Spread sensitivity of tranche denoted by 'delta'

One of the key factors that affect the tranche premium is obviously the spreads of the credits in the underlying portfolio. We measure the spread sensitivity of each tranche relative to each credit via a measure called 'delta'. The spread delta of a credit (for a particular tranche) is defined as the ratio of the tranche MTM to the credit MTM for a 1bp change in the credit spread. The riskier a tranche, the greater the sensitivity to individual spread movements, i.e. the equity tranche delta for each credit is greater than the senior tranche for the same credit.

Delta is also a function of spreads

A long or short position in a tranche can be hedged for spread movements by taking the opposite position in the underlying credit (hedge notional = delta *x* credit notional). This particular hedge would be spread neutral (i.e. MTM ~ 0) for small changes in the credit spread. Deltas lie between 0 and 1 and are themselves a function of the underlying credit spreads. As a result, investors who want to remain delta-neutral would have to continuously rebalance their deltas[15].

More practical to delta-hedge with underlying index…

The underlying portfolio typically has over 100 credits (125 each for CDX IG and iTraxx Europe) which makes this sort of individual credit delta-hedging extremely cumbersome. A more practical (though less accurate) hedge is to use the underlying index itself to hedge a tranche position. For example a long position in

[15] See Volume 2, Chapter 5 for in-depth discussion of tranche deltas.

the 0-3% CDX tranche can be hedged via a short position in the CDX index based on the tranche leverage. The tranche **leverage** is the relative notional amount of the index required to hedge the MTM in the tranche for a 1bps move in the underlying index[16].

…but less accurate

The advantage of this hedging method is that the standardized indices are extremely liquid[17]. An index hedge, however, is an average delta hedge. For a senior tranche, the tighter names (which have more of impact on senior tranche relative to equity tranche and therefore have higher deltas) would be underhedged. The wider names (which have less of an impact on senior tranche relative to equity tranche and therefore have lower deltas) would be overhedged. Table 9 highlights the tranche deltas relative to the underlying index, i.e. a long position in the 7-10% tranche of 5y CDX can be delta-hedged by buying protection on 1.5x the tranche notional of the underlying 5y CDX IG index.

The longer maturity of the 10y iTraxx shifts the allocated losses towards the mezz/senior part of the capital structure, thus lowering the riskiness (leverage) of the 10y equity tranche relative to 5y equity tranche.

Table 9: Leverage for Tranches of CDX IG and iTraxx Europe

CDX IG (North America)	5y Mid	10y Mid	iTraxx (Europe)	5y Mid	10y Mid
0-3%	20x	6.25x	0-3%	22.5x	8x
3-7%	5.75x	10x	3-6%	5.5x	10.5x
7-10%	1.5x	4.5x	6-9%	2x	4.25x
10-15%	0.8x	2.25x	9-12%	1x	2x
15-30%	0.35x	0.8x	12-22%	0.45x	1x

Underlying 5y CDX: 45 mid; 10y CDX: 67 mid; 5y iTraxx 36 mid; 10y iTraxx 58 mid. Levels as of 30th January 2006.
Source: Merrill Lynch

Tranche prices usually reflect "exchange of deltas at mid"

Investors who buy protection on a tranche could delta-hedge by selling protection on the underlying index at the bid. The dealer who sells protection on the tranche could delta-hedge by buying protection on the underlying index at the offer. One way to bypass this bid-offer is to "**exchange deltas at the mid**". Therefore both parties receive better hedging terms as they not only trade the tranche but also buy and sell the delta-hedge with each other at the mid level. The more important practical consequence of exchanging deltas at mid is that the investor benefits from more attractive pricing on the tranche since the dealer uses the mid level of the underlying index to price the tranches. This is especially beneficial to the investor for more leveraged tranches where the tranche bid-offer would be noticeably wider without exchange of deltas at the mid. The levels shown in Table 6 assume deltas are exchanged at the mid.

■ Index Roll

Index roll every six months…

One of the features of the CDS indices (both CDX and iTraxx) is the concept of an index "roll". Every six months (20 March and 20 September), credits that have deteriorated in quality are replaced by a new set of credits. For example, in the last roll on 20 March, six names of the CDX IG were replaced because they were downgraded to sub-investment grade.

…could lower off-the-run tranche liquidity

Following an index roll, the on-the-run tranches are typically more liquid than the off-the-run tranches and investors may choose to roll into the new tranches. However, such a tranche roll could be quite expensive. Investors could, on the other hand, ignore the roll and continue to hold the off-the-run tranche. This investment, however, could be less liquid than on-the-run alternative.

Post the most recent roll in September 2005, we have observed good liquidity for the off-the-run (CDX IG4) tranches relative to the on-the-run (CDX IG5) tranches.

[16] Tranche leverage = Notional size of delta-hedged portfolio/Notional size of the tranche.
[17] The CDX.NA.IG trades with a relatively tight bid-ask spread (0.5 bps) on a daily notional of billions of $.

High Yield Based Tranches

Differences with IG tranche market

Using the CDX HY[18] as the underlying portfolio, the market currently trades five standardized tranches. These five tranches complete the entire capital structure and are highlighted in Table 10. There are a few key differences with the IG tranche market (all discussed below in the report):

- Different standardized attachment points.
- Extensive use of upfront pricing conventions.
- Lower tranche deltas than are typical in the IG market.
- Higher implied tranche correlation than in IG market but lower base correlations (as a function of detachment points). However, base correlations are higher for HY when compared to tranche expected loss.

Table 10: CDX HY Tranches

	5 y Levels			Correlation	
Tranche	Bid (bps)	Offer (bps)	Delta	Base	Tranche Implied
0-10%	82% (all upfront)	83% (all upfront)	1.5x	23.09%	23.09%
10-15%	51.125% (all upfront)	52.625% (all upfront)	3.5x	22.41%	21.57%
15-25%	443	463	3x	33.63%	7.68%
25-35%	83	93	0.7x	52.95%	17.32%
35-100%	16	22	0x	N/A	N/A

Underlying 5y CDX HY at 102.5 mid. Levels as of 30th January 2006
Source: Merrill Lynch; Delta calculated using base correlations

■ Mechanics

Two tranches trade on fully upfront basis

The most junior tranches, i.e. the 0-10% and the 10-15%, trade on an entirely upfront basis. This compares to the high grade 0-3% equity tranche which trades upfront plus 500 bps running. For example an investor who sold protection on the 0-10% tranche would receive 82% of the notional upfront and nothing on a running basis. The other three tranches trade entirely on a running basis.

The mechanics of the HY tranches are similar to those of the IG tranches. For the HY tranches that trade on a running basis, the premiums are paid quarterly until the notional of the tranches gets fully written down following credit events. For each credit event that affects the tranche (i.e. when portfolio losses exceed the attachment point), the notional of the tranche is reduced by the loss amount and the premium payments are made on this reduced notional (see the case study on CKC in a later section).

Documentation is standardized

Like the CDX IG tranches, the 2003 ISDA Credit Derivatives Definitions also govern the HY tranches. The underlying CDX HY has two credit events: Bankruptcy and Failure to Pay (similar to the credit events in the HY single-name CDS market). The tranches are cash-settled. Standard confirmations have been in place for since second half of 2004 and serve to lower basis risk as an investor with a tranche exposure to one dealer can offset it via a trade with another dealer.

[18] Each time we refer to CDX.NA.HY index we refer to the current "on-the-run" series – currently Series 5.

Subordination & Probability of Triggering Notional Losses

Tranches with lower attachment points are riskier tranches. Table 11 highlights credit assessments for different attachment points for the current CDX HY index. These assessments are not officially provided by S&P but have been estimated by Merrill Lynch using S&P's CDO Evaluator ratings model.

Table 11: Credit Assessment for Tranched CDX HY Portfolio

Attachment point	Credit Assessment
26.47%	AAA
24.35%	AA
22.99%	A
19.83%	BBB
16.07%	BB
13.35%	B

Source: Merrill Lynch; Estimated using S&P's CDO Evaluator (v3.0) ratings model. Calculations as of 31 January 2006.

Table 12 highlights the number of defaults required to absorb the full notional exposure for each tranche. For example, the 0-10% tranche would require 15 defaults for a full loss of notional assuming a 30% recovery. Similarly the 15-25% tranche would require 42 defaults at a recovery of 40% for full loss of notional. The 35-100% tranche is not shown as it would obviously require all 100 credits to default for full notional loss (assuming zero recovery).

Table 12: Number of Defaults for Full Loss of Notional

CDX HY	# of Defaults for Full Loss of Notional (By Recovery)			
(100 credits)	20%	30%	40%	50%
0-10%	12.5	14.3	16.7	20.0
10-15%	18.8	21.4	25.0	30.0
15-25%	31.3	35.7	41.7	50.0
25-35%	43.8	50.0	58.3	70.0

Source: Merrill Lynch

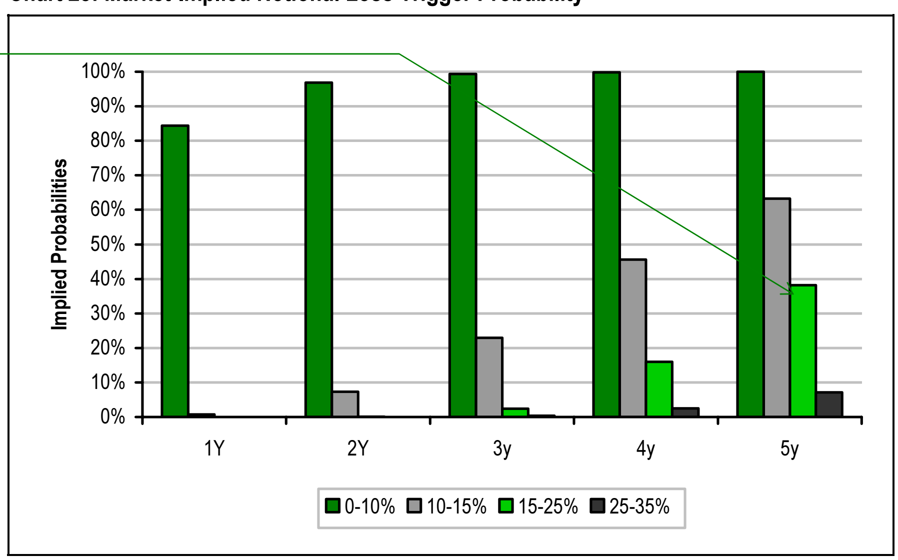
What is the probability of triggering notional loss for each tranche?

The speculative grade default rate is of key concern to investors who are exposed to HY tranches. The tranche spreads can be used to imply the probabilities of the tranche notional being hit over the 5y term. For example, assuming a 40% recovery, the 10-15% tranche requires at least 17 defaults before **triggering** a default payment from the protection seller. Given the spread on the underlying credits and the default correlation implied in the 10-15% market spread, it is then possible to compute the implied default probability of the trigger event by computing the probability of having at least 17 defaults at each year.

Assuming a recovery of 40%, Chart 28 highlights the probabilities of triggering notional losses for four of the tranches: Investors who have strong views about speculative grade default rates can use these tranches to appropriately express their views relative to those implied by the market.

The probability of losses hitting the 15-25% tranche over a 5y period is about 39%

Chart 28: Market-Implied Notional Loss Trigger Probability



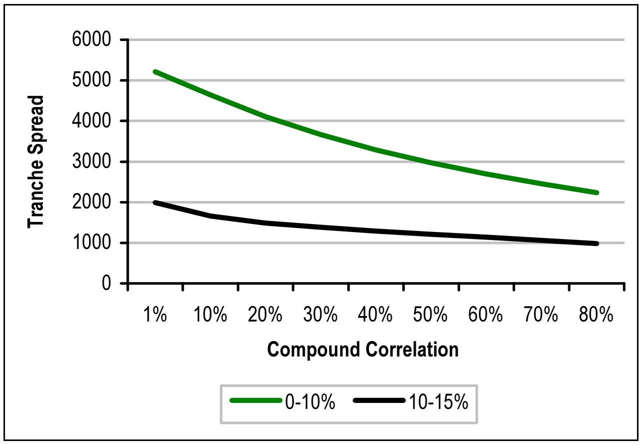
Source: Merrill Lynch. Assume a 40% recovery and a different correlation for each tranche as reported in Table 10. Assume quarterly payments on Act/360 day-count fraction basis.

Correlation Sensitivity

Long correlation tranches trade on an upfront basis

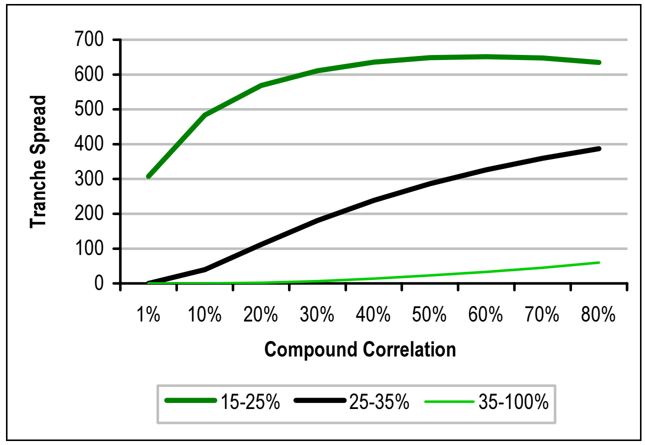
Chart 29 highlights the correlation sensitivity of the junior most tranches, i.e. 0-10% and 10-15%. Both tranches are long correlation, i.e. they have a positive MTM if the default correlation increases with all other factors remaining constant. Chart 30 shows the correlation sensitivity of the other three tranches. The senior most tranches, i.e. 25-35% and 35-100% are clearly short correlation. While the "mezzanine" 15-25% tranche is also short correlation at low levels of correlation, it is less sensitive at higher levels as seen in the chart.

Chart 29: Long Correlation Tranches



Source: Merrill Lynch; Assume spreads shown in Table 10.

Chart 30: Short Correlation Tranches



Source: Merrill Lynch; Assume spreads shown in Table 10.

Spreads can be used to derive implied correlations for each tranche. Base correlations (another correlation measure used in the market) can also be implied from these spreads and has the benefit of overcoming some of the limitations of the tranche-implied measure[19]. Table 10 highlights both tranche implied as well as base correlations at the current spread levels.

Spread Sensitivity (Deltas and Leverage)

Tranches provide attractive leverage

Like IG tranches, dealers also quote HY tranches assuming an "exchange of deltas at the mid". Delta hedging with individual credits is cumbersome and inefficient for a portfolio of 100 credits. In addition, liquidity is sparse for many high yield single-name CDS making it even more difficult to delta hedge individual credits. Though not an entirely accurate hedge, it is quicker and more efficient to hedge with the underlying CDX HY index itself. The index is relatively liquid and currently trades with a bid-ask spread of about 0.25 points or 6bps.

The notional amount of the index hedge is determined by the tranche leverage which is defined as the relative notional amount of the index required to hedge the MTM of the tranche for a 1bps move in the underlying index[20] (see Table 13). For example, a long position in the 15-25% tranche can be delta-hedged by buying protection on 3x the notional of the underlying CDX HY index. The tranche leverage that is normally quoted is derived within a base correlation framework.

Table 13: Tranche Leverage

Tranche	Leverage
0-10%	1.5x
10-15%	3.5x
15-25%	3x
25-35%	0.7x
35-100%	0x

Source: Merrill Lynch; Leverage quoted with a base correlation framework; as of 30th Jan 2006

Limiting our analysis to the equity part of the capital structure, we note a lower leverage of the 0-10% CDX HY tranche compared to the equity 0-3% CDX IG counterparty. This is mainly due to the **higher width** of the HY tranche (10% vs. 3%) which reduces the tranche sensitivity with respect to (parallel) spread movements of the underlying reference entities.

[19] See Volume 2, Chapter 9for more details on Base Correlation.
[20] See Volume 2, Chapter 5 for a detailed discussion of leverage

Interest Rate Sensitivity

Upfront traded tranches have a lower sensitivity to credit spread movement…

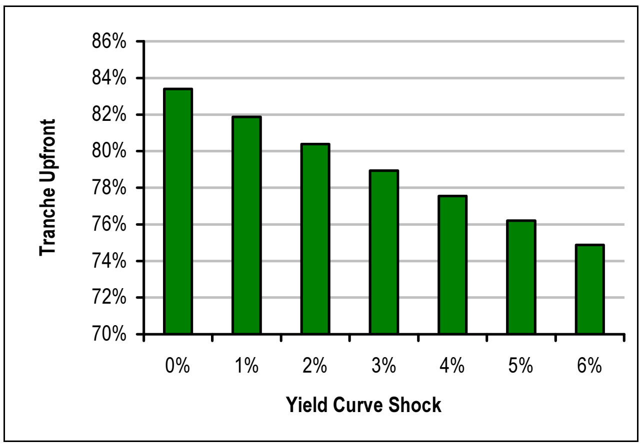
Unlike most of the unfunded credit derivatives products traded in the market equity CDO tranches are usually exchanged on an upfront basis, i.e. the buyer pays the full (discounted) protection premium at trade inception.

Compared to tranches that trade on a running basis, upfront traded tranches are less sensitive to credit curve movements[21]. The upfront payment convention, however, is sensitive to changes in underlying interest rates. Therefore, upfront buyers/sellers are exposed to an additional risk of unexpected shocks in the interest rate curve used to discount the stream of protection payments.

…but bring additional interest rate risk

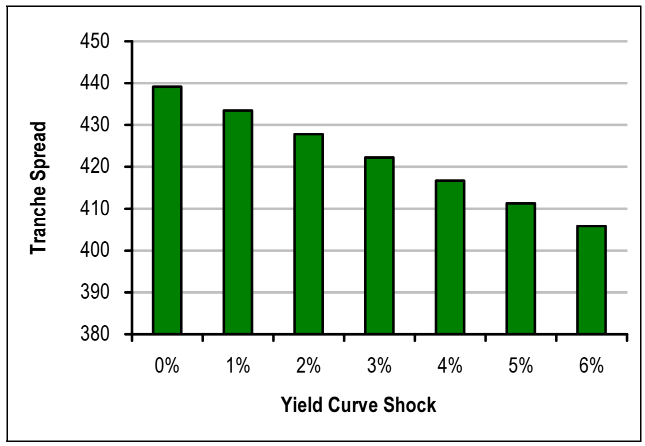
From a protection buyer's point of view and unchanged credit curves for the index underlying CDS, a sudden widening of the interest rate curve will decrease the discounted value of the protection bought and imply a negative mark-to-market. Chart 31 and Chart 32 illustrate the sensitivity of the 0-10% upfront premium and the 15-25% running premium for different shocks on the current swap curve.

Chart 31: 0-10% (Upfront) is More Sensitive to Interest Rates…



Source: Merrill Lynch; Swap curve as of 30th January 2006; Assume an instantaneous parallel shift of the curve.

Chart 32: …than 15-25% (Running Premium)



Source: Merrill Lynch; Swap curve as of 30th January 2006; Assume an instantaneous parallel shift of the curve.

Case Study: CKC Default – The Impact of Default on Standardized Tranches (and the Underlying Index)

Cash settlement is a sensible option

The CDX family of indices experienced its first credit event when CKC filed for bankruptcy on 18th May 2005. Though CDX HY index contracts require physical settlement, the market agreed to cash settle starting with CKC. The proposal to cash settle CKC was driven by the following key factors:

1. Large number of CDX HY index and tranche contracts outstanding which were difficult to quantify due to the OTC nature of the market.
2. Small amount of deliverable debt outstanding – $ 500mn of CKC 10.75% '31 was the only senior unsecured deliverable bond.
3. Since CKC is only 1% of the CDX HY index, the size of each transaction is relatively small.
4. Physical delivery would be particular troublesome in the tranche market – different recovery rates would imply different attachment and detachment points for different contracts of the same tranche. We could expect liquidity of these tranche contracts to drop dramatically if physically settled.

For these reasons, CKC was cash settled. An auction on 14th June 2005 determined a settlement price of 43.625 for all index and tranche settlements. This process was used as a blueprint for the other defaults that followed in the second

[21] See Volume 1, Chapter 4.

half of 2005 including Delta, Northwest, DPH and Calpine (see Table 13). Agreeing to a new protocol for each default is relatively troublesome for both dealers and investors. As a result, the market expects cash settlement to be included in the tranche documentation staring from the next roll in March 2006.

Table 14: Credit Events in 2005

Credit	Credit Event (Date)	Cash Recovery
CKC	Bankruptcy (18th May 05)	43.625%
Delta	Bankruptcy (14th Sep 05)	28%
Northwest	Bankruptcy (14th Sep 05)	18%
DPH	Bankruptcy (8th Oct 05)	63.375%
Calpine	Bankruptcy (20th Dec 05)	19.125%

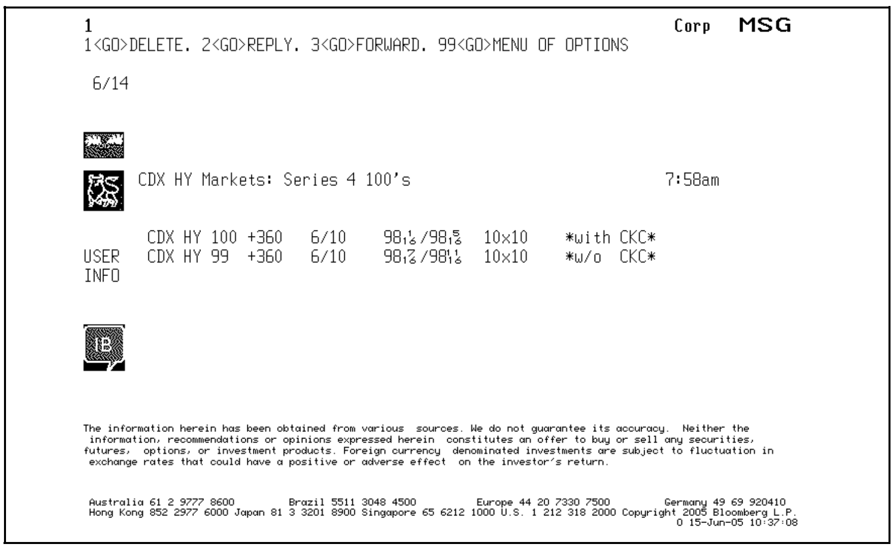
Source: Merrill Lynch

Trading the CDX HY Indices ex CKC

CDX HY 4 was a 99-name index post CKC default

The cash settlement applied to all the series of the CDX HY family as well as the prior iBoxx and Trac-X that include CKC. Since CKC's filing, the on-the-run CDX HY.4 had been trading (a) with CKC and (b) without CKC. Chart 33 shows quotes for both indices as of 14th June 2005.

Chart 33: On-the-run CDX HY quote with and without CKC



Source: Merrill Lynch; Bloomberg

Quoted notional is adjusted by 0.99 factor

CDX HY 99 refers to the CDX HY Series 4 index without CKC. The index trades with the same fixed coupon of +360bps. However, the index has a **notional factor of 0.99**. As a result, we observe the following for the CDX HY 99 (using the quotes in Chart 33):

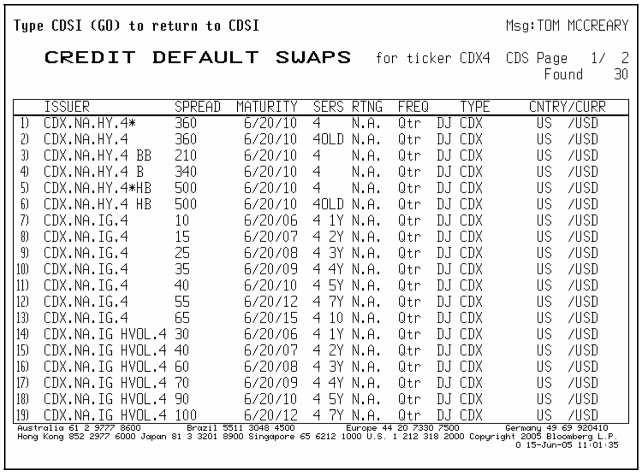
- A **market** quote of \$10mn notional for CDX HY 99 corresponds to an **actual** notional of \$9.9mn (= \$10mn*0.99).
- The running spread on a \$10mn notional (quoted) is therefore 360bps * \$9.9mn = \$356,400.
- A seller of protection would receive the above running spread as well as an upfront payment of $1^5/_{16}$ (= $100 - 98^{11}/_{16}$) on the actual notional of \$9.9mn i.e. \$129,937.50 upfront.
- A buyer of protection would pay the above running spread as well as an upfront payment of $1^9/_{16}$ (= $100 - 98^7/_{16}$) on the actual notional of \$9.9mn, i.e. \$154,687.50 upfront.

Following the auction, dealers are trade only this 99-name index. The original 100-name index with a notional of \$10mn now trades as a 99-name index with a notional of \$9.9mn.

The new 99-name index is represented as **CDX HY.4*** on Bloomberg as shown in Chart 34 below. Since CKC was not a member of the BB or B indices, these indices are unchanged. This Bloomberg screen can be accessed directly by typing: **CDX4 CDS Corp <GO>**. As before, the screen in Chart 35 can then be accessed by choosing the index and typing: **CDSW <GO>**.

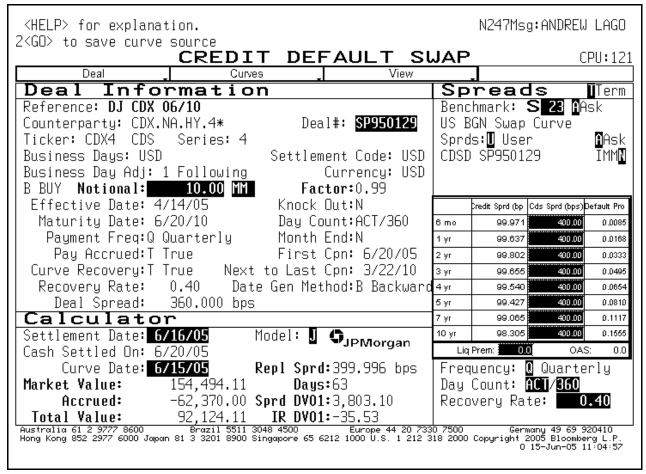
For a detailed description of index composition, mechanics and rules please see Volume 1, Chapter 8.

Chart 34: CDX4 CDS Corp <GO> on Bloomberg



Source: Merrill Lynch; Bloomberg

Chart 35: Choose the Index; CDSW<GO>



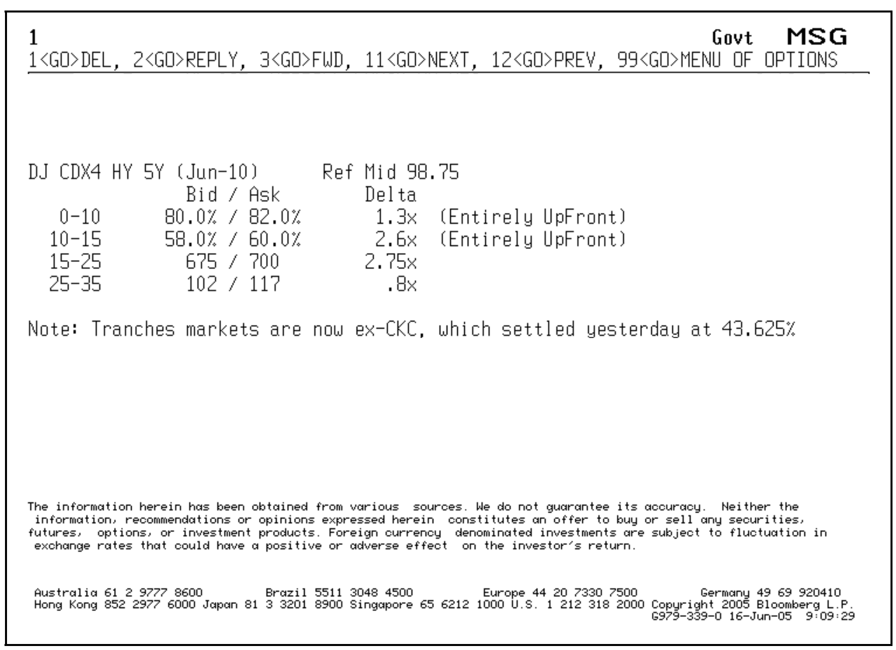
Source: Merrill Lynch; Bloomberg

■ Trading the CDX HY Tranches ex CKC

Old tranche contracts are fungible with new tranches (ex CKC)

Interest in the HY tranche market dried up post CKC's filing on 18th May. However, the resolution of the CKC settlement renewed investor interest in this market. The cash settlement applied to all 5y CDX HY tranches that include CKC. The CDX HY 4 tranches trade without CKC as shown in Chart 36.

Chart 36: On-the-Run CDX HY Tranche Quotes Ex CKC



Source: Merrill Lynch; Bloomberg

Old tranches (with CKC) are fungible with new tranches (ex CKC). We highlight some of the key points regarding the quotes for the new tranches ex CKC:

- Tranches continue to be **quoted as before**, i.e. 0-10, 10-15, 15-25, etc.
- The underlying index is the **99-name index** ex CKC (or CDX HY 99).
- The **new equity tranche is 0-9.53157%** for the 99-name index.
  - With a 1% index exposure, CKC's recovery of 43.625 represents a loss of 0.56375% on the initial 100-name index portfolio. This loss is fully absorbed by the equity tranche implying an adjusted detachment point for the equity of 9.43625% = (10-0.56375).
  - However, the underlying index is now a 99-name index which implies an actual detachment point of 9.53157% (= 9.43625*100/99).
- A **market** quote of $10mn notional on the 0-10 (quoted) tranche corresponds to an **actual** notional of $9.43625 on the 0-9.53157 tranche. This implies a underlying index notional of $99mn (= $9.43625/9.53157%)
- Along with attachment points, tranche widths will also change for the other tranches. For example, the quoted 10-15% tranche now has a width of 5.05% (= 5*100/99). Similarly the quoted 15-25% or the 25-35% tranches have a width of 10.1% (= 10%*100/99).

Table 15 highlights the new attachment and detachment points for all tranches. The actual notionals are unchanged from the quoted levels for all tranches except the 0-10% and 35-100%. The table also illustrates the $ premium that is received by the seller of protection on quoted $10mn "notionals".

Table 15: Selling Protection on CDX HY Tranches ex CKC

Attachment & Detachment Points		Notional		Premium		Remaining
Quoted	Actual	Quoted (mn)	Actual (mn)	Bid	Actual* $ (mn)	Subordination** (mn)
0-10	0 – 9.53157	$10	$9.43625	80%	$7.549	$0
10-15	9.53157 – 14.58207	$5	$5	58%	$2.9	$9.43625
15-25	14. 58207 – 24. 68308	$10	$10	675bps	$0.675	$14.43625
25-35	24.68308 – 34.78409	$10	$10	102bps	$0.102	$24.43625
35-100	34.78409 – 100	$65	$64.56375	28bps	$0.1807785	$34.43625
0-100	0-100	$100	$99			

* Actual Premium ($ mn) = Bid * Actual Notional ($ mn)
** Remaining Subordination ($ mn) = $99mn * Actual Attachment Point (%)
Source: Merrill Lynch

Refer to important disclosures on page 196.

5. Trading Correlation

Bespoke tranche market investors are mainly long-only “real money” accounts focused on yield enhancement. Standardized tranche market participants, however, are typically correlation traders who are relative value players looking to acquire cheap convexity, volatility or correlation. Such traders typically combine different tranches, CDS indices and single name default swaps to isolate these risks and tailor their payoffs accordingly. They are typically short-term participants and include hedge funds, principal finance groups and dealers.

This chapter discusses the risk management “greeks” such as delta and gamma that characterize correlation trading.

Long or Short Correlation?

Tranche exposure determines a view on correlation . . .

Fundamentally, an investor who buys or sells protection on a tranche is taking a view on correlation. An investor's view on correlation via exposure to the equity or senior tranche is summarized in Table 16. We highlight the following:

An investor can be **long correlation** by being either **long the equity** tranche or **short the senior** tranche.

An investor can be **short correlation** by being either **short the equity** tranche or **long the senior** tranche.

The sensitivity of a mezzanine tranche to correlation is function of its attachment point and size as well as its underlying credit spreads and lies in between the equity and senior tranches.

Table 16: Tranche Exposure to Reflect Correlation View

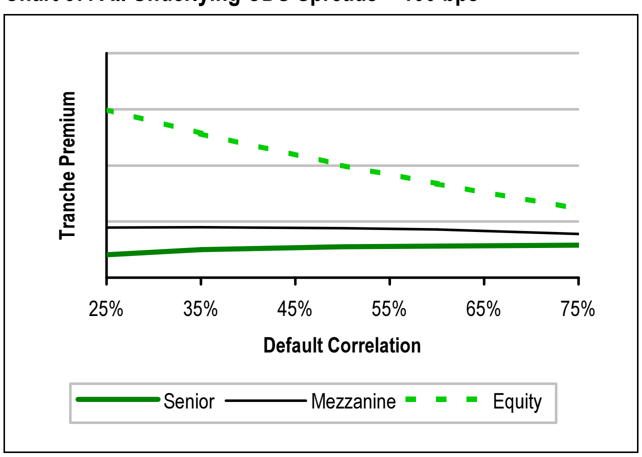
	Equity Tranche Exposure	Senior Tranche Exposure
Long Correlation	LONG*	SHORT
Short Correlation	SHORT*	LONG

Long = Sell Protection; Short = Buy Protection
Source: Merrill Lynch;

. . . that can change over time

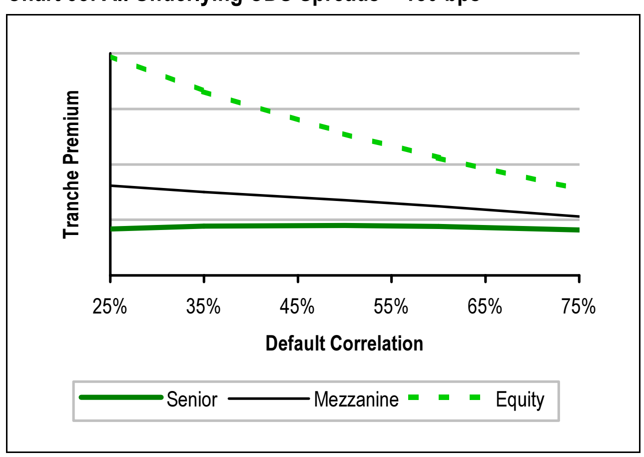
This sensitivity to correlation can change through time as a function of spreads, defaults and time decay. For instance, if all spreads in the underlying portfolio tighten significantly, a (correlation neutral) mezzanine tranche would begin to take on the correlation sensitivity of a senior tranche i.e. as correlation *increased*, the premium on the mezzanine tranche would *rise* and vice versa. Similarly, if all spreads widen significantly (see Chart 37 and Chart 38), the mezzanine tranche would begin to take on the correlation sensitivity of an equity tranche, i.e. as correlation increased, the premium on the mezzanine tranche would fall and vice-versa. However, the exact behavior of a mezzanine tranche would be a function of the magnitude of the spread change and the attachment points of the mezzanine tranche. The charts below highlight the typical behavior of these tranches. We explore correlation sensitivity in more detail in a later section.

Chart 37: All Underlying CDS Spreads = 100 bps



Source: Merrill Lynch

Chart 38: All Underlying CDS Spreads = 150 bps



Source: Merrill Lynch

Delta Hedging

Dealers typically manage the spread risk of a tranche by using single-name CDS as an offsetting hedge. The amount of CDS protection bought or sold on credits in the CDO portfolio is defined by its Delta. The directionality and magnitude of Delta can be more difficult to understand however, and depends on many factors including subordination, spread, correlation and time.

Managing the Risks of Correlation Products

Dealers need to manage the multiple risks in correlation products

An 'ideal' hedge for a dealer who has shorted (bought protection on) a single-tranche of a CDO would be to enter into an identical offsetting transaction. This would involve selling protection on an identical tranche of the same underlying portfolio. However, due to the bespoke nature of the single-tranche CDO market, the availability of such an offsetting transaction is very unlikely. As a result, a dealer will focus on **managing the spread risk of a single-tranche position** (first-order risk) via the underlying single-name CDS market, and apply a trading-book approach to managing other risks such as implied correlation etc.

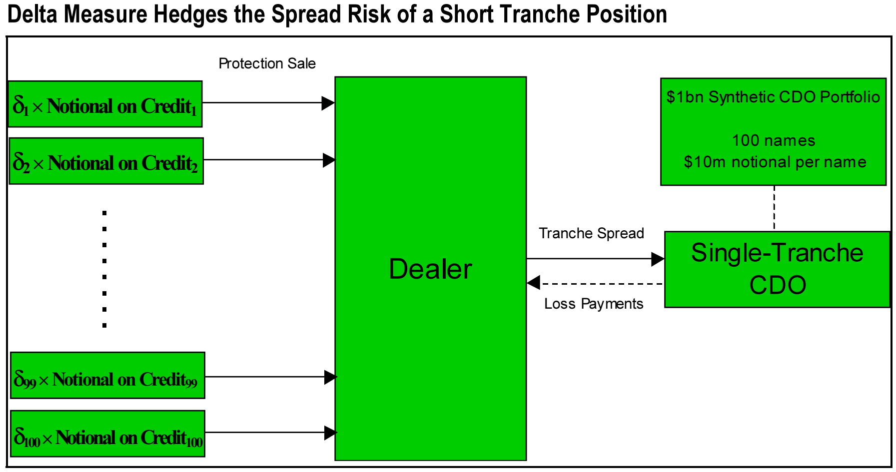
However, dealers can enter into offsetting transactions to hedge the risk of standardized tranches of CDS indices. Moreover, these standardized tranches can also be used as an approximate offsetting hedge for a bespoke tranche, given that there may well be a reasonable overlap in reference entities of both CDO pools. Though such a hedge is not a perfect hedge, it is typically the most common.

Defining Delta

Delta hedging is used to manage spread risk

All single-tranche positions are subject to MTM movements as credit spreads on names in the underlying portfolio move about over the life of the transaction. To hedge the spread risk of a *short position* in a tranche, a dealer needs to *sell protection* on each of the underlying credits in the portfolio according to the **Delta** (δ) measure. Similarly, to hedge the spread risk of a *long position* in a tranche, a dealer needs to *buy protection* on each of the underlying names in the portfolio according to the Delta measure. In Chart 39, we show the hedging behavior of a dealer who is short a single-tranche CDO.

Chart 39: Selling Protection on each name in the Underlying Portfolio According to the Delta Measure Hedges the Spread Risk of a Short Tranche Position



Source: Merrill Lynch

We define the Delta of a credit in the underlying portfolio as the amount of protection the dealer sells (buys) on that name to hedge the MTM risk of a short (long) tranche position due to movements in the credit spread of that name.

More formally, we can define the Delta of a credit as the ratio of the spread sensitivity of the tranche position to the spread sensitivity of that credit:

$$\text{Delta of Credit} = \frac{\text{Change in MTM of Tranche}}{\text{Change in MTM of Credit}}$$

This relationship defines the percentage of the notional amount of the credit that needs to be sold (bought) in order to hedge a short (long) tranche position.

For example, assume that a credit in the underlying portfolio, originally with a spread of 100bps, widens by 1bp. This spread widening translates into an absolute MTM change of €4,540 on the credit. If this spread widening also resulted in an absolute MTM change on the short tranche position of €2,107, then the Delta for that credit would be equal to **46.4%** (=2,107/4,540). This means that to hedge the MTM risk of the tranche to small moves in the credit spread of that name, the dealer would sell protection on 46.4% of the notional of that credit. If the credit had a notional value of €10m in the CDO pool, the amount of protection sold would be €4,640,000.

Deltas are calculated by "brute force"

There are no explicit formulas to calculate Delta. Instead, Deltas are generally calculated using “brute force” by shifting the credit spreads on each individual name in the CDO portfolio by a small amount (1-10bps), and then calculating the resultant MTM change of the tranche. **Deltas range from 0%-100% for a credit in a CDO.** The ‘Delta’ of a single-name CDS is 100% and the Delta of a 0%-100% tranche of a CDO is 100%.

Note that we calculate Delta for small spread changes in the underlying credits. For larger spread changes, a delta-hedged tranche is not totally immune to spread movements. In other words, a delta-hedged CDO position is still subject to *Spread Convexity* for larger spread moves, and as a result the Deltas will need to be dynamically rebalanced throughout the life of the transaction.

Dynamic Hedging

Deltas need to be rebalanced to remain fully hedged for spread movements . . .

In the context of correlation products, dynamic hedging is the process of delta-hedging a single-tranche CDO over time. Dealers will typically manage the spread risk of a single-tranche position by dynamically rebalancing the Delta hedges as spreads move. However there are practical limits to such a process. First, there is a cost associated with rebalancing the hedges. Whilst underlying spread curves move on a daily basis, rebalancing will typically be on a less frequent basis as buying and selling small portions of credit risk can become relatively expensive when bid-offers are taken into account. The more frequent the rebalancing of Deltas, the more expensive the hedging cost. Tight bid-offer spreads in the single-name CDS market would help mitigate the cost of rebalancing.

Second, it may not always be possible to transact single-name hedges in the precise Delta amounts that are produced from analytical models. Third, CDO collateral portfolios are constructed to be diverse in nature so that more senior tranches obtain higher credit ratings. However, not every name referenced in the portfolio will have the same liquidity in the underlying single-name CDS market. This may cause a conflict between the need to rebalance hedges on a frequent basis and the availability of liquid CDS contracts for which to hedge.

Other Risks of Correlation Products

. . . but still exposed to other risks

Delta-hedging immunizes the dealer against small movements in the credit spreads of names in the underlying portfolio. However, a delta-hedged tranche is not a completely risk-free position for a dealer as there are a number of other risk factors that need to be managed. These additional risks are implied correlation risk, risk of a sudden and unexpected default in the portfolio, time-decay, spread convexity and recovery rate risk. As a result, dealers are motivated to do trades that reduce these risk factors such as placing other tranches of an existing CDO with other investors or, where possible, entering into similar offsetting transactions.

Delta Sensitivities

Deltas are sensitive to several factors

There are many sensitivities of Delta that can at first appear less straightforward. Deltas of individual credits[22] depend on the following parameters and will change as these parameters change through the life of the transaction:

- **Attachment point (i.e. Subordination) and Width of the Tranche**
- **CDS Spread of Underlying Credits and Relative Spread Movements**
- **Time Remaining until Maturity**
- **Correlation with other Credits in the Portfolio**
- **Recovery Rate assumption of the Credits**

In order to understand the directionality and magnitude of Deltas we explore these sensitivities with two CDO examples.

Table 17: Discrete CDO Portfolio and Tranching

Property	Quantity
Underlying Portfolio	
Number of credits	50
Notional Per Credit	€10m
Total Portfolio Size	€500m
CDS Spread on *each* credit	100 bps
Recovery Rate on *each* credit	35%
Default Correlation	25%
Tranching	
Equity	0%-4%
Mezzanine	4%-8%
Senior	8%-12%
Super Senior	12%-100%

Source: Merrill Lynch

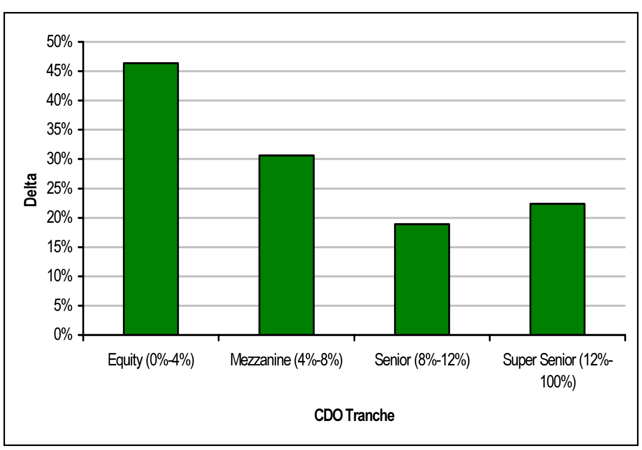
We define a **Discrete CDO** example with underlying collateral pool and tranching as detailed in Table 17. We also define a **Continuous CDO** on the same collateral pool where each tranche has a 1% width and attachment points vary by 1% increments.

As the CDO collateral pool is homogenous with respect to spread, recovery and correlation assumptions, the Delta of each credit will be identical for any given tranche. However, as actual collateral pools tend to be much more heterogeneous with respect to credit spreads, Deltas will typically be different for each credit.

Delta as a Function of Subordination

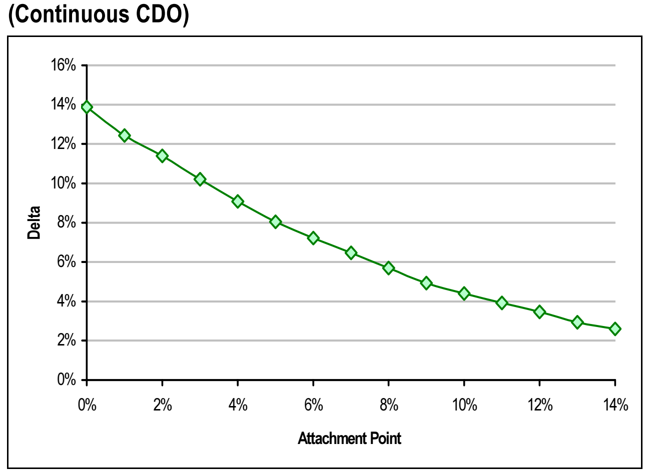
Tranches lower in the capital structure are more risky as there is less subordination to act as a 'buffer' against defaults. As the equity (first-loss) tranche is the riskiest tranche in the capital structure, credits in the underlying portfolio have the highest equity-tranche Delta relative to the more senior tranches. For more senior tranches in the capital structure, the Delta of a credit is lower.

Chart 40: Delta as a Function of Subordination (Discrete CDO)



Source: Merrill Lynch

Chart 41: Delta as a Function of Subordination (Continuous CDO)



Source: Merrill Lynch

A credit's Delta is *higher* for a tranche with *less subordination*, and vice-versa.

Deltas decrease with increase in subordination . . .

As Delta can be thought of as representing the probability of losses in a tranche, tranches with less subordination have a higher Delta. Note however, that in the Discrete CDO example the super-senior tranche actually has a higher Delta than

[22] For an individual credit, the credit Delta is different for different tranches, i.e. the equity tranche Delta of a credit is different from its senior tranche Delta.

Refer to important disclosures on page 196.

the senior tranche as the tranche width is significantly larger. However, as a percentage of the notional size of the tranche ("leverage"), Delta for the super-senior tranche is lower than the Delta for the senior tranche. We talk more about the concept of Leverage later in this chapter.

Delta as a Function of Credit Spreads

. . . but depend on the tranche as a function of individual credit spreads

To show the dependence of Delta on individual credit spreads, we disperse the credit spreads in the underlying collateral pool of Table 17 such their average-spread remains at 1%. Spreads are adjusted to be linearly increasing in this instance.

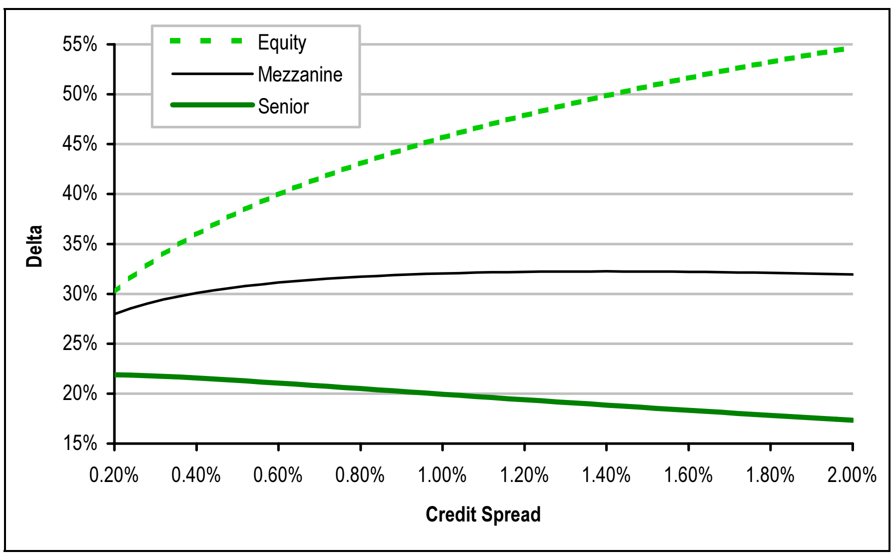
One way to conceptualize Delta is to think of ordering all of the credits in the underlying portfolio by their time to default. For constant recovery rate and correlation assumptions across all names, this will simply be a reflection of the credit spread of the name: **credits with a higher spread are expected to default before credits with a lower spread**. If a credit whose Delta we are calculating is towards the front of this 'queue' (i.e. higher spread) it is more likely to cause losses to the equity tranche and so its **equity tranche Delta will be higher**. If the credit is further back in this queue (i.e. lower spread) then its **equity tranche Delta will be lower**. Moreover, as credits further down the queue are likely to have a later default time, they are more likely to cause losses to the more senior tranches in the capital structure. As a result, the **senior tranche Delta of the credit will rise**.

For the equity tranche, a *higher* credit spread is associated with a *higher* Delta for that credit (relative to the average Delta of that tranche), and vice-versa.

For the senior tranche, a *higher* credit spread is associated with a *lower* Delta for that credit (relative to the average Delta of that tranche), and vice-versa.

For the mezzanine tranche, the Delta has less directionality with respect to credit spreads.

Chart 42: Deltas as a Function of Credit Spread



Assume Discrete CDO.
Source: Merrill Lynch

This relationship is shown in Chart 42 where the equity tranche Deltas are higher for wider spread credits and the senior tranche Deltas are lower for wider spread credits. These results also apply to the directionality of Delta when credit spreads in the portfolio move throughout the life of the transaction:

For the equity tranche, Deltas on individual credits will *rise* as their credit spreads *widen* and vice-versa[23].

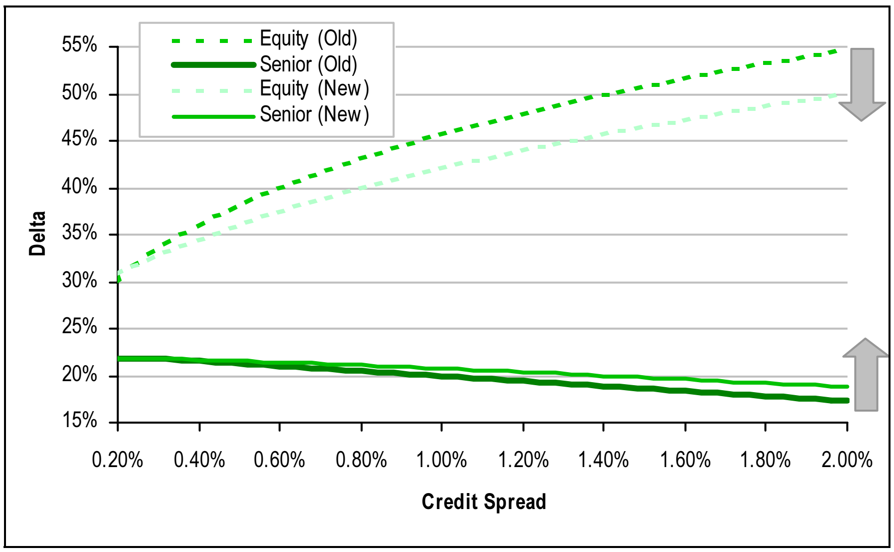
For the senior tranche, Deltas on individual credits will *fall* as their credit spreads *widen* and vice-versa.

Delta as a Function of Cumulative Spread Moves

Deltas move differently for different tranches as all spreads move together . . .

The **cumulative** spread movements of all the credits in the portfolio will also impact how the Deltas change. We analyze how Deltas move in this instance by shifting all credit spreads in the above example wider by 25bps. As can be seen in Chart 43 this has the impact of *decreasing* the equity tranche Deltas for *each* individual credit, but *increasing* the senior tranche Deltas for *each* individual credit. This is because shifting all credit spreads wider simultaneously has the impact of decreasing the probability of a small number of losses (lowering the relative risk of the equity tranche), whilst increasing the probability of a larger number of losses (increasing the relative risk of the senior tranche). The mezzanine tranche Deltas are less impacted.

Chart 43: Behavior of Deltas as ALL Spreads in the underlying Portfolio are Shifted Wider



Assume Discrete CDO.
Source: Merrill Lynch

Similarly, when all credit spreads in the underlying portfolio are shifted tighter then the equity tranche Delta *increases* for *each* individual credit and the senior tranche Delta *decreases* for *each* individual credit. This is because shifting all credit spreads tighter simultaneously has the impact of increasing the probability of a small number of losses (increasing the risk of the equity tranche), whilst decreasing the probability of a larger number of losses (decreasing the risk of the senior tranche). The mezzanine tranche Deltas are less impacted.

We note however, that the **shape of the distribution of Deltas remains the same after cumulative spread movements**: For the equity tranche, wider spread credits still have a higher Delta than lower spread credits. For the senior tranche, wider spread credits still have a lower Delta than lower spread credits.

[23] As the change in spread of any one credit impacts the weighted-average spread of the portfolio, there will be a small impact on the Deltas of the remaining credits. For example, shifting one credit from 100bps to 105bps increases its equity tranche Delta but the equity tranche Deltas of the other names in the portfolio fall marginally, and vice-versa.

Refer to important disclosures on page 196.

The equity tranche Deltas of *each* credit *decrease* as credit spreads across the entire portfolio widen and the equity tranche Deltas of *each* credit *increase* as credit spreads across the entire portfolio tighten.

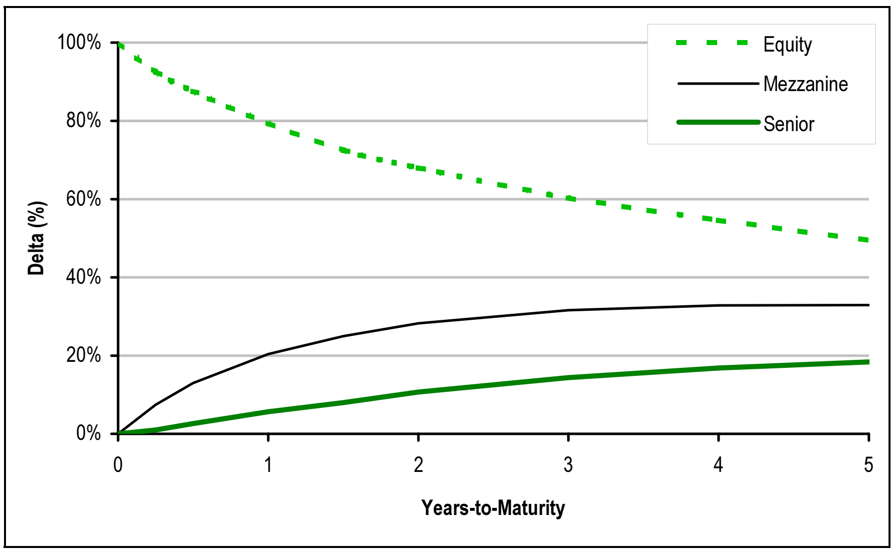
The senior tranche Deltas of each credit *increase* as credit spreads across the whole portfolio widen and the senior tranche Deltas of *each* credit *decrease* as credit spreads across the whole portfolio tighten.

Delta as a Function of Time

. . . or as time to maturity decreases . . .

Deltas will also change due to the passage of time even if credit spreads remain unchanged. Assuming no defaults in the underlying portfolio, the Delta of the equity tranche will *increase to 100%* as time to maturity approaches and accordingly, the Deltas of the Mezzanine and Senior tranches will *decrease to 0%.* This is because as maturity approaches, the more senior tranches become relatively less risky compared to the equity tranche as there is less time for defaults to accumulate such that the notionals of the mezzanine and senior tranches are reduced. The equity tranche becomes relatively more risky in the capital structure, and as a result its Delta tends to 100%.

Chart 44: Deltas over Time: Equity Tranche Delta tends to 100% as it Becomes Relatively Riskier



Assume Discrete CDO.
Source: Merrill Lynch

Delta as a Function of Correlation

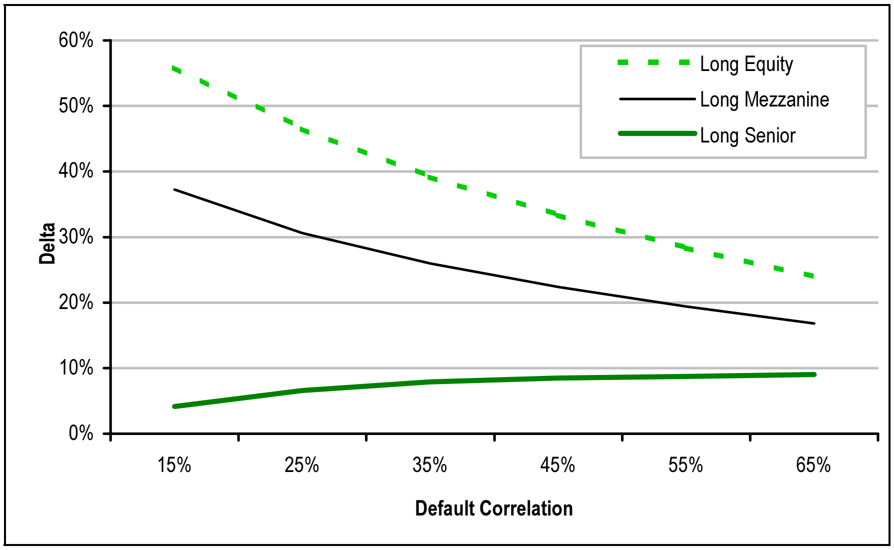
. . . or correlation changes

Deltas are also impacted by the changes in the underlying default correlation. If we increase the underlying default correlation assumption, we have the following conclusions:

- As correlation increases, the equity tranche Delta of a credit *decreases* and the senior tranche Delta *increases*.
- For every default correlation assumption, equity-tranche Deltas are higher than senior-tranche Deltas. Mezzanine-tranche Deltas remain in between.

As the correlation of the portfolio increases, more of the risk is parceled to the senior tranche implying an increase in senior tranche Delta and a consequent decrease in equity tranche Delta.

Chart 45: How Delta Changes as Default Correlation in the Portfolio Changes



Assume Discrete CDO. We have chosen a senior tranche with greater subordination to show the upward sloping nature of the senior curve.
Source: Merrill Lynch

Leverage "Lambda" of a Tranche

Leverage is a credit risk measure . . .

The **Leverage** (or "Lambda") of a tranche is an important spread risk measure. We define Leverage as equal to the **notional size of a tranche's Delta-hedge portfolio divided by the tranche's notional size**.

For example, in Table 17, each credit in the equity tranche has a Delta of 46.4%. As there are 50 names each of €10m in the CDO collateral pool, we have the following:

- The Delta-hedge portfolio has a notional size of €231.85mn (50*€10mn*46.4%).
- The equity tranche has a notional size of €20mn (=4%*€500mn).
- The Leverage of the equity tranche is 11.6x (=231.85/20).

Table 18: Calculating Leverage ("Lambda") for Discrete CDO Case

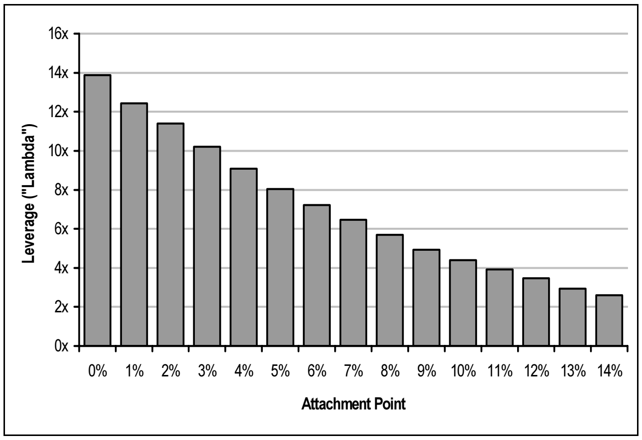
Tranche	Delta (%)	Notional Size of Delta-Hedge Portfolio (€)	Tranche Notional Size (€)	Leverage (Delta-Hedge Portfolio ÷ Tranche Notional)
Equity	46.4%	231,850,000	20,000,000	11.6x
Mezzanine	30.6%	153,100,000	20,000,000	7.7x
Senior	**18.9%**	94,420,020	20,000,000	4.7x
Super Senior	**22.4%**	111,800,000	440,000,000	0.3x

Source: Merrill Lynch

. . . that scales Delta by notional size of tranche

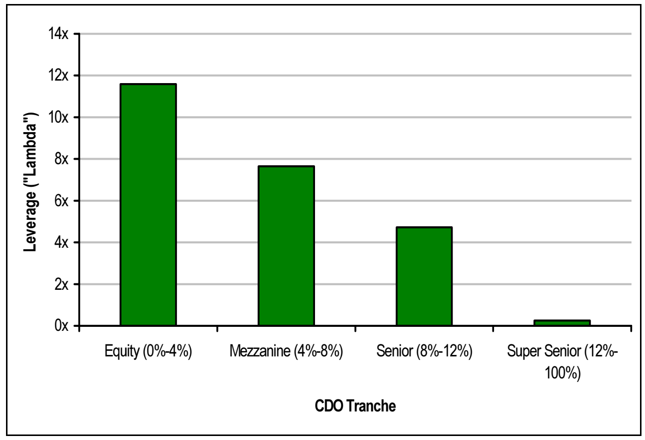
Effectively, Leverage 'scales' the Delta by the notional size of the tranche, i.e. the Delta of the super-senior tranche is actually larger than the Delta of the senior tranche, but its Leverage is smaller (Chart 47). Leverage gives an indication of how total risk is parceled out between the different tranches – more leveraged tranches are those for which the spread risk is high in relation to the notional. Delta and Leverage are both good measures of the credit risk of a tranche.

Chart 46: Leverage ("Lambda") for a Continuous CDO



Source: Merrill Lynch

Chart 47: Leverage ("Lambda") for a Discrete CDO



Source: Merrill Lynch

■ Delta-Hedging a Standardized Tranche with a CDS Index

Leverage measure used to delta-hedge standardized tranche via CDS index

While underlying CDS will typically be used to delta-hedge a bespoke single-tranche position, standardized tranches of CDS indices can be delta-hedged via the underlying index itself. Suppose that a dealer buys protection on the 7%-10% tranche of 5y CDX index. The Delta-hedging calculations are as follows:

- 7%-10% tranche notional is $30mn as total portfolio size assumed is $1bn.
- Suppose that 5y CDX 7%-10% tranche has Leverage of 1.5x.
- Implied Delta portfolio is $45mn ($30mn*1.5).

Note that this hedge is an average delta-hedge. For a senior tranche where underlying spreads are dispersed as in CDX, the tighter names (with higher than average Deltas) would be underhedged whereas the wider names (with lower than average Deltas) would be overhedged. The reverse would be true for hedging an equity tranche position with the index.

■ Importance of Delta

Delta is also important outside of correlation trading

Whilst Delta is of practical importance in managing the spread risk of a single-tranche, investors in bespoke CDO tranches may conclude that Delta is not so important. However, Delta plays an important role outside of pure correlation trading for two reasons:

➢ Adding CDOs to a Portfolio of Bonds or Loans

It is not immediately clear how the risk/reward profile of a portfolio of bonds or loans changes by adding a bespoke CDO tranche to the portfolio. However, given that the Delta is a credit risk equivalent metric, CDO tranches can be 'replaced' by their Delta-equivalent credit portfolios. This provides a basis for which aggregate exposure to any one credit can be summed-up over the portfolio.

➢ Substitution Mechanics in Managed Synthetic Deals

Deltas are contractually used in managed single-tranche deals to determine the economics of substitution. Substitutions are typically permitted for reasons of credit deterioration or improvement of names in the underlying portfolio. The loss or gain from substitution will be a function of several factors:

- The Difference Between the Spreads of the Outgoing and Incoming Credits
- The Delta of both the Outgoing and Incoming Credits
- Remaining Spread Duration of the Tranche
- Notional Amount of the Credits

Correlation Sensitivity (Rho)

Tranche sensitivities to correlation can change with time

As mentioned previously, different tranches along the capital structure have different sensitivities to movements in correlation. Equity tranche investments are typically *long* correlation positions (tranche premium decreases as correlation increases), senior tranche investments are *short* correlation (tranche premium increases as correlation increases), and mezzanine tranche investments are fairly correlation invariant depending on the respective attachment points. Over time however, tranche sensitivities to correlation can change if, for instance, spreads in the underlying portfolio move meaningfully wider or tighter, or if defaults erode the subordination beneath a tranche.

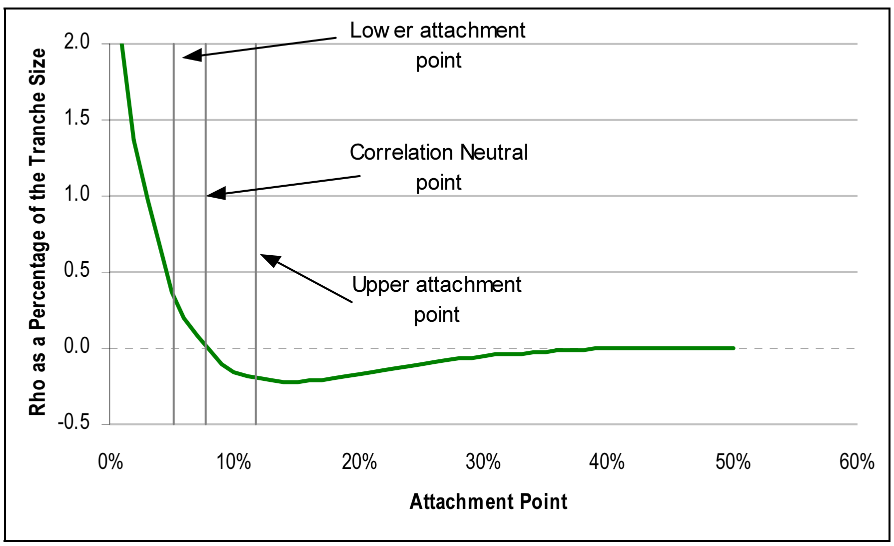
We define Rho as the MTM change of a tranche for a 1% change in the default correlation used to price the tranche. From the perspective of an investor in the equity tranche and using the discrete CDO example in Table 17, if we adjust our initial correlation assumption of 25% to 26%, the Equity tranche MTM for the investor increases by about €229k i.e. **Rho is positive for the equity tranche**. However, the same adjustment in correlation for an investor in the senior tranche results in a negative MTM of about €-31k i.e. **Rho is negative for the senior tranche**.

Some of the factors impacting Rho include:

- Position in the Tranche i.e. Long or Short Tranche
- Tranche Subordination
- Average level of Credit Spreads in the underlying portfolio
- Time to Maturity of the Transaction
- Actual level of Correlation

Long Correlation positions (long equity or short senior) have a *positive* Rho.
Short Correlation positions (short equity or long senior) have a *negative* Rho.

Chart 48: Rho as a Function of Subordination (Continuous CDO Example)



We divide Rho by the Notional size of each Continuous CDO Tranche (€5mn).
Source: Merrill Lynch.

In Chart 48 we show Rho as a function of subordination for the Continuous CDO example from the perspective of an investor who has long exposure. Rho is positive for the equity tranche and then tends to a negative value for more senior tranches. Note that Rho tends to zero for very high attachment points. In between the equity and senior tranches, there lies a C*orrelation Neutral Point*, where Rho is zero. We can define attachment points around this Correlation Neutral Point to construct a **Correlation Neutral Mezzanine Tranche**.

We note also that a CDS and a portfolio of CDS (such as a CDS index) have no sensitivity to correlation. As a result, a Delta-hedged tranche has the same correlation sensitivity as the tranche itself. Hence, correlation sensitive tranches can be combined with underlying CDS and index positions without altering the correlation behavior of the strategy.

Spread Convexity

Spread convexity for a credit product represents the curvature of its MTM as a function of the underlying spreads. The spread convexity across tranches is significantly different reflecting the different spread risks associated with each tranche. In addition, the spread convexity can be relatively large for a tranche compared to a single-name CDS or a CDS index. It is this difference in spread convexity across tranches and the underlying single-name CDS that makes delta-hedged portfolios exposed to spread convexity risk.

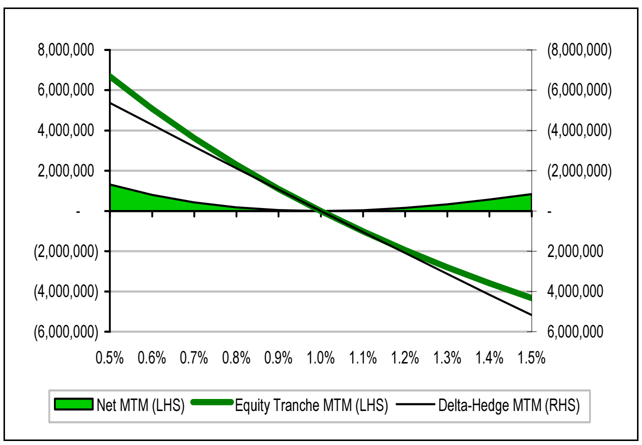
■ Behavior of Delta-Hedged Tranches

Equity and senior tranches have different spread sensitivity relative to delta-hedged portfolio

Chart 49 and Chart 50 highlight the MTM behavior of a delta-hedged long equity tranche and a delta-hedged long senior tranche respectively as the underlying CDS spreads shift uniformly away from their initial spread of 100 bps. The shaded portion of the graphs represents the net MTM. We observe the following:

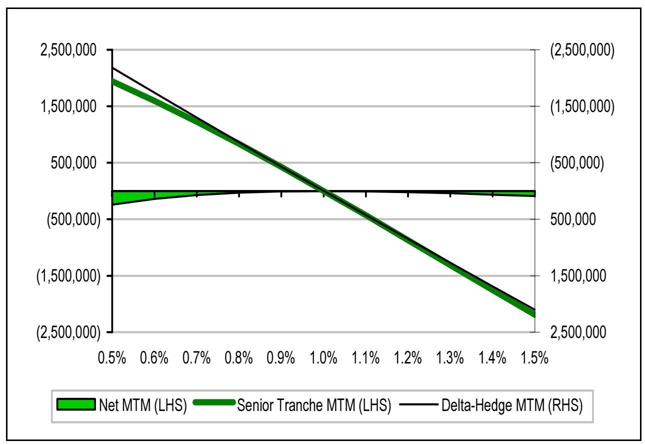
- For a uniform shift in spreads, the equity tranche is more convex than its delta-hedge resulting in a net positive P&L.
- For a uniform shift in spreads, the senior tranche is less convex than its delta-hedge resulting in a net negative P&L. In fact, the senior tranche in the chart below is concave.

Chart 49: Delta-Hedge Long Equity Tranche as a Function of Uniform Shift in All Spreads



Source: Merrill Lynch

Chart 50: Delta-Hedged Long Senior Tranche as a Function of Uniform Shift in All Spreads



Source: Merrill Lynch

We infer from the above charts that while **delta-hedged tranches are MTM neutral for small spread changes they are sensitive to larger CDS spread moves**. This is because Delta is itself a function of spreads and changes as spreads move. A constant rebalancing of deltas is required for the delta-hedged position to remain spread-neutral. Rebalancing of deltas leads to gains and losses based on the investor's exposure.

In order to understand the intuition behind spread convexity and the resulting direction of the MTM of a delta-hedged position, let us consider the behavior of two portfolios: (a) a delta-hedged long equity tranche (long correlation) and (b) a delta-hedged long senior tranche (short correlation). We assume that all spreads move together (either all up or all down) (see Table 19).

Table 19: Delta-Hedged Portfolio P&L for Uniform Change in All Spreads

	All Spreads Widen	All Spreads Tighten
Long Correlation		
Sell Protection on Equity Tranche	- MTM	+ MTM
Buy CDS (Delta notional)	+ MTM	- MTM
Change in Delta of All Credits	-	+
Average Delta Over Spread Movement	-	+
Effective Delta Hedge	Overhedged	Underhedged
Net P&L (or net MTM)	+ MTM	+ MTM
Short Correlation		
Sell Protection on Senior Tranche	- MTM	+ MTM
Buy CDS (Delta notional)	+ MTM	- MTM
Change in Delta	+	-
Average Delta Over Spread Movement	+	-
Effective Delta Hedge	Underhedged	Overhedged
Net P&L (or net MTM)	- MTM	- MTM

Assume all spreads in delta-hedged portfolio are initially trading at similar levels.
Source: Merrill Lynch

For a delta-hedged long equity portfolio, an **increase in all the CDS spreads together** implies a lower Delta (see earlier section). Therefore, the average Delta for the tranche decreases over the spread increase implying that the tranche is overhedged. Therefore, the hedge MTM is greater in magnitude than the MTM of the tranche. Since the MTM on the hedge is positive, we infer that the net MTM is positive. From the perspective of maintaining a hedged portfolio and rebalancing Deltas, the investor would need to sell additional CDS at higher spreads thus locking in a profit. We use similar intuition to understand the P&L of any other delta-hedged portfolio for a uniform shift in spreads.

In Table 19 we have assumed that all CDS spreads shift uniformly in the same direction (or *positive* spread correlation). Consider the situation when spreads are uncorrelated, e.g. **one CDS spread moves and the others remain constant**. Table 20 highlights the net P&L of a delta-hedged portfolio as one credit either widens or tightens while the other spreads remain constant.

Table 20: Delta-Hedged Portfolio P&L for Change in One Credit Spread

	One Spread Widens	One Spread Tightens
Long Correlation		
Sell Protection on Equity Tranche	- MTM	+ MTM
Buy CDS (Delta notional)	+ MTM	- MTM
Change in Delta of Credit *	+	-
Average Delta Over Spread Movement	+	-
Effective Delta Hedge	Underhedged	Overhedged
Net P&L (or net MTM)	- MTM	- MTM
Short Correlation		
Sell Protection on Senior Tranche	- MTM	+ MTM
Buy CDS (Delta notional)	+ MTM	- MTM
Change in Delta of Credit *	-	+
Average Delta Over Spread Movement for the Credit	-	+
Effective Delta Hedge	Overhedged	Underhedged
Net P&L (or net MTM)	+ MTM	+ MTM

* The other credit deltas also change. However, since those credit spreads remain constant, they do not impact the MTM of the CDS leg and as a result do not contribute to the net P&L of the delta-hedged portfolio.
Assume all spreads in delta-hedged portfolio are initially trading at similar levels.
Source: Merrill Lynch

In the case of a delta-hedged long equity tranche scenario, we note that the Delta increases (decreases) for the one credit that widens (tightens)[24]. The other deltas also change with a move in one spread but the MTM of the Delta hedge is only due to the CDS whose spread has moved. Thus the other deltas are not required in the MTM analysis in Table 20. The intuition used to generate the net P&L is similar to that described above.

Realized Correlation

As spreads of the underlying credits move the resulting gain or loss of the delta-hedged tranche is a function of the "realized" correlation. **Realized correlation is defined as the observed (spread) correlation between the credits in a portfolio <u>relative to</u> the assumed underlying (default) correlation of the portfolio**. This assumed underlying default correlation is used to price the tranche and determine the deltas for each credit.

Realized correlation can be either positive or negative . . .

The same movement in spreads can represent either positive or negative realized correlation depending on the correlation of the underlying portfolio. For example, if all spreads move together with an observed correlation of 60%, then the realized correlation would be positive relative to a tranche that is priced at a correlation of less than 60%. The realized correlation, however, would be negative relative to a tranche priced at a correlation of more than 60%.

. . . and tends to have a positive bias in a tight spread environment

If we ignore company-specific events, credits spreads tend to be positively correlated for small spread moves resulting in a positive realization of correlation[25]. This positive spread correlation bias tends to get reinforced in the current environment of tight spreads. However, a default (or a distressed widening) by one of the credits combined with a limited movement in spreads of the other credits in the portfolio would be reflected as a negative realization of correlation.

Using this terminology, Table 19 and Table 20 highlight positive and negative realizations of correlation respectively. We observe that the same delta-hedged tranche does not always generate a positive P&L when spreads change, i.e. it is **not unambiguously long convexity**. However, a delta-hedged tranche is **unambiguously long or short correlation** depending on the subordination of the tranche and whether the investor is long or short the tranche. Net P&L of a delta-hedged tranche is clearly a function of realized correlation as shown in the tables above.

We observe the following convexity biases:

A delta-hedged tranche that is **long correlation** will generate a <u>gain</u> for a positive realized correlation and a <u>loss</u> for a negative realized correlation.

A delta-hedged tranche that is **short correlation** will generate a <u>loss</u> for a positive realized correlation and a <u>gain</u> for a negative realized correlation.

Gamma/iGamma/nGamma

Gamma is a second-order risk measure

In order to quantify the convexity biases described above, we define the following three convexity measures **for delta-hedged portfolios** that take into account positively correlated, uncorrelated and negatively correlated spread moves respectively:

- **Gamma**: is defined as the portfolio convexity corresponding to a uniform relative shift in all the underlying CDS spreads.
- **iGamma** (individual Gamma): is defined as the portfolio convexity resulting from one CDS spread moving independently of the others, i.e. one spread moves and the others remain constant.
- **nGamma** (negative Gamma): is defined as the portfolio convexity corresponding to a uniform relative shift in underlying CDS spreads, with half of the credits widening and half of the credits tightening by a uniform amount.

[24] The reasoning is explained earlier in this chapter.
[25] Assume a default correlation of around 20% in this example.

The spread movements relating to **Gamma reflect a positive realization of correlation** whereas the spread movements corresponding to the **iGamma and nGamma reflect a negative realization of correlation**. In other words:

Delta-hedged investors who are **long correlation (**and therefore generate a **gain from positive realized correlation)** will also be **long Gamma** and **short iGamma and nGamma**.

Delta-hedged investors who are **short correlation (**and therefore generate a **gain from negative realized correlation)** will also be **short Gamma** and **long iGamma** and **nGamma**.

Table 21 highlights the Gamma exposures for delta-hedged investors who are either long or short correlation. We see that Gamma of a long senior position can be reduced (or hedged) with a long equity position. Such as Gamma reduction strategy is particularly attractive when Delta hedging becomes expensive, e.g. in volatile market conditions (assuming the spreads move together).

Table 21: Correlation & Convexity

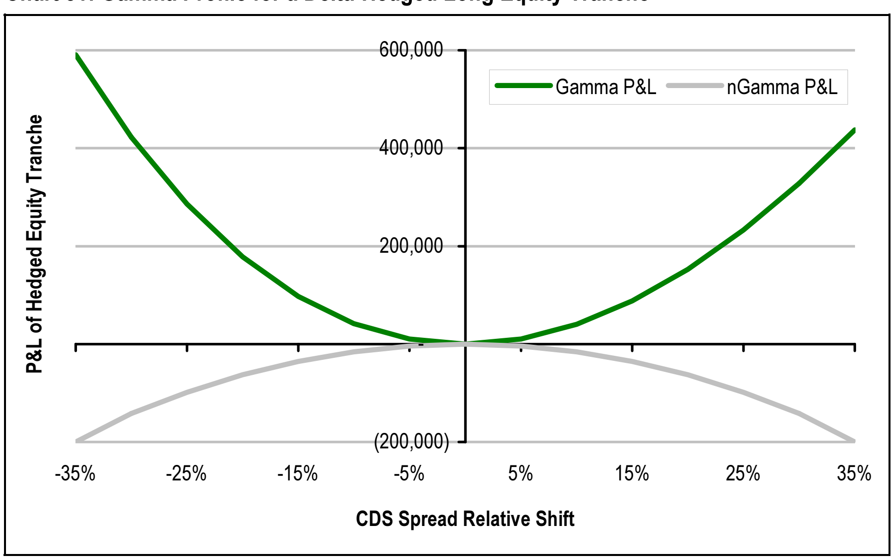
Correlation View	Delta-Hedged Tranche Exposure	Convexity Gamma	Convexity iGamma	Convexity nGamma
Long Correlation	Long Equity/Short Senior	Long	Short	Short
Short Correlation	Short Equity/Long Senior	Short	Long	Long

Source: Merrill Lynch

Convexity of Equity Tranche

Delta-hedged investors who are long an equity tranche are also long correlation. Therefore as spreads moves they will gain (lose) from any positive (negative) realization of correlation. In other words they are long Gamma and short iGamma and nGamma. Chart 51 illustrates two of the three measures for this delta-hedged portfolio.

Chart 51: Gamma Profile for a Delta-Hedged Long Equity Tranche



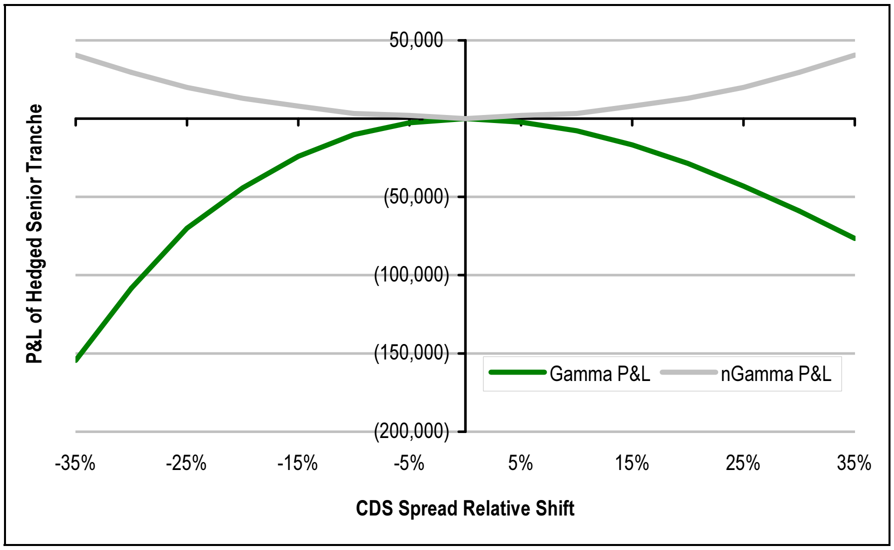
Assume Discrete CDO tranche.
Source: Merrill Lynch

Refer to important disclosures on page 196.

Convexity of Senior Tranche

Delta-hedged investors who are long a senior tranche are also short correlation. Therefore, as spreads moves they will benefit (lose) from any negative (positive) realization of correlation. In other words they are short Gamma and long iGamma and nGamma. Chart 52 illustrates two of the three measures for this delta-hedged portfolio. Note that Gamma (nGamma) is more convex (concave) for a delta-hedged long equity tranche relative to a delta hedged short senior tranche.

Chart 52: Gamma Profile of a Delta-Hedged Long Senior Tranche

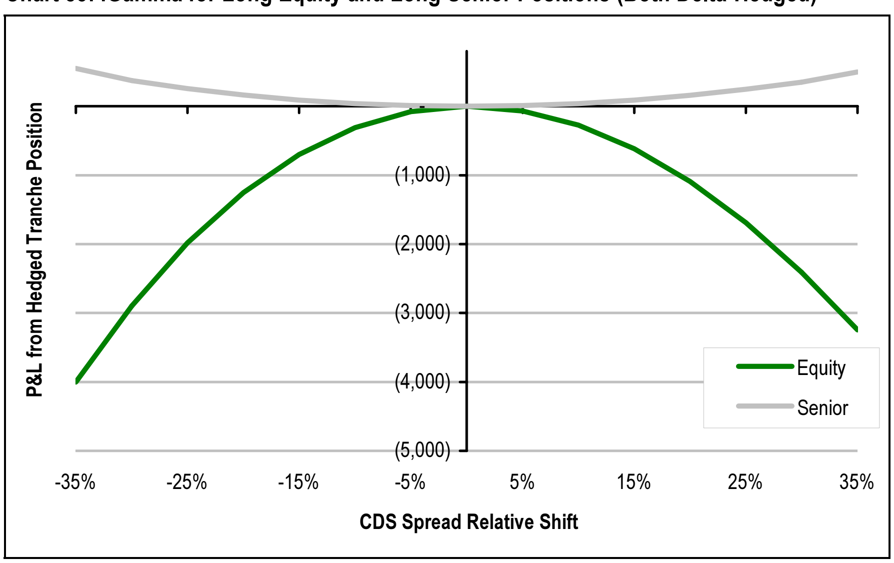


Assume Discrete CDO tranche.
Source: Merrill Lynch

iGamma

Chart 53 highlights the iGamma P&L for the long delta-hedged positions in both the equity and senior tranches. Note that the magnitude of iGamma is smaller than Gamma or nGamma. Also iGamma for a delta-hedged short equity tranche is more convex than the iGamma for the delta-hedged long senior tranche.

Chart 53: iGamma for Long Equity and Long Senior Positions (Both Delta-Hedged)



Assume Discrete CDO.
Source: Merrill Lynch

Instantaneous Default Risk

Another risk factor in correlation products is the risk of an instantaneous default to a recovery value. For unhedged tranches this risk is high, for delta-hedged tranches this risk is somewhat lower but can still be sizeable depending on the tranche in question. The P&L following an instantaneous default depends on whether the tranche is long or short correlation.

Default-to-Recovery

Hedged or unhedged tranche positions are exposed to instantaneous default risk . . .

Unhedged tranche positions are sensitive to (rare) instantaneous defaults and the idiosyncratic risk of the underlying portfolio. For unhedged tranche positions, this is a first-order risk, for hedged tranche positions this is a second-order risk.

We define the Default-to-Recovery or DTR as the net P&L of a tranche position (hedged or unhedged) resulting from an instantaneous default of a credit, keeping all other credit spreads in the portfolio unchanged.

. . . which is typically a negative realization of correlation

An instantaneous default is typically a **negative realization of correlation**, unless spreads on the rest of the undefaulted names in the portfolio widen substantially once default occurs. In fact, DTR can be viewed as the most severe form of iGamma so a tranche that is short iGamma is also short DTR, and a tranche that is long iGamma is also long DTR. Therefore, DTR should typically benefit delta-hedged tranches that are short correlation. In other words:

- **Short correlation** Delta-Hedged tranches are **long DTR**.
- **Long correlation** Delta-Hedged tranches are **short DTR**.

We examine the impact of DTR with and without spread changes for the Discrete CDO example. In Table 22 we compute the net P&L after a DTR in the underlying portfolio for a long Delta-hedged equity position and a long Delta-hedged senior position. We assume that defaults recover 35% i.e. the loss amount is €6.5m.

The DTR for the Delta-hedged long equity position is negative, although the magnitude of the loss *falls* if spreads simultaneously *widen*, and the magnitude of the loss *rises* if spreads simultaneously *tighten*.

The DTR for a Delta-hedged long senior position is positive, although the magnitude of the gain *falls* if spreads simultaneously *widen*, and the magnitude of the gain *rises* if spreads simultaneously *tighten*.

For a DTR of one credit accompanied by a simultaneous shift in spreads of the remaining credits, we observe the following:

- If spreads on the remaining credits widen, the realized correlation would be higher. Therefore, a *long correlation* position would *lose less* and a *short correlation* position would *gain less*.
- If spreads on the remaining credits tighten, the realized correlation would be lower. Therefore, a *long correlation* position would *lose more* while a *short correlation* position would *gain more*.

For a long equity tranche position, following a DTR with no simultaneous spread change the investor has to pay the loss payments of €6.5m under the portfolio default swap. However, the investor has a positive P&L from settling the single-name CDS hedge, but this is less than the loss payment under the portfolio default swap as the Delta of the credit is less than 100%. In addition, the investor also incurs a negative MTM on the tranche position as the tranche width is reduced.

For a long senior tranche position, following a DTR with no simultaneous spread change, the investor is not exposed to any loss payments on the portfolio default swap, as this is only an exposure for an investor in the equity (first-loss) tranche. The investor again has a positive P&L from settling the single-name CDS hedge (although the magnitude of this payment is lower than in the equity tranche example as a senior tranche Delta is smaller than an equity tranche Delta). Finally, the investor again incurs a negative MTM on the tranche position as subordination is reduced.

Table 22: P&L From Instantaneous Default

		P&L Of Delta-Hedged Long Position	
One Credit	All Other Credits	Equity (0%-4%) Tranche (mn)	Senior (8%-12%) Tranche (mn)
Defaults	50 bps Widening	(2.86)	0.24
Defaults	25 bps Widening	(4.13)	0.42
Defaults	15 bps Widening	(4.57)	0.49
DTR	No Spread Change	(5.14)	0.60
Defaults	15 bps Tightening	(5.57)	0.69
Defaults	25 bps Tightening	(5.76)	0.73
Defaults	50 bps Tightening	(5.79)	0.74

Source: Merrill Lynch

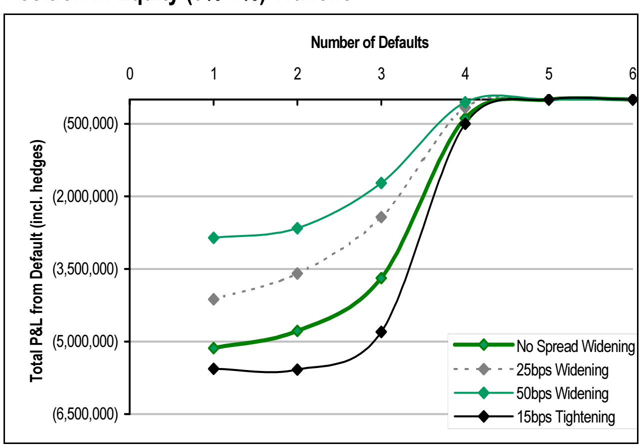
As a result of the different mechanics, the **Equity DTR is about 9x larger than the Senior DTR**. Hence, **selling (buying) protection on a senior tranche can be an effective way to hedge DTR exposure to a long (short) equity position**

We also note that mezzanine tranches are more neutral with respect to DTR. In the same way that we can construct a correlation neutral mezzanine tranche, it is possible to construct attachment points such that a mezzanine tranche has zero exposure to DTR.

DTR P&L For Multiple Defaults

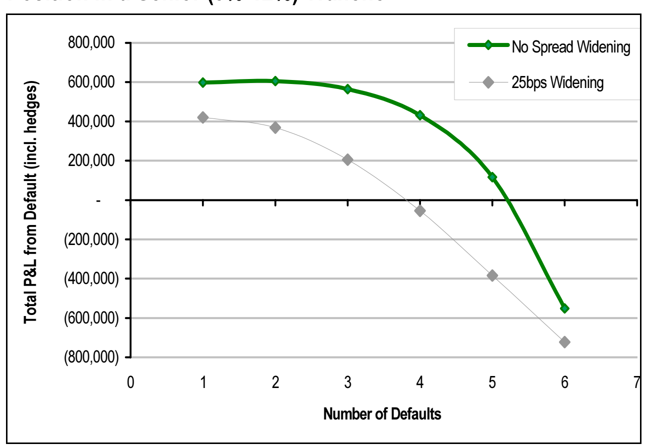
We can further investigate the DTR dependency of a delta-hedged tranche by looking at the P&L profile following multiple defaults with and without simultaneous spread changes. In Chart 54 and Chart 55 we plot the marginal net P&L of the delta-hedged equity and delta-hedged senior tranches following multiple defaults in the underlying portfolio.

Chart 54: Marginal DTR P&L, Including Hedges, of a Long Position in Equity (0%-4%) Tranche



Source: Merrill Lynch

Chart 55: Marginal DTR P&L, Including Hedges, of a Long Position in a Senior (8%-12%) Tranche



Source: Merrill Lynch

For a delta-hedged long equity tranche position, the marginal P&L impact is usually always negative, unless spreads simultaneously widen significantly, but decreases as multiple defaults occur and the equity tranche is increasingly eroded. This scenario represents a low correlation event. If remaining spreads simultaneously widen, it represents a slightly higher correlation event and the P&L on each default is less negative since the position is long correlation.

For a delta-hedged long senior tranche position, the marginal P&L impact is usually always positive, unless spreads widen significantly, but decreases as multiple defaults occur and the senior tranche begins to behave more like a mezzanine tranche. This scenario represents a low correlation event. If remaining spreads simultaneously widen, it represents a slightly higher correlation event and the P&L on each default is less positive since the position is short correlation.

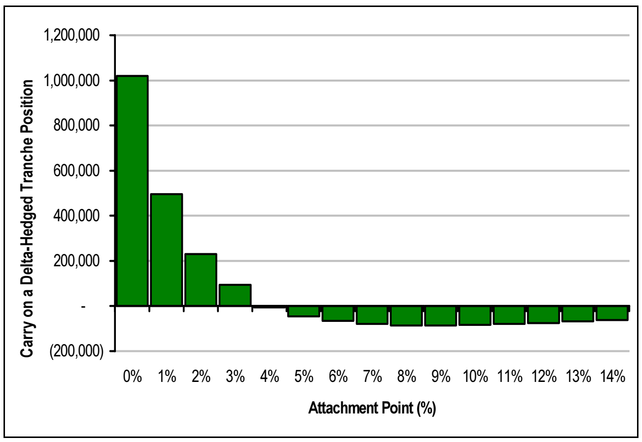
■ Implications of DTR on the Carry of a Delta-Hedged Tranche

A delta-hedged position is not a carry neutral trade for equity or senior tranches

The carry on the **delta-hedged equity tranche** is typically positive for an investor who is long the tranche and delta-hedged by buying protection on the underlying credits. (Conversely, this represents a negative carry position for a dealer who has bought protection on the tranche and delta-hedged by selling protection on the underlying names.) However, the carry position for the investor becomes negative for tranches that are more senior in the capital structure (Chart 56).

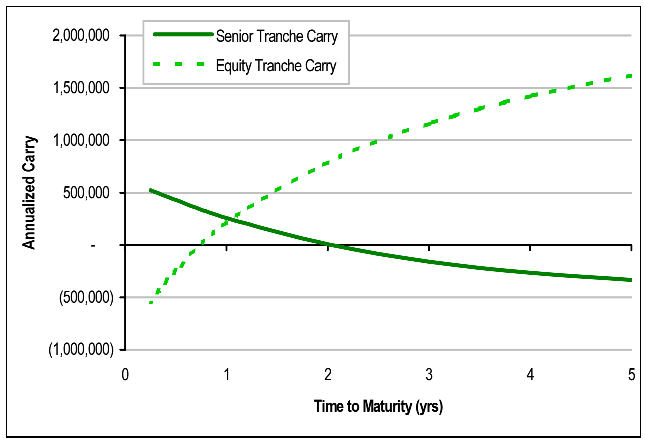
A delta-hedged tranche is not a risk-free position i.e. delta-hedging a tranche is not a perfect static hedge over the life of the transaction. An investor still has exposure to DTR. As was shown above, DTR has a *negative* P&L impact for a long position in the Equity tranche and a *positive* P&L impact for a long position in the senior tranche[26]. **Thus a delta-hedged tranche position should not be expected to be a carry neutral trade**.

Chart 56: Carry P&L on a Long Position in a Delta-Hedged CDO as a Function of Subordination (Continuous CDO Example)



Source: Merrill Lynch

Chart 57: Senior and Equity Tranche Carry as a Function of Time to Maturity



Source: Merrill Lynch. Long position in Delta-hedged tranches.

The carry changes over time

Moreover, the net carry will change over time due to time decay and the rebalancing of Deltas. For instance, a long position in a 5yr delta-hedged senior tranche initially has a negative carry. However, as time to maturity approaches (and assuming no defaults in the underlying portfolio), the Deltas tend to 0% and so the carry on the trade increases. Similarly, a long position in a 5yr delta-hedged equity tranche initially has positive carry but as time to maturity approaches, the Deltas tend to 100% and the carry on the trade turns negative (Chart 57).

A delta-hedged tranche position that is *long correlation* is s*hort DTR* and hence is a *positive carry* trade at inception.

A delta-hedged tranche position that is *short correlation* is l*ong DTR* and hence is a *negative carry* trade at inception.

[26] To hedge against this default risk, an investor could instead buy protection on the individual credits in the full notional amount of that credit. However, this would then be an overhedge for the spread risk of the tranche.

6. Tranche Relative Value Tools

Liquidity of standardized tranches has driven investors to look for relative value across the capital structure. In this section we present five alternative relative value metrics to identify relative value opportunities in the correlation market:

1) **Relative Loss Allocation (RLA)**
2) **Short correlation net carry**
3) **Synthetic Super Senior Implied Value**
4) **Carry Neutral Multiples**
5) **Relative Trading Bands**

We also present a detailed rating based approach to value synthetic CDO tranches that can be efficiently blended with RLA analysis to provide an additional tool to screen relative value opportunities.

Relative Loss Allocation

RLA is an efficient tool to analyze value distribution across tranches

We have often used the concept of **Relative Loss Allocation** ("RLA") as a useful tool to screen and spot relative value opportunities within the tranche space. The RLA indicator is measured as **the ratio of:**

- **the tranche default leg and;**
- **the default leg of the underlying portfolio**.

For example the 10y 7-10% RLA of 5.7% indicates that the expected loss "absorbed" by the 7-10% tranche represents 5.7% of the total expected loss of the underlying portfolio.

Table 23: 7-10% CDX.NA.IG Tranche RLA Calculation

	CDX IG4			CDX 7-10% Tranche			
Maturity	Spread	DV01	Default Leg	Spread	DV01	Default Leg	RLA
5y	49	4.28	2.10%	26	4.34	0.03%	1.61%
7y	58	5.76	3.40%	51	5.89	0.09%	2.65%
10y	72	7.66	5.51%	133	7.84	0.31%	5.68%

Source: Merrill Lynch. Default Leg =spread x DV01. Calculation as of September, 15th 2005.

Once the current tranche spread is used to compute the **market-implied** RLA, the next step consists of comparing it with the two **rating-implied** RLA thresholds computed as follows:

- Given the credit rating of each issuer, extract the corresponding migration based survival curves from the historical default rates[27].
- For each tranche, find the two correlation values (r^{max} and r^{min}) that correspond to the upper (highest tranche breakeven spread) and lower (lowest tranche breakeven spread) tranche thresholds[28].

[27] Please refer to the appendix for a detailed illustration of a rating based model for computing single name CDS curves as well as valuing single tranche CDOs.
[28] Typically r^{max} equals 100% for senior and 0% for equity tranches. This procedure is specifically designed for mezzanine tranches, which has a non monotonic spread vs. correlation relationship.

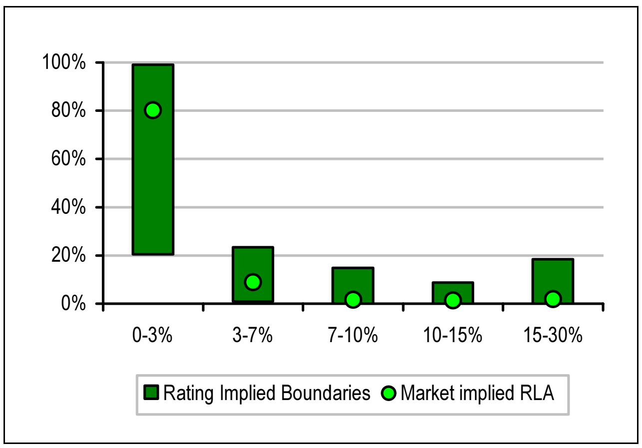
- Compute the tranche default leg[29] (for r^{max} and r^{min}) using the rating based default probabilities[30] and the standard one-factor gaussian copula model.
- Scale each default leg by the tranche width.
- Compute the tranche relative loss allocation (RLA) as the ratio between (a) the tranche scaled default leg and (b) the index default leg.

Chart 58, Chart 59, Chart 60 and Chart 61 show the RLA indicators (market and rating implied) for the CDX IG and HY tranches computed using the information coming from the rating agencies historical default matrices.

In our opinion spread-based RLAs should lie within the ratings-based RLA thresholds (computed using rating agencies information). The charts highlight that with the exception of the 10y 3-7% all standardized tranche RLAs lie within the rating implied thresholds. This implies that selling 10y 3-7% protection is relatively attractive.

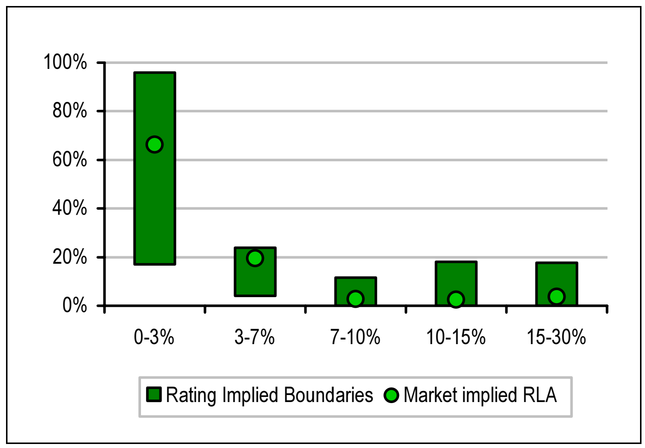
This last observation, combined with a proper assessment of the technical forces driving the supply/demand flow for each tranche, makes the RLA approach a powerful tool to screen and identify relative value opportunities.

Chart 58: CDX.NA.IG 5y Tranche Relative Loss Allocation



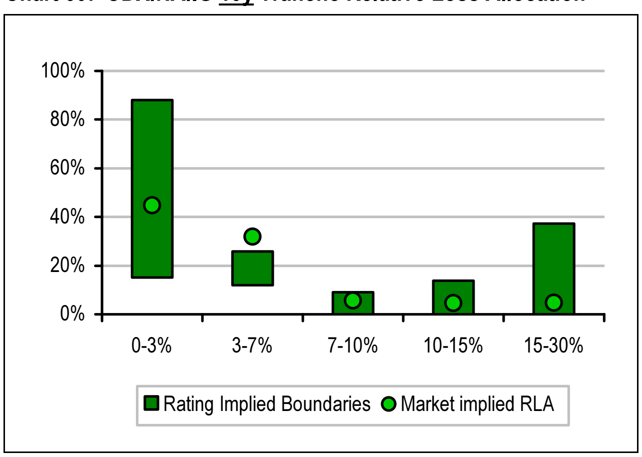
Source: Merrill Lynch calculation on S&P rating data as of Sep, 1st 2005.

Chart 59: CDX.NA.IG 7y Tranche Relative Loss Allocation



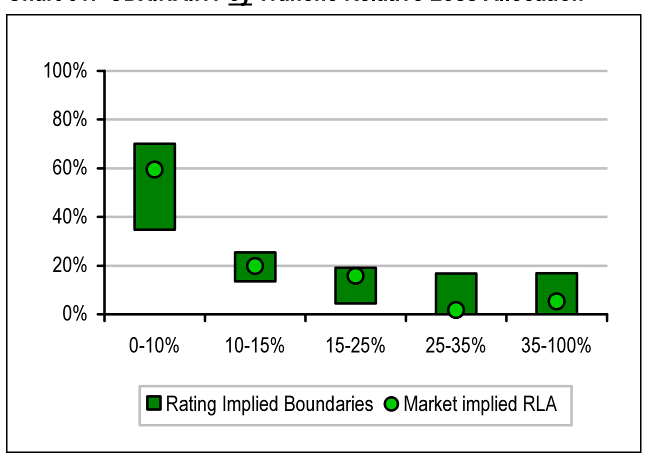
Source: Merrill Lynch calculation on S&P rating data as of Sep, 1st 2005.

Chart 60: CDX.NA.IG 10y Tranche Relative Loss Allocation



Source: Merrill Lynch calculation on S&P rating data as of Sep, 1st 2005.

Chart 61: CDX.NA.HY 5y Tranche Relative Loss Allocation



Source: Merrill Lynch calculation on S&P rating data as of Sep, 1st 2005.

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[29] We define the default leg as the discounted expected value of losses affecting the tranche. Given the parity condition at trade inception (premium leg = default leg) the default leg can be easily computed by multiplying the tranche spread by its DV01.

[30] See note 27.

Short Correlation Positive Carry

Short correlation positions, because of their positive payoff upon default, typically trade at negative carry. For example, buying protection on a junior-mezzanine tranche and selling protection on a more senior tranche on a delta-hedge basis generally implies a negative periodic carry for the package holder (see Table 24).

Table 24: Delta Hedge Tranche Combination Carry Table for CDX.IG5 Tranches

	Tranche	Running Spread	Delta	Buy Protection 0-3%	3-7%	7-10%	10-15%	15-30%
Sell Protection	0-3%	500	20.00x	**0 bps**	37 bps	12 bps	6 bps	2 bps
	3-7%	103 / 107	5.75x	-142 bps	**-4 bps**	1 bps	0 bps	-1 bps
	7-10%	22 / 26	1.50x	-207 bps	-23 bps	**-4 bps**	-2 bps	-2 bps
	10-15%	10 / 14	0.80x	-250 bps	-35 bps	-7 bps	**-4 bps**	-3 bps
	15-30%	4 / 7	0.35x	-271 bps	-41 bps	-9 bps	-5 bps	**-3 bps**

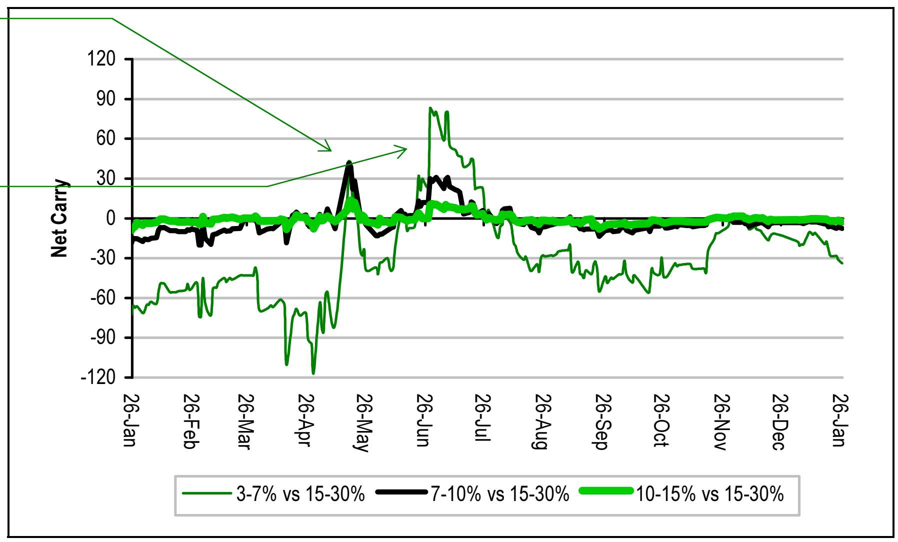
Source: Merrill Lynch. Data as of February, 7th 2006.

Tracking this indicator can provide a valuable relative value tool; as shown in Chart 62 over the course of 2005 we observed few instances where the short correlation net carry has been positive. However, divergence between theoretical and realized deltas can jeopardize the performance of the trade especially when unwound before maturity.

After the correlation shakeout in mid May, value squeezed from mezz tranches into equity and super senior tranches…

Hedging of bespokes and protection selling pressure on equity transferred value once again into the senior region

Chart 62: CDX 5y Short Correlation Net Carry (Delta-hedged tranches)

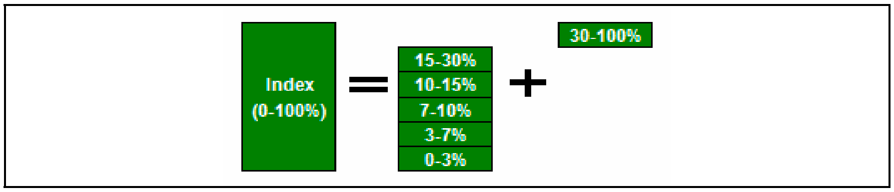


Source: Merrill Lynch

Synthetic Super Senior Implied Value

The investment grade structured credit market include five tranche with attachment points up to 30% and 22% in North America and Europe respectively.

Chart 63: CDX IG Super Senior 30-100% Implied Super Senior Value



Source: Merrill Lynch

The following no-arbitrage argument, can be used to compute the implied super senior spread for the 30-100% CDX.NA and the 22-100% iTraxx tranche:

$$PV(Index) = \sum_{t=1}^{5} PV(tranche_t) \times WIDTH(tranche_t) + PV(tranche_{SS}) \times WIDTH(tranche_{ss}).$$

By recalling that:

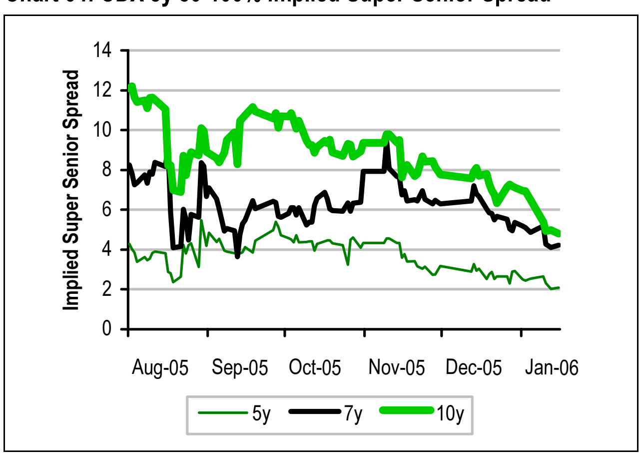
$$PV(Index) = SPREAD(Index) \times DV01(Index)$$
$$PV(Tranche) = SPREAD(Tranche) \times DV01(Tranche)$$

we can then establish the following formula in order to compute the implied super senior spread:

$$SPREAD(tranche_{SS}) = \frac{PV(Index) - \sum_{t=1}^{5} PV(tranche_t) \times WIDTH(tranche_t)}{WIDTH(tranche_{ss}) \times DV01(tranche_{ss})}$$

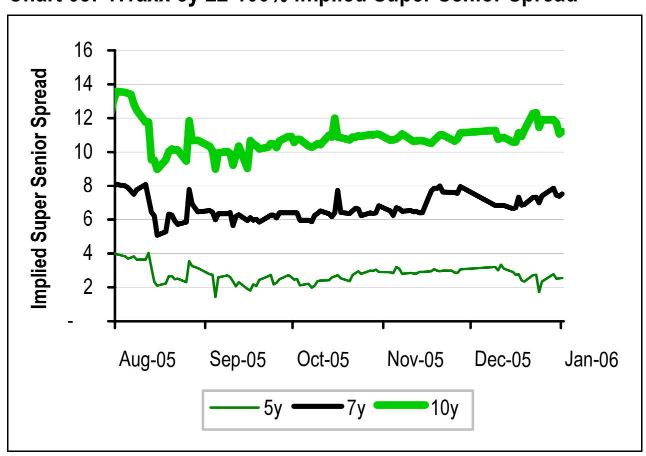
In **Error! Reference source not found.** and **Error! Reference source not found.** we show the historical super senior spread for the CDX and iTraxx super senior tranches.

Chart 64: CDX 5y 30-100% implied Super Senior Spread



Source: Merrill Lynch

Chart 65: iTraxx 5y 22-100% implied Super Senior Spread



Source: Merrill Lynch

Tracking the pattern of the implied super senior spread can offer valuable insights in order to understand how tranche losses/values are allocated over the different parts of the capital structure.

Carry-Neutral Multiples

The next two sections are extracted from a "Protection Strategies" report published on 6th July 2005 by Jonsson et al.

Definition

Carry-Neutral Multiples equal the spread on a given tranche divided by the spread on either (a) the index or (b) the detaching tranche

Carry-Neutral Multiple analysis quantifies relative trading values of tranches from a historical point of view. We define **Carry-Neutral Multiple** as the spread of a given tranche, divided by the spread on either:

(a) the next senior-ranking (detaching) tranche or

(b) the index

For the 3-6% tranche, its carry-neutral **tranche multiple** equals the **ratio** of its premium to the 6-9% premium, where as its carry-neutral **index multiple** equals the ratio of its premium to the index premium.[31] The current observation is then compared to its one-year historical average and standard deviation (across all on-the-run series) to obtain a **z-score**.

Carry-Neutral Tranche Multiples

Carry-neutral tranche ratios are similar to bond subordination premiums, but measure spread ratios rather than spread differences.

Carry-Neutral Tranche Multiples are conceptually similar to bond subordination premiums. In bank capital markets[32], investors asses the attractiveness of Lower Tier 2 or Tier 1 instruments relative to senior bonds and Upper Tier 2 securities, respectively. The Carry-Neutral Tranche Multiple measures the subordination premium in terms of the **ratio** of two spreads, whereas bond subordination premiums represent their **difference**. Therefore, the method can also be extended to assess the relative value of any tranche vis-à-vis any other tranche, e.g., the 12-22% vs. the 3-6% tranche.

Table 25 illustrates that, currently:

- The **0-3% tranche is trading too wide relative to the 3-6% tranche**, or 3.0x standard deviations wider than the average. Thus, the 3-6% tranche can also be viewed as trading too tight to the 0-3% tranche.
- The **9-12% tranche is too tight relative to the 12-22% tranche**, or 2.0x standard deviations below the average. By the same token, the 12-22% tranche could be viewed as trading too wide relative to the 9-12% tranche.

Currently, the 0-3% tranche is 3.0x standard deviations wider than the average spread over the 3-6% tranche

The 9-12% tranche, however, is 2.0x standard deviations tighter than the average spread over the 12-22% tranche

Table 25: iTraxx Europe 5Y Carry-Neutral Tranch Multiples

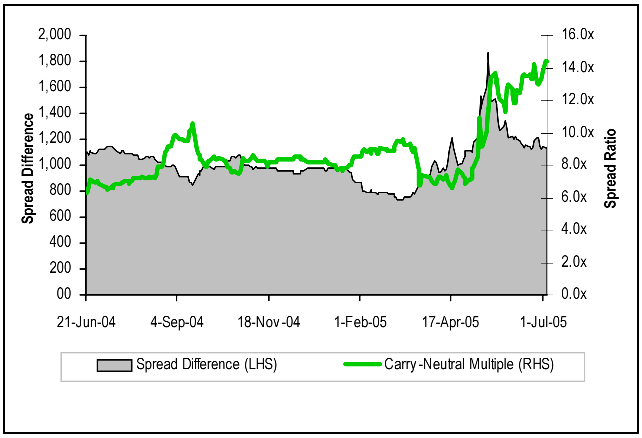
Statistics	0-3 vs. 3-6% Tranche	3-6 vs. 6-9% Tranche	6-9 vs. 9-12% Tranche	9-12 vs. 12-22% Tranche
Current				
Carry-Neutral Multiple	14.40	2.64	1.83	1.40
Z-Score	**3.01**	(0.94)	0.44	**(2.01)**
Historical				
Average	8.81	2.94	1.72	1.87
Maximum	14.43	4.15	2.25	2.41
Minimum	6.53	2.28	1.33	1.23
Standard Deviation	1.86	0.32	0.25	0.24

Source: Merrill Lynch. Based on observations between 6 July 2004 and 5 July 2005.

[31] Carry Neutral Multiple analysis for the 0-3% tranche is highly subjective since the tranche is quoted on an up-front *plus* running basis. Our analysis, however, requires the use of full running premium. Actual quotes on a full running basis, however, would likely differ from those implied by the market quote.

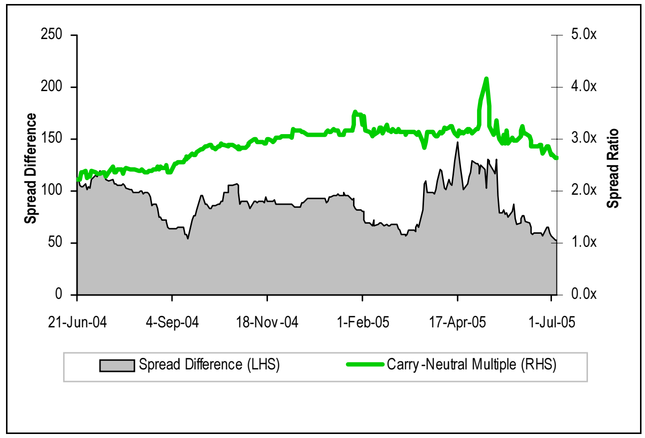
[32] For the analogy between tranche or CDO markets and bank capital, please see our 1 November 2004 report "*Capital Structure Investing in UK Banks*".

Chart 66: 0-3% Carry-Neutral Tranche Multiple (vs. 3-6%)



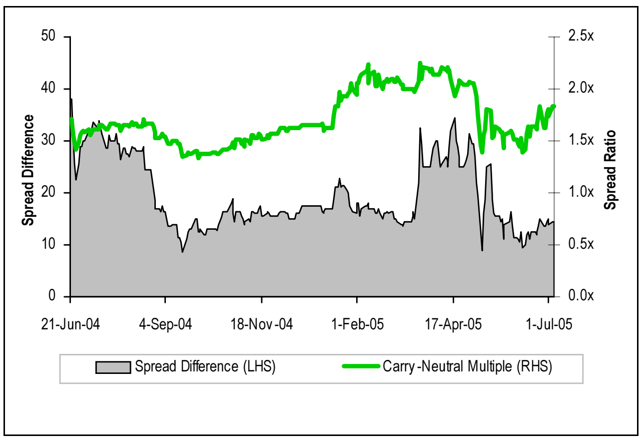
Source: Merrill Lynch

Chart 67: 3-6% Carry-Neutral Tranche Multiple (vs. 6-9%)



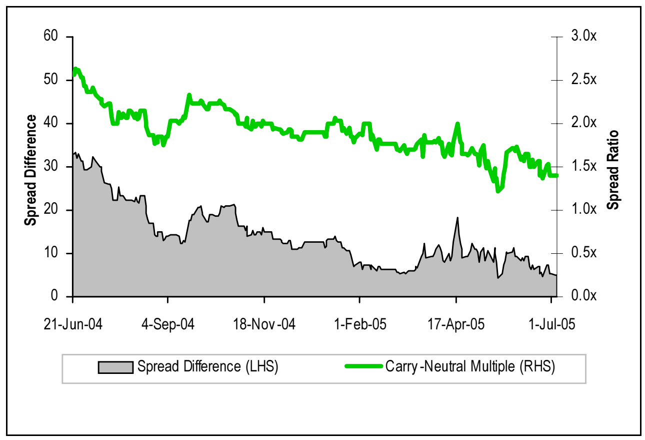
Source: Merrill Lynch

Chart 68: 6-9% Carry-Neutral Tranche Multiple (vs. 9-12%)



Source: Merrill Lynch

Chart 69: 9-12% Carry-Neutral Tranche Multiple (vs. 12-22%)



Source: Merrill Lynch

■ Carry-Neutral Index Multiples

Carry-Neutral Index Multiples quantify how a tranche is trading vs. the index

Applying the same historical average, standard deviation and z-score analysis, **Carry-Neutral Index Multiples** can establish whether a tranche is trading wide (or tight) relative to the index, on a historical basis. For example, a carry-neutral **ratio** for the 3-6% tranche of 2.17x simply implies that the tranche premium equals that multiple of the index premium (Table 26). Currently:

Except for the equity tranche, tranches are currently tight on a historical basis relative to the index

- Tranches other than the 0-3% equity tranche are currently trading comparatively tight relative to the index.
- The **3-6% tranche** is trading **2.6 standard deviations below the average** vs. the index.
- The **9-12% tranche** is trading at its lowest index multiple over the past year.

Refer to important disclosures on page 196.

Both the 3-6% tranche and the 9-12% tranche are trading at their lowest multiples relative to the index

Relative to the index, the 0-3% tranche is trading wider than average, whereas all other tranches are trading tighter than the average

Table 26: iTraxx Europe 5Y Carry-Neutral Index Multiples

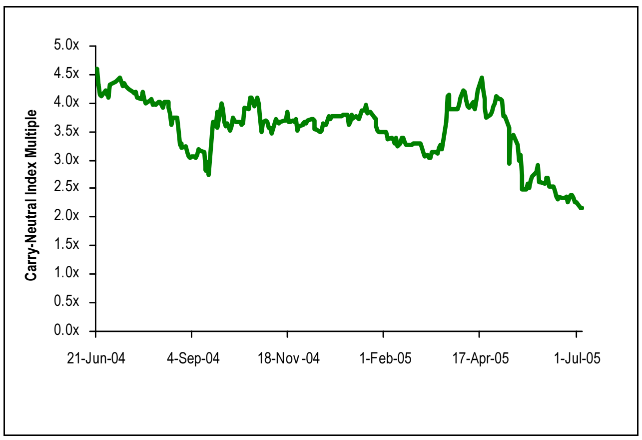
Statistics	0-3% Tranche vs. Index	3-6% Tranche vs. Index	6-9% Tranche vs. Index	9-12% Tranche vs. Index	12-22% Tranche vs. Index
Current					
Carry-Neutral Multiple	31.19	2.17	0.82	0.45	0.32
Z-Score	0.87	**(2.59)**	(1.56)	(1.43)	(0.94)
Historical					
Average	30.08	3.52	1.21	0.72	0.38
Maximum	34.48	4.46	1.90	1.21	0.52
Minimum	27.87	**2.17**	0.71	**0.45**	0.25
Standard Deviation	1.29	0.52	0.25	0.19	0.07

Source: Merrill Lynch. Based on observations between 6 July 2004 and 5 July 2005.

Quantifying the Obvious?

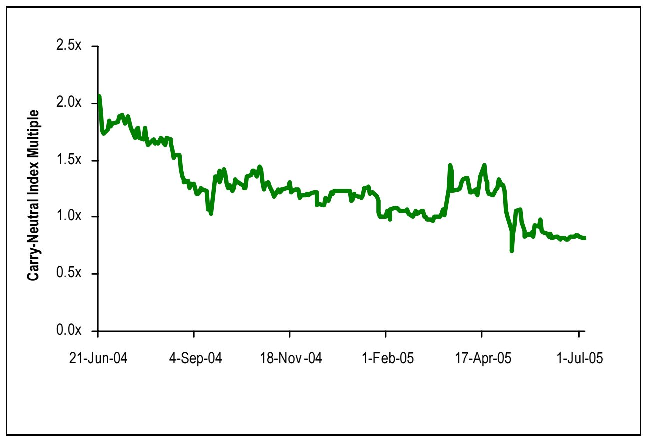
Our conclusions above may seem obvious post the recent correlation shakeout in early May, i.e. equity tranche is trading wider than average whereas the mezz/senior tranches are trading tighter than average. Though the measure above does reflect the lower correlation regime, more importantly, this tool provides a new **quantitative basis** to identify tranche cheapness or richness.

Chart 70: Historical Carry-Neutral Index Multiple for 3-6% Tranche



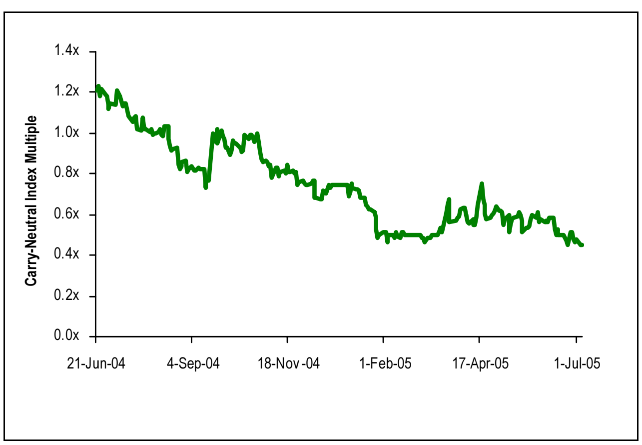
Source: Merrill Lynch

Chart 71: Historical Carry-Neutral Index Multiple for 6-9% Tranche



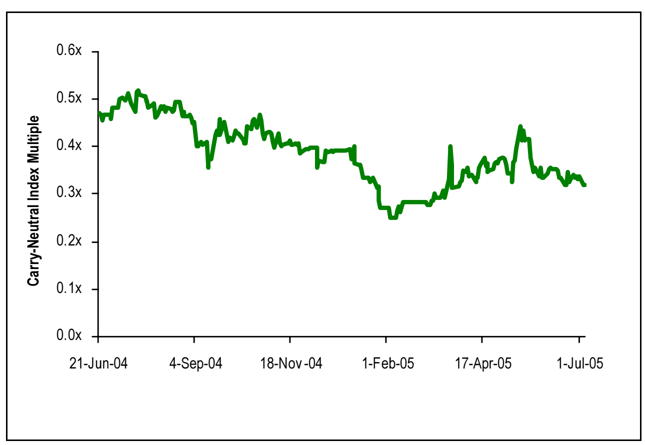
Source: Merrill Lynch

Chart 72: Historical Carry-Neutral Index Multiple for 9-12% Tranche



Source: Merrill Lynch

Chart 73: Historical Carry-Neutral Index Multiple for 12-22% Tranche



Source: Merrill Lynch

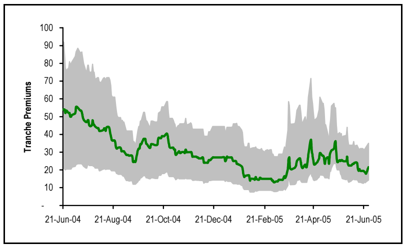
Relative Trading Bands

A tranche should always trade in a band defined by adjacent junior- and senior-ranking tranches of equal width

By extension, one can view the trading levels of any tranche in relation to two adjacent tranches. Where a tranche is trading relative to two adjacent tranches can also be expressed in terms of averages and standard deviations. For example, the 6-9% tranche should **always** trade tighter than the 3-6% tranche and wider than the 9-12% tranche. (The 9-12% tranche would also generally be expected to trade within the spread range defined by the 12-22% and 6-9% tranches.)[33] Within this trading band, however, the 6-9% tranche could either trade **too close** to:

- the **3-6% tranche** (in which case it should be comparatively more attractive to protection sellers than the lower-ranking tranche), or
- the **9-12% tranche** (in which case it should be comparatively more attractive to protection buyers than the higher-ranking tranche).

Chart 74: The 6-9% Tranche Should Always Trade in the Range Defined by the 9-12% Tranche (Bottom) and the 3-6% Tranche (Top) Premiums



Source: Merrill Lynch

Similar analysis could be applied to the base correlation curve, where one would analyze how far correlation of any tranche is relative to the base correlation on two adjacent tranches.

■ The 3-6% Tranche: Too Close to the 6-9% Tranche

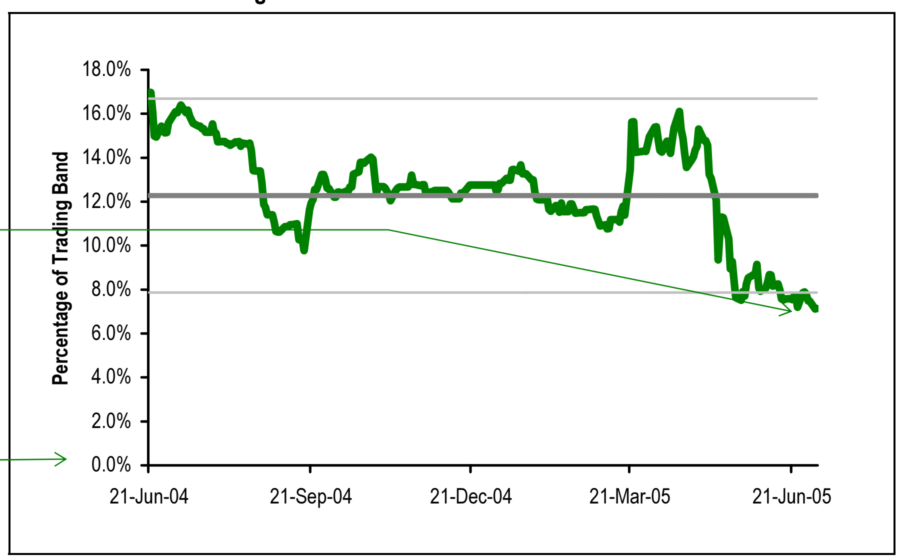
The 3-6% has traded on average at 12.3% of the spread between the 0-3% and 6-9% tranches

Over the last twelve-month period, the 3-6% tranche has traded on average at **12.3%** of the spread difference between the 6-9% and 0-3% tranche. Currently, it is trading **2.3 standard deviations below** the one-year average, or too close to the 6-9% tranche (Chart 75), suggesting its relative value for protection buyers.

Chart 75: Relative Trading Bands for the 3-6% Tranche

*Currently, the 3-6% tranche is trading at **only 7.1%** of the spread, or **2.3 standard deviations below average**, suggesting it is comparatively cheap for protection buyers vs. the 6-9% tranche*

If the 3-6% tranche traded flat to the 6-9% tranche, it would trade at 0% of the spread difference between the 0-3% and 6-9% tranche



Source: Merrill Lynch. Denotes at what percentage the 3-6% tranche trades of the difference between the 0-3% and 6-9% tranche premiums. Lines denote 1y average and average ± 2 standard deviations.

■ The 6-9% Tranche: Too Close to the 3-6% Tranche

The 6-9% has traded on average at 17.4% of the spread between the 3-6% and 9-12% tranches

Over the last twelve-month period, the 6-9% tranche has traded on average at **17.4%** of the spread difference between the 9-12% and 3-6% tranche. Currently, it is trading **1.4 standard deviations** above that average, suggesting it is comparatively close to the 3-6% tranche (Chart 76). These results are in line with the conclusions of the trading band analysis for the 3-6% tranche above.

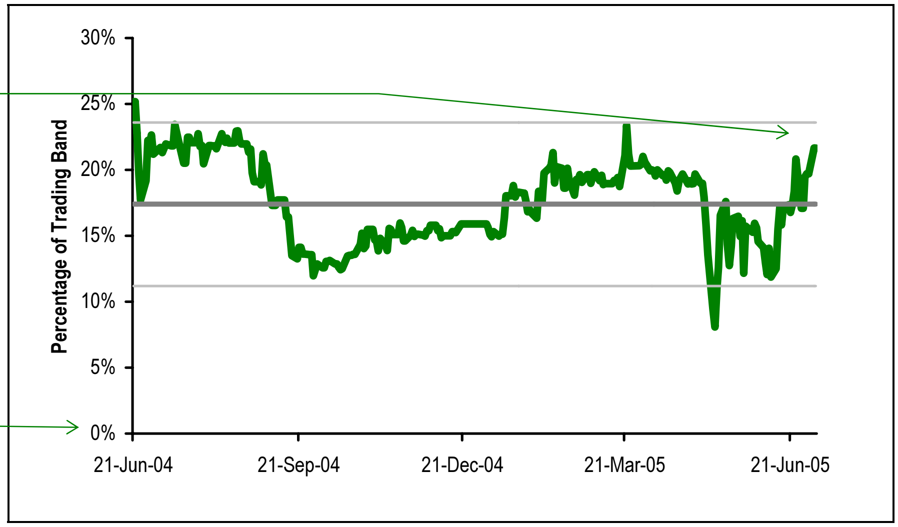
[33] Note, the 0-3%, 3-6%, 6-9% and 9-12% tranches are of equal width and therefore expose the protection seller to identical maximum principal loss. However, the width of the 12-22% tranche is significantly greater, exposing the protection seller to potentially higher principal loss.

Refer to important disclosures on page 196.

Currently, the 6-9% tranche is trading at 1.4 standard deviations above that average, or closer to the 3-6% tranche than the 9-12% tranche

If the 6-9% traded flat to the 9-12% tranche, it would trade at 0% of the spread difference between the 3-6% and 9-12% tranche.

Chart 76: Relative Trading Bands for the 6-9% Tranche



Source: Merrill Lynch. Denotes at what percentage the 6-9% tranche trades of the difference between the 3-6% and 9-12% tranche premiums. Lines denote average and average ± 2 standard deviations.

■ The 9-12% Tranche: Too Close to the 12-22% Tranche

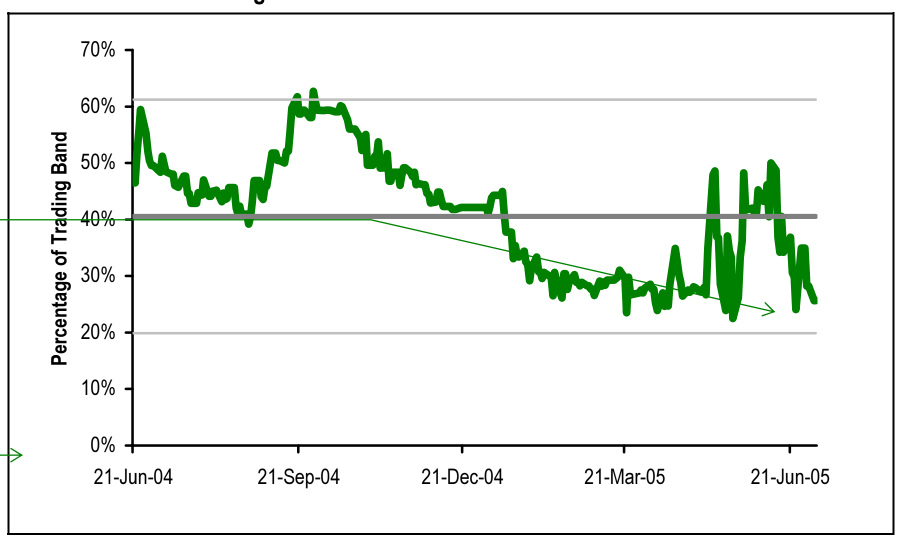
The 9-12% tranche has traded on average at 40.6% of the spread between the 6-9% and 9-12% tranches

Over the same period, the 9-12% tranche has traded on average at **40.6%** of the spread difference between the 12-22% and 6-9% tranche. Currently, however, it is trading **1.4 standard deviation** below the average, or much closer to the 12-22% tranche than 6-9% tranche (Chart 77).

Currently, the 9-12% tranche is trading at 1.4 standard deviations below that average, or closer to the 12-22% tranche than the 6-9% tranche

If the 9-12% traded flat to the 12-22% tranche, it would trade at 0% of the spread difference between the 6-9% and 12-22% tranche

Chart 77: Relative Trading Bands for the 9-12% Tranche



Source: Merrill Lynch. Denotes at what percentage the 9-12% tranche trades of the difference between the 6-9% and 12-22% tranche premiums. Lines denote average and average ± 2 standard deviations.

Appendix: A Rating Based Model to Value Single Tranche CDOs

The Standard Approach to Value Index Tranches

Within the standard one-factor approach, we assume that, for each credit, the firm value is driven by two main (independent and normally distributed) drivers:

- A **systematic** or common factor.
- An **idiosyncratic** noise or firm-specific factor.

As a consequence the individual risk process of an obligor is defined by (linearly) combining the two risk components:

$$V_i = \sqrt{\rho_i} Z + \sqrt{1-\rho_i}\,\varepsilon_i$$

- V_i : risk driver of the i^{th} credit in the portfolio
- Z : value of the common (systematic) factor
- ε_i : value of the idiosyncratic risk for the i^{th} credit in the portfolio
- ρ_i : sensitivity of the i^{th} credit to the common factor.

Default occurs when the risk driver process hits the default threshold

Along the lines of the classic Merton's structural framework, the one-factor model postulates that a company defaults when the risk driver falls below a certain default barrier *k*. Hence[34], for a specific realization $Z=z$ of the systematic factor, the (conditional) probability *p* of the i^{th} obligor defaulting (i.e. hitting the default barrier) is given by the following expression:

The conditional default probability depends on the realization of the common factor and the asset correlation

$$p_z^i = N\left[\frac{k_i - \sqrt{\rho_i}\,z}{\sqrt{1-\rho_i}}\right]$$

where, $N(x)$ is the standard normal distribution function.

Given the normality assumption for the firm value risk process V_i and the default probability d_i for the individual obligor *i*, the default threshold k_i is given by the following expression:

$$k_i = N^{-1}(d_i) \qquad \textbf{(1)}$$

where $N^{-1}()$ is the inverse of the standard normal distribution function.

Within a risk neutral framework the default probability for the individual obligor is driven by two main factors:

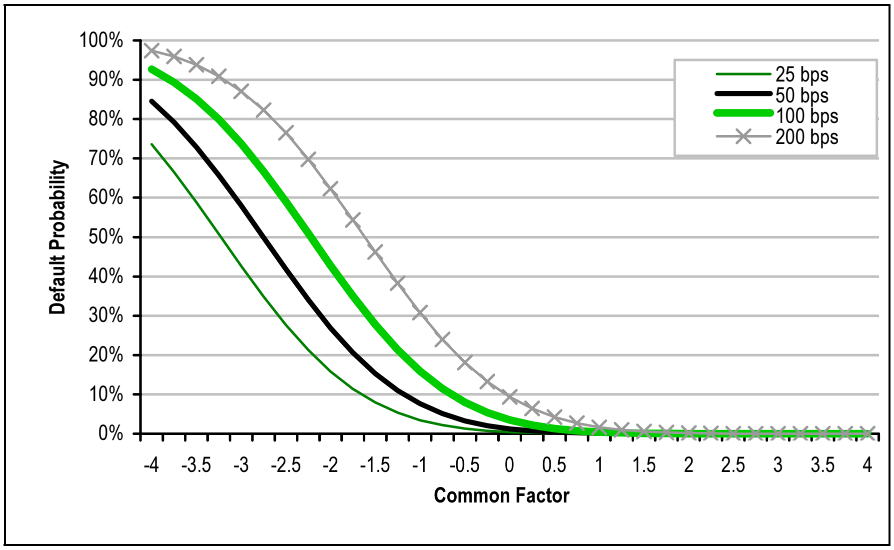
1. the **CDS spread** of the issuer[35], and;
2. the assumed **recovery** of the issuer upon default.

Chart 78 shows the individual default probability as a function of the common factor *Z* for different CDS spread levels:

[34] See Volume 2, Chapter 8.

[35] If the CDS term structure is flat, then the (risk neutral) default probability is simply given by $d_i = 1 - e^{-\left(\frac{spread_i}{1-R}\cdot t\right)}$ where $spread_i$ is the CDS spread for the i^{th} credit and *R* is the recovery value.

Chart 78: Market-implied One-Factor Conditional Default Probability



Source: Merrill Lynch. Assume 20% correlation, 40% recovery and 5y time horizon.

Starting from the collection of individual default probabilities defined above, it is then possible[36] to recover the conditional portfolio default distribution $\Pi_{n,z}^{k}$ of having k defaults in an n-credit portfolio by using the following recursive algorithm[37]:

$$\Pi_{n+1,z}^{k} = \Pi_{n,z}^{k} \cdot \left(1 - p_z^{n+1}\right) + \Pi_{n,z}^{k-1} \cdot p_z^{n+1}.$$

The unconditional distribution is obtained by averaging over all the possible values of the common factor

The unconditional distribution is then recovered by averaging the conditional default probability over all the possible value of the (normally distributed) common factor Z:

$$\Pi_{n+1}^{k} = E\left(\Pi_{n+1,z}^{k}\right) = \int_{-\infty}^{+\infty} \Pi_{n+1,z}^{k} \varphi(z) dz$$

where $\varphi()$ is the standard normal density function.

As illustrated in another section[38], once the portfolio loss distribution is computed, it is then straightforward to determine the tranche breakeven spread.

The Rating Based Approach to Value Index Tranches

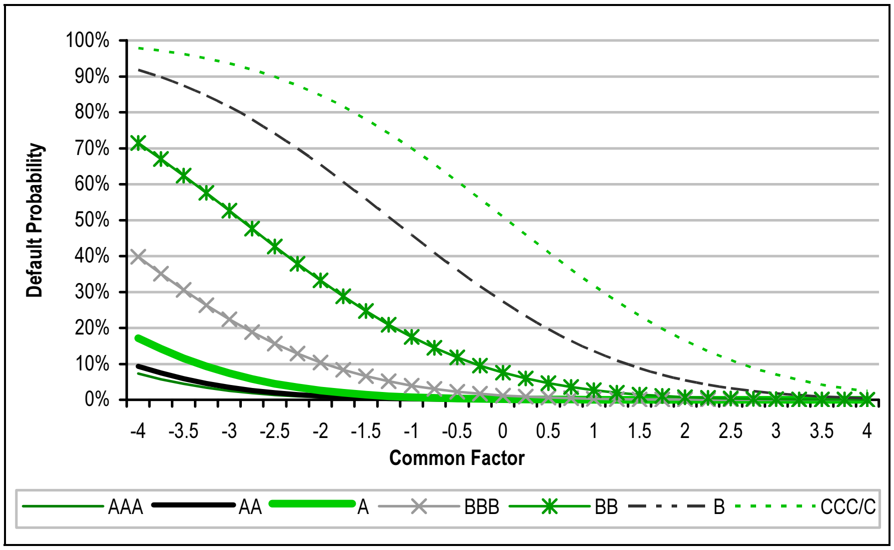
Ratings data can be used to compute the individual default probabilities…

With regard to formula (1), we observe that the default barrier used to compute the conditional default probability is a function of the individual default probability of the reference entity. The individual default probability is usually estimated either from **market observable information** (CDS spreads, bond prices or equity) or **historical default rates** (as provided by the main rating agencies).

Using the second approach, in Chart 79 we show the conditional default probabilities computed for different rating classes.

[36] Assume equal notional and recovery for each name in the portfolio.
[37] See Volume 2, Chapter 8.
[38] See Volume 2, Chapter 8.

Chart 79: Rating-Implied One-Factor Conditional Default Probability



Source: Merrill Lynch calculation on data provided in the S&P Annual Global Corporate Default Study, January 2005. Assume 20% correlation, and 5y time horizon.

..and then recompute the tranche spread

Once the individual conditional probabilities are available, the procedure to compute the tranche breakeven spread then proceeds as outlined in the previous section.

Incorporating Credit Migration

A further refinement to the rating based methodology described above consists of including the information embedded in the **ratings migration matrices** that are periodically published by the major rating agencies.

Transition matrices offer useful insights to analyze historical rating path of rated credits

A migration (or, equivalently, transition) matrix measures the likelihood of a generic credit "migrating" from a given rating class to another one. For example, as reported in **Error! Reference source not found.**, the probability of a generic AA corporate credit being downgraded one notch to AA- is 7.9%. Similarly the probability of a A+ credit remaining in the same rating category is 78.84%.

Chart 80: Average One-Year Transition Rates by Rating Modifier 1981 to 2004 (%)

From/To	AAA	AA+	AA	AA-	A+
AAA	87.44	4.04	2.66	0.66	0.17
AA+	2.22	77.46	11.04	3.36	0.47
AA	0.59	1.15	81.48	7.9	2.73
AA-	0.05	0.23	3.17	77.32	9.88
A+	0	0.03	0.6	3.95	78.84
A	0.05	0.09	0.43	0.71	4.82
A-	0.09	0.04	0.09	0.38	0.81
BBB+	0.02	0.06	0.04	0.12	0.4
BBB	0.02	0.02	0.1	0.07	0.27
BBB-	0.02	0	0.07	0.16	0.18
BB+	0.15	0.04	0	0.11	0.07

Source: S&P Annual Global Corporate Default Study, January 2005.

Several attempts have been made to include rating transition matrix into the valuation process of credit risky assets.

We propose a specific ad-hoc procedure that uses credit migration data to value single tranche synthetic CDOs. The suggested approach can be implemented using a three step procedure:

1. Compute the migration based portfolio probabilities

Let $p_i^{a,b}$ be the probability that credit *i* migrates from rating class *a* to *b.*

We define the **portfolio upgrade probability** p^+ as the normalized average of the upgrade probability for each individual credit, namely:

$$p^+ = \frac{\sum_{i=1}^{n}\sum_{j=a_i+1}^{AAA} p_i^{a_i,j}}{\sum_{i=1}^{n}\sum_{j=a_i+1}^{AAA} p_i^{a_i,j} + \sum_{i=1}^{n} p_i^{a_i,a_i} + \sum_{i=1}^{n}\sum_{j=a_i-1}^{D-1} p_i^{a_i,j}}$$

The portfolio no-change and downgrade probabilities can be equivalently defined as follows:

$$p^0 = \frac{\sum_{i=1}^{n} p_i^{a_i,a_i}}{\sum_{i=1}^{n}\sum_{j=a_i+1}^{AAA} p_i^{a_i,j} + \sum_{i=1}^{n} p_i^{a_i,a_i} + \sum_{i=1}^{n}\sum_{j=a_i-1}^{D-1} p_i^{a_i,j}}$$

$$p^- = \frac{\sum_{i=1}^{n}\sum_{j=a_i-1}^{D-1} p_i^{a_i,j}}{\sum_{i=1}^{n}\sum_{j=a_i+1}^{AAA} p_i^{a_i,j} + \sum_{i=1}^{n} p_i^{a_i,a_i} + \sum_{i=1}^{n}\sum_{j=a_i-1}^{D-1} p_i^{a_i,j}}$$

Intuitively we can think of p^+, p^0 and p^- as being the probability of 1) upgrade, 2) remaining in the same rating cluster, and 3) downgrade, relative to a single (synthetic) asset with an initial rating equal to the average rating of the portfolio.

2. Compute the migration based hazard rates

In the usual reduced form framework, the survival probability s_i is usually expressed as a function of a hazard function, *h,* defined as follows:

$$s_i(t) = \exp\left[-\int_0^t h_i(s)ds\right]$$

Given the evenly (usually yearly) distributed default rate statistics published by the major rating agencies, the expression above can be simplified to:

$$s_i(t) = \exp\left[-\sum_{k=1}^{t} h_i(k)\right]$$

Therefore, once the chosen default rate matrix is chosen, it is then possible to recover the implied hazard *h(t)* for a generic interval *(t-1,t)* via the following recursive relationship:

$$h_i(t) = -\log[s_i(t)] - \sum_{k=1}^{t-1} h(k) \qquad \textbf{(2)}$$

h(3)=-log(1-0.91%)-
-(0.28%+0.342%)=0.292%

Table 27: Calibrating Hazard Rate from Default Rate Data

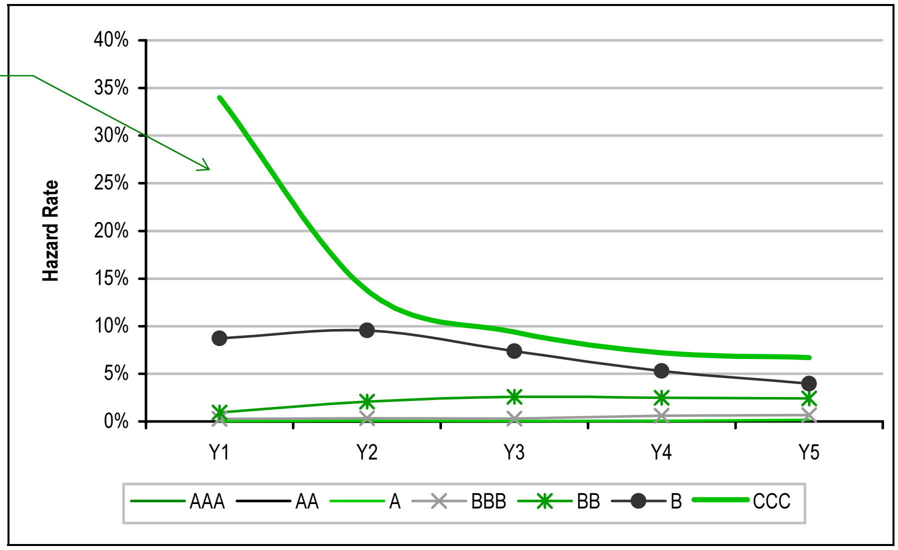
BBB Rating	Y1	Y2	Y3	Y4	Y5
Cumulative Default Rate	0.28%	0.62%	0.91%	1.52%	2.17%
Hazard Rate	0.280%	0.342%	0.292%	0.618%	0.662%

Source: Merrill Lynch calculations on S&P data from the Annual Global Corporate Default Study, January 2005

In Chart 81 we show the implied hazard rate function for different rating classes.

Speculative credits exhibit a downward sloped curve

Chart 81: Rating-Based Implied Hazard Function



Source: Merrill Lynch calculations on S&P data from the Annual Global Corporate Default Study, January 2005

In order to include the information coming from the transition matrix we introduce a **migration based hazard rate function** computed, under two different scenarios, as follows:

- **Upgrade**

The adjusted survival rate under the upgrade scenario is computed by taking the weighted average of the survival probability over all the possible upgrade migration states, i.e.:

$$s_i^+(t) = \frac{\sum_{r=a_i+1}^{AAA} \exp\left[-\sum_{k=1}^{t-1} h_i^+(k) - \underbrace{h_i^r(t)}_{\substack{\text{Hazard Rate for} \\ \text{a "r" rated credit}}}\right] \times \underbrace{p_i^{a_i,r}}_{\substack{\text{Migration probability} \\ \text{from rating "}a_i\text{" to "r"}}}}{\sum_{r=a_i+1}^{AAA} p_i^{a_i,r}}$$

Given the adjusted survival probability, we can then use formula **(2)** to back out the **implied upgrade migration based hazard rate** as follows:

$$h_i^+(t) = -\log\left[s_i^+(t)\right] - \sum_{k=1}^{t-1} h_i^+(k)$$

- **Downgrade**

The adjusted survival rate under the downgrade scenario is computed by taking the weighted average of the survival probability over all the possible downgrade migration states, i.e.:

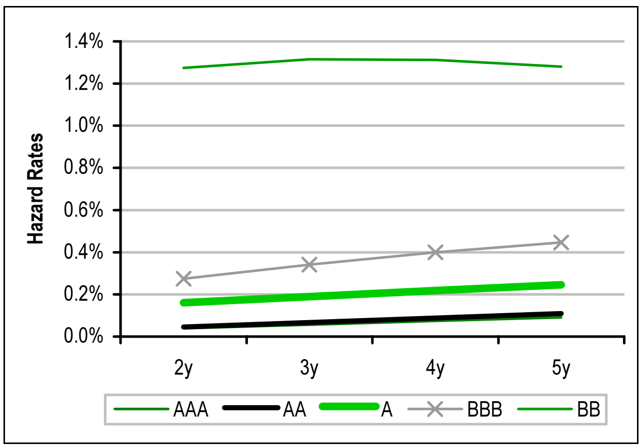
$$s_i^-(t) = \frac{\sum_{r=a_i-1}^{D-1} \exp\left[-\sum_{k=1}^{t-1} h_i^-(k) - \underbrace{h_i^r(t)}_{\substack{\text{Hazard Rate for} \\ \text{a "r" rated credit}}}\right] \times \underbrace{p_i^{a_i,r}}_{\substack{\text{Migration probability} \\ \text{from rating "}a_i\text{" to "r"}}}}{\sum_{r=a_i-1}^{D-1} p_i^{a_i,r}}$$

Given the adjusted survival probability, we can then use formula **(2)** to back out the **implied downgrade migration based hazard rate** as follows:

$$h_i^-(t) = -\log\left[s_i^-(t)\right] - \sum_{k=1}^{t-1} h_i^-(k)$$

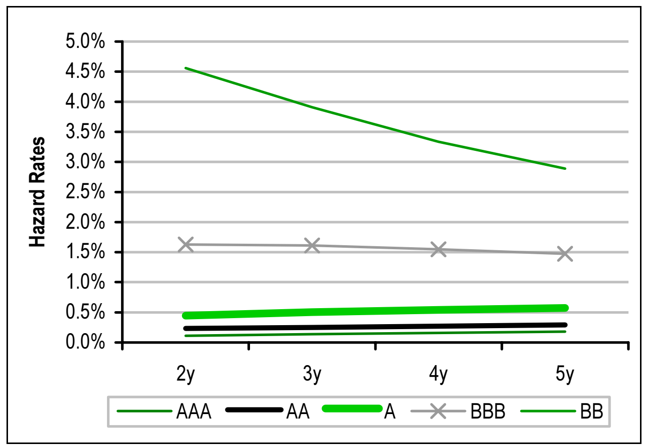
Chart 82 and Chart 83 show the implied migration based hazard rate function under the two different scenarios:

Chart 82: Implied Upgrade Migration based Hazard Rate



Source: Merrill Lynch calculations on S&P data from the Annual Global Corporate Default Study, January 2005

Chart 83: Implied Downgrade Migration based Hazard Rate



Source: Merrill Lynch calculations on S&P data from the Annual Global Corporate Default Study, January 2005

3. Compute the Migration Based Tranche Expect Loss

The tranche expected loss is mainly a function of:

1. Maturity
2. Attachment and Detachment Point
3. Individual Default Probability
4. Individual Recovery
5. Single Factor Correlation

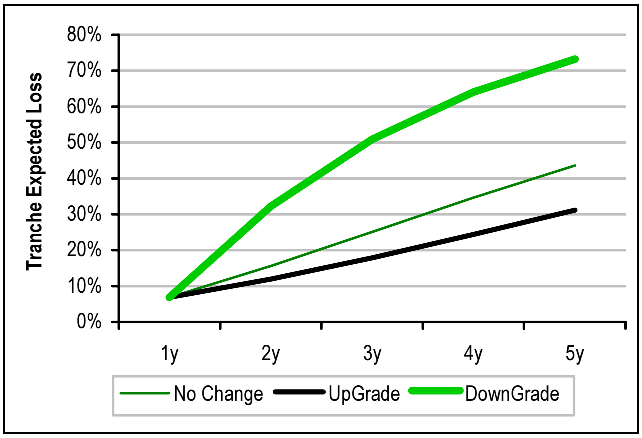
The migration based expected loss takes into account transition dynamics of each underlying credit

Given the three-dimensional hazard rate vector (upgrade, downgrade and no change) computed in the previous section, we then define the **Migration Based Tranche Expected Loss** as the weighted average of the upgrade, downgrade and no-change expected loss defined as follows:

$$EL(t) = \underbrace{EL^+(t)}_{\substack{\text{Expected Loss} \\ \text{using the Upgrade} \\ \text{Hazard Function}}} \cdot \underbrace{p^+}_{\substack{\text{Portfolio Upgrade} \\ \text{Probability}}} + EL^0(t) \cdot p^0 + EL^-(t) \cdot p^-$$

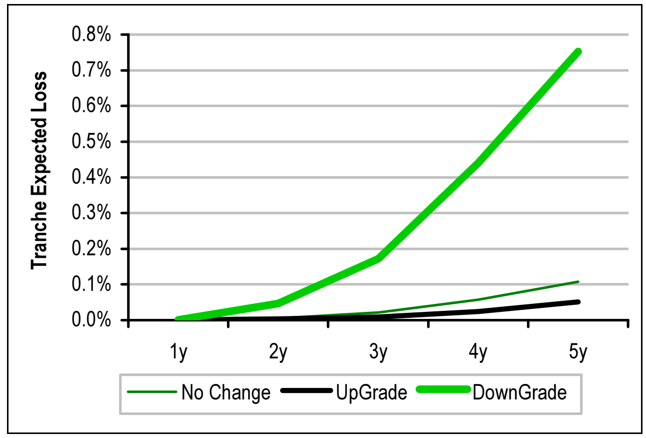
Chart 84 and Chart 85 show the term structure of expected losses for the 5y 0-3% and 15-30% CDX.NA.IG tranches under the three different states. Once the expected loss at each payment date is computed, we can then easily calculate the tranche spread as well as the other required sensitivity measures.

Chart 84: 0-3% Migration Based Expected Loss Term Structure



Source: Merrill Lynch calculations on S&P data from the Annual Global Corporate Default Study, January 2005. Assume 40% fixed recovery and 10% single factor correlation.

Chart 85: 15-30% Migration Based Expected Loss Term Structure



Source: Merrill Lynch calculations on S&P data from the Annual Global Corporate Default Study, January 2005. Assume 40% fixed recovery and 30% single factor correlation.

In Chart 86, Chart 87, Chart 88 and Chart 89 we exhibit the RLA indicators (market and rating implied) for the CDX IG and HY tranches computed **with** the information coming from the migration matrices[39].

Neither investment grade nor high yield standardized tranches seem to be affected by the inclusion of migration matrices.

However it is interesting to note how the upper rating implied threshold for equity tranche generally **shifts down** when transition matrices are included in the computation.

Among other factors, this can be explained by looking at the average historical migration behavior of a generic BBB credit[40]. The historical transition probability of being downgraded below BBB is 6.01% which is higher than the probability (4.35%) of remaining in the investment grade space (above BBB)[41]. Including this information in the valuation process for our specific example implies a reallocation of loss (or risk) from the junior tranches toward the more senior tranches, thus lowering the relative contribution of the junior tranches with respect to the overall portfolio risk.

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[39] Several authors have pointed out the importance of the preliminary adjustment of the transition matrix in order to "smooth" some of the inconsistencies (e.g. higher rated credits with default probability higher than lower rated ones, higher probability of migrating to more distant rating classes than closer classes) usually encountered in historical averages of migration data. For our purposes we used the regularization algorithm proposed by Jarrow-Lando-Turnbull (1997) which essentially splits the procedure into two steps:

1. find a suitable generator matrix **Q** from the historical matrix **M.** The entries q_{ij} of the matrix **Q** can be obtained using the following formula:

$$q_{ij} = \begin{cases} \log(m_{ij}) & \text{if } i = j \\ m_{ij} \log(m_{ij}) / (m_{ii} - 1) & \text{if } i \neq j \end{cases}$$

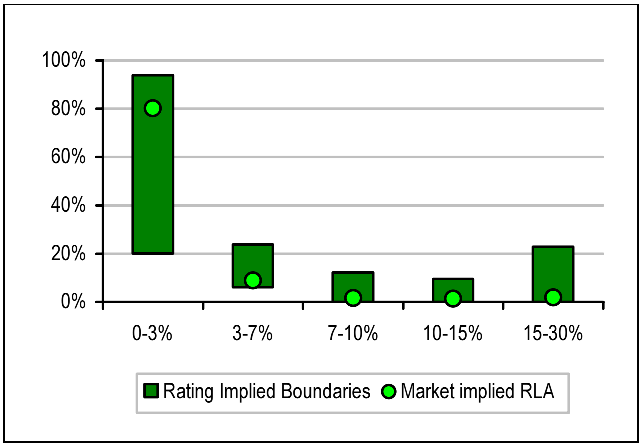
2. compute the adjusted migration matrix $\mathbf{M}^{\mathbf{adj}}$ via the following relationship: $\mathbf{M}^{\mathbf{adj}} = \exp(\mathbf{Q})$. Unfortunately Excel does not support the matrix exponential operator **exp**. However, both Matlab (function: `expm`) and Mathematica (function: `MatrixExp`) easily perform the required calculation.

[40] The WARF (computed using S&P data) of the CDX.NA.IG 4 Index is about 322 as of September, 14th. This corresponds to a weighted rating of about BBB (which represents about 55% of credits included in the CDX.NA.IG 4).

[41] Merrill Lynch calculations on S&P data from the Annual Global Corporate Default Study, January 2005

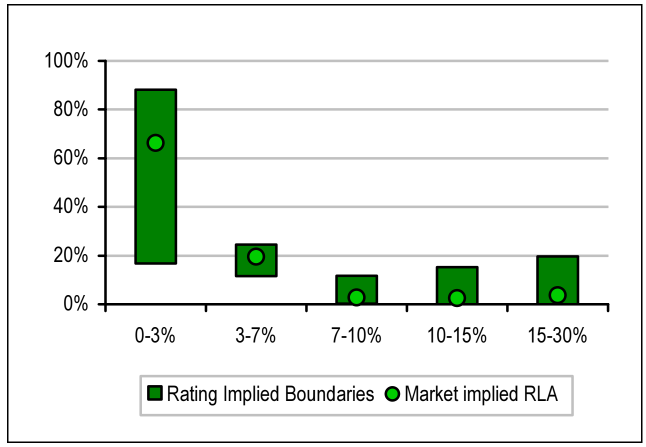
Refer to important disclosures on page 196.

Chart 86: CDX.NA.IG 5y Tranche Relative Loss Allocation



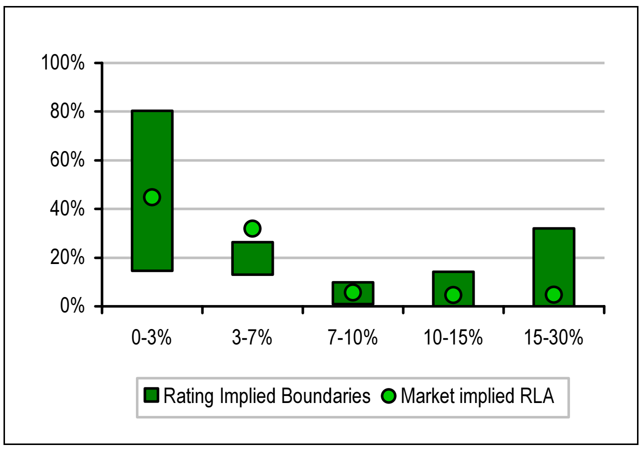
Source: Merrill Lynch calculation on S&P rating data as of Sep, 1st 2005.

Chart 87: CDX.NA.IG 7y Tranche Relative Loss Allocation



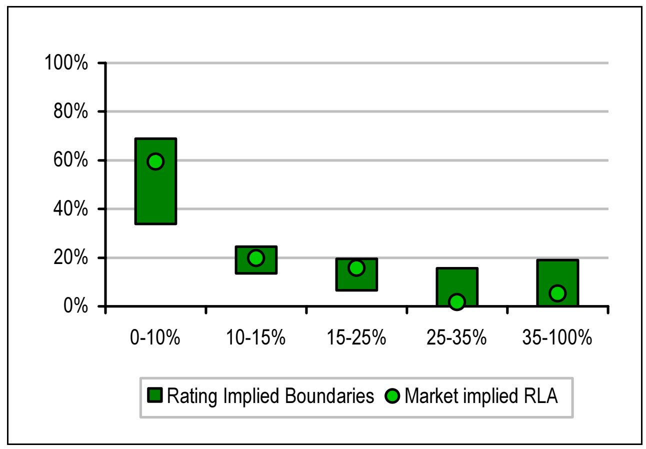
Source: Merrill Lynch calculation on S&P rating data as of Sep, 1st 2005.

Chart 88: CDX.NA.IG 10y Tranche Relative Loss Allocation



Source: Merrill Lynch calculation on S&P rating data as of Sep, 1st 2005.

Chart 89: CDX.NA.HY 5y Tranche Relative Loss Allocation



Source: Merrill Lynch calculation on S&P rating data as of Sep, 1st 2005.

7. Correlation Strategies

The liquid standardized tranche market can be used by investors to efficiently implement complex credit correlation positions. In this chapter we present the following five strategies extracted from our regular publications ("Protection Strategies"):

- **Carry vs. Correlation: Optimizing the Trade** (13th Sept 2004): illustrates the use of tranches to achieve desired risk-reward profile by combining different tranches of the same maturity.
- **Increasing Value in Super Senior** (27th July 2005): illustrates the use of Relative Loss Allocation to identify transfer of value in the capital structure.
- **Value in CDX 15-30% vs. Mezz/Senior** (6th July 2005): illustrates the risk-reward of Short Correlation Net Positive Carry trade.
- **10y vs. 5y Tranche Curve Flatteners** (8th February 2005): highlights risk-reward of curve trades in the tranche markets.
- **FTD Strategies Using Top 10 Credit Picks** (23rd January 2006): demonstrates the use of Merrill Lynch credit analyst recommendation to create FTD baskets. Also highlights how FTDs can be used potentially to hedge jump-to-default risk in equity tranches.

Carry vs. Correlation: Optimizing the Trade

This section is extracted from a "Protection Strategies" report published on 13th September 2004 by Kakodkar et al.

Market spreads have trended tighter over the summer with relatively low volatility. The CDX index is currently trading at a 4-month low of 56bps after hitting a peak of about 68bps in mid-May. The direction of spreads remains difficult to predict but there is no clear catalyst for spread widening. Overall, short of a catastrophic event, this is an environment where credit spreads should remain tight and spread volatility subdued. In this environment, we recommend trades with the following features:

Credit environment seems benign with no catalyst for spread widening

- Positive carry: to benefit from the tight spreads and low volatility;
- Long correlation: to benefit from credit spreads widening (or tightening) together to reflect increasing (or decreasing) macro credit risk.

Carry & Correlation Sweet Spot: Hedge CDX 0-3% with 7-10%

Trades with healthy positive carry most attractive

Benign credit fundamentals and tight spreads suggest that spreads will remain relatively unchanged over the next three months. In this environment, tranche trades that generate a healthy positive carry (to offset any time decay) would look most appealing. Over a three-month horizon we recommend the following trade:

- <u>**CDX IG**</u>: Sell 0-3% @ 38.5% upfront + 500bps running & buy 3x notional of 7-10% @ 110bps. Upfront plus positive carry of 170bps.

Hedge long equity with 7-10% in North America

In addition to the positive carry, this trade also benefits from slightly positive time decay over the next three months. This delta-hedged trade is also long correlation. However, in the event of spread widening, the gamma is less attractive than the CDX trade discussed below (long equity hedged with mezzanine). On the other hand, since time decay and carry work in favor of the trade during the next three months, any **gamma upside is self-financing** during this period.

Bullish Correlation? Hedge Equity with Mezzanine

Bullish correlation?

Given technicals in the tranche markets, we believe that hedging a long equity position with the mezzanine tranche is the most attractive way to set up a delta-hedged long correlation trade. Investors who are **bullish on correlation** should consider the following:

Hedge long equity with mezzanine tranches

- <u>**CDX IG**</u>: Sell 0-3% @ 38.5% upfront + 500bps running & buy 1.5x notional of 3-7% @ 269bps. Upfront plus positive carry of 97bps.
- <u>**iTraxx Europe**</u>: Sell 0-3% @ 20% upfront + 500bps running & buy 3.6x notional of 3-6% @ 110bps. Upfront plus positive carry of 104bps.

Refer to important disclosures on page 196.

Both are long correlation delta-hedged trades with a positive carry (in addition to the points upfront). The trade is long gamma, i.e. benefits from spreads widening (or tightening) together. These trades have the following **two key risks**:

But beware the risks!

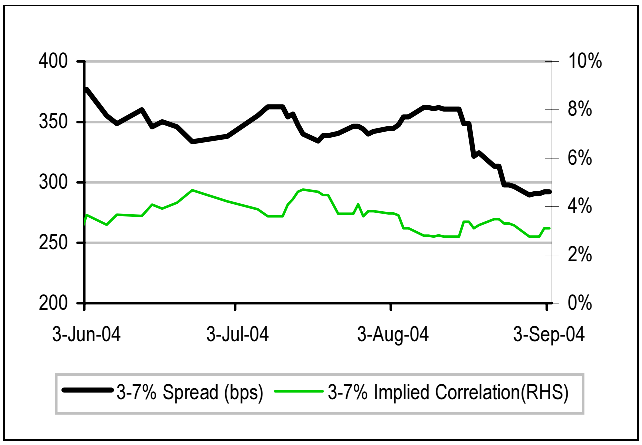
- If spreads remain unchanged over the next few months, the time decay of these trades could weigh heavily on the P&L of this trade.
- Even if all spreads widen out over the next three months, it is possible that additional issuance of bespoke would lead to further protection selling of the mezzanine tranches, thus lowering the net positive gamma. .

Tranche Technicals

Recent tightening of mezzanine tranche…

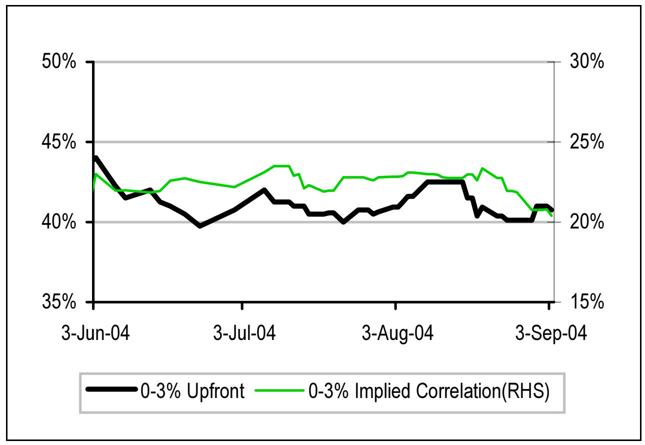
Chart 90 highlights the significant tightening in the 3-7% over the last few weeks (more than that expected due to the tightening of the underlying CDX index). A key reason for this tightening has been the increase in issuance of bespoke (or customized) single tranche synthetic CDOs. Dealers who bought protection on these bespoke tranches have used the 3-7% as a hedge (in addition to the CDX index as well as the individual single-name CDS) causing its spread to tighten significantly. We have seen similar activity in the European tranche market – the 3-6% mezzanine tranche (along with the iTraxx Europe index) has tightened considerably over the last few weeks.

Chart 90: CDX 3-7% Tranche Spreads & Implied Correlation



Source: Merrill Lynch; All mid levels

Chart 91: CDX 0-3% Tranche Spreads & Implied Correlation



Source: Merrill Lynch; All mid levels

…and decrease in equity implied correlation

Over the same period we also observe that there has been more protection buying of the equity tranche driving implied correlations down to lowest levels since the beginning of May (see Chart 91). In our opinion, this is primarily due to investors unwinding long equity positions following the tightening of the index in the last few weeks. We believe this is a good opportunity to be long correlation by selling 0-3% protection.

…is driven primarily by single-tranche issuance

While fundamentals remain unchanged, technicals could cause spreads to tighten further. Single-tranche synthetic CDO issuance has increased as **investors turn to structured products for additional yield** and could continue in a relatively benign credit environment. Unlike the 2003 boom in the bespoke tranche market, this time around we find that besides 5y tranches, investors are also looking to invest in 10y bespoke tranches. Our European strategists have highlighted this in a recent publication[42]. We believe similar technicals hold in the US.

Bullish Correlation? Mezzanine Offers Best Value

Long correlation trades would benefit if spreads move together

If credit spreads widen across the board due to global concerns (e.g. terrorism disruption) long correlation trades should benefit. Similarly they would also benefit from any market wide spread tightening. Given the historically tight credit

[42] See "European Credit Strategist" by Martin/Southgate, 25th August 2004.

markets, we believe that the **most likely catalyst for spread tightening would be a bid for more single tranche synthetic CDOs**.

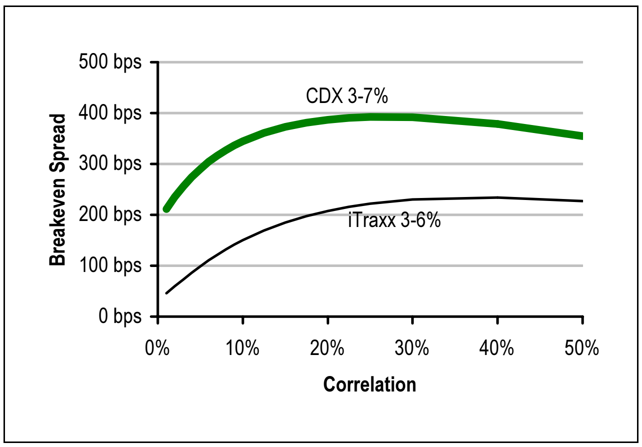
…but use the mezzanine tranche to delta hedge

Investors who are bullish correlation should consider the following two trades. :

- **CDX IG**: Sell 0-3% @ 38.5% upfront + 500bps running & buy 1.5x notional of 3-7% @ 269bps. Upfront plus positive carry of 97bps.
- **iTraxx Europe**: Sell 0-3% @ 20% upfront + 500bps running & buy 3.6x notional of 3-6% @ 110bps. Upfront plus positive carry of 104bps.

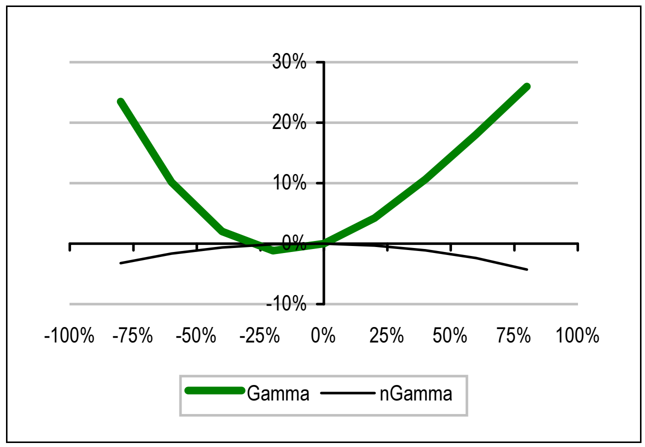
These are delta-hedged long correlation trades that generate a positive carry (in addition to the upfront receipts) **Buying mezzanine protection is also long correlation at the current level of spreads** (see Chart 92). The delta-hedged trades are **long Gamma** i.e. benefit when spreads move together. However, the trades are also **short nGamma** i.e. experience negative MTM when 50% of credits widen and the other 50% tighten. See Chart 93 for Gamma and nGamma profiles.

Chart 92: Mezzanine Tranche Spread vs. Correlation



Source: Merrill Lynch

Chart 93: Gamma & nGamma Profile for CDX Tranche Trade



Gamma is the MTM if all underlying credits index widen by same % amount (x-axis). nGamma is the MTM if 50% of underlying credits in the index widen by the same % amount (x-axis).and the other 50% tighten by that amount.
Source: Merrill Lynch

CDX Tranche Trade Analysis

CDX mezzanine more attractive than the index or senior tranches

Long correlation trades can also be established by using the index or any of the senior tranches to delta hedge the long equity position. However, investors who want to put on bullish correlation trades, the mezzanine (3-7%) is the most attractive hedging option. We analyzed **four hedging options** (index, 3-7%, 7-10% and 10-15%) based on the following **five metrics**:

(a) **Gamma**: the 3-7% option looked the most attractive especially in an environment when all credits widen together instantaneously (see Chart 94).

(b) **nGamma**: the index hedge looked most attractive followed by the 3-7% option. (see Chart 95). Note that the scale of losses is much smaller than the gain in the Gamma profile.

(c) **DTR** (default-to-recovery or instantaneous risk of default): Chart 96 highlights that following an instantaneous default, the 3-7% hedge is the option with the lowest negative MTM after a jump to default.

(d) **Time Decay**: If spreads remain unchanged, Chart 97 highlights that, over a three month period, the 3-7% has the highest time decay (MTM).

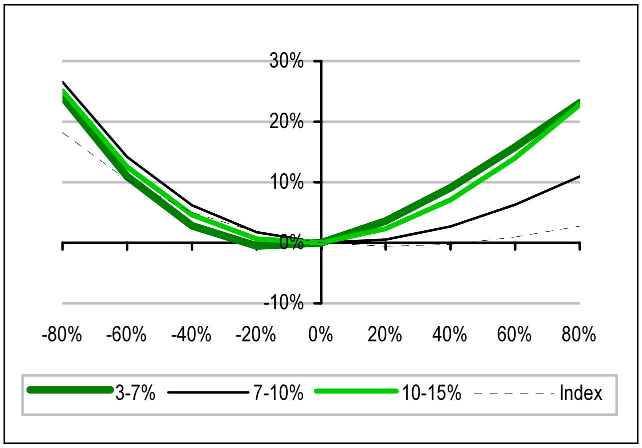
(e) **Carry**: The senior tranche options have the most attractive carry. However, all trades receive 38.5 points upfront. (see Table 28).

Table 28: Carry for Delta-hedged Long Correlation Trades

Long 0-3% Hedged with…	Spread (Implied Correlation)	Leverage	Notional/ Notional of 0-3%	Carry (ex Upfront)
3-7%	269 (4.1%)	8x	1.5x	97
7-10%	110 (19.4%)	4x	3x	170
10-15%	41.5(21.8%)	1.5x	8x	168
CDX IG Index	56.5	1x	12x	-178

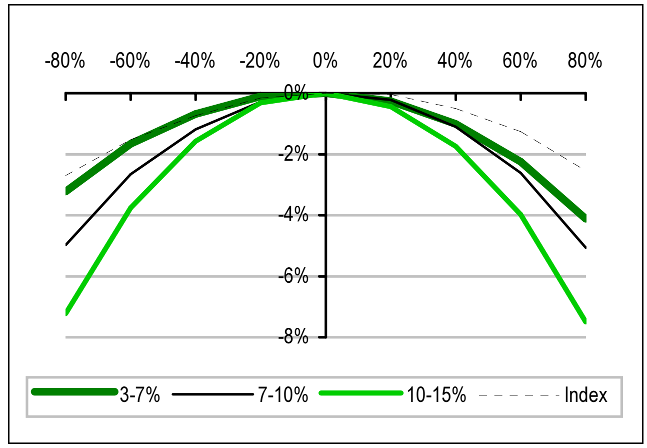
Leverage of 0-3% = 12x
Source: Merrill Lynch

Chart 94: Gamma Profile for Different Long Correlation Trades



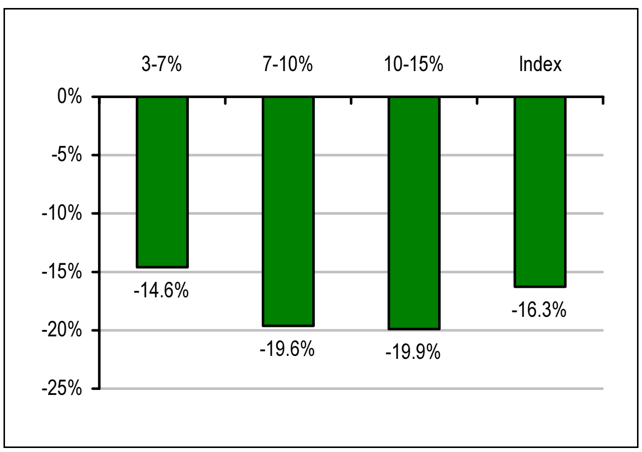
All trades involve selling 0-3% protection and buying protection on one of the above
Y-axis: % of notional of equity tranche
Source: Merrill Lynch

Chart 95: nGamma Profile for Different Long Correlation Trades



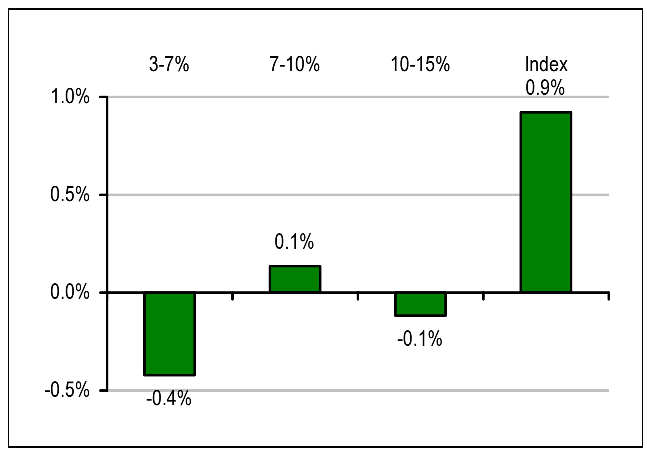
All trades involve selling 0-3% protection and buying protection on one of the above
Y-axis: % of notional of equity tranche
Source: Merrill Lynch

Chart 96: DTR for Different Correlation Trades



All trades involve selling 0-3% protection and buying protection on one of the above
Y-axis: % of notional of equity tranche
Source: Merrill Lynch

Chart 97: Time Decay after Three Months



All trades involve selling 0-3% protection and buying protection on one of the above
Y-axis: % of notional of equity tranche
Source: Merrill Lynch

iTraxx Europe Trade Analysis

iTraxx Europe mezzanine also the most attractive

We performed the same analysis for the European trade:

(a) **Gamma**: the 3-6% option looked the most attractive especially in an environment when all credits widen together instantaneously (see Chart 98).

(b) **nGamma**: the index hedge looked most attractive followed by the 3-6% option. (see Chart 99). However, the scale of losses is much smaller than the gain in the Gamma profile.

(c) **DTR** (default-to-recovery or instantaneous risk of default): Chart 100 highlights that following an instantaneous default, the 3-6% hedge is the option with the lowest negative MTM after a jump to default.

(d) **Time Decay**: If spreads remain unchanged, Chart 101 highlights that, over a three month period, the 3-6% has the highest time decay (MTM).

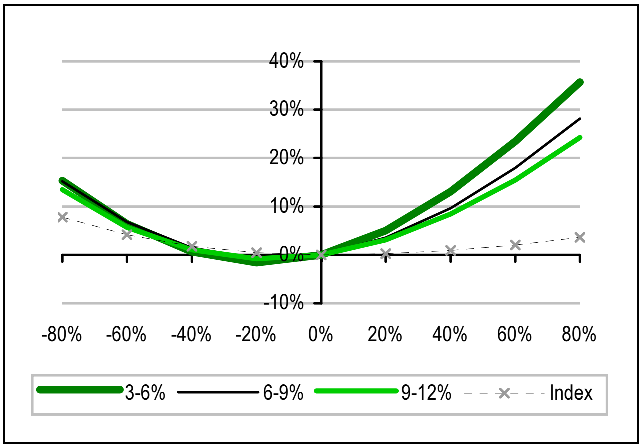
(e) **Carry**: The 6-9% tranche has the most attractive carry. However, all trades receive 20 points upfront. (see Table 29).

Table 29: Carry for Delta-hedged Long Correlation Trades

Long 0-3% Hedged with...	Spread (Implied Correla-tion)	Leve-rage	Notional/ Notional of 0-3%	Carry (ex Upfront)
3-6%	110 (4.9%)	5.2x	3.6x	104
6-9%	44 (14.6%)	2.2x	8.6x	122
9-12%	30 (22.9%)	1.4x	13.5x	95
ITraxx Index	34.25	1x	18.9x	-147

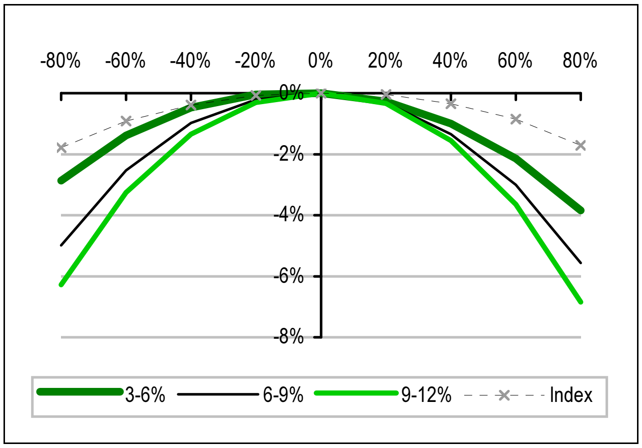
Leverage of 0-3% = 18.9x
Source: Merrill Lynch

Chart 98: Gamma Profile for Different Long Correlation Trades



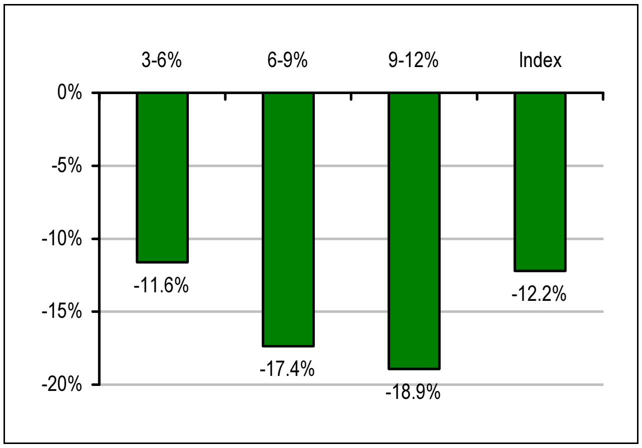
All trades involve selling 0-3% protection and buying protection on one of the above
Y-axis: % of notional of equity tranche
Source: Merrill Lynch

Chart 99: nGamma Profile for Different Long Correlation Trades



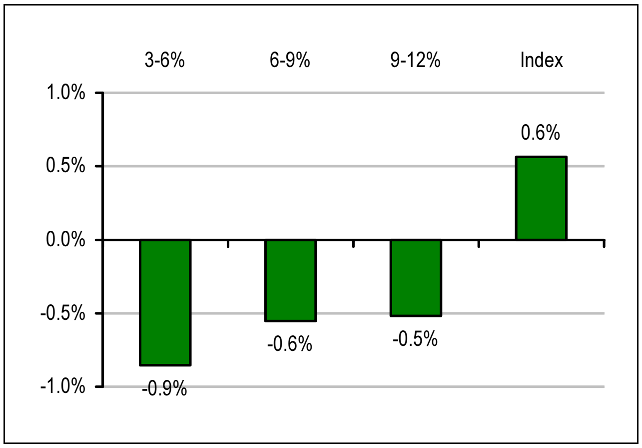
All trades involve selling 0-3% protection and buying protection on one of the above
Y-axis: % of notional of equity tranche
Source: Merrill Lynch

Chart 100: DTR for Different Correlation Trades



All trades involve selling 0-3% protection and buying protection on one of the above
Y-axis: % of notional of equity tranche
Source: Merrill Lynch

Chart 101: Time Decay after Three Months



All trades involve selling 0-3% protection and buying protection on one of the above
Y-axis: % of notional of equity tranche
Source: Merrill Lynch

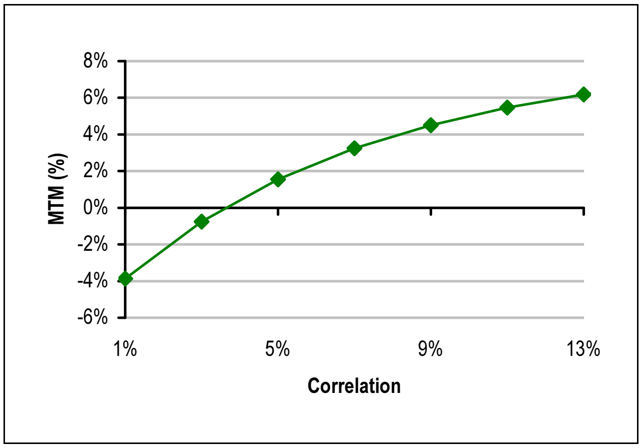
Key Risks

But beware the risks

Based on the above observation, correlation bulls would prefer to hedge long equity with a mezzanine. This trade has the following **key risks**:

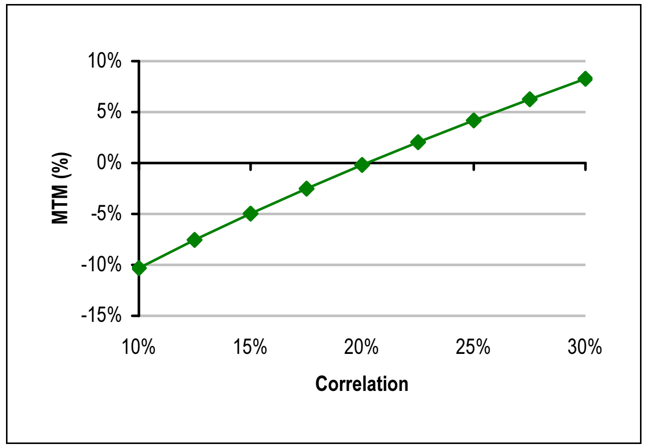
- If spreads remain unchanged over the next three months, the **time decay** on either of these trades will eat heavily into the P&L of the trade (Table 30).
- If spreads widen out, there could be **additional protection selling of the mezzanine** due to the issuance of more bespoke single-tranches (see Chart 102 for 3-7% MTM). This would lower any benefit from being long gamma.
- Implied correlations of the equity tranche may continue its downward trend if **more long equity positions are unwound** (see Chart 103 for 0-3% MTM).
- **Instantaneous default risk** is also a key risk. Long correlation trades have a positive carry to compensate for the large instantaneous default risk. Amongst the four hedging options, the mezzanine hedge generates the lowest DTR.

Chart 102: MTM of Short 3-7% (CDX) Leg vs. Correlation



Assume index level is fixed.
Source: Merrill Lynch

Chart 103: MTM of Long 0-3% (CDX) Leg vs. Correlation



Assume index level is fixed.
Source: Merrill Lynch

In Table 30 we highlight different scenarios for the European and North American trades.

Table 30: Scenario Analysis for <u>3-Month Horizon</u> Excluding the Carry

ITraxx Trade (Carry = 104bps per annum)			CDX Trade (Carry = 97bps per annum)		
0-3% MTM	**3-6% MTM**	**Net MTM***	**0-3% MTM**	**3-7% MTM**	**Net MTM****
Spreads and Correlation Unchanged			**Spreads and Correlation Unchanged**		
0.56%	-0.39%	-0.85%	0.92%	-0.90%	-0.42%
Technicals on 3-6% (correlation to 4%, breakeven spread of 78bps)			**Technicals on 3-7% (correlation to 3.5%, breakeven at 244bps)**		
0.56%	-0.88%	-2.62%	0.92%	-1.37%	-1.14%
Technicals on the 0-3% (correlation to 19.5%)			**Technicals on the 0-3% (correlation to 19%)**		
-0.14%	-0.39%	-1.55%	-0.04%	-0.90%	-1.38%
Correlation Increases: 6% mezzanine and 22.5% equity			**Correlation Increases: 6% mezzanine and 22% equity.**		
1.60%	0.18%	2.24%	2.54%	0.39%	3.13%
Index Moves to 40, correlation unchanged			**Index Moves to 65, correlation unchanged**		
-4.25%	1.80%	2.22%	-4.19%	3.76%	1.45%
Index Moves to 25bps, correlation unchanged			**Index moves to 45, correlation unchanged**		
9.47%	-2.84%	0.95%	8.92	-6.19%	-0.36%

Source: Merrill Lynch; **Net MTM expressed as a percent of equity notional; * Net MTM = 0-3%MTM + 3.6 * 3-6% MTM; ** Net MTM = 0-3%MTM + 1.5 * 3-7% MTM**

■ Positive Carry: Hedge CDX 0-3% with 7-10%

No clear catalyst for spread widening

Table 30 clearly highlights that in an environment where spreads remain unchanged over the next three months, the time decay of the long correlation trades described above would eat heavily into the P&L of the trade. In our opinion, there is **no clear catalyst for spread widening during this period** making this scenario quite likely**.** Therefore, in this environment, trades that have a healthy positive carry (that offset any time decay) would look most appealing.

Prefer healthy positive carry to offset any time decay

Over the next three months we recommend the following trade:

- <u>**CDX IG**</u>: Sell 0-3% @ 38.5% upfront + 500bps running & buy 3x notional of 7-10% @ 110bps. Upfront plus positive carry of 170bps.

Hedging CDX equity with 7-10% most appealing

Table 28 and Chart 97 highlight that this trade has the **most appealing carry and time decay** among the four options. While the carry is the most positive, the time decay is slightly positive over the next three months. In the event that spreads remain unchanged over this period, this trade has a positive return unlike the trades where the mezzanine is used as a hedge.

An additional upside is that this trade is also long correlation and would benefit from any spread widening over the next three months. The gamma profile of this trade (Chart 94) indicates that it is less attractive than the prior CDX trade (long equity hedged with 3-7%). However, it is worth noting that any **gamma upside** over the next three months is self-financing due to the positive carry and time decay.

Table 31 highlights the net MTM of the trade under different scenarios. We highlight the following:

- If spreads are unchanged over the next three months, this trade returns +0.56% net of the carry compared to -0.18% using the 3-7% hedge.
- If the index widens out to 65bps (15% increase) over the next three months, the MTM is +0.41% (net of carry) compared to +1.69% (net of carry) for the 3-7% hedge.
- If the index tightens to 45bps (20% decrease) over the next three months, the MTM is +2.74% (net of carry) compared to -0.12% (net of carry) for the 3-7% hedge.

We believe this trade is clearly attractive for investors who are looking to take advantage of the current environment of tight spreads while at the same time benefiting from any upside due to spread widening or tightening.

Table 31: Scenario Analysis for 3-Month Horizon - CDX Equity Hedged with 7-10% (Excludes the Carry)

CDX Trade (Carry = 170bps per annum)		
0-3% MTM	**7-10% MTM**	**Net MTM***
Spreads and Correlation Unchanged		
0.92%	-0.26%	0.14%
Technicals on the 0-3% (correlation to 19%)		
-0.04%	-0.26%	-0.83%
Correlation Increases: 21% 7-10% and 22% equity.		
2.54%	0.12%	2.20%
Index moves to 65, correlation unchanged		
-4.19%	1.39%	-0.02%
Index moves to 45, correlation unchanged		
8.92%	-2.20%	2.31%

*** Net MTM expressed as a percent of equity notional; Net MTM = 0-3%MTM + 3 * 7-10% MTM;**
Source: Merrill Lynch

Increasing Value in Super Senior (CDX 30-100%)

This section is extracted from a "Protection Strategies" report published on 27th July 2005 by Kakodkar et al.

Over the last twelve weeks, we have observed a significant shift in value out of the mezzanine and into the junior and super-senior tranches. CDX 30-100% super senior is at relative wides. Investors can sell protection on CDX index and buy protection on 0-30% to effectively take exposure at the following **implied** levels:

- **5y 30-100% @ 6.6 bps** (implied)
- **7y 30-100% @ 12.7 bps (implied)**
- **10y 30-100% @ 14.9 bps** (implied)

Since CDX 30-100% is relatively illiquid on an outright basis, investors can capture value more efficiently via replication. The risks associated with this strategy are different compared to risk of selling super-senior protection outright.

■ The Need to Hedge: Thinner Belly & Fatter Wings

Dealers rush to hedge bespokes…

Volatility in the correlation market in May led to significant underperformance of long correlation trades particularly in the 5y space. Idiosyncratic spread widening by GM and F precipitated two events:

- A "rush to hedge" by dealers holding (correlation) unhedged bespoke positions at the mezz/senior subordination level (long correlation positions).

Refer to important disclosures on page 196.

- A “rush to unwind” long correlation trades by hedge funds and prop desks (they had bought equity protection and sold mezzanine protection).

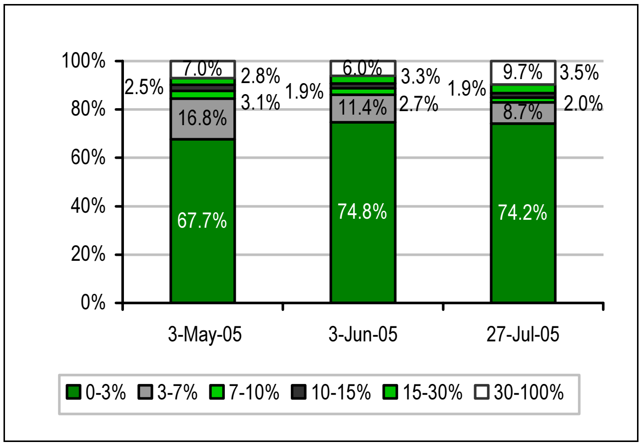
…by selling mezz or buying equity/super-senior

We believe the **“rush to hedge” was the stronger force** in the market. The main outcome of the hedging activity was:

- Protection selling pressure on the “belly”, i.e. mezzanine tranches
- Protection buying pressure on the “wings”, i.e. equity and super senior tranches in order to fill up the capital structure (and hedge correlation risk)

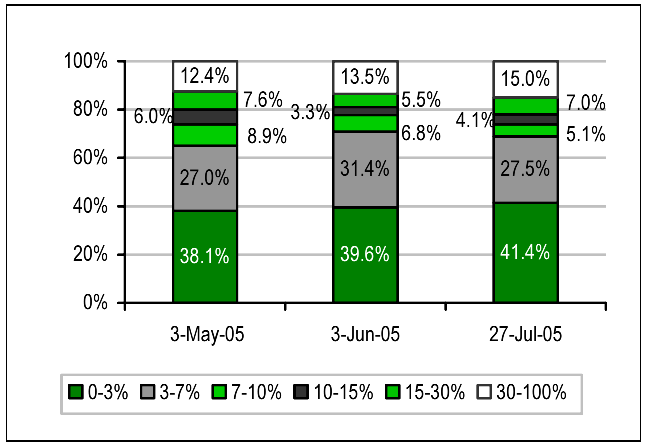
Hedging of bespoke tranches by dealers makes them better buyers of equity/super-senior protection and better sellers of mezzanine protection. This has been a key reason for the transfer of value out of the tranche belly and into the tranche wings. There is a risk that more hedging activity continues to drive value into the wings.

Chart 104: Expected Loss Allocation for 5y CDX IG4 Tranches



Source: Merrill Lynch

Chart 105: Expected Loss Allocation for 10y CDX IG4 Tranches



Source: Merrill Lynch

Value has transferred out of tranche belly into the wings

Chart 104 and Chart 105 illustrate this shift in value over the last twelve weeks for 5y and 10y CDX IG4 tranches. In the initial phase (during May):

- 5y value was transferred mainly out of 3-7% and 7-10% into the 0-3%.
- 10y value was transferred out of senior tranches mainly into the 3-7% (used as a proxy for 5y equity to hedge bespokes).

In the second phase (June and July):

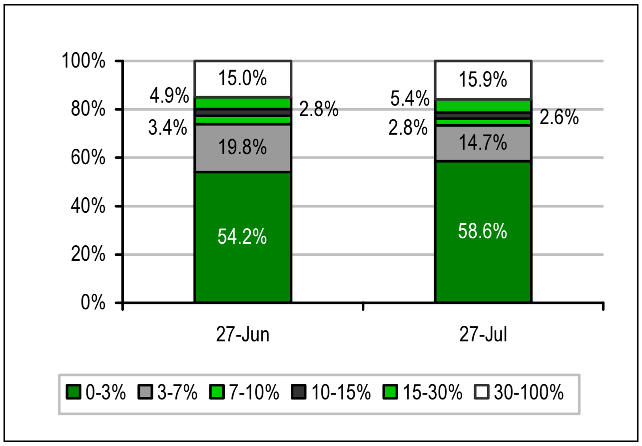
- 5y value continued to move out of 3-7% but this time it was transferred into the super-senior tranches particularly the 30-100%.
- 10y value has shifted out of 3-7% and 7-10% and into the wings, i.e. 0-3% and 10-100%.

7y has been very active with similar technicals

We have recently observed **significant activity in the 7y tranches** which are trading with an increasing amount of liquidity. Similar technical forces are at work in the 7y market, which, in our opinion, is probably a better hedge for most bespoke maturities. Over the last month, 7y CDX tranches have also experienced protection buying pressure on the wings and protection selling pressure on the belly (see Chart 106).

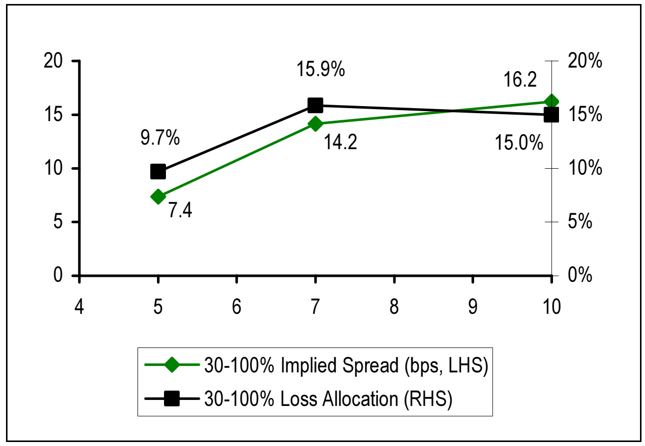
Chart 107 highlights current implied 30-100% spreads (mid) and the corresponding loss allocation for this tranche for the three liquid maturities. The implied correlation for the 7y 30-100% is higher than that for the 5y and 10y (which are trading at similar implied correlations). This explains why the 7y 30-100% may look relatively more attractive than the other two maturities.

Chart 106: Expected Loss Allocation for 7y CDX IG4 Tranches



Source: Merrill Lynch

Chart 107: Current Implied 30-100% (Mid) vs. Maturity

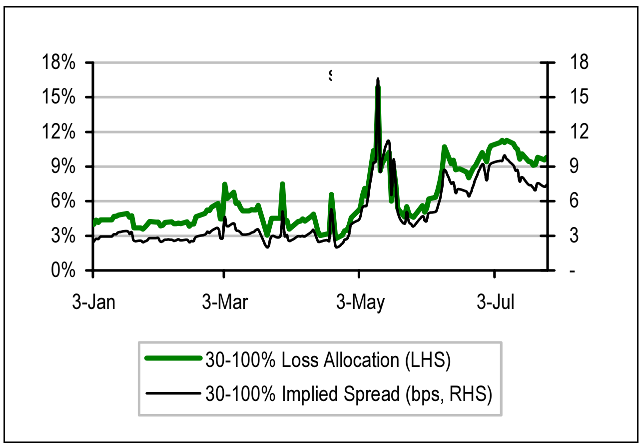


Source: Merrill Lynch

Implied super senior spreads are at relative wides

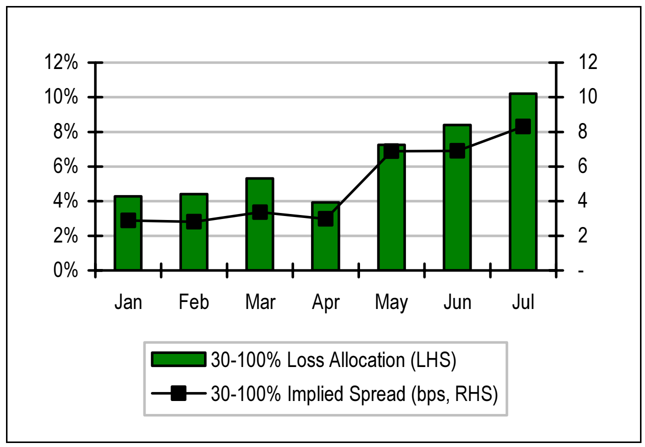
Chart 108 to Chart 111 highlight CDX 30-100% implied spreads during 2005. Implied super senior spreads peaked in mid-May during the correlation shakeout. However, 5y and 10y 30-100% are currently at relative wides. 5y average monthly implied spread is at its highest level in 2005 whereas the 10y peaked in June. We cannot show similar charts for 7y super-senior due to a lack of sufficient data.

Chart 108: Daily Implied Levels for 5y CDX 30-100%



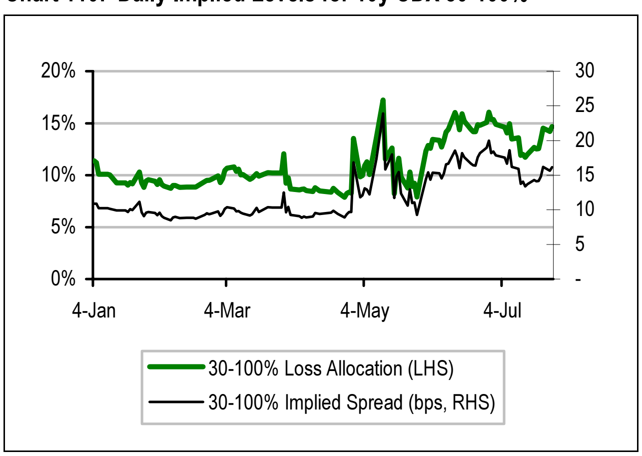
Source: Merrill Lynch

Chart 109: Monthly Average Implied Levels for 5y CDX 30-100%



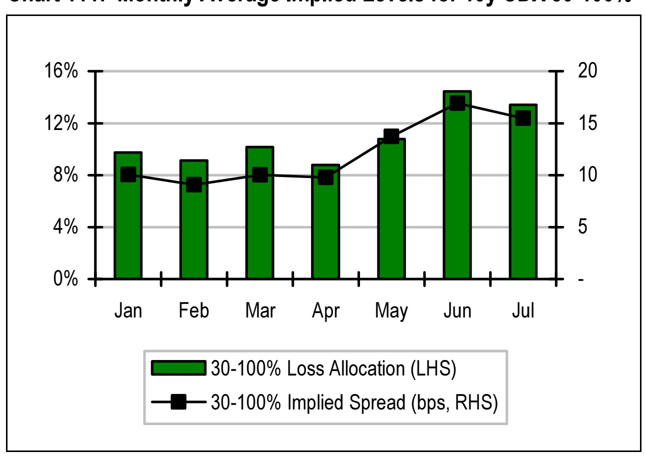
Source: Merrill Lynch

Chart 110: Daily Implied Levels for 10y CDX 30-100%



Source: Merrill Lynch

Chart 111: Monthly Average Implied Levels for 10y CDX 30-100%



Source: Merrill Lynch

Risk & Reward

Exposure to the CDX 30-100% super senior tranche can essentially be taken via two different strategies:

- Replication: Selling the CDX index and buying 0-30% (0.3x index notional)
- Outright: Selling the 30-100% CDX tranche (0.7x index notional)

Despite the apparent similarity, these two strategies have different risks related to jump-to-default, sensitivity to spread movements and time decay. In our analysis, we assume both strategies are done at the same spread (unlikely in practice).

Low jump-to-default risk

Jump-to-Default

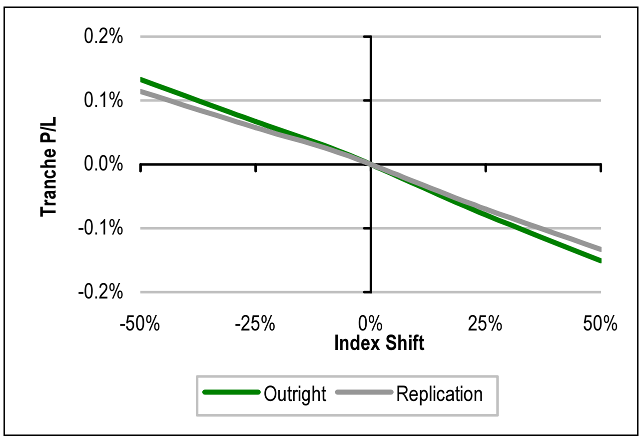
Given the high level of subordination and tranche width, the 30-100% tranche is not significantly affected by an instantaneous default of a credit in the underlying index. This is true for both the outright and the replication strategy.

Gamma

Replicating strategy less exposed to spread volatility…

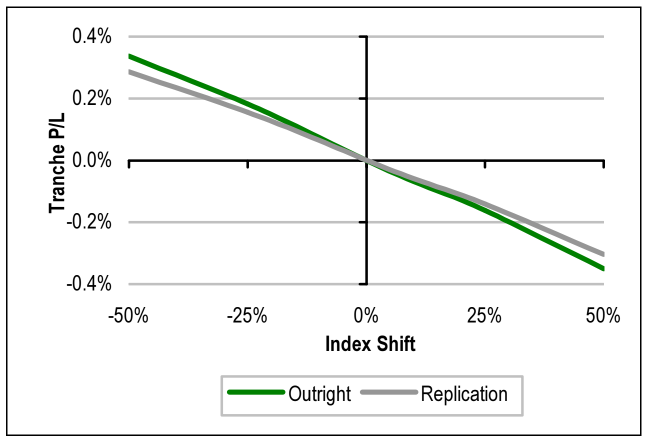
The high attachment point and width of the tranche imply a relatively low exposure sto parallel spreads movement of the underlying portfolio of credits. Chart 112, Chart 113 and Chart 114 highlight the higher sensitivity[43] of the 30-100% outright exposure when compared to the equivalent exposure obtained via replication.

Chart 112: Gamma Profile for 5y CDX 30-100% Tranche



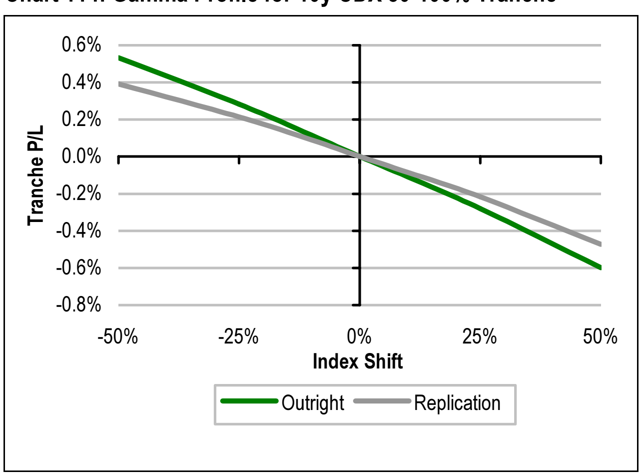
Source: Merrill Lynch

Chart 113: Gamma Profile for 7y CDX 30-100% Tranche



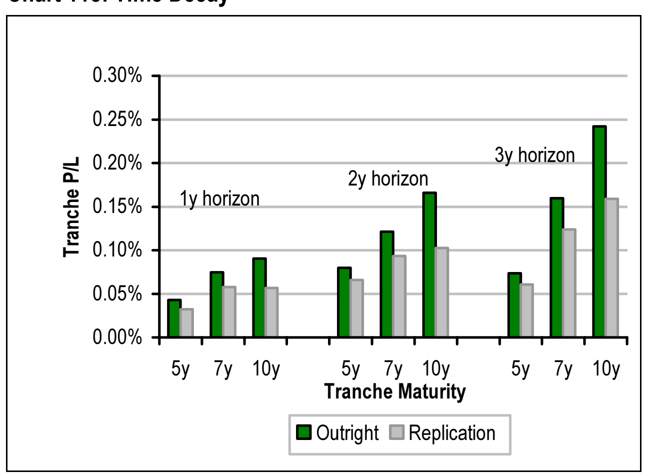
Source: Merrill Lynch

Chart 114: Gamma Profile for 10y CDX 30-100% Tranche



Source: Merrill Lynch

Chart 115: Time Decay



Source: Merrill Lynch

[43] If spreads widen, senior tranches usually exhibit an increasing credit spread delta. See Volume 2, Chapter 5.

Time decay (Theta)

…but benefits less from time decay

Chart 114 shows the positive theta for the 30-100% tranche for different maturities. Compared to the replicating strategy, the outright position has a more attractive theta. Short equity tranches generally have the highest negative theta in the capital structure. The inclusion of a short equity tranche in the replicating strategy is the key reason for its less attractive time decay profile.

Liquidity

Bid-ask spread can jeopardize the profitability of the trade

The 30-100% is relatively illiquid on an outright basis. Unexpected market tensions or index rolls can further reduce the liquidity of this tranche. A lack of liquidity would mean a higher bid-ask spread which could result in a potential loss for an investor looking to unwind the trade.

Running vs. Upfront

Prefer to pay running

The 0-3% tranche is typically traded on an upfront basis which implies that the 0-30% may also be traded on an upfront basis. A replication strategy would require the investor to pay a considerable amount of capital upfront to buy equity protection. Investors would, however, prefer to pay on a running basis. Dealers are increasingly making more markets in equity tranches on a running basis.

Value in CDX 15-30% vs. Mezz/Senior

This section is extracted from a "Protection Strategies" report published on 6th July 2005 by Kakodkar et al.

CDX 15-30% delta-hedged with the mezzanine or senior tranche offers attractive value. This trade usually has a negative carry but can currently be established at a positive carry (see Chart 95). We believe this mispricing is due to dealers buying protection on the "wings", i.e. equity or super senior tranches, to hedge their short bespoke mezz tranche positions.

In a market characterized by technicals, the biggest risk in this trade is that the empirical deltas are quite different from the model deltas as we have observed over the last few months.

Super senior vs. mezz/senior looks attractive

Table 32: Trade Description: Hedge Ratios and Carry

Buy 5y Protection			Sell 5y Protection			
Tranche	Offer (bps)	Notional (mn)	Tranche	Bid (bps)	Notional (mn)	Carry (bps)
CDX IG4						
3-7%	138	1x	15-30%	14.5	15x	79.5
7-10%	43	1x	15-30%	14.5	5x	29.5
10-15%	26	1x	15-30%	14.5	2.5x	10.25

Source: Merrill Lynch

■ Risk & Reward

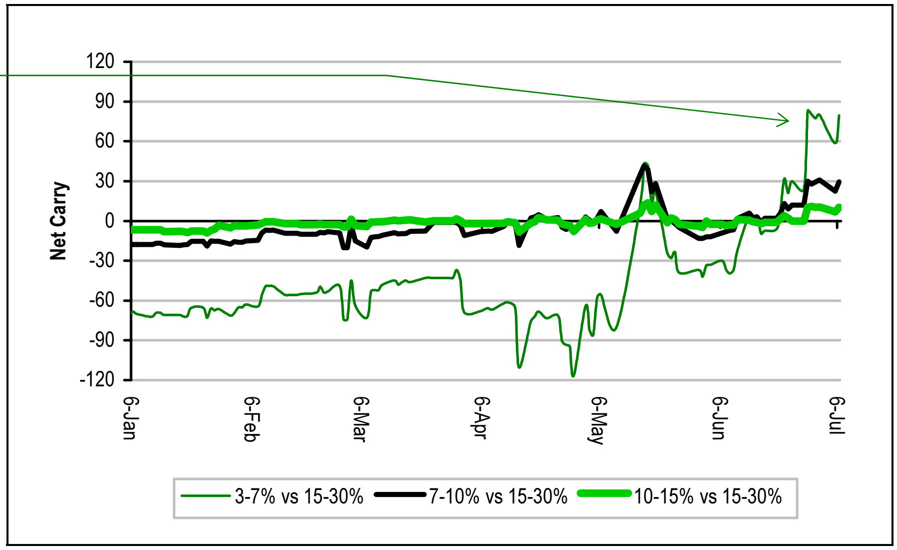
Positive carry, long DTR, long to flat gamma

All of the above delta-hedged trades have a **positive carry**. These trades also benefit upon credit default (**long DTR**) and are generally **flat or long gamma** (positive convexity), i.e. benefit when all spreads move wider or tighter together. They also offer reasonable protection against idiosyncratic events, i.e. limited losses when volatility affects only a subset of credits. The biggest risk is that the **realized deltas** differ from the theoretical deltas used in the trades above.

Positive carry: The carry for each of the above trades has historically been negative (see Chart 116) except post the recent shakeout in the tranche market. The carry for all these delta-hedged trades is now distinctly positive and seems to be mispriced once again. We believe this is due to dealers buying protection on 15-30% to hedge their bespoke mezz tranche positions.

Positive carry in May quickly disappeared but has now reappeared.

Chart 116: Carry of Delta-Hedged Trades in 5y CDX IG4 Tranches

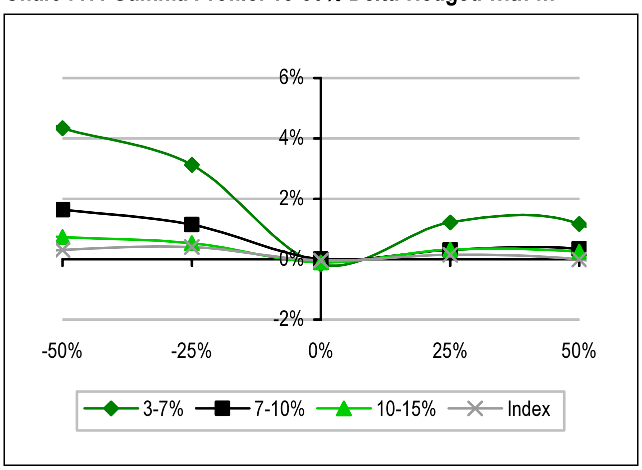


Source: Merrill Lynch

Gamma/Convexity: The **gamma** profile for these trades is generally positive (see Chart 117). From a correlation perspective, a long correlation leg (short mezz/senior) counters the short correlation leg (long 15-30%). The trade has a very limited exposure to **time decay**, i.e. the mark-to-market change due to the curve roll down is negligible.

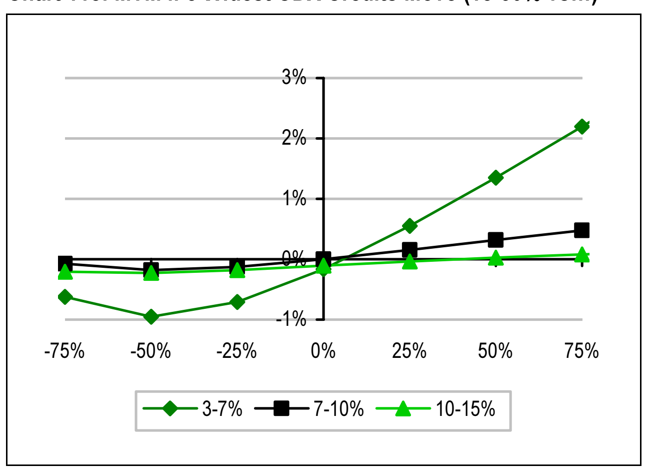
Chart 118 captures the exposure of the trade **to idiosyncratic risk**. It shows the net MTM of the trades following a spread shock of the 5 widest credits in the CDX index. We observe that the exposure to idiosyncratic risk is relatively limited. We believe this is an important property of the trade, especially after the recent turmoil that affected the correlation market.

Chart 117: Gamma Profile: 15-30% Delta-Hedged with …



* We have assumed a 3-month time horizon; Analysis includes carry
Source: Merrill Lynch

Chart 118: MTM if 5 Widest CDX Credits Move (15-30% vs…)*



* GMAC, FMCC, MYG, LEA & AXL. Analysis includes carry and assumes 3-month horizon.
Source: Merrill Lynch

Risk lies in realized deltas being different from theoretical

Theoretical vs. Realized Deltas: Divergence between theoretical and realized deltas is a key risk in each of these delta-hedged trades. In the last few months, the empirical (or realized) deltas have diverged significantly from the theoretical values particularly for the 3-7% and 7-10% tranches (see Table 33).

A key reason for this recent divergence has been technical flows due to the unwind of long correlation trades, i.e. there was protection buying pressure on the equity tranche and protection selling pressure on the mezz/senior tranches. In April and May the MTM on long correlation trades was as bad as -12%[44].

5y CDX IG widened only 2bps in June. 5y 15-30% was expected to widen only 1bp but widened 4bps instead (all levels mid-to-mid).

Table 33: Model vs. Realized Deltas for 5y CDX Tranches in 2005

	3-7%		7-10%		10-15%		15-30%	
Month	Model	Empirical	Model	Empirical	Model	Empirical	Model	Empirical
January	6.7x	7.2x	2.7x	2.6x	1.0x	1.4x	0.4x	0.4x
February	6.8x	10.6x	2.8x	3.8x	1.1x	0.8x	0.4x	0.3x
March	6.4x	5.5x	2.6x	0.8x	1.1x	0.6x	0.4x	0.4x
April	6.8x	4.4x	2.8x	-0.7x	1.3x	0.3x	0.5x	0.1x
May	7.0x	69.0x	2.5x	7.1x	1.2x	4.6x	0.4x	-0.4x
June	6.5x	-18.9x	2.0x	-6.3x	1.0x	0.3x	**0.5x**	**2.0x**

Source: Merrill Lynch

10y vs. 5y Tranche Curve Flatteners

This section is extracted from a "Protection Strategies" report published on 8th February 2005 by Kakodkar et al.

An interesting phenomenon over the last few months has been the steepening of the 10y-5y slope of the underlying CDX IG index (see Chart 119). Since mid 2004 the 5y index has tightened faster than the 10y index. A key reason has been the technical tightening due to the synthetic CDO bid (more demand for 5y vs. 10y). As spread volatility and the perceived risk of jump-to-defaults remains low we believe demand for 10y risk relative to 5y will continue to rise at least over the next three months.

10y-5y curve steepening could attract 10y synthetic CDO bid

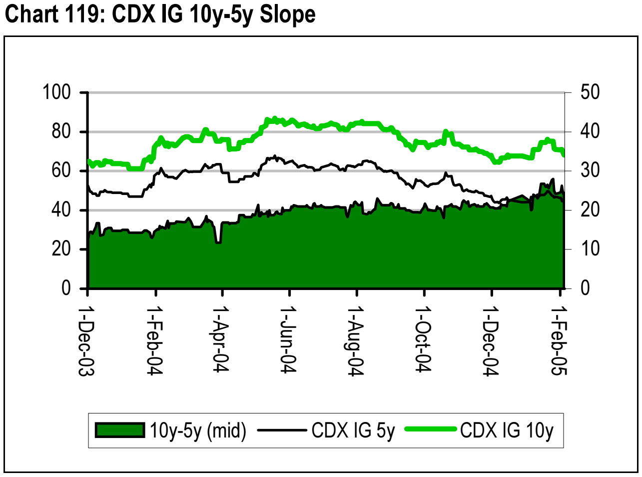
In our opinion, there is no clear catalyst for spread widening during this period. However, any sporadic curve steepening in the short-term could be accompanied by a flurry of activity in the 10y single-tranche synthetic CDO (STCDO) market. Recent widening of the CDX index in mid January was followed by STCDO issuance and the consequent tightening of index and senior tranche spreads. In addition, the 10y-5y slope of the tranches also steepened since end 2004 (Chart 120).

Curve flatteners look attractive

Given the CDO technicals and movement of tranche spreads, investors may be inclined to set up curve flatteners via the index tranche market. In particular, we believe that the following two trades look relatively attractive:

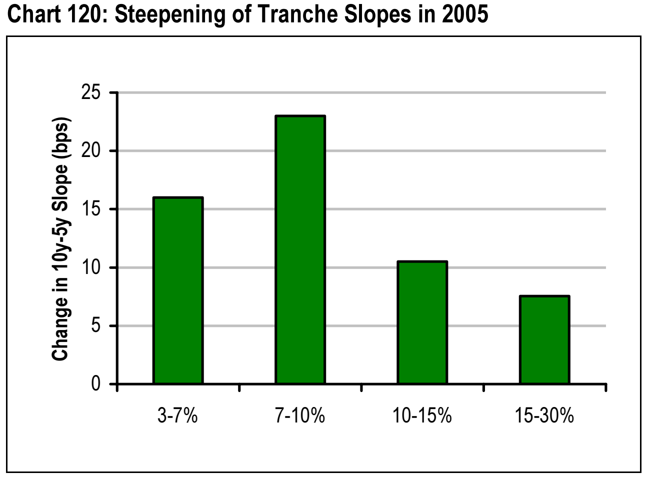
- **Sell 10y 3-7% / Buy 5y 3-7%**
- **Sell 10y 3-7% / Buy 5y 7-10%**

Chart 119: CDX IG 10y-5y Slope



Source: Merrill Lynch

Chart 120: Steepening of Tranche Slopes in 2005



Equity slope increased by 2.125% (upfront) over the same period.
Source: Merrill Lynch

[44] See "Global Tranche Returns", dated 2nd May 2005 & 1st June 2005

10y markets are increasingly more liquid

Investors searching for yield have extended beyond the most liquid 5y maturity for CDS related products. This includes selling protection on an increasing amount of 10y single-tranche synthetic CDOs (STCDOs). The resulting demand by dealers to hedge 10y bespoke tranches has also driven the liquidity of the standardized 10y CDX IG tranche market.

Sell 10y Tranche, Buy 5y Tranche

We analyze curve flattener trades using the CDX IG 10y & 5y tranche market. The improved liquidity in 10y CDX IG tranches makes this trade easy to execute. We examine six curve flattening trades as highlighted in Table 34.

Table 34: Curve Flatteners via CDX IG Tranches

Sell 10y Protection		Buy 5y Protection		
CDX IG Tranche	Bid (bps)	CDX IG Tranche	Offer (bps)	Carry (bps)
0-3%	56% (upfront)	0-3%	30.75% (upfront)	25.25% (upfront)
3-7%	478	3-7%	175	303
7-10%	176	7-10%	60	116
10-15%	82	10-15%	21	61
3-7%	478	7-10%	60	418
7-10%	176	3-7%	175	1

Source: Merrill Lynch

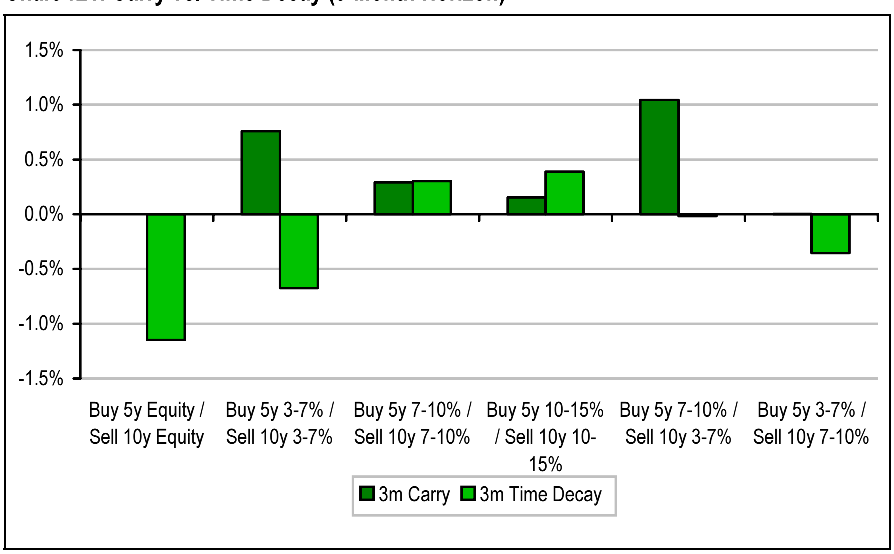
Equal notional curve flatteners are positive carry but…

Besides benefiting from a flattening of the 10y-5y CDX IG curve, each trade also generates a **positive carry** as shown in the above table (the 10y 7-10% vs 5y 3-7% is almost flat carry). The equity tranche trade has no carry but receives a positive upfront amount. However, each trade is exposed to different levels of risk related to **jump-to-default** and **time decay**.

…exposed to other risks

In Chart 121 we compare the 3-month time decay and the carry for each trade. The equity 10y-5y flattener has the least attractive time decay (and a zero carry) whereas the 7-10% and the 10-15% flatteners have positive time decays (but relatively low carry).

Chart 121: Carry vs. Time Decay (3-Month Horizon)



Source: Merrill Lynch

Alternative hedge ratios could also be used e.g. DV01 hedged

All six trades suggested above are **equal notional** trades and most are bullish trades, i.e. they would benefit from parallel spread tightening. Investors who want to exploit the CDX curve but who are not bullish on spreads could use alternative heavier hedge ratios. For example, a **DV01-hedged**[45] 3-7% flattener where investor sells $10mn protection on 10y 3-7% would require buying about $20mn protection on 5y 3-7% (lower annual carry of 128bps).

■ Trade Analysis (Assumptions)

Assume 3-month horizon

We make the following assumptions:

- **Equal tranche notional**: We assume both legs of each trade have the same tranche notional, i.e. each trade is fully hedged for default in the first 5 years. However, the MTM of the 10y will differ from the 5y resulting in a net exposure to jump-to-default risk. Note that equal tranche notional trades with 3-7% and 7-10% as the two legs will not be fully hedged for default due to the difference in notional exposure for each underlying credit.
- **3m horizon**: We analyze all scenarios after a three month holding period.
- **Curve Flattens**: We assume 10y-5y CDX IG slope flattens due to:
    - 5y constant and 10y tightens (more likely in our opinion)
    - 10y constant and 5y widens (less likely)
- **Curve Steepens**: We assume IG 5y tightens or IG 10y widens.
- **Parallel Spread Movement**: Most trades are bullish and exposed to parallel spread moves of the 5y and 10y CDX IG.

Another factor that can cause the tranche curve to flatten (even if the underlying index remains constant) is the change in 10y-5y implied correlations. Recently we have seen a divergence of implied correlations between 10y and 5y tranches with similar attachment points. For example any mean reversion in the divergence of the 10y and 5y equity correlations would flatten the 10y-5y equity tranche curve and benefit an equity curve flattener trade. This could occur, for example, if there is a significant amount of 10y single-tranche synthetic CDO activity. We have not examined this scenario in this report.

Table 35: Equal Notional Tranche Curve Flatteners (3-Month Horizon)

Sell 10y Protection	Buy 5y Protection	Curve Flattening Impact	Carry	Time Decay	Relative Bullishness*
0-3%	0-3%	+ +	Upfront	- - -	- -
3-7%	3-7%	+ + +	+ + +	- -	+ +
7-10%	7-10%	+	+ +	+	+ +
10-15%	10-15%	+ +	+	+ +	+ +
3-7%	7-10%	+ + +	+ + +	-	+ + +
7-10%	3-7%	+	+ / -	- -	-

* Assume parallel spread moves over 3 months.
Source: Merrill Lynch

[45] Tranche MTM = Leverage x Index MTM; For 1bp index move, Index MTM = 1bp * Index DV01;
Notional 10y/ Notional 5y = (Leverage 10y * 10y Index DV01)/ Leverage 5y * 5y Index DV01)
This ratio is the DV01 hedge ratio of the trade.

Scenario 1: CDX IG 5y Constant, 10y Tightens

More likely flattening scenario

In this scenario we assume the 10y index tightens whereas the 5y index remains constant. In our opinion this is the more likely scenario either as investors take on more 10y risk or due to a 10y STCDO bid as the 10y-5y CDX IG curve steepens.

The MTM on the 5y tranche leg on each trade will be unaffected by the change in spreads of the 10y index and will therefore represent its time decay of the 5y tranche over three months.

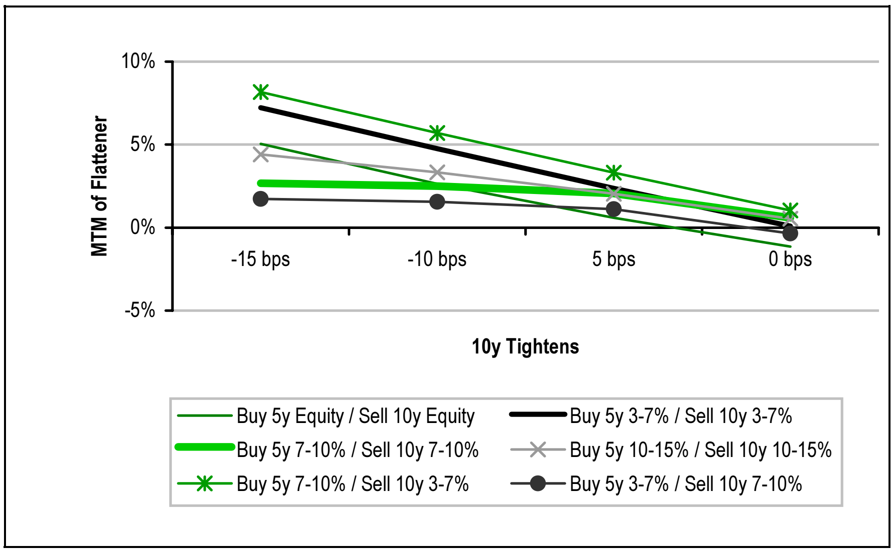
10y 3-7%/5y 7-10% outperforms followed by 3-7% and 10-15% flatteners

Chart 122 highlights the following:

- Sell 10y 3-7%/ Buy 5y 7-10% is the most attractive flattener.
- The 3-7% and 10-15% flatteners also perform well.
- The 7-10% flattener also looks attractive for small changes in the 10y index.

Both trades where investor sells 10y 3-7% benefit from the attractive carry whereas the 10-15% flattener benefits from the positive time decay over 3 months. During this period, the 7-10% flattener has an attractive combined carry and time decay (see Chart 121).

Chart 122: MTM of Tranche Flatteners – CDX IG 10y Tightens, 5y Unchanged (Includes 3-Month Carry)



Source: Merrill Lynch

Scenario 2: CDX IG 5y Widens, 10y Constant

10y 7-10% / 5y 3-7% does well followed by 3-7% flattener

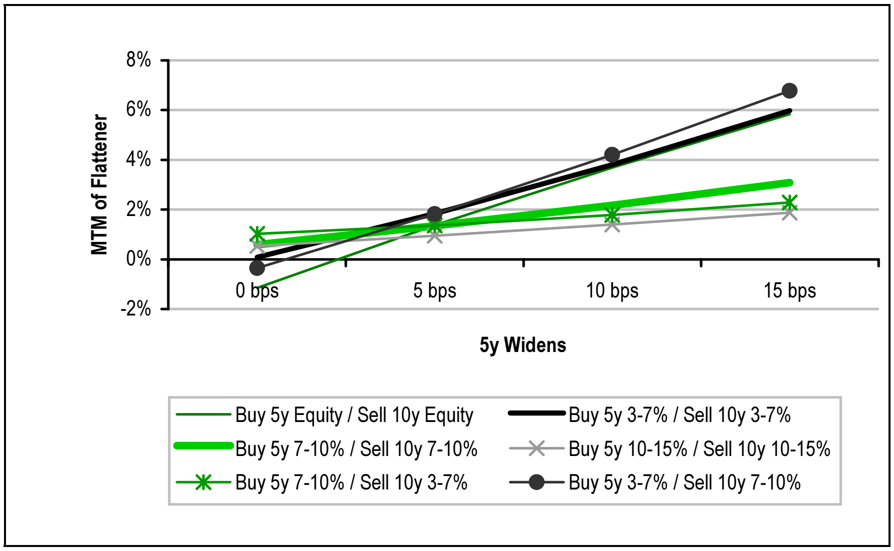
In this scenario, we assume that the 5y index widens while the 10y index remains unchanged. In our opinion this is the less likely scenario.

From Chart 123 we observe the following :

- Sell 10y 7-10%/Buy 5y 3-7% is the best performer (despite the negligible carry and negative time decay over 3 months)
- The 3-7% and the equity flatteners are also good performers

For small changes, however, the equity tranche underperforms the other flatteners due to its relatively large time decay over 3 months.

Chart 123: MTM of Tranche Flatteners – CDX IG 5y Widens, 10y Unchanged (Includes 3-Month Carry)



Source: Merrill Lynch

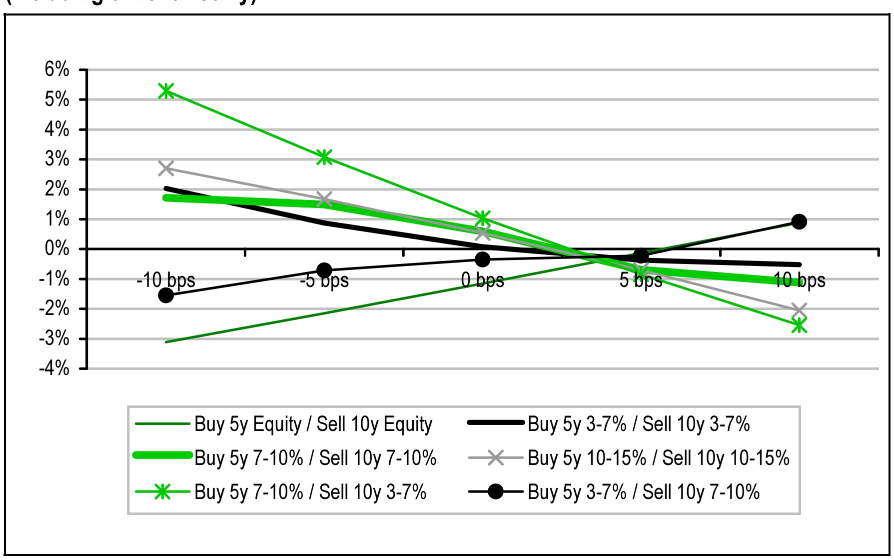
Scenario 3: Parallel Spread Movements

Most trades are essentially bullish

Most of the flatteners are long spread duration i.e. essentially bullish, and would benefit from parallel spread tightening (Table 35 on front page highlights the relative bullishness of each trade). Chart 124 highlights the relative performance of each trade under parallel widening and tightening scenarios over three months. The 10y 3-7%/5y 7-10% flattener is the most bullish of the six trades during this period.

10y 3-7/5y 7-10% flattener is most leveraged to parallel spread moves

Chart 124: MTM of Tranche Flatteners – Parallel Movement of CDX IG 5y & 10y Spreads (Including 3-Month Carry)



Source: Merrill Lynch

Table 36: Tranche Leverage

CDX IG 10y Tranche	Tranche Leverage	CDX IG 5y Tranche	Tranche Leverage
0-3%	6.50	0-3%	16.0
3-7%	8.25	3-7%	7.0
7-10%	4.50	7-10%	3.0
10-15%	2.50	10-15%	1.25
3-7%	8.25	7-10%	3.0
7-10%	4.5	3-7%	7.0

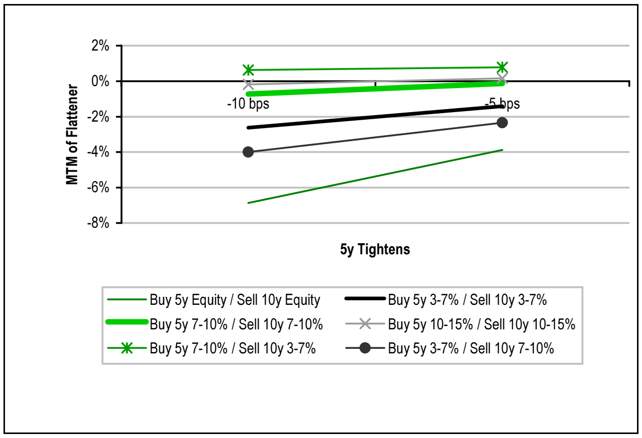
DV01 of 10y CDX IG = 7.9; DV01 of 5y CDX IG = 4.6
Source: Merrill Lynch

Scenario 4: CDX 10y-5y Curve Steepens

What happens if CDX curve steepens?

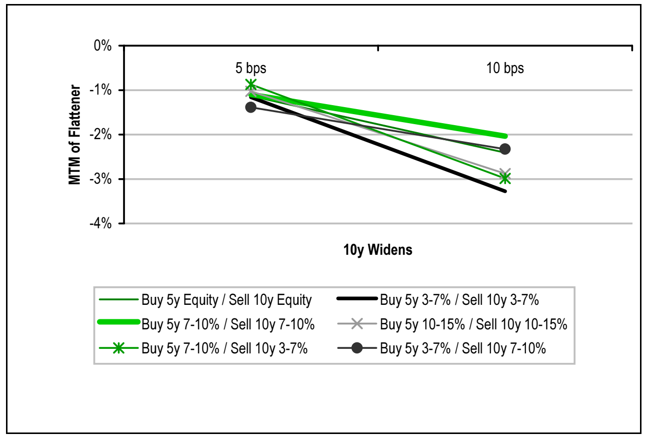
In the event that the 10y-5y curve steepens over the next three months, we would expect a negative MTM for each of the trades. Chart 125and Chart 126 highlight the impact of curve steepening. Equity flattener has the worst performance if 5y tightens whereas 3-7% flattener is the worst performer if 10y widens.

Chart 125: MTM of Flatteners - CDX 5y Tightens, 10y Flat (Includes 3-Month Carry)



Source: Merrill Lynch

Chart 126: MTM of Flattenr – CDX 5y Flat, 10y Widens (Includes 3-Month Carry)



Source: Merrill Lynch

FTD Strategies Using Top 10 Credit Picks

This section is extracted from a "Protection Strategies" report published on 23rd January 2006 by Kakodkar et al.

Merrill Lynch's credit research team recently published a list of top 10 North American longs and shorts for 2006[46]. In this report we highlight FTD baskets to efficiently express long and short views of our top picks. Investors who are long CDX IG5 equity tranche can also use FTDs to hedge spread jump or jump-to-default risk.

We recommend the following three trades:

- **Sell 5y FTD protection @ 260 bps (DHI, ETP, SFI, JCP, EPD)**
- **Buy 5y FTD protection @ 247 bps (APC, XL, WM, SLE, CSC) outright**
- **Sell 5y CDX IG5 equity to fund 5y Underweight FTD protection**

Overweight FTD Basket

FTD baskets are attractive instruments that can enhance yield by taking a leveraged view on a basket of credits on which investors have a favorable outlook. Long investors can maximize the spread pickup by creating baskets that are both diversified (typically implies lower default correlation) as well as relatively homogeneous in spread dispersion (the widest credit in a non-homogeneous baskets would typically dictate the basket price)[47].

Table 37: Top 10 Overweights

Ticker	Sector
DHI	Capital Goods-Homebuilders
EXC	Utilities/Pipelines
ETP	Utilities/Pipelines
LIBMUT	Insurance
SFI	REITs
FALCF	Metals/Mining
JCP	Retail
EPD	Oil/Gas
HCP	REITs
AIG/il	Finance

Source: Merrill Lynch

Table 37 represents our Top 10 Overweight picks by our credit analysts. We suggest selling protection on the following diversified 5y FTD basket:

- **DHI, ETP, SFI, JCP, EPD @ 260 bps**

The basket is diversified and represents attractive relative value – it yields about 70% sum of the spreads which represents about 3.7x the average credit spread of the basket.

46 See "Credit Strategies" by Rooney/Charles, 10th Jan 2006.
47 For more details on FTD baskets, see Volume 2, Chapter 3.

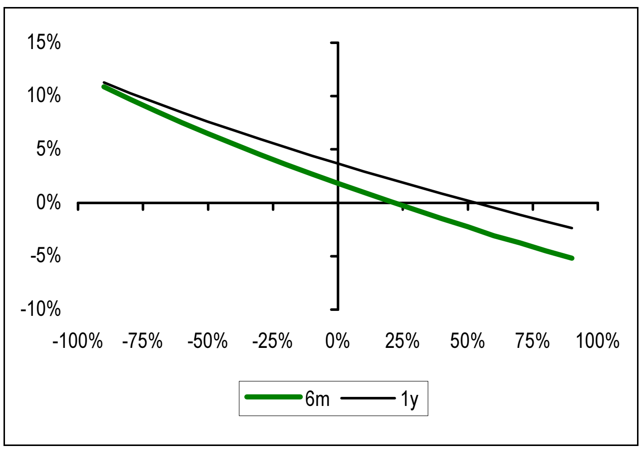
Table 38: Overweight FTD Basket

Ticker	Rating	5y CDS
DHI	Baa3/BB+	85
ETP	NA/BBB-	75
SFI	Baa3/BBB-	60
JCP	Ba1/BB+	61
EPD	Baa3/BB+	70
Total Aggregate CDS Spread		351
Average CDS Spread		70
Max CDS Spread		85
Min CDS Spread		60
Indicative FTD Spread		**260**
FTD/Average CDS		3.7x
FTD/Max CDS		3.1x
FTD/Total CDS		74%

Source: Merrill Lynch

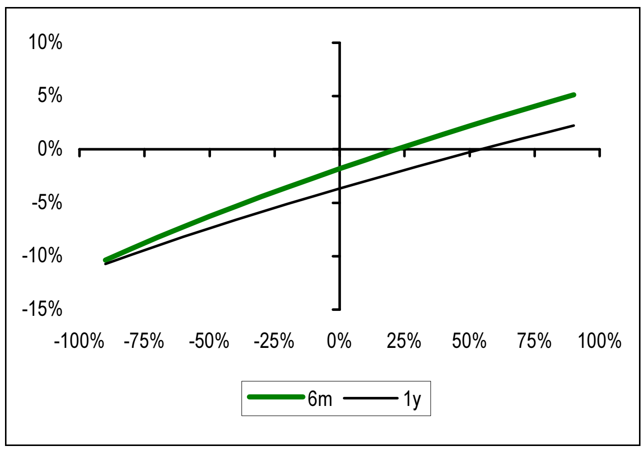
This basket would benefit from any spread tightening. We highlight the MTM of the basket for a 6m and 1y holding period in Chart 127 assuming all the underlying spreads widen or tighten by the same % amount. These scenarios assume a constant correlation.

Chart 127: MTM of OW FTD vs. % Change in Underlying Spreads



Source: Merrill Lynch. MTM includes carry and assumes constant pairwise correlation. Using percentage uniform curve shifts from current spreads.

Chart 128: MTM of UW FTD vs. % Change in Underlying Spreads



Source: Merrill Lynch. MTM includes carry and assumes constant pairwise correlation. Using percentage uniform curve shifts from current spreads.

Table 39: Top 10 Underweights

Ticker	Sector
APC	Oil/Gas
XL	Insurance
WM	Banking
BWA	Auto Parts
GCI	Cable/Media
SLE	Consumer Products
SVU	Retail
CSC	Electronics/Tech
MDC	Capital Goods-homebuilders
ABS	Retail

Source: Merrill Lynch

Underweight FTD Basket

Investors can also buy FTD protection as a cheaper way to short names on which they have a bearish outlook. We suggest the following basket of 5 names from the top 10 Underweights highlighted in Table 39:

- **APC, XL, WM, SLE, CSC @ 247 bps**

The cost of buying protection on the FTD basket is about 86% of the sum of the spreads and is cheaper than buying protection on each name individually. Chart 128 above highlights the MTM of the FTD for a 6m and 1y holding period.

Table 40: Underweight FTD Basket

Ticker	Rating	5y CDS
APC	Baa1/BBB+	31
XL	A3/A-	36
WM	A3/A-	35
SLE	A3/BBB+	68
CSC	A3/A	118
Total Aggregate CDS Spread		287
Average CDS Spread		57
Max CDS Spread		118
Min CDS Spread		31
Indicative FTD Spread		**247**
FTD/Average CDS		4.3x
FTD/Max CDS		2.1x
FTD/Total CDS		86%

Source: Merrill Lynch

Sell 0-3% CDX IG5 and Buy 5y FTD

Use Underweight FTD to hedge jump risk in long CDX 0-3%

All 5 names in the Underweight basket are members of CDX IG5 index. Investors who have established long IG5 equity tranche positions due to their current relative attractiveness can use the running spread of 500bps to fund protection on the Underweight FTD basket. Knowing that IG5 0-3% notional of $15mn represents a notional of $4mn for each individual credit, we suggest the following:

- **Buy 5y Underweight FTD protection @ 247 bps ($4mn notional, pay $98,800)**
- **Sell 5y CDX IG5 0-3% protection @ 37.125% upfront + 500bps running ($15mn notional, receive $750,000 running)**
- **Investor receives 37.125%% upfront and a carry of $651,200**

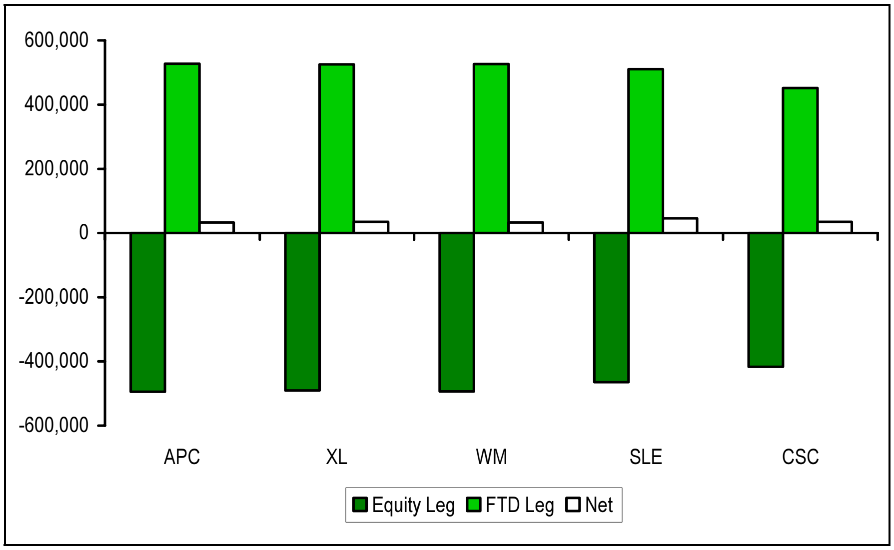
FTD provides a default hedge for one credit

The FTD protection provides a hedge against the jump-to-default and is cheaper than buying protection on all five names individually. However, the FTD basket protection would terminate following the first credit event and the investor would no longer be protected on the remaining credits. The investor is also exposed to the risk of simultaneous defaults in the underlying portfolio. The positive carry from the trade can be used to either buy protection on more baskets or individual credits. However, we believe that the Underweight FTD basket is more exposed to spread widening than actual defaults.

In the event of a default of one of the names in the FTD, the investor is fully hedged. The worst case scenario for this strategy occurs when a credit that is not included in either of the baskets defaults while the spreads of credits in the baskets remain either unchanged or (worse) tighten. In the best case all credits (except those in the baskets) tighten while the credits in the basket remain unchanged or (better still) widen out.

Chart 129 highlights the MTM impact if each of the credits in the Underweight FTD basket above jump to 500bps (assuming all other spreads are unchanged).

Chart 129: MTM of Equity Tranche and UW FTD Basket if any Credit Widens to 500bps



Source: Merrill Lynch. Assume a flat curve when credit widens to 500bps. Assume notional of $15mn for 0-3% and $4mn for FTD. Assume an instantaneous jump to 500bps.

8. Modeling Correlation Products

The proliferation of structured credit products such as Single Tranche CDOs and CDO squareds has forced market participants to develop reliable pricing and hedging algorithms to efficiently manage the different sources of risk (correlation, spread risk, jump-to-default etc.) embedded in the product.

Copula functions, introduced in early 2000, still constitute the key building block used in order to estimate the portfolio loss distribution needed to compute the expected risk profile of a correlation based structure.

Through time, academics and practioneers have been actively attempting to develop efficient numerical schemes to speed up the pricing process as well as the computation of the hedging measures.

This section will introduce the concept of copula functions and present the three main techniques commonly used to construct the portfolio loss distribution from the individual credit survival curves:

- Monte Carlo Simulations
- Fast Fourier Transform
- Recursive Algorithm

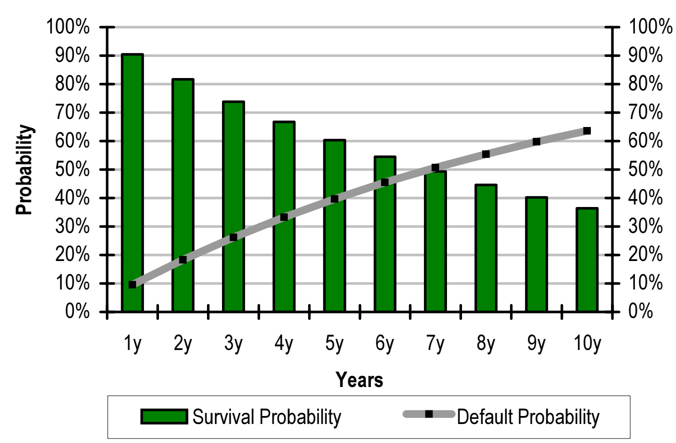
We will then illustrate how to use the information from the estimated loss distribution to evaluate first-to-default baskets as well as single tranche CDOs.

From Single-Name to Multi-Name

Bootstrap procedure to compute single-name survival curve is relatively straightforward…

A common approach to compute default or survival probabilities[48] for an individual credit is to use a bootstrap procedure based on CDS quotes and an exogenous (that is, not derived by the model itself) assumption for the recovery rate. We have discussed this method in some detail in Volume 1, Chapter 2. Chart 130 highlights this term structure for a generic issuer.

Chart 130: Survival Curve and Default Probabilities



Assume flat CDS curve of 300 bps from years 1-10 and a recovery of 50%.
Source: Merrill Lynch

[48] Survival Probability = 1 - Default Probability

The survival curve can be used to compute the probability of default (or survival) at each point in time for the issuer. For example, Chart 130 implies that the probability of not triggering a credit event before 3 years (or the probability of surviving at least 3 years) is approximately equal to 74%.

We use the following notation:

- Let τ_j denote the time until default for the j^{th} issuer.
- Let $S_j(t)$ denote the survival probability function at each point in time, t.
- Let $F_j(t)$ denote the default probability function at each point in time t.

The probability that the issuer will survive at least t years is given by:

$$S_j(t) = \Pr(\tau_j > t)$$

The probability that issuer will default within t years is given by:

$$F_j(t) = \Pr(\tau_j \leq t)$$

We can easily see that:

$$S_j(t) = 1 - F_j(t).$$

…but computing a joint survival curve for multi-name credit derivatives is not

Aggregating individual survival curves into a joint survival curve for all names in the portfolio is the fundamental problem with multiname credit derivatives.

Let us suppose we have a basket default swap referencing a portfolio of three issuers. We can then ask the following questions:

- How do we compute a "portfolio" based or joint survival probability $S_{portfolio}(t)$ such that all credits survive at least the next two years?
- How do we measure the likelihood of joint occurrences of default within a two year horizon?

We use the following added notation:

- Let $S_{portfolio}(t)$ denote the joint survival probability function at time, t.
- Let $F_{portfolio}(t)$ denote the joint default probability function at time t.

The probability that all three credits in the basket will survive at least 2 years is given by:

$$S_{portfolio}(2y) = 1 - F_{portfolio}(2y) = \Pr(\tau_1 > 2y, \tau_2 > 2y, \tau_3 > 2y)$$

Unfortunately **there is no explicit solution to this problem**, that is, it is not possible to retrieve this multivariate cumulative probability from market quotes. This happens for two main reasons:

- Lack of reliable data due to the lack of liquid traded portfolio assets such as NTD baskets.
- The dependence on default correlation which is a difficult parameter to determine (as explained earlier in the report).

Therefore, our problem can be summarised as follows:

Individual CDS based information

Portfolio based information

$$\begin{aligned} S_1(t) &= \Pr(\tau_1 > t) \\ S_2(t) &= \Pr(\tau_2 > t) \qquad \Rightarrow \qquad S_{portfolio}(t) = \Pr(\tau_1 > t, \tau_2 > t, \tau_3 > t) \\ S_3(t) &= \Pr(\tau_3 > t) \end{aligned}$$

Copula Functions To The Rescue

Linking Individual Distributions to One Joint Distribution

Copula functions provide an efficient link between multiple single-name survival curves to one multi-name survival curve

The number of joint default events increases exponentially as a function of the number of credits. As a result, for typical multiname portfolios of 50-100 credits, the computation of joint default probabilities becomes increasingly complex and requires efficient numerical methods. This is where copula functions come to the rescue.

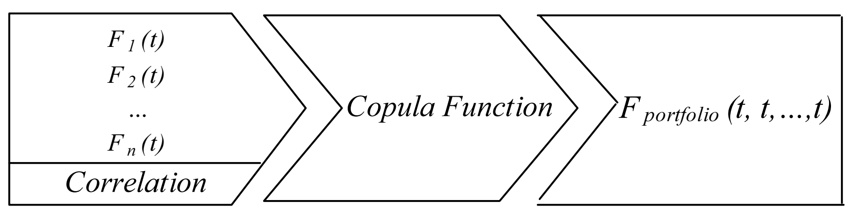
In fact, **copulas provide an efficient way to link multiple unidimensional survival curves with a single multidimensional survival curve**. This fact derives from an important theorem, attributed to Sklar (1959), which, in our context, can be stated as follows:

Theorem (Sklar): Let $F_{portfolio}(t)$ be the "portfolio" default distribution function and $F_j(t)$ the default distribution function for the generic j^{th} issuer with $j=1,2,\ldots,n,$ where n stands for the number of obligors referenced by the contract. Then there exists an n-dimensional **copula function** C such that:

$$F_{portfolio}(t) = \Pr(\tau_1 < t, \tau_2 < t, \ldots, \tau_n < t) = C\left[F_1(t), F_2(t), \ldots, F_n(t)\right]$$

Sklar's theorem provides the core idea of dependency modeling via copula functions. It states that for any "portfolio" multidimensional distribution function, the single obligors' distribution functions $F_j(t)$ and the dependency structure (represented by the default correlation) can be **separated**, via a copula function. Conversely, the theorem shows that, starting from the univariate survival curves for each obligor, it is possible to **aggregate** them into a multidimensional survival curve by using a copula function.

Chart 131: Linking Individual Curves



Source: Merrill Lynch

Because of its analytical tractability and the small number of parameters required, **Gaussian (or normal) copula represents the current market standard** in modeling portfolio credit risk sensitive positions and their relative trades. The types of portfolios include CDOs as well as NTD-basket default swaps and notes.

Inverted Distribution

Before providing an explicit expression for the Gaussian copula, we need to introduce the concept of the inverted distribution function to understand the simulation algorithm that is presented in the next section.

We know that $F(t)$ maps time t to the probability of default. Now suppose we have the opposite situation, i.e. we are given the probability of default and asked to compute the implied time t. **The function that maps the cumulative default probability to the corresponding time t, is called the inverted distribution function** and is a key concept in the copula framework.

Using F^{-1} to denote the inverted distribution function, we have:

$$t = F^{-1}[F(t)]$$

From Sklar's Theorem, we derive the following relationship:

$$C[F_1(t), \ldots, F_n(t)] = F_{portfolio}\{F_1^{-1}[F_1(t)], \ldots, F_n^{-1}[F_n(t)]\}$$

Gaussian Copula

Gaussian Copula is the market standard

Let N^{-1} be the inverted **one-dimensional** Gaussian or normal distribution function for an issuer. The **multidimensional** Gaussian copula C^G is given by:

$$C^G[F_1(t), \ldots, F_n(t)] = N_R\{N^{-1}[F_1(t)], \ldots, N^{-1}[F_n(t)]\}$$

where N_R is the multidimensional Gaussian distribution function with correlation matrix R. From Sklar's Theorem, we have the following:

$$F_{portfolio}(t) = N_R\{N^{-1}[F_1(t)], \ldots, N^{-1}[F_n(t)]\}$$

Simulation Algorithm

How can we simulate the time-until-default random variables taking into account both the individual creditworthiness (individual measure) and the default correlation (portfolio measure)?

David X. Li, in his paper "*On Default Correlation: A Copula Function Approach*" dated 1999, offers an interesting way to perform this task using the copula framework we discussed above:

1. Simulate the *n* random variables $Y_1, Y_2, \ldots, Y_n$ from a multidimensional normal distribution function[49] with correlation matrix R.
2. Map back the Y's to the τ's using $\tau_j = F_j^{-1}[N(Y_j)]$

The tricky part of this algorithm is represented by the correct specification of the single-name distribution function $F_j(t)$. We are going to assume that default distribution functions follow an exponential distribution with parameter h[50], i.e.

$$F(x) = 1 - e^{-hx}$$

[49] This task consists in a decomposition of the correlation matrix R (such as the Cholesky decomposition) and the subsequent generation of correlated normal random variables. We refer the reader to Press et al., (1993), "*Numerical Recipes in C: The Art of Scientific Computing*".

[50] The parameter h is one of the main building block of modern credit risk management models. It is commonly called *hazard rate* and is defined as $h(t) = \lim_{\Delta t \to 0} \frac{\Pr[t < \tau \le t + \Delta t \mid \tau > t]}{\Delta t}$. It represents the probability that the issuer will survive for an additional infinitesimal period Δt conditional on the fact that it has survived until time t. By assuming a flat term structure of credit default swap quotes, it is possible to show that the relation $h = \frac{CDS\ spread}{1 - \mathrm{Recovery}}$ holds. In more sophisticated models h is a stochastic variable usually recovered from market data by assuming a certain specification of the "jump" process used to model the occurrence of defaults.

Refer to important disclosures on page 196.

This basic assumption allows us to provide an explicit solution for step 2 which is given by the following expression[51]:

$$\tau_j = -\frac{1}{h}\ln\left[1 - N(Y_j)\right]$$

This is an important result as it allows us to obtain the sample path of the default arrival time for each issuer by taking into account both the key inputs:

- Individual creditworthiness via the factor *h* which depends on the underlying CDS curve of the issuer, and
- Correlation matrix *R* which affects the sampling procedure of the random variables *Y's*.

A Simple Application

We use the above methodology to compute the portfolio default distribution function for a portfolio with the following characteristics:

- Reference pool of 5 credits;
- Constant pairwise correlation of 50%;
- Flat CDS spread of 200bps; and
- Recovery rate of 50% for each credit.

Table 41 highlights the output of the simulation algorithm by showing default arrival times for each of the issuers; for example, looking at the first simulation run, we can see that the first default occurs in year 58 by issuer five.

The first default occurs in year 58 by issuer 5

Table 41: Copula Simulation of Default Arrival Time (Years)

Simulation #	Issuer 1	Issuer 2	Issuer 3	Issuer 4	Issuer 5	First to Default Time
1	122.3	100.6	112.9	61.4	57.8	57.8
2	149.1	6.6	98.4	128.8	112.6	6.6
3	4.6	16.3	68.0	65.7	23.5	4.6
4	326.9	347.8	63.0	424.3	166.4	63
5	74.4	137.2	15.2	77.4	60.8	15.2
…	…	…	…	…	…	…
…	…	…	…	…	…	…
9996	35.4	69.8	80.7	32.2	29.8	29.8
9997	177.1	230.8	173.6	517.0	423.2	173.6
9998	25.7	74.8	260.9	44.7	81.8	25.7
9999	11.2	424.5	140.2	10.3	87.9	10.3
10000	10.5	4.2	46.8	10.5	1.3	1.3

Constant pairwise correlation of 50%, hazard rate *h*=1%, five reference entities, 10,000 simulation runs.
Source: Merrill Lynch.

Using multiple simulations, we plot the portfolio distribution function for different values of correlation (20%,50%, 90%) and seniority (1^{st} and 2^{nd} to default) as shown in Chart 132 and Chart 133.

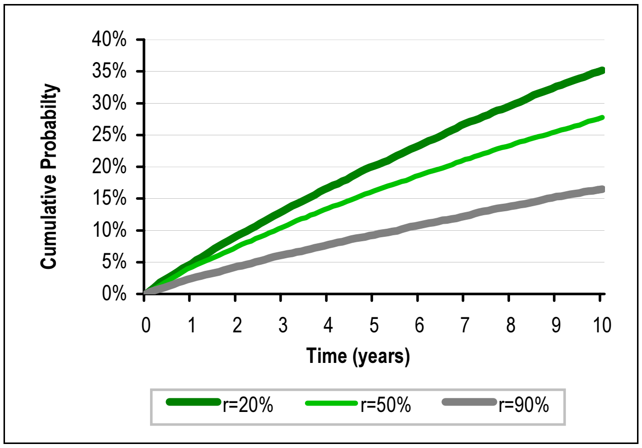
For example, in the 50% correlation scenario, we see that the probability that at least **one** default occurs within 5 years is around 16%, while the probability of having at least **two** default within 5 years is about 5.25%.

Once the whole portfolio distribution function is computed, it is then possible to calculate the corresponding portfolio losses for each attachment point and fix the breakeven spread for each CDO tranche.

[51] We can easily work out this expression by using the definition of inverted distribution function: if $F(x) = 1 - e^{-hx} = W$, where W lies between 0 and 1. Then, we have

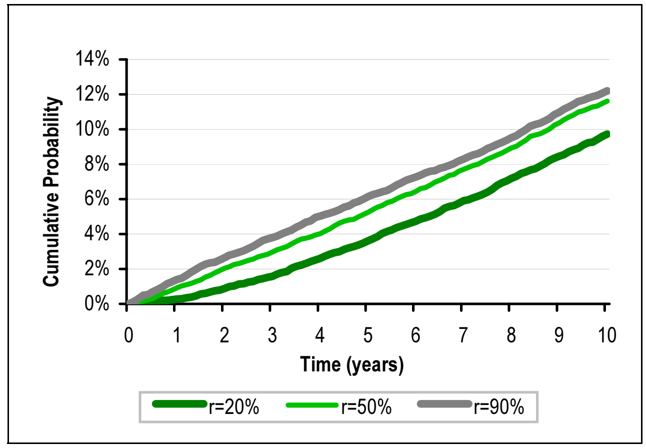
$$F^{-1}(W) = -\frac{1}{h}\ln(1 - W).$$

Chart 132: First to Default Distribution Function



Constant pairwise correlation, hazard rate *h*=1%, five reference entities, 50% recovery, 50,000 simulation runs.
Source: Merrill Lynch

Chart 133: Second to Default Distribution Function



Constant pairwise correlation, hazard rate *h*=1%, five reference entities, 50% recovery, 50,000 simulation runs.
Source: Merrill Lynch

From Numerical Simulations To Factor Models

Although easy to implement, Monte Carlo simulations can be very slow

As previously illustrated, the basic inputs in the Monte Carlo approach include:

- The **individual credit spread** or default probability for each obligor,
- The **pairwise (credit vs. credit) correlation matrix** of the portfolio.

Leaving aside the problematic interpretation of the second input (asset, equity or spread correlation as a proxy for the "desired" default correlation matrix), the basic Monte Carlo setting suffers from two main drawbacks:

- For an accurate estimate of spreads, this technique requires a large number of simulations. Given the large size of a typical underlying portfolio (usually over 100 credits) this procedure can, therefore, be excessively time consuming.
- Estimation of the default correlation matrix. With *N* obligors, the *NxN* pairwise correlation matrix requires $N \cdot (N-1)/2$ estimates[52].

Factor models lower the number of estimates…

A common trick to reduce the number of estimates is to introduce **"factors"** that can capture and describe the dependency structure among credits. Thus, instead of analyzing the dependency of each pair of credits, **factor models** replace this **credit-vs.-credit** approach with a **credit-vs.-common factors** approach.

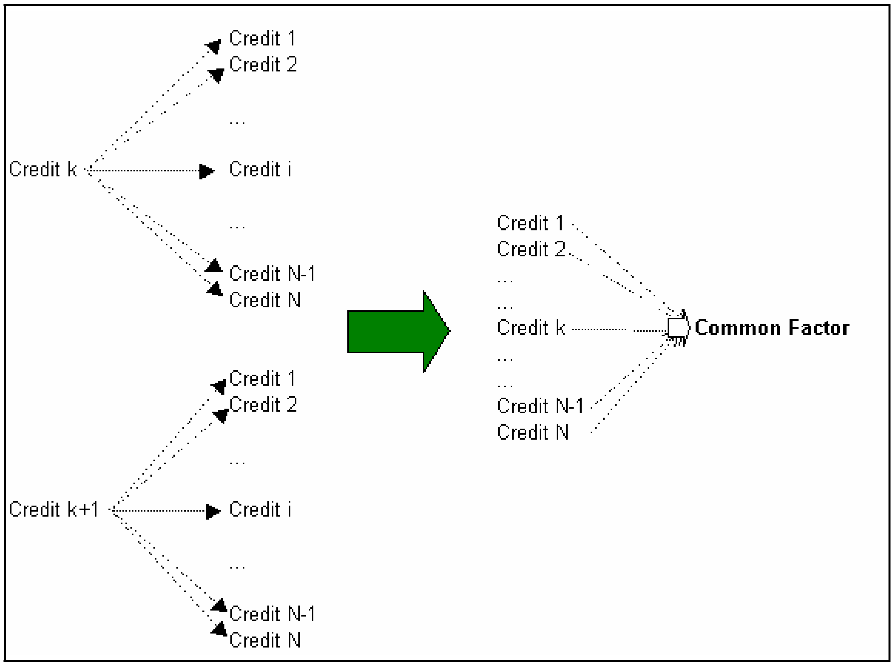
…by using a "credit vs. common factor" instead of "credit vs. credit" approach

For a reasonably small number *M* of common factors, it is possible to reduce the calibration problem to the estimation of "only" *MxN* dependency parameters[53]. For example, for a 100-name portfolio and only one common factor, the number of estimates reduce from 4950 (number of entries in a pairwise correlation matrix) to 100 (the correlation of each credit with respect to the common factor). The logic behind the procedure is represented in the Chart 134:

[52] For a 100-credit portfolio, the full pairwise correlation matrix would require 4950 estimates.
[53] For simplicity we assume the number of common factors *M* to be smaller than the number of credit *N* in the portfolio.

Refer to important disclosures on page 196.

Chart 134: From pairwise to common factor dependency



Source: Merrill Lynch

Factor models allow for efficient computation of portfolio loss distributions

A key consequence of using factor models is that it simplifies the computation of expected losses of the underlying portfolio[54]. Reduction in the dimensionality also lowers the computational time significantly when compared to the numerical simulation approach. Computing portfolio loss distributions with the factor model approach consists of the following three steps:

1. Model the firm value based on the number of factors. In our discussion below, we use a one-factor model.
2. Estimate the conditional default probability for a given realization of the factors.
3. Compute the expected loss of the portfolio based on the specific characteristics of the underlying portfolio (in terms of size and homogeneity).

Step 1: Modeling the Firm Value

The firm value is driven by two sources of risk

Using a factor model approach specified above, the next step consists of modeling the firm value process for each underlying credit. With a one-factor approach, we assume that the firm value[55] of each credit is driven by two main components:

Systematic risk (common factor)…

- A **systematic** or common factor, which can be interpreted as an indicator of the general state of the business cycle (e.g. stock index, bond index, gross domestic product)

…and idiosyncratic risk (firm-specific)

- An **idiosyncratic** noise or firm-specific factor that is an indicator of events strictly linked to the credit itself (e.g. market share, quality of management, relative position in a sector compared to peers).

[54] Depending on the complexity of the underlying portfolio, the computation of the expected loss might require the adoption of numerical techniques (i.e. numerical integration) which can be efficiently implemented in a typical spreadsheet environment.
[55] Within the classical Merton setting, firm value is modeled by :

$M_t^{\,i} = M_0 \exp((r - \frac{1}{2}\sigma^2)t + \sigma V_i \sqrt{t})$, σ -is the volatility of the firms assets and r is the risk-free rate, V_i will be defined on the next page

We assume that both the systematic and the idiosyncratic components are **normally distributed** with zero mean and variance equal to one. Moreover, by definition of systematic and idiosyncratic risk, the two factors are assumed to be **independent.**

Under the **one-factor** approach defined above, the individual risk process of an obligor is defined by (linearly) combining the two risk sources as shown below:

$$V_i = \beta_i Z + w_i \varepsilon_i$$

- V_i : risk driver of the i^{th} credit in the portfolio
- Z : value of the common (systematic) factor
- ε_i : value of the idiosyncratic risk for the i^{th} credit in the portfolio
- β_i is the sensitivity of the i^{th} credit to the common factor, and
- w_i is the idiosyncratic weight for the i^{th} credit.

Under assumptions of normality and independence of the distributions of the two risk sources (systematic and idiosyncratic) this simplifies[56] to:

$$V_i = \sqrt{\rho_i} Z + \sqrt{1-\rho_i}\,\varepsilon_i \text{ (where } \rho_i = \beta_i^2\text{)} \tag{1}$$

We derive two key inferences from this expression:

1. ρ_i drives the **relative contribution** of the two sources of risk into the value of the firm. If ρ_i is equal to 0, then the general state of the economy (common factor) does not affect the value of the i^{th} obligor. If ρ_i is equal to 1, then company specific linked events do not govern the fate of the company.
2. ρ_i can be interpreted as the correlation of the i^{th} obligor to the common factor. By further exploiting the normal independent distributions of systematic and idiosyncratic risks, we can derive the pairwise correlations[57] r_{ij} as:

$$r_{ij} = \sqrt{\rho_i}\sqrt{\rho_j} = \sqrt{\rho_i \rho_j}\ .$$

Thus, if the correlations of the i^{th} and of the jth obligor to the common factor are equal to 20% and 30% respectively, then the **pairwise correlation** of the two credits is approximately equal to 24.5%. Note that if the correlations to the common factor are identical, the pairwise correlation is also equal to this value.

[56] Since Z and ε_i are standard (0,1) normal independent random variables, then their sum (and hence, V_i) is still a normal random variable. Therefore, the expectation and the variance of V_i are given by $E[V_i] = \beta_i E[Z] + w_i E[\varepsilon_i] = 0$ and $Var[V_i] = \beta_i^2 Var[Z] + w_i^2 Var[\varepsilon_i] + 2\beta_i w_i Cov[Z,\varepsilon_i] = \beta_i^2 + w_i^2$. By imposing the variance of V_i equal to 1, we can express w_i in terms of β_i i.e.

$w_i = \sqrt{1-\beta_i^2} = \sqrt{1-\rho_i}$ with $\rho_i = \beta_i^2$.

[57] $Corr[V_i, V_j] = \dfrac{Cov[V_i, V_j]}{\sqrt{Var[V_i]}\sqrt{Var[V_j]}} = \sqrt{\rho_i}\sqrt{\rho_j} E[Z^2] = \sqrt{\rho_i \rho_j}$.

Default occurs when the firm value process hits the default threshold

Step 2: Estimating Conditional Default Probability

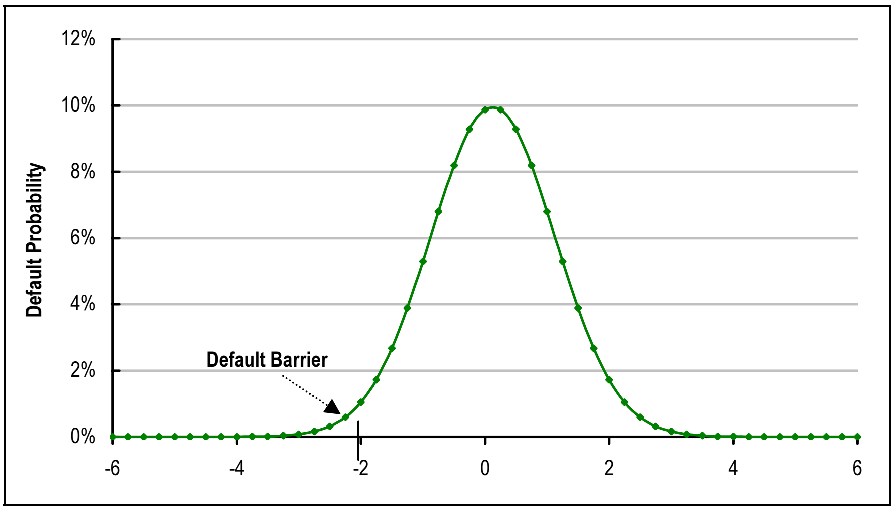
Along the lines of the classic Merton's structural framework, the one-factor model postulates that a company defaults when the firm value falls below a certain default barrier.

Given the normality assumption for the firm value risk process V_i and the default probability[58] d_i for the individual obligor i, the default threshold k_i is given by the following expression:

$$k_i = N^{-1}(d_i)$$

where $N^{-1}()$ is the inverse of the standard normal distribution function[59]. For example given a 5y default probability of 2.275%, the corresponding default barrier is equal to -2.

Chart 135: Default Barrier



Source: Merrill Lynch

For a specific realization $Z=z$ of the systematic factor, the i^{th} obligor will default as soon as its risk process hits the default barrier, i.e.

$$V_i(Z=z) \le k_i$$

Substituting from equation (1) we obtain:

$$\sqrt{\rho_i}z + \sqrt{1-\rho_i}\varepsilon_i \le k_i,$$

which implies:

$$\varepsilon_i \le \frac{k_i - \sqrt{\rho_i}z}{\sqrt{1-\rho_i}}$$

Thus for a specific realization of the common factor Z, this expression suggests that a credit defaults when the idiosyncratic component hits a level determined by the following two factors:

[58] The default probability can be either taken from rating agencies or calibrated from market data. If the second approach is chosen and the CDS term structure is flat, the default probability at time t is equal to: $\text{default probability(t)} = 1 - \exp(-ht)$ with

$$h \approx \frac{spread}{1-\text{recovery}}.$$

[59] NORMSINV() in Microsoft Excel.

- The **individual default barrier** k_i and
- The **correlation of the obligor with the common factor** Z.

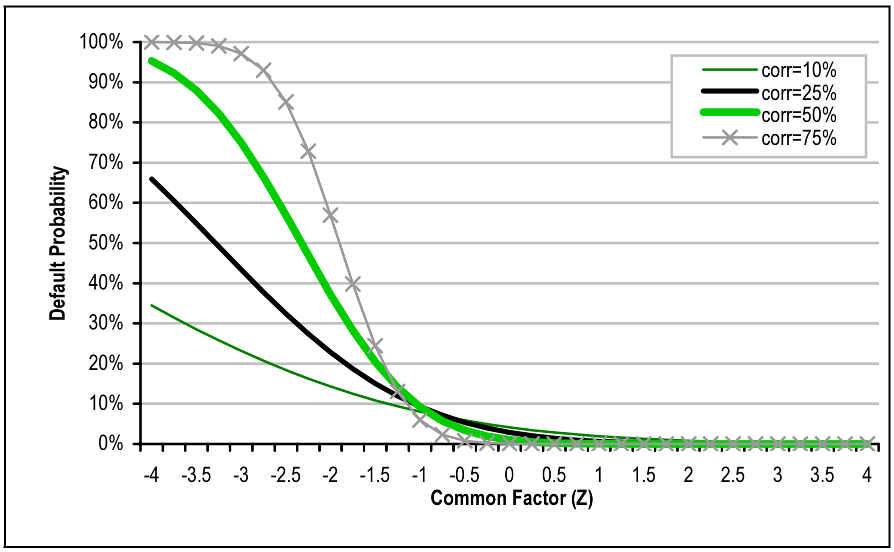
The conditional default probability depends on the realization of the common factor and the asset correlation

Since ε_i, is normally distributed, the **conditional default probability** for the i^{th} obligor is given by:

$$\Pr\left[\varepsilon_i \le \frac{k_i - \sqrt{\rho_i}Z}{\sqrt{1-\rho_i}} \middle| Z = z\right] = N\left[\frac{k_i - \sqrt{\rho_i}z}{\sqrt{1-\rho_i}}\right] \tag{2}$$

where N is the standard normal distribution function[60]. In Chart 136 we highlight the conditional default probability for different values of correlation and realizations of the common factor.

Chart 136: Conditional Default Probability in a 1-Factor Default Model



Source: Merrill Lynch. Assume 5% default probability which corresponds to a normal default barrier of –1.645.

From the chart we observe the following:

- During bear market trends (corresponding to negative realizations of the common factor Z), the default probability increases.
- Higher values of correlation amplify the impact of negative realizations of the common factor and imply a greater likelihood of default.

Step 3: Computing the Expected Portfolio Loss

The complexity of the final step…

The final step is to determine the expected loss of the underlying portfolio of the synthetic CDO. Though factor models provide an appealing framework to perform this task the mathematical complexity and analytical tractability of the model itself will vary according to the specific properties of the underlying portfolio of credits.

We suggest two main measures to assess the complexity of a portfolio of obligors:

…is a function of specific characteristics of the underlying portfolio

- **Portfolio Size**: The number of credits in the underlying portfolio of the synthetic CDO.
- **Portfolio Homogeneity**: The uniformity of the underlying credits with respect to parameters such as recovery, default probability and correlation to common factors.

[60] NORMSDIST() is a function in Microsoft Excel.

Based on the specific characteristics of the underlying portfolio, we illustrate, in the following sections, how factor models can serve as an efficient tool to determine the portfolio loss distribution of a synthetic CDO.

Case 1: Finite Size, Homogeneous Portfolio

If the underlying portfolio of credits shows uniformity in terms of **default probabilities** (individual default risk equal to the average one), **correlation** (same sensitivity to common shocks) and **recovery** rates, it is then possible to derive an explicit expression for the portfolio loss distribution. In other words, we have a closed form solution that does not use time-consuming simulation techniques

The conditional default probability is constant for all underlying credits

Since all credits are homogeneous, we can drop the credit subscript in equation 2 and restate the expression for the conditional default probability as follows:

$$\Pr\left[V \le k \middle| Z = z\right] = \Pr\left[\varepsilon \le \frac{k - \sqrt{\rho}Z}{\sqrt{1-\rho}} \middle| Z = z\right] = N\left[\frac{k - \sqrt{\rho}z}{\sqrt{1-\rho}}\right] \quad (3)$$

Given the binary nature of the default process (an obligor either defaults or survives), and the constant conditional default probability, the probability of having x number of defaults can be obtained by binomial distribution[61]:

$$\Pr\left[\# defaults = x \middle| Z = z\right] = \binom{N}{x}\left(\Pr\left[V \le k \middle| Z = z\right]\right)^x \left(1 - \Pr\left[V \le k \middle| Z = z\right]\right)^{N-x}$$

In order to obtain the unconditional default probability, we compute the expectation of the conditional probability over all the possible value of the common factor, Z[62]:

$$\Pr\left[\# defaults = x\right] = E\left[\Pr\left[\# defaults = x \middle| Z = z\right]\right]$$

Unconditional default probability can be computed without simulations

By exploiting the normality of the common factor distribution and by using the expression of the conditional default probability developed earlier we can write the explicit expression for the portfolio default distribution, which is equal[63] to:

$$\Pr\left[\# defaults = x\right] = \int_{-\infty}^{\infty} \binom{N}{x}\left(N\left[\frac{k - \sqrt{\rho}z}{\sqrt{1-\rho}}\right]\right)^x \left(1 - N\left[\frac{k - \sqrt{\rho}z}{\sqrt{1-\rho}}\right]\right)^{N-x} \phi(z)dz$$

The computation of the above expression **does not** require the adoption of a Monte Carlo simulation procedure. However, the integral must be solved numerically. This can be done relatively easily with the use of a typical spreadsheet application.

Computation of the associated loss distribution is straightforward. Given a constant recovery rate assumption, R, for the underlying credits in the portfolio, the probability of having a percentage portfolio loss L (following x defaults each with a loss-given-default of 1-R) is given by:

$$\Pr\left[Loss = L\right] = \Pr\left[\# defaults = x\right]$$

---

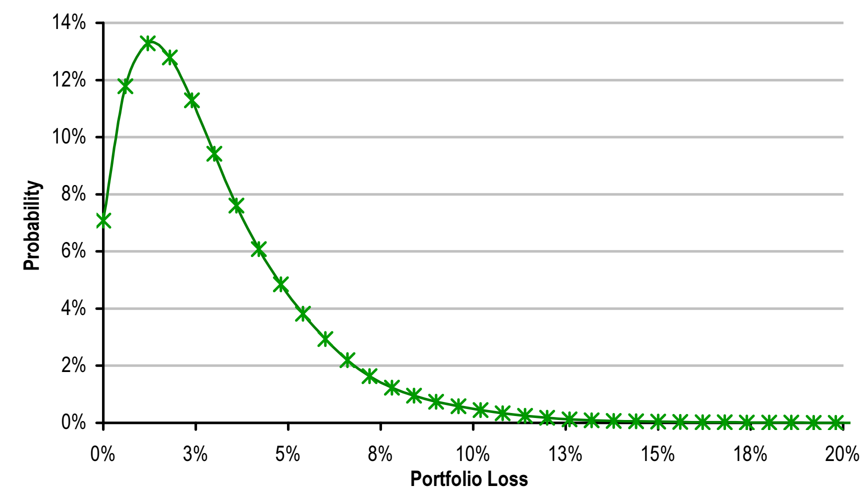
[61] In a sequence of n identical (homogeneous), independent trials, the binomial distribution is particularly useful in modeling the probability of having x success out of n trials. In our specific example, **homogeneity** is guaranteed by the constant default probability (among obligors) and **independence** is satisfied by the conditional independence of the firm value processes.

[62] Given two random variables X and Y, we denote the conditional expectation of X as $E\left[X \middle| Y\right]$. The unconditional expectation can be recovered by applying the following property: $E\left[X\right] = E\left[E\left[X \middle| Y\right]\right]$.

[63] We denote with φ the density function of the standard normal distribution.

Computing the above expression for different values of x (number of defaults, ranging from 0 to N number of credits) produces a chart similar to the one shown below:

Chart 137: Portfolio Loss Distribution



Source: Merrill Lynch. Assume a 100-name portfolio with an average 5y spread of 60bps, 40% recovery and 10% correlation. We numerically solved the involved indefinite integral by applying Gauss-Hermite quadrature.

Case 2: Infinite (Large) Size, Homogeneous Portfolio

Large pool approximation can simplify the computation

If the number of reference entities of the underlying portfolio becomes reasonably large (as it effectively is for a typical CDO), the expression for the portfolio loss probability, developed above, can be further simplified. This allows for computation of the portfolio loss distribution without resorting to either MonteCarlo simulations or numerical schemes to solve the associated indefinite integral.

This simplification is based on the assumption that, if the portfolio is relatively large, the expected fraction of credits of the underlying portfolio defaulting over a specific time horizon should be roughly equal to the individual default probability of the underlying credits (assumed constant due to homogeneity)

For a specific realization of the common factor, Z=z, assume A is the percentage fraction of defaulted credits within a large portfolio. Therefore:

$$\underbrace{E\left[A \middle| Z = z\right]}_{\text{Conditional Expected Default Fraction}} \rightarrow \underbrace{N\left[\frac{k-\sqrt{\rho}z}{\sqrt{1-\rho}}\right]}_{\text{Conditional Default Probability in a one-factor model}}$$

Note that we have substituted the expected portion of defaulted securities with the conditional individual default probability reported in equation 3.

Therefore, in order to derive the unconditional default probability, we have:

$$F(\alpha) = \Pr(A \le \alpha) = \Pr\left(N\left[\frac{k-\sqrt{\rho}Z}{\sqrt{1-\rho}}\right] \le \alpha\right) = \Pr\left(Z > \frac{k - N^{-1}(\alpha)\sqrt{1-\rho}}{\sqrt{\rho}}\right)$$

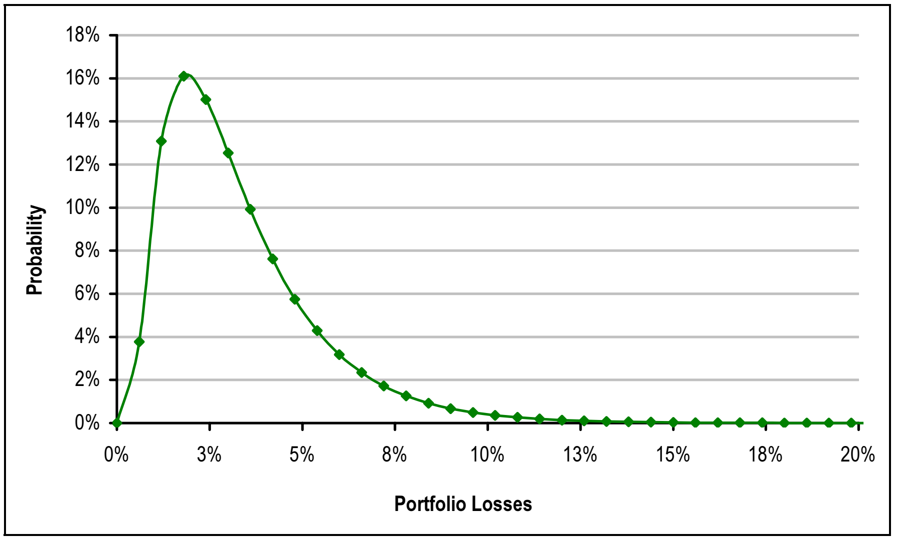
Given the symmetry[64] of the normal distribution of the common factor Z we then have:

[64] $Pr(Z>z)=Pr(Z<-z)$

$$F(\alpha) = \Pr\left(Z > \frac{k - N^{-1}(\alpha)\sqrt{1-\rho}}{\sqrt{\rho}}\right) = N\left[\frac{1}{\sqrt{\rho}}\left(N^{-1}(\alpha)\sqrt{1-\rho} - k\right)\right].$$

By computing the distribution function $F(\alpha)$ for different default fractions α we can construct the portfolio loss distribution whose chart is reported below:[65]:

Chart 138: Infinite Size Portfolio Loss Distribution



Source: Merrill Lynch. Assume a 100-name portfolio with an individual 5y default probability of 5% (corresponding to a default threshold of about –1.64) and 10% correlation.

Easy to implement but poor approximation in most applications

The big advantage of using the above equation relies in its simple implementation. Users can easily compute the portfolio loss distribution with the use of pre-defined functions in a typical spreadsheet application[66]. Nonetheless, this model presents some limitations, among which we stress:

- The model works only for very large portfolios (at least 100-200 credits). That means that it not possible to use this approach to evaluate basket default swaps (first-to-default, second-to-default, etc), which are usually written on smaller portfolios.
- Since the model deals with the portfolio default **fraction** (represented as a continuous event) rather than the **absolute** number of defaults (treated as discrete events), the model cannot produce consistent probabilities for low numbers of default (zero or one). This is evident by comparing the curve shape in Chart 137 and Chart 138. The probability of low defaults (zero or one) in the large portfolio approximation is not consistent with the (correct) corresponding figure produced in the finite size portfolio model.

■ Case 3: Finite Size Portfolio, Not Homogeneous

Diversified portfolios are the most difficult to model but also the most common

Finite size portfolios with a pool of diversified credits represent the common example in the CDO market. Unfortunately, they are also the most difficult to model. Computing the loss distribution for a pool of obligors with different individual default probabilities and recoveries is not an easy task and requires the introduction of rather advanced mathematical techniques to describe the complexity of the portfolio.

[65] It is possible to go from the default fraction distribution to the loss distribution, by fixing the size of the portfolio and the recovery (constant) of the underlying credits. For example, if we have a 125 names portfolio with a constant recovery of 35% and we are interested in computing the loss corresponding to a default fraction α=5% we have loss=5%*125*(1-0.35)=4.06%.
[66] In Excel, the user only has to use the built in normal distribution function or NORMSDIST() and the normal inverse or NORMSINV().

To this end, a common tool used to characterize the distribution of a generic random variable is the **characteristic function (CF).** Given a random variable ***X*** and a number *t* in the neighborhood of 0, we define the CF as $\psi(t) = E\left[e^{itX}\right]$, $i = \sqrt{-1}$. One of the biggest advantages of this approach is that the CF uniquely characterizes the distribution of the random variable. For example, the expectation (mean) of the random variable *X* is simply given by the first derivative of $\psi(t)$ evaluated at t=0.[67]

Advanced numerical techniques are needed to solve the problem

Therefore, provided we can obtain the CF of the portfolio loss, we can then derive its distribution function. In practice, deriving the distribution from the CF can be rather cumbersome. Fortunately, in our case, we can implement this via specific numerical techniques.

First we derive an expression for the portfolio loss CF ψ . Let LGD_1, $LGD_2,\ldots,LGD_n$ denote the notional exposure (loss) given default with respect to each credit in the portfolio[68]. The conditional CF of the total LGD for the portfolio is given by:

$$\psi_z(t) = E\left[e^{itLGD}\middle|Z\right] = E\left[e^{it\left(LGD_1+LGD_2+\ldots+LGD_n\right)}\middle|Z\right]$$

Using the independence[69] of the individual exposures LGD_I conditional on the realization of common factor *Z*, we have:

$$\psi_z(t) = E\left[e^{it\left(LGD_1+LGD_2+\ldots+LGD_n\right)}\middle|Z\right] = E\left[e^{itLGD_1}\middle|Z\right]\cdot E\left[e^{itLGD_2}\middle|Z\right]\cdot\ldots\cdot E\left[e^{itLGD_n}\middle|Z\right]$$

Given the individual **conditional** default probability $\Pr\left[V_i \le k_i \middle| Z = z\right] = p_z(i)$, for each obligor, the random variable LGD_i will be equal to $(1-R_i)$ with probability $p_z(i)$ if there is a default, and equal to 0 with probability 1- $p_z(i)$ if no default occurs. Hence, with respect to the generic product term $E\left[e^{tLGD_i}\middle|Z\right]$ we have:

$$E\left[e^{itLGD_i}\middle|Z\right] = e^{it(1-R_i)}p_z(i) + e^{it(1-R_i)\cdot 0}\cdot\left[1-p_z(i)\right]$$

Combining the two equations above, we have:

$$\psi_z(t) = \prod_{i=1}^{n} 1 + p_z(i)\cdot\left[e^{it(1-R_i)}-1\right]$$

The **unconditional portfolio CF** is then obtained by averaging the above expression over all the possible values of the common shock *Z*, yielding to the following expression:

$$\psi(t) = E\left[\psi_z(t)\right] = \int_{-\infty}^{\infty}\prod_{i=1}^{n} 1 + p_z(i)\cdot\left[e^{it(1-R_i)}-1\right]\varphi(z)dz$$

As in the previous cases, the integral must be solved numerically. Furthermore, from the portfolio CF we then need to derive the expression for the portfolio loss distribution.

---

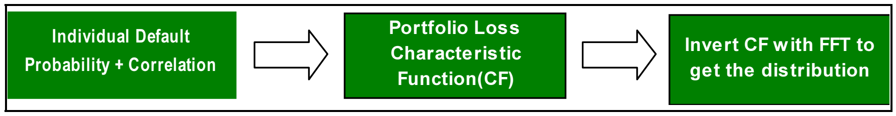
[67] $E[X] = \frac{1}{i}\frac{\partial\psi(t)}{\partial t}\bigg|_{t=0}$

[68] Since the credits are not homogeneous, we assume that each individual credit has its own specific default probability and recovery rate R_i assumption.

[69] Given two <u>independent</u> random variables *X* and *Y*, we have $E[XY] = E[X]\cdot E[Y]$.

This can be implemented by using the **Fast Fourier Transform** (FFT) technique[70].

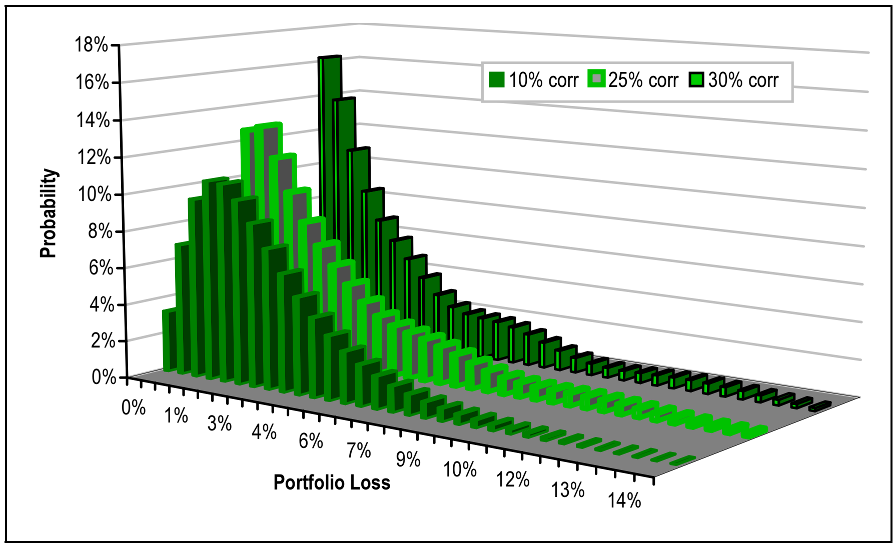
Chart 139: Fast Fourier Transform approach to compute the loss distribution



Source: Merrill Lynch

As an example let us consider the portfolio of 125-credit comprising the CDX.NA.IG index. In the chart below we consider the portfolio loss distribution for a 5y[71] period using three different values of the (constant) common factor correlation.

Chart 140: Portfolio Loss Distribution computed via the FFT approach



Source: Merrill Lynch. CDX.NA.IG members CDS spread as of August 31st 2004. Average spread equal to 64.5 bps. Assume a recovery of 40% and flat individual credit curves centered on the 5y CDS quotes.

Leaving aside the computation complexity, this approach represents an extremely powerful tool for pricing CDO written on diversified portfolios with a high degree of accuracy and speed. Assuming uniform recovery, 125 names CDO can be priced within a few seconds. When compared to a traditional MonteCarlo approach, even the sensitivity parameters (to spreads and correlation shocks) can be computed in a relatively short time.

As an alternative to the FFT approach outlined above, a simple **tree model** can efficiently be used to compute the portfolio distribution of a pool of correlated assets.

A simple tree representation can efficiently describe the default process of a credit portfolio

To this end let us start with a portfolio referencing three credits, and assume that we know the specific realization, z, of the systemic factor *Z*. The conditional default probabilities are given by p_z^1, p_z^2 and p_z^3.

[70] For a description of the properties and the application of Fast Fourier Transform we refer to Press et al. (1992) "*Numerical Recipes in C: The Art of Scientific Computing*", Cambridge University Press. Additional references for the application of the FFT in finance are provided at the end of the report.

[71] Any loss distribution will depend on the specific time frame considered. In our example, given the 5y CDS quotes, we compute the implied 5y default probabilities for each reference entity and plug them into the valuation model.

Let us then denote with $\Pi^{k}_{n,z}$ the conditional aggregate probability of having k defaults in an n-credit portfolio.

In order to fully understand the valuation mechanism, let us a start with a simple portfolio which includes only one credit (for example credit 1). As expected, given this simple portfolio, the probability $\Pi^{1}_{1,z}$ of having one default equals p^{1}_{z} whereas the probability $\Pi^{0}_{1,z}$ of having no default is equal to $1 - p^{1}_{z}$.

Let us now include in our initial simple portfolio an additional credit (for example credit 2). For a two-credit portfolio, three outcomes are possible:

1. No default;
2. One credit defaults (either credit 1 or credit 2);
3. Both credits default (credit 1 and credit 2).

Computing the probability of having either zero or two defaults is trivial. Given the conditional independence of the default processes for the two credits, these probabilities are equal to $\left(1 - p^{1}_{z}\right)\cdot\left(1 - p^{2}_{z}\right)$ and $p^{1}_{z} \cdot p^{2}_{z}$ respectively.

The probability of having one default can be computed by taking into account the following two potential scenarios:

- The first credit defaults and the second does not
- The second credit defaults and the first does not.

The first scenario has a probability of occurrence equal to $\Pi^{1}_{1,z} \cdot \left(1 - p^{2}_{z}\right)$, which is computed by multiplying the probability of the first original credit defaulting whilst having the new asset surviving.

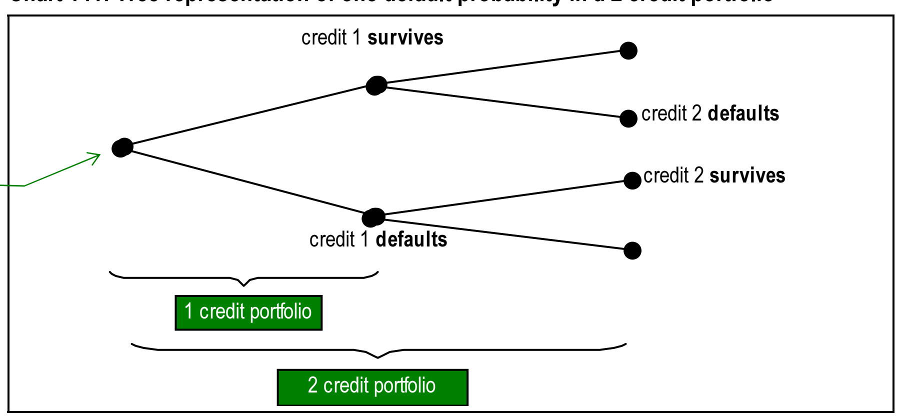
The second scenario probability is given by $\Pi^{0}_{1,z} \cdot p^{2}_{z}$, which is the product of the survival probability of the first original asset and the default probability of the second new asset.

This tree represents the default processes of the two credits.

*The probability of having **one default** is obtained by summing the probability of two scenarios:*

1. *Credit 1 **defaults** / Credit 2 **survives***
2. *Credit 2 **defaults** / Credit 1 **survives***

Chart 141: Tree representation of one default probability in a 2 credit portfolio



Source: Merrill Lynch

Clearly the two scenarios above are mutually exclusive and hence we can sum up the probabilities of occurrence for the two scenarios, yielding to:

$$\Pi^{1}_{2,z} = \Pi^{1}_{1,z} \cdot \left(1 - p^{2}_{z}\right) + \Pi^{0}_{1,z} \cdot p^{2}_{z}$$

This expression, therefore, represents the probability of one credit defaulting in a two-credit portfolio.

Let us extend this analysis and now include a third credit (i.e. credit 3) in our current two-credit portfolio. At this point we have four possible outcomes:

1. No default;
2. One credit defaults (either credit 1, credit 2 or credit 3);
3. Two credits default (either credit 1 and 2, credit 1 and 3 or credit 2 and 3);
4. All three credits default.

The probabilities of no defaults or all three credits defaulting (outcomes 1 and 4) are trivial to compute. Deriving the default probabilities of outcomes 2 and 3 require particular attention. The probability of having one default will be given by the sum of the probabilities of these two following scenarios:

- One default in our original two credit portfolio and no default for the third additional security
- No default in the initial two credit portfolio and a default for the third new asset

The first scenario has a probability of occurrence equal to $\Pi_{2,z}^{1} \cdot \left(1 - p_z^3\right)$ whereas the second scenario has a probability of $\Pi_{2,z}^{0} \cdot p_z^3$. Therefore the probability of one default for our three name portfolio is equal to:

$$\Pi_{3,z}^{1} = \Pi_{2,z}^{1} \cdot \left(1 - p_z^3\right) + \Pi_{2,z}^{0} \cdot p_z^3 .$$

We can use a similar argument to derive the probability of two defaults in a three-credit portfolio (outcome 3) as follows:

$$\Pi_{3,z}^{2} = \Pi_{2,z}^{2} \cdot \left(1 - p_z^3\right) + \Pi_{2,z}^{1} \cdot p_z^3 .$$

A recursive algorithm allows to quickly computing the conditional default distribution…

Based on our example we can then extend our analysis to any generic credit portfolio by establishing a general rule which allows the user to compute the conditional portfolio default distribution using the following recursive algorithm:

$$\Pi_{n+1,z}^{k} = \Pi_{n,z}^{k} \cdot \left(1 - p_z^{n+1}\right) + \Pi_{n,z}^{k-1} \cdot p_z^{n+1} .$$

The unconditional distribution is then recovered by averaging the conditional default probability over all the possible value of the (normally distributed) common factor *Z*:

$$\Pi_{n+1}^{k} = E\left(\Pi_{n+1,z}^{k}\right) = \int_{-\infty}^{+\infty} \Pi_{n+1,z}^{k} \varphi(z) dz$$

…the unconditional distribution is obtained by averaging over the possible values of the common factor

where $\varphi()$ is the standard normal density function. The above integral can be computed using any standard numerical integration scheme.

As a practical application, let us consider a homogeneous portfolio of five reference entities, each one with a 5y CDS spread set at 50bps and recovery equal to 35% respectively.

The steps needed to compute the portfolio default distribution can be outlined as follows:

1. Fix a specific realization z for the common factor Z
2. Given $Z=z$, compute the conditional individual default probabilities p_z^i for each name in the portfolio
3. Establish the recurrence algorithm as follows:
   - 3.1. Form a two asset portfolio and compute the probability of having zero, one and two defaults
   - 3.2. Given the previous two asset portfolio, add a new credit, form a three-asset portfolio and compute the probability of having zero, one, two and three defaults using the information in 3.1

3.3. Repeat the portfolio construction procedure until all the credits are included in the analysis

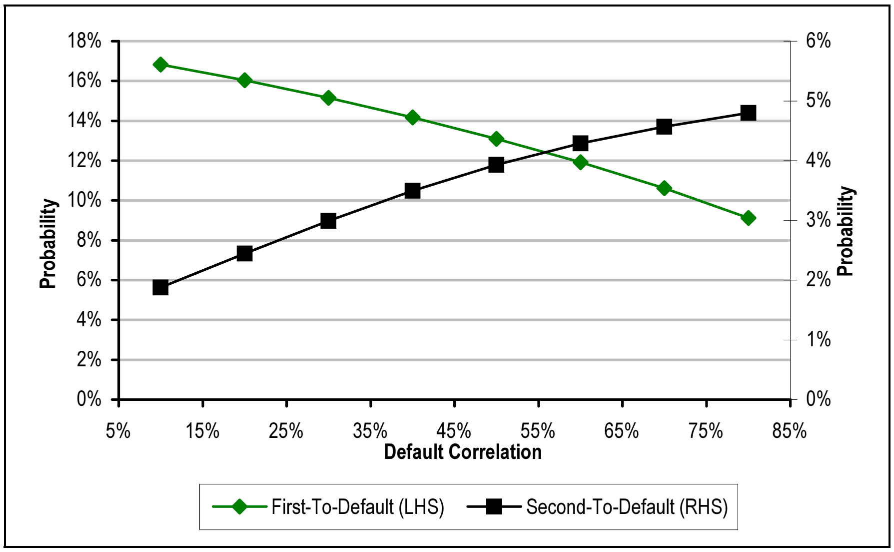
4. Repeat step 3 for all the possible values of the common factor Z and compute the average by using the normal density function of Z.

Once the default distribution is obtained, it is then possible to compute the probability of having at least k defaults by simply adding the relative probabilities. For example, in a five-credit portfolio, the first-to-default (at least 1 default) and second-to-default (at least two defaults) probabilities will be given by the following two expressions:

$$P^{FTD} = \sum_{k=1}^{5} \Pi_5^k = 1 - \Pi_5^0 \, , \; P^{STD} = \sum_{k=2}^{5} \Pi_5^k = 1 - \sum_{k=0}^{1} \Pi_5^k \, .$$

In Chart 142 we show the first and second-to-default distribution for the above five-credit portfolio.

Chart 142: First and Second-to-Default Probability using a Recursive Scheme



Source: Merrill Lynch.

Single Tranche Synthetic CDO Pricing

The Tranche Loss Function

In order to evaluate the fair spread of a specific CDO tranche, we need to introduce the concept of a **tranche loss function** and relate it to the portfolio loss experienced within a generic time interval.

If a certain percentage portfolio loss l occurs, the impact on the tranche holder's position will be driven by the attachment and detachment point of the tranche. If the attachment (lower threshold) point L^- is 3% and the detachment (upper threshold) L^+ is 7%, the tranche loss can be represented as given in the following table:

If losses on the portfolio are equal to 6%, tranche holders bear a 3% loss (75% of the capital invested)

If losses on the portfolio are lower than the tranche attachment point, tranche loss is zero

If losses on the exceed 7% tranche losses are capped to 4% (100% of the amount invested) since the investor covers only the 3-7% range

Table 42: 3-7% Tranche Loss Profile

Portfolio Loss	Portfolio Loss Absorbed by the Tranche (A)	Tranche Loss = (A) / Tranche Width
0%	0%	0%
1%	0%	0%
2%	0%	0%
3%	0%	0%
4%	1%	25%
5%	2%	50%
6%	3%	75%
7%	4%	100%
8%	4%	100%
9%	4%	100%
10%	4%	100%

Source: Merrill Lynch. We define the **tranche width** as the difference between the upper and lower thresholds.

Formally we can write down the expression of the tranche loss function as follow:

$$TL_t^{L^-,L^+}(l) = \frac{\max\left[\min\left(l, L^+\right) - L^-, 0\right]}{L^+ - L^-}$$

At any time t, this function, given any portfolio loss l, will provide the corresponding loss suffered by the tranche holder. For example let us suppose that the lower and upper tranche exhaustion points are L^-=3% and L^+=7% respectively. If the portfolio loss $l=5\%$, then the loss absorbed by the tranche is equal to 2% and, therefore, the tranche loss is equal to 2%/(7%-3%)=50%. The expected tranche loss is crucial to determine the equilibrium spread of a tranche. It is obvious that the higher the expected tranche loss, the higher the spread required by the tranche buyer.

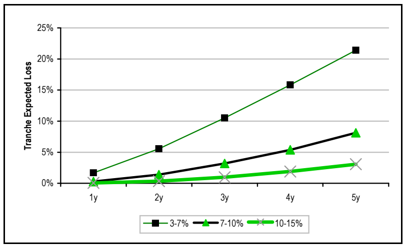
Within the framework outlined for factor models, computing the **expected tranche loss** at each point of time, is straightforward. For each portfolio loss l, we only need to multiply the tranche loss function by the corresponding **loss probability derived by applying a factor model**.

Table 43: Expected Loss for a 3-7% tranche

Portfolio Loss (less than)	Loss Probability	Tranche Loss	Tranche Loss x Frequency
3%	5.4%	0%	0.00%
4%	3.8%	25%	0.95%
5%	2.7%	50%	1.37%
6%	2.0%	75%	1.52%
7%	1.5%	100%	1.51%
…	…	…	…
…	…	…	…
59%	0.0%	100%	0.00%
60%	0.0%	100%	0.00%
Cumulative Expected Tranche Loss over 5y			**18.75%**

Source: Merrill Lynch. Assume 40% recovery

Chart 143: Tranche Expected Loss



Source: Merrill Lynch. We assume a 125 credits portfolio, average spread of 60bps, 40% recovery and 22% correlation.

Formally, we define the tranche expected loss at a given time t as follows:

$$E\left[TL_t^{L^-,L^+}\right] = \sum_l TL_t^{L^-,L^+}(l) * \Pr\left[Loss = l\right].$$

The next step consists of determining the expected tranche loss for any given time. This step is crucial in computing the tranche breakeven spread, which, depends on the outstanding notional at each payment date[72].

By applying the formula above for each year for a generic 5 years period, assuming annual payments, we obtain a chart similar to the one reported in Chart 143.

The tranche premium and default legs

Given the expected term structure of the tranche loss distribution, it is then possible to proceed with the estimation of the breakeven spread. Similar to the more traditional single name CDS contracts, an unfunded single tranche synthetic CDO can be represented by two legs:

- **Premium Leg:** it represents the premium payments, paid periodically by the protection buyer to the tranche holder (protection seller). Unlike a vanilla CDS where premium payments cease upon the default of the reference entity, the premium in a single tranche synthetic CDO continues to be paid on the amortized outstanding notional.
- **Default Leg**: it represents the cash flows exchanged upon each sequential default of the credits included in the reference pool. Upon the occurrence of a credit event, given the recovery of each defaulted entity, the original notional of the tranche is reduced by the loss given default of the defaulted entity[73]. Moreover, the tranche holder (protection seller) pays to the protection buyer an amount which depends on the recovery and the exposure with respect to the defaulted obligor.

With regard to the premium leg, we can formally express it as[74]:

$$PL^{L^-,L^+} = \sum_{t=1}^{n} spread \cdot Discount\ Factor(t) \cdot \left\{Notional - E\left[TL_t^{L^-,L^+}\right]\right\}.$$

This expression shows that the premium leg is computed by discounting back the tranche premium adjusted for the notional outstanding at each payment date.

As far as the default leg is concerned, the computation effort is heavier, since theoretically the default leg should be computed by averaging the present value of payments over all the set of possible default times. In practice a discrete set of default dates is chosen (daily, monthly or yearly spaced) and it is assumed that a default can occur only upon the specified set of default dates. If we impose that a default can occur only upon the set of payment dates, we can then write down the expression of the default as follows:

$$DL^{L^-,L^+} = \sum_{t=1}^{n} Discount\ Factor(t) \cdot \left\{E\left[TL_t^{L^-,L^+}\right] - E\left[TL_{t-1}^{L^-,L^+}\right]\right\}$$

[72] We define the outstanding tranche notional as the original notional invested reduced by the (relevant) losses accumulated in the portfolio.
[73] The original notional is reduced only if the cumulative losses exceed the tranche attachment point. If the portfolio cumulative losses are lower than the attachment point, the tranche notional is not affected.
[74] n is the total number of premium payments, which depends on the maturity of the tranche and the payment frequency. As an example, for a 5y tranche with quarterly payments, the total number of payments will be equal to 20.

The default leg is therefore expressed as the discounted value of the marginal tranche loss[75] over each payment interval.

The breakeven spread is computed by equating the premium and the default tranche legs

The breakeven spread is then computed by exploiting the initial equivalence relationship between the premium and default leg i.e. compute breakeven spread, Spread* such that: $Spread^* \Rightarrow \text{Premium Leg} = \text{Default Leg}$

Therefore we obtain:

$$Spread^* = \frac{\sum_{t=1}^{n} Discount\ Factor(t) \cdot \left\{ E\left[TL_t^{L^-,L^+}\right] - E\left[TL_{t-1}^{L^-,L^+}\right] \right\}}{\sum_{t=1}^{n} Discount\ Factor(t) \cdot \left\{ Notional - E\left[TL_t^{L^-,L^+}\right] \right\}} \tag{4}$$

The tranche expected loss is the key ingredient in the pricing formula

We can summarize the steps required to determine the tranche fair spread as follows:

1. Given the complexity of the initial portfolio (size and diversity) apply the appropriate factor model to compute the **portfolio loss distribution.** For any portfolio loss bucket, this distribution will provide the corresponding probability.
2. Given the attachment and detachment point of the tranche, compute the tranche loss function for all the relevant portfolio loss levels.
3. Combine the probability obtained in step 1 with the tranche loss function determined in step 2. to calculate the expected tranche loss at a specific time *t*.
4. Repeat 1., 2. and 3. for all the time points required.
5. Given the term structure of the expected tranche loss, compute the risk free discount factors and apply equation (4) to obtain the breakeven spread.

The above algorithm is efficiently shown in Table 44. The final step is the calculation of the breakeven spread. This is done by dividing the default leg value (5.4%) by the premium leg[76] (4.22) yielding a 7-10% tranche breakeven spread of 128 bps.

The marginal loss between 1y and 2y is given by the difference of the 2y and 1y cumulative losses, i.e. 0.7%=0.8%-0.1%

PV of 1bp multiplied by the outstanding notional

The outstanding notional is given by the difference of 100% and the cumulative expected loss, i.e. 96%=100%-4.1%

Table 44: Calculation Prospectus for a 7-10% tranche

Payment Dates	Cumulative Expected Loss	Discount Factor	Marginal Expected Loss	Outstanding Notional	PV of Premium Leg	PV of Default Leg
1y	0.1%	0.9524	0.1%	100%	95%	0.1%
2y	0.8%	0.9070	0.7%	99%	90%	0.6%
3y	2.1%	0.8638	1.3%	98%	85%	1.1%
4y	4.1%	0.8227	2.0%	96%	79%	1.6%
5y	6.5%	0.7835	2.4%	93%	73%	1.9%
Premium and Default Legs Values					**4.22**	**5.4%**

Source: Merrill Lynch. Assume a 125 credits portfolio, average spread of 60bps, 40% recovery and 18.8% correlation. Annual payments on a 30/360 day-count basis are assumed.

[75] For any payment period (*t-1,t*), we define the marginal loss as the additional loss between time *t-1* and time *t*. For instance, if the cumulative losses after 3 and 4 years are 18% and 25% respectively, the marginal loss specific to the *(3y,4y)* time interval is equal to 25%-18%=7%.

[76] In the table above the premium leg is computed by using a notional spread of 1bp.

Pricing a Basket Default Swap

Once the default distribution is computed for all the relevant time points required[77], the valuation of a first-to-default (or any other combination such as second-to-default, third-to-default, etc.) closely mimics the pricing procedure of a traditional vanilla CDS.

A basket default swap can be decomposed into two legs…

An unfunded basket default swap can be decomposed into two legs:

- **Premium Leg:** it represents the premium payments, periodically paid by the protection buyer to the basket holder (protection seller) either until the maturity of the contract or until a default event is triggered, if earlier.
- **Default Leg**: it represents the cash flows exchanged following a credit event. If the basket seniority is *k* (i.e. *k*=1 for first-to-default, *k*=2 for second-to-default) the premium payment will cease upon the k^{th} default in the reference pool and the contract will be settled (usually physically) based on the recovery value of the defaulted credit.

With regard to the premium leg, we can formally express it as[78]:

$$PL^{KTD} = \sum_{i=1}^{n} spread \cdot \alpha_i \cdot D(t_i) \cdot \left[1 - P^{KTD}(t_i)\right].$$

where $P^{KTD}(t_i)$ represents the probability of having at least *k* defaults from the current valuation date to the generic payment date t_i, $D(t_i)$ the discount factor, α_i is the number of accrual days within each payment period and *spread* is the premium of the basket. The above expression shows that the premium leg is computed by discounting the basket premium and weighting each term by the corresponding "**basket survival probability**".

As far as the default leg is concerned, the computational effort is heavier, since theoretically the default leg should be computed by averaging the present value of payments over all the set of possible values for the k^{th} default time.[79] As an approximation to the actual value let us consider a discrete set of dates (daily, monthly or yearly spaced) and assume that the relevant k^{th} default occurs halfway through each payment period. Given this assumption we can then write down the expression of the default as follows[80]:

$$DL^{KTD} = (1-R)\sum_{i=1}^{n} \alpha_i \cdot D\left(\frac{t_i + t_{i-1}}{2}\right) \cdot \left[P^{KTD}(t_i) - P^{KTD}(t_{i-1})\right]$$

[77] If the maturity of the basket is 5 years and the premium is paid on a quarterly basis, then the default distribution has to be computed for each of the 20 quarterly payment dates.
[78] *n* is the total number of premium payments, which depends on the maturity of the contract and the payment frequency. As an example, for a 5y deal with quarterly payments, the total number of payments will be equal to 20.
[79] Assuming homogeneous recovery, the value of default leg can be written as:

$$DL^{KTD} = (1-R) \cdot \int_0^T D(t) dP^{KTD}(t)$$

[80] Please refer to Volume 1, Chapter 2 for the derivation of the expression of the default and premium legs of a vanilla CDS.

The above expression expresses the default leg as the discounted value of the loss given default (based on the recovery value *R*) weighted by the "**basket marginal default probability**"[81] over each payment interval.

The breakeven spread is computed by equating the premium and the default basket legs

The breakeven spread is then computed by exploiting the initial equivalence relationship between the premium and default leg:

$$Spread^* \Rightarrow \text{Premium Leg} = \text{Default Leg}$$

Thus we obtain:

$$Spread^* = \frac{(1-R)\sum_{i=1}^{n}\alpha_i \cdot D\left(\frac{t_i+t_{i-1}}{2}\right)\cdot\left[P^{KTD}(t_i)-P^{KTD}(t_{i-1})\right]}{\sum_{i=1}^{n}\alpha_i \cdot D(t_i)\cdot\left[1-P^{KTD}(t_i)\right]}.$$

The default distribution is the key ingredient in the pricing formula

The denominator in this equation represents the present value of 1bp or the risky DV01 of the basket.

We can now summarize the steps needed to determine the basket breakeven or fair spread as follows:

1. Apply the tree-based algorithm illustrated earlier to compute the portfolio default distribution.
2. Given the seniority *k* of the basket, compute the k^{th}-to-default survival/default probabilities.
3. Repeat steps 1 and 2 for all the time points (i.e. payment dates) required.
4. Given the term structure of the k^{th}-to-default survival/default probabilities, compute the risk free discount factors and apply the formula above to obtain the breakeven spread.

Assuming annual premium payments, the above algorithm is outlined in the table below:

FTD probabilities are computed using the tree based model explained earlier

Given the marginal default probability, the default leg between 1y and 2y is obtained by discounting back the loss given default, i.e.

Default leg (1y,2y)=

(95%-90%)x0.94x(1-35%)=2.991%

Table 45: Calculation Prospectus for a First-to-Default Basket

Payment Dates	(1) FTD Survival Probability	(2) Discount Factors	(1) x (2) Risky Discount Factors	Default Leg
1y	95%	0.97	0.92	3.469%
2y	90%	0.94	0.84	2.991%
3y	85%	0.92	0.78	2.659%
4y	81%	0.89	0.72	2.391%
5y	77%	0.86	0.67	2.163%
Risky DV01 and Default Leg			**3.93**	**14%**

Source: Merrill Lynch. Assume a 5-credit portfolio, average spread of 80bps (25,50,75,100 and 150bps respectively), 35% recovery and 30% correlation. Annual payments on a 30/360 day-count basis are assumed.

The spread is then computed by dividing the default leg value (14%) by the premium leg[82] (3.93) yielding to a breakeven spread of about 350bps.

[81] For any payment period (t_{i-1}, t_i), we define the **marginal basket default probability** as the probability that the k-th default happens between time t_{i-1} and time t_i. For instance, if the cumulative default probability between the third and fourth year are 18% and 25% respectively, the marginal basket default probability specific to the *(3y,4y)* time interval is equal to 25%-18%=7%.

[82] In the table above the premium leg is computed by using a notional spread of 1bp.

The mark-to-market of an existing transaction depends on the breakeven spread and the DV01 of the basket

Given the breakeven spread and the risky DV01 of the basket, the computation of the **mark-to-market of an existing transaction** is relatively straightforward. For example, let us assume that an investor sold 6y protection on a first-to-default basket one year ago on a notional of $10,000,000 at an annual contractual spread of 380bps. If valued as of today (with 5y maturity), the current mark-to-market will be simply given by the resulting annuity (old contractual spread minus current breakeven spread) multiplied by the current 5y risky DV01, that is, (380bps – 350bps) x $3,930= $117,800.

Hedging a basket default swap

Pricing the basket represents the first step in managing the risk

The initial breakeven spread of the basket represents a crucial point in the investment decision process. However as time evolves, the value of the basket will fluctuate, reflecting changing economic conditions. For example, as the market perception on the credit worthiness of any underlying credit changes, so will the breakeven basket spread thus affecting the mark-to-market position of each counterparty. To neutralize (small) changes in the CDS spreads and their reflection on the mark-to-market of the trade, a possible strategy involves the purchase/sale of each the underlying CDS. The quantity of protection one would have to buy or sell on each single name CDS is commonly measured by delta (Δ)[83]. In the rest of the section we will illustrate how, under a set of simplifying assumptions, one can derive semi-analytical expressions for deltas.

Deltas relate mark-to-market volatilities of the basket and the hedging portfolio

To this end let us rewrite the current value of a generic default swap as a function of the current spread c_i of the i^{th} credit, as follows:

$$V^{KTD}(c_i) = s^{KTD} \cdot PL^{KTD}(c_i) - DL^{KTD}(c_i)$$

where s^{KTD} is the contractual coupon of the swap. Within this context, the delta is defined as the **nominal amount of protection on the i^{th} credit one would have to hold in order to offset the variation in the value of the default swap** due to the change in the spread c_i , i.e.

$$dV(c_i)^{KTD} = \Delta_i \cdot dV(c_i)^{CDS(i)}$$

A common approach to evaluate the delta consists in shifting the credit spreads on each individual name in the basket portfolio by a small amount (1-10bps), and then calculating the resultant mark-to-market change of the basket and the corresponding individual CDS. This can prove to be a frustrating exercise especially if a MonteCarlo based model is adopted. Alternatively, using basic results from ordinary calculus[84] it is possible to evaluate the change in the value of the k^{th}-to-default swap due to a small (for example 1bp) move in the individual CDS spread c_i. directly. In particular:

$$V^{KTD}(c_i + 1bp) - V^{KTD}(c_i) \approx \frac{\partial V^{KTD}(c^*)}{\partial c_i} \cdot 1bp$$

$$V^{CDS}(c_i + 1bp) - V^{CDS}(c_i) \approx \frac{\partial V^{CDS}(c^*)}{\partial c_i} \cdot 1bp$$

where we set $c^* = c_i + \frac{1}{2} bps$

[83] Please refer to Volume 2, Chapter 3, for an explanation of delta-hedging applied to basket default swaps.

[84] Under certain technical conditions, given a suitable function *f(x),* we have

$$f(x+h) - f(x) \approx \frac{\partial f(c)}{\partial x} h$$ where *c* is some value between *x* and *x+h.*

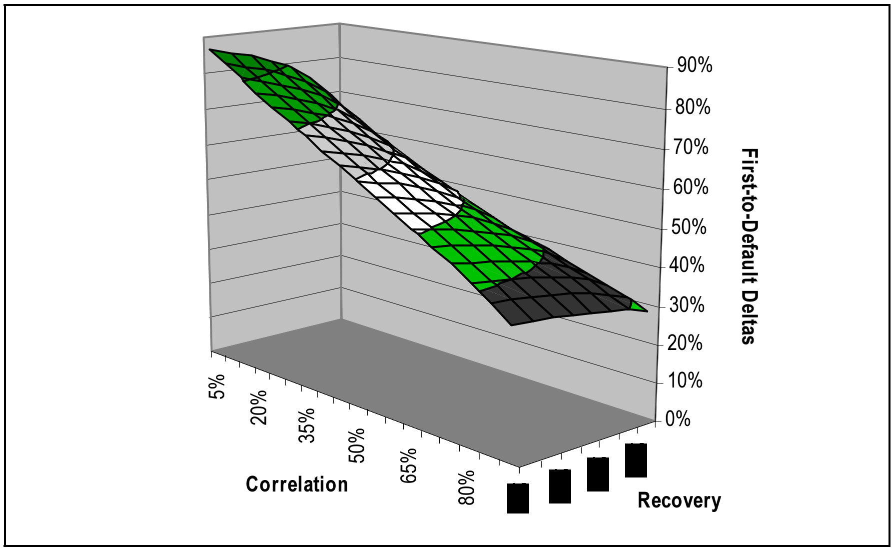
Under this specification the delta takes the following form:

$$\Delta_i = \frac{\dfrac{\partial V^{KTD}(c^*)}{\partial c_i}}{\dfrac{\partial V^{CDS}(c^*)}{\partial c_i}}$$

First-to-default delta decreases for higher correlation values as more risk is allocated to the senior part of the capital structure

In the following chart we show how deltas behave for a typical 5y first-to-default basket for a range of different values of **correlation** and **recovery**. As expected, deltas decrease with respect to correlation, since, for higher correlation values the likelihood of having multiple defaults increases and, therefore, risk is transferred from the junior part (first-to-default) to the more senior portion (second-to-default) of the portfolio. The dependency on the recovery is less pronounced and we observe that, especially for lower correlation, a higher recovery rate decreases the basket delta[85].

Chart 144: First-to-Default Deltas vs. Correlation and Recovery



Source: Merrill Lynch. Assume a 5 name basket with an underlying average CDS Spread at 100bps and 5y maturity.

Under certain assumptions deltas can be computed quickly in a semi-explicit form

If a flat CDS term structure is chosen it is then possible to **further simplify** the above expression and compute the partial derivative of the basket value function in a semi-explicit form.

[85] Let us recall the expression for the (risk neutral) default probability assuming a flat spread CDS term structure: $d_i = 1 - e^{\left(-\frac{spread_i}{1-R} \cdot t\right)}$. From this expression is clear that, for increasing recoveries, the default probability increases as well. This increased riskiness of the underlying portfolio, in turn, increases the expected number of defaults thus increasing the amount of protection (delta) to be allocated in the more senior part of the underlying portfolio.

9. What Correlation?

Leaving aside the philosophical debate on the "right" correlation (equity, spread, default or asset correlation) to use in pricing portfolio credit structures, market participants have agreed on a common way of communicating tranche prices by using the concept of implied compound correlation, i.e. use a model (the standard guassian copula) to back out the correlation parameter that best matches the tranche traded spread. Despite its intuitive meaning (especially when compared to the concept of implied volatility for equity or interest rate options) the compound correlation presents several drawbacks, chief among which is the non-uniqueness and non-existance for mezzanine tranches. In order to overcome these limitations, base correlation has emerged as the standard correlation parameter to exchange pricing information on standardized tranches.

In this chapter we introduce the concept of compound and base correlation and present some of the key issues in extending the concept of base correlation to price bespoke transactions.

Compound Correlation and Correlation Skew

Demand/supply technicals and model…

All variables that affect premiums of standardized tranches are typically known except the default correlation. Therefore, for given a market premium, an implied correlation (also knows as **compound correlation)** for a tranche can be derived via a model. The market standard is still the Gaussian Copula model.

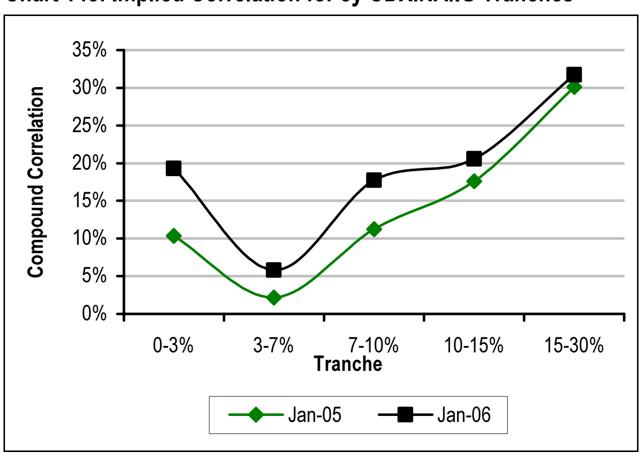
…gives rise to correlation skew

Theoretically all tranches should be trading at identical implied correlations since the underlying portfolio for all tranches is identical. However, in practice we observe different implied correlations for different tranches. This variation in implied correlation across tranches is known as the **correlation skew.**

Skew has also changed over time

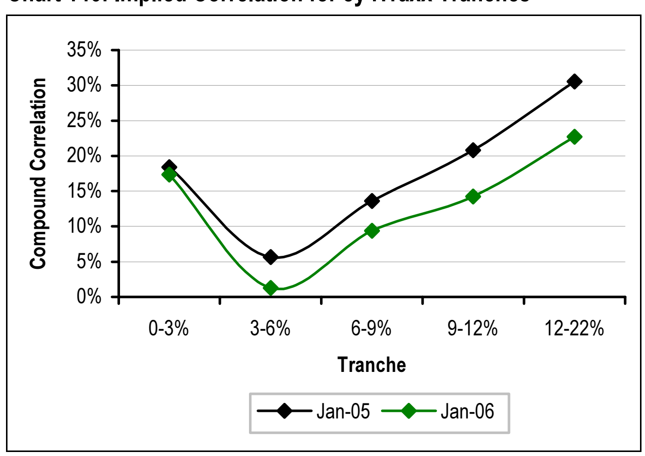
Chart 145 highlights the skew for tranches of the 5y CDX.NA.IG. The skew is sometimes also referred to as the "correlation smile" due to its shape. The chart also highlights that **implied correlations have changed significantly for the equity and mezzanine tranches over the last year**. We believe that the skew and its change over time is driven primarily by a combination of **demand/supply technicals** as well as the **model** that is used to price the tranches and calibrate the market data.

Chart 145: Implied Correlation for 5y CDX.NA.IG Tranches



Source: Merrill Lynch. Computations as of 10th January 2006.

Chart 146: Implied Correlation for 5y iTraxx Tranches



Source: Merrill Lynch. Computations as of 10th January 2006.

Demand & Supply

Preference for particular tranches of risk has created demand/supply technicals that have been the key drivers of correlation skew.

Less preference for equity and super-senior

- **Equity tranche**: Equity or first-loss tranches are exposed to significant instantaneous default risk (or 'idiosyncratic' risk). In order to compensate for this risk, investors tend to demand higher premiums (or lower implied correlations, other things being equal) to make the tranche attractive. The correlation shakeout occurred in May 05 has increased the idiosyncratic risk perception even further, resulting in a significant drop in the implied correlation quoted in the market over the last 6 months.
- **Super-senior tranche**: Spread tightening has lowered the relative attraction of super-senior tranches. In addition, super-senior investors always seek a minimum coupon of about 10bps for taking on catastrophic risk even though from a pricing perspective the super-senior tranche does not have much value. Increasing tranche premiums to make them more attractive leads to higher implied correlations, other things being equal.

- *More preference for mezzanine tranches to hedge bespoke risk*

- **Mezzanine/senior tranche**: The standardized tranche market has been dominated by dealers who have used these tranches to hedge their exposure to bespoke single-tranches. Since most bespoke tranches are typically mezzanine/senior in nature, dealers who have bought protection on these bespoke tranches have hedged their risk by selling protection on mezzanine/senior standardized tranches. As a result implied correlations are relatively lower for these tranches and have also fallen over time with the issuance of more bespoke tranches.

Model Based

Gaussian copula is standard model

A Gaussian copula model is the standard model that is used by the market to calibrate the tranche market spreads. Its limitations are also responsible for the existence of the correlation skew.

- **Expected defaults**: Using the Gaussian copula model, we would expect the 5y CDX.NA.IG portfolio to experience about 4 defaults over 5 years (assume 40% recovery). The 0-3% tranche should fully absorb this loss[86] without eating into the mezzanine tranche. As a result, the model makes the mezzanine and senior tranches look more attractive and tends to lower implied correlations for these tranches.
- **Flat correlation**: The use of flat correlation number in the Gaussian copula model tends to assign more value to the mezzanine tranche and very little value to the super-senior tranches. However, to attract investors, correlation needs to be skewed to provide more value to the super-senior tranche.

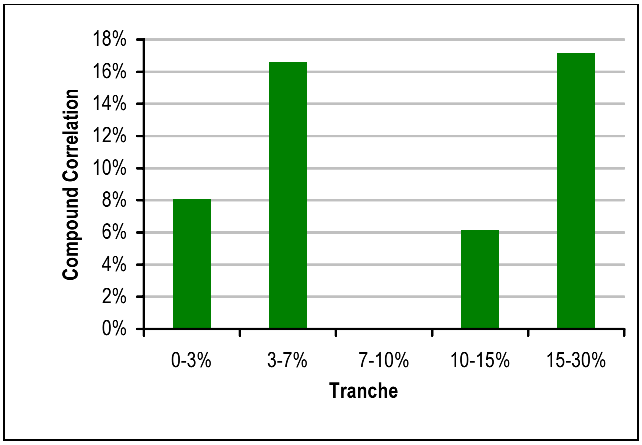
Given the level of expected defaults and the subsequent preference for mezzanine/senior bespoke tranches, we expect the skew to be centered around the standardized mezzanine tranche which is what we observe for the 5y and 10y tranches.

Skew for 10y tranches…

The use of a Gaussian copula model should generate similar skews for longer maturity tranches (e.g. 10y) or for different portfolios. Due to the larger number of expected defaults, we would expect the skew for the 10y standardised tranches to be shifted towards a higher subordination. We also expect a similar effect on 5y tranches on a portfolio with a wider average spread than the index.

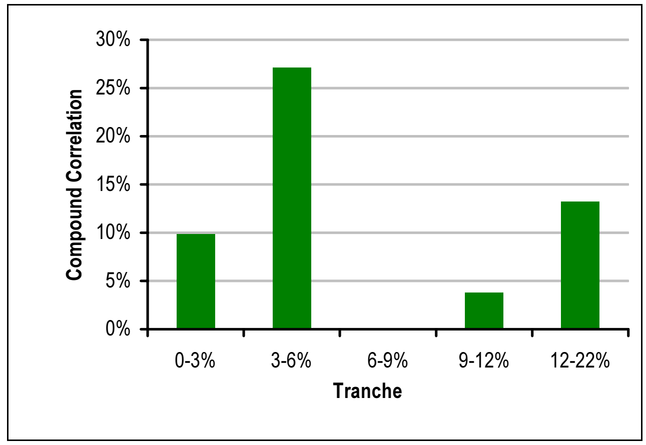
[86] Loss per default = (1-R)*Credit Notional = (1-0.4)*(1/125) = 0.6*0.8% =0.48%; Loss for 4 defaults = 4*0.48% = 1.92% < 3% detachment point of equity tranche.

Chart 147: Implied Correlation for 10y CDX.NA.IG Tranches



Source: Merrill Lynch. Data as of Jan, 24th 2006.

Chart 148: Implied Correlation for 10y iTraxx Europe Tranches



Source: Merrill Lynch. Data as of Jan, 24th 2006.

…highlights shortcoming of tranche implied correlation measure

Chart 147 and Chart 148 highlight the skew for 10y tranches. These charts **illustrate one of the problems of using this measure of implied correlation, i.e. the implied correlation may either not exist or have a non-unique solution.** Currently there are no admissible (non negative) correlation values that match the traded 10y 7-10% tranche.

Base Correlations

Base correlations are first-loss tranches corresponding to standardized detachment points

One of the methods that is commonly used in the market to address the above issues is the use of **base correlations**. Base correlations are defined as the implied correlations of the **first-loss** tranches where the attachment point of each tranche is zero and the detachment point is the detachment point of each of the standardized tranches. Table 46 highlights the tranche implied (compound) correlations and the implied base correlations for the standardized tranches.

Table 46: Base Correlations & Tranche Implied Correlations for 5y Standardized Tranches

	North America					Europe					
	Bid	Ask	Mid	Tranche Correlation	Base Correlation		Bid	Ask	Mid	Tranche Correlation	Base Correlation
0-3%	35 5/8	36 1/8	35.88%	10.18%	10.18%	0-3%	26 ¼	26 ¾	26.5%	12.7%	12.7%
3-7%	106	109.5	108	2.18%	25.07%	3-6%	83	86	84.5	3.57%	24.42%
7-10%	26.5	28.5	27.5	11.46%	33.24%	6-9%	27	30	28.5	12.08%	32.52%
10-15%	12	14	13	17.72%	44.15%	9-12%	12	14	13	16.78%	39.43%
15-30%	6	7	6.5	30.83%	67.10%	12-22%	5.75	6.75	6	24.89%	56.94%

Source: Merrill Lynch; As of 10th Jan 2006; All 0-3% tranches quoted on Upfront + 500bps running basis.

Overcomes the key limitations of tranche implied correlations

Base correlations overcome the key limitations of tranche implied correlations:

- Base correlations can be used to price tranches with customized attachment and detachment points of either standardized or bespoke underlying portfolios.
- Base correlations are unique across the capital structure.

Base correlations can be computed with relative ease using the same model that is used to derive tranche implied correlations. We also highlight in the following sections how base correlations can be used to price non-standard tranches.

Deriving Base Correlations of Standardized Tranches

Use bootstrapping method to derive base correlations from market spreads

Base correlations can be derived directly from the market spreads of the standardized tranches using standard bootstrapping techniques. The base correlation of the first-loss 0-3% tranche is obviously the same as its tranche implied correlation. However, the 0-7% tranche can be derived in the following standard manner:

- Price the 0-7% tranche as a combination of the 0-3% and the 3-7% assuming both tranches have a premium equal to that of the 3-7% market premium.
- Price of 3-7% tranche using the 3-7% market premium is zero.
- Price of 0-7% tranche is therefore equal to the price of 0-3% with the same premium as the 3-7% market premium, using the 0-3% base correlation (also the tranche implied correlation in this case). The price will be positive since the 3-7% premium is smaller than the 0-3% market premium.
- Use this price for the 0-7% and the standard Gaussian Copula model to imply the base correlation of the 0-7% tranche.

In the same manner, we can imply the 0-10% base correlation by pricing the 0-10% tranche as a combination of the 0-7% tranche and the 7-10% assuming both tranches have a premium equal to that of the 7-10% market premium. This procedure is then repeated to derive 0-15% and 0-30% base correlations.

In order to illustrate the implementation of the above procedure, let us compute the base correlation for the 0-7% tranche. As explained above, we start by computing the price of the 0-3% tranche using the 3-7% tranche premium; this value, expressed in terms of the 0-3% notional, equals 49.19%.

..Scaling each intermediate step by the corresponding notional is crucial..

The next step consists in computing the implied 0-7% correlation which matches the value of the 0-7% tranche values using the 3-7% premium. With this regard, we first need to re-state the 0-7% tranche value by **scaling** the above number (the 0-3% mark-to-market using the 3-7% premium) by the appropriate notional. This can be easily done by multiplying the value of the 0-3% tranche with the 3-7% premium by the ratio of the width of the 0-7% tranche (7%) and the width of the 0-3% tranche (3%). The required calculations are reported below:

Table 47: Notional Adjusted Mark-to-Market

0-7% Tranche Value Notional Scaled	
0-3% Tranche Value using the 3-7% premium	49.19%
Width Ratio	3% / 7% = 0.43x
0-7% Tranche Value using the 3-7% premium	21.15%

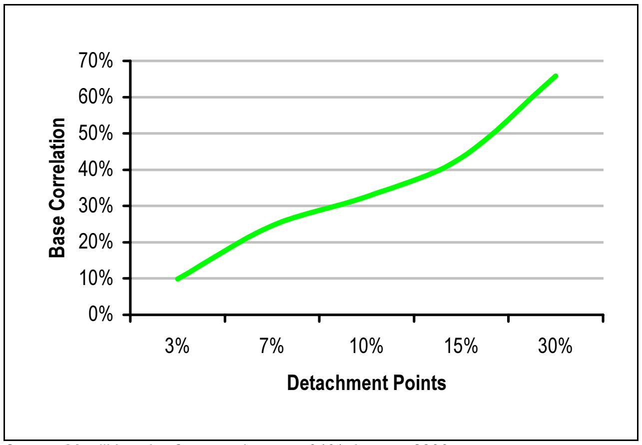
Source: Merrill Lynch. Computations as of 10th January 2006.

0-7% Notional Scaled Value
49.19% x (3/7)=21.15%

Once the notional-adjusted 0-7% mark-to-market has been computed, we then apply the standard Gaussian Copula Model to find the value of correlation matching the tranche value. This can be computed by a simple trial-and-error procedure yielding to an implied figure equal to 25.07%.

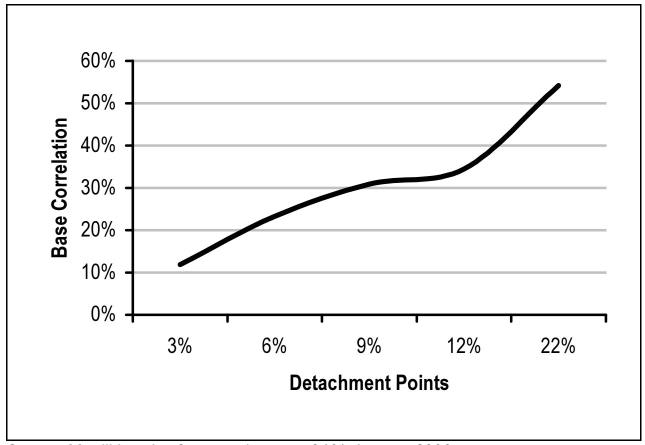
Once the 0-7% base correlation is computed, we can recursively compute the base correlation for the remaining tranches. With regard to the 0-10% tranche, we first need to compute the value of the 0-7% tranche using the 7-10% premium and the 0-7% base correlation. This implies a 0–7% mark-to-market of 24.27%. Given the width ratio (0-7% width / 0-10% width=0.7x) we can then compute the notional adjusted 0-10% mark-to-market which is equal to 16.98% (24.27% x 0.7). Given this value we can apply again the Gaussian Copula model and solve for the implied 0-10% base which, as reported in Table 46, equals 33.24%

Chart 149: CDX NA IG5 5y Base Correlation Skew



Source: Merrill Lynch. Computations as of 10th January 2006.

Chart 150: iTraxx 5y Base Correlation Skew



Source: Merrill Lynch. Computations as of 10th January 2006.

Pricing Non-Standard Tranches of Standardized Indices

Correlation skew makes it difficult to price non-standard tranche of the index

One of the key problems with tranche implied correlations is the pricing of non-standard tranches such as a 4-8% tranche. For the standardized tranche market in North America and Europe, we observe that tranche implied correlations are skewed in the form of a correlation "smile" (see Chart 145 and Chart 146). Due to this skew, it is not very clear what correlation input should be used to derive this price.

Pricing non-standard tranches is more intuitive with base correlations

Base correlations provide a more intuitive way to estimate this correlation input and price the tranche. Consider the pricing of a 4-8% tranche of the CDX.NA.IG. A long position in a 4-8% tranche is essentially comprised of two first-loss tranches:

- long position in the 0-8%
- short position in the 0-4%

Each of these first-loss or base tranches needs a default correlation input to be priced. This input can be derived from the base correlations of the standardized tranches. Chart 149 and Chart 150 highlight that base correlations for the standardized tranches increase for increasing detachment points. As a result, the base correlation of a 0-8% CDX tranche, for example, can be derived by interpolating on the curve between 7% and 10%. Similarly we can price the 0-4% tranche by interpolating on the curve between 3% and 7%. The 4-8% tranche can be priced in the following four steps:

- Interpolate between 0-7% and 0-10% to get the base correlation of the 0-8% tranche.
- Interpolate between 0-3% and 0-7% to get the base correlation of the 0-4% tranche.
- Use the standard Gaussian Copula model to price both tranches.
- The 4-8% tranche price is the difference in 0-8% and 0-4% prices adjusted for the notional of each tranche

With respect to the above example, the table below shows the required calculations:

Refer to important disclosures on page 196.

*We use the two **width ratios** to scale each intermediate step by the appropriate notional*

Notional Scaled 4-8% DV01

(3.69x2)-(3.08x1)=4.29

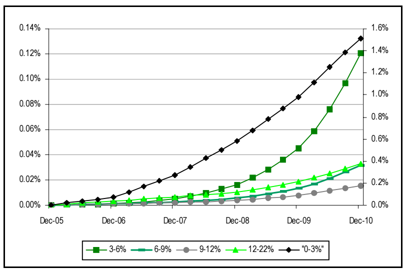
Table 48: Non-Standard 4-8% Tranche Breakeven Spread Calculation

	Tranche #1	Tranche #2	Off-The-Run Tranche
Attachment Point	0%	0%	4%
Detachment Point	4%	8%	8%
Base Correlation	13.55%	27.25%	
Width Ratios	4%/4%=1x	8%/4%=2x	
Risky DV01	3.08	3.69	4.29
Default Leg	0.41	0.21	0.02
Breakeven Spread	1315 bps	579 bps	**51 bps**

Source: Assume an spread for the CDX.NA.IG 5 Index at 45bps, 5y maturity and 40% average recovery. Computations as of 10th January 2006.

Base Tranches, Loss Surface and Implied Distribution

Chart 151: Term structure of expected losses for iTraxx tranches



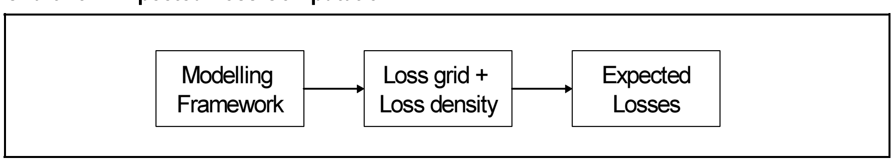
Source: Merrill Lynch. iTraxx tranches. X-axis – time tenors, Y-axis-expected loss in percent of index.

The value of tranches is to a large extent driven by the term structure of the expected losses. From the market we observe a term structure (typically 5,7,10Y) of tradable tranche levels. Observing a term structure of tranche premiums is equivalent to observing a term structure of expected losses. The shape of the implied term structure is model specific. Different models make different assumption about dependence structure of default times which results in different term structure behavior of loss density. In Chart 151 we illustrate the implied term structure of expected losses for the iTraxx tranche calibrated to the standardized market using the CBM[87] modeling framework.

■ From loss surface to default distribution

In the typical setting, we generate expected losses by considering a certain modeling framework that specifies the grid for possible losses and respective probabilities.

Chart 152: Expected Loss Computation



Source: Merrill Lynch

However, it is possible to parameterize expected losses directly as is assumed in the base correlation methodology. Since expected loss on any tranche could be represented as linear combination of expected losses on two equity (base) tranches, base tranches form a convenient building block. In more technical terms expected loss on the equity tranche with detachment point of δ is given by:

$$e(\delta,t) = \mathrm{E}\left[\min\left(\delta, L_t\right)\right] = \int_0^{\delta} x \cdot f(x)dx + \delta \cdot \mathrm{P}\left(L_t > \delta\right) \quad ^{88}$$

Differentiating twice this expression with respect to strike k it is then possible to recover loss density. In the general setting, expected loss on the tranche over a given time horizon is driven by the uncertainty on the number of defaults and recovery rates for the defaulted issuers. To simplify the setting we will assume that recovery rate is deterministic and homogeneous across obligors in the portfolio. We will further assume that a "**strike structure**" of losses (i.e. a

[87] CBM – composite basket model is discussed in the appendix of this report
[88] *E(X)* is the expectation of random variable *X*, *f(x)* - is the loss density, *t*-is the time period, and L_t - is portfolio loss random variable.

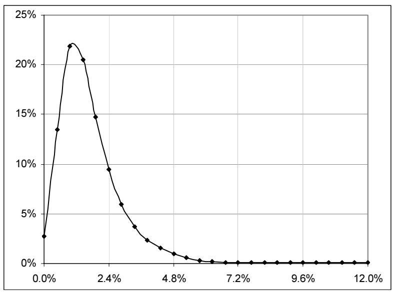
collection of expected losses on base tranches with detachment points equal to the integer multiples of loss given default - l^*) is known. Given these assumptions we can write:

$$e(k \cdot l^*, t) = \mathrm{E}\left[\min\left(L_t, k \cdot l^*\right)\right] = l^* \cdot \left[\sum_{i=1}^{k-1} i \cdot p(i) + k \cdot \sum_{i=k}^{N} p(i)\right]$$ [89]

By combining the expression above for two contiguous tranches with detachment points $k \cdot l^*$ and $(k-1) \cdot l^*$ respectively, we can establish the following recursive formula:

$$\mathrm{P}\left(L_t \geq k \cdot l^*\right) = \left[\sum_{i=k}^{N} p(i)\right] = \frac{e(k \cdot l^*, t) - e\left[(k-1) \cdot l^*, t\right]}{l^*}$$

Chart 153: Implied probability density



Source: Merrill Lynch. iTraxx tranches. X-axis-portfolio loss, Y-axis- oss probability.

In turn the loss density is given by:

$$p(k) = \mathrm{P}\left(L_t \geq k \cdot l^*\right) - \mathrm{P}\left(L_t \geq (k-1) \cdot l^*\right).$$

In Chart 153 opposite we illustrate the implied loss density generated by a collection of expected losses. The shape of the implied density depends on the second and third order properties of the expected loss strike structure.

Implied loss density is a useful criterion when one needs to interpolate and extrapolate either expected losses or model parameters that characterize expected loss structure. The key criterion that interpolation/extrapolation scheme must satisfy is that resulting implied density is non-negative everywhere. Additional conditions could be included in the interpolation scheme that characterizes loss density shape and smoothness.

Interpolation/Extrapolation in Standardized Markets

In the standardized markets, prices are only available for a limited number of base tranches. To value a tranche with bespoke attachment and detachment points or to model dynamics of the loss distribution we would like to specify the full strike structure of expected losses. To generate the full grid of expected losses we can interpolate / extrapolate either losses directly or model parameters (for example base correlations). There are many possible interpolation / extrapolation schemes, some of which result in plausible loss densities. While interpolation schemes tend to produce similar results, extrapolation schemes could produce divergent behavior for base correlation curves. To illustrate these points we sub-divided the modeling problem in the interpolation and extrapolation sections. In the following sections we would like to explore different alternatives and illustrate possible issues associated with each method.

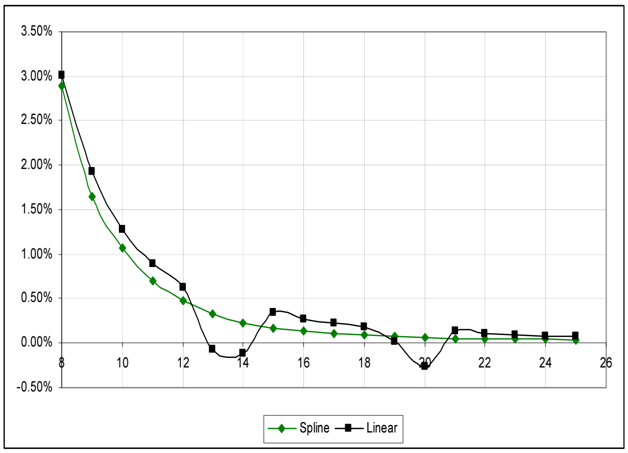
■ Interpolating Base Correlations

Within base correlation framework one postulates that losses on the base tranches at different strikes and maturities are priced at different GC (Gaussian Copula) correlations. Thus the methodology explicitly parameterizes the expected loss surface and, as explained in the previous section, implicitly specifies a loss distribution. For the purpose of this illustration we restrict ourselves to a single maturity example.

[89] *p(i)* – is probability of *i* defaults over time period *t*.

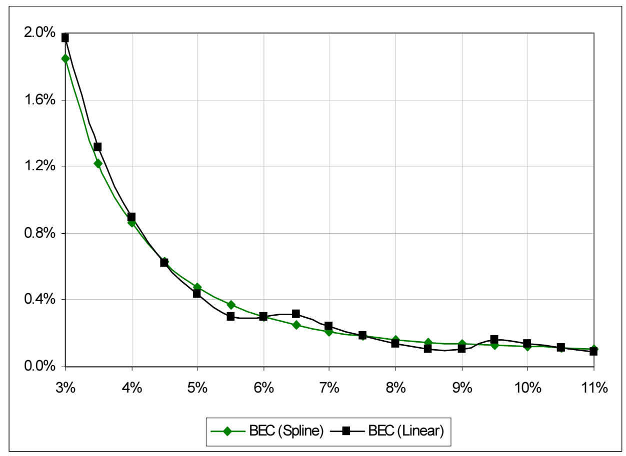
In the investment grade "correlation" markets (Itraxx/CDX) base correlations, as a function of the base tranche detachment point, typically exhibit an upward sloping shape. At first inspection it seems that the shape of the curve is nearly linear. **However, when looking at the shape of the implied density generated by linearly interpolating base correlations, we observe that the implied probabilities can be negative (**Chart 154**)**. This in turn translates into the oscillatory behavior of the mini-tranche (or tranchlet) premiums. Negative probabilities and resulting non-strictly decreasing tranchlet premiums present arbitrage opportunities. In particular, as clear from Chart 155, it is possible to lock a risk free profit by selling protection on the more senior tranche and buy protection on the adjacent junior tranche (both with the same width) while enjoying a positive carry.

Chart 154: Probability density implied by different interpolators



Source: Merrill Lynch. X-axis-number of defaults, Y-axis-probability.

Chart 155: Itraxx tranchlets break even coupons



Source: Merrill Lynch. X-axis-detachment, Y-axis-breakeven coupon in %.

A smoother interpolation of the base correlation curve is required in order to produce more plausible results. The smoothness (second order behavior) of the base correlation curve is intimately related to the shape of the implied loss density. For illustration we present interpolation results using **tension spline**. When restricted to the region of interpolation, spline based interpolation generally produces plausible results, however from our experience it is not sufficient to preclude possibility of negative density. Arbitrage considerations need to be directly incorporated in the interpolator. This could be implemented with a shape preserving splines. Arbitrage considerations are naturally incorporated in terms of restrictions on the possible shape of the base curve. However, the inclusion of these conditions is not straightforward and could be numerically challenging. An alternative approach is to model loss density directly by interpolating expected losses or default legs.

■ Loss (Default Leg) Interpolation

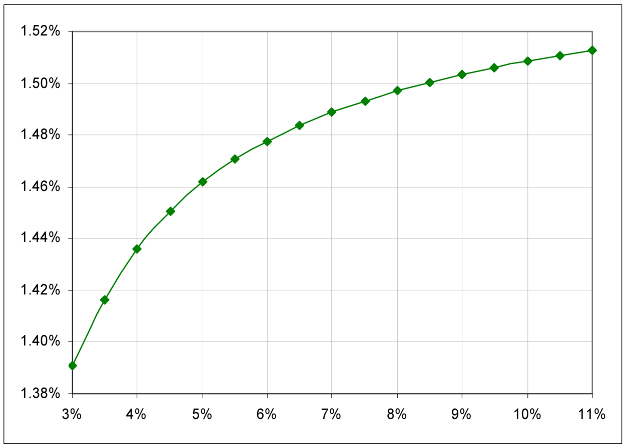
As we mentioned earlier, arbitrage requirements are summarized in the non-negativity of the loss density. On one hand we can relate smoothness of the base correlation curve to the shape of the loss density, alternatively we can relate the smoothness of the expected loss curve/default leg (as a function of strike) to the shape of the loss density. We believe that modeling expected loss curve is both intuitive and computationally tractable. Shape preserving splines could be used to model loss curve. We can readily incorporate arbitrage considerations via shape restriction on the loss curve. Smoothness of the implied density is controlled with a smoothness of the loss curve (spline). Once losses are interpolated base correlation could be recovered.

Both interpolation schemes (of base and loss curves) typically yield similar results. Significant differences may arise when the base correlation and loss curves are steep. Steep curves result in a greater scope for specifying admissible shapes for the base correlation or loss curve. This situation, for example, was recently observed for the CDX tranches.

Interpolation schemes do not incorporate full spread characteristics of portfolio…

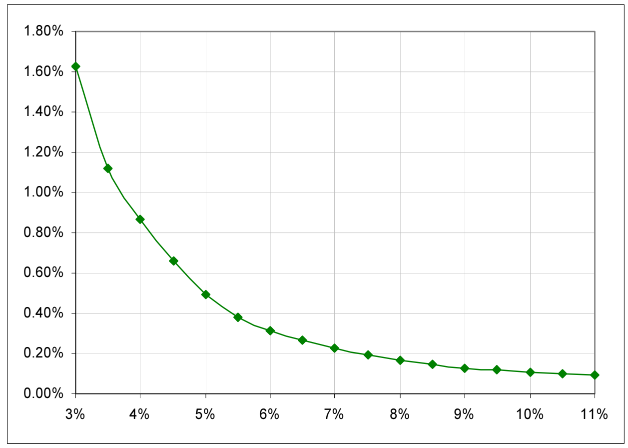
To further restrict number of possible shapes we can attempt to specify local behavior for the loss density. Intuitively restrictions on the loss density could be driven by the portfolio spread distribution and perceived level of dependence among the obligors. Unfortunately, we do not see an intuitive way of incorporating this information in the interpolation routine.

Chart 156: Interpolated Default Leg (shape preserving spline)



Source: Merrill Lynch. X-axis-detachment point, Y-axis-expected loss in %.

Chart 157: Implied break even coupon



Source: Merrill Lynch. X-axis-detachment point, Y-axis-breakeven coupon.

Interpolation with Calibrated Models

…a model can be used to produce a consistent base correlation curve

As an **alternative** to the interpolation schemes proposed so far, a specific **model** can be selected in order to:

- Reconcile the base correlation skew (calibrate to the observed tranche levels with a single set of parameters)
- Estimate the shape of the loss density in the region of interest and value bespokes.

A calibrated model naturally incorporates any spread characteristics for a specific portfolio. In that respect we hope that the results produced with this approach are more robust than previously mentioned interpolation schemes.

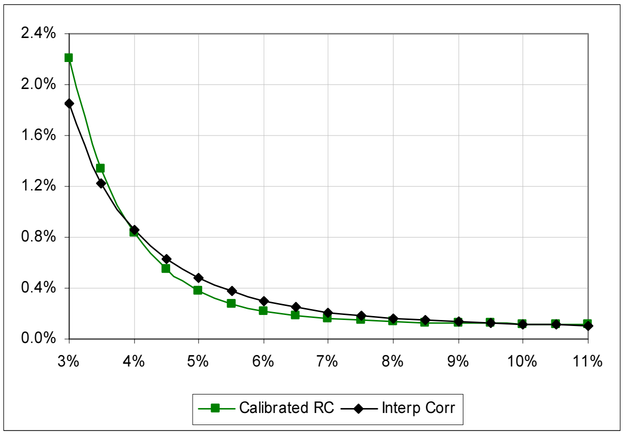
Over the past few years a number of models have been proposed that try to match the base correlation skew implicit in market prices. Most of the models assume a low-dimensional factor setting, with dependence structure between default times captured via dependence of latent variables.

To our knowledge, there is no market-wide consensus yet on which model is preferable. In this note the Random Correlation (RC) model and the Composite Basket Model (CBM) constitute the frameworks that we have chosen to generate the loss distribution. The procedure can be summarized as follows:

- Calibrate the model parameters from the available set of standardized tranche prices.
- Given the model parameters value bespoke or generate loss distribution.

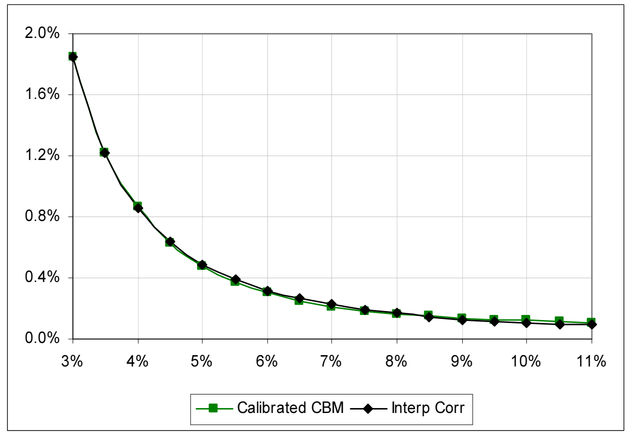
In Chart 158 and Chart 159 we illustrate tranchlet pricing for two models.

Chart 158: Calibrated RC vs Interpolated Base Curve



Source: Merrill Lynch. Calculations are with respect to iTraxx tranches. X-axis-detachment, Y-axis-breakeven coupon.

Chart 159: Calibrated CBM vs. Interpolated Base Curve



Source: Merrill Lynch. Calculations are with respect to iTraxx tranches. . X-axis-detachment, Y-axis-breakeven coupon.

From these charts we observe that different modeling assumptions, with nearly identical quality of calibration, can result in a different behavior for the loss density and consequently different shape for the implied base correlation curves and tranchlet prices.

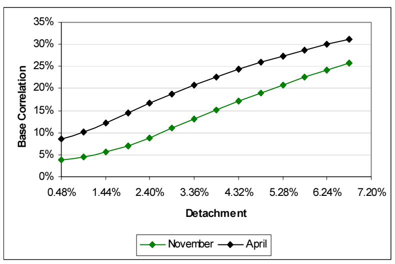
Different interpolation/extrapolation schemes may result in the different behavior for the loss density. The extrapolation task magnifies this uncertainty.

Extrapolation - More Questions than Answers…

There is no clear magic recipe for extrapolation either with base correlations or expected losses. Observable tranche levels do not provide us with a full parameterization of the loss density and constraints of positive density are not strict enough, leaving us with a multitude of possible specifications.

For example, when extrapolating base correlations for subordination levels below the 3% detachment point, should the base correlation curve be flat, downward sloping, upward sloping, or some combination of them? In many scenarios we can easily satisfy arbitrage constraints, however the shape of implied density could take various forms depending on the extrapolated base curve. We can attempt to introduce additional constraints to narrow the range of possible shapes.

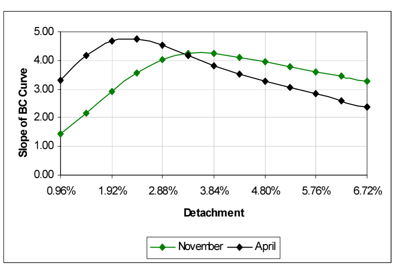
Chart 160: Model implied base curves



Source: Merrill Lynch. Itraxx Portfolio. X-axis-detachment, Y-axis-correlation.

For example, we can try to incorporate in the extrapolation routine changes in the level of the idiosyncratic risk priced into the tranche market. This could be relevant given the tranche re-pricing in May post GM. In April, Itraxx Equity tranche (0-3%) had 68% of index losses allocated to it and Itraxx Mezzanine (3-6%) had 12% while in November Itraxx Equity had 83% of losses and Itraxx Mezzanine had just 7% of losses. In other words more risk is currently assigned to equity part of capital structure. Intuitively, we would expect a flatter correlation skew in the latter case and steeper in the former. By using the CBM model, Chart 160 and Chart 161 confirmed our intuition.

Chart 161: Slope of base curve

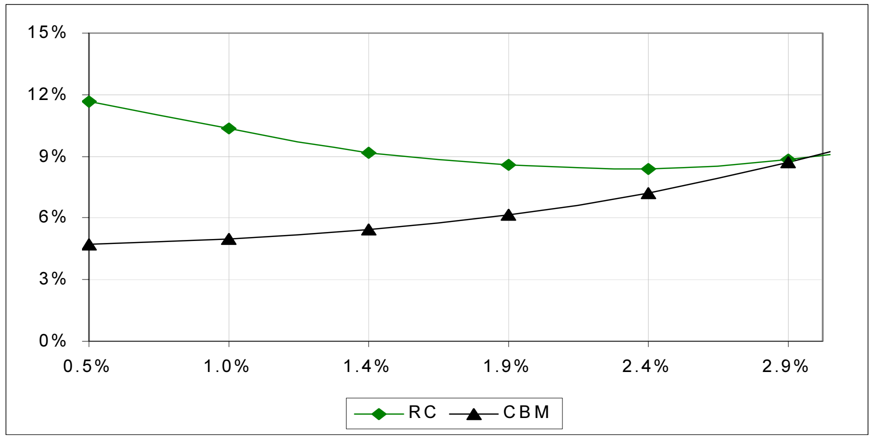


Source: Merrill Lynch. Itraxx Portfolio. X-axis-detachment, Y-axis-correlation.

Chart 161 also illustrates that the slope of the base correlation curve changed significantly in the lower region of the capital structure. However, incorporating this information into the extrapolation procedure remains a difficult task.

As an alternative to direct extrapolation we can calibrate and use a model. A model would naturally incorporate idiosyncratic and systemic risk allocations via calibration. However, as we saw in the previous section, models with nearly identical calibrations can produce different shapes of the loss distribution. This is particularly pronounced in the lower part of the capital structure. In Chart 162 we illustrate base correlation curves generated by two models. The behavior is significantly different. Which of the parameterization is "better"? We will leave it for the market to decide.

Chart 162: Implied base correlation curves for two models in the lower region.



Source: Merrill Lynch. X-axis-detachment, Y-axis-correlation.

Appendix: Model Details and Brief Review

Stochastic Correlation

The random correlation model we used in our analysis postulates that the value process of a company is described by the standard one-factor representation[90] thereby assuming that the firm value of each credit is driven by two main components:

- A **systematic** or common factor, which can be interpreted as an indicator of the general state of the business cycle (eg, stock index, bond index, gross domestic product)
- An **idiosyncratic** noise or firm-specific factor that is an indicator of events strictly linked to the credit itself (eg, market share, quality of management, relative position in a sector compared to peers).

We assume that both systematic and idiosyncratic components are **normally distributed** with zero mean and variance equal to one. Moreover, by definition of systematic and idiosyncratic risk, the two factors are assumed to be **independent.** Within this setting we can express the value process of a generic issuer as:

$$V_i = \rho_i Z + \sqrt{1-\rho_i^2}\,\varepsilon_i$$

- V_i : value process of the i^{th} credit in the portfolio
- Z : value of the common (systematic) factor
- ε_i : value of the idiosyncratic risk for the i^{th} credit in the portfolio
- ρ_i : sensitivity of the i^{th} credit to the common factor**. Assumed to be a [0,1] random variable independent** from V_i, Z and ε_i **.**

Given the independence between the random correlation parameter ρ_i, the value process V_i and the common factor Z, V_i still follows a normal distribution.

In a previous report, we have shown that the individual default probability, conditional on the common factor Z and a constant correlation $\rho = \overline{\rho}$ can be expressed as:

[90] Please refer to Volume 2, Chapter 8 for a detailed description of factor models applied to portfolio credit derivatives products.

Refer to important disclosures on page 196.

$$p^i_{z,\bar{\rho}} = N\left[\frac{k_i - \sqrt{\bar{\rho}_i}\, z}{\sqrt{1-\bar{\rho}_i}}\right]$$

Where k is the default threshold calibrated to the individual CDS spread curve.

In the case of random correlation, we can generalize the expression above by taking the expected value of the conditional default probability over the possible admissible values of the correlation component, namely:

$$p^i_z = \int_0^1 N\left[\frac{k_i - \sqrt{\rho_i}\, z}{\sqrt{1-\rho_i}}\right] dF(\rho_i)$$

Burtschell et al. (2005) have recently proposed the following specification of the random discrete correlation component ρ_i:

$$\rho_i = (1-B_i)(1-B_s)\rho + B_s$$

with $i=1,\dots,n$ and B_i and B_s being Bernoulli random variables independent from V_i, Z and ε_i taking either value 0 or 1 with $p=Prob(B_i=1)$ and $p_s=Prob(B_s=1)$.

We refer to Burtschell et al.[91] for implementation and empirical evidence of the proposed model.

Composite Basket Model (CBM)

The CBM model postulates that the default probability of a given company is driven by three main independent factors:

1. Systemic Risk (e.g. sector the company operates in)
2. Idiosyncratic Risk
3. Market Variable (i.e. the *copula* term which capture the market consensus on the general state of the economy).

1. and 2. are modeled as Poisson processes, i.e.

$$\tau_s = \inf_t \left\{ \exp\left(- \int_0^t h_s(u)du \right) \le U \right\}$$ with U being an independent uniform [0,1]

$$\tau_i^{idiosyncratic} = \inf_t \left\{ \exp\left(- \int_0^t h_i(u)du \right) \le U_i \right\}$$ with U_i being an independent uniform [0,1]

whereas the copula term is described through the standard one-factor representation, namely:

$$\tau_i^{copula} = \inf_t \left\{ \exp\left(- \int_0^t h_i(u)du \right) \le N(V_i) \right\}$$

In the proposed setting the default time is modeled as:

$$\tau_i = \min\left(\tau_s, \tau_i^{idiosyncratic}, \tau_i^{copula}\right).$$

In case of flat CDS curves, the following decomposition of the credit spread holds:

$$s_i = s_s + s_i^{idiosyncratic} + s_i^{copula}.$$

We refer to Tavares et al.[92] for a more detailed description of the model.

[91] Burtschell, X., Gregory, J. and Laurent, J.P., "*Beyond the Gaussian Copula: Stochastic and Local Correlation",* (2005), available at www.defaultrisk.com.

[92] Tavares, P., Nguyen, T., Chapovsky, A. and Vaysburd, I., "*Composite Basket Model",* (2004), available at www.defaultrisk.com.

10. Synthetic CDO^2

This chapter is extracted from a report published on 9th July 2004 by Batchvarov et al.

Market Background

In an environment of tight spreads, investors search for value and yield is increasingly directed to the highly structured debt instrument. From a relative value perspective investors often compare plain vanilla corporate debt instruments (industrials and financials bonds) and structured debt instruments (ABS, CDO, etc.). A commonly asked question is, for example: what is a better choice a single-A corporate bond, or a single-A tranche of a CDO comprising a number of triple-B corporate bonds, or a single A tranche of a CDO with triple-B tranches of corporate CDOs? While a certain level of spread differential between the two is warranted, a large spread differential often raises questions and tips the balance in favor of one instrument or the other. This question can also be modified to reflect the ratings composition of underlying pool and different attachment points for ST CDO (single tranche CDO) and ST CDO^2 (single tranche CDO square).

In early 2003, for example, the investor base favored plain vanilla corporates, as it preferred the ability to select individual bonds in the relatively uncertain credit environment at that time. Rapidly contracting corporate spreads exacerbated the discrepancy between the pricing of plain vanilla corporate bonds/ CDS and the CDOs derived from pools of such bonds/ CDS, and shifted investor interest towards the CDO sector – investor appetite for CDOs rose in the second half of 2003. With the stabilization, at a low level, of corporate bond and CDS spreads, the execution of fully funded CDOs based on such pools became less economically viable, and the market's interest shifted in the direction of single tranche CDOs, which was a predominant structured during 2003. Should spreads widen again, we expect a revival in the investor interest in ST CDO.

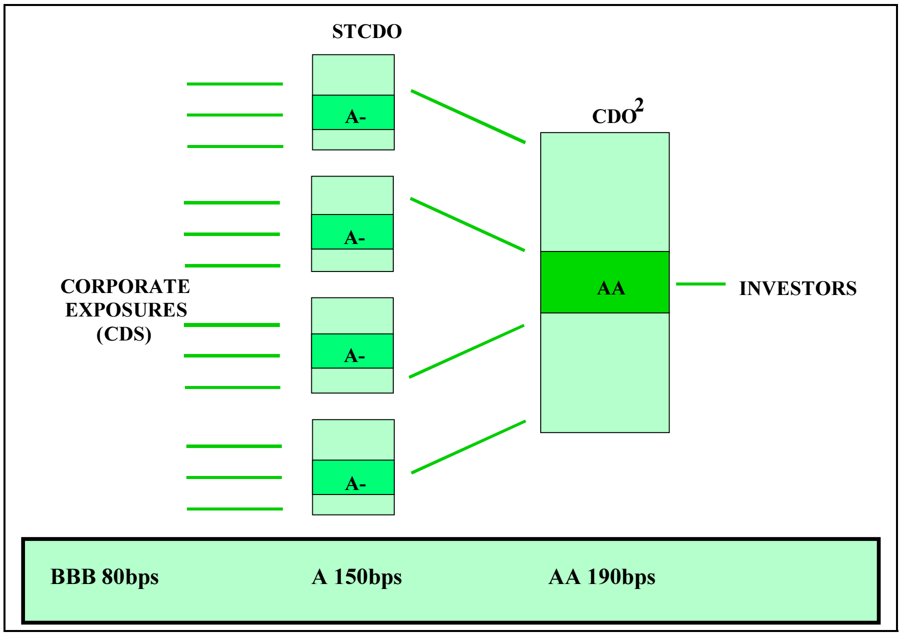
During 2004, low spreads and relative stability in corporate credits shifted investor interest further towards even more structured instruments – the CDO^2 emerged. In a European context, the CDO^2 is constructed on the basis of single tranche of synthetic CDOs, and may itself be a single tranche CDO of CDOs.

CDO^2 Structure

Chart 163 below highlights the structure of a synthetic European CDO^2. We note several aspects of that structure:

- the CDO^2 is effectively repackaging of single tranche CDOs specifically created for the purposes of the transaction.
- each single tranche CDO has a rated tranche, in this case with a rating of single-A, but can be tranched with a higher or lower rating, and the respective single tranches are pooled to create the collateral of the CDO^2.
- the ratings of the single tranche CDOs determine the level of subordination required to achieve a certain rating level of the single tranche of the CDO^2.
- the particular attachment point, determines the rating and depends on the investor's interest in a specific risk profile.
- the individual single tranche CDOs comprising the CDO^2 pool are backed by pools of several hundred single name CDS.
- given the number of single name CDS (about a hundred) in each single tranche CDO and the limited universe of traded CDS, from which they are extracted, it is inevitable that a certain number of single name CDS will be repeated in the single tranche CDO pools, hence the issue of credit exposure overlap (concentration) across the CDS pools.

Chart 163: CDO² Schematic: Structure and Indicative Spreads (bps over Libor)



Source: Merrill Lynch

- Given that the performance of the single tranche CDO^2 is derived from the performance of the pool of single tranche CDOs, which in turn depends on the performance of the related pools of CDS, ultimately the performance of the CDO^2 is effectively derived from the performance of the combined pools of CDS. From a credit analytical point of view the CDO^2 is backed by a large pool of CDS, characterized by the number of names, credit quality of the names in the combined pool and derived average pool credit quality, concentration by single name/ industry in the combined pool, etc.
- Both ST CDOs and ST CDO^2 introduce a leverage/ de-leverage play for the investor. In the case of ST CDO the investor is the ST CDO^2, while in the case of ST CDO^2, the investor is the ultimate institutional investor.

We discuss some of these points further below:

The Issue of Leverage

There are different views in the market with respect to leverage across the CDO capital structure:

- Some market participants are of the view that all debt tranches (including the BB tranche) are not levered due to the fact that there is no upside in returns – the only levered tranche, then, is the equity piece. In the case where the subordination of a particular tranche, say the lower mezzanine, has been eroded due to losses in the collateral pool (the equity is written down), such a tranche may then be viewed as distressed debt rather than levered equity.
- Others, we among them, are of the view that all senior tranches (with the exception of the senior most one) are levered to some degree – partially levered depending on the tranches above them, and partially de-levered depending on the tranches below them. The levered/de-levered ratio changes the more junior tranches are written down. As the subordination level below a tranche declines to zero, the tranche becomes fully levered. Structures that allow for amortization of the senior tranches alter their leverage ratios over time due to such amortization.

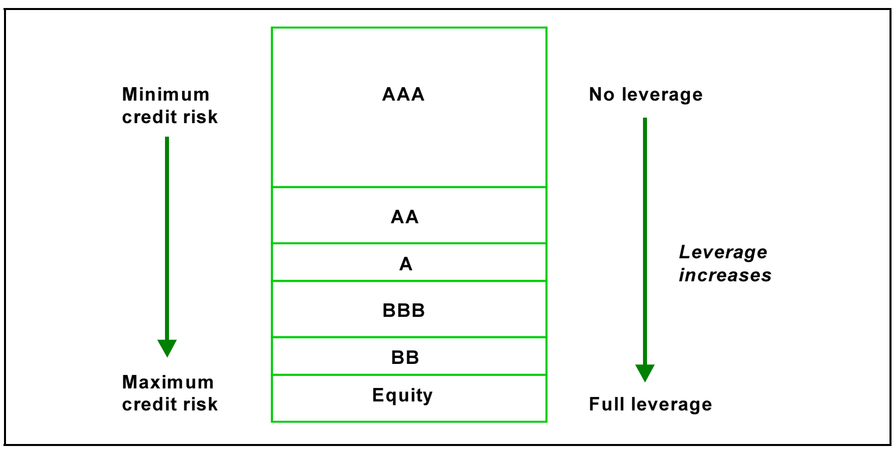
While the second view is theoretically and intuitively acceptable, it is often difficult to show a numerical representation of the leverage explanation. However, we provide what can be a reference framework for such calculation below using the example of synthetic indices and the calculation of deltas for tranches of such indices.

One final point on the issue of leverage – given that investors compare tranches of CDOs with corporate bonds, there is the misperception that the former (in their mezzanine debt tranches) are leveraged, while the latter are not. We need to emphasize the word 'misperception' here. Corporate bonds in their most common form of senior unsecured exposures in many cases are similar to mezzanine CDO debt in their levered/de-levered position – senior unsecured bonds may be junior to senior secured lending and senior to junior unsecured lending, tier 1 and tier 2 capital for banks. Nevertheless, the positions senior to the senior unsecured debt of a company, however, are more often than not smaller in relative terms than the positions of the senior and super-senior tranches relative to the rest of the CDO capital structure. Therefore, senior unsecured corporate debt can, like CDO debt or tranches, also be viewed as partially levered although to a lesser degree (see Chart 164 and Chart 165).

An Attempt to Quantify Leverage

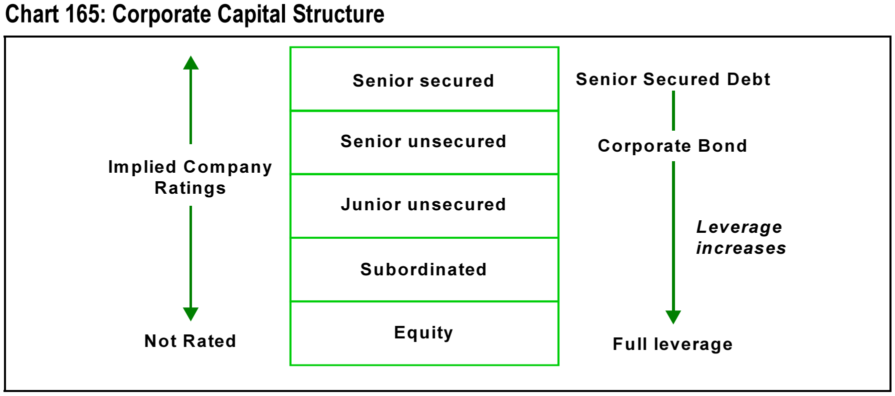
A standardised and an increasingly liquid market for CDO tranches based on credit default swaps has recently emerged. In Europe, these tranches are structured on the iTraxx CDS index. In the US, these tranches are structured on the CDX.NA.IG CDS index. The traded market premiums of these tranches can be used to calculate the leverage (or delta) of each tranche in the capital structure relative to the CDS index itself. This Leverage measure gives an indication of how sensitive the tranche is to spread changes in the underlying CDS index. For the base case, it is assumed that the CDS index has 1x leverage.

Chart 164: Typical CDO structure



Source: Merrill Lynch

Chart 165: Corporate Capital Structure



Source: Merrill Lynch

Depending on the tranche in question, its leverage can be less than 1x for very senior tranches in the capital structure, but greater than 1x for more subordinate tranches in the capital structure. In this regard, more senior tranches can be viewed as 'deleveraged' relative to the CDS index, and more subordinate tranches can be viewed as 'leveraged' relative to the CDS index.

However, this again depends on whether one views the underlying senior unsecured corporate obligations as leveraged in the first place.

The Issue of Investment Risk Profile

Investing in CDO^2 requires understanding of the CDO^2 itself and the effect that the addition of these investments may have on the existing portfolio of the investor. We suggest some general points for consideration, and we underline the difficulty of putting them into quantitative terms.

A key consideration is related to the type of risk that a given tranche of a CDO^2 represents. The underlying combined CDS pool and the existing portfolio of the investor will have a certain level of diversification in the individual exposures – diversification reduces the idiosyncratic risk of the portfolio and increases its systemic risk. Tranching a diversified portfolio reallocates these risks to the different tranches of the CDO, the senior tranches representing more systemic risk and the junior tranches – more idiosyncratic risk.

When determining the impact of a CDO^2 tranche on an investor's portfolio, several questions need to be asked:

- *First,* is there a significant overlap between the CDO^2 pool of corporate exposures and the exposures in the investor's portfolio? In that regard, single name, geographic and industry exposure distributions should be carefully considered.
- *Second,* which tranche does the investor choose and primarily what type of risk this tranche represents? That depends on the tranche's seniority and degree of systemic / idiosyncratic risk it has in comparison to the rest of investor's portfolio.
- *Third,* what degree of leverage is the investor willing to take through the CDO tranche investment as compared to the degree of leverage he can achieve on his own in his portfolio through repo or other ways? Even if there is a degree of an overlap between the names in the CDS pool and in the investor portfolio, it is worth considering a CDO tranche in the context of the leverage it is offering. It is uncommon that an investor can achieve such leverage on its own.
- *Fourth,* what is the degree of protection available to the investor, i.e. attachment point or level of subordination? It is usually expressed in the number of defaults in the portfolio (at a given recovery rate) that investors can 'afford' without incurring losses in their investment.

Credit Risk Balance within CDO^2

The last point needs further discussion. The degree of subordination afforded to an investor in a single tranche $CD0^2$ depends on the following factors:

- attachment point (and associated rating) of the single tranche CDOs constituting the CDO^2 pool.
- the correlation of the underlying CDS in the combined pool.
- the level of overlap in the CDS pool.

We consider the following case: If we keep constant the correlation and overlap in the CDS pool, and focus on the rating of the single tranche CDOs, the investor has the following two main choices, assuming that the investor wants the same high rating (say, double-A) for the ST CDO^2:

- select low attachment points for the underlying ST CDOs (lower rated or non-rated), which will result in a higher attachment point for the CDO^2

- select high attachment points for the underlying STCDO (highly rated), which will lead to low attachment points for the CDO^2.

In our opinion there is no right or wrong answer regarding which structure is better. Some investors prefer 'optically' having more credit enhancement on the position they are holding, while others are more concerned with the rating of the collateral supporting the bond. In this regard we note that lower rated underlying suggests more idiosyncratic risk in the master CDO pool, and higher rated underlying suggests more systemic risk in the master CDO pool.

The overlap effect in the CDS pool on the CDO^2 diminishes the higher the attachment point for the ST CDOs. Alternatively, the lower the attachment point for the ST CDOs, the stronger the overlap effect of the CDS pool on the CDO^2 – higher idiosyncratic risk at the ST CDO tranche level should suggest higher attachment point for a given high rating of the ST CDO^2. In that regard, the attachment point may be affected by the number of ST CDOs for any given overlap level.

While that makes sense theoretically, from a CDO^2 ratings analytical point of view the type of risk of the underlying tranches is somewhat irrelevant, given the look-through approach – that is, the underlying CDO tranches are disregarded and the CDO^2 is analyzed on the basis of the combined pool of exposures from each individual CDO pool.

The differences in the ST CDO pool composition, however, will have an impact on the pay to the investor even at the same attachment point for the ST CDO^2. Lower rated ST CDOs will generate higher fees, which should compensate investors for the higher risk and offset to some degree the losses that may take place in the pool (depending on the timing of their occurrence).

Case Study: CDO^2

In order to illustrate the behavior of CDO^2 we constructed a Master CDO portfolio based on 10 single tranche CDOs (ST CDOs), each backed by one hundred investment grade CDS with each name being 1% of the respective CDO pool. The combined pool backing the 10 ST CDOs represents 234 separate single-name CDS. That corresponds to the maximum frequency of occurrence of any single name CDS of 6 out of 10 portfolios.

The ST CDOs are tranched in two ways:

- Case 1: 'A-' rated tranche between 3.5% to 6.5%, and
- Case 2: unrated tranche of 2% to 5%.

Different rating levels are considered for the CDO^2.

Default Frequency Effects

Default simulations are run for the respective ST CDOs under three scenarios related to which defaults of CDS affect:

1. *the most frequently occurring assets* – that is, assets that are repeated six times in the respective CDO pools
2. *assets with the widest spread* - that is, defaults of the assets demonstrating lowest credit quality expressed in wider spreads by the market
3. *random asset default*.

Under all three scenarios, we attempt to answer the question: how many defaults can occur before the respective tranche of the master CDO incurs a loss, given a constant recovery assumption? The results of the simulations are reflected in Tables 1 and 2.

Further on, in Table 51, we demonstrate the default resilience of a ST CDO, similar to the ones included in the CDO^2. We note that the results are presented in a similar away – how many defaults can occur before the respective tranche suffers a loss?

We note, however, that the comparison between Table 49 and Table 51 for example, should reflect the fact that the credit enhancement to achieve similar rated tranches in the two different types of deals is different.

We conclude that:

- The results speak in favor of the CT CDO^2: it can sustain roughly twice as many defaults at different rating levels before suffering a loss than the ST CDO.
- Most conservative default scenario for CDO^2 is the one assuming that the CDS that appear most frequently in the ST CDOs default first. Conversely, the most lax scenario is the one assuming random default.

Table 49: CDO^2 – Case 1: Underlying CDO Tranches 3.5% to 6.5%, Rated A-, Recovery Assumption 40%

		Number of defaults required to cause loss at a given rating level		
Rating	Subordination	Defaulting most frequently occurring assets first	Defaulting widest spread assets first	Using a random default simulation
AAA	26.50%	12	17	17
AA	15.00%	11	13	15
A	10.75%	11	13	14
BBB	5.50%	10	12	13
Equity	0.00%	7	8	9

Source: Merrill Lynch

Table 50: CDO^2 – Case 2: Underlying CDO Tranches 2% to 5%, Unrated, Recovery Assumption 40%

		Number of defaults required to cause loss at a given rating level		
Rating	Subordination	Defaulting most frequently occurring assets first	Defaulting widest spread assets first	Using a random default simulation
AAA	71.00%	12	16	18
AA	53.50%	11	13	15
A	43.50%	10	12	14
BBB	29.00%	8	10	12
Equity	0.00%	4	6	5.2

Source: Merrill Lynch

Table 51: ST CDO Default Analysis, Recovery 40%

	Underlying pool: 100 IG Credits, 1% Notional per name	
Rating	Subordination	No. of defaults required to cause loss at a given rating:
AAA	6.00%	11
AA	4.75%	8
A	4.50%	8
BBB	3.50%	6
Equity	0.00%	1

Source: Merrill Lynch

Correlation Effects

We also considered the impact of correlation on the loss distribution for CDO^2 under the two different attachment point cases of underlying ST CDO. We compare the loss distribution curves under 10%, 20%, 30% and 40% correlation assumption with the loss distribution curves for ST CDO under the same correlation assumptions.

We conclude that the shape of the loss distributions is different:

- *ST CDO loss distribution* resembles a normal distribution with moderate correlation – it has highest probability of average losses, and low probability of high or zero losses. (see Chart 174 to Chart 177)
- *CDO^2 loss distribution* heavily weighted towards zero or low loss probabilities - the likelihood of zero losses is very high, while the probabilities of average and high losses is very low. (see Chart 166 to Chart 173)

We also observe that the tails of both distributions extend and get fatter with the increase in correlation. The degree of lengthening of the tails is different, however. ST CDO distribution shows low probabilities of loss rate in the single and low double-digit levels. The CDO^2 loss distribution comprises very low probabilities of very high loss levels, extending up to 60%.

Correlation among CDS and the degree of overlap across the portfolios of ST CDOs will jointly determine the behavior of CDO^2. High correlation, close to 100%, or very high overlap will make the CDO^2 loss distribution similar to that of ST CDO. As the correlation diminishes and the overlap is held constant the CDO^2 loss distribution assumes the shape shown in the case studies here. Similar results can be achieved by keeping correlation constant and varying the degree of overlap in the ST CDO portfolios of CDS.

Tranche Thickness

The loss distributions discussed above are related to the pools backing the CDO^2 and the STCDO. The potential loss of a given tranche, however, will depend on the attachment point and detachment point of that tranche, i.e. its thickness. In our case study we do not explore this effect, as we focus on the risk of the CDO^2 experiencing a loss. We keep the size of the tranche fixed at 3% - one can further explore the issue of a loss to a tranche relative to its size.

Final Remarks

This report should provide sufficient background for an investor to understand the risks and mechanics of given ST CDO and ST CDO^2 transactions. In a simplistic way, if ST CDO is the first derivative of the corporate bonds or CDS pool, ST CDO^2 is a second derivative of that same pool.

Hence, the performance of the two types of deals will depend on the performance of that underlying pool, but to a different degree – that difference is clearly expressed in:

- the higher number of defaults that a pool can sustain before a given tranche of ST CDO^2 experiences a loss in comparison to the number of defaults for ST CDO.
- the higher probability of zero losses for ST CDO^2 in comparison to ST CDO, as reflected in their respective loss distributions. In our example the probability of zero loss for ST CDO^2 is often higher by a multiple of 2-3 times.
- the relatively higher probability of high losses for ST CDO^2 is reflected in the much longer tail of the loss distribution in comparison with ST CDO.

All of the above suggests that ***ST CDO^2*** provides better protection to investors under normal and moderately worse credit conditions and leaves them moderately exposed to more extreme systemic market situations. ST CDO^2 effectively allows investors to take 'the cliff risk', that is no losses up to a point of about 90% of the cases, after which the loss deterioration is fairly fast.

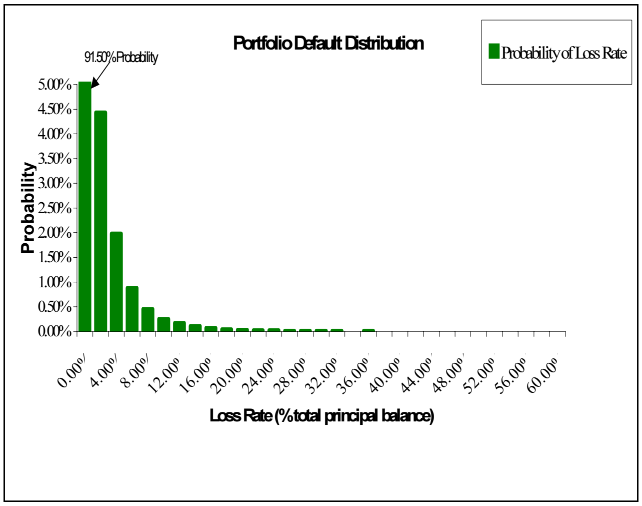
ST CDO, on the other hand, leaves investors more evenly exposed to losses under virtually all market conditions. In about a quarter of the cases investors will have no loss, and in all other cases they will have some loss ranging from small to moderate.

Refer to important disclosures on page 196.

The higher yield CDO² offers investors can be viewed as a compensation for the rapid deterioration in returns beyond a certain point ('cliff effect'). Whether that yield is sufficient is a matter of investor's judgement regarding the probability of 'extreme' events and their timing. In the meantime, the yield remains there for picking.

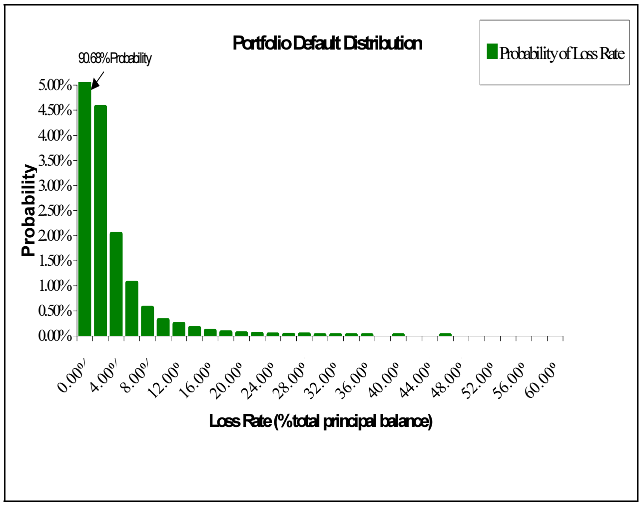
Case 1: Correlation Effects on CDO2 Portfolio Loss Distribution

Chart 166: Underlying CDO Tranches 3.5% to 6.5%; 10% Correlation



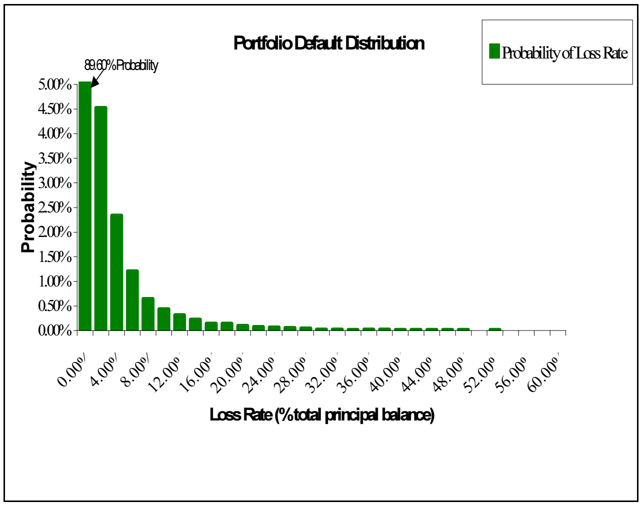
Source: Merrill Lynch

Chart 167: Underlying CDO Tranches 3.5% to 6.5%; 20% Correlation



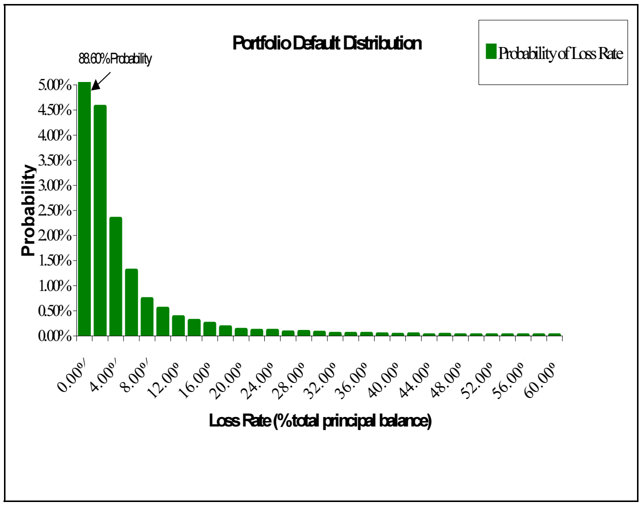
Source: Merrill Lynch

Chart 168: Underlying CDO Tranches 3.5% to 6.5%; 30% Correlation



Source: Merrill Lynch

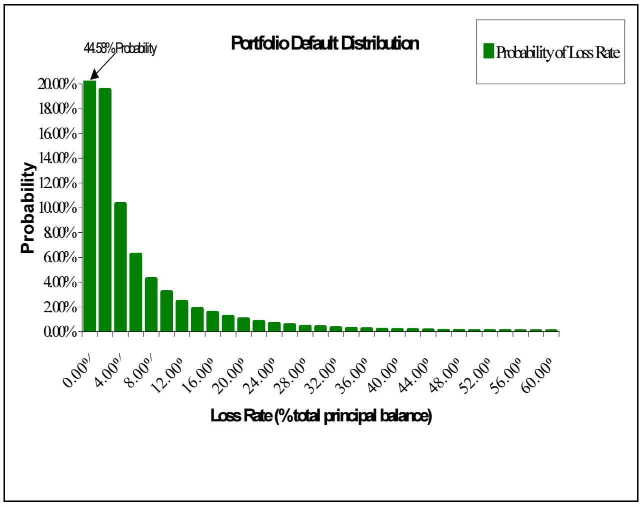
Chart 169: Underlying CDO Tranches 3.5% to 6.5%; 40% Correlation



Source: Merrill Lynch

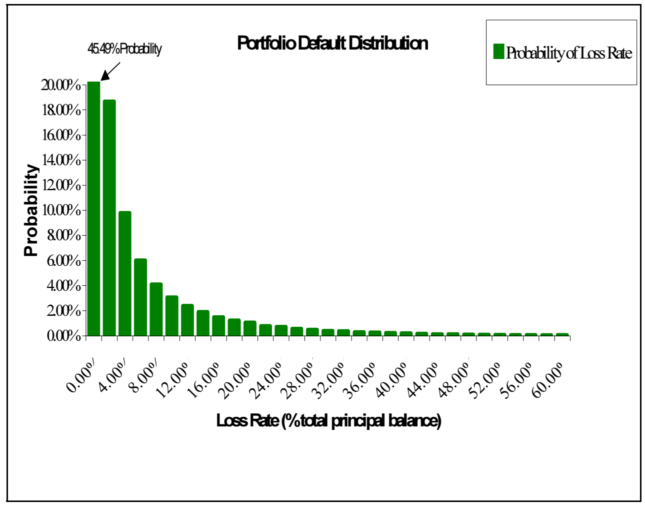
Case 2: Correlation Effect on CDO^2 Portfolio Loss Distribution

Chart 170: Underlying CDO Tranches 2% to 5%; 10% Correlation



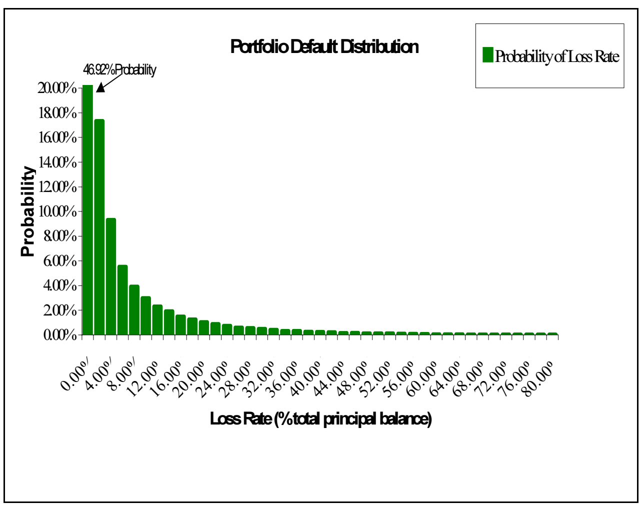
Source: Merrill Lynch

Chart 171: Underlying CDO Tranches 2% to 5%; 20% Correlation



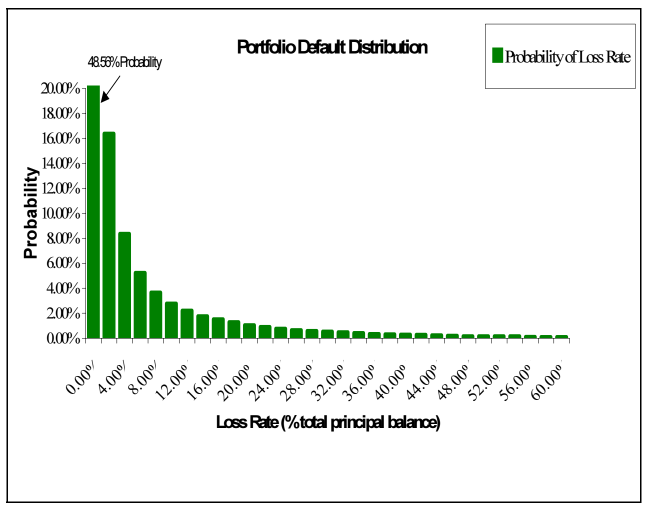
Source: Merrill Lynch

Chart 172: Underlying CDO Tranches 2% to 5%; 30% Correlation



Source: Merrill Lynch

Chart 173: Underlying CDO Tranches 2% to 5%; 40% Correlation

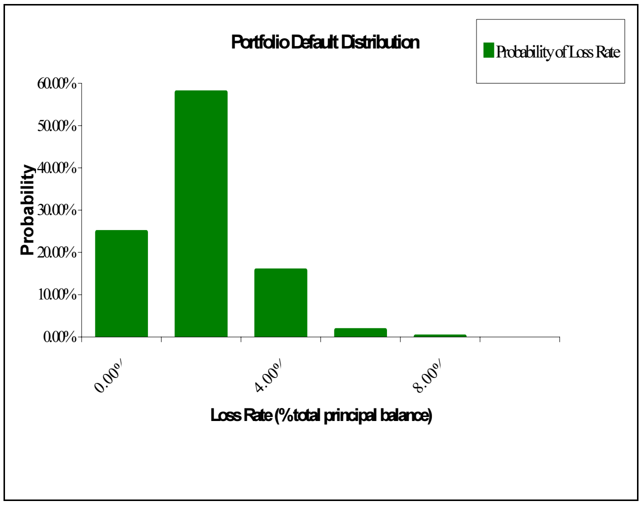


Source: Merrill Lynch

Refer to important disclosures on page 196.

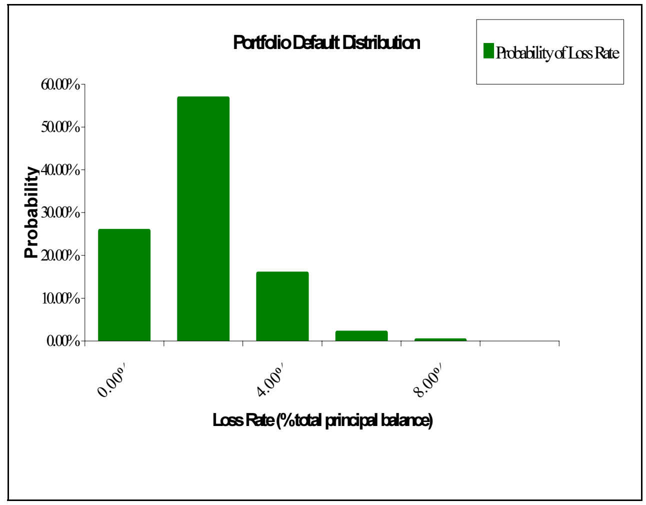
Correlation Effects on ST CDO Portfolio Default Distribution

Chart 174: Single Tranche of A- CDS; 10% Correlation



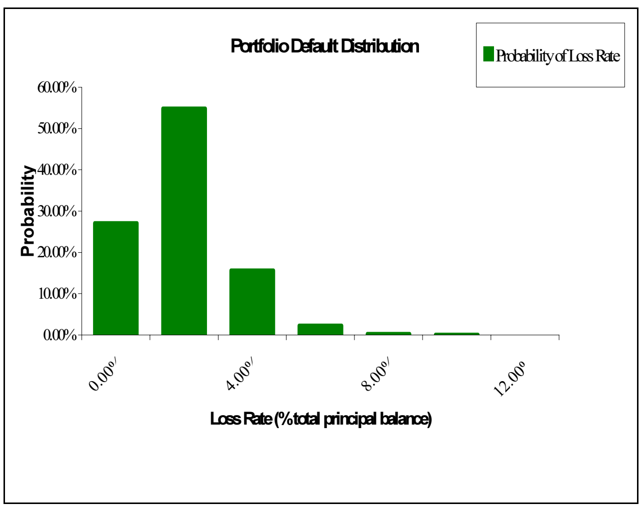
Source: Merrill Lynch

Chart 175: Single Tranche of A- CDS; 20% Corr



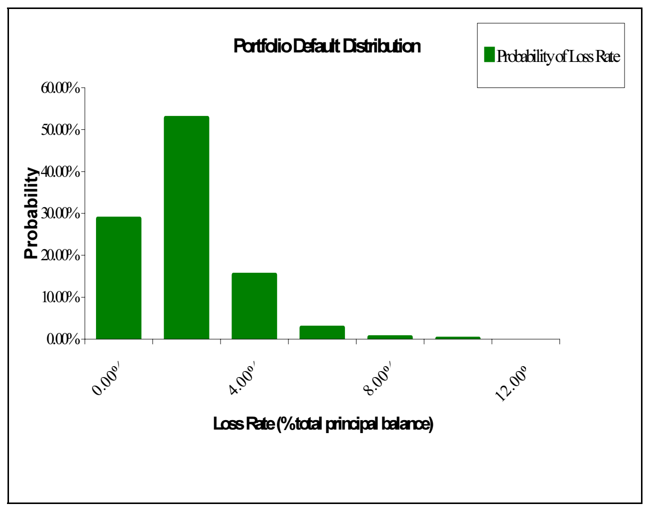
Source: Merrill Lynch

Chart 176: Single Tranche of A- CDS; 30% Correlation



Source: Merrill Lynch

Chart 177: Single Tranche of A- CDS; 40% Correlation



Source: Merrill Lynch

11. Leveraged Super Senior

This chapter is extracted from a report published on 30th August 2005 by Gibson et al.

LSS Notes Leverage Remote Default Risk

Leveraged Super Senior (LSS) notes apply leverage outside the CDO deal to allow investors to achieve a higher yield. The LSS note structure is a means to attract a broader investor base by offering a portion of the super-senior tranche in the form of a cash note[93]. The notional of the LSS note is a fraction of the notional of the super senior tranche. From that perspective, the LSS tranche receives the benefit of the cash flows allocated to the full super senior tranche, thus allowing it to pay a considerably higher spread. However, at the same time, its exposure to loss is capped at its size. In other words, the LSS note investment has limited recourse to the funded amount. Should losses accumulate to exceed the LSS notional, this discrepancy, unless corrected with triggers, would be absorbed by the protection buyer.

LSS trades provide leverage outside the deal

By leveraging the 5–12-bp premium of a super senior (SS) tranche by a factor of 10, for example, LSS notes provide outsized yield for an investor's AAA portfolio. In this 10x leverage example, the investor would fund 10% (essentially on margin) of the super senior tranche notional. In return for limited recourse leverage (only at risk for the margin), performance triggers require the investor to partially deleverage the transaction (e.g., fund the margin account by an additional 10%) or unwind the trade at a loss. The triggers are based on either the market value or collateral losses exceeding a threshold usually consistent with a downgrade below AAA. These trigger points are typically struck well out-of-the-money since the initial attachment point can be twice as much, or more, as that required to maintain the AAA rating.

■ Triggers Mitigate the Protection Buyer's Gap Risk

Triggers are MTM- or loss-based

The protection buyer is exposed to the risk that gapping spreads will result in a tranche mark-to-market exceeding the LSS note investor's tranche notional. To protect against such a scenario, the protection buyer desires the structure to deleverage (increase the funded amount) or unwind, should the probability of such a scenario rise significantly.

To gauge that probability and protect against such risk, one of the following triggers are used:

- **Tranche mark-to-market (MTM).** Tranche MTM is seemingly the most relevant trigger, yet the most difficult to verify because valuations are subjective, especially in a stressed scenario. To maintain a cushion against full loss, it may be set at, say, 50% of the market value loss of the tranche.
- **Weighted average spread (WAS) on the portfolio.** Although pool WAS is simply a proxy for tranche MTM, its value is more easily verified. We note that WAS trigger levels are a function of time to maturity and of loss rate. They increase as time to maturity diminishes and decrease as losses accumulate[94].
- **Losses (of subordination/collateral).** This trigger is based on actual cumulative collateral losses and varies as a function of remaining maturity of the LSS tranche. Although the activation of this trigger level can be easily verified, it is further removed from tranche MTM than pool WAS. We note that the loss trigger level rises with maturity, usually.

The trigger levels are set in advance, often in the form of a matrix, which links trigger level, leverage and remaining time to maturity. Whether a trigger is hit or not is often based on calculations provided by parties external to the transaction in order to avoid potential conflicts of interest. Typically, a third-party calculation agent would be used for the spread trigger, while the market value of the super-senior tranche is often estimated based on a dealer poll.

[93] We note that the structure is also packaged in unfunded form for derivative investors.
[94] However, in some cases, the WAS trigger may be flat.

Refer to important disclosures on page 196.

Trigger Tightness is a Function of Dealer's Basis Risk

In all cases, the protection buyer (dealer) relies on triggers to mitigate its risk that the tranche MTM exceeds the funded notional (or margin). Therefore, the buyer would prefer tranche MTM as the trigger because there is no uncertainty as to its exposure once the trigger is breached. By contrast, with WAS- or loss-based triggers, the protection buyer has meaningful basis risk when it attempts to hedge its true tranche MTM exposure. Consequently, such triggers are typically structured more tightly than tranche MTM triggers to enable a cushion for this uncertainty.

If Trigger is Breached, Investor has Option to Delever or Unwind

Triggers would likely partially deleverage the deal

If a trigger is breached, in most structures, investors have a choice between:

- Unwinding the transaction at the current tranche MTM, or
- Partially deleveraging the transaction by posting additional margin/funding

We believe that investors would typically choose to deleverage, rather than realize a loss. The exception would be if the investor does not have adequate cash or prefers to realize a loss and invest in a higher yielding alternative. For instance, upon breach of a trigger, a new LSS note would very likely have higher carry than the deleveraged trade because:

- Spreads are likely much wider; and
- The carry on the existing trade is reduced (usually by half) due to the posting of additional funding.

Typically, if a trigger is breached, the LSS note investor will opt to double up on its initial margin. For each successive deleveraging event, structures often prescribe a more generous set of triggers (from the perspective of the investor). The breach of subsequent trigger schedules would require posting further margin usually equal to the initial funded amount.

Triggers Are Typically Set at AAA Attachment or Higher

Triggers are struck well out-of-the-money

For most LSS notes, the trigger is set at or slightly above the required attachment point for the junior-most AAA tranche:

- In loss-based trigger structures, one solves for the trigger level simply by subtracting the minimum AAA attachment point for a given tenor from the initial attachment point.
- For MTM-based triggers, one backs in to the WAS or tranche MTM that would approximate the AAA attachment point.

Setting triggers at the AAA breakeven is intuitive for investors and greatly facilitates rating methodology, which we discuss in the last section.

Given the position of the LSS tranche high in the capital structure of the CDO, the likelihood of principal loss to that tranche is quite small. The probability of loss is usually much lower than AAA if expressed in ratings' terms (given that the LSS tranche benefits from a portion of AAA-rated subordination beneath it). Therefore, the LSS investor should be more concerned with the risk of a trigger being hit, rather than a principal loss.

Lastly, we note that the trigger levels are designed in conjunction with the overall deal structure and pool composition and are reviewed by the rating agencies when assigning a rating to the transaction.

Trigger Valuation

We calculate low probabilities for hitting triggers

The geared nature of an LSS tranche potentially exposes dealers to large losses should portfolio losses exceed the exposure of the tranche investor. In this section, we discuss trigger valuation using unfunded super senior tranche examples.

Assume that an investor buys $10m of an LSS (say, the 20-70% tranche with 10x leverage). This implies an actual notional of $100mn ($10mn x 10) of the super-senior. Also, let us assume that the cumulative loss suffered by the portfolio is 22%. The loss on the LSS can be computed using the following simple formula:

LSS Loss = Leverage x (Portfolio Loss – Attachment Point) / Tranche Width

In our example, the LSS would lose 40% (= 10 x 2% / 50%). A further 3% portfolio loss (which brings the cumulative loss up to 25%), would exhaust the LSS tranche notional (LSS loss = 100%) leaving the dealer unhedged against any further loss.

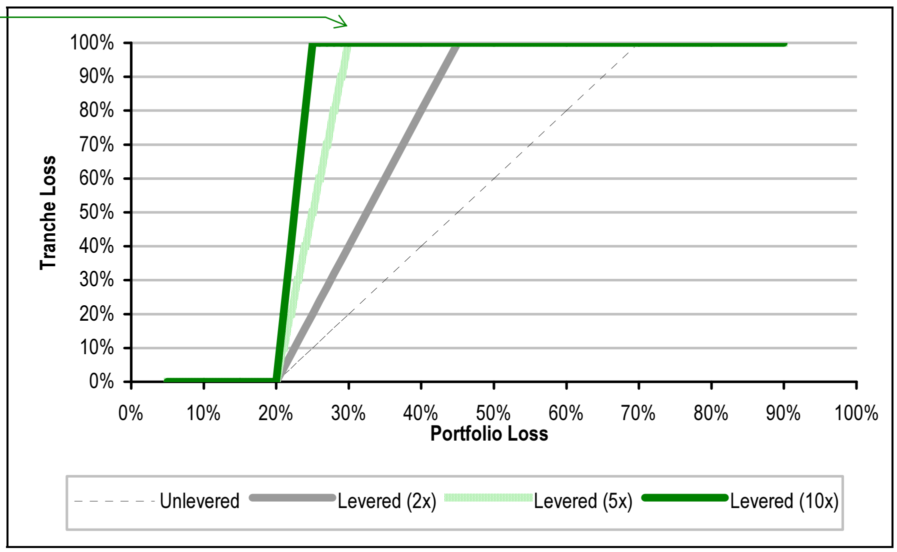
Given a portfolio, the degree of protection offered by an LSS is mainly a function of:

- The **tranche attachment point**
- The **tranche width** (the higher the tranche width, the lower the relative impact of an additional default)
- The **leverage factor** (the higher the tranche leverage, the higher the relative impact of an additional default)

Chart 178 shows the loss profile for different leverage assumptions on a 20-70% tranche.

For a given portfolio loss rate, the leverage drives the speed at which the LSS is exhausted

Chart 178: Unlevered and Levered Tranche Loss Profile



Source: Merrill Lynch.

Two key issues affect the valuation of the structure:

1. Difference between the **theoretical tranche** (e.g. 20-70%) and the **effective tranche** (e.g. 20-25% with 10x leverage or 20-30% with 5x leverage); and
2. **Risk of exhausting** the LSS leaving the dealers unhedged (gap risk).

As already discussed in the previous section, in order to mitigate these factors, a specific set of early unwind (or deleveraging) triggers is usually designed and included in these transactions. Three types of triggers exist for typical LSS structures: a) Portfolio loss triggers; b) Weighted average portfolio spread (WAS) triggers, and c) LSS mark-to-market triggers.

If the chosen trigger is hit, the investor can either unwind the LSS completely or deleverage the trade, i.e. increase the notional exposure of the LSS. In the latter case, the unfunded investor would continue to receive the same $ amount but a smaller effective spread due to the higher notional exposure.

Portfolio Loss Triggers

Portfolio loss triggers are based on actual losses experienced by the underlying portfolio. Table 52illustrates typical loss triggers for the 30-100% CDX tranches. This assumes the 125-name CDX IG4 index as the underlying portfolio.

Table 52: Loss Trigger Example (5y CDX 30-100%, 10x Levered)

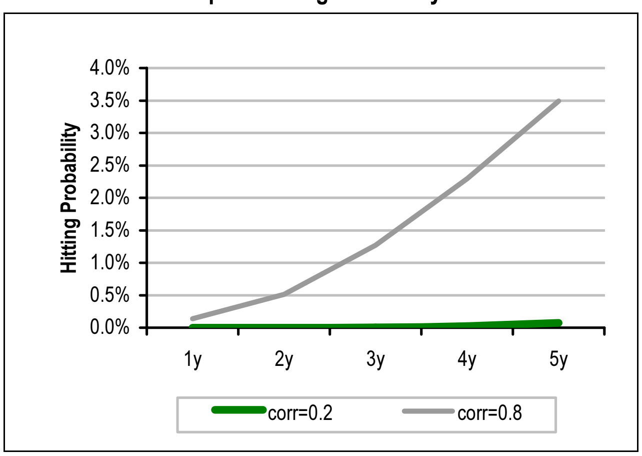
Time from Trade Date (Up to and Including)	Loss Trigger
1st year	16%
2nd year	18%
3rd year	20%
4th year	21%
5th year	21.5%

Source: Merrill Lynch; Assumes underlying portfolio is the 125-name CDX IG4.

For example, if, at anytime in the 4th year from the trade date, cumulative losses in the CDX IG4 portfolio exceed 21%, the transaction would deleverage or unwind at the prevailing market conditions (underlying CDS spreads, correlation).

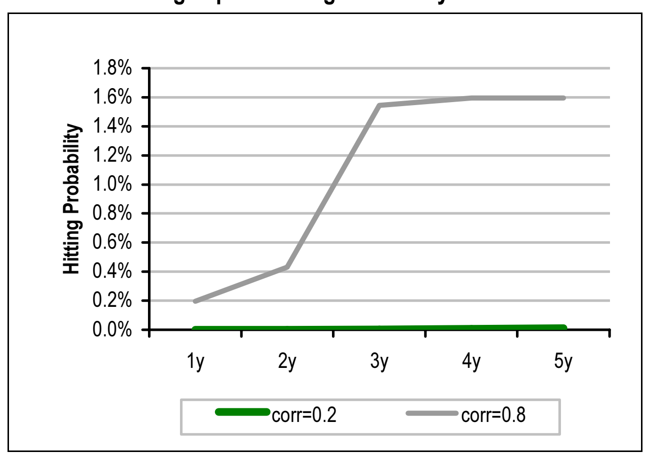
By adopting the standard Gaussian copula model, it is then possible to assess the **market-implied and rating-implied** probability of hitting the Table 52 loss triggers (Chart 179 and Chart 180). The main benefit of this trigger mechanism is that the LSS will deleverage or unwind only if there are actual defaults. Spread volatility, demand technicals and liquidity issues do not contribute to early unwinding or deleveraging of the LSS.

Chart 179: Market Implied Hitting Probability



Source: Merrill Lynch. Assume a 10x levered 30-100% CDX.NA.IG tranche.

Chart 180: Rating Implied Hitting Probability



Source: Merrill Lynch calculation on S&P data. Individual ratings as of Aug, 29th 2005. Assume a 10x levered 30-100% CDX.NA.IG tranche.

Weighted Average Portfolio Spread Triggers

An alternative trigger mechanism is based on the volatility of the underlying average CDS spread. It is usually specified via a spread-loss matrix similar to those shown in Chart 181 and Chart 182. For a given cumulative loss rate and leverage (for example 1% and 10x leverage respectively), if the notional weighted average portfolio spread exceeds 320 bp between 3y to 4y time to maturity, then the transaction is delevered or unwound at the prevailing market levels.

Chart 181: Loss-Spread Trigger Matrix Example (10x leverage)

Cumulative portfolio losses	Time to Maturity: 5y	4y	3y	2y	1y
0%	300 bps	350 bps	…	…	500 bps
1%	275 bps	320 bps			
2%	…	…	…	…	…
…	…	…	…	…	…
…	…	…	…	…	…
10%	60 bps	82 bps	…	…	250 bps

Average Portfolio Spread

Source: Merrill Lynch

Chart 182: Loss-Spread Trigger Matrix Example (5x leverage)

Cumulative portfolio losses	Time to Maturity: 5y	4y	3y	2y	1y
0%	400 bps	460 bps	…	…	610 bps
1%	316 bps	350 bps			
2%	…	…	…	…	…
…	…	…	…	…	…
…	…	…	…	…	…
10%	71 bps	97 bps	…	…	283 bps

Average Portfolio Spread

Source: Merrill Lynch

As seen in the charts above, the weighted average portfolio spread threshold is affected by three main drivers:

1. **Time to Maturity:** Spread threshold **increases as time to maturity decreases**. For a given portfolio loss rate, the probability of multiple defaults hitting the loss barrier decreases with time thus increasing the LSS tolerance versus higher spread movement[95].
2. **Portfolio Loss Rate:** Spread threshold **decreases for higher portfolio losses**. At any point during the life of the transaction losses in the underlying portfolio reduce the tranche subordination and, therefore, increase the risk of exhausting the tranche protection. As a consequence, the tranche tolerance against spread volatility decreases.
3. **Leverage:** Spread threshold **decreases for higher leverage**. Higher leverage increases the risk of exhausting the tranche. This, in turn, reduces the tranche tolerance against spread movements.

Tranche Mark-to-Market Triggers

The last trigger mechanism is based on the mark-to-market of the tranche. If the value of the tranche falls below a certain threshold, the tranche is deleveraged or unwound at the prevailing market conditions.

This method is the least attractive for investors as it requires a market value for a super-senior tranche which is typically not very liquid. Marking this tranche to market can therefore be a challenging task. A dealer poll system is often adopted to determine the super-senior spread.

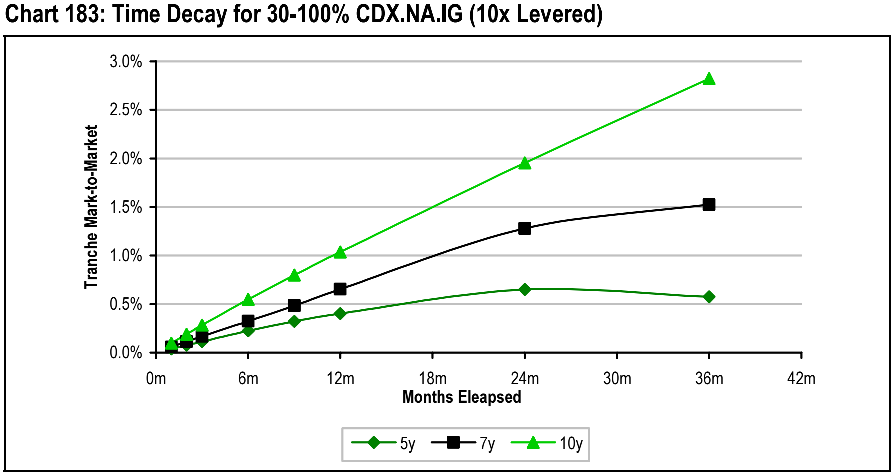
Time Decay

As a seller of protection, the LSS investor also benefits from a positive time decay. We define time decay as the **change in the LSS mark-to-market due to the passage of time**, under the assumption that:

- The spread curve of each reference credit is constant; and
- The correlation remains constant.

Time decay measures the MTM impact on the tranche due to the "curve roll-down" of each of the credits in the underlying portfolio. Chart 183 exhibits the time decay profile of the 5y, 7y and 10y CDX IG4 30-100% tranche with a leverage of 10x.

Chart 183: Time Decay for 30-100% CDX.NA.IG (10x Levered)



Source: Merrill Lynch

95 Given the standard formula used to compute the mark-to-market at any time t for a tranche expiring in T [**Tranche MTM(t)= (Tranche Spread at inception – Tranche Current Spread) x DV01(t,T) + Actual Tranche Losses**], we observe that, as t approaches the maturity T, the tranche DV01 will decrease thus lowering the impact of a spread widening in the overall tranche mark-to-market.

Refer to important disclosures on page 196.

LSS versus CDO Squared: External versus Internal Leverage

If we track the development of the CDO market, there seems to be a certain logical progression of introducing more leverage as spreads tighten and investor search for yield. The move from fully funded to partially funded transactions through to single-tranche CDOs and the repackaging of single tranche CDOs into CDO Squared followed the logic of building up leverage internal to the CDO structure. With the introduction of LSS note, the tack changes through the introduction of external leverage to parts of the CDO structure combined with performance triggers.

Table 53 provides a brief comparison between the two forms of leveraged investments on the basis of CDOs: LSS and CDO Squared.

Table 53: Comparison between LSS and CDO Squared

	LSS	CDO Squared
Nature of leverage	Leverage external to CDO structure	Leverage internal to CDO structure
Changes in leverage	Leverage established at closing, but can be reduced upon trigger events by building up reserve account	Leverage established at closing and changes only in case of losses in the reference pool
Option to delever or unwind	Investor may have an option to delever (by increasing reserve account) or unwind transaction upon trigger activation	Investor has no option to change leverage and transaction unwind dependant on settlement negotiation
Risks to investor	Investor assumes remote default risk, but higher market-to-market risk (MTM payment in case of unwind) and/or margin posting risk	Investor assumes default/ recovery risk only, usually level of defaults exceeding historical averages
Rating	Rated AAA; rating depends on probability of trigger activation and/ or losses reaching attachment point	Rated from AAA to BBB; rating depends on probability of losses reaching attachment point

Source: Merrill Lynch

Rating Agency Approaches to Rating LSS Notes

Agency unwind assumption may imply conservative ratings

Although no official methodology/criteria have been published, S&P, Moody's and Fitch have outlined their initial approach to rating IG synthetic LSS notes in recent publications[96]. As discussed above, LSS notes usually face the risk of trigger breach where losses accumulate to the AAA breakeven attachment point. Thus, the focus of the rating analysis shifts to assessing the probability of breaching the trigger. As the agencies assume that the breached trigger will result in an unwind, the rating is conservative to the extent one believes that the investor will instead deleverage and thus avoid locking in a loss.

- **S&P**

S&P has outlined their approach to rating leveraged super senior CDO tranches when the trigger is either loss-based or spread-based. S&P addresses both the default risk and the risk of breaching the trigger when assigning the rating.

Loss-Based Trigger

The rating approach involves using a version of CDO Evaluator to estimate the probabilities of losses reaching the trigger levels in each period. Whether or not deleveraging is allowed, either outcome implies either cash payments by the protection seller or an unwind where the protection seller is likely to receive less

[96] S&P report "Approach to Rating Leveraged Super Senior CDO Notes", 22 August 2005 Moody's presentation "Moody's Approach to Leveraged Super Senior", 7 July 2005, Fitch Ratings report "Leveraged Super-Senior Credit Default Swaps", 26 July 2005

than par. Thus breaching the loss trigger is treated as a default event in simulation runs. The number of these events determines the overall probability of breaching the trigger and is compared to the default table to arrive at the corresponding rating, usually triple-A.

Spread-Based Trigger

S&P chose to model the timing and magnitude of portfolio losses and determine the probability of the spread trigger being breached. The losses are assumed to be driven by two factors (global and industry) and modeled within an extended structural model. Two factors allow for correlation between obligors in line with the S&P correlation assumptions used in the CDO Evaluator. The model parameters are calibrated to fit CDO Evaluator assumptions as well.

Portfolio average spread is modeled as a mean-reverting (Vacisek) process and is calibrated on bond index and CDS historic data. After the losses are simulated they are correlated with the average spread process; however, S&P notes that the effect of the correlation between average spread and the global and industry factors on the probability of the LSS default is very small relative to the effect of changing the volatility in the spread model. As a result, the focus is placed on stressing the volatility of the spread rather than fine-tuning the correlation between the portfolio spread and the factors.

Unlike in Moody's and Fitch approach, the ratings transition is not modeled based on the finding that the effect of shortening maturity is likely to dominate the effect of even a very stressful rating transition scenario. Thus the average portfolio spread is modeled as a constant-maturity spread, thereby offsetting the effect of a potential negative rating transition scenario.

■ Fitch Ratings

The rating of LSS notes addresses the probability of the transaction being terminated upon breaching the unwind trigger (in addition to other factors). Although deals typically allow for additional cash injections by the investor to "deleverage" the transaction and avoid its termination when spreads widen significantly, Fitch's approach is to assume that no such action will take place. Fitch has developed models for both loss-based and WAS-based structures.

Loss-Based Model

LSS notes with loss-based triggers are modeled within Fitch's STCDO-squared VECTOR model treating each trigger as an individual inner CDO (building block). Each of the inner CDOs is set up with the attachment point and maturity consistent with the trigger level. In order for the LSS note to get the AAA rating, losses may not exceed the AAA attachment point at both the master and inner CDO level.

WAS-Based Model

Fitch has also developed a model for WAS-based structures. The model takes into account the four most important factors affecting portfolio spread – losses, rating transition, spread movements (market and individual) and the correlation between spreads and rating transition. As inputs, the model takes rating transition matrices from Fitch rating transition studies and calibrated correlations, i.e. standard correlations adjusted for smaller time intervals and for the dependence between time periods due to the fact that rating at the end of a time period will depend on the rating at the beginning of the time period.

Rating transitions are modeled for each name in the portfolio at each time interval. The corresponding spreads are aggregated into the pool WAS. Fitch notes that if a portfolio has a large number of names (e.g., a typical portfolio of 200 names), then the effect of individual spread movements is not significant, and it is more worthwhile to concentrate on market spread movement instead of trying to model each individual spread movement.

The WAS is modeled as a mean-reverting process (Gaussian diffusion model), which is calibrated using the data on corporate spreads and CDS spreads between Sep-98 and Oct-04. Earlier data would not likely produce substantial spread widening and thus may underestimate the probability of WAS triggers being breached.

Once both rating transitions and average spreads are simulated, the model then derives the default rate on the portfolio and the probability of breaching the WAS trigger.

The model assumes a direct relationship between the spreads and the rating transition, which is a reasonable assumption given that rating is a measure of the credit quality and is historically linked to the credit spread performance. However, there may be cases when the spreads widen without accompanying change in credit fundamentals. If, indeed, technicals do activate the spread trigger, an unwind could force an investor loss where the portfolio suffers no or few losses.

■ Moody's

Moody's approach is very similar to Fitch's in spirit. Rating changes are modeled periodically, using average transition matrices since 1983 (including defaults). The rating changes are correlated, assuming 3% and 15% correlation (these are derived from the analysis of co-movement of upgrades and downgrades). For each rating category, average spread is modeled separately. Monte-Carlo simulation is used to calculate daily spreads/losses for each rating category.

Spreads are assumed to follow a mean-reverting (Vacisek) model, which is calibrated using two sets of data: aggregate bond spreads for each category from 1991-99 and CDS spreads since 2001.

Again, once both rating transitions and average spread are simulated, the model then derives the default rate on the portfolio and the probability of breaching the WAS trigger.

Regulatory Capital Treatment

As mentioned earlier, the super-senior tranches of CDOs were usually placed with monoline and multi-line insurers. The financial regulators in different jurisdictions have different requirements for the CDO super-senior position. For example, US bank originators of CDOs are not required to hedge the super senior tranche and are assigned 0% risk weight against such positions; whereas, in Europe, banks are required to hedge such risk transacting with another bank or a monoline counterparty. Subsequently, the bank attracts a 20% risk weighting based on the substitution approach, which considers risk weight for the hedging counterparty. An OECD financial institution attracts 20% risk weight.

We understand that there is no pending change in the regulatory capital treatment in the US, except maybe for the largest, internationally diversified banks. In Europe, banks will implement new regulatory capital guidelines under BIS2 from Jan 1, 2007 (the transition period starts from January 1, 2006). Under BIS2, the risk weights for securitisation positions are commensurate with their ratings (under the Standardised Approach, RBA) and with ratings, granularity and seniority (under the Ratings Based Approach). Under the Standardised Approach, the risk weight for a AAA is 20% regardless of how many AAAs there are in the structure. Under the RBA approach, CDOs are considered granular securitisations. Hence, the risk weighting of a CDO tranche will depend on its rating and seniority: the most senior AAA will attract 7% risk weight and all AAAs below it will attract 12%.

12. CDO Combination Notes

This chapter is extracted from a report published on 7th July 2005 by Davletova et al.

A combo note is a stand-alone, usually rated, note, combining debt or debt/ equity tranches of a given CDO. It allows investors to find the most suitable risk/reward profile and to express a view on the future performance of the CDO underlying pool. Combo note returns are a function of defaults: their volume, timing and clustering.

A combination note is usually created by combining two or more different CDO debt tranches or a debt and an equity CDO tranche within a single structure. That new structure is treated as a stand-alone instrument within a given CDO and is often rated as such. The proportion of each tranche in a combination note is determined by desired risk/return characteristics of a combination note and/or its desired rating. It should be in demand by investors, who find the equity payback attractive, but do not like the fact that it is not rated and are not willing to take the higher principal payback risk, associated with a direct investment in equity tranches.

Combination notes are used mostly in cash CDO transactions, particularly in leveraged loan CLO deals, but they also appear in CDO of ABS and some of the synthetic CDOs. The growing popularity of combination notes is due to a number of opportunities they provide:

- Combination notes are structured to offer investor risk/return profiles that are not available within the original CDO structure. For example, combining double-A tranche with double-B tranche may create a triple-B combination note, which may not have been issued in the original structure. This may benefit an investor, who otherwise would not be able to buy a triple-B exposure.
- A note may be individually tailored to suit investors' risk preferences by selecting which tranches to combine and in what proportion.
- Combo notes that include an equity tranche allow investors to benefit from the potential upside of an equity investment, without taking the direct risk of such equity investment.
- Combo-notes can be rated, thus addressing many investor requirements for a desired rating.
- Combo notes help address investors' risk capital considerations.
- Combo notes allow broadening of the investor base for a range of CDO products.

Main Combo Note Types

Combo notes could be structured in a variety of ways. We describe the typical structures below.

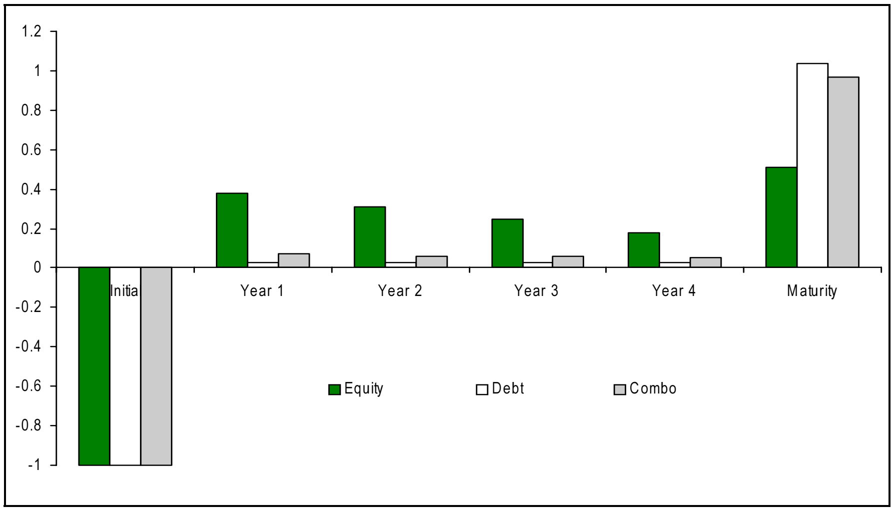
A Combination of One or More Debt Tranches with an Equity Tranche

This is the most common CDO combo structure. Investment in the CDO equity could be attractive due to the high returns if default rate is low. However, as the cash flow payments to the CDO equityholders are subject to the default risk of the CDO underlying collateral pool, direct investment in CDO equity may not be suitable for all investors. Combo note technology allows alternative way to benefit from equity type returns and at the same time maintain desired risk profile. For example, a note could be structured around original CDO capital structure with the notional being allocated between CDO equity and a debt tranche (tranches). The note could be rated, with the rating depending on the relative equity exposure and the coupon on the combo note, if any. Higher equity allocations will result in a lower rating relative to the full notional allocation in the debt tranche.

Refer to important disclosures on page 196.

In Chart 184 we illustrate cash flows to holders of debt tranche, equity tranche and a combination note in case of a hypothetical example of a synthetic CDO[97] of corporate names with a combo note consisting of 10% equity and 90% senior debt tranche under 2% annual default rate assumption.

Chart 184: Cash Flows to Static Synthetic CDO*



Source: Merrill Lynch
* Evenly distributed defaults; cash flows are shown per €1 invested; 2% Annual Default Rate

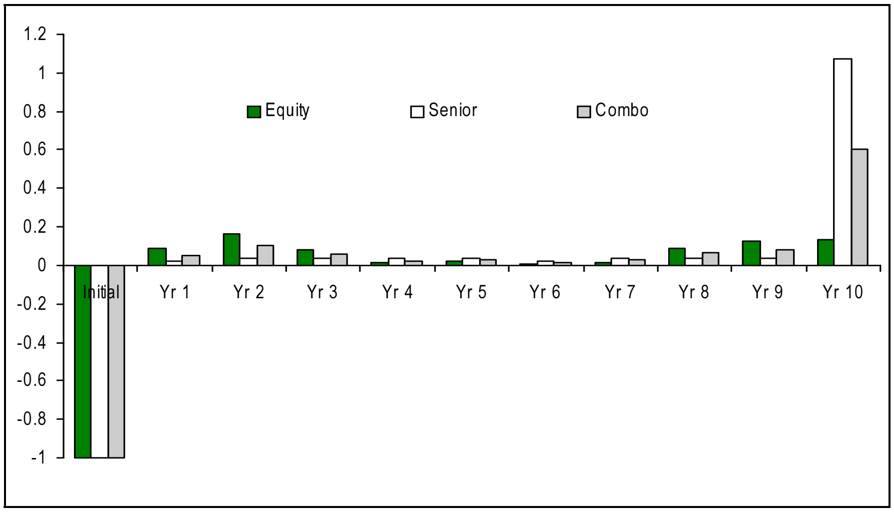
In the second case - a hypothetical example of a cash flow CDO of leveraged loans[98] the combination note consists of 50% equity and 50% triple-B exposures. As Chart 185 illustrates, the equity tranche has a cash flow profile, which is not as heavily concentrated towards the end of the deal's life, as is the case with the debt tranche's cash flows. The last cash flow to equityholders represents only 12% of the total cash flows, much lower than 76% that debtholders receive on the last payment day (or 54% for the combination note).

It may be worthwhile contrasting the two cases. Unlike the synthetic CDO example above, cash CDO cash flows are more volatile due to uncertainty of the asset cash flows (versus synthetic CDO cash flows) and the presence of the certain structural features. For example, cash flow CDOs typically have overcollateralisation / interest coverage tests, which, if breached, may trigger an interest diversion from the equity tranche to redeem the notes in order of seniority (or sometimes by redeeming first the more expensive tranches, if there is a turbo feature). Another variation in the cash flows of a cash deal may be triggered by the existence and trigger if an equity IRR cap.

While for the purposes of the illustration of the effect of defaults on combo note returns we use a synthetic CDO example, the results are also applicable to cash CDO combos. For them, though, the return profiles will be modified to reflect the structural features mentioned above.

[97] The portfolio composition is similar to iTraxx. The combo note's size is 3% of portfolio's notional, five-year transaction. For this example we priced equity (0-3%) at 9.73% and senior (12-22%) at .13%.
[98] AAA/A/BBB floating-rate liability structure (65/15/10%) with 10% equity and spreads on rated tranches of 30/80/175bp over Euribor (2.1%). We also assume a constant reinvestment yield on loans of 5.1% (2.1% plus 300bp spread), 75% recovery (i.e. mostly senior secured loans), and 25% annual prepayment rate on loans.

Chart 185: Cash Flows to Cash CDO *



Source: Merrill Lynch
* Evenly distributed defaults; cash flows are per €1 invested; 4% annual default rate

Combo notes with equity participation usually come in two forms, reflecting the cash-flow structure of the debt component. In Table1 we summarise key features of variable and stated coupon structures.

Table 54: Comparison of Debt Components with Stated Coupon and Variable Coupon

Variable Coupon Structure	Stated Coupon Structure
Debt component is issued either with **zero or with below stated coupon**. Usually, par of the debt component is equal to the combo note's principal. The excess of the note's principal over the debt discount is invested in the CDO equity. Alternatively, the debt component par may be below the combo note's principal.	Debt component is issued as a vanilla note with stated coupon. Combo note's notional is allocated between equity and debt to achieve desired risk/return profile.
Rated notional is usually equal to combo note's principal and any promised coupon	Rated notional is usually equal to combo note's principal and any promised coupon. Rated notional amortises with the cash flows from both interest and principal payments if any.
Current cash flows are generated by the coupon of the debt component if any and equity distributions.	Current cash flows are generated by the coupon of the debt component and equity distributions.
Under a low-defaults scenario, at maturity debt component will yield par and any remaining equity distributions, thus total cash flow at maturity may be higher than par.	Under a low-defaults scenario the note will return par at maturity.
Structure generates back-loaded cash flows.	Structure generates front-loaded cash flows.

Source: Merrill Lynch

For the variable-coupon structure equity exposure is equal to the combo note's principal less discounted zero coupon debt exposure. The size of the discount and hence allocation to equity is a function of the rating of a debt tranche. For a given debt tranche, variable coupon and stated coupon structures with the same equity exposure will have similar return characteristics.

Refer to important disclosures on page 196.

A Combination of Two Debt Tranches

Combining positions in two rated tranches within a single structure can produce a variety of risk/return profiles. This flexibility could be appealing to investors with a particular rating or return criteria. As with equity participation notes these structures could have a variety of cash flow profiles reflecting investor's preferences. Combining two debt tranches is used much less frequently than combo notes with equity exposures.

Other variations

As an alternative to the rated tranche/equity combo investors may consider a more defensive structure where highly rated assets (reference assets) are purchased in the zero coupon form (discount), such that the value accrues to par at maturity of the deal. Remaining notional is invested in the CDO equity to provide possible upside in returns.

Voting Rights

Cash flow CDO combination notes retain all the voting rights of their components, i.e. combination notes are treated as the corresponding class notes, and in the proportion of the principal of the components to the total principal on this class of notes. Having said that, the voting rights are unaffected by the principal on combo note being reduced by the amount of interest received. They will be affected if the underlying class of notes is amortised, for example, if a combo note had a triple-A class component and that class was paid down due to OC test failure (or any other early amortisation situation), then the combo note's principal and voting right are reduced. We note that for synthetic CDO combo notes, where no actual re-packaging occurs, the issue of the voting rights is not relevant.

CDO Combo Note Returns

The risk-return characteristics of a combination note are driven by the performance of its constituents and their relative exposures. Consequently, all the factors that are affecting note's components will have direct impact on the performance of the note. Given that in most cases one of the components is equity, here we will concentrate on a combination note comprising an equity tranche and a debt (senior) tranche.

Direct investment in the equity exposes an investor to the first layer of losses in the portfolio. It is an effective but risky way to express a view on the over-all default rate and timing of defaults. Combo notes, on the other hand, offer a more defensive way to leverage the same view via a rated structure.

By choosing the equity/debt tranche allocation in the combo note an investor may achieve the desired risk/return profile not available within original CDO. To increase his return an investor can increase notes' exposure to the riskier of two allocations. On the other hand, more defensive position would keep higher allocation in the more senior debt. For example, if an investor has a view that the default rate will stay within one standard deviation of historical average and defaults will be back-loaded then a more aggressive allocation into equity could be appropriate, as it will benefit from the early high payouts.

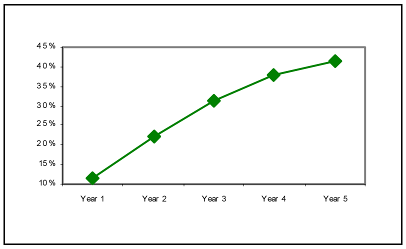
'Loading' the Defaults

Ultimate performance of the combo structure will be driven by:

- the realised default rate,
- how clustered the defaults are (which is affected by correlation or co-dependence among credits) and
- the timing of defaults.

Different default timing profiles will produce substantial variations in the note's payoff. Equity returns are extremely sensitive to the timing of defaults. For example, as shown in the side chart, at the same cumulative level of default rate equity IRR may vary from 10% to 40% depending in which year the defaults take place. Clearly, this will have a pronounced effect on the performance of the combo note. We address this by considering three scenarios for the default timing: front loaded, evenly spaced and back loaded.

Chart 186: Equity IRR vs Year of Defaults



Source: Merrill Lynch

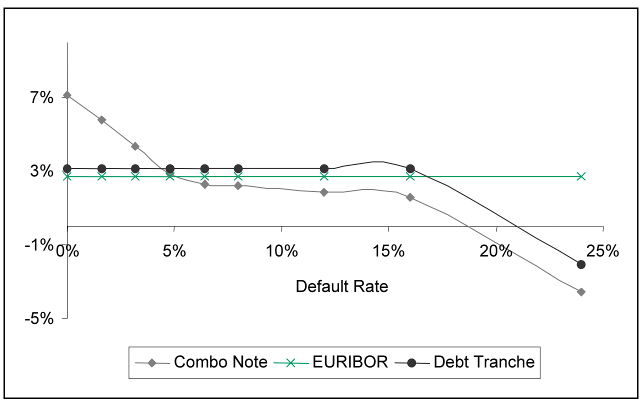
High or low, but evenly spread defaults

To illustrate some of these issues and their implications for CDO combination notes, we will use a hypothetical example of a synthetic CDO[99] described in the previous section and show IRR on both its components and the combination note for different default rate scenarios.

As shown in Chart 187 and Chart 188 when defaults are evenly distributed between deal's inception and maturity combo note IRR curve lies mid-way between IRR profiles of its components. As we increase the proportion of the equity component, the IRR curve will become steeper (higher leverage) and closer to the CDO equity IRR profile.

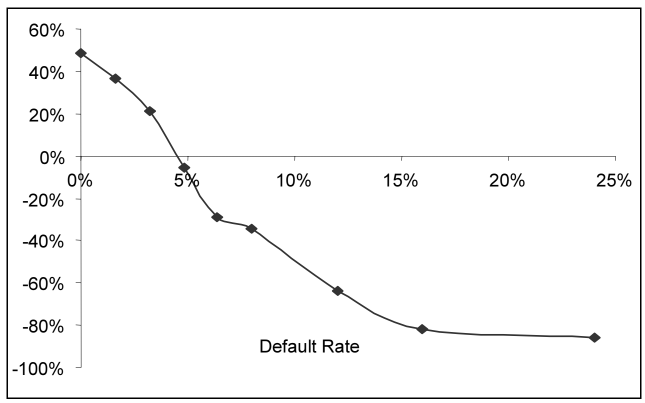
An investment in a combo note will outperform direct CDO equity exposure when defaults are at moderate to high levels, and underperform at low default levels. In that respect, returns on the combo note will have much lower variability than the direct equity investment, but higher than the direct debt investment.

Chart 187: IRR for Evenly-Spaced Defaults *



Source: Merrill Lynch
*Synthetic CDO example

Chart 188: Equity IRR for Evenly-Spaced Defaults *



Source: Merrill Lynch
*Synthetic CDO example

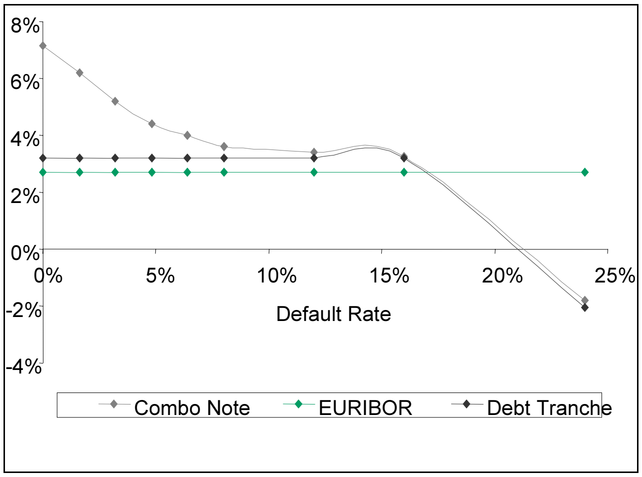
Unevenly distributed defaults

When defaults arrive in the last few years of the deal, combo note's performance could be better than a safer investment in the senior note only due to equity's significant outperformance. The equity cash flows are much more front-loaded, i.e. its cash flow profile looks much less like a bullet-type profile of debt tranches which receive as much as 70-80% of their total cash flows towards the end of the deal's life. Thus **the more back-loaded the defaults are, irrespective of whether they are lump-summed or evenly distributed across the back-end, the better the CDO equity component performs** and the combo note will participate in the early equity income distribution. In this case, relatively aggressive allocation in the equity part of capital structure will result in a more significant outperformance versus the debt tranche (Chart 189 and Chart 190).

[99] The portfolio composition is similar to iTraxx. The combo note's size is 3% of portfolio's notional, five-year fully funded transaction. For this example we priced equity (0-3%) at 9.73% and senior (12-22%) at 0.13%.

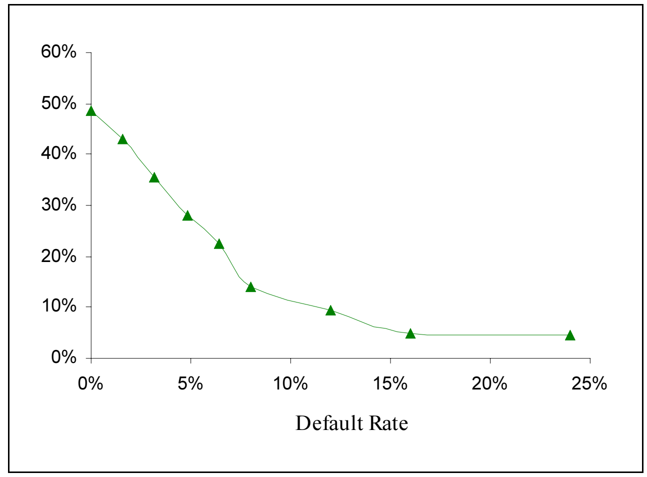
Refer to important disclosures on page 196.

Chart 189: IRR for Back-Loaded Defaults*



Source: Merrill Lynch
*Synthetic CDO example

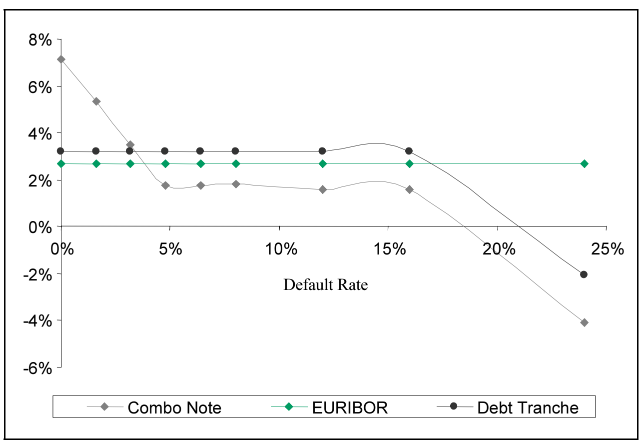
Chart 190: Equity IRR for Back-Loaded Defaults*



Source: Merrill Lynch
*Synthetic CDO example

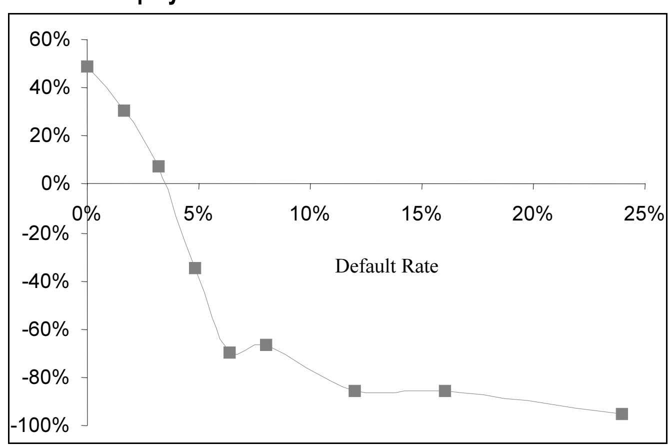
When defaults are front-loaded (Chart 191 and Chart 192), the relative performance depends on the overall cumulative level of defaults. If defaults are low, the combo note still outperforms the debt tranche (and underperforms equity), but at moderate to high levels of defaults the equity tranche gets written off early in the deal's life and will not receive enough residual cashflows to compensate for the principal losses.

Chart 191: IRR for Front-Loaded Defaults*



Source: Merrill Lynch
*Synthetic CDO example

Chart 192: Equity IRR for Front-Loaded Defaults*



Source: Merrill Lynch
*Synthetic CDO example

In summary:

- the CDO combination note will offer better IRR performance than the CDO debt tranche, if the defaults do not occur during the first few years of the deal.
- The combo note should also provide higher IRR if defaults do occur in the first few years but the level of defaults is low.
- If the defaults concentrate in the early years of the CDO life, then the debt tranches offer the better returns.

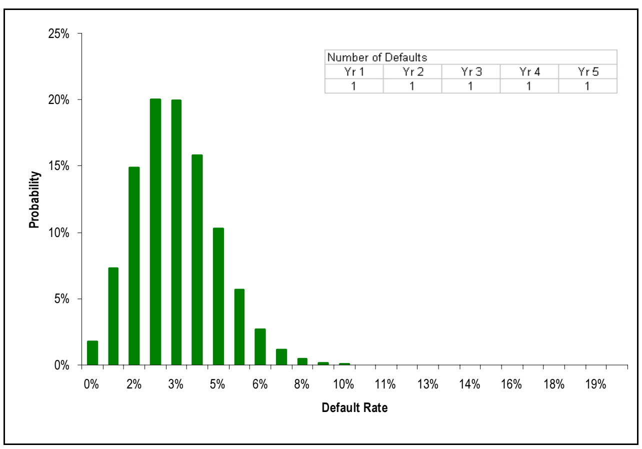
The above observations are a key element in constructing investor's approach to CDOs and related investment strategy. An investor needs to take a view on the timing of defaults and the severity of defaults over time, and on that basis decide which investment strategy makes most sense or what combination of debt/ equity combination to seek, when considering a CDO combo note. For example, if an investor expects defaults to be occur towards the end of the deal's life, then she should consider an investment in CDO equity or in a combo note with high proportion of CDO equity. If, however, an investor expects moderate to high level of defaults, concentrated in the first few years of the deal's life, then the investor may be well advised to consider a combo note with a high proportion of debt or she, in the extreme to be on the safe side, is better off buying a debt tranche only.

■ What about correlation?

Analysis of possible default scenarios is usually comprised of two components: investigating possible probability distributions for overall default rate and attempt to capture clustering of defaults in certain periods relative to the overall default rate. As we showed above, the overall default rate and timing of the defaults (front-loaded, back-loaded or evenly distributed) is a major factor for the performance of CDO and consequently combo structures.

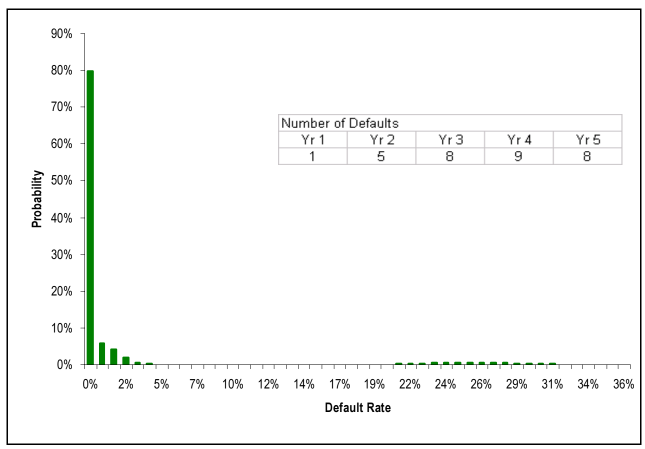
From the non-technical perspective one may associate realization of clustered defaults with high level of "correlation" among obligors[100]. For popular class of models for portfolio credit risk correlation could be thought of as a hidden parameter that influences the degree of co-dependence (or likelihood of joint occurrence) of defaults in the portfolio[101]. In these models one can simulate possible realizations of default times for a collection of names and then estimate the correlation structure, in the statistical sense.

Chart 193: Default Rate Distribution, Correlation 0%*



Source: Merrill Lynch
The shape of distribution of default rates is driven by the credit risk profile of the individual issuers and dependence assumption, in our case measured by level of correlation.*

Chart 194: Default Rate Distribution, Correlation 90%*



Source: Merrill Lynch
The shape of distribution of default rates is driven by the credit risk profile of the individual issuers and dependence assumption, in our case measured by level of correlation.*

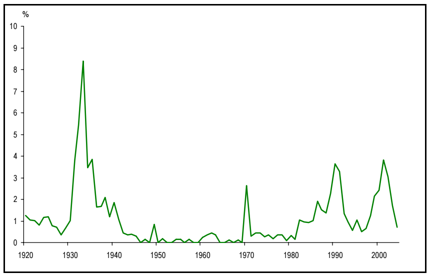
For a large number of simulation runs it will get closer and closer to the correlation structure initially specified within the model[102]. We illustrate the impact of correlation on the probability distribution of overall default rate and likely distribution of defaults over the observation period (in our case 5 years). At two extremes we have a portfolio which has either a very high level of inter-dependence among obligors (i.e. correlation is high) or credits do not share any common source for possible default (i.e. the correlation is low).

---

[100] Due to rare nature of defaults correlation, in the statistical sense, is hard to estimate from the empirical point of view. On the other hand a common practice is to specify a dependence structure that can produce observable patterns of defaults, with correlation being the parameter that controls the level of dependence.
[101] We refer to the Gaussian dependence. Although a multitude of models have been proposed, Gaussian copula is a market standard for portfolio credit risk modeling.
[102] This is a technical point and is a consequence of the modeling approach.

Refer to important disclosures on page 196.

Chart 195: Corporate Annual Default Rate



Source: Moody's

High correlation is an extreme situation, where default risk of every obligor to the large extent is driven by a common factor. This will result in a systematic behavior of the defaults (for the given observation period), either everybody/large group survives or most of the credits default. In Chart 194 we illustrate the impact high correlation has on the probability distribution (5-year horizon) of default rates for the synthetic CDO example. In the high correlation environment, overall default rate is likely to be either very low or very high with defaults being not evenly distributed (1 default in yr 1, 5 defaults in year 2, 9 defaults in year 4). These phenomena (i.e. clustering of defaults, and hence increased co-dependence among obligors) manifests itself in the considerable variation in the annualized corporate default rates (see side chart). Unfortunately such developments cannot be observed in many asset classes included in CDOs as the incidence and history of defaults are limited – European leveraged loans, US trust preferreds, structured finance securities, etc.

At the other end of the spectrum is a world where default risk of every obligor is driven purely by idiosyncratic factors. The consequence is that the overall default rate is likely to be close to the expected[103] (1 default per year in our example, as shown in Chart 193) [104]. In particular, in the above example there is an 81% probability that the default rate will stay within one standard deviation of the expected.

The impact of correlation should be definitely considered by investors as it has a direct influence on the payoff profile of the investment.

Rating Agencies' Approach

The rating approach for a combo note is derived from the existing rating approaches for CDOs, but exhibits several differences. As with every investment, CDO combo notes are subject to a range of potential regulatory capital and accounting treatments. In that respect some level of uncertainty remains both at national and at a company level.

When looking at the rating assigned to the combo note it is important to pay attention to whether the note is rated to principal only or to principal and coupon. The former rating addresses the likelihood of receiving principal by maturity date, while the latter rating reflects the probability of receiving both the principal and interest by maturity date. Therefore, they may not be always directly comparable to corporate bond ratings or even senior CDO tranche rating, which usually carry a non-PIKable interest and thus their rating assesses the likelihood of receiving ultimate principal and timely interest.

We note that in line with rating agencies' CDO rating methodologies, CDO combo notes are rated on the basis of a simulation of the cash flows of the CDO collateral pool. CDO combo notes' ratings are not a weighted average rating of its component parts.

■ Standard & Poor's

S&P rates combo-notes to the principal balance outstanding, rather than a hypothetical rated balance, and to all promised interest, if a certain coupon was promised by the note. Only if a coupon is stated as contingent (i.e. not promised), may S&P rate the note to principal only.

Standard & Poor's uses CDO Cash Flow method for rating combination notes. The approach [105] is essentially the same as the one used for rated tranches. First,

[103] Expected default rate is a function of the individual credit risk profile of the obligors in the portfolio and is independent of assumption on co-dependence.
[104] We take the average number of defaults each year for all scenarios with cumulative default rate within 1 standard deviation (1.6%) from the mean of 3.16%.
[105] "CDO Combination Securities-Standard & Poor's Global Rating Methodologies", Standard & Poor's, 7 July 2003

the cash flows on all components of a combo note are modelled. Then these cash flows are applied to the combination note, and if the note fulfils the principal and/or interest pay obligations to the noteholders at the required rating, then this rating can be assigned to the combo note.

For example, if a combo note consists of a double-A tranche and a double-B tranche, and the desired rating is single-A, first the cash flows for both tranches are modelled at single-A default level. As a double-B tranche generates higher interest than required on a combo note the excess interest may be applied against losses from a more conservative single-A scenario. To ensure single-A rating, the cash flows must be sufficient to pay principal and/or interest promised to noteholders.

■ Moody's[106]

Moody's rating addresses the expected loss to investors by the maturity date. Th rating process consists of the following steps.

1) Using the recovery rate and other stresses applicable for the required rating on a combo note, model cash flows are simulated for each component of the combo note and for each default scenario. Both interest and principal cash flows included.

2) Compute the amount of promised principal/interest on the note
For each default scenario compute what is promised to the investor taking into account all the structural features (PIK etc.)

3) Discount received cash flows and promised cash flows using the same interest rate used in step 1. The difference between the two is the present value of the loss.

4) Compute the expected loss: take probability weighted average of the ratios of present value of loss to present value of promised cash flows

5) Assign the rating by comparing the expected loss of the combo note to benchmark bullet bonds of each rating and selecting the one with the matching expected duration of the promised cash flows.

■ Fitch[107]

Fitch's approach to rating combination notes is similar to S&P's. Cash flow analysis is performed for the components of the combo note, at the stress level of the required rating. The future performance of the combo note is evaluated probabilistically under various stress scenarios (default timing, interest rate) and the required rating is granted only if cash flows are sufficient to cover the principal and stated interest on the combo note within a degree of certainty.

Regulatory Capital Treatment of CDO Combo-Notes

The regulatory capital treatment of combo note is, to put it mildly, an enigma at present. To our knowledge, there is no clear reference in the BIS2 Guidelines, nor have we found a specific pronouncement by any national regulator to-date. In the absence of specific regulatory guidelines we can only explore the potential regulatory capital treatment to be applied to combo notes. Such treatment will reflect the answers to several questions.

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[106] "Using the Structured Note methodology to Rate CDO Combo-Notes", Moody's Investors Service, 26 February 2004
[107] "Fitch's Criteria for Rating CDO Equity and Combination Notes", Fitch Ratings, 26 July 2001

Refer to important disclosures on page 196.

Questions to answer

What is a combo note and how is it structured?

There are, for the purposes of this discussion, two types of combo notes:

- a combo note as a combination of the equity tranche and a senior or a mezzanine tranche of a given CDO
- a combo note as a combination of the equity tranche of a CDO and highly-rated government securities – this variation is also known as principal protected notes. Such combination can be executed with a highly rated corporate security, as well.

In the above cases we are talking about a static combination of a riskier and less risky security. If such combination is dynamically and regularly adjusted the principal protected note attains the characteristics of what is widely known as CPPI (Constant Proportion Portfolio Insurance). A corollary question is whether the combo note in any or all of the above formats should be treated as a securitisation position and, if not, what should it be treated as.

In the case of the first type (a combination of an equity and another tranche of a given CDO), the treatment of the combo note as a securitisation seems fairly obvious. The second type combo note, as a mix of CDO equity and government bonds, can also be viewed as a structure with some form of subordination: 'senior', risk-free government bond and a 'junior', risky CDO equity – however, the elements of such subordination do not belong to the same stand alone structure, but are rather drawn from two different sectors of the fixed income market. The same view can be taken with regards the CPPI structure, but managed on the basis of a predetermined formula. Management or dynamic element, as such, does not negate the potential treatment of a note under the securitisation framework – eg managed cash or synthetic CDOs.

If the combo note, in one of its variations above, is not treated as a securitisation positions, a question arises as to what an instrument it should be viewed as, for the purposes of regulatory capital.

How is the combo note rated? What is the rating promise?

Any rating can be generally assigned on the basis of:

- timely repayment of principal only
- timely repayment of principal and interest
- availability: public or private.

In other words, the rating can incorporate different aspects of the repayment profile of the security or have different level of availability and transparency.

Regulatory Capital Treatment

The different combinations of answers to the above questions could lead to significantly different regulatory capital treatment of a combo note. For the purposes of this discussion we focus only on a combo note, which represents a re-packaging of the equity and mezzanine (or senior piece) of the same CDO transaction.

When considering how the variety of approaches in the assessment of the regulatory capital of a combo note, we start with the aspects, where some clarity exists.

- *First, availability and type of rating assigned to a combo note.*

According to BIS2, in order for a rating to be used for the purposes of establishing the risk weighting of a securitisation exposure, such rating must be publicly available and must address both principal and interest repayment. Principal-only ratings are acceptable, but will required deduction from capital

or 1250% risk weighting of the respective exposure. If the rating meets such conditions, then the risk weighting can be derived from the following tables under the standardized and RBA approaches respectively:

As the combo note is purchased by an investor, then the last column of Table 55 below should be used, suggesting a risk weight of 100% for a triple-B combo note under the standardized approach and under the RBA approach the middle column of Table 56 can be used suggesting a weighting of 50%/ 75% / 100% for BBB+ / BBB/ BBB- respectively.

Table 55: Standardised Approach Risk Weightings

	Sovereign and Central Banks*	Bank** Options 1 and 2***	Corporate	Securitisation for Originator	Securitisation for Investor
AAA to AA-	0	20 / 20	20	20	20
A+ to A-	20	50 / 50	50	50	50
BBB+ to BBB-	50	100 / 50	100	100	100
BB+ to BB-	100	100 / 100	100	Deduction	350
B+ and B-	100	100 / 100	150	Deduction	Deduction
Below B-	150	150 / 150	150	Deduction	Deduction
Unrated	100	100 / 50	100	Deduction****	Deduction****

Notes:
** To determine the credit risk of sovereigns supervisors may recognise the country scores assigned by Export Credit Agencies if published and assigned in accordance with IECD methodology. Risk weightings for BIS, IMF, ECB and EU are 0%.*
*** Risk weightings for banks are also applicable for non-central government public sector entities as well as to securities firms*
**** Option 1 allows for all banks incorporated in a given country to be assigned a risk weight one category less favourable than that assigned to claims on the sovereign of that country with a cap of 100% for banks in sovereign rated BB+ to B- and unrated. Option 2 allows for risk weightings based on the external credit assessment of the banks themselves with a cap of 50% and more favourable treatment is given to claims with an original maturity of three months or less with a floor of 20%. The national supervisor will choose which Option is to be used by the standardised banks under its jurisdiction.*

*****Exceptions to the overall rule requiring deduction of unrated securitisation exposures apply to the most senior tranches in a securitisation, second loss or better position exposures in ABCP programs, and eligible liquidity facilities.*
Source: BIS CP3

Table 56: RBA Risk Weightings for Securitisation Exposures

	RW for senior positions and eligible senior IAA exposures (%) (1)	**Base RW (%) (2)**	**RW for tranches backed by non-granular pool (%) (3)**
External rating	**Effective number of loans N >= 6**	**All cases not covered in the preceding and following columns' RW**	**Effective number of loans N < 6**
Aaa	7	12	20
Aa	8	15	25
A+	10	18	35
A	12	20	35
A-	20	35	35
BBB+	35	50	50
BBB	60	75	75
BBB-	100	100	100
BB+	250	250	250
BB	425	425	425
BB-	650	650	650
Below BB- and Unrated	Deduction	Deduction	Deduction
Short - Term Ratings			
A-1 / P-1	7	12	20
A-2 / P-2	12	20	35
A-3 / P-3	60	75	75
All other ratings/ Unrated	Deduction	Deduction	Deduction

Source: BIS

Refer to important disclosures on page 196.

- *Second, acceptability of the combo note as a securitisation exposure*

The above suggested risk weights are valid, if and only if the combo note is accepted to be a securitisation exposure. Given that we limit our discussion to a combo note, packaging together the equity and mezzanine (senior) tranche of a given CDO transaction, its treatment as a securitisation exposure should not be a problem. Moreover, given the preceding analysis in this report, we believe that the combo note should be treated as a unique whole, a second-order CDO (first one being the repackaging of corporate or structured finance bonds into a CDO, and the second one – the repackaging of the tranches of that CDO into a combo note).

We can see how some regulators may be tempted to argue that the combo note should be split into its component parts, and each part risk weighted individually: That will bring the risk weighting of the CDO equity to 1250% (or deduction), while the risk weighting of the senior/ mezzanine tranche will be derived from the respective tables above. However, we believe such treatment will not be justified:

- on the one hand, the combo note has a distinctly different risk profile from the CDO tranches repackaged in its creation, as we demonstrated in the preceding analysis;
- on the other hand, looking through the combo note and splitting it into its components may create a precedent for the treatment of other securitisation positions, which could lead to the revision of substantial parts of BIS2 guidelines.

Another temptation may be to question the rating of the combo note on the grounds that the coupon payment is small – combo notes, when rated on the basis of interest and principal repayment, generally command small coupons. Such treatment will generate another set of problems:

- on the one hand, how can one impose what the appropriate coupon for any given rating level is and should not any rating be questionable of the basis of the coupon the rated bond pays?
- On the other hand, this could question the whole approach of the regulatory reliance on external ratings for regulatory capital purposes. If the rated obligation is clearly stated (principal and a specified amount of interest) and the meaning of the rating is clearly conveyed to investors (it is publicly accessible and included in the rating transition studies), then such rating should not be acceptable in one set of situations, but objectionable in another set of situations.

Conclusion

While there is no regulatory clarity with regards to the capital treatment of CDO combo notes in the context of BIS2, ***we argue that a combo note should be treated as a securitisation exposure and the risk weighting assigned should correspond to the rating of such exposure, as and when its rating meets the regulatory requirements.*** As we discuss above, a deviation from such treatment of a combo note created through the repackaging of the equity and a senior or mezzanine tranche of a given CDO would create precedents and regulatory inconsistency.

Accounting Treatment

The accounting treatment of a combo note depends on whether it is considered a financial liability or not and whether it has derivative exposures.

The definitions of financial liability, equity, derivative and embedded derivative highlighted below

IAS32 defines a ***financial liability*** as a contractual obligation to deliver cash or another financial asset to another entity or to exchange financial assets or financial liabilities with another entity under potentially unfavorable conditions. Even if the financial obligation is contingent upon an uncertain future event, the obligation is classified as a financial liability. Therefore, instruments that meet the definition of a financial liability include perpetual preferred shares (contractual dividend is payable if sufficient distributable reserves are available) and redeemable preference shares (in which the holder has a put option back to the issuer for predetermined monetary amount, which may be settled either in cash or in a variable number of shares).

In contrast, IAS 32 defines ***equity*** a contract that evidences a residual interest in the entity's assets after deducting all liabilities. Equity may include preferred stock, where the dividend is not mandatory and there is no obligatory maturity.

Next, a ***derivative*** is defined as a financial instrument or other contract embracing <u>all three</u> of the following characteristics: a/ its value changes in response to the change in a specified variable (a financial variable, e.g., interest rate, financial instrument price, credit rating or credit index, commodity price, foreign exchange rate, etc. or a non-financial variable, which however cannot be specific to a party to the contract); b/ it requires no initial net investment or an initial net investment that is smaller than would be required for other types of contracts that would be expected to have a similar response to changes in market factors, i.e. allows leverage, and c/ it is settled at a future date.

Finally, an ***embedded derivative*** is a component of a hybrid (combined) instrument that also includes a non-derivative host contract – with the effect that some of the cash flows of the combined instrument vary in a way similar to a stand-alone derivative. An embedded derivative causes some or all of the cash flows that otherwise will be required by the contract to be modified according to a specified variable (examples given above under the definition of derivative). A derivative that is attached to a financial instrument but is contractually transferable independently of that instrument or has a different counterparty from that instrument, is not an embedded derivative, but a separate financial instrument.

An embedded derivative may not be bifurcated from the underlying cash instrument, when it is driven by those factors that drive the value of the host contract (underlying instrument). E.g. a capped floating rate note does not require bifurcation of the cap.

On that basis, we understand that:

- a combo note based on the tranches of a cash CDO should be treated as a financial liability and accounted for using one of the four categories of financial accounting: held for trading, held to maturity, available for sale, loans and receivables. As it is widely known, each of these treatments has different implications for where the mark-to-market and permanent impairment flow and how the note is recorded on the balance sheet.
- A combo note based on the tranches of synthetic CDO may be viewed as one having embedded derivatives, which calls for bifurcation between the derivative and the host contract. That could be a fairly complex exercise in the context of a combo note, and if it is considered too difficult or too misleading to do, the combo note may then be treated as a combined whole and accounted for under 'held for trading' category.

Finally

The discussion of the accounting and regulatory capital treatment of a combo note repackaging the equity and senior (mezzanine) tranches of a given CDO is just that: a discussion. Individual investors should seek guidance from their national regulators and auditors as to the treatments applicable in the respective jurisdiction.

Refer to important disclosures on page 196.

13. CPPI for Debt Investors

This chapter is extracted from a report published on 14th July 2005 by Davletova et al.

The Basics Elements of Constant-Proportion Portfolio Insurance

The difficulty in predicting market movements over a long period of time and investors' desire to limit or altogether eliminate losses in case of downside market movements have prompted the developments of techniques, known as Portfolio Insurance. They allow investors to limit the negative effect and take advantage of the positive effects of the movements in different market sectors. The theoretical underpinning of such techniques is the modern option pricing theory. Two techniques, which fall within the family of Portfolio Insurance are Option-Based Portfolio Insurance (OBPI), which as the name suggests uses options, and the Constant-Proportion Portfolio Insurance (CPPI), which allows for dynamic asset allocation over time, in order to achieve the above goals.

CPPI techniques were examined in detail in a Merrill Lynch research paper *Dynamic Protection, Constant Proportion Portfolio Insurance: How It Works from Feb 6, 2003*, as they relate to the equity markets. Here we highlight key point of this technique as it is applied to debt market instruments.

■ OBPI – Making Use of Puts

One way to implement portfolio insurance is by using a put option. Option-based portfolio insurance is designed to expose an investor to limited downside risk while allowing some participation in upside markets. The investor buys a portfolio of risky assets as well as an European put option to sell the portfolio at a given price (put strike price). Whatever the value of the risky asset is at maturity, the portfolio value will be at least equal to the strike of the put. Thus the option-based portfolio insurance allows an investor to receive a guaranteed fixed amount at maturity (subject, of course, to counterparty risk).

The drawback of this approach is that in many cases it is not possible to construct the desired portfolio using an option strategy because:

- option contracts are not traded in the markets for every financial instrument.
- the strike prices and expiration dates are standardised and may not meet the needs of the portfolio manager.
- the size of the contract may be too big compared with the normally traded sizes or the size desired by the investor.
- different types of investors face certain limitations on the use of derivatives.

Fortunately, any option can be replicated through an adequate dynamic portfolio strategy[108], which forms the basis for the constant-proportion portfolio insurance technique.

[108] The key feature of the modern option pricing theory is that the price behaviour of an option is very similar to a portfolio of the underlying stock (risky asset) and cash or zero coupon bonds (risk free asset) that is revised over time. That is, there exists a replicating portfolio strategy, involving a risky asset and a risk-free asset only, which creates returns identical to those of an option. According to the standard option pricing theory, by continuously adjusting a portfolio consisting of a risky and risk free assets, an investor can exactly replicate the return of a desired option contract (Bird, Cunningham, Dennis, Tippett, 1990). In reality a continuous adjustment is impracticable and trading takes place only in discrete times. If trading take place reasonably frequently, hedging errors could be relatively small and uncorrelated with the market returns but, on the other hands, the transaction costs may have a considerable impact on the strategy profitability. Discrete time rebalancing implies that CPPI cannot replicate the option without hedging errors; thus hedging errors should be considered when the investor re-balances the portfolio.

CPPI Does Not Use Derivatives

As already stated, CPPI replicates the outcome of a put option strategy through the dynamic rebalancing of a portfolio. To illustrate it, we consider a simple portfolio structure, when the portfolio contains two different elements: a risky asset (e.g. government, corporate, commodity, etc.) represented by an index and a risk-free asset (zero coupon government bond, generally). The allocation between the two assets changes in accordance with a specified set of rules:

- An investor is promised to receive a pre-specified amount of principal at maturity.
- The minimum value the portfolio should have in order to be able to produce the guaranteed principal at maturity is called the ***floor.***
- The value of the portfolio less the floor is called the ***cushion***.
- The amount allocated to the risky asset (the risky asset exposure) is determined by multiplying the cushion by a predetermined ***multiplier***, as follows:

Risky Asset Exposure = Multiplier x (Portfolio Value – Floor)

- The remaining funds are invested in the risk-free asset, usually a zero-coupon bond or another liquid money market instrument.

For example, let us consider a portfolio of €100, a floor of €75 and a multiplier equal to 2. The initial cushion is €25 and the initial investment in the risky asset is €50 (€25x2). The initial mix of total portfolio is €50 risky and €50 risk-free assets.

- Let us assume that the risky asset falls 10%; the value of the risky asset falls from €50 to €45. At this point the total portfolio value is €95 and the cushion is €20 (Portfolio Value €95 – Floor €75). According to the CPPI rules the investor needs to rebalance the portfolio and the new risky position will be €40 (Multiplier 2 * Cushion €20), which requires a sale of €5 of the risky asset (Actual Risk Asset €45 – Desired Risky Asset €40) and the investment of proceeds in the risk-free asset.
- If the risky asset falls again, more of it will be sold.
- On the other hand, if the value of the risky asset increases, the investor will begin to increase its proportion in the portfolio (note that in this example we do not allow for leverage, which issue we will address later):
- In the above example if the risky asset grows by 10%, its value will increase from €50 to €55 and the total portfolio value will be €105. The cushion will be €30 (Portfolio Value €105 – Floor €75).
- According to the CPPI rule the investor needs to rebalance the portfolio by increasing the value of the risky asset from €55 to €60 (Multiplier 2 * Cushion €30) and reducing the risk-free exposure (from €50 to €45) in the portfolio.

The Floor

The floor can be defined as the minimum value the portfolio should have in order to be able to produce the guaranteed principal at maturity. It works as a trigger point, which if reached will cause the portfolio to be invested 100% in the risk-free asset until maturity. In this situation the investor may receive the promised principal, but he may lose some or all of his interest on that principal, and the overall return on his investment may fall below the risk-free rate (the rate on the risk-free asset).

If the investor is guaranteed the full or partial protection of the invested capital only (the principal is equal to or less than the invested capital), the floor value at any time should be equal to the promised amount discounted at risk-free-rate. In case of partial protection the floor may be also fixed at the promised amount of principal If

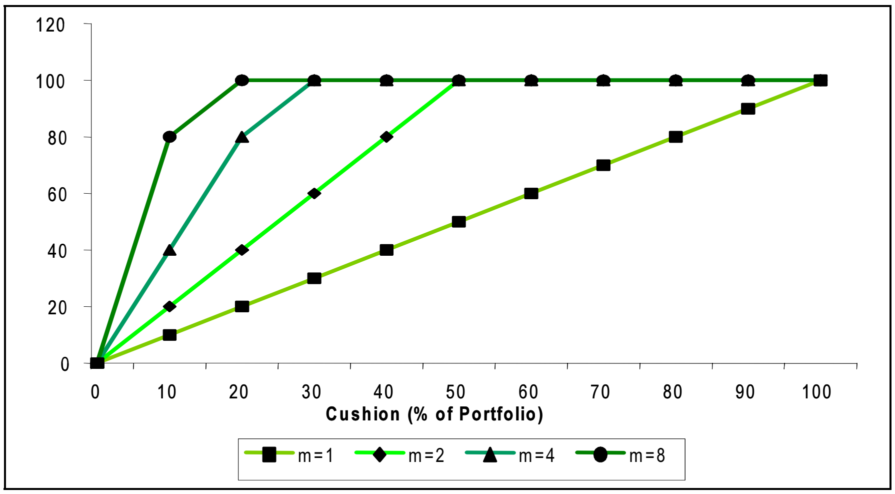
Refer to important disclosures on page 196.

the investor is guaranteed the full protection of the invested capital plus the specified return (spread), the floor at any time should be equal to the invested capital, discounted at the risk-free rate plus the spread.

The Multiplier

The multiplier represents the sensitivity of the risky asset exposure to the value of the cushion. It affects directly the performance of the portfolio and the probability of the portfolio reaching the floor. The multiplier determines the extent of the rebalancing needed after a change in the cushion. As shown on Chart 196, a cushion of 20% and a multiplier of 1 suggests risky assets of 20% in the portfolio (and 80% risk free assets). For a multiple of 2, that percentage increases to 40%, and so on. We note that a multiple of 8 produces a risky asset of 100%, not 160%, given that in this example, no leverage is allowed.

Chart 196: Risky Asset Exposure for Different Multipliers and No Leverage



Source: Merrill Lynch

The multiplier depends on the risk tolerance of the investors: the higher the multiplier, the riskier the investor's strategy and vice versa[109]. The higher the multiplier, the larger the rebalancing and the higher the probability of a gain when the market rallies, but at the same time the higher the probability that the portfolio reaches the floor in case of a market downturn (ie the faster the shift into risky assets in an upward market movement, and the slower the shift into risk free assets in a downward market movement).

As shown above, the multiplier is the key element of the strategy. The traditional CPPI consider a constant multiplier (Constant-Proportion strategy). However, there are many versions of CPPI strategies, which introduce a dynamic multiplier. In fact, for a desired portfolio performance it would be useful for the multiplier to change in accordance with the risky asset volatility:

- For example, if an investor has a low risk profile, the multiplier could change in the opposite direction to the risky asset's volatility (the higher the risky asset's volatility, the lower the multiplier, and vice versa).
- On the other hand, if the investor has a high risk tolerance, the multiplier could increase along with the volatility of risky asset and vice versa.

In the first case, the investor will not take large profit if the risky asset appreciates, but the probability of reaching the floor (and the portfolio being locked in the risk-free asset only) is lower in a market downturn. In the second case, the investor will succeed in taking profit from a risky asset's upward movements, but the probability of hitting the floor in a downward market movement also rises. Thus, there is a trade-off that has to be addressed by the investor depending on his risk

[109] Black, Perold, 1992

tolerance and related asset allocation profile. Various volatility-related variables can be used to adjust the multiplier such as current, implicit or historical volatility, but this aspect goes beyond the purpose of this paper and we assume the multiplier to be constant.

The Leverage

When the multiplier and the floor are chosen in such a way that the investment in the risky asset exceeds the portfolio value, the strategy will require selling short the risk-free asset or borrowing an amount equal to the shortfall (i.e. Multiplier * Cushion - Portfolio Value). Generally short-selling strategy is the easier to implement, although it is a riskier one.

When no leverage is allowed, the risky asset exposure is limited to the total asset value. In this case the asset allocation rule should look like

Risky Asset Exposure = min (Portfolio, Multiplier x (Portfolio– Floor))

It is important to distinguish whether leverage is allowed or not. If leverage is allowed, then the manager can invest more than the value of portfolio in the risky assets (or sell short the risk-free asset). As a result the CPPI with leverage may outperform the risky asset in a rallying market, while the CPPI with no leverage may only perform on par with the risky asset in the same situation.

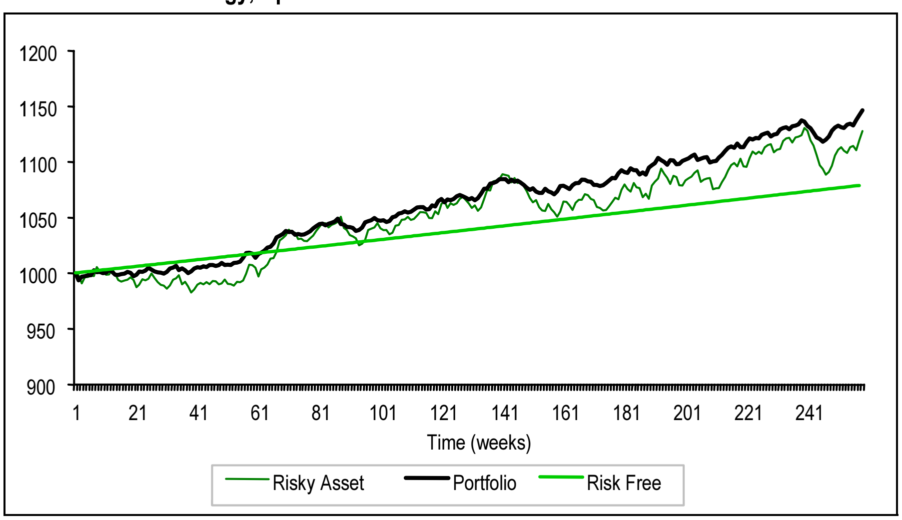
Application of CPPI

When does the CPPI strategy work well?

The examples of how the CPPI strategy works in upward and downward markets are shown in Chart 197 and Chart 198, respectively. We consider the following inputs: initial portfolio value of 1000, principal protected at maturity 950, multiplier of 4, risk-free rate of 1.56% and no leverage. The floor at the beginning is the PV of 950 at 1.56% (or 880.52). The strategy is rebalanced in discrete times. The risky asset index has been simulated using a lognormal process, which is typically what corporate and government bond indices follow.

Chart 197 shows that in upward markets with a well defined trend, the CPPI strategy allows the portfolio to benefit from the positive market performance and, at the same time, to limit the effect of negative performance that may happen in short periods within the long upward trend. The strategy outperforms both the risky and risk free assets.

Chart 197: CPPI Strategy, Upward Market Trend

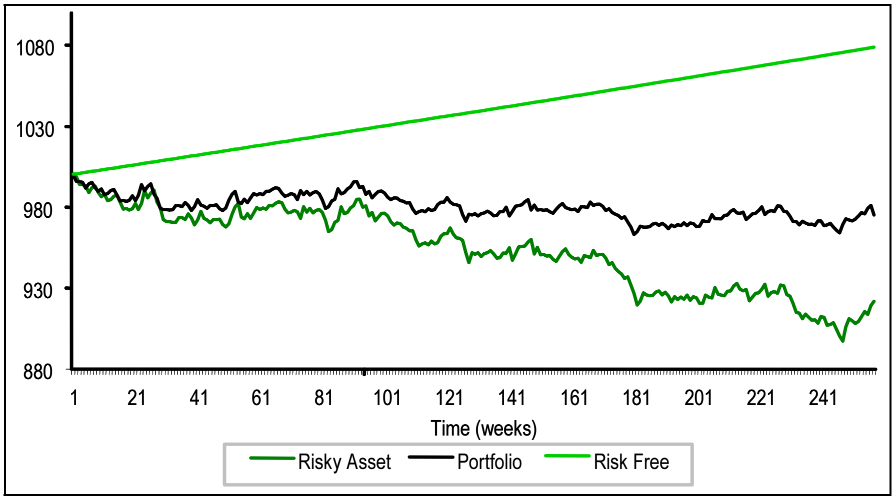


Source: Merrill Lynch

Chart 198 shows that in a downward market the CPPI strategy will outperform the risky asset, because the proportion of risk-free asset will increase and the portfolio will be influenced less by the decline in the risky asset performance. In this example, although the portfolio outperforms the risky asset, we can see that the CPPI strategy still underperforms the risk-free asset.

As mentioned earlier, once the portfolio value falls below the floor (which grows at risk free rate above), the entire portfolio will be invested in the risk-free asset until maturity. We highlight this aspect of the CPPI strategy: in such a situation the investor will not be able to take profit from the possible future rallies of the risky asset, which reduces the efficacy of the strategy. On the other hand, a continuous rebalancing may not be feasible because of the high transaction costs that the investor is likely to face.

Chart 198: CPPI Strategy, Downward Market Trend



Source: Merrill Lynch

Next we summarise briefly the expected performance of the CPPI technique under different market conditions:

- In a market with a well-defined trend, CPPI performs well. In a **bull market**, the strategy leads the investor to buy more of the risky asset as it rises and thus participate in its appreciation. In a **bear market** the portfolio performance should at least guarantee that the principal is preserved, Such a strategy will put more and more into the risk-free asset as the risky asset value declines, reducing the exposure to the risky asset to zero, as the portfolio approaches the floor.
- In **oscillating markets**, such as a mean-reverting one with low volatility, the strategy performance will depend on the frequency of reversion and the frequency of rebalancing. We distinguish between low frequency reversion and high frequency reversion. Only if the rebalancing happens more frequently than the reversion will the CPPI capture the market movement, but in this case it is possible that the strategy faces higher transaction costs.
- In cases when the market experiences jumps (the latter tend to be characteristic of the commodity, interest rate, exchange rate markets), we distinguish between jumps-up or jumps-down. If the market moves sharply up, the portfolio benefits from the upside of the performance of the risky assets. In a downward jump, the CPPI strategy faces the risk that the portfolio value falls to or below the floor rapidly and as a result the portfolio is allocated entirely into risk-free assets. This can occur under the condition of slow downward movement as discussed on Chart 198, however, the jump down can cause the portfolio to 'freeze' (performance revert to risk free asset only) rapidly and unexpectedly.

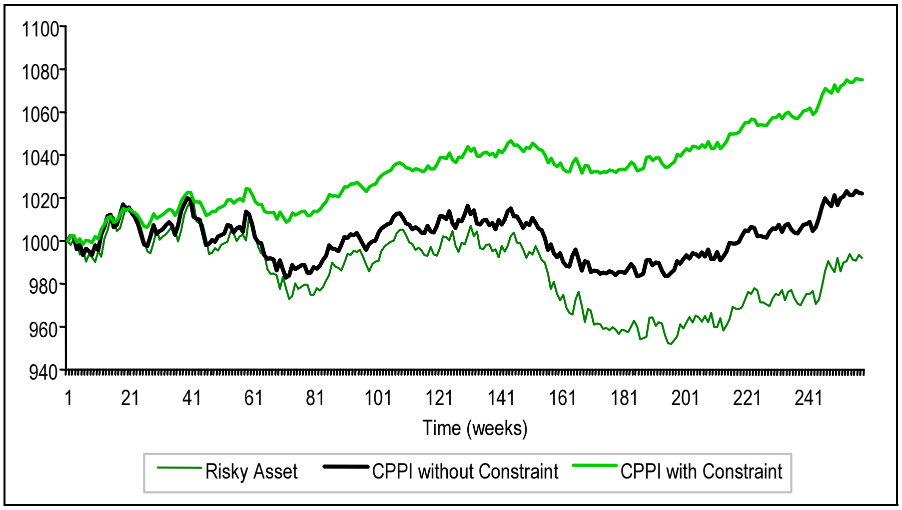
- In markets, experiencing spikes (say exceptional up and down movements in short time intervals) the risk to the CPPI strategy is that rebalancing occurs at the time of the spike.

Exposure Constraints

So far we have discussed the CPPI strategy with no constraints. It means that the risky asset exposure can be as high as 100% of the portfolio with no leverage, or more with leverage. Sometimes the CPPI is implemented with additional constraints: for example, the risky exposure may not be allowed to exceed a given percentage of the portfolio (say, 50%).

To illustrate the effect of the constraints on Chart 199 we show the performance of the risky asset and the CPPI strategy with and without the 50% constraint and the floor growing at a risk-free rate.

Chart 199: CPPI with and without the 50% Exposure Constraint



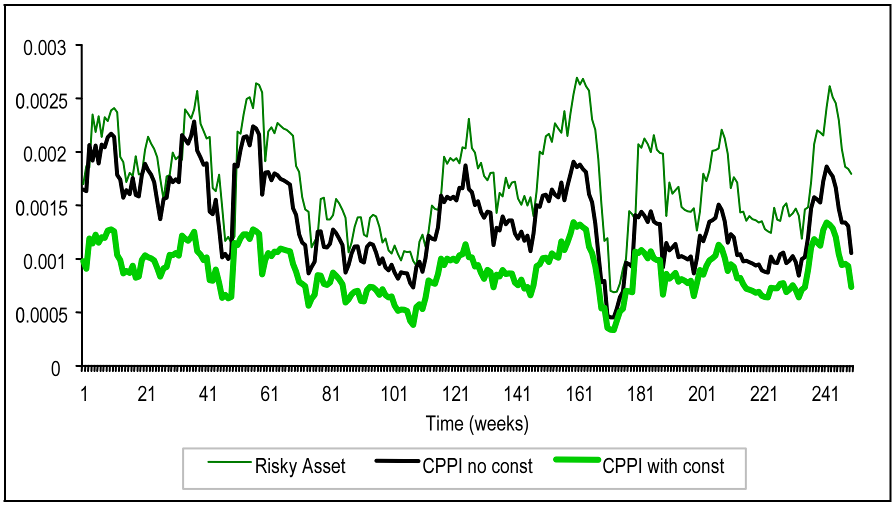
**The floor grows at a constant rate*
Source: Merrill Lynch

There is an advantage and a drawback to using a constraint on the risky asset exposure. The **advantage** is that CPPI with exposure constraints is more defensive than in the one without constraints because there is a minimum proportion of risk free asset, which limits investor exposure to downward market movement. This strategy may perform well, when the investor has a low risk profile. The **drawback** is that CPPI with constraints will perform worse than the one without constraints, when the market is growing.

Obviously the CPPI strategy with constraints has also the effect of reducing the portfolio volatility (see Chart 200, Exponentially Weighted Moving Average Volatility[110]). As we can see the CPPI strategy reduces portfolio volatility with respect to the market volatility, but such reduction is even higher for CPPI strategy with exposure constraints.

[110] When we calculate volatility using the customary methods (i.e. standard deviation) we don't take into account the order of observations. Additionally, all observations have equal weights in the formulas. But the most recent data about asset's movements is more important for volatility than more dated data. That is why, the recently recorded statistical data should be given more weight than older data. One of the models that operate off of this assumption is the Exponentially Weighted Moving Average Volatility.

Chart 200: Exponentially Weighted Moving Average Volatility*



Source: Merrill Lynch

Which asset classes can the CPPI strategy be applied to?

The CPPI is a strategy that may guarantee the return of invested capital at maturity and at the same time succeed in reducing the volatility of portfolio returns. CPPI is the most popular and efficient form of the portfolio insurance techniques and many types of deals offering exposure to various risky assets have been structured using it. Its main advantages are that it does not involve investment in options, is suitable for portfolios consisting in all types of marketable securities and is relatively simple to understand and implement. Using CPPI, protected securities benefit from the return on the risky assets with minimal risk of losing capital at maturity.

Generally, the CPPI strategy can be applied to many assets classes:

- equity, (for detailed discussion please refer to the report mentioned earlier)
- fixed income, e.g. corporate bond portfolios – as mentioned, their performance is characterised by trends, under which conditions CPPI works well.
- commodities - energy, non-energy and agricultural commodities markets are characterised by short periods of extremely high volatility, spikes and jumps. CPPI can be used to reduce portfolio volatility and achieve principal repayment.
- structured products, e.g. CDO equity – CPPI can be used in the context of structuring principal guaranteed investments.
- hedge funds or funds of funds – depending on their strategies, they can exhibit high or low volatility, as well as high dependence on manager performance. CPPI can help address volatility and manager issues, as well as principal protection.
- long/short combinations, e.g. long/short synthetic corporate exposures – CPPI can provide principal protection to such structures.

References

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Black, F., Perold, A.F., 1992. Theory of Constant Portfolio Insurance. Journal of Economic Dynamics and Control 16, 403-426

Black, F., Scholes, M., 1973. The pricing of options and corporate liabilities. Journal of Political Economy 81, 637-654

14. CDS Options

CDS options serve as versatile instruments to either enhance yield or provide downside protection in the event of spread movement. We believe that this development of credit volatility as a new asset class is driven by several factors:

- The non-linear payoff of CDS options enables investors to construct more flexible risk-return profiles when used with underlying single-name CDS, index or bonds.
- CDS options are a natural product for investors who are looking for additional yield or leverage to express a particular view in a relatively low volatility environment.
- Liquidity in single-name CDS and index market (CDX and iTraxx) has enabled the dealer community to hedge CDS options more efficiently.

Investor demand has been increasing for swaptions[111] on both single-names as well as in the increasingly liquid CDS index. We believe that with the increasing liquidity of CDX/iTraxx tranches, demand for swaptions on these tranches will also further develop in the future.

CDS Option Basics

Payers & Receivers – an interest rate derivative terminology

A CDS option represents the right, but not the obligation, to buy or sell protection on a specified credit entity at a specified spread (the strike spread) at a specified date in the future (the option expiration date). There are two types of CDS options:

- **Right to Buy Protection (also referred to as a "Payer" option)**
- **Right to Sell Protection (also referred to as a "Receiver" option)**

The naming convention for CDS options (**Payers** and **Receivers**) has been borrowed from the interest rate derivatives and not the equity options market. As these options can either be bought or sold investors can do four basic trades. Table 61 on the front page summarises these four basic option strategies. The charts below represent payoff diagrams for these strategies at expiration.

■ Payer Options

Payer = Right to Buy Protection

Buy payer (Chart 201):

- Investor pays the option premium and buys the right to buy protection.
- The investor benefits if the underlying spreads widen above the strike spread on expiration date.
- Upon exercise the investor will buy protection at the lower strike price.
- The investor is expressing a bearish credit view.

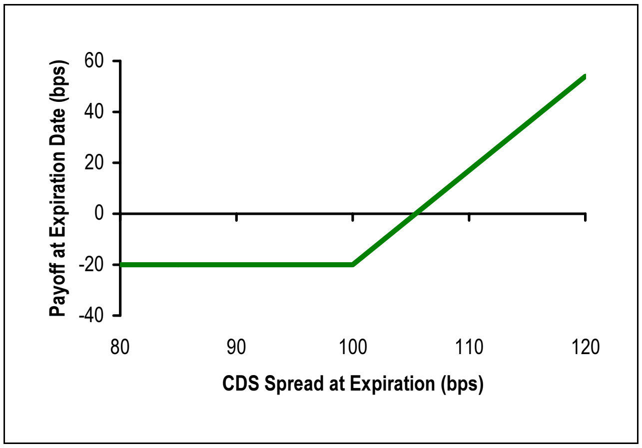
Sell payer (Chart 202):

- Investor receives the option premium for selling the right to buy protection.
- Investor benefits if the underlying credit spread remains stable or tightens and the option buyer does not exercise.
- However, if spreads widen above the strike on expiration, the option seller would be obliged to sell protection at the lower strike spread.
- The investor is expressing a bullish credit view.

[111] We use "CDS options" and "swaptions" terminology interchangeably as is the custom in the market. Both refer to exactly the same product.

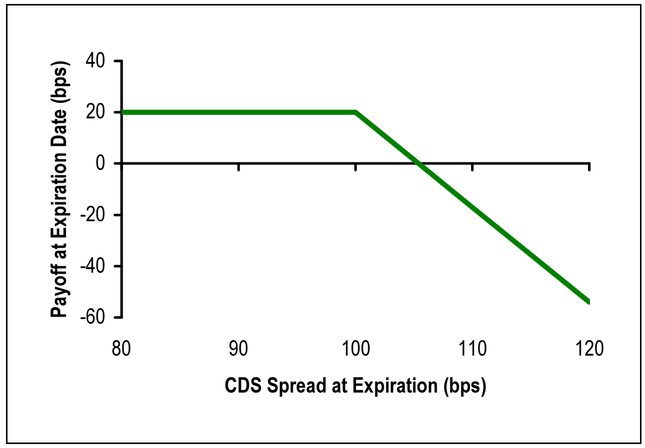
Refer to important disclosures on page 196.

Chart 201: Buy Payer (Buy the Right to Buy Protection) at 100bps



Payoff = (CDS spread at expiration – Strike spread) * Forward Duration – Upfront
Source: Merrill Lynch

Chart 202: Sell Payer (Sell the Right to Buy Protection) at 100bps



Payoff = Upfront – (CDS spread at expiration – Strike spread) * Forward Duration
Source: Merrill Lynch

Receiver Options

Receiver = Right to Sell Protection

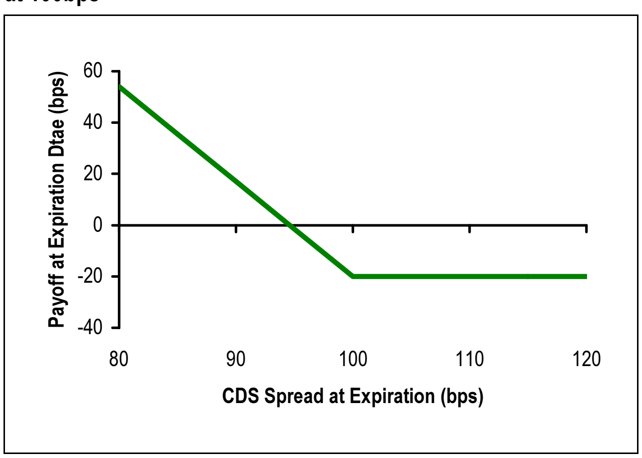
Buy receiver (Chart 203):

- Investor pays the option premium and buys the right to sell protection.
- The investor benefits if underlying spreads tighten below the strike spread on expiration date.
- Upon exercise the investor will sell protection at the higher strike price.
- The investor is expressing a bullish credit view.

Sell receiver (Chart 204):

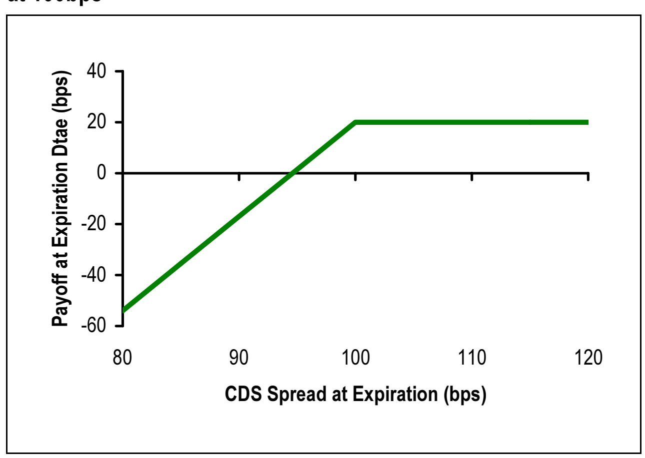
- Investor receives the option premium for selling the right to sell protection.
- The investor benefits if spreads remain stable or widen and the option buyer does not exercise.
- However, if spreads tighten below the strike on the expiration date, the option seller would be obliged to buy protection at the higher strike spread.
- The investor is expressing a bearish credit view.

Chart 203: Buy Receiver (Buy the Right to Sell Protection) at 100bps



Payoff = (Strike spread – CDS spread at expiration) * Forward Duration – Upfront
Source: Merrill Lynch

Chart 204: Sell Receiver (Sell the Right to Sell Protection) at 100bps



Payoff = Upfront – (Strike spread – CDS spread at expiration) * Forward Duration
Source: Merrill Lynch

Other Swaption Features

Underlying can be single-name or index

CDS options can be broadly divided into three categories of the underlying:

- Single-name swaptions, i.e. options to buy or sell protection on an individual credit such as GMAC or FMCC.
- Index swaptions, e.g. options to buy or sell protection on the iTraxx index.
- Embedded options e.g. cancellable CDS/extendable CDS.

Unlike corporate bond options which are struck on the price or yield of the bond, CDS options are struck on the CDS spread. CDS option investors are therefore exposed primarily to spread risk unlike the exposure to both spread and interest rate risk faced by corporate bond option investors.

Options are typically European

CDS options are typically European in nature, i.e. they can be exercised only on the expiration date. American options are less common.

3-month and 6-month expirations are most liquid

The most liquid options are 3-month and 6-month options though longer-dated options can also priced. The option is most commonly bought or sold on the 5y CDS (from the transaction date). Option expirations that match standard maturities of the CDS index or single-name CDS have better liquidity.

What happens if there is a Credit Event?

Which Credit Events Apply?

Credit Events depend on the terms of the underlying instrument

Determination of whether a Credit Event has occurred depends on the documentation of the CDS underlying the option. Thus there are significant differences based on regional market practice. For example, single-name US contracts will typically be based on Modified Restructuring, whilst single-name European contracts would typically feature the Modified-Modified Restructuring Credit Event.

With index options, things get a little more complicated. North American CDX indices trade on No-Restructuring unlike the index members which typically trade on Mod-R. The European iTraxx indices, however, trade on Mod-Mod-R similar to the underlying members.

If a Credit Event is Triggered

First, 2 notices must be delivered

If a Credit Event occurs during the life of a swaption a number of different procedures can apply depending on the specific terms of the transaction. However, as a first stage either the option buyer or seller must deliver to the other a Credit Event Notice and a Notice of Publicly Available Information (as defined in the confirmation of the underlying default swap transaction). These notices must be effective before the expiration of the option contract.

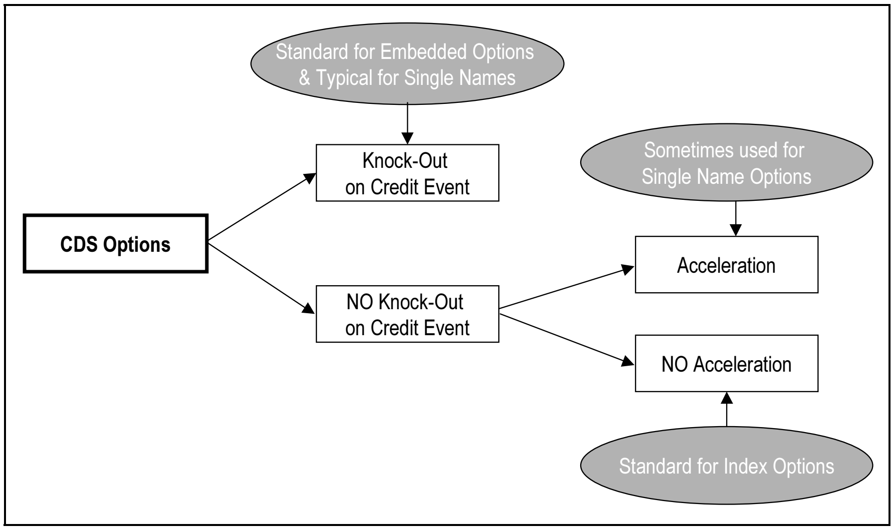
Knock-Out and Acceleration

CDS options have knock-out and acceleration features

CDS options also have two special features called **knock-out** and **acceleration** that apply when a Credit Event has been triggered. A knock-out feature in a CDS option refers to a situation where the option is terminated following a Credit Event on the underlying credit before the expiration date of the option. There is no breakage cost associated with a knock-out. CDS options are quoted both with knock-out or without knock-out. Single name CDS options are more commonly quoted with knock-out. If the option does not have a knock-out feature then the option may either accelerate or not accelerate.

Refer to important disclosures on page 196.

Chart 205: CDS Option Usage of Knock-Outs and Acceleration



Source: Merrill Lynch

Table 57: Knock-Out & Acceleration Features of CDS Options

CDS Option Category	Knock-out on Credit Event	Acceleration
Single-name swaption	Yes/No*	Yes (if no knock-out)
Index/Tranche swaption	No	No
Embedded options	Yes	Not applicable

* More commonly quoted with Knock-Out
Source: Merrill Lynch

Option acceleration means that the option is exercised immediately following a Credit Event, i.e. before the scheduled expiration date. Alternatively, if an option with no knock-out does not accelerate the option holder must wait until the expiration date to exercise the option. Chart 205 and Table 57 highlight the knockout and acceleration features for different categories of CDS options.

If a single-name swaption does not knock-out following a Credit Event, then it typically includes acceleration. Upon delivery of the Credit Event Notice and Notice of Publicly Available Information, the protection buyer in the underlying CDS contract has the right (only applicable for Payer) to deliver the Notice of Physical Settlement and thereby trigger settlement in accordance with the provisions of the CDS.

The Right to Buy protection without knock-out should therefore be more valuable than a similar option with a knock-out feature. A Right to Sell protection option is extinguished following a Credit Event and therefore the knock-out feature has effectively no value for this option.

Index swaptions have typically no knock-outs or acceleration

We note that index swaptions neither knock-out nor accelerate unlike single-name swaptions. Therefore if a name in the index has a Credit Event, the option holder must wait until the option's expiration date to exercise. Also, an embedded option such as a cancellable option in a CDS typically knocks out upon a credit event.

Option Pricing & Risk Parameters

Forward Spread

Forward spread determines whether the option is in or out of the money

The option strike spread can be any spread. However, the forward spread determines whether the option is in or out of the money. In option terminology, this spread is also referred to as the at-the-money (ATM) forward[112]. We use three rules of thumb to compute forward spreads:

- If the shorter-dated and longer-dated spreads are **equal** (i.e. flat curve), the forward spread is also equal to this spread.
- If the shorter-dated spread is **lower** than the longer-dated spread (i.e. upward-sloping curve), the forward spread is higher than the longer-dated spread.
- If the shorter-dated spread is **greater** than the longer-dated spread (i.e. inverted curve), the forward spread is lower than the shorter-dated spread.

[112] Please refer to Appendix for a detailed explanation of forward CDS.

Table 58: CVC CDS Term Structure

Maturity	Premium
1y	153 bps
2y	208 bps
3y	254 bps
5y	342 bps
7y	365 bps
10y	381 bps

Source: Mark-It. Mid levels as of 31st January 2006

If the dealer uses the forward spread as the strike spread, the theoretical price of the payer option is equal to the price of a similar maturity receiver option for the same underlying bond and strike price. This relationship is implied from the "Put-Call" parity relationship[113] for options. **This relationship, however, holds only if there is a knock-out feature in the options**.

As an example let's compute the ATM forward spread for a 1y swaption to buy 4y CDS on CVC on expiration date. Table 58 shows the CDS term structure for CVC. The ATM forward rate for a 4y CDS starting after 1 year can be computed using a simple procedure that is summarised in the appendix at the end of this chapter.

Essentially the 5y spread is the duration-weighted average of the 1y spread and the 4y forward spread. Since we know 1y and 5y spreads as well as the durations of the 1y, 5y and 4y forward[114], we can calculate the 4y forward spread. In our specific example the 4y forward spread after 1y is equal to 399bps[115].

Pricing Factors

Table 59: Sensitivity of CDS Option Premium

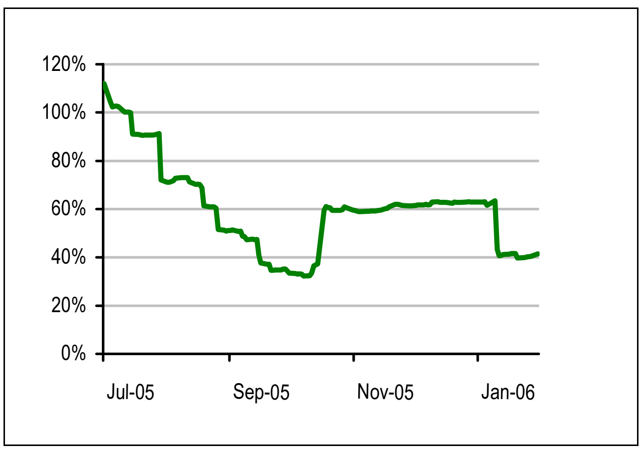
	Effect on Premium if Factor Rises	
Pricing Factor	Right to Buy	Right to Sell
Underlying CDS Spread	+	–
Strike Spread	–	+
Time till Expiration	+	+
Volatility	+	+

Source: Merrill Lynch

CDS options are a function of the following factors: underlying CDS spread, strike spread, time to expiration and spread volatility. Table 59 highlights the sensitivity of the option premium as these factors change.

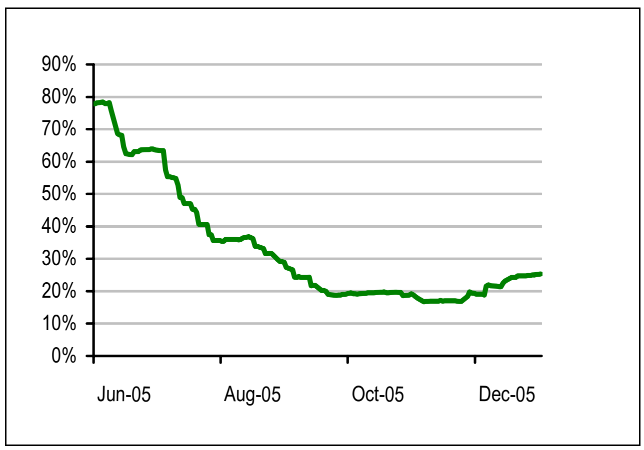
All the above factors except volatility can be directly observed. *Spread* volatility is used to price CDS options. One method of estimating spread volatility is from historical spreads. Investors can also use CDS option prices to derive *implied* spread volatilities which can be compared to volatility estimates to identify potentially rich or cheap CDS options. The charts below show historical spread volatility for two credits.

Chart 206: FMCC Spread Volatility (60-Day, 5y CDS)



Source: Merrill Lynch, Mark-It

Chart 207: FRTEL Spread Volatility (60-Day, 5y CDS)



Source: Merrill Lynch, Mark-it

113 Call Premium – Put Premium = PV(Forward Spread – Strike Spread);
if Forward spread = Strike spread, then Call Premium = Put Premium
or Receiver Premium = Payer Premium (assumes knock-out)

114 Duration of 4y forward CDS 1y from today = Duration of 5y CDS – Duration of 1y CDS

115 Assume 5y CDS duration = 4.05 and 1y CDS duration = 0.93. Then we have:
399 = (342*4.05 – 153*0.93)/(3.12)

Risk Parameters

Dealers use "greeks" to manage the CDS option risk. Table 60 highlights these risk parameters.

Table 60: Managing the "Greeks"

"Greek"	Description
Delta	Change in option price for small change in CDS spread. Approximately the probability that option will be in the money.
Gamma	Change in option delta for small change in CDS spread. Measures the convexity of the option wth reference to underlying spread. Positive for long CDS options. Same for payer and receiver with the same strike.
Vega	Change in option price for small change in volatility. Same for payers and receivers with same underlying CDS and strike. Higher for longer-dated and near-the-money options.
Theta	Change in option price as a result of passage of time (or time decay). Negative for long options and more significant for short-dated near-the-money options.

Source: Merrill Lynch

Dealers delta-hedge to manage spread risk

Delta is a first-order risk that is commonly hedged by most dealers. If the underlying is a liquid single-name CDS (i.e. with a relatively small bid-ask) the dealer may be able to recoup the option premium by dynamically hedging the CDS option with the underlying CDS. The following outcomes are possible in the process of dynamic hedging:

- If the observed spread volatility is the same as implied by the option price, the option buyer will make back the option premium by dynamically hedging in a frictionless market.
- If the observed volatility is more than the implied volatility, the option buyer would make more than the option premium.
- If the observed volatility is less than the implied volatility, the option buyer would make not make enough to cover the cost of the option.

CDS Option Strategies

Table 61: Basic CDS Option Strategies

	Buy Payer	Sell Payer	Buy Receiver	Sell Receiver
Option Type	Buy the right to buy protection	Sell the right to buy protection	Buy the right to sell protection	Sell the right to sell protection
Upfront Premium	Pay	Receive	Pay	Receive
Spread view	Bearish	Stable/Bullish	Bullish	Stable/Bearish
Spread volatility exposure	Long	Short	Long	Short
Exercised by	Investor	Counterparty	Investor	Counterparty
Exercised if	Spreads widen above strike	Spreads widen above strike	Spreads tighten below strike	Spreads tighten below strike
On exercise	Buy protection at strike (less than market spread)	Sell protection at strike (less than market spread)	Sell protection at strike (more than market spread)	Buy protection at strike (more than market spread)
Loss	Upfront premium (if market spread less than strike)	Negative MTM increases as spreads increase over strike	Upfront premium (if market spread more than strike)	Negative MTM increases as spreads decrease below strike
Gain	Positive MTM increases as spreads increase over strike	Upfront premium (if market spread less than strike)	Positive MTM increases as spreads decrease below strike	Upfront premium (if market spread more than strike)

Source: Merrill Lynch

In the current environment of tight spreads CDS options can be used to both enhance yield as well as to provide downside protection in the event of spread widening. CDS options can also be used as leveraged tools to strategically establish long or short credit positions. We highlight these strategies below.

Targeted Short (Sell Receiver)

Sell receiver: get paid while committing to buy protection at tighter spread

In the current environment of relatively tight spreads some investors are keen to express a bearish view on particular credits. However with the volatility continuing to remain low investors may expect spreads to tighten even further before establishing an attractive short credit position. Such investors should consider **selling receiver options at tighter strikes as targeted short strategies**.

For example, if an investor wants to buy protection on the North American CDS HY index CDX.HY.5 but expects spreads to tighten further before widening, the investor can do the following: sell the Right to Sell CDX.HY.5 struck at 320bps on 20th June 2006 (expiration date). Assume spot spread is 332bps, investor receives 39 cents upfront and breakeven spread at expiration is 308bps[116].

- If CDX.HY.5 spread remains stable or widen on expiration the option is not exercised and the investor keeps the option premium.
- If CDX.HY.5 spread is less than the strike at expiration the investor would be obliged to buy protection at 320bps. The investor therefore gets paid a premium for locking-in to a committed strategy. However, the risk is that the spreads fall substantially below the breakeven resulting in a large MTM loss.

Targeted Long (Sell Payer)

Sell payer: get paid while committing to sell protection at wider spreads

Investors who believe credits would be attractive at wider spreads should consider **selling payer options at wider strike spreads as targeted long strategies**. For example, an investor who finds the CDX.HY.5 index attractive at 380bps (332bps as of 31st January 2006) could sell the Right to Buy 5y CDX.HY.5 @ 380bps on 20th June 2006 (expiration date) for 94 cents upfront. The breakeven spread is about 407bps.

- If 5y CDX.HY.5 spread remains stable or tightens at expiration the option would not be called and the investor would keep the premium.
- If 5y CDX.HY.5 spread is more than the strike at expiration the investor would be obliged to sell protection at 380bps. Once again, the investor gets paid for locking-in to a committed strategy. The risk is that spreads widen significantly above the breakeven resulting in a large MTM loss.

Leveraged Short (Buy Payer)

Buy payer: leverage a bearish credit view

Investors looking to leverage a bearish outlook on spreads could consider buying the Right to Buy protection. For example, let's assume an investor believes that the 5y CDS of a hypothetical credit is trading tight at 40bps and its spread will widen significantly by September 2006. This view can be expressed by buying an option that is struck at, say, 60bps with a Sep-06 expiration date for 22 cents upfront. In Sep-06, the investor will have the Right to Buy protection to Mar-11 for 60bps.

Options with knock-outs do not provide protection before the option expiration. They are therefore cheaper and provide greater leverage. They also make more sense for investors who are more concerned about spread widening rather than default before the option expiration date.

Table 62 highlights the net payoff at different spreads on the option expiration date. The table clearly demonstrates the leverage obtained from spreads widening.

[116] Breakeven spread = Strike – Upfront premium/Risky BPV of CDS at expiration

Refer to important disclosures on page 196.

Table 62: Payoff Scenario for Buying the Right to Buy Protection to Mar-11 in Sep-06 ($10mn Notional)

Spread at Expiration (bps)	Payoff at Expiration* ($ '000)	Initial Cost ($ '000)	Profit ($ '000)	Profit/ Cost
<60 (Strike)	0	22	-22	-100%
65	21	22	-1	-5%
75	63	22	41	186%
85	105	22	83	377%
95	147	22	125	568%

* Payoff at Expiration = (CDS Spread at Expiration – Strike Spread) * Risky BPV of CDS at expiration.
Assume Risky BPV of CDS at expiration is 4.2 in this example.
Source: Merrill Lynch

Leveraged Long (Buy Receiver)

Sell payer: leverage a bullish credit view

This trade is similar to the above trade except that the investor is buying the Right to Sell protection. The trade would benefit if spreads tighten. In the current environment of relatively tight spreads investors may not find this leveraged long trade attractive. However, a delta-hedged version of this trade can prove beneficial for credits that are expected to be relatively volatile. We illustrate via a delta-hedged trade with a hypothetical credit, XYZ.

Assume an investor did the following trade on 1st March:

- Buy the Right to Sell XYZ 20th March@260bps on 20th June for 130 cents upfront. For a notional of $18mn, the option costs $234k.
- Initial delta is 28%: buy protection on $5mn single-name XYZ @292bps.

Table 63 highlights the delta hedges over time as underlying single-name XYZ spreads change. Given this volatility, an investor would have recouped about 80% of the initial premium over the period of 19 days.

Table 63: Delta-Hedging a Buy Receiver Option (Notional = $18mn; Initial Cost = 130 cents upfront or $234k)

Date	5y CDS Spread	Delta	Action: Buy (Sell)	Delta Position	Weighted-Average Spread	Trading P/L ($)	Unrealized P/L($)	Accrued Coupon ($)
1-Mar	292	28%	5,000,000	5,000,000	292	-	-	-
4-Mar	375	0%	(5,000,000)	0	-	182,600	-	(1,217)
4-Mar	325	17%	3,000,000	3,000,000	325	-	-	-
11-Mar	355	11%	(1,000,000)	2,000,000	325	13,200	26,400	(1,896)
19-Mar	320	17%	1,000,000	3,000,000	323	-	(4,400)	(1,444)
					Net as of 19th March	**195,800**	**(4,400)**	**(4,557)**
							P&L Recouped*	**79.8%**
							Current Option Premium (@78 cents upfront) on $18mn notional	140,400
							Net P/L (Trading + Unrealized + Accrued Coupon + Current Option Premium – Initial Cost of Option)	**93,243**
							Return (% of Upfront Cost)	**40%**

* P&L Recouped = 195,800-4,400-4,557 = $186,843 = 79.8% of initial premium of $234,000;
Risky bpv assumption = 4.4
Source: Merrill Lynch

Long or Short Volatility via Straddles

Straddles can express long/short volatility views

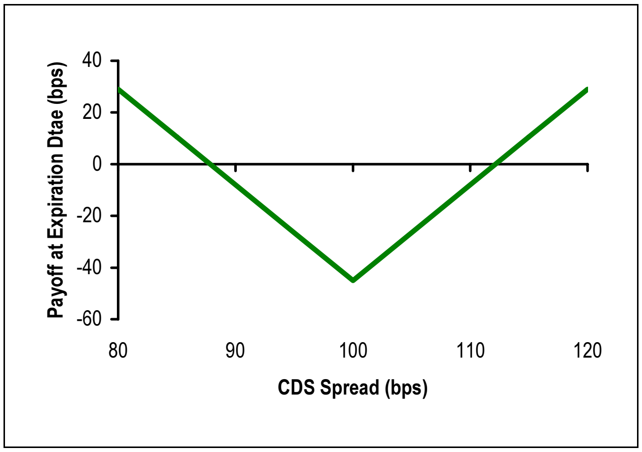
Investors who wish to express a strong view on underlying spread volatility can do so via long or short straddles.

A **long straddle** consists of a long payer option and a long receiver option at the same strike. The investor benefits if spread volatility increases but loses the combined premium if the volatility decreases.

In a low volatility environment a **short straddle** (sell a payer option and simultaneously sell a receiver option at the same strike) may be a more attractive strategy. An investor who expects spread volatility to decline would benefit from this trade.

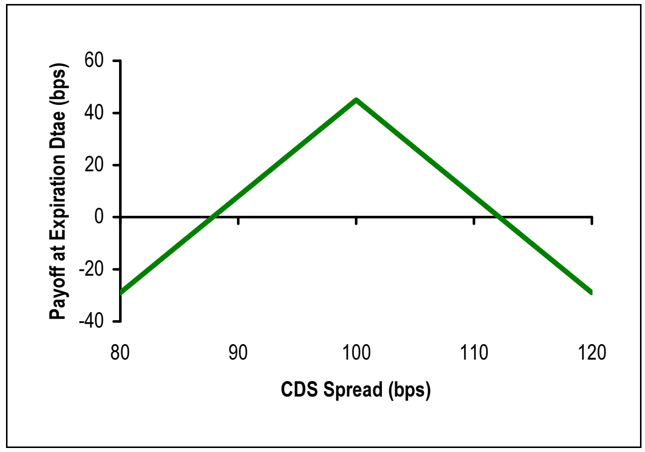
A slightly different variation of this would be a long or short **strangle** where the investor would choose different strikes to either lower the cost (long strangle) or to lower the yield (short strangle).

Chart 208: Long Straddle at 100 bps



Source: Merrill Lynch

Chart 209: Short Straddle at 100 bps



Source: Merrill Lynch

Combining CDS Options with CDS Positions.

Investors can use the above strategies in combination with long or short CDS positions to generate more complicated trading strategies. These include:

- Writing "covered calls" (enhance yield but limit the upside) : Sell CDS + Sell Receiver
- Buy downside protection on short CDS: Sell CDS + Buy Payer
- Lower the carry cost of a long CDS position (but limit the upside): Buy CDS + Sell Payer
- Limit downside on long CDS: Buy CDS + Buy Receiver

Index Tranche Options

Tranche Options allow investors to access non linear payoffs…

The investors' hunt for non linear payoffs in the correlation market has fueled the development for a market of European options written on the standardized CDX/iTraxx tranches. Since the second half of 2005, we have observed an increasingly active market for 3m, 6m, 9m and 12m options for all the standard attachment/detachment points excluding the equity tranche.

The tranche option payoff resembles the one computed for single name CDS options, i.e.:

Payoff = (Tranche Spread at Expiration – Strike Spread) x Tranche Risky BPV at expiration

Like index options, tranche swaptions neither knock-out nor accelerate unlike single-name swaptions. Therefore if a name in the index has a Credit Event, the tranche option holder must wait until the option's expiration date to exercise.

The lack of a general modeling framework able to capture the complexity of the product still poses problems not only for valuation purposes but also for the computation of hedging parameters.

…however further work is needed to better capture the complexity of the product

The usual Gaussian Copula (GC) approach used to price standardized and bespoke single tranche CDOs allows users to easily compute the terminal default distribution of a credit portfolio by modeling the correlated default time of each

Refer to important disclosures on page 196.

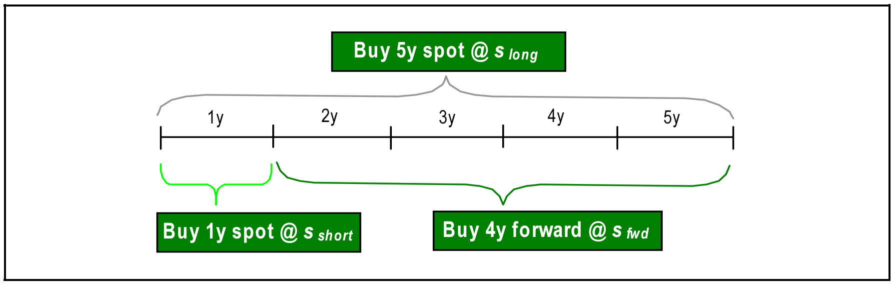
issuer. However the GC approach does not provide any information on the dynamics of the tranche/single name spreads over time, making the GC model an unsuitable candidate to model options[117].

Appendix: Calculation of Forward Spread

The ATM forward spread is calculated by using a simple no-arbitrage argument given the CDS curve for a particular credit or index. Consider the following two protection strategies:

1. Buy 5y CDS @ s_{long}
2. Buy 1y CDS @ s_{short} + Buy 4y CDS 1y Forward @ s_{fwd}

Chart 210: Payment Flows Under The Two Strategies



Source: Merrill Lynch

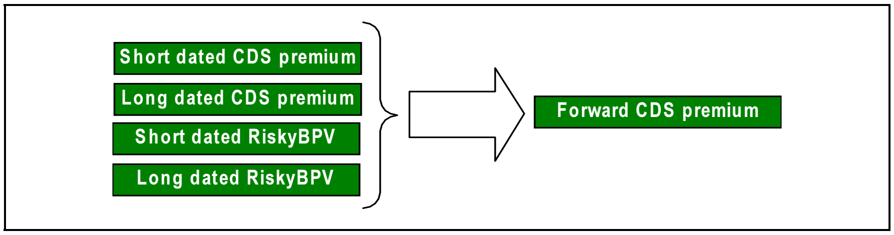
Two strategies with the same payoff must have the same initial price

Upon default, the contingent payment due from the protection seller to the buyer, is the same under the two strategies, since, in both cases, the protection buyer is fully hedged from the spot date until the expiry date, i.e. 5 years. Therefore, the expected payments stream paid by the protection buyer, must be exactly the same in both cases[118]. In other words, the present value of both strategies should be identical, i.e. PV(5y CDS) = PV(1y CDS) + PV(4y Fwd CDS after 1y). We can show that

$$s_{fwd} = \frac{s_{long} \cdot RiskyBPV_{long} - s_{short} \cdot RiskyBPV_{short}}{RiskyBPV_{long} - RiskyBPV_{short}} = \frac{s_{long} - s_{short} \cdot \Phi}{1 - \Phi}$$

where *RiskyBPV* is the risky basis point value computed from the spot interest rate swap and CDS curves and Φ is the ratio between the short dated and long dated Risky BPV.

Chart 211: Forward CDS rate computation



Source: Merrill Lynch

117 In that respect several authors have started exploring a theoretical consistent framework to model loss dynamics over time. We refer to Sidenius, J., Piterbarg, V. and Andersen, L., "*A New Framework for Dynamic Credit Portfolio Loss Modeling*", (2005) and Albanese, C., Chen, O. and Dalessandro, A. "*Dynamic Credit Correlation Modeling*", (2005), available at www.defaultrisk.com for further details.

118 If not, investors would go long the "cheap" strategy and short the "expensive" one. In this case, investors would be perfectly hedged over the investment horizon, whilst receiving a risk-free cash amount given by the difference of the expected present values of premium payments from the two strategies.

15. CMCDS

This chapter is extracted from a report published on 23rd April 2004 by Kakodkar et al.

Investors who are naturally long credit or optimistic about improving credit fundamentals may be concerned about spread widening risk at tight spread levels. In a low volatility environment, however, the timing of any spread widening is difficult to predict. Traditional defensive (or bearish) strategies, if timed incorrectly, could prove to be both ineffective and expensive in terms of carry.

Constant maturity credit default swaps (or CMCDS) provide an interesting alternative for investors wishing to balance strong fundamentals with stretched valuations. This could be relatively attractive for the following reasons:

- CMCDS serves as an effective hedge against spread widening for long credit investors.
- CMCDS has a significantly lower MTM than a vanilla CDS in the event of parallel spread widening.
- CMCDS benefits from curve steepening environment.
- A short CMCDS position can be combined with a long CDS position to isolate spread risk while hedging default risk

CMCDS Basics

CMCDS premium resets periodically…

A CMCDS is similar to a vanilla CDS in terms of risk transfer, i.e. the CMCDS protection buyer transfers credit risk to the CMCDS protection seller. However, unlike a vanilla CDS, the CMCDS premium is not fixed but resets periodically.

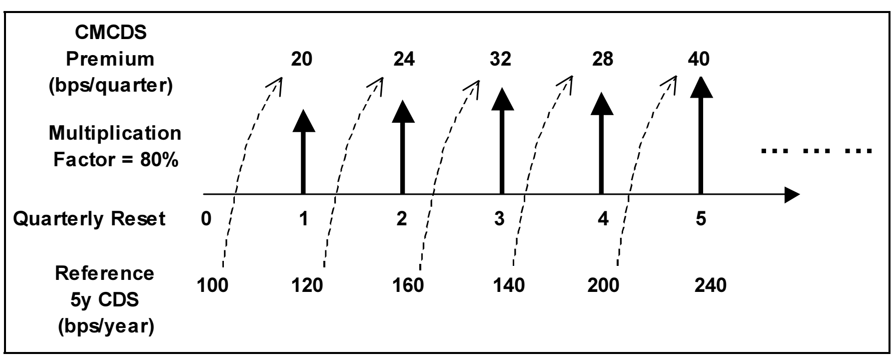
…based on a reference constant maturity…

The CMCDS premium is determined by a reference constant maturity rate (say, 5y CDS premium) that is reset periodically (say, every quarter). For example, a CMCDS premium may be equal to 80% of the 5y CDS rate reset every quarter. On each reset date, the prevailing 5y CDS premium is determined and the CMCDS premium is reset to 80% of this prevailing rate (see Chart 212). The prevailing reset rate may be determined either through a single calculation agent or a dealer poll.

…and a multiplication factor

The bid/ask for the CMCDS is quoted in terms of the multiplication factor (also called reset percentage). This factor is determined at inception and remains constant throughout the life of the CMCDS. For e.g. CMCDS may be quoted as 80%/90%, i.e. premium for a CMCDS protection seller is reset to 80% of 5y CDS bid and that for a CMCDS protection buyer is reset to 90% of 5y CDS offer.

Chart 212: CMCDS Cash Flows for CMCDS Protection Seller



Source: Merrill Lynch

Refer to important disclosures on page 196.

Multiplication Factor

Multiplication factor depends on slope of the CDS curve

The multiplication factor (80% in the example above) is based on the CDS term structure of the underlying credit. For an upwardly sloping term structure, 5y implied forward CDS premiums on the reset dates are higher than the current 5y CDS premium. Therefore the multiplication factor is less than one as the seller of protection is expected to get higher premiums at future resets.

Similarly if the term structure is inverted, the 5y implied forward CDS premiums on reset dates are expected to decrease. The multiplication factor in this case is greater than one as the seller of protection is expected to get lower premiums at future resets. We illustrate this behaviour in a later section.

Cap Rate

Premium is capped at reset

If the credit quality of the underlying Reference Entity deteriorates significantly, the vanilla CDS would typically trade on an upfront basis. The CMCDS therefore specifies a cap rate (say, 700 bps). If the prevailing 5y CDS premium is above the cap on a reset date, the CMCDS premium is set at the cap rate multiplied by the multiplication factor.

CMCDS Example

The floating CDS rate is reset at the beginning of each payment interval

Each payment is equal to the prevailing spot CDS rate (in our case 100bps) times the multiplication factor (80%) times the appropriate daycount convention (1/4 in case of quarterly payments on a 30/360 basis):

*100bps*80%*0.25=20bps*

Consider a CMCDS with the specifications shown in Table 64. The floating rate for the first period is set to 80% of initial 5y CDS (100bps) or 80bps per annum. For a protection seller, this equates to 20bps premium received at the end of the first quarter (which is also the first reset date). Chart 212 above shows one possible progression of CMCDS cash flows over the next few reset periods.

Table 64: Example of CMCDS Specifications

Specification	
CMCDS Maturity	5 years (20th June 2003 – same maturity as on-the-run 5y CDS)
Reference Constant Maturity	5y CDS
Initial 5y CDS Premium	100bps per year (25 bps per quarter)
Multiplication Factor	80%
Cap Rate	700bps
Reset Frequency	Every quarter

Source: Merrill Lynch

Table 65 below highlights a sample of indicative CMCDS bids for European and US credits. The CMCDS have quarterly resets and a 5y reference constant maturity.

Table 65: Indicative CMCDS Bids

Ticker	Indicative 5y CDS Bid	Indicative 5y CMCDS Bid	Initial Reset Premium
US Credits			
WYE	60	60%	36
GMAC	145	69%	100
CZN	335	80%	268
European Credits			
BAPLC	58	60%	35
PRTP	126	71.5%	90
ISYSLN	490	81.5%	399

CMCDS resets are quarterly. The cap rate is 700 bps except for ISYSLN where the cap rate is 900bps.
Source: Merrill Lynch

Forward Rates – A Key Pricing Ingredient

Forward rates are key for pricing CMCDS

Unlike the fixed spread of a vanilla CDS, a CMCDS is characterized by a floating spread that is reset periodically to the prevailing spot rate of a fixed maturity CDS. At trade inception only the premium for the first period is known. **All future premiums are estimated from the implied forward CDS curve of the issuer thus making forward rates a key pricing factor for CMCDS**.

Curve Shape & Liquidity

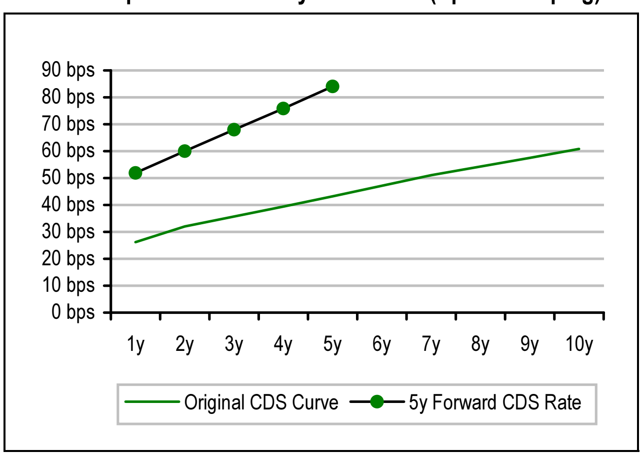
CMCDS valuation uses the entire curve

The shape of the CDS term structure determines the forward CDS curve. The crucial determinant is simply the slope of the curve between the shorter-dated and longer-dated CDS. We highlight the following three rules of thumb:

- If the shorter-dated and longer-dated spreads are **equal** (i.e. flat curve), the forward CDS spread is also equal to this spread.
- If the shorter-dated spread is **lower** than the longer-dated spread (i.e. upward-sloping curve), the forward CDS spread is higher than the longer-dated CDS spread.
- If the shorter-dated spread is **greater** than the longer-dated spread (i.e. inverted curve), the forward CDS spread is lower than the shorter-dated CDS spread.

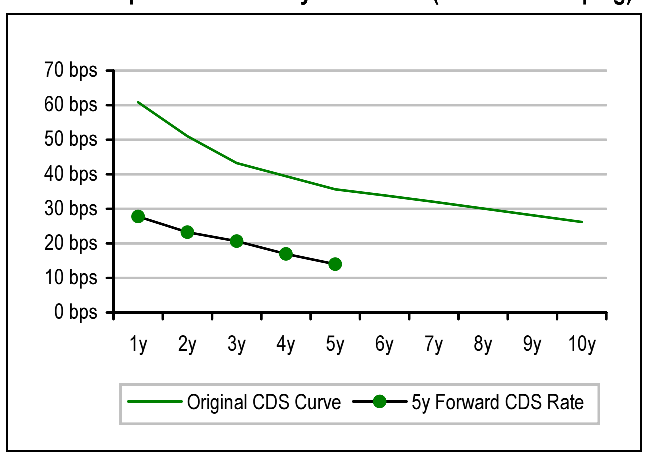
Chart 213 and Chart 214 highlight the forward spreads for upward and downward sloping curves respectively. We use a simple formula to derive CDS forward spreads[119].

Chart 213: Spot vs. Forward 5y CDS Curve (Upward Sloping)



Source: Merrill Lynch. Assume annual frequency, 30/360 convention and 35% recovery.

Chart 214: Spot vs. Forward 5y CDS Curve (Downward Sloping)



Source: Merrill Lynch. Assume annual frequency, 30/360 convention and 35% recovery.

[119] Forward spreads can be approximated by:

$$S_{fwd} = \frac{S_{long\_dated} \cdot RiskyBPV_{long\_dated} - S_{short\_dated} \cdot RiskyBPV_{short\_dated}}{RiskyBPV_{long\_dated} - RiskyBPV_{short\_dated}}$$

If 2y CDS is 125bps with a risky bpv of 1.94, and 7y CDS is 175bps with a risky bpv of 5.86. Then the 5y CDS after two years, is given by:

$$Spread(2,5) = \frac{175bps \cdot 5.86 - 125bps \cdot 1.94}{5.86 - 1.94} = 200bps$$

Not all credits have liquid CDS term structures

CMCDS valuation requires a CDS term structure up to the term of the CMCDS contract plus the reference constant maturity. For example, the valuation of a 5y CMCDS based on the 5y CDS as a reference would require an underlying CDS term structure for ten years. We therefore expect typical CMCDS (5y maturity with reference constant maturity of 5 years) to be more easily priced for credits that have relatively liquid CDS term structure out to ten years.

Value of Spread Cap

Forward rates also affect the value of the spread cap…

The cap rate provides an upper limit to the premium paid by the protection buyer (at each reset period) in the event of credit deterioration. This spread cap is equivalent to a series of Payer (or Right to Buy protection) options[120] bought by the CMCDS protection buyer on each reset date and which expire on the next reset date. Each of these Payer options has a strike equal to the cap rate.

…which is significant only when reference rate is close to cap rate

The spread cap depends on several factors including forward rates, forward spread volatility, CMCDS maturity, reference constant maturity as well as the credit risk of the underlying reference entity. As long as reference constant maturity spreads are substantially lower than the cap rate, the value of the spread cap is relatively small. The spread cap becomes significant only when the credit deteriorates considerably and the reference constant maturity spread is close to the cap rate.

Deriving the Multiplication Factor

Multiplication factor depends on the slope of the curve…

What's the impact of forward rates on the multiplication or percentage factor? Clearly higher (lower) implied forward rates increase (decrease) the expected premium paid by the buyer, thus lowering (increasing) the required multiplication factor to be applied.

…and can be derived from a simple arbitrage argument

The multiplication factor can be derived via a simple **arbitrage argument**. Since the payment upon default for both the CMCDS and vanilla CDS is **equal** (the protection seller pays 100- Recovery), the amount paid by the protection buyer (the *premium leg*) must be the same for both the instruments.

Let *Spread(t,m)* be the future spot premium of a *m* years maturity CDS at time *t*. We can then express the present value of the premium leg of the CMCDS as follows:

$$\text{Premium Leg}^{\text{CMCDS}} = \sum_{t=1}^{n} MF \cdot Spread(t-1,m) \cdot DiscFact(t) \cdot Surv\,\Pr(t)$$

Similarly, the premium leg for a vanilla CDS is simply given by the (weighted) discounted sum of the constant spread paid by the protection buyer, i.e.

$$\text{Premium Leg}^{\text{CDS}} = \sum_{t=1}^{n} Spread \cdot DiscFact(t) \cdot Surv\,\Pr(t)$$

Given our arbitrage argument we then have:

$$\text{Premium Leg}^{\text{CMCDS}} = \text{Premium Leg}^{\text{CDS}}$$

Therefore:

$$\sum_{t=1}^{n} MF \cdot Spread(t-1,m) \cdot DiscFact(t) \cdot Surv\,\Pr(t) = \sum_{t=1}^{n} Spread \cdot DiscFact(t) \cdot Surv\,\Pr(t)$$

This implies:

[120] For more details on Payer options, see Volume 2, Chapter 14.

$$MF = \frac{\sum_{t=1}^{n} Spread \cdot DiscFact(t) \cdot Surv\Pr(t)}{\sum_{t=1}^{n} Spread(t-1,m) \cdot DiscFact(t) \cdot Surv\Pr(t)}$$

MF is less than (more than) 100% for upward (downward) sloping curve

The projected premium payments of the CMCDS are set equal to the implied forward rates. Thus if the CDS term structure is **upward sloping**, the implied forward rates will be greater than the constant CDS spread , making the above ratio (i.e. the multiplication factor MF) **smaller** than 100%. On the other hand, if the curve is **downward sloping**, the constant CDS spread will be less than the implied forward rates, and the ratio will be **greater** than 100%.

In practice, a further adjustment, generally known as "convexity adjustment", is applied to the forward rates. This convexity adjustment is based on the observation that the duration of the underlying CDS does not change linearly with the spread itself, i.e. higher the spread, shorter the duration. The adjustment ensures that the following two contracts have the same value: 1) a forward starting default swap whose coupon is determined on the forward date; and 2) a forward starting default swap where the coupon is calculated today.

Table 66 highlights multiplication factors for illustrative upward and downward sloping CDS term structures.

Table 66: Illustrative Upward and Downward Sloping CDS Term Structures

CDS Maturity	Downward Sloping	Upward Sloping
1y	165 bps	21 bps
2y	162 bps	28 bps
3y	156 bps	32 bps
5y	141 bps	40 bps
7y	139 bps	47 bps
10y	132 bps	56 bps
Multiplication Factor	**115%**	**63%**

Source: Merrill Lynch and Mark-It Partners. Assume 30% recovery, CMCDS is 5y maturity and reference constant maturity is 5y vanilla CDS.

CMCDS Spread Sensitivity

The entire CDS curve comes into play

As explained earlier, unlike a vanilla CDS, the valuation of a typical CMCDS contract depends on a full CDS term structure that extends beyond the maturity of the CMCDS. For **parallel** changes in the underlying curve, we observe that the CMCDS structure is less sensitive than a vanilla CDS. For example, the spread DV01 (or risky bpv) for a 5y CMCDS (with a 5y reference constant maturity) is much smaller than the spread DV01 of a 5y CDS of the same credit[121]. Table 67 highlights the sensitivity of CMCDS and CDS for parallel spread movements.

Table 67: CDS vs. CMCDS Sensitivity

	Parallel Spread Widening		Parallel Spread Tightening	
	CMCDS	CDS	CMCDS	CDS
Carry	↑	Unchanged	↓	Unchanged
Probability of Default	↑	↑	↓	↓
CDS Duration	↓	↓↓	↑	↑↑

Source: Merrill Lynch

[121] Assuming a flat CDS curve of 100bps, a 5y CDS has a spread DV01 of about 4.5 compared to a spread DV01 of about 0.8 for a 5y CMCDS with 5y reference constant maturity and multiplication factor of 80%.

Refer to important disclosures on page 196.

Table 68: FMCC CDS Curve

Maturity	FMCC CDS Curve (bps)
1y	83
2y	124
3y	149
5y	191
7y	200
10y	212

Source: Merrill Lynch & Mark-It Partners; Mid levels as of April 12, 2004.

The P&L impact of any **non-parallel** spread widening or tightening, however, depends on the portion of the curve that experiences the widening or tightening. Default likelihood is driven primarily by the relative curve movement of maturities up to that of the CMCDS. The expected premium received/paid, however, is dependent on the forward rates. While forward rates are a function of the entire CDS curve, any changes in the term structure beyond the CMCDS maturity would affect only the forward rates and not the likelihood of default.

We illustrate the spread sensitivity of the CMCDS via different scenarios using **Ford Motor Credit (FMCC)** as the underlying credit. FMCC is one of the credits that has a relatively liquid CDS curve out to a maturity of ten years (Table 68).

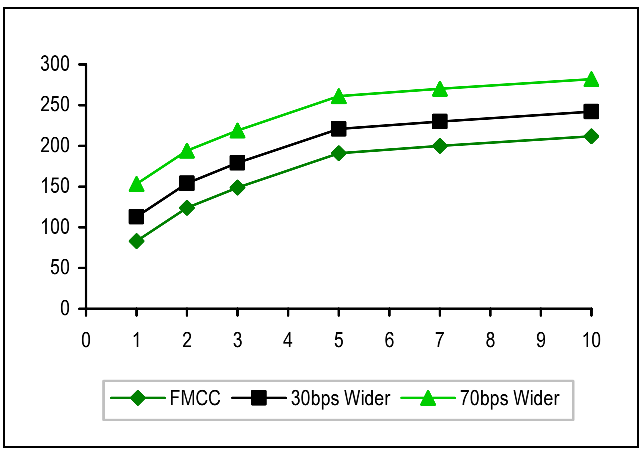
Parallel Shifts in the CDS Curve

Parallel Widening

CMCDS is less sensitive than CDS for parallel widening…

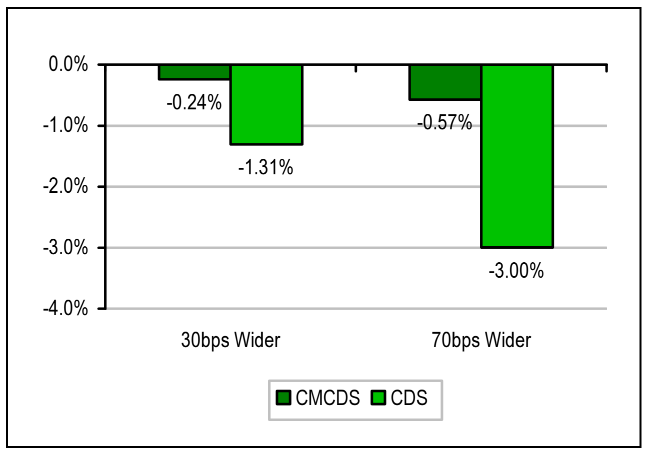
A parallel widening increases the likelihood of default for both CMCDS as well as vanilla CDS. This would result in a negative MTM for a protection seller in either of these two instruments. However, the parallel spread widening also increases the implied forward rates. As a result, for a CMCDS protection seller, the **negative MTM due to the increased likelihood of default is mitigated by the expectation of higher premiums at future resets**. This lowers the MTM volatility of the CMCDS. Chart 216 highlights the relatively small MTM loss for a CMCDS protection seller compared to a seller of vanilla CDS. We assume an instantaneous shift in the CDS curve (see Chart 215).

Chart 215: Parallel Widening of CDS Curve



Source: Merrill Lynch and Mark-It Partners. Assume an instantaneous shift of the original CDS curve

Chart 216: MTM for CMCDS vs. CDS (Long Credit Risk)



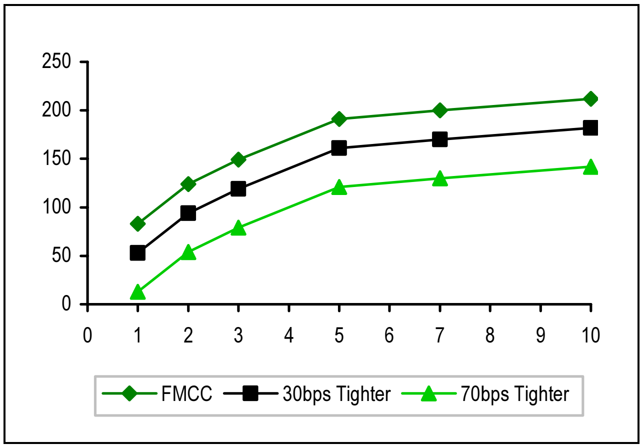
Source: Merrill Lynch and Mark-It Partners. CMCDS is 5y and indexed to 5y vanilla CDS spread. CDS is 5y.

Parallel Tightening

…or parallel tightening

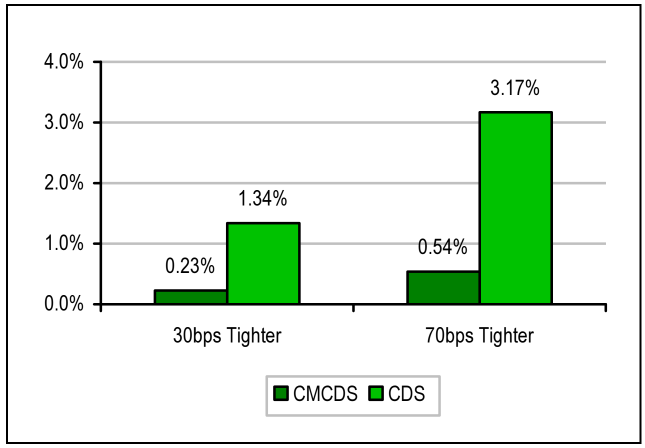
A parallel tightening lowers the likelihood of default for both CMCDS and vanilla CDS resulting in a positive MTM for a protection seller in either of these instruments. In addition, the implied forward rates decrease due to the parallel tightening. As a result, for a CMCDS protection seller, **the positive MTM from the decrease in likelihood of default is lowered by the expectation of lower premiums at future resets**. Once again, this lowers the MTM volatility for CMCDS. Chart 218 highlights this relatively small MTM gain for a CMCDS protection seller compared to a seller of vanilla CDS. We assume an instantaneous shift in the CDS curve (see Chart 217).

Chart 217: Parallel Tightening of CDS Curve



Source: Merrill Lynch and Mark-It Partners. Assume an instantaneous shift of the original CDS curve

Chart 218: MTM for CMCDS vs. CDS (Long Credit Risk)



Source: Merrill Lynch and Mark-It Partners. CMCDS is 5y and indexed to 5y vanilla CDS spread. CDS is 5y.

Non-Parallel Shifts in CDS Curve

For non-parallel shifts in the CDS curve, the MTM of the CMCDS **depends on whether the curve has shifted in the front-end or in the back-end**. We assume the back-end to include all maturities on the curve beyond the reference constant maturity (5y in the following discussion). The front-end includes all maturities up to, but not including, the reference constant maturity (or the 5y in this case).

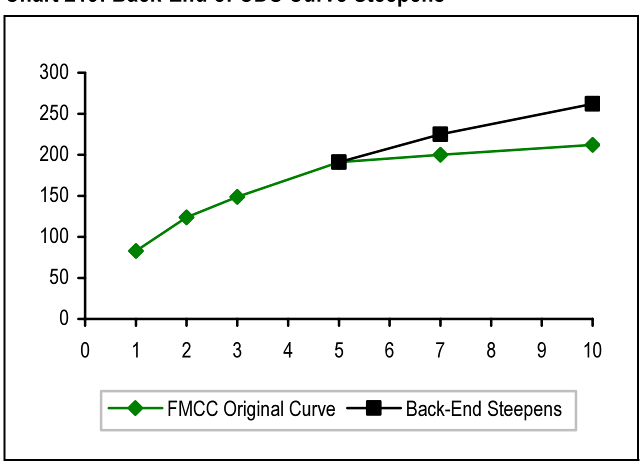
The CMCDS is more sensitive to movements in the front-end of the curve as these affect not only the implied forward rates but also the likelihood of default. The back-end of the curve does not have any impact on the likelihood of default for either the CMCDS or the CDS but affects the implied forward rates which only affects the CMCDS.

Curve Steepening

Curve steepening benefits seller of CMCDS protection

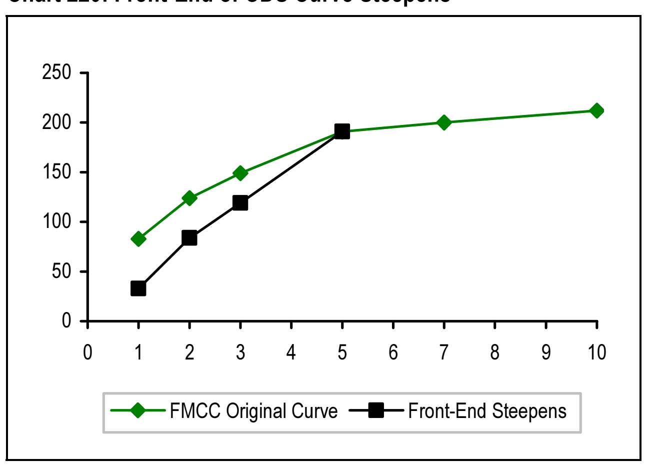
In our initial scenario we assume the CDS curve steepens in the back-end as shown in Chart 219. The CMCDS benefits from this back-end spread widening due to the subsequent increase in the implied forward rates. This is reflected in a positive MTM for the CMCDS (see Chart 222). On the other hand, the MTM for a seller of 5y CDS protection is unaffected by a change in the back-end of the curve.

Chart 219: Back-End of CDS Curve Steepens



Source: Merrill Lynch and Mark-It Partners. Assume an instantaneous shift of the original CDS curve; Assume 7y widens by 25bps and 10y widens by 50bps

Chart 220: Front-End of CDS Curve Steepens



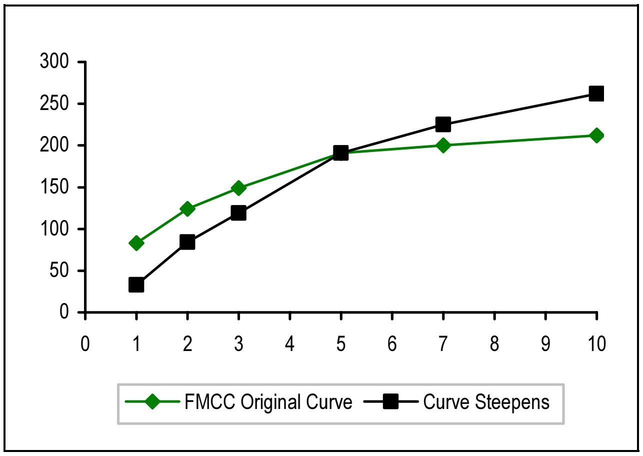
Source: Merrill Lynch and Mark-It Partners. Assume an instantaneous shift of the original CDS curve. Assume 1y tightens by 50bs, 2y tightens by 40bps and 3y tightens by 30bps.

Refer to important disclosures on page 196.

If the front-end of the curve steepens as shown in Chart 220, the CMCDS benefits from an increase in the implied forward rates. This generates a positive MTM impact on the CMCDS as it increases the expected premium at future resets (see Chart 222). Both CMCDS and CDS would benefit from the lower probability of default due to the steeper front-end. However, since this is an instantaneous shift, the MTM due to this effect is zero for both these instruments. We have assumed that 5y spread is unchanged as the curve steepens.

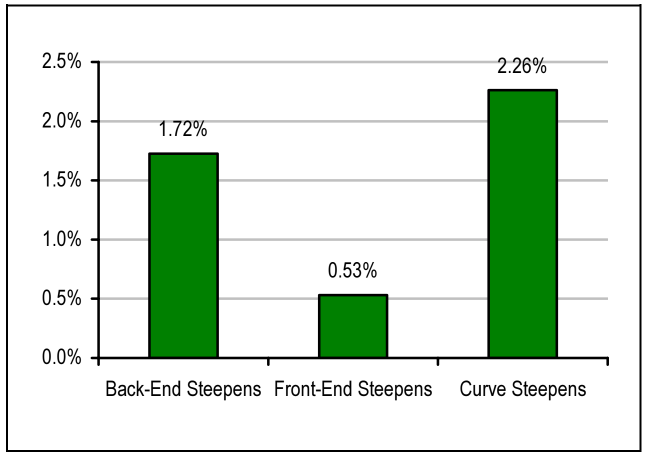
Based on the two scenarios described above, we would expect a full steepening of the CDS curve to be significantly more beneficial to the seller of CMCDS protection relative to a seller of CDS protection. Chart 222 shows that for the curve steepening scenario shown in Chart 221, the positive MTM for the CMCDS is significantly higher than the CDS MTM (zero). This assumes that the curve shifts instantaneously with the 5y CDS remaining unchanged.

Chart 221: CDS Curve Steepens



Source: Merrill Lynch and Mark-It Partners. Assume an instantaneous shift of the original CDS curve.

Chart 222: MTM for CMCDS (Long Credit Risk)



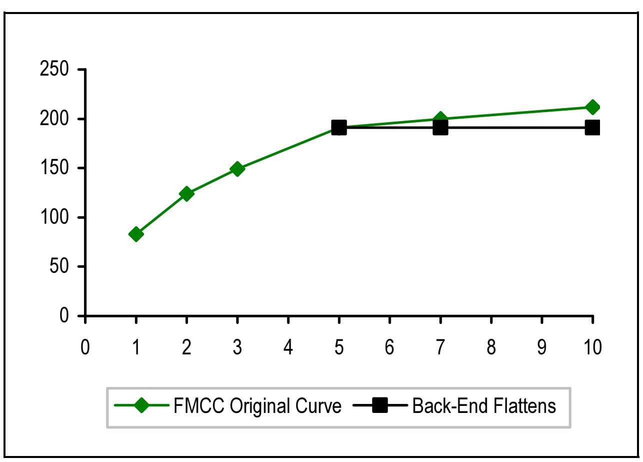
Source: Merrill Lynch and Mark-It Partners. CMCDS is 5y and indexed to 5y vanilla CDS spread. CDS is 5y.

Curve Flattening

Curve flattening has negative impact on CMCDS

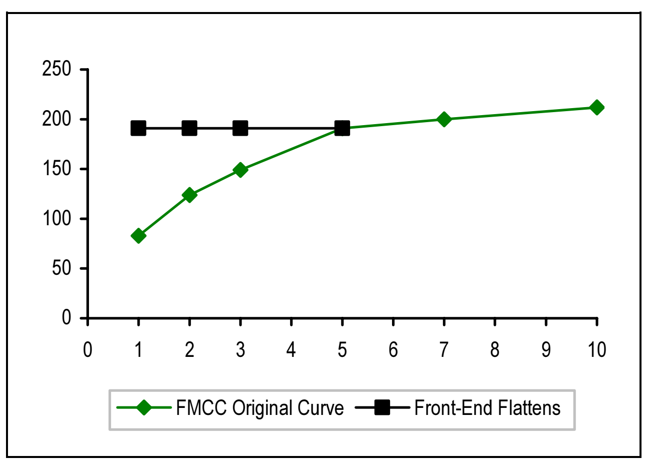
If the back-end of the curve flattens, the CMCDS has a negative MTM due to the lower implied forward rates. The back-end of the curve has no impact on the likelihood of default of either the CMCDS or the CDS. As a result, there is no effect on the MTM of the CDS due to this move. Chart 226 highlights the MTM impact for the back-end curve flattening shown in Chart 223.

Chart 223: Back-End of CDS Curve Flattens



Source: Merrill Lynch and Mark-It Partners. Assume an instantaneous shift of the original CDS curve. Assume 7y and 10y tighten to the 5y level.

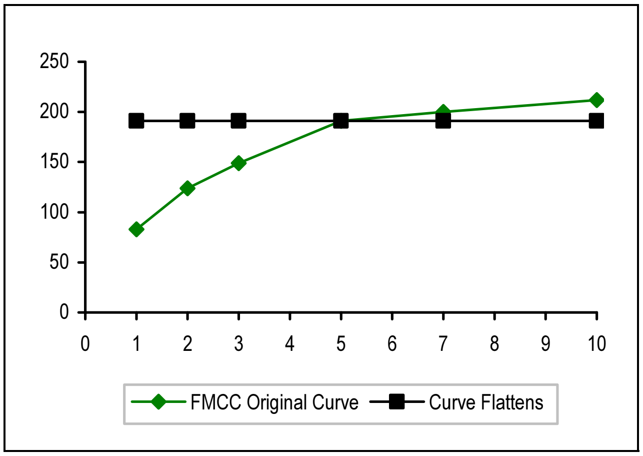
Chart 224: Front-End of CDS Curve Flattens



Source: Merrill Lynch and Mark-It Partners. Assume an instantaneous shift of the original CDS curve. Assume the 1y, 2y and 3y widen to the 5y level.

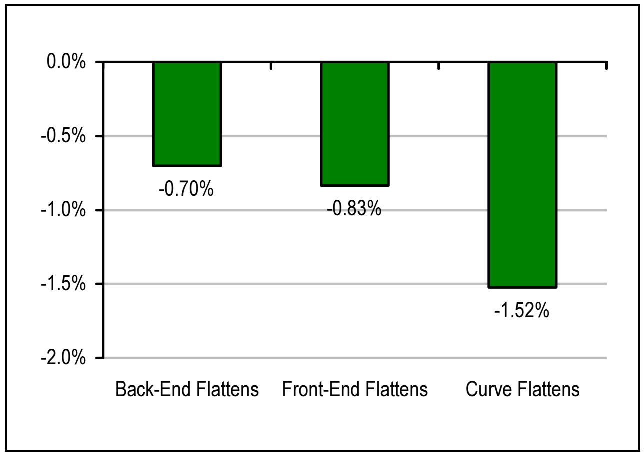
Alternatively, if the front-end of the curve flattens (see Chart 224) the MTM of the CMCDS is negative (see Chart 226) due to the decrease in forward rates which decreases the expected premium of the CMCDS. Both CMCDS and CDS should have a negative impact from the increase in default probability due to the curve flattening in the front-end. However since it is an instantaneous shift and we assume that 5y is unchanged the MTM due to this effect is zero for both instruments.

Chart 225: CDS Curve Flattens



Source: Merrill Lynch and Mark-It Partners. Assume an instantaneous shift of the original CDS curve

Chart 226: MTM for CMCDS (Long Credit Risk)



Source: Merrill Lynch and Mark-It Partners. CMCDS is 5y and indexed to 5y vanilla CDS spread. CDS is 5y.

A combination of the front-end and back-end flattening scenarios above would result in a curve flattening for the entire CDS curve. Based on our observations above, we would expect the MTM for the CMCDS to be more negative than that for the CDS. Chart 226 shows that for the curve flattening scenario shown in Chart 225 the MTM for the CMCDS is more negative than the CDS MTM (zero). This assumes that the curve shifts instantaneously with the 5y CDS remaining unchanged.

Who Should Consider CMCDS?

■ Hedging Spread Risk but not Default

Investors who are long credit, not concerned about defaults…

We infer from the above sensitivity analysis that selling CMCDS protection makes sense for investors who hold the following credit specific views:

1. Long credit, i.e. believe credit fundamentals are improving;
2. Want to lower MTM loss if spreads widen in parallel fashion;
3. Not concerned about defaults;
4. Believe the credit curve could steepen.

…but concerned with parallel spread widening

The CMCDS protection seller is effectively hedged for parallel spread moves of the entire CDS curve. However, the results are different when the curve moves in a non-parallel fashion. As shown earlier, the worst MTM scenario for a seller of CMCDS protection occurs when the curve flattens. Investors who expect the curve to flatten would be better off buying CMCDS protection compared to buying vanilla CDS protection. Investors who are more concerned about parallel spread widening or who believe curve steepening is the more likely scenario would be more inclined to sell protection on a CMCDS.

Isolating Spread Risk but Hedging Default

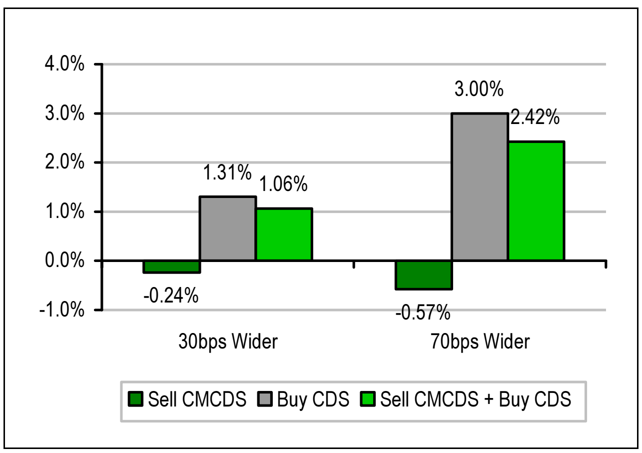
CMCDS can be combined with CDS to isolate spread risk

Investors who have an outright bearish view (i.e. spreads will widen with the potential for default) can express this view by buying protection outright on vanilla CDS. Others, however, may be of the opinion that while spreads could widen, actual default is highly unlikely. These investors would instead prefer exposure only to spread widening (and not default) at a lower cost. Selling CMCDS protection and simultaneously buying CDS protection can create such an exposure.

The investor has an initial negative carry on the trade if the curve is upwardly sloping (the multiplication factor is less than 1)[122]. If spreads widen out the carry on the trade could switch to positive if the CMCDS reset premium exceeds the initial CDS premium. Under a parallel spread widening scenario, any MTM gain from the long CDS position is slightly dampened by the much smaller MTM loss of the short CMCDS position (see Chart 227 and Chart 228). However, default risk is completely hedged though upon default any positive MTM from spread widening would not be realised.

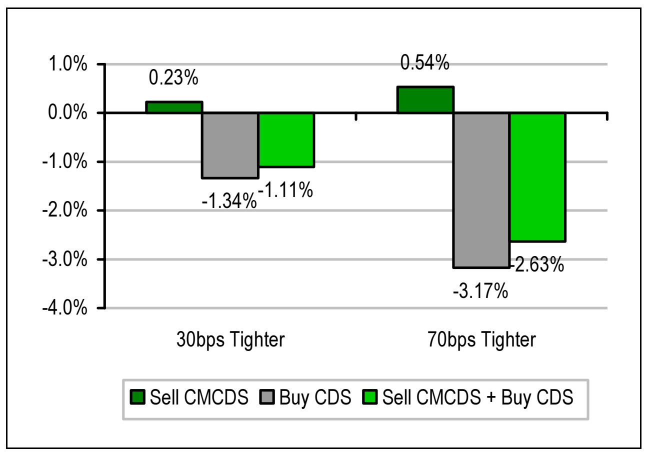
This trade is attractive for investors who have strong views that spreads will widen but believe that the risk of default is relatively small. The CMCDS in combination with a CDS enables them to leverage this view without talking on any exposure to instantaneous default risk.

Chart 227: Parallel Widening of FMCC Curve



Source: Merrill Lynch & Mark-It Partners. Assume an instantaneous shift of the original CDS curve.

Chart 228: Parallel Tightening of FMCC Curve



Source: Merrill Lynch & Mark-It Partners. Assume an instantaneous shift of the original CDS curve.

[122] Initial negative carry = MF*5y CDS – 5y CDS = - (1-MF)*5y CDS; MF < 1

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