Zoo: A framework for the verification of concurrent OCaml 5 programs using separation logic

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Abstract

The release of OCaml 5, which introduced parallelism into the language, drove the need for safe and efficient concurrent data structures. New libraries like Saturn [31] aim at addressing this need. From the perspective of formal verification, this is an opportunity to apply and further state-of-the-art techniques to provide stronger guarantees.

We present a framework for verifying fine-grained concurrent OCaml 5 algorithms. Following a pragmatic approach, we support a limited but sufficient fragment of the language whose semantics has been carefully formalized to faithfully express such algorithms. Source programs are translated to a deeply-embedded language living inside Rocq where they can be specified and verified using the Iris [28] concurrent separation logic.

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1 Introduction

Designing concurrent algorithms, in particular fine-grained concurrent algorithms, is a notoriously difficult task. Similarly, the formal verification of such algorithms is also difficult. It typically involves finding and reasoning about non-trivial linearization points [21, 29, 53, 54, 11].

In recent years, concurrent separation logic [5] has enabled significant progress in this area. In particular, the development of Iris [28], a state-of-the-art mechanized *higher-order* concurrent separation logic with *user-defined ghost state*, has nourished a rich and successful line of works [29, 53, 54, 11, 6, 27, 48, 38, 37, 17, 43, 41, 40], dealing with external [54] and future-dependent [29, 53, 11] linearization points, relaxed memory [38, 37, 17, 43] and automation [41, 40].

Most of these works [29, 53, 54, 6, 27, 48, 41, 40] and many others [19, 45, 52, 35] rely on HeapLang [51], the canonical Iris language. HeapLang is a concurrent, imperative, untyped, call-by-value functional language. To the best of our knowledge, it is currently the closest language to OCaml 5 in the Iris ecosystem—we review the existing frameworks in Section 2. It has been extended to handle weak memory [38] and algebraic effects [18].

Although HeapLang is theoretically expressive enough to represent OCaml programs, our experience showed that it is fairly impractical when it comes to verifying large OCaml libraries. Indeed, it lacks basic abstractions such as algebraic data types (tuples, mutable and immutable records, variants) and mutually recursive functions. It also has very few standard data structures that can be directly reused. This view, we believe, is shared by many people in the Iris community. Our first motivation in this work is therefore to fill this gap by providing a more practical OCaml-like verification language: ZooLang. This language consists in a subset of OCaml 5 extended with atomic record fields and equipped with a formal semantics and a program logic based on Iris. We were influenced by the

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Perennial [8, 9, 10, 11] framework, which achieved similar goals for the Go language with a focus on crash-safety. As in Perennial, we also provide a translator from OCaml to ZooLang: `ocaml2zoo`. We call the resulting framework Zoo.

Another, maybe less obvious, shortcoming of HeapLang is the soundness of its semantics with respect to OCaml, in other words how faithful it is to the original language. One ubiquitous—particularly in lock-free algorithms relying on low-level atomic primitives—and subtle point is *physical equality*. In Section 5, we show that (1) HeapLang's semantics for physical equality is not compatible with OCaml and (2) OCaml's informal semantics is actually too imprecise to verify basic concurrent algorithms. To remedy this, we propose a new formal semantics for physical equality and structural equality. We hope this work will influence the way these notions are specified in OCaml.

In summary, we make the following contributions:

1. We present ZooLang, a convenient subset of OCaml 5 formalized in Rocq (Sections 3 and 4). ZooLang comes with a program logic based on Iris and supports proof automation through Diaframe [41, 40].
2. We provide a translator from OCaml to ZooLang: `ocaml2zoo` (Section 3).
3. We formalize physical equality (Section 5) and structural equality (Section 6) in a faithful way. The careful analysis of these notions suggests a new OCaml feature: *generative constructors*.
4. We extend OCaml with *atomic record fields* and *atomic arrays* to ease the development of fine-grained concurrent algorithms (Section 7).
5. We verify realistic use cases (Section 5) involving physical equality: (1) Treiber stack [7], (2) a thread-safe wrapper around a file descriptor using reference-counting from the `Eio` [36] library.

2 Related work

The idea of applying formal methods to verify OCaml programs is not new. Generally speaking, there are mainly two ways:

2.1 Semi-automated verification

The verified program is annotated by the user to guide the verification tool: preconditions, postconditions, invariants, *etc*. Given this input, the tool generates proof obligations that are mostly automatically discharged. One may further distinguish two types of semi-automated systems: *foundational* and *non-foundational*.

In *non-foundational* automated verification, the tool and the external solvers it may rely on are part of the trusted computing base. It is the most common approach and has been widely applied in the literature [50, 42, 26, 20, 3, 22, 34, 46], including to OCaml by Cameleer [44], which uses the Gospel specification language [13] and Why3 [22].

In *foundational* automated verification, the proofs are checked by a proof assistant like Rocq, meaning the automation does not have to be trusted. To our knowledge, it has been applied to C [47] and Rust [23].

2.2 Non-automated verification

The verified program is translated, manually or in an automated way, into a representation living inside a proof assistant. The user has to write specifications and prove them.

The representation may be primitive, like Gallina for Rocq. For pure programs, this is rather straightforward, *e.g.* in `hs-to-coq` [49]. For imperative programs, this is more challenging. One solution is to use a monad, *e.g.* in `coq-of-ocaml` [14], but it does not support concurrency.

The representation may be embedded, meaning the semantics of the language is formalized in the proof assistant. This is the path taken by some recent works [12, 24, 8, 16] harnessing the power of separation logic. In particular, CFML [12] and Osiris [16] target OCaml. However, CFML does not support concurrency and is not based on Iris. Osiris, still under development, is based on Iris but does not support concurrency.

3 Zoo in practice

```
Rocq term         t
constructor       C
projection        proj
record field      fld
identifier        s, f   ∈    String
integer           n      ∈    ℤ
boolean           b      ∈    𝔹
binder            x      ::=  <> | s
unary operator    ⊕      ::=  ~ | -
binary operator   ⊗      ::=  + | - | * | `quot` | `rem` | `land` | `lor` | `lsl` | `lsr`
                          |   <= | < | >= | > | = | ≠ | == | !=
                          |   and | or
expression        e      ::=  t | s | #n | #b
                          |   fun: x1 ... xn => e | rec: f x1 ... xn => e
                          |   let: x := e1 in e2 | e1 ;; e2
                          |   let: f x1 ... xn := e1 in e2 | letrec: f x1 ... xn := e1 in e2
                          |   let: ‘C x1 ... xn := e1 in e2 | let: x1, ..., xn := e1 in e2
                          |   ⊕e | e1 ⊗ e2
                          |   if: e0 then e1 (else e2)?
                          |   for: x := e1 to e2 begin e3 end
                          |   §C | ‘C (e1, ..., en) | (e1, ..., en) | e.<proj>
                          |   [] | e1 :: e2
                          |   ‘C {e1, ..., en} | {e1, ..., en} | e.{fld} | e1 <-{fld} e2
                          |   ref e | !e | e1 <- e2
                          |   match: e0 with br1 | ... | brn (|_ (as s)? => e)? end
                          |   e.[fld] | Xchg e1 e2 | CAS e1 e2 e3 | FAA e1 e2
                          |   Proph | Resolve e0 e1 e2
branch            br     ::=  C (x1 ... xn)? (as s)? => e
                          |   [] (as s)? => e | x1 :: x2 (as s)? => e
toplevel value    v      ::=  t | #n | #b
                          |   fun: x1 ... xn => e | rec: f x1 ... xn => e
                          |   §C | ‘C (v1, ..., vn) | (v1, ..., vn)
                          |   [] | v1 :: v2
```

Figure 1 ZooLang syntax (omitting mutually recursive toplevel functions)

```
type 'a t =
  'a list Atomic.t

let create () =
  Atomic.make []

let rec push t v =
  let old = Atomic.get t in
  let new_ = v :: old in
  if not @@ Atomic.compare_and_set t old new_ then (
    Domain.cpu_relax () ;
    push t v
  )

let rec pop t =
  match Atomic.get t with
  | [] ->
      None
  | v :: new_ as old ->
      if Atomic.compare_and_set t old new_ then (
        Some v
      ) else (
        Domain.cpu_relax () ;
        pop t
      )
```

Figure 2 Implementation of a concurrent stack

In this section, we give an overview of our framework. We also provide a minimal example[1] demonstrating its use.

3.1 Language

The core of Zoo is ZooLang: a concurrent, imperative, untyped, functional programming language fully formalized in Rocq. Its semantics has been designed to match OCaml's.

ZooLang comes with a program logic based on Iris: reasoning rules expressed in separation logic (including rules for the different constructs of the language) along with Rocq tactics that integrate into the Iris proof mode [33, 32]. In addition, it supports Diaframe [41, 40], enabling proof automation.

The ZooLang syntax is given in Figure 1[2], omitting mutually recursive toplevel functions that are treated specifically. Expressions include standard constructs like booleans, integers, anonymous functions (that may be recursive), `let` bindings, sequence, unary and binary operators, conditionals, `for` loops, tuples. In any expression, one can refer to a Rocq term representing a ZooLang value (of type `val`) using its Rocq identifier. ZooLang is a deeply

[1] Non-anonymous link
[2] More precisely, it is the syntax of the surface language, including Rocq notations.

embedded language: variables (bound by functions and `let`) are quoted, represented as strings.

Data constructors (immutable memory blocks) are supported through two constructs : §C represents a constant constructor (*e.g.* `§None`), ‘C (e_1, ... ,e_n) represents a non-constant constructor (*e.g.* ‘`Some(` e `)`). Unlike OCaml, ZooLang has projections of the form e.<*proj*> (*e.g.* (e_1,e_2).<1>), that can be used to obtain a specific component of a tuple or data constructor. ZooLang supports shallow pattern matching (patterns cannot be nested) on data constructors with an optional fallback case.

Mutable memory blocks are constructed using either the untagged record syntax {e_1, ... ,e_n} or the tagged record syntax ‘C {e_1, ... ,e_n}. Reading a record field can be performed using e.{*fld*} and writing to a record field using e_1 <-{*fld*} e_2. Pattern matching can also be used on mutable tagged blocks provided that cases do not bind anything—in other words, only the tag is examined, no memory access is performed. References are also supported through the usual constructs : `ref` e creates a reference, `!`e reads a reference and e_1 `<-` e_2 writes into a reference. The syntax seemingly does not include constructs for arrays but they are supported through the `Array` standard module (*e.g.* `array_make`).

Parallelism is mainly supported through the `Domain` standard module (*e.g.* `domain_spawn`). Special constructs (`Xchg`, `CAS`, `FAA`), described in Section 4.4, are used to model atomic references.

The `Proph` and `Resolve` constructs are used to model *prophecy variables* [29], as described in Section 4.5.

3.2 Translation from OCaml to ZooLang

While ZooLang lives in Rocq, we want to verify OCaml programs. To connect them, we provide a tool to automatically translate OCaml source files[3] into Rocq files containing ZooLang code: `ocaml2zoo`. This tool can process entire `dune` projects, including many libraries.

The supported OCaml fragment includes: tuples, variants, records (including inline records), shallow `match`, atomic record fields, unboxed types, toplevel mutually recursive functions.

Consider, for example, the OCaml implementation of a concurrent stack [7] in Figure 2. The `push` function is translated into:

```
Definition stack_push : val :=
  rec: "push" "t" "v" =>
    let: "old" := !"t" in
    let: "new_" := "v" :: "old" in
    if: ~ CAS "t".[contents] "old" "new_" then (
      domain_yield () ;;
      "push" "t" "v"
    ).
```

3.3 Specifications and proofs

Once the translation to ZooLang is done, the user can write specifications and prove them in Iris. For instance, the specification of the `stack_push` function could be:

[3] Actually, `ocaml2zoo` processes binary annotation files (`.cmt` files).

```
Lemma stack_push_spec t ι v :
  <<<
    stack_inv t ι | ∀∀ vs, stack_model t vs
  >>>
    stack_push t v @ ↑ι
  <<<
    stack_model t (v :: vs) | RET (); True
  >>>.
Proof. ... Qed.
```

Here, we use a *logically atomic specification* [15], which has been proven [4] to be equivalent to *linearizability* [25] in sequentially consistent memory models.

Similarly to Hoare triples, the two assertions inside curly brackets represent the precondition and postcondition for the caller. For this particular operation, the postcondition is trivial. The stack-inv t precondition is the stack invariant. Intuitively, it asserts that t is a valid concurrent stack. More precisely, it enforces a set of logical constraints—a concurrent protocol—that t must respect at all times.

The other two assertions inside angle brackets represent the *atomic precondition* and *atomic postcondition*. They specify the linearization point of the operation: during the execution of `stack_push`, the abstract state of the stack held by stack-model is atomically updated from vs to $v :: vs$; in other words, v is atomically pushed at the top of the stack.

4 Zoo features

In this section, we review the main features of Zoo, starting with the most generic ones and then addressing those related to concurrency.

4.1 Algebraic data types

Zoo is an untyped language but, to write interesting programs, it is convenient to work with abstractions like algebraic data types. To simulate tuples, variants and records, we designed a machinery to define projections, constructors and record fields.

For example, one may define a list-like type with:

```
Notation "'Nil'"  := (in_type "t" 0) (in custom zoo_tag).
Notation "'Cons'" := (in_type "t" 1) (in custom zoo_tag).
```

Given this incantation, one may directly use the tags `Nil` and `Cons` in data constructors using the corresponding ZooLang constructs:

```
Definition map : val :=
  rec: "map" "fn" "t" =>
    match: "t" with
    | Nil =>
        §Nil
    | Cons "x" "t" =>
        let: "y" := "fn" "x" in
        ‘Cons( "y", "map" "fn" "t" )
    end.
```

The meaning of this incantation is not really important, as such notations can be generated by `ocaml2zoo`. Suffice it to say that it introduces the two tags in the `zoo_tag` custom entry, on which the notations for data constructors rely. The `in_type` term is needed to distinguish the tags of distinct data types; crucially, it cannot be simplified away by Rocq, as this could lead to confusion during the reduction of expressions.

Similarly, one may define a record-like type with two mutable fields `f1` and `f2`:

```
Notation "'f1'" := (in_type "t" 0) (in custom zoo_field).
Notation "'f2'" := (in_type "t" 1) (in custom zoo_field).

Definition swap : val :=
  fun: "t" =>
    let: "f1" := "t".{f1} in
    "t" <-{f1} "t".{f2} ;;
    "t" <-{f2} "f1".
```

4.2 Mutually recursive functions

Zoo supports non-recursive (`fun:` $x_1 \ldots x_n$ `=>` e) and recursive (`rec:` f $x_1 \ldots x_n$ `=>` e) functions but only *toplevel* mutually recursive functions. Indeed, it is non-trivial to properly handle mutual recursion: when applying a mutually recursive function, a naive approach would replace the recursive functions by their respective bodies, but this typically makes the resulting expression unreadable. To prevent it, the mutually recursive functions have to know one another so as to replace by the names instead of the bodies. We simulate this using some boilerplate that can be generated by `ocaml2zoo`. For instance, one may define two mutually recursive functions `f` and `g` as follows:

```
Definition f_g := (
  recs: "f" "x" => "g" "x"
  and:  "g" "x" => "f" "x"
)%zoo_recs.

(* boilerplate *)
Definition f := ValRecs 0 f_g.
Definition g := ValRecs 1 f_g.
Instance : AsValRecs' f 0 f_g [f;g]. Proof. done. Qed.
Instance : AsValRecs' g 1 f_g [f;g]. Proof. done. Qed.
```

4.3 Standard library

To save users from reinventing the wheel, we provide a standard library—more or less a subset of the OCaml standard library. Currently, it mainly includes standard data structures like: array (`Array`), resizable array (`Dynarray`), list (`List`), stack (`Stack`), queue (`Queue`), double-ended queue, mutex (`Mutex`), condition variable (`Condition`).

Each of these standard modules contains ZooLang functions and their verified specifications. These specifications are modular: they can be used to verify more complex data structures. As an evidence of this, lists [1] and arrays [2] have been successfully used in verification efforts based on Zoo.

4.4 Concurrent primitives

ZOO supports concurrent primitives both on atomic references (from `Atomic`) and atomic record fields (from `Atomic.Loc`[4]) according to the table below. The OCAML expressions listed in the left-hand column translate into the ZOO expressions in the right-hand column. Notice that an atomic location `[%atomic.loc` e`.`f`]` (of type `_ Atomic.Loc.t`) translates directly into e`.[`f`]`.

OCAML	ZOO
`Atomic.get` e	`!`e
`Atomic.set` e_1 e_2	e_1 `<-` e_2
`Atomic.exchange` e_1 e_2	`Xchg` e_1`.[contents]` e_2
`Atomic.compare_and_set` e_1 e_2 e_3	`CAS` e_1`.[contents]` e_2 e_3
`Atomic.fetch_and_add` e_1 e_2	`FAA` e_1`.[contents]` e_2
`Atomic.Loc.exchange [%atomic.loc` e_1`.`f`]` e_2	`Xchg` e_1`.[`f`]` e_2
`Atomic.Loc.compare_and_set [%atomic.loc` e_1`.`f`]` e_2 e_3	`CAS` e_1`.[`f`]` e_2 e_3
`Atomic.Loc.fetch_and_add [%atomic.loc` e_1`.`f`]` e_2	`FAA` e_1`.[`f`]` e_2

One important aspect of this translation is that atomic accesses (`Atomic.get` and `Atomic.set`) correspond to plain loads and stores. This is because we are working in a sequentially consistent memory model: there is no difference between atomic and non-atomic memory locations.

4.5 Prophecy variables

Lockfree algorithms exhibit complex behaviors. To tackle them, IRIS provides powerful mechanisms such as *prophecy variables* [29]. Essentially, prophecy variables can be used to predict the future of the program execution and reason about it. They are key to handle *future-dependent linearization points*: linearization points that may or may not occur at a given location in the code depending on a future observation.

ZOO supports prophecy variables through the `Proph` and `Resolve` expressions—as in HEAPLANG, the canonical IRIS language. In OCAML, these expressions correspond to `Zoo.proph` and `Zoo.resolve`, that are recognized by `ocaml2zoo`.

5 Physical equality

5.0.0.1 Example 1: physical equality.

Consider, for example, the OCAML implementation of a concurrent stack [7] in Figure 2. Essentially, it consists of an atomic reference to a list that is updated atomically using the `Atomic.compare_and_set` primitive. While this simple implementation—it is indeed one of the simplest lock-free algorithms—may seem easy to verify, it is actually more subtle than it looks.

Indeed, the semantics of `Atomic.compare_and_set` involves *physical equality*: if the content of the atomic reference is physically equal to the expected value, it is atomically updated to the new value. Comparing physical equality is tricky and can be dangerous—this is why *structural equality* is often preferred—because the programmer has few guarantees

[4] The `Atomic.Loc` module is part of the PR that implements atomic record fields.

```
type state =
  | Open of Unix.file_descr
  | Closing of (unit -> unit)

type t =
  { mutable ops: int [@atomic];
    mutable state: state [@atomic];
  }

let make fd =
  { ops= 0; state= Open fd }

let closed =
  Closing (fun () -> ())
let close t =
  match t.state with
  | Closing _ ->
      false
  | Open fd as prev ->
      let close () = Unix.close fd in
      let next = Closing close in
      if Atomic.Loc.compare_and_set [%atomic.loc t.state] prev next then (
        if t.ops == 0
        && Atomic.Loc.compare_and_set [%atomic.loc t.state] next closed
        then
          close () ;
        true
      ) else (
        false
      )
```

Figure 3 `Rcfd.close` function from `Eio` [36]

about the *physical identity* of a value. In particular, the physical identity of a list, or more generally of an inhabitant of an algebraic data type, is not really specified. The only guarantee is: if two values are physically equal, they are also structurally equal. Apparently, we don't learn anything interesting when two values are physically distinct. Going back to our example, this is fortunately not an issue, since we always retry the operation when `Atomic.compare_and_set` returns `false`.

Looking at the standard runtime representation of OCaml values, this makes sense. The empty list is represented by a constant while a non-empty list is represented by pointer to a tagged memory block. Physical equality for non-empty lists is just pointer comparison. It is clear that two pointers being distinct does not imply the pointed memory blocks are.

From the viewpoint of formal verification, this means we have to carefully design the semantics of the language to be able to reason about physical equality and other subtleties of concurrent programs. Essentially, the conclusion we can draw is that the semantics of physical equality and therefore `Atomic.compare_and_set` is non-deterministic: we cannot determine the result of physical comparison just by looking at the abstract values.

5.0.0.2 Example 2: when physical identity matters.

Consider another example given in Figure 3: the `Rcfd.close`[5] function from the `Eio` [36] library. Essentially, it consists in protecting a file descriptor using reference counting. Similarly, it relies on atomically updating the `state` field using `Atomic.Loc.compare_and_set`[6]. However, there is a complication. Indeed, we claim that the correctness of `close` derives from the fact that the `Open` state does not change throughout the lifetime of the data structure; it can be replaced by a `Closing` state but never by another `Open`. In other words, we want to say that 1) this `Open` is *physically unique* and 2) `Atomic.Loc.compare_and_set` therefore detects whether the data structure has flipped into the `Closing` state. In fact, this kind of property appears frequently in lock-free algorithms; it also occurs in the `Kcas` [30] library[7].

Once again, this argument requires special care in the semantics of physical equality. In short, we have to reveal something about the physical identity of some abstract values. Yet, we cannot reveal too much—in particular, we cannot simply convert an abstract value to a concrete one (a memory location)—, since the OCaml compiler performs optimizations like sharing of immutable constants, and the semantics should remain compatible with adding other optimizations later on, such as forms of hash-consing.

In Zoo, a value is either a bool, an integer, a memory location, a function or an immutable block. To deal with physical equality in the semantics, we have to specify what guarantees we get when 1) physical comparison returns `true` and 2) when it returns `false`.

We assume that the program is semantically well typed, if not syntactically well typed, in the sense that compared values are loosely compatible: a boolean may be compared with another boolean or a location, an integer may be compared with another integer or a location, an immutable block may be compared with another immutable block or a location. This means we never physically compare, *e.g.*, a boolean and an integer, an integer and an immutable block. If we wanted to allow it, we would have to extend the semantics of physical comparison to account for conflicts in the memory representation of values.

For booleans, integers and memory locations, the semantics of physical equality is plain equality. Let us consider the case of abstract values (functions and immutable blocks).

If physical comparison returns `true`, the semantics of OCaml tells us that these values are structurally equal. This is very weak because structural equality for memory locations is not plain equality. In fact, assuming only that, the stack of Section 1 and many other concurrent algorithms relying on physical equality would be incorrect. Indeed, for *e.g.* a stack of references (`'a ref`), a successful `Atomic.compare_and_set` in `push` or `pop` would not be guaranteed to have seen the exact same list of references; the expected specification of Section 3 would not work. What we want and what we assume in our semantics is plain equality. Hopefully, this should be correct in practice, as we know physical equality is implemented as plain comparison.

If physical comparison returns `false`, the semantics of OCaml tells us essentially nothing: two immutable blocks may have distinct identities but same content. However, given this semantics, we cannot verify the `Rcfd` example of Section 1. To see why, consider the first `Atomic.compare_and_set` in the `close` function. If it fails, we expect to see a `Closing` state because we know there is only one `Open` state ever created, but we cannot prove it. To address it, we take another step back from OCaml's semantics by introducing the `Reveal` construct. When applied to an immutable memory block, `Reveal` yields the same block

[5] https://github.com/ocaml-multicore/eio/blob/main/lib_eio/unix/rcfd.ml
[6] Here, we make use of atomic record fields that were recently introduced in OCaml.
[7] https://github.com/ocaml-multicore/kcas/blob/main/doc/gkmz-with-read-only-cmp-ops.md

annotated with a logical identifier that can be interpreted as its abstract identity. The meaning of this identifier is: if physical comparison of two identified blocks returns `false`, the two identifiers are necessarily distinct. The underling assumption that we make here—which is hopefully also correct in the current implementation of OCaml—is that the compiler may introduce sharing but not unsharing.

The introduction of `Reveal` can be performed automatically by `ocaml2zoo` provided the user annotates the data constructor (*e.g.* `Open`) with the attribute `[@zoo.reveal]`. For `Rcfd.make`, it generates:

```
Definition rcfd_make : val :=
  fun: "fd" =>
    { #0, Reveal ‘Open( "fd" ) }.
```

Given this semantics and having revealed the `Open` block, we can verify the `close` function. Indeed, if the first `Atomic.compare_and_set` fails, we now know that the identifiers of the two blocks, if any, are distinct. As there is only one `Open` block whose identifier does not change, it cannot be the case that the current state is `Open`, hence it is `Closing` and we can conclude.

6 Structural equality

7 OCaml extensions for fine-grained concurrent programming

Over the course of this work, we studied efficient fine-grained concurrent OCaml programs written by experts. This revealed various limitations of OCaml in these domains, that those experts would work around using unsafe casts, often at the cost of both readability and memory-safety; and also some mismatches between their mental model of the semantics of OCaml and the mental model used by the OCaml compiler authors. We worked on improving OCaml itself to reduce these work-arounds or semantic mismatches.

7.1 Atomic record fields

7.1.1 Before

OCaml 5 offers a type `'a Atomic.t` of atomic references exposing sequentially-consistent atomic operations. Data races on non-atomic mutable locations has a much weaker semantics and is generally considered a programming error. For example, the Michael-Scott concurrent queue [39] relies on a linked list structure that could be defined as follows:

```
type 'a node =
| Nil
| Cons of { value : 'a; next : 'a node Atomic.t }
```

Performance-minded concurrency experts dislike this representation, because `'a Atomic.t` introduces an indirection in memory: it is represented as a pointer to a block containing the value of type `'a`. Instead, they use something like the following:

```
type 'a node =
| Nil
| Cons of { mutable next: 'a node; value: 'a }
```

```
let as_atomic : 'a node -> 'a node Atomic.t option = function
  | Nil -> None
  | (Next _) as record -> Some (Obj.magic record : 'a node Atomic.t)
```

Notice that the `next` field of the `Cons` constructor has been moved first in the type declaration. Because the OCaml compiler respects field-declaration order in data layout, a value `Cons { next; value }` has a similar low-level representation to a reference (atomic or not) pointing at `next`, with an extra argument. The code uses `Obj.magic` to unsafely cast this value to an atomic reference, which appears to work as intended.

`Obj.magic` is a shunned unsafe cast (the OCaml equivalent of `unsafe` or `unsafePerformIO`). It is very difficult to be confident about its usage given that it may typically violate assumptions made by the OCaml compiler and optimizer. In the example above, casting a two-fields record into a one-argument atomic reference may or may not be sound—but it gives measurable performance improvements on concurrent queue benchmarks. (TODO: benchmark to quantify the improvement.)

It is possible to statically forbid passing `Nil` to `as_atomic` to avoid error handling, by turning `'a node` into a GADT indexed over it a type-level representation of its head constructor. Examples of this pattern can be found in the `Kcas` library by Vesa Karvonen. It is difficult to write correctly and use, in particular as unsafe casts can sometimes hide type-errors in the intended static discipline.

Note that this unsafe approach only works for the first field of a record, so it is not applicable to records that hold several atomic fields, such as the toplevel record storing atomic `front` and `back` pointers for the concurrent queue.

7.1.2 Atomic fields proposal

We proposed a design for atomic record fields as an OCaml language change proposal: RFC #39[8]. Declaring a record field atomic simply requires an `[@atomic]` attribute—and could eventually become a proper keyword of the language.

```
(* re-implementation of atomic references *)
type 'a atomic_ref = { mutable contents : 'a [@atomic]; }

(* concurrent linked list *)
type 'a node =
| Nil
| Cons of { value: 'a; mutable next : 'a node [@atomic]; }

(* bounded SPSC circular buffer *)
type 'a bag = {
  data : 'a Atomic.t array;
  mutable front: int [@atomic];
  mutable back: int [@atomic];
}
```

The design difficulty is to express atomic operations on atomic record fields. For example, if `buf` has type `'a bag` above, then one naturally expects the existing notation `buf.front` to

[8] Non-anonymous link

perform an atomic read and `buf.front <- n` to perform an atomic write. But how would one express exchange, compare-and-set and fetch-and-add? We would like to avoid adding a new primitive language construct for each atomic operation.

Our proposed implementation[9] introduces a built-in type `'a Atomic.Loc.t` for an atomic location that holds an element of type `'a`, with a syntax extension `[%atomic.loc <expr>.<field>]` to construct such locations. Atomic primitives operate on values of type `'a Atomic.Loc.t`, and they are exposed as functions of the module `Atomic.Loc`.

For example, the standard library exposes

```
val Atomic.Loc.fetch_and_add : int Atomic.Loc.t -> int -> int
```

and users can write:

```
let preincrement_front (buf : 'a bag) : int =
  Atomic.Loc.fetch_and_add [%atomic.loc buf.front] 1
```

where `[%atomic.loc buf.front]` has type `int Atomic.Loc.t`. Internally, a value of type `'a Atomic.Loc.t` can be represented as a pair of a record and an integer offset for the desired field, and the `atomic.loc` construction builds this pair in a well-typed manner. When a primitive of the `Atomic.Loc` module is applied to an `atomic.loc` expression, the compiler can optimize away the construction of the pair—but it would happen if there was an abstraction barrier between the construction and its use.

Note: the type `'a Atomic.t` of atomic references exposes a function

```
val Atomic.make_contended : 'a -> 'a Atomic.t
```

that ensures that the returned atomic value is allocated with enough alignment and padding to sit alone on its cache line, to avoid performance issues caused by false sharing. Currently there is no such support for padding of atomic record fields (we are planning to work on this if the support for atomic fields gets merged in standard OCaml), so the less-compact atomic references remain preferable in certain scenarios.

7.2 Atomic arrays

On top of our atomic record fields, we have implemented support for atomic arrays, another facility commonly requested by authors of efficient concurrent programs. Our previous example of a concurrent bag of type `'a bag` used a backing array of type `'a Atomic.t array`, which contains more indirections than may be desirable, as each array element is a pointer to a block containing the value of type `'a`, instead of storing the value of type `'a` directly in the array.

Our implementation of atomic arrays[10] builds on top of the type `'a Atomic.Loc.t` we described in the previous section, and it relies on two new low-level primitives provided by the compiler:

```
val Atomic_array.index : 'a array -> int -> 'a Atomic.Loc.t
val Atomic_array.unsafe_index : 'a array -> int -> 'a Atomic.Loc.t
```

[9] Non-anonymous link
[10] Non-anonymous link

The function `index` takes an array and an integer index within the array, and returns an atomic location into the corresponding element after performing a bound check. `unsafe_index` omits the boundcheck—additional performance at the cost of memory-safety—and allows to express the atomic counterpart of the unsafe operations `Array.unsafe_get` and `Array.unsafe_set`. The atomic primitives of the module `Atomic.Loc` can then be used on these indices; our implementation implements a library module on top of these primitives to provide a higher-level layer to the user, with direct array operations such as:

```
val Atomic_array.exchange : 'a Atomic_array.t -> int -> 'a -> 'a
val Atomic_array.unsafe_exchange : 'a Atomic_array.t -> int -> 'a -> 'a
```

8 Conclusion and future work

The development of Zoo is still ongoing. While it is not yet available on `opam`, it can be installed and used in other Rocq projects. We provide a minimal example demonstrating its use.

Zoo supports a limited fragment of OCaml that is sufficient for most of our needs. Its main weakness so far is its memory model, which is sequentially consistent as opposed to the relaxed OCaml 5 memory model. It also lacks exceptions and algebraic effects, that we plan to introduce in the future.

Another interesting direction would be to combine Zoo with semi-automated techniques. Similarly to Why3, the simple parts of the verification effort would be done in a semi-automated way, while the most difficult parts would be conducted in Rocq.

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