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Zoo: A framework for the verification of concurrent OCAML 5 programs using separation logic

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The release of OCAML 5, which introduced parallelism into the language, drove the need for safe and efficient concurrent data structures. New libraries like Saturn aim at addressing this need. From the perspective of formal verification, this is an opportunity to apply and further state-of-the-art techniques to provide stronger guarantees.

We present Zoo, a framework for verifying fine-grained concurrent OCAML 5 algorithms. Following a pragmatic approach, we defined a limited but sufficient fragment of the language to faithfully express these algorithms: Zoolang. We formalized its semantics carefully via a deep embedding in the Rocq proof assistant, uncovering subtle aspects of physical equality. We provide a tool to translate source OCAML programs into Zoolang syntax inside Rocq, where they can be specified and verified using the Iris concurrent separation logic. To illustrate the applicability of Zoo, we verified a subset of the standard library and a collection of fined-grained concurrent data structures from the Saturn and Eio libraries.

In the process, we also extended OCAML to more efficiently express certain concurrent programs.

1 Introduction

OCaml 5.0 was released on December 15th 2022, the first version of the OCaml programming language to support parallel execution of OCaml threads by merging the Multicore OCaml runtime [Sivaramakrishnan, Dolan, White, Jaffer, Kelly, Sahoo, Parimala, Dhiman and Madhavapeddy 2020]. It provided basic support in the language runtime to start and stop coarse-grained threads ("domains" in OCaml parlance) and support for strongly sequential atomic references in the standard library. The third-party library domainslib offered a simple scheduler for a pool of tasks, used to benchmark the parallel runtime. A world of parallel and concurrent software was waiting to be invented.

Shared-memory concurrency is a difficult programming domain, and existing ecosystems (C++, Java, Haskell, Rust, Go...) took decades to evolve comprehensive libraries of concurrent abstractions and data structures. In the last couple years, a handful of contributors to the OCaml system have been implementing basic libraries for concurrent and parallel programs in OCAML, in particular Saturn [Karvonen and Morel 2025], a library of lock-free thread-safe data structures (stacks, queues, a work-stealing dequeue, a skip list, a bag and a hash table), Eio [Madhavapeddy and Leonard 2025], a library of asynchronous IO and structured concurrency, and Kcas [Karvonen 2025a], a library offering a software-transactional-memory abstraction for users to build safe yet efficient thread-safe data structures.

Concurrent algorithms and data structures are extremely difficult to reason about. Their implementations tend to be fairly short, a few dozens of lines. There is only a handful of experts able to write such code, and many potential users. They are difficult to test comprehensively. These characteristics make them ideally suited for mechanized program verification.

We embarked on a mission to mechanize correctness proofs of OCAML concurrent algorithms and data structures as they are being written, in contact with their authors, rather than years later. In the process, we not only gained confidence in these complex new building blocks, but we also improved the OCAML language and its verification ecosystem.

OCAML language features. When studying the new codebases of concurrent and parallel data structures, we found a variety of unsafe idioms, working around expressivity or performance limitations with the OCAML language support for lock-free concurrent data structures. In particular, the support for atomic references in the OCAML library proved inadequate, as idiomatic concurrent

1:2 Anon.

 data-structures need the more expressive feature of *atomic record fields*. We designed an extension of OCAML with atomic record fields, implemented it as a an experimental compiler variant, and succeeded in getting it integrated in the upstream OCAML compiler: it should be available as part of OCaml 5.4, which is not yet released at the time of writing.

Verification tools for concurrent programs. The state-of-the-art approach for mechanized verification of fine-grained concurrent algorithms is to use IRIS [Jung, Krebbers, Jourdan, Bizjak, Birkedal and Dreyer 2018], a state-of-the-art mechanized higher-order concurrent separation logic with user-defined ghost state. Its expressivity allows to precisely capture delicate invariants, and to reason about the linearization points of fine-grained concurrent algorithms (including external [Vindum, Frumin and Birkedal 2022] and future-dependent [Chang, Jung, Sharma, Tassarotti, Kaashoek and Zeldovich 2023; Jung, Lepigre, Parthasarathy, Rapoport, Timany, Dreyer and Jacobs 2020; Vindum and Birkedal 2021] linearization). Iris provides a generic mechanism to define programming languages and program logics for them. Much of the existing Iris concurrent verification work has been performed in Heaplang, the exemplar Iris language; a concurrent, imperative, untyped, call-by-value functional language.

To the best of our knowledge, it is currently the closest language to OCAML 5 in the IRIS ecosystem — we review the existing frameworks in Section 11. We started our verification effort in Heaplang, but it eventually proved impractical to verify realistic OCAML libraries. Indeed, it lacks basic abstractions such as algebraic data types (tuples, mutable and immutable records, variants) and mutually recursive functions. Verifying OCAML programs in Heaplang requires difficult translation choices and introduces various encodings, to the point that the relation between the source and verified programs can become difficult to maintain and reason about. It also has very few standard data structures that can be directly reused. This view, we believe, is shared by many people in the Iris community.

We created a new IRIS language, Zoolang, that can better express concurrent OCaml programs. Its feature set grew over time as we applied it to more verification scenarios, and we now believe that it allows practical verification of fine-grained concurrent OCaml 5 programs — including the use of our atomic record fields which were co-designed with Zoolang. We were influenced by the Perennial framework [Chajed, Tassarotti, Kaashoek and Zeldovich 2019], which achieved similar goals for the Go language with a focus on crash-safety. As in Perennial, we also provide a translator from (a subset of) OCaml to Zoolang: ocaml2zoo. We start from OCaml code and call our translator to obtain a deep Zoolang embedding inside Rocq; we can use lightweight annotations to guide the translation. Inside Rocq we define specifications using Iris, and prove them correct with respect to the Zoolang version, which is syntactically very close to the original OCaml source. We call the resulting framework Zoo.

One notable current limitation of Zoolang is that it assumes a sequentially-consistent memory model, whereas OCaml offers a weaker memory model [Dolan, Sivaramakrishnan and Madhavapeddy 2018]. We made the choice to ensure that we supported practical verification in a sequentially-consistent setting first; in the future we plan to equip Zoolang with the OCaml memory model as formalized in Cosmo [Mével, Jourdan and Pottier 2020]. We discuss the impact of this difference in ??.

Specified OCAML semantics. Our IRIS mechanization of ZooLang defines an operational semantics and a corresponding program logic. Our users on the other hand run their program through the standard OCAML implementation, which is not verified and does not have a precise formal specification. To bridge this formal-informal gap as well as reasonably possible, we carefully audit our ZooLang semantics to ensure that they coincide with OCAML's.

In doing so we discovered a hole in state-of-the-art language semantics for program verification (not just for OCAML), which is the treatment of *physical equality* (pointer quality). Physical equality is typically exposed to language users as an efficient but under-specified equality check, as the physical identity of objects may or may not be preserved by various compiler transformations. It is an essential aspect of concurrent programs, as it underlies the semantics of important atomic instructions such as compare_and_set. We found that the current informal semantics in OCAML is incomplete, it does not allow to reason on programs that use structured data which mix mutable and immutable constructors. Existing formalizations of physical equality in verification frameworks typically restrict it to primitive datatypes, but idiomatic concurrent programs do not fit within this restriction. We propose a precise specification of physical equality in Zoo that scales to the verification of all the concurrent programs we encountered.

Worse, our discussions with the maintainers of the OCaml implementation showed that implementors guarantee weaker properties of physical equalities than users assume, in particular they may allow *unsharing*, which makes some existing concurrent programs incorrect. We propose a small new language feature for OCaml, per-constructor unsharing control, which we also integrate in our Zoolang translation, to fix affected programs and verify them. Finally, we discussed these subtleties with authors who axiomatize physical equality within Rocq for the purpose of efficient extraction, and we found out that some subtleties we discovered could translate into incorrectness in their axiomatization, requiring careful restrictions.

Verification results. We verified a small library for Zoolang, typically a subset of the OCami standard library. It can serve as building blocks to define our concurrent data structures. (The lack of such a reusable standard library is a current limitation of Heaplang.) We verified a specific component of the Eio library, whose author Thomas Leonard had pointed to us as being delicate to reason about and worth mechanizing. Finally, we verified a large subset of the Saturn library: stacks, queues (list-based and stack-based), and finally the Chase-Lev work-stealing queue [Chase and Lev 2005]. The Saturn implementation of these lock-free data structures are used by the concurrent schedulers proposed in the OCami 5 library ecosystem, notably domainslib and picos. (The main Saturn concurrent structures missing from our verification are a skip-list and a hashtable.) Several of these data structures contained verification challenges, which we will describe in the relevant section.

Contributions. In summary, we claim the following contributions:

- (1) Zoo, a program verification framework aimed at practical verification of concurrent OCAML programs, mechanized in Rocq. The language ZooLang comes with a program logic expressed in the IRIS concurrent separation logic. A translator ocam12zoo generates the Rocq embedding from source OCAML programs, and works well with OCAML tooling (dune support).
- (2) The verification (in a sequentially-consistent model) of important structures coming from Saturn, the OCAML 5 library of lock-free data structures. In particular we present a precise concurrent invariant for the Chase-Lev work-stealing queue, which gives stronger specifications than its previous formalizations.

 Gabriel/It would be nice to emphasize here a "proof technique" for concurrent verification.
 - **Gabriel**{It would be nice to emphasize here a "proof technique" for concurrent verification that is novel in Clément's work. (Some novel usage setup for prophecy variables?)}
- (3) The extension of OCAML with atomic record fields, which after significant design, implementation and discussion work have now been integrated into upstream OCAML.

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(4) The identification of blind spots in existing specifications of *physical equality* and a new specification that is precise enough to reason about compare-and-set in the various programs we considered.

In the process we identified a potential bug in existing OCAML programs related to *unsharing*, and we propose a small language extension to let users selectively disable unsharing.

2 Zoo in practice

2.1 Language

 The core of Zoo is ZooLang: a concurrent, imperative, untyped, functional programming language fully formalized in Rocq. Its semantics has been designed to match OCami's.

Zoolang comes with a program logic based on IRIS: reasoning rules expressed in separation logic (including rules for the different constructs of the language) along with Rocq tactics that integrate into the IRIS proof mode [Krebbers, Jourdan, Jung, Tassarotti, Kaiser, Timany, Charguéraud and Dreyer 2018]. In addition, it supports DIAFRAME [Mulder, Krebbers and Geuvers 2022], enabling proof automation.

The ZooLang syntax is given in Figure 1¹, omitting mutually recursive toplevel functions that are treated specially. Expressions include standard constructs like booleans, integers, anonymous functions (that may be recursive), applications, **let** bindings, sequence, unary and binary operators, conditionals, **for** loops, tuples. In any expression, one can refer to a Rocq term representing a ZooLang value (of type val) using its Rocq identifier. ZooLang is deeply embedded: variables (bound by functions and **let**) are quoted as strings.

Data constructors (immutable memory blocks) are supported: C represents a constant constructor (e.g. §None), and 'C (e_1 , ..., e_n) represents a non-constant constructor (e.g. 'Some(e)). Projections of the form $e \cdot proj$ (include on tuples: $(x, y) \cdot 1$) can be used to obtain a specific component of a tuple or data constructor. ZooLANG supports shallow pattern matching (patterns cannot be nested) on data constructors with an optional fallback case.

Mutable memory blocks are constructed using either the untagged record syntax $\{e_1, \ldots, e_n\}$ or the tagged record syntax ' $C\{e_1, \ldots, e_n\}$. Reading a record field can be performed using e. $\{fld\}$ and writing to a record field using $e_1 < -\{fld\} e_2$. Pattern matching can also be used on mutable tagged blocks provided that cases do not bind anything—in other words, only the tag is examined, no memory access is performed. Mutable references are supported: ref e creates a reference, !e reads a reference and $e_1 < -e_2$ writes into a reference. There is no built-in syntax for arrays, hey are supported through the **Array** standard library module (e.g. array_make).

Note that Zoolang follows OCaml in sometimes eschewing orthogonality to provide more compact memory representations: constructors are *n*-ary instead of taking a tuple as parameter, and the tagged record syntax is distinct from a constructor taking a mutable record as parameter. In each case the simplifying encoding would introduce an extra indirection in memory, which is absent from the Zoolang semantics. Performance-conscious experts care about these representation choices, and we care about faithfully modeling their programs.

Parallelism is mainly supported through the **Domain** standard library module, including domain-local storage. Atomic operations are provided as built-in constructs (Xchg, CAS, FAA; see Section 3.4).

The Proph and Resolve constructs model *prophecy variables* [Jung, Lepigre, Parthasarathy, Rapoport, Timany, Dreyer and Jacobs 2020], see Section 3.5.

2.2 Translation from OCAML to ZOOLANG

¹More precisely, it is the syntax of the surface language, including Roco notations.

```
Roco term
                                                                                                                t
197
                                                                                                                C
                                        constructor
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                                        projection
                                                                                                                proj
                                        record field
                                                                                                                fld
200
                                        identifier
                                                                                                                s, f
                                                                                                                                                             String
                                                                                                                                            \in
                                        integer
                                                                                                                                             \in
                                                                                                                                                             \mathbb{Z}
                                                                                                                n
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                                        boolean
                                                                                                                b
                                                                                                                                            \in
                                                                                                                                                             \mathbb{B}
                                                                                                                                          ::=
                                        binder
                                                                                                                                                           <> | s
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                                                                                                                x
                                                                                                                                         ::=
                                                                                                                                                             ~ | -
                                        unary operator
                                                                                                                \oplus
                                                                                                                                                             + | - | * | 'quot' | 'rem' | 'land' | 'lor' | 'lsl' | 'lsr'
                                        binary operator
                                                                                                                                          ::=
206
                                                                                                                                                             <= | < | >= | > | == | !=
                                                                                                                                                             and | or
                                        expression
                                                                                                                                          := t \mid s \mid \#n \mid \#b
                                                                                                                                                             fun: x_1 ... x_n \Rightarrow e \mid \text{rec}: f x_1 ... x_n \Rightarrow e \mid e_1 e_2
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                                                                                                                                                             let: x := e_1 \text{ in } e_2 \mid e_1;; e_2
211
                                                                                                                                                             let: f x_1 ... x_n := e_1 \text{ in } e_2 \mid \text{letrec: } f x_1 ... x_n := e_1 \text{ in } e_2
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                                                                                                                                                             let: 'C x_1 ... x_n := e_1 \text{ in } e_2 \mid \text{let: } x_1, ..., x_n := e_1 \text{ in } e_2
                                                                                                                                                             \oplus e \mid e_1 \otimes e_2
                                                                                                                                                             if: e_0 then e_1 (else e_2)?
                                                                                                                                                              for: x := e_1 to e_2 begin e_3 end
                                                                                                                                                              SC \mid C(e_1, \ldots, e_n) \mid (e_1, \ldots, e_n) \mid e < proj >
                                                                                                                                                              [] | e_1 :: e_2
                                                                                                                                                               C\{e_1,\ldots,e_n\} \mid \{e_1,\ldots,e_n\} \mid e.\{fld\} \mid e_1 < -\{fld\} \mid e_2 < -\{fl
                                                                                                                                                              ref e \mid !e \mid e_1 < -e_2
                                                                                                                                                             match: e_0 with br_1 | \dots | br_n (| (as s)^? \Rightarrow e)^? end
                                                                                                                                                             e \cdot [fld] \mid Xchg e_1 e_2 \mid CAS e_1 e_2 e_3 \mid FAA e_1 e_2
                                                                                                                                                             Proph | Resolve e_0 e_1 e_2
                                                                                                                                                        C(x_1...x_n)^? (as s)^? => e
                                         branch
224
                                                                                                                                                             [] (as s)^? => e | x_1 :: x_2 (as s)^? => e
                                                                                                                                            := t \mid \#n \mid \#b
                                         toplevel value
                                                                                                                                                             fun: x_1 \dots x_n \Rightarrow e \mid \text{rec} : f x_1 \dots x_n \Rightarrow e
                                                                                                                                                              SC \mid C(v_1, \ldots, v_n) \mid (v_1, \ldots, v_n)
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                                                                                                                                                              [] | v_1 :: v_2
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```

Fig. 1. Zoolang syntax (omitting mutually recursive toplevel functions)

While Zoolang lives in Rocq, we want to verify OCaml programs. To connect them we provide the tool ocaml2zoo to translate OCaml source files into Rocq files containing Zoolang code. This tool can process entire dune projects, and support several libraries provided together or as dependencies of the project.

The supported OCAML fragment includes: tuples, variants, records and inline records, shallow match, atomic record fields, unboxed types, toplevel mutually recursive functions.

In Figure 2 we include the OCAML implementation of a simple lock-free concurrent stack [Treiber 1986], and its automatic translation to ZooLang, demonstrating that readability is preserved.

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²Actually, ocaml2zoo processes binary annotation files (.cmt files).

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```
type 'a t = 'a list Atomic.t
                                   Definition stack_create : val :=
let create () = Atomic.make []
                                     fun: <> => ref [].
let rec push t v =
                                   Definition stack_push : val :=
                                     rec: "push" "t" "v" =>
  let old = Atomic.get t in
                                       let: "old" := !"t" in
  let new_ = v :: old in
                                       let: "new_" := "v" :: "old" in
  if not (Atomic.compare_and_set
                                       if: ~ CAS "t".[contents] "old" "new_"
            t old new_)
                                       then (
  then (
    Domain.cpu_relax () ;
                                         domain_cpu_relax () ;;
                                         "push" "t" "v"
    push t v
  )
                                   Definition stack_pop : val :=
let rec pop t =
                                     rec: "pop" "t" =>
  match Atomic.get t with
                                       match: !"t" with
  | [] -> None
  | v :: new_ as old ->
                                       | [] => §None
      if Atomic.compare_and_set
                                       | ("v" :: "new_") as "old" =>
                                           if: CAS "t".[contents] "old" "new_"
           t old new_
                                           then 'Some( "v" )
      then Some v
      else (
                                           else (
        Domain.cpu_relax () ;
                                             domain_yield () ;;
                                             "pop" "t"
        pop t
      )
                                       end.
```

Fig. 2. A concurrent stack in OCAML and its automatic ZooLang translation

2.3 Specifications and proofs

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293 294 Once the translation to ZooLang is done, the user can write specifications and prove them in Iris. For instance, the specification of the stack_push function could be:

```
Lemma stack_push_spec t \iota v : 
 <<< stack_inv t \iota | \forall \forall vs, stack_model t vs >>> 
   stack_push t v @ \uparrow \iota 
 <<< stack_model t (v :: vs) 
  | RET (); True >>>. 
 Proof. ... Qed.
```

Here, we use a *logically atomic specification* [da Rocha Pinto, Dinsdale-Young and Gardner 2014], which has been proven [Birkedal, Dinsdale-Young, Guéneau, Jaber, Svendsen and Tzevelekos 2021] to be equivalent to *linearizability* [Herlihy and Wing 1990] in sequentially consistent memory models.

Similarly to Hoare triples, the specification is formed of a precondition and a postcondition, represented in angle brackets. But each is split in two parts, a *public* or *atomic* condition, and a *private* condition. Following standard IRIS notations, the private conditions are on the outside (first line of the precondition, last line of the postcondition) and the atomic conditions are inside.

For this particular operation, the private postcondition is trivial. The private precondition $stack_inv t$ is the stack invariant. Intuitively, it asserts that t is a valid concurrent stack. More precisely, it defines a concurrent protocol that t must respect at all times.

The atomic pre- and post-conditions specify the linearization point of the operation: during the execution of $stack_push$, the abstract state of the stack held by $stack_model$ is atomically updated from vs to v: vs when v is atomically pushed at the top of the stack.

3 Zoo features

 In this section, we review the salient features of Zoo, which we found lacking when we attempted to use HeapLang to verify real-world OCaml programs. Providing a better Iris language than HeapLang for this problem domain is a contribution of our work; others are welcome to use Zoolang for their own verification effort, or to reuse our designs by integrating specific features into their own Iris language.

3.1 Algebraic data types

ZooLang is an untyped language but, to write interesting programs, it is convenient to work with abstractions like algebraic data types. To simulate tuples, variants and records, we designed a machinery to define projections, constructors and record fields.

For example, one may define a list-like type with:

```
Notation "'Nil'" := (in_type "t" 0) (in custom zoo_tag).
Notation "'Cons'" := (in_type "t" 1) (in custom zoo_tag).
```

Users do not need to write this incantation directly, as they are generated by ocam12zoo from the OCAML type declarations. Suffice it to say that it introduces the two tags in the zoo_tag custom entry, on which the notations for data constructors rely. The in_type term is needed to distinguish the tags of distinct data types; crucially, it cannot be simplified away by Rocq, as this could lead to confusion during the reduction of expressions.

One may then directly use the tags Nil and Cons in data constructors:

Similarly, one can define a record-like type with two mutable fields f1 and f2:

```
Notation "'f1" := (in_type "t" 0) (in custom zoo_field).

Notation "'f2'" := (in_type "t" 1) (in custom zoo_field).

Definition swap : val :=
fun: "t" =>
let: "f1" := "t".{f1} in
"t" <-{f1} "t".{f2} ;; "t" <-{f2} "f1".
```

3.2 Mutually recursive functions

Zoo supports non-recursive (fun: $x_1 ldots x_n => e$) and recursive (rec: $f(x_1 ldots x_n => e)$) functions but only *toplevel* mutually recursive functions. It is non-trivial to properly handle mutual recursion:

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when applying a mutually recursive function, a naive approach would replace calls to sibling functions by their respective bodies, but this typically makes the resulting expression unreadable and can create proof-checking performance issues during verification. To prevent it, the mutually recursive functions have to know one another to preserve their names during β -reduction. We simulate this using some boilerplate that can be generated by ocaml2zoo. For instance, one may define two mutually recursive functions f and g as follows:

```
Definition f_g := (
  recs: "f" "x" => "g" "x"
  and: "g" "x" => "f" "x"
)%zoo_recs.

(* boilerplate *)
Definition f := ValRecs 0 f_g.
Definition g := ValRecs 1 f_g.
Instance : AsValRecs' f 0 f_g [f;g]. Proof. done. Qed.
Instance : AsValRecs' g 1 f_g [f;g]. Proof. done. Qed.
```

3.3 Standard library

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391 392 To save users from reinventing the wheel, we provide a standard library—more or less a subset of the OCAML standard library. Currently, it mainly includes standard data sequential structures like: array (Array), resizable array (Dynarray), list (List), stack (Stack), queue (Queue), double-ended queue. It also includes mutexes (Mutex) and condition variable (Condition).

Each of these standard modules contains ZooLang functions and their verified specifications. These specifications are modular: they have been used to verify more complex data structures.

3.4 Fine-grained concurrent primitives

Zoo supports concurrent primitives both on atomic references (from **Atomic**) and atomic record fields (from **Atomic**. Loc³) according to the table below. The OCAML expressions listed in the left-hand column translate into the Zoo expressions in the right-hand column. Notice that an atomic location [%atomic.loc e.f] (of type _ **Atomic.Loc.**t) translates directly into e.[f].

```
OCAML
                                                              Zoo
                                                              ! e
Atomic.get e
Atomic. set e_1 e_2
                                                              e_1 < -e_2
                                                             Xchg e_1. [contents] e_2
Atomic.exchange e_1 e_2
                                                             CAS e_1. [contents] e_2 e_3
Atomic.compare_and_set e_1 e_2 e_3
                                                             FAA e_1.[contents] e_2
Atomic.fetch_and_add e_1 e_2
                                                             Xchg e_1.[f] e_2
Atomic.Loc. exchange [%atomic.loc e_1.f] e_2
Atomic.Loc.compare_and_set [%atomic.loc e_1.f] e_2 e_3
                                                             CAS e_1. [f] e_2 e_3
Atomic.Loc. fetch_and_add [%atomic.loc e_1.f] e_2
                                                             FAA e_1. [f] e_2
```

One notable aspect of this translation is that atomic accesses (Atomic.get and Atomic.set) correspond to plain loads and stores. This is because we are working in a sequentially consistent memory model: there is no difference between atomic and non-atomic memory locations.

3.5 Prophecy variables

Lock-free algorithms exhibit complex behaviors. To tackle them, IRIS provides powerful mechanisms such as *prophecy variables* [Jung, Lepigre, Parthasarathy, Rapoport, Timany, Dreyer and Jacobs

³The **Atomic.Loc** module is part of the PR that implements atomic record fields (see Section 4).

2020]. Essentially, prophecy variables can be used to predict the future of the program execution and reason about it. They are key to handle *future-dependent linearization points*: linearization points that may or may not occur at a given location in the code depending on a future observation.

Zoo supports prophecy variables through the Proph and Resolve expressions—as in Heaplang, the canonical Iris language. In OCAML, these expressions correspond to **Zoo**. proph and **Zoo**. resolve, that are recognized by ocaml2zoo.

4 OCAML extensions for fine-grained concurrent programming

Over the course of this work, we studied efficient fine-grained concurrent OCAML programs written by experts. This revealed various limitations of OCAML in these domains, that those experts would work around using unsafe casts, often at the cost of both readability and memory-safety; and also some mismatches between their mental model of the semantics of OCAML and the mental model used by the OCAML compiler authors. We worked on improving OCAML itself to reduce these work-arounds or semantic mismatches.

4.1 Atomic record fields

 OCAML 5 offers a type 'a **Atomic**.t of atomic references exposing sequentially-consistent atomic operations. Data races on non-atomic mutable locations has a much weaker semantics and is generally considered a programming error. For example, the Michael-Scott concurrent queue [Michael and Scott 1996] relies on a linked list structure that could be defined as follows:

```
type 'a node = Nil | Cons of { value : 'a; next : 'a node Atomic.t }
```

Performance-minded concurrency experts dislike this representation, because 'a **Atomic**.t introduces an indirection in memory: it is represented as a pointer to a block containing the value of type 'a. Instead, they use something like the following:

Notice that the next field of the **Cons** constructor has been moved first in the type declaration. Because the OCAML compiler respects field-declaration order in data layout, a value **Cons** { next; value } has a similar low-level representation to a reference (atomic or not) pointing at next, with an extra argument. The code uses **Obj**. magic to unsafely cast this value to an atomic reference, which appears to work as intended.

Obj. magic is a shunned unsafe cast (the OCAML equivalent of unsafe or unsafePerformIO). It is very difficult to be confident about its usage given that it may typically violate assumptions made by the OCAML compiler and optimizer. In the example above, casting a two-fields record into a one-argument atomic reference may or may not be sound—but it gives measurable performance improvements on concurrent queue benchmarks.

It is possible to statically forbid passing **Nil** to as_atomic to avoid error handling, by turning 'a node into a GADT indexed over a type-level representation of its head constructor. Examples of this pattern can be found in the Kcas [Karvonen 2025a] library by Vesa Karvonen. It is difficult to write correctly and use, in particular as unsafe casts can sometimes hide type-errors in the intended static discipline.

Note that this unsafe approach only works for the first field of a record, so it is not applicable to records that hold several atomic fields, such as the toplevel record storing atomic front and back pointers for the concurrent queue.

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4.1.1 Our atomic fields proposal. In May-June 2024 we proposed a design for atomic record fields as an OCAML language change proposal: RFC #39⁴. Declaring a record field atomic simply requires an <code>[@atomic]</code> attribute—and could eventually become a proper keyword of the language.

```
(* re-implementation of atomic references *)
type 'a atomic_ref = { mutable contents : 'a [@atomic]; }

(* concurrent linked list *)
type 'a node = Nil | Cons of { value: 'a; mutable next : 'a node [@atomic]; }

(* bounded SPSC circular buffer *)
type 'a bag =
    { data : 'a Atomic.t array;
    mutable front: int [@atomic];
    mutable back: int [@atomic]; }
```

The design difficulty is to express atomic operations on atomic record fields. For example, if buf has type 'a bag above, then one naturally expects the existing notation buf.front to perform an atomic read and buf.front <- n to perform an atomic write. But how would one express exchange, compare-and-set and fetch-and-add? We would like to avoid adding a new primitive language construct for each atomic operation.

Two different designs have been proposed for atomic operations on atomic record fields.

(1) The original, "full" design in our proposal introduces a built-in type ('r, 'a) Atomic.Field.t that denotes a field/index of type 'a within a record of type 'r, with a syntax extension [%atomic.loc <field>] to construct such field description, and atomic primitives in a module Atomic.Field, that need both the record value of type 'r and the field description. For example, the standard library exposes:

```
val Atomic.Field.fetch_and_add :
    'a -> ('a, int) Atomic.Field.t -> int -> int
and users can write:
let preincrement_front (buf : 'a bag) : int =
    Atomic.Loc.fetch_and_add buf [%atomic.field front] 1
where [%atomic.field front] has type ('a bag, int) Atomic.Loc.t. Internally, a value of type ('r, 'a) Atomic.Field.t is just an integer offset locating the field within the record.
```

(2) An alternative "simple" design, which was proposed by Basile Clément, introduces a built-in type 'a Atomic.Loc.t for an atomic location that holds an element of type 'a, with a syntax extension [%atomic.loc <expr>.<field>] to construct such locations. Atomic primitives operate on values of type 'a Atomic.Loc.t and they are exposed as functions of the module Atomic.Loc. For example, the standard library exposes:

```
val Atomic.Loc.fetch_and_add :
   int Atomic.Loc.t -> int -> int
and users can write:
let preincrement_front (buf : 'a bag) : int =
   Atomic.Loc.fetch_and_add [%atomic.loc buf.front] 1
where [%atomic.loc buf.front] has type int Atomic.Loc.t. Internally, a value of type
'a Atomic.Loc.t can be represented as a pair of a record and an integer offset for the
desired field, and the atomic.loc construction builds this pair in a well-typed manner.
```

⁴De-anonymizing link: https://github.com/ocaml/RFCs/pull/39

 When a primitive of the **Atomic.Loc** module is applied to an atomic.loc expression, the compiler can optimize away the construction of the pair—but it would happen if there was an abstraction barrier between the construction and its use.

The simple design has the strong advantage of exposing simpler types to users. The full design involves more complex types, but a simpler data representation that does not rely on specific compiler optimizations to generate efficient code, even across abstraction boundaries. As remarked by Leo White, the simpler type 'a **Atomic.Loc.**t can be reconstructed as a dependent pair of a 'r and a ('r, 'a) **Atomic.Field.**t, which is expressible in OCAML as a GADT:

```
type 'a loc = Loc : 'r * ('r, 'a) Atomic.Field.t -> 'a loc
```

In August and September 2024 we implemented the simple design as an experimental version of the OCAML compiler. We got a full code review by Olivier Nicole. When we approached the OCAML maintainers to integrate the feature upstream, they asked us to reconsider implementing the full design, based on feedback in this direction by Vesa Karvonen (author of Kcas and Saturn).

We implemented the full design between November 2024 and January 2025. We found that it is harder to implement in the type-checker. The extension form [%atomic.loc buf.front] of the simple design has typing rules that are very similar to a field access buf.front. On the other hand, [%atomic.loc front] interacts in a non-trivial way with the OCAML machinery for type-based disambiguation of record fields—several records with a field named front can co-exist in the typing environment.

For technical reasons, there is also a non-trivial interaction with the type-checking of inline record types (record types that are not defined by themselves but only as the argument of a sum type constructor), which currently prevents from using this approach with inline records, who are unfortunately common in efficient mutable data structures. In February and March 2025 we worked with OCAML maintainers experts to try to lift these limitations and proposed implementation changes for inline records, but we failed to reach a consensus on the best design.

In April and May, we managed to build consensus among OCAML maintainers that the simple design, with a simpler user-facing design and a simpler implementation, was a good compromise in view of the significant difficulties of the full design with inline records. Our implementation of the simple design was reviewed again, we wrote user documentation, and it was integrated upstream in May 2025, to be included in the upcoming release of OCaml 5.4.

Limitation: no support for cache contention. The type **Atomic**. t comes with a function

```
val Atomic.make_contended : 'a -> 'a Atomic.t
```

that ensures that the returned atomic reference is allocated with enough alignment and padding to sit alone on its cache line, to avoid performance issues caused by false sharing. Currently there is no such support for padding of atomic record fields (we are planning to work on this if the support for atomic fields gets merged in standard OCAML), so the less-compact atomic references remain preferable in certain scenarios.

4.2 Atomic arrays

On top of our atomic record fields, we have implemented experimental support for atomic arrays, another facility commonly requested by authors of efficient concurrent programs. Our previous example of a concurrent bag of type 'a bag used a backing array of type 'a **Atomic.t array**, which contains more indirections than may be desirable, as each array element is a pointer to a block containing the value of type 'a, instead of storing the value of type 'a directly in the array.

1:12 Anon.

Our implementation of atomic arrays⁵ builds on top of the type 'a **Atomic.Loc.**t we described in the previous section, and it relies on two new low-level primitives provided by the compiler:

```
val Atomic_array.index : 'a array -> int -> 'a Atomic.Loc.t
val Atomic_array.unsafe_index : 'a array -> int -> 'a Atomic.Loc.t
```

The function index takes an array and an integer index within the array, and returns an atomic location into the corresponding element after performing a bound check. unsafe_index omits the boundcheck—additional performance at the cost of memory-safety—and allows to express the atomic counterpart of the unsafe operations <code>Array.unsafe_get</code> and <code>Array.unsafe_set</code>. The atomic primitives of the module <code>Atomic.Loc</code> can then be used on these indices; our implementation implements a library module on top of these primitives to provide a higher-level layer to the user, with direct array operations such as:

```
val Atomic_array.exchange : 'a Atomic_array.t -> int -> 'a -> 'a
val Atomic_array.unsafe_exchange : 'a Atomic_array.t -> int -> 'a -> 'a
```

5 Physical equality

 The notion of *physical equality* is ubiquitous in fine-grained concurrent algorithms. It appears not only in the semantics of the == operator, but also in the semantics of the **Atomic**. compare_and_set primitive, which atomically sets an atomic reference to a desired value if its current content is physically equal to an expected value. This primitive is commonly used to try committing an atomic operation in a retry loop, as in the push and pop functions of Figure 2.

5.1 Physical equality in HEAPLANG

In HEAPLANG, this primitive is provided but restricted. Indeed, its semantics is only defined if either the expected or the desired value fits in a single memory word in the HEAPLANG value representation: literals (booleans, integers and pointers⁶) and literal injections⁷; otherwise, the program is stuck. In practice, this restriction forces the programmer to introduce an indirection [Iris development team 2025; Jung, Lepigre, Parthasarathy, Rapoport, Timany, Dreyer and Jacobs 2020; Vindum and Birkedal 2021] to physically compare complex values, *e.g.* lists. Furthermore, when the semantics is defined, values are compared using their Rocq representations; physical equality boils down to Rocq equality.

5.2 Physical equality in OCAML

In OCAML, physical equality is more tricky and often considered dangerous. *Structural equality*, which we describe in Section 6, should be the preferred way of comparing values. However, physical equality is typically much faster than structural equality, as it basically compiles to only one assembly instruction. Also, the **Atomic**. compare_and_set requires the comparison to be atomic, which is the case for physical equality but not structural equality.

In particular, the semantics of physical equality is *non-deterministic*. To see why, consider the case of *immutable blocks* representing constructors and immutable records (as opposed to *mutable blocks* representing mutable records), *e.g.* **Some** 0. The physical comparison of two seemingly identical immutable blocks, according to the Rocq representation (essentially a tag and a list of fields), may return false. Indeed, at runtime, a non-empty immutable block is represented by a pointer to a tagged memory block. In this case, physical equality is just pointer comparison. It is clear that two

⁵Non-anonymous link: https://<clickable>

⁶HEAPLANG allows arbitrary pointer arithmetic and therefore inner pointers. This is forbidden in both OCAML and ZOOLANG, as any reachable value has to be compatible with the garbage collector.

⁷HEAPLANG has no primitive notion of constructor, only pairs and injections (left and right).

pointers being distinct does not imply the pointed memory blocks are. In other words, we cannot determine the result of physical comparison just by looking at the abstract values.

The question is then: what guarantees do we get when physical equality returns true and when it returns false? Given such guarantees, denoted by val_physeq and val_physneq, the non-deterministic semantics is reflected in the logic through the following specification:

```
Lemma physeq_spec v1 v2 :
    {{{ True }}}
    v1 == v2
    {{{ b, RET #b; 「(if b then val_physeq else val_physneq) v1 v2¬ }}}
Proof. ... Qed.
```

The OCAML manual documents a partial specification for physical equality, which is precise for basic types such as references, but does not clearly extend to structured values containing a mix of immutable and mutable constructors. The only guarantee that it provides for all values is: if two values are physically equal, they are also structurally equal. This means we don't learn anything when two values are physically distinct.

In the following, we will explore both cases, looking at the optimizations that the compiler or the runtime system may perform. We will show that the aforementioned guarantee is arguably not sufficient to verify interesting concurrent programs and attempt to establish stronger guarantees.

5.3 When physical equality returns true

 Let us go back to the concurrent stack of Figure 2 and more specifically the push function. To prove the atomic specification given in Section 2, we rely on the fact that, if **Atomic**.compare_and_set returns true, we actually observe the same list of values in the sense of Rocq equality. However, assuming only structural equality as per OCAML's specification of physical equality, this cannot be proven. To see why, consider, *e.g.*, a stack of references ('a ref). As structural equality is indeed *structural*, it traverses the references without comparing their *physical identities*. In other words, we cannot conclude the references are *exactly* the same. Hence, we cannot prove the specification.

This conclusion might seem surprising and counterintuitive. Indeed, we know that physical equality essentially boils down to a comparison instruction, so we should be able to say more. Departing from OCAML's imprecise specification, let us attempt to establish stronger guarantees. We assume the following classification of values: booleans, integers, mutable blocks (pointers), immutable blocks, functions.

The easy cases are mutable blocks and functions. Each of these two classes is disjoint from the others. We can reasonably assume that, when physical equality returns true and one of the compared values belongs to either of these classes, the two values are actually the same in Rocq. As far as we are aware, there is no optimization that could break this.

Booleans, integers and empty immutable blocks are represented by immediate integers through an encoding. This encoding induces conflicts: two seemingly distinct values in Rocq may have the same encoding. For example, the following tests all return true (**Obj**.repr is an unsafe primitive revealing the memory representation of a value):

```
let test1 = Obj.repr false == Obj.repr 0 (* true *)
let test2 = Obj.repr None == Obj.repr 0 (* true *)
let test3 = Obj.repr [] == Obj.repr 0 (* true *)
```

The semantics of unrestricted physical equality has to reflect these conflicts. In our experience, restricting compared values similarly to typing is quite burdensome; the specification of polymorphic data structures using physical equality has to be systematically restricted. In summary,

1:14 Anon.

when physical equality on immediate values returns true, it is guaranteed that they have the same encoding.

Finally, let us consider the case of non-empty immutable blocks. At runtime, they are represented by pointers to tagged memory blocks. At first approximation, it is tempting to say that physically equal immutable blocks really are definitionally equal in ROCQ. Alas, this is not true. To explain why, we have to recall that the OCAML compiler and the runtime system (e.g., through hash-consing) may perform *sharing*: immutable blocks containing physically equal fields may be shared. For example, the following tests may return true:

```
let test1 = Some 0 == Some 0 (* true *)
let test2 = [0;1] == [0;1] (* true *)
```

 On its own, sharing is not a problem. However, coupled with representation conflicts, it can be surprising. Indeed, consider the any type defined as:

```
type any = Any : 'a -> any
The following tests may return true:
let test1 = Any false == Any 0 (* true *)
let test2 = Any None == Any 0 (* true *)
let test3 = Any [] == Any 0 (* true *)
```

Now, going back to the push function of Figure 2, we have a problem. Given a stack of any, it is possible for the **Atomic**.compare_and_set to observe a current list (e.g., [Any 0]) physically equal to the expected list (e.g., [Any false]) while these are actually distinct in Rocq. In short, the expected specification of Section 2 is incorrect. To fix it, we would need to reason *modulo physical equality*, which is non-standard and quite burdensome.

We believe this really is a shortcoming, at least from the verification perspective. Therefore, we propose to extend OCAML with *generative immutable blocks*⁸. These generative blocks are just like regular immutable blocks, except they cannot be shared. Hence, if physical equality on two generative blocks returns true, these blocks are definitionally equal in Rocq. At user level, this notion is materialized by *generative constructors*. For instance, to verify the expected push specification, we can use a generative version of lists:

```
type 'a list =
    | Nil
    | Cons of 'a * 'a list [@generative]
```

5.4 When physical equality returns false

Most formalizations of physical equality in the literature do not give any guarantee when physical equality returns false. Many use-cases of physical equality, in particular retry loops, can be verified with only sufficient conditions on true. However, in some specific cases, more information is needed.

Consider the **Rcfd** module from the Eio [Madhavapeddy and Leonard 2025] library, an excerpt of which is given in Figure 3⁹. Thomas Leonard, its author, suggested that we verify this real-life example because of its intricate logical state. However, we found out that it is also relevant regarding the semantics of physical equality. Essentially, it consists in wrapping a file descriptor in a thread-safe way using reference-counting. At creation in the make function, the wrapper starts in the **Open** state. At some point, it can switch to the **Closing** state in the close function and can

⁸Non-anonymous link: https://<clickable>

⁹We make use of atomic record fields as introduced in Section 4.1.

```
type state =
687
       | Open of Unix.file_descr
688
       | Closing of (unit -> unit)
689
690
     type t =
691
       { mutable ops: int [@atomic];
692
         mutable state: state [@atomic]; }
694
     let make fd = { ops = 0; state = Open fd }
695
696
     let closed = Closing (fun () -> ())
     let close t =
       match t.state with
       | Closing _ -> false
       | Open fd as prev ->
           let next = Closing (fun () -> Unix.close fd) in
           if Atomic.Loc.compare_and_set [%atomic.loc t.state] prev next then (
              if t.ops == 0
              && Atomic.Loc.compare_and_set [%atomic.loc t.state] next closed
              then close ();
              true
           ) else false
```

Fig. 3. Rcfd module from Eio [Madhavapeddy and Leonard 2025] (excerpt)

never go back to the **Open** state. Crucially, the **Open** state does not change throughout the lifetime of the data structure.

The interest of **Rcfd** lies in the close function. First, the function reads the state. If this state is **Closing**, it returns false; the wrapper has been closed. If this state is **Open**, it tries to switch to the **Closing** state using **Atomic.Loc.** compare_and_set; if this attempt fails, it also returns false. In this particular case, we would like to prove that the wrapper has been closed, or equivalently that **Atomic.Loc.** compare_and_set cannot have observed **Open**. Intuitively, this is true because there is only one **Open**.

Obviously, we need some kind of guarantee related to the *physical identity* of **Open** when **Atomic.Loc.** compare_and_set returns false. If **Open** were a mutable block, we could argue that this block cannot be physically distinct from itself; no optimization we know of would allow that. Unfortunately, it is an immutable block, and immutable blocks are subject to more optimizations. In fact, something surprising but allowed¹⁰ by OCAML can happen: *unsharing*, the dual of sharing. Indeed, any immutable block can be unshared, that is reallocated. For example, the following test may theoretically return false:

```
let x = Some 0
let test = x == x (* false *)
```

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Going back to **Rcfd**, we have a problem: in the second branch, the **Open** block corresponding to prev could be unshared, which would make **Atomic.Loc.**compare_and_set fail. Hence, we cannot prove the expected specification; in fact, the program as it is written has a bug.

¹⁰This has been confirmed by OCAML experts developing the FLAMBDA backend.

1:16 Anon.

To remedy this unfortunate situation, we propose to reuse the notions of generative immutable blocks, that we introduced to prevent sharing, to also forbid unsharing by the OCAML compiler – we implemented this in an experiment branch of OCAML.

In our semantics, each generative block is annotated with a *logical identifier*¹¹ representing its physical identity, much like a pointer for a mutable block. If physical equality on two generative blocks returns false, the two identifiers are necessarily distinct. Given this semantics, we can verify the close function. Indeed, if **Atomic.Loc.** compare_and_set fails, we now know that the identifiers of the two blocks, if any, are distinct. As there is only one **Open** block whose identifier does not change, it cannot be the case that the current state is **Open**, hence it is **Closing**. We can verify this function after adding the following annotation:

5.5 Summary

 In summary, we give the following specification to physical equality in Zoolang, which also serves as a precise specification of physical equality of a practical fragment of OCAML:

- On values whose low-level representation is an immediate integer, physical equality is immediate equality.
- On values whose low-level representation are mutable blocks at some location, or generative immutable blocks with some identity, physical equality is equality of locations or identities.
- On values whose low-level representation are immutable blocks, physical-equality is underspecified, but it implies that the blocks have the same tags and their arguments are in turn physically equal.
- Two values that fall into different categories above are never physically equal.

6 Structural equality

Structural equality is also supported. More precisely, it is not part of the semantics of the language but axiomatized on top of it¹². The reason is that it is in fact difficult to specify for arbitrary values. In general, we have to compare graphs—which implies structural comparison may diverge.

Accordingly, the specification of $v_1 = v_2$ requires the (partial) ownership of a *memory footprint* corresponding to the union of the two compared graphs, giving the permission to traverse them safely. If it terminates, the comparison decides whether the two graphs are isomorphic (modulo representation conflicts, as described in Section 5). In IRIS, this gives:

Obviously, this general specification is not very convenient to work with. Fortunately, for abstract values (without any mutable part), we can prove a much simpler variant saying that structural equality boils down to physical equality:

¹¹Actually, for practical reasons, we distinguish identified and unidentified generative blocks.

 $^{^{12}}$ We could also have implemented it in ZooLang, but that would require more low-level primitives.

```
785     Lemma structeq_spec_abstract v1 v2 :
786     val_abstract v1 →
787     val_abstract v2 →
788     {{{ True }}}
789     v1 = v2
790     {{{ b, RET #b; 「(if b then val_physeq else val_physneq) v1 v2¬ }}}
791     Proof. ... Qed.
```

7 Standard data structures

To save users from reinventing the wheel, we provide a library of verified standard data structures — more or less a subset of the OCAML standard library. Most of these data structures¹³ are completely reimplemented in Zoo and axiom-free, including the **Array**¹⁴ module.

Sequential data structures. We provide verified implementations of various sequential data structures: array, dynamic array (vector), list, stack, queue (bounded and unbounded), double-ended queue. We claim that the proven specifications are modular and practical. In fact, most of these data structures have already been used to verify more complex ones — we present some in Section 8 and Section 10. Especially, we developed an extensive collection of flexible specifications for the iterators of the **Array** and **List** modules. Remarkably, our formalization of **Array** features different (fractional) predicates to express the ownership of either an entire array, a slice or even a circular slice — we use it to verify algorithms involving circular arrays, *e.g.* Chase-Lev working-stealing queue [Chase and Lev 2005] as presented in Section 10.4.

Concurrent data structures. We provide verified implementations of various concurrent data structures: domain¹⁵ (including domain-local storage), mutex, semaphore, condition variable, write-once variable (also known as *ivar*), atomic array. Note that there is currently no **Atomic_array** module in the OCAML standard library, but we are planning to propose it.

8 Persistent data structures

To further demonstrate the practicality of Zoo, we verified a collection of persistent data structures. This includes purely functional data structures such as persistent stack and queue, but also efficient imperative implementations of persistent array [Conchon and Filliâtre 2007], store [Allain, Clément, Moine and Scherer 2024] and union-find [Allain, Clément, Moine and Scherer 2024].

Currently, verification of purely functional programs relies on the regular Zoolang translation, *i.e.* on a deeply embedded representation. However, this approach is quite cumbersome. In the future, it would be desirable to be able to verify them directly in Rocq, through a translation to Gallina. Similarly to Hacspec [Haselwarter, Hvass, Hansen, Winterhalter, Hritcu and Spitters 2024], this new translation would come with a generated proof of equivalence with the Zoolang representation.

9 Rcfd: Parallelism-safe file descriptor

As mentioned in Section 5, the **Rcfd** module from the **Eio** library is particularly interesting in several respects. Not only does it justify the introduction of generative constructors in OCAML, but it also demonstrates the use of **Iris** for expressing realistic concurrent protocols.

 $^{^{13}}$ For practical reasons, to make them completely opaque, we chose to axiomatize a few functions from the **Domain** and **Random** modules. They could trivially be realized in Zoo.

¹⁴Our implementation of the **Array** module is compatible with the standard one. In particular, it uses the same low-level value representation.

¹⁵Domains are the units of parallelism in OCAML 5.

1:18 Anon.

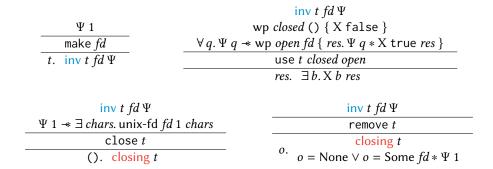


Fig. 4. Rcfd specification (excerpt)

Specification. The **Rcfd** module provides a parallelism-safe file descriptor (FD) relying internally on reference-counting. Interestingly, it is used in Eio in two different ways, more precisely two different ownership regimes: 1) any thread can try to access or close the FD; 2) any thread can try to access the FD but only the owner thread can close it — and is responsible for closing it. To verify all uses, the specification of **Rcfd** has to support both ownership regimes. However, due to space constraints, we consider a simplified specification given in Figure 4. The full verified specification can be found in the mechanization.

The specification features four operations¹⁶.

 make creates a new object t of type **Rcfd**. t wrapping a given FD fd, yielding the (persistent) invariant inv t fd Ψ , where Ψ is an arbitrary fractional predicate. Crucially, the user must provide the full predicate Ψ 1, which is stored in the invariant. Once it is created, a wrapped FD can be accessed through the use operation and closed through the close operation.

use requires the invariant along with the weakest preconditions of the *closed* function, that is called if the FD has been flagged as closed, and *open* function, that is called if the FD is still open. To control the postconditions of the specification and the weakest preconditions, the user can choose an arbitrary predicate X parameterized by a boolean indicating whether the *closed* (false) or the *open* (true) was called. Furthermore, the *open* function is given a fraction of Ψ during its execution, thereby accessing the FD.

close requires the invariant and proving that the full ownership of Ψ entails the full ownership of the FD fd, which is necessary to call <code>Unix.close</code>. It yields closing t, a persistent resource witnessing that t has been flagged as closed. Actually, the wrapped FD is not closed immediately. It will be closed only once it is possible, meaning all ongoing calls to use owning a fraction of the FD end.

Alternatively, instead of closing the FD, remove tries to retrieve the full ownership of Ψ . To achieve it, it exploits the same mechanism as close — flagging t as closed as witnessed by closing t — but also waits until all use calls are done.

Logical state. Thomas Leonard, the author of Rcfd, suggested verifying it to make sure the informal concurrent protocol he described in the OCAML interface was correct. This protocol introduces a notion of monotonic logical state — modeled in Iris using a specific resource algebra [Timany and Birkedal 2021] — to describe the evolution of a FD. Originally, there were four logical states but we found that only three are necessary for the verification: open, closing/users and closing/no-users.

¹⁶We omitted two non-essential operations: is_open and peek.

 In the **open** state, the FD is available for use, meaning any thread can access it through use. Physically, this corresponds to the **Open** constructor.

When some thread flags the FD as closed through close or remove, the state transitions from **open** to **closing/users**. Crucially, there can only be one such thread. In this state, the FD is not really closed yet because of ongoing use operations. Physically, this logical transition corresponds to switching from the **Open** to the **Closing** constructor using **Atomic**.Loc.compare_and_set.

Once all use operations have finished, when the reference-count reaches zero, it is time to actually 'close' the FD by calling the function carried by the **Closing** constructor. However, this has to be done only once. The 'closing' thread is the one that succeeds in updating the **Closing** constructor (to a new one carrying a no-op function) using again **Atomic.Loc.** compare_and_set. At this point, the state transitions from **closing/users** to **closing/no-users** and the wrapper no longer owns the FD.

Generative contructors. As explained in Section 5, the <code>Open</code> constructor has to be generative to prevent <code>unsharing</code>. In fact, the <code>Closing</code> constructor also has to be generative to prevent <code>sharing</code>: when the 'closing' thread wins the <code>Atomic.Loc.compare_and_set</code>, it must be true that the 'closing' function has not changed.

10 Saturn: A library of standard lock-free data structures

We verified a collection of standard lock-free data structures from the Saturn [Karvonen and Morel 2025], Eio [Madhavapeddy and Leonard 2025] and Picos [Karvonen 2025b] libraries. It includes stacks, queues (list-based, array-based and stack-based), bags, and work-stealing queues. These data structures are meant to be used as is or adapted to fit specific needs. To cover a wide range of use cases, we often provide specialized variants: bounded or unbounded, single-producer (SP) or multi-producer (MP), single-consumer (SC) or multi-consumer (MC).

Due to space constraints, we focus on the most important algorithms and refrain from showing the corresponding (non-trivial) IRIS invariants, which are mechanized in Roco.

10.1 Stacks

We verified three variants of the Treiber stack [Treiber 1986]: 1) unbounded MPMC (the standard one), 2) bounded MPMC, 3) closable unbounded MPMC. This last variant features a closing mechanism: at some point, some thread can decide to close the stack, retrieving the current content and preventing others from operating on it. For example, we used it to represent a set of vertex successors in the context of a concurrent graph implementation (not presented in this paper).

As explained in Section 5, the three verified stacks use generative constructors to prevent sharing. One may ask whether it would be easier to use a mutable version of lists instead. From the programmer's perspective, this is unsatisfactory because 1) the compiler will typically emit warnings complaining that the mutability is not exploited and 2) it does not really reflect the intent, *i.e.* we want precise guarantees for physical equality, not modify the list. From the verification perspective, this is also unsatisfactory because the mutable representation is more complex to write and reason about: pointers and points-to assertions versus pure Roco list.

Although verified stacks may seem like a not-so-new contribution, it is, as far as we know, the first verification of realistic OCAML implementations. For comparison, the exemplary concurrent stacks verified in IRIS [Iris development team 2025] all suffer from the same flaw: they need to introduce indirections (pointers) to be able to use the compare-and-set primitive.

1:20 Anon.

- 10.2 List-based queues
- 10.3 Stack-based queues
- 10.4 Work-stealing queues
- 11 Related work

 In general there are two approaches to practical program verification:

11.1 Non-automated verification

The verified program is translated, manually or in an automated way, into a representation living inside a proof assistant. The user has to write specifications and prove them.

The representation may be primitive, like Gallina for Rocq. For pure programs, Gabriel[this is rather straightforward]{I disagree, I believe that hs-to-coq is scientifically problematic as the translation is unsound for higher-order functions and infinite data.}, e.g. in hs-to-coq [Spector-Zabusky, Breitner, Rizkallah and Weirich 2018]. For imperative programs, this is more challenging. One solution is to use a monad, e.g. in coq-of-ocaml [Claret 2025], but it does not support concurrency.

The representation may be embedded, meaning the semantics of the language is formalized in the proof assistant. This is the path taken by some recent works [Chajed, Tassarotti, Kaashoek and Zeldovich 2019; Charguéraud 2023; Daby-Seesaram, Madiot, Pottier, Seassau and Yoon 2024; Gondelman, Hinrichsen, Pereira, Timany and Birkedal 2023] harnessing the power of separation logic. In particular, CFML [Charguéraud 2023] and Osiris [Daby-Seesaram, Madiot, Pottier, Seassau and Yoon 2024] target OCAML. However, CFML does not support concurrency and is not based on Iris. Osiris, still under development, is based on Iris but does not support concurrency.

At the time of writing, HEAPLANG is thus the most appropriate tool to verify concurrent OCAML programs. We discussed limitations of HEAPLANG in the introduction, and ZOOLANG is our proposal to improve on this. Conversely, one notable limitation of ZOOLANG today is its lack of support for OCAML's relaxed memory model.

11.2 Semi-automated verification

In semi-automated verification approaches, the verified program is annotated by the user to guide the verification tool: preconditions, postconditions, invariants, *etc.* Given this input, the verification tool generates proof obligations that are mostly automatically discharged. One may further distinguish two types of semi-automated systems: *foundational* and *non-foundational*.

In non-foundational automated verification, the tool and the external solvers it may rely on are part of the trusted computing base. It is the most common approach and has been widely applied in the literature [Astrauskas, Bilý, Fiala, Grannan, Matheja, Müller, Poli and Summers 2022; Denis, Jourdan and Marché 2022; Filliâtre and Paskevich 2013; Jacobs, Smans, Philippaerts, Vogels, Penninckx and Piessens 2011; Lattuada, Hance, Cho, Brun, Subasinghe, Zhou, Howell, Parno and Hawblitzel 2023; Müller, Schwerhoff and Summers 2017; Pulte, Makwana, Sewell, Memarian, Sewell and Krishnaswami 2023; Swamy, Chen, Fournet, Strub, Bhargavan and Yang 2013], including to OCAML by CAMELEER [Pereira and Ravara 2021], which uses the Gospel specification language [Charguéraud, Filliâtre, Lourenço and Pereira 2019] and Why3 [Filliâtre and Paskevich 2013].

In *foundational* automated verification, the proofs are checked by a proof assistant like Rocq, meaning the automation does not have to be trusted. To our knowledge, it has been applied to C [Sammler, Lepigre, Krebbers, Memarian, Dreyer and Garg 2021] and Rust [Gäher, Sammler, Jung, Krebbers and Dreyer 2024].

Zoo is a non-automated verification framework—except for our use Diaframe for local automation of separation logic reasoning. We would be interested in moving towards more automation in the future.

11.3 Physical equality

 There is some literature in proof-assistant research on reflecting physical equality from the implementation language into the proof assistant, for optimization purposes: for example, exposing OCAML's physical equality as a predicate in Rocq lets us implement some memoization and sharing techniques in Rocq libraries. However, axiomatizing physical equality in the proof assistant is difficult, and can result in inconsistencies.

The earlier discussions of this question that we know come from Jourdan's thesis [Jourdan 2016] (chapter 9), also presented more succintly in [Braibant, Jourdan and Monniaux 2014]. This work introduces the Jourdan condition, that physical equality implies equality of values. [Boulmé 2021] extends the treatment of physical equality in Rocq, integrating it in an "extraction monad" to control it more safely. There is also a discussion of similar optimizations in Lean in [Selsam, Hudon and de Moura 2020].

The correctness of the axiomatization of physical equality depends on the type of the values being compared: axiomatizations are typically polymorphic on any type A, but their correctness depends on the specific A being considered. For example, it is easy to correctly characterize physical on natural numbers, and other non-dependent types arising in Rocq verification projects. One difficulty in Heaplang and Zoolang is that they are untyped languages, their representation of \emptyset and false has the same type. But our remark that structural equality (in OCaml) does not necessarily coincide with definitional equality (in Rocq) also applies to other Rocq types: our examples with an existential Any constructor (see Section 5) can be reproduced with Σ -types.

12 Conclusion and future work

We presented Zoo, a framework for the verification of concurrent OCAML 5 programs. While it is not yet available on opam, it can be installed and used in other Roco projects. We provide a minimal example 17 demonstrating its use.

Zoo has already been used to verify sequential imperative algorithms [anonymous] and is currently being used to verify a library of lock-free data structures. Its main weakness so far is its memory model, which is sequentially consistent as opposed to the relaxed OCAML 5 memory model. It also lacks exceptions and algebraic effects, that we plan to introduce in the future.

Another interesting direction would be to combine Zoo with semi-automated techniques. Similarly to Why3, the simple parts of the verification effort would be done in a semi-automated way, while the most difficult parts would be conducted in Rocq.

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¹⁷Non-anonymous link: https://<clickable>

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