A verified parallel scheduler for OCaml 5

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We present the implementation and mechanized verification of a realistic parallel scheduler for OCaml 5 using the Iris-based Zoo framework. Similarly to `Domainslib`, it relies on a work-stealing strategy to perform load balancing but also supports other scheduling strategies thanks to its flexible interface. We provide basic benchmarks demonstrating that its performance is on par with other schedulers from the OCaml ecosystem.

As part of this effort, we verify the Chase-Lev work-stealing deque, as implemented in the `Saturn` library. We show that it features a subtle external and future-dependent linearization point. To deal with it, we introduce new abstractions for reasoning about prophecy variables in Iris.

1 Introduction

The OCaml programming language migrated from a sequential to a multicore runtime in OCaml 5, imported from the Multicore OCaml research project [Sivaramakrishnan, Dolan, White, Jaffer, Kelly, Sahoo, Parimala, Dhiman and Madhavapeddy 2020] and first released as OCaml 5.0 in 2022. The sequential runtime had a "big runtime lock" guaranteeing that at most one OCaml thread would run at any point in time. The multicore runtime supports "domains" (heavyweight threads) that can run OCaml code in parallel, operating on a shared heap and cooperating for garbage collection. It is designed to offer a M:N threading model with M lightweight tasks (or threads, fibers) are mapped to N domains, with N no larger than the number of CPU cores[1]. The implementation of lightweight tasks and their scheduler is left to be done in userland, as an OCaml library running on top of runtime-provided domains.

The authors of the Multicore OCaml runtime implemented the `Domainslib` library for CPU-bound tasks, initially to write benchmarks to test the scalability of the OCaml runtime. It uses a work-stealing scheduler and is state-of-the-art in the OCaml library ecosystem. Other lightweight task libraries include `Moonpool`, which was implemented independently, and `Eio` which focuses on efficient asynchronous I/O. All of those schedulers have been used in performance-sensitive scenarios, benchmarked and optimized, notably using lock-free data structures implemented in OCaml 5.

In this work we present `Parabs`, a verified implementation of a state-of-the-art scheduler for lightweight tasks, following the overall design of `Domainslib`, with some implementation choices inspired by the `Taskflow` C++ library [Huang, Lin, Lin and Lin 2022]. This verification builds on top of the Zoo framework [Allain and Scherer 2026], which supports the formal verification of a subset of OCaml 5 in the Iris program logic [Jung, Krebbers, Jourdan, Bizjak, Birkedal and Dreyer 2018], mechanized within the Rocq proof assistant. Our verification effort includes a new mechanized verification of the Chase-Lev work-stealing queue [Chase and Lev 2005], with stronger invariants than had previously appeared in the literature. Our scheduler supports two different scheduling strategies, one using standard randomized work-stealing [Blumofe and Leiserson 1999],

[1]The multicore runtime of OCaml performs frequent stop-the-world pause in its garbage collector, to facilitate the migration of existing programs using the OCaml foreign function interface — this design does not require adding a read barrier. A consequence of these stop-the-world events is that runtime performance declines sharply if a domain is paused by the operating system, so it is critical to limit the number of domains to available cores: domains are a heavier, less composable abstraction than "kernel threads" in typical M:N models. In consequence, they must be controlled by end-used applications, possibly via a concurrency framework that hides them entirely, and software libraries should not implicitly spawn new domains, they need to use a more lightweight task abstraction.

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the other using work-stealing with *private* deques [Acar, Charguéraud and Rainey 2013]. On top of the scheduler, we expose an API closely resembling `Domainslib`, but also a *task graph* abstraction which implements the DAG-calculus of Acar, Charguéraud, Rainey and Sieczkowski [2016].

This work is focused on formal verification but we did write and run relatively simple benchmarks to validate experimentally that the performance of our verified implementation, `Parabs`, is comparable to `Domainslib`; in our tests it is equal or faster.

Contributions. Our contributions include:

(1) A new mechanized verification of the Chase-Lev work-stealing queue [Chase and Lev 2005], with finer-grained invariants than had previously appeared in the literature. In particular, our invariant lets us reason about the failure case of the `pop` operation, which was missing from earlier formalizations and is essential to prove termination of the work-stealing scheduler when all task queues are exhausted.

(2) A fully verified implementation of a parallel scheduler in OCaml 5, `Parabs`, which provides a verified alternative to the state-of-the-art `Domainslib` library. To the best of our knowledge, this is the first verified implementation of a parallel work-stealing task scheduler (for any language) using realistic implementation techniques. Our experimental evaluation on a set of simple benchmarks shows that `Parabs` has comparable or better performance than `Domainslib`, and better performance than `Moonpool`.

(3) A verified implementation of the DAG-calculus interface for parallel task graphs proposed by Acar, Charguéraud and Rainey [2013].

(4) At the level of Iris proof techniques, we developed extensions of prophecy variables. To reason about the linearization points of our concurrent data structures we needed to introduce "wise" and "multiplexed" prophecy variables, which are reusable building blocks and could be useful for the verification of other concurrent data structures, in any Iris formalization of any programming language.

Artifact. The Zoo verification framework supports a fragment of OCaml called ZooLang, with formal semantics defined as an Iris program logic, and a partial translator from OCaml to ZooLang programs deeply embedded within Rocq. Our developments are thus available as OCaml libraries that are readily available for usage, and their Rocq specifications, invariants and proofs. For reasons of space we cannot possibly hope to describe them in full in the paper, so we focus on the most readily reusable parts: specifications, and key invariants and proof techniques. For more details, we view our code and mechanized proofs as an integral part of this submission; they are available at https://anonymous.4open.science/r/zoo-A236, publicly available and open-source.[2] To ease in-depth exploration, the paper contains direct references to the implementation and the proof as picture/icon links, for example and .

2 Iris arsenal

Separation logic. Iris is a concurrent separation logic [O'Hearn 2007; O'Hearn, Reynolds and Yang 2001; Reynolds 2002] fully mechanized in the Rocq proof assistant [Krebbers, Jourdan, Jung, Tassarotti, Kaiser, Timany, Charguéraud and Dreyer 2018]. As such, it features basic connectives like separation conjunction $*$ and separating implication $-\!\!*$.

Persistent assertions. In Iris, assertions are affine: using a resource consumes it, removes it from the proof context. Some assertions, however, are *persistent*. Once a persistent assertion holds, it

[2]For ease of development we followed the Zoo approach of working in a mono-repository, so we have an experimental version of Zoo with our developments added, as well as some cross-cutting improvements to the pre-existing support libraries and tactics.

holds forever; using it does not consume it. This enables *duplication* ($P \vdash P * P$) and *sharing*. In particular, pure (meta-level) assertions embedded into the logic are persistent.

Formally, persistence is defined in terms of the *persistence modality*:

$$\mathsf{persistent}\ P \triangleq P \vdash \Box P$$

Informally, $\Box P$ means P holds without asserting any exclusive ownership; in other words, it only expresses knowledge. Naturally, $\Box P$ is persistent.

Ghost state. One of the most important features of Iris is its *user-defined higher-order ghost state*, a very flexible form of ghost state. Ghost updates, of the form $\mathrel{|}\!\!\Rrightarrow P$, allow updating the ghost state during the proof; they are purely logical, hence not visible in the program.

Sequential specification. Sequential specifications take the form of Hoare triples:

$$\begin{array}{c} P \\ \hline e \\ \hline \Phi \end{array} \qquad\qquad \begin{array}{c} \mathsf{stack\text{-}model}\ t\ vs \\ \hline \mathsf{stack\_push}\ t\ v \\ \hline ().\ \mathsf{stack\text{-}model}\ t\ (v :: vs) \end{array}$$

where P is an Iris assertion, e an expression and Φ an Iris predicate over values.

Informally, this triple says: if the precondition P holds, we can safely execute e and, if the execution terminates, the returned value satisfies the postcondition Φ. It is a persistent resource, allowing executing e many times.

Weakest precondition. Hoare triples are defined using the more primitive notion of *weakest precondition* $\mathsf{wp}\ e\ \{\Phi\}$. Informally, it says that: once only, we can execute e and, if the execution terminates, the returned value satisfies the postcondition Φ. Contrary to Hoare triples, it can depend on exclusive ownership and therefore is not persistent.

Atomic specification. To specify concurrent operations, we use the notion of *logical atomicity* [da Rocha Pinto, Dinsdale-Young and Gardner 2014]. An operation is said to be logically atomic if it appears to take effect atomically at some point during its execution; this point is called the *linearization point* of the operation. Birkedal, Dinsdale-Young, Guéneau, Jaber, Svendsen and Tzevelekos showed that this notion implies *linearizability* [Herlihy and Wing 1990] in a sequentially consistent memory model.

In Iris, logical atomicity takes the form of *atomic specifications*:

$$\begin{array}{c} P_{priv} \\ \hdashline \overline{x}.\ P_{pub} \\ \hline e \\ \hline \overline{y}.\ Q \\ \hdashline \Phi \end{array} \qquad\qquad \begin{array}{c} \mathsf{stack\text{-}inv}\ t \\ \hdashline vs.\ \mathsf{stack\text{-}model}\ t\ vs \\ \hline \mathsf{stack\_push}\ t\ v \\ \hline \mathsf{stack\text{-}model}\ t\ (v :: vs) \\ \hdashline ().\ \mathsf{True} \end{array}$$

P_{priv} and Φ are standard *private* pre- and postcondition for the user of the specification, similarly to Hoare triples. P_{pub} and Q are *public* pre- and postcondition; they specify the linearization point of the operation. Quantifiers $\overline{x}$ represent the *demonic nature* of P_{pub}: the exact state at the linearization point, given by P_{pub}, is unknown until it happens. Quantifiers $\overline{y}$ represent the *angelic nature* of Q: at the linearization point, the operation can choose how to instantiate the new state Q.

In sum, the atomic specification says: if the private precondition P_{priv} holds, we can safely execute e and, if the execution terminates, (1) the returned value satisfies the private postcondition Φ and (2) at some point during the execution, the state was atomically updated from P_{pub} to Q.

$$\textsc{prophet-model-exclusive}\quad \frac{\mathsf{model}\ \mathit{pid}\ \mathit{prophs}_1 \qquad \mathsf{model}\ \mathit{pid}\ \mathit{prophs}_2}{\mathsf{False}}$$

$$\textsc{wp-proph}\quad \frac{\mathsf{True}}{\frac{\mathsf{Proph}}{\mathit{pid}.\ \exists\ \mathit{prophs}.\ \mathsf{model}\ \mathit{pid}\ \mathit{prophs}}}$$

$$\textsc{wp-resolve}\quad \frac{\mathsf{to\text{-}val}\ e = \mathsf{None} \qquad \mathsf{model}\ \mathit{pid}\ \mathit{prophs} \qquad \begin{array}{c}\mathsf{atomic}\ e \\ \mathsf{wp}\ e \left\{ \begin{array}{l} \mathit{res}.\ \forall\ \mathit{prophs}'. \\ \quad \mathit{prophs} = (\mathit{res}, v) :: \mathit{prophs}' \twoheadrightarrow \\ \quad \mathsf{model}\ \mathit{pid}\ \mathit{prophs}' \twoheadrightarrow \\ \quad \Phi\ \mathit{res} \end{array} \right\}\end{array}}{\mathsf{wp}\ \mathsf{Resolve}\ e\ \mathit{pid}\ v\ \{\Phi\}}$$

Fig. 1. Reasoning rules for primitive prophets

3 Prophecy variables

In 2020, Jung, Lepigre, Parthasarathy, Rapoport, Timany, Dreyer and Jacobs introduced *prophecy variables* in Iris. Essentially, prophecy variables — or *prophets*, as we will call them in this section — can be used to predict the future of the program execution and reason about it. They are key to handle *future-dependent linearization points* [Dongol and Derrick 2014]: linearization points that may or may not occur at a given location in the code depending on a future observation.

In the program, prophecies take the form of two primitives: `Proph` and `Resolve`. To reason about them in the logic, Jung, Lepigre, Parthasarathy, Rapoport, Timany, Dreyer and Jacobs [2020] proposed two abstraction layers, which we recall in Sections 3.1 and 3.2. To verify the Chase-Lev work-stealing deque (see Section 4), we needed to introduce two additional layers, presented in Sections 3.3 and 3.4.

3.1 Primitive prophet

The first layer consists of *primitive prophets*. These prophets are primitive in the sense that they simply reflect the semantics of `Proph` and `Resolve` in the program logic. The corresponding reasoning rules are given in Figure 1.

The assertion model *pid prophs* represents the exclusive ownership of the prophet with identifier *pid*; *prophs* is the list of prophecies that must still be resolved.

wp-proph says that `Proph` allocates a new prophet with some unknown prophecies to be resolved. wp-resolve says that `Resolve` *e pid v atomically* resolves the next prophecy of prophet *pid*: we learn that the prophecies before resolution *prophs* is non-empty and its head is the pair (res, v) where *res* is the evaluation of *e*.

3.2 Typed prophet

The second layer consists of *typed prophets*. They are very similar to primitive prophets except prophecies are now typed. The corresponding reasoning rules, given in Figure 2, are essentially the same as before. The prophet must provide a type τ along with two functions of-val and to-val. to-val converts an inhabitant of τ to a value; typed-prophet-resolve-spec relies on it to enforce that the prophecies are well-typed. of-val attempts to convert a value to τ; it is used internally. of-val and to-val must be compatible: of-val (to-val *proph*) = Some *proph*.

TYPED-PROPHET-MODEL-EXCLUSIVE

$$\frac{\mathsf{model}\ \mathit{pid}\ \mathit{prophs}_1 \qquad \mathsf{model}\ \mathit{pid}\ \mathit{prophs}_2}{\mathsf{False}}$$

TYPED-PROPHET-PROPH-SPEC

$$\begin{array}{c} \mathsf{True} \\ \hline \mathsf{Proph} \\ \hline \mathit{pid}.\ \exists\ \mathit{prophs}.\ \mathsf{model}\ \mathit{pid}\ \mathit{prophs} \end{array}$$

TYPED-PROPHET-RESOLVE-SPEC

$$\frac{\begin{array}{c}\mathsf{atomic}\ e \qquad \mathsf{to\text{-}val}\ e = \mathsf{None} \\ v = \mathit{prophet}.\mathsf{to\text{-}val}\ \mathit{proph} \qquad \mathsf{model}\ \mathit{pid}\ \mathit{prophs} \qquad \mathsf{wp}\ e \left\{ \begin{array}{l} w.\ \forall\ \mathit{prophs}'. \\ \quad \mathit{prophs} = \mathit{proph} :: \mathit{prophs}' \mathrel{-\!\!*} \\ \quad \mathsf{model}\ \mathit{pid}\ \mathit{prophs}' \mathrel{-\!\!*} \\ \quad \Phi\ w \end{array} \right\}\end{array}}{\mathsf{wp}\ \mathsf{Resolve}\ e\ \mathit{pid}\ v\ \{\Phi\}}$$

Fig. 2. Reasoning rules for typed prophets

$$\mathsf{persistent}\ (\mathsf{full}\ \gamma\ \mathit{prophs}) \qquad \mathsf{persistent}\ (\mathsf{snapshot}\ \gamma\ \mathit{past}\ \mathit{prophs}) \qquad \mathsf{persistent}\ (\mathsf{lb}\ \gamma\ \mathit{lb})$$

WISE-PROPHET-MODEL-EXCLUSIVE

$$\frac{\mathsf{model}\ \mathit{pid}\ \gamma_1\ \mathit{past}_1\ \mathit{prophs}_1 \qquad \mathsf{model}\ \mathit{pid}\ \gamma_2\ \mathit{past}_2\ \mathit{prophs}_2}{\mathsf{False}}$$

WISE-PROPHET-FULL-GET

$$\frac{\mathsf{model}\ \mathit{pid}\ \gamma\ \mathit{past}\ \mathit{prophs}}{\mathsf{full}\ \gamma\ (\mathit{past} +\!\!+ \mathit{prophs})}$$

WISE-PROPHET-FULL-VALID

$$\frac{\mathsf{model}\ \mathit{pid}\ \gamma\ \mathit{past}\ \mathit{prophs}_1 \qquad \mathsf{full}\ \gamma\ \mathit{prophs}_2}{\mathit{prophs}_2 = \mathit{past} +\!\!+ \mathit{prophs}_1}$$

WISE-PROPHET-FULL-AGREE

$$\frac{\mathsf{full}\ \gamma\ \mathit{prophs}_1 \qquad \mathsf{full}\ \gamma\ \mathit{prophs}_2}{\mathit{prophs}_1 = \mathit{prophs}_2}$$

WISE-PROPHET-SNAPSHOT-GET

$$\frac{\mathsf{model}\ \mathit{pid}\ \gamma\ \mathit{past}\ \mathit{prophs}}{\mathsf{snapshot}\ \gamma\ \mathit{past}\ \mathit{prophs}}$$

WISE-PROPHET-SNAPSHOT-VALID

$$\frac{\mathsf{model}\ \mathit{pid}\ \gamma\ \mathit{past}_1\ \mathit{prophs}_1 \qquad \mathsf{snapshot}\ \gamma\ \mathit{past}_2\ \mathit{prophs}_2}{\exists\ \mathit{past}_3.\ \mathit{past}_1 = \mathit{past}_2 +\!\!+ \mathit{past}_3 * \mathit{prophs}_2 = \mathit{past}_3 +\!\!+ \mathit{prophs}_1}$$

WISE-PROPHET-LB-GET

$$\frac{\mathsf{model}\ \mathit{pid}\ \gamma\ \mathit{past}\ \mathit{prophs}}{\mathsf{lb}\ \gamma\ \mathit{prophs}}$$

WISE-PROPHET-LB-VALID

$$\frac{\mathsf{model}\ \mathit{pid}\ \gamma\ \mathit{past}\ \mathit{prophs} \qquad \mathsf{lb}\ \gamma\ \mathit{lb}}{\exists\ \mathit{past}_1\ \mathit{past}_2.\ \mathit{past} = \mathit{past}_1 +\!\!+ \mathit{past}_2 * \mathit{lb} = \mathit{past}_2 +\!\!+ \mathit{prophs}}$$

WISE-PROPHET-PROPH-SPEC

$$\begin{array}{c} \mathsf{True} \\ \hline \mathsf{Proph} \\ \hline \mathit{pid}.\ \exists\ \gamma\ \mathit{prophs}.\ \mathsf{model}\ \mathit{pid}\ \gamma\ []\ \mathit{prophs} \end{array}$$

WISE-PROPHET-RESOLVE-SPEC

$$\frac{\begin{array}{c}\mathsf{atomic}\ e \qquad \mathsf{to\text{-}val}\ e = \mathsf{None} \qquad v = \mathit{prophet}.\mathsf{to\text{-}val}\ \mathit{proph} \\ \mathsf{model}\ \mathit{pid}\ \gamma\ \mathit{past}\ \mathit{prophs} \qquad \mathsf{wp}\ e \left\{ \begin{array}{l} w.\ \forall\ \mathit{prophs}'. \\ \quad \mathit{prophs} = \mathit{proph} :: \mathit{prophs}' \mathrel{-\!\!*} \\ \quad \mathsf{model}\ \mathit{pid}\ \gamma\ (\mathit{past} +\!\!+ [\mathit{proph}])\ \mathit{prophs}' \mathrel{-\!\!*} \\ \quad \Phi\ w \end{array} \right\}\end{array}}{\mathsf{wp}\ \mathsf{Resolve}\ e\ \mathit{pid}\ v\ \{\Phi\}}$$

Fig. 3. Reasoning rules for wise prophets

3.3 Wise prophet

The third layer consists of *wise prophets*. These prophets *remember* past prophecies. The corresponding reasoning rules are given in Figure 3.

The exclusive assertion model *pid* γ *past* *prophs* represents the ownership of the prophet with identifier *pid*; γ is the logical name of the prophet; *past* is the list of prophecies resolved so far; *prophs* is the list of prophecies that must still be resolved.

The persistent assertion full γ *prophs* represents the list of all (resolved or not) prophecies associated to the prophet with name γ, as stated by WISE-PROPHET-FULL-VALID.

The persistent aassertion snapshot γ *past* *prophs* represents a snapshot of the state of the prophet with name γ at some point in the past. WISE-PROPHET-SNAPSHOT-VALID allows to relate the current state of model to the past state of snapshot.

The persistent assertion lb γ *lb* represents a lower bound on the non-resolved prophecies of the prophet with name γ. In particular, as stated by WISE-PROPHET-LB-VALID, the list of currently non-resolved prophecies carried by model is always a suffix of *lb*.

WISE-PROPHET-RESOLVE-SPEC is the same as before, except we also update the list of resolved prophecies after resolution.

3.4 Multiplexed prophet

The fourth layer consists of *multiplexed prophets*. Essentially, they allow to combine different prophets, each operating at a fixed index. They were made to handle the case when a single prophet is used to make independent predictions, as in Section 4. The corresponding reasoning rules are given in Figure 4.

The predicates and rules are basically the same as before, except that (1) model now carries sequences of lists of prophecies — one past and one future per index — and (2) full, snapshot and lb are parameterized with an index.

Importantly, the third argument provided to `Resolve` in WISE-PROPHETS-RESOLVE-SPEC must be a pair of an index and a prophecy value. Resolution happens only at the given index, meaning the prophecies at other indices are unchanged.

Note that we could generalize this abstraction to non-integer keys. In other words, we could replace sequences with functions of type $X \to \tau$, where τ is the prophecy type, and indices with inhabitants of X. In practice, however, we never needed such generalization.

4 Chase-Lev work-stealing deque

Work-stealing. Randomized *work stealing* [Blumofe and Leiserson 1999] is the standard strategy for parallel task scheduling. It has been implemented in many libraries, including Cilk [Blumofe, Joerg, Kuszmaul, Leiserson, Randall and Zhou 1996; Frigo, Leiserson and Randall 1998], TBB, OpenMP, `Taskflow` [Huang, Lin, Lin and Lin 2022], `Tokio` and `Domainslib` [Multicore OCaml development team 2025].

The idea of work-stealing, illustrated in Figure 5, is the following. Each domain owns a deque-like data structure, called *work-stealing deque*, to store its tasks. Locally, each domain treats its deque as a stack, operating at the back end. When a domain runs out of tasks, it becomes a thief: it tries to steal a task from the deque of another randomly selected “victim” domain, operating at the front end. Multiple thieves may concurrently attempt to steal tasks from a single deque.

Work-stealing deque. The most popular work-stealing deque algorithm is the Chase-Lev deque [Chase and Lev 2005; Lê, Pop, Cohen and Nardelli 2013]; it is lock-free and unbounded. We verified the implementation from the `Saturn` library [Karvonen and Morel 2025] along with two other

$$\text{persistent}\ (\text{full}\ \gamma\ i\ \mathit{prophs}) \qquad \text{persistent}\ (\text{snapshot}\ \gamma\ i\ \mathit{past}\ \mathit{prophs}) \qquad \text{persistent}\ (\text{lb}\ \gamma\ i\ \mathit{lb})$$

WISE-PROPHETS-MODEL-EXCLUSIVE

$$\frac{\text{model}\ \mathit{pid}\ \gamma_1\ \mathit{pasts}_1\ \mathit{prophss}_1 \qquad \text{model}\ \mathit{pid}\ \gamma_2\ \mathit{pasts}_2\ \mathit{prophss}_2}{\text{False}}$$

WISE-PROPHETS-FULL-GET

$$\frac{\text{model}\ \mathit{pid}\ \gamma\ \mathit{pasts}\ \mathit{prophss}}{\text{full}\ \gamma\ i\ (\mathit{pasts}\ i \mathbin{+\!\!+} \mathit{prophss}\ i)}$$

WISE-PROPHETS-FULL-VALID

$$\frac{\text{model}\ \mathit{pid}\ \gamma\ \mathit{pasts}\ \mathit{prophss} \qquad \text{full}\ \gamma\ i\ \mathit{prophs}}{\mathit{prophs} = \mathit{pasts}\ i \mathbin{+\!\!+} \mathit{prophss}\ i}$$

WISE-PROPHETS-FULL-AGREE

$$\frac{\text{full}\ \gamma\ i\ \mathit{prophs}_1 \qquad \text{full}\ \gamma\ i\ \mathit{prophs}_2}{\mathit{prophs}_1 = \mathit{prophs}_2}$$

WISE-PROPHETS-SNAPSHOT-GET

$$\frac{\text{model}\ \mathit{pid}\ \gamma\ \mathit{pasts}\ \mathit{prophss}}{\text{snapshot}\ \gamma\ (\mathit{pasts}\ i)\ (\mathit{prophss}\ i)}$$

WISE-PROPHETS-SNAPSHOT-VALID

$$\frac{\text{model}\ \mathit{pid}\ \gamma\ \mathit{pasts}\ \mathit{prophss} \qquad \text{snapshot}\ \gamma\ i\ (\mathit{pasts}\ i)\ (\mathit{prophss}\ i)}{\exists\ \mathit{past}'.\ \mathit{pasts}\ i = \mathit{past} \mathbin{+\!\!+} \mathit{past}' \ast \mathit{prophs} = \mathit{past}' \mathbin{+\!\!+} \mathit{prophss}\ i}$$

WISE-PROPHETS-LB-GET

$$\frac{\text{model}\ \mathit{pid}\ \gamma\ \mathit{pasts}\ \mathit{prophss}}{\text{lb}\ \gamma\ i\ (\mathit{prophss}\ i)}$$

WISE-PROPHETS-LB-VALID

$$\frac{\text{model}\ \mathit{pid}\ \gamma\ \mathit{pasts}\ \mathit{prophss} \qquad \text{lb}\ \gamma\ i\ \mathit{lb}}{\exists\ \mathit{past}_1\ \mathit{past}_2.\ \mathit{pasts}\ i = \mathit{past}_1 \mathbin{+\!\!+} \mathit{past}_2 \ast \mathit{lb} = \mathit{past}_2 \mathbin{+\!\!+} \mathit{prophss}\ i}$$

WISE-PROPHETS-PROPH-SPEC

$$\frac{\dfrac{\text{True}}{\text{Proph}}}{\mathit{pid}.\ \exists\ \gamma\ \mathit{prophss}.\ \text{model}\ \mathit{pid}\ \gamma\ (\lambda\_.\ [])\ \mathit{prophss}}$$

WISE-PROPHETS-RESOLVE-SPEC

$$\frac{\begin{array}{c}\text{atomic}\ e \qquad \text{to-val}\ e = \text{None} \qquad v = \mathit{prophet}.\text{to-val}\ \mathit{proph} \qquad \text{model}\ \mathit{pid}\ \gamma\ \mathit{pasts}\ \mathit{prophss} \\ \text{wp}\ e \left\{ \begin{array}{l} w.\ \forall\ \mathit{prophs}. \\ \quad \mathit{prophss}\ i = \mathit{proph} :: \mathit{prophs} \mathrel{-\!\!*} \\ \quad \text{model}\ \mathit{pid}\ \gamma\ (\text{alter}\ (\cdot \mathbin{+\!\!+} [\mathit{proph}])\ i\ \mathit{pasts})\ (\mathit{prophss}\,[i \mapsto \mathit{prophs}]) \mathrel{-\!\!*} \\ \quad \Phi\ w \end{array} \right\}\end{array}}{\text{wp Resolve}\ e\ \mathit{pid}\ (i, v)\ \{\Phi\}}$$

Fig. 4. Reasoning rules for multiplexed prophets

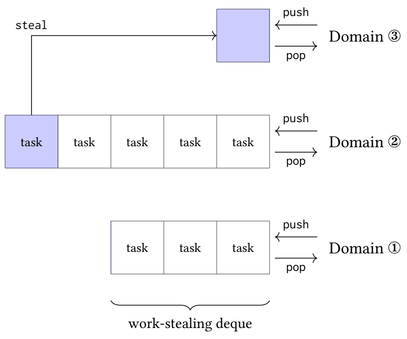

Fig. 5. Work stealing

$$\mathsf{persistent}\ (\mathsf{inv}\ t)$$

INF-WS-DEQUE-MODEL-EXCLUSIVE
$$\frac{\mathsf{model}\ t\ vs_1 \qquad \mathsf{model}\ t\ vs_2}{\mathsf{False}}$$

INF-WS-DEQUE-OWNER-EXCLUSIVE
$$\frac{\mathsf{owner}\ t\ ws_1 \qquad \mathsf{owner}\ t\ ws_2}{\mathsf{False}}$$

INF-WS-DEQUE-OWNER-MODEL
$$\frac{\mathsf{owner}\ t\ ws \qquad \mathsf{model}\ t\ vs}{\mathsf{suffix}\ vs\ ws}$$

INF-WS-DEQUE-CREATE-SPEC
$$\begin{array}{c}
\mathsf{True} \\ \hline
\texttt{create ()} \\ \hline
t.\ \mathsf{inv}\ t\ * \\
\mathsf{model}\ t\ []\ * \\
\mathsf{owner}\ t\ []
\end{array}$$

INF-WS-DEQUE-SIZE-SPEC
$$\begin{array}{c}
\mathsf{inv}\ t\ * \\
\mathsf{owner}\ t\ ws \\ \hdashline
vs.\ \mathsf{model}\ t\ vs \\ \hline
\texttt{size}\ t \\ \hline
\mathsf{suffix}\ vs\ ws\ * \\
\mathsf{model}\ t\ vs \\ \hdashline
res.\ res = \mathsf{length}\ vs\ * \\
\mathsf{owner}\ t\ vs
\end{array}$$

INF-WS-DEQUE-IS-EMPTY-SPEC
$$\begin{array}{c}
\mathsf{inv}\ t\ * \\
\mathsf{owner}\ t\ ws \\ \hdashline
vs.\ \mathsf{model}\ t\ vs \\ \hline
\texttt{is\_empty}\ t \\ \hline
\mathsf{suffix}\ vs\ ws\ * \\
\mathsf{model}\ t\ vs \\ \hdashline
res.\ res = \mathsf{decide}\ \ (vs = [])\ * \\
\mathsf{owner}\ t\ vs
\end{array}$$

INF-WS-DEQUE-POP-SPEC
$$\begin{array}{l}
\qquad\mathsf{inv}\ t\ * \\
\qquad\mathsf{owner}\ t\ ws \\ \hdashline
\qquad vs.\ \mathsf{model}\ t\ vs \\ \hline
\qquad\texttt{pop}\ t \\ \hline
o\ ws'.\ \mathsf{suffix}\ vs\ ws\ * \\
\quad \mathbf{match}\ o\ \mathbf{with} \\
\quad \mid \mathsf{None} \Rightarrow \\
\qquad vs = []\ *\ ws' = []\ * \\
\qquad \mathsf{model}\ t\ [] \\
\quad \mid \mathsf{Some}\ v \Rightarrow \\
\qquad \exists\ vs'. \\
\qquad vs = vs' +\!\!+ [v]\ *\ ws' = vs'\ * \\
\qquad \mathsf{model}\ t\ vs' \\
\quad \mathbf{end} \\ \hdashline
\qquad res.\ res = o\ * \\
\qquad \mathsf{owner}\ t\ ws'
\end{array}$$

INF-WS-DEQUE-PUSH-SPEC
$$\begin{array}{c}
\mathsf{inv}\ t\ * \\
\mathsf{owner}\ t\ ws \\ \hdashline
vs.\ \mathsf{model}\ t\ vs \\ \hline
\texttt{push}\ t\ v \\ \hline
\mathsf{suffix}\ vs\ ws\ * \\
\mathsf{model}\ t\ (vs +\!\!+ [v]) \\ \hdashline
().\ \mathsf{owner}\ t\ (vs +\!\!+ [v])
\end{array}$$

INF-WS-DEQUE-STEAL-SPEC
$$\begin{array}{c}
\mathsf{inv}\ t \\ \hdashline
vs.\ \mathsf{model}\ t\ vs \\ \hline
\texttt{steal}\ t \\ \hline
\mathsf{model}\ t\ (\mathsf{tail}\ vs) \\ \hdashline
res.\ res = \mathsf{head}\ vs
\end{array}$$

Fig. 6. **`Inf_ws_deque`**: Specification

variants: a bounded variant, used in the `Moonpool` [Cruanes 2025] and `Taskflow` [Huang, Lin, Lin and Lin 2022] libraries, and an idealized infinite-array-based variant.

Remarkably, the three variants essentially share the same logical states. In particular, although they do not behave exactly the same way, the original and the idealized versions follow a similar concurrent protocol, involving external and future-dependent linearization.

4.1 Infinite work-stealing deque

4.1.1 Specification. The specification of the infinite-array-based version is given in Figure 6. It features three predicates: inv, model and owner.

The persistent assertion inv t represents the knowledge that t is a valid deque. It is returned by `create` (INF-WS-DEQUE-CREATE-SPEC) and required by all operations.

$$
\begin{array}{c}
\textsc{ws-deque-steal-spec-weak} \\
\mathsf{inv}\ t \\
\hdashline
vs.\ \mathsf{model}\ t\ vs \\
\hline
\texttt{steal}\ t \\
\hline
\begin{array}{l}
o.\ \textbf{match}\ o\ \textbf{with} \\
\quad \mid \mathsf{None} \Rightarrow \\
\qquad \mathsf{model}\ t\ vs \\
\quad \mid \mathsf{Some}\ v \Rightarrow \\
\qquad \exists\ vs'. \\
\qquad vs = v :: vs' \ * \\
\qquad \mathsf{model}\ t\ vs' \\
\textbf{end}
\end{array} \\
\hdashline
res.\ res = o
\end{array}
\qquad
\begin{array}{c}
\textsc{ws-deque-pop-spec-weak} \\
\mathsf{inv}\ t\ * \\
\mathsf{owner}\ t \\
\hdashline
vs.\ \mathsf{model}\ t\ vs \\
\hline
\mathsf{pop}\ t \\
\hline
\begin{array}{l}
o.\ \textbf{match}\ o\ \textbf{with} \\
\quad \mid \mathsf{None} \Rightarrow \\
\qquad \mathsf{model}\ t\ vs \\
\quad \mid \mathsf{Some}\ v \Rightarrow \\
\qquad \exists\ vs'. \\
\qquad vs = vs' +\!\!+ [v] \ * \\
\qquad \mathsf{model}\ t\ vs' \\
\textbf{end}
\end{array} \\
\hdashline
res.\ res = o\ * \\
\mathsf{owner}\ t
\end{array}
$$

Fig. 7. **Ws_deque**: Weak specification (excerpt)

The exclusive assertion model t vs represents the ownership of the content of the deque vs. It it returned by `create` and accessed atomically by all operations.

The exclusive assertion owner t ws represents the owner of the deque; ws is an upper bound on the current content of the deque (INF-WS-DEQUE-OWNER-MODEL). It is returned by `create` and used by all private operation: `size` (INF-WS-DEQUE-SIZE-SPEC), `is_empty` (INF-WS-DEQUE-IS-EMPTY-SPEC), `push` (INF-WS-DEQUE-PUSH-SPEC) and pop (INF-WS-DEQUE-POP-SPEC). The only public operation is `steal` (INF-WS-DEQUE-STEAL-SPEC), which does not require owner.

Note that the public postconditions of the private operations are quite verbose. This is due to the fact that owner is passed to the operation and therefore cannot be combined with model through INF-WS-DEQUE-OWNER-MODEL to get information about the content of the deque; instead, we provide such information in the public postcondition. We need this expressivity in practice to verify a wrapper with better liveness properties.

4.1.2 Weak specification. Jung, Lee, Choi, Kim, Park and Kang [2023][3] also worked on the verification of the Chase-Lev work-stealing deque. However, we argue that the specification they prove, given in Figure 7, is unsatisfactory. Indeed, contrary to our specification, WS-DEQUE-STEAL-SPEC-WEAK and WS-DEQUE-POP-SPEC-WEAK say nothing about the observed content of the deque when the operation fails.

In practice, these weaker specifications, especially that of pop, are not sufficient to reason about the *termination* of a work-stealing scheduler. In Section 5, we show how our strong specifications are lifted all the way up to the scheduler.

Another point we would like to make is that weakening the specification does make the verification simpler, but one may argue that the most subtle and interesting part of it is lost.

4.1.3 Implementation. The implementation relies on (1) an infinite array, (2) a *monotonic* front index for the thieves, and (3) a back index reserved to the owner of the deque.

In general, we can divide the infinite array as in Figure 8. The first part, between 0 and the front index, corresponds to the *persistent* history of stolen values. The second part, between the two

[3]See also the master thesis of Choi [2023].

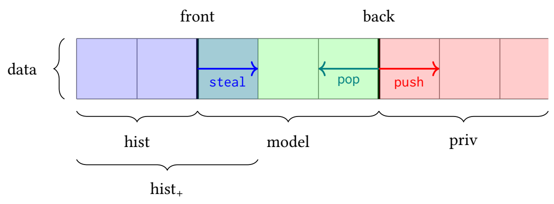

Fig. 8. **Inf_ws_deque**: Physical state

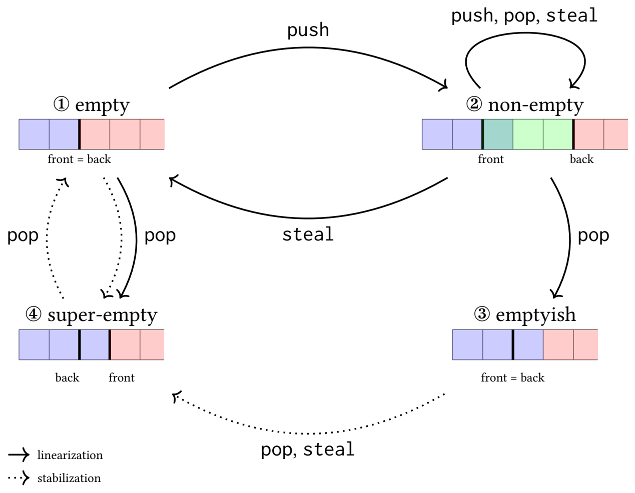

Fig. 9. **Inf_ws_deque**: Logical state

indices, corresponds to the logical content of the deque, as represented by model. The last part, beyond the back index, corresponds to the private section of the array, reserved to the owner.

Given this representation, the algorithm proceeds as follows. push t v writes v into the first private cell and atomically increments the back index, thereby publishing the value. Symmetrically, pop t atomically decrements the back index and returns the value of the cell it just privatized. `steal` t is much more careful: (1) it reads the front and the back indices; (2) if the deque looks empty, it fails; (3) otherwise, it attempts to advance the front index; (4) if the update succeeds, the value at the front index is returned; (5) otherwise, it starts over.

The above description overlooked one crucial aspect: what happens at the limit, when pop and `steal` compete for the last value in the deque? In that case, the deque must be *stabilized*: pop also attempts to advance the front index before incrementing the back index — whether it wins the update or not — thereby equalizing the two indices.

4.1.4 Logical states. Figure 9 tells the same story as above in terms of four *logical states*: (1) in the stable "empty" state, the deque is indeed empty, as indicated by the two equal indices; (2) in the stable "non-empty" state, the model is non-empty, meaning thieves may compete for the first value; (3) in the unstable "emptyish" state, the thieves and the owner compete for the same value; (4) in the unstable "super-empty" state, some operation won the value and the deque is waiting to be stabilized by the owner.

Let us now focus on the “emptyish” state. In this physical configuration, it makes sense to say that the model of the deque should be empty. In fact, is has to be empty: if a `steal` operation observed this state, it would conclude that the deque is empty — except under a weak specification. But then, if the model should be empty, which operation was linearized during the transition to the “emptyish” state? We have no choice: it should be the winner of the front update, *i.e.* the operation which triggers the transition to the “super-empty” state. In conclusion, we have to predict the winner at each index using a multiplexed prophecy variable (see Section 3).

4.2 Bounded work-stealing deque

In the bounded variant, the infinite array is replaced with a finite circular array. As a consequence, the convenient infinite representation goes away and tedious reasoning about circular array slices is required. However, the logical states and transitions as well as the prophecy mechanism are essentially the same.

It is an open question whether we could factorize part of the verification through a well-chosen abstraction that could be instantiated both with infinite and circular arrays. One certainty is that this is not possible without slightly altering the implementation of the infinite variant: in `steal`, the front cell is read after performing the update in the infinite variant, which would be incorrect in the finite variant since the owner is allowed to overwrite the value.

4.3 Dynamic work-stealing deque

In the original algorithm, the owner may dynamically resize the circular array. More precisely, it can change the array at will provided that the public part (between the two indices) is preserved. Thus, while only one array is stored in the deque, there can be many different circular arrays alive at the same time, *i.e.* accessible by thieves.

While the invariant of Choi [2023] requires additional ghost state to keep track of the arrays and maintain their compatibility, the precision of our notion of logical state allows to only maintain compatibility between the current array and the array read by the next winner (if any).

5 `Parabs`: A library of parallel abstractions

We present the verified `Parabs` library, offering parallel abstractions atop a task scheduler. While it was originally based on `Domainslib` [Multicore OCaml development team 2025], it evolved as a more ambitious project aimed at unifying various existing paradigms and scheduling strategies. It was designed with a focus on *flexibility*, letting users choose the scheduling strategy and build their own scheduler. One of the motivations of this design is to provide a framework to easily develop and experiment parallel infrastructures in OCaml 5.

6 Overview

Figure 10 gives an overview of `Parabs`; solid edges represent module dependencies while dashed edges represent interface implementations. Essentially, the library is made of four abstraction levels built on top of each other: **`Ws_deques`**, **`Ws_hub`**, **`Pool`** and **`Future`** / **`Vertex`**.

The **`Pool`** module provides a task scheduler; internally, it maintains a pool of domains. Its design is inspired by `Domainslib`, `Taskflow` [Huang, Lin, Lin and Lin 2022] and `Moonpool` [Cruanes 2025]. As of today, it supports three scheduling strategies: (1) standard randomized work-stealing [Blumofe and Leiserson 1999] with public deques (as presented in Section 4), (2) randomized work-stealing with private deques [Acar, Charguéraud and Rainey 2013], (3) a simple “first-in first-out” strategy with one shared queue. In addition, it should be possible to implement other scheduling strategies (see Section 16), *e.g.* work sharing.

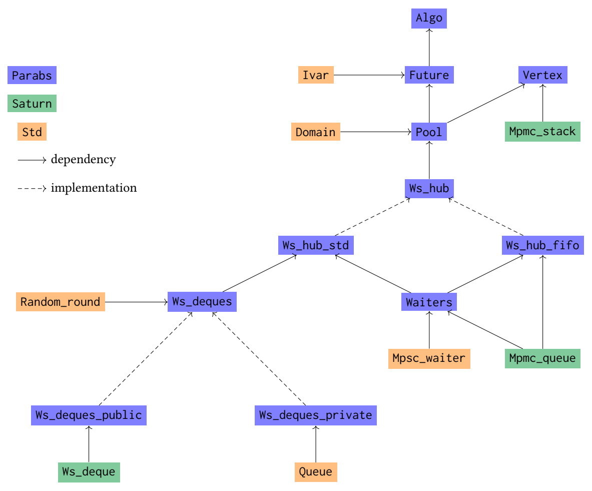

Fig. 10. Overview of the Parabs library

On top of **`Pool`**, the **`Vertex`** module provides a *task graph* abstraction. More precisely, it is an implementation of *DAG-calculus* [Acar, Charguéraud, Rainey and Sieczkowski 2016] — we present it in Section 13.

Remarkably, the three upper levels implemented on top of **`Ws_deques`** should be OCaml functors. Unfortunately, ZooLang does not currently support functors; therefore, only one branch of the tree of Figure 10 is active at a time.

7 Work-stealing deques

At the first level, **`Ws_deques`** provides a generic interface for a set of work-stealing deques, abstracting over the underlying scheduling strategy. It currently has two realizations: **`Ws_deques_public`** (Section 7.1) and **`Ws_deques_private`** (Section 7.2).

7.1 Public deques

The first realization, **`Ws_deques_public`**, implements the standard work-stealing strategy with *public deques*. More precisely, it simply relies on a shared array of Chase-Lev work-stealing deques (see Section 4). These deques are public in the sense that both their owner and the thieves can access it directly — which requires synchronization.

7.2 Private deques

The second realization, **`Ws_deques_private`**, implements the *receiver-initiated* work-stealing algorithm proposed by Acar, Charguéraud and Rainey [2013][4]. Their idea is to reduce synchronization costs in the fast path of local (owner-only) operations by essentially introducing an indirection. They show that this work-stealing strategy performs well for *fine-grained* parallel programs, *i.e.* when task sizes are small, especially irregular graph computations.

Instead of stealing directly from public deques, thieves follow a protocol: (1) having selected a victim, a thief attempts to send a request by atomically updating the *request cell* of the victim; (2) if the update fails, the thief starts over with another victim, otherwise it awaits a response by

[4]They also propose a *sender-initiated* algorithm that we have not implemented.

repeatedly checking its *response cell*; (3) if the response is negative, the thief starts over, otherwise it returns the task transferred by the victim.

Symmetrically, busy domains regularly poll their request cell and respond accordingly through response cells. Crucially, tasks are stored in private, non-concurrent deques that are only accessed by their owner. In addition, each domain has a *status cell* indicating whether it is (1) blocked, meaning it has no task to share, or (2) non-blocked, meaning it may have tasks to share; before sending a request, thieves check that their victim is non-blocked.

8 Waiters

In the realizations of the second level, described in the next section, we use a *sleep-based mechanism* to adapt the number of active thieves. The idea is to put to sleep desperate thieves who do not find work after a number of failed steal attempts. In practice, doing so can improve the overall system performance, especially when tasks are scarce.

To manage sleeping thieves, we use the **`Waiters`** module. Following the design of `Taskflow` [Huang, Lin, Lin and Lin 2022], it implements a *two-phase commit protocol*[5] — `Domainslib`[6] relies on a similar mechanism, although it is not as clear-cut.

9 Work-stealing hub

At the second level, **`Ws_hub`** provides a generic interface for a set of tasks supporting work-stealing operations — a so-called "work-stealing hub". It currently has two realizations: **`Ws_hub_std`** (Section 9.1) and **`Ws_hub_fifo`** (Section 9.2).

9.1 Work-stealing strategy

The first realization, **`Ws_hub_std`**, implements the standard randomized work-stealing strategy. Under the hood, any work-stealing algorithm may be used, provided that it fits into the **`Ws_hub`** interface; in particular, it can instantiated with both realization of **`Ws_deques`**.

9.2 FIFO strategy

The second realization, **`Ws_hub_fifo`**, implements a simple "first-in first-out" scheduling strategy. All workers push and pop tasks from a shared concurrent queue taken from `Saturn`; thieves also attempts to pop from the queue. `Moonpool` adopted a similar strategy[7].

As explained by Cruanes[8], the point of this strategy is to provide better *latency* than work-stealing — as demanded by certain applications like network servers — at the cost of a lower throughput. Indeed, contrary to work-stealing, older tasks have priority over younger tasks.

However, this strategy may also have undesirable consequences. For example, in divide-and-conquer algorithms, this strategy corresponds to *breadth-first* search, whereas work-stealing corresponds to *depth-first* search. On large problems, the former may be unsustainable; on some benchmarks (see Section 14), especially for small cutoffs, `Moonpool` saturates the memory.

10 Pool

At the third level, **`Pool`** implements a task scheduler on top of a given realization of **`Ws_hub`**. It offers essentially the same functionalities as `Domainslib` with a few notable differences. (1) Exceptions raised by tasks are not caught and therefore not re-raised properly by the scheduler since ZooLang does not currently support them. (2) Since ZooLang does not support algebraic

[5]https://www.1024cores.net/home/lock-free-algorithms/eventcounts
[6]https://github.com/ocaml-multicore/domainslib/blob/main/lib/multi_channel.ml
[7]https://github.com/c-cube/moonpool/blob/main/src/core/fifo_pool.ml
[8]https://github.com/c-cube/moonpool/blob/main/src/core/fifo_pool.mli

$$\mathsf{persistent}\ (\mathsf{inv}\ t\ sz) \qquad \mathsf{persistent}\ (\mathsf{obligation}\ t\ P) \qquad \mathsf{persistent}\ (\mathsf{finished}\ t)$$

$$\overset{\textsc{pool-inv-agree}}{\dfrac{\mathsf{inv}\ t\ sz_1 \qquad \mathsf{inv}\ t\ sz_2}{sz_1 = sz_2}} \qquad \overset{\textsc{pool-obligation-finished}}{\dfrac{\mathsf{obligation}\ t\ P \qquad \mathsf{finished}\ t}{\triangleright \Box P}}$$

$$\textsc{pool-create-spec}\quad \begin{array}{c} 0 \leq sz \\ \hline \mathtt{create}\ sz \\ \hline t.\ \mathsf{inv}\ t\ sz\ * \\ \mathsf{model}\ t \end{array}$$

$$\textsc{pool-run-spec}\quad \begin{array}{l} \mathsf{model}\ t\ * \\ \quad \forall\ ctx\ scope. \\ \quad \mathsf{context}\ t\ ctx\ scope \mathrel{-\!\!*} \\ \quad \mathsf{wp}\ task\ ctx\ \left\{ v.\ \mathsf{context}\ t\ ctx\ scope * \Psi\ v \right\} \\ \hline \qquad\qquad \mathtt{run}\ t\ task \\ \hline \qquad\qquad v.\ \mathsf{model}\ t * \Psi\ v \end{array}$$

$$\textsc{pool-kill-spec}\quad \begin{array}{c} \mathsf{model}\ t \\ \hline \mathtt{kill}\ t \\ \hline ().\ \mathsf{finished}\ t \end{array}$$

$$\textsc{pool-size-spec}\quad \begin{array}{l} \mathsf{inv}\ t\ sz\ * \\ \mathsf{context}\ t\ ctx\ scope \\ \hline \qquad \mathtt{size}\ ctx \\ \hline res.\ res = sz\ * \\ \quad \mathsf{context}\ t\ ctx\ scope \end{array}$$

$$\textsc{pool-async-spec}\quad \begin{array}{l} \mathsf{context}\ t\ ctx\ scope\ * \\ \quad \forall\ ctx\ scope. \\ \quad \mathsf{context}\ t\ ctx\ scope \mathrel{-\!\!*} \\ \quad \mathsf{wp}\ task\ ctx\ \left\{ \_.\ \mathsf{context}\ t\ ctx\ scope * \triangleright \Box P \right\} \\ \hline \qquad\qquad \mathtt{async}\ ctx\ task \\ \hline \qquad\quad ().\ \mathsf{context}\ t\ ctx\ scope\ * \\ \qquad\quad \mathsf{obligation}\ t\ P \end{array}$$

$$\textsc{pool-wait-until-spec}\quad \begin{array}{l} \mathsf{context}\ t\ ctx\ scope\ * \\ \{\,\mathsf{True}\,\}\ pred\ ()\ \{\,b.\ \mathbf{if}\ b\ \mathbf{then}\ P\ \mathbf{else}\ \mathsf{True}\,\} \\ \hline \qquad \mathtt{wait\_until}\ ctx\ pred \\ \hline \quad ().\ \mathsf{context}\ t\ ctx\ scope * P \end{array}$$

Fig. 11. **Pool**: Specification

effects [Sivaramakrishnan, Dolan, White, Kelly, Jaffer and Madhavapeddy 2021] neither, the interface is slightly more involved (see *execution contexts* in Section 10.1).

Moreover, this limitation imposes a *child-stealing* strategy, as opposed to a *continuation-stealing* strategy that would require capturing the continuation of a computation.

Also, this makes it difficult to implement a `yield` operation[9], *i.e.* an operation that yields control to the scheduler, letting it reschedule the current task later.

10.1 Specification

The specification is given in Figure 11. It features five predicates: inv, model, context, finished and obligation.

The persistent assertion inv t vsz represents the knowledge that t is a valid scheduler; sz is the number of worker domains. It is returned by `create` (POOL-CREATE-SPEC) and required only by `size` (POOL-SIZE-SPEC). Its only purpose is to record the immutable characteristics of the scheduler.

The assertion model t represents the ownership of scheduler t. It is returned by `create` (POOL-CREATE-SPEC) and required by external operations (POOL-RUN-SPEC, POOL-KILL-SPEC). For example, `run` t *task* submits *task* to scheduler t; it returns both model and the output predicate of *task*.

[9]`Domainslib` does not currently provide a `yield` operation but it can be easily implemented.

The assertion context *t ctx scope* represents the ownership of *execution context ctx* attached to scheduler *t*; *scope* is a purely logical parameter connecting input and output context, which is necessary in the proof. Any task execution happens under such a context (POOL-RUN-SPEC, POOL-ASYNC-SPEC, POOL-WAIT-UNTIL-SPEC). In particular, all internal operations require and return context. For example, `async` *ctx task* submits *task* asynchronously while executing under context *ctx*; *task* must be shown to execute safely under any context attached to the same scheduler (POOL-ASYNC-SPEC).

The persistent assertion finished *t* represents the knowledge that scheduler *t* has finished, meaning all submitted tasks were executed. It can be obtained by calling `kill` (POOL-KILL-SPEC).

The persistent assertion obligation *t P* represents a proof obligation attached to scheduler *t*. It allows retrieving *P* once *t* has finished executing (POOL-OBLIGATION-FINISHED). Obligations are obtained by submitting tasks through `async` (POOL-ASYNC-SPEC).

10.2 Implementation

Worker domains. The implementation relies on a pool of worker domains and a work-stealing hub. Each worker runs the following loop: (1) get a task using **Ws_hub**.`pop_steal`; (2) if it fails, the scheduler has been killed and so the worker stops, otherwise execute the task in the context of the current worker; (3) start over.

Blocking. Care must be taken to block and unblock work-stealing deques properly. When the scheduler is killed, it is crucial that workers block their deque before stopping; otherwise, the scheduler may never terminate because of a running worker waiting forever for a response from a stopped but unblocked worker. Also, the main domain, from which tasks can be submitted externally through `run`, must unblock when it is executing tasks and block when it is not.

Awaiting. `wait_until` runs a loop similar to that of the worker domains described above; the wait is *active* in the sense that the domain participate in the execution of tasks. Consequently, `wait_until` calls can be nested. This can be a problem in practice because it increases the call stack size in an arbitrary way, potentially causing stack overflow.

Instead, `Domainslib` leverages algebraic effects: awaiting a future captures the continuation and stores it into the future; when the future is resolved, it resubmits all the waiting tasks. This avoids any stack issue and is probably more efficient, since no polling is necessary.

Shutdown. In `Domainslib`, scheduler shutdown consists in submitting special tasks through the main domain; when a worker finds such a task, it quickly stops. However, this simple mechanism has at least two drawbacks: (1) it introduces an indirection for every regular task, which may be expensive; (2) it works well under standard work-stealing but is more difficult to implement under other scheduling strategies, especially work-stealing with private deques (see Section 7.2). Consequently, we use an alternative mechanism implemented at the level of **Ws_hub**: a shared flag, regularly checked in **Ws_hub**.`steal` and **Ws_hub**.`pop_steal`, is set when the scheduler is killed.

11 Futures

At the fourth level, **Future** implements futures[10], a standard abstraction for representing the future result of an asynchronous task.

$$\text{persistent}\ (\mathsf{inv}\ pool\ t\ depth\ \Psi\ \Xi) \qquad \text{persistent}\ (\mathsf{obligation}\ pool\ depth\ P) \qquad \text{persistent}\ (\mathsf{result}\ t\ v)$$

$$\textsc{future-result-agree}\quad \frac{\begin{array}{l}\mathsf{result}\ t\ v_1\\ \mathsf{result}\ t\ v_2\end{array}}{v_1 = v_2}$$

$$\textsc{future-inv-result}\quad \frac{\begin{array}{c}\mathsf{inv}\ pool\ t\ depth\ \Psi\ \Xi\\ \mathsf{result}\ t\ v\end{array}}{\Rrightarrow \triangleright \square\ \Xi\ v}$$

$$\textsc{future-inv-finished}\quad \frac{\begin{array}{c}\mathsf{inv}\ pool\ t\ depth\ \Psi\ \Xi\\ \mathsf{pool.finished}\ pool\end{array}}{\triangleright^{2 \cdot depth+1}\ \exists\ v.\ \mathsf{result}\ t\ v}$$

$$\textsc{future-consumer-divide}\quad \frac{\begin{array}{c}\mathsf{inv}\ pool\ t\ depth\ \Psi\ \Xi\\ \mathsf{consumer}\ t\ \mathrm{X}\\ \forall\ v.\ \mathrm{X}\ v \mathrel{-\!\!*} \mathop{\Huge *}_{\mathrm{X} \in \mathrm{X}s} \mathrm{X}\ v\end{array}}{\Rrightarrow \mathop{\Huge *}_{\mathrm{X} \in \mathrm{X}s} \mathsf{consumer}\ t\ \mathrm{X}}$$

$$\textsc{future-inv-result-consumer}\quad \frac{\begin{array}{c}\mathsf{inv}\ pool\ t\ depth\ \Psi\ \Xi\\ \mathsf{result}\ t\ v\\ \mathsf{consumer}\ t\ \mathrm{X}\end{array}}{\Rrightarrow \triangleright^{2}\ \mathrm{X}\ v}$$

$$\textsc{future-obligation-finished}\quad \frac{\begin{array}{c}\mathsf{obligation}\ pool\ depth\ P\\ \mathsf{pool.finished}\ pool\end{array}}{\triangleright^{2 \cdot depth+2}\ \square\ P}$$

$$\textsc{future-async-spec}\quad \frac{\begin{array}{l}\mathsf{pool.context}\ pool\ ctx\ scope\ *\\ \quad \forall\ ctx\ scope.\\ \quad \mathsf{pool.context}\ pool\ ctx\ scope \mathrel{-\!\!*}\\ \quad \mathsf{wp}\ task\ ctx \left\{\begin{array}{l} v.\ \mathsf{pool.context}\ pool\ ctx\ scope\ *\\ \quad \triangleright\ \Psi\ v\ *\\ \quad \triangleright\ \square\ \Xi\ v\end{array}\right\}\end{array}}{\dfrac{\mathtt{async}\ ctx\ task}{\begin{array}{l} t.\ \mathsf{pool.context}\ pool\ ctx\ scope\ *\\ \quad \mathsf{inv}\ pool\ t\ 0\ \Psi\ \Xi\ *\\ \quad \mathsf{consumer}\ t\ \Psi\end{array}}}$$

$$\textsc{future-wait-spec}\quad \frac{\begin{array}{l}\mathsf{pool.context}\ pool\ ctx\ scope\ *\\ \mathsf{inv}\ pool\ t\ depth\ \Psi\ \Xi\end{array}}{\dfrac{\mathtt{wait}\ ctx\ t}{\begin{array}{l} v.\ \pounds\ 2\ *\\ \quad \mathsf{pool.context}\ pool\ ctx\ scope\ *\\ \quad \mathsf{result}\ t\ v\end{array}}}$$

$$\textsc{future-iter-spec}\quad \frac{\begin{array}{l}\mathsf{pool.context}\ pool\ ctx\ scope\ *\\ \mathsf{inv}\ pool\ t\ depth\ \Psi\ \Xi\ *\\ \quad \forall\ ctx\ scope\ v.\\ \quad \mathsf{pool.context}\ pool\ ctx\ scope \mathrel{-\!\!*}\\ \quad \mathsf{result}\ t\ v \mathrel{-\!\!*}\\ \quad \mathsf{wp}\ task\ ctx\ v\\ \quad \left\{\begin{array}{l} ().\ \mathsf{pool.context}\ pool\ ctx\ scope\ *\\ \quad \triangleright\ \square\ P\end{array}\right\}\end{array}}{\dfrac{\mathtt{iter}\ ctx\ t\ task}{\begin{array}{l} ().\ \mathsf{pool.context}\ pool\ ctx\ scope\ *\\ \quad \mathsf{obligation}\ pool\ depth\ P\end{array}}}$$

$$\textsc{future-map-spec}\quad \frac{\begin{array}{l}\mathsf{pool.context}\ pool\ ctx\ scope\ *\\ \mathsf{inv}\ pool\ t_1\ depth\ \Psi_1\ \Xi_1\ *\\ \quad \forall\ ctx\ scope\ v_1.\\ \quad \mathsf{pool.context}\ pool\ ctx\ scope \mathrel{-\!\!*}\\ \quad \mathsf{result}\ t_1\ v_1 \mathrel{-\!\!*}\\ \quad \mathsf{wp}\ task\ ctx\ v_1\\ \quad \left\{\begin{array}{l} v_2.\ \mathsf{pool.context}\ pool\ ctx\ scope\ *\\ \quad \triangleright\ \Psi_2\ v_2\ *\\ \quad \triangleright\ \square\ \Xi_2\ v_2\end{array}\right\}\end{array}}{\dfrac{\mathtt{map}\ ctx\ t_1\ task}{\begin{array}{l} t_2.\ \mathsf{pool.context}\ pool\ ctx\ scope\ *\\ \quad \mathsf{inv}\ pool\ t_2\ (depth + 1)\ \Psi_2\ \Xi_2\ *\\ \quad \mathsf{consumer}\ t_2\ \Psi_2\end{array}}}$$

Fig. 12. **Future**: Specification

11.1 Specification

The specification is given in Figure 12. It features four predicates: inv, result, consumer and obligation.

[10]Futures are called *promises* in `Domainslib`. In fact, the two notions are often used in conjunction to represent the two sides of the same object.

`async` allows submitting a task asynchronously while executing under a context (FUTURE-SYNC-SPEC), returning a *future* representing the result of the task. To actually get the result, one must call `wait` (FUTURE-WAIT-SPEC). `iter` *ctx fut task* attaches callback *task* to *fut* (FUTURE-ITER-SPEC) and `map` *ctx* fut_1 *task* creates a new future to be resolved after fut_1 (FUTURE-MAP-SPEC).

The persistent assertion inv *pool t depth* Ψ Ξ represents the knowledge that t is a valid future attached to pool *pool* such that: (1) Ψ is the *non-persistent output predicate* satisfied by the produced value; (2) Ξ is the *persistent output predicate* satisfied by the produced value. *depth* is the depth of t in the forest formed by all futures.

The persistent assertion result t v represents the knowledge that future t has been resolved to value v. Using FUTURE-INV-RESULT, it can also be combined with inv to obtain the persistent output predicate. After the pool has finished, it is guaranteed that all futures have been resolved (FUTURE-INV-FINISHED).

The assertion consumer t X represents the right to consume X once future t has been resolved. Indeed, using FUTURE-INV-RESULT-CONSUMER, it can be combined with inv and result to obtain X. When t is created, this assertion is produced with the full non-persistent predicate (FUTURE-ASYNC-SPEC, FUTURE-MAP-SPEC); then, it can be divided into several parts (FUTURE-CONSUMER-DIVIDE).

The persistent assertion obligation *pool depth* P represents a proof obligation emitted by `iter` (FUTURE-ITER-SPEC). It allows retrieving P once *pool* has finished (FUTURE-OBLIGATION-FINISHED).

One notable aspect of this specification is that resolution of the future — as indicated by result — is separated from the division of the output predicates — as achieved by consumer.

11.2 Implementation

Futures are implemented using *ivars* (concurrent write-once variables), as implemented and verified in the Zoo standard library. `async` creates an ivar and calls **`Pool`**`.async` to resolve it asynchronously. `wait` calls **`Pool`**`.wait_until` to wait *actively* until the ivar is resolved and returns the resulting value.

12 Parallel iterators

On top of **`Future`**, we implemented and verified standard parallel iterators that are particularly useful for benchmarks (see Section 14): `for_`, `for_each`, `fold` and `find`.

13 Vertex

At the fourth level, **`Vertex`** implements *DAG-calculus* [Acar, Charguéraud, Rainey and Sieczkowski 2016], *i.e.* a task graph abstraction. `Taskflow` offers similar, although much more developed, abstractions. The longer term goal is to support the more practical `Taskflow` interface, including static, dynamic, module and condition tasks.

The raison d'être of these works is to represent more interesting dependency relations than is possible using standard parallel primitives (`fork`/`join`, futures, *etc.*) in order to express irregular parallel computations, *e.g.* those for graph problems.

This takes the form of a simple and elegant programming model: a parallel computation is seen as a graph where vertices represent basic sequential computations and edges represent dependencies between vertices. A vertex can be executed only when its predecessors, *i.e.* dependencies, are finished. Crucially, the structure of the graph is not static: while executing, a vertex may create new vertices and edges. Naturally, with great expressivity comes great responsibility: care must be taken not to introduce cycles in the graph, although the model does allow looping on a vertex.

$$\text{persistent}\ (\text{inv}\ t\ P\ R) \qquad \text{persistent}\ (\text{ready}\ \mathit{iter}) \qquad \text{persistent}\ (\text{finished}\ t)$$

$$\text{persistent}\ (\text{predecessor}\ t\ \mathit{iter})$$

$$\textsc{vertex-model-exclusive}\quad \frac{\begin{array}{l}\text{model}\ t\ \mathit{task}_1\ \mathit{iter}_1\\ \text{model}\ t\ \mathit{task}_2\ \mathit{iter}_2\end{array}}{\text{False}}$$

$$\textsc{vertex-model-finished}\quad \frac{\begin{array}{c}\text{model}\ t\ \mathit{task}\ \mathit{iter}\\ \text{finished}\ t\end{array}}{\text{False}}$$

$$\textsc{vertex-output-divide}\quad \frac{\begin{array}{c}\text{inv}\ t\ P\ R\\ \text{output}\ t\ Q\\ Q \mathrel{-\!\!*} \mathop{\Huge *}_{Q \in Qs} Q\end{array}}{\Rrightarrow \mathop{\Huge *}_{Q \in Qs} \text{output}\ t\ Q}$$

$$\textsc{vertex-predecessor-finished}\quad \frac{\begin{array}{l}\text{predecessor}\ t\ \mathit{iter}\\ \text{ready}\ \mathit{iter}\end{array}}{\text{finished}\ t}$$

$$\textsc{vertex-inv-finished}\quad \frac{\begin{array}{l}\text{inv}\ t\ P\ R\\ \text{finished}\ t\end{array}}{\Rrightarrow \triangleright \Box R}$$

$$\textsc{vertex-inv-finished-output}\quad \frac{\begin{array}{l}\text{inv}\ t\ P\ R\\ \text{finished}\ t\\ \text{output}\ t\ Q\end{array}}{\Rrightarrow \triangleright^2 Q}$$

$$\textsc{vertex-create-spec}\quad \begin{array}{l}\hline \text{True} \\ \hline \texttt{create}\ \mathit{task} \\ \hline t.\ \exists\ \mathit{iter}.\\ \quad \text{inv}\ t\ P\ R \ast\\ \quad \text{model}\ t\ (\text{option.get}\ (\texttt{fun: <> => ()})\ \mathit{task})\ \mathit{iter} \ast\\ \quad \text{output}\ t\ P\end{array}$$

$$\textsc{vertex-task-spec}\quad \begin{array}{l}\text{model}\ t\ \mathit{task}\ \mathit{iter}\\ \hline \texttt{task}\ t\\ \hline \mathit{res}.\ \mathit{res} = \mathit{task} \ast\\ \quad \text{model}\ t\ \mathit{task}\ \mathit{iter}\end{array}$$

$$\textsc{vertex-set-task-spec}\quad \begin{array}{c}\text{model}\ t\ \mathit{task}_1\ \mathit{iter}\\ \hline \texttt{set\_task}\ t\ \mathit{task}_2\\ \hline ().\ \text{model}\ t\ \mathit{task}_2\ \mathit{iter}\end{array}$$

$$\textsc{vertex-precede-spec}\quad \begin{array}{l}\text{inv}\ t_1\ P_1\ R_1 \ast\\ \text{inv}\ t_2\ P_2\ R_2 \ast\\ \text{model}\ t_2\ \mathit{task}\ \mathit{iter}\\ \hline \texttt{precede}\ t_1\ t_2\\ \hline ().\ \text{model}\ t_2\ \mathit{task}\ \mathit{iter} \ast\\ \quad \text{predecessor}\ t_1\ \mathit{iter}\end{array}$$

$$\textsc{vertex-release-spec}\quad \begin{array}{l}\text{pool.context}\ \mathit{pool}\ \mathit{ctx}\ \mathit{scope} \ast\\ \text{inv}\ t\ P\ R \ast\\ \text{model}\ t\ \mathit{task}\ \mathit{iter} \ast\\ \quad \forall\ \mathit{pool}\ \mathit{ctx}\ \mathit{scope}\ \mathit{iter}'.\\ \quad \text{pool.context}\ \mathit{pool}\ \mathit{ctx}\ \mathit{scope} \mathrel{-\!\!*}\\ \quad \text{ready}\ \mathit{iter} \mathrel{-\!\!*}\\ \quad \text{model}\ t\ \mathit{task}\ \mathit{iter}' \mathrel{-\!\!*}\\ \quad \text{wp}\ \mathit{task}\ \mathit{ctx} \left\{\begin{array}{l}().\ \exists\ \mathit{task}.\\ \quad \text{pool.context}\ \mathit{pool}\ \mathit{ctx}\ \mathit{scope} \ast\\ \quad \text{model}\ t\ \mathit{task}\ \mathit{iter}' \ast\\ \quad \triangleright P \ast\\ \quad \triangleright \Box R\end{array}\right\}\\ \hline \texttt{release}\ \mathit{ctx}\ t\\ \hline ().\ \text{pool.context}\ \mathit{pool}\ \mathit{ctx}\ \mathit{scope}\end{array}$$

Fig. 13. **`Vertex`**: Specification

13.1 Specification

The specification is given in Figure 13. It features no less than six predicates: inv, model, ready, output, finished and predecessor.

The persistent assertion inv *t P R* represents the knowledge that *t* is a valid vertex; *P* is the *non-persistent* output while *R* is the *persistent* output. It is returned by `create` (VERTEX-CREATE-SPEC) and required by most operations.

The exclusive assertion model *t task iter* represents the ownership of vertex *t*. It is returned by `create` (VERTEX-CREATE-SPEC). *task* is the current computation attached to *t*; it can accessed using `task` (VERTEX-TASK-SPEC) and `set_task` (VERTEX-SET-TASK-SPEC). *iter* is the current *logical iteration* of *t*. Indeed, a vertex may be executed several times; more precisely, a vertex task returns a boolean indicating whether the vertex should be re-executed.

The persistent assertion ready *iter* represents the knowledge that the iteration identified by *iter* has started — it may be finished and obsoleted by subsequent iterations.

The assertion output *t Q* represents the right to consume *Q* from the non-persistent output of *t* once the latter has finished executing. It is returned by `create` (VERTEX-CREATE-SPEC) with the full non-persistent output and can then be divided using VERTEX-OUTPUT-DIVIDE.

The persistent assertion finished *t* represents the knowledge that vertex *t* has finished executing. It allows retrieving both the persistent (VERTEX-INV-FINISHED) and non-persistent (VERTEX-INV-FINISHED-OUTPUT) output of *t*.

The persistent assertion predecessor *t iter* represents the knowledge that iteration *iter* has predecessor *t*, *i.e.* *iter* can only run once vertex *t* has finished (VERTEX-PREDECESSOR-FINISHED). It can be obtained through `precede` (VERTEX-PRECEDE-SPEC), including while the target vertex is executing; in other words, a vertex may add dependencies to itself so that its next iteration only starts when the new dependencies have finished.

The most important operation is `release` (VERTEX-RELEASE-SPEC), which declares a vertex ready for execution, provided that its dependencies (more precisely, those of the corresponding iteration) have finished. The current task must be shown to execute safely in any execution context given back the possession of the vertex and produce the two outputs.

13.2 Implementation

Our implementation is very close to that of Acar, Charguéraud, Rainey and Sieczkowski [2016]. The representation of a vertex consists of: (1) the current task, (2) an atomic counter corresponding to the number of unfinished predecessors, (3) a closable concurrent stack from `Saturn` corresponding to the successors. When creating a new edge through `precede`, the target is added to the successors of the source and the counter of the target is incremented. After executing, a vertex atomically closes its successors and decrements their counter, releasing those with zero remaining predecessors.

Actually, a vertex counter does not exactly correspond to the number of predecessors. Before the vertex is released for the first time and during its execution, there is one phantom predecessor preventing premature release; it is removed by `release`.

14 Benchmarks

We ran several concurrent benchmarks exercising three scheduler implementations: our own `Parabs` scheduler, the reference `Domainslib` library, and its alternative `Moonpool`. The benchmark results validate our qualitative claim that the performance of `Parabs` is on par with that of `Domainslib`; we found that it is as efficient, and even slightly faster on some benchmarks.

Due to space constraints, we do not present our benchmark results here. They can be found in Appendix A.

15 Related work

Chase-Lev work-stealing deque. Jung, Lee, Choi, Kim, Park and Kang [2023][11] were the first to achieve foundational verification of the Chase-Lev work-stealing deque, including safe memory reclamation schemes. Before, Lê, Pop, Cohen and Nardelli [2013] presented a pen-and-paper proof of the correctness of an ARM implementation and Mutluergil and Tasiran [2019] verified an idealized implementation based on an infinite array using CIVL [Kragl and Qadeer 2021].

As explained in Section 4, however, Jung, Lee, Choi, Kim, Park and Kang [2023] only verify a weak specification, too weak to prove the termination of our scheduler. We verify a strong specification but, contrary to Lê, Pop, Cohen and Nardelli [2013], we rely on a sequentially consistent memory model; extending our work to relaxed memory is left for future work (see Section 16).

Parallel scheduler. To the best of our knowledge, `Parabs` is the first realistic scheduler to be verified in Iris. Previous works cover toy implementations, not suitable for real-world usage; in contrast, our implementation is close to state-of-the-art schedulers and offers comparable performance according to our preliminary experiments.

De Vilhena and Pottier [2021] verify a simple cooperative scheduler based on algebraic effects, as a case study for their Iris-based program logic. This scheduler does not support parallelism; it runs fibers inside a single domain. Their notion of future/promise is rudimentary; it only supports persistent output predicates. However, their work, especially the way they formalize the scheduler's effects, will be of particular interest when introducing algebraic effects into ZooLang and `Parabs`.

Ebner, Martínez, Rastogi, Dardinier, Frisella, Ramananandro and Swamy [2025] verify a parallel scheduler with the same interface as `Domainslib`, which also serves as a case-study for their program logic. However, their implementation is extremely simplified: a task list protected by a mutex. Their notion of future/joinable is also somewhat rudimentary.

16 Future work

Relaxed memory model. The main limitation of our work is inherited from Zoo: it relies on a sequentially consistent memory model whereas OCaml 5 has a relaxed memory model [Dolan, Sivaramakrishnan and Madhavapeddy 2018]. This simplification endangers the soundness of our specifications. Transitioning to relaxed memory by merging Zoo with Cosmo [Mével and Jourdan 2021; Mével, Jourdan and Pottier 2020] involves introducing memory views, which complicates specifications and invariants.

Language features. `Parabs` suffers from the lack of a number of language features unsupported by Zoo. With functors, we could make the `Parabs` library completely modular. With exceptions, we could catch and re-raise exceptions in **`Pool`** and **`Vertex`**. With algebraic, we could get rid of evaluation contexts in **`Pool`** and use continuation-stealing.

Extensions. In the future, we would like to extend the library in several directions: (1) develop the interface of futures, similarly to `Moonpool`[12]; (2) support the different task types of `Taskflow`, aiming at a more practical **`Vertex`** interface.

Other designs. We could experiment other designs. For instance, one of the two designs of `Moonpool` relies on a bounded work-stealing deque combined with a master queue. In the literature, many other scheduling strategies were proposed: continuation-stealing [Schmaus, Pfeiffer, Schröder-Preikschat, Hönig and Nolte 2021; Williams and Elliott 2025], steal-half work-stealing [Hendler and Shavit 2002], split work-stealing [Cartier, Dinan and Larkins 2021; Custódio, Paulino and Rito

[11]See also the master thesis of Choi [2023].
[12]https://github.com/c-cube/moonpool/blob/main/src/core/fut.mli

2023; Dinan, Larkins, Sadayappan, Krishnamoorthy and Nieplocha 2009; Rito and Paulino 2022; van Dijk and van de Pol 2014], idempotent work-stealing [Michael, Vechev and Saraswat 2009].

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A Benchmarks

In this section, we present simple benchmarks to assess the performance of `Parabs` relatively to `Domainslib` [Multicore OCaml development team 2025] and `Moonpool` [Cruanes 2025] on simple workloads. Benchmarking parallel schedulers is subtle and difficult; we have not tried here to validate and study experimentally all our implementation choices, or to cover the wide range of parallel workloads, but to validate a simple qualitative claim:

> For CPU-bound tasks, `Parabs` has comparable throughput to `Domainslib`, a state-of-the-art scheduler used in production in the OCaml 5 library ecosystem.

In fact our results validate a stronger qualitative claim: the performance of `Parabs` are equal or better than `Domainslib`, with a 10% speedup in some cases.

A.1 Setting

A.1.1 Machine. The benchmark results were produced on a 12-core AMD Ryzen 5 7640U machine, set at a fixed frequency of 2GHz.

A.1.2 Parameters. For each benchmark, we pick an input parameter that gives long-enough computation times on our test machine, typically between 200ms and 2s. We use the `hyperfine` tool and run each benchmark ten time. All benchmark were run with two parameters varying:

- `DOMAINS`, the number of domains used for computation;
- `CUTOFF`, representing an input size or chunk size below which a sequential baseline is used.

For each benchmark, we show:

- per-cutoff results with a fixed value `DOMAINS` = 6, which should be enough to experience scaling issues while not suffering from CPU contention;
- per-domain results with a `CUTOFF` value that is chosen to work well for all implementations for this benchmark.

Remark: Large cutoff values tend to work well for benchmarks with homogeneous-enough tasks, as they effectively amortize the scheduling costs. The advantage of having schedulers that also perform well on small cutoffs are two-fold. First, this typically indicate that they will adapt to irregular tasks (but: our benchmarks do not perform an in-depth exploration of irregular workloads). Second, this can alleviate the burden of asking users to choose cutoff sizes (by widening the range of values that perform well), an activity which requires cumbersome hand-tuning and can limit performance portability.

A.1.3 Scheduler implementations. Each benchmark is written on top of a simple scheduler interface, for which the following implementations are provided:

- `domainslib` uses the `Domainslib` library;
- `parabs` uses our `Parabs` library;
- `moonpool-fifo` uses the `Moonpool` scheduler with a global FIFO queue of task;
- `moonpool-ws` uses the `Moonpool` scheduler with a work-stealing pool of tasks, which is described as better for throughput
- `sequential` is a baseline implementation with no parallelism, all tasks run sequentially on a single domain.

We used the latest software versions currently available: `Domainslib` 0.5.2, and `Moonpool` 0.9.

A.1.4 Benchmarks.

`fibonacci`. A parallel implementation of Fibonacci extended with a sequential cutoff: below the cutoff value, a sequential implementation is used.

`iota`. This benchmark uses a parallel-for to write a default value in each cell of an array. We expect significant variations due to the `CUTOFF` parameter.

`for_irregular`. This benchmark uses a parallel-for loop with irregular per-element workload: as a first approximation, the i-th iteration computes `fibonacci` i; this cost grows exponentially in i, so the majority of computation work is concentrated on the largest loop indices.

`lu`. This benchmark performs the LU factorization of a random matrix of floating-point values. It consists in $O(N)$ repetitions of a parallel-for loop of $O(N)$ iterations, where each iteration performs $O(N)$ sequential work.

`matmul`. This benchmark computes matrix multiplication with a very simple parallelization strategy — only the outer loop is parallelized. In other word, there is a parallel-for loop with $O(N)$ iterations, where each iteration performs $O(N^2)$ sequential work work.

A.2 Results

A.2.1 Pre-benchmarking expectations. Our expectation before running the benchmarks is that `Parabs` has the same performance as `Domainslib`, and that they are both more efficient than `Moonpool` (which uses a central pool of jobs instead of per-domain deques).

Because `Moonpool` has a less optimized scheduler, we expect scheduling overhead to be an issue for small `CUTOFF` values.

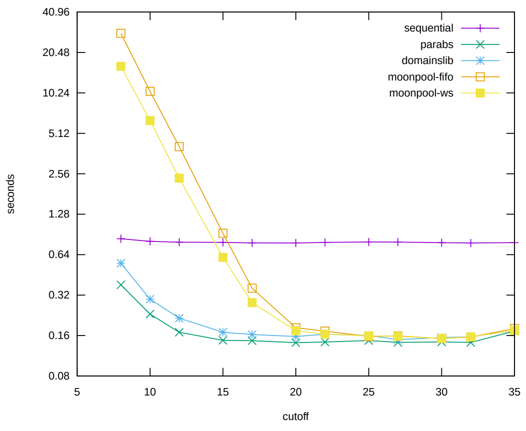
On all schedulers, the performance for larger `CUTOFF` values should be good if the benchmark has homogeneous/regular tasks, and it should be worse if the benchmark has heterogeneous/irregular tasks.

A.2.2 Per-benchmark results. Figure 14 and Figure 14 contain the full results, with per-cutoff and per-domain plots for each benchmarks. Notice that while the per-domain plot always use linear axes, the per-cutoff plots often use logarithmic plot axes, to preserve readability when performance difference become very large for small cutoff values, and to express large ranges of possible cutoff choices.

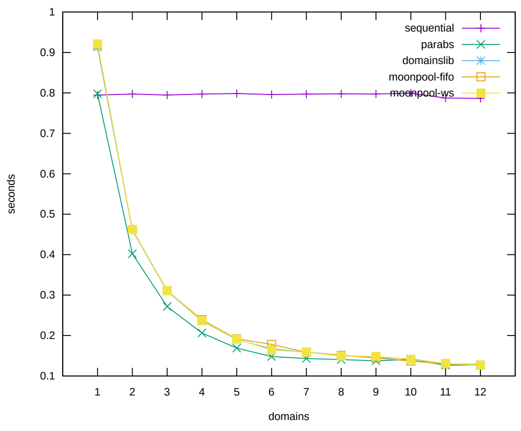
`fibonacci`. In the per-cutoff results (logarithmic scale), we see that all schedulers start to behave badly when the `CUTOFF` becomes small enough, with exponentially-decreasing performance after a certain drop point. For `Moonpool`, performance drops around `CUTOFF` = 20. The FIFO and work-stealing variants have similar profiles, with work-stealing performing noticeably better. For `Parabs` and `Domainslib`, performance drops around `CUTOFF = 12`. `Parabs` performs noticeably better for small-enough cutoff values. In fact, even for the sequential scheduler we observe a small performance drop: the task-using version creates closures and performs indirect calls, so it is noticeably slower (by a constant factor) than the version used below the sequential cutoff.

Note: we observe very large memory usage with `Moonpool` at smaller cutoff values — when computing fibonacci 40, attempting to run the benchmark with `CUTOFF = 5` fails with out-of-memory errors on a machine with 32Gio of RAM. This seems to come from the FIFO architecture which runs the oldest and thus biggest task first, and thus stores an exponential number of smaller tasks in the queue.

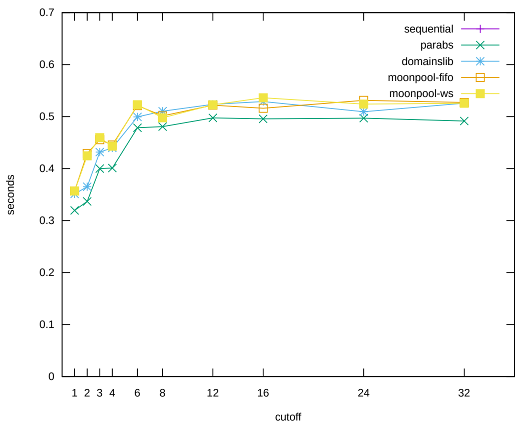
Per-domain results (linear scale): we studied per-domain performance on a `CUTOFF` = 25 point where all implementations behave well. For this value we see that `parabs` and `domainslib` perform similarly, and both `moonpool` implementations are measurably slower. Performance becomes very close for larger number of domains (`DOMAINS` $\geq$ 7).



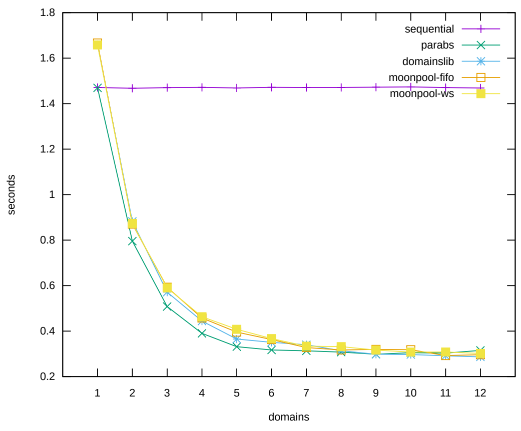
(a) `fibonacci`: varying `CUTOFF`



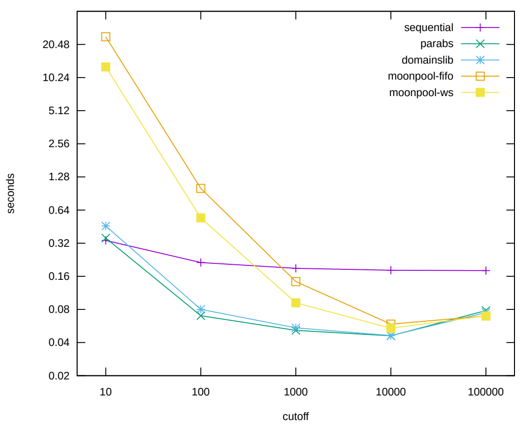
(b) `fibonacci`: varying `DOMAINS`



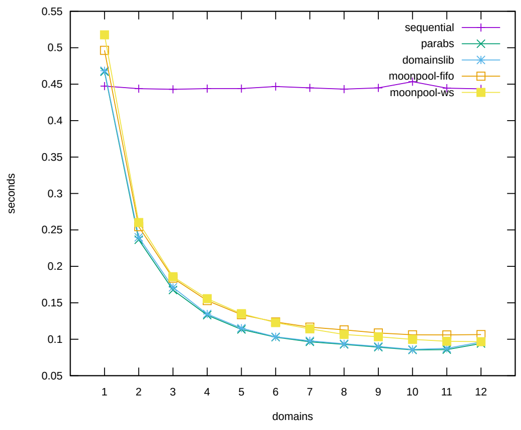
(c) `for_irregular`: varying `CUTOFF`



(d) `for_irregular`: varying `DOMAINS`

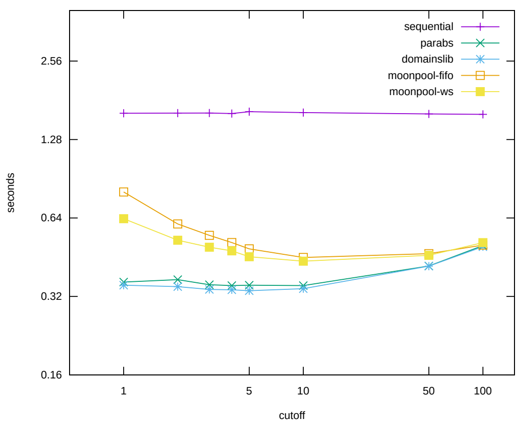


(e) `iota`: varying `CUTOFF`

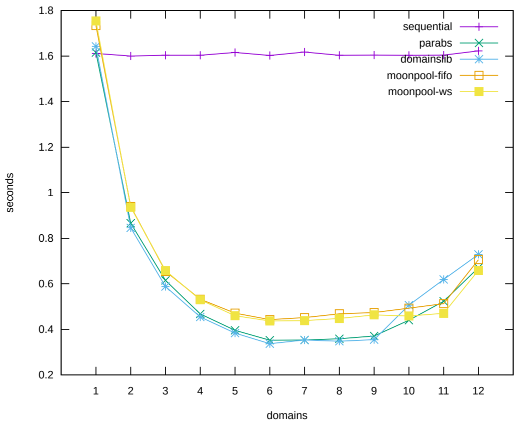


(f) `iota`: varying `DOMAINS`

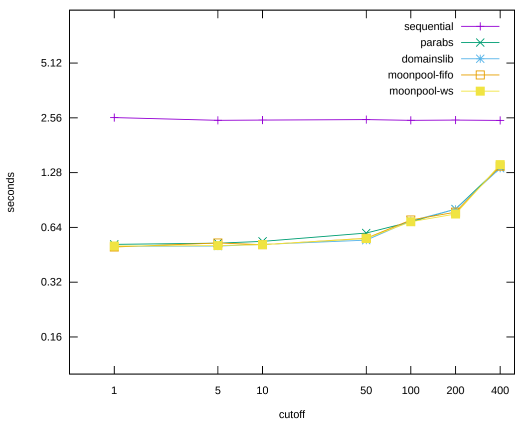
Fig. 14. Benchmarks (1/2)



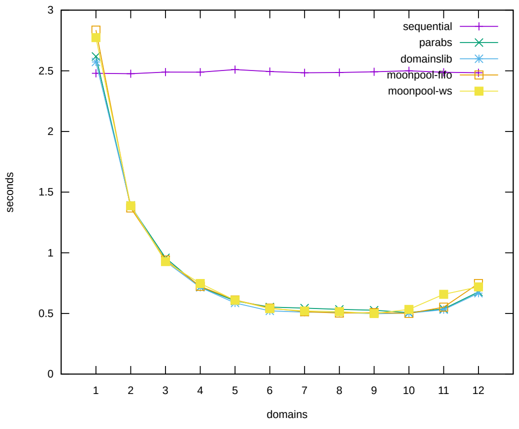
(g) `lu`: varying `CUTOFF`



(h) `lu`: varying `DOMAINS`



(i) `matmul`: varying `CUTOFF`



(j) `matmul`: varying `DOMAINS`

Fig. 14. Benchmarks (2/2)

`for_irregular`. This benchmark is designed to behave poorly with large `CUTOFF` values. We indeed observe better noticeably performance with `CUTOFF` = 1 than with larger values, across all schedulers — for example `domainslib` is 50% slower with `CUTOFF` = 8.

In the per-cutoff results we observe that `parabs` performs best on this benchmark, then `domainslib`, then `moonpool`.

In the per-domain results (with `CUTOFF` = 1) we see that `parabs` performs noticeably better than the other implementations for relatively low domain counts, and they become comparable around `DOMAINS` ≥ 7.

`iota`. Each iteration of parallel-for in `iota` is immediate, so as expected we observe a large sensitivity to the choice of `CUTOFF`, with `parabs` and `domainslib` performing much better than `moonpool` on smaller `CUTOFF` values.

In the per-domains result we see that `domainslib` and `parabs` have similar performance, noticeably better than the `moonpool` implementations.

lu. The performance is relatively stable over most choices of CUTOFF. The per-domain results are similar across all benchmarks after controlling for the one-domain shift of Moonpool.

Remark: we observe a marked decline in performance, across all schedulers, when the number of domains becomes close to the number of available cores, around DOMAINS $\geq$ 10. We believe that this comes from the high-allocation rate of this benchmark (10.2GiB/s) causing frequent minor collections, and thus stop-the-world pauses, with some domains temporarily suspended by the operating system. In other words, the slowdown comes from the OCaml runtime, not from the scheduler implementations. The allocations can be avoided in this benchmark by optimizing more agressively to eliminate float boxing, but this phenomenon is likely to occur for other high-allocation OCaml programs so we chose to preserve it.

matmul. The performance is stable across a wide range of CUTOFF values. The parallel-loop performs 500 iterations, so CUTOFF values closer to 500 prevent parallelization and bring performance closer to the sequential scheduler.

The per-domain performance is remarkably similar under all schedulers: our implementation of matrix multiplication has a coarse-grained parallelization strategy where the choice of scheduler makes no difference.

A.2.3 Result summary. Overall, Parabs has the same qualitative performance as Domainslib. In fact it performs measurably better (around 10% better for some domain values) on the benchmarks fibonacci and for_irregular, which have irregular tasks; and it has qualitatively the same performance otherwise.