

Zoo: A framework for the verification of concurrent OCaml 5 programs using separation logic

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The release of OCaml 5, which introduced parallelism in the OCaml runtime, drove the need for safe and efficient concurrent data structures. New libraries like Saturn address this need. This is an opportunity to apply and further state-of-the-art program verification techniques.

We present Zoo, a framework for verifying fine-grained concurrent OCaml 5 algorithms. Following a pragmatic approach, we defined a limited but sufficient fragment of the language to faithfully express these algorithms: ZooLang. We formalized its semantics carefully via a deep embedding in the Rocq proof assistant, uncovering subtle aspects of physical equality. We provide a tool to translate source OCaml programs into ZooLang syntax embedded inside Rocq, where they can be specified and verified using the Iris concurrent separation logic. To illustrate the applicability of Zoo, we verified a subset of the standard library and a collection of fined-grained concurrent data structures from the Saturn and Eio libraries.

In the process, we also extended OCaml to more efficiently express certain concurrent programs.

1 Introduction

OCaml 5.0 was released on December 15th 2022, the first version of the OCaml programming language to support parallel execution of OCaml threads by merging the Multicore OCaml runtime [Sivaramakrishnan, Dolan, White, Jaffer, Kelly, Sahoo, Parimala, Dhiman and Madhavapeddy 2020]. It provided basic support in the language runtime to start and stop coarse-grained threads called “domains”, and support for strongly sequential atomic references in the standard library. The third-party library `domainslib` offered a simple scheduler for a pool of tasks, used to benchmark the parallel runtime. A world of parallel and concurrent software was waiting to be invented.

Shared-memory concurrency is a difficult programming domain, and existing ecosystems (C++, Java, Haskell, Rust, Go...) took decades to evolve comprehensive libraries of concurrent abstractions and data structures. In the last couple years, a handful of contributors to the OCaml ecosystem have been implementing basic libraries for concurrent and parallel programs in OCaml, in particular Saturn [Karvonen and Morel 2025], a library of lock-free parallelism-safe data structures (stacks, queues, a work-stealing deque, a skip list, a bag and a hash table), Eio [Madhavapeddy and Leonard 2025], a library of asynchronous IO and structured concurrency, and Kcas [Karvonen 2025a], a library offering a software-transactional-memory abstraction.

Concurrent algorithms and data structures are extremely difficult to reason about. Their implementation tend to be fairly short, a few dozens of lines. There is only a handful of experts able to write such code, and many potential users. They are difficult to test comprehensively. These characteristics make them ideally suited for mechanized program verification.

We embarked on a mission to mechanize correctness proofs of OCaml concurrent algorithms and data structures as they are being written, in contact with their authors, rather than years later. In the process, we not only gained confidence in these complex new building blocks, but we also improved the OCaml language and its verification ecosystem.

OCaml language features. When studying the new codebases of concurrent and parallel data structures, we found a variety of unsafe idioms, working around expressivity or performance limitations with the OCaml language support for lock-free concurrent data structures. In particular, the support for *atomic references* in the OCaml library proved inadequate, as idiomatic concurrent

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data-structures need the more expressive feature of *atomic record fields*. We designed an extension of OCaml with atomic record fields, implemented it as an experimental compiler variant, and succeeded in getting it integrated in the upstream OCaml compiler: it should be available as part of OCaml 5.4, which is not yet released at the time of writing.

Verification tools for concurrent programs. The state-of-the-art approach for mechanized verification of fine-grained concurrent algorithms is Iris [Jung, Krebbers, Jourdan, Bizjak, Birkedal and Dreyer 2018], a mechanized *higher-order* concurrent separation logic with *user-defined ghost state*. Its expressivity allows to precisely capture delicate invariants, and to reason about the linearization points of fine-grained concurrent algorithms, including external [Vindum, Frumin and Birkedal 2022] and future-dependent [Chang, Jung, Sharma, Tassarotti, Kaashoek and Zeldovich 2023; Jung, Lepigre, Parthasarathy, Rapoport, Timany, Dreyer and Jacobs 2020; Vindum and Birkedal 2021] linearization. Iris provides a generic mechanism to define programming languages and program logics for them. Most existing Iris concurrent verification work has been performed in HeapLang, the exemplar Iris language; a concurrent, imperative, untyped, call-by-value functional language.

To the best of our knowledge, it is currently the closest language to OCaml 5 in the Iris ecosystem — we review the existing frameworks in Section 12. We started our verification effort in HeapLang, but it eventually proved impractical to verify realistic OCaml libraries. Indeed, it lacks basic abstractions such as algebraic data types (tuples, mutable and immutable records, variants) and mutually recursive functions. Verifying OCaml programs in HeapLang requires difficult translation choices and introduces various encodings, to the point that the relation between the source and verified programs can become difficult to maintain and reason about. It also has very few standard data structures that can be directly reused. These limitations are well-known in the Iris community.

We created a new Iris language, ZooLang, that can better express concurrent OCaml programs. Its feature set grew over time as we applied it to more verification scenarios, and we now believe that it allows practical verification of fine-grained concurrent OCaml 5 programs — including the use of our atomic record fields which were co-designed with ZooLang. We were influenced by the Perennial framework [Chajed, Tassarotti, Kaashoek and Zeldovich 2019], which achieved similar goals for the Go language with a focus on crash-safety. As in Perennial, we also provide a translator from (a subset of) OCaml to ZooLang: `ocaml2zoo`. We start from OCaml code and call our translator to obtain a deep ZooLang embedding inside Rocq; we can use lightweight annotations to guide the translation. Inside Rocq we define specifications using Iris, and prove them correct with respect to the ZooLang version, which is syntactically very close to the original OCaml source. We call the resulting framework Zoo.

One notable current limitation of ZooLang is that it assumes a sequentially consistent memory model, whereas OCaml 5 has a relaxed memory model [Dolan, Sivaramakrishnan and Madhavapeddy 2018]. We wanted to ensure that we supported practical verification in a sequentially consistent setting first. In the future, we plan to adopt the OCaml memory model as formalized in Cosmo [Mével, Jourdan and Pottier 2020].

Specified OCaml semantics. Our Iris mechanization of ZooLang defines an operational semantics and a corresponding program logic. Our users on the other hand run their program through the standard OCaml implementation, which is not verified and does not have a precise formal specification. To bridge this formal-informal gap as well as reasonably possible, we carefully audit our ZooLang semantics to ensure that they coincide with OCaml's.

In doing so we discovered a hole in state-of-the-art language semantics for program verification (not just for OCaml), which is the treatment of *physical equality* (pointer equality). Physical equality is typically exposed to language users as an efficient but under-specified equality check, as the physical identity of objects may or may not be preserved by various compiler transformations It

is an essential aspect of concurrent programs, as it underlies the semantics of important atomic instructions such as `compare_and_set`. We found that the current informal semantics in OCaml is incomplete, it does not allow to reason on programs that use structured data which mix mutable and immutable constructors. Existing formalizations of physical equality in verification frameworks typically restrict it to primitive datatypes, but idiomatic concurrent programs do not fit within this restriction. We propose a precise specification of physical equality in Zoo that scales to the verification of all the concurrent programs we encountered.

Worse, our discussions with the maintainers of the OCaml implementation showed that implementors guarantee weaker properties of physical equalities than users assume, in particular they may allow *unsharing*, which makes some existing concurrent programs incorrect. We propose a small new language feature for OCaml, per-constructor unsharing control, which we also integrate in our ZooLang translation, to fix affected programs and verify them. Finally, we discussed these subtleties with authors who axiomatize physical equality within Rocq for the purpose of efficient extraction, and we found out that some subtleties we discovered could translate into incorrectness in their axiomatization, requiring careful restrictions.

Verification results. We verified a small library for ZooLang (essentially a subset of the OCaml standard library). It can serve as building blocks to define our concurrent data structures. We verified a specific component of the Eio library, whose author, Thomas Leonard, had pointed to us as being delicate to reason about and worth mechanizing. Finally, we verified a large subset of the Saturn library. Several of these data structures contained verification challenges, which we will describe in the relevant section. The main Saturn concurrent structures remaining unverified are a skiplist and a hash table; we have verified its work-stealing deque [Chase and Lev 2005] but do not discuss it here for space reasons.

Contributions. In summary, we claim the following contributions:

- (1) Zoo, a practical program verification framework aimed at concurrent OCaml programs, mechanized in Rocq. The language ZooLang comes with a program logic expressed in the Iris concurrent separation logic. A translator `ocaml2zoo` generates Rocq embeddings from source OCaml programs, and works well with OCaml tooling (dune support).
- (2) The verification (in a sequentially consistent model) of important structures coming from Saturn, the OCaml 5 library of lock-free data structures. Our implementations and invariants sometimes improve over the Iris state of the art for those data structures.
- (3) The extension of OCaml with atomic record fields, which after significant design, implementation and discussion work have now been integrated into upstream OCaml.
- (4) The identification of blind spots in existing specifications of *physical equality*, and a new specification precise enough to reason about `compare-and-set` in concurrent programs. In the process we identified a potential bug in existing OCaml programs related to *unsharing*, and we propose a small language extension to let users selectively disable unsharing.

Artifact. We include an anonymous artifact containing (1) our experimental fork of the OCaml compiler implementing atomic arrays (Section 4.2) and generative constructors (Section 5.3), (2) the source of the `ocaml2zoo` translator, and (3) the Zoo repository, which includes ZooLang and its metatheory (theories/zoo), verified OCaml source (in lib/) and corresponding proofs (in theories/). In particular, the output of `ocaml2zoo` is included; for example `mpmc_stack_1.ml` in `lib/zoo_saturn/` is translated into `mpmc_stack_1__code.v` in `theories/zoo_saturn/`.

```

type 'a t = 'a list Atomic.t
let create () = Atomic.make []

let rec push t v =
  let old = Atomic.get t in
  let new_ = v :: old in
  if not (Atomic.compare_and_set
    t old new_)
  then (
    Domain.cpu_relax () ;
    push t v
  )

let rec pop t =
  match Atomic.get t with
  | [] -> None
  | v :: new_ as old ->
    if Atomic.compare_and_set
      t old new_
    then Some v
    else (
      Domain.cpu_relax () ;
      pop t
    )

Definition stack_create : val :=
  fun: <> => ref [].


Definition stack_push : val :=
  rec: "push" "t" "v" =>
    let: "old" := !"t" in
    let: "new_" := "v" :: "old" in
    if: ~ CAS "t".[contents] "old" "new_"
    then (
      domain_yield () ;;
      "push" "t" "v"
    ).




Definition stack_pop : val :=
  rec: "pop" "t" =>
    match: !"t" with
    | [] => $None
    | ("v" :: "new_") as "old" =>
      if: CAS "t".[contents] "old" "new_"
      then 'Some( "v" )
      else (
        domain_yield () ;;
        "pop" "t"
      )
  end.

```

Fig. 1. A concurrent stack in OCaml and its automatic ZooLang translation

2 Zoo in practice

The core of Zoo is ZooLang : a concurrent, imperative, untyped, functional programming language fully formalized in Rocq. Its semantics has been designed to match OCaml.

ZooLang comes with an Iris-based program logic , proved correct with respect to its small-step operational semantics . The reasoning rules are expressed in separation logic (including rules for the different constructs of the language) along with Rocq tactics  that integrate into the Iris proof mode [Krebbers, Jourdan, Jung, Tassarotti, Kaiser, Timany, Charguéraud and Dreyer 2018]. In addition, it supports Diaframe [Mulder and Krebbers 2023; Mulder, Krebbers and Geuvers 2022], enabling proof automation.

2.1 Translation from OCaml to ZooLang

While ZooLang lives in Rocq, we want to verify OCaml programs. To connect them we provide the tool `ocaml2zoo` to translate OCaml source files¹ into Rocq files containing ZooLang code. This tool can process entire dune projects, and supports library dependencies.

The supported OCaml fragment includes: tuples, variants, records and inline records, shallow **match**, atomic record fields, unboxed types, toplevel mutually recursive functions.


¹Actually, `ocaml2zoo` processes binary annotation files (.cmt files).

| | | |
|-----------------|-----------|--|
| Rocq term | t | |
| constructor | C | |
| projection | $proj$ | |
| record field | fld | |
| identifier | s, f | $\in \text{String}$ |
| integer | n | $\in \mathbb{Z}$ |
| boolean | b | $\in \mathbb{B}$ |
| binder | x | $::= \langle \rangle \mid s$ |
| unary operator | \oplus | $::= \sim \mid -$ |
| binary operator | \otimes | $::= + \mid - \mid * \mid \text{'quot'} \mid \text{'rem'} \mid \text{'land'} \mid \text{'lor'} \mid \text{'lsl'} \mid \text{'lsr'}$ $\mid \leq \mid < \mid \geq \mid > \mid = \mid \neq \mid == \mid !=$ $\mid \text{and} \mid \text{or}$ |
| expression | e | $::= t \mid s \mid \#n \mid \#b$ $\mid \text{fun: } x_1 \dots x_n \Rightarrow e \mid \text{rec: } f x_1 \dots x_n \Rightarrow e \mid e_1 e_2$ $\mid \text{let: } x := e_1 \text{ in } e_2 \mid e_1 ;; e_2$ $\mid \text{let: } f x_1 \dots x_n := e_1 \text{ in } e_2 \mid \text{letrec: } f x_1 \dots x_n := e_1 \text{ in } e_2$ $\mid \text{let: } 'C x_1 \dots x_n := e_1 \text{ in } e_2 \mid \text{let: } x_1, \dots, x_n := e_1 \text{ in } e_2$ $\mid \oplus e \mid e_1 \otimes e_2$ $\mid \text{if: } e_0 \text{ then } e_1 \text{ (else } e_2)^?$ $\mid \text{for: } x := e_1 \text{ to } e_2 \text{ begin } e_3 \text{ end}$ $\mid \$C \mid 'C (e_1, \dots, e_n) \mid (e_1, \dots, e_n) \mid e. \langle proj \rangle$ $\mid [] \mid e_1 :: e_2$ $\mid 'C \{e_1, \dots, e_n\} \mid \{e_1, \dots, e_n\} \mid e. \{fld\} \mid e_1 \leftarrow \{fld\} e_2$ $\mid \text{ref } e \mid !e \mid e_1 \leftarrow e_2$ $\mid \text{match: } e_0 \text{ with } br_1 \mid \dots \mid br_n (_ (as s)^? \Rightarrow e)^? \text{ end}$ $\mid e. [fld] \mid Xchg e_1 e_2 \mid CAS e_1 e_2 e_3 \mid FAA e_1 e_2$ $\mid \text{Proph} \mid \text{Resolve } e_0 e_1 e_2$ |
| branch | br | $::= C (x_1 \dots x_n)^? (as s)^? \Rightarrow e$ $\mid [] (as s)^? \Rightarrow e \mid x_1 :: x_2 (as s)^? \Rightarrow e$ |
| toplevel value | v | $::= t \mid \#n \mid \#b$ $\mid \text{fun: } x_1 \dots x_n \Rightarrow e \mid \text{rec: } f x_1 \dots x_n \Rightarrow e$ $\mid \$C \mid 'C (v_1, \dots, v_n) \mid (v_1, \dots, v_n)$ $\mid [] \mid v_1 :: v_2$ |

Fig. 2. ZooLang syntax (omitting mutually recursive toplevel functions)

In [Figure 1](#) we include the OCaml implementation of a simple lock-free concurrent stack [[Treiber 1986](#)], and its automatic translation to ZooLang, demonstrating that readability is preserved. Readability is important as users constantly see program fragments during interactive verification.

2.2 The full language

The ZooLang syntax  is given in [Figure 2](#)², omitting mutually recursive toplevel functions that are treated specially. Expressions include standard constructs like booleans, integers, anonymous functions (that may be recursive), applications, **let** bindings, sequence, unary and binary operators, conditionals, **for** loops, tuples. In any expression, one can refer to a Rocq term representing a

²More precisely, it is the syntax of the surface language, including Rocq notations.

ZooLang value (of type `val`) using its Rocq identifier. ZooLang is deeply embedded: variables (bound by functions and **let**) are quoted as strings.

Data constructors (immutable memory blocks) are supported: `$C` represents a constant constructor (e.g. `$None`), and `'C (e1, ..., en)` represents a non-constant constructor (e.g. `'Some(e)`). Projections of the form `e.<proj>` (including on tuples: `(x, y).<1>`) can be used to obtain a specific component of a tuple or data constructor. ZooLang supports shallow pattern matching (patterns cannot be nested) on data constructors with an optional fallback case.

Mutable memory blocks are constructed using either the untagged record syntax `{e1, ..., en}` or the tagged record syntax `'C {e1, ..., en}`. Reading a record field can be performed using `e.{fld}` and writing to a record field using `e1 <- {fld} e2`. Pattern matching can also be used on mutable tagged blocks provided that cases do not bind anything — in other words, only the tag is examined, no memory access is performed. Mutable references are supported: `ref e` creates a reference, `!e` reads a reference and `e1 <- e2` writes into a reference. There is no built-in syntax for arrays; they are supported through the **Array** standard library module (e.g. `array_make`).

Note that ZooLang follows OCaml in sometimes eschewing orthogonality to provide more compact memory representations: constructors are n -ary instead of taking a tuple as parameter, and the tagged record syntax is distinct from a constructor taking a mutable record as parameter. In each case the simplifying encoding would introduce an extra indirection in memory, which is absent from the ZooLang semantics. Performance-conscious experts care about these representation choices, and we care about faithfully modeling their programs.

Parallelism is mainly supported through the **Domain** standard library module, including domain-local storage. Atomic operations are provided as built-in constructs (`Xchg`, `CAS`, `FAA`; see [Section 3.3](#)).

The `Proph` and `Resolve` constructs model *prophecy variables* [Jung, Lepigre, Parthasarathy, Rapoport, Timany, Dreyer and Jacobs 2020], see [Section 3.4](#).

2.3 Specifications and proofs

Once the translation to ZooLang is done, the user can write specifications and prove them in Iris. For instance, the specification of the `stack_push` function from [Figure 1](#) could be:

```
Lemma stack_push_spec t ι v :
  <<< stack_inv t ι
    | ∀ v s, stack_model t v s >>>
    stack_push t v @ ↑ι
  <<< stack_model t (v :: v s)
    | RET (); True >>>.
```

It uses a *logically atomic specification* [da Rocha Pinto, Dinsdale-Young and Gardner 2014], which has been proven [Birkedal, Dinsdale-Young, Guéneau, Jaber, Svendsen and Tzevelekos 2021] to be equivalent to *linearizability* [Herlihy and Wing 1990] in sequentially consistent memory models.

Similarly to **Hoare triples**, the specification is formed of a precondition and a postcondition, represented in angle brackets. But each is split in two parts, a *public* or *atomic* condition, and a *private* condition. Following standard Iris notations, the private conditions are on the outside (first line of the precondition, last line of the postcondition) and the atomic conditions are inside.

For this particular operation, the private postcondition is trivial. The private precondition `stack_inv t` is the stack invariant. Intuitively, it asserts that `t` is a valid concurrent stack. More precisely, it defines a concurrent protocol that `t` must respect at all times.

The atomic pre- and post-conditions specify the linearization point of the operation: during the execution of `stack_push`, the abstract state of the stack held by `stack_model` is atomically updated from `vs` to `v :: vs` when `v` is atomically pushed at the top of the stack.

3 Zoo features

In this section, we review the salient features of Zoo, including those that we found lacking when we attempted to use HeapLang to verify real-world OCaml programs. Providing a better Iris language than HeapLang for this problem domain is a contribution of our work; others are welcome to use ZooLang for their own verification effort, or to reuse our designs by integrating specific features into their own Iris language.

3.1 Algebraic data types

ZooLang is an untyped language but, to write interesting programs, it is convenient to work with abstractions like algebraic data types. To simulate tuples, variants and records, we designed a machinery to define projections, constructors and record fields.

For example, one may define a list-like type with:

Notation `"Nil"` := (in_type "t" 0) (in custom zoo_tag).

Notation `"Cons"` := (in_type "t" 1) (in custom zoo_tag).

Users do not need to write this incantation directly, as they are generated by `ocaml2zoo` from the OCaml type declarations. Suffice it to say that it introduces the two tags in the `zoo_tag` custom entry, on which the notations for data constructors rely. The `in_type` term is needed to distinguish the tags of distinct data types; crucially, it cannot be simplified away by Rocq, as this could lead to confusion during the reduction of expressions.

One may then directly use the tags `Nil` and `Cons` in data constructors:

Definition `map` : val := rec: "map" "fn" "t" =>
 match: "t" with
 | Nil => \$Nil
 | Cons "x" "t" =>
 let: "y" := "fn" "x" in
 'Cons("y", "map" "fn" "t")
 end.

Similarly, one can define a record-like type with two mutable fields `f1` and `f2`:

Notation `"f1"` := (in_type "t" 0) (in custom zoo_field).

Notation `"f2"` := (in_type "t" 1) (in custom zoo_field).

Definition `swap` : val := fun: "t" =>
 let: "f1" := "t".{f1} in
 "t" <-{f1} "t".{f2} ;; "t" <-{f2} "f1".

3.2 Mutually recursive functions

Zoo supports non-recursive (`fun: $x_1 \dots x_n \Rightarrow e$`) and recursive (`rec: $f \ x_1 \dots x_n \Rightarrow e$`) functions but only *oplevel* mutually recursive functions. It is non-trivial to properly handle mutual recursion: when applying a mutually recursive function, a naive approach would replace calls to sibling functions by their respective bodies, but this typically makes the resulting expression unreadable and can create proof-checking performance issues during verification. To prevent it, the mutually recursive functions have to know one another to preserve their names during β -reduction. For instance, one may define two mutually recursive functions `f` and `g` as follows. `ocaml2zoo` generates some additional boilerplate to control the recursive unfolding.

Definition `f_g` := (
 recs: "f" "x" => "g" "x"


```
and: "g" "x" => "f" "x"
)%zoo_recs.
```

3.3 Fine-grained concurrent primitives


Zoo supports concurrent primitives both on atomic references (from **Atomic**) and atomic record fields (from **Atomic.Loc**, see [Section 4.1](#)) according to the table below. The OCaml expressions listed on the left translate into the Zoo expressions on the right.

| OCaml | Zoo |
|--|---|
| Atomic .get e | ! e |
| Atomic .set e_1 e_2 | $e_1 <- e_2$ |
| Atomic .exchange e_1 e_2 | Xchg e_1 . <code>[contents]</code> e_2 |
| Atomic .compare_and_set e_1 e_2 e_3 | CAS e_1 . <code>[contents]</code> e_2 e_3 |
| Atomic .fetch_and_add e_1 e_2 | FAA e_1 . <code>[contents]</code> e_2 |
| Atomic.Loc .exchange [%atomic.loc e_1 . <code>f</code>] e_2 | Xchg e_1 . <code>[f]</code> e_2 |
| Atomic.Loc .compare_and_set [%atomic.loc e_1 . <code>f</code>] e_2 e_3 | CAS e_1 . <code>[f]</code> e_2 e_3 |
| Atomic.Loc .fetch_and_add [%atomic.loc e_1 . <code>f</code>] e_2 | FAA e_1 . <code>[f]</code> e_2 |

One notable aspect of this translation is that atomic accesses (**Atomic**.get and **Atomic**.set) correspond to plain loads and stores. This is because we are working in a sequentially consistent memory model: there is no difference between atomic and non-atomic memory locations.

3.4 Prophecy variables

Lock-free algorithms exhibit complex behaviors. To tackle them, Iris provides powerful mechanisms such as *prophecy variables* [[Jung, Lepigre, Parthasarathy, Rapoport, Timany, Dreyer and Jacobs 2020](#)]. Essentially, prophecy variables can be used to predict the future of the program execution and reason about it. They are key to handle *future-dependent linearization points*: linearization points that may or may not occur at a given location in the code depending on a future observation.

Zoo supports prophecy variables through the Proph and Resolve expressions — as in HeapLang, the canonical Iris language. In OCaml , these expressions correspond to **Zoo**.proph and **Zoo**.resolve, that are recognized by ocaml2zoo.

4 OCaml extensions for fine-grained concurrent programming

Over the course of this work, we studied efficient fine-grained concurrent OCaml programs written by experts. This revealed various limitations of OCaml in these domains, that those experts would work around using unsafe casts, often at the cost of both readability and memory-safety; and also some mismatches between their mental model of the semantics of OCaml and the mental model used by the OCaml compiler authors. We worked on improving OCaml itself to reduce these work-arounds or semantic mismatches.

4.1 Atomic record fields

OCaml 5 offers a type 'a **Atomic**.t of atomic references exposing sequentially consistent atomic operations. Data races on non-atomic mutable locations has a much weaker semantics and is generally considered a programming error. For example, the Michael-Scott concurrent queue [[Michael and Scott 1996](#)] relies on a linked list structure that could be defined as follows:

```
type 'a node = Nil | Cons of { value : 'a; next : 'a node Atomic.t }
```

Performance-minded concurrency experts dislike this representation, because 'a **Atomic**.t introduces an indirection in memory: it is represented as a pointer to a block containing the value of type 'a. Instead, they use something like the following:


```

type 'a node = Nil | Cons of { mutable next: 'a node; value: 'a }
let as_atomic : 'a node -> 'a node Atomic.t option = function
  | Nil -> None
  | (Cons _) as record -> Some (Obj.magic record : 'a node Atomic.t)

```

Notice that the next field of the `Cons` constructor has been moved first in the type declaration. Because the OCaml compiler respects field-declaration order in data layout, a value `Cons { next; value }` has a similar low-level representation to a reference (atomic or not) pointing at next, with an extra argument. The code uses `Obj.magic` to unsafely cast this value to an atomic reference, which appears to work as intended.

`Obj.magic` is a shunned unsafe cast (the OCaml equivalent of `unsafe` or `unsafePerformIO`). It is very difficult to be confident about its usage given that it may typically violate assumptions made by the OCaml compiler and optimizer. In the example above, casting a two-fields record into a one-argument atomic reference may or may not be sound — but it gives measurable performance improvements on concurrent queue benchmarks.

It is possible to statically forbid passing `Nil` to `as_atomic` to avoid error handling, by turning `'a node` into a GADT indexed over a type-level representation of its head constructor. For instance, this pattern can be found in the Kcas [Karvonen 2025a] library. It is difficult to write and use correctly, in particular as unsafe casts can sometimes hide errors in the intended static discipline.

Note that this unsafe approach only works for the first field of a record, so it is not applicable to records that hold several atomic fields, such as the toplevel record storing atomic front and back pointers for the concurrent queue.

4.1.1 Our atomic fields proposal. In May-June 2024 we proposed a design for atomic record fields as an OCaml language change proposal³. Declaring a record field atomic simply requires an `[@atomic]` attribute — and could eventually become a proper keyword of the language.

```

(* re-implementation of atomic references *)
type 'a atomic_ref = { mutable contents : 'a [@atomic]; }

(* concurrent linked list *)
type 'a node = Nil | Cons of { value: 'a; mutable next : 'a node [@atomic]; }

(* bounded SPSC circular buffer *)
type 'a bag = { data : 'a Atomic.t array;
  mutable front: int [@atomic];
  mutable back: int [@atomic]; }

```

The design difficulty is to express atomic operations on atomic record fields. For example, if `buf` has type `'a bag` above, then one naturally expects the existing notation `buf.front` to perform an atomic read and `buf.front <- n` to perform an atomic write. But how would one express exchange, compare-and-set and fetch-and-add? We would like to avoid adding a new primitive language construct for each atomic operation.

Two different designs have been proposed for atomic operations on atomic record fields.

- (1) The original, “full” design in our proposal introduces a built-in type `('r, 'a) Atomic.Field.t` that denotes a field/index of type `'a` within a record of type `'r`, with a syntax extension `[%atomic.loc <field>]` to construct such field description, and atomic primitives in a module `Atomic.Field`, that need both the record value of type `'r` and the field description. For example, the standard library exposes:

³<https://github.com/ocaml/RFCs/pull/39>

```
val Atomic.Field.fetch_and_add : 'a -> ('a, int) Atomic.Field.t -> int -> int
```

and users can write:

```
let preincrement_front (buf : 'a bag) : int =
  Atomic.Loc.fetch_and_add buf [%atomic.field front] 1
```

where [%atomic.field front] has type ('a bag, int) Atomic.Loc.t. Internally, a value of type ('r, 'a) Atomic.Field.t is just an integer offset locating the field within the record.

- (2) An alternative “simple” design, which was proposed by Basile Clément⁴, introduces a built-in type 'a Atomic.Loc.t for an atomic location that holds an element of type 'a, with a syntax extension [%atomic.loc <expr>.<field>] to construct such locations. Atomic primitives operate on values of type 'a Atomic.Loc.t and they are exposed as functions of the module Atomic.Loc. For example, the standard library exposes:

```
val Atomic.Loc.fetch_and_add : int Atomic.Loc.t -> int -> int
```

and users can write:

```
let preincrement_front (buf : 'a bag) : int =
  Atomic.Loc.fetch_and_add [%atomic.loc buf.front] 1
```

where [%atomic.loc buf.front] has type int Atomic.Loc.t. Internally, a value of type 'a Atomic.Loc.t can be represented as a pair of a record and an integer offset for the desired field, and the atomic.loc construction builds this pair in a well-typed manner. When a primitive of the Atomic.Loc module is applied to an atomic.loc expression, the compiler can optimize away the construction of the pair — but it would happen if there was an abstraction barrier between the construction and its use.

The simple design has the strong advantage of exposing simpler types to users. The full design involves more complex types, but a simpler data representation that does not rely on specific compiler optimizations to generate efficient code, even across abstraction boundaries. As remarked by Leo White⁵, the simpler type 'a Atomic.Loc.t can be reconstructed as a dependent pair of a 'r and a ('r, 'a) Atomic.Field.t, which is expressible in OCaml as a GADT:

```
type 'a loc = Loc : 'r * ('r, 'a) Atomic.Field.t -> 'a loc
```

In August and September 2024 we implemented the simple design, and submitted it for inclusion in the OCaml compiler⁶. We got a full code review by Olivier Nicole⁷. When we approached the OCaml maintainers to integrate the feature upstream, they asked us to reconsider implementing the full design, based on feedback in this direction by Vesa Karvonen (author of Kcas and Saturn)⁸.

We implemented the full design between November 2024 and January 2025⁹. We found that it is harder to implement in the type-checker. The extension form [%atomic.loc buf.front] of the simple design has typing rules that are very similar to a field access buf.front. On the other hand, [%atomic.loc front] interacts in a non-trivial way with the OCaml machinery for type-based disambiguation of record fields — when several records exist with a field named front.

For technical reasons, there is also a non-trivial interaction with the type-checking of inline record types (record types that are not defined by themselves but only as the argument of a sum type constructor), which currently prevents from using this approach with inline records, who are unfortunately common in efficient mutable data structures. In February and March 2025 we worked

⁴<https://github.com/ocaml/RFCs/pull/39#issuecomment-2147862938>

⁵<https://github.com/ocaml/RFCs/pull/39#issuecomment-2168612623>

⁶<https://github.com/ocaml/ocaml/pull/13404>

⁷<https://github.com/ocaml/ocaml/pull/13404#pullrequestreview-2291958008>

⁸<https://github.com/ocaml/ocaml/pull/13404#issuecomment-2441751885>

⁹<https://github.com/ocaml/ocaml/pull/13404#issuecomment-2441751885>

with OCaml maintainers experts to try to lift these limitations and proposed implementation changes for inline records¹⁰, but we failed to reach a consensus on the best design.

In April and May, we managed to build consensus among OCaml maintainers that the simple design, with a simpler user-facing design and a simpler implementation, was a good compromise in view of the significant difficulties of the full design with inline records. Our implementation of the simple design was reviewed again, we wrote user documentation, and it was integrated upstream in May 2025, to be included in the upcoming release of OCaml 5.4.

Limitation: no support for cache contention. The type `Atomic.t` comes with a function

```
val Atomic.make_contended : 'a -> 'a Atomic.t
```

that ensures that the returned atomic reference is allocated with enough alignment and padding to sit alone on its cache line, to avoid performance issues caused by false sharing. Currently there is no such support for padding of atomic record fields (we are planning to work on this), so the less-compact atomic references remain preferable in certain scenarios.

4.2 Atomic arrays

On top of our atomic record fields, we have implemented experimental support for atomic arrays, another facility commonly requested by authors of efficient concurrent programs. Our previous example of a concurrent bag of type `'a bag` used a backing array of type `'a Atomic.t array`, which contains more indirections than may be desirable, as each array element is a pointer to a block containing the value of type `'a`, instead of storing the value of type `'a` directly in the array.

Our implementation of atomic arrays¹¹ builds on top of the type `'a Atomic.Loc.t` we described in the previous section, and it relies on two new low-level primitives provided by the compiler:

```
val Atomic_array.index : 'a array -> int -> 'a Atomic.Loc.t
val Atomic_array.unsafe_index : 'a array -> int -> 'a Atomic.Loc.t
```

The function `index` takes an array and an integer index within the array, and returns an atomic location into the corresponding element after performing a bound check. `unsafe_index` omits the boundcheck — additional performance at the cost of memory-safety — and allows to express the atomic counterpart of the unsafe operations `Array.unsafe_get` and `Array.unsafe_set`. The atomic primitives of the module `Atomic.Loc` can then be used on these indices; our implementation implements a library module on top of these primitives to provide a higher-level layer to the user, with direct array operations such as:

```
val Atomic_array.exchange : 'a Atomic_array.t -> int -> 'a -> 'a
val Atomic_array.unsafe_exchange : 'a Atomic_array.t -> int -> 'a -> 'a
```

5 Physical equality

The notion of *physical equality* is ubiquitous in fine-grained concurrent algorithms. It appears not only in the semantics of the `(==)` operator, but also in the semantics of the `Atomic.compare_and_set` primitive, which atomically sets an atomic reference to a desired value if its current content is physically equal to an expected value. This primitive is commonly used to try committing an atomic operation in a retry loop, as in the push and pop functions of Figure 1.

5.1 Physical equality in HeapLang

In HeapLang, this primitive is provided but restricted. Indeed, its semantics is only defined if either the expected or the desired value fits in a single memory word in the HeapLang value representation:

¹⁰<https://github.com/ocaml/ocaml/pull/13770>

¹¹https://github.com/clef-men/ocaml/tree/atomic_array

literals (booleans, integers and pointers¹²) and literal injections¹³; otherwise, the program is stuck. In practice, this restriction forces the programmer to introduce an indirection [Iris development team 2025; Jung, Lepigre, Parthasarathy, Rapoport, Timany, Dreyer and Jacobs 2020; Vindum and Birkedal 2021] to physically compare complex values, e.g. lists. Furthermore, when the semantics is defined, values are compared using their Rocq representations; physical equality boils down to Rocq equality.

5.2 Physical equality in OCaml

In OCaml, *physical* equality is tricky. *Structural* equality $v1 = v2$, which we describe in Section 6, would be the preferred way of comparing values, and using physical equality $v1 == v2$ is often an unintentional mistake. However, physical equality is typically much faster than structural equality, as it compiles to only one assembly instruction instead of traversing the value. Also, the `Atomic.compare_and_set` requires the comparison to be atomic, ruling out structural equality.

Physical equality is in a counter-intuitive situation: it is very simple to *implement* (in the OCaml compiler, or in an interpreter, etc.) but difficult to *specify* precisely. To make verification practical, we need a specification at the level of *source* OCaml (or Zoo) programs, using a high-level representation of values, as close to the source as reasonably possible, that we call *abstract* values. On the other hand, its implementation typically work with *low-level* values, and its observable behavior depends on compiler transformations that happen in-between the two abstraction levels. This difficulty can result in dangerous gaps between the programming language used to write code and the semantics used for its verification.

ZooLang has a grammar of values, and most operations are specified by defining how they compute with ZooLang values. Its definition may look as follows in Rocq (simplified):

| | |
|-----------------------------|--|
| Inductive literal := | Inductive val := |
| Bool (b : bool) | Lit (lit : literal) |
| Int (n : nat) | Recs (i : nat) |
| Loc (l : location). | (recs : list (binder * binder * expr)) |
| | Block (tag : nat) (vs : list val). |

The value `Cons(42, $Nil)` is represented in Rocq as `Block 1 [Lit (Int 42), Block 0 []]`. Notice that immutable blocks are represented in Rocq using the `Block` constructor directly, and *not* as a location (`Loc`) allocated on the heap. We use locations only for *mutable* records. We would say that our representation of ZooLang values is high-level or *abstract*, as close to the surface syntax as reasonably possible. This distinction is important to make verification pleasant in practice, by reducing the number of locations and heap indirections that the programmer needs to work with during verification. A ZooLang tuple is directly a tuple, etc., and this design decision of using high-level values is important to the verification experience — in addition, assuming full ownership of arguments of immutable blocks would be incorrect.

It is tempting to specify, as HeapLang does, that physical equality decides equality between abstract values. This specification makes sense for immediate values (integers, booleans, constant constructors), and for mutable records which are compared by location. But it is incorrect on immutable blocks, and HeapLang essentially does not specify its behavior on those values. Yet programmers use physical equality on immutable blocks in practice, as in our example of a Treiber stack of Figure 1.

Defining physical equality as Rocq equality of abstract values is problematic in opposite ways:

¹²HeapLang allows arbitrary pointer arithmetic and therefore inner pointers. This is forbidden in both OCaml and ZooLang, as any reachable value has to be compatible with the garbage collector.

¹³HeapLang has no primitive notion of constructor, only pairs and injections (left and right).

- (1) Some distinct abstract values are physically equal in OCaml, for example `0` and `false`. Their type differ, but it possible to store them in an existential type where they can be compared for physical equality:

```
type any = Any : 'a -> any
let test = Any false == Any 0 (* maybe true *)
```

This shows that even on immediate values, specifying physical equality as equality of abstract values is convenient but incorrect in practice.

- (2) A deeper problem is that some definitionally equal abstract values may be physically distinct. Consider for example the case of *immutable blocks* representing constructors and immutable records (as opposed to *mutable blocks* representing mutable records), e.g. `Some 0`. The physical comparison of `Some 0` and `Some 0` may return either `true` or `false`: we cannot determine the result of physical comparison just by looking at the abstract values.

To solve these problems we treat physical equality on abstract values is *non-deterministic* — even though the comparison instruction that implements it on low-level values is perfectly deterministic. The question is then: what guarantees do we get when physical equality returns `true` and when it returns `false`? Given such guarantees, denoted by $v_1 \approx v_2$ and $v_1 \not\approx v_2$, the non-deterministic semantics is reflected in the logic through the following specification:

$$\frac{\text{True}}{v_1 == v_2} \quad b. \text{ if } b \text{ then } v_1 \approx v_2 \text{ else } v_1 \not\approx v_2$$

The OCaml manual documents a partial specification for physical equality, which is precise for basic types such as integers or integer references, but does not clearly extend to structured values containing a mix of immutable and mutable constructors, which are present in the programs we verify. The only guarantee that it provides for all values is: if two values are physically equal, they are also structurally equal. For values that contain immutable constructors, we do not learn anything when they are physically distinct.

We will now explore the specifications of the `true` and `false` return cases. We describe our program verification requirements to suggest a precise enough semantics: if it does not say enough, we cannot prove our programs correct. In the other direction we describe some optimizations that OCaml implementations may perform (gathered through our discussion with OCaml maintainers), in particular the current compiler, that may rule out certain stronger specifications as incorrect.

Remark. It is tempting to state that physical equality implies equality of Rocq representations, but incorrect in general as we have seen. Existing work on the modelling of OCaml physical equality within proof assistants have typically made this simplification, restricting the set of supported values to preserve soundness. We discussed our examples with authors of such earlier formalizations, and this has sometimes uncovered soundness issues on their side, as we discuss in [Section 12.3](#).

5.3 When physical equality returns `true`

Let us go back to the concurrent stack of [Figure 1](#) and more specifically the push function. Its atomic specification, given in [Section 2.3](#), states that if we push a value v onto a stack whose current model is vs , then it atomically becomes a stack of model $v :: vs$. To prove this specification we rely on the fact that, if `Atomic.compare_and_set` returns `true`, the current list must be equal to vs , in the sense of Rocq equality for Zoo values. This equality is strictly stronger than structural equality on mutable types, so we need a more precise specification than provided by the OCaml manual.

Zoo supports the following fragment of OCaml values: booleans, integers, mutable blocks (pointers), immutable blocks, functions.

The easy cases are mutable blocks and functions. Each of these two classes is disjoint from all others. We can reasonably assume that, when physical equality returns `true` and one of the compared values belongs to either of these classes, the two values are actually the same in Rocq. As far as we are aware, there is no optimization that could break this.

Booleans, integers and empty immutable blocks (constant constructors) are all represented as immediate integers in OCaml's low-level representation. This encoding induces conflicts: two abstract values that are distinct in Rocq may have the same low-level representation. The semantics of unrestricted physical equality has to reflect these conflicts: on those values that have an immediate representation, our specification does not state that physical equality includes equality of abstract values, it introduces a (simplified) notion of low-level representation and only states that those representations are equal.

Finally, let us consider the case of non-empty immutable blocks. At runtime, they are represented by pointers to tagged memory blocks. At first approximation, it is tempting to say that physically equal immutable blocks really are definitionally equal in Rocq. Alas, this is not true. To explain why, we have to recall that the OCaml compiler may perform *sharing*: immutable blocks containing physically equal fields may be shared. For example, the following tests may return `true`:

```
let test1 = Some 0 == Some 0 (* maybe true *)
let test2 = [0;1] == [0;1] (* maybe true *)
```

On its own, sharing is not a problem. However, coupled with representation conflicts, it can be surprising. Indeed, consider the any type we introduced previously:

```
type any = Any : 'a -> any
```

The following tests may return `true` (they do with `ocamlOPT`, not `ocamlC`):

```
let test1 = Any false == Any 0 (* maybe true *)
let test2 = Any None == Any 0 (* maybe true *)
let test3 = Any [] == Any 0 (* maybe true *)
```

Now, going back to the push function of Figure 1, we have a problem. Given a stack of any, it is possible for the `Atomic.compare_and_set` to observe a current list (e.g. the one-element list `[Any 0]`) physically equal to the expected list (e.g., `[Any false]`) while these are actually distinct in Rocq. In short, the expected specification of Section 2.3 is incorrect: we may not get $v :: vs$ back in the model, but a list $v :: vs'$ where vs' is physically equal to vs but not the same abstract value. To fix this discrepancy, we would need to weaken all our specifications to be formulated *modulo physical equality*, which is non-standard and quite burdensome.

We believe this really is a shortcoming, at least from the verification perspective. Therefore, we extended OCaml with *generative immutable blocks*¹⁴. These generative blocks are just like regular immutable blocks, except they cannot be shared. Hence, if physical equality on two generative blocks returns `true`, these blocks are definitionally equal in Rocq. At user level, this notion is materialized by *generative constructors*. For instance, to verify the expected push specification, we can use a generative version of lists:

```
type 'a glist =
| Nil
| Cons of 'a * 'a glist [@generative]
```

We modified the Zoo translator to support those generative constructors, and modified our implementation of Treiber stack to use the type `'a glist` instead of `'a list`, so that we could finally prove the expected, convenient specification. The `[@generative]` attribute is ignored by the

¹⁴https://github.com/clef-men/ocaml/tree/generative_constructors


```

type state =
  | Open of Unix.file_descr
  | Closing of (unit -> unit)

type t =
  { mutable ops: int [@atomic];
    mutable state: state [@atomic]; }

let make fd = { ops = 0; state = Open fd }

let closed = Closing (fun () -> ())
let close t =
  match t.state with
  | Closing _ -> false
  | Open fd as prev ->
    let close () = Unix.close fd in
    let next = Closing close in
    if Atomic.Loc.compare_and_set [%atomic.loc t.state] prev next then (
      if t.ops == 0
      && Atomic.Loc.compare_and_set [%atomic.loc t.state] next closed
      then close () ;
      true
    ) else false

```

Fig. 3. **Rcfd** module from **Eio** (excerpt)

standard OCaml compiler, and we have an experimental version that disables sharing for generative constructors.

5.4 When physical equality returns **false**

Most formalizations of physical equality in the literature do not give any guarantee when physical equality returns **false**. Many use-cases of physical equality, in particular retry loops, can be verified with only sufficient conditions on **true**. However, in some specific cases, more information is needed.

Consider the **Rcfd** module from the **Eio** library, an excerpt of which is given in Figure 3¹⁵. Thomas Leonard, its author, suggested that we verify this real-life example because of its intricate logical state. However, we found out that it is also relevant regarding the semantics of physical equality. Essentially, it consists in wrapping a file descriptor in a thread-safe way using reference-counting. At creation in the **make** function, the wrapper starts in the **Open** state. At some point, it can switch to the **Closing** state in the **close** function, and will remain **Closing** forever. Crucially, the **Open** state changes at most once to **Closing**, never to another **Open**.

The interest of **Rcfd** lies in the **close** function. First, the function reads the state. If this state is **Closing**, it returns **false**; the wrapper has been closed. If this state is **Open**, it tries to switch to the **Closing** state using **Atomic.Loc.compare_and_set**; if this attempt fails, it also returns **false**. In this particular case, we would like to prove that the wrapper has been closed, or equivalently that **Atomic.Loc.compare_and_set** cannot have observed **Open**. Intuitively, if we observed a different value then it must be **Closing**.

Obviously, we need some kind of guarantee related to the *physical identity* of **Open** when **Atomic.Loc.compare_and_set** returns **false**. If **Open** were a mutable block, we could argue that this block cannot be physically distinct from itself; no optimization we know of would allow that.

¹⁵We make use of *atomic record fields* as introduced in Section 4.1.

Unfortunately, it is an immutable block, and immutable blocks are subject to more optimizations. In fact, something surprising but allowed¹⁶ by OCaml can happen: *unsharing*, the dual of sharing. Indeed, any immutable block can be unshared, that is reallocated as if its definition was inlined. For example, the following test may theoretically return *false*:

```
let x = Some 0
let test = x == x (* maybe false *)
```

Going back to *Rcfd*, we have a problem: in the second branch, the *Open* block corresponding to *prev* could be unshared, which would make *Atomic.Loc.compare_and_set* fail. Hence, we cannot prove the expected specification; in fact, the program as it is written has a bug.

To remedy this unfortunate situation, we propose to reuse the notions of generative immutable blocks, that we introduced to prevent sharing, to also forbid unsharing by the OCaml compiler — we implemented this in our experiment compiler branch.

Supporting this requires enriching the Zoo semantics so that each generative block is annotated with a *logical identifier*¹⁷ representing its physical identity, much like a pointer for a mutable block. If physical equality on two generative blocks returns *false*, the two identifiers are necessarily distinct. Given this semantics, we can verify the *close* function. Indeed, if *Atomic.Loc.compare_and_set* fails, we now know that the identifiers of the two blocks, if any, are distinct. As the concurrent protocol has only one *Open* block whose identifier does not change, it cannot be the case that the current state is *Open*, hence it is *Closing*. We can verify this function after adding the following annotation:

```
type state =
  | Open of Unix.file_descr [ @generative ]
  | Closing of (unit -> unit)
```

5.5 Summary

In summary, we extended our abstract values with generative immutable blocks, and give the following specification to physical equality in ZooLang. It can also serve as a precise specification of physical equality of a practical fragment of OCaml.

- On values whose low-level representation is an immediate integer, physical equality is immediate equality.
- On mutable blocks at some location, or generative immutable blocks with some identity, physical equality is equality of locations or identities.
- On immutable blocks, physical-equality is under-specified, but it implies that the blocks have the same tags and their arguments are in turn physically equal.
- Two values that fall into different categories above are never physically equal.

These informal rules are described precisely in Figure 4. We introduce the grammar v of ZooLang values, and a grammar \hat{v} of *low-level* values. We describe the lowering $[v]$ of values into low-level values: integers, booleans and constant constructors are lowered into “immediate” machine integers. We introduce two predicates on low-level values: $\hat{v}_1 == vlow_2$ holds when \hat{v}_1 and \hat{v}_2 *must* be physically equal, and $\hat{v}_1 \approx \hat{v}_2$ when they *may* be physically equal (depending on under-specified representation choices). Generative constructors must be physically equal when they have the same generative identifier.¹⁸ Non-generative constructors may be physically equal (or not) when they have the same tag and their arguments may be physically equal.


¹⁶OCaml maintainers developing the Flambda optimiser have confirmed that it may perform unsharing in certain cases.

¹⁷Actually, for practical reasons, we distinguish identified and unidentified generative blocks.

¹⁸In this case we also ask that their tags and arguments be equal as Rocq values, to avoid having to establish this invariant in the operational semantics before we can reason about these relations.


| | | |
|---|--|---|
| block generativity | $gen ::= nongen \mid gen \text{ } bid$ | |
| generative identifier | bid | |
| block tag | $tag \in \mathbb{Z}$ | |
| values | $v ::= b \in \mathbb{B} \mid n \in \mathbb{Z} \mid \ell \mid recs \dots \mid block \text{ } gen \text{ } tag \text{ } vs$ | |
| low values | $\hat{v} ::= imm \in \mathbb{Z} \mid \ell \mid recs \dots \mid block \text{ } gen \text{ } tag \text{ } \hat{vs} \quad \text{with } \hat{vs} \neq \emptyset$ | |
| $\lfloor n \rfloor$ | $:= n$ | $\lfloor recs \dots \rfloor := recs \dots$ |
| $\lfloor true \rfloor$ | $:= 1$ | $\lfloor block \text{ } gen \text{ } tag \text{ } \emptyset \rfloor := tag$ |
| $\lfloor false \rfloor$ | $:= 0$ | $\lfloor block \text{ } gen \text{ } tag \text{ } vs \rfloor := block \text{ } gen \text{ } tag \text{ } \lfloor vs \rfloor \quad \text{when } vs \neq \emptyset$ |
| $\frac{imm_1 = imm_2}{imm_1 == imm_2} \quad \frac{bid_1 = bid_2 \quad tag_1 = tag_2 \quad vs_1 = vs_2}{block \text{ } (gen \text{ } bid_1) \text{ } tag_1 \text{ } vs_1 == block \text{ } (gen \text{ } bid_2) \text{ } tag_2 \text{ } vs_2}$ | | |
| $\frac{\hat{v}_1 == \hat{v}_2}{\hat{v}_1 \approx \hat{v}_2} \quad \frac{}{recs \dots \approx recs \dots} \quad \frac{tag_1 = tag_2 \quad vs_1 \approx vs_2}{block \text{ } nongen \text{ } tag_1 \text{ } vs_1 \approx block \text{ } nongen \text{ } tag_2 \text{ } vs_2}$ | | |
| $v_1 \approx v_2 := \lfloor v_1 \rfloor \approx \lfloor v_2 \rfloor$ | | |
| $v_1 \not\approx v_2 := \neg(\lfloor v_1 \rfloor == \lfloor v_2 \rfloor)$ | | |

Fig. 4. Values, low-level values, physical equality

Finally, we define similarity and non-similarity relations on (high-level) values: two values are similar $v_1 \approx v_2$ when their representations may be physically equal, and they are non-similar $v_1 \not\approx v_2$ when their representations cannot be physically equal. We prove  that for any v_1, v_2 , then either $v_1 \approx v_2$ or $v_1 \not\approx v_2$ holds, but it may be the case that for certain values both $v_1 \approx v_2$ and $v_1 \not\approx v_2$ hold, namely for pure non-generative constructors, for which physical equality is underspecified.

Our verification work uncovered correctness bugs in existing OCaml concurrent code due to unsharing. We propose to extend the language with a `@generative` annotation for immutable constructors whose physical identity we want to reason about when implementing a fine-grained concurrent data structure. A rule of thumb would be to use it for all non-constant constructors involved in `Atomic.compare_and_set` operations.

6 Structural equality

Structural equality  is also supported. More precisely, it is not part of the semantics of the language but implemented using low-level primitives¹⁹. The reason is that it is in fact difficult to specify for arbitrary values. In general, we have to compare graphs — which implies structural comparison may diverge.

Accordingly, the specification of $v_1 = v_2$ requires the (partial) ownership of a *memory footprint* corresponding to the union of the two compared graphs, giving the permission to traverse them safely. If it terminates, the comparison decides whether the two graphs are bisimilar (modulo

¹⁹In OCaml, these primitives correspond to the unsafe functions `Obj.is_int`, `Obj.tag`, `Obj.size` and `Obj.field`.

representation conflicts, as described in [Section 5](#)). In Iris, this gives:

$$\begin{array}{c}
 \text{val-traversable } \textit{footprint } v_1 * \\
 \text{val-traversable } \textit{footprint } v_2 * \\
 \text{structeq-footprint } \textit{footprint} \\
 \hline
 v_1 = v_2 \\
 \hline
 b. \text{ if } b \text{ then val-structeq } \textit{footprint } v_1 \text{ } v_2 \text{ else val-structneq } \textit{footprint } v_1 \text{ } v_2
 \end{array}$$

Obviously, this general specification is not very convenient to work with. Fortunately, for abstract values (without any mutable part), we can prove a much simpler variant saying that structural equality coincides with physical equality in absence of generative or mutable constructors:

$$\begin{array}{c}
 \text{val-abstract } v_1 * \\
 \text{val-abstract } v_2 \\
 \hline
 v_1 = v_2 \\
 \hline
 b. \text{ if } b \text{ then } v_1 \approx v_2 \text{ else } v_1 \not\approx v_2
 \end{array}$$

7 Standard data structures

To save users from reinventing the wheel, we provide a library of verified standard data structures 🦊 🦋 — more or less a subset of the OCaml standard library. Most of these data structures²⁰ are completely reimplemented in Zoo and axiom-free, including the [Array](#)²¹ module.

Sequential data structures. We provide verified implementations of various sequential data structures: array 🦊 🦋, dynamic array 🦊 🦋 (vector), list 🦊 🦋, stack 🦊 🦋, queue (bounded 🦊 🦋 and unbounded 🦊 🦋), double-ended queue 🦊 🦋. We claim that the proven specifications are modular and practical. In fact, most of these data structures have already been used to verify more complex ones — we present some in [Section 8](#) and [Section 10](#). Especially, we developed an extensive collection of flexible specifications for the iterators of the [Array](#) and [List](#) modules. Remarkably, our formalization of [Array](#) features different (fractional) predicates to express the ownership of either an entire array, a slice or even a circular slice — we used it to verify algorithms involving circular arrays, e.g. Chase-Lev working-stealing queue [[Chase and Lev 2005](#)].

Concurrent data structures. We provide verified implementations of various concurrent data structures: domain²² 🦊 🦋 (including domain-local storage), mutex 🦊 🦋, semaphore 🦊 🦋, condition variable 🦊 🦋, write-once variable 🦊 🦋 (also known as *ivar*), atomic array 🦊 🦋. Note that there is currently no [Atomic_array](#) module in the OCaml standard library, but we are planning to propose it.

8 Persistent data structures

To further demonstrate the practicality of Zoo, we verified a collection of persistent data structures 🦊 🦋. This includes purely functional data structures such as persistent stack 🦊 🦋 and queue 🦊 🦋, but also efficient imperative implementations of persistent array 🦊 🦋 from [Conchon and Filliâtre \[2007\]](#), store 🦊 🦋 and union-find 🦊 🦋 from [Allain, Clément, Moine and Scherer \[2024\]](#).

Currently, verification of purely functional programs relies on the regular ZooLang translation, i.e. on a deeply embedded representation. However, we found this approach is cumbersome. In the future, it would be desirable to be able to verify them directly in Rocq, through a translation

²⁰For practical reasons, to make them completely opaque, we chose to axiomatize a few functions from the [Domain](#) and [Random](#) modules. They could trivially be realized in Zoo.

²¹Our implementation of the [Array](#) module is compatible with the standard one. In particular, it uses the same low-level value representation.

²²Domains are the units of parallelism in OCaml 5.

$$\begin{array}{c}
\frac{\Psi \ 1}{\text{make } fd} \\
\hline
t. \text{ inv } t \text{ } fd \ \Psi
\end{array}
\qquad
\frac{
\begin{array}{c}
\text{inv } t \text{ } fd \ \Psi \\
\text{wp closed } () \{ X \text{ false } \} \\
\forall q. \Psi \ q \multimap \text{wp open } fd \{ \text{res}. \Psi \ q \multimap X \text{ true } \text{res} \}
\end{array}
}{
\text{use } t \text{ closed open}
}
\qquad
\frac{}{\text{res}. \exists b. X \ b \ \text{res}}$$

$$\frac{
\begin{array}{c}
\text{inv } t \text{ } fd \ \Psi \\
\Psi \ 1 \multimap \exists \text{chars}. \text{unix-fd } fd \ 1 \ \text{chars}
\end{array}
}{
\text{close } t
}
\qquad
\frac{
\begin{array}{c}
\text{inv } t \text{ } fd \ \Psi \\
\text{remove } t
\end{array}
}{
o.
}$$

$$\frac{}{(). \text{ closing } t}
\qquad
\frac{}{o = \text{None} \vee o = \text{Some } fd \ * \ \Psi \ 1}$$

Fig. 5. Rcfd specification (excerpt)

to Gallina. Similarly to Hacspecc [Haselwarter, Hvass, Hansen, Winterhalter, Hritcu and Spitters 2024], this new translation would come with a generated proof of equivalence with the ZooLang representation.

9 Rcfd: Parallelism-safe file descriptor

As mentioned in Section 5, the **Rcfd** module  from the **Eio** library is particularly interesting in several respects. Not only does it justify the introduction of generative constructors in OCaml, but it also demonstrates the use of Iris for expressing realistic concurrent protocols.

Specification. The **Rcfd** module provides a parallelism-safe file descriptor (FD) relying internally on reference-counting. Interestingly, it is used in **Eio** in two different ways, more precisely two different ownership regimes: (1) any thread can try to access or close the FD; (2) any thread can try to access the FD but only the owner thread can close it — and is responsible for closing it. To verify all uses, the specification of **Rcfd** has to support both ownership regimes. However, due to space constraints, we consider a simplified specification given in Figure 5. The full verified specification can be found in the mechanization. The specification features four operations²³:

make creates a new object t of type **Rcfd**. t wrapping a given FD fd , yielding the (persistent) invariant $\text{inv } t \text{ } fd \ \Psi$, where Ψ is an arbitrary fractional predicate. Crucially, the user must provide the full predicate $\Psi \ 1$, which is stored in the invariant. Once it is created, a wrapped FD can be accessed through the **use** operation and closed through the **close** operation.

use requires the invariant along with the weakest preconditions of the **closed** function, that is called if the FD has been flagged as closed, and **open** function, that is called if the FD is still open. To control the postconditions and the weakest preconditions, the user can choose an arbitrary predicate X parameterized by a boolean indicating whether the **closed** (**false**) or the **open** (**true**) was called. The **open** function is given a fraction of Ψ , thereby accessing the FD.

close requires the invariant and proving that the full ownership of Ψ entails the full ownership of the FD fd , which is necessary to call **Unix**. **close**. It yields **closing** t , a persistent resource witnessing that t has been flagged as closed. Actually, the wrapped FD is not closed immediately. It will be closed only once it is possible, meaning all ongoing calls to **use** owning a fraction of the FD end.

Alternatively, instead of closing the FD, **remove** tries to retrieve the full ownership of Ψ . To achieve it, it exploits the same mechanism as **close** — flagging t as closed as witnessed by **closing** t — but also waits until all use calls are done.

²³We omitted two non-essential operations: **is_open** and **peek**.

Logical state. Thomas Leonard, the author of `Rcfd`, suggested verifying it to make sure the informal concurrent protocol he described in the OCaml interface was correct. This protocol introduces a notion of monotonic logical state — modeled in Iris using a specific resource algebra [Timany and Birkedal 2021] — to describe the evolution of a FD. Originally, there were four logical states but we found that only three are necessary for the verification: **open**, **closing/users** and **closing/no-users**.

In the **open** state, the FD is available for use, meaning any thread can access it through use. Physically, this corresponds to the `Open` constructor.

When some thread flags the FD as closed through `close` or `remove`, the state transitions from **open** to **closing/users**. Crucially, there can only be one such thread. In this state, the FD is not really closed yet because of ongoing use operations. Physically, this logical transition corresponds to switching from the `Open` to the `Closing` constructor using `Atomic.Loc.compare_and_set`.

Once all use operations have finished, when the reference-count reaches zero, it is time to actually “close” the FD by calling the function carried by the `Closing` constructor. This has to be done only once. The “closing” thread is the one that succeeds in updating the `Closing` constructor (to a new one carrying a no-op function) using `Atomic.Loc.compare_and_set`. At this point, the state transitions from **closing/users** to **closing/no-users** and the wrapper no longer owns the FD.

Generative constructors. As explained in Section 5, the `Open` constructor has to be generative to prevent *unsharing*. In fact, the `Closing` constructor also has to be generative to prevent *sharing*, otherwise two calls could have a shared value of next and believe they both won the second update.

10 Saturn: A library of standard lock-free data structures

We verified a collection of standard lock-free data structures 🦊 🚩 from the Saturn, Eio and Picos [Karvonen 2025b] libraries. It includes stacks, queues (list-based, array-based and stack-based) and bags. These data structures are meant to be used as is or adapted to fit specific needs. To cover a wide range of use cases, we provide specialized variants: bounded or unbounded, single-producer (SP) or multi-producer (MP), single-consumer (SC) or multi-consumer (MC).

Due to space constraints, we focus on the most important algorithms and refrain from showing the corresponding (non-trivial) Iris invariants, which are mechanized in Rocq.

10.1 Stacks

We verified three variants of the Treiber stack [Treiber 1986]: unbounded MPMC 🦊 🚩, bounded MPMC 🦊 🚩, closable unbounded MPMC 🦊 🚩. This last variant features a closing mechanism: at some point, some thread can decide to close the stack, retrieving the current content and preventing others from operating on it. For example, we used it to represent a set of vertex successors in the context of a concurrent graph implementation (not presented in this paper).

As explained in Section 5, the three verified stacks use generative constructors to prevent sharing. One may ask whether it would be easier to use a mutable version of lists instead. From the programmer’s perspective, this is unsatisfactory because (1) the compiler will typically emit warnings complaining that the mutability is not exploited and (2) it does not really reflect the intent, *i.e.* we want precise guarantees for physical equality, not modify the list. From the verification perspective, this is also unsatisfactory because the mutable representation is more complex to write and reason about: pointers and points-to assertions versus pure Rocq list.

Although verified stacks may seem like a not-so-new contribution, it is, as far as we know, the first verification of realistic OCaml implementations. For comparison, the exemplary concurrent stacks verified in Iris [Iris development team 2025] all suffer from the same flaw: they need to introduce indirections (pointers) to be able to use the compare-and-set primitive.

10.2 List-based queues

We verified four variants of the Michael-Scott queue [Michael and Scott 1996]: unbounded MPMC 🐘🐘, bounded MPMC 🐘🐘, unbounded MPSC 🐘🐘 and unbounded SPMC 🐘🐘. The SPMC queue is used to implement one of the bags — a relaxed queue guaranteeing only per-producer ordering.

In the Iris literature, Vindum and Birkedal [2021] established contextual refinement of the Michael-Scott queue while Mulder and Krebbers [2023] proved logical atomicity. However, we had to redesign and extend the invariant for several reasons.

Efficient implementation. The Michael-Scott essentially consists of a singly linked list of nodes that only grows over time. The previously verified implementations, implemented in HeapLang, use a double indirection to represent the list [Vindum and Birkedal 2021, Figure 2]. Similarly to the Treiber stack, this is made so as to be able to use the compare-and-set primitive of HeapLang. Vindum and Birkedal [2021] write:

A node is a pointer to either none or some of a pair of a value and a pointer to the next node. The pointer serves to make nodes comparable by pointer equality such that pointers to nodes can be changed with CAS.

In OCaml, this would correspond to introducing extra atomic references (`Atomic.t`) between the nodes. Using atomic record fields, we can represent the list more efficiently, without the extra indirection. However, there is one subtlety: in this new representation, we need to clear the outdated nodes so that their value is no longer reachable and can be garbage-collected, in other words to prevent a memory leak. This subtlety is not discussed in the original implementation [Michael and Scott 1996] designed for non-garbage-collected languages, but it is folklore; it is implemented in Saturn (`saturn#64`) and we verified that it preserves correctness.

To deal with this representation in separation logic, we introduce the notion of *explicit chain* that allows decoupling the chain structure formed by the nodes and the content of the nodes. Concretely, the assertion $xchain\ dq\ \ell s\ dst$ 🐘 represents a chain linking locations ℓs and ending at value dst ; dq is a discardable fraction [Vindum and Birkedal 2021] that controls the ownership of the chain. This notion is very flexible as it is independent of the rest of the structure. As a matter of fact, we used it and its generalization to doubly linked list more broadly, to verify other algorithms. All the variants of Michael-Scott we verified rely on it. In particular, it was quite straightforward to extend the invariant of the bounded queue, where nodes carry more (mutable and immutable) information.

External linearization point. Our work also revealed another interesting aspect that is not addressed in the literature, as far as we know. None of the previously verified implementations deal with the `is_empty` operation, that consists in reading the sentinel node and checking whether it has a successor. If it has no successor, it is necessarily the last node of the chain, hence the queue is empty. If it does have a successor, `is_empty` returns `false`, meaning we must have observed a non-empty queue. However, this last part is more tricky than it may seem. Indeed, it may happen that (1) we read the sentinel while the queue is empty, (2) other operations fill and empty again the queue so that the sentinel is outdated, (3) we read the successor of the former sentinel while the queue is still empty. The crucial point here is that `is_empty` is linearized when the first push operation filled the queue. In other words, the linearization point of `is_empty` is triggered by another operation; this is called an *external linearization point*. To handle this in the proof, we introduced a mechanism in the invariant to transfer the Iris resource materializing the linearization point²⁴ from `is_empty` to push and vice versa.

²⁴This resource is known as an *atomic update*. Mulder and Krebbers [2023] provide a good description.

10.3 Stack-based queues

A standard way to implement a sequential queue is to use two stacks: producers push onto the *back stack* while consumers pop from the *front stack*, stealing and reversing the back stack when needed. Based on this simple idea, Vesa Karvonen²⁵ developed a new lock-free concurrent queue. We verified the MPMC variant 🦊🚗 used in Picos and the closable MPSC variant 🦊🚗 used in Eio.

As in the sequential implementation, the two stacks are mainly immutable. Both stacks are updated using compare-and-set so we use generative constructors to reason about physical equality.

Similarly again, producers and consumers work concurrently on separate stacks, limiting interference. The key difference compared to the sequential version is that the algorithm has to deal with the concurrent back stack reversal in a lock-free manner. Essentially, the concurrent protocol — and therefore the Iris invariant — includes a *destabilization* phase during which a new back stack pointing to the former one awaits to be *stabilized*, which happens when the reversed former back stack becomes the new front stack. In practice, the synchronization is a bit tricky and relies on the indices of the elements.

11 Memory safety

Concurrency creates tensions between performance and memory-safety. The OCaml maintainers intend to maintain memory-safety for all OCaml programs, including racy concurrent programs. They have tried to imprint this focus on memory-safety to library authors as well.

Performance-sensitive OCaml libraries are sometimes written using unsafe primitives, that may break memory-safety if used incorrectly. It is the responsibility of code authors to ensure that the preconditions of those unsafe primitives are satisfied to ensure safety. Unfortunately, the addition to parallel code execution in OCaml 5 broke the safety of some existing code: it adds more possible interleaving and may invalidate safety reasoning.

11.1 Dynarray

In January 2023, Gabriel Scherer²⁶ proposed the addition of **Dynarray**, a module of (sequential) resizable arrays, to the OCaml standard library.

A concurrent safety problem. Resizable arrays are implemented as a record with two mutable fields, a size field that stores the current length of the resizable array, and a data field storing the “backing array” a (non-resizable) array of size at least size elements. When a user adds a new element to the end of a resizable array, it typically suffices to write the new element at index size in the backing array, and then increment the size field. But when the size field reaches the actual length of the backing array (which we call the “capacity” of the resizable array), we first need to allocate a new, larger backing array, to copy the values from the old to the new backing array, and to overwrite the data field with the new backing array.

This operation of adding a new element is performance-critical for resizable arrays. After the size check and potential resizing, sequentially we know that the backing array has enough space, and we can write the new element using an `unsafe_set` primitive that avoids a redundant bound-check on array access. This optimization can provide speedups of up to 20% in certain scenarios.

```
let push a x = (* sequential version *)
  let size = a.size in
  if Array.length a.data = size then reserve a (size + 1);
```

²⁵<https://github.com/ocaml-multicore/saturn/pull/112>

²⁶<https://github.com/ocaml/ocaml/pull/11882>


```
a.size <- size + 1;
Array.unsafe_set a.data size x
```

Unfortunately, the safety reasoning becomes incorrect in presence of parallelism: another thread could mutate the backing array after the size check and before the unsafe write, for example `Dynarray.reset` which sets the size to 0 and replaces the backing array by an empty array. Writing to a (non-resizable) array outside its bounds without bound checks breaks memory safety, so this implementation was memory-safe under OCaml 4 but it becomes memory-unsafe under OCaml 5. On the other hand, `Dynarray` is explicitly documented as a sequential data structure, so it is the user responsibility to respect this precondition by using appropriate synchronization (for example a mutex) to prevent data races. Some performance-obsessed users would argue that if *other* users fail in their responsibility of ensuring sequential access, then they do not deserve memory-safety. The OCaml maintainers and standard library authors consider on the other hand that memory-safety should be preserved even in this case: it can only be lifted for operations that are explicitly marked as unsafe, and hopefully have simple, checkable preconditions. So they decided to change the optimization of `Dynarray` to guarantee memory-safety even in presence of racy concurrent usage.

The proposed implementation reads the data field to get the backing array *b* and performs the resizing check. If no resizing is necessary, then an unsafe write is performed as before (on the backing array *b*, without re-reading the data field). In the infrequent case where resizing is necessary, the operation retries afterwards. (A bound-checking write would suffice for safety.)

```
let rec push a x = (* memory-safe under concurrency *)
  let {size; data} = a in
  if Array.length data >= size
  then ( reserve a (size + 1); push a x )
  else ( a.size <- size + 1; Array.unsafe_set arr size x )
```

A concurrent verification problem. We verified (a representative fragment of) the proposed `Dynarray` implementation 🐘 🚗, proving that it is functionally correct under sequential usage, and that it does preserve memory-safety even under concurrent usage. The verified fragment is now part of the Zoo standard library, and can be used in further verification projects.

Informally, the verification relies on two different invariants:

- Functional correctness relies on a *strong* invariant, which may not be preserved under concurrent usage; for example that the content of the size field is smaller than the size of the backing array.
- Memory-safety relies on a *weak* invariant that does not suffice to prove correctness, but is preserved under concurrent usage. For this `Dynarray` implementation, the weak invariant is that the backing array remains well-typed and that the content of the size field is non-negative.

To informally check the two desired properties, it suffices to check that the strong invariant is indeed a sequential invariant, and that implies functional correctness; and separately that the weak invariant is preserved by all operations on the data structure.

In our mechanized verification, the strong invariant becomes a *model* of the data structure, and we verify separation-logic triples where each operation can assume unique ownership of the model in input, and has to return ownership of a valid model in output, as is standard. For example, the “functional correctness” lemma for the push function is the following:

$$\frac{\frac{\text{model } t \text{ vs}}{\text{push } t \text{ } v}}{(). \text{ model } t \text{ (vs } \# [v])}$$

This lemma establishes correctness in a purely-sequential usage, but also under any concurrent usage where correct synchronization is used to guarantee unique ownership of the model.


The formal counterpart of our weak invariant is a *semantic type*, following the approach of the RustBelt project [Jung, Jourdan, Krebbers and Dreyer 2018], which also deals with unsafe fragments within a language intended to be safe. Note that the RustBelt semantic types are derived from Rust types, where mutating operations get a mutable borrow and thus unique ownership (for a time). In contrast, our semantic types carry no ownership of the values they manipulate, so they are stored entirely as invariants, and all interactions with the structure must be shown to preserve this invariant atomically. The invariant must be robust against any interference coming from any other function called in parallel on the same structure.

The definition of the semantic type for dynarrays, and the statement of memory safety (or in fact semantic typing) for push look as follows:

$$\llbracket t \ \tau \rrbracket (v) \triangleq \begin{array}{l} \exists \ell. t = \ell * \\ \boxed{\begin{array}{l} \exists sz \text{ cap } data. \\ 0 \leq sz * \\ \ell.size \mapsto sz * \\ \ell.data \mapsto data * \\ \llbracket \text{array} (\text{element } \tau) \text{ cap} \rrbracket data \end{array}} \end{array} \quad \frac{\frac{\llbracket t \ \tau \rrbracket t * \llbracket \tau \rrbracket v}{\text{push } t \ v}}{(). \text{ True}}$$

The predicate τ is the semantic type for the elements of the array — it must hold for each value of the backing array and for the new value v .

11.2 Saturn: Single-producer or Single-consumer queues

Similar problems of memory-safety occur in the concurrent data structures of the Saturn library. For example, we explained (Section 10.2) that the Saturn implementation of the Michael-Scott MPMC queue is careful to “erase” values stored in the queue to avoid memory leaks. This erasure is performed by writing a non-type-safe dummy value, `Obj.magic ()`. We define a semantic type  for the MPMC queue which is essentially its concurrent invariant, and prove that this unsafe idiom does not endanger memory-safety.

On the other hand, the efficient implementations of single-consumer or single-producer queues in Saturn do *not* respect the general OCaml recommendation of favoring safety over performance: if a library user uses a single-consumer structure from two consumers in racy ways, they lose memory safety. Our correctness results imply memory-safety when the single-consumer or single-producer protocol is respected; when the caller does own its end of the structure uniquely, as our precondition requires. But we cannot prove memory-safety with only persistent preconditions, it does not hold.

The Saturn authors also provide “safe” variants of their data structures, which are slightly slower but memory-safe even for unintended concurrent usage. This typically adds an indirection, such as using the type `'a option` instead of `'a`, which provides a type-safe `None` value for dummies.

12 Related work

We have amply covered related work on verification of concurrent fine-grained data structures in the main body of the paper. We focus here on the related work on program verification and the question of physical equality.

12.1 Non-automated verification

The verified program is translated, manually or in an automated way, into a representation living inside a proof assistant. The user has to write specifications and prove them.

Translating into the native language of the proof assistant, such as Gallina for Rocq, is challenging as it is hard to faithfully preserve the semantics of the source language, which typically has non-terminating functions for example. Monadic translations should support it, but faithfully encoding all impure behaviors is challenging, and tools typically provide a best-effort translation [Claret 2025; Spector-Zabusky, Breitner, Rizkallah and Weirich 2018] that is only approximately sound.

The representation may be embedded, meaning the semantics of the language is formalized in the proof assistant. This is the path taken by some recent works (for example Gondelman, Hinrichsen, Pereira, Timany and Birkedal [2023]) harnessing the power of separation logic. In particular, (1) CFML [Charguéraud 2023], (2) Osiris [Seassau, Yoon, Madiot and Pottier 2025] and (3) DRFCaml [Georges, Peters, Elbeheiry, White, Dolan, Eisenberg, Casinghino, Pottier and Dreyer 2025] target OCaml.

(1) CFML does not support concurrency and is not based on Iris.

(2) Osiris is based on Iris but does not support concurrency. Its design philosophy is more perfectionist than pragmatic, especially in its treatment of evaluation order, at the cost of a complex program logic. The relatively small number of verified examples suggests that it is not yet ready for practical verification at scale.

(3) DRFCaml is based on Iris and does support concurrency. It is mostly an extension of HeapLang with features (modalities and stack regions) entirely orthogonal to our work; in particular, it also assumes a sequentially consistent memory model. The crucial difference is that it forbids data races on non-atomic locations, which makes it compatible with OCaml 5 thanks to the DRF property [Dolan, Sivaramakrishnan and Madhavapeddy 2018] but is too restrictive to verify legal concurrent programs, including some that we verified.

12.2 Semi-automated verification

In semi-automated verification approaches, the verified program is annotated by the user to guide the verification tool: preconditions, postconditions, invariants, *etc.* Given this input, the verification tool generates proof obligations that are mostly automatically discharged. One may further distinguish two types of semi-automated systems: *foundational* and *non-foundational*.

In *non-foundational* automated verification, the tool and external solvers it may rely on are part of the trusted computing base. It is the most common approach and has been widely applied in the literature [Astrauskas, Bilý, Fiala, Grannan, Matheja, Müller, Poli and Summers 2022; Denis, Jourdan and Marché 2022; Filliâtre and Paskevich 2013; Jacobs, Smans, Philippaerts, Vogels, Penninckx and Piessens 2011; Lattuada, Hance, Cho, Brun, Subasinghe, Zhou, Howell, Parno and Hawblitzel 2023; Müller, Schwerhoff and Summers 2017; Pulte, Makwana, Sewell, Memarian, Sewell and Krishnaswami 2023; Swamy, Chen, Fournet, Strub, Bhargavan and Yang 2013], including to OCaml by Cameleer [Pereira and Ravara 2021], which uses the Gospel specification language [Charguéraud, Filliâtre, Lourenço and Pereira 2019] and Why3 [Filliâtre and Paskevich 2013].

In *foundational* automated verification, proofs are checked by a proof assistant so the automation does not have to be trusted. To our knowledge, it has been applied to C [Sammler, Lepigre, Krebbers, Memarian, Dreyer and Garg 2021] and Rust [Gäher, Sammler, Jung, Krebbers and Dreyer 2024].

Zoo is mostly non-automated — except for our use Diaframe for local automation of separation logic reasoning. We would be interested in moving towards more automation in the future.

12.3 Physical equality

There is some literature in proof-assistant research on reflecting physical equality from the implementation language into the proof assistant, for optimization purposes: for example, exposing OCaml's physical equality as a predicate in Rocq lets us implement some memoization and sharing

techniques in Rocq libraries. However, axiomatizing physical equality in the proof assistant is difficult, and can result in inconsistencies.

The earlier discussions of this question that we know come from Jourdan’s thesis [Jourdan 2016] (chapter 9), also presented more succinctly in [Braibant, Jourdan and Monniaux 2014]. This work introduces the Jourdan condition, that physical equality implies equality of values. [Boulmé 2021] extends the treatment of physical equality in Rocq, integrating it in an “extraction monad” to control it more safely. There is also a discussion of similar optimizations in Lean in [Selsam, Hudon and de Moura 2020].

The correctness of the axiomatization of physical equality depends on the type of the values being compared: axiomatizations are typically polymorphic on any type A , but their correctness depends on the specific A being considered. For example, it is easy to correctly characterize physical equality on natural numbers, and other non-dependent types arising in Rocq verification projects. One difficulty in HeapLang and ZooLang is that they are untyped languages, their representation of \emptyset and `false` has the same type. But our remark that structural equality (in OCaml) does not necessarily coincide with definitional equality (in Rocq) also applies to other Rocq types: our examples with an existential `Any` constructor (see Section 5) can be reproduced with Σ -types.

13 Conclusion

We presented Zoo, a framework for the verification of concurrent OCaml 5 programs. It has been used to verify sequential imperative algorithms and a library of lock-free data structures. While it is not yet available on opam, it can be installed and used in other Rocq projects.

14 Future work

Relaxed memory model. Currently, the most important limitation of ZooLang is that it assumes a sequentially consistent memory model, whereas OCaml 5 has a relaxed memory model [Dolan, Sivaramakrishnan and Madhavapeddy 2018]. As a result, our semantics does not capture all observable behaviors and therefore our correctness results are compromised. This choice has a pragmatic justification: we wanted to ensure that we could scale up verification of concurrent algorithms in the simpler setting of sequential consistency before moving to relaxed memory.

It should be noted that moving to relaxed memory is much simpler than for other languages like C because the OCaml 5 memory model is comparatively not very relaxed. Indeed, Mével, Jourdan and Pottier [2020] propose an Iris-based program logic for Multicore OCaml [Sivaramakrishnan, Dolan, White, Jaffer, Kelly, Sahoo, Parimala, Dhiman and Madhavapeddy 2020] which Mével and Jourdan [2021] use to verify a fine-grained concurrent queue; they show that it is possible to adapt specifications and proofs in non-trivial but relatively straightforward way. This suggests that the transition is feasible and would not throw away our work; we plan to do it in the future.

Language features. ZooLang has been designed from the start for pragmatic verification of advanced concurrent data structures; this informed the choice of feature coverage and the semantics design. To accommodate other uses, more features are needed and therefore should be supported: exceptions, algebraic effects [Sivaramakrishnan, Dolan, White, Kelly, Jaffer and Madhavapeddy 2021], modules, functors, threads²⁷. Algebraic effects have been formalized by de Vilhena and Pottier [2021], who propose an Iris-based program logic; accordingly, we do not anticipate specific difficulties in introducing them in ZooLang.

²⁷<https://ocaml.org/manual/5.4/api/Thread.html>

Automation. Another interesting direction would be to combine Zoo with semi-automated techniques. Similarly to Why3, the simple parts of the verification effort would be done in a semi-automated way, while the most difficult parts would be conducted in Rocq.

Merging new features. Contrary to atomic record fields, atomic arrays and generative constructors have not yet been integrated into OCaml. We plan to discuss them with OCaml maintainers in the future; as both features are not very invasive, we hope that it will be easy to gather consensus.

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