Verification of Chase-Lev work-stealing deque

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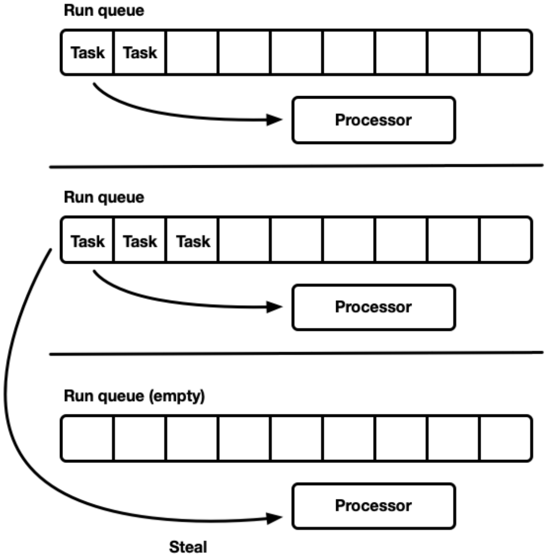
Verification of a scheduler

```
let rec fib pool n =
  if n < 2 then 1 else
  let r1 = async pool (fun () -> fib_par (n - 1)) in
  let r2 = async pool (fun () -> fib_par (n - 2)) in
  await pool r1 + await pool r2
```

Work-stealing



Work-stealing algorithms

1. Frigo, Leiserson & Randall (1998)
    - at the core of Cilk 5
    - lock
2. Arora, Blumofe & Plaxton (2001)
    - no lock
    - one fixed size array (not circular), can overflow
3. Hendler, Lev & Shavit (2004)
    - no lock
    - list of small size arrays, no overflow
    - memory leak?
4. <u>Chase & Lev (2005)</u>
    - no lock
    - circular arrays, no overflow

Why is it interesting?

- demonstration of Iris on a (simplified) real-life concurrent data structure
- rich ghost state to enforce a subtle protocol
  - logical state $\neq$ physical state
  - external future-dependent linearization point
- use of (typed) prophecy variables (with memory)

The rest of this talk

- specification using logically atomic triples
- rough idea of how the data structure works
- why we need prophecy variables (with memory)

Specification — `chaselev_make`

$$\frac{\{\,\text{True}\,\}}{\dfrac{\texttt{chaselev\_make ()}}{\left\{\lambda\, t \cdot\ \text{chaselev-inv}\ t\ \iota\ *\ \text{chaselev-model}\ t\ []\ *\ \text{chaselev-owner}\ t\right\}}}$$

Specification — `chaselev_make`

$$\frac{\{\,\text{True}\,\}}{\dfrac{\texttt{chaselev\_make ()}}{\left\{\, \lambda\, t \cdot\ \text{chaselev-inv}\ t\ \iota\ * \ \text{chaselev-model}\ t\ []\ *\ \text{chaselev-owner}\ t \,\right\}}}$$

enforces a protocol (using an Iris invariant)

Specification — `chaselev_make`

$$\frac{\{\,\text{True}\,\}}{\dfrac{\texttt{chaselev\_make ()}}{\left\{\,\lambda\, t \cdot\ \text{chaselev-inv}\ t\ \iota\ \ast\ \text{chaselev-model}\ t\ []\ \ast\ \text{chaselev-owner}\ t\,\right\}}}$$

asserts the list of values that the deque (logically) contains

Specification — `chaselev_make`

$$\frac{\{\ \mathrm{True}\ \}}{\dfrac{\texttt{chaselev\_make ()}}{\left\{\ \lambda\, t \cdot\ \mathrm{chaselev\text{-}inv}\ t\ \iota\ \ast\ \mathrm{chaselev\text{-}model}\ t\ []\ \ast\ \mathrm{chaselev\text{-}owner}\ t\ \right\}}}$$

gives the owner exclusive access to his end of the deque

Specification — `chaselev_push`

$$\dfrac{\left\{\ \text{chaselev-inv}\ t\ \iota\ \ast\ \text{chaselev-owner}\ t\ \right\}}{\dfrac{\left\langle\ \forall\, vs \cdot\ \text{chaselev-model}\ t\ vs\ \right\rangle}{\dfrac{\texttt{chaselev\_push}\ t\ v,\ \uparrow\iota}{\dfrac{\left\langle\ \exists \cdot\ \text{chaselev-model}\ t\ (vs +\!\!+ [v])\ \right\rangle}{\left\{\ \lambda\,\texttt{()} \cdot\ \text{chaselev-owner}\ t\ \right\}}}}}$$

Specification — `chaselev_push`

Specification of a concurrent operation ($\simeq$ transaction):
standard triple + logically atomic triple

$$\frac{\dfrac{\dfrac{\dfrac{\{\, P \,\}}{\langle\, \forall\, \overline{x} \cdot P_{\text{lin}} \,\rangle}}{e,\ E}}{\langle\, \exists\, \overline{y} \cdot Q_{\text{lin}} \,\rangle}}{\{\, \lambda\, res \cdot Q \,\}}$$

P : private precondition

Q : private postcondition

P_{lin} : public precondition

Q_{lin} : public postcondition

Specification — `chaselev_push`

For a concurrent data structure:

$$\frac{\frac{\frac{\frac{\{\, \text{???-inv} \cdots * P \,\}}{\langle\, \forall\, \overline{x} \cdot \text{???-model} \cdots \rangle}}{e,\ E}}{\langle\, \exists\, \overline{y} \cdot \text{???-model} \cdots \rangle}}{\{\, \lambda\, res \cdot Q \,\}}$$

Specification — `chaselev_push`

$$\left\{\ \text{chaselev-inv}\ t\ \iota\ \ast\ \text{chaselev-owner}\ t\ \right\}$$

$$\left\langle\ \forall\, vs \cdot\ \text{chaselev-model}\ t\ vs\ \right\rangle$$

$$\texttt{chaselev\_push}\ t\ v,\ \uparrow\iota$$

$$\left\langle\ \exists \cdot\ \text{chaselev-model}\ t\ (vs +\!\!+ [v])\ \right\rangle$$

$$\left\{\ \lambda\,\texttt{()} \cdot\ \text{chaselev-owner}\ t\ \right\}$$

owner's operation

Specification — `chaselev_push`

$$\left\{ \text{chaselev-inv } t\ \iota \ast \text{chaselev-owner } t \right\}$$

$$\left\langle \forall\, vs \cdot\ \text{chaselev-model } t\ vs \right\rangle$$

$$\texttt{chaselev\_push}\ t\ v,\ \uparrow \iota$$

$$\left\langle \exists \cdot\ \text{chaselev-model } t\ (vs \mathbin{+\!\!+} [v]) \right\rangle$$

$$\left\{ \lambda\, () \cdot\ \text{chaselev-owner } t \right\}$$

some value v is atomically pushed at the owner's end

Specification — `chaselev_pop`

$$\dfrac{\dfrac{\left\{\ \text{chaselev-inv}\ t\ \iota\ \ *\ \ \text{chaselev-owner}\ t\ \right\}}{\left\langle\ \forall\, vs \cdot\ \ \text{chaselev-model}\ t\ vs\ \right\rangle}}{\dfrac{\texttt{chaselev\_pop}\ t,\ \uparrow\iota}{\dfrac{\left\langle\ \exists\, o \cdot\ \ \bigvee\left[\begin{array}{l} vs = [] * o = \texttt{NONE} * \text{chaselev-model}\ t\ [] \\ \exists\, v, vs' \cdot vs = vs' +\!\!+ [v] * o = \texttt{SOME}\ v * \text{chaselev-model}\ t\ vs' \end{array}\right]\ \right\rangle}{\left\{\lambda\, o \cdot\ \ \text{chaselev-owner}\ t\ \right\}}}}$$

Specification — `chaselev_pop`

$$\left\{\ \text{chaselev-inv}\ t\ \iota\ \ *\ \ \text{chaselev-owner}\ t\ \right\}$$
$$\left\langle\ \forall\, vs \cdot\ \ \text{chaselev-model}\ t\ vs\ \right\rangle$$
$$\texttt{chaselev\_pop}\ t,\ \uparrow \iota$$
$$\left\langle\ \exists\, o \cdot\ \ \bigvee\left[\begin{array}{l} vs = [\,] * o = \texttt{NONE} * \text{chaselev-model}\ t\ [\,] \\ \exists\, v, vs' \cdot vs = vs' +\!\!+ [v] * o = \texttt{SOME}\ v * \text{chaselev-model}\ t\ vs' \end{array}\right]\ \right\rangle$$
$$\left\{\ \lambda\, o \cdot\ \ \text{chaselev-owner}\ t\ \right\}$$

owner's operation

Specification — `chaselev_pop`

$$\dfrac{\left\{\ \text{chaselev-inv}\ t\ \iota\ \ *\ \ \text{chaselev-owner}\ t\ \right\}}{\dfrac{\left\langle\ \forall\, vs \cdot\ \ \text{chaselev-model}\ t\ vs\ \right\rangle}{\dfrac{\texttt{chaselev\_pop}\ t,\ \uparrow \iota}{\dfrac{\left\langle\ \exists\, o \cdot\ \ \bigvee \begin{bmatrix} vs = [] * o = \texttt{NONE} * \text{chaselev-model}\ t\ [] \\ \exists\, v, vs' \cdot vs = vs' +\!\!+ [v] * o = \texttt{SOME}\ v * \text{chaselev-model}\ t\ vs' \end{bmatrix}\ \right\rangle}{\left\{\ \lambda\, o \cdot\ \ \text{chaselev-owner}\ t\ \right\}}}}}$$

either 1) some value v is atomically popped at the owner's end
or 2) the deque is seen empty

Specification — `chaselev_steal`

$$\left\{\ \text{chaselev-inv}\ t\ \iota\ \right\}$$

$$\left\langle \forall\, vs \cdot\ \ \text{chaselev-model}\ t\ vs \right\rangle$$

$$\texttt{chaselev\_steal}\ t,\ \uparrow\iota$$

$$\left\langle\ \exists\, o \cdot\ \ \bigvee \left[\begin{array}{l} vs = [\,] * o = \texttt{NONE} * \text{chaselev-model}\ t\ [\,] \\ \exists\, v, vs' \cdot vs = v :: vs' * o = \texttt{SOME}\ v * \text{chaselev-model}\ t\ vs' \end{array}\right]\ \right\rangle$$

$$\{\, \lambda\, o \cdot\ \text{True} \,\}$$

Specification — `chaselev_steal`

$$\frac{\left\{\ \text{chaselev-inv}\ t\ \iota\ \right\}}{\dfrac{\left\langle \forall\, vs \cdot\ \ \text{chaselev-model}\ t\ vs \right\rangle}{\dfrac{\texttt{chaselev\_steal}\ t,\ \uparrow\iota}{\dfrac{\left\langle \exists\, o \cdot\ \ \bigvee \begin{bmatrix} vs = [] * o = \texttt{NONE} * \text{chaselev-model}\ t\ [] \\ \exists\, v, vs' \cdot vs = v :: vs' * o = \texttt{SOME}\ v * \text{chaselev-model}\ t\ vs' \end{bmatrix} \right\rangle}{\{\, \lambda\, o \cdot \text{True} \,\}}}}}$$

either 1) some value v is atomically popped at the thieves' end
or 2) the deque is seen empty

Thank you for your attention!