Verification of Chase-Lev work-stealing deque

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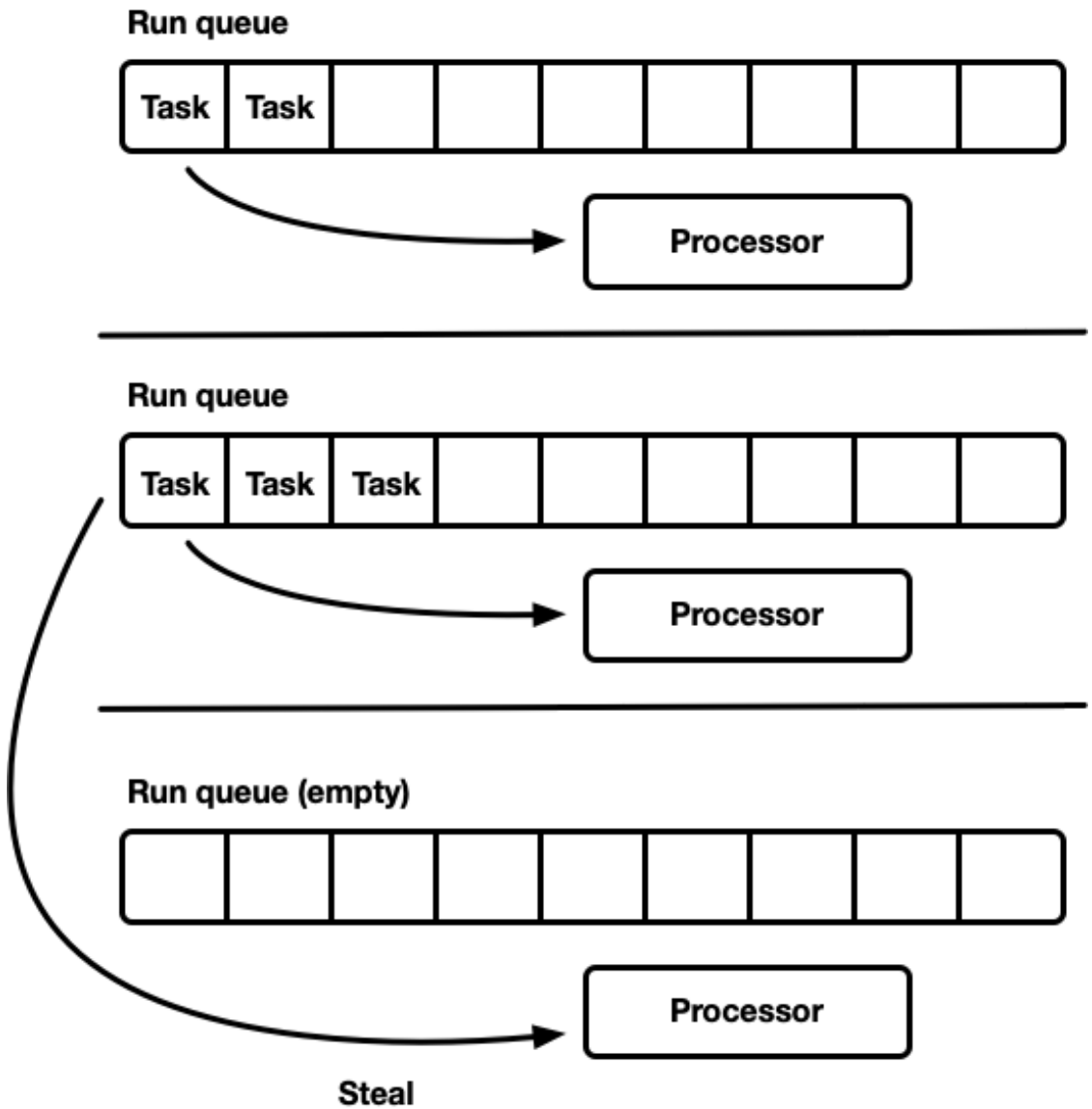
Verification of a scheduler

```
let rec fib pool n =
  if n < 2 then 1 else
  let r1 = async pool (fun () -> fib_par (n - 1)) in
  let r2 = async pool (fun () -> fib_par (n - 2)) in
  await pool r1 + await pool r2
```

Work-stealing



Work-stealing algorithms

1. Frigo, Leiserson & Randall (1998)
   - at the core of Cilk 5
   - lock
2. Arora, Blumofe & Plaxton (2001)
   - no lock
   - one fixed size array (not circular), can overflow
3. Hendler, Lev & Shavit (2004)
   - no lock
   - list of small size arrays, no overflow
   - memory leak?
4. Chase & Lev (2005)
   - no lock
   - circular arrays, no overflow

Why is it interesting?

- demonstration of Iris on a (simplified) real-life concurrent data structure
- rich ghost state to enforce a subtle protocol
  - logical state $\neq$ physical state
  - external future-dependent linearization point
- use of (typed) prophecy variables (with memory)

The rest of this talk

- specification using logically atomic triples
- rough idea of how the data structure works
- why we need prophecy variables (with memory)

Specification — `chaselev_make`

$$\frac{\{\,\text{True}\,\}}{\texttt{chaselev\_make ()}}$$

$$\left\{\,\lambda\, t \cdot\ \text{chaselev-inv}\ t\ \iota\ *\ \text{chaselev-model}\ t\ []\ *\ \text{chaselev-owner}\ t\,\right\}$$

Specification — `chaselev_make`

$$\frac{\{\,\text{True}\,\}}{\texttt{chaselev\_make ()}}$$

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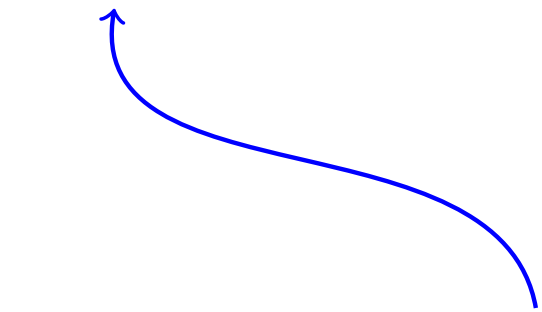

enforces a protocol (using an Iris invariant)

Specification — `chaselev_make`

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asserts the list of values that the deque (logically) contains

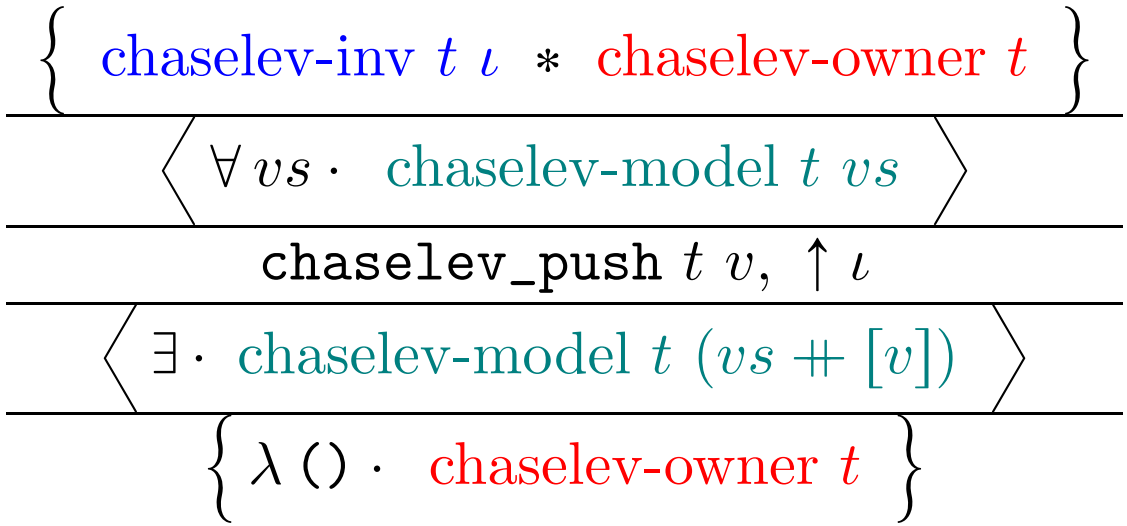
Specification — `chaselev_make`

$$\frac{\{\,\text{True}\,\}}{\texttt{chaselev\_make ()}}$$

$$\left\{\,\lambda\, t \cdot\ \text{chaselev-inv}\ t\ \iota\ *\ \text{chaselev-model}\ t\ []\ *\ \text{chaselev-owner}\ t\,\right\}$$

gives the owner exclusive access to his end of the deque

Specification — `chaselev_push`



Specification — `chaselev_push`

Specification of a concurrent operation ($\simeq$ transaction):
standard triple + logically atomic triple

$$\frac{\frac{\frac{\frac{\{\, P \,\}}{\langle\, \forall\, \overline{x} \cdot\ P_{\text{lin}} \,\rangle}}{e,\ \mathcal{E}}}{\langle\, \exists\, \overline{y} \cdot\ Q_{\text{lin}} \,\rangle}}{\{\, \lambda\, res \cdot\ Q \,\}}$$

P : private precondition

Q : private postcondition

P_{lin} : public precondition

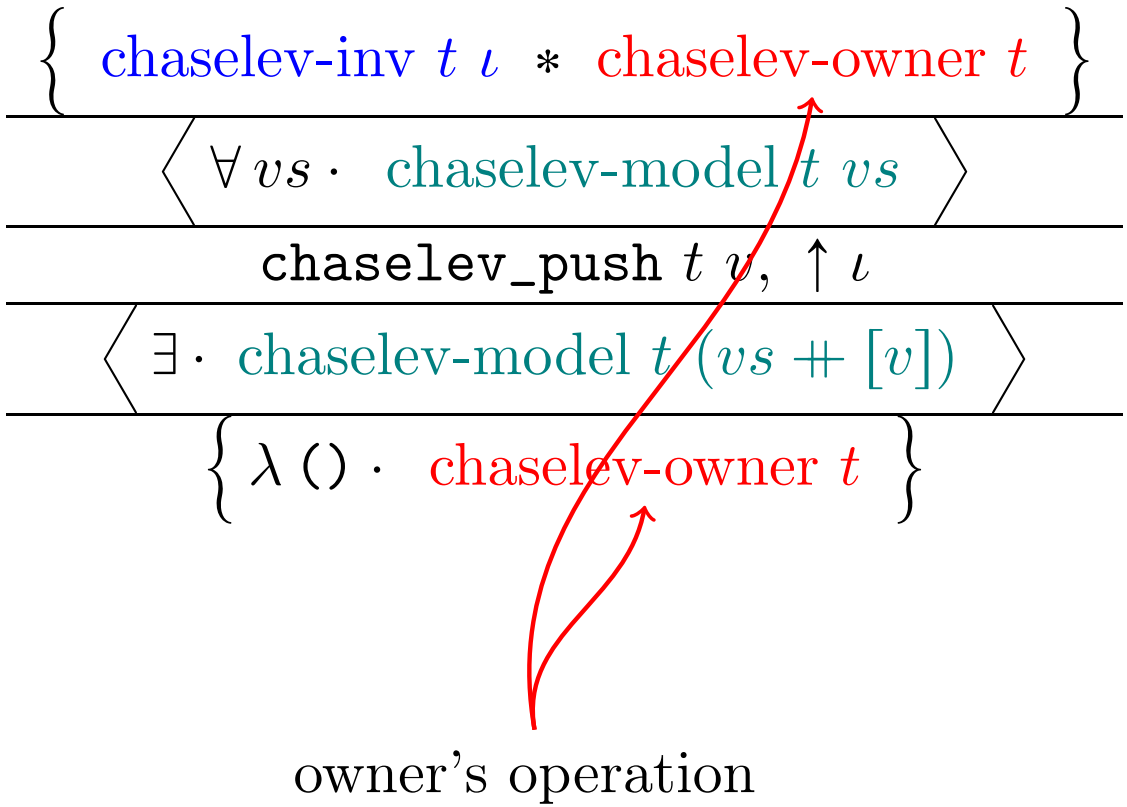
Q_{lin} : public postcondition

Specification — `chaselev_push`

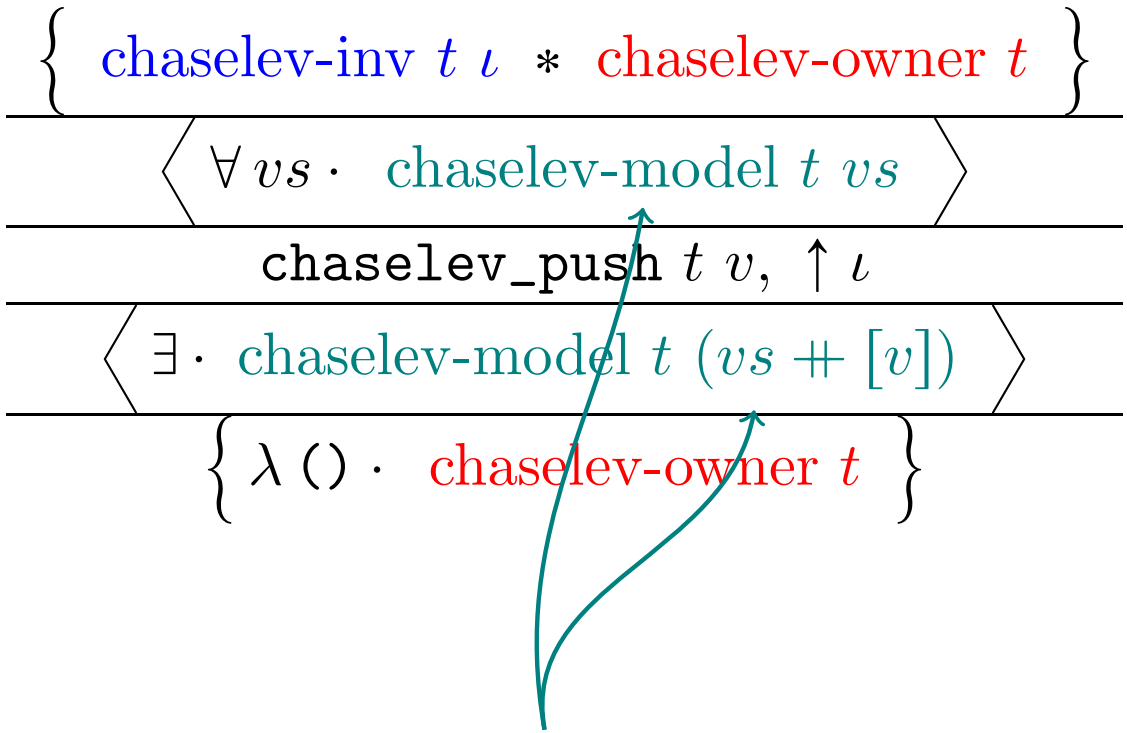
For a concurrent data structure:

$$\frac{\{\ \text{???-inv}\cdots * P\ \}}{\frac{\langle\ \forall\, \overline{x}\cdot\ \text{???-model}\cdots\ \rangle}{\frac{e,\ \mathcal{E}}{\frac{\langle\ \exists\, \overline{y}\cdot\ \text{???-model}\cdots\ \rangle}{\{\ \lambda\, res\cdot\ Q\ \}}}}}$$

Specification — `chaselev_push`



Specification — `chaselev_push`



v is atomically pushed at the owner's end

Specification — `chaselev_pop`

$$\left\{ \text{chaselev-inv } t\ \iota \ *\ \text{chaselev-owner } t \right\}$$

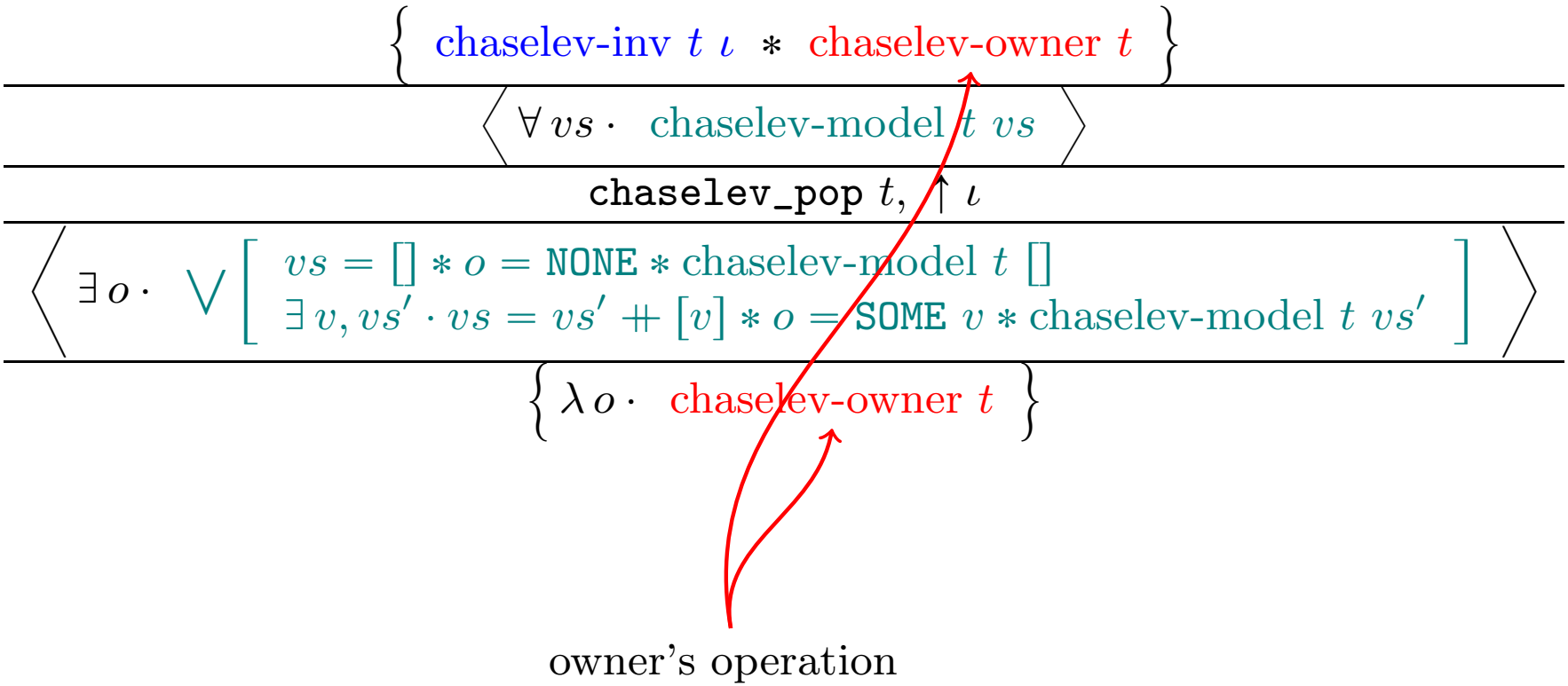
$$\left\langle \forall\, vs \cdot \ \text{chaselev-model } t\ vs \right\rangle$$

$$\texttt{chaselev\_pop}\ t,\ \uparrow \iota$$

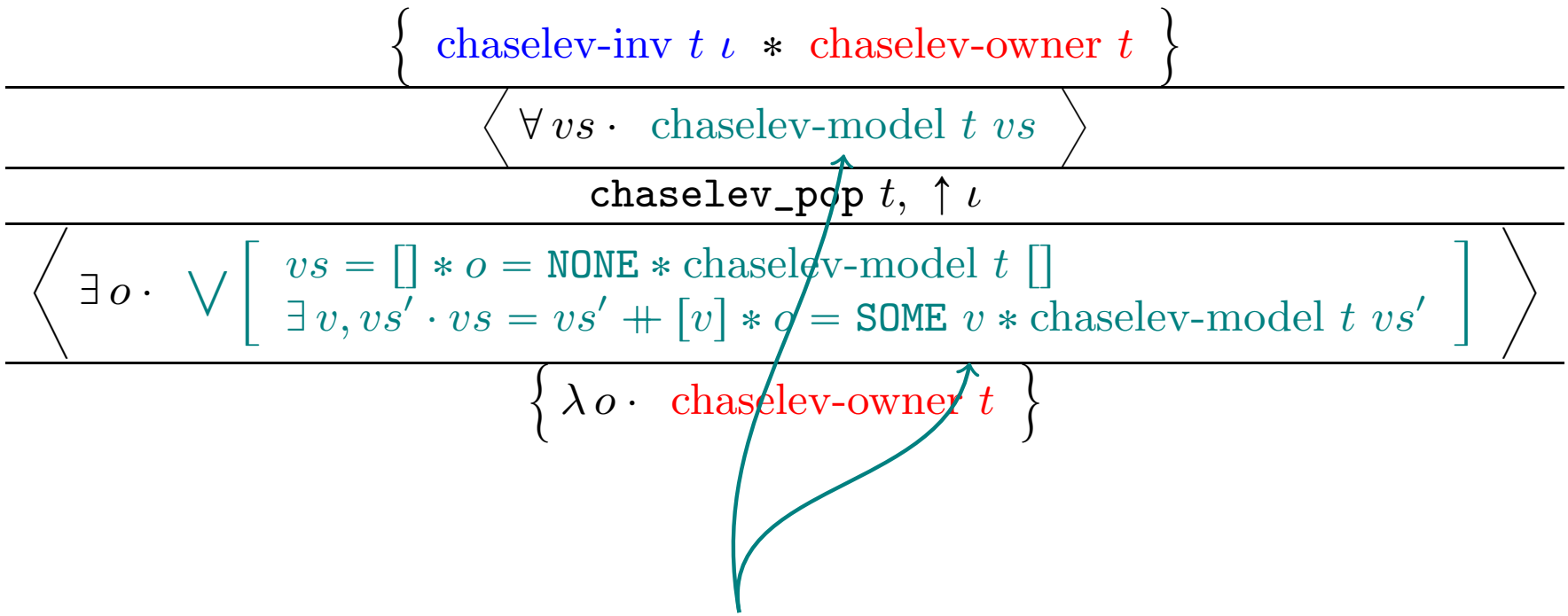
$$\left\langle \exists\, o \cdot \ \bigvee \left[\begin{array}{l} vs = [] * o = \texttt{NONE} * \text{chaselev-model } t\ [] \\ \exists\, v, vs' \cdot vs = vs' +\!\!+ [v] * o = \texttt{SOME}\ v * \text{chaselev-model } t\ vs' \end{array} \right] \right\rangle$$

$$\left\{ \lambda\, o \cdot \ \text{chaselev-owner } t \right\}$$

Specification — `chaselev_pop`



Specification — `chaselev_pop`



either 1) some value v is atomically popped at the owner's end
or 2) the deque is seen empty

Specification — `chaselev_steal`

$$\left\{ \text{chaselev-inv } t\ \iota \right\}$$

$$\left\langle \forall\, vs \cdot \ \text{chaselev-model } t\ vs \right\rangle$$

$$\texttt{chaselev\_steal}\ t,\ \uparrow \iota$$

$$\left\langle \exists\, o \cdot \ \bigvee \left[\begin{array}{l} vs = [] * o = \texttt{NONE} * \text{chaselev-model } t\ [] \\ \exists\, v, vs' \cdot vs = v :: vs' * o = \texttt{SOME}\ v * \text{chaselev-model } t\ vs' \end{array} \right] \right\rangle$$

$$\{ \lambda\, o \cdot \ \text{True} \}$$

Specification — `chaselev_steal`

$$\left\{ \text{chaselev-inv } t\ \iota \right\}$$

$$\left\langle \forall\, vs \cdot \quad \text{chaselev-model } t\ vs \right\rangle$$

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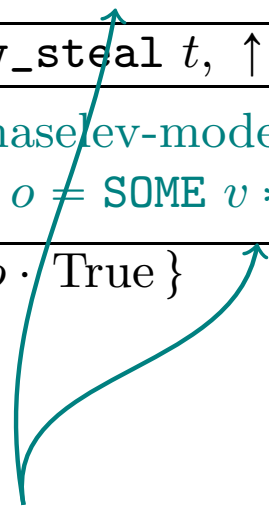
$$\{\, \lambda\, o \cdot \text{True} \,\}$$

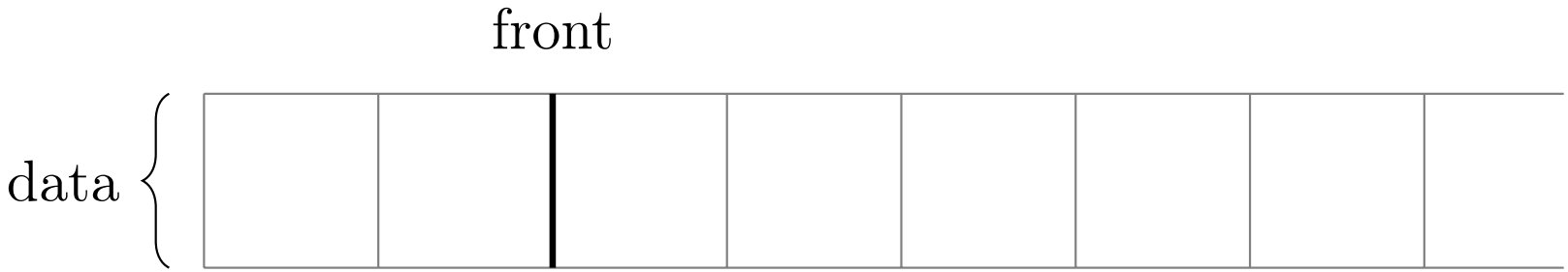


either 1) some value v is atomically popped at the thieves' end
or 2) the deque is seen empty

Physical state

data: infinite array storing all values

Physical state



data: infinite array storing all values

front: *monotone* index for thieves' end

Physical state

data

data: in

front: m

ChaselevFrontValid

γ.front $\bullet\ front_1$ $\quad$ γ.front $\circ\ front_2$

$$front2 \leqslant front1$$

ChaselevFrontUpdate

$front \leqslant front'$ $\quad$ γ.front $\bullet\ front$

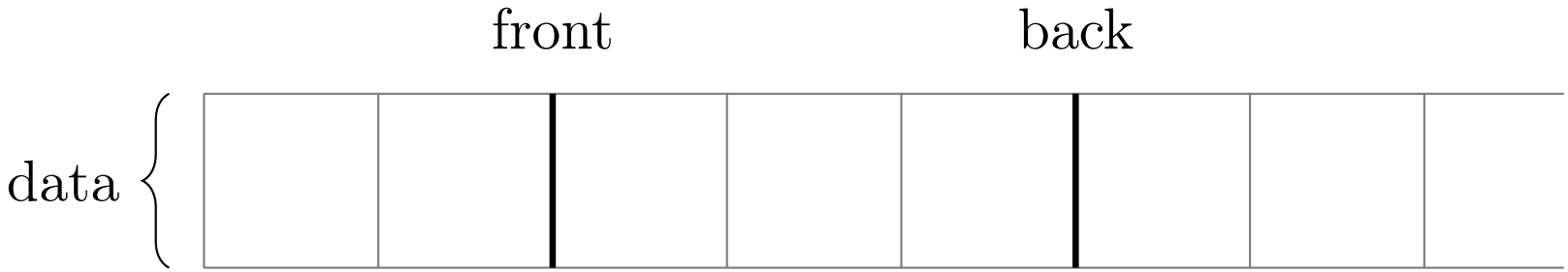
$$\gamma\text{.front}\ \bullet\ front'$$

ChaselevFrontFragGet

γ.front $\bullet\ front$

$$\gamma\text{.front}\ \circ\ front$$

Physical state

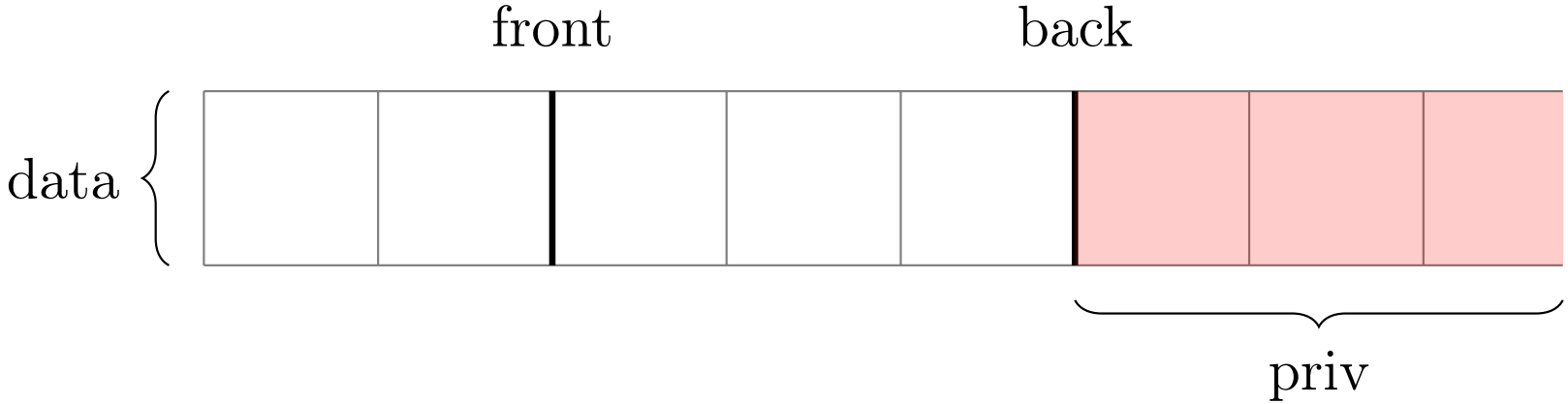


data: infinite array storing all values

front: *monotone* index for thieves' end

back: index for owner's end

Physical state



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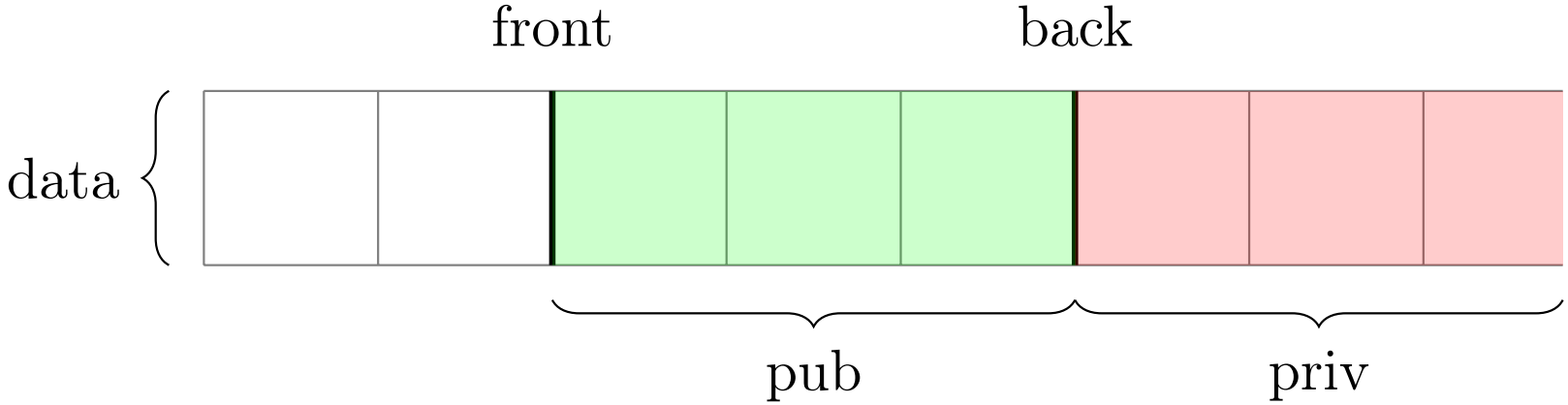
front: *monotone* index for thieves' end

back: index for owner's end

priv: list of private values (controlled by owner)

Physical state



data: infinite array storing all values

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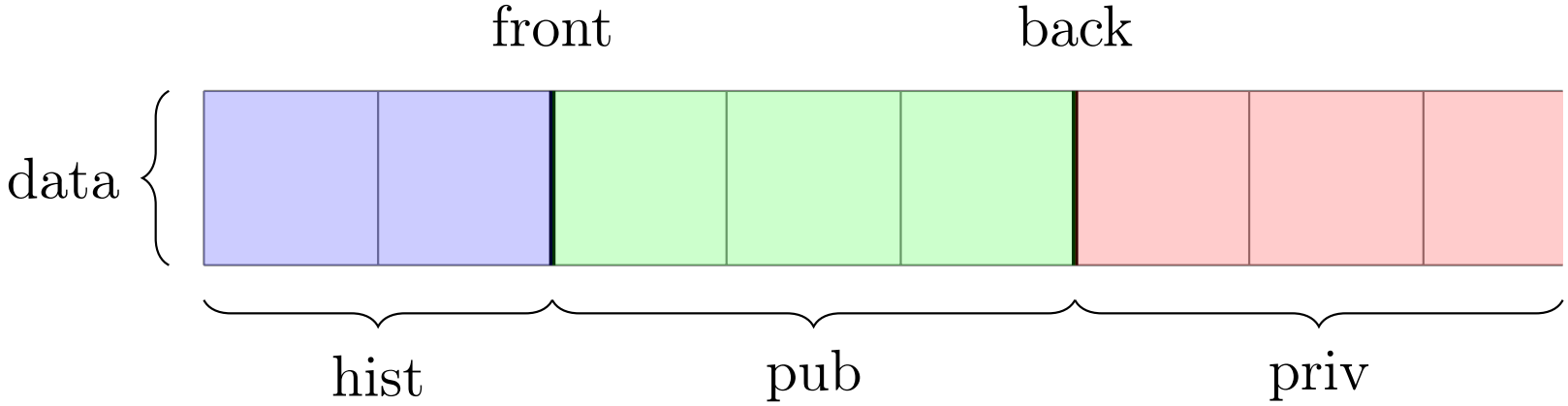
priv: list of private values (controlled by owner)

pub: list of public values (= model)

Physical state



data: infinite array storing all values

front: *monotone* index for thieves' end

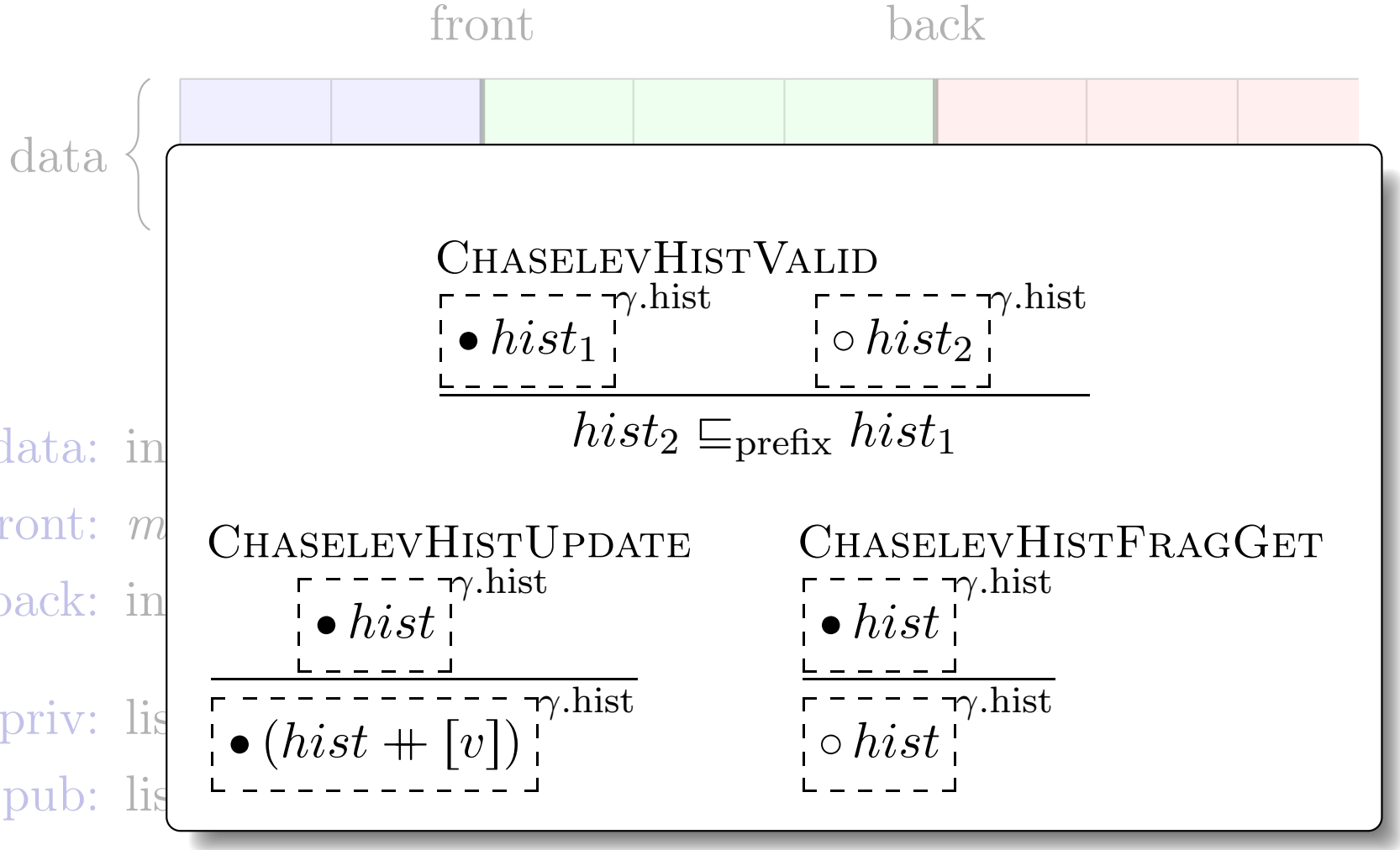
back: index for owner's end

priv: list of private values (controlled by owner)

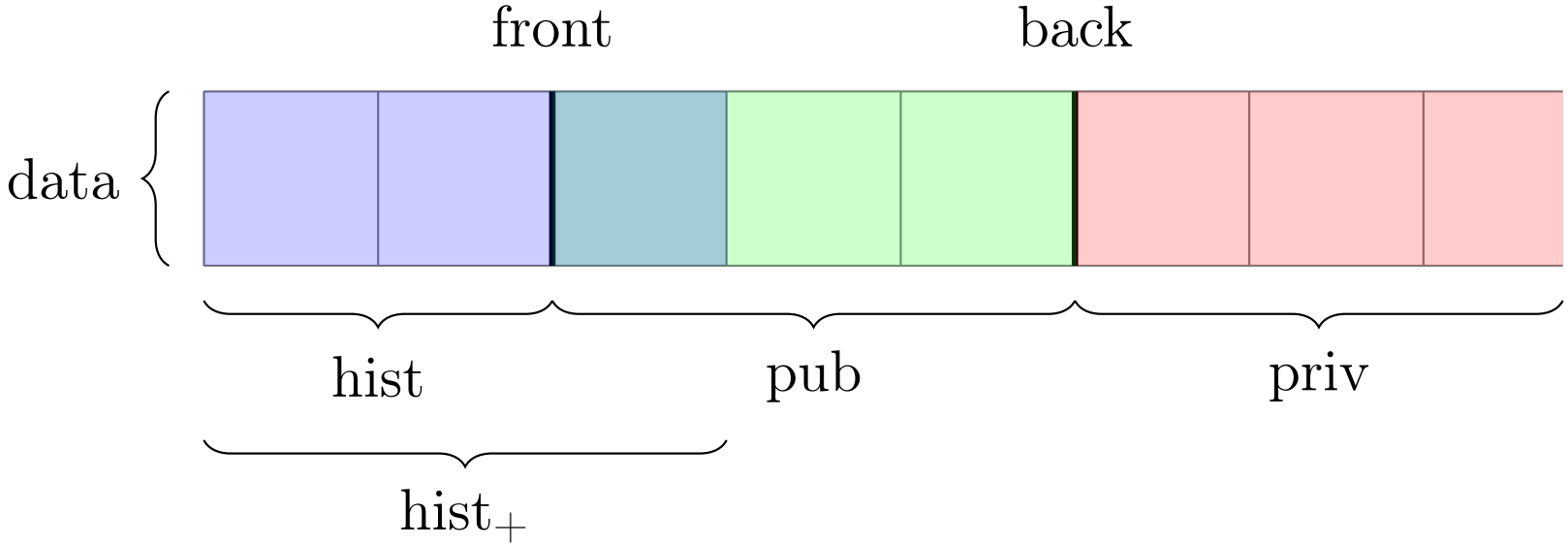
pub: list of public values (= model)

hist: *monotone* list of history values

Physical state



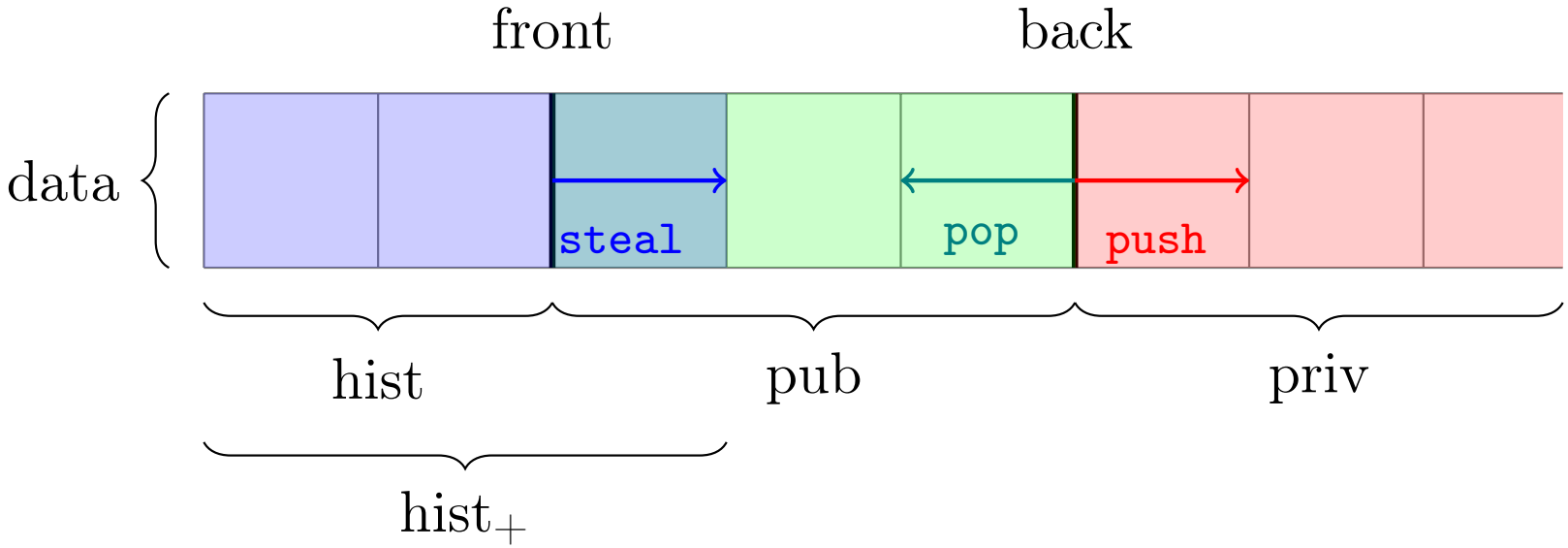
Physical state



data: infinite array storing all values
front: *monotone* index for thieves' end
back: index for owner's end

priv: list of private values (controlled by owner)
pub: list of public values (= model)
hist: *monotone* list of history values
hist$_+$: *monotone* list of extended history values

Physical state



data: infinite array storing all values
front: *monotone* index for thieves' end
back: index for owner's end

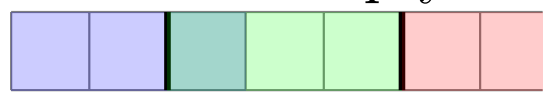
priv: list of private values (controlled by owner)
pub: list of public values (= model)
hist: *monotone* list of history values
hist_+: *monotone* list of extended history values

Logical state

① empty

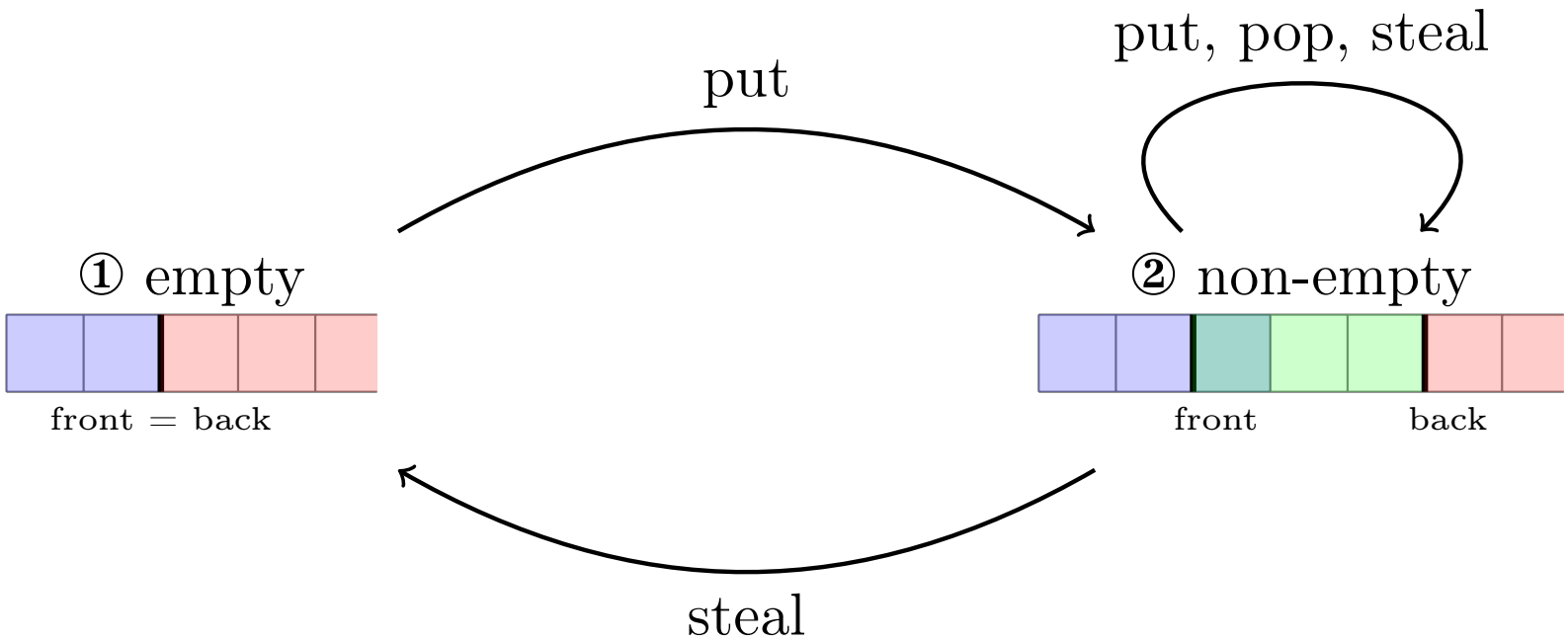
front = back

② non-empty

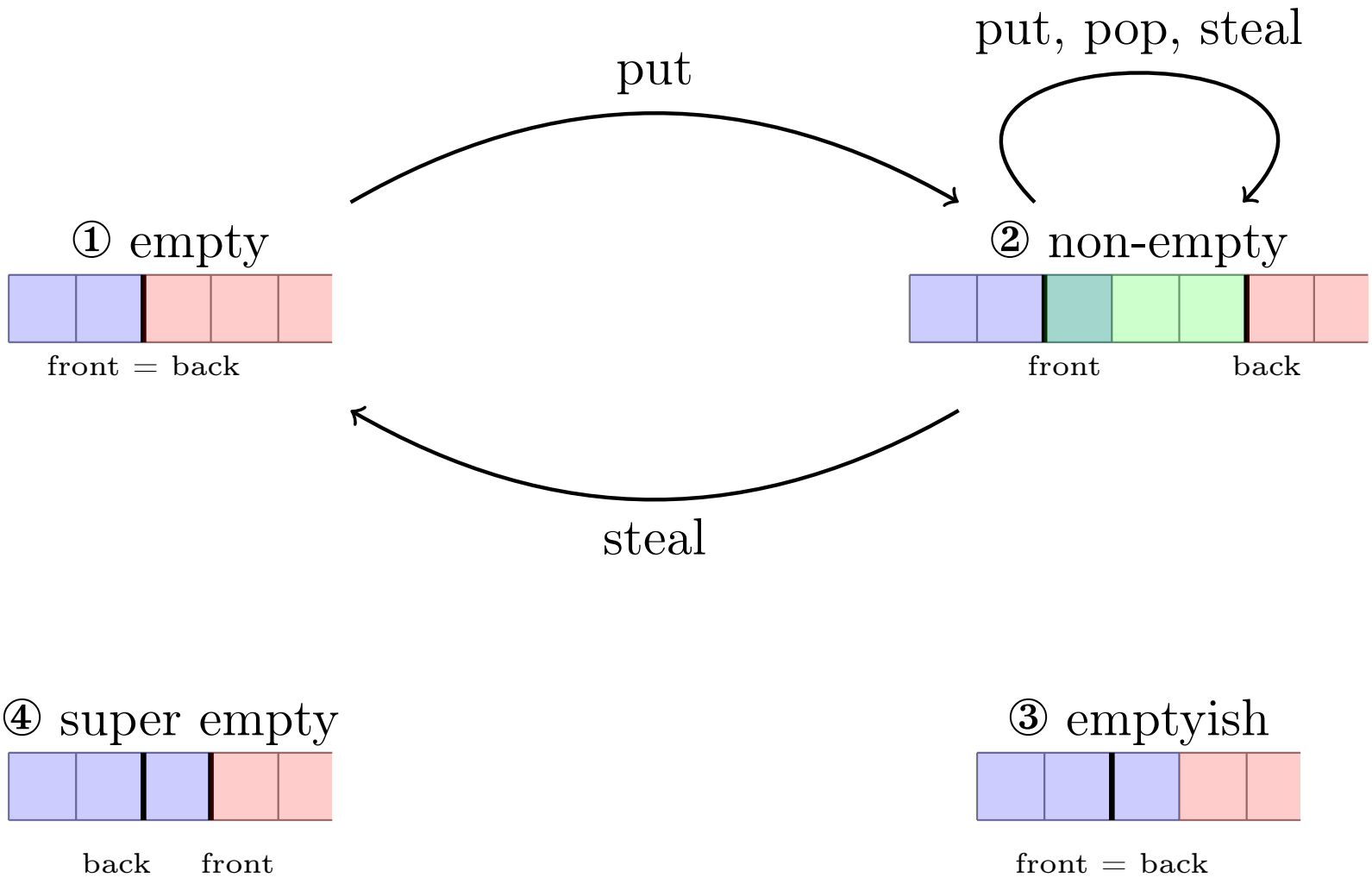


front back

Logical state



Logical state



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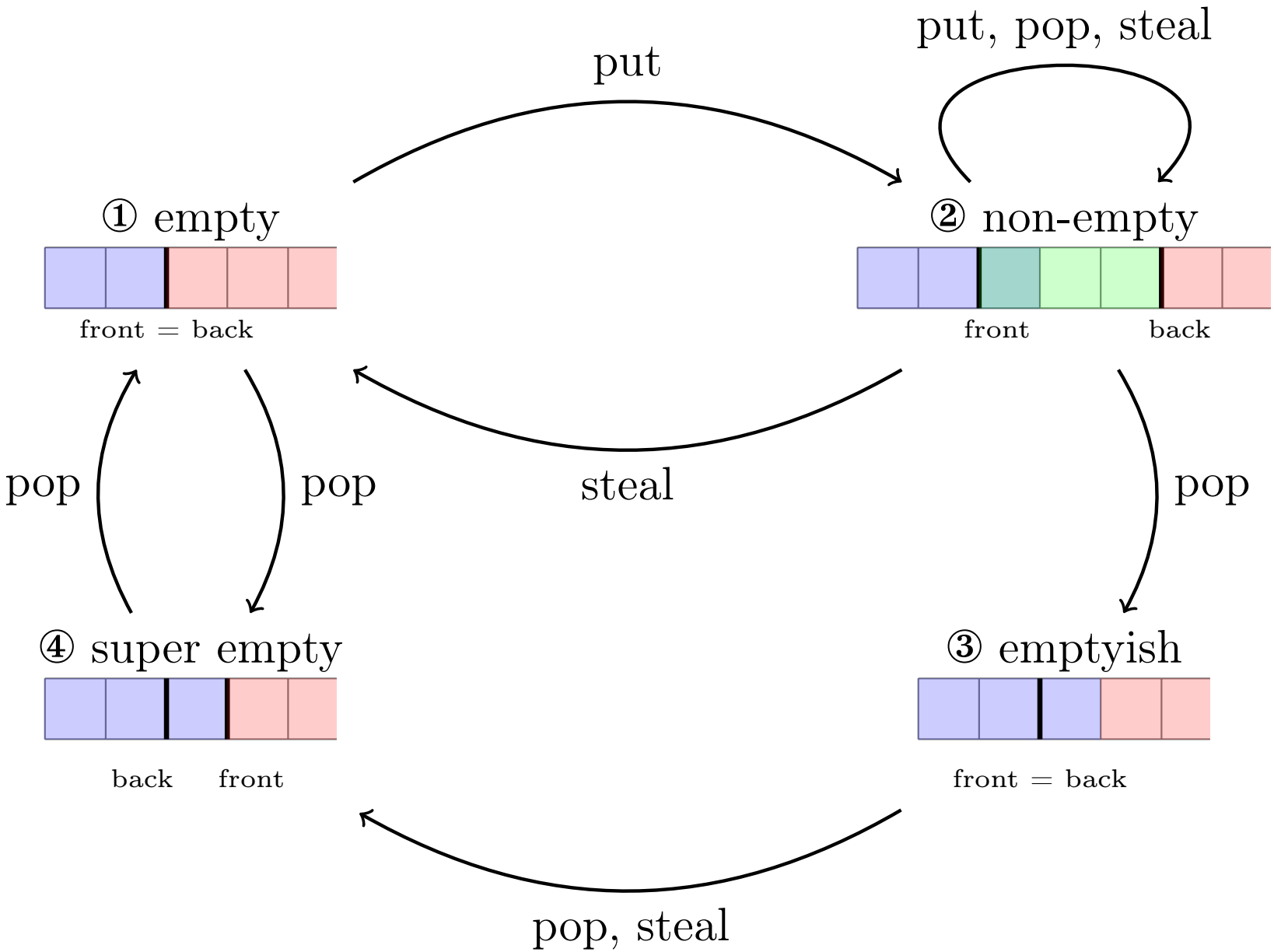

Thank you for your attention!