The IA|BE 2020 mortality projection for the Belgian population

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Working group mortality - IA|BE and LRisk - KU Leuven

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LRISK

Presenter + Q&A moderators

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(*) This presentation reflects the personal views of the author and not the views of his employer.

The basic principles of IA|BE 2015 and IA|BE 2020

Recap

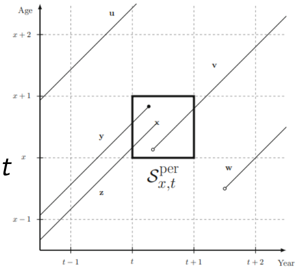
The mortality project of IA|BE opts for:

- biological reasonableness
- robustness and consistency
- good performance on in sample statistical measures as well as out-of-time back-tests
- ability to generate future scenarios of mortality
- reproducibility and full transparency
- ...

More detailed discussion in Antonio, Devriendt et al. (2017, EAJ), KAG 2014, 2016, 2018 and 2020.

Notation

Recap



$q_{x,t}$ is the probability that a person who is alive on January 1 of year t and who was born on January 1 of year $t - x$, will not be alive on January 1 of year $t + 1$.

Exact age definition.

$q_{x,t}$ is called the mortality rate:

modelled via $q_{x,t} = 1 - \exp(-\mu_{x,t})$,

where $\mu_{x,t}$ is the force of mortality or hazard rate.

IA|BE 2015 is a multi-population mortality model

Recap

Multi-population models construct $M_{\text{com}} + M_{\text{country}}$,
where M_{com} is common and M_{country} is country-specific.

The augmented common factor model by Li & Lee (2005, Demography):

- across a set of countries:

$$\ln \mu_{x,t}^{(i)} = \underbrace{A_x}_{\text{common}} + \underbrace{\alpha_x^{(i)}}_{\text{country}} + \underbrace{B_x \cdot K_t}_{\text{common}} + \underbrace{\beta_x^{(i)} \cdot \kappa_t^{(i)}}_{\text{country}},$$

  for a specific country (i)

- twice a Lee & Carter (1992, JASA) specification.

"Collect and include new data points, re-calibrate the model parameters, and - if necessary - incorporate any methodological changes."

Starting point is the IA|BE 2015 model, documentation of all assumptions, calibration and simulation details is part of our mission.

Antonio, Devriendt et al. (2015, European Actuarial Journal). Producing the Dutch and Belgian mortality projections: a stochastic multi-population standard.

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Data collection

We select European countries with GDP per capita above the average of the Euro area (in 2018).

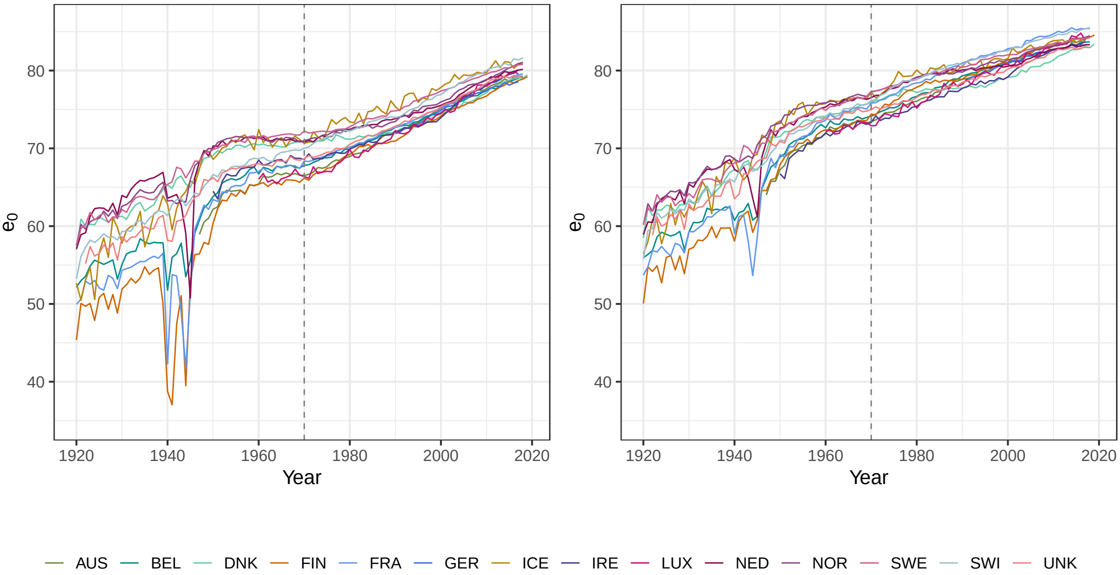
This results in a set of 14 countries:

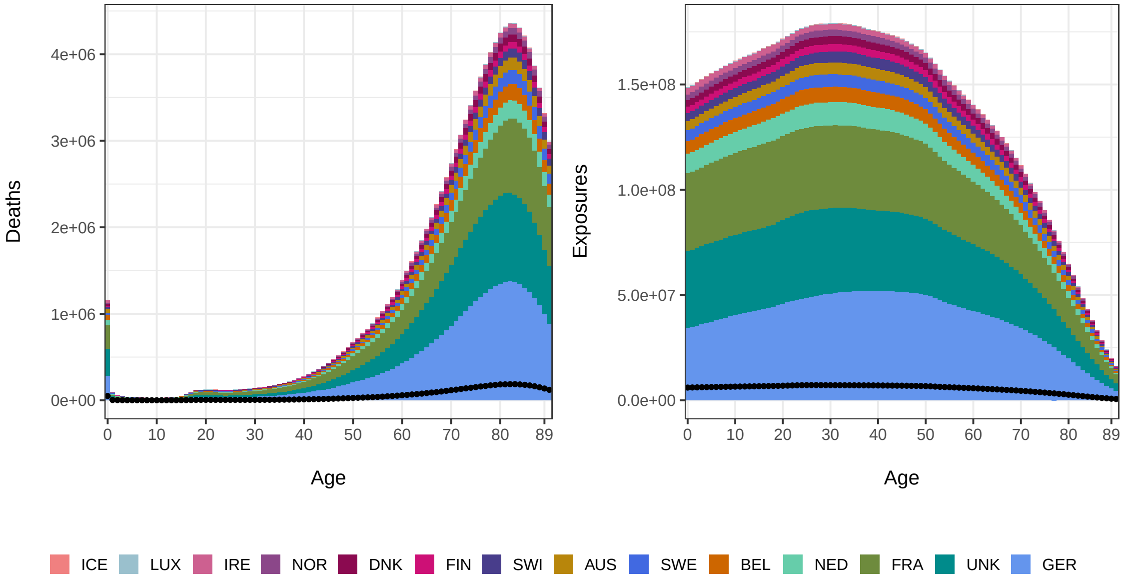
> Belgium, The Netherlands, Luxembourg, Germany, France, UK, Ireland, Iceland, Norway, Sweden, Finland, Denmark, Switzerland, Austria.

For these countries we obtain data on deaths and exposures from 1970 until 2018 from HMD (www.mortality.org) and Eurostat (https://ec.europa.eu/eurostat).

For Belgium, we add deaths and exposures in 2019 from Statbel (https://statbel.fgov.be/en).

The data - evolution in period life expectancy (source: HMD)




Deaths
4e+06
3e+06
2e+06
1e+06
0e+00
0
10
20
30
40
50
60
70
80
89
Age
Exposures
1.5e+08
1.0e+08
5.0e+07
0.0e+00
0
10
20
30
40
50
60
70
80
89
Age
ICE
LUX
IRE
NOR
DNK
FIN
SWI
AUS
SWE
BEL
NED
FRA
UNK
GER

From IA|BE 2015 to IA|BE 2020: changes in data collection

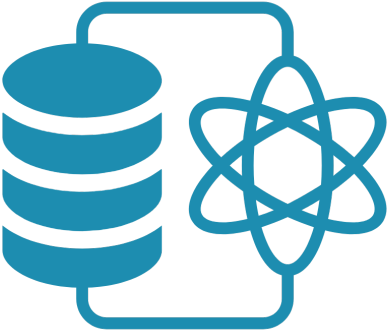
Two changes:

(1) England & Wales replaced by UK (new!)

(2) before 1990 we use West-Germany and from 1990 on we use Germany (new!).

Data sources: (mind the Lexis definitions!)

- HMD
- Eurostat (new!)
- Statbel.



Model specification and calibration

Lee & Carter model for EU mortality and country-specific deviation:

$$\begin{aligned}
\ln \mu_{x,t}^{(\mathrm{BEL})} &= \ln \mu_{x,t}^{(\mathrm{EU})} + \ln \tilde{\mu}_{x,t}^{(\mathrm{BEL})} \\
\ln \mu_{x,t}^{(\mathrm{EU})} &= A_x + B_x K_t \\
\ln \tilde{\mu}_{x,t}^{(\mathrm{BEL})} &= \alpha_x + \beta_x \kappa_t
\end{aligned}$$

with constraints

$$\sum_t K_t = \sum_t \kappa_t = 0 \text{ and } \sum_x B_x^2 = \sum_x \beta_x^2 = 1 \quad \text{(new!)}.$$

We use a Poisson assumption for the number of deaths and Maximum Likelihood Eestimation (MLE) for estimating the parameters.

Three-step approach with Newton-Raphson as tool: (per gender)

1. Estimate A_x, B_x, K_t through POI likelihood for EU mortality $\mu_{x,t}^{(\mathrm{EU})}$ with $x \in \{0, \ldots, 90\}$ and $t \in \{t_{\mathrm{start}}, \ldots, 2018\}$:

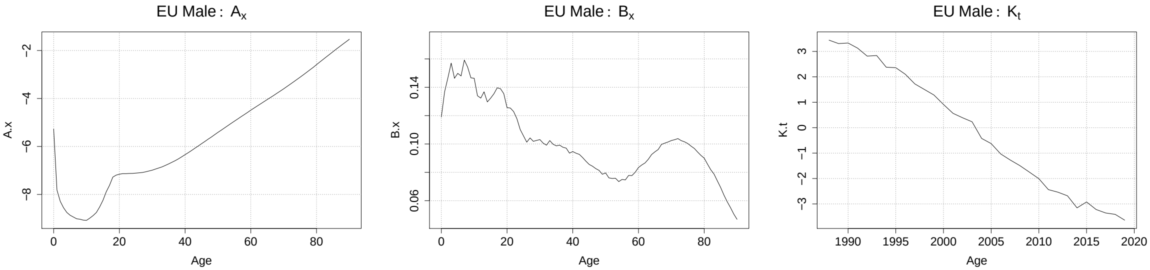
$$\prod_x \prod_t \left(E_{x,t}^{(\mathrm{EU})} \cdot \mu_{x,t}^{(\mathrm{EU})}\right)^{d_{x,t}^{(\mathrm{EU})}} \cdot \exp\left(-E_{x,t}^{(\mathrm{EU})} \cdot \mu_{x,t}^{(\mathrm{EU})}\right) / \left(d_{x,t}^{(\mathrm{EU})}!\right).$$

2. Extrapolate (linearly) K_t to $t = 2019$.

3. Estimate $\alpha_x, \beta_x, \kappa_t$ through POI conditional likelihood for country mortality $\tilde{\mu}_{x,t}^{(\mathrm{BEL})}$ with $x \in \{0, \ldots, 90\}$ and $t \in \{t_{\mathrm{start}}, \ldots, 2019\}$:

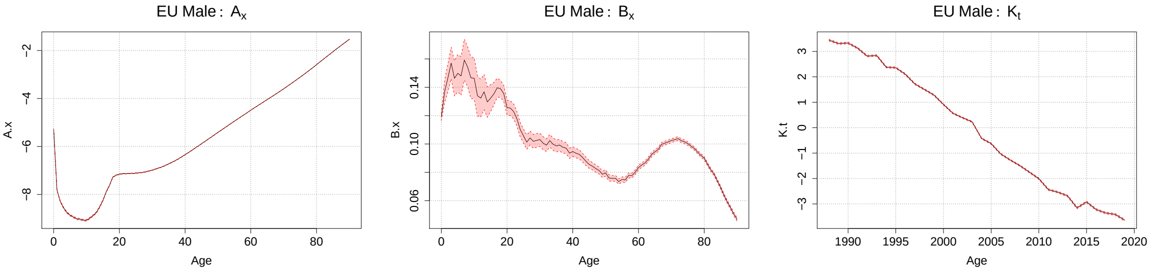
$$\prod_x \prod_t \left(E_{x,t}^{(\mathrm{BEL})} \mu_{x,t}^{(\mathrm{EU})} \cdot \tilde{\mu}_{x,t}^{(\mathrm{BEL})}\right)^{d_{x,t}^{(\mathrm{BEL})}} \cdot \exp\left(-E_{x,t}^{(\mathrm{BEL})} \mu_{x,t}^{(\mathrm{EU})} \cdot \tilde{\mu}_{x,t}^{(\mathrm{BEL})}\right) / \left(d_{x,t}^{(\mathrm{BEL})}!\right).$$

IA|BE 2020: males, fitted parameters, EU level



Note: calibration period 1988 - 2019 extensively motivated in Antonio, Devriendt and Robben (2020).

IA|BE 2020: males, fitted parameters, EU level



Poisson bootstrap used to add parameter uncertainty (new!) (99% pointwise intervals, based on 10 000 simulations).

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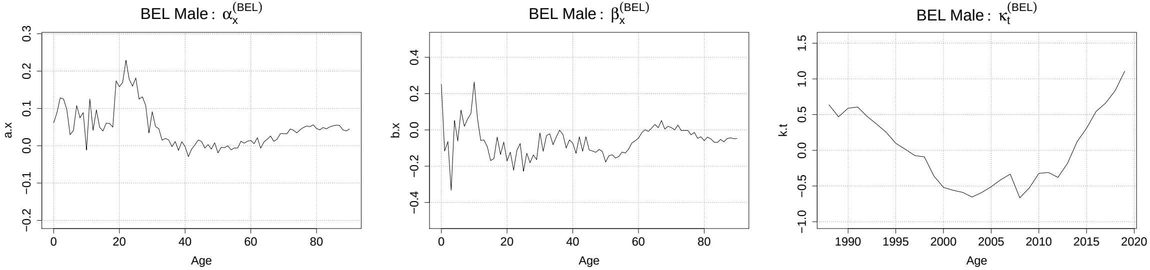

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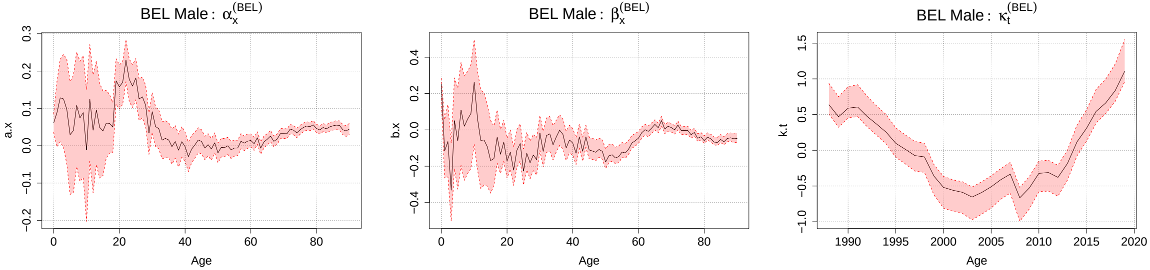
IA|BE 2020: males, fitted parameters, Belgian deviation



Poisson bootstrap used to add parameter uncertainty (new!) (99% pointwise intervals, based on 10 000 simulations).

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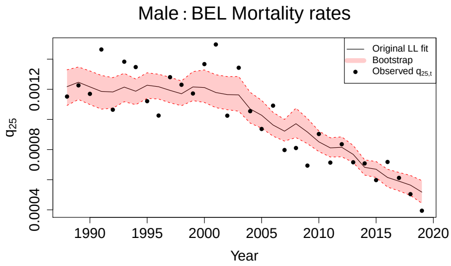
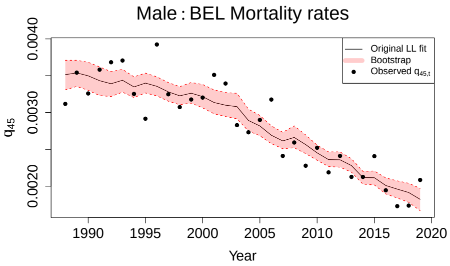
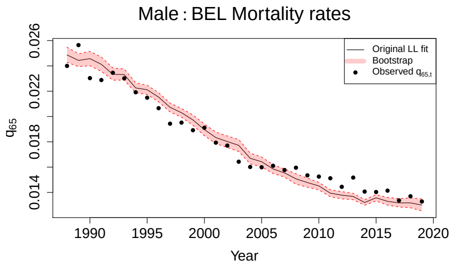
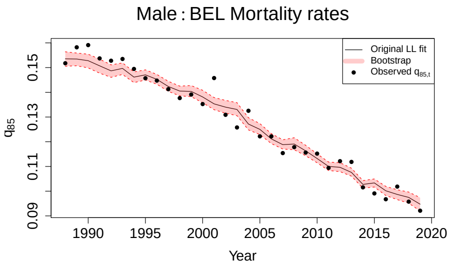
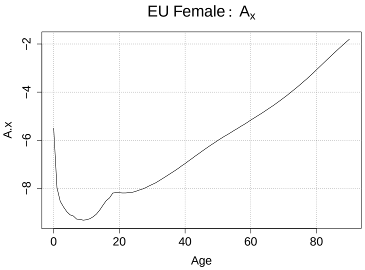
IA|BE 2020: males, in sample $q_{x,t}$ for $x = 25, 45, 65, 85$

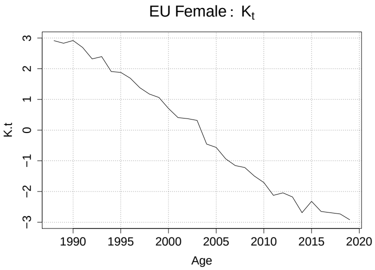






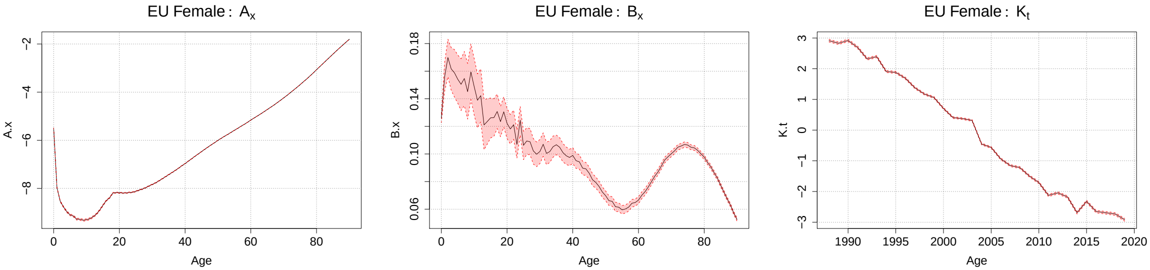






Note: calibration period 1988 - 2018 extensively motivated in Antonio, Devriendt and Robben (2020).

IA|BE 2020: females, fitted parameters, EU level



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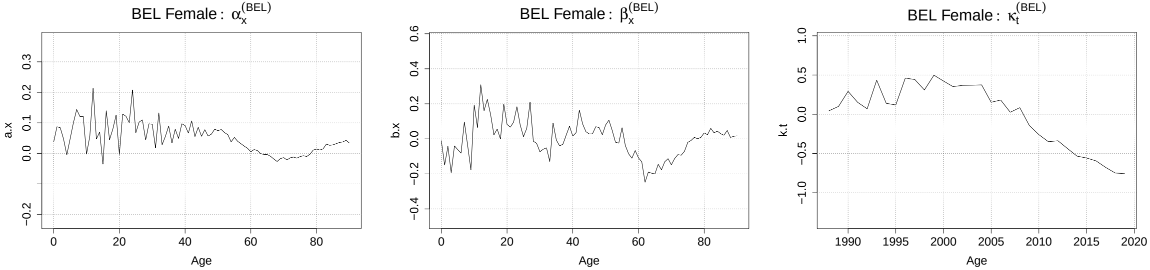
IA|BE 2020: females, fitted parameters, Belgian deviation



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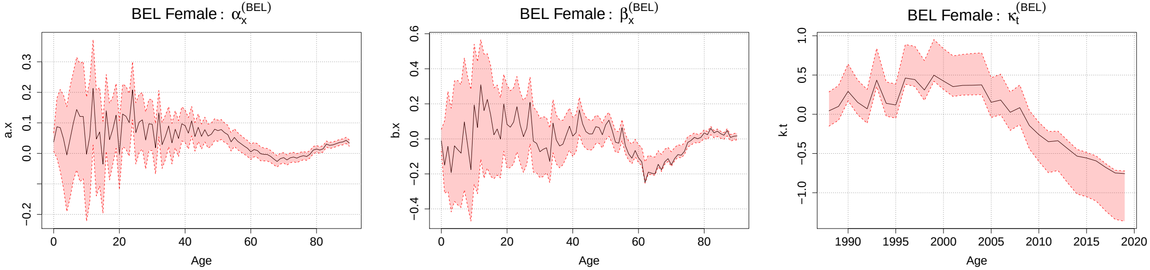

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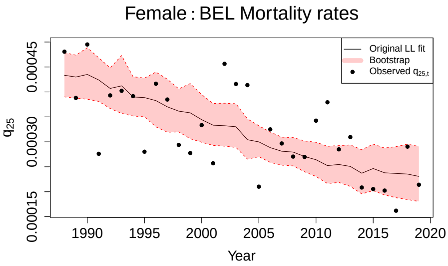
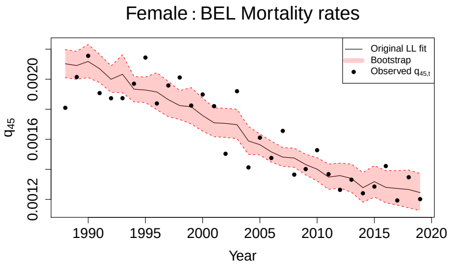
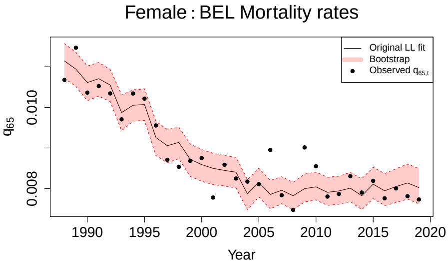
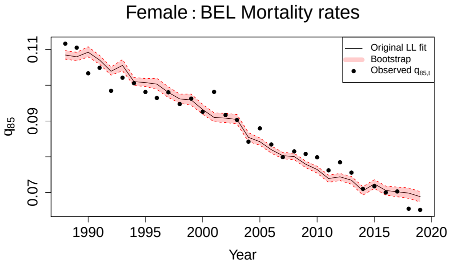
IA|BE 2020: females, in sample $q_{x,t}$ for $x = 25, 45, 65, 85$









Time dynamics

in the IA|BE 2020 model

Bivariate time series model estimated for K_t and κ_t (per gender):

$$
\begin{aligned}
K_{t+1} &= K_t + \theta + \epsilon_{t+1} \\
\kappa_{t+1} &= c + \phi \cdot \kappa_t + \delta_{t+1}.
\end{aligned}
$$

We combine $(\epsilon_t^{(M)}, \delta_t^{(M)}, \epsilon_t^{(F)}, \delta_t^{(F)})$ and assume i.i.d 4-variate normally distributed noise terms with mean (0,0,0,0) and covariance matrix $\boldsymbol{C}$.

Estimate with MLE for $\boldsymbol{C}$.

Simulate new noise terms (see further) to obtain projections for $\mu_{x,t}^{(\text{BEL})}$ and $q_{x,t}^{(\text{BEL})}$.

From IA|BE 2015 to IA|BE 2020: differences in time dynamics

IA|BE 2020 incorporates correlation between K_t the European trend and $\kappa_t^{(\text{BEL})}$ the country-specific deviation from this trend

- for males and females, jointly (new!)
- hence, a (new!) 4-variate distribution for error terms $(\epsilon_t^{(\text{M})}, \delta_t^{(\text{M})}, \epsilon_t^{(\text{F})}, \delta_t^{(\text{F})})$.

From IA|BE 2015 to IA|BE 2020: differences in time dynamics

IA|BE 2020 uses AR(1) with intercept (new!):

- AR(1) parameter in the κ_t process no longer depends on the linear identifiability constraint imposed on κ_t
- κ_t may converge to a non-zero value, thus an extra gap, besides the age effect α_x, between the long term projected mortality rates for Belgium and Europe
- stability of the AR(1) process and sensitivity analysis with respect to AR(k) process extensively investigated.

For each future scenario i and with t running from $t_{\max} = 2019$ to T (e.g. 2060):

1 start with the published $(K^M_{t_{\max}}, \kappa^M_{t_{\max}}, K^F_{t_{\max}}, \kappa^F_{t_{\max}})$ and time series parameter estimates

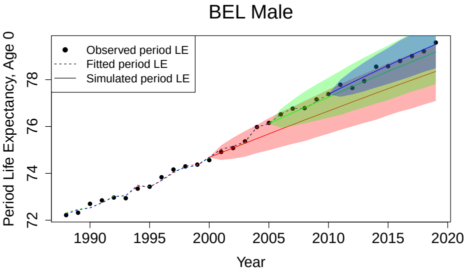
simulate error terms $(\epsilon^M_t, \delta^M_t, \epsilon^F_t, \delta^F_t)$ and retrieve future $(K^{M,i}_t, \kappa^{M,i}_t, K^{F,i}_t, \kappa^{F,i}_t)$ from the time series specifications

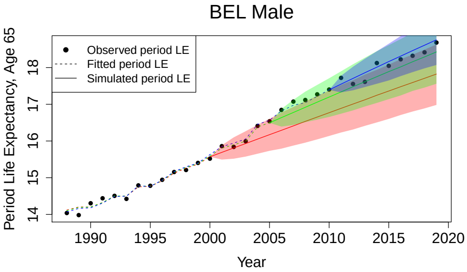
2 combine with the published age specific parameters (A_x, B_x, α_x and β_x) and obtain future $\mu^i_{x,t}$ or $q^i_{x,t}$

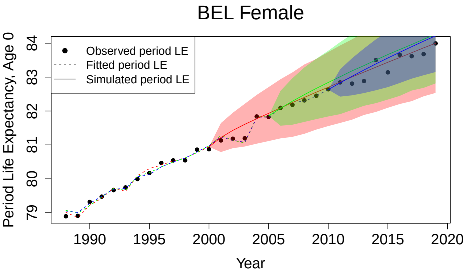
3 close each generated period table (i, t) for old ages, say $x \in \{91, \ldots, 120\}$, using law of Kannistö (1992) calibrated on ages $\{80, 81, \ldots, 90\}$.

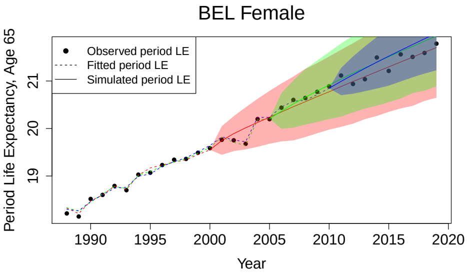
Results, including back-tests

IA|BE 2020: back-test period LE

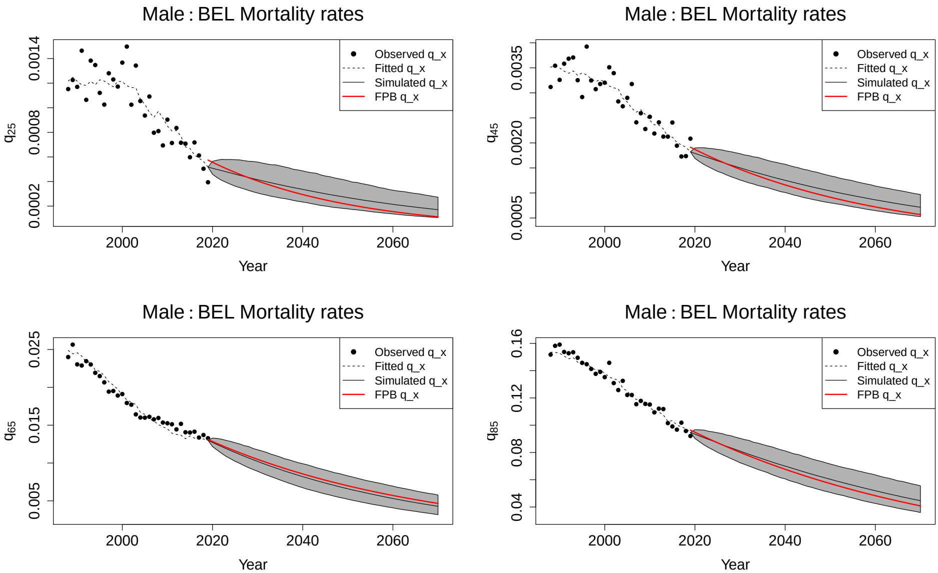


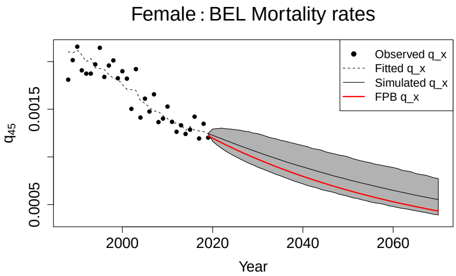


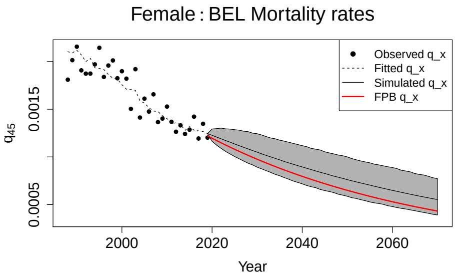


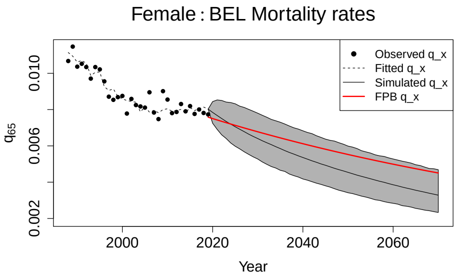


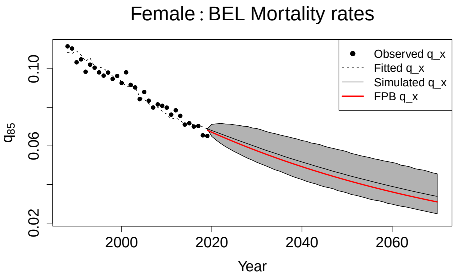
IA|BE 2020: mortality rates $q_{x,t}$



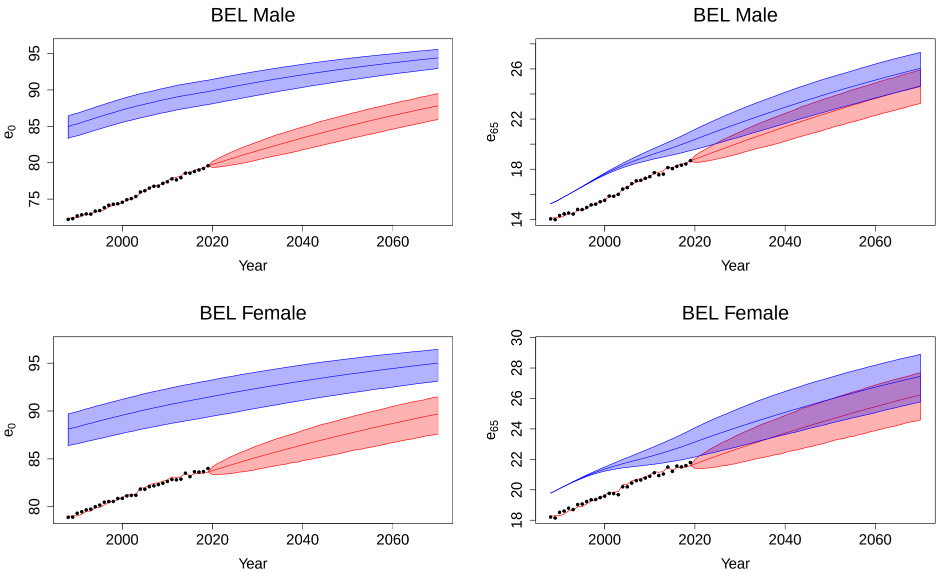

Female : BEL Mortality rates
Observed q_x
Fitted q_x
Simulated q_x
FPB q_x
q45
0.0005
0.0015
2000
2020
2040
2060
Year


Female : BEL Mortality rates
Observed q_x
Fitted q_x
Simulated q_x
FPB q_x
q45
0.0005
0.0015
2000
2020
2040
2060
Year


Female : BEL Mortality rates
Observed q_x
Fitted q_x
Simulated q_x
FPB q_x
q65
0.002
0.006
0.010
2000
2020
2040
2060
Year

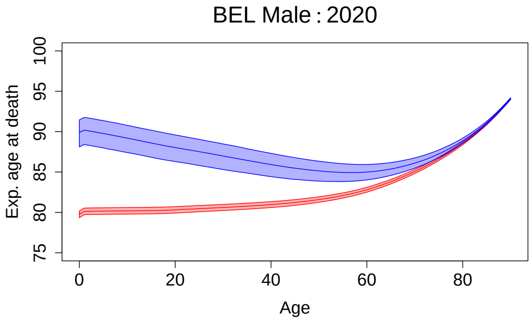

Female : BEL Mortality rates
Observed q_x
Fitted q_x
Simulated q_x
FPB q_x
q85
0.02
0.06
0.10
2000
2020
2040
2060
Year

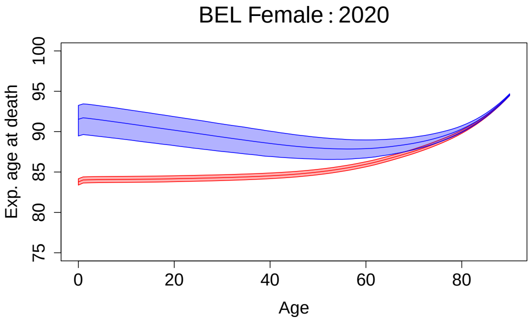
IA|BE 2020: period and cohort life expectancy

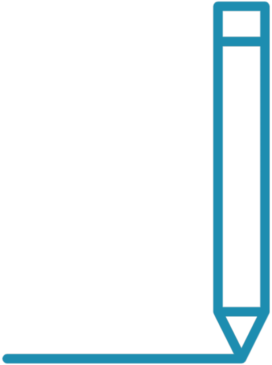


year		males		females	
		0	65	0	65
2020	Best Est.	89.91	20.38	91.54	23.14
	[$q_{0.5}$; q_{50}; $q_{99.5}$]	[88.11;89.89;91.46]	[19.57;20.37;21.17]	[89.46;91.53;93.25]	[22.15;23.14;24.07]
	FPB	(90.07;90.25)	(20.11;20.56)	(91.28;91.53)	(22.92;23.38)
2040	Best Est.	92.08	22.94	93.15	25.09
	[$q_{0.5}$; q_{50}; $q_{99.5}$]	[90.35;92.08;93.52]	[21.65;22.94;24.14]	[91.12;93.14;94.82]	[23.63;25.09;26.46]
	FPB	(92.09;92.36)	(22.80;23.26)	(92.80;93.08)	(24.82;25.28)
2060	Best Est.	93.73	25.11	94.45	26.74
	[$q_{0.5}$; q_{50}; $q_{99.5}$]	[92.18;93.72;94.97]	[23.69;25.11;26.39]	[92.50;94.45;95.97]	[25.06;26.74;28.18]
	FPB	(93.62;93.90)	(25.00;25.48)	(94.06;94.34)	(26.45;26.92)

IA|BE 2020: expected age at death - period and cohort







That’s a wrap!

That's a wrap - going from IA|BE 2015 to IA|BE 2020

IA|BE 2015 was calibrated on EU 1970 - 2009 and BE 1970 - 2013, with a Li & Lee model.

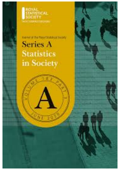
IA|BE 2020 now uses EU 1988 - 2018 and BE 1988 - 2019, including some methodological changes.

Our report evaluates step-by-step the impact of these methodological changes and the impact of the collection of new data points when going from IA|BE 2015 to IA|BE 2020.

Cohort LE 2020	males		females	
	0	65	0	65
IA|BE 2015	88.71	19.92	92.85	23.63
IA|BE 2020	89.91	20.38	91.54	23.14

Applications and portfolio specific mortality

References to scientific literature



- Antonio, Devriendt et al. (2015, European Actuarial Journal). Producing the Dutch and Belgian mortality projections: a stochastic multi-population standard

  with applications on e.g. calculating life expectancies, cash flow valuation for stylized portfolios, . . .

- Van Berkum, Antonio & Vellekoop (2020, JRSS A). Quantifying longevity gaps using micro-level lifetime data

  capture portfolio-specific mortality with population model (e.g. IA|BE 2020) as baseline.

Both papers are available via open access.

Thank you for your attention!

A general class of models ('LifeMetrics models'):

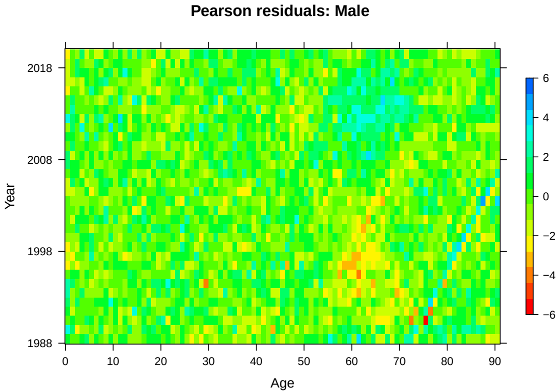
$$\log \mu_{x,t} = \beta_x^{(1)} \kappa_t^{(1)} \gamma_{t-x}^{(1)} + \ldots + \beta_x^{(N)} \kappa_t^{(N)} \gamma_{t-x}^{(N)}$$

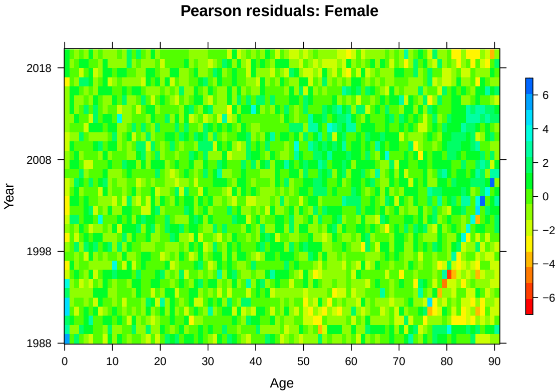
or

$$\text{logit } q_{x,t} = \beta_x^{(1)} \kappa_t^{(1)} \gamma_{t-x}^{(1)} + \ldots + \beta_x^{(N)} \kappa_t^{(N)} \gamma_{t-x}^{(N)}$$

where

- $\beta_x^{(k)}$ = age effect for component k
- $\kappa_t^{(k)}$ = period effect for component k
- $\gamma_{t-x}^{(k)}$ = cohort effect for component k.


Pearson residuals: Male
Year
Age
2018
2008
1998
1988
0
10
20
30
40
50
60
70
80
90
6
4
2
0
−2
−4
−6


Pearson residuals: Female
Year
Age
2018
2008
1998
1988
0
10
20
30
40
50
60
70
80
90
6
4
2
0
−2
−4
−6

	$\hat{\theta}^{(BEL)}$	$\hat{c}^{(BEL)}$	$\hat{\phi}^{(BEL)}$		$\hat{\boldsymbol{C}}^{(BEL)}$			
					ϵ_t^M	δ_t^M	ϵ_t^F	δ_t^F
Male	-0.2285	0.0140	0.9682	ϵ_t^M	0.0291	0.0014	0.0353	0.0058
				δ_t^M	0.0014	0.0249	-0.0016	-0.0096
Female	-0.1882	-0.0240	0.9226	ϵ_t^F	0.0353	-0.0016	0.0458	0.0089
				δ_t^F	0.0058	-0.0096	0.0089	0.0211