LEE is not preserved under bisimulation collapse of chart interpretations of star expressions (including 1)

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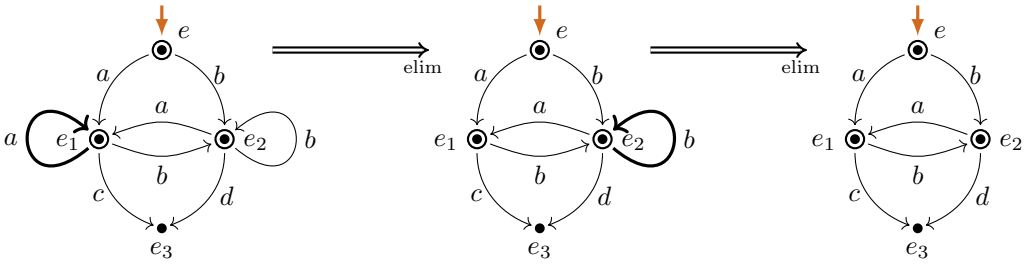
The chart translation for the process semantics of (general) star expressions with deadlock 0, empty step 1, choice $+$, concatentation $\cdot$, and unary star iteration $(\cdot)^*$ is defined by means of the transition system specification:

$$\frac{}{1\!\downarrow} \qquad \frac{e_i\!\downarrow}{(e_1+e_2)\!\downarrow} \qquad \frac{e_1\!\downarrow \qquad e_2\!\downarrow}{(e_1\cdot e_2)\!\downarrow} \qquad \frac{}{(e^*)\!\downarrow}$$

$$\frac{}{a \xrightarrow{a} 1} \qquad \frac{e_i \xrightarrow{a} e_i'}{e_1+e_2 \xrightarrow{a} e_i'} \qquad \frac{e_1 \xrightarrow{a} e_1'}{e_1\cdot e_2 \xrightarrow{a} e_1'\cdot e_2} \qquad \frac{e_1\!\downarrow \qquad e_2 \xrightarrow{a} e_2'}{e_1\cdot e_2 \xrightarrow{a} e_2'} \qquad \frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e'\cdot e^*}$$

Interpretations of star expressions are charts in the more general sense that immediate termination is now possible at arbitrary vertices (as opposed to only in the special vertex $\surd$ as in the submission). As a consequence, condition (L3) of for a chart $\mathcal{L}$ to be a loop chart has to be adapted (from 'not containing $\surd$' in the special case) to: Immediate termination is only permitted at the start vertex of $\mathcal{L}$. The definitions of the properties LEE and LLEE are then based on the adapted definition of loop (sub-)chart.

The chart translation $\mathcal{C}(e)$ of the star expression $e := (a\cdot(1+c\cdot 0)+b\cdot(1+d\cdot 0))^*$ is the chart on the left below with $e_1 := (1\cdot(1+c\cdot 0))\cdot e$, $e_2 := (1\cdot(1+d\cdot 0))\cdot e$, $e_3 := (1\cdot 0)\cdot e$, and where permitted immediate termination in a vertex is indicated by a double circle.



$\mathcal{C}(e)$ is a bisimulation collapse. But $\mathcal{C}(e)$ does not satisfy LEE: $\mathcal{C}(e)$ contains two loop subcharts that can be eliminated successively, see the picture above, where the self-loop-transitions that induce loop subcharts that are eliminated in the two steps are emphasized. The resulting chart on the left does not contain loop subcharts any more, because taking, for example, a transition from e_1 to e_2 as an entry-transition does not yield a loop subchart, because in the induced subchart immediate termination is not only possible at the start vertex, in contradiction to the (adaped form, see above) of (L3). But while the resulting chart on the left does not contain a loop subchart any more, it still permits an infinite trace. Hence $\mathcal{C}(e)$ does not satisfy LEE.

This example shows:

(i) The chart translation of star expressions (with 1 and $(\cdot)^*$) does not satisfy LEE in general.

(ii) The bisimulation collapse of star expressions (with 1 and and $(\cdot)^*$) does not satisfy LEE in general.