LEE is not preserved under bisimulation collapse of chart interpretations of star expressions with 1 and unary star (part of LICS authors' response)

March 20, 2020

Abstract

Added responses to questions and remarks by a reviewer, including the illustration of the example of the chart translation that violates the property LEE for a star expression with 1.

1 LEE may fail for chart interpretation of star expressions with 1

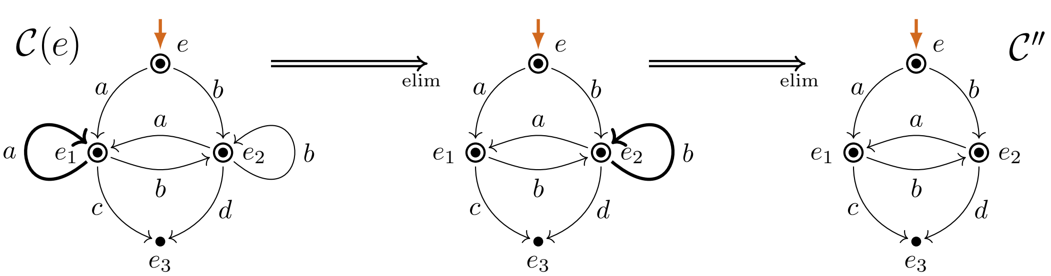
The chart translation for the process semantics of (general) star expressions with deadlock 0, empty step 1, choice +, concatentation $\cdot$, and unary star iteration $(\cdot)^*$ is defined by means of the transition system specification:

$$\frac{}{1{\downarrow}} \qquad \frac{e_i{\downarrow}}{(e_1 + e_2){\downarrow}} \qquad \frac{e_1{\downarrow} \qquad e_2{\downarrow}}{(e_1 \cdot e_2){\downarrow}} \qquad \frac{}{(e^*){\downarrow}}$$

$$\frac{}{a \xrightarrow{a} 1} \qquad \frac{e_i \xrightarrow{a} e_i'}{e_1 + e_2 \xrightarrow{a} e_i'} \qquad \frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2} \qquad \frac{e_1{\downarrow} \qquad e_2 \xrightarrow{a} e_2'}{e_1 \cdot e_2 \xrightarrow{a} e_2'} \qquad \frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e' \cdot e^*}$$

Interpretations of (general) star expressions are charts in the more general sense that immediate termination is now possible at arbitrary vertices (as opposed to only in the special vertex $\surd$ as in the submission). As a consequence, condition (L3) of for a chart $\mathcal{L}$ to be a loop chart has to be adapted (from 'not containing $\surd$' in the special case) to: Immediate termination is only permitted at the start vertex of $\mathcal{L}$. The definitions of the properties LEE and LLEE are then based on the adapted definition of loop (sub-)chart.

The chart translation $\mathcal{C}(e)$ of the star expression $e := (a \cdot (1 + c \cdot 0) + b \cdot (1 + d \cdot 0))^*$ is the chart on the left below with $e_1 := (1 \cdot (1 + c \cdot 0)) \cdot e$, $e_2 := (1 \cdot (1 + d \cdot 0)) \cdot e$, $e_3 := (1 \cdot 0) \cdot e$, and where permitted immediate termination in a vertex is indicated by a double circle.



$\mathcal{C}(e)$ is a bisimulation collapse. But it does not satisfy LEE: $\mathcal{C}(e)$ contains two loop subcharts induced by the cycling transitions at e_1 and e_2 that can be eliminated successively, see the picture above, where the loop-entry transitions that are eliminated in the two steps are emphasized. The resulting chart $\mathcal{C}''$

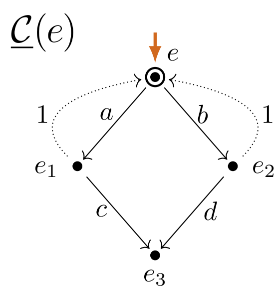
does not contain loop subcharts any more, because taking, for example, a transition from e_1 to e_2 as an entry-transition does not yield a loop subchart, because in the induced subchart immediate termination is not only possible at the start vertex e_1 but also in the body vertex e_2, in contradiction to the (adapted form, see above) of (L3). But while $\mathcal{C}''$ does not contain a loop subchart any more, it still has an infinite trace. It follows that $\mathcal{C}(e)$ does not satisfy LEE.

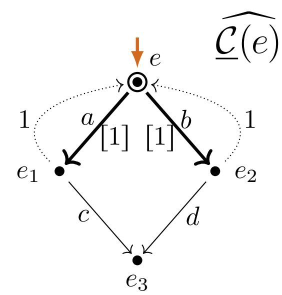
This example shows:

(i) The chart translation of star expressions (with 1 and $(\cdot)^*$) does not satisfy LEE in general.

(ii) The bisimulation collapse of the chart translation of star expressions (with 1 and $(\cdot)^*$) does not satisfy LEE in general.

A remedy, mentioned in the author's response, point (3.a.), is to define a translation of star expressions into 1-charts, that is, charts with special 1-transitions. For the example of the star expression e above and its chart translation $\mathcal{C}(e)$ the following 1-chart translation $\underline{\mathcal{C}}(e)$ that satisfies LEE, and a LLEE-witness $\widehat{\underline{\mathcal{C}}(e)}$ of $\underline{\mathcal{C}}(e)$ can be used:





Here the dotted transitions indicate 1-transitions. Note that in $\underline{\mathcal{C}}(e)$ only e permits immediate termination, but, unlike in $\mathcal{C}(e)$, not e_1 and e_2. The chart translation $\mathcal{C}(e)$ of e then arises as the 'induced chart' of the 1-chart translation $\mathcal{C}(e)$ with induced transitions that correspond to paths that start with a (potentially empty) 1-transition path and a final proper action transition. For example the looping transition from e_1 to e_1 in $\mathcal{C}(e)$ arises as the induced transition in $\underline{\mathcal{C}}(e)$ that is the path that consists of the 1-transition from e_1 to e and the a-transition from e to e_1.

2 Responses to other remarks and questions

- *In Definition 3.8, what is the motivation of the definition of the directly loops-back-to relation ${}_{d}\!\curvearrowleft$?*

  The definition of ${}_{d}\!\curvearrowleft$ is motivated by the property of the irreflexive (see Lemma A.9) loops-back-to relation $\curvearrowleft$ to linearly order the $\curvearrowleft$-successors of vertices:

  $$w \curvearrowleft v_1 \;\wedge\; w \curvearrowleft v_2 \;\;\Longrightarrow\;\; v_1 \curvearrowleft v_2 \;\vee\; v_1 = v_2 \;\vee\; v_2 \curvearrowleft v_1 \,. \tag{1}$$

  (This property also extends to the transitive closure $\curvearrowleft^+$ of $\curvearrowleft$, and the reflexive–transitive closure $\curvearrowleft^*$ of $\curvearrowleft$, which are partial orders by Lemma A.10.) In view of (1), ${}_{d}\!\curvearrowleft$ defines the direct, or immediate, $\curvearrowleft$-successor. The alternative definition that reviewer 4 suggests:

  $$w \;{}_{d}\!\curvearrowleft\; v \;\;:\Longleftrightarrow\;\; w \curvearrowleft v \wedge \forall u \big[\, w \curvearrowleft u \;\Longrightarrow\; v \rightarrow^*_{\text{bo}} u \,\big]$$

  is also possible. We have chosen to define ${}_{d}\!\curvearrowleft$ purely from $\curvearrowleft$ in order to focus on ${}_{d}\!\curvearrowleft$ as the direct- or immediate-successor relation with respect to $\curvearrowleft$ in view of (1).

  ▻ We will highlight (1) as a motivation for the definition of ${}_{d}\!\curvearrowleft$.

- *Use of* RSP$^{\circledast}$ *in the proofs of Lemma 5.7 and Proposition 5.8, but not for Lemma 5.4 and Proposition 5.5.*

  Reviewer 4 is completely right in observing this fact from the proofs.

  ▻ We will mention this fact explicitly.

- *In Definition 6.1, what is the motivation of the definition of the entry/body-labeling* $\hat{\mathcal{C}}^{(w_1)}_{w_2}$ *of the connect-*w_1*-through-to-*w_2 *chart* $\mathcal{C}^{(w_1)}_{w_2}$ *with respect to a entry/body-labeling* $\hat{\mathcal{C}}$ *of* $\mathcal{C}$*?*

  For understanding the definition of the entry/body-labeling $\hat{\mathcal{C}}^{(w_1)}_{w_2}$ it is not important to already understand the context in which it will be used later. Only that it defines an entry/body-labeling of the connect-w_1-through-to-w_2 chart $\mathcal{C}^{(w_1)}_{w_2}$ in the sense of Definition 3.2. I.p., that every transition of $\mathcal{C}^{(w_1)}_{w_2}$ gets <u>precisely one</u> marking-labeled version in $\hat{\mathcal{C}}^{(w_1)}_{w_2}$. But clearly, the definition of $\mathcal{C}^{(w_1)}_{w_2}$ has been chosen in such a way that later the correctness of transformations I, II, and III can be shown.

  The definition of $\hat{\mathcal{C}}^{(w_1)}_{w_2}$ is 'conservative' in the sense that it gives <u>precedence</u> to the marking labels of already existing transitions in $\hat{\mathcal{C}}$ over the marking labels of redirections of transitions. A transition $\tau = \langle v, \langle a, l \rangle, w_1 \rangle$ in $\hat{\mathcal{C}}$ (whose underlying transition $\langle v, a, w_1 \rangle$ of $\mathcal{C}$ gives rise to the redirected transition $\langle v, a, w_1 \rangle$ in the connect-w_1-through-to-w_2 chart $\mathcal{C}^{(w_1)}_{w_2}$) is used as the redirected transition $\tau_{w_2} = \langle v, \langle a, l \rangle, w_2 \rangle$ for $\hat{\mathcal{C}}^{(w_1)}_{w_2}$ <u>only if</u> there is not already a transition $\tau' = \langle v, \langle a, l' \rangle, w_2 \rangle$ present in $\hat{\mathcal{C}}$ where τ' has the same source, action label, and target as τ_{w_2}, but possibly a different marking label l'. Otherwise the redirection τ_{w_2} is not added to $\hat{\mathcal{C}}^{(w_1)}_{w_2}$, but the already existing transition τ' of the entry/body-labeling $\hat{\mathcal{C}}$ is kept.

  This definition of the entry/body-labeling $\hat{\mathcal{C}}^{(w_1)}_{w_2}$ for $\mathcal{C}^{(w_1)}_{w_2}$ prevents the formation of two different transitions $\tau_{w_2} = \langle v, \langle a, l \rangle, w_2 \rangle$ and $\tau' = \langle v, \langle a, l' \rangle, w_2 \rangle$ of the same transition $\langle v, a, w_2 \rangle$ of $\mathcal{C}^{(w_1)}_{w_2}$ but with different marking labels $l \neq l'$. That is not permitted for entry/body-labelings, see Definition 3.2.

  The definition of $\hat{\mathcal{C}}^{(w_1)}_{w_2}$ has, due to its respect for marking labels of undirected transitions, useful properties that we exploit in the correctness proof.

  ▻ We will mention the conservativity of the definition of $\hat{\mathcal{C}}^{(w_1)}_{w_2}$ with respect to the use of marking labels for unredirected transitions in relation to marking labels of redirected transitions.