LEE is not preserved under bisimulation collapse
of chart interpretations of star expressions with 1 and unary star

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The chart translation for the process semantics of (general) star expressions with deadlock 0, empty step 1, choice $+$, concatentation $\cdot$, and unary star iteration $(\cdot)^*$ is defined by means of the transition system specification:

$$\frac{}{1\downarrow} \qquad \frac{e_i\downarrow}{(e_1 + e_2)\downarrow} \qquad \frac{e_1\downarrow \quad e_2\downarrow}{(e_1 \cdot e_2)\downarrow} \qquad \frac{}{(e^*)\downarrow}$$

$$\frac{}{a \xrightarrow{a} 1} \qquad \frac{e_i \xrightarrow{a} e_i'}{e_1 + e_2 \xrightarrow{a} e_i'} \qquad \frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2} \qquad \frac{e_1\downarrow \quad e_2 \xrightarrow{a} e_2'}{e_1 \cdot e_2 \xrightarrow{a} e_2'} \qquad \frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e' \cdot e^*}$$

Interpretations of (general) star expressions are charts in the more general sense that immediate termination is now possible at arbitrary vertices (as opposed to only in the special vertex $\sqrt{}$ as in the submission). As a consequence, condition (L3) of for a chart $\mathcal{L}$ to be a loop chart has to be adapted (from 'not containing $\sqrt{}$' in the special case) to: Immediate termination is only permitted at the start vertex of $\mathcal{L}$. The definitions of the properties LEE and LLEE are then based on the adapted definition of loop (sub-)chart.

The chart translation $\mathcal{C}(e)$ of the star expression $e := (a \cdot (1 + c \cdot 0) + b \cdot (1 + d \cdot 0))^*$ is the chart on the left below with $e_1 := (1 \cdot (1 + c \cdot 0)) \cdot e$, $e_2 := (1 \cdot (1 + d \cdot 0)) \cdot e$, $e_3 := (1 \cdot 0) \cdot e$, and where permitted immediate termination in a vertex is indicated by a double circle.
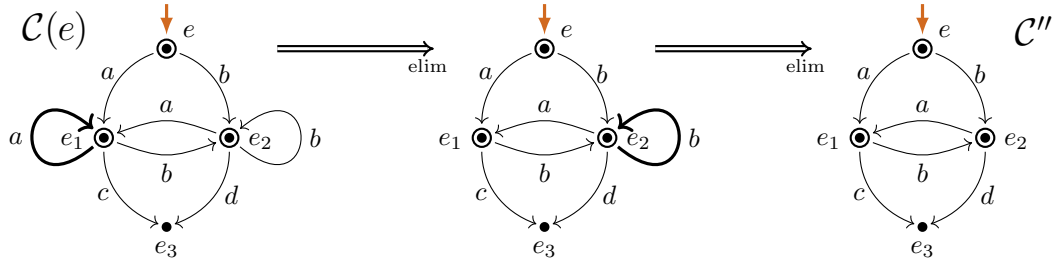
$\mathcal{C}(e)$ is a bisimulation collapse. But it does not satisfy LEE: $\mathcal{C}(e)$ contains two loop subcharts induced by the cycling transitions at e_1 and e_2 that can be eliminated successively, see the picture above, where the loop-entry transitions that are eliminated in the two steps are emphasized. The resulting chart $\mathcal{C}''$ does not contain loop subcharts any more, because taking, for example, a transition from e_1 to e_2 as an entry-transition does not yield a loop subchart, because in the induced subchart immediate termination is not only possible at the start vertex e_1 but also in the body vertex e_2, in contradiction to the (adapted form, see above) of (L3). But while $\mathcal{C}''$ does not contain a loop subchart any more, it still has an infinite trace. It follows that $\mathcal{C}(e)$ does not satisfy LEE.

This example shows:

(i) The chart translation of star expressions (with 1 and $(\cdot)^*$) does not satisfy LEE in general.

(ii) The bisimulation collapse of the chart translation of star expressions (with 1 and $(\cdot)^*$) does not satisfy LEE in general.