LEE is not preserved under bisimulation collapse of chart interpretations of star expressions with 1 and unary star (part of LICS authors' response)

March 20, 2020

Abstract

Added responses to questions and remarks by a reviewer, including the illustration of the example of the chart translation that violates the property LEE for a star expression with 1.

1 LEE may fail for chart interpretation of star expressions with 1

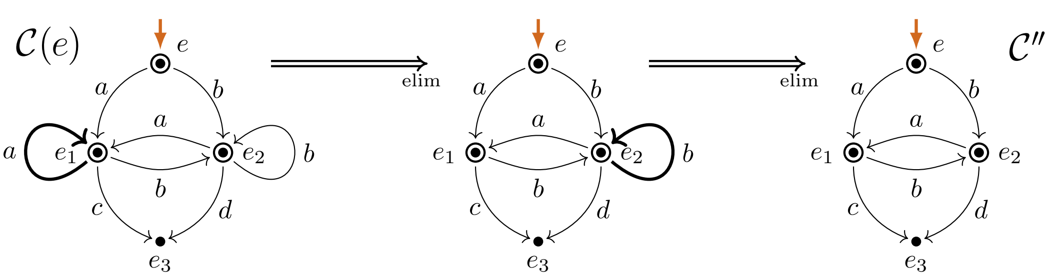
The chart translation for the process semantics of (general) star expressions with deadlock 0, empty step 1, choice +, concatentation $\cdot$, and unary star iteration $(\cdot)^*$ is defined by means of the transition system specification:

$$\frac{}{1{\downarrow}} \qquad \frac{e_i{\downarrow}}{(e_1+e_2){\downarrow}} \qquad \frac{e_1{\downarrow} \qquad e_2{\downarrow}}{(e_1\cdot e_2){\downarrow}} \qquad \frac{}{(e^*){\downarrow}}$$

$$\frac{}{a \xrightarrow{a} 1} \qquad \frac{e_i \xrightarrow{a} e_i'}{e_1+e_2 \xrightarrow{a} e_i'} \qquad \frac{e_1 \xrightarrow{a} e_1'}{e_1\cdot e_2 \xrightarrow{a} e_1'\cdot e_2} \qquad \frac{e_1{\downarrow} \qquad e_2 \xrightarrow{a} e_2'}{e_1\cdot e_2 \xrightarrow{a} e_2'} \qquad \frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e'\cdot e^*}$$

Interpretations of (general) star expressions are charts in the more general sense that immediate termination is now possible at arbitrary vertices (as opposed to only in the special vertex $\surd$ as in the submission). As a consequence, condition (L3) of for a chart $\mathcal{L}$ to be a loop chart has to be adapted (from 'not containing $\surd$' in the special case) to: Immediate termination is only permitted at the start vertex of $\mathcal{L}$. The definitions of the properties LEE and LLEE are then based on the adapted definition of loop (sub-)chart.

The chart translation $\mathcal{C}(e)$ of the star expression $e := (a\cdot(1+c\cdot 0)+b\cdot(1+d\cdot 0))^*$ is the chart on the left below with $e_1 := (1\cdot(1+c\cdot 0))\cdot e$, $e_2 := (1\cdot(1+d\cdot 0))\cdot e$, $e_3 := (1\cdot 0)\cdot e$, and where permitted immediate termination in a vertex is indicated by a double circle.



$\mathcal{C}(e)$ is a bisimulation collapse. But it does not satisfy LEE: $\mathcal{C}(e)$ contains two loop subcharts induced by the cycling transitions at e_1 and e_2 that can be eliminated successively, see the picture above, where the loop-entry transitions that are eliminated in the two steps are emphasized. The resulting chart $\mathcal{C}''$

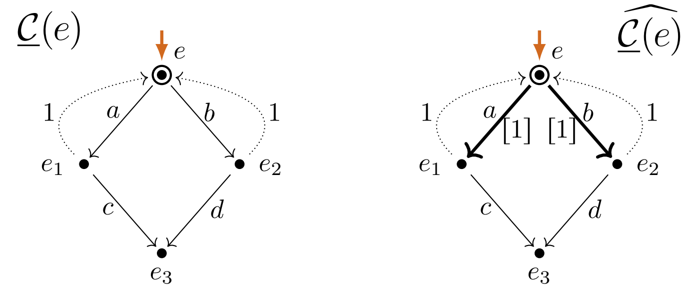
does not contain loop subcharts any more, because taking, for example, a transition from e_1 to e_2 as an entry-transition does not yield a loop subchart, because in the induced subchart immediate termination is not only possible at the start vertex e_1 but also in the body vertex e_2, in contradiction to the (adapted form, see above) of (L3). But while $\mathcal{C}''$ does not contain a loop subchart any more, it still has an infinite trace. It follows that $\mathcal{C}(e)$ does not satisfy LEE.

This example shows:

(i) The chart translation of star expressions (with 1 and $(\cdot)^*$) does not satisfy LEE in general.

(ii) The bisimulation collapse of the chart translation of star expressions (with 1 and $(\cdot)^*$) does not satisfy LEE in general.

A remedy, mentioned in the author's response, point (3.a.), is to define a translation of star expressions into 1-charts, that is, charts with special 1-transitions. For the example of the star expression e above and its chart translation $\mathcal{C}(e)$ the following 1-chart translation $\underline{\mathcal{C}}(e)$ that satisfies LEE, and a LLEE-witness $\widehat{\underline{\mathcal{C}}(e)}$ of $\underline{\mathcal{C}}(e)$ can be used:



Here the dotted transitions indicate 1-transitions. Note that in $\underline{\mathcal{C}}(e)$ only e permits immediate termination, but, unlike in $\mathcal{C}(e)$, not e_1 and e_2. The chart translation $\mathcal{C}(e)$ of e then arises as the 'induced chart' of the 1-chart translation $\mathcal{C}(e)$ with induced transitions that correspond to paths that start with a (potentially empty) 1-transition path and a final proper action transition. For example the looping transition from e_1 to e_1 in $\mathcal{C}(e)$ arises as the induced transition in $\underline{\mathcal{C}}(e)$ that is the path that consists of the 1-transition from e_1 to e and the a-transition from e to e_1.

2 Responses to other remarks and questions

- *How to facilitate easy comprehension of the dynamics behind the collapse procedure in Section 6?*

  For gaining an intuitive understanding of the dynamics of finding, in a LLEE-chart that is not a bisimulation collapse, a pair of distinct, bisimilar vertices (a 'bisimilarity redundancy') that satisfies one of the conditions (C1), (C2), or (C3) in Proposition 6.4, the 5 illustrations in the proof of this proposition in the appendix can be helpful. These pictures concern (only) the different positions of the vertices of a bisimilarity redundancy with an scc (= a joint loops-back-to part). For the the case of a bisimilarity redundancy with vertices in different scc's, which the proof deals with first, a similar easy pictures can be drawn.

  ▻ In order to support facilitate comprehension, we can center the explanations for the proof of Proposition 6.4 around these pictures, and favor intuitive explanations.

  For obtaining an quick understanding of the dynamics of the transformations I, II, and III that remove 'bisimilarity redundancies' of kind (C1), (C2), and (C3), respectively, an additional option is to use similar pictures.

  ▻ We can provide schematic pictures for the transformations I, II, and III that illustrate the intuitions on which their definitions are based.

$\rhd$ The use of such pictures will also have our preference for talks on our result.

- *In Definition 3.8, what is the motivation of the definition of the directly loops-back-to relation* ${}_{d}\!\curvearrowleft$ *?*

  The definition of ${}_{d}\!\curvearrowleft$ is motivated by the property of the irreflexive (see Lemma A.9) loops-back-to relation $\curvearrowleft$ to linearly order the $\curvearrowleft$-successors of vertices:

$$w \curvearrowleft v_1 \;\wedge\; w \curvearrowleft v_2 \;\;\Longrightarrow\;\; v_1 \curvearrowleft v_2 \;\vee\; v_1 = v_2 \;\vee\; v_2 \curvearrowleft v_1 \,. \tag{1}$$

  (This property also extends to the transitive closure $\curvearrowleft^{+}$ of $\curvearrowleft$, and the reflexive–transitive closure $\curvearrowleft^{*}$ of $\curvearrowleft$, which are partial orders by Lemma A.10.) In view of (1), ${}_{d}\!\curvearrowleft$ defines the direct, or immediate, $\curvearrowleft$-successor. The alternative definition that reviewer 4 suggests:

$$w \;{}_{d}\!\curvearrowleft\; v \;\;:\Longleftrightarrow\;\; w \curvearrowleft v \wedge \forall u\big[\, w \curvearrowleft u \;\Longrightarrow\; v \to_{\text{bo}}^{*} u \,\big]$$

  is also possible. We have chosen to define ${}_{d}\!\curvearrowleft$ purely from $\curvearrowleft$ in order to focus on ${}_{d}\!\curvearrowleft$ as the direct- or immediate-successor relation with respect to $\curvearrowleft$ in view of (1).

  $\rhd$ We will highlight (1) as a motivation for the definition of ${}_{d}\!\curvearrowleft$.

- *Use of* RSP$^{\circledast}$ *in the proofs of Lemma 5.7 and Proposition 5.8, but not for Lemma 5.4 and Proposition 5.5.*

  Reviewer 4 is completely right in observing this fact from the proofs.

  $\rhd$ We will mention this fact explicitly.

- *In Definition 6.1, what is the motivation of the definition of the entry/body-labeling* $\hat{\mathcal{C}}^{(w_1)}_{w_2}$ *of the connect-*w_1*-through-to-*w_2 *chart* $\mathcal{C}^{(w_1)}_{w_2}$ *with respect to a entry/body-labeling* $\hat{\mathcal{C}}$ *of* $\mathcal{C}$ *?*

  For understanding the definition of the entry/body-labeling $\hat{\mathcal{C}}^{(w_1)}_{w_2}$ it is not important to already understand the context in which it will be used later. Only that it defines an entry/body-labeling of the connect-w_1-through-to-w_2 chart $\mathcal{C}^{(w_1)}_{w_2}$ in the sense of Definition 3.2. I.p., that every transition of $\mathcal{C}^{(w_1)}_{w_2}$ gets precisely one marking-labeled version in $\hat{\mathcal{C}}^{(w_1)}_{w_2}$. But clearly, the definition of $\mathcal{C}^{(w_1)}_{w_2}$ has been chosen in such a way that later the correctness of transformations I, II, and III can be shown.

  The definition of $\hat{\mathcal{C}}^{(w_1)}_{w_2}$ is 'conservative' in the sense that it gives precedence to the marking labels of already existing transitions in $\hat{\mathcal{C}}$ over the marking labels of redirections of transitions. A transition $\tau = \langle v, \langle a, l\rangle, w_1\rangle$ in $\hat{\mathcal{C}}$ (whose underlying transition $\langle v, a, w_1\rangle$ of $\mathcal{C}$ gives rise to the redirected transition $\langle v, a, w_1\rangle$ in the connect-w_1-through-to-w_2 chart $\mathcal{C}^{(w_1)}_{w_2}$) is used as the redirected transition $\tau_{w_2} = \langle v, \langle a, l\rangle, w_2\rangle$ for $\hat{\mathcal{C}}^{(w_1)}_{w_2}$ only if there is not already a transition $\tau' = \langle v, \langle a, l'\rangle, w_2\rangle$ present in $\hat{\mathcal{C}}$ where τ' has the same source, action label, and target as τ_{w_2}, but possibly a different marking label l'. Otherwise the redirection τ_{w_2} is not added to $\hat{\mathcal{C}}^{(w_1)}_{w_2}$, but the already existing transition τ' of the entry/body-labeling $\hat{\mathcal{C}}$ is kept.

  This definition of the entry/body-labeling $\hat{\mathcal{C}}^{(w_1)}_{w_2}$ for $\mathcal{C}^{(w_1)}_{w_2}$ prevents the formation of two different transitions $\tau_{w_2} = \langle v, \langle a, l\rangle, w_2\rangle$ and $\tau' = \langle v, \langle a, l'\rangle, w_2\rangle$ of the same transition $\langle v, a, w_2\rangle$ of $\mathcal{C}^{(w_1)}_{w_2}$ but with different marking labels $l \neq l'$. That is not permitted for entry/body-labelings, see Definition 3.2.

  The definition of $\hat{\mathcal{C}}^{(w_1)}_{w_2}$ has, due to its respect for marking labels of undirected transitions, useful properties that we exploit in the correctness proof.

  $\rhd$ We will mention the conservativity of the definition of $\hat{\mathcal{C}}^{(w_1)}_{w_2}$ with respect to the use of marking labels for unredirected transitions in relation to marking labels of redirected transitions.