Modeling Terms by Graphs with Structure Constraints

(Two Illustrations)

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Overview

- timeline of my career
- research
  - Milner's problems for star expressions
  - maximal sharing for functional programs
- teaching
  - master course Models of Computation
  - first-year course Logic and Sets
- management
- plans and collaborations
  - the λ-calculus with generalized letrec
  - with group, department, other departments

Maximal sharing

Process interpretation of regular expressions

- Milner's questions for star expressions
  - results and partial answers
- Modeling expressible process graphs
  - examples

Language interpretation $[\![\cdot]\!]_L$

$$\mathbf{0} \xmapsto{[\![\cdot]\!]_L} \text{empty set } \varnothing$$

$$\mathbf{1} \xmapsto{[\![\cdot]\!]_P} \{\epsilon\} \qquad (\epsilon \text{ the empty word})$$

$$a \xmapsto{[\![\cdot]\!]_P} \{a\}$$

$$e + f \xmapsto{[\![\cdot]\!]_L} \text{union of } [\![e]\!]_L \text{ and } [\![f]\!]_L$$

$$e \cdot f \xmapsto{[\![\cdot]\!]_L} \text{element-wise concatenation of } [\![e]\!]_L \text{ and } [\![f]\!]_L$$

$$e^* \xmapsto{[\![\cdot]\!]_L} \text{set of words over } [\![e]\!]_L$$

Process interpretation $[\![\cdot]\!]_{P}$

$$\mathbf{0} \xmapsto{[\![\cdot]\!]_{P}} \text{deadlock } \boldsymbol{\delta}\text{, no termination}$$

$$\mathbf{1} \xmapsto{[\![\cdot]\!]_{P}} \text{empty process } \boldsymbol{\epsilon}\text{, then terminate}$$

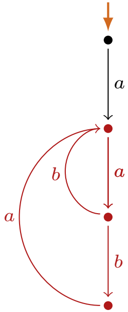
$$a \xmapsto{[\![\cdot]\!]_{P}} \text{atomic action } a\text{, then terminate}$$

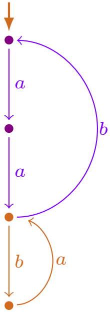
$$e + f \xmapsto{[\![\cdot]\!]_{P}} \text{alternative composition between } [\![e]\!]_{P} \text{ and } [\![f]\!]_{P}$$

$$e \cdot f \xmapsto{[\![\cdot]\!]_{P}} \text{sequential composition of } [\![e]\!]_{P} \text{ and } [\![f]\!]_{P}$$

$$e^{*} \xmapsto{[\![\cdot]\!]_{P}} \text{unbounded iteration of } [\![e]\!]_{P}\text{, option to terminate}$$

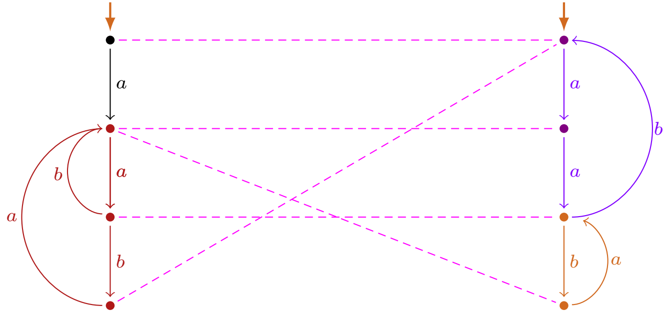
Process interpretation of regular expressions





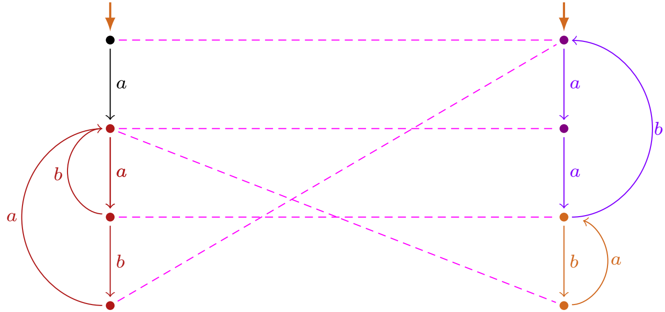
$$[\![a \cdot (a \cdot (b + b \cdot a))^* \cdot 0]\!]_{\boldsymbol{P}} \qquad [\![(a \cdot a \cdot (b \cdot a)^* \cdot b)^* \cdot 0]\!]_{\boldsymbol{P}}$$

Process interpretation of star expressions



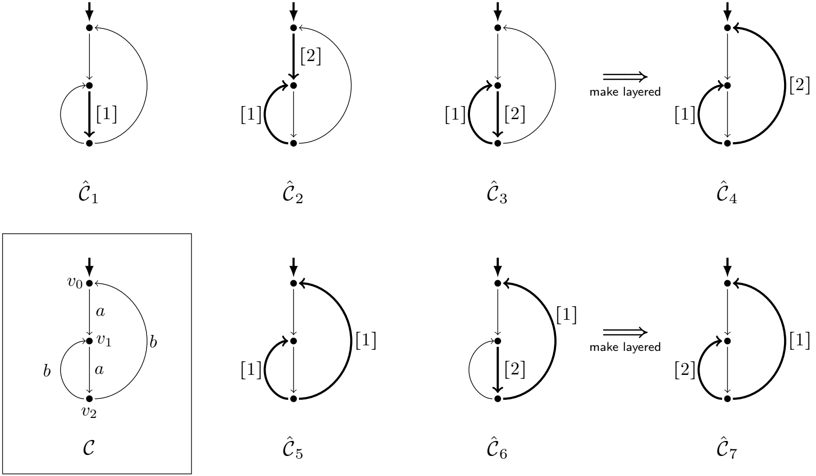
$$[\![a(a(b+ba))^{*}0]\!]_{\boldsymbol{P}} \quad \underline{\leftrightarrow} \quad [\![(aa(ba)^{*}b)^{*}0]\!]_{\boldsymbol{P}}$$

Process interpretation of star expressions

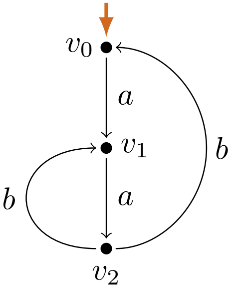


$$a(a(b+ba))^*0 \quad \underline{\leftrightarrow}_{P} \quad (aa(ba)^*b)^*0$$

7 LEE-witnesses



Layered LEE-witness



loop–branch labeling: marking transitions $\xrightarrow{a}$ as

- entry steps $\xrightarrow{\langle a,[n]\rangle}$
- branch steps $\xrightarrow{\langle a,\mathsf{br}\rangle}$

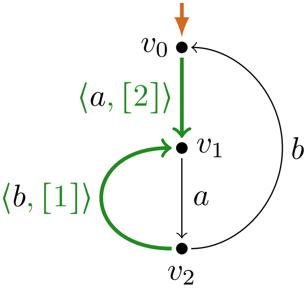
Definition

A loop–branch labeling is a layered LEE-witness, if:

- no infinite $\rightarrow_{\mathsf{br}}$ paths from start vertex,
- $\forall n \in \mathbb{N}\ \forall v \in V \big(\mathcal{L}(v, \rightarrow_{[n]}, \rightarrow_{\mathsf{br}})$ is loop chart $\big)$.

$\mathcal{L}(v, \rightarrow_{[n]}, \rightarrow_{\mathsf{br}}) :=$ subchart induced
by entry steps $\rightarrow_{[n]}$ from v
followed by branch steps $\rightarrow_{\mathsf{br}}$,

Layered LEE-witness



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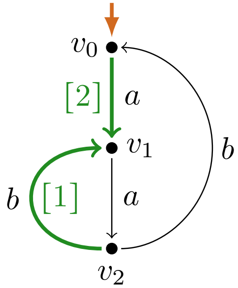

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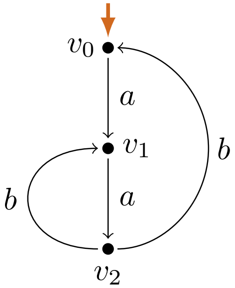
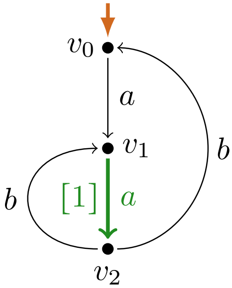
Extraction (example)



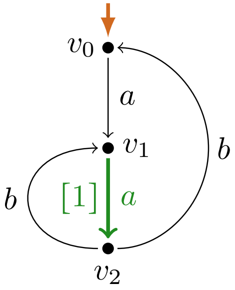
Extraction (example)



layered
LEE-witness

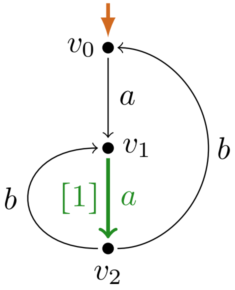
Extraction (example)

$$s(v_0) = \quad 0^* \cdot a \cdot s(v_1)$$



layered
LEE-witness

Extraction (example)

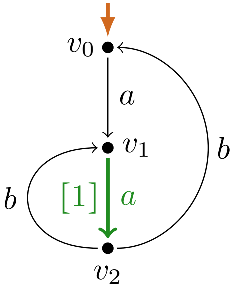


layered
LEE-witness

$$s(v_0) = \quad 0^* \cdot a \cdot s(v_1)$$

$$s(v_2) = \quad \big(a \cdot t(v_2, v_1)\big)^* \cdot 0$$

Extraction (example)



layered
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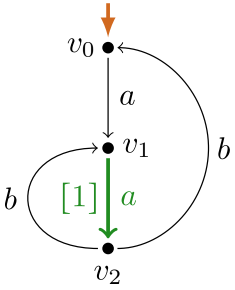
$$t(v_2, v_1) = \quad 0^* \cdot \big(b \cdot t(v_1, v_1) + b \cdot t(v_0, v_1)\big)$$

Extraction (example)



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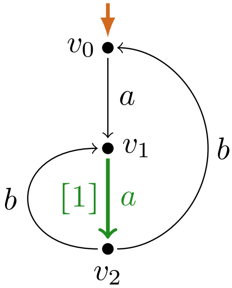
$$s(v_2) = \big(a \cdot t(v_2, v_1)\big)^* \cdot 0$$

$$t(v_2, v_1) = 0^* \cdot \big(b \cdot t(v_1, v_1) + b \cdot t(v_0, v_1)\big)$$

$$t(v_1, v_1) = 1$$

Extraction (example)



layered
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$$s(v_0) = 0^* \cdot a \cdot s(v_1)$$

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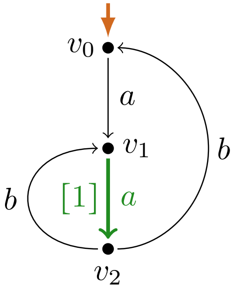
$$t(v_1, v_1) = 1$$
$$t(v_0, v_1) = 0^* \cdot a \cdot t(v_1, v_1)$$

Extraction (example)



layered
LEE-witness

$$s(v_0) = 0^* \cdot a \cdot s(v_1)$$

$$s(v_2) = \left(a \cdot t(v_2, v_1)\right)^* \cdot 0$$

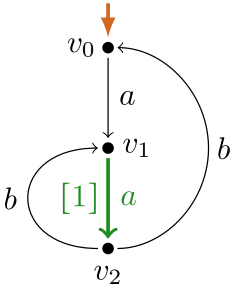
$$t(v_2, v_1) = 0^* \cdot \left(b \cdot t(v_1, v_1) + b \cdot t(v_0, v_1)\right)$$

$$t(v_1, v_1) = 1$$

$$t(v_0, v_1) = 0^* \cdot a \cdot t(v_1, v_1)$$

$$= 0^* \cdot a \cdot 1$$

Extraction (example)



layered
LEE-witness

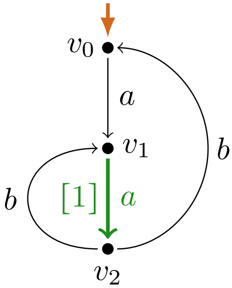
$$s(v_0) = 0^* \cdot a \cdot s(v_1)$$

$$s(v_2) = \big(a \cdot t(v_2, v_1)\big)^* \cdot 0$$

$$t(v_2, v_1) = 0^* \cdot \big(b \cdot t(v_1, v_1) + b \cdot t(v_0, v_1)\big)$$

$$\begin{aligned} t(v_1, v_1) &= 1 \\ t(v_0, v_1) &= 0^* \cdot a \cdot t(v_1, v_1) \\ &= 0^* \cdot a \cdot 1 \\ &=_{\mathsf{Mil}^-} a \end{aligned}$$

Extraction (example)



layered
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$$s(v_0) = 0^* \cdot a \cdot s(v_1)$$

$$s(v_2) = \big(a \cdot t(v_2, v_1)\big)^* \cdot 0$$

$$\begin{aligned} t(v_2, v_1) &= 0^* \cdot \big(b \cdot t(v_1, v_1) + b \cdot t(v_0, v_1)\big) \\ &=_{\mathsf{Mil}^-} 0^* \cdot \big(b \cdot 1 + b \cdot a\big) \end{aligned}$$

$$\begin{aligned} t(v_1, v_1) &= 1 \\ t(v_0, v_1) &= 0^* \cdot a \cdot t(v_1, v_1) \\ &= 0^* \cdot a \cdot 1 \\ &=_{\mathsf{Mil}^-} a \end{aligned}$$

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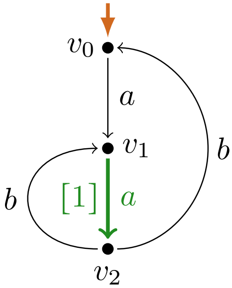
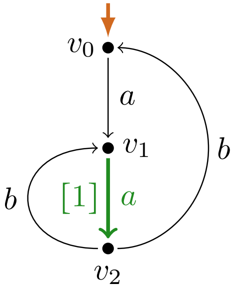


layered
LEE-witness

$$
\begin{aligned}
s(v_0) &= 0^* \cdot a \cdot s(v_1) \\
s(v_2) &= \big(a \cdot t(v_2, v_1)\big)^* \cdot 0 \\
t(v_2, v_1) &= 0^* \cdot \big(b \cdot t(v_1, v_1) + b \cdot t(v_0, v_1)\big) \\
&=_{\mathsf{Mil}^-} 0^* \cdot \big(b \cdot 1 + b \cdot a\big) \\
&=_{\mathsf{Mil}^-} b + b \cdot a \\
t(v_1, v_1) &= 1 \\
t(v_0, v_1) &= 0^* \cdot a \cdot t(v_1, v_1) \\
&= 0^* \cdot a \cdot 1 \\
&=_{\mathsf{Mil}^-} a
\end{aligned}
$$

Extraction (example)



layered
LEE-witness

$$s(v_0) = 0^* \cdot a \cdot s(v_1)$$

$$\begin{aligned}
s(v_2) &= \big(a \cdot t(v_2, v_1)\big)^* \cdot 0 \\
&=_{\mathsf{Mil}^-} \big(a \cdot (b + b \cdot a)\big)^* \cdot 0 \\
t(v_2, v_1) &= 0^* \cdot \big(b \cdot t(v_1, v_1) + b \cdot t(v_0, v_1)\big) \\
&=_{\mathsf{Mil}^-} 0^* \cdot \big(b \cdot 1 + b \cdot a\big) \\
&=_{\mathsf{Mil}^-} b + b \cdot a \\
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t(v_0, v_1) &= 0^* \cdot a \cdot t(v_1, v_1) \\
&= 0^* \cdot a \cdot 1 \\
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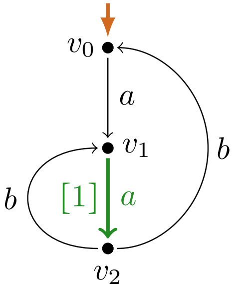
\end{aligned}$$

Extraction (example)



layered
LEE-witness

$$
\begin{aligned}
s(v_0) &= 0^* \cdot a \cdot s(v_1) \\
&=_{\mathsf{Mil}^-} a \cdot s(v_1) \\[2ex]
s(v_2) &= \big(a \cdot t(v_2, v_1)\big)^* \cdot 0 \\
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t(v_2, v_1) &= 0^* \cdot \big(b \cdot t(v_1, v_1) + b \cdot t(v_0, v_1)\big) \\
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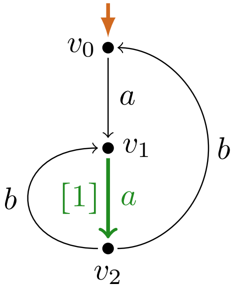
\end{aligned}
$$

Extraction (example)



layered
LEE-witness

$$
\begin{aligned}
s(v_0) &= 0^* \cdot a \cdot s(v_1) \\
&=_{\mathsf{Mil}^-} a \cdot s(v_1) \\
&=_{\mathsf{Mil}^-} a \cdot \big(a \cdot (b + b \cdot a)\big)^* \cdot 0 \\
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&=_{\mathsf{Mil}^-} \big(a \cdot (b + b \cdot a)\big)^* \cdot 0 \\
t(v_2, v_1) &= 0^* \cdot \big(b \cdot t(v_1, v_1) + b \cdot t(v_0, v_1)\big) \\
&=_{\mathsf{Mil}^-} 0^* \cdot \big(b \cdot 1 + b \cdot a\big) \\
&=_{\mathsf{Mil}^-} b + b \cdot a \\
t(v_1, v_1) &= 1 \\
t(v_0, v_1) &= 0^* \cdot a \cdot t(v_1, v_1) \\
&= 0^* \cdot a \cdot 1 \\
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$$