

Introduction to Model Checking

(Preview of Core Module)

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Model Checking

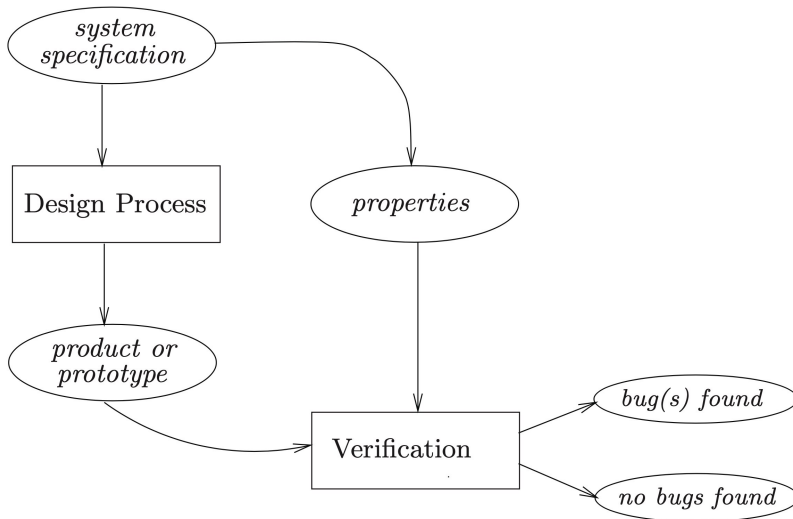
... is an effective automatable technique:

- ▶ *to expose potential software design errors;*
- ▶ *that, given a finite-state model of a system and a formal property, systematically checks whether this property holds for that model.*

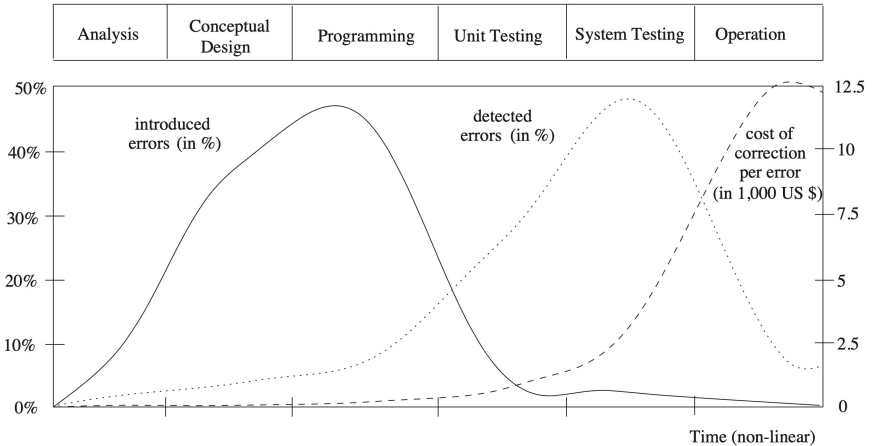
Strengths:

- ▶ widely applied in industry
for: embedded systems, software engineering, hardware design
- ▶ supports **partial verification** (of system parts)
- ▶ provides **diagnostic information** for debugging
- ▶ has sound **mathematical underpinning** (logic and process theory)

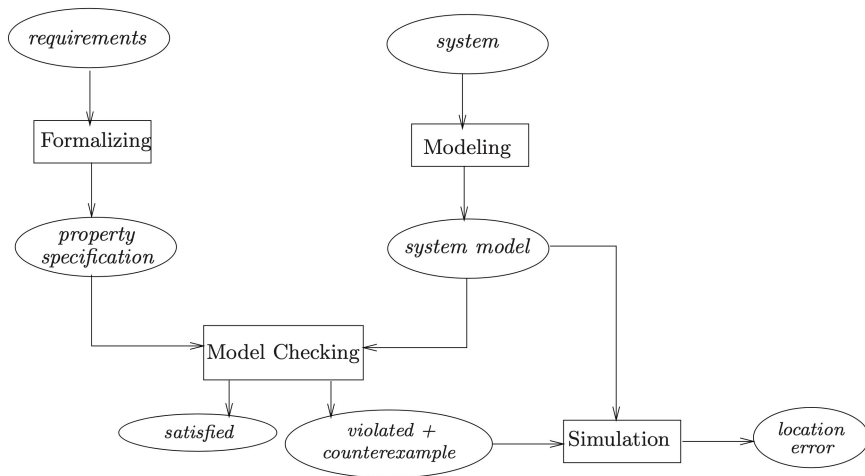
Hard-/Software Verification (traditionally)



Error introduction, detection, and repair costs



Model checking



Example

Programs [Inc](#), [Dec](#), and [Reset](#) cooperate, and use a shared variable x :

```
proc Inc
  while true
    do
      if  $x < 200$ 
        then  $x := x + 1$ 
      fi
    od
```

```
proc Dec
  while true
    do
      if  $x > 0$ 
        then  $x := x - 1$ 
      fi
    od
```

```
proc Reset
  while true
    do
      if  $x = 200$ 
        then  $x := 0$ 
      fi
    od
```

Example

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```
proc Reset
  while true
  do
    if  $x = 200$ 
    then  $x := 0$ 
    fi
  od
```

Question: Is $0 \leq x \leq 200$ always guaranteed?

Modeling (by labeled transition systems)

proc [Inc](#)

while true

do

if $x < 200$

then $x := x + 1$

fi

od

proc [Dec](#)

while true

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if $x > 0$

then $x := x - 1$

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proc [Reset](#)

while true

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od

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proc [Inc](#)

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1: **if** $x < 200$

2: **then** $x := x + 1$

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Modeling (by labeled transition systems)

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while true

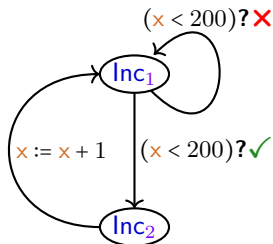
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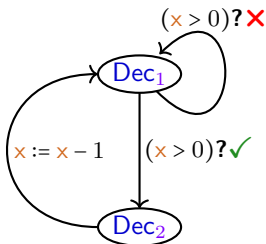
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proc Reset

while true

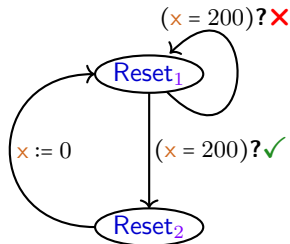
do

1: **if** $x = 200$

2: **then** $x := 0$

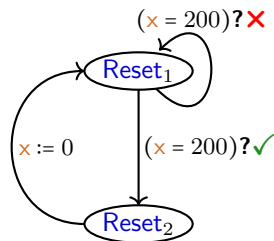
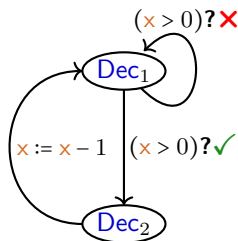
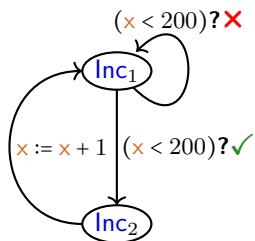
fi

od



Labeled transition systems (LTSs)

Formalizing properties (in temporal logic)



$$Inc_1 \parallel Dec_1 \parallel Reset_1 \stackrel{?}{\models} \Box(0 \leq x \wedge x \leq 200) \quad (\text{Linear-TL formula})$$

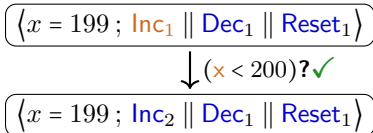
Counterexample (offending execution trace)

$$\langle x = 199 ; \text{Inc}_1 \parallel \text{Dec}_1 \parallel \text{Reset}_1 \rangle$$

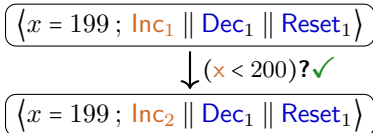
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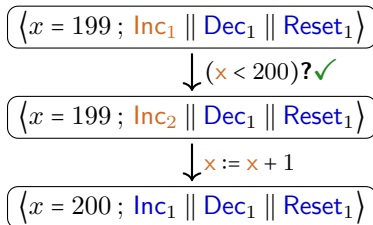
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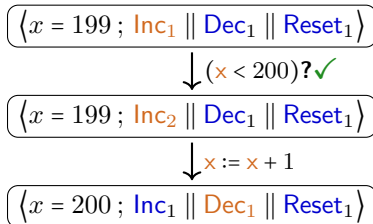
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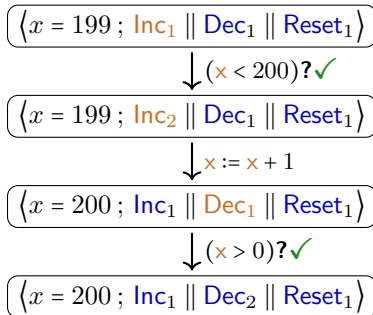
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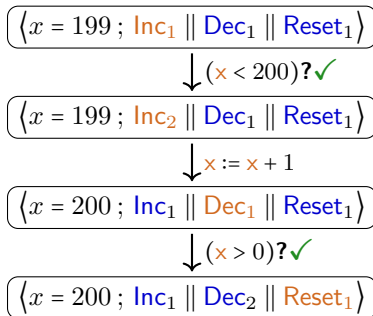
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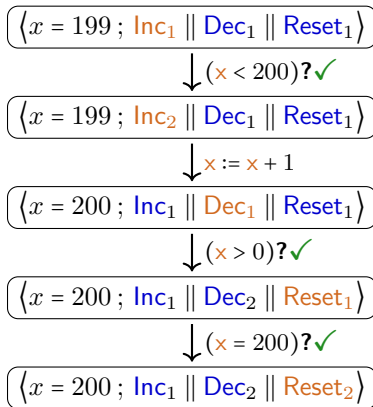
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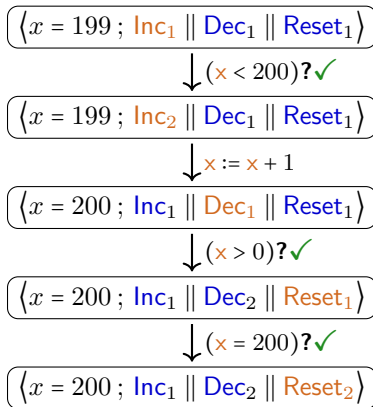
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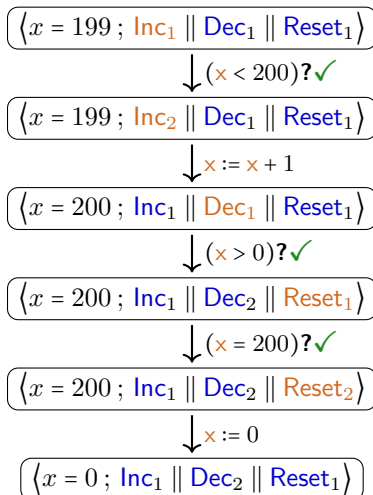
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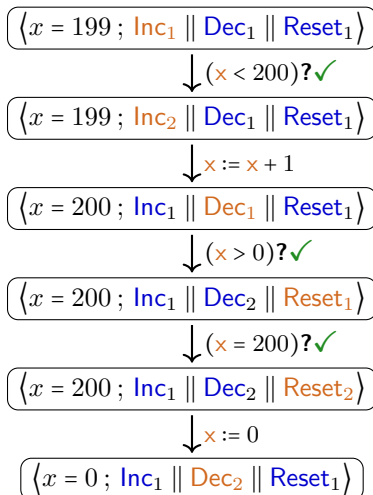
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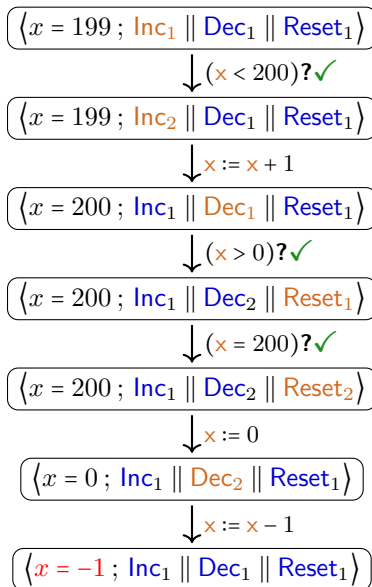
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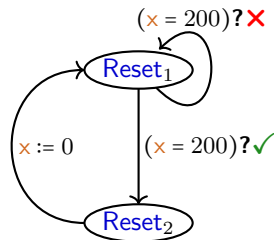
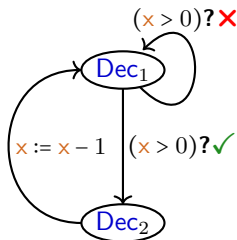
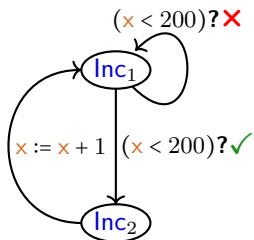
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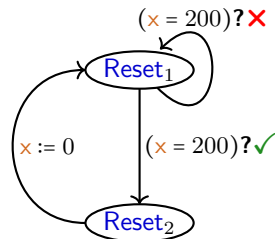
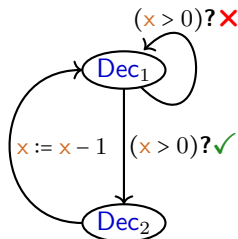
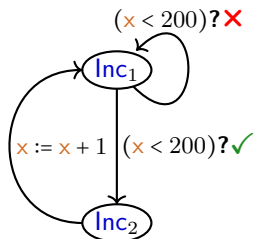


Formalizing properties (in temporal logic)



$\text{Inc}_1 \parallel \text{Dec}_1 \parallel \text{Reset}_1 \not\models \Box(0 \leq x \wedge x \leq 200)$ (Linear-TL formula)

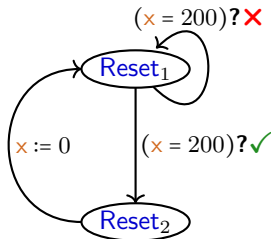
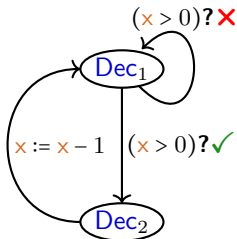
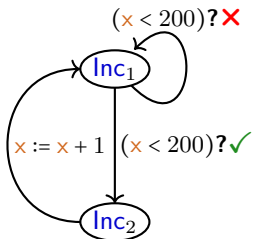
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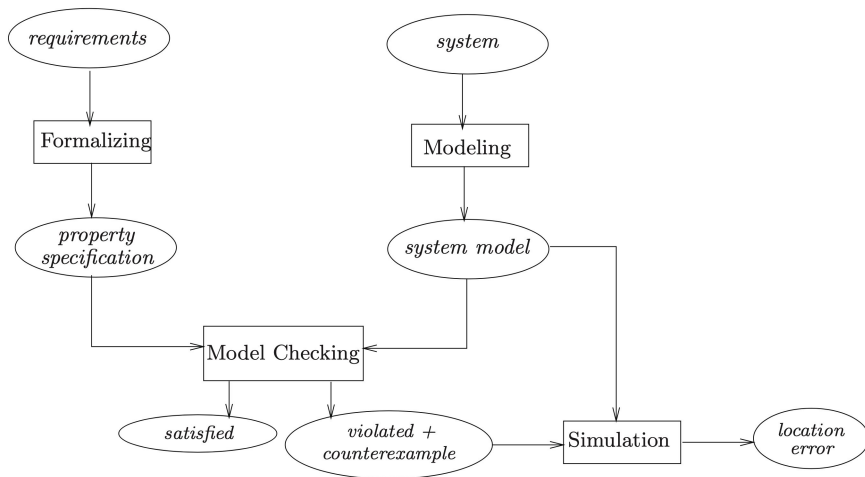
$Inc_1 \parallel Dec_1 \parallel Reset_1 \models \Diamond(x < 0)$ (LTL formula)

$Inc_1 \parallel Dec_1 \parallel Reset_1 \not\models \forall \Box(0 \leq x \wedge x \leq 200)$ (Computation-Tree-L formula)

$Inc_1 \parallel Dec_1 \parallel Reset_1 \models \exists \Box(0 \leq x \wedge x \leq 200)$ (CTL formula)

$Inc_1 \parallel Dec_1 \parallel Reset_1 \models \forall \Box \exists \Diamond(x < 0)$ (CTL formula)

Model checking

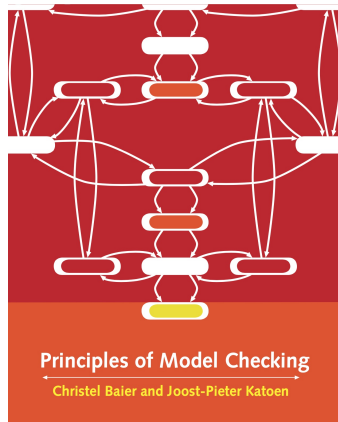


Any [such] verification is only as good as the model of the system.

Topics of the module

- ▶ modeling systems by **labeled transition systems (LTSs)**
- ▶ **fairness**
- ▶ **Linear Temporal Logic (LTL)**
 - ▶ model checking formulas
 - ▶ express properties by **Büchi automata**
 - ▶ model check LTSs and properties via **product automata**
- ▶ **Computation Tree Logic (CTL)**
- ▶ **partial** model checking
 - ▶ partially known systems (state properties/states/transitions)
- ▶ analysing system behavior with the **mCRL2 model-checker toolbox**

Book



- pdf available:

https://is.ifmo.ru/books/_principles_of_model_checking.pdf

Organization

Lectures (Clemens 5/Emilio 2)

- ▶ presentations on blackboard
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- ▶ February (first/second week)

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 - ▶ small verification project (of an algorithm, e.g. in [mCRL2](#))
 - ▶ presentation about a paper
 - ▶ written exam?

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Thank you – we are looking forward to the course!