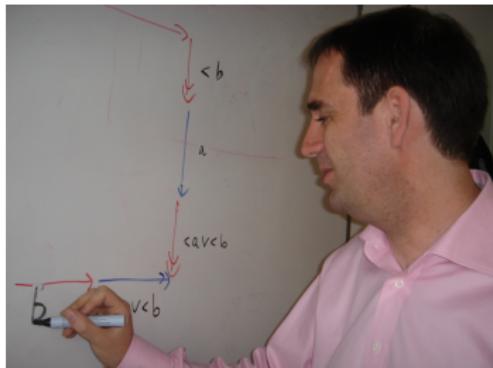


# hogere orde en eerste orde

ter ere van Vincent van Oostrom (afscheid van de UU)



Clemens Grabmayer

VU Amsterdam, Informatica

Universiteit Utrecht

2 July 2014

# overzicht

- ▶ wat ik van Vincent heb geleerd, en blijf leren:
  - ▶ hogere-orde herschrijven
  - ▶ optimale reductie in de  $\lambda$ -calculus (Lambdascope)
  - ▶ hogere-orde naar eerste-orde vertaling:  
 $\lambda$ -calculus naar orthogonale TRS
- ▶ toepassingen:
  - ▶ maximal sharing (met Jan Rochel)
  - ▶ geneste termgrafen (met Vincent)
- ▶ andere leermomenten:
  - ▶ paradoxaal interveniëren
  - ▶ diep
  - ▶ gewoon
  - ▶ het Vincents
  - ▶ moeten?
  - ▶ university management
  - ▶ voetbal

# tijdlijn

onderzoeksprojecten:

2007–2009 NWO project *Infinity* (UU/VU/CWI)

2009–2012 NWO proj. *Realising Optimal Sharing* (UU–Phil/UU–CS)

- ▶ VvO (projectleider)
- ▶ Doaitse Swierstra
- ▶ Jan Rochel, CG

onderwijs samen met Vincent:

- ▶ datastructuren (bachelor CKI)
- ▶ models of computation (master CAI)

## paradoxaal interveniëren (V)

- ▶ een ‘typische Vincent opmerking’
  - ▶ volgt vaak letterlijke betekenissen van woorden, of bijbetekenissen
  - ▶ gebruikt betekenissen waar anderen helemaal niet aan denken
  - ▶ gevuld door: ‘oooh, je bedoelde eigenlijk . . .’, ‘zeg dat dan’
  - ▶ is speels, maar verruimt (in een oopopslag) het referentiekader
  - ▶ en leidt daarmee tot gezamenlijke nieuwe energie, soms ook inzichten

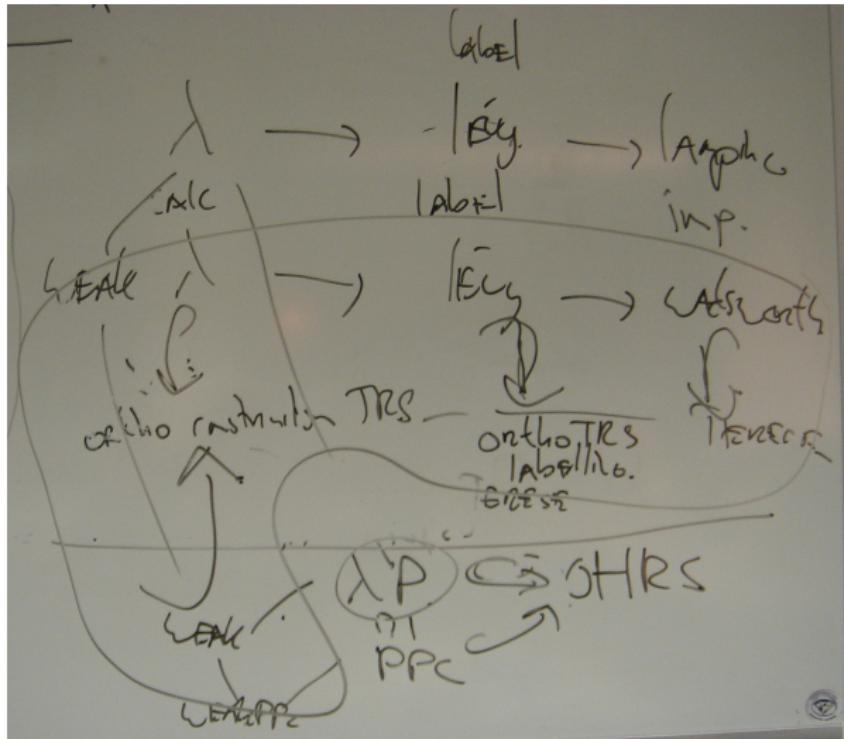
# paradoxaal interveniëren (psychotherapie)

Alfred Adler (1870–1937)



- ▶ in gezinstherapie
- ▶ doel van de therapeut: weerstand van de client elimineren
- ▶ therapeut
  - ▶ neemt letterlijk wat de client zegt
  - ▶ bemoedigt de client om door te gaan met het symptomatisch gedrag
  - ▶ suggereert het tegenovergestelde van wat logica/rede zou ingegeven
- ▶ wordt op ethisch-verantwoorde manier toegepast
- ▶ de onredelijk lijkende suggestie dwingt de client om na te denken
- ▶ wat ook de gekozen reactie is, de client besefte dat zij/hij meer controle heeft dan gedacht

# gradaties van optimale reductie [VvO, plaatje 2008/10]



# gradaties van optimale reductie

calculus (rewrite relation)	labelling	graph rewriting implementation	sharing notion
$\lambda$ -calculus ( $\beta$ -reduction $\rightarrow_\beta$ )	Lévy labelling '78	Lamping '89 Kathail '90 Abdadi/Gonthier/ /Levy '92 Asperti/Guerrini '93 VvO '03 (Terese)	context sharing
$\lambda$ -calculus (weak- $\beta$ red. $\rightarrow_{w\beta}$ )	Blanc/Lévy/ Maranget '05/'07	Wadsworth '71 Shivers/Wand '04	extended-scope sharing
orthogonal TRS (induced rewrite relation $\rightarrow$ )	VvO '03 (Terese)	Staples '80 VvO '03 (Terese)	subterm sharing

# hogere orde term herschrijven



Vincent en Femke van Raamsdonk

# hogere orde term herschrijven



Yuri Gurevich



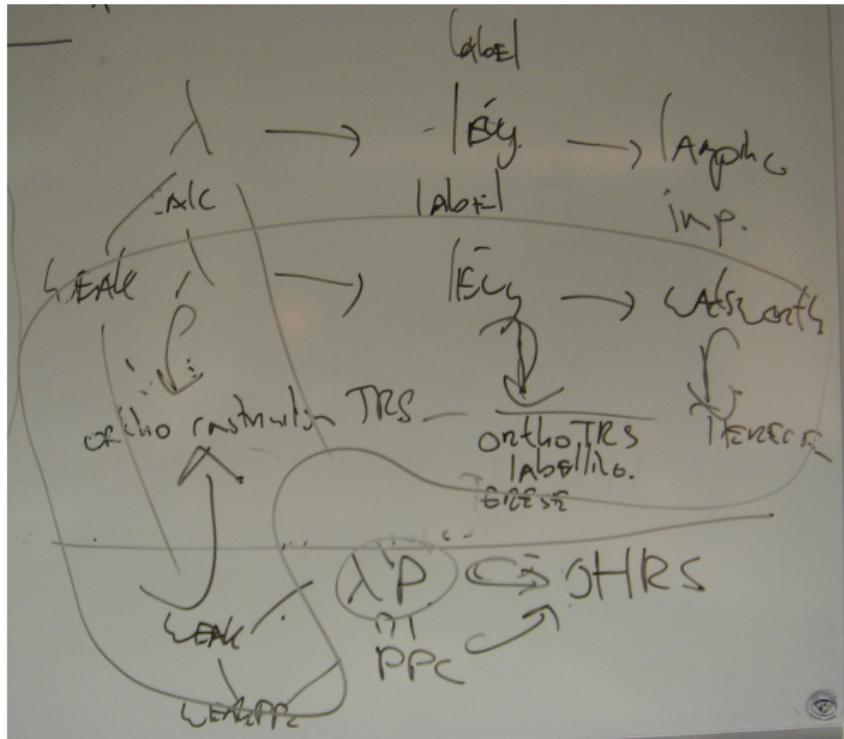
# hogere orde term herschrijven

herschrijven: 'equationeel redeneren, één kant op, naar het antwoord toe'

hogere-orde herschrijven: herschr. van syntactische objecten met binding

- ▶ prototypische voorbeeld:  $\lambda$ -calculus ( $\lambda$ )
- ▶ verschillende formaten zoals:
  - ▶ Combinatory Reduction Systems (CRSs) [Klop]
  - ▶ Higher-Order Rewrite Systems (HRSs) [Nipkow]
- ▶ HRS-introductie voor filosofen geschreven door Vincent in:  
G, Joop Leo, Vincent van Oostrom, Albert Visser: *On the termination of Russell's description elimination algorithm*, RSL'2011.

# gradaties van optimale reductie [VvO, plaatje 2008/10]



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# gradaties van optimale reductie

optimale evaluatie in  $\lambda$ -calculus vermeidt:

- ▶ onnodig werk
- ▶ dubbel werk

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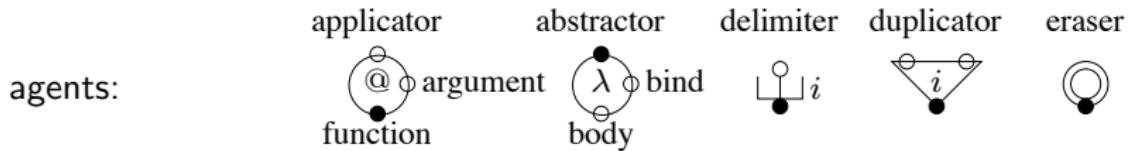
# gradaties van optimale reductie

optimale evaluatie in  $\lambda$ -calculus vermeidt:

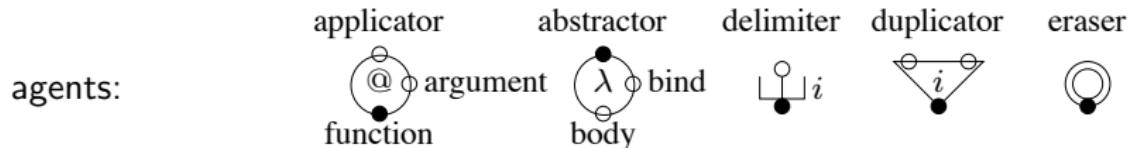
- ▶ onnodig werk
- ▶ dubbel werk

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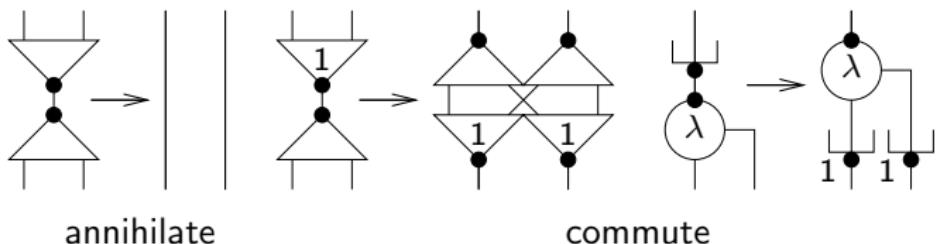
# Lambdascope [VvO, van de Looij, Zwitserlood, 2003]



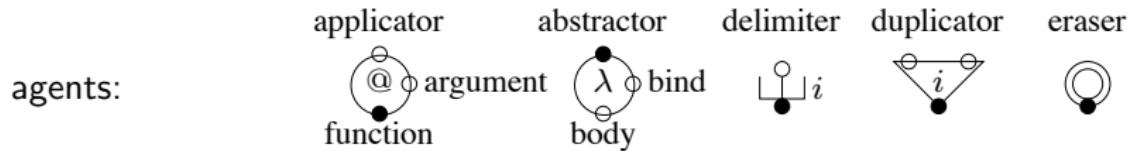
# Lambdascope [VvO, van de Looij, Zwitserlood, 2003]



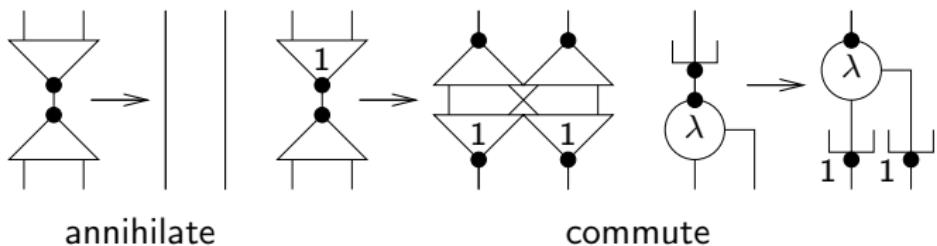
basic rules:



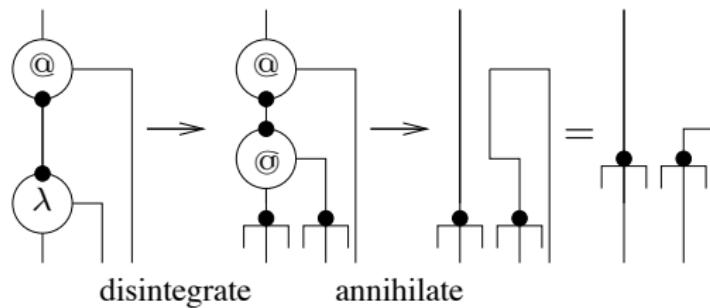
# Lambdascope [VvO, van de Looij, Zwitserlood, 2003]



basic rules:



$\beta$ -rule:



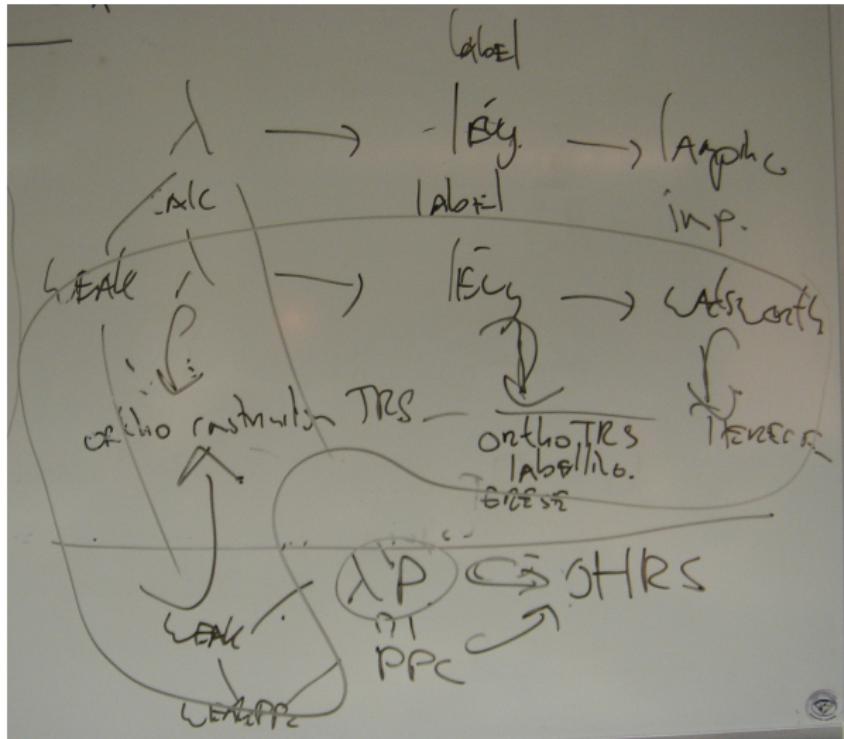
# graaf herschrijf-tool Jan Rochel



graph rewrite tool on Hackage:

<http://hackage.haskell.org/package/graph-rewriting-0.7.5>

# gradaties van optimale reductie [VvO, plaatje 2008/10]



# gradaties van optimale reductie

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## review [VvO, 2005]

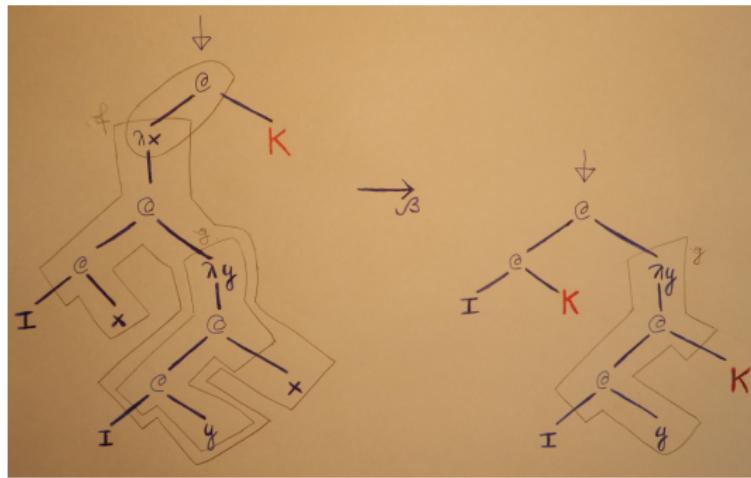
[REDACTED] That being said, by the  $\lambda$ -calculus considered being weak, its theory becomes closer to first-order TRSs than to  $\lambda$ -calculus; a rather useful connection (corroborated by *ad hoc* first-order implementations like supercombinators), which the authors do not mention. Therefore, since for TRSs it is known that dags suffice, it is not that surprising that they suffice for the weak  $\lambda$ -calculus as well. Even stronger, I think that this connection could and should have been exploited also technically (see the Interpretation in TRSs below). It would not only show that the results are not as difficult as the authors suggest now, but it could also help to improve the presentation [REDACTED]

## weak $\beta$ -step

$$(\lambda x. I\,x\,(\lambda y. I\,y\,x))\,K \rightarrow_{w\beta} I\,K\,(\lambda y. I\,K)$$

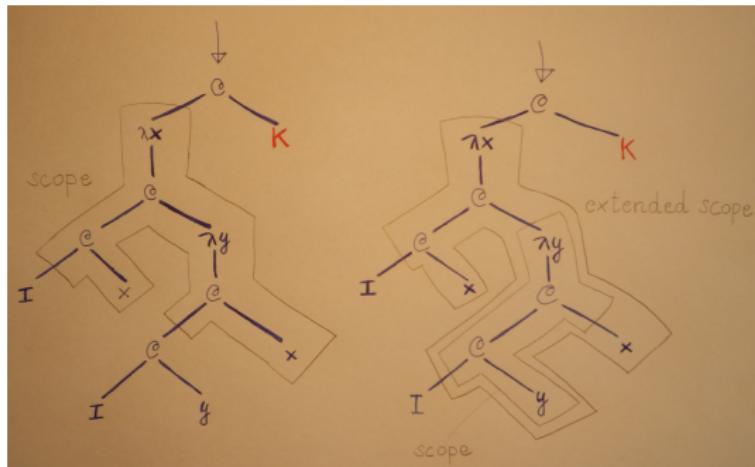
## weak $\beta$ -step

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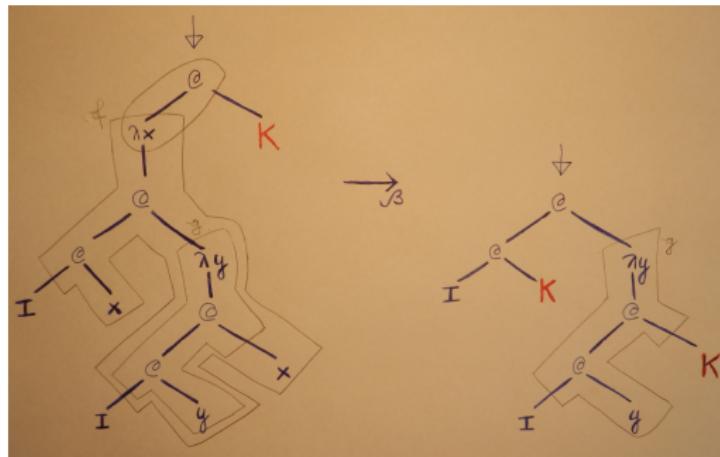
# scope versus extended scope

$$(\lambda x. I x (\lambda y. I y x)) K$$



# oTRS vertaling van $\lambda$ -calculus [VvO, review, 2005]

$$(\lambda x.Ix(\lambda y.Iyx))K \rightarrow_{w\beta} IK(\lambda y.IK)$$



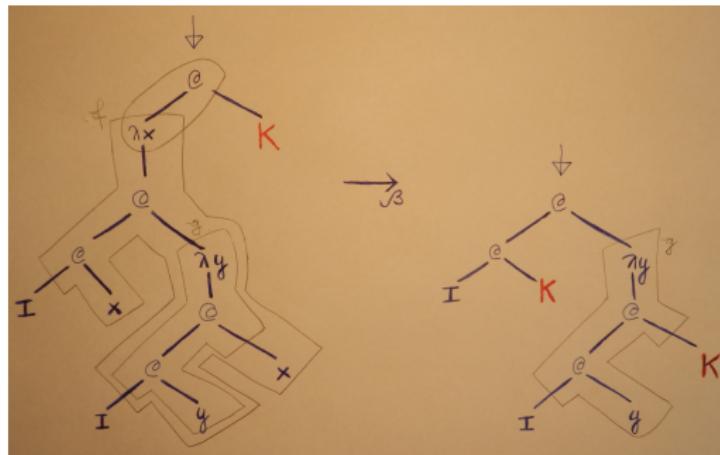
$f(w_1, w_2) X \rightarrow x_1 X g(w_2, X)$
$g(z_1, z_2) Y \rightarrow z_1 Y z_2$
$f(I, I) K$

$f(w_1, w_2) X \rightarrow x_1 X g(w_2, X)$
$g(z_1, z_2) Y \rightarrow z_1 Y z_2$
$IKg(I, K)$

orthogonal term rewrite system (applicative notation)

# oTRS vertaling van $\lambda$ -calculus [VvO, review, 2005]

$$(\lambda x. I x (\lambda y. I y x)) K \rightarrow_{w\beta} I K (\lambda y. I K)$$



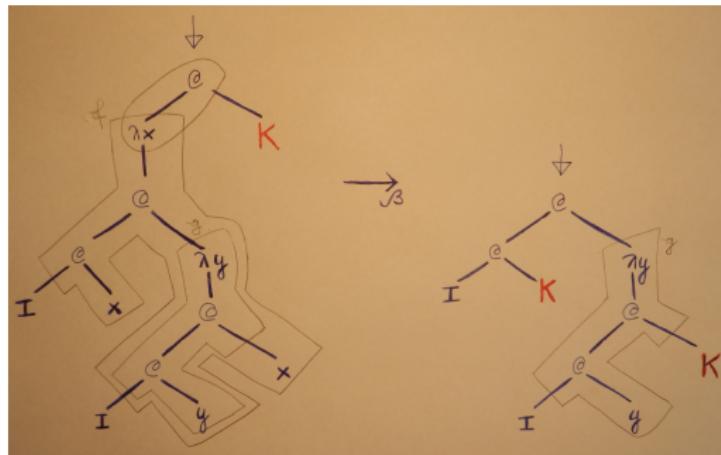
$\text{@}(f(w_1, w_2), X) \rightarrow \text{@}(\text{@}(x_1, X), g(w_2, X))$
$\text{@}(g(z_1, z_2), Y) \rightarrow \text{@}(\text{@}(z_1, Y), z_2)$
$\text{@}(f(I, I), K)$

$\text{@}(f(w_1, w_2), X) \rightarrow \text{@}(\text{@}(x_1, X), g(w_2, X))$
$\text{@}(g(z_1, z_2), Y) \rightarrow \text{@}(\text{@}(z_1, Y), z_2)$
$\text{@}(\text{@}(I, K), g(I, K))$

orthogonal term rewrite system

# oTRS vertaling van $\lambda$ -calculus [VvO, review, 2005]

$$(\lambda x. I x (\lambda y. I y x)) K \rightarrow_{w\beta} I K (\lambda y. I K)$$



$F w_1 w_2 x \rightarrow w_1 x (g w_2 x)$
$G z_1 z_2 y \rightarrow z_1 y z_2$
$F I I K$

$F w_1 w_2 x \rightarrow w_1 x (g w_2 x)$
$G z_1 z_2 y \rightarrow z_1 y z_2$
$I K (G I K)$

super combinator system (result of **fully-lazy lambda-lifting**)

# gradaties van optimale reductie

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# diep

- ▶ 'ik begrijp echt niet wat mensen bedoelen met **diepe** resultaten'
- ▶ zijn deze diep omdat:
  - ▶ geloof mij maar?
  - ▶ ik heb zo een gevoel?
- ▶ acceptabel:  
'A verklaart B, en is daarom **dieper** of **verklarender** dan B'
- ▶ 'Was sich überhaupt sagen lässt, lässt sich klar sagen; [...]'  
(Wittgenstein, TLP)

# gewoon

- ▶ 'let maar eens op als mensen *gewoon* zeggen
  - meestal klopt er dan iets niet'
- ▶ empirisch feit



- ▶ mogelijke oorzaken? — behoefte van mensen om
  - ▶ niet verder na te denken
  - ▶ ergens bij te horen



# het Vincents

- ▶ variant<sup>1</sup> van het Nederlands

---

<sup>1</sup>van Dale: variant = 'vorm die enigszins afwijkt van de gewone'

# het Vincents

- ▶ variant<sup>1</sup> van het Nederlands
- ▶ er is maar een enkeling die het correct spreekt
- ▶ correctheid wordt duidelijk beargumenteerd
- ▶ Nederlands als daarover maar goed genoeg was nagedacht
  - ▶ bijv.: gegevens worden 'aangehecht' bij een e-mail niet 'bijgesloten'
  - ▶ fout: 'per omgaande' voor situaties zonder analogie met de postbode
- ▶ gekenmerkt door grote kennis van en groot interesse voor streektalen
- ▶ altijd inspirerend:  
speurt betekenissen van uitdrukkingen nauwkeurig na

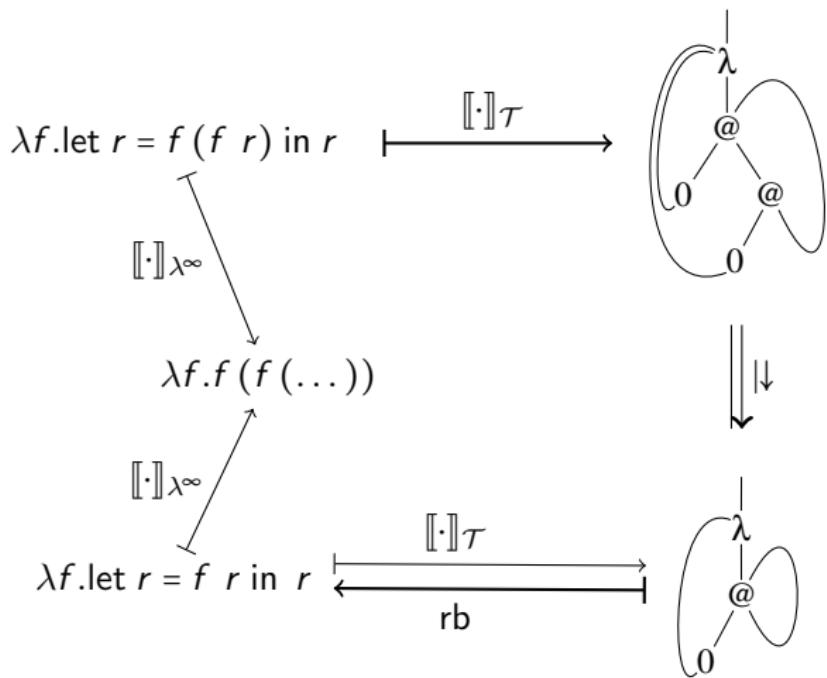
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<sup>1</sup>van Dale: variant = 'vorm die enigszins afwijkt van de gewone'

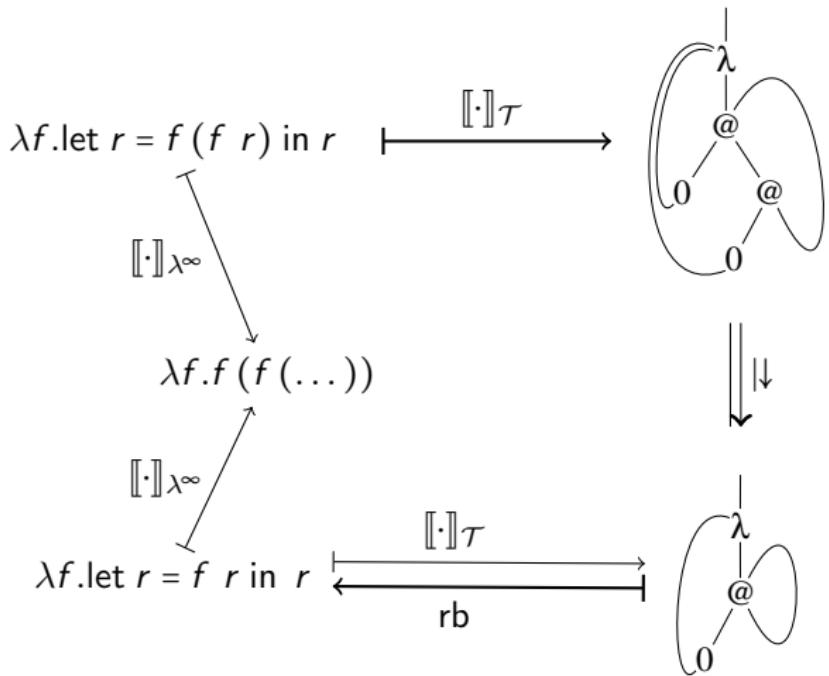
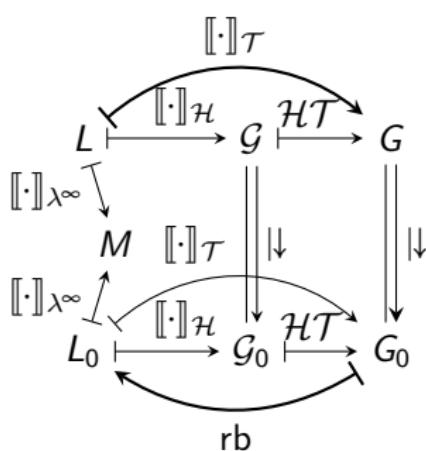
# moeten?

- ▶ echt?
- ▶ waarom?
- ▶ van wie? wie zegt dat?
- ▶ wie gaat over de deadline? is deze redelijk?
- ▶ woord beter niet gebruiken?
- ▶ mogelijk om een ander woord te gebruiken?  
soms ja:
  - ▶ 'dienen te'
  - ▶ A 'zou het graag zien' als N gebeurt

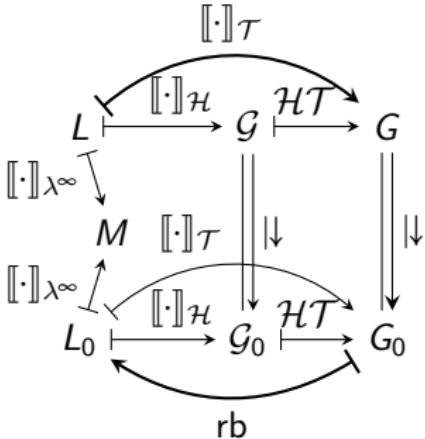
# maximal sharing [G/Rochel, ICFP'14]



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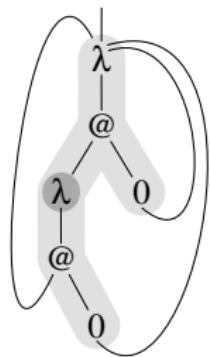


1. term graph translation  $\llbracket \cdot \rrbracket$ .  
of program  $L$  into:
  - a. **higher-order** term graph  $G = \llbracket L \rrbracket_H$
  - b. **first-order** term graph  $G = \llbracket L \rrbracket_T$
2. bisimulation collapse  $\Downarrow$   
of f-o term graph  $G$  into  $G_0$
3. readback rb  
of f-o term graph  $G_0$   
into program  $L_0 = \text{rb}(G_0)$ .

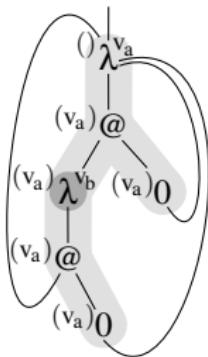
- ▶ tool on Hackage:  
<http://hackage.haskell.org/package/maxsharing/>
- ▶ paper (ICFP'14) and report (arXiv):  
G/Rochel: *Maximal Sharing in the  $\lambda$ -calculus with letrec.*

# higher-order and first-order term graph interpretations

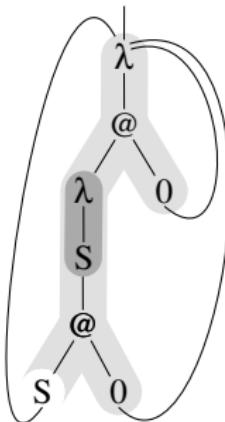
let  $f = \lambda x.(\lambda y.f\ x)\ x$  in  $f$



naive term graph  
interpretation



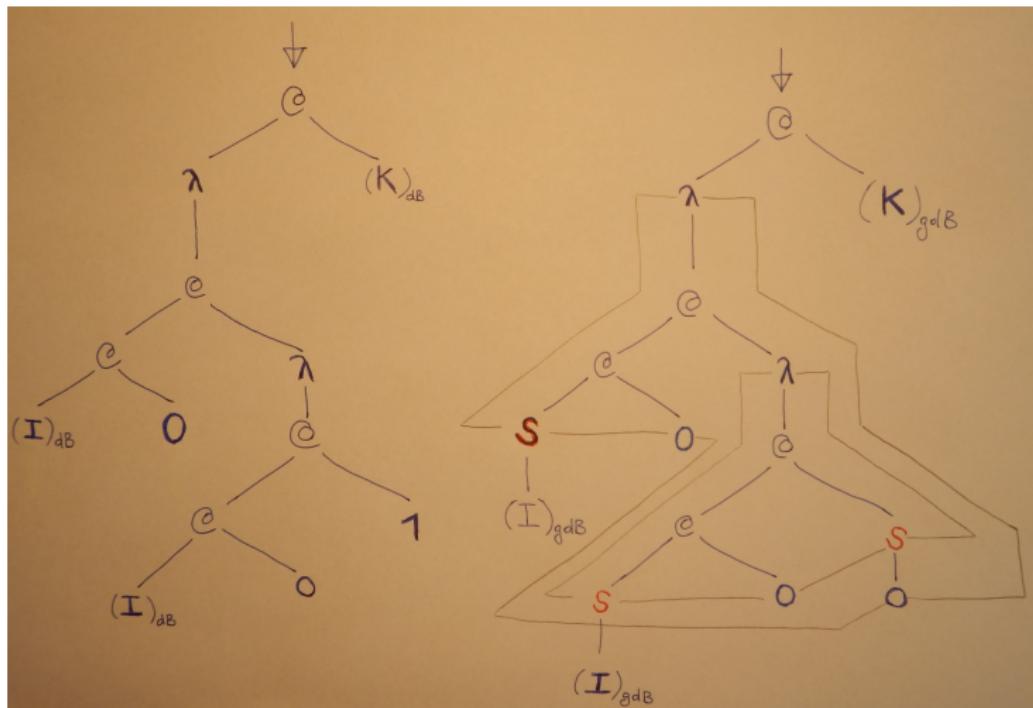
$\lambda$ -h-o-term-graph  
interpretation



$\lambda$ -term-graph  
interpretation

# generalized de Bruijn index form [Patterson/Bird, VvO/Hendriks]

$$(\lambda x.Ix(\lambda y.Iy x))K$$



$$\lambda(I)_{dB} 0 (\lambda(I)_{dB} 0 1)$$

$$\lambda(S(I)_{gdB}) 0 (\lambda(S(I)_{gdB}) 0 (S 0))$$

# nested

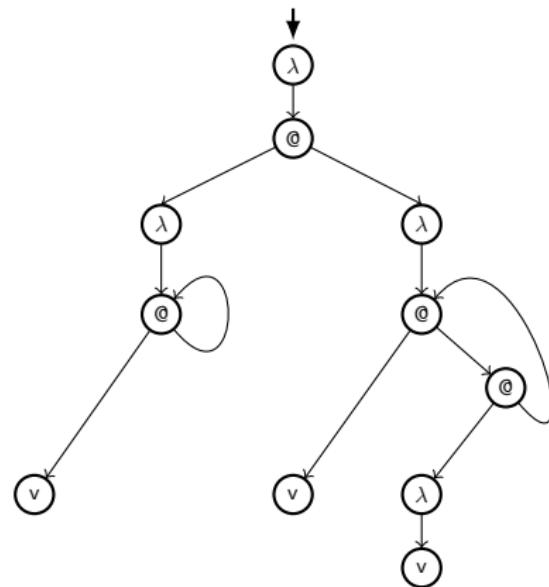
'a group of objects made to fit close together or one within another'



$$x = \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \dots}}}}$$

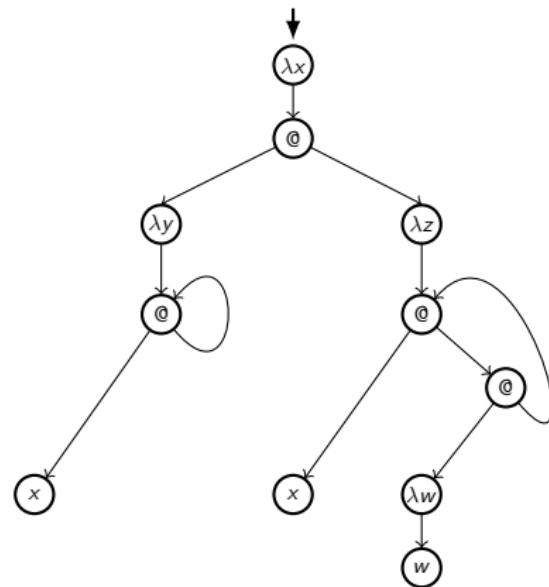
```
for i = 0 to 9 do
    for j = 0 to 9 do
        for k = 0 to 9 do
            sum = sum + i*100
                + j*10 + k + 1;
```

## nested scopes

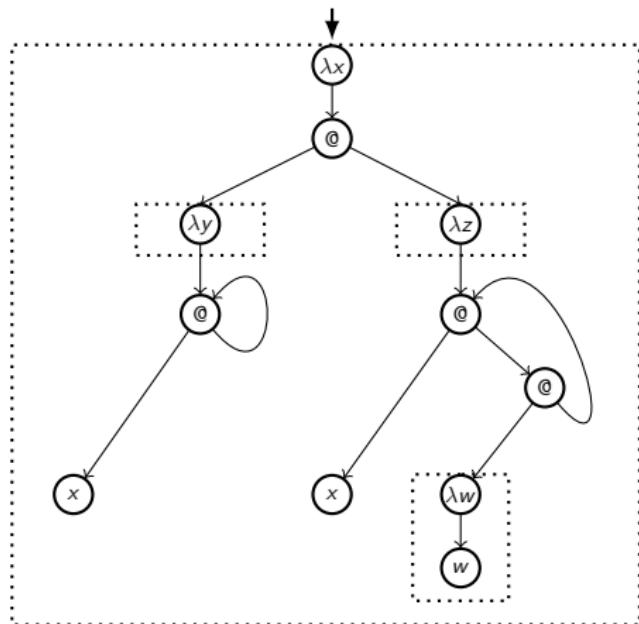


term graph with  $\Sigma = \{\lambda/1, @/2, v/0\}$

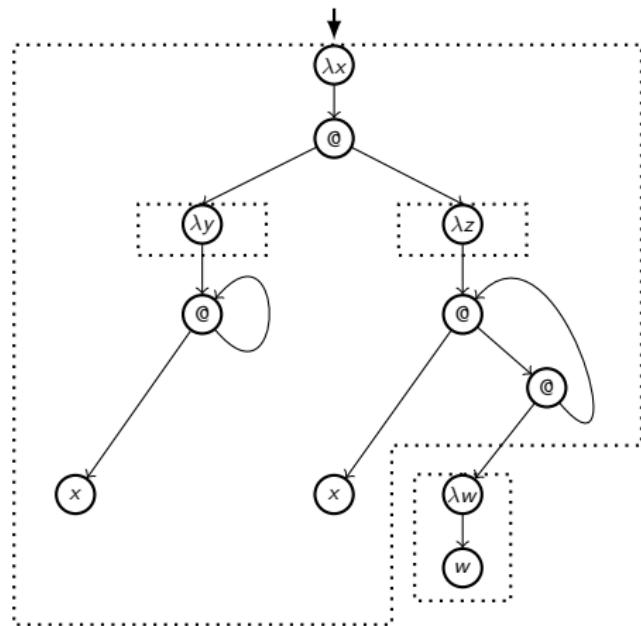
## nested scopes


$$\lambda x.(\lambda y.\text{let } \alpha = x \alpha \text{ in } \alpha) (\lambda z.\text{let } \beta = x (\lambda w.w) \beta \text{ in } \beta)$$

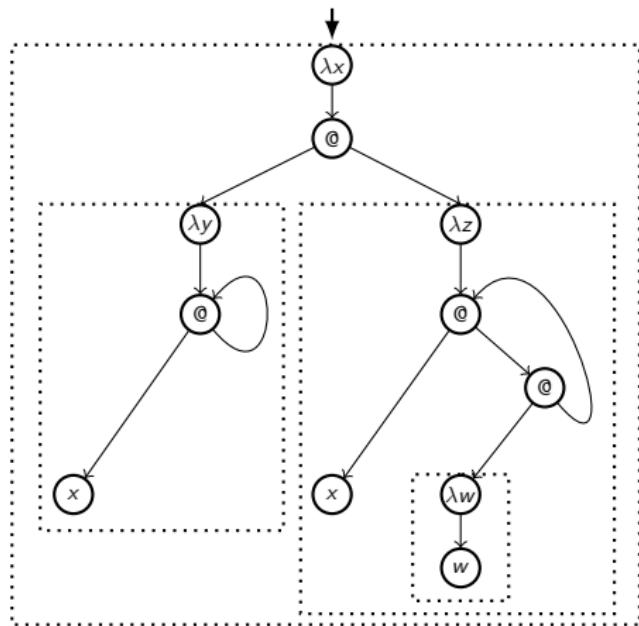
## nested scopes


$$\lambda x.(\lambda y.\text{let } \alpha = x \alpha \text{ in } \alpha) (\lambda z.\text{let } \beta = x (\lambda w.w) \beta \text{ in } \beta)$$

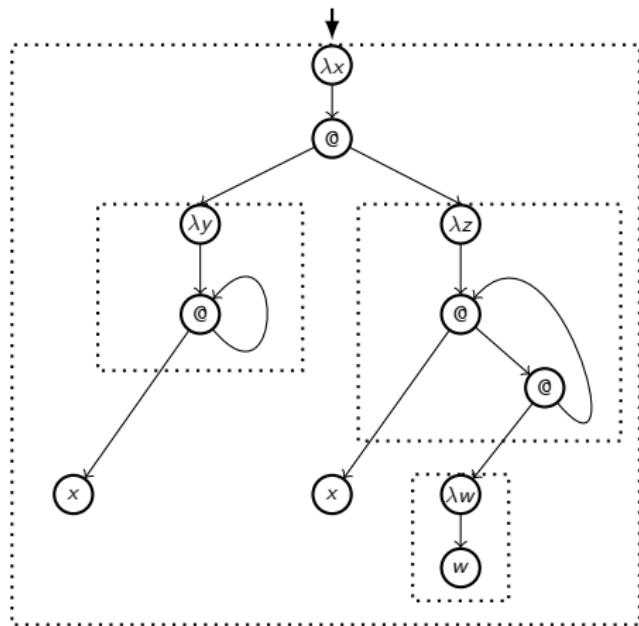
## nested scopes


$$\lambda x.(\lambda y.\text{let } \alpha = x \alpha \text{ in } \alpha) (\lambda z.\text{let } \beta = x (\lambda w.w) \beta \text{ in } \beta)$$

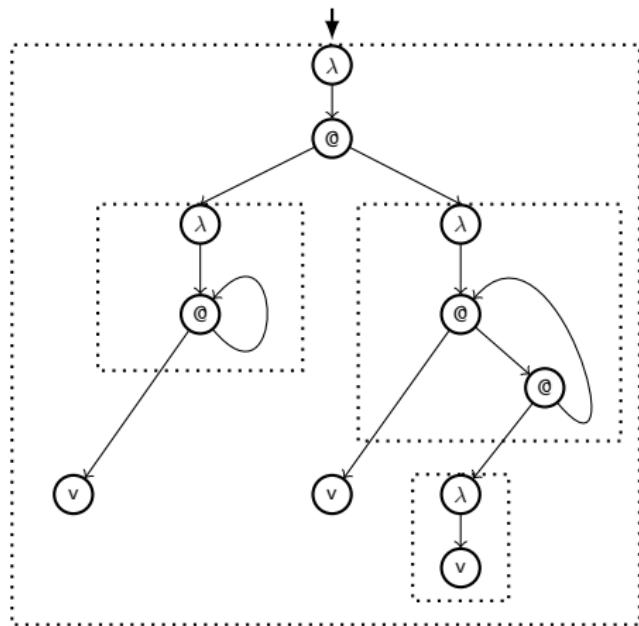
## nested scopes


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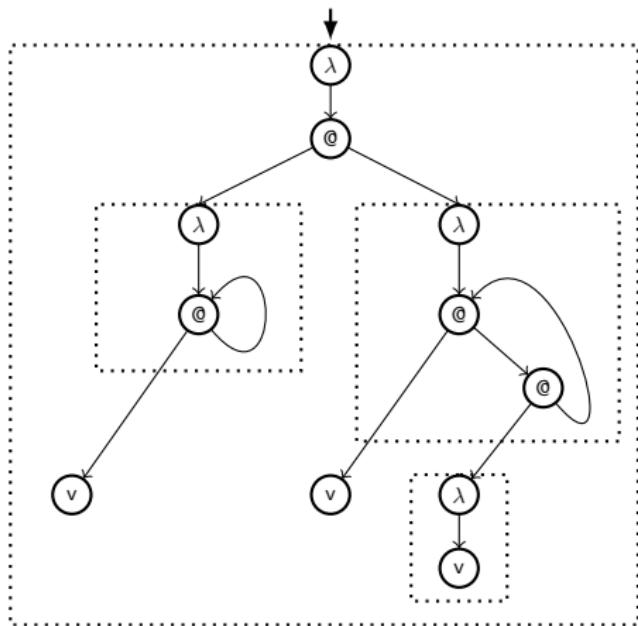
## nested scopes


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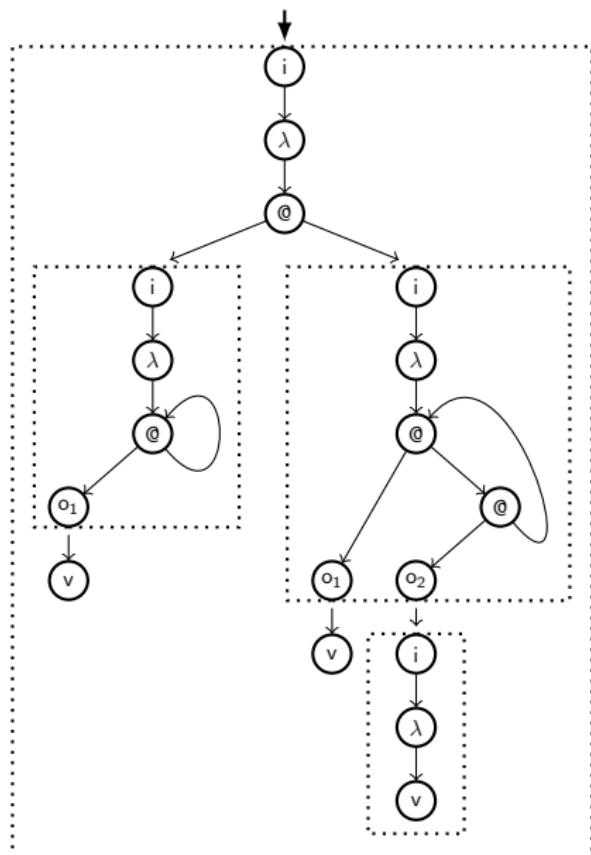
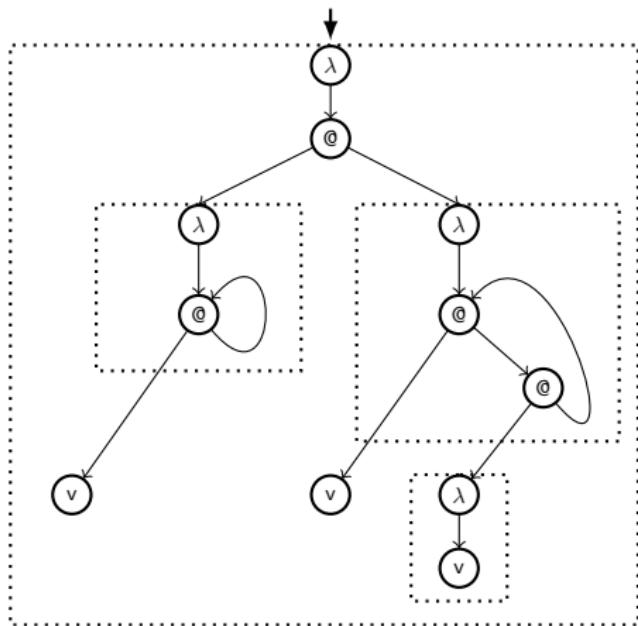
## nested scopes


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# nested scopes



nested scopes → nested term graph



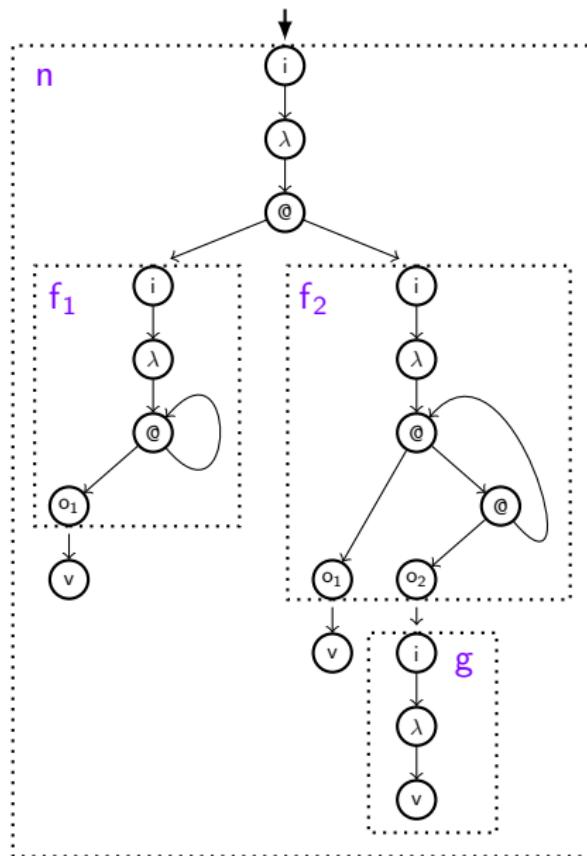
# nested term graph

gletrec

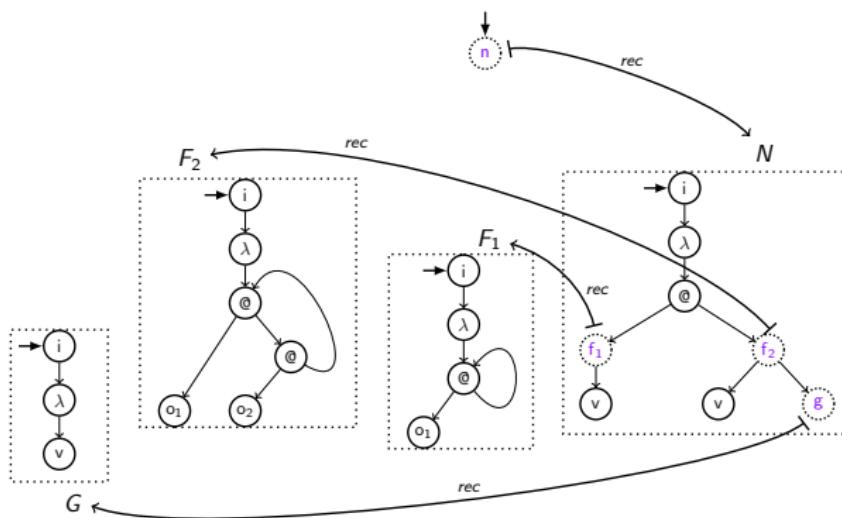
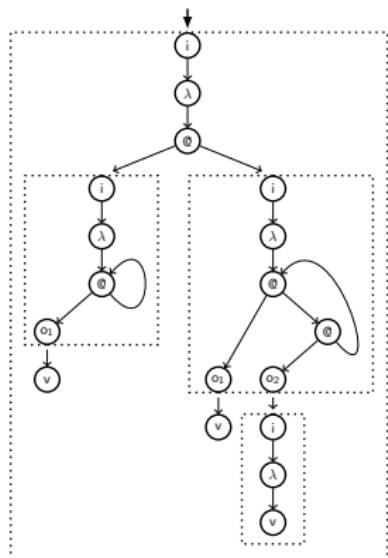
$$\begin{aligned} n() &= \lambda x. f_1(x) f_2(x, g()) \\ f_1(X_1) &= \lambda x. \text{let } \alpha = X_1 \alpha \text{ in } \alpha \\ f_2(X_1, X_2) &= \lambda y. \text{let } \beta = X_1 (X_2 \beta) \text{ in } \beta \\ g() &= \lambda z. z \end{aligned}$$

in

$n()$



# nested term graph

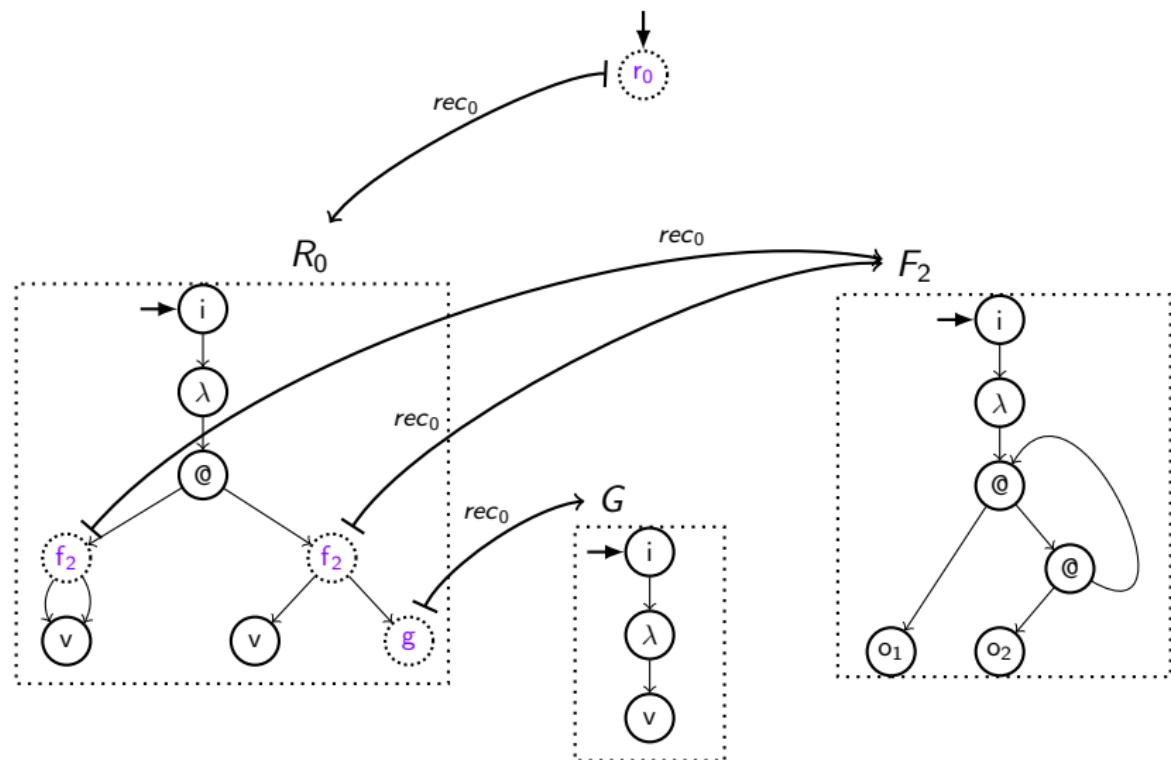


# signature

A *signature for nested term graphs* (*ntg-signature*) is a signature  $\Sigma$  that is partitioned into:

- ▶ *atomic* symbol alphabet  $\Sigma_{\text{at}}$
- ▶ *nested* symbol alphabet  $\Sigma_{\text{ne}}$
- ▶ *interface symbols* alphabet  $IO = I \cup O$ 
  - ▶  $I = \{i\}$  with  $i$  unary
  - ▶  $O = \{o_1, o_2, o_3, \dots\}$  with  $o_i$  nullary

# recursive graph specification



$$\Sigma_{at} = \{\lambda/1, @/2, v/0\}, \Sigma_{ne} = \{r_0/0, f_2/2, g/0\}, I = \{i/1\}, O = \{o_1/0, o_2/0, \dots\}.$$

# recursive graph specification

## Definition

Let  $\Sigma$  be an ntg-signature.

A *recursive graph specification* (a *rgs*)  $\mathcal{R} = \langle \text{rec}, r \rangle$  consists of:

- *specification function*

$$\text{rec} : \Sigma_{\text{ne}} \longrightarrow \text{TG}(\Sigma \cup \text{IO})$$

$$f \longmapsto \text{rec}(f/k) = F \in \text{TG}(\Sigma \cup \{i, o_1, \dots, o_k\})$$

where  $F$  contains precisely one vertex labeled by  $i$ , the root, and one vertex each labeled by  $o_i$ , for  $i \in \{1, \dots, k\}$ ;

- nullary *root symbol*  $r \in \Sigma_{\text{ne}}$ .

# recursive graph specification

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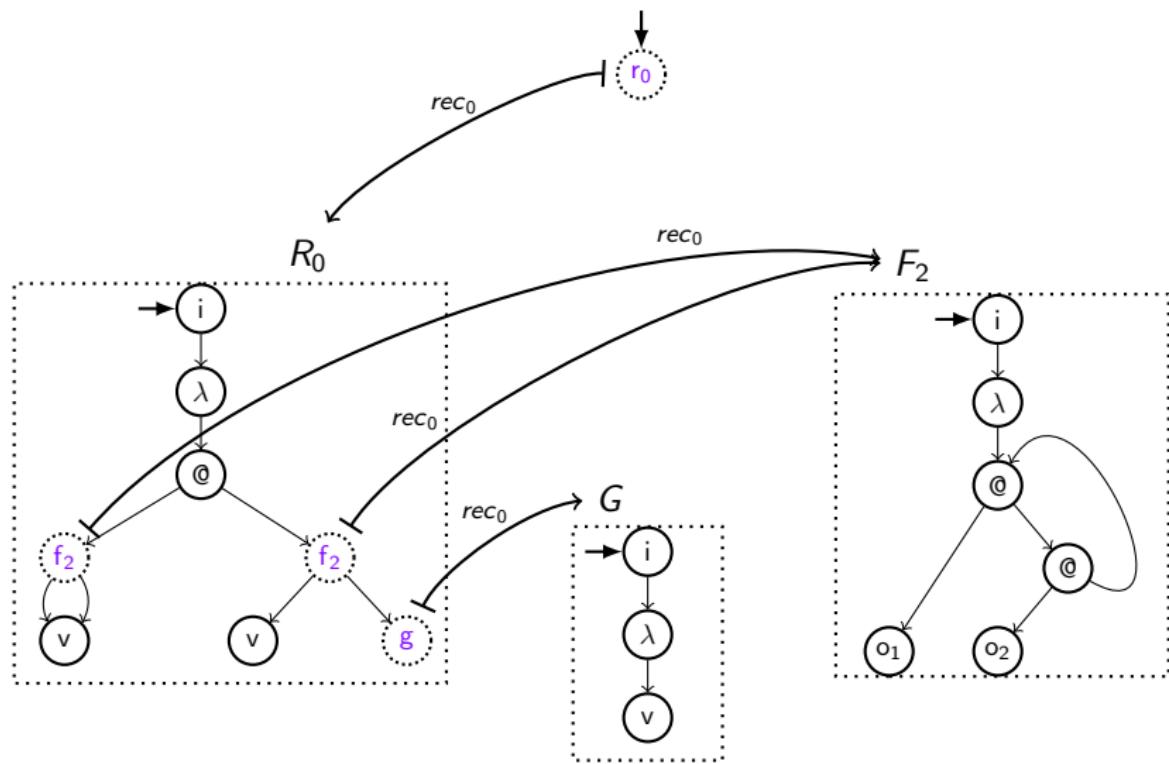
- nullary *root symbol*  $r \in \Sigma_{\text{ne}}$ .

rooted *dependency ARS*  $\multimap$  of  $\mathcal{R}$ :

- ▶ objects: nested symbols in  $\Sigma_{\text{ne}}$
- ▶ steps: for all  $f, g \in \Sigma_{\text{ne}}$ :

$$p : f \multimap g \iff g \text{ occurs in the term graph } \text{rec}(f) \text{ at position } p$$

# recursive graph specification



dependency ARS:  $f_2 \multimap^{\circ} r_0 \multimap g$  is a dag (but not a tree).

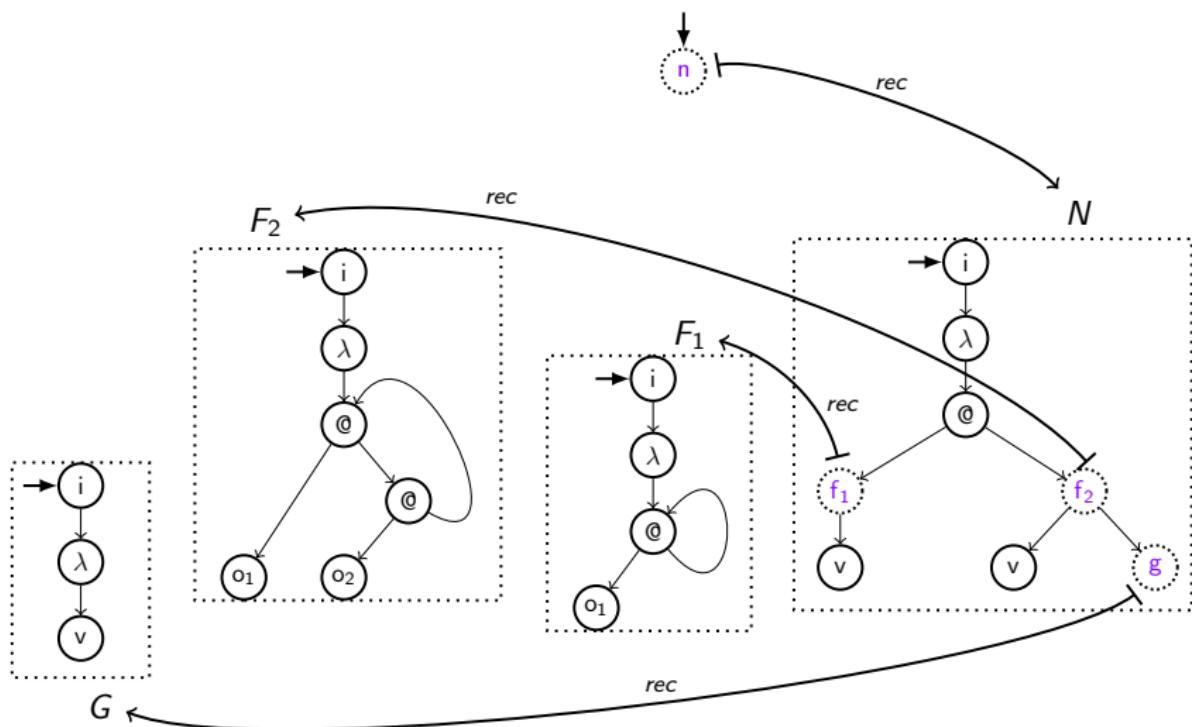
# nested term graph

## Definition

Let  $\Sigma$  be an ntg-signature.

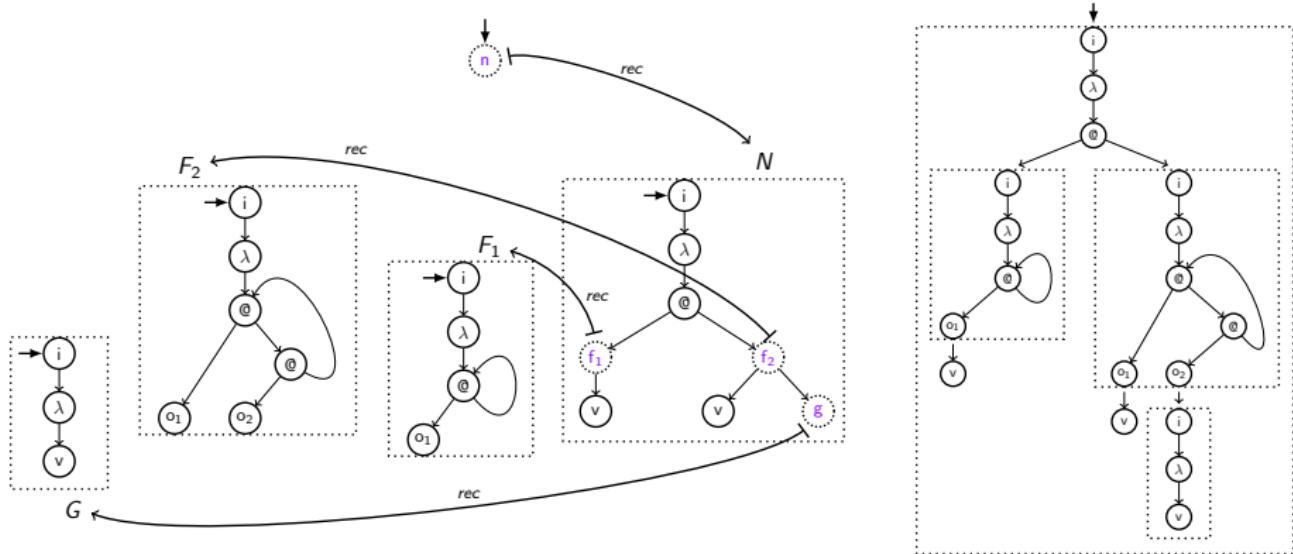
A *nested term graph* over  $\Sigma$  is an rgs  $\mathcal{N} = \langle \text{rec}, r \rangle$  such that the rooted dependency ARS  $\multimap$  is a **tree**.

# nested term graph



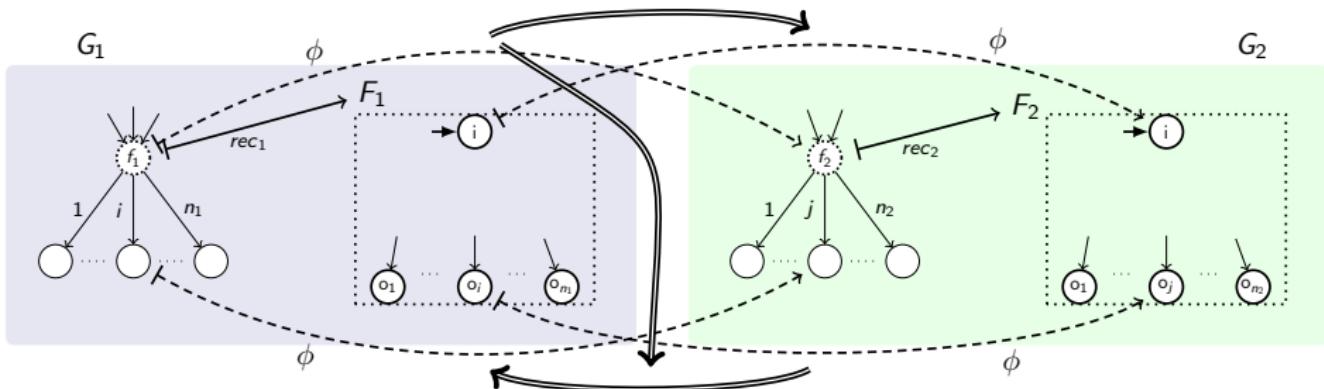
dependency ARS:  $f_1 \multimap r_0$   $\begin{cases} \multimap f_2 \\ \multimap g \end{cases}$  is a tree.

# nested term graph



dependency ARS:  $f_1 \multimap r_0 \circ \begin{matrix} f_2 \\ \circ \\ g \end{matrix}$  is a tree.

# bisimulation clause



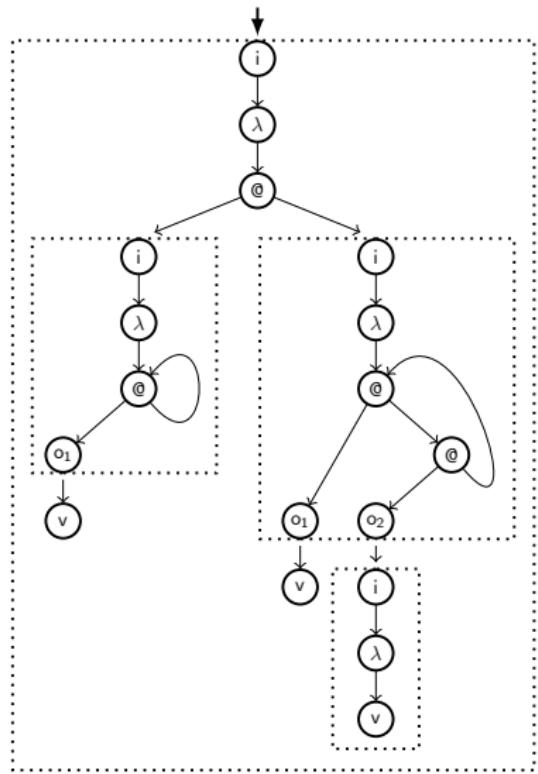
## Theorem

Let  $\Sigma$  be an ntg-signature, and  $\Sigma' = \Sigma \cup I \cup \{o_2, i_r/1, o_r/1\}$ .  
There are functions

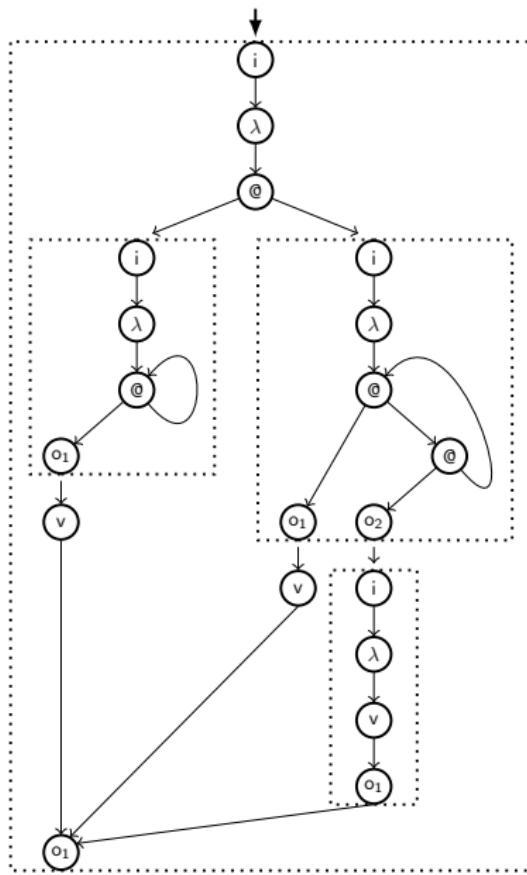
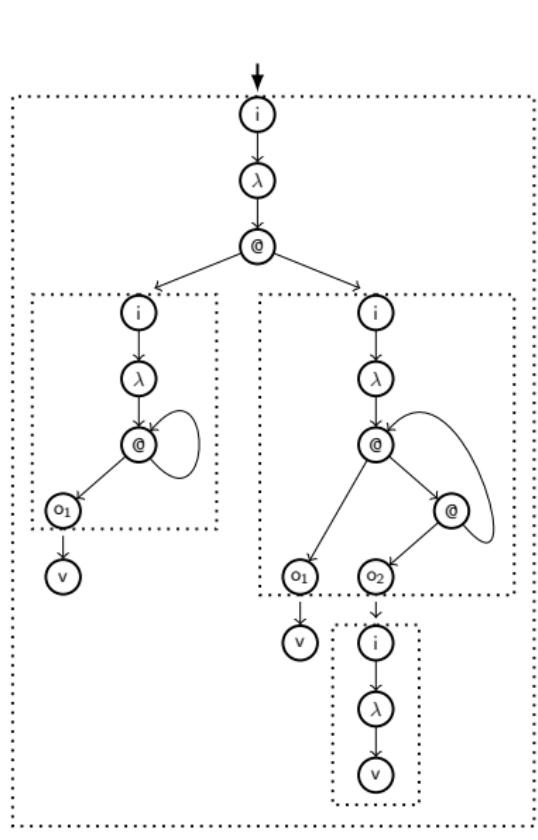
$T : NG(\Sigma) \rightarrow TG(\Sigma')$  and  $N : TG(\Sigma') \rightarrow NG(\Sigma)$  such that:

- ①  $N \circ T = id_{NG(\Sigma)}$  (i.e.  $T$  is a retraction of  $N$ ,  $N$  is a section of  $T$ )
- ②  $T$  and  $N$  preserve and reflect  $\preceq$ .
- ③  $T$  and  $N$  are efficiently computable.

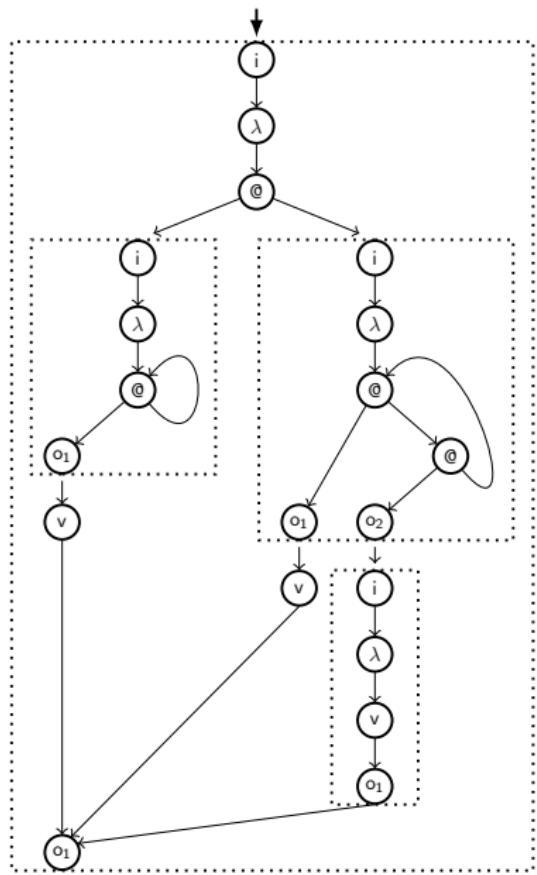
# implementation by first-order term graph



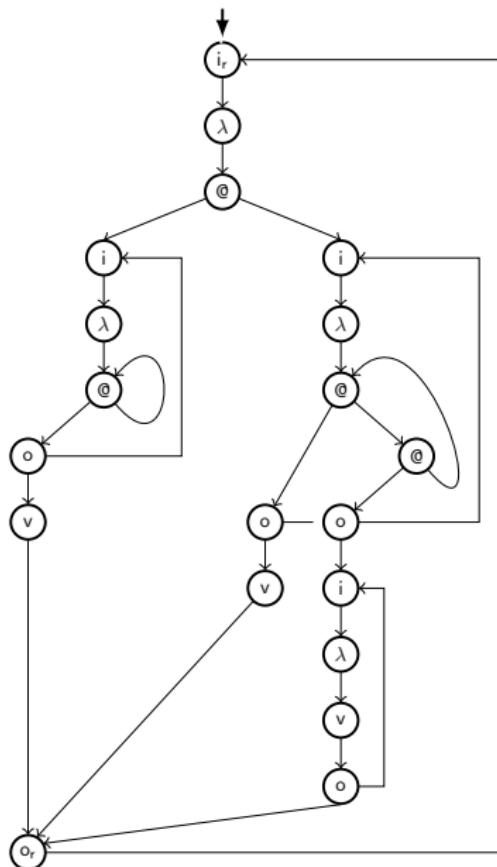
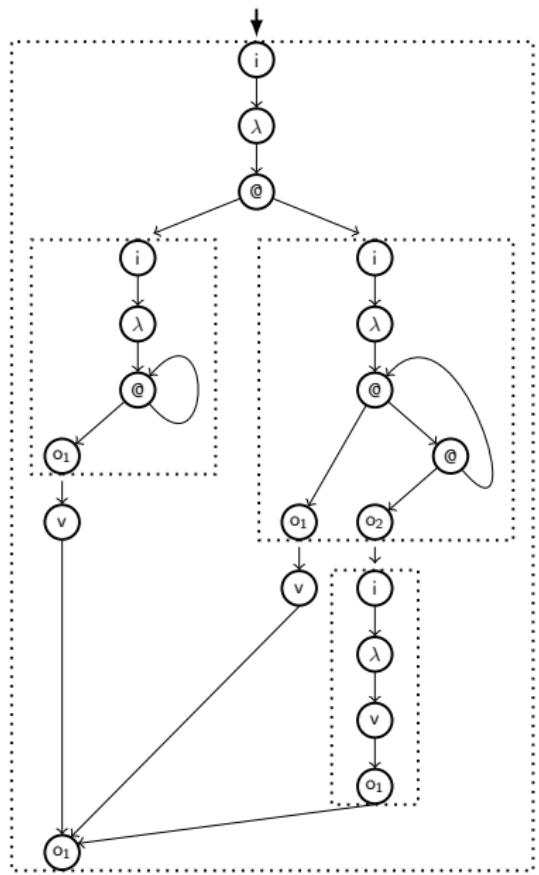
# implementation by first-order term graph



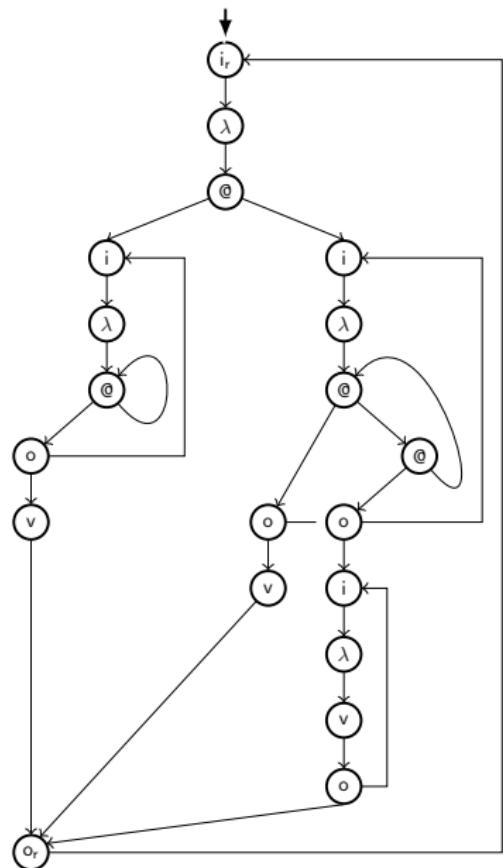
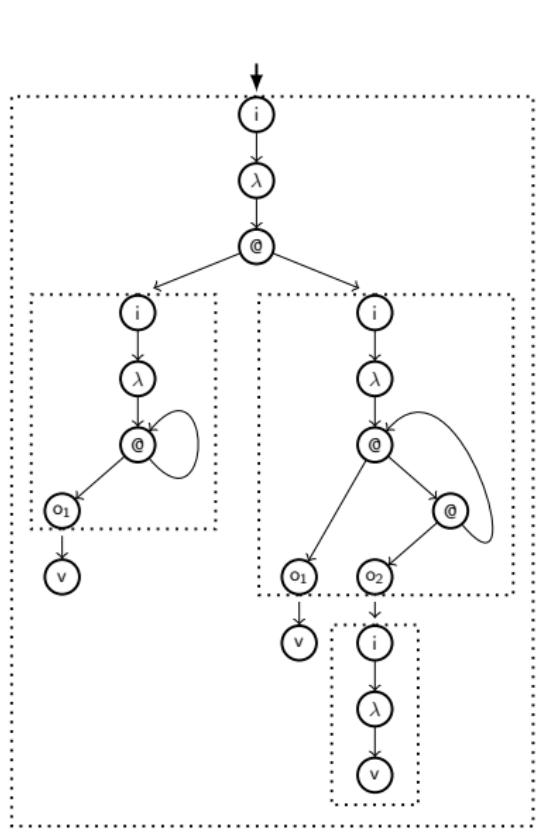
# implementation by first-order term graph



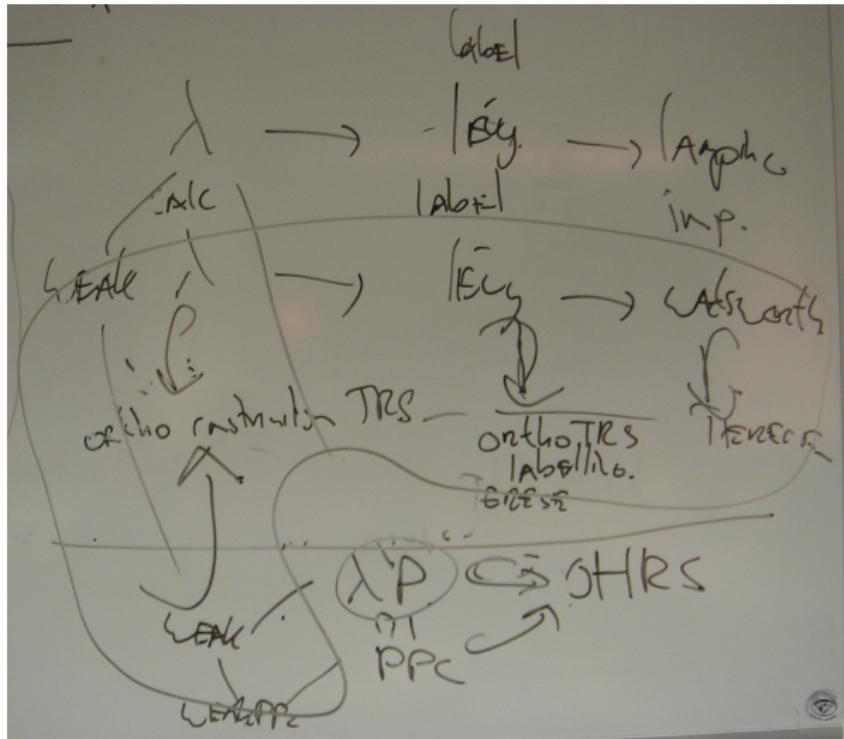
# implementation by first-order term graph



# implementation by first-order term graph



# gradaties van optimale reductie [VvO, plaatje 2008/10]



# gradaties van optimale reductie

calculus (rewrite relation)	labelling	graph rewriting implementation	sharing notion
$\lambda$ -calculus ( $\beta$ -reduction $\rightarrow_\beta$ )	Lévy labelling '78	Lamping '89 Kathail '90 Abdadi/Gonthier/ /Levy '92 Asperti/Guerrini '93 VvO '03 (Terese)	context sharing
$\lambda$ -calculus (weak- $\beta$ red. $\rightarrow_{w\beta}$ )	Blanc/Lévy/ Maranget '05/'07	Wadsworth '71 Shivers/Wand '04	extended-scope sharing
orthogonal TRS (induced rewrite relation $\rightarrow$ )	VvO '03 (Terese)	Staples '80 VvO '03 (Terese)	subterm sharing

## bijkomende gradaties van optimale reductie

calculus (rewrite relation)	labelling	graph rewriting implementation	sharing notion
$\lambda$ -calculus $(\beta\text{-reduction } \rightarrow_\beta)$	Lévy labelling '78	Lamping '89 Kathail '90 Abdadi/Gonthier/ /Levy '92 Asperti/Guerrini '93 VvO '03 (Terese)	context sharing
	?	term/port graph implementation	scope sharing
	?	nested term graph implementation	extended scope sharing
$\lambda$ -calculus $(\text{weak-}\beta\text{ red. } \rightarrow_{w\beta})$	Blanc/Lévy/ Maranget '05/'07	Wadsworth '71 Shivers/Wand '04	extended-scope sharing
orthogonal TRS (induced rewrite relation $\rightarrow$ )	VvO '03 (Terese)	Staples '80 VvO '03 (Terese)	subterm sharing

# university management

*'Traditionally, a university was regarded as an institution whose primary function was the furtherance of learning and knowledge. Money was needed to maintain the infrastructure and pay the staff, but the money was a means to an end, not an end in itself.'*

*'However, it seems that this quaint notion is now rejected in favour of a model of a university whose success is measured in terms of its income, not in terms of its intellectual capital.'*

*'Management [...] just doesn't seem to get a very basic fact about running a university: Its academic staff are vital for the university's goal of achieving academic excellence. They need to be fostered, not bullied.'*

Dorothy Bishop  
*'The university as Big Business'*

22–06–2014

# voetbal



*Alles wat ik zeker weet over moraal en verplichtingen van de mens, heb ik aan voetbal te danken.*

*Albert Camus*

# voetbal



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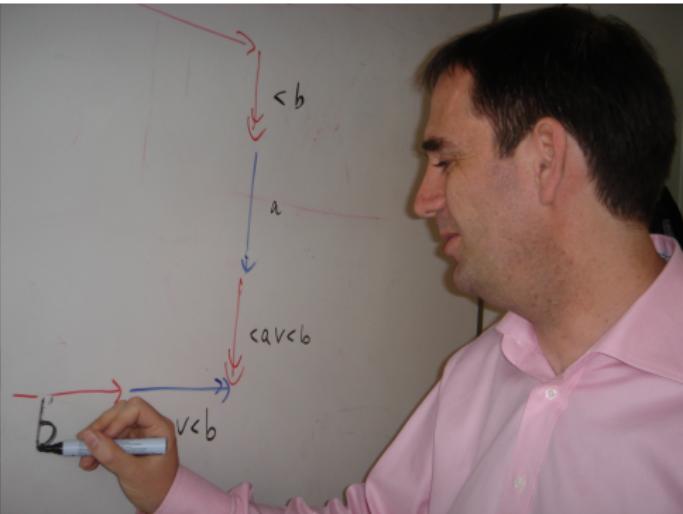


*Alles wat ik zeker weet over moraal en verplichtingen van de mens, heb ik aan voetbal te danken.*

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mijn dank & beste wensen voor de toekomst

- ▶ aan Vincent



- ▶ aan de disciplinegroep Theoretische Filosofie
- ▶ aan de studenten CKI en CAI