Productivity Practicum

Jörg Endrullis   Clemens Grabmayer   Dimitri Hendriks

5th International School on Rewriting
Utrecht, July 8, 2010

The Stream of Factorials

exercise

1. Define in PSF the stream of factorials:

$$\underline{1} : \underline{1} : \underline{2} : \underline{6} : \underline{24} : \ldots$$

   where $\underline{n} := \mathsf{S}^n(0)$.

2. Draw the pebbleflow net corresponding to your specification (use the online tool if you want).
3. Reduce your net stepwise to a source of pebbles.

The Fine Mechanics of I/O sequences

exercise

Let f be a unary stream function defined by:

$$\mathsf{f}(xs) \to \mathsf{zip}(\mathsf{even}(xs), \mathsf{odd}(xs))$$

1. What is the I/O sequence corresponding to this function?
2. Is $\mathsf{M} \to 0 : \mathsf{f}(\mathsf{M})$ productive?

Now consider the slightly changed function g defined by:

$$\mathsf{g}(xs) \to \mathsf{zip_c}(\mathsf{even_c}(xs), \mathsf{odd}(xs)) \quad \text{where}$$

$$\mathsf{zip_c}(x : xs, y : ys) \to x : y : \mathsf{zip_c}(xs, ys)$$

$$\mathsf{even_c}(x : y : xs) \to x : \mathsf{even_c}(xs)$$

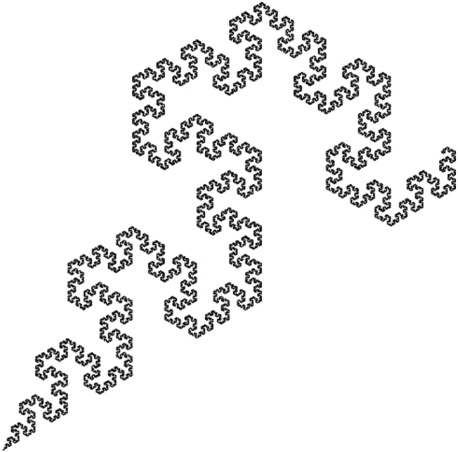
3. Compute the I/O sequence corresponding to g.
4. Is there an extensional difference between f and g?
5. And operational? Is $\mathsf{N} \to 0 : \mathsf{g}(\mathsf{N})$ productive?

The Mephisto Walz

The Mephisto Walz is the iterative fixed point of the morphism

$$0 \to 001 \qquad\qquad 1 \to 110$$

on the starting word 0.



The Mephisto Walz

exercise

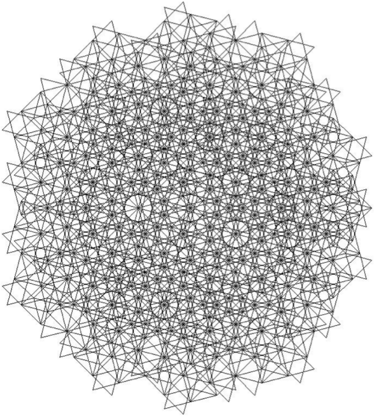
- Define the Mephisto Waltz W in PSF.
- Define the stream Z formed by the lengths of strings of 1's between consecutive zeros in W.
- Is your specification of Z productive?
- Is it data-obliviously productive?
- What do you expect ProPro to answer?

$$W \to 0 : 0 : 1 : 0 : 0 : 1 : 1 : 1 : 0 : \ldots$$
$$Z \to 0 : 1 : 0 : 3 : \ldots$$

The Fibonacci Word

The Fibonacci word is the infinite bitstream obtained by iterating the following morphism on the starting word 0:

$$0 \rightarrow 01 \qquad\qquad 1 \rightarrow 0$$



turtle graphics, Hans Zantema

The Fibonacci Word

exercise

- Give a stream specification for the Fibonacci word.
- What format (pure, pure$^+$, flat, . . .) does your specification fall under?
- Determine the data-oblivious lower bound of the occurring stream functions (e.g. by giving the correspoding I/O sequence).
- Is your specification data-obliviously productive?
- Check your results with ProPro.

The Towers of Hanoi

The Towers of Hanoi

- three vertical pegs A, B, C
- N disks of different diameters stacked on peg A
- rule: no disk is stacked on a smaller one
- step: transfer topmost disk from one peg to another
- goal: move the entire stack from peg A to peg B (or to C)

exercise

Define in PSF a stream H of moves (pairs of pegs) such that the first $2^N - 1$ moves of H transfer the top N disks from peg A to B if N is odd, and from A to C if N is even.

The Towers of Hanoi

Hints:

- Express the prefix transferring $1 + N$ disks from A to B (C) in terms of the prefix transferring N disks from A to C (resp. B).
- Instead of recursing on the top N disks (numbered $1, \ldots, N$) recurse on the lower N disks (numbered $2, \ldots, N + 1$) !
- What is the sequence of moves for the smallest disk?
  How is it hidden in H?

$$\begin{pmatrix} 1 & & \\ 2 & & \\ 3 & \_ & \_ \end{pmatrix} \xrightarrow{AB} \begin{pmatrix} & & \\ 2 & & \\ 3 & 1 & \_ \end{pmatrix} \xrightarrow{AC} \begin{pmatrix} & & \\ & & \\ 3 & 1 & 2 \end{pmatrix} \xrightarrow{BC} \begin{pmatrix} & & \\ & & 1 \\ 3 & \_ & 2 \end{pmatrix} \xrightarrow{AB} \begin{pmatrix} & & \\ & & 1 \\ \_ & 3 & 2 \end{pmatrix} \xrightarrow{CA} \begin{pmatrix} & & \\ & & \\ 1 & 3 & 2 \end{pmatrix} \xrightarrow{CB} \begin{pmatrix} & & \\ & 2 & \\ 1 & 3 & \_ \end{pmatrix} \xrightarrow{AB} \begin{pmatrix} & 1 & \\ & 2 & \\ \_ & 3 & \_ \end{pmatrix}$$

$$\begin{pmatrix} 2 & & \\ 3 & \_ & \_ \end{pmatrix} \qquad \xrightarrow{AC} \begin{pmatrix} & & \\ 3 & \_ & 2 \end{pmatrix} \qquad \xrightarrow{AB} \begin{pmatrix} & & \\ \_ & 3 & 2 \end{pmatrix} \qquad \xrightarrow{CB} \qquad \begin{pmatrix} & 2 & \\ \_ & 3 & \_ \end{pmatrix}$$