From Compressing Lamba-Letrec Terms to Recognizing Regular-Expression Processes

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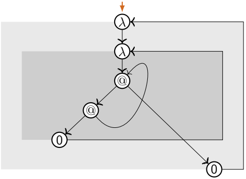

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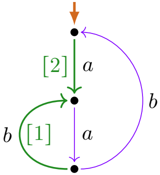

Overview

1. Maximal sharing of functional programs
   - higher-order λ-term graphs

2. Process interpretation of regular expressions
   - LEE-witnesses: graph labelings based on a loop–condition LEE

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   - from terms in the λ-calculus with letrec to:
     - higher-order λ-term graphs
     - first-order λ-term graphs
     - λ-NFAs, and λ-DFAs
   - minimization / readback / efficiency / Haskell implementation
2. Process interpretation of regular expressions
     - LEE-witnesses: graph labelings based on a loop–condition LEE

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   - Milner's questions, known results
   - structure-constrained process graphs:
     - LEE-witnesses: graph labelings based on a loop–condition LEE
     - preservation under bisimulation collapse
   - readback: from graph labelings to regular expressions

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- Comparison desiderata

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λ-calculus with letrec under unfolding semantics

Not available: term graph interpretation that is studied under $\underline{\leftrightarrow}$

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Regular expressions under process semantics (bisimilarity $\underline{\leftrightarrow}$)

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Desired: reason with graphs that are $P(\cdot)$-expressible modulo $\underline{\leftrightarrow}$ (at least with 'sufficiently many')

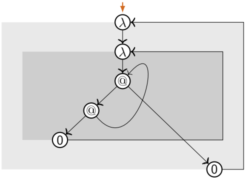
understand incompleteness by a structural graph property

structure constraints (L'Aquila)

Maximal sharing of functional programs

(joint work with Jan Rochel)

$$\begin{tikzcd}
L \arrow[r, mapsto, bend left, "\text{interpret}"] & G \arrow[d, Rightarrow, "\text{collapse}"] \\
L_0 & G_0 \arrow[l, mapsto, bend left, "\text{readback}"]
\end{tikzcd}$$



Maximal sharing: example (fix)

$$\begin{tikzcd}
L \arrow[rr, mapsto, bend left, "\text{interpret}"] \arrow[d, mapsto, "\text{unfold}"'] & & G \arrow[dd, Rightarrow, "\text{collapse}"] \\
M & & \\
L_0 \arrow[u, mapsto, "\text{unfold}"] \arrow[rr, mapsto, bend left, "\text{interpret}"] & & G_0 \arrow[ll, mapsto, bend left, "\text{readback}"]
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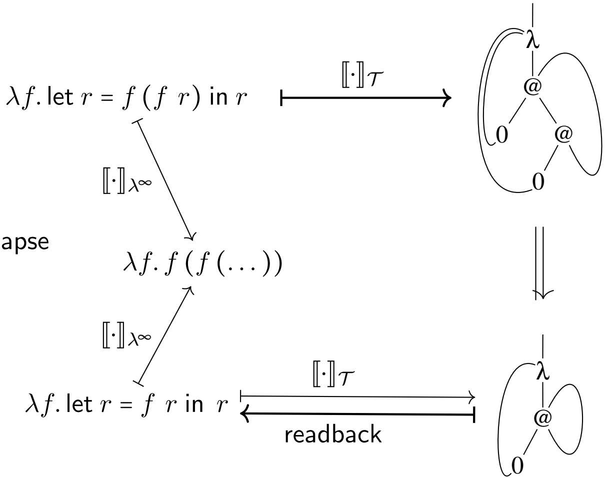

$\lambda f.\,\mathsf{let}\ r = f\,(f\ r)\ \mathsf{in}\ r \xmapsto{[\![\cdot]\!]_{\mathcal{T}}}$

$[\![\cdot]\!]_{\lambda^\infty}$

$\lambda f.\,f\,(f\,(\ldots))$

$[\![\cdot]\!]_{\lambda^\infty}$

$\lambda f.\,\mathsf{let}\ r = f\ r\ \mathsf{in}\ r$

$[\![\cdot]\!]_{\mathcal{T}}$

readback

Maximal sharing: the method

$$\begin{tikzcd} L \arrow[r, mapsto, "\llbracket\cdot\rrbracket_{\mathcal{H}}"] & \mathcal{G} \end{tikzcd}$$

1. term graph interpretation $\llbracket\cdot\rrbracket$.
   of $\boldsymbol{\lambda}_{\mathsf{letrec}}$-term L as:
   a. higher-order term graph
      $\mathcal{G} = \llbracket L\rrbracket_{\mathcal{H}}$

Maximal sharing: the method

$$\begin{tikzcd} L \arrow[r, mapsto, "⟦\cdot⟧_{\mathcal{H}}"] & \mathcal{G} \arrow[r, mapsto] & G \end{tikzcd}$$

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$$\begin{tikzcd} L \arrow[r, mapsto, "\llbracket \cdot \rrbracket_{\mathcal{H}}"] \arrow[rr, mapsto, bend left, "\llbracket \cdot \rrbracket_{\mathcal{T}}"] & \mathcal{G} \arrow[r, mapsto] & G \end{tikzcd}$$

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2. bisimulation collapse $|\!\!\downarrow$
   of f-o term graph G into G_0

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Maximal sharing: the method

$$\begin{tikzcd}
L \arrow[r, mapsto, "{[\![\cdot]\!]_{\mathcal{H}}}"] \arrow[rr, mapsto, bend left=40, "{[\![\cdot]\!]_{\mathcal{T}}}"] & \mathcal{G} \arrow[r, mapsto] \arrow[d, Rrightarrow] & G \arrow[d, Rrightarrow] \\
L_0 & \mathcal{G}_0 \arrow[r, mapsto] & G_0 \arrow[ll, mapsto, bend left=40, "\text{rb}"]
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   of f-o term graph G_0
   yielding program $L_0 = \text{rb}(G_0)$.

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Interpretation

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Running example

instead of:

$\lambda f.\,\mathsf{let}\ r = f\,(f\,r)\ \mathsf{in}\ r \quad \longmapsto_{\text{max-sharing}} \quad \lambda f.\,\mathsf{let}\ r = f\,r\ \mathsf{in}\ r$

we use:

$\lambda x.\,\lambda f.\,\mathsf{let}\ r = f\,(f\,r\,x)\,x\ \mathsf{in}\ r \quad \longmapsto_{\text{max-sharing}} \quad \lambda x.\,\lambda f.\,\mathsf{let}\ r = f\,r\,x\ \mathsf{in}\ r$

$L \quad \longmapsto_{\text{max-sharing}} \quad L_0$

Graph interpretation (example 1)

$L_0 = \lambda x.\, \lambda f.\, \text{let } r = f\, r\, x \text{ in } r$

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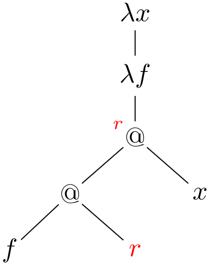

syntax tree

Graph interpretation (example 1)

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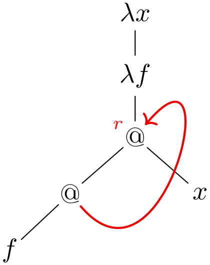

syntax tree (+ recursive backlink)

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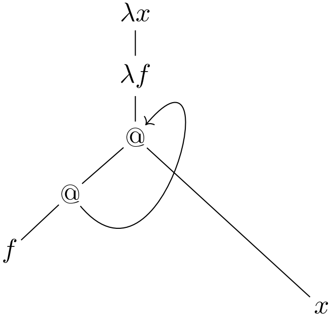

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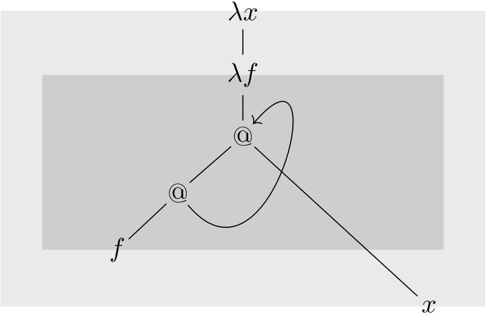

syntax tree (+ recursive backlink, + scopes)

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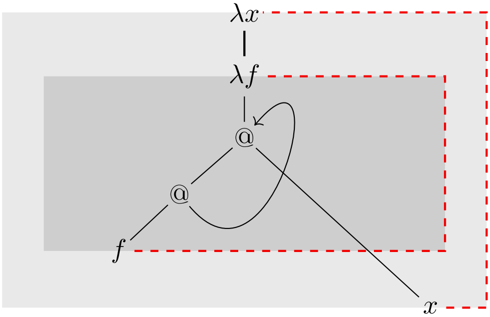

syntax tree (+ recursive backlink, + scopes, + binding links)

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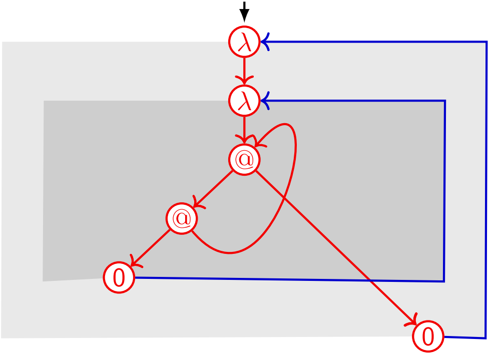

first-order term graph with binding backlinks (+ scope sets)

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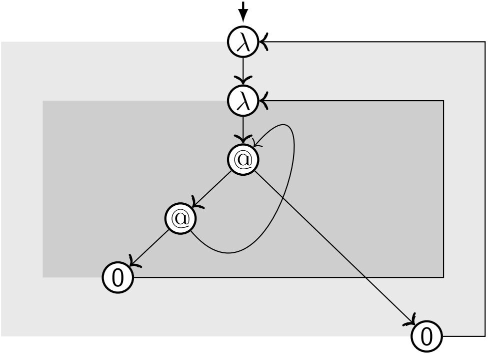

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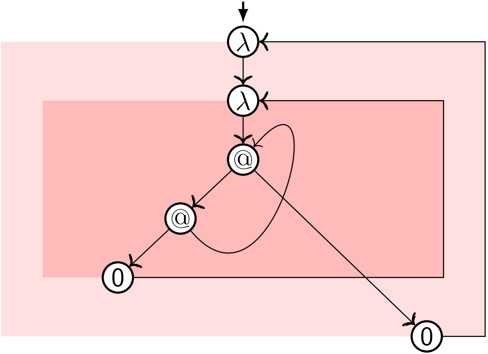

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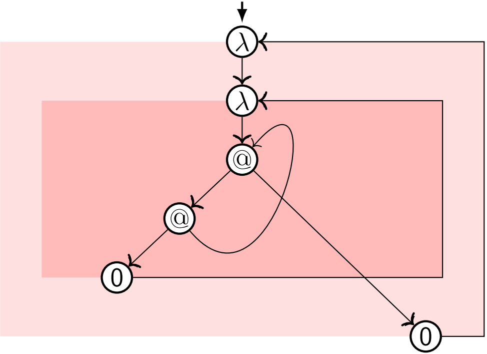

higher-order term graph (with scope sets, Blom [2003])

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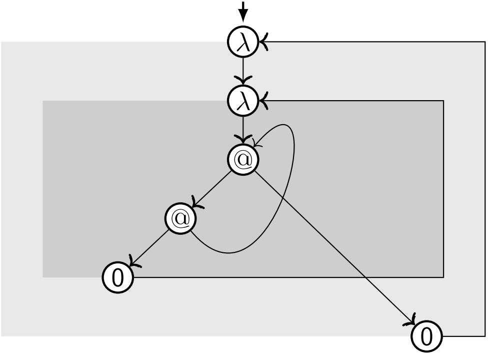

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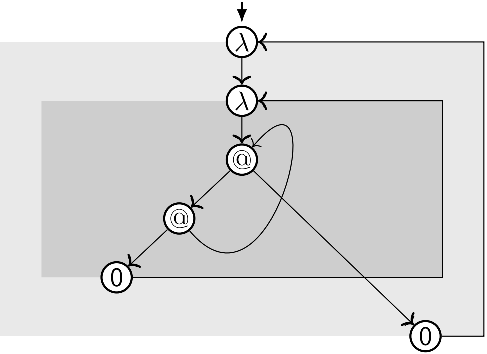

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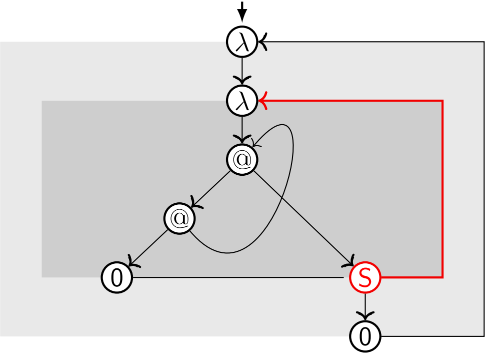

first-order term graph with scope vertices with backlinks (+ scope sets)

Graph interpretation (example 1)

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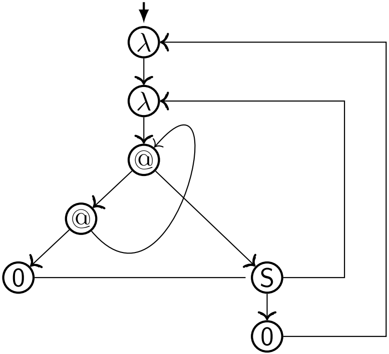

first-order term graph with scope vertices with backlinks

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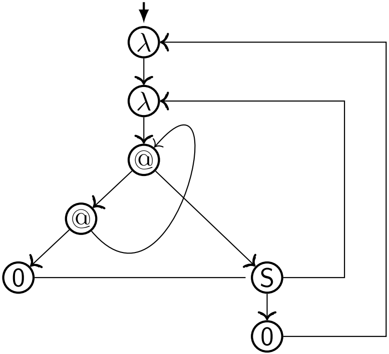

λ-term-graph $[\![L_0]\!]_{\mathcal{T}}$

Graph interpretation (example 1)

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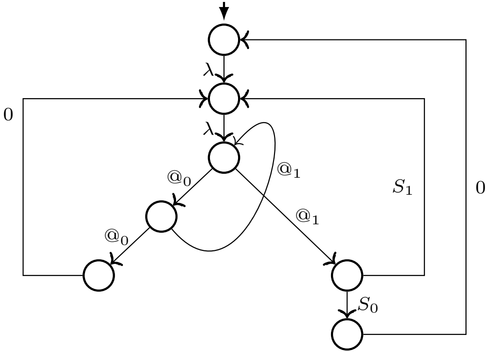

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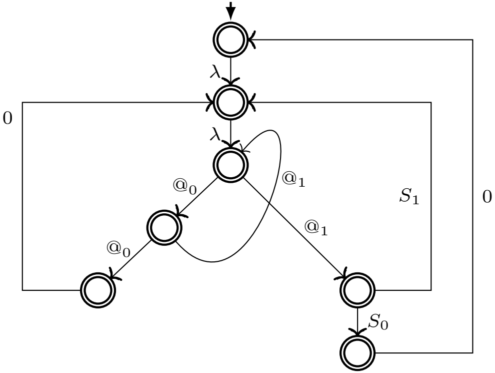

λ-NFA

Graph interpretation (example 1)

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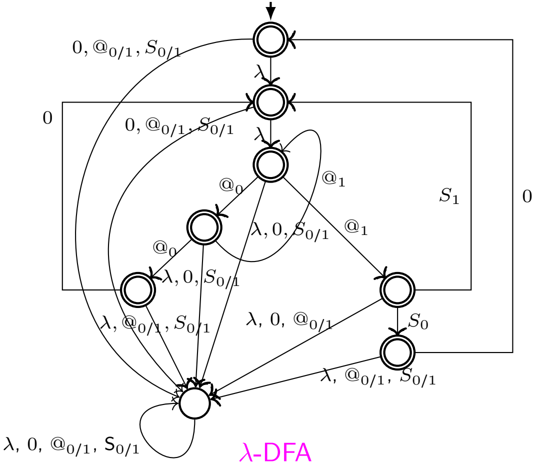

λ-DFA

Graph interpretation (example 2)

$L = \lambda x.\, \lambda f.\, \text{let } r = f\,(f\,r\,x)\,x \text{ in } r$

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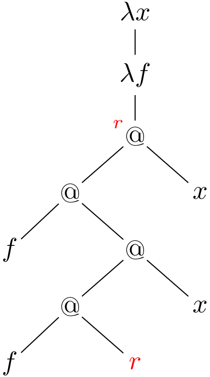

syntax tree

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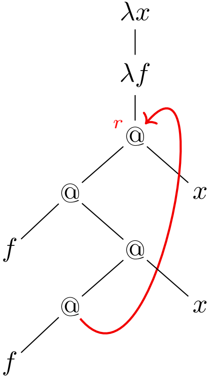

syntax tree (+ recursive backlink)

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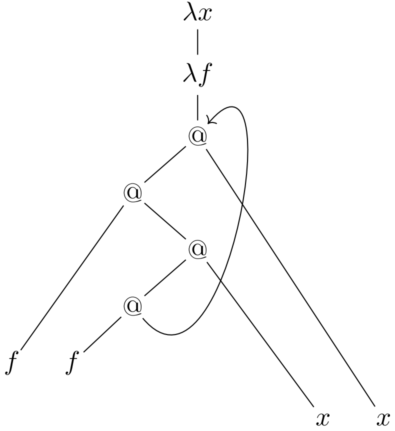

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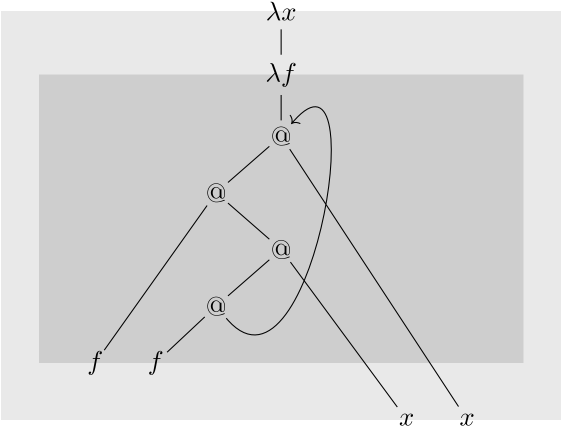

syntax tree (+ recursive backlink, + scopes)

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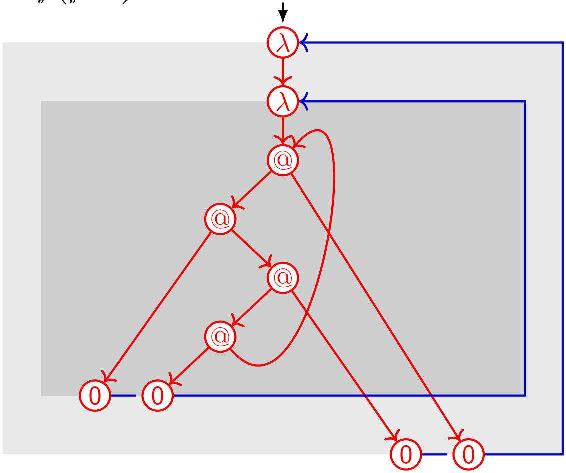

first-order term graph with binding backlinks (+ scope sets)

Graph interpretation (example 2)

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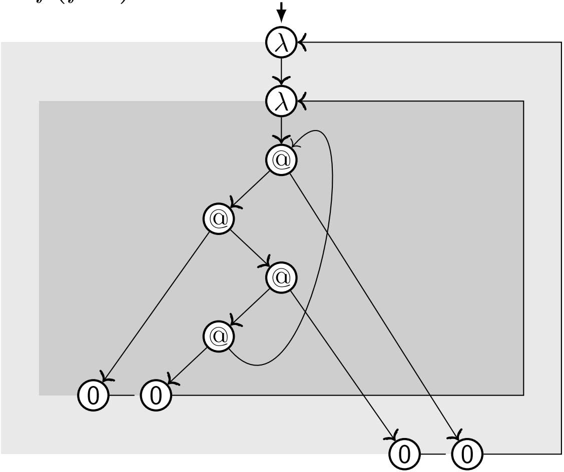

λ-higher-order-term-graph $[\![L]\!]_{\mathcal{H}}$

Graph interpretation (example 2)

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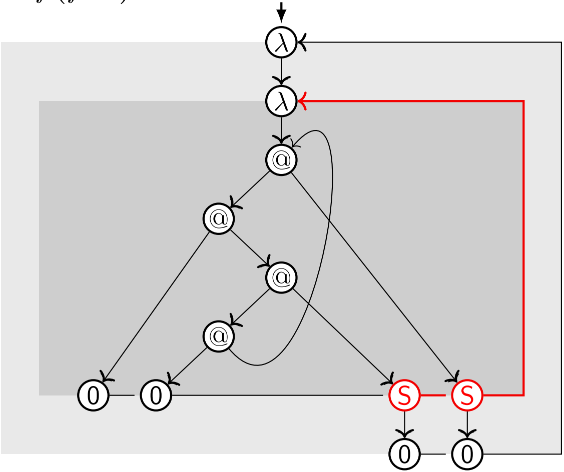

first-order term graph with scope vertices with backlinks (+ scope sets)

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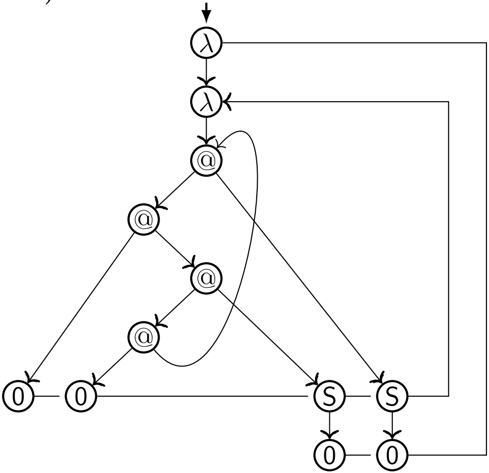
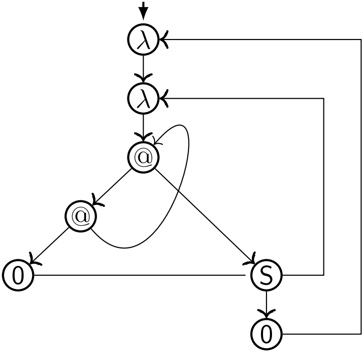
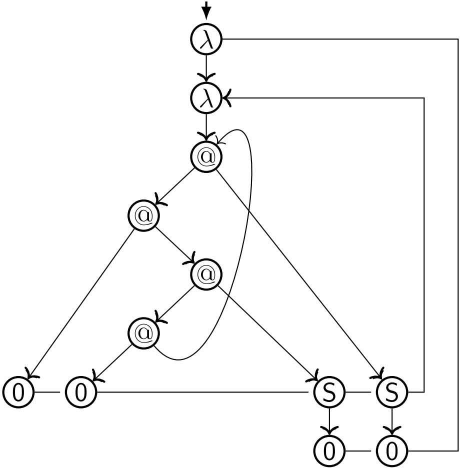


λ-term-graph $[\![L]\!]_{\mathcal{T}}$

Graph interpretation (examples 1 and 2)





$[\![L_0]\!]_{\mathcal{T}}$

$[\![L]\!]_{\mathcal{T}}$

Interpretation $[\![\cdot]\!]_{\mathcal{T}}$: properties (cont.)

interpretation $\boldsymbol{\lambda}_{\text{letrec}}$-term $L \longmapsto \lambda$-term-graph $[\![L]\!]_{\mathcal{T}}$

- defined by induction on structure of L
- similar analysis as fully-lazy lambda-lifting
- yields eager-scope λ-term-graphs: $\sim$ minimal scopes

Theorem

For $\boldsymbol{\lambda}_{\text{letrec}}$-terms L_1 and L_2 it holds: Equality of infinite unfolding coincides with bisimilarity of λ-term-graph interpretations:

$$[\![L_1]\!]_{\lambda^\infty} = [\![L_2]\!]_{\lambda^\infty} \iff [\![L_1]\!]_{\mathcal{T}} \leftrightarrows [\![L_2]\!]_{\mathcal{T}}$$

Interpretation $[\![\cdot]\!]_{\mathcal{T}}$: properties (cont.)

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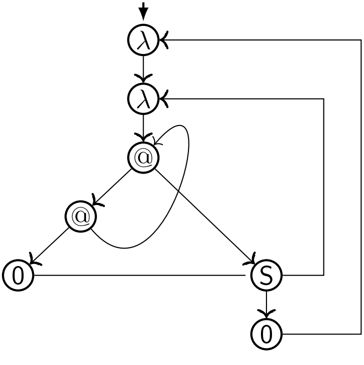
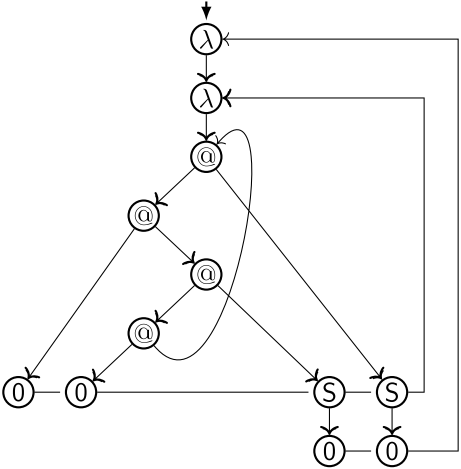
$$[\![L_1]\!]_{\lambda^\infty} = [\![L_2]\!]_{\lambda^\infty} \quad\Longleftrightarrow\quad [\![L_1]\!]_{\mathcal{T}} \leftrightarrows [\![L_2]\!]_{\mathcal{T}}$$

Collapse

$$\begin{tikzcd}
L \arrow[r, mapsto, bend left, "\text{interpret}"] & G \arrow[d, Rightarrow, "\text{collapse}"] \\
L_0 & G_0 \arrow[l, mapsto, bend left, "\text{readback}"]
\end{tikzcd}$$

Bisimulation check between λ-term-graphs

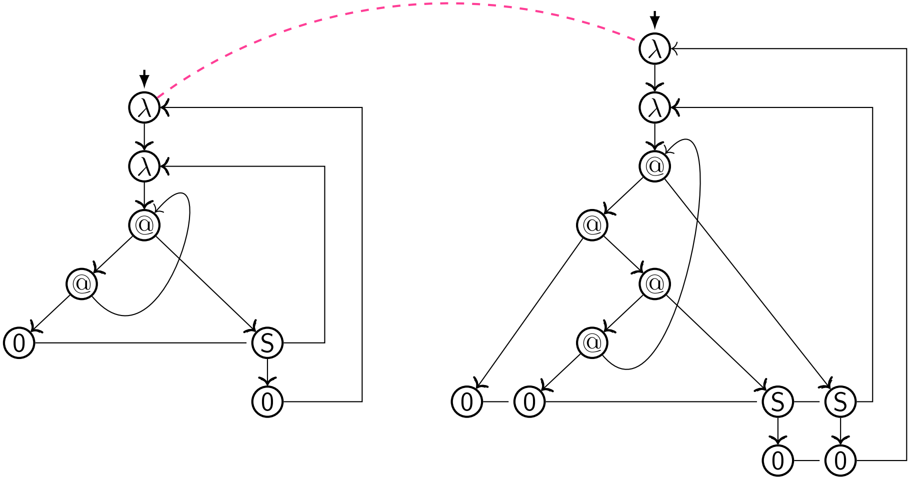




$[\![L_0]\!]_{\mathcal{T}}$

$[\![L]\!]_{\mathcal{T}}$

Bisimulation check between λ-term-graphs



$[\![L_0]\!]_{\mathcal{T}}$ $[\![L]\!]_{\mathcal{T}}$

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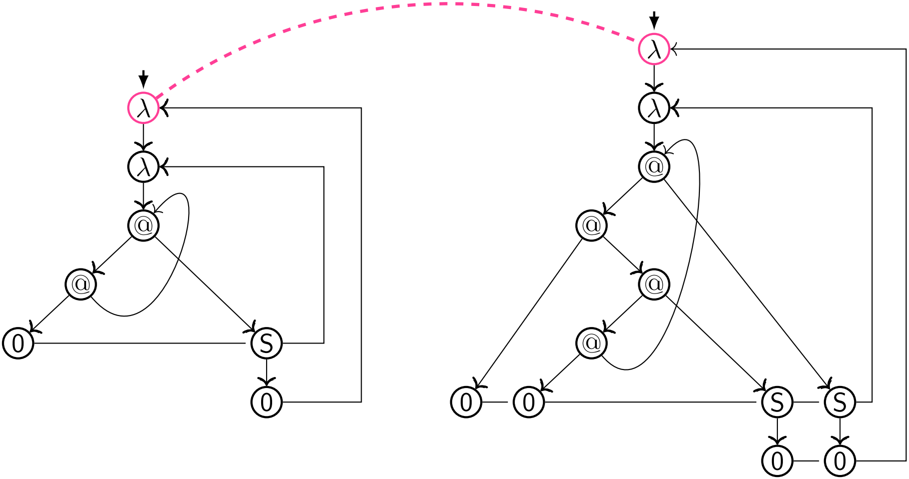

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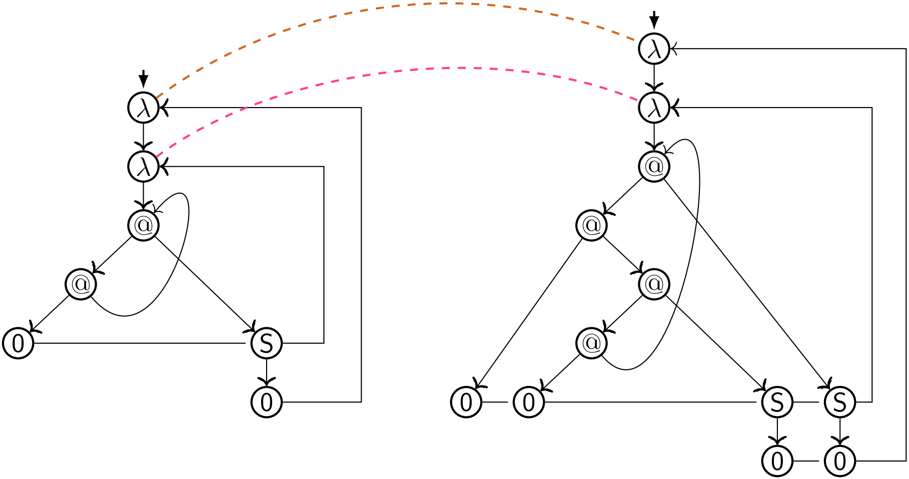

$[\![L_0]\!]_{\mathcal{T}}$ $[\![L]\!]_{\mathcal{T}}$

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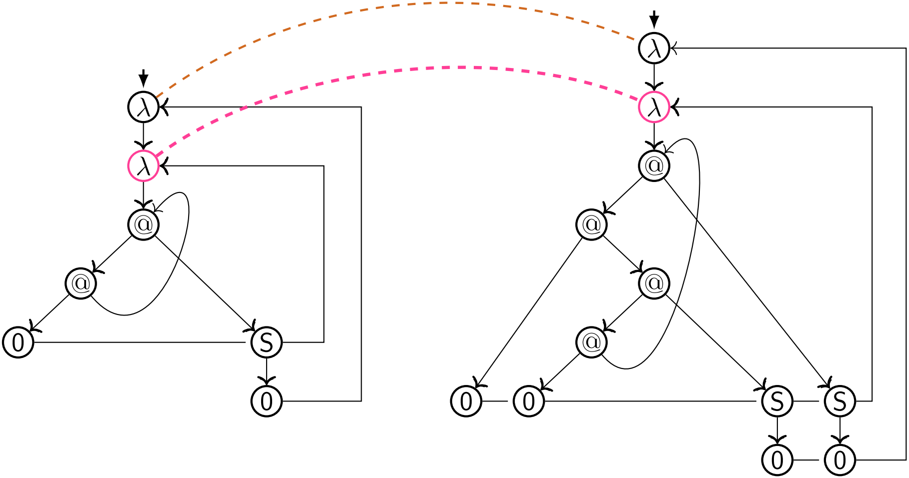

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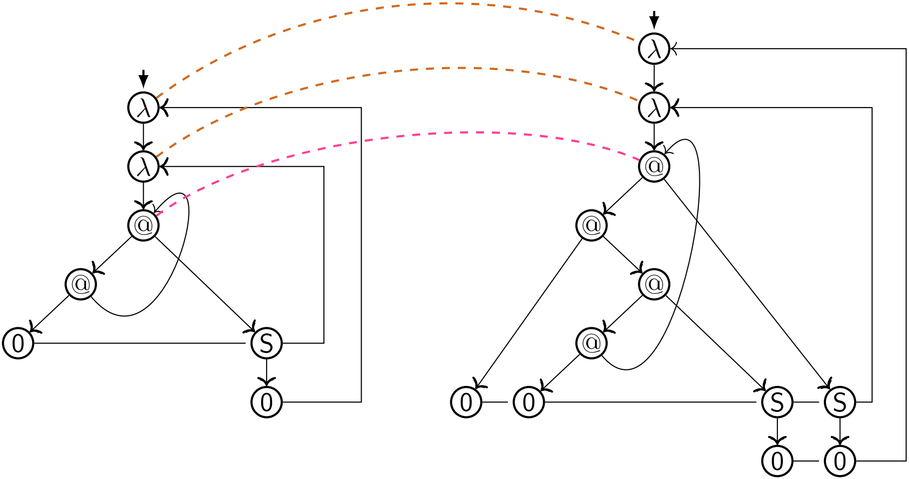

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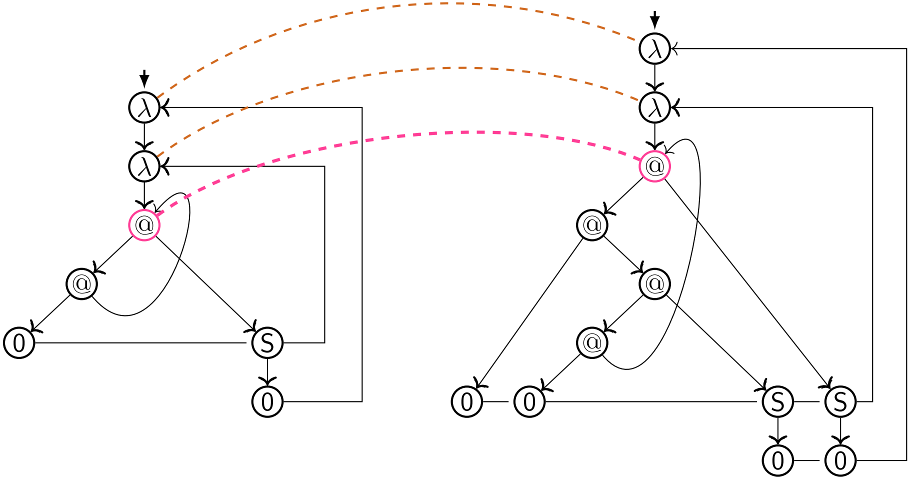

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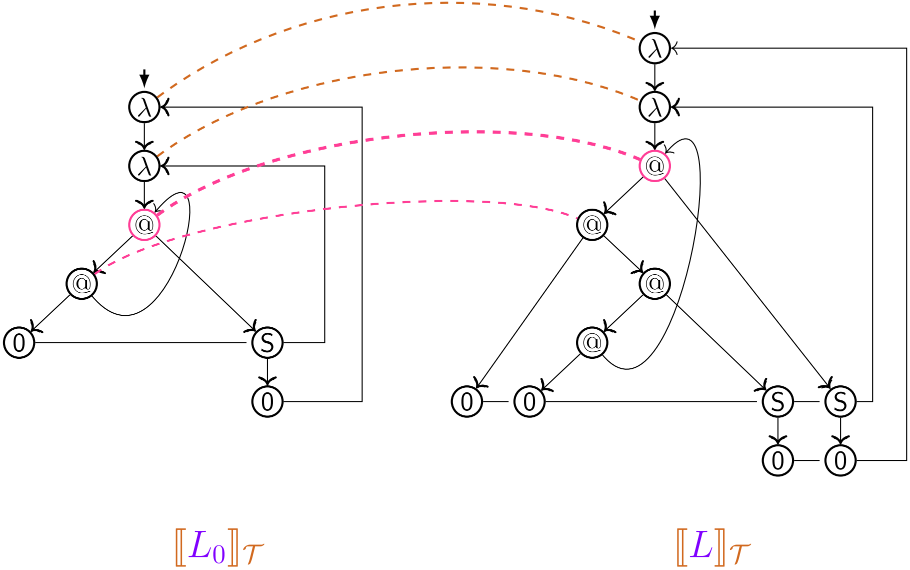

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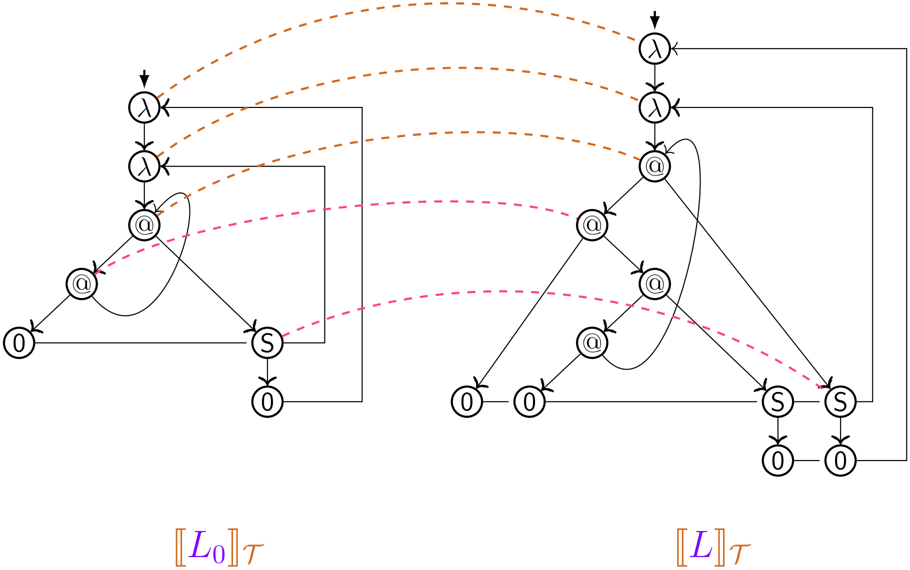

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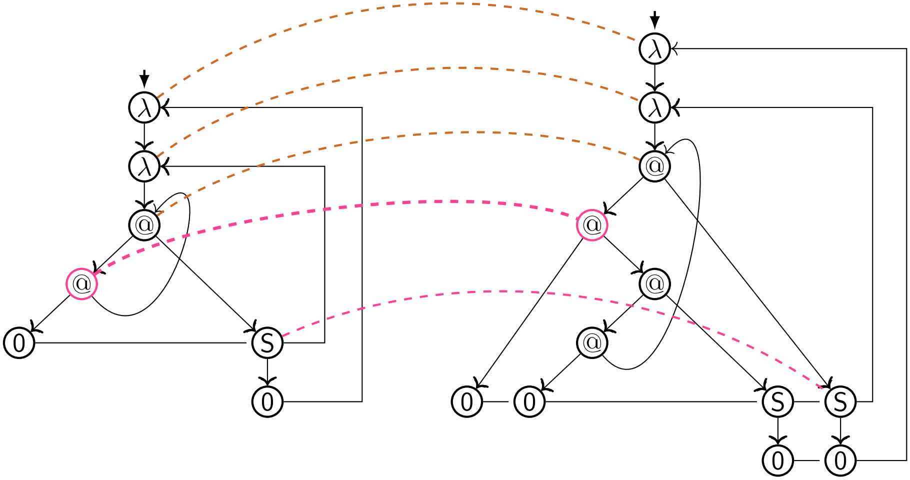

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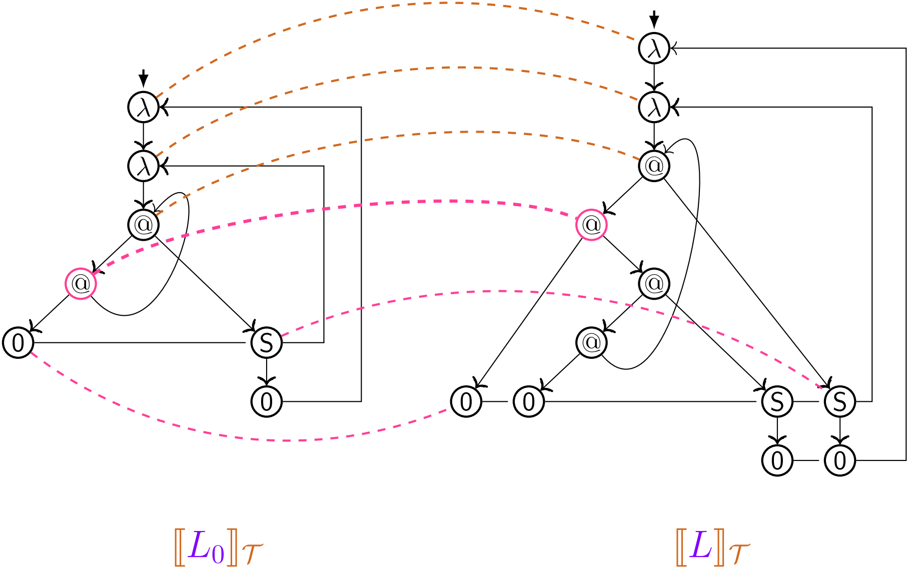

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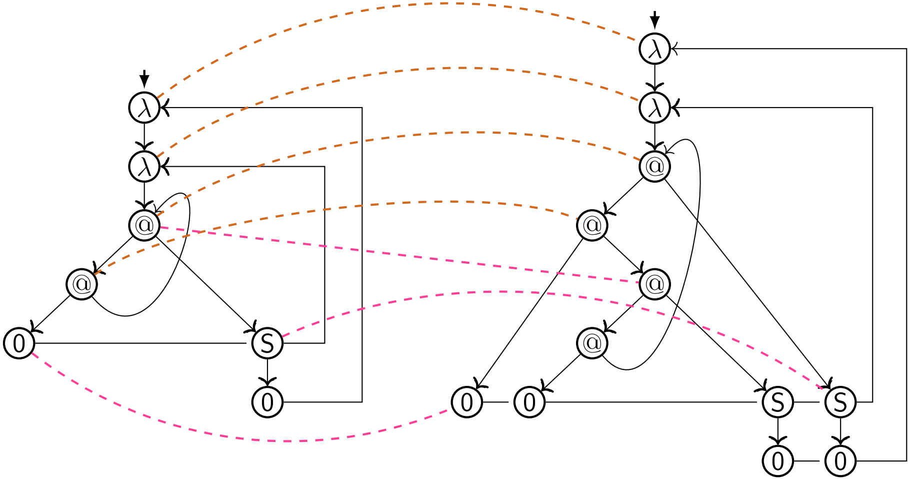

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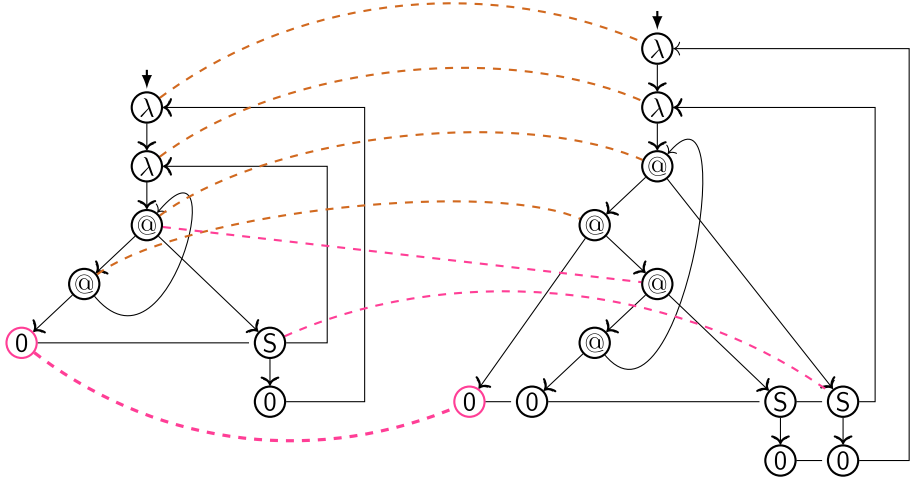

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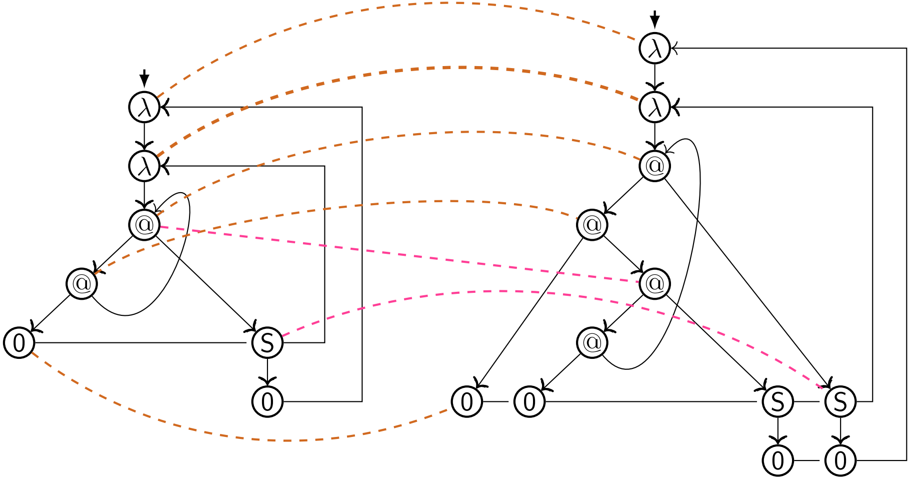

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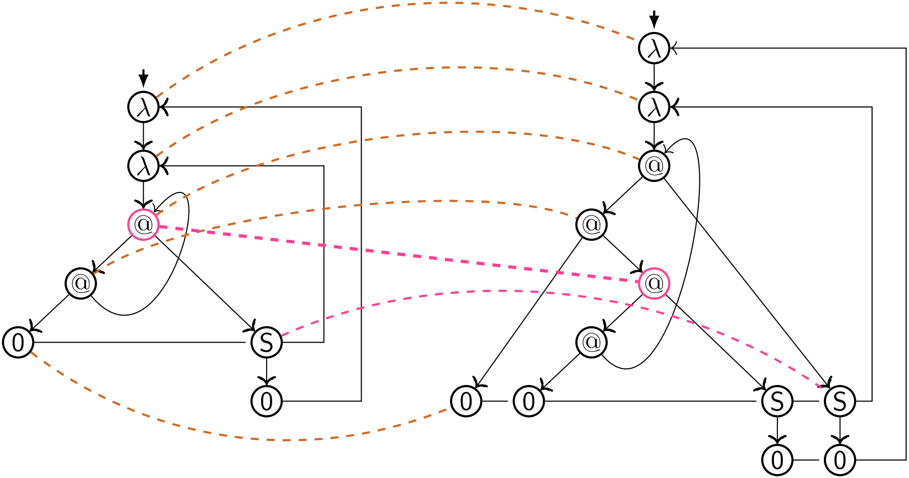

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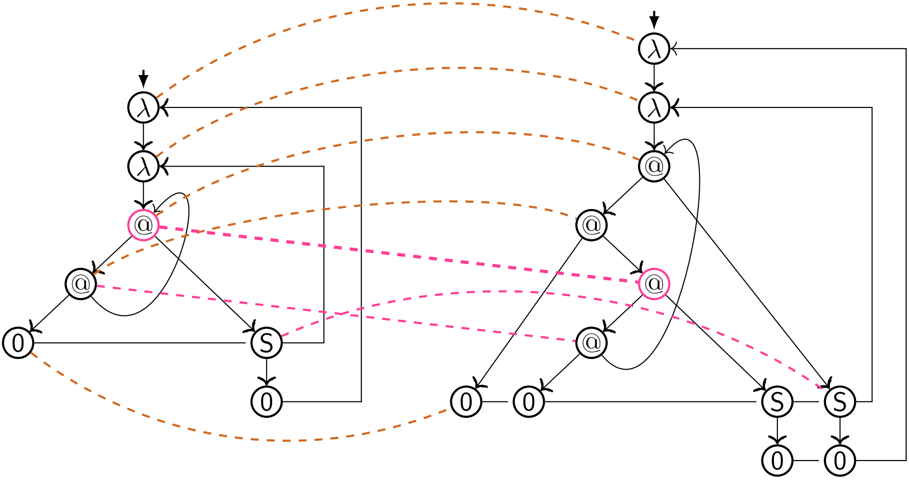

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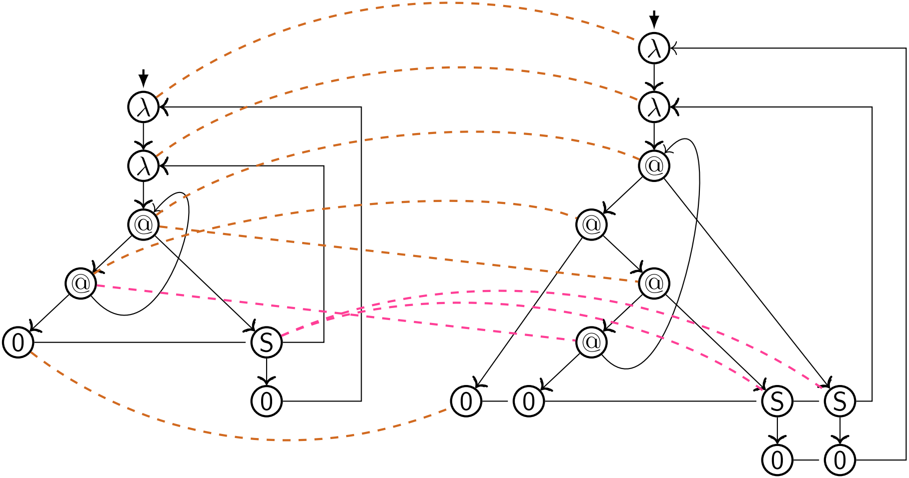

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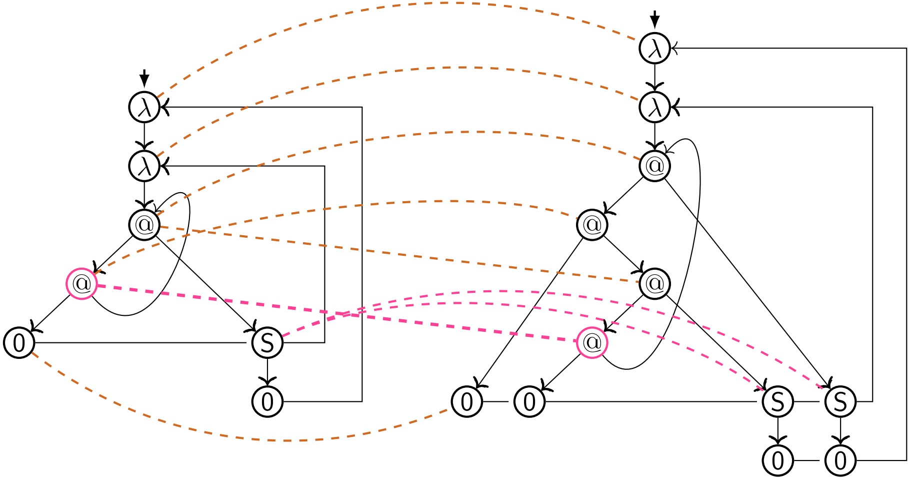

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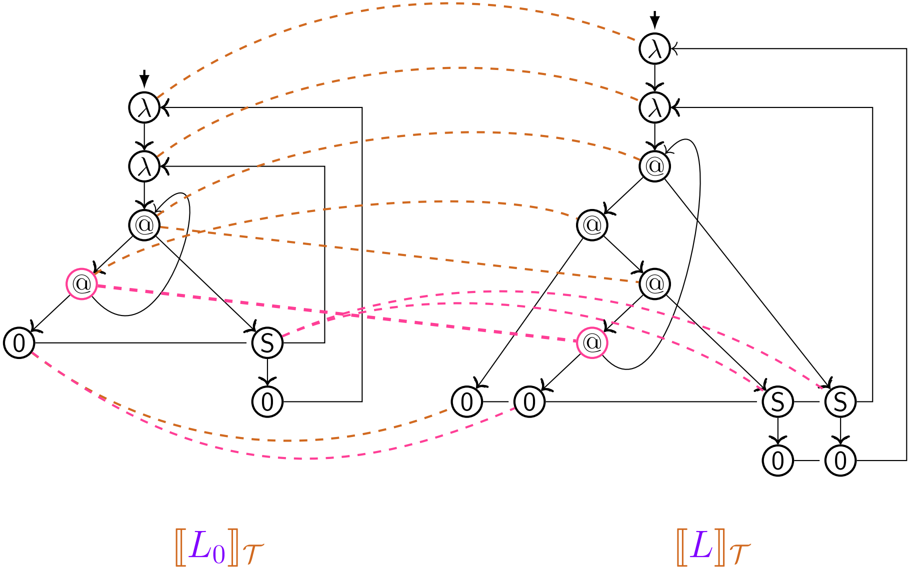

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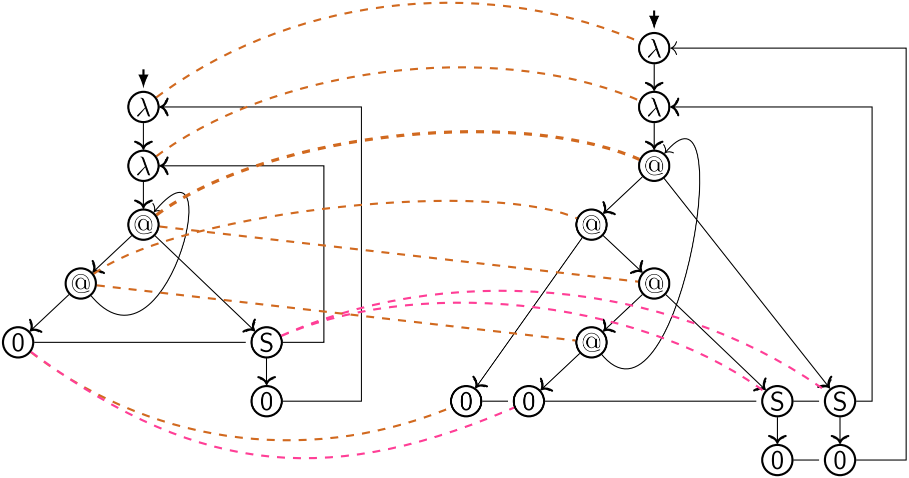

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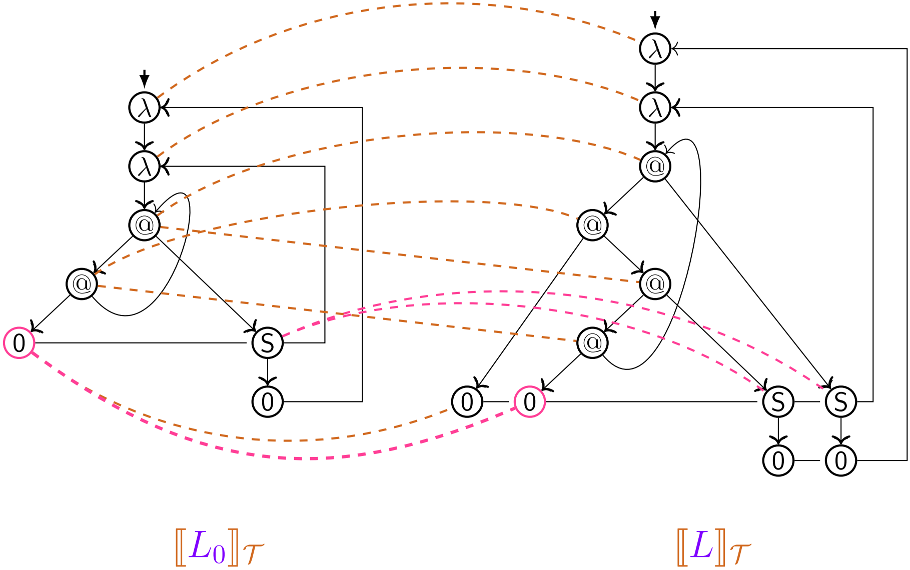

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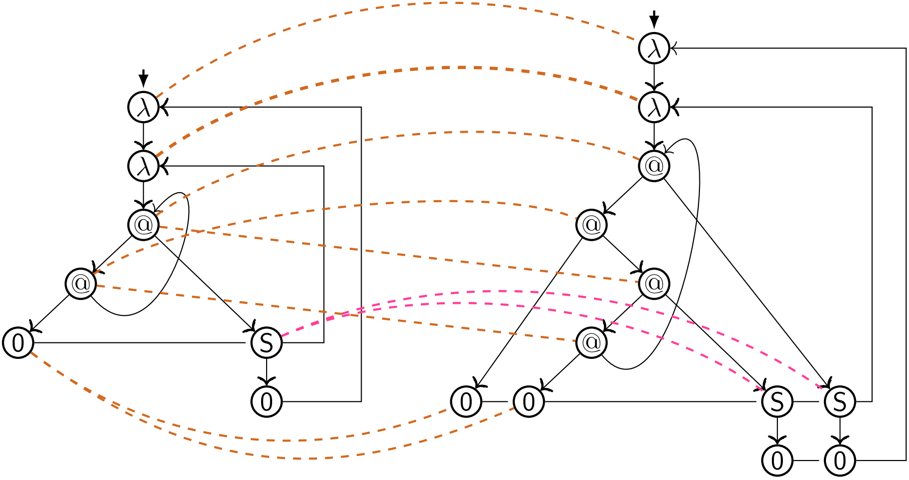

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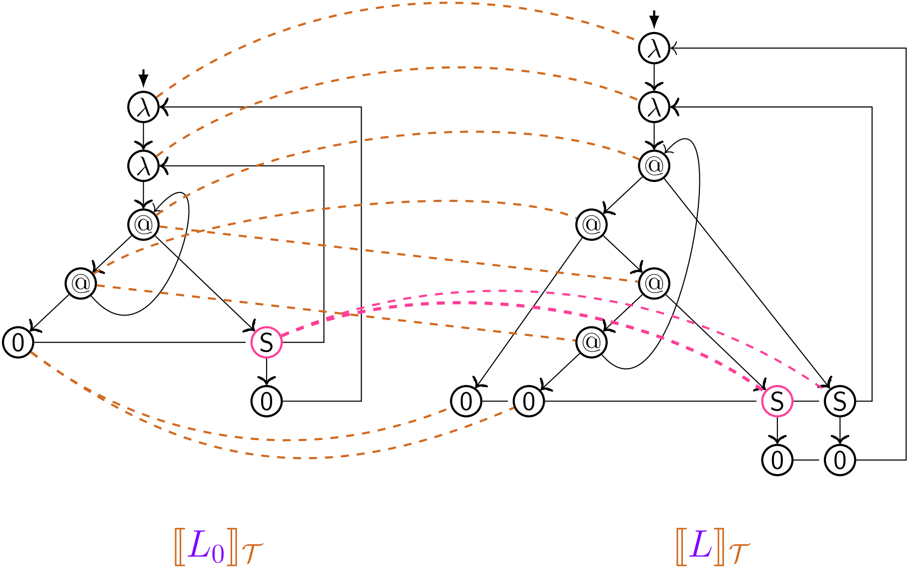

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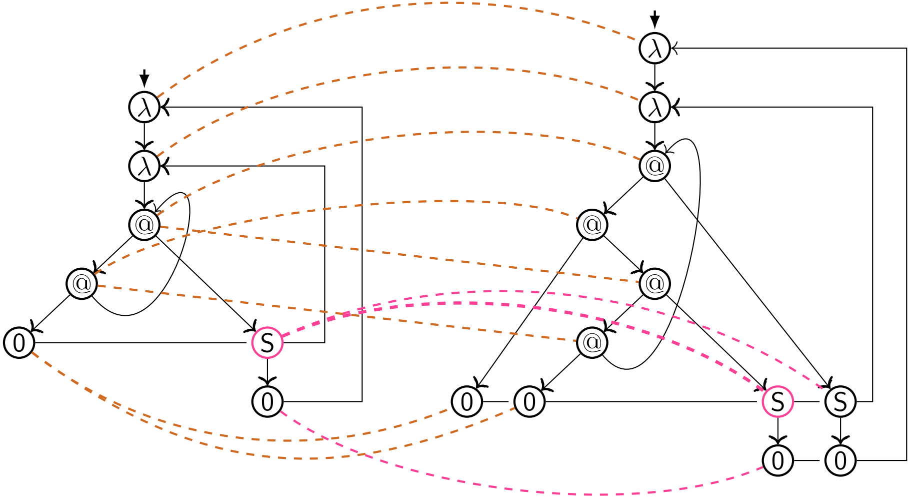

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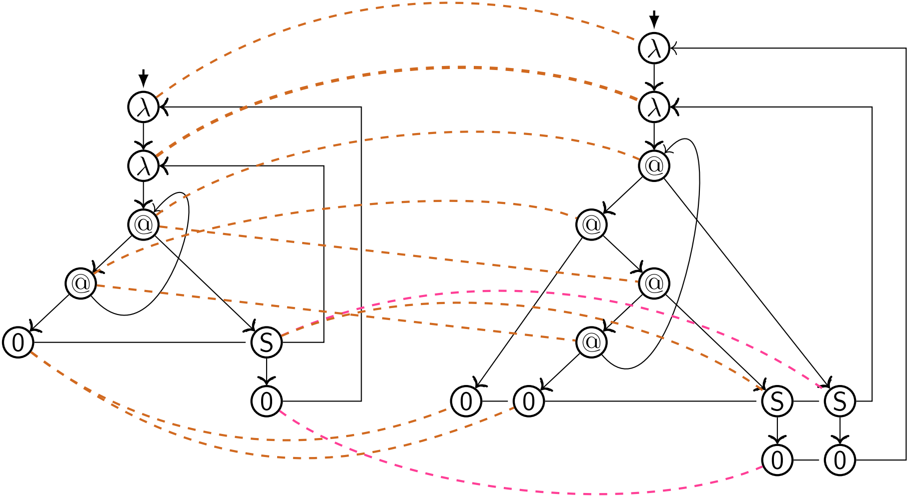

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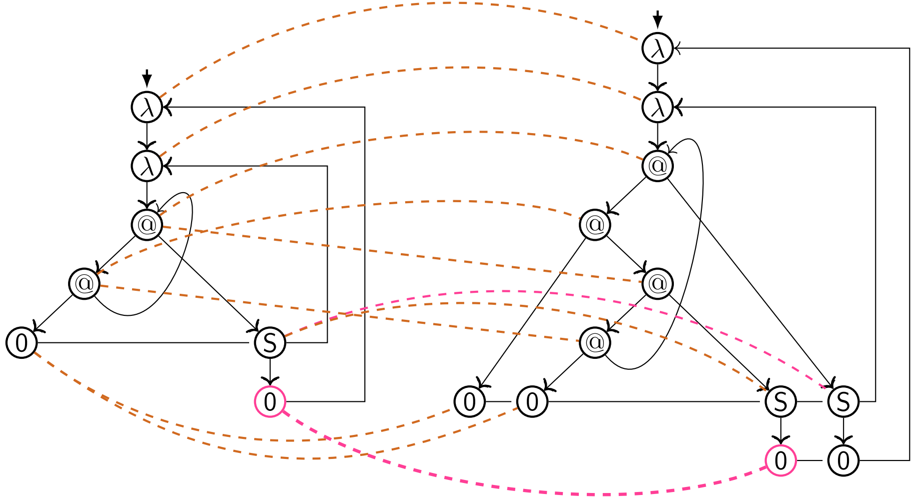

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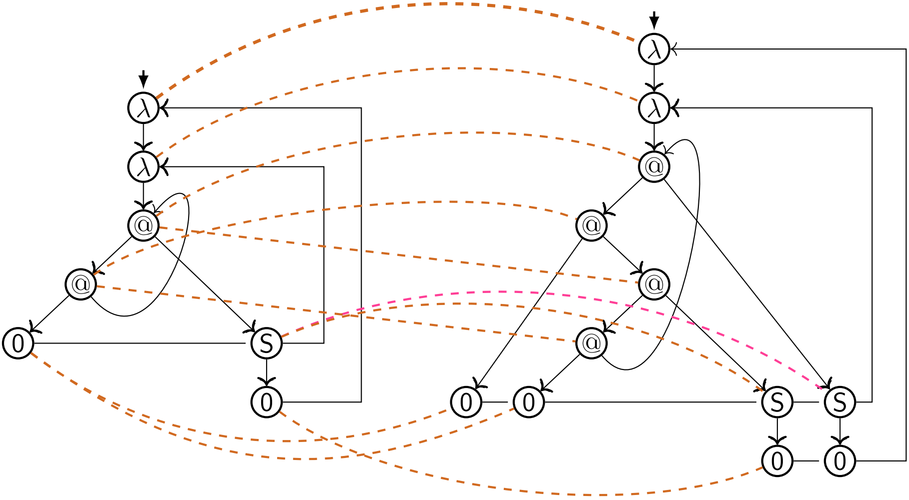

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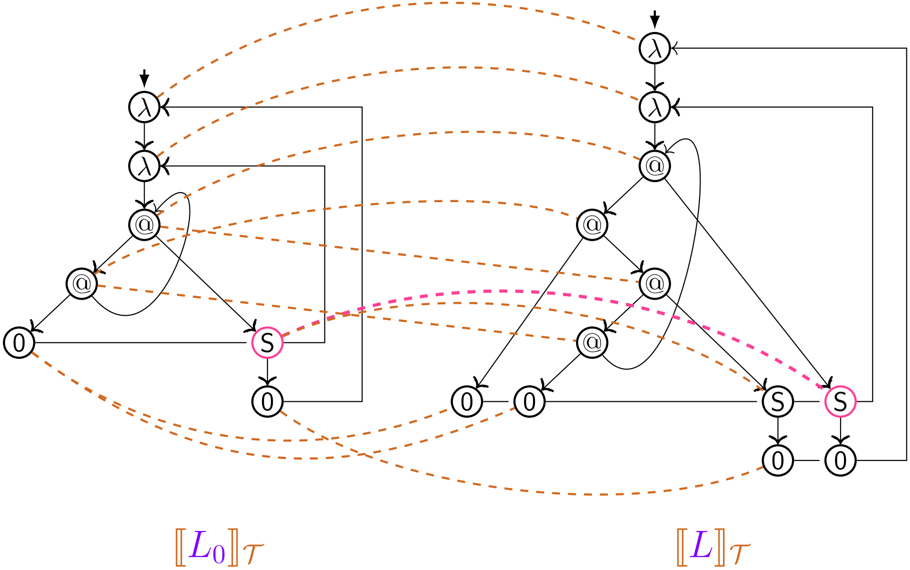

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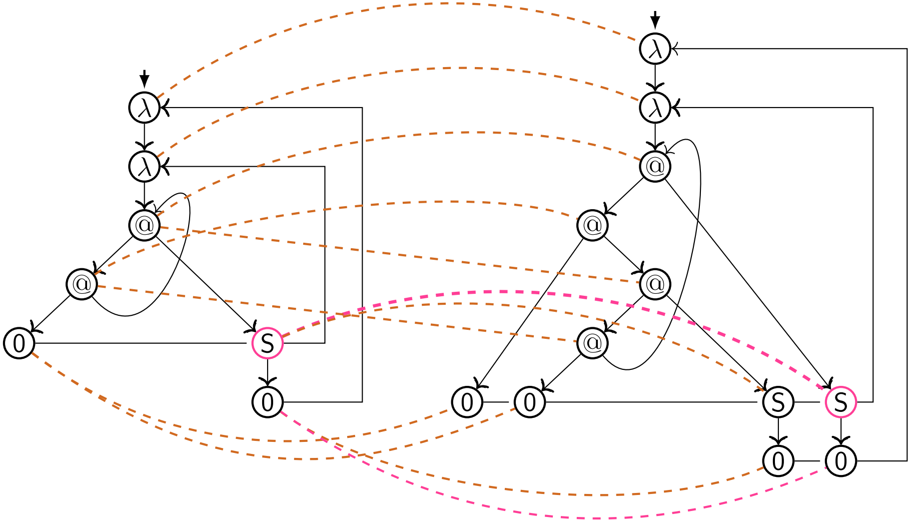

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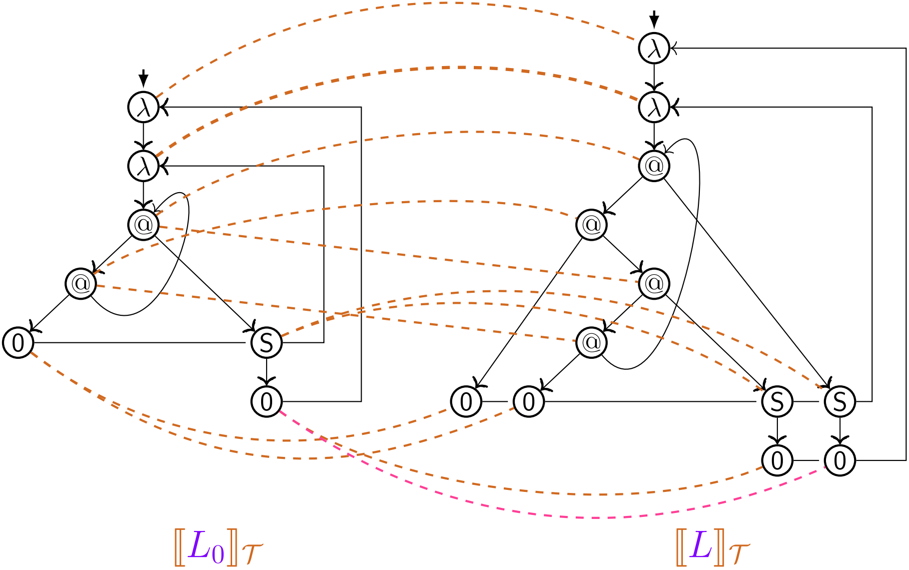

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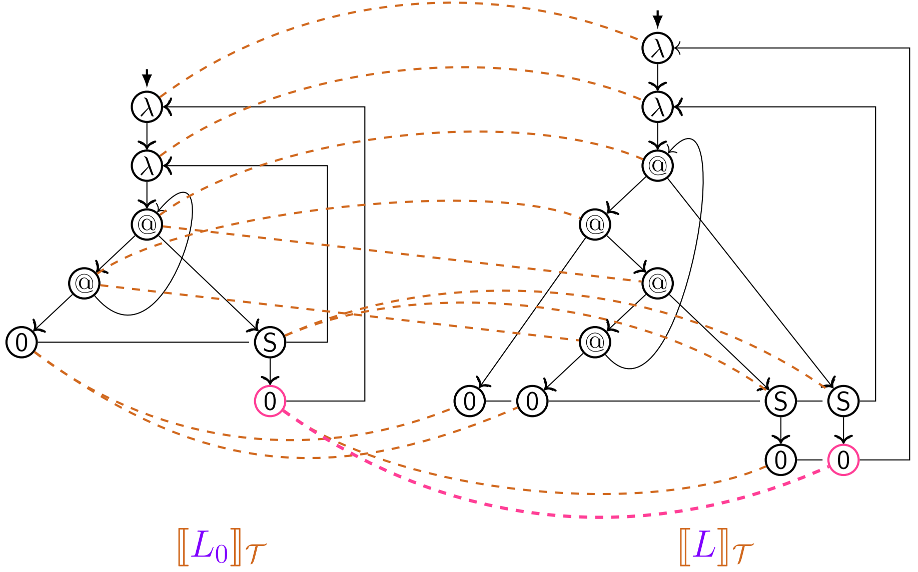

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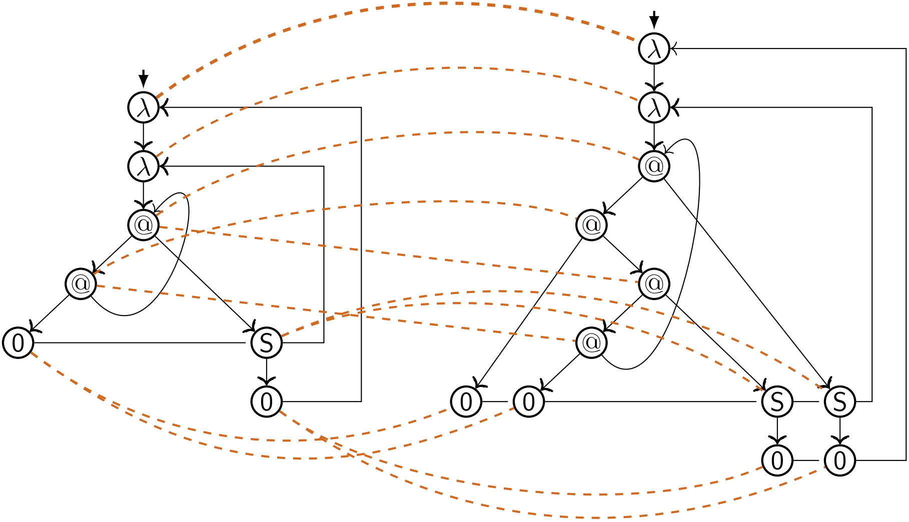
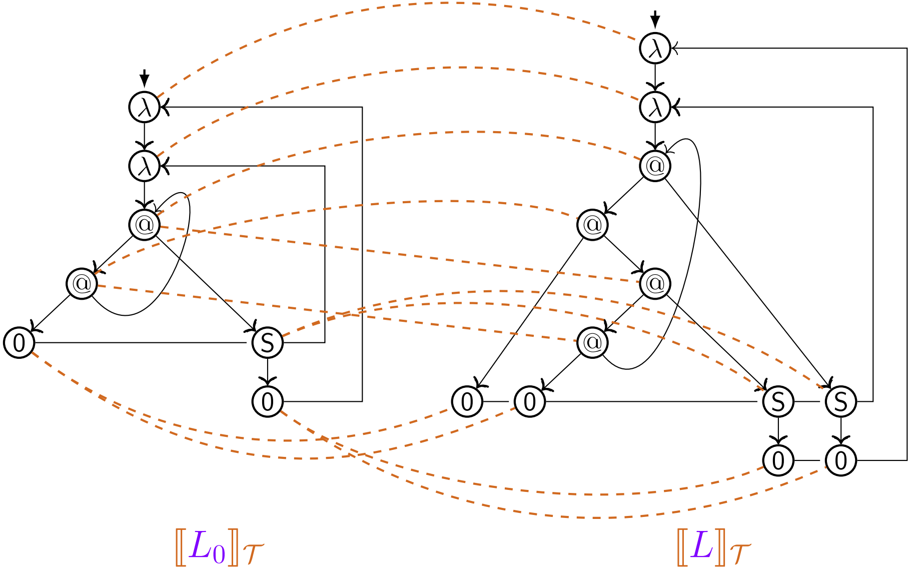
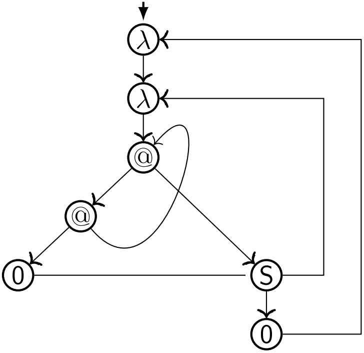


$[\![L_0]\!]_{\mathcal{T}}$ $[\![L]\!]_{\mathcal{T}}$

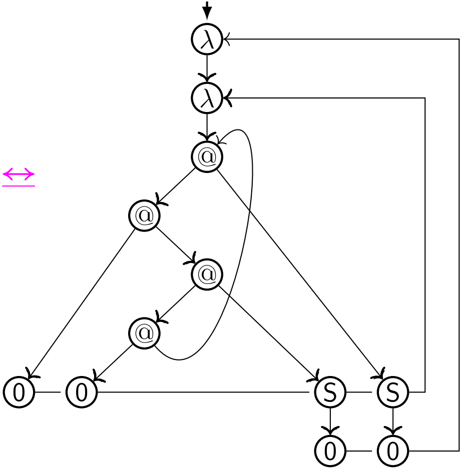
bisimulation between λ-term-graphs



bisimilarity between λ-term-graphs

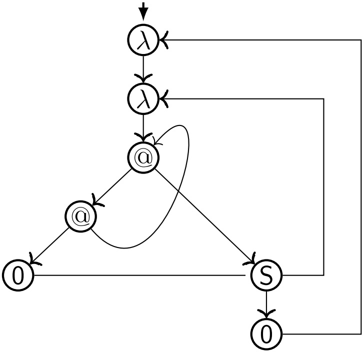


$\underline{\leftrightarrow}$

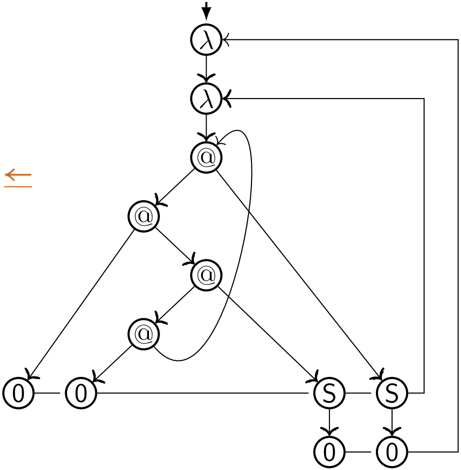


$$[\![L_0]\!]_{\mathcal{T}} \quad \underline{\leftrightarrow} \quad [\![L]\!]_{\mathcal{T}}$$

functional bisimilarity and bisimulation collapse



$\underline{\leftarrow}$



$[\![L_0]\!]_{\mathcal{T}}$ $\underline{\leftarrow}$ $[\![L]\!]_{\mathcal{T}}$

Bisimulation collapse: property

Theorem

The class of eager-scope λ-term-graphs
is closed under functional bisimilarity $\mathrel{\underline{\rightarrow}}$.

$\Longrightarrow$ For a $\boldsymbol{\lambda}_{\mathsf{letrec}}$-term L

the bisimulation collapse of $[\![L]\!]_{\mathcal{T}}$ is again an eager-scope λ-term-graph.

Readback

$$\begin{tikzcd}
L \arrow[r, mapsto, bend left, "\text{interpret}"] & G \arrow[d, Rightarrow, "\text{collapse}"] \\
L_0 & G_0 \arrow[l, mapsto, bend left, "\text{readback}"]
\end{tikzcd}$$

Readback

defined with property:

$$\begin{tikzcd} L & G \text{ eager-scope} \arrow[l, bend left, mapsto, "\text{rb}"] \end{tikzcd}$$

Readback

defined with property:

$$\begin{tikzcd} L \arrow[r, bend left, mapsto, "\llbracket \cdot \rrbracket_{\mathcal{T}}"] & G \ \text{eager-scope} \arrow[l, bend left, mapsto, "\text{rb}"] \end{tikzcd}$$

Readback

defined with property:

$$\begin{tikzcd} L \arrow[r, mapsto, bend left, "{[\![\cdot]\!]_{\mathcal{T}}}"] & G \arrow[l, mapsto, bend left, "\mathsf{rb}"] \ \text{eager-scope} \end{tikzcd}$$

Theorem

For all eager-scope λ-term-graphs G:

$$([\![\cdot]\!]_{\mathcal{T}} \circ \mathsf{rb})(G) \simeq G$$

The readback rb *is a right-inverse of $[\![\cdot]\!]_{\mathcal{T}}$ modulo isomorphism $\simeq$.*

Readback

defined with property:

$$\begin{tikzcd} L \arrow[r, mapsto, bend left, "{[\![\cdot]\!]_{\mathcal{T}}}"] & G \arrow[l, mapsto, bend left, "\mathsf{rb}"] & \text{eager-scope} \end{tikzcd}$$

Theorem

For all eager-scope λ-term-graphs G:

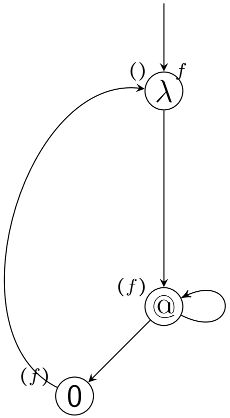
$$([\![\cdot]\!]_{\mathcal{T}} \circ \mathsf{rb})(G) \simeq G$$

The readback rb *is a right-inverse of* $[\![\cdot]\!]_{\mathcal{T}}$ *modulo isomorphism* $\simeq$.

idea:

1. construct a spanning tree T of G
2. using local rules, in a bottom-up traversal of T synthesize $L = \mathsf{rb}(G)$

Readback: example (fix)



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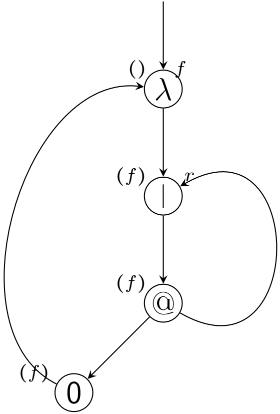

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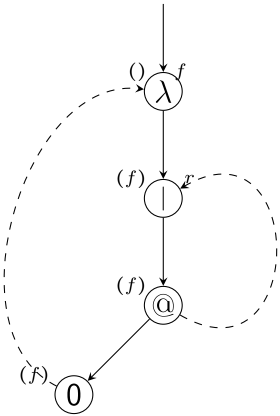

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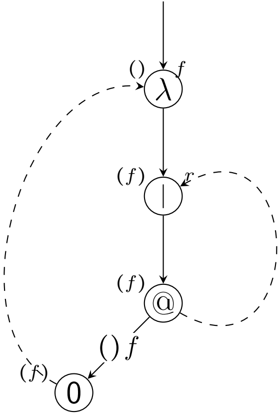

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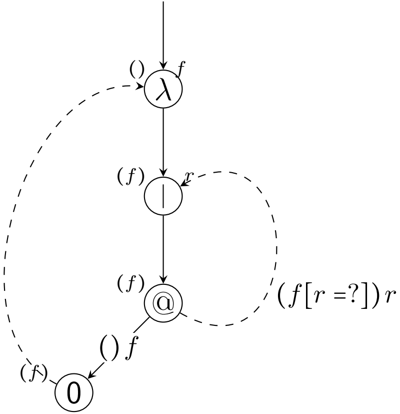

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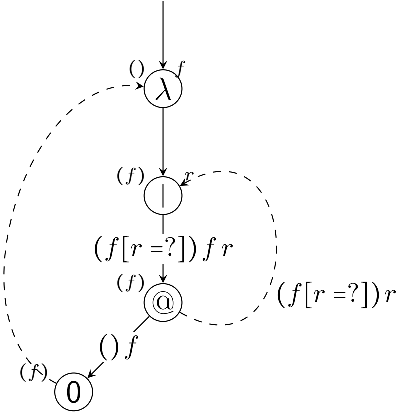

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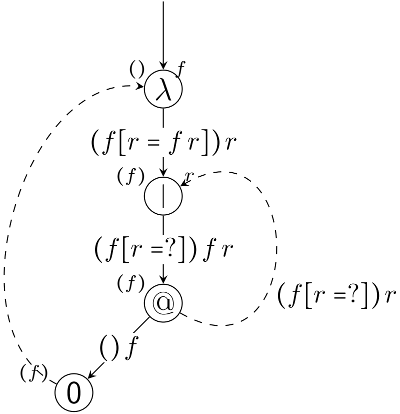

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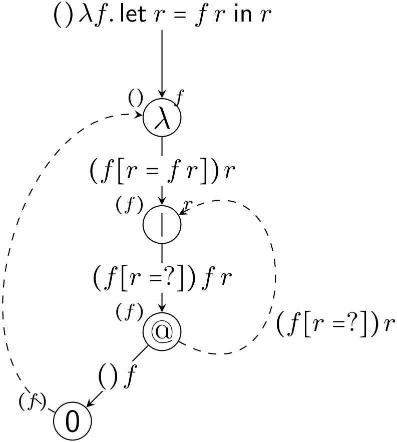

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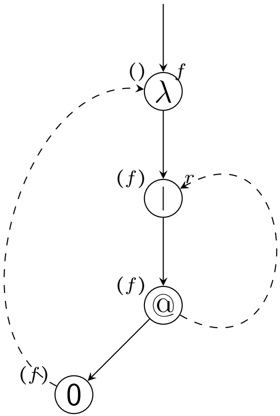

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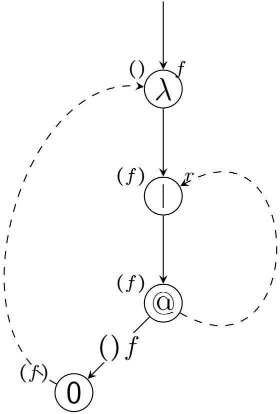

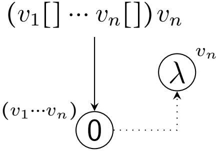

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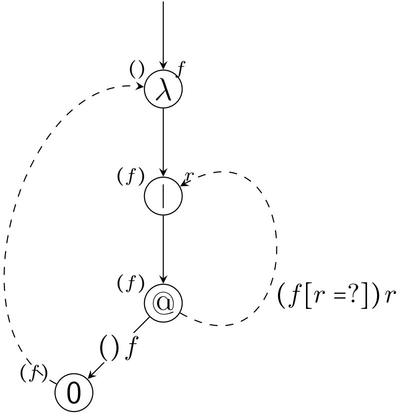
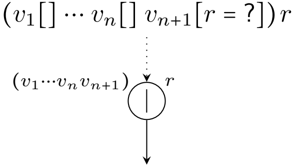
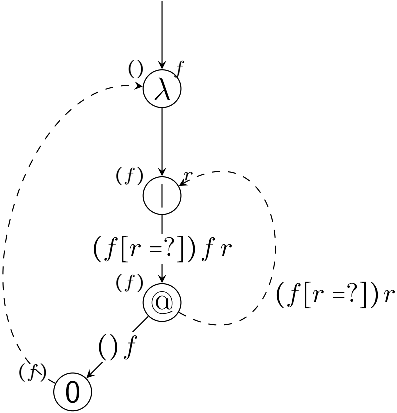
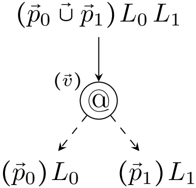


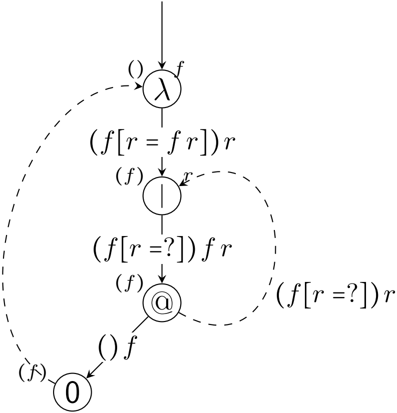

$(v_1[]\cdots v_n[]\; v_{n+1}[r = ?])\,r$
$(v_1\cdots v_n v_{n+1})$ r
$|$

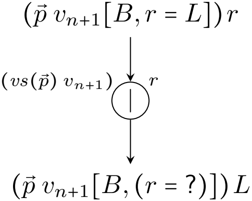
Readback: example (fix)





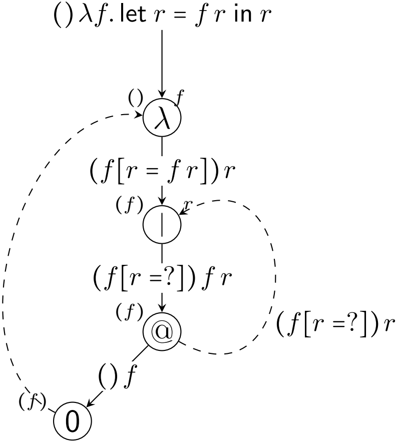
Readback: example (fix)

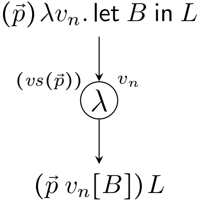




$(\vec{p}\;v_{n+1}[B, r = L])\,r$
$(vs(\vec{p})\;v_{n+1})$
r
$|$
$(\vec{p}\;v_{n+1}[B, (r = ?)])\,L$

Readback: example (fix)





Maximal sharing: complexity

$$\begin{tikzcd}
L \arrow[r, mapsto, bend left, "{[\![\cdot]\!]_{\mathcal{T}}}"] & G \arrow[d, "|\!\!\downarrow"] \\
L_0 & G_0 \arrow[l, mapsto, bend left, "\mathsf{rb}"]
\end{tikzcd}$$

1. interpretation

   of $\boldsymbol{\lambda}_{\mathsf{letrec}}$-term L with $|L| = n$

   as λ-term-graph $G = [\![L]\!]_{\mathcal{T}}$

   - in time $O(n^2)$, size $|G| \in O(n^2)$.

2. bisimulation collapse $|\!\!\downarrow$

   of f-o term graph G into G_0

   - in time $O(|G| \log |G|) = O(n^2 \log n)$

3. readback rb

   of f-o term graph G_0

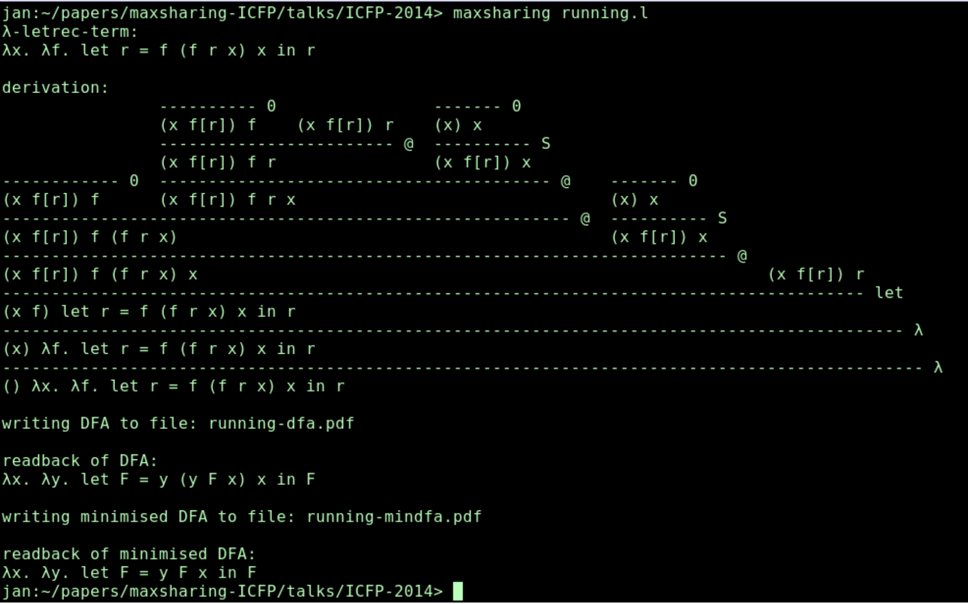
   yielding $\boldsymbol{\lambda}_{\mathsf{letrec}}$-term $L_0 = \mathsf{rb}(G_0)$.

   - in time $O(|G| \log |G|) = O(n^2 \log n)$

Theorem

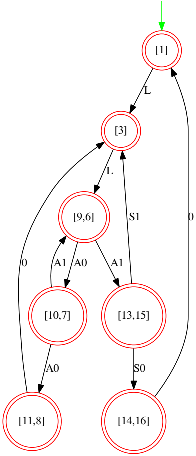
Computing a maximally compact form $L_0 = (\mathsf{rb} \circ |\!\!\downarrow \circ [\![\cdot]\!]_{\mathcal{T}})(L)$ of L for a $\boldsymbol{\lambda}_{\mathsf{letrec}}$-term L requires time $O(n^2 \log n)$, where $|L| = n$.

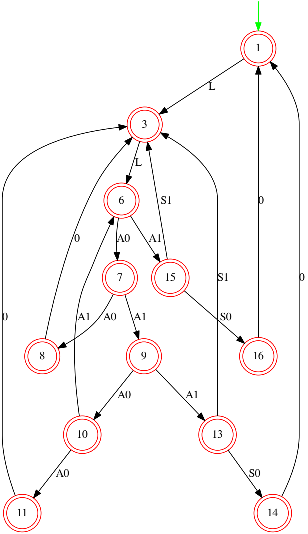
Demo: console output



```
jan:~/papers/maxsharing-ICFP/talks/ICFP-2014> maxsharing running.l
λ-letrec-term:
λx. λf. let r = f (f r x) x in r

derivation:
                ---------- 0                ------- 0
                (x f[r]) f    (x f[r]) r    (x) x
                ------------------------ @  ---------- S
                (x f[r]) f r                (x f[r]) x
------------ 0  ---------------------------------------- @    ------- 0
(x f[r]) f      (x f[r]) f r x                                (x) x
---------------------------------------------------------- @  ---------- S
(x f[r]) f (f r x)                                            (x f[r]) x
-------------------------------------------------------------------------- @
(x f[r]) f (f r x) x                                                          (x f[r]) r
---------------------------------------------------------------------------------------- let
(x f) let r = f (f r x) x in r
-------------------------------------------------------------------------------------------- λ
(x) λf. let r = f (f r x) x in r
---------------------------------------------------------------------------------------------- λ
() λx. λf. let r = f (f r x) x in r

writing DFA to file: running-dfa.pdf

readback of DFA:
λx. λy. let F = y (y F x) x in F

writing minimised DFA to file: running-mindfa.pdf

readback of minimised DFA:
λx. λy. let F = y F x in F
jan:~/papers/maxsharing-ICFP/talks/ICFP-2014> █
```

Demo: generated λ-NFAs



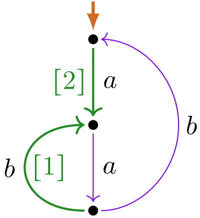


Resources (maximal sharing)

- tool maxsharing on `hackage.haskell.org`
- papers and reports
  - Maximal Sharing in the Lambda Calculus with Letrec
    - ICFP 2014 paper
    - accompanying report arXiv:1401.1460
  - Term Graph Representations for Cyclic Lambda Terms
    - TERMGRAPH 2013 proceedings
    - extended report arXiv:1308.1034
  - Vincent van Oostrom, CG: Nested Term Graphs
    - TERMGRAPH 2014 post-proceedings in EPTCS 183
- thesis Jan Rochel
  - Unfolding Semantics of the Untyped λ-Calculus with letrec
    - Ph.D. Thesis, Utrecht University, 2016

Process interpretation of regular expressions

(work started with Wan Fokkink)



Regular expressions *(S.C. Kleene, 1951)*

Definition

The set $\mathrm{Reg}(A)$ of regular expressions over alphabet A is defined by the grammar:

$$e, f \;::=\; 0 \mid 1 \mid a \mid (e + f) \mid (e \cdot f) \mid (e^{\star}) \qquad \text{(for } a \in A\text{).}$$

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Note, here:

- symbol 0 instead of $\varnothing$
- symbol 1 used (often dropped, definable as $0^{\star}$)
- no complementation operation $\overline{e}$
  - which is not expressible under language interpretation

Language semantics $[\![\cdot]\!]_L$ of reg. expr's *(Copi–Elgot–Wright, 1958)*

$\mathbf{0} \xmapsto{L}$ empty language $\varnothing$

$\mathbf{1} \xmapsto{L} \{\epsilon\}$ (ϵ the empty word)

$a \xmapsto{L} \{a\}$

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$e + f \xmapsto{L}$ union of $L(e)$ and $L(f)$

$e \cdot f \xmapsto{L}$ element-wise concatenation of $L(e)$ and $L(f)$

$e^* \xmapsto{L}$ set of words formed by concatenating words in $L(e)$,
and adding the empty word ϵ

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$[\![e]\!]_L := L(e)$ (language defined by e)

Process semantics of regular expressions $[\![\cdot]\!]_P$ *(Milner, 1984)*

$\mathbf{0} \xmapsto{P}$ deadlock $\boldsymbol{\delta}$, no termination

$\mathbf{1} \xmapsto{P}$ empty-step process $\boldsymbol{\epsilon}$, then terminate

$a \xmapsto{P}$ atomic action a, then terminate

Process semantics of regular expressions $[\![\cdot]\!]_P$ *(Milner, 1984)*

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$e + f \overset{P}{\longmapsto}$ (*choice*) execute $[\![e]\!]_P$ or $[\![f]\!]_P$

$e \cdot f \overset{P}{\longmapsto}$ (*sequentialization*) execute $[\![e]\!]_P$, then $[\![f]\!]_P$

$e^{*} \overset{P}{\longmapsto}$ (*iteration*) repeat (terminate or execute $[\![e]\!]_P$)

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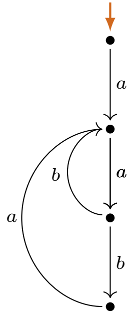
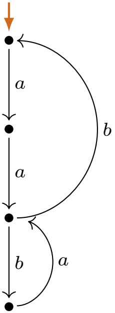
$e^* \quad \overset{P}{\longmapsto}$ (*iteration*) repeat (terminate or execute $[\![e]\!]_P$)

$[\![e]\!]_P \quad := \quad [P(e)]_{\underline{\leftrightarrow}}$ (bisimilarity equivalence class of process $P(e)$)

Process interpretation of regular expressions

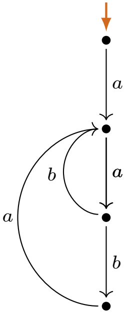




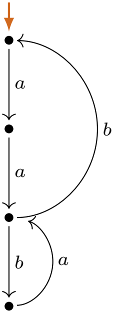
$P(a(a(b+ba))^*0)$

$P((aa(ba)^*b)^*0)$

Process interpretation of regular expressions

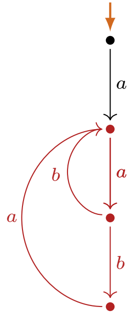


$P(a \cdot (a \cdot (b + b \cdot a))^{*} \cdot 0)$

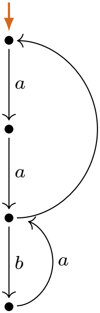


$P((a \cdot a \cdot (b \cdot a)^{*} \cdot b)^{*} \cdot 0)$

Process interpretation of regular expressions

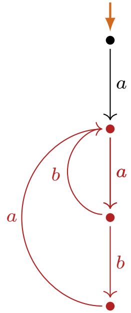


$\mathsf{P}(a \cdot (a \cdot (b + b \cdot a))^{*} \cdot 0)$

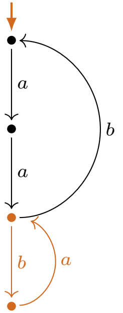


$\mathsf{P}((a \cdot a \cdot (b \cdot a)^{*} \cdot b)^{*} \cdot 0)$

Process interpretation of regular expressions



$$P(a \cdot (a \cdot (b + b \cdot a))^* \cdot 0)$$



$$P((a \cdot a \cdot (b \cdot a)^* \cdot b)^* \cdot 0)$$

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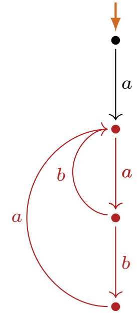
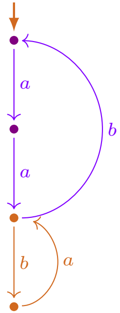
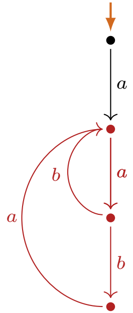
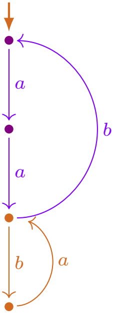




$P(a \cdot (a \cdot (b + b \cdot a))^{*} \cdot 0)$ $P((a \cdot a \cdot (b \cdot a)^{*} \cdot b)^{*} \cdot 0)$

Process interpretation of regular expressions





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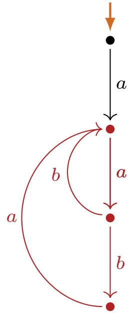
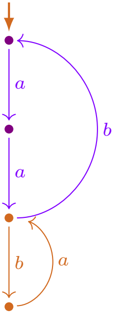


$P(a(a(b+ba))^{*}0)$



$P((aa(ba)^{*}b)^{*}0)$

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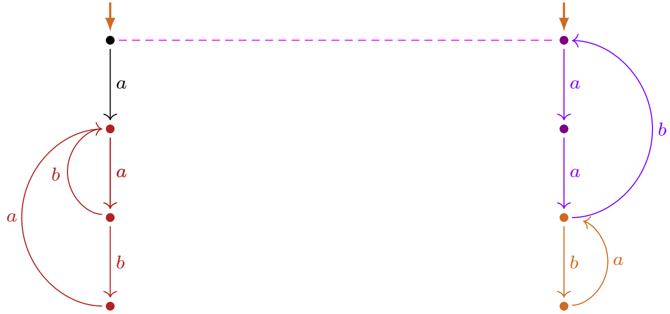

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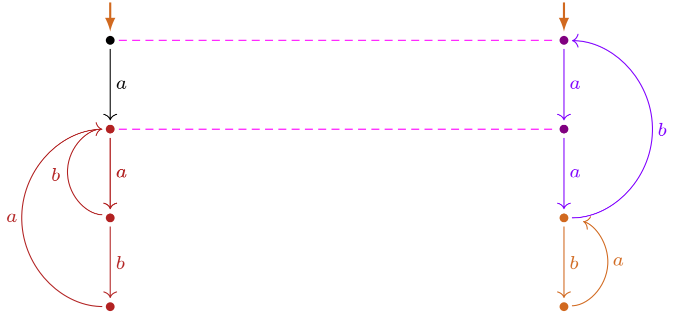

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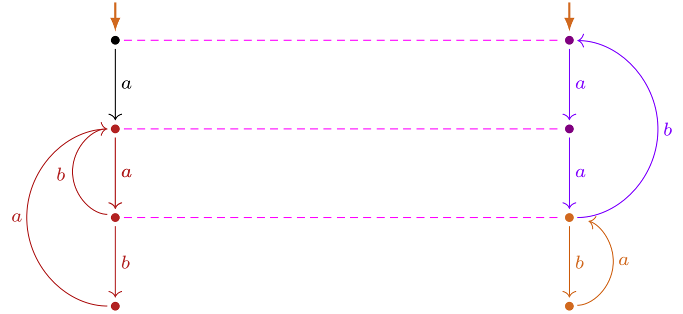

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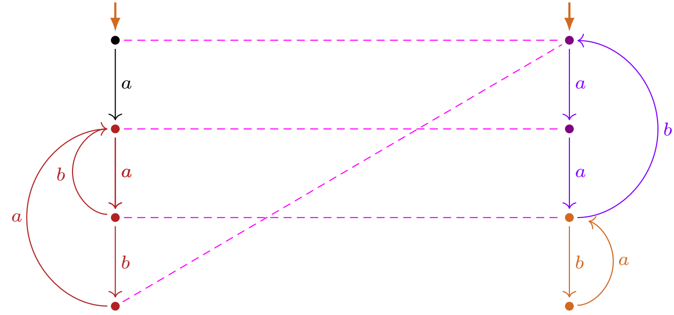

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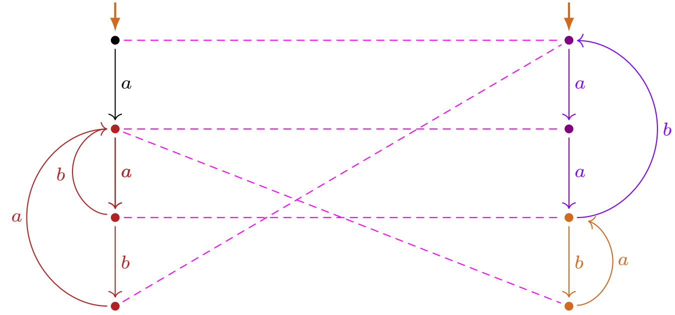

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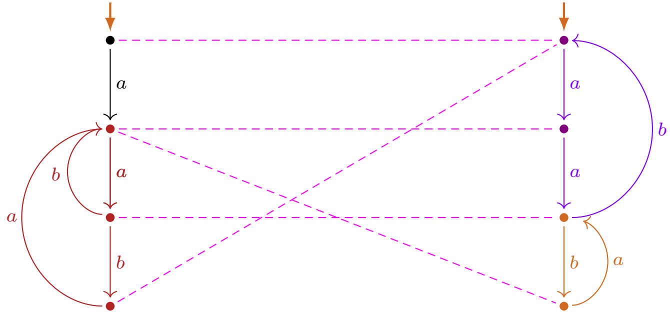
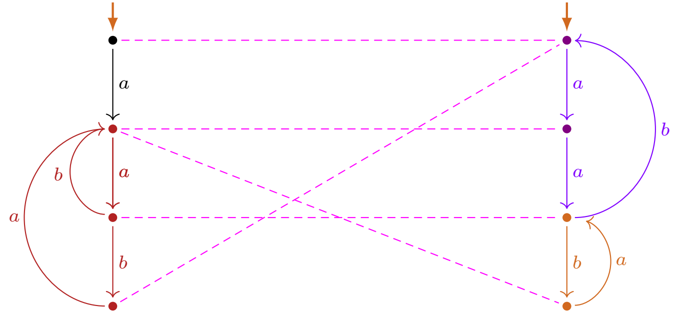


$$P(a(a(b+ba))^{*}0) \quad \underline{\leftrightarrow} \quad P((aa(ba)^{*}b)^{*}0)$$

Process interpretation of regular expressions



$$a(a(b+ba))^{*}0 \quad \underline{\leftrightarrow}_{P} \quad (aa(ba)^{*}b)^{*}0$$

Process graphs and NFAs

Definition

A process graph over actions in A is a tuple $G = \langle V, v_{\mathsf{s}}, T, E \rangle$ where:

- V is a set of *vertices*,
- $v_{\mathsf{s}} \in V$ is the *start vertex*,
- $T \subseteq V \times A \times V$ the set of *transitions*,
- $E \subseteq V \times \{\downarrow\}$ the set of *termination extensions*.

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Here we only consider finite and start-vertex connected process graphs.

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Correspondence with NFAs

With the finiteness restriction, process graphs can be viewed as:

- nondeterministic finite-state automata (NFAs),

that are studied under bisimulation, not under language equivalence.

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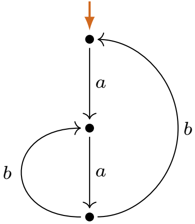
Antimirov (1996): NFA-definition of $P(\cdot)$ via partial derivatives.

Expressible process graphs (under bisimulation $\underline{\leftrightarrow}$)



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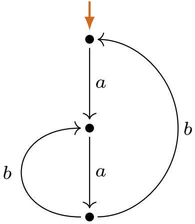
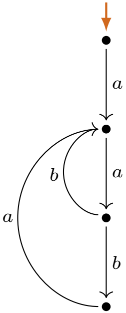
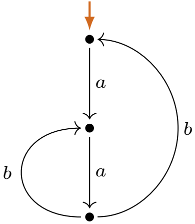


$? \in im(P(\cdot))\ ?$

Expressible process graphs (under bisimulation $\underline{\leftrightarrow}$)



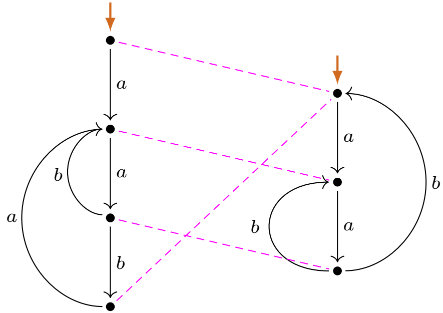


$\in im(P(\cdot))$

$P(\cdot)$-expressible

? $\in im(P(\cdot))$?

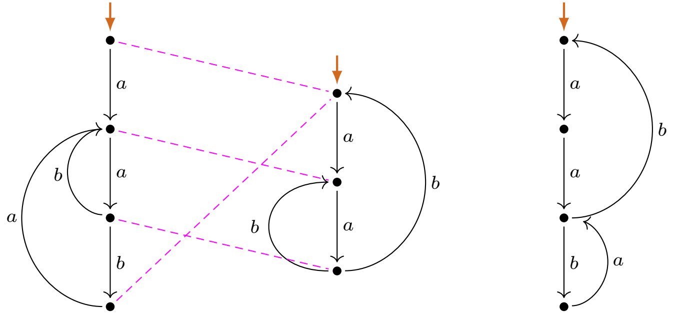
Expressible process graphs (under bisimulation $\underline{\leftrightarrow}$)



$\in im(P(\cdot))$ $\quad$ $? \in im(P(\cdot))\ ?$

$P(\cdot)$-expressible

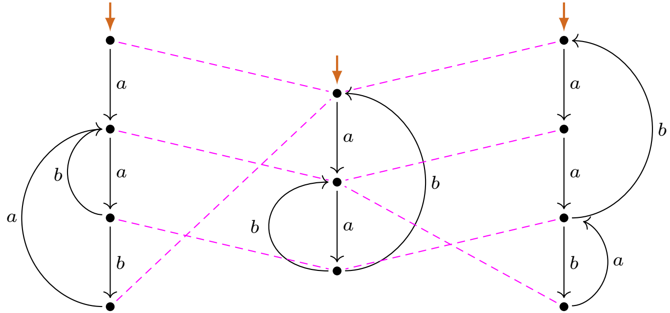
Expressible process graphs (under bisimulation $\underline{\leftrightarrow}$)



$\in im(P(\cdot))$ $\qquad$ $?\in im(P(\cdot))\,?$ $\qquad$ $\in im(P(\cdot))$

$P(\cdot)$-expressible $\qquad$ $P(\cdot)$-expressible

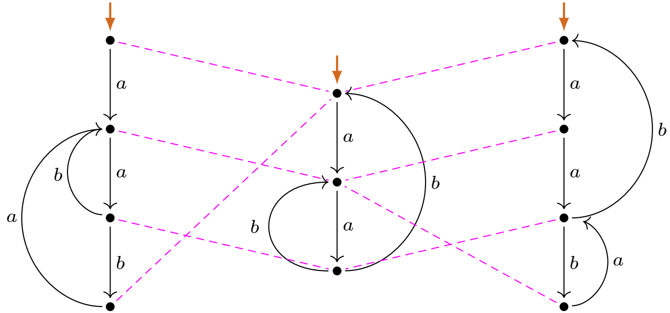
Expressible process graphs (under bisimulation $\underline{\leftrightarrow}$)



$\in im(P(\cdot))$ $\quad$? $\in im(P(\cdot))$? $\quad$ $\in im(P(\cdot))$

$P(\cdot)$-expressible $\quad$ $P(\cdot)$-expressible

Expressible process graphs (under bisimulation $\underline{\leftrightarrow}$)



$\in im(P(\cdot))$

$P(\cdot)$-expressible

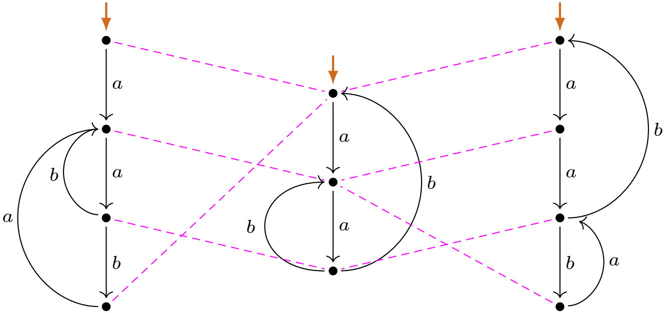
? $\in im(P(\cdot))$?

$P(\cdot)$-expressible modulo $\underline{\leftrightarrow}$

$\in im(P(\cdot))$

$P(\cdot)$-expressible

Expressible process graphs (under bisimulation $\underline{\leftrightarrow}$)



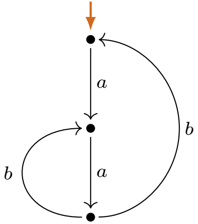
$\in im(P(\cdot))$ $P(\cdot)$-expressible $[\![\cdot]\!]_P$-expressible

$? \in im(P(\cdot))\,?$ $P(\cdot)$-expressible modulo $\underline{\leftrightarrow}$ $[\![\cdot]\!]_P$-expressible

$\in im(P(\cdot))$ $P(\cdot)$-expressible $[\![\cdot]\!]_P$-expressible

Properties of P and $[\![\cdot]\!]_P$

- Not every finite-state process is $P(\cdot)$-expressible.



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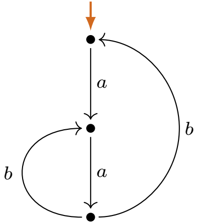

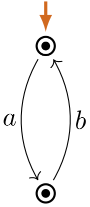

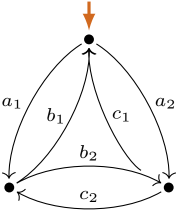

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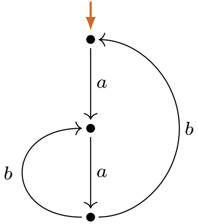

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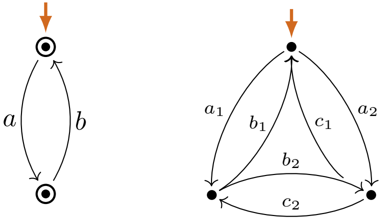

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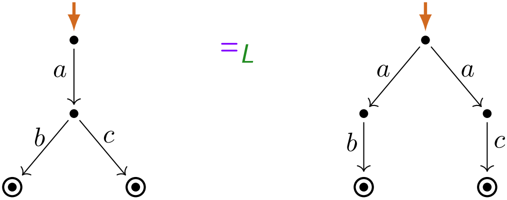

$$a \cdot (b + c) \quad =_L \quad a \cdot b + a \cdot c$$

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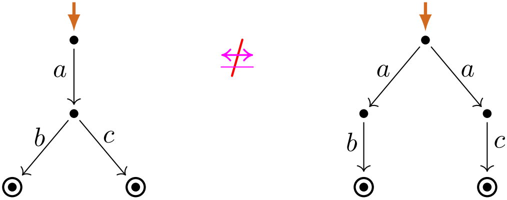

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Complete axiomatization of $=_L$ *(Aanderaa/Salomaa, 1965/66)*

Axioms:

(B1)	$e + (f + g) = (e + f) + g$	(B7)	$e \cdot 1 = e$
(B2)	$(e \cdot f) \cdot g = e \cdot (f \cdot g)$	(B8)	$e \cdot 0 = 0$
(B3)	$e + f = f + e$	(B9)	$e + 0 = e$
(B4)	$(e + f) \cdot g = e \cdot g + f \cdot g$	(B10)	$e^* = 1 + e \cdot e^*$
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Inference rules: equational logic *plus*

$$\frac{e = f \cdot e + g}{e = f^* \cdot g}\ \text{FIX} \quad (\text{if } \underbrace{\{\epsilon\} \notin L(f)}_{\textit{non-empty-word property}})$$

Sound and unsound axioms with respect to $\underline{\leftrightarrow}_P$

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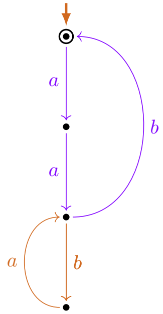
- $\text{Mil}^- +$ one of two stronger rules (than RSP^*) is complete *(G, 2006)*

Well-behaved form, looping palm trees



$P((aa(ba)^{*}b)^{*})$

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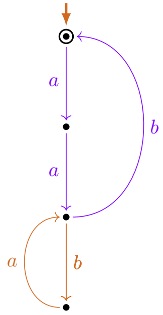

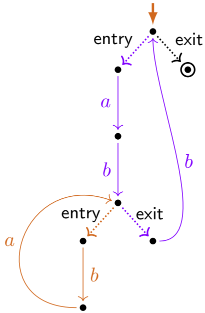

well-behaved form

(Corradini, Baeten)

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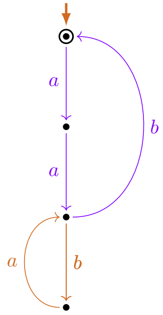

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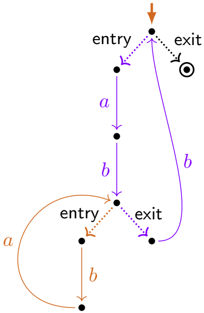

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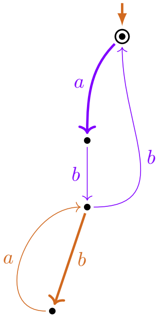

looping palm tree

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Loop chart

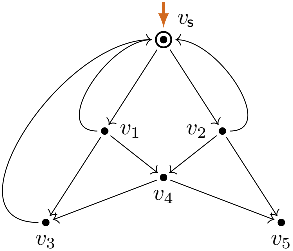
Definition

A process graph is a loop chart if:

L-1.

L-2.

L-3.



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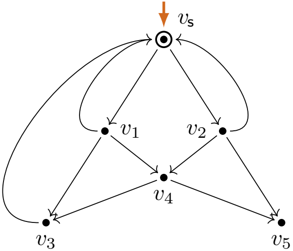

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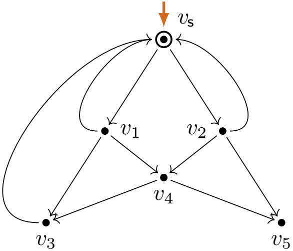

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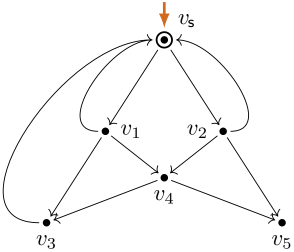

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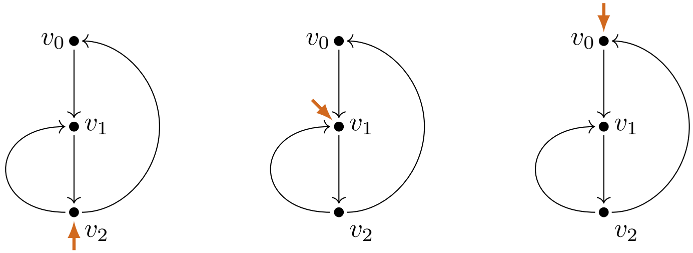

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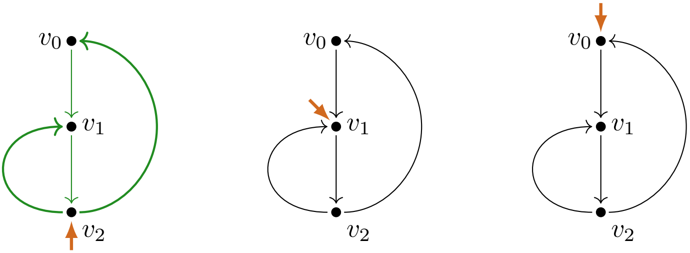

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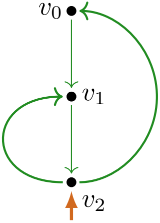

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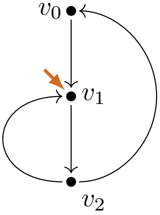

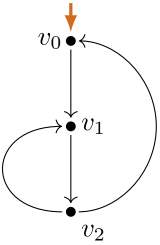

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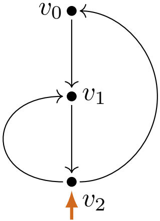

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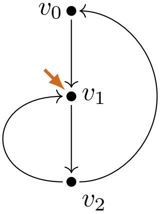

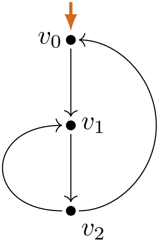

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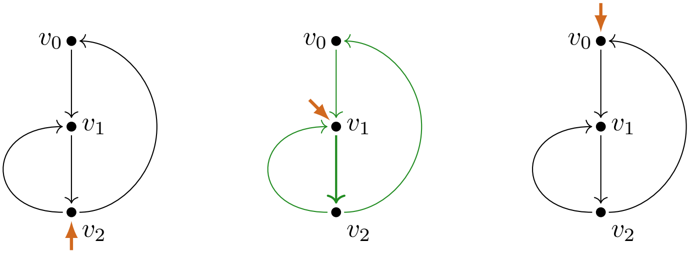

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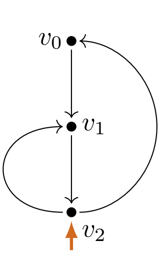

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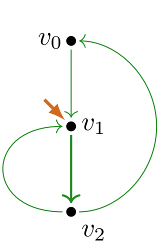

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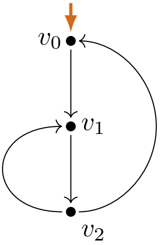

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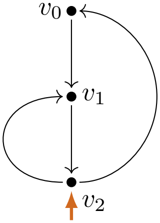

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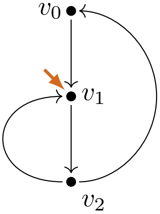

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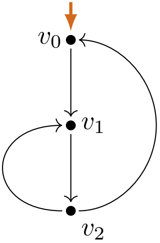

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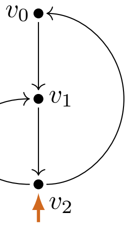

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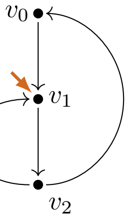

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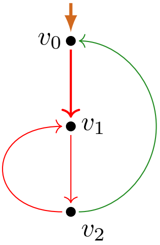

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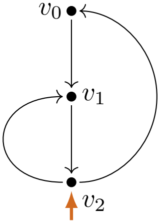
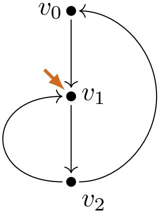
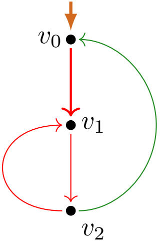


loop chart



loop chart



no loop chart

Loop chart

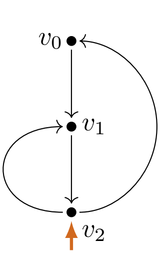
Definition

A process graph is a loop chart if:

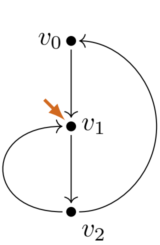
L-1. There is an infinite path from the start vertex.

L-2. Every infinite path from the start vertex returns to it.

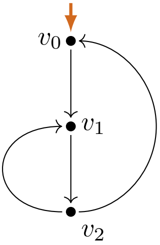
L-3. Termination is only possible at the start vertex.



loop chart

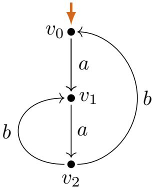


loop chart

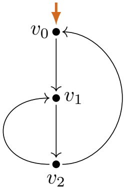


no loop chart

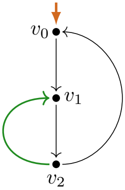
Loop elimination



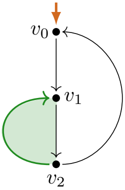
Loop elimination



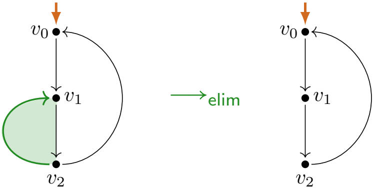
Loop elimination



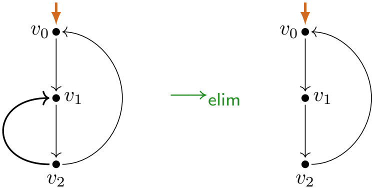
Loop elimination



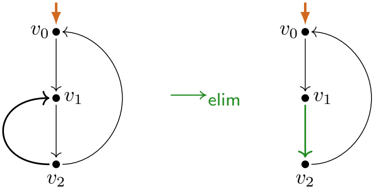
Loop elimination



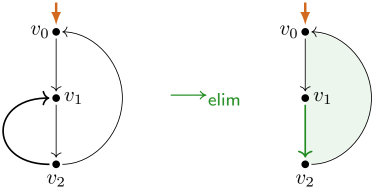
Loop elimination



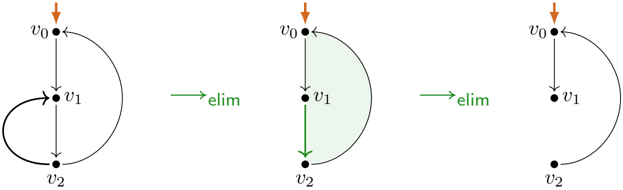
Loop elimination



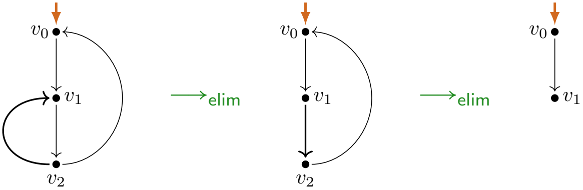
Loop elimination



Loop elimination

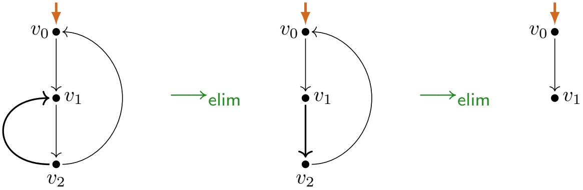

v_0
v_1
v_2
$\longrightarrow_{\text{elim}}$
v_0
v_1
v_2
$\longrightarrow_{\text{elim}}$
v_0
v_1
v_2

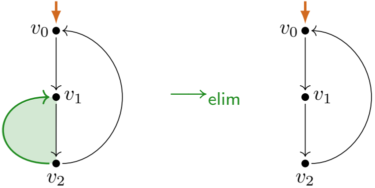
Loop elimination


v_0
v_1
v_2
$\longrightarrow_{\text{elim}}$
v_0
v_1
v_2
$\longrightarrow_{\text{elim}}$
v_0
v_1

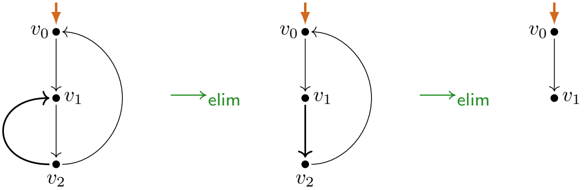
)

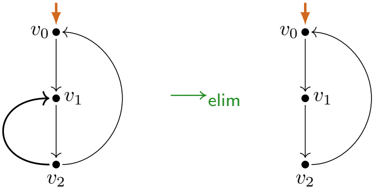
Loop elimination


v_0
v_1
v_2
$\longrightarrow_{\mathsf{elim}}$
v_0
v_1
v_2
$\longrightarrow_{\mathsf{elim}}$
v_0
v_1

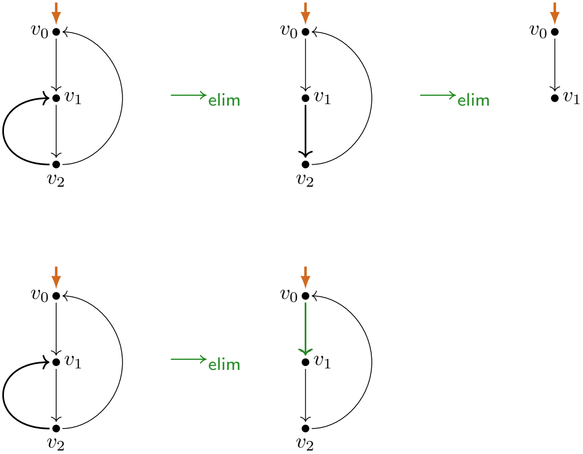


Loop elimination

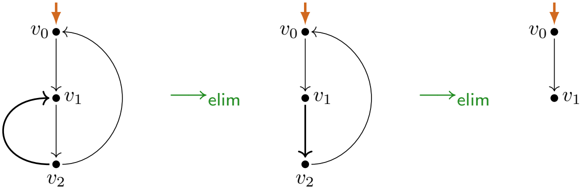

v_0
v_1
v_2
$\longrightarrow_{\text{elim}}$
v_0
v_1
v_2
$\longrightarrow_{\text{elim}}$
v_0
v_1

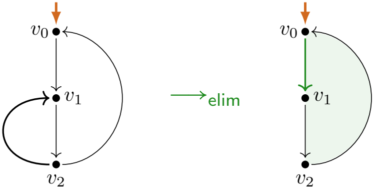


Loop elimination

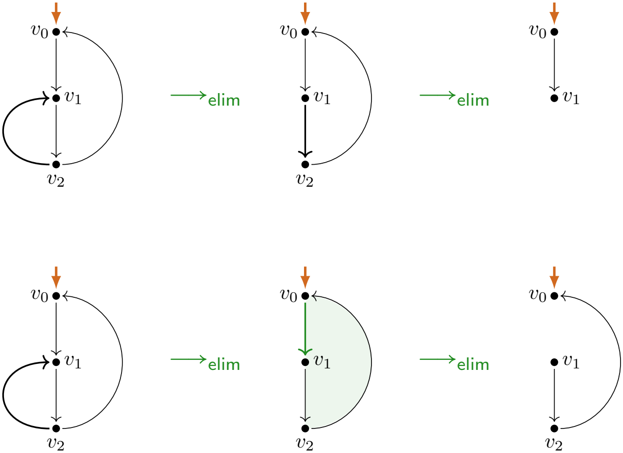

v_0
v_1
v_2
$\longrightarrow_{\text{elim}}$
v_0
v_1
v_2
$\longrightarrow_{\text{elim}}$
v_0
v_1
v_0
v_1
v_2
$\longrightarrow_{\text{elim}}$
v_0
v_1
v_2

Loop elimination

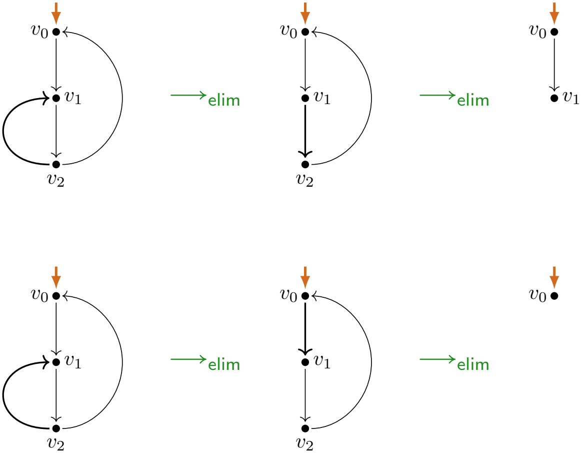

v_0
v_1
v_2
$\longrightarrow_{\text{elim}}$
v_0
v_1
v_2
$\longrightarrow_{\text{elim}}$
v_0
v_1



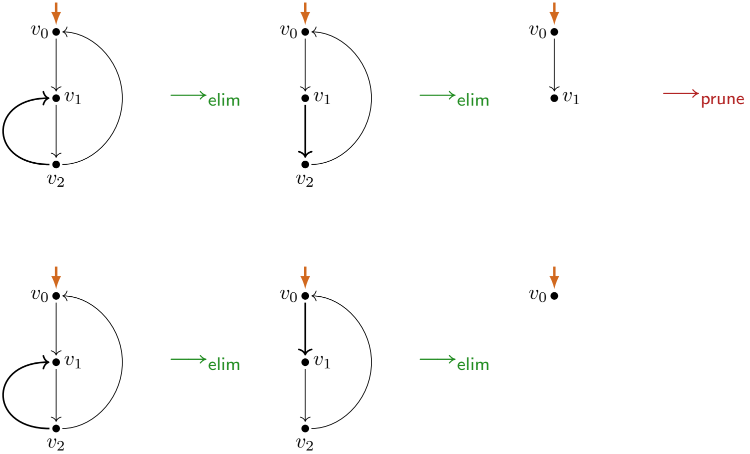
Loop elimination


v_0
v_1
v_2
$\longrightarrow_{\text{elim}}$
v_0
v_1
v_2
$\longrightarrow_{\text{elim}}$
v_0
v_1
v_0
v_1
v_2
$\longrightarrow_{\text{elim}}$
v_0
v_1
v_2
$\longrightarrow_{\text{elim}}$
v_0
v_1
v_2

Loop elimination


v_0
v_1
v_2
$\longrightarrow_{\text{elim}}$
v_0
v_1
v_2
$\longrightarrow_{\text{elim}}$
v_0
v_1
v_0
v_1
v_2
$\longrightarrow_{\text{elim}}$
v_0
v_1
v_2
$\longrightarrow_{\text{elim}}$
v_0

Loop elimination


v_0
v_1
v_2
$\longrightarrow_{\text{elim}}$
v_0
v_1
v_2
$\longrightarrow_{\text{elim}}$
v_0
v_1
$\longrightarrow_{\text{prune}}$
v_0
v_0
v_1
v_2
$\longrightarrow_{\text{elim}}$
v_0
v_1
v_2
$\longrightarrow_{\text{elim}}$
v_0

Loop elimination, and properties

$\xrightarrow{}_{\text{elim}}$: eliminate a transition-induced loop by:

- removing the loop-entry transition(s)
- garbage collection

$\xrightarrow{}_{\text{prune}}$: remove a transition to a deadlocking state

Loop elimination, and properties

$\longrightarrow_{\text{elim}}$: eliminate a transition-induced loop by:

- removing the loop-entry transition(s)
- garbage collection

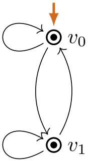
$\longrightarrow_{\text{prune}}$: remove a transition to a deadlocking state

Lemma

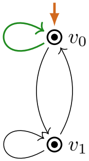
(i) $\longrightarrow_{\text{elim}}$ *is terminating.*

(ii) $\longrightarrow_{\text{elim}} \cup \longrightarrow_{\text{prune}}$ *is terminating and confluent.*

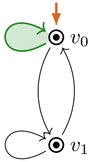
Loop elimination



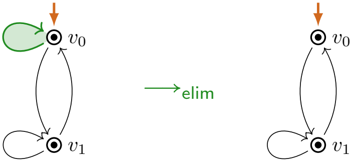
Loop elimination



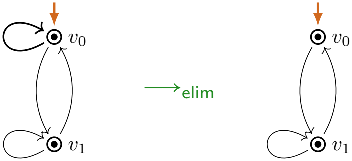
Loop elimination



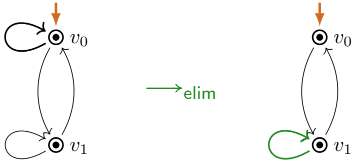
Loop elimination



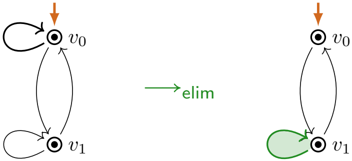
Loop elimination



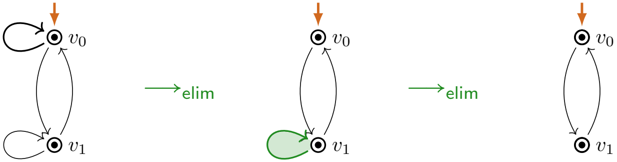
Loop elimination



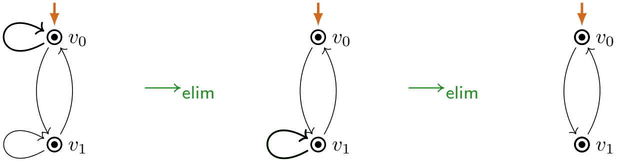
Loop elimination



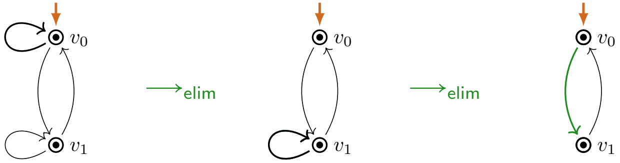
Loop elimination


v_0
v_1
$\longrightarrow_{\text{elim}}$
v_0
v_1
$\longrightarrow_{\text{elim}}$
v_0
v_1

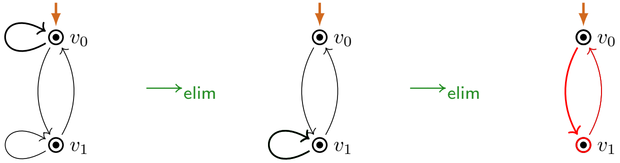
Loop elimination


v_0
v_1
$\longrightarrow_{\text{elim}}$
v_0
v_1
$\longrightarrow_{\text{elim}}$
v_0
v_1

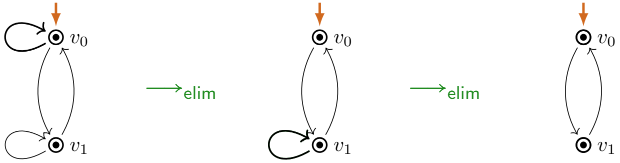
Loop elimination


v_0
v_1
$\longrightarrow_{\text{elim}}$
v_0
v_1
$\longrightarrow_{\text{elim}}$
v_0
v_1

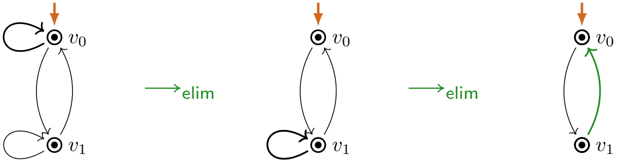
Loop elimination


v_0
v_1
$\longrightarrow_{\text{elim}}$
v_0
v_1
$\longrightarrow_{\text{elim}}$
v_0
v_1

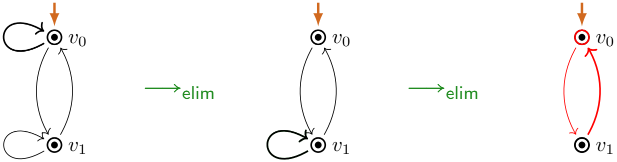
Loop elimination


v_0
v_1
$\longrightarrow_{\text{elim}}$
v_0
v_1
$\longrightarrow_{\text{elim}}$
v_0
v_1

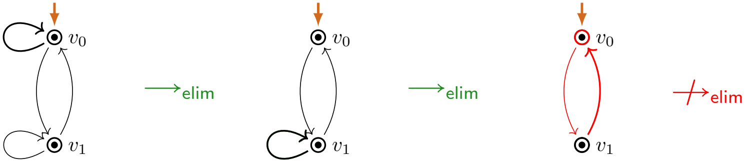
Loop elimination


v_0
v_1
$\longrightarrow_{\text{elim}}$
v_0
v_1
$\longrightarrow_{\text{elim}}$
v_0
v_1

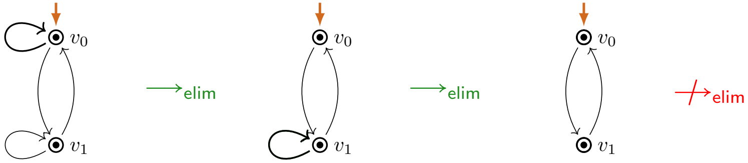
Loop elimination



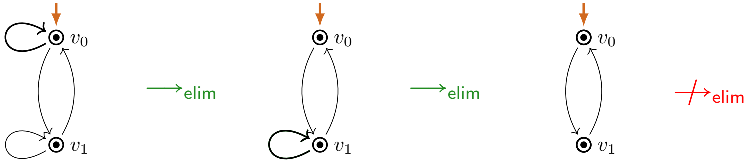
Loop elimination

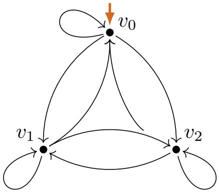

v_0
v_1
$\longrightarrow_{\text{elim}}$
v_0
v_1
$\longrightarrow_{\text{elim}}$
v_0
v_1
$\not\longrightarrow_{\text{elim}}$

Loop elimination

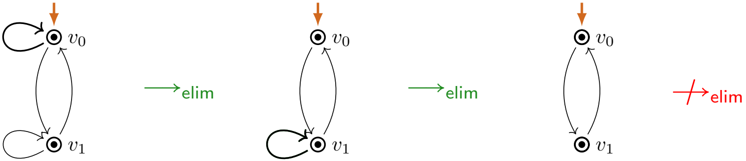

v_0
v_1
$\longrightarrow_{\text{elim}}$
v_0
v_1
$\longrightarrow_{\text{elim}}$
v_0
v_1
$\not\longrightarrow_{\text{elim}}$

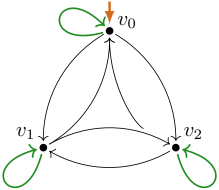
Loop elimination


v_0
v_1
$\longrightarrow_{\text{elim}}$
v_0
v_1
$\longrightarrow_{\text{elim}}$
v_0
v_1
$\not\longrightarrow_{\text{elim}}$

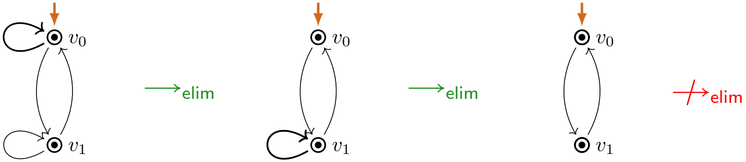


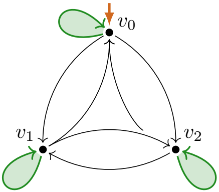
Loop elimination


v_0
v_1
$\longrightarrow_{\text{elim}}$
v_0
v_1
$\longrightarrow_{\text{elim}}$
v_0
v_1
$\not\longrightarrow_{\text{elim}}$

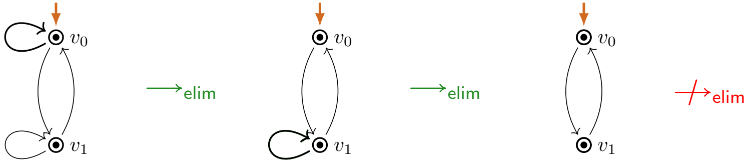


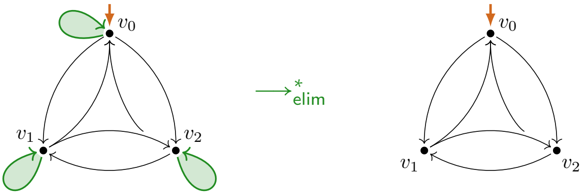
Loop elimination


v_0
v_1
$\longrightarrow_{\text{elim}}$
v_0
v_1
$\longrightarrow_{\text{elim}}$
v_0
v_1
$\not\longrightarrow_{\text{elim}}$

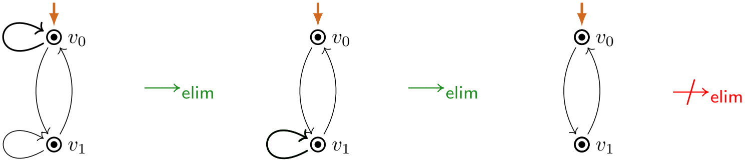


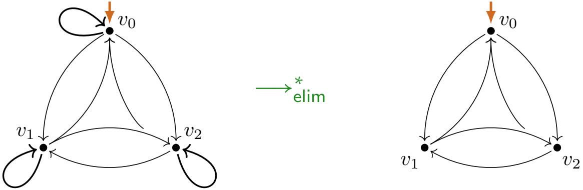
Loop elimination


v_0
v_1
$\longrightarrow_{\text{elim}}$
v_0
v_1
$\longrightarrow_{\text{elim}}$
v_0
v_1
$\not\longrightarrow_{\text{elim}}$

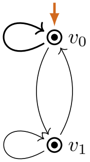


Loop elimination

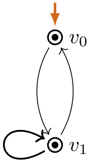

v_0
v_1
$\longrightarrow_{\text{elim}}$
v_0
v_1
$\longrightarrow_{\text{elim}}$
v_0
v_1
$\not\longrightarrow_{\text{elim}}$



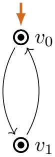
Loop elimination



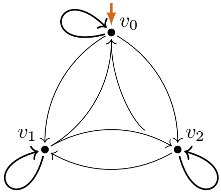
$\longrightarrow_{\text{elim}}$



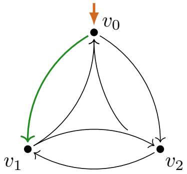
$\longrightarrow_{\text{elim}}$



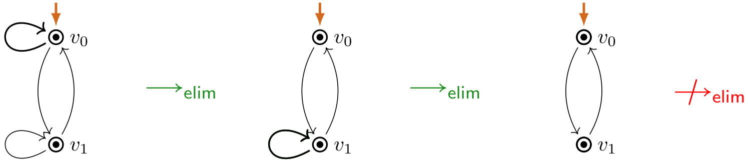
$\not\longrightarrow_{\text{elim}}$

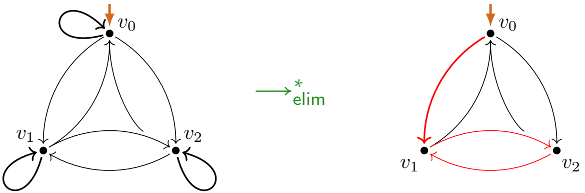


$\longrightarrow^{*}_{\text{elim}}$

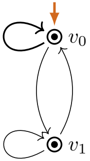


Loop elimination

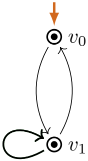

v_0
v_1
$\longrightarrow_{\text{elim}}$
v_0
v_1
$\longrightarrow_{\text{elim}}$
v_0
v_1
$\not\longrightarrow_{\text{elim}}$



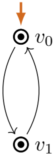
Loop elimination



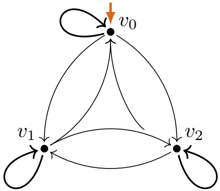
$\longrightarrow_{\text{elim}}$



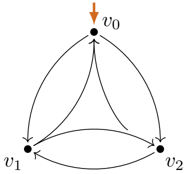
$\longrightarrow_{\text{elim}}$



$\not\longrightarrow_{\text{elim}}$



$\longrightarrow^{*}_{\text{elim}}$



$\not\longrightarrow_{\text{elim}}$

Structure property LEE

Definition

A process graph G satisfies LEE (*loop existence and elimination*) if:

$$\exists\, G_0 \,\big(\, G \longrightarrow^{*}_{\text{elim}} G_0 \not\longrightarrow_{\text{elim}} \\ \wedge\; G_0 \text{ has no infinite trace} \,\big)\;.$$

Structure property LEE

Definition

A process graph G satisfies LEE (*loop existence and elimination*) if:

$$\exists\, G_0 \left(G \longrightarrow^{*}_{\text{elim}} G_0 \not\longrightarrow_{\text{elim}} \right.$$
$$\left. \wedge\; G_0 \text{ has no infinite trace} \right) .$$

Lemma (by using confluence properties)

For every process graph G the following are equivalent:

(i) LEE(G).

(ii) *There is an $\longrightarrow_{\text{elim}}$ normal form without an infinite trace.*

Structure property LEE

Definition

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For every process graph G the following are equivalent:

(i) LEE(G).

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(iii) *There is an $\longrightarrow_{\text{elim},\text{prune}}$ normal form without an infinite trace.*

Structure property LEE

Definition

A process graph G satisfies LEE (*loop existence and elimination*) if:

$$\exists G_0 \left(G \longrightarrow^{*}_{\text{elim}} G_0 \not\longrightarrow_{\text{elim}} \right.$$
$$\left. \wedge\; G_0 \text{ has no infinite trace} \right) .$$

Lemma (by using confluence properties)

For every process graph G the following are equivalent:

(i) LEE(G).

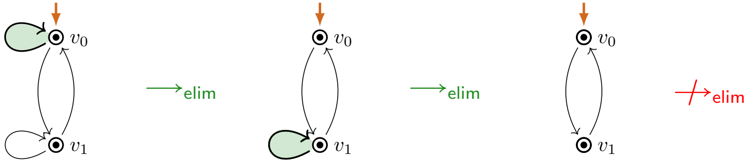
(ii) *There is an $\longrightarrow_{\text{elim}}$ normal form without an infinite trace.*

(iii) *There is an $\longrightarrow_{\text{elim,prune}}$ normal form without an infinite trace.*

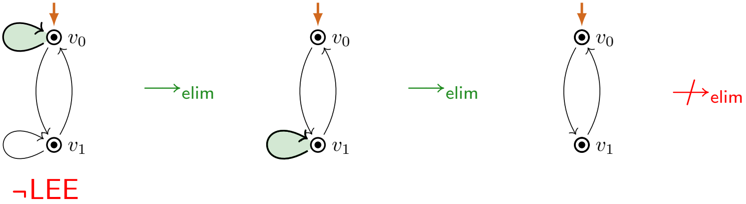
(iv) *Every $\longrightarrow_{\text{elim}}$ normal form is without an infinite trace.*

(v) *Every $\longrightarrow_{\text{elim,prune}}$ normal form is without an infinite trace.*

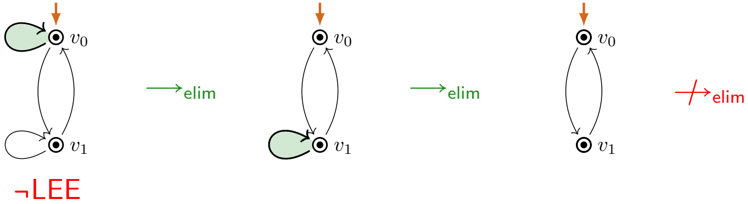
LEE fails

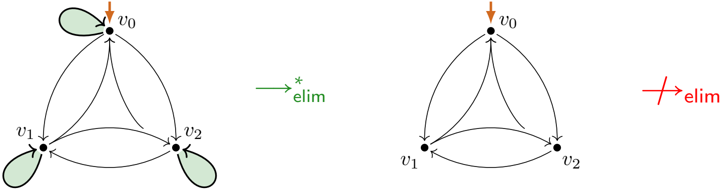

v_0
v_1
$\longrightarrow_{\mathsf{elim}}$
v_0
v_1
$\longrightarrow_{\mathsf{elim}}$
v_0
v_1
$\not\longrightarrow_{\mathsf{elim}}$

LEE fails

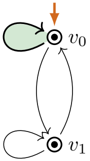

v_0
v_1
$\neg$LEE
$\longrightarrow_{\text{elim}}$
v_0
v_1
$\longrightarrow_{\text{elim}}$
v_0
v_1
$\not\longrightarrow_{\text{elim}}$

LEE fails

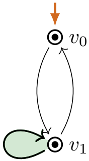

v_0
v_1
$\longrightarrow_{\text{elim}}$
v_0
v_1
$\longrightarrow_{\text{elim}}$
v_0
v_1
$\not\longrightarrow_{\text{elim}}$
¬LEE


v_0
v_1
v_2
$\longrightarrow^{*}_{\text{elim}}$
v_0
v_1
v_2
$\not\longrightarrow_{\text{elim}}$

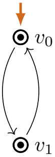
LEE fails



$\longrightarrow_{\text{elim}}$

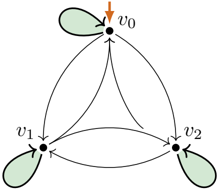


$\longrightarrow_{\text{elim}}$

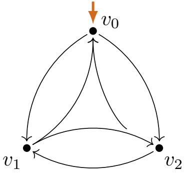


$\not\longrightarrow_{\text{elim}}$

$\neg$LEE



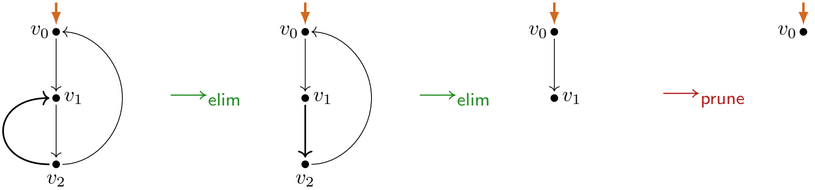
$\longrightarrow^{*}_{\text{elim}}$



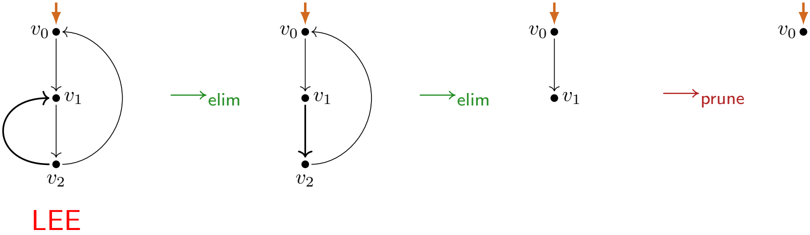
$\not\longrightarrow_{\text{elim}}$

$\neg$LEE

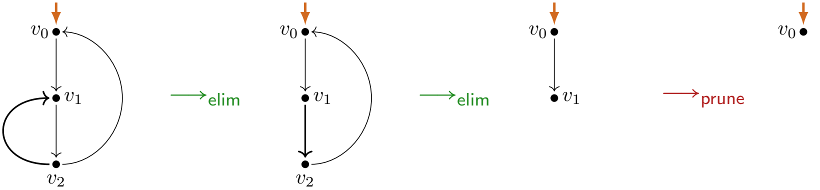
LEE holds



LEE holds

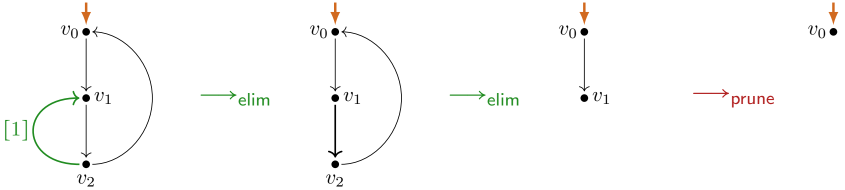

v_0
v_1
v_2
LEE
$\longrightarrow_{\text{elim}}$
v_0
v_1
v_2
$\longrightarrow_{\text{elim}}$
v_0
v_1
$\longrightarrow_{\text{prune}}$
v_0

LEE holds / Recording loop elimination


v_0
v_1
v_2
$\longrightarrow_{\text{elim}}$
v_0
v_1
v_2
$\longrightarrow_{\text{elim}}$
v_0
v_1
$\longrightarrow_{\text{prune}}$
v_0

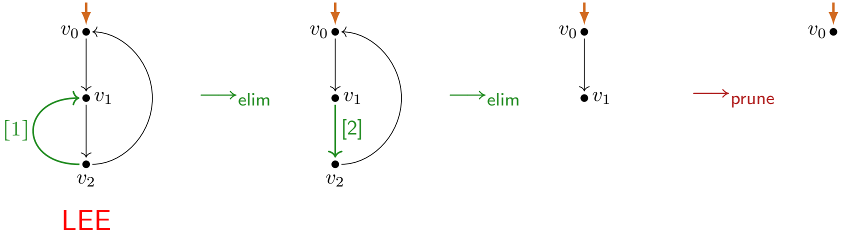
LEE

LEE holds / Recording loop elimination

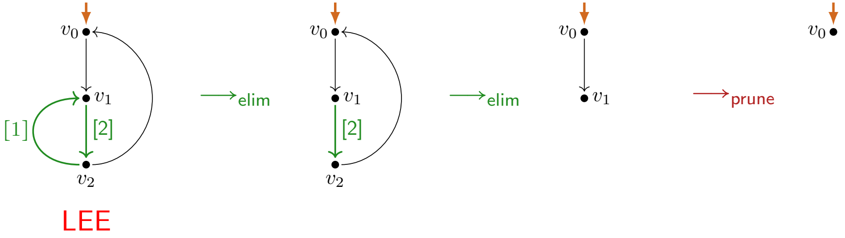

v_0
v_1
v_2
[1]
$\longrightarrow_{\text{elim}}$
v_0
v_1
v_2
$\longrightarrow_{\text{elim}}$
v_0
v_1
$\longrightarrow_{\text{prune}}$
v_0

LEE

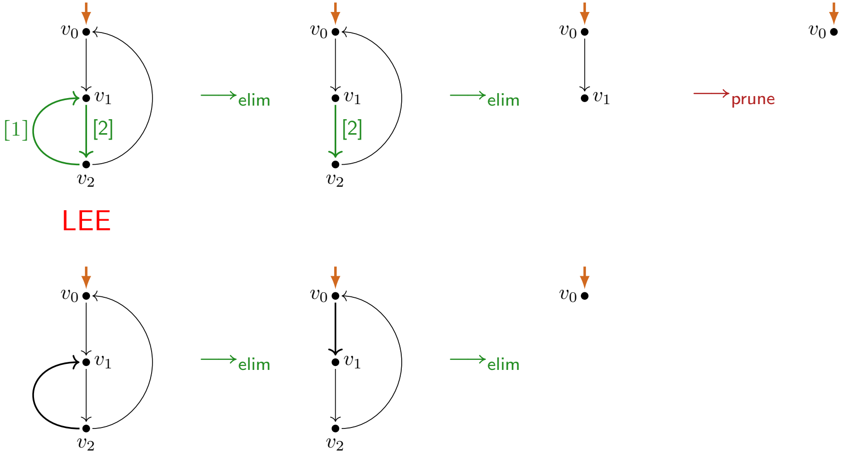
LEE holds / Recording loop elimination


v_0
v_1
v_2
[1]
LEE
$\longrightarrow_{\text{elim}}$
v_0
v_1
v_2
[2]
$\longrightarrow_{\text{elim}}$
v_0
v_1
$\longrightarrow_{\text{prune}}$
v_0

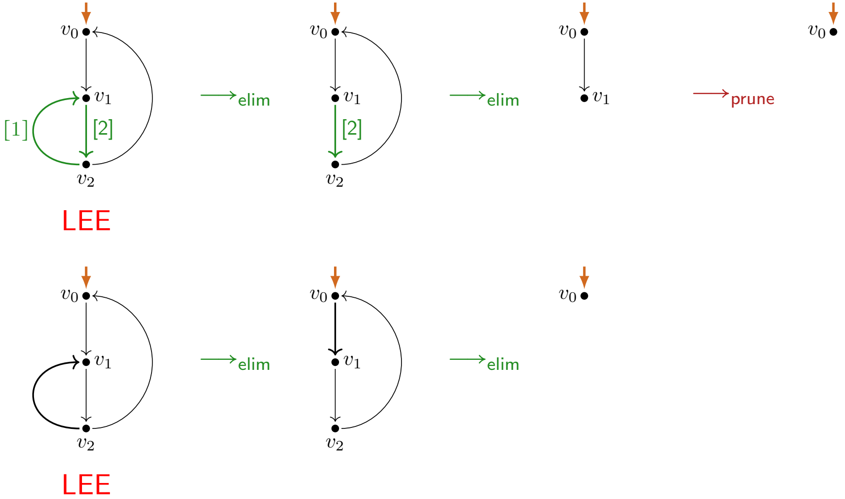
LEE holds / Recording loop elimination


v_0
v_1
v_2
[1]
[2]
LEE
$\longrightarrow_{\text{elim}}$
v_0
v_1
v_2
[2]
$\longrightarrow_{\text{elim}}$
v_0
v_1
$\longrightarrow_{\text{prune}}$
v_0

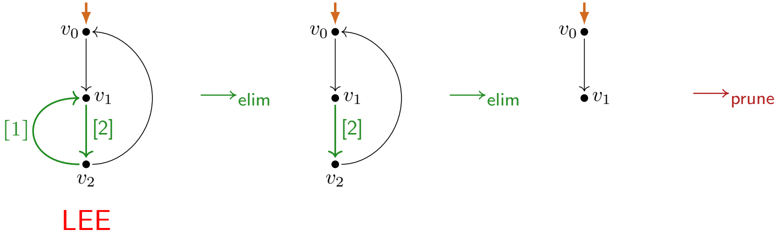
LEE holds / Recording loop elimination

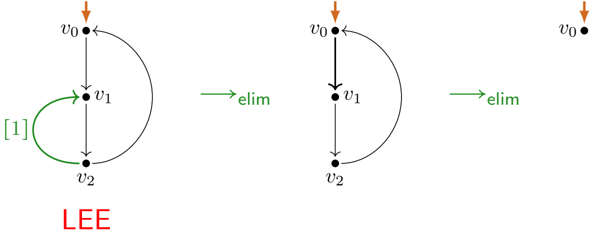


LEE holds / Recording loop elimination

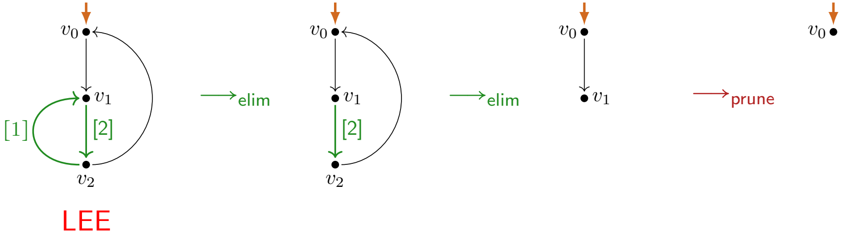


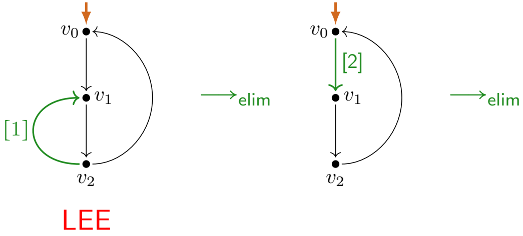
LEE holds / Recording loop elimination



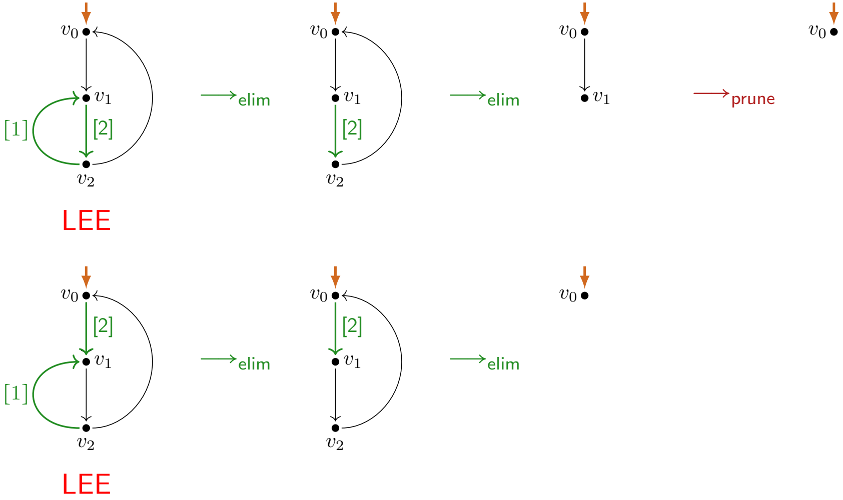


LEE holds / Recording loop elimination

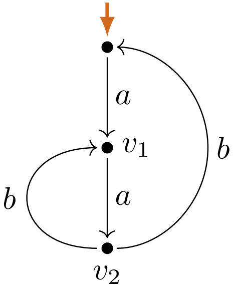




LEE holds / Recording loop elimination

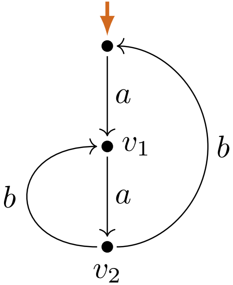


LEE-witness



LEE-witness

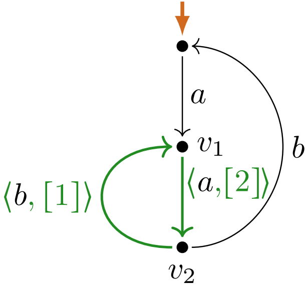
loop–branch labeling: marking transitions $\xrightarrow{a}$ as:



LEE-witness

loop–branch labeling: marking transitions $\xrightarrow{a}$ as:

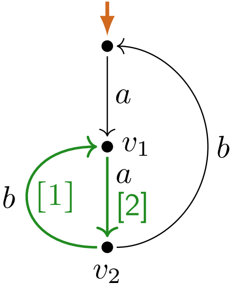
- entry steps $\xrightarrow{\langle a,[n]\rangle}$ for $n \in \mathbb{N}$,



LEE-witness

loop–branch labeling: marking transitions $\xrightarrow{a}$ as:

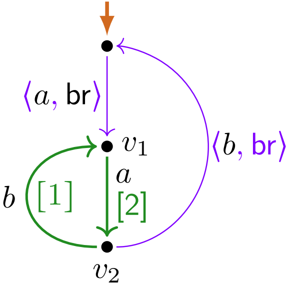
- entry steps $\xrightarrow{\langle a,[n]\rangle}$ for $n \in \mathbb{N}$, written $\xrightarrow{a}_{[n]}$,



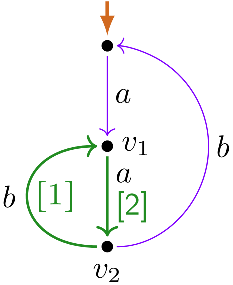
LEE-witness

loop–branch labeling: marking transitions $\xrightarrow{a}$ as:

- entry steps $\xrightarrow{\langle a,[n]\rangle}$ for $n \in \mathbb{N}$, written $\xrightarrow{a}_{[n]}$,
- branch steps $\xrightarrow{\langle a,\mathsf{br}\rangle}$,



LEE-witness



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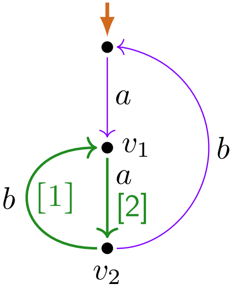
- branch steps $\xrightarrow{\langle a,\mathsf{br}\rangle}$, written $\xrightarrow{a}_{\mathsf{br}}$ or $\xrightarrow{a}$.

LEE-witness



loop–branch labeling: marking transitions $\xrightarrow{a}$ as:

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- branch steps $\xrightarrow{\langle a,\mathsf{br}\rangle}$, written $\xrightarrow{a}_{\mathsf{br}}$ or $\xrightarrow{a}$.

Definition

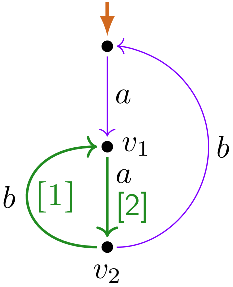
A loop–branch labeling is a LEE-witness, if:

L1.

L2.

L3.

LEE-witness



loop–branch labeling: marking transitions $\xrightarrow{a}$ as:

- entry steps $\xrightarrow{\langle a,[n]\rangle}$ for $n \in \mathbb{N}$, written $\xrightarrow{a}_{[n]}$,
- branch steps $\xrightarrow{\langle a,\text{br}\rangle}$, written $\xrightarrow{a}_{\text{br}}$ or $\xrightarrow{a}$.

Definition

A loop–branch labeling is a LEE-witness, if:

L1.

L2.

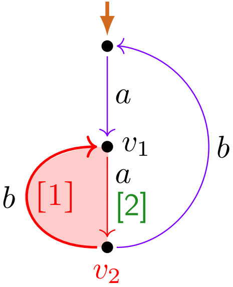
L3.

$\mathcal{L}(v, \rightarrow_{[n]}, \rightarrow_{\text{br},[>n]})$:= subchart induced
by entry steps $\rightarrow_{[n]}$ from v
followed by branch steps $\rightarrow_{\text{br}}$
or entry steps $\rightarrow_{[m]}$ with $m > n$,
until v is reached again

LEE-witness

loop–branch labeling: marking transitions $\xrightarrow{a}$ as:

- entry steps $\xrightarrow{\langle a,[n]\rangle}$ for $n \in \mathbb{N}$, written $\xrightarrow{a}_{[n]}$,
- branch steps $\xrightarrow{\langle a,\mathsf{br}\rangle}$, written $\xrightarrow{a}_{\mathsf{br}}$ or $\xrightarrow{a}$.



$\mathcal{L}(v_2, \to_{[1]}, \to_{\mathsf{br},[>1]})$

Definition

A loop–branch labeling is a LEE-witness, if:

L1.

L2.

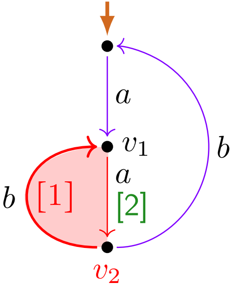
L3.

$\mathcal{L}(v, \to_{[n]}, \to_{\mathsf{br},[>n]})$:= subchart induced
by entry steps $\to_{[n]}$ from v
followed by branch steps $\to_{\mathsf{br}}$
or entry steps $\to_{[m]}$ with $m > n$,
until v is reached again

LEE-witness

loop–branch labeling: marking transitions $\xrightarrow{a}$ as:

- entry steps $\xrightarrow{\langle a,[n]\rangle}$ for $n \in \mathbb{N}$, written $\xrightarrow{a}_{[n]}$,
- branch steps $\xrightarrow{\langle a,\mathsf{br}\rangle}$, written $\xrightarrow{a}_{\mathsf{br}}$ or $\xrightarrow{a}$.



$\mathcal{L}(v_2, \rightarrow_{[1]}, \rightarrow_{\mathsf{br},[>1]})$
is loop subchart

Definition

A loop–branch labeling is a LEE-witness, if:

L1.

L2.

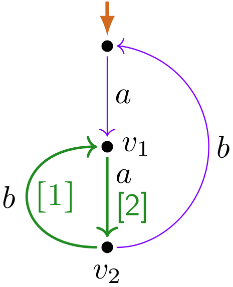
L3.

$\mathcal{L}(v, \rightarrow_{[n]}, \rightarrow_{\mathsf{br},[>n]})$:= subchart induced
by entry steps $\rightarrow_{[n]}$ from v
followed by branch steps $\rightarrow_{\mathsf{br}}$
or entry steps $\rightarrow_{[m]}$ with $m > n$,
until v is reached again

LEE-witness

loop–branch labeling: marking transitions $\xrightarrow{a}$ as:

- entry steps $\xrightarrow{\langle a,[n]\rangle}$ for $n \in \mathbb{N}$, written $\xrightarrow{a}_{[n]}$,
- branch steps $\xrightarrow{\langle a,\mathsf{br}\rangle}$, written $\xrightarrow{a}_{\mathsf{br}}$ or $\xrightarrow{a}$.



Definition

A loop–branch labeling is a LEE-witness, if:

L1.

L2.

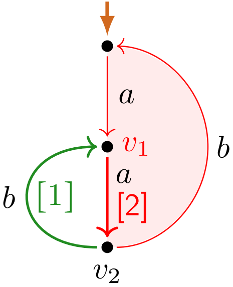
L3.

$\mathcal{L}(v, \rightarrow_{[n]}, \rightarrow_{\mathsf{br},[>n]})$:= subchart induced
by entry steps $\rightarrow_{[n]}$ from v
followed by branch steps $\rightarrow_{\mathsf{br}}$
or entry steps $\rightarrow_{[m]}$ with $m > n$,
until v is reached again

LEE-witness

loop–branch labeling: marking transitions $\xrightarrow{a}$ as:

- entry steps $\xrightarrow{\langle a,[n]\rangle}$ for $n \in \mathbb{N}$, written $\xrightarrow{a}_{[n]}$,
- branch steps $\xrightarrow{\langle a,\mathsf{br}\rangle}$, written $\xrightarrow{a}_{\mathsf{br}}$ or $\xrightarrow{a}$.



$\mathcal{L}(v_1, \rightarrow_{[2]}, \rightarrow_{\mathsf{br},[>2]})$

Definition

A loop–branch labeling is a LEE-witness, if:

L1.

L2.

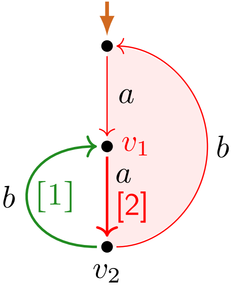
L3.

$\mathcal{L}(v, \rightarrow_{[n]}, \rightarrow_{\mathsf{br},[>n]})$:= subchart induced
by entry steps $\rightarrow_{[n]}$ from v
followed by branch steps $\rightarrow_{\mathsf{br}}$
or entry steps $\rightarrow_{[m]}$ with $m > n$,
until v is reached again

LEE-witness

loop–branch labeling: marking transitions $\xrightarrow{a}$ as:

- entry steps $\xrightarrow{\langle a,[n]\rangle}$ for $n \in \mathbb{N}$, written $\xrightarrow{a}_{[n]}$,
- branch steps $\xrightarrow{\langle a,\text{br}\rangle}$, written $\xrightarrow{a}_{\text{br}}$ or $\xrightarrow{a}$.



$\mathcal{L}(v_1, \rightarrow_{[2]}, \rightarrow_{\text{br},[>2]})$
is loop subchart

Definition

A loop–branch labeling is a LEE-witness, if:

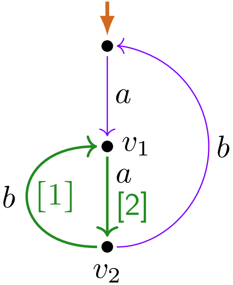
L1. $\forall n \in \mathbb{N} \forall v \in V \left(\begin{array}{c} v \rightarrow_{[n]} \Rightarrow \mathcal{L}(v, \rightarrow_{[n]}, \rightarrow_{\text{br},[>n]}) \\ \text{is a loop subchart} \end{array} \right)$.

L2.

L3.

$\mathcal{L}(v, \rightarrow_{[n]}, \rightarrow_{\text{br},[>n]})$:= subchart induced
by entry steps $\rightarrow_{[n]}$ from v
followed by branch steps $\rightarrow_{\text{br}}$
or entry steps $\rightarrow_{[m]}$ with $m > n$,
until v is reached again

LEE-witness



loop–branch labeling: marking transitions $\xrightarrow{a}$ as:

- entry steps $\xrightarrow{\langle a,[n]\rangle}$ for $n \in \mathbb{N}$, written $\xrightarrow{a}_{[n]}$,
- branch steps $\xrightarrow{\langle a,\mathsf{br}\rangle}$, written $\xrightarrow{a}_{\mathsf{br}}$ or $\xrightarrow{a}$.

Definition

A loop–branch labeling is a LEE-witness, if:

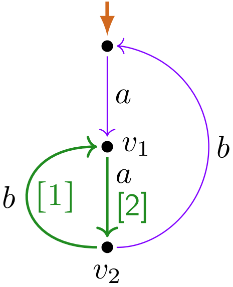
L1. $\forall n \in \mathbb{N} \forall v \in V \left(v \rightarrow_{[n]} \Rightarrow \begin{array}{c}\mathcal{L}(v, \rightarrow_{[n]}, \rightarrow_{\mathsf{br},[>n]}) \\ \text{is a loop subchart}\end{array} \right)$.

L2.

L3.

$\mathcal{L}(v, \rightarrow_{[n]}, \rightarrow_{\mathsf{br},[>n]})$:= subchart induced
by entry steps $\rightarrow_{[n]}$ from v
followed by branch steps $\rightarrow_{\mathsf{br}}$
or entry steps $\rightarrow_{[m]}$ with $m > n$,
until v is reached again

LEE-witness



loop–branch labeling: marking transitions $\xrightarrow{a}$ as:

- entry steps $\xrightarrow{\langle a,[n]\rangle}$ for $n \in \mathbb{N}$, written $\xrightarrow{a}_{[n]}$,
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Definition

A loop–branch labeling is a LEE-witness, if:

L1. $\forall n \in \mathbb{N} \forall v \in V \left(\begin{array}{r} v \rightarrow_{[n]} \Rightarrow \mathcal{L}(v, \rightarrow_{[n]}, \rightarrow_{\text{br},[>n]}) \\ \text{is a loop subchart} \end{array} \right)$.

L2. No infinite $\rightarrow_{\text{br}}$ path from start vertex.

L3.

$\mathcal{L}(v, \rightarrow_{[n]}, \rightarrow_{\text{br},[>n]})$:= subchart induced
by entry steps $\rightarrow_{[n]}$ from v
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LEE-witness

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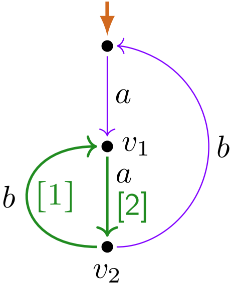

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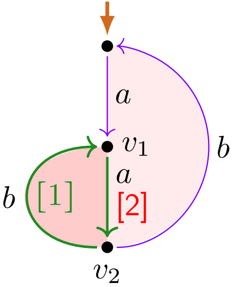

$\mathcal{L}(v_2, \rightarrow_{[1]}, \rightarrow_{\mathsf{br},[>1]})$

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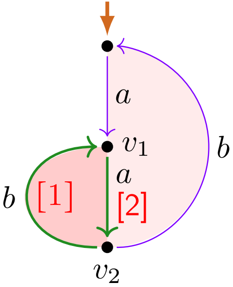

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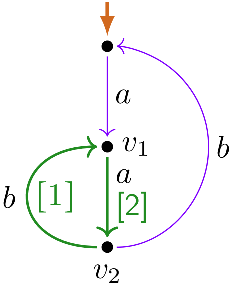

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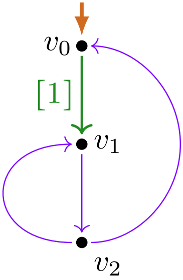

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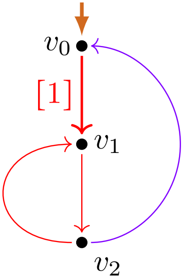

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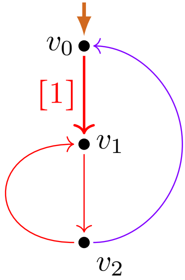


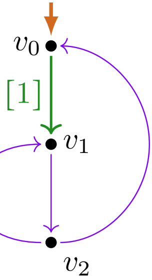
no!

(L1.) violated:

$\mathcal{L}(v_0, \to_{[1]}, \to_{\mathsf{br},[>1]})$

nota loop chart

LEE-witness ?



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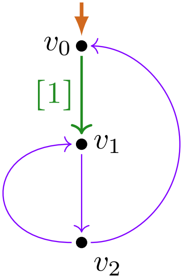
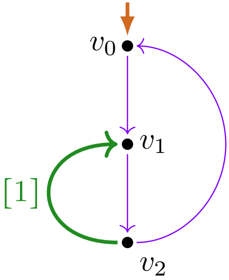
LEE-witness ?





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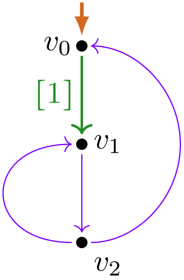
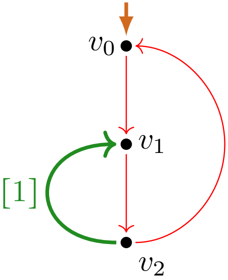
LEE-witness ?





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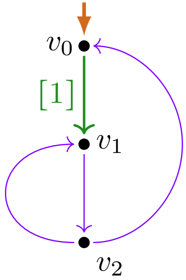
LEE-witness ?



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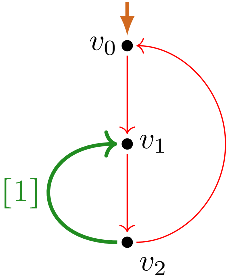
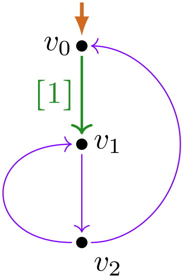


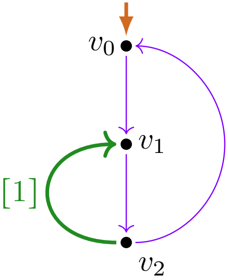
no!

(L2.) violated:

infinite $\to_{\text{br}}$ path
from start vertex

LEE-witness ?





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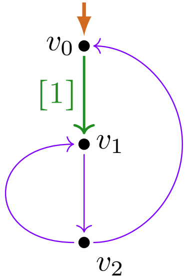

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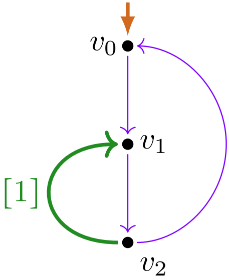
not a loop chart



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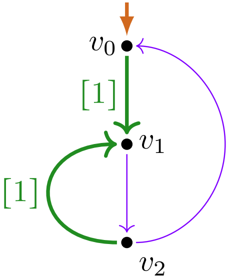
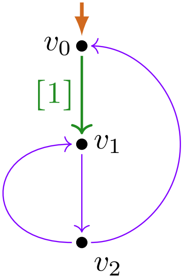


LEE-witness ?



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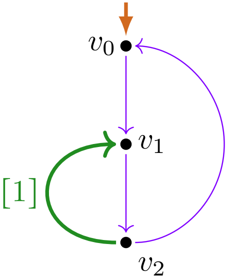

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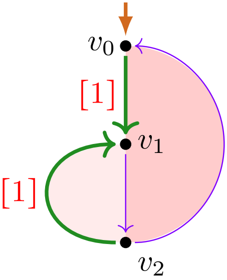

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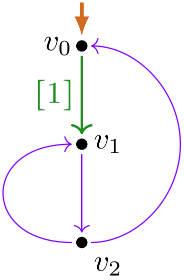

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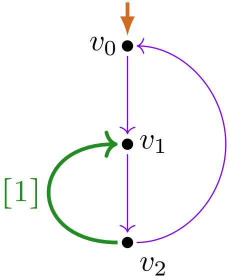

no!

(L2.) violated:

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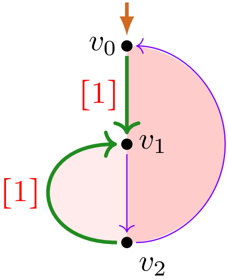


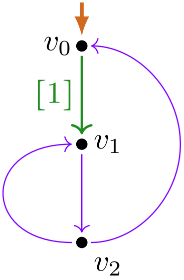
no!

(L3.) violated:

overlapping loop charts

have samelevel

LEE-witness ?

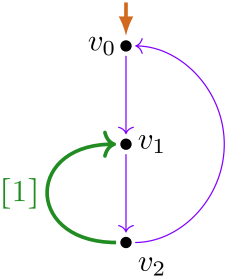


no!

(L1.) violated:

$\mathcal{L}(v_0, \to_{[1]}, \to_{\mathsf{br},[>1]})$

not a loop chart



no!

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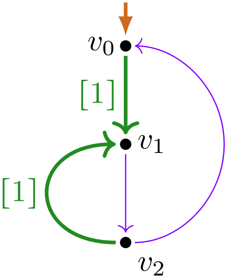

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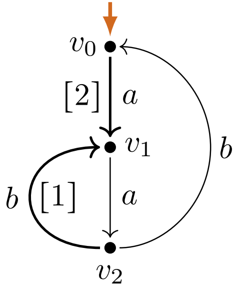
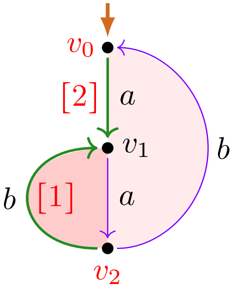


LEE-witness



$\mathcal{L}(v_2, \rightarrow_{[1]}, \rightarrow_{\mathsf{br},[>1]})$

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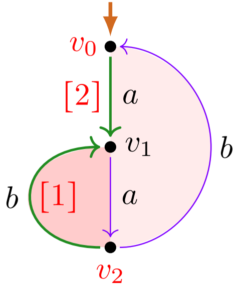
Layered LEE-witness



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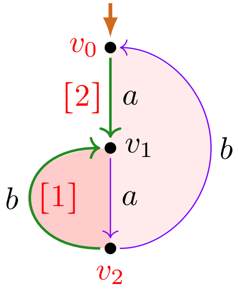

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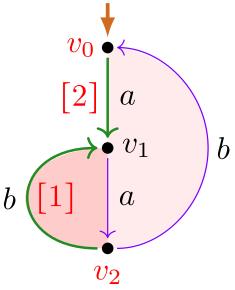
Layered LEE-witness



$\mathcal{L}(v_2, \to_{[1]}, \to_{\text{br}})$

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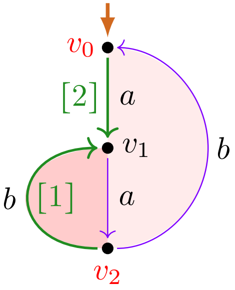
Layered LEE-witness



$\mathcal{L}(v_2, \rightarrow_{[1]}, \rightarrow_{\text{br}})$
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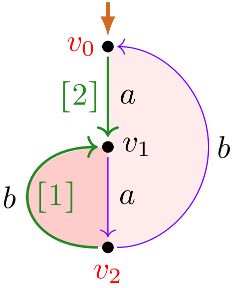
l-L2. No infinite $\rightarrow_{\text{br}}$ path from start vertex.

l-L3. A loop subchart generated by a vertex contained in another generated loop subchart has lower level.

$\mathcal{L}(v, \rightarrow_{[n]}, \rightarrow_{\text{br}})$:= subchart induced
by entry steps $\rightarrow_{[n]}$ from v
followed by branch steps $\rightarrow_{\text{br}}$

Layered LEE-witness



$\mathcal{L}(v_2, \rightarrow_{[1]}, \rightarrow_{\mathsf{br}})$

$\mathcal{L}(v_0, \rightarrow_{[2]}, \rightarrow_{\mathsf{br}})$

layered
LEE-witness

loop–branch labeling: marking transitions $\xrightarrow{a}$ as:

- entry steps $\xrightarrow{\langle a,[n]\rangle}$ for $n \in \mathbb{N}$, written $\xrightarrow{a}_{[n]}$,
- branch steps $\xrightarrow{\langle a,\mathsf{br}\rangle}$, written $\xrightarrow{a}_{\mathsf{br}}$ or $\xrightarrow{a}$.

Definition

A loop–branch labeling is a layered LEE-witness, if:

I-L1. $\forall n \in \mathbb{N} \forall v \in V \left(v \rightarrow_{[n]} \Rightarrow \begin{array}{r} \mathcal{L}(v, \rightarrow_{[n]}, \rightarrow_{\mathsf{br}}) \\ \text{is a loop subchart} \end{array} \right)$.

I-L2. No infinite $\rightarrow_{\mathsf{br}}$ path from start vertex.

I-L3. A loop subchart generated by a vertex contained in another generated loop subchart has lower level.

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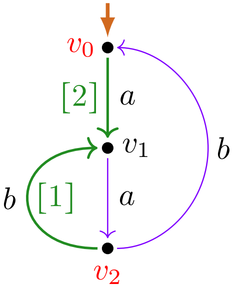
Layered LEE-witness



$\mathcal{L}(v_2, \to_{[1]}, \to_{\text{br}})$

$\mathcal{L}(v_0, \to_{[2]}, \to_{\text{br}})$

layered LEE-witness

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Definition

A loop–branch labeling is a layered LEE-witness, if:

l-L1. $\forall n \in \mathbb{N} \forall v \in V \Big(v \to_{[n]} \Rightarrow \mathcal{L}(v, \to_{[n]}, \to_{\text{br}})$ is a loop subchart $\Big)$.

l-L2. No infinite $\to_{\text{br}}$ path from start vertex.

l-L3. A loop subchart generated by a vertex contained in another generated loop subchart has lower level.

$\mathcal{L}(v, \to_{[n]}, \to_{\text{br}})$:= subchart induced
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LEE versus LEE-witness

Theorem

For every process graph G :

$$\mathsf{LEE}(G) \iff G \text{ has a } \mathsf{LEE}\text{-witness.}$$

LEE versus LEE-witness

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Proof.

$\Rightarrow$: record loop elimination

LEE versus LEE-witness

Theorem

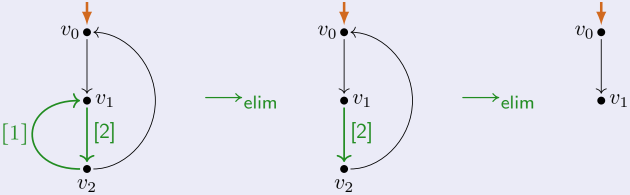
For every process graph G :

$$\mathsf{LEE}(G) \iff G \textit{ has a } \mathsf{LEE}\textit{-witness.}$$

Proof.

$\Rightarrow$: record loop elimination

$\Leftarrow$: carry out loop-elimination as indicated in the LEE-witness, in *inside–out* direction, e.g.:



LEE and (layered) LEE-witness

Lemma

Every layered LEE-witness is a LEE-witness.

Lemma

Every LEE-witness $\widehat{G}$ of a process graph G
can be transformed by an effective procedure (cut-elimination-like)
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Every LEE-witness $\widehat{G}$ of a process graph G can be transformed by an effective procedure (cut-elimination-like) into a layered LEE-witness $\widehat{G}'$ of G.

Lemma

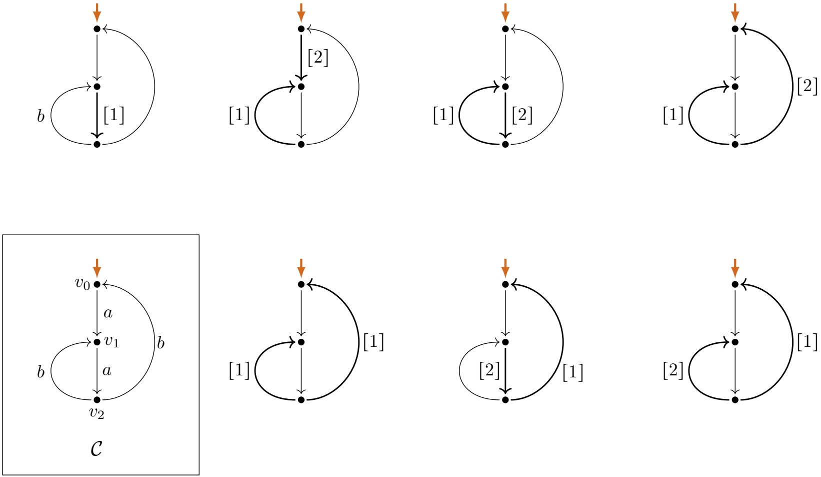
For every process graph G the following are equivalent:

(i) LEE(G).

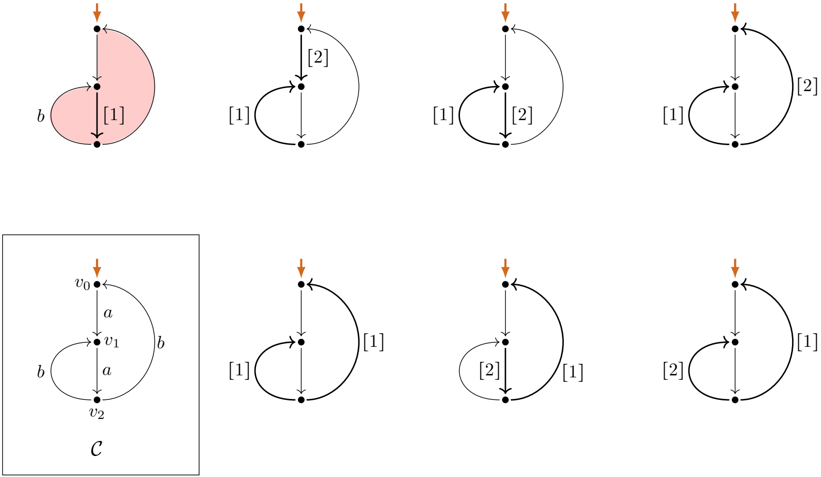
(ii) *G has a LEE-witness.*

(iii) *G has a layered LEE-witness.*

7 LEE-witnesses



7 LEE-witnesses



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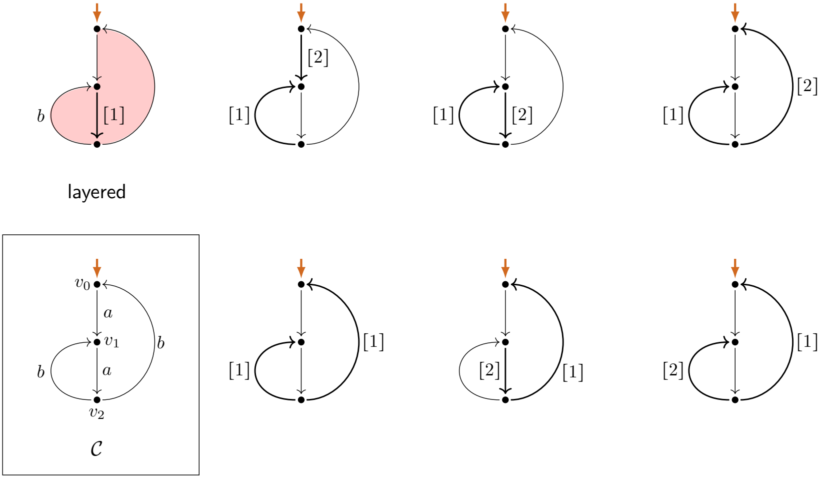

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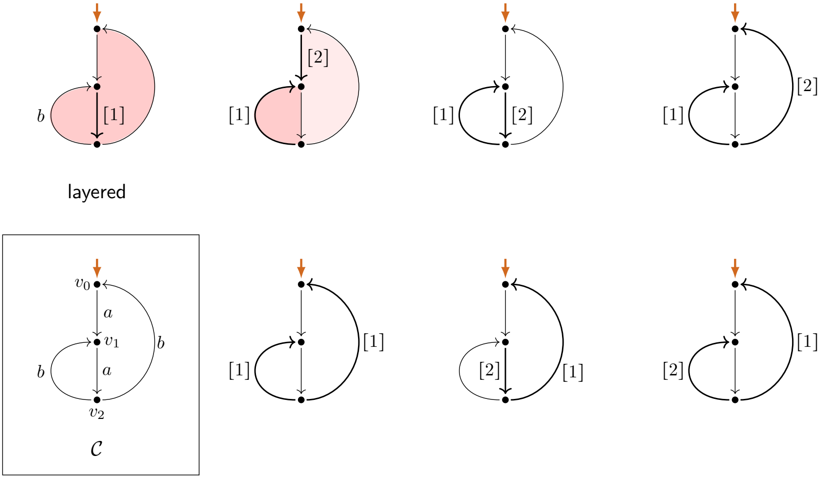

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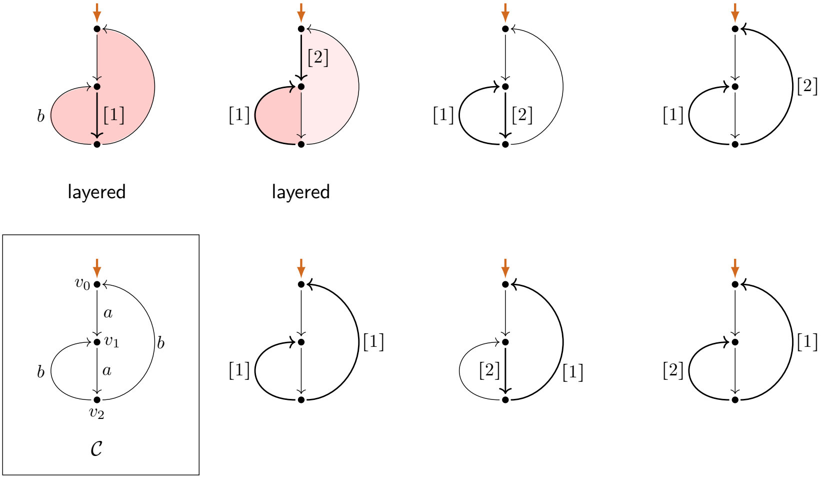

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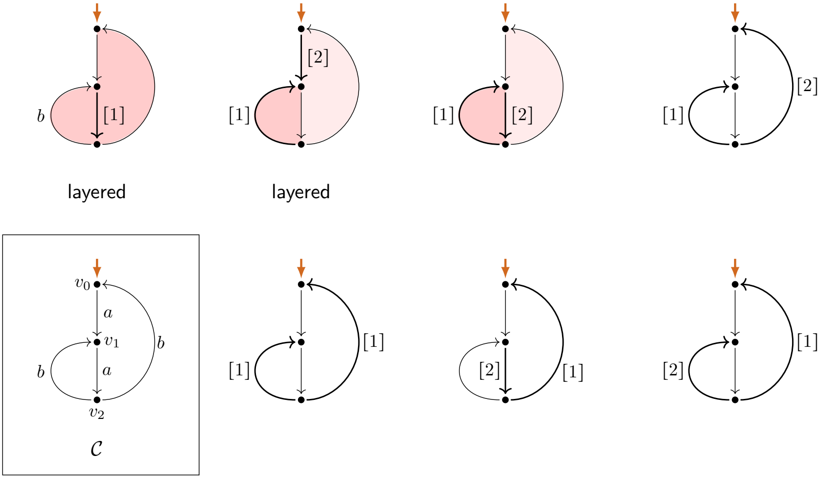
7 LEE-witnesses



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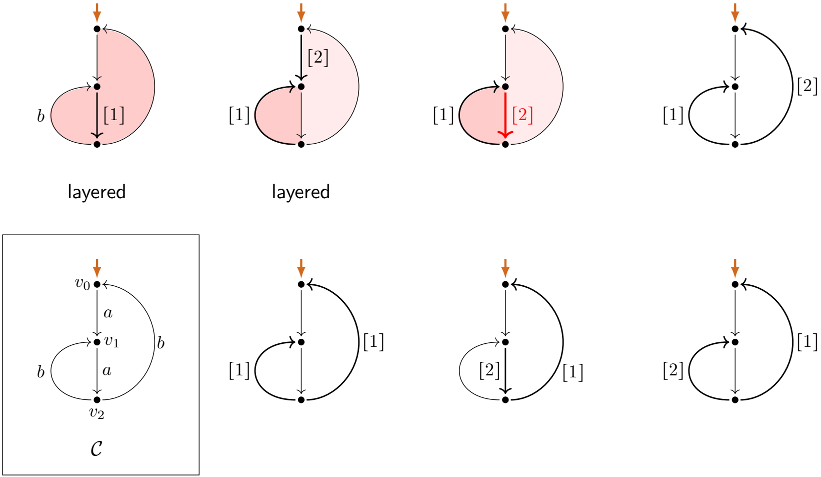

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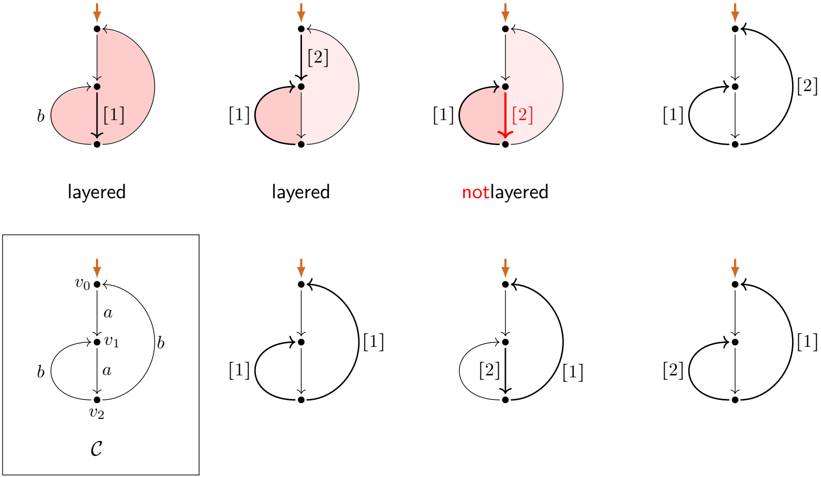

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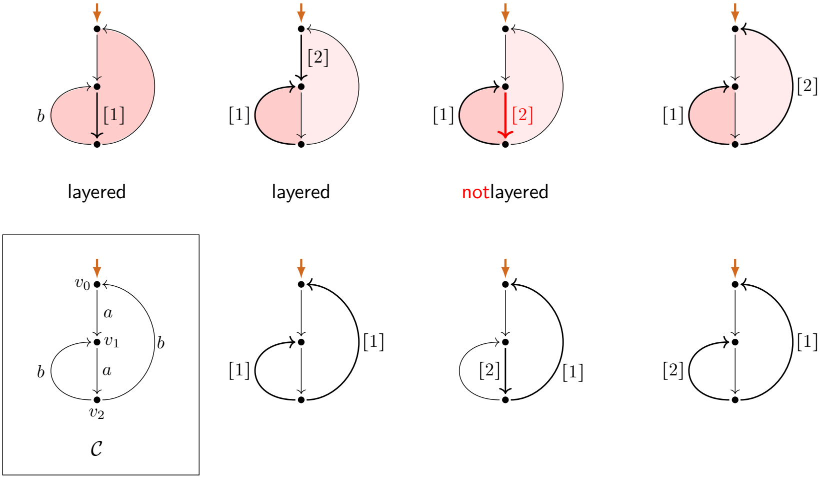

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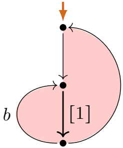

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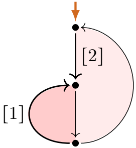

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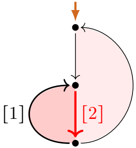

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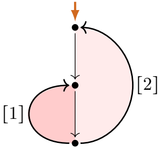

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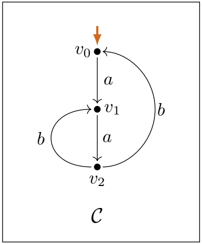

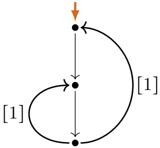

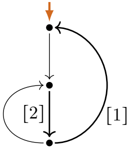

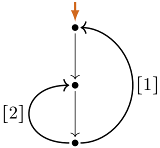

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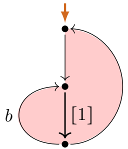

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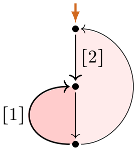

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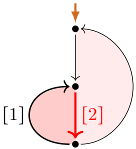

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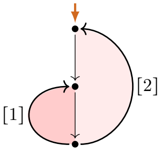

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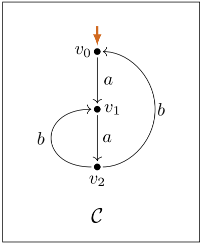

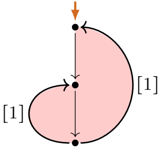

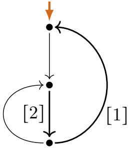

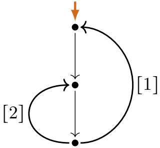

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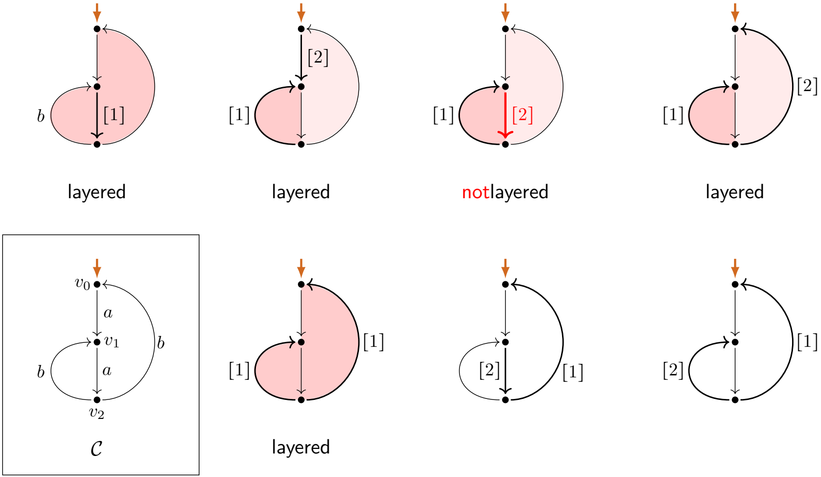

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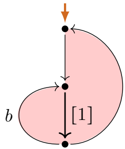
7 LEE-witnesses



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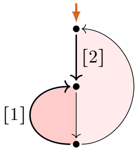

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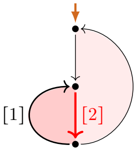

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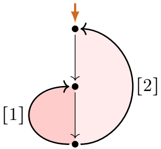

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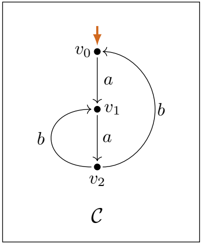

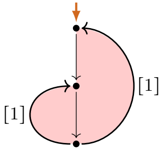

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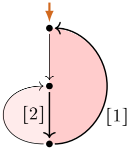

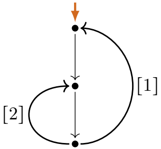

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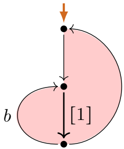

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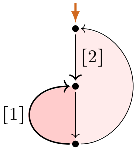

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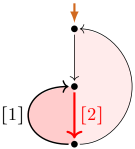

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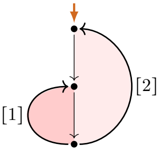

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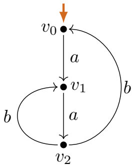

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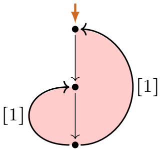

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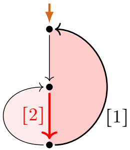

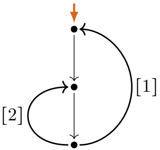

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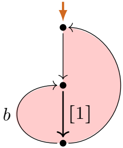

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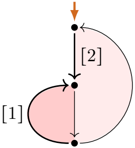

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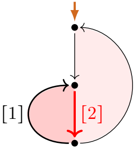

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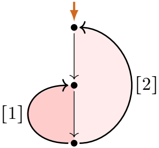

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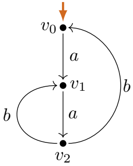

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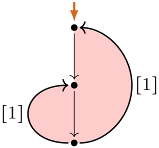

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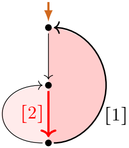

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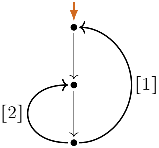

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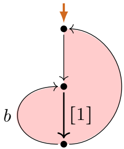

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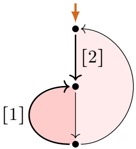

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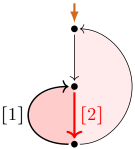

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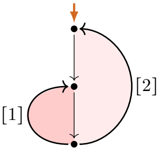

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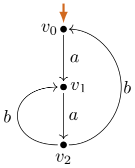

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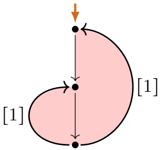

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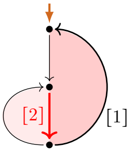

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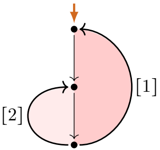

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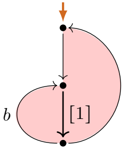

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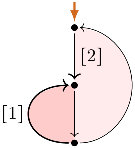

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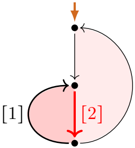

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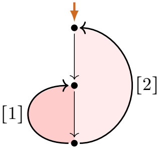

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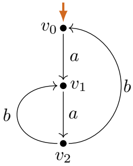

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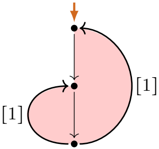

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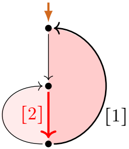

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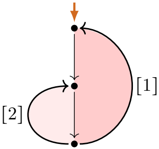

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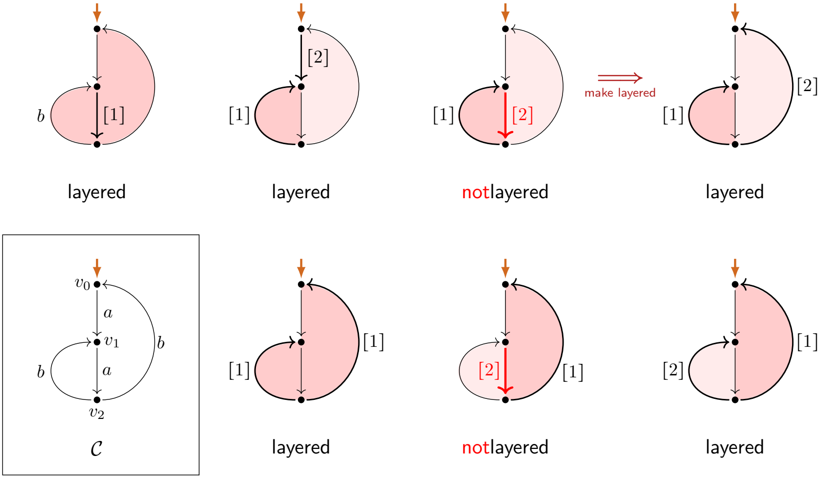
7 LEE-witnesses



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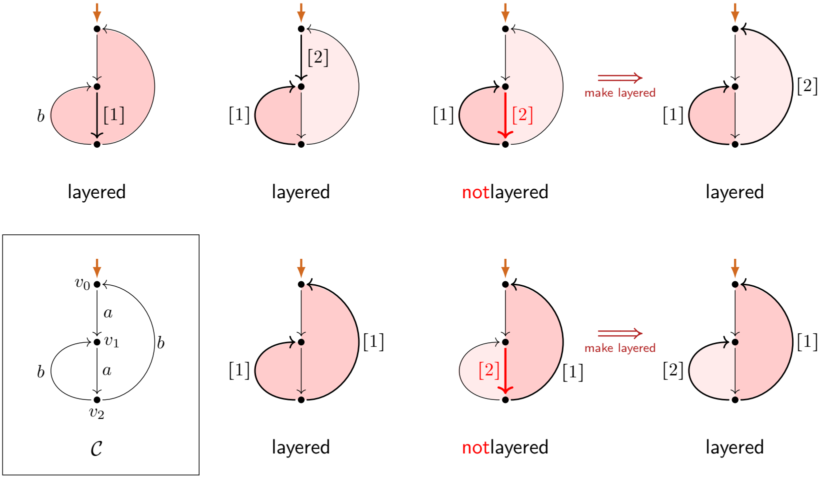
7 LEE-witnesses



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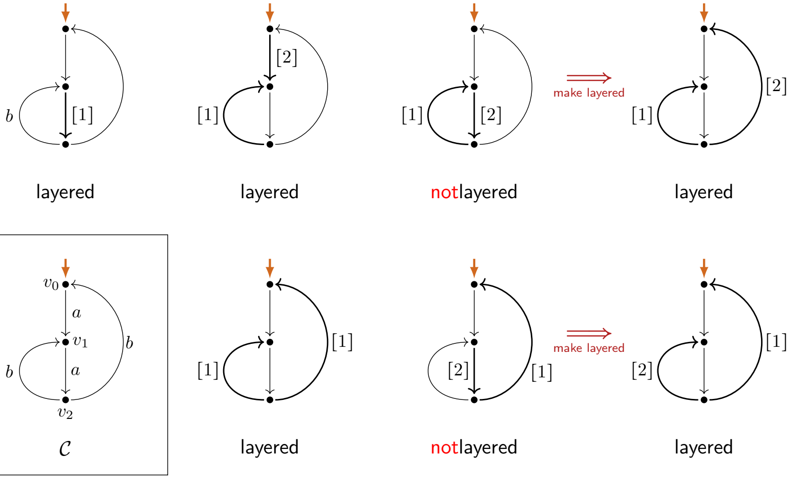
7 LEE-witnesses



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LEE under bisimulation?

LEE under bisimulation

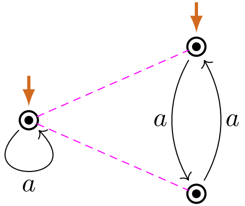
Observation

- LEE is not invariant under bisimulation.

LEE under bisimulation

Observation

- LEE is not invariant under bisimulation.

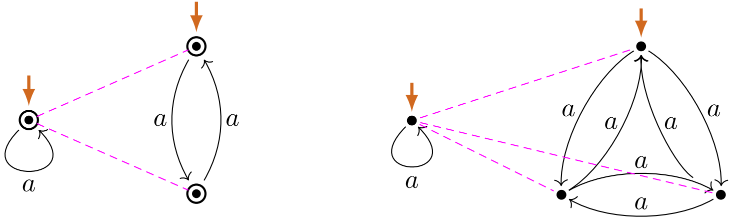


LEE ¬LEE

LEE under bisimulation

Observation

- LEE is not invariant under bisimulation.

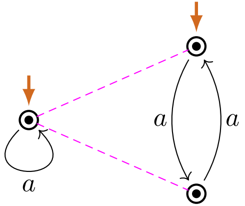


LEE ¬LEE LEE ¬LEE

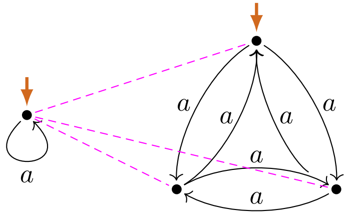
LEE under bisimulation

Observation

- LEE is not invariant under bisimulation.
- LEE is not preserved by converse functional bisimulation.



LEE ¬LEE



LEE ¬LEE

LEE under functional bisimulation

Lemma

(i) LEE *is preserved by functional bisimulations:*

$$\mathsf{LEE}(G_1) \wedge G_1 \mathrel{\underline{\rightarrow}} G_2 \implies \mathsf{LEE}(G_2)\ .$$

LEE under functional bisimulation

Lemma

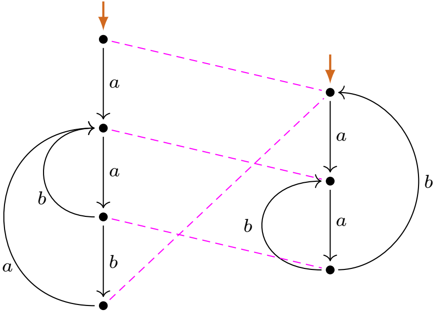
(i) LEE *is preserved by functional bisimulations:*

$$\mathsf{LEE}(G_1) \wedge G_1 \mathrel{\underline{\rightarrow}} G_2 \implies \mathsf{LEE}(G_2)\ .$$

Proof (Idea).

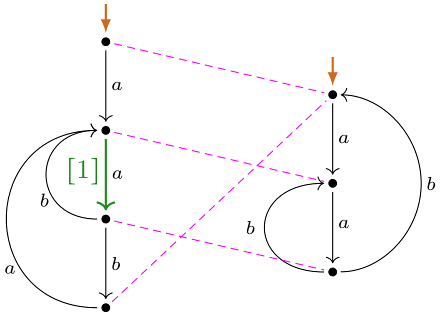
Use loop elimination in G_1 to carry out loop elimination in G_2.

Collapsing LEE-witnesses



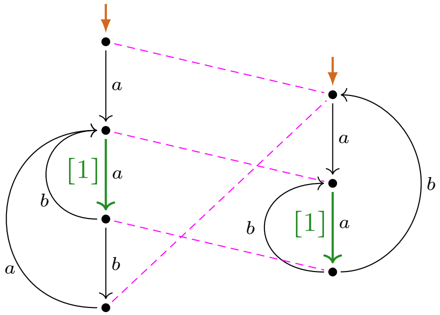
$P(a(a(b + ba))^*0)$

Collapsing LEE-witnesses



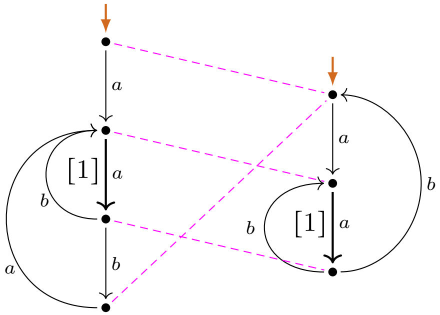
$P(a(a(b+ba))^*0)$

Collapsing LEE-witnesses



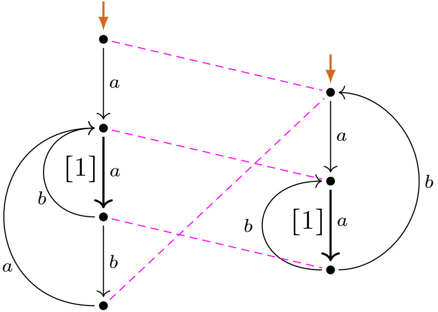
$P(a(a(b+ba))^*0)$

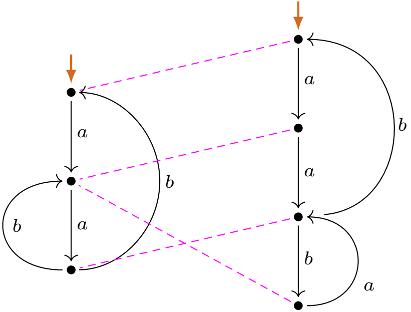
Collapsing LEE-witnesses



$P(a(a(b+ba))^{*}0)$

Collapsing LEE-witnesses

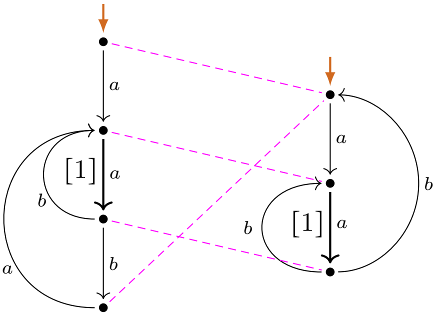




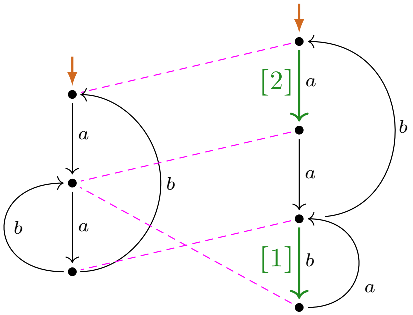
$P(a(a(b+ba))^*0)$

$P((aa(ba)^*b)^*0)$

Collapsing LEE-witnesses

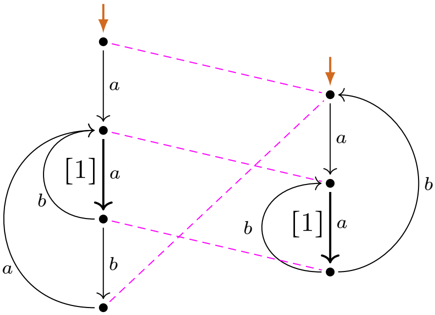


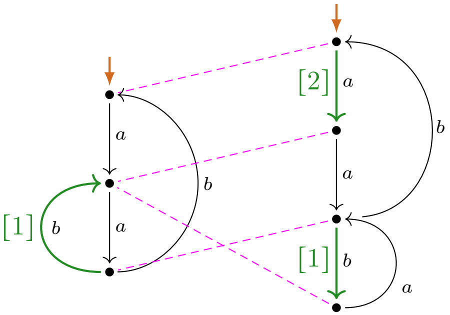
$P(a(a(b+ba))^*0)$



$P((aa(ba)^*b)^*0)$

Collapsing LEE-witnesses

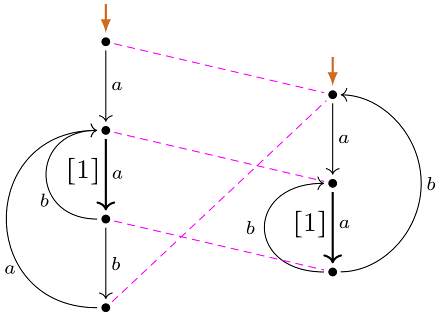


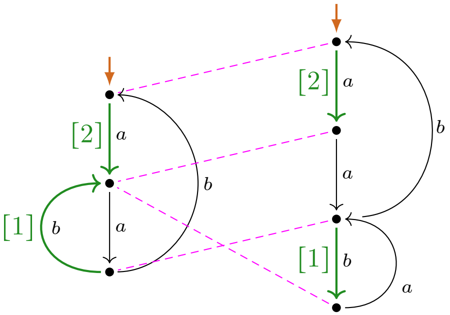


$P(a(a(b+ba))^{*}0)$

$P((aa(ba)^{*}b)^{*}0)$

Collapsing LEE-witnesses

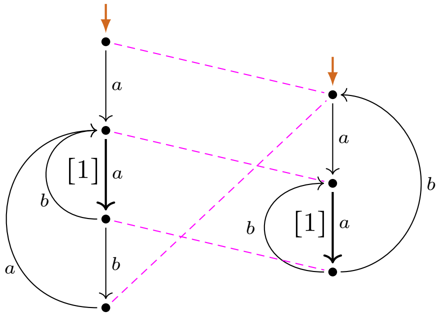


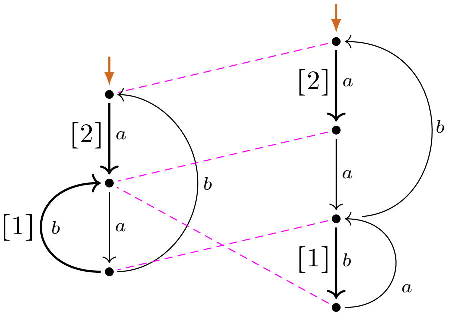


$P(a(a(b+ba))^*0)$

$P((aa(ba)^*b)^*0)$

Collapsing LEE-witnesses





$P(a(a(b+ba))^*0)$

$P((aa(ba)^*b)^*0)$

LEE under functional bisimulation

Lemma

(i) LEE *is preserved by functional bisimulations:*

$$\mathsf{LEE}(G_1) \wedge G_1 \mathrel{\underline{\rightarrow}} G_2 \implies \mathsf{LEE}(G_2)\ .$$

Idea of Proof for (i)

Use loop elimination in G_1 to carry out loop elimination in G_2.

LEE under functional bisimulation / bisimulation collapse

Lemma

(i) LEE *is preserved by functional bisimulations:*

$$\mathsf{LEE}(G_1) \wedge G_1 \mathrel{\underline{\rightarrow}} G_2 \implies \mathsf{LEE}(G_2)\ .$$

(ii) LEE *is preserved from a process graph to its bisimulation collapse:*

$$\mathsf{LEE}(G) \wedge C \textit{ is bisimulation collapse of } G \implies \mathsf{LEE}(C)\ .$$

Idea of Proof for (i)

Use loop elimination in G_1 to carry out loop elimination in G_2.

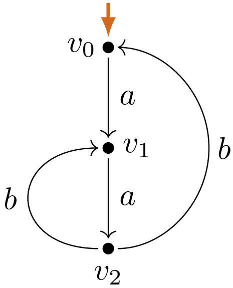
Readback

Lemma

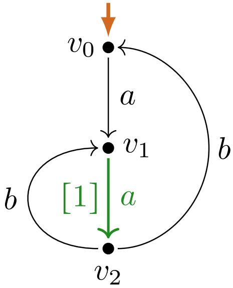
Process graphs with LEE are $P(\cdot)$-expressible:

$$\mathsf{LEE}(G) \implies \exists e \in \mathsf{Reg}(A) \left(G \mathrel{\underline{\leftrightarrow}} P(e) \right) .$$

Readback from layered LEE-witness (example)

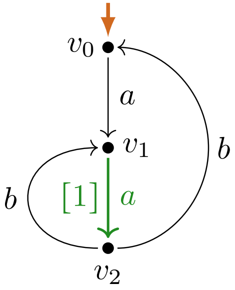


Readback from layered LEE-witness (example)



layered
LEE-witness

Readback from layered LEE-witness (example)

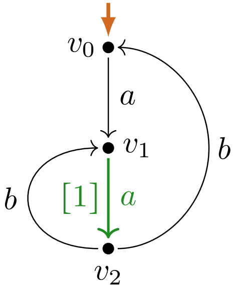


layered
LEE-witness

$$
\begin{aligned}
s(v_0) &= 0^* \cdot a \cdot s(v_1) \\
&=_{\mathsf{Mil}^-} a \cdot s(v_1) \\
&=_{\mathsf{Mil}^-} a \cdot \big(a \cdot (b + b \cdot a)\big)^* \cdot 0 \\
s(v_1) &= \big(a \cdot s(v_2, v_1)\big)^* \cdot 0 \\
&=_{\mathsf{Mil}^-} \big(a \cdot (b + b \cdot a)\big)^* \cdot 0 \\
s(v_2, v_1) &= 0^* \cdot \big(b \cdot s(v_1, v_1) + b \cdot s(v_0, v_1)\big) \\
&=_{\mathsf{Mil}^-} 0^* \cdot \big(b \cdot 1 + b \cdot a\big) \\
&=_{\mathsf{Mil}^-} b + b \cdot a \\
s(v_1, v_1) &= 1 \\
s(v_0, v_1) &= 0^* \cdot a \cdot s(v_1, v_1) \\
&= 0^* \cdot a \cdot 1 \\
&=_{\mathsf{Mil}^-} a
\end{aligned}
$$

Readback from layered LEE-witness (example)

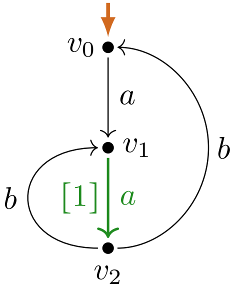
$$s(v_0) = \quad 0^* \cdot a \cdot s(v_1)$$



layered
LEE-witness

Readback from layered LEE-witness (example)

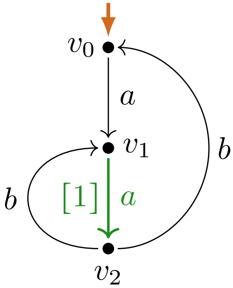
$$s(v_0) = \quad 0^* \cdot a \cdot s(v_1)$$



layered
LEE-witness

$$s(v_1) = \quad \big(a \cdot s(v_2, v_1)\big)^* \cdot 0$$

Readback from layered LEE-witness (example)



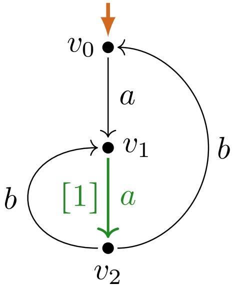
layered
LEE-witness

$$s(v_0) = \quad 0^* \cdot a \cdot s(v_1)$$

$$s(v_1) = \quad \big(a \cdot s(v_2, v_1)\big)^* \cdot 0$$

$$s(v_2, v_1) = \quad 0^* \cdot \big(b \cdot s(v_1, v_1) + b \cdot s(v_0, v_1)\big)$$

Readback from layered LEE-witness (example)



layered
LEE-witness

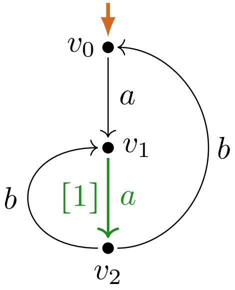
$$s(v_0) = \quad 0^* \cdot a \cdot s(v_1)$$

$$s(v_1) = \quad \big(a \cdot s(v_2, v_1)\big)^* \cdot 0$$

$$s(v_2, v_1) = \quad 0^* \cdot \big(b \cdot s(v_1, v_1) + b \cdot s(v_0, v_1)\big)$$

$$s(v_1, v_1) = \quad 1$$

Readback from layered LEE-witness (example)



layered
LEE-witness

$$s(v_0) = 0^* \cdot a \cdot s(v_1)$$

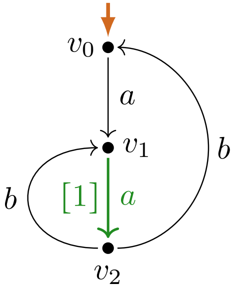
$$s(v_1) = \big({\color{green}a} \cdot s(v_2, v_1)\big)^* \cdot 0$$

$$s(v_2, v_1) = 0^* \cdot \big(b \cdot s(v_1, v_1) + b \cdot s(v_0, v_1)\big)$$

$$s(v_1, v_1) = 1$$

$$s(v_0, v_1) = 0^* \cdot a \cdot s(v_1, v_1)$$

Readback from layered LEE-witness (example)



layered
LEE-witness

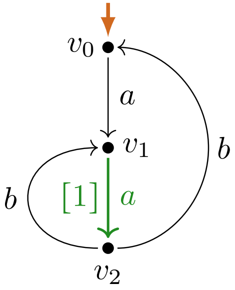
$$s(v_0) = 0^* \cdot a \cdot s(v_1)$$

$$s(v_1) = \big(a \cdot s(v_2, v_1)\big)^* \cdot 0$$

$$s(v_2, v_1) = 0^* \cdot \big(b \cdot s(v_1, v_1) + b \cdot s(v_0, v_1)\big)$$

$$\begin{aligned} s(v_1, v_1) &= 1 \\ s(v_0, v_1) &= 0^* \cdot a \cdot s(v_1, v_1) \\ &= 0^* \cdot a \cdot 1 \end{aligned}$$

Readback from layered LEE-witness (example)



layered
LEE-witness

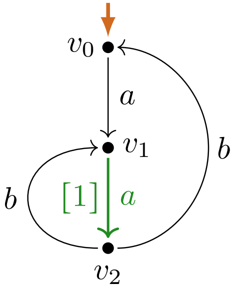
$$s(v_0) = 0^* \cdot a \cdot s(v_1)$$

$$s(v_1) = \big(a \cdot s(v_2, v_1)\big)^* \cdot 0$$

$$s(v_2, v_1) = 0^* \cdot \big(b \cdot s(v_1, v_1) + b \cdot s(v_0, v_1)\big)$$

$$\begin{aligned} s(v_1, v_1) &= 1 \\ s(v_0, v_1) &= 0^* \cdot a \cdot s(v_1, v_1) \\ &= 0^* \cdot a \cdot 1 \\ &=_{\mathsf{Mil}^-} a \end{aligned}$$

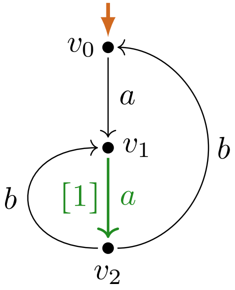
Readback from layered LEE-witness (example)



layered
LEE-witness

$$
\begin{aligned}
s(v_0) &= 0^* \cdot a \cdot s(v_1) \\
s(v_1) &= \big(a \cdot s(v_2, v_1)\big)^* \cdot 0 \\
s(v_2, v_1) &= 0^* \cdot \big(b \cdot s(v_1, v_1) + b \cdot s(v_0, v_1)\big) \\
&=_{\mathsf{Mil}^-} 0^* \cdot \big(b \cdot 1 + b \cdot a\big) \\
s(v_1, v_1) &= 1 \\
s(v_0, v_1) &= 0^* \cdot a \cdot s(v_1, v_1) \\
&= 0^* \cdot a \cdot 1 \\
&=_{\mathsf{Mil}^-} a
\end{aligned}
$$

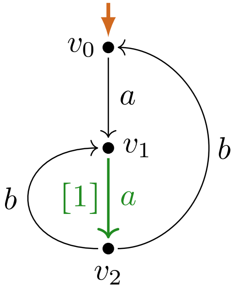
Readback from layered LEE-witness (example)



layered
LEE-witness

$$
\begin{aligned}
s(v_0) &= \quad 0^* \cdot a \cdot s(v_1) \\
s(v_1) &= \quad \big(a \cdot s(v_2, v_1)\big)^* \cdot 0 \\
s(v_2, v_1) &= \quad 0^* \cdot \big(b \cdot s(v_1, v_1) + b \cdot s(v_0, v_1)\big) \\
&=_{\mathsf{Mil}^-} 0^* \cdot \big(b \cdot 1 + b \cdot a\big) \\
&=_{\mathsf{Mil}^-} b + b \cdot a \\
s(v_1, v_1) &= \quad 1 \\
s(v_0, v_1) &= \quad 0^* \cdot a \cdot s(v_1, v_1) \\
&= \quad 0^* \cdot a \cdot 1 \\
&=_{\mathsf{Mil}^-} a
\end{aligned}
$$

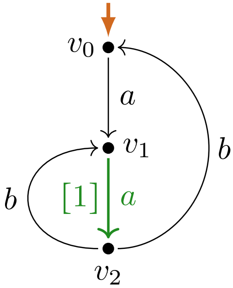
Readback from layered LEE-witness (example)



layered
LEE-witness

$$
\begin{aligned}
s(v_0) &= 0^* \cdot a \cdot s(v_1) \\
s(v_1) &= \big(a \cdot s(v_2, v_1)\big)^* \cdot 0 \\
&=_{\mathsf{Mil}^-} \big(a \cdot (b + b \cdot a)\big)^* \cdot 0 \\
s(v_2, v_1) &= 0^* \cdot \big(b \cdot s(v_1, v_1) + b \cdot s(v_0, v_1)\big) \\
&=_{\mathsf{Mil}^-} 0^* \cdot \big(b \cdot 1 + b \cdot a\big) \\
&=_{\mathsf{Mil}^-} b + b \cdot a \\
s(v_1, v_1) &= 1 \\
s(v_0, v_1) &= 0^* \cdot a \cdot s(v_1, v_1) \\
&= 0^* \cdot a \cdot 1 \\
&=_{\mathsf{Mil}^-} a
\end{aligned}
$$

Readback from layered LEE-witness (example)



layered LEE-witness

$$
\begin{aligned}
s(v_0) &= 0^* \cdot a \cdot s(v_1) \\
&=_{\mathsf{Mil}^-} a \cdot s(v_1)
\end{aligned}
$$

$$
\begin{aligned}
s(v_1) &= \big(a \cdot s(v_2, v_1)\big)^* \cdot 0 \\
&=_{\mathsf{Mil}^-} \big(a \cdot (b + b \cdot a)\big)^* \cdot 0 \\
s(v_2, v_1) &= 0^* \cdot \big(b \cdot s(v_1, v_1) + b \cdot s(v_0, v_1)\big) \\
&=_{\mathsf{Mil}^-} 0^* \cdot \big(b \cdot 1 + b \cdot a\big) \\
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s(v_1, v_1) &= 1 \\
s(v_0, v_1) &= 0^* \cdot a \cdot s(v_1, v_1) \\
&= 0^* \cdot a \cdot 1 \\
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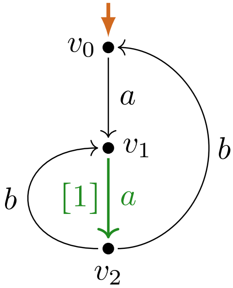
\end{aligned}
$$

Readback from layered LEE-witness (example)



layered
LEE-witness

$$\begin{aligned}
s(v_0) &= 0^* \cdot a \cdot s(v_1) \\
&=_{\mathsf{Mil}^-} a \cdot s(v_1) \\
&=_{\mathsf{Mil}^-} a \cdot \big(a \cdot (b + b \cdot a)\big)^* \cdot 0 \\
s(v_1) &= \big(a \cdot s(v_2, v_1)\big)^* \cdot 0 \\
&=_{\mathsf{Mil}^-} \big(a \cdot (b + b \cdot a)\big)^* \cdot 0 \\
s(v_2, v_1) &= 0^* \cdot \big(b \cdot s(v_1, v_1) + b \cdot s(v_0, v_1)\big) \\
&=_{\mathsf{Mil}^-} 0^* \cdot \big(b \cdot 1 + b \cdot a\big) \\
&=_{\mathsf{Mil}^-} b + b \cdot a \\
s(v_1, v_1) &= 1 \\
s(v_0, v_1) &= 0^* \cdot a \cdot s(v_1, v_1) \\
&= 0^* \cdot a \cdot 1 \\
&=_{\mathsf{Mil}^-} a
\end{aligned}$$

1-return-less regular expressions

Lemma

Process graphs with LEE are $P(\cdot)$-expressible:

$$\mathsf{LEE}(G) \implies \exists e \in \mathrm{Reg}(A) \left(G \underline{\leftrightarrow} P(e) \right) .$$

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- $(a \cdot (1 + b))^*$

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Characterization of expressibility$^{\cancel{\mathbf{1r}}\backslash\star}$ modulo $\underline{\leftrightarrow}$

Theorem

For every process graph G with bisimulation collapse C the following are equivalent:

(i) G *is* $[\![\cdot]\!]_{P}^{\cancel{\mathbf{1r}}\backslash\star}$*-expressible modulo* $\underline{\leftrightarrow}$.

(ii) $\mathsf{LEE}(C)$.

(iii) C *has a* LEE*-witness.*

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Milners characterization question:

Q1. Which structural property of finite process graphs

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Milners characterization question restricted:

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Milners characterization question restricted, and adapted:

$\mathrm{Q1}''$. Which structural property of collapsed finite process graphs characterizes $[\![\cdot]\!]_P^{\cancel{\mathbf{1r}}\backslash\star}$-expressibility modulo $\underline{\leftrightarrow}$?

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Answering Milners characterization question restricted, and adapted:

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- The loop-existence and elimination property LEE.

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Answering Milners characterization question restricted, and adapted:

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- The loop-existence and elimination property LEE.

Also yields: efficient decision method of $[\![\cdot]\!]_P^{\cancel{\mathbf{1r}}\backslash\star}$-expressibility modulo $\underline{\leftrightarrow}$.

Structure constrained finite process graphs

graphs with LEE / a (layered) LEE-witness

Benefits of the class of process graphs with LEE:

- is closed under $\underline{\rightarrow}$
- forth-/back-correspondence with 1-return-less regular expressions

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$\subsetneqq$ graphs whose collapse satisfies LEE

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Structure constrained finite process graphs

loop–exit palm trees $\subsetneqq$ $\llbracket\cdot\rrbracket_{P}^{\cancel{\mathbf{1r}}\backslash\star}$-expressible graphs

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Application to Milner's questions yields partial results:

Q1: characterization/efficient decision of $[\![\cdot]\!]_{P}^{\cancel{\mathbf{1r}}\backslash\star}$-expressibility modulo $\underline{\leftrightarrow}$

Q2: alternative compl. proof of Mil on 1-return-less expressions (C/DN/L)

Comparison results: structure-constrained graphs

λ-calculus with letrec under $=_{\lambda^\infty}$

Not available: graph interpretation that is studied under $\underline{\leftrightarrow}$

Regular expressions under $\underline{\leftrightarrow}_P$

Given: graph interpretation $P(\cdot)$, studied under bisimulation $\underline{\leftrightarrow}$

- not closed under $\underline{\rightarrow}$, and $\underline{\leftrightarrow}$, incomplete under $\underline{\leftrightarrow}$

Comparison results: structure-constrained graphs

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Not available: graph interpretation that is studied under $\underline{\leftrightarrow}$

Defined: int's $[\![\cdot]\!]_{\mathcal{H}}/[\![\cdot]\!]_{\mathcal{T}}$ as higher-order/first-order λ-term graphs

- closed under $\underline{\to}$ (hence under collapse)
- back-/forth correspondence with λ-calculus with letrec
  - efficient translation and readback
  - translation is inverse of readback

Regular expressions under $\underline{\leftrightarrow}_{P}$

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Regular expressions under $\underline{\leftrightarrow}_P$

Given: graph interpretation $P(\cdot)$, studied under bisimulation $\underline{\leftrightarrow}$

- not closed under $\underline{\rightarrow}$, and $\underline{\leftrightarrow}$, incomplete under $\underline{\leftrightarrow}$

Defined: class of process graphs with LEE / (layered) LEE-witness

- closed under $\underline{\rightarrow}$ (hence under collapse)
- back-/forth correspondence with 1-return-less expr's
- contains the collapse of a process graph G
  $\iff$ G is $[\![\cdot]\!]_{P}^{\cancel{\mathbf{1r}}\backslash\star}$-expressible modulo $\underline{\leftrightarrow}$