

# Core Course Preview

## Introduction to Model Checking

<https://clegra.github.io/mc/mc.html>

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# Model Checking

... is an effective automatable technique:

- ▶ *to expose potential software design errors;*
- ▶ *that, given a finite-state model of a system and a formal property, systematically checks whether this property holds for that model.*

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- ▶ widely applied in industry
  - ▶ embedded systems, software engineering, hardware design, symbolic AI
- ▶ supports **partial verification** (of system parts)
- ▶ provides **diagnostic information** for debugging
- ▶ has sound **mathematical underpinning** (**logic** and **process theory**)

# Model Checking

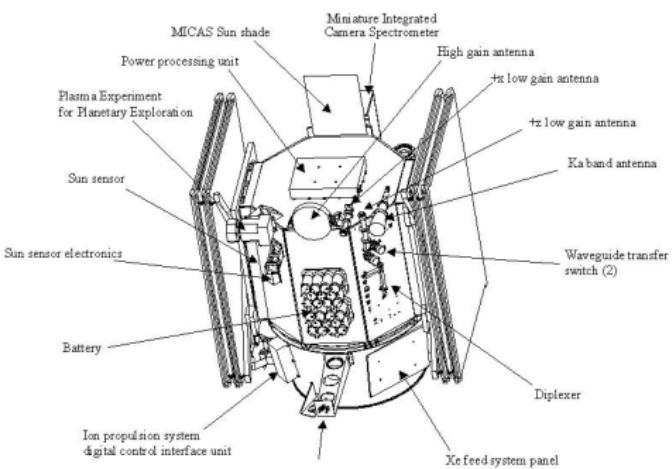
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**Course Goals** are introduction to:

- ▶ Theory:
  - ▶ **modeling** systems by **labeled transition systems**,
  - ▶ expressing **properties** by **temporal-logic formulas**
  - ▶ **model-checking algorithms**
- ▶ Practice: use the **Maude** system for examples

# Deep Space 1 (NASA)



- ▶ Flyby of asteroid 9969 Braille (1999)
- ▶ Entered the coma of Comet Borrelly (2001)
- ▶ Model checking discovered 5 concurrency errors

# Example (program concurrency/non-determinism)

Programs `Inc`, `Dec`, and `Reset` cooperate, and use a shared variable `x`:

```
proc Inc
  while true
    do
      if x < 200
        then x := x + 1
      fi
    od
```

```
proc Dec
  while true
    do
      if x > 0
        then x := x - 1
      fi
    od
```

```
proc Reset
  while true
    do
      if x = 200
        then x := 0
      fi
    od
```

# Example (program concurrency/non-determinism)

Programs `Inc`, `Dec`, and `Reset` cooperate, and use a shared variable `x`:

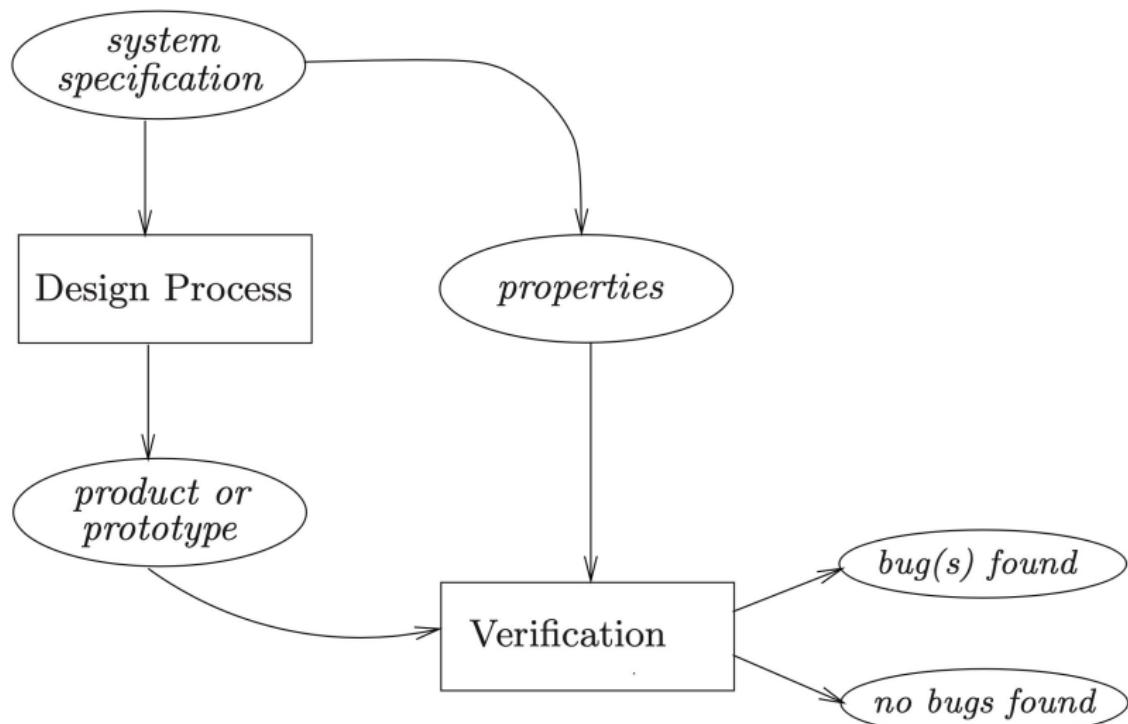
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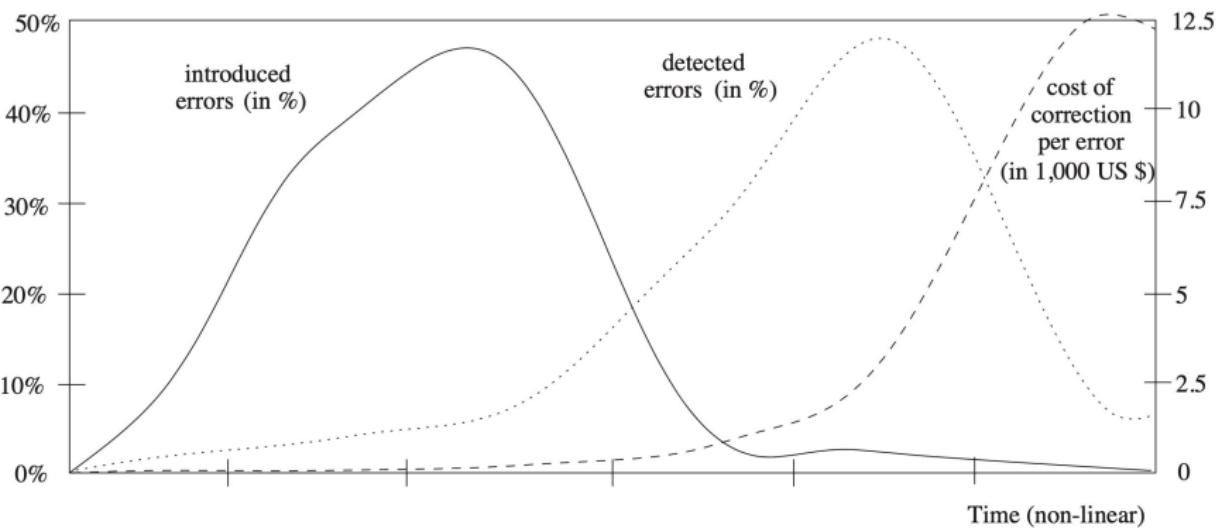
Question: Is  $0 \leq x \leq 200$  always guaranteed?

# Hard-/Software Verification (traditionally)

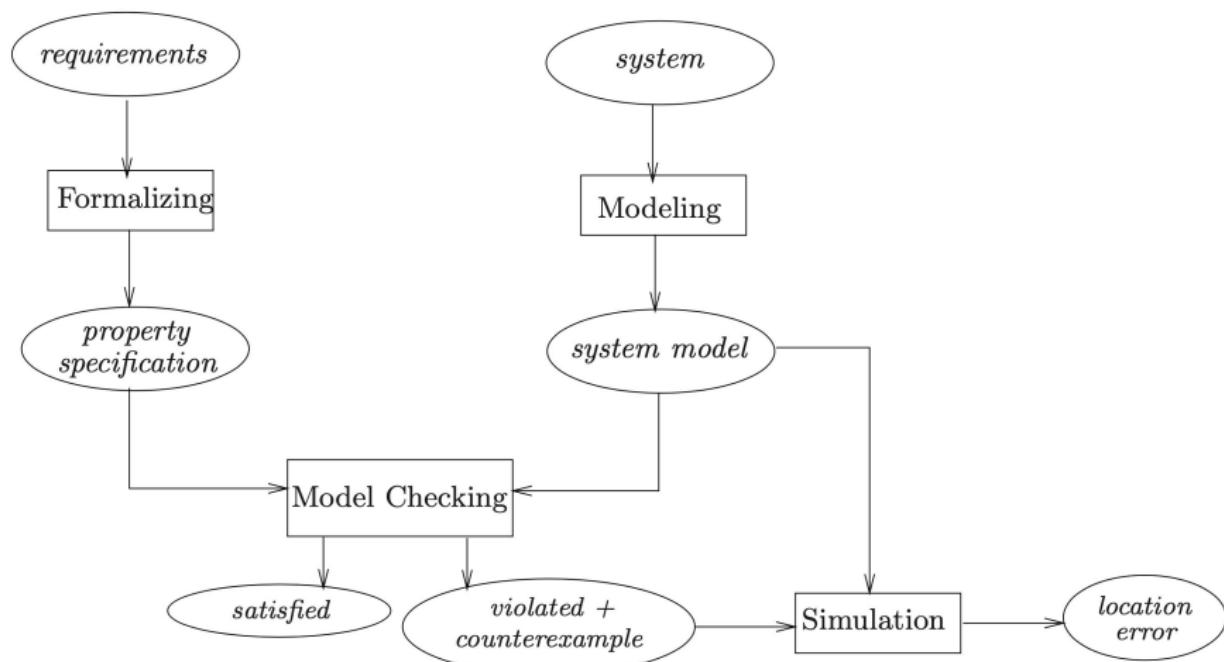


# Error introduction, detection, and repair costs

Analysis	Conceptual Design	Programming	Unit Testing	System Testing	Operation
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# Model checking



# Modeling (by program graphs)

**proc** Inc

**while** true

**do**

**if**  $x < 200$

**then**  $x := x + 1$

**fi**

**od**

**proc** Dec

**while** true

**do**

**if**  $x > 0$

**then**  $x := x - 1$

**fi**

**od**

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**while** true

**do**

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**then**  $x := 0$

**fi**

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# Modeling (by program graphs)

**proc Inc**

**while** true

**do**

1:   **if**  $x < 200$

2:   **then**  $x := x + 1$

**fi**

**od**

**proc Dec**

**while** true

**do**

1:   **if**  $x > 0$

2:   **then**  $x := x - 1$

**fi**

**od**

**proc Reset**

**while** true

**do**

1:   **if**  $x = 200$

2:   **then**  $x := 0$

**fi**

**od**

# Modeling (by program graphs)

```
proc Inc
```

```
  while true
```

```
    do
```

```
1:      if  $x < 200$ 
```

```
2:      then  $x := x + 1$ 
```

```
    fi
```

```
  od
```

```
proc Dec
```

```
  while true
```

```
    do
```

```
1:      if  $x > 0$ 
```

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2:      then  $x := x - 1$ 
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```

```
proc Reset
```

```
  while true
```

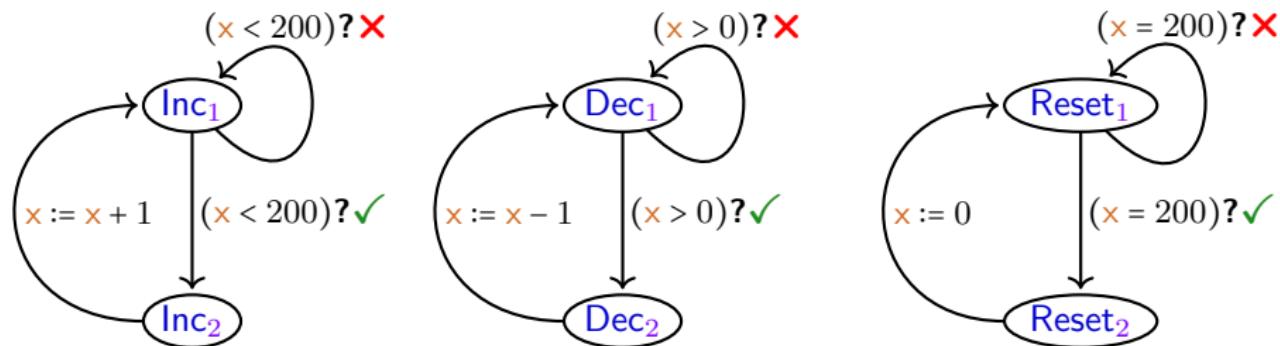
```
    do
```

```
1:      if  $x = 200$ 
```

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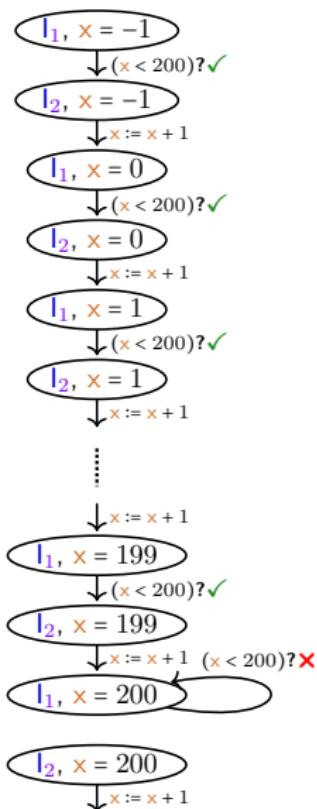
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    fi
```

```
  od
```

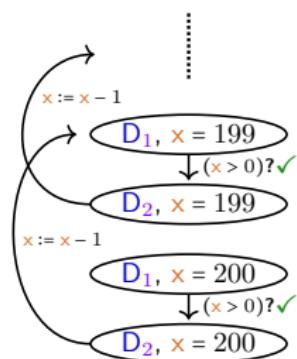
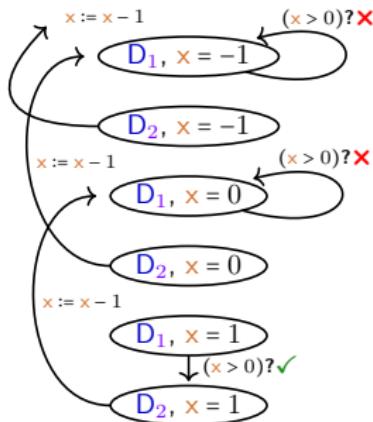


Program graphs (PG)

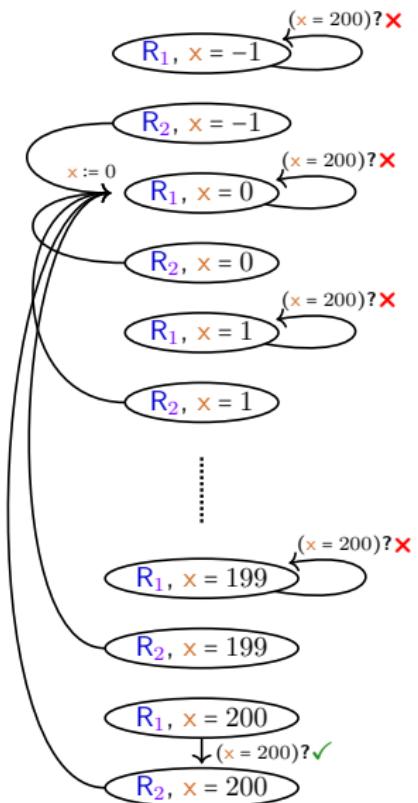
## Modeling (by labeled transition systems, with state space explosion)



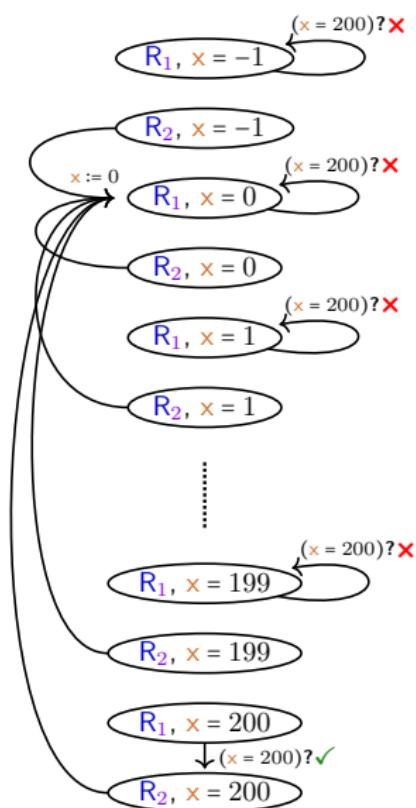
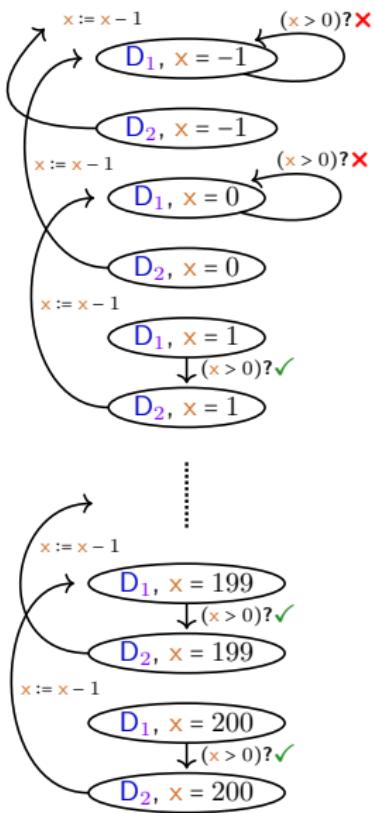
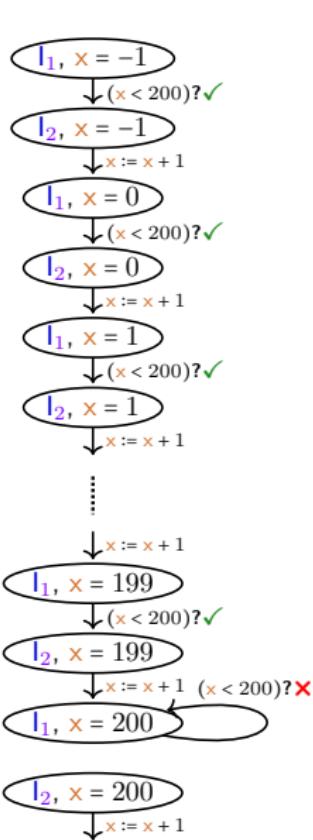
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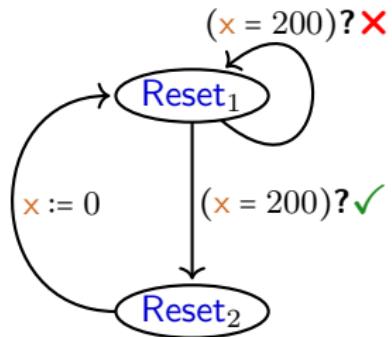
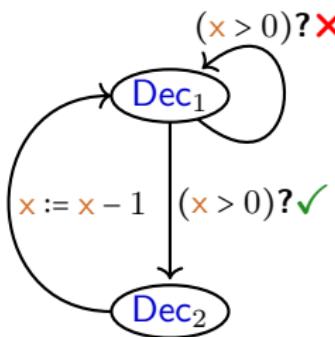
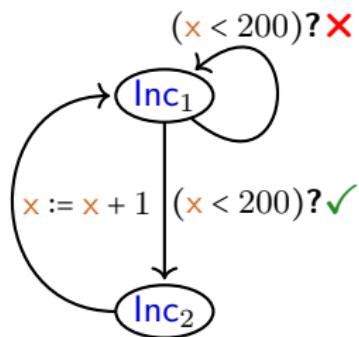
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## Modeling (by labeled transition systems, with state space explosion)



# Formalizing properties (in temporal logic)



$\text{Inc}_1 \parallel \text{Dec}_1 \parallel \text{Reset}_1 \stackrel{?}{\models} \square(0 \leq x \wedge x \leq 200)$  (Linear-TL formula)

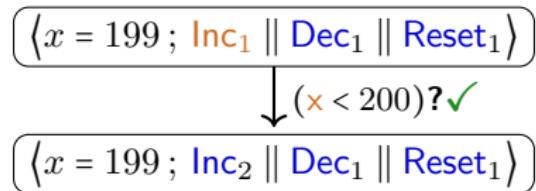
# Counterexample (offending execution trace)

$$\langle x = 199 ; \text{Inc}_1 \parallel \text{Dec}_1 \parallel \text{Reset}_1 \rangle$$

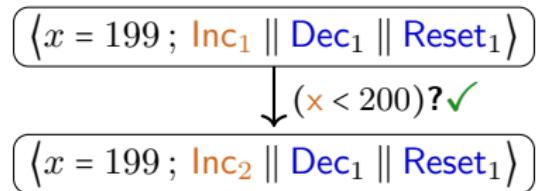
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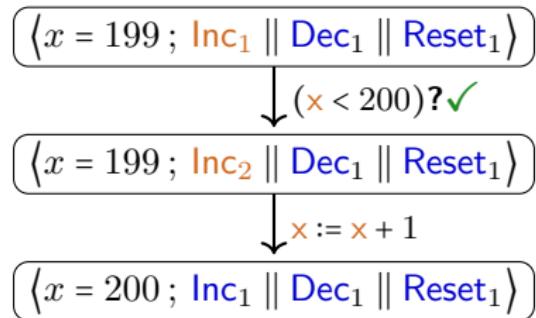
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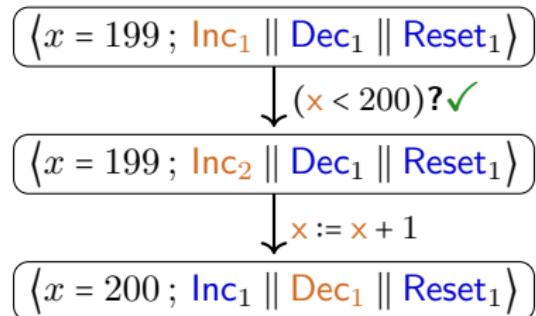
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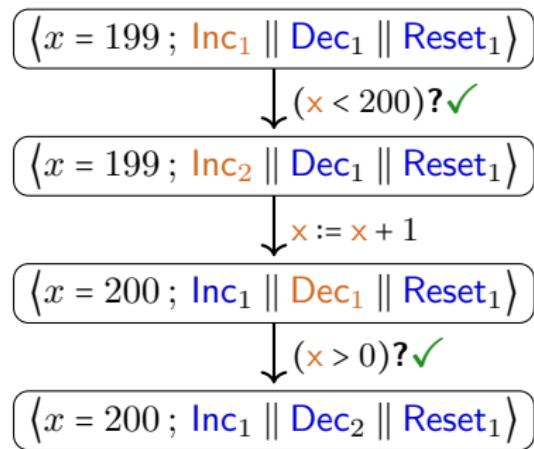
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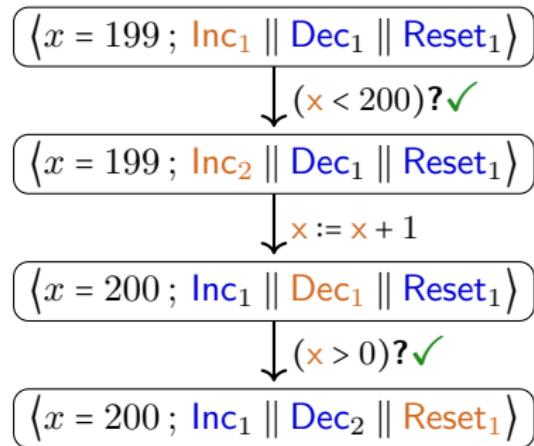
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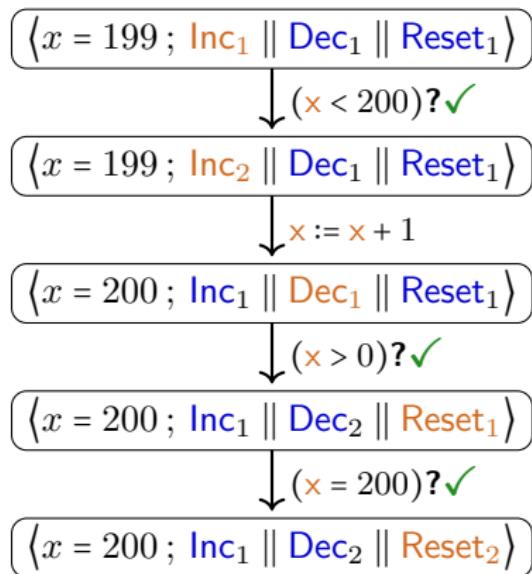
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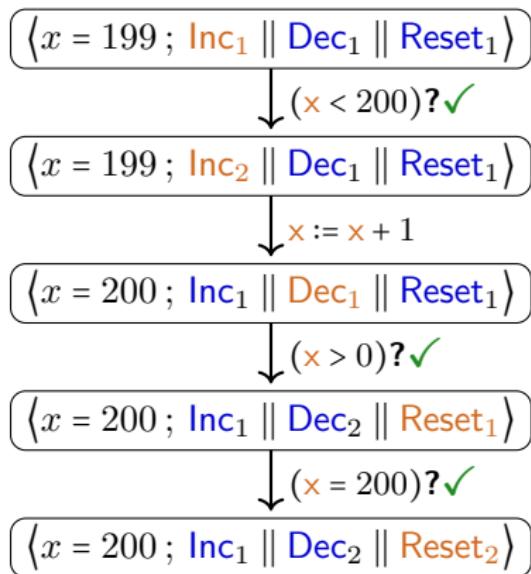
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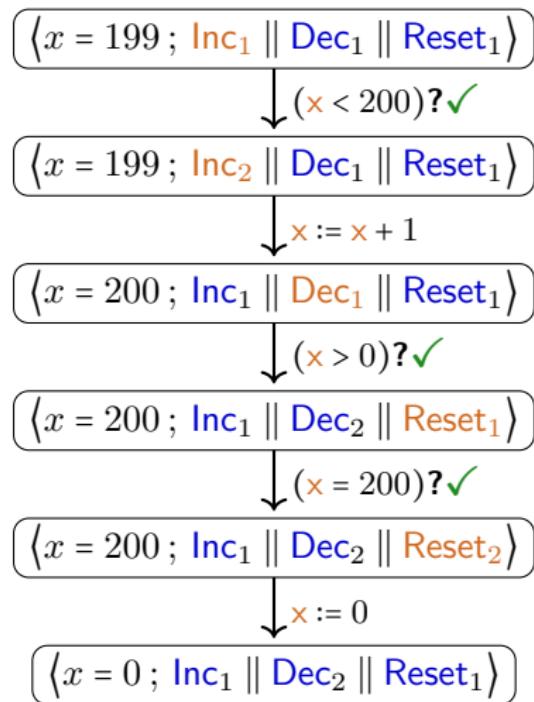
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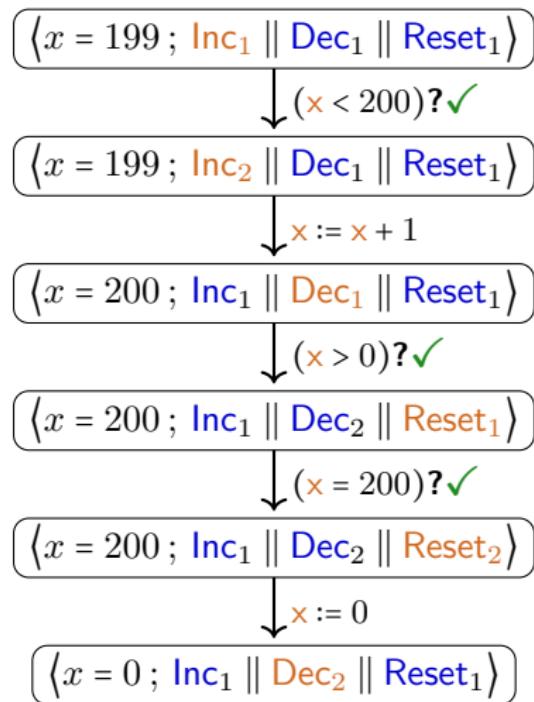
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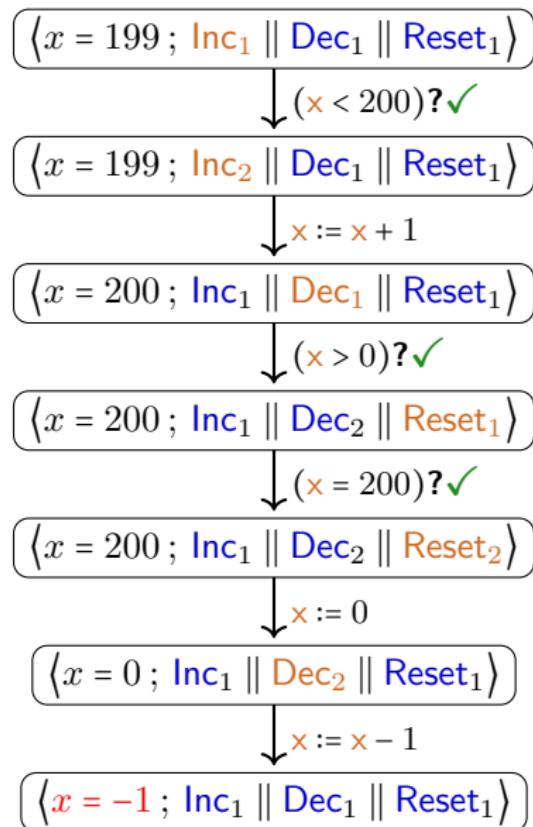
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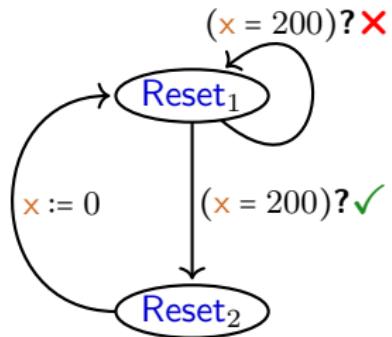
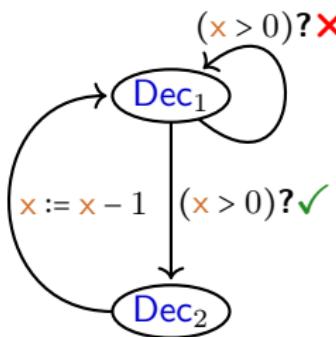
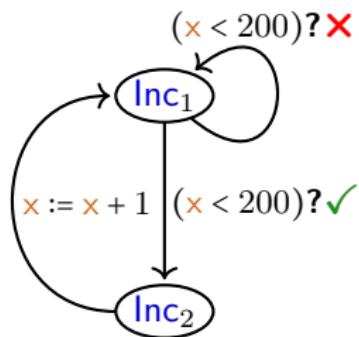
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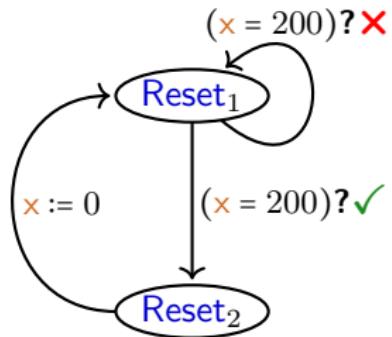
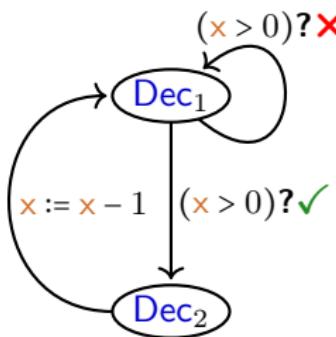
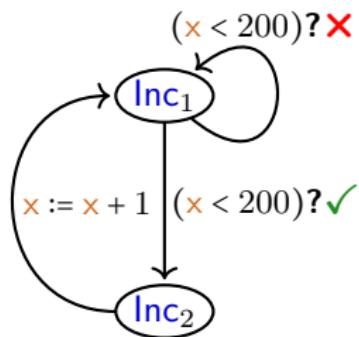


# Formalizing properties (in temporal logic)



$\text{Inc}_1 \parallel \text{Dec}_1 \parallel \text{Reset}_1 \neq \square(0 \leq x \wedge x \leq 200)$  (Linear-TL formula)

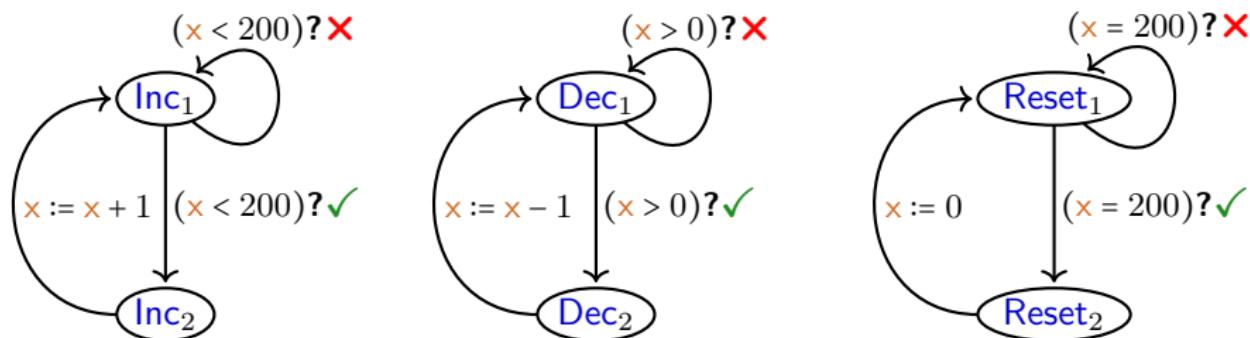
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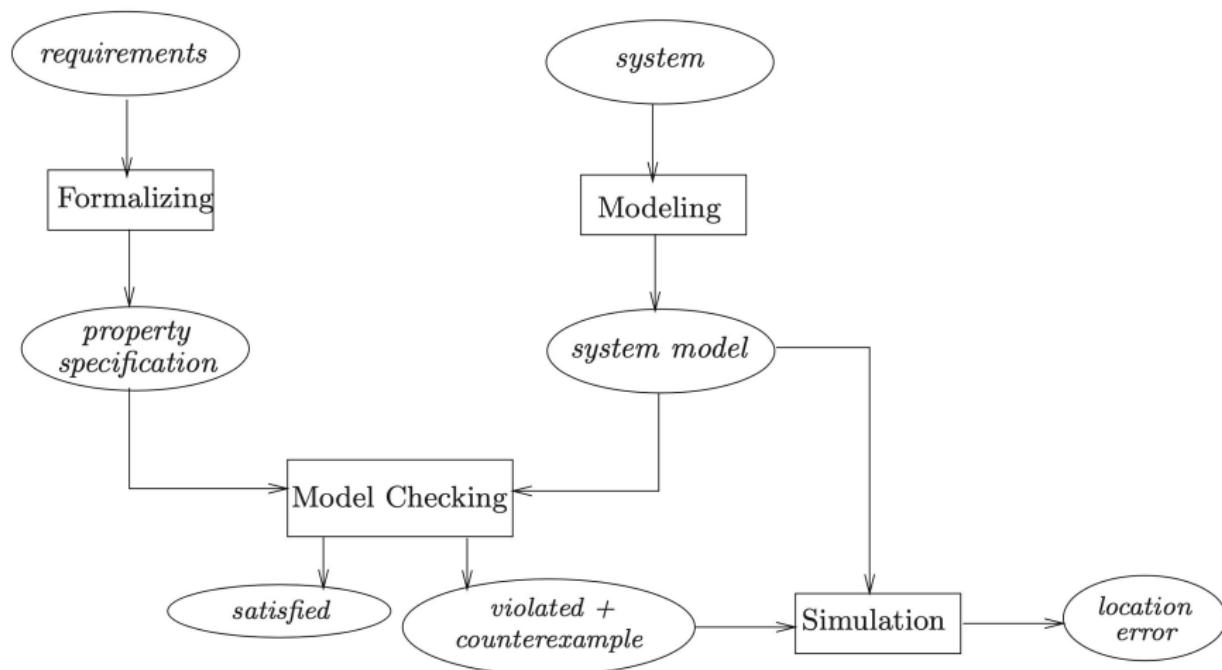
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 $\text{Inc}_1 \parallel \text{Dec}_1 \parallel \text{Reset}_1 \models \Diamond(x < 0)$  (LTL formula)  
 $\text{Inc}_1 \parallel \text{Dec}_1 \parallel \text{Reset}_1 \not\models \forall \Box(0 \leq x \wedge x \leq 200)$  (Computation-Tree-L formula)  
 $\text{Inc}_1 \parallel \text{Dec}_1 \parallel \text{Reset}_1 \models \exists \Box(0 \leq x \wedge x \leq 200)$  (CTL formula)  
 $\text{Inc}_1 \parallel \text{Dec}_1 \parallel \text{Reset}_1 \models \forall \Box \exists \Diamond(x < 0)$  (CTL formula)

# Model checking



*Any [such] verification is only as good as the model of the system.*

# Possible Maude code (idea)

```
crl [Inc1a]    : Inc1 x => Inc2 x  if x < 200  
rl  [Inc2]     : Inc2 x => Inc1 x + 1  
crl [Inc1b]    : Inc1 x => Inc1 x  if not(x < 200)
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eq initial = { Dec1 Inc1 Reset1 199 }
```

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```

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eq initial = { Dec1 Inc1 Reset1 199 }

ceq (S1 S2 S3 x |= counterge0) = true  if (0 < x \vee x = 0)
ceq (S1 S2 S3 x |= counterlt0) = true  if (x < 0)
ceq (S1 S2 S3 x |= counterle200) = true  if (x < 200 \vee x = 200)

```

# Maude output

```
Maude> red modelCheck(initial, <> counterlt0)
reduce in COUNTERS-CHECK : modelCheck(initial, <> counterlt0)
result ModelCheckResult:
result Bool : true
```

# Maude output

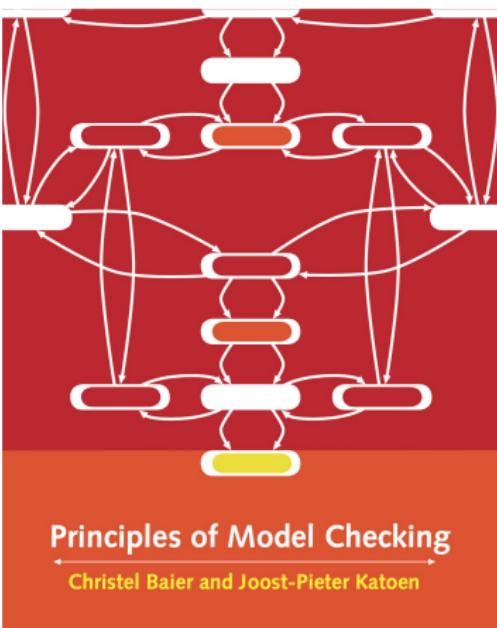
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Maude> red modelCheck(initial, <> counterlt0)
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```
Maude> red modelCheck(initial, [](counterge0 /\ counterle200)
reduce in COUNTERS-CHECK :
          modelCheck(initial, [](counterge0 /\ counterle200)
result ModelCheckResult:
counterexample({Inc1 Dec1 Reset1 199}
              {Inc2 Dec1 Reset1 199}
              {Inc1 Dec1 Reset1 200}
              {Inc1 Dec2 Reset1 200}
              {Inc1 Dec2 Reset2 200}
              {Inc1 Dec2 Reset1 0}
              {Inc1 Dec1 Reset1 -1})
```

# Topics of the course

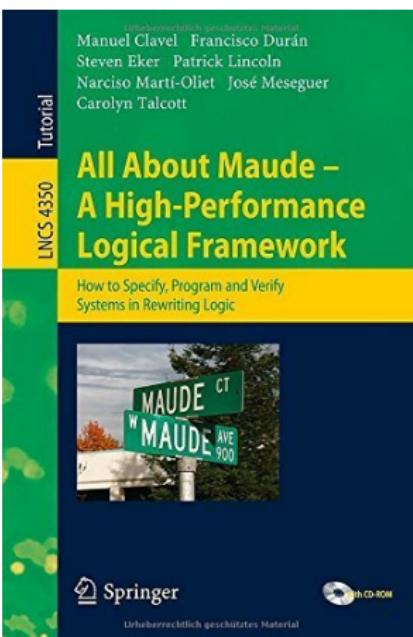
- ▶ modeling systems by labeled transition systems (LTSs)
- ▶ safety, liveness, and fairness properties
- ▶ Linear Temporal Logic (LTL)
  - ▶ model checking formulas
    - ▶ express properties by Büchi automata
    - ▶ model check LTSs and properties via product automata
- ▶ Computation Tree Logic (CTL)
- ▶ partial model checking
  - ▶ partially known systems (state properties/states/transitions)
- ▶ learning Maude and its model-checker

# Book



- ▶ pdf available:  
[https://is.ifmo.ru/books/\\_principles\\_of\\_model\\_checking.pdf](https://is.ifmo.ru/books/_principles_of_model_checking.pdf)

# Book Maude



- ▶ pdf available:

<https://maude.cs.illinois.edu/w/images/0/0d/Maude-book.pdf>

# Organization

Lectures (Emilio 3 / Clemens 4)

- ▶ January 19–January 28 (7 meetings)
- ▶ blackboard presentations
- ▶ notes after the lecture (last year's notes available)
- ▶ Maude examples at the end of each lecture

Webpage

- ▶ <https://clegra.github.io/mc/mc.html>



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- ▶ options:
  - ▶ verification project (of an algorithm, etc.) in Maude
  - ▶ presentation about a paper
  - ▶ written exam

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Thank you – we are looking forward to the course!