

Core Course

Introduction to Model Checking

<https://clegra.github.io/mc/mc.html>

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Model Checking

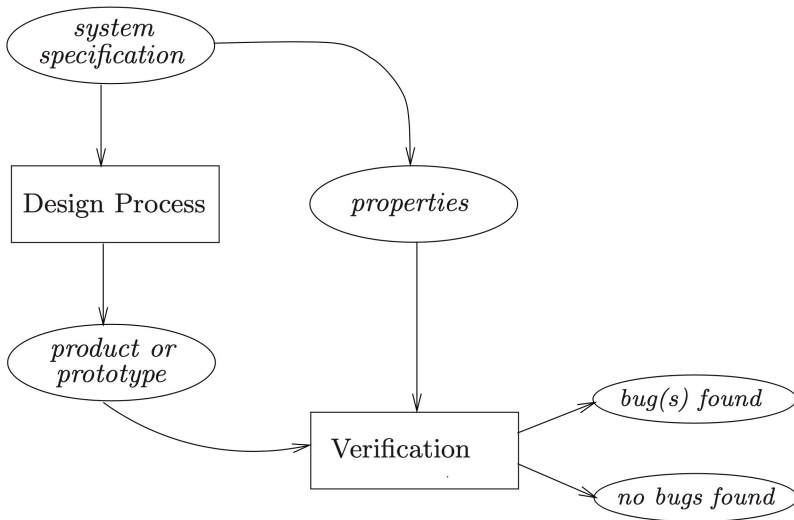
... is an effective automatable technique:

- ▶ *to expose potential software design errors;*
- ▶ *that, given a finite-state model of a system and a formal property, systematically checks whether this property holds for that model.*

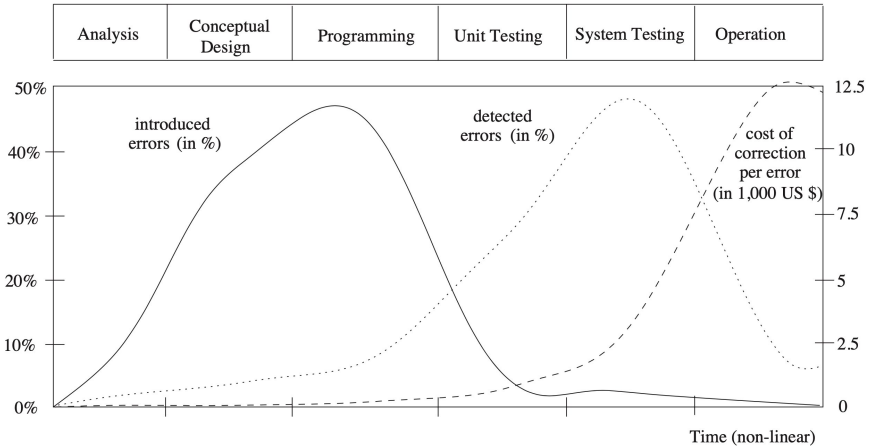
Strengths:

- ▶ widely applied in industry
for: embedded systems, software engineering, hardware design, explainable AI
- ▶ supports **partial verification** (of system parts)
- ▶ provides **diagnostic information** for debugging
- ▶ has sound **mathematical underpinning** (logic and process theory)

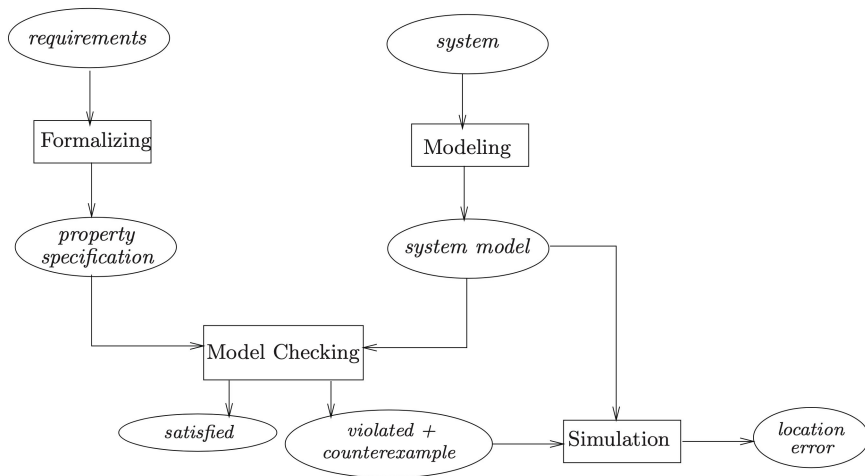
Hard-/Software Verification (traditionally)



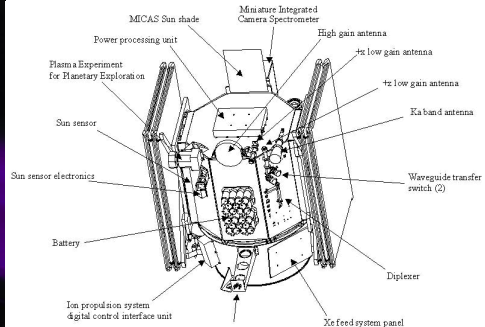
Error introduction, detection, and repair costs



Model checking



Deep Space 1



- ▶ Flyby of asteroid 9969 Braille (1999)
- ▶ Entered the coma of Comet Borrelly (2001)
- ▶ Model checking discovered **5 concurrency errors**

Example (program concurrency/non-determinism)

Programs [Inc](#), [Dec](#), and [Reset](#) cooperate, and use a shared variable x :

```
proc Inc
  while true
    do
      if  $x < 200$ 
        then  $x := x + 1$ 
      fi
    od
```

```
proc Dec
  while true
    do
      if  $x > 0$ 
        then  $x := x - 1$ 
      fi
    od
```

```
proc Reset
  while true
    do
      if  $x = 200$ 
        then  $x := 0$ 
      fi
    od
```

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proc Reset
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  do
    if  $x = 200$ 
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    fi
  od
```

Question: Is $0 \leq x \leq 200$ always guaranteed?

Modeling (by labeled transition systems)

proc [Inc](#)

while true

do

if $x < 200$

then $x := x + 1$

fi

od

proc [Dec](#)

while true

do

if $x > 0$

then $x := x - 1$

fi

od

proc [Reset](#)

while true

do

if $x = 200$

then $x := 0$

fi

od

Modeling (by labeled transition systems)

proc Inc

while true

do

1: **if** $x < 200$

2: **then** $x := x + 1$

fi

od

proc Dec

while true

do

1: **if** $x > 0$

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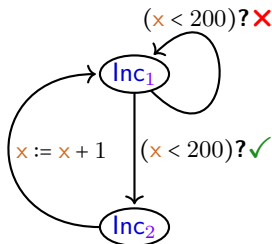
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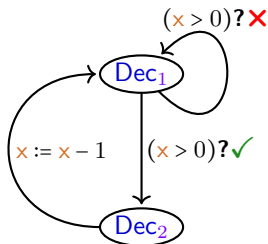


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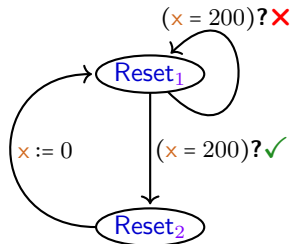


proc Reset

while true

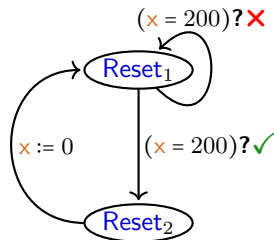
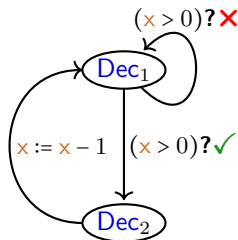
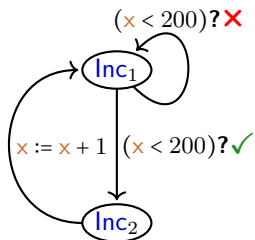
do

1: **if** $x = 200$
 2: **then** $x := 0$
fi
od



Labeled transition systems (LTSs)

Formalizing properties (in temporal logic)



$$Inc_1 \parallel Dec_1 \parallel Reset_1 \stackrel{?}{\models} \Box(0 \leq x \wedge x \leq 200) \quad (\text{Linear-TL formula})$$

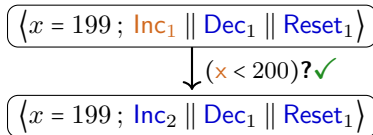
Counterexample (offending execution trace)

$$\langle x = 199 ; \text{Inc}_1 \parallel \text{Dec}_1 \parallel \text{Reset}_1 \rangle$$

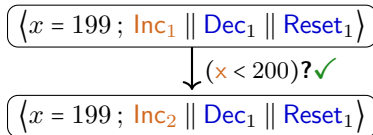
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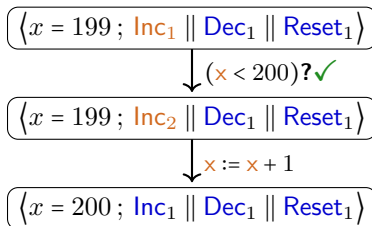
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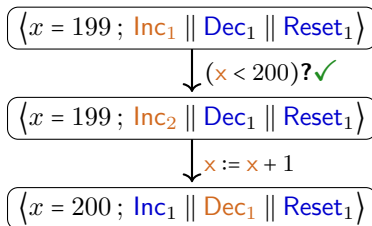
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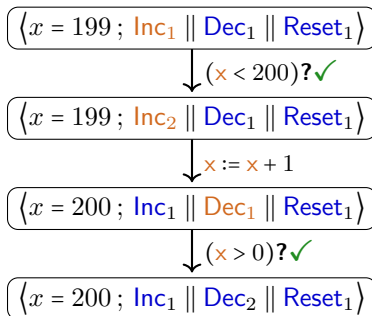
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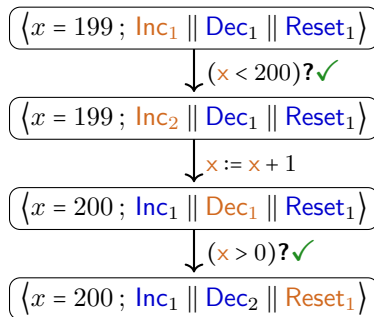
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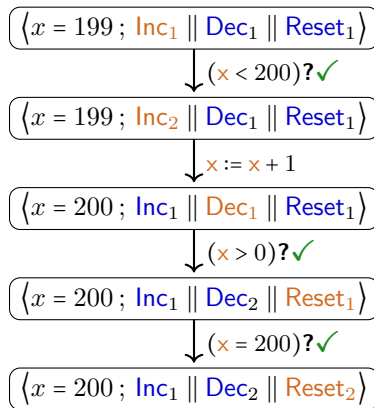
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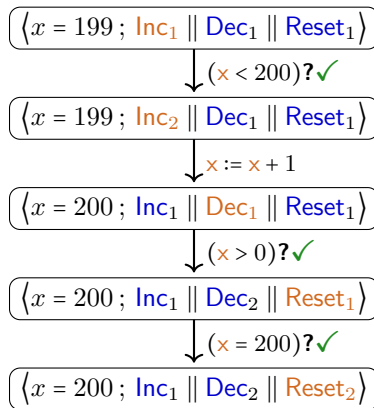
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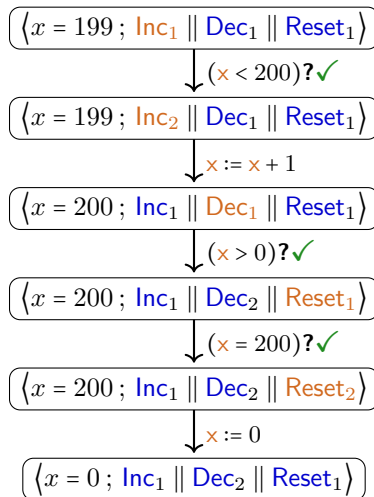
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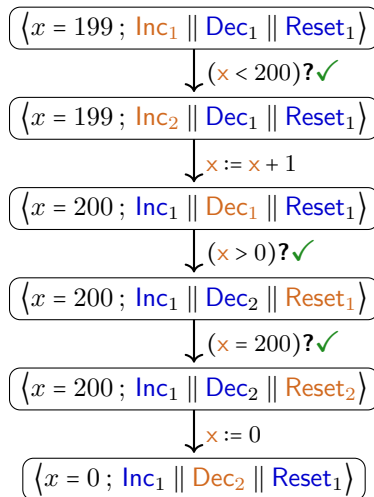
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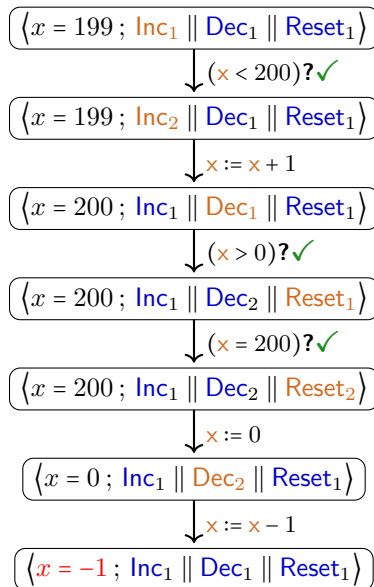
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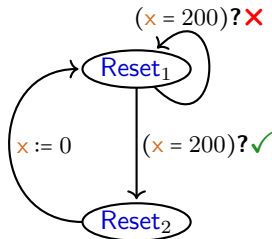
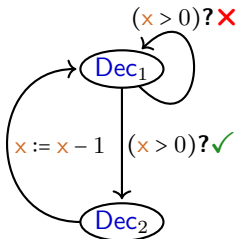
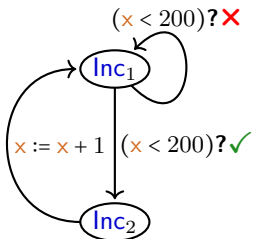
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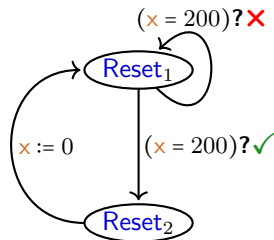
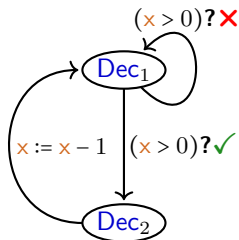
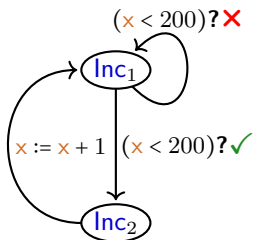


Formalizing properties (in temporal logic)



$\text{Inc}_1 \parallel \text{Dec}_1 \parallel \text{Reset}_1 \not\models \Box(0 \leq x \wedge x \leq 200)$ (Linear-TL formula)

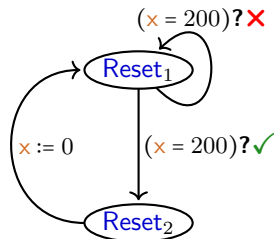
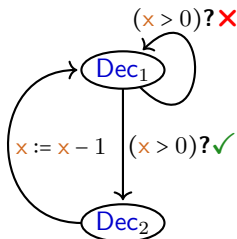
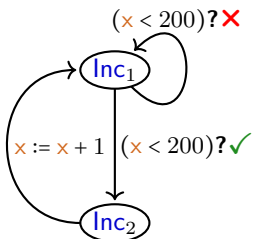
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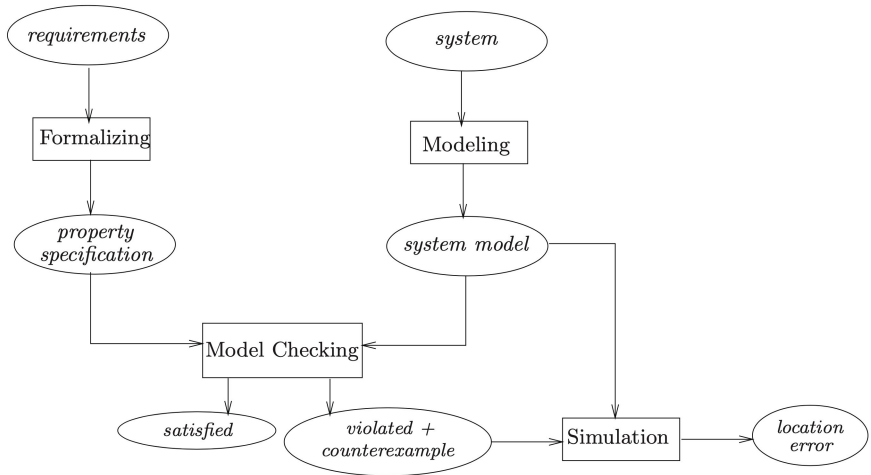
$Inc_1 \parallel Dec_1 \parallel Reset_1 \models \Diamond(x < 0)$ (LTL formula)

$Inc_1 \parallel Dec_1 \parallel Reset_1 \not\models \forall \Box(0 \leq x \wedge x \leq 200)$ (Computation-Tree-L formula)

$Inc_1 \parallel Dec_1 \parallel Reset_1 \models \exists \Box(0 \leq x \wedge x \leq 200)$ (CTL formula)

$Inc_1 \parallel Dec_1 \parallel Reset_1 \models \forall \Box \exists \Diamond(x < 0)$ (CTL formula)

Model checking

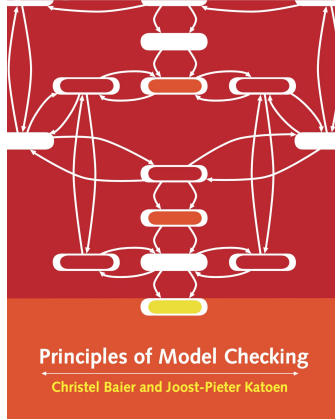


Any [such] verification is only as good as the model of the system.

Topics of the module

- ▶ modeling systems by [labeled transition systems \(LTSs\)](#)
- ▶ [fairness](#)
- ▶ [Linear Temporal Logic \(LTL\)](#)
 - ▶ model checking formulas
 - ▶ express properties by [Büchi automata](#)
 - ▶ model check LTSs and properties via [product automata](#)
- ▶ [Computation Tree Logic \(CTL\)](#)
- ▶ [partial](#) model checking
 - ▶ partially known systems (state properties/states/transitions)
- ▶ analysing system behavior with the [mCRL2 model-checker toolbox](#)

Book



- pdf available:
https://is.ifmo.ru/books/_principles_of_model_checking.pdf

Organization

Lectures (Emilio 2/Clemens 5)

- ▶ presentations on blackboard
- ▶ notes after the lecture (notes 2025/26 available)
- ▶ January 19 – January 28 (7 lectures)

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Exam

- ▶ options:
 - ▶ small verification project (of an algorithm, e.g. in [mCRL2](#))
 - ▶ presentation about a paper
 - ▶ written exam?

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Thank you – we are looking forward to the course!