

Introduction to Model Checking

<https://clegra.github.io/mc.html>

Lecture 1

Clemens Grabmayer

<https://clegra.github.io>

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<https://cs.gssi.it/emilio.tuosto/>

Department of Computer Science



January 26, 2026

Organization

Lectures (Clemens 4 / Emilio 3)

- ▶ this week:
 - ▶ Monday, 10.30–12.30, room B, Aurora
 - ▶ Tuesday, Thursday, Friday, **14.30–16.30**, conference room -1, Zenith

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Exam (more later)

- ▶ options:
 - ▶ small verification project (of an algorithm, e.g. in [Maude](#))
 - ▶ presentation about a paper
 - ▶ written exam?

Topics of the course

- ▶ modeling systems by **labeled transition systems (LTSs)**
- ▶ **safety**, **liveness**, and **fairness** properties

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 - ▶ model checking formulas
 - ▶ express properties by Büchi automata
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- ▶ Seeing Maude examples model-checker
- ▶ (partial model checking)
 - ▶ (partially known systems (state properties/states/transitions))

Model Checking

... is an effective automatable technique:

- ▶ *to expose potential software design errors;*
- ▶ *that, given a finite-state model of a system and a formal property, systematically checks whether this property holds for that model.*

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 - ▶ embedded systems, software engin., hardware design, **explainable AI**
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- ▶ provides **diagnostic information** for debugging
- ▶ has sound **mathematical underpinning** (**logic** and **process theory**)

Model Checking

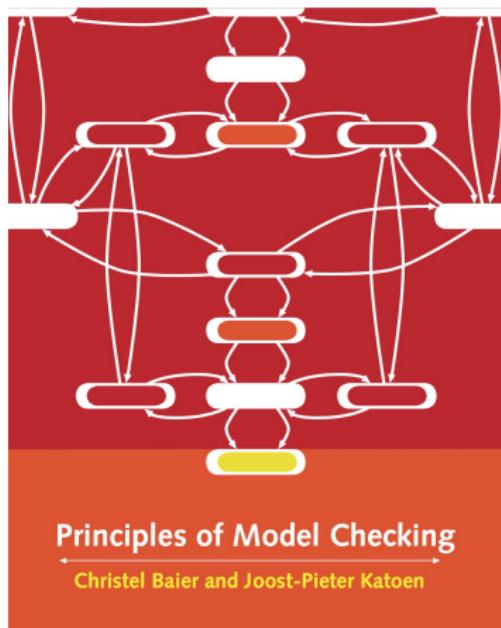
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Course Goals are introduction to:

- ▶ Theory:
 - ▶ **modeling** systems by **labeled transition systems**,
 - ▶ expressing **properties** by **temporal-logic formulas**
 - ▶ **model-checking algorithms**
- ▶ Practice related: see **Maude examples**

Book



- ▶ pdf online available
- ▶ we study chapters 1–3, 5, 6

Lectures

1. Introduction (preview extended, LTSs)
2. Modeling by labeled transition systems
 - ▶ executions; traces; non-determinism; examples
3. Linear-Time Behaviour and Properties
 - ▶ invariant, safety, and liveness (and fairness) properties
4. Linear Temporal Logic (LTL)
 - ▶ syntax and semantics; interpretation of LTSs; examples
5. LTL (continued)
 - ▶ model checking of LTL formulas, and fairness in LTL
 - ▶ Maude examples
6. Computation Tree Logic (CTL)
 - ▶ syntax and semantics, examples
7. Extensions of CTL, and Outlook
 - ▶ expressibility differences with LTL
 - ▶ model checking formulas in CTL
 - ▶ (μ -calculus | partial model-checking | Maude examples)

Software correctness

- ▶ Software is ubiquitous \implies software is valuable

Software correctness

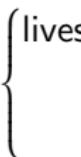
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- ▶ Deductive reasoning
 - $\begin{cases} + & \text{also possible for infinite-state system} \end{cases}$

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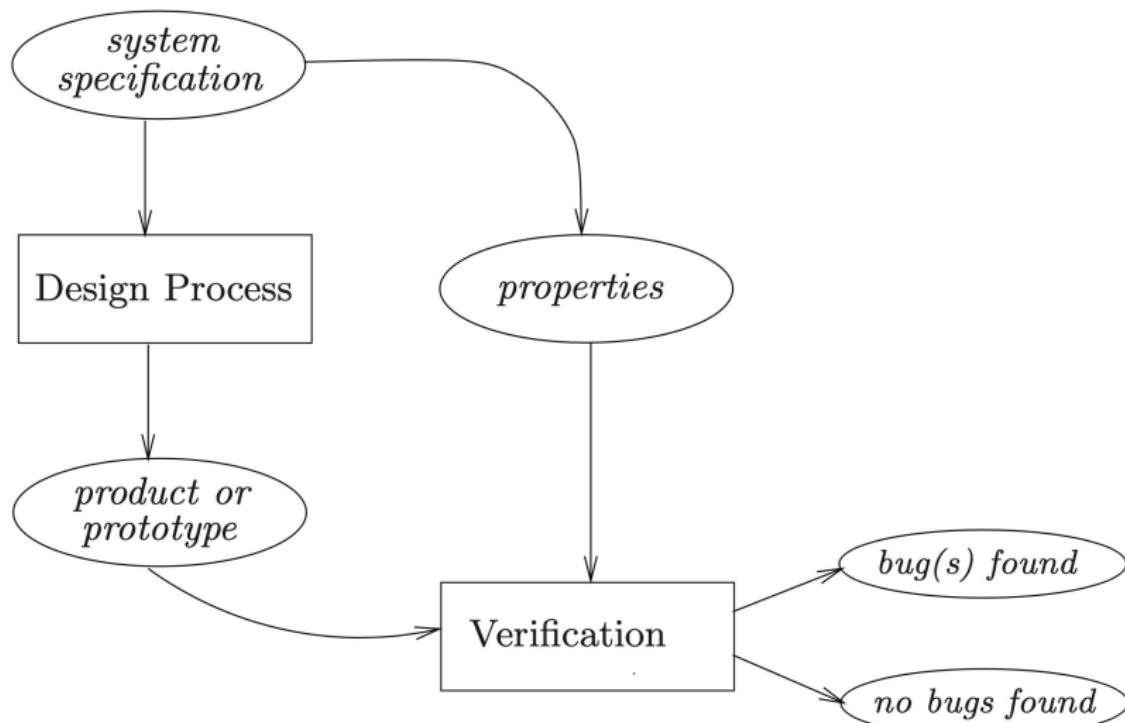
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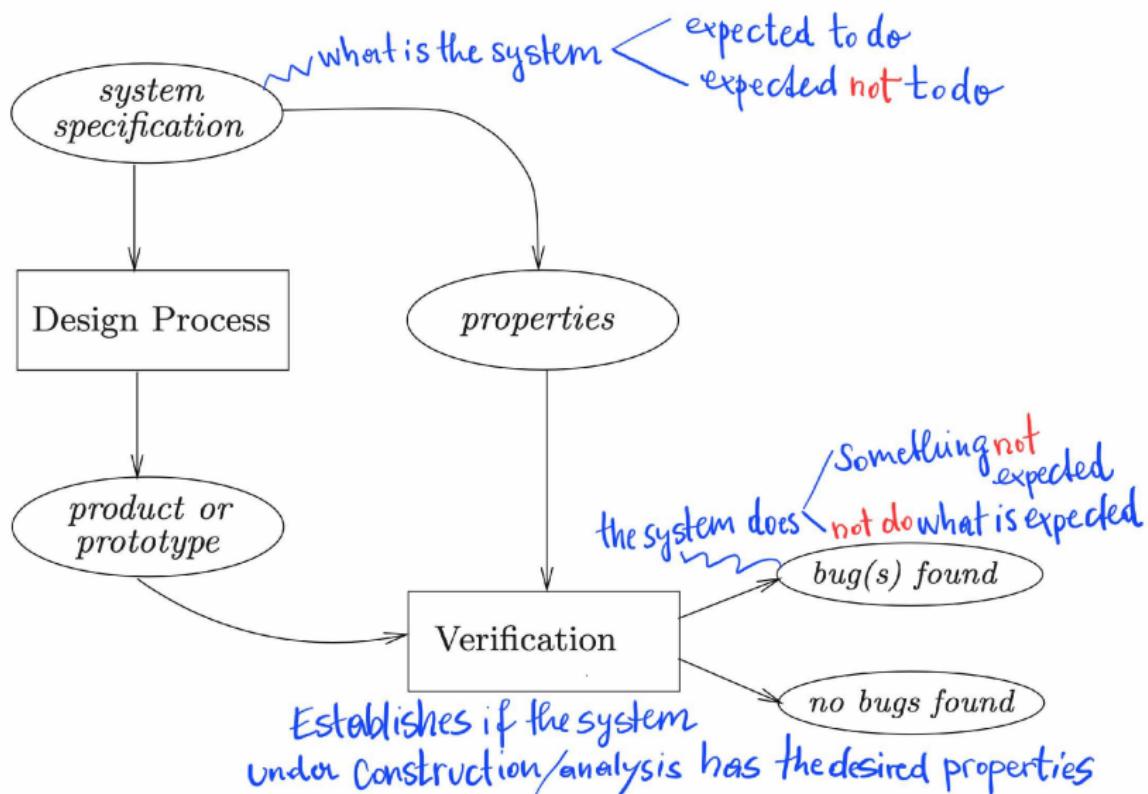
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Hard-/Software Verification (traditionally)



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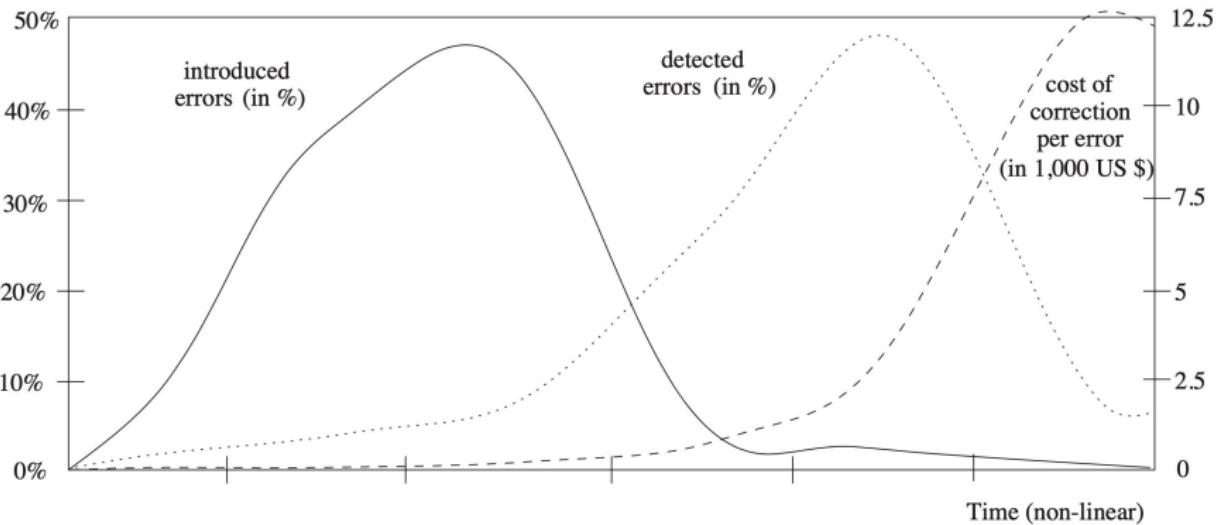
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- ▶ Software testing
 - { + applicable to all sorts of software
 - { + test generation can (partly) be automated
 - { - exhaustive testing of all execution paths is infeasible
 - { - When do we stop?
- ▶ Catching software errors: **the sooner the better.**

Software lifecycle, error introduction/detection, repair costs

Analysis	Conceptual Design	Programming	Unit Testing	System Testing	Operation
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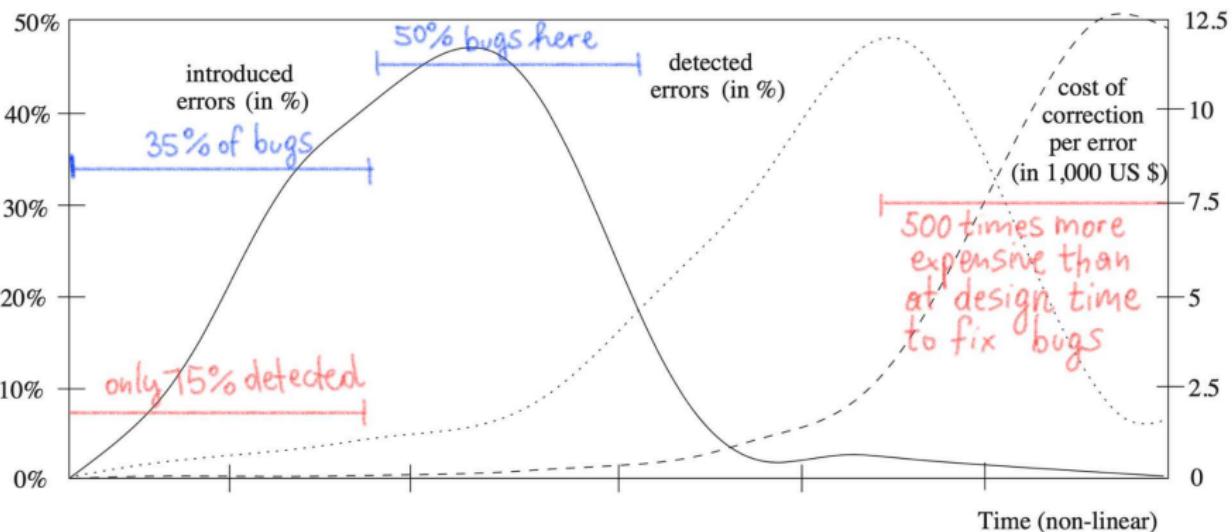


Liggesmeyer et al: "Qualitätssicherung technischer Systeme . . ." [4, 1998]

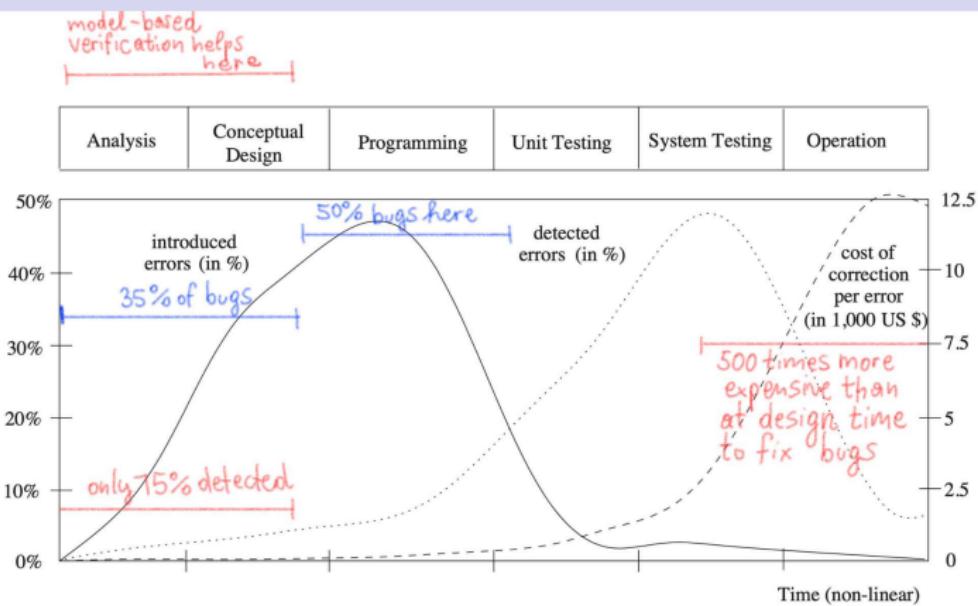
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model-based verification helps here

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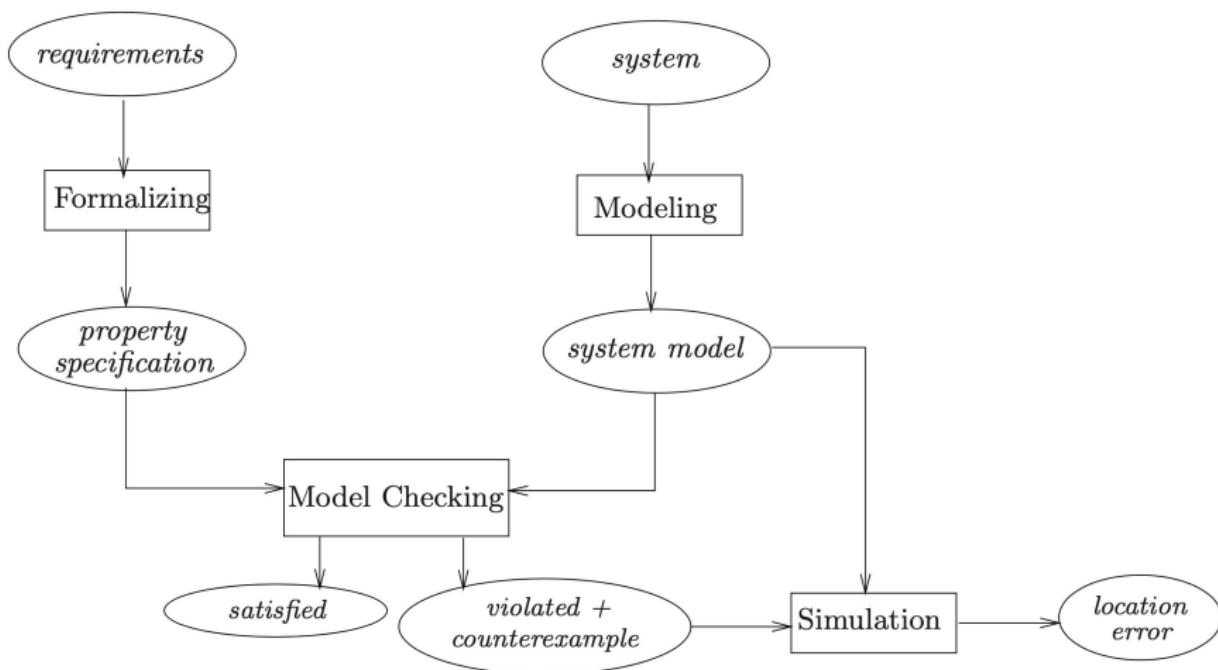


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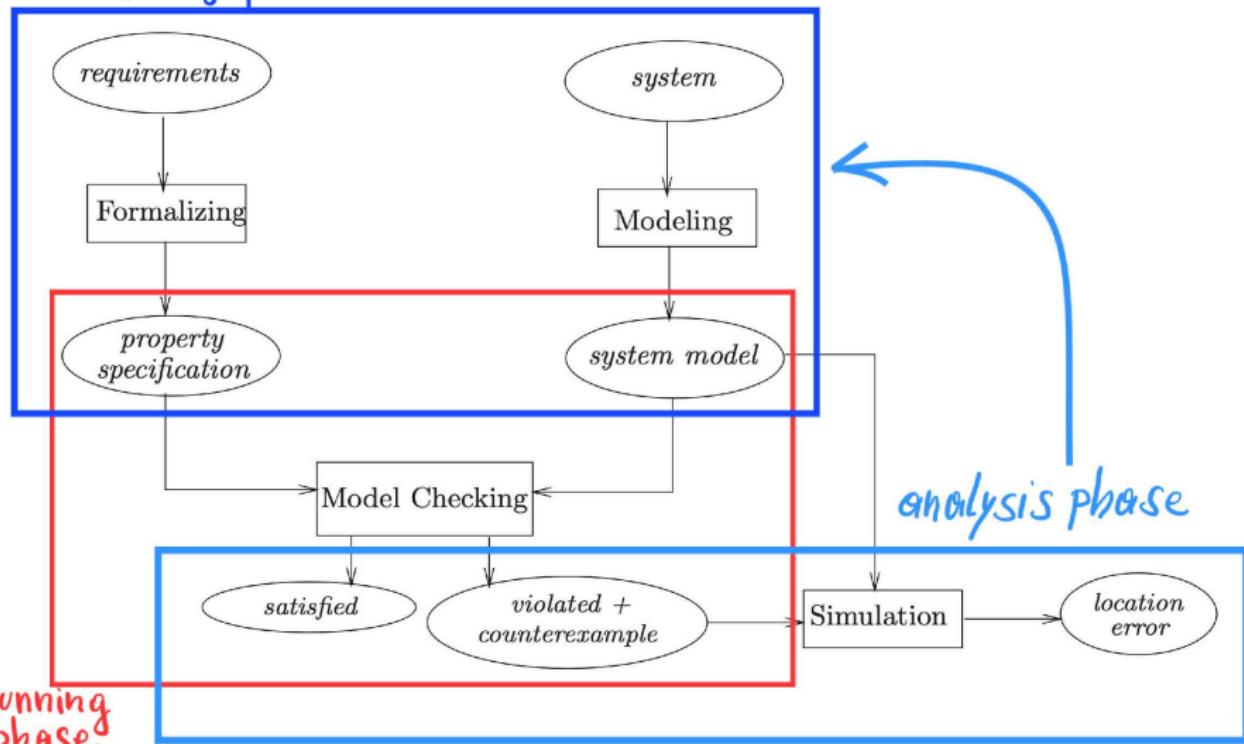
"In software and hardware design of complex systems, more time and effort are spent on verification than on construction. Techniques are sought to reduce and ease the verification efforts while increasing their coverage. Formal methods offer a large potential to obtain an early integration of verification in the design process, to provide more effective verification techniques, and to reduce the verification time."

Model checking



Model checking

modeling phase



running
phase

analysis phase

Glancing at temporal logic

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 a and b do not occur at the same time,

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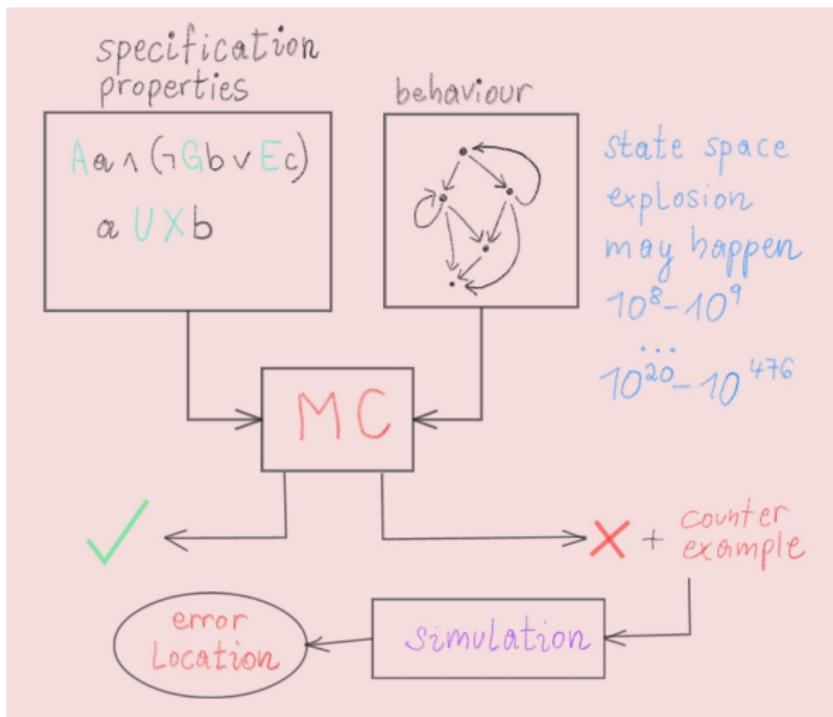
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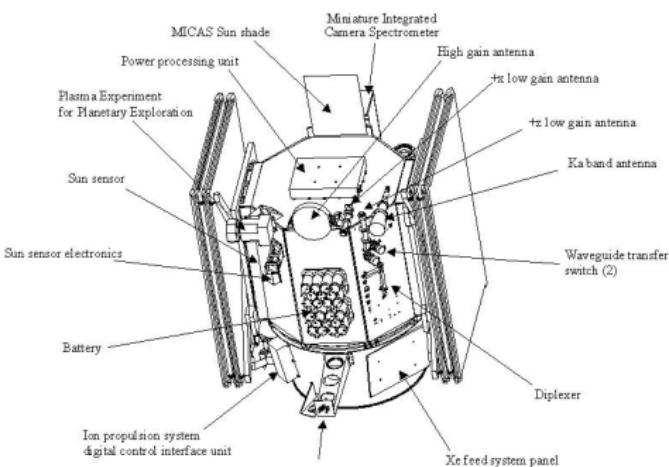
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$\Box(\neg(a \wedge b)) \stackrel{\wedge}{=} \text{for all time moments,}$
 $\quad \quad \quad \text{events } a \text{ and } b \text{ do not occur at the same time,}$
 $= \text{it will never happen that}$
 $\quad \quad \quad \text{events } a \text{ and } b \text{ occur at the same time.}$

Glancing at temporal logic (its use for model checking)



Deep Space 1 (NASA)



- ▶ Flyby of asteroid 9969 Braille (1999)
- ▶ Entered the coma of Comet Borrelly (2001)
- ▶ Model checking discovered 5 concurrency errors

Example (program concurrency/non-determinism)

Programs `Inc`, `Dec`, and `Reset` cooperate, and use a shared variable `x`:

```
proc Inc
  while 0 ≤ x ≤ 200
    do
      if x < 200
        then x := x + 1
      fi
    od
```

```
proc Dec
  while 0 ≤ x ≤ 200
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      if x > 0
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 Is $0 \leq x \leq 200$ always guaranteed?

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proc Reset

while true

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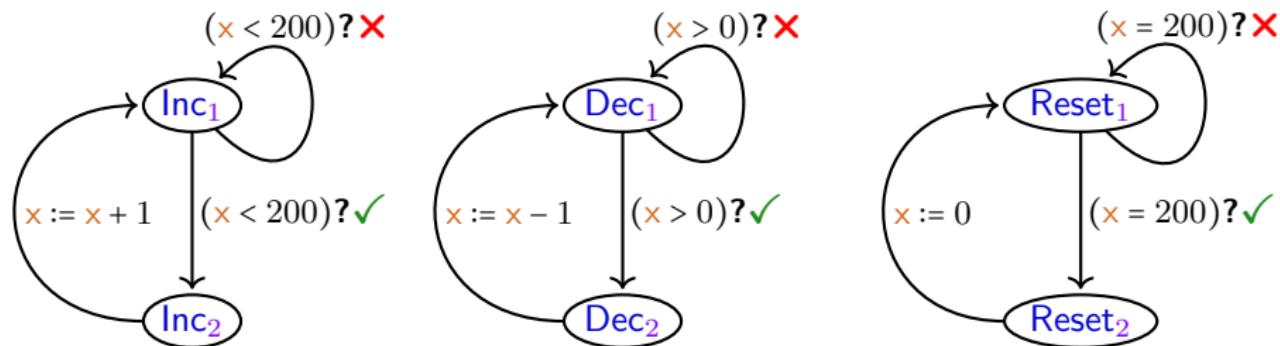
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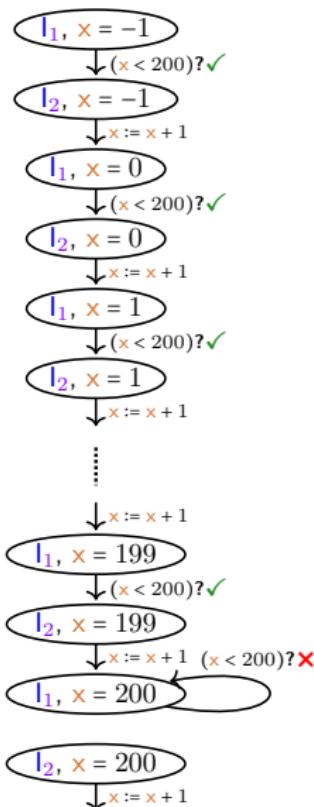
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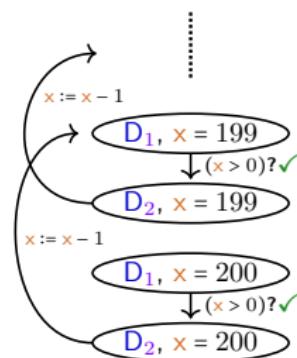
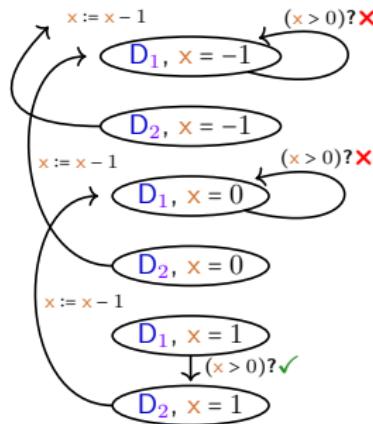
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Program graphs (PG)

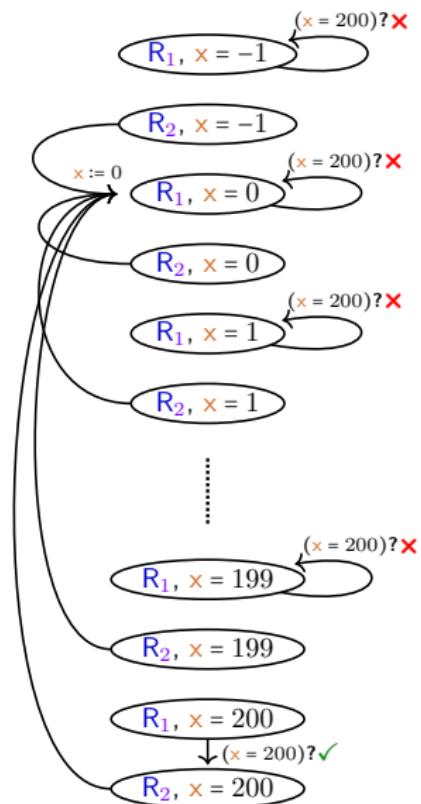
Modeling (by labeled transition systems, with state space explosion)



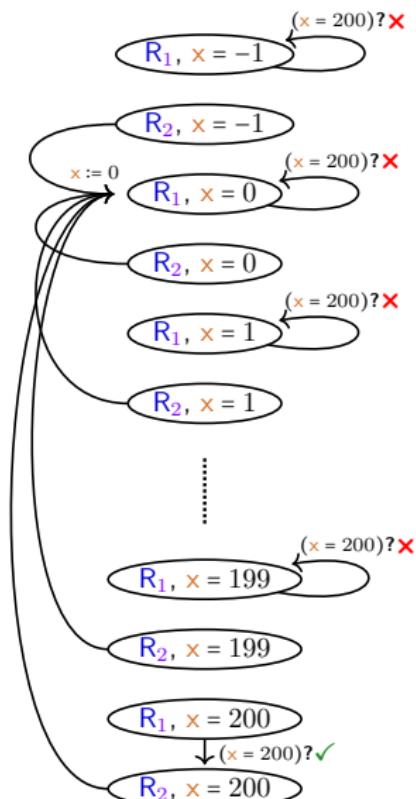
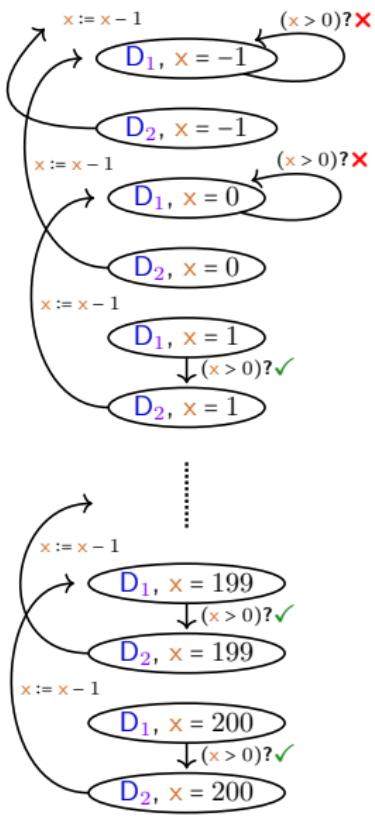
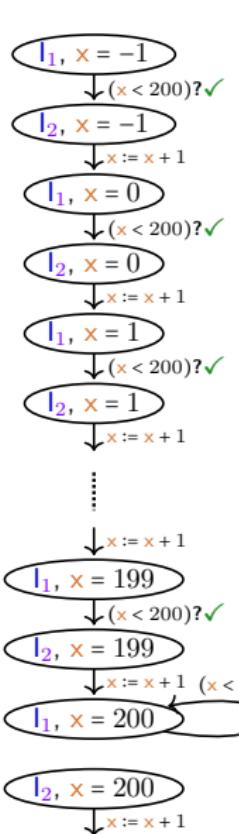
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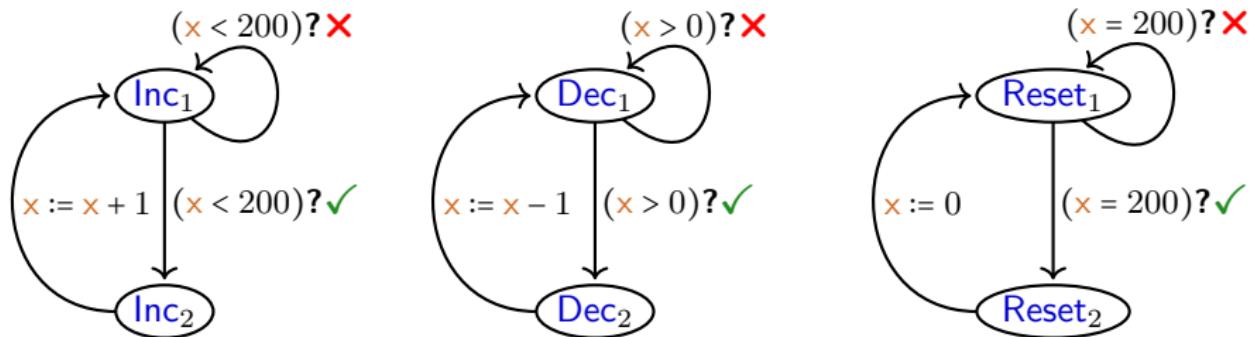
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Formalizing properties (in temporal logic)



We assume $x := 0$ initially.

$$\text{Inc}_1 \parallel \text{Dec}_1 \parallel \text{Reset}_1 \stackrel{?}{\models} \square(0 \leq x \wedge x \leq 200) \quad (\text{Linear-TL formula})$$

Counterexample (offending execution trace)

$$\langle x = 199 ; \text{Inc}_1 \parallel \text{Dec}_1 \parallel \text{Reset}_1 \rangle$$

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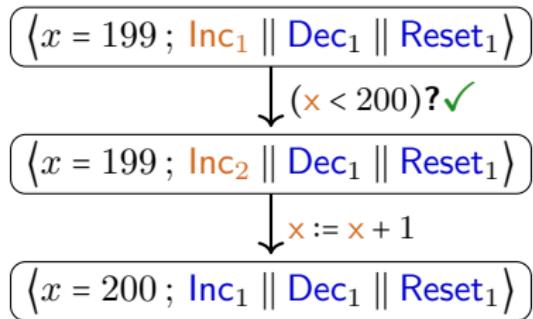
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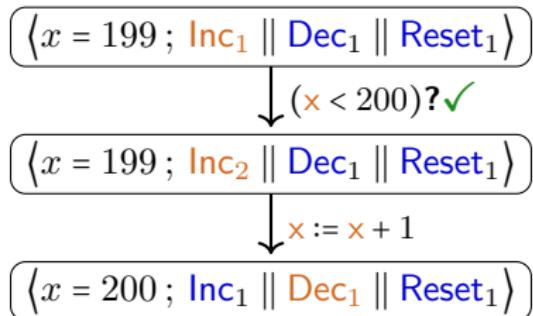
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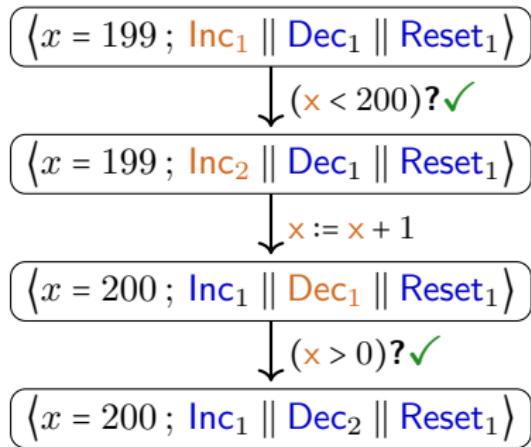
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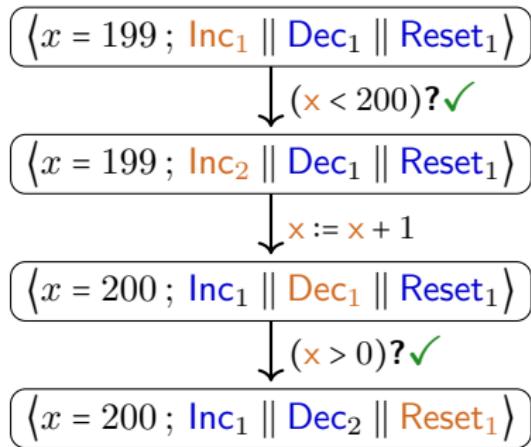
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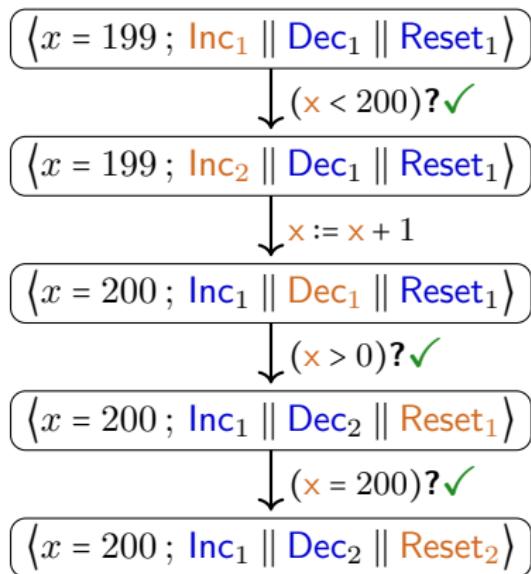
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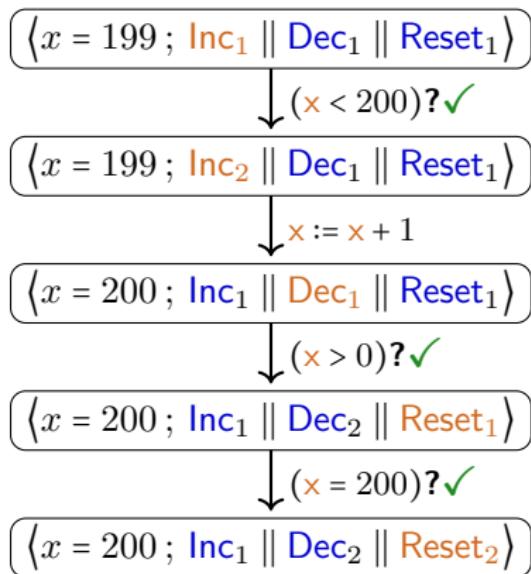
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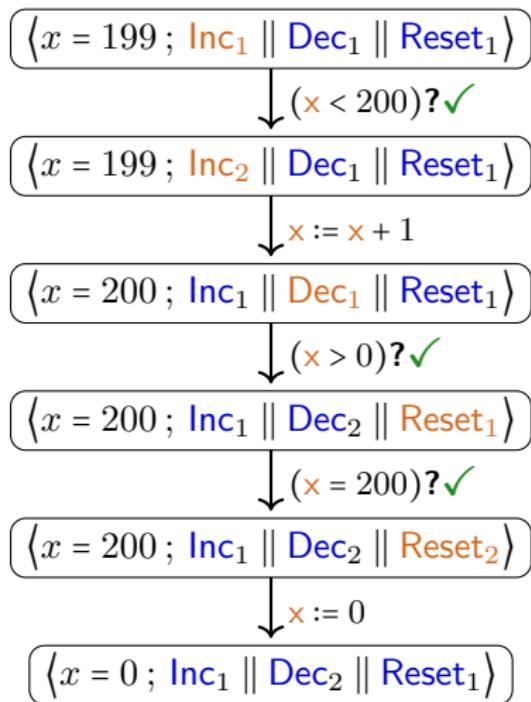
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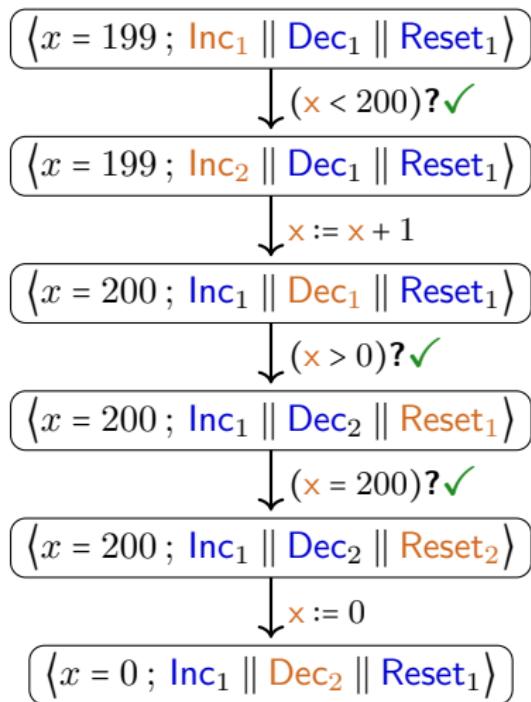
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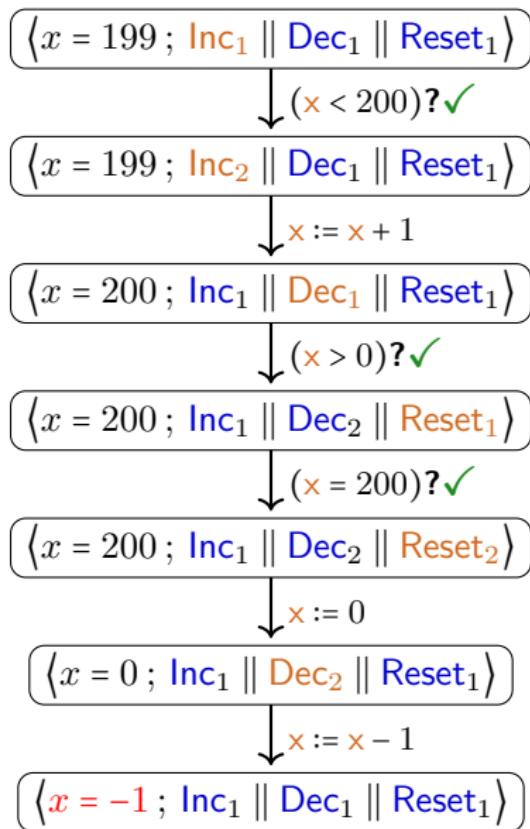
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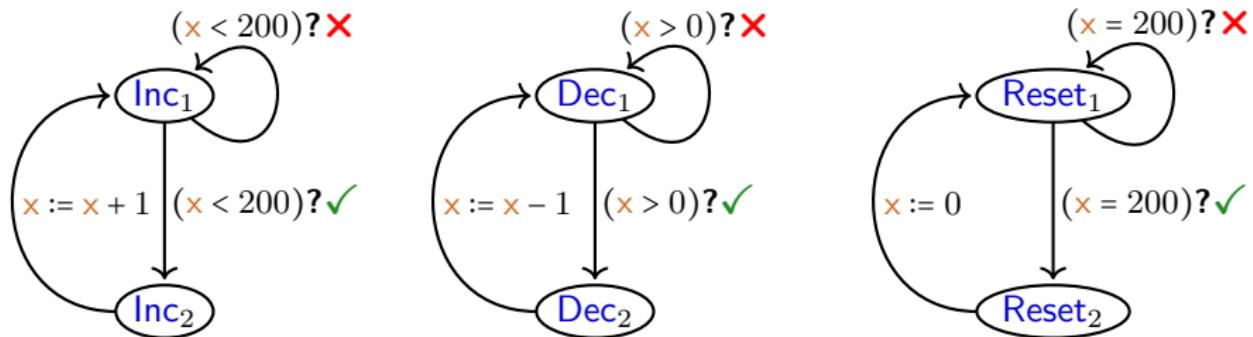
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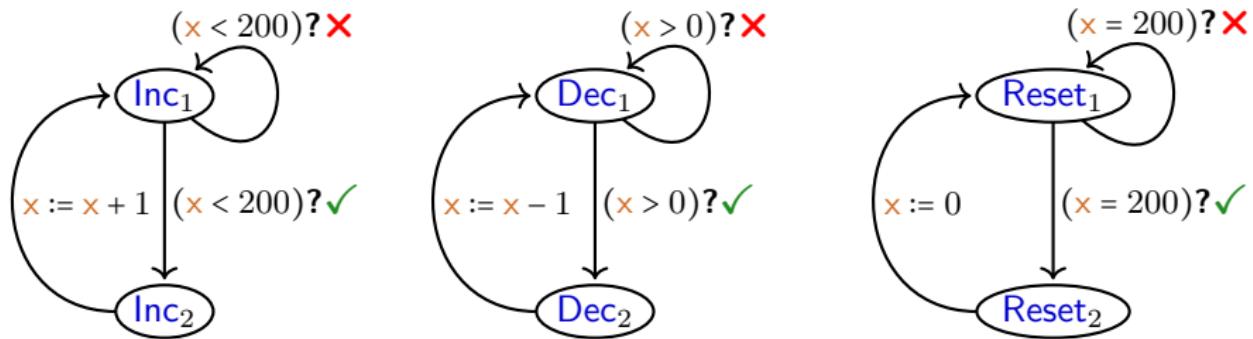
Formalizing properties (in temporal logic)



We assume $x := 0$ initially.

$$\text{Inc}_1 \parallel \text{Dec}_1 \parallel \text{Reset}_1 \stackrel{?}{\models} \square(0 \leq x \wedge x \leq 200) \quad (\text{Linear-TL formula})$$

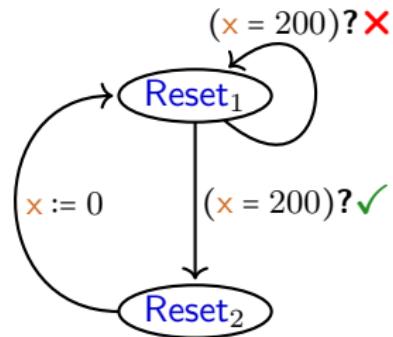
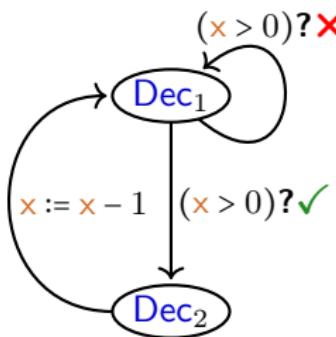
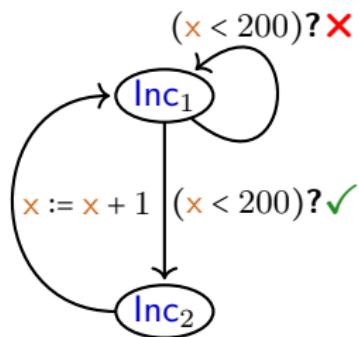
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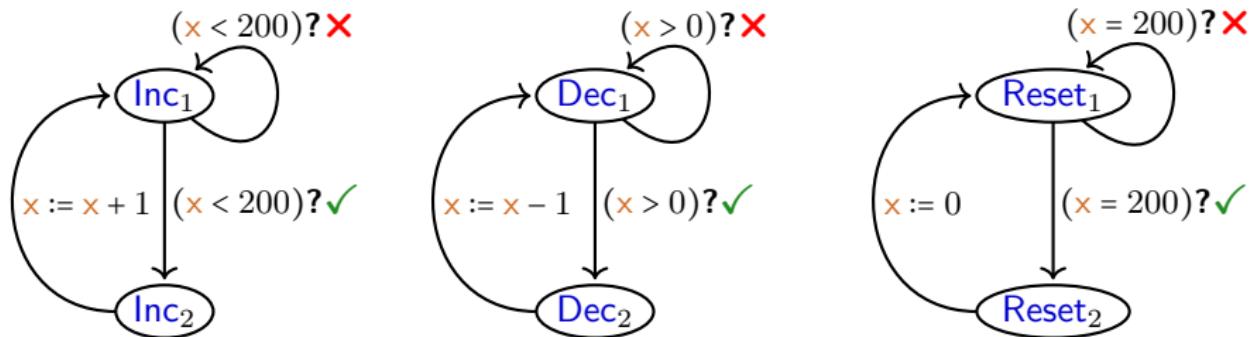


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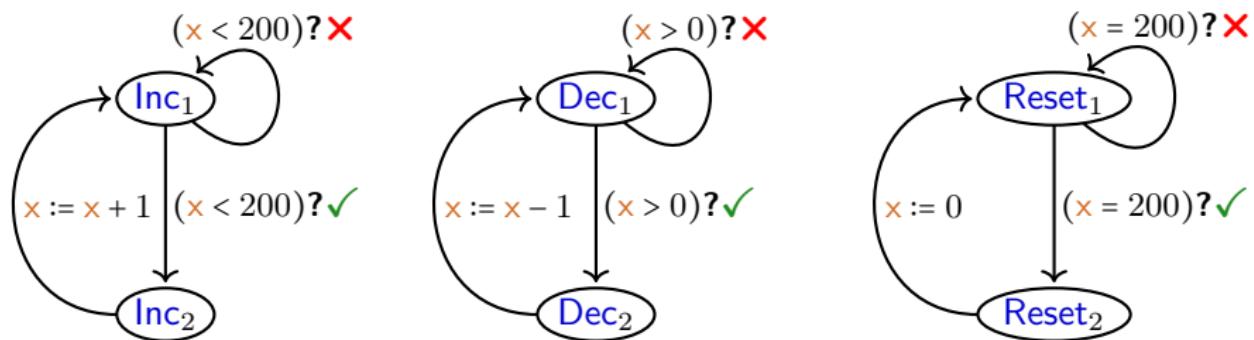


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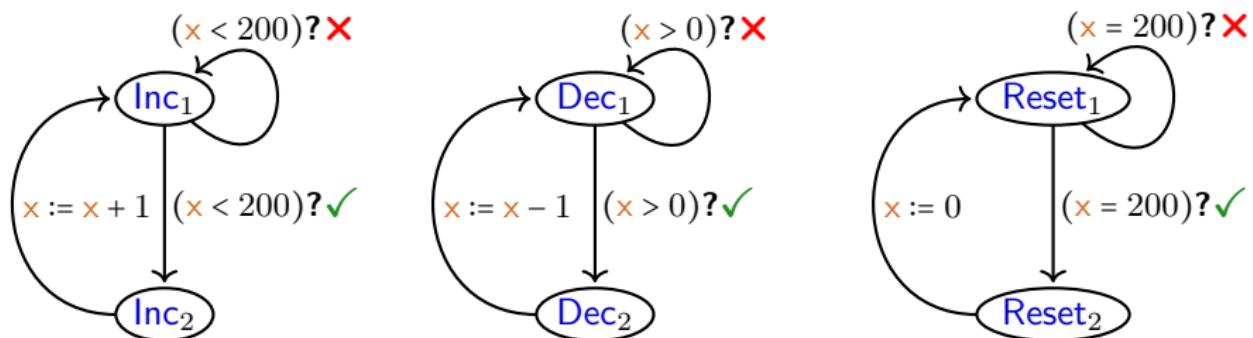
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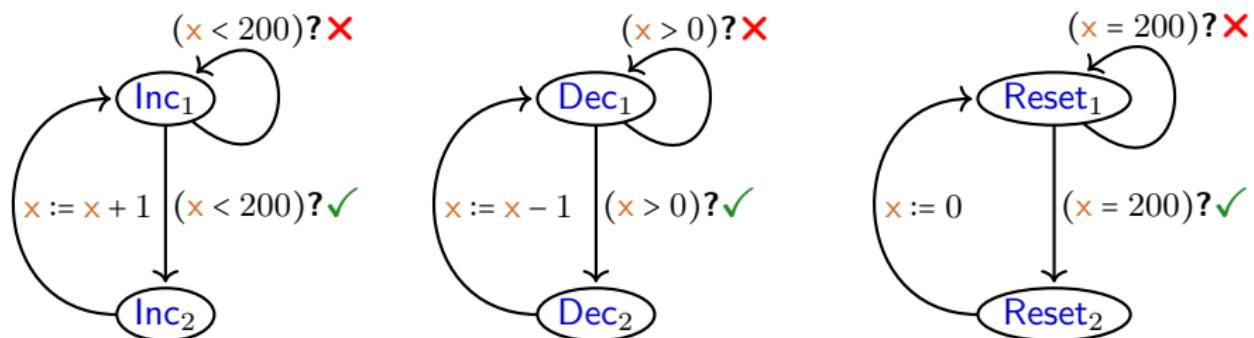
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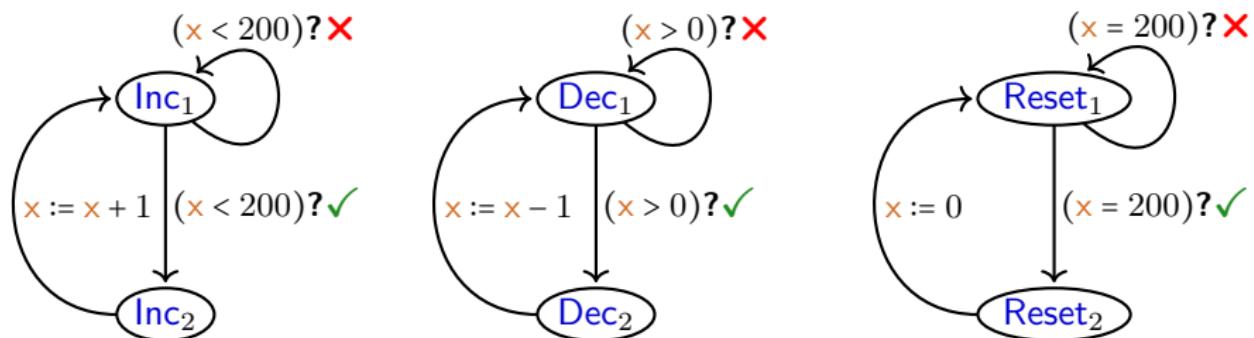
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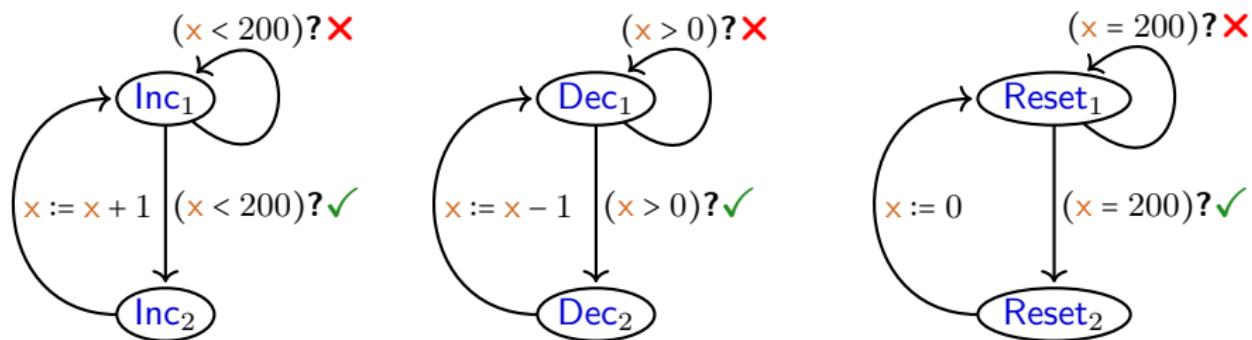
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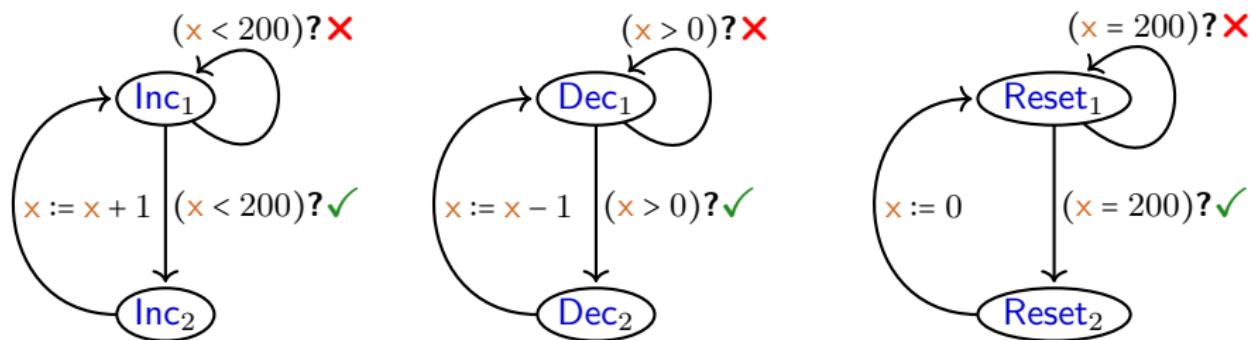
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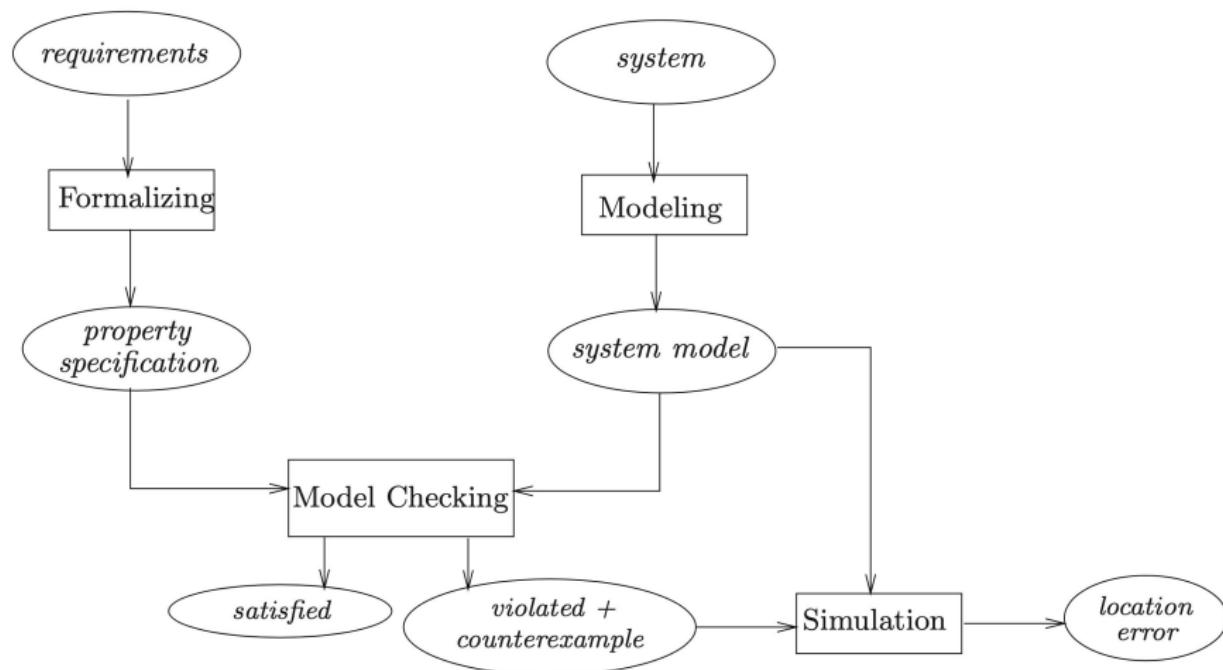
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Model checking: validation



Any [such] verification is only as good as the model of the system.

Modeling: Validation versus Verification

VALIDATION

Are we building the **right thing**?



Is the **design faithfully capturing
the requirements**?

VERIFICATION

Are we building the **thing right**?



Does the **design satisfy the
expected properties**?

Model-checking: strengths and weaknesses

Strengths:

- ▶ it is a **general** technique
- ▶ supports **partial** verification,
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Weaknesses:

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- ▶ does not permit to directly check
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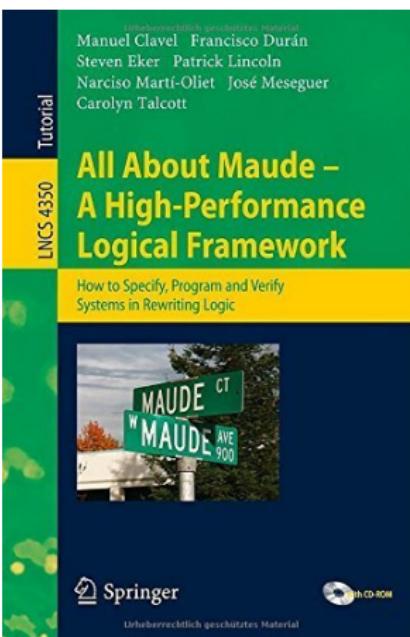
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See Baier and Katoen [1] for more references.

Book Maude



- ▶ pdf available:

<https://maude.cs.illinois.edu/w/images/0/0d/Maude-book.pdf>

Lectures

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2. Modeling by labeled transition systems
 - ▶ executions; traces; non-determinism; examples
3. Linear-Time Behaviour and Properties
 - ▶ invariant, safety, and liveness (and fairness) properties
4. Linear Temporal Logic (LTL)
 - ▶ syntax and semantics; interpretation of LTSs; examples
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 - ▶ model checking of LTL formulas, and fairness in LTL
 - ▶ Maude examples
6. Computation Tree Logic (CTL)
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crl [Inc1a]    : Inc1 x => Inc2 x  if x < 200  
rl  [Inc2]     : Inc2 x => Inc1 (x + 1)  
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ceq (S1 S2 S3 x |= counterle200) = true  if (x < 200 \vee x = 200)

```

Maude output (simplified)

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reduce in COUNTERS-CHECK : modelCheck(initial, <> counterlt0)
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counterexample({Inc1 Dec1 Reset1 199}
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2. Modeling by labeled transition systems
 - ▶ executions; traces; non-determinism; examples
3. Linear-Time Behaviour and Properties
 - ▶ invariant, safety, and liveness (and fairness) properties
4. Linear Temporal Logic (LTL)
 - ▶ syntax and semantics; interpretation of LTSs; examples
5. LTL (continued)
 - ▶ fairness in LTL
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Exam options

Together we decide on a topic.

Study an article, or a book chapter.

or

Dive deeper into a proof from the lecture.

or

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or

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You give a 25-minute presentation about what you found.

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Exam examples

Presentations about articles:

- ▶ *Progress, Justness, and Fairness* (2019) by R. van Glabbeek, P. Höfner
- ▶ *Comparison of Model Checking Tools for Information Systems* (2010) by M. Frappier et al. (focusing on SPIN and NuSMV)

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-  M. Clavel, F. Durán, S. Eker, P. Lincoln, N. Martí-Oliet, J. Meseguer, and C. Talcott.
All About Maude - A High-Performance Logical Framework: How to Specify, Program, and Verify Systems in Rewriting Logic.
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