Closing the Image of the Process Interpretation of 1-Free Regular Expressions Under Bisim. Collapse

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Overview

- 1-free regular expressions (with unary/binary star)
- process interpretation/semantics of regular expressions
  - expressible/not expressible process graphs

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- loop existence and elimination property (LEE) (G/Fokkink, 2020)
  - interpretation/extraction correspondences with 1-free reg. expr's
  - LEE is preserved under bisimulation collapse
- Q: Image of process interpretation of 1-free regular expressions closed under bisimulation collapse?

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- process interpretation / semantics of regular expressions
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  - interpretation/extraction correspondences with 1-free reg. expr's
  - LEE is preserved under bisimulation collapse
- Q: Image of process interpretation of 1-free regular expressions closed under bisimulation collapse?
  - A1: No. — But . . .
- compact process interpretation
- refined expression extraction
  - A2: compact process interpretation is image-closed under collapse
- outlook: consequences

Regular Expressions

Definition (~ *Copi–Elgot–Wright, 1958*)

Regular expressions over alphabet A with unary Kleene star:

$$e, e_1, e_2 \; ::= \; \mathbf{0} \;\mid\; \boldsymbol{a} \;\mid\; e_1 + e_2 \;\mid\; e_1 \cdot e_2 \;\mid\; e^{*}$$ (for $\boldsymbol{a} \in A$).

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- symbol $\mathbf{0}$ instead of $\varnothing$, symbol $\mathbf{1}$ instead of $\{\varnothing\}$ and ϵ
- with unary Kleene star $*$: $\mathbf{1}$ is definable as $\mathbf{0}^*$

Regular Expressions

Definition (~ *Kleene, 1951*, ~ *Copi–Elgot–Wright, 1958*)

Regular expressions over alphabet A with unary / binary Kleene star:

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Definition

1-free regular expressions over alphabet A with binary Kleene star:

$$f, f_1, f_2 ::= \mathbf{0} \mid \boldsymbol{a} \mid f_1 + f_2 \mid f_1 \cdot f_2 \mid f_1{}^{\circledast} f_2 \qquad (\text{for } \boldsymbol{a} \in A).$$

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Process interpretation $P(\cdot)$ of regular expressions *(Milner, 1984)*

$\mathbf{0} \stackrel{P}{\longmapsto}$ deadlock $\boldsymbol{\delta}$, no termination

$\mathbf{1} \stackrel{P}{\longmapsto}$ empty-step process $\boldsymbol{\epsilon}$, then terminate

$a \stackrel{P}{\longmapsto}$ atomic action a, then terminate

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$e_1 + e_2 \xmapsto{P}$ *(choice)* execute $P(e_1)$ or $P(e_2)$

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$e_1{}^{\circledast} e_2 \xmapsto{P}$ (*iteration-exit*) repeat (terminate or execute $P(e_1)$), then $P(e_2)$

Process semantics $[\![\cdot]\!]_P$ of regular expressions *(Milner, 1984)*

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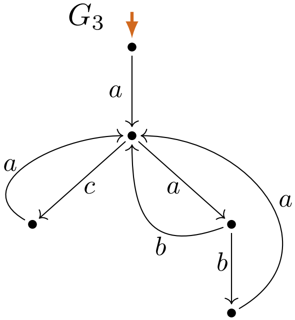
$[\![e]\!]_P \;:=\; [P(e)]_{\underline{\leftrightarrow}}$ (bisimilarity equivalence class of process $P(e)$)

P-expressibility and $[\![\cdot]\!]_P$-expressibility (examples)



$$P\Big(\overbrace{(a\cdot(c\cdot a + a\cdot(b + b\cdot a))^*)\cdot 0}^{f}\Big)$$

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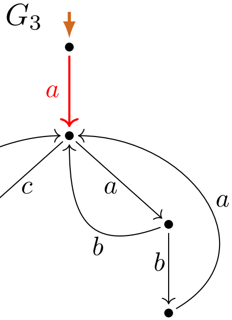

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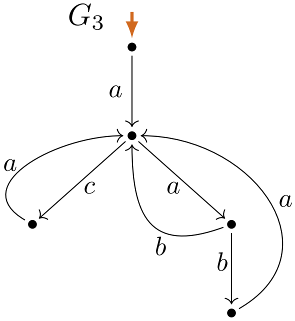

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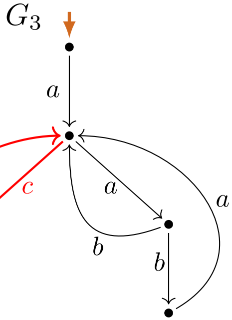

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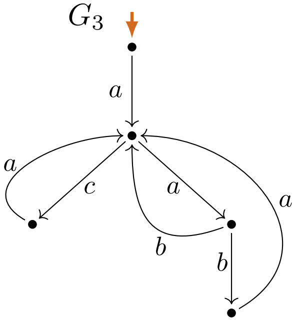

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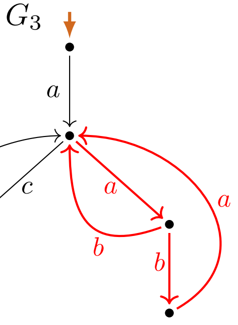

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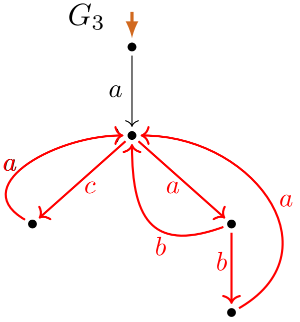

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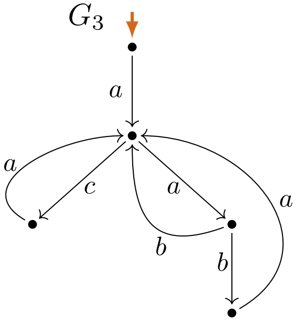

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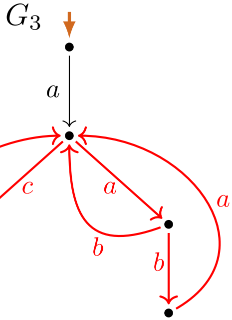

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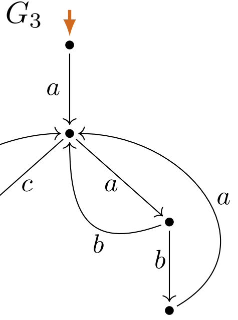

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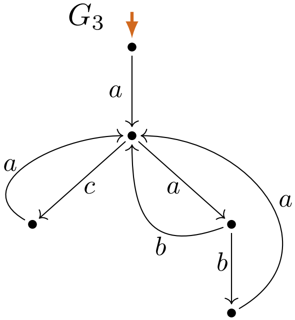

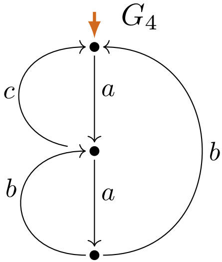

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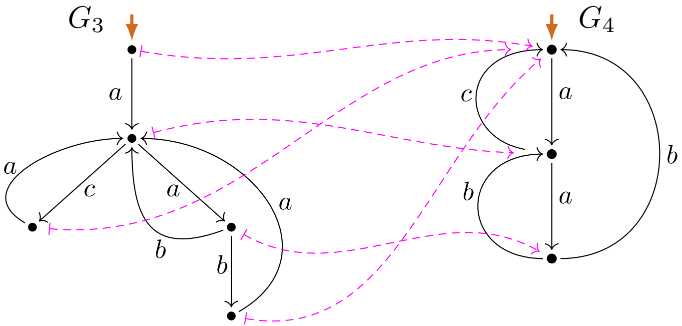

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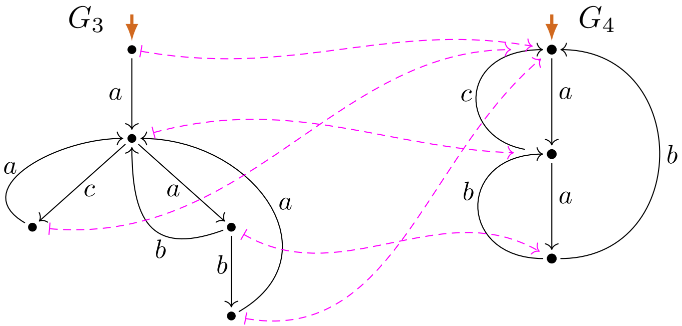

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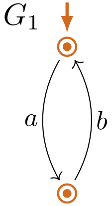

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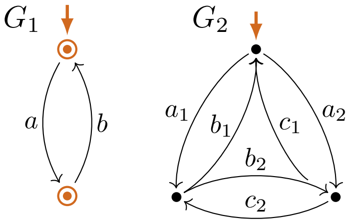

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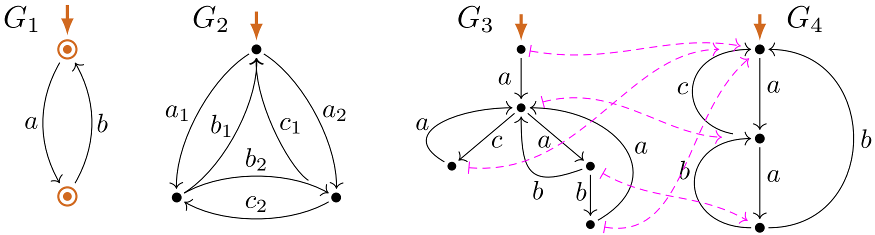

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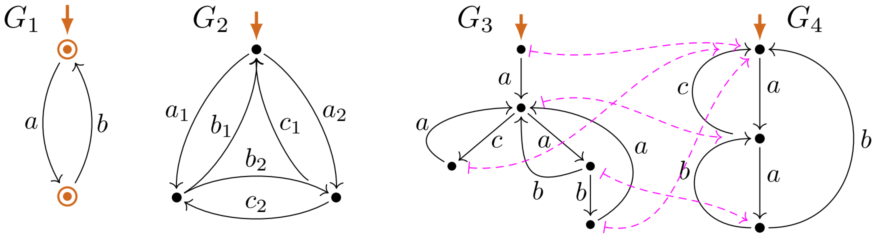

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$[\![\cdot]\!]_P$-expressible

?

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Process interpretation P (formal definition)

Definition (Transition system specification $\mathcal{T}$)

$$\frac{}{a \xrightarrow{a} 1} \qquad \frac{e_i \xrightarrow{a} e_i'}{e_1 + e_2 \xrightarrow{a} e_i'} \; (i \in \{1,2\})$$

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Definition

The process (graph) interpretation $P(e)$ of a regular expression e:

$P(e) :=$ labeled transition graph generated by e by derivations in $\mathcal{T}$.

Interpretation/extraction correspondences with LEE

($\Leftarrow$ G/Fokkink 2020, G 2021)

(Int)$_{P}^{(\cancel{1})}$: *P-expressible graphs have the structural property* LEE

Process **i**nterpretations $P(e)$ of 1-free regular expressions e are finite process graphs that satisfy LEE.

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From every finite process graph G with LEE a regular expression e can be **e**xtracted such that $G \,\underline{\leftrightarrow}\, P(e)$.

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From every finite process graph G with LEE
a regular expression e can be **e**xtracted
such that $G \underline{\leftrightarrow} P(e)$.

(Coll): LEE *is preserved under collapse*

The class of finite process graphs with LEE
is closed under bisimulation **c**ollapse.

Loop charts (interpretations of innermost iterations)

Definition

A chart is a loop chart if:

(L1) There is an infinite path from the start vertex.

(L2) Every infinite path from the start vertex returns to it.

(L3) Termination is only possible at the start vertex.

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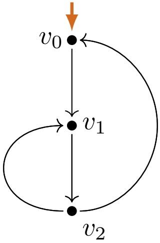

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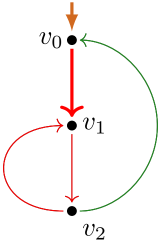

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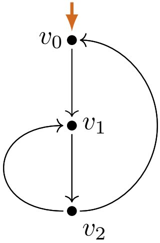

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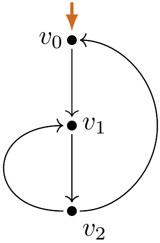

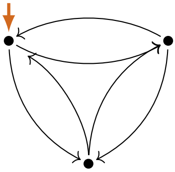

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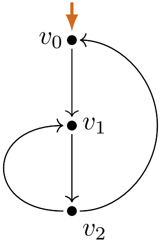

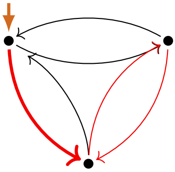

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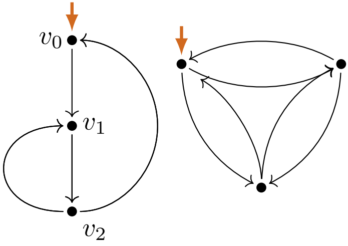

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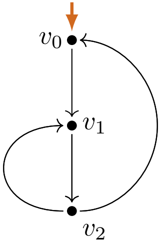

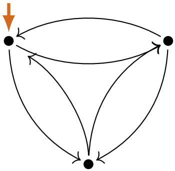

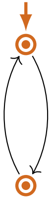

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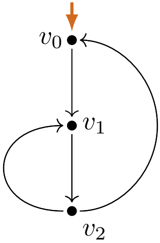

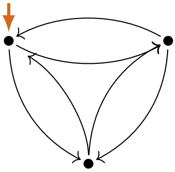

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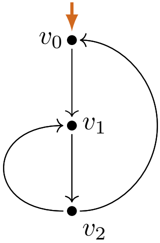

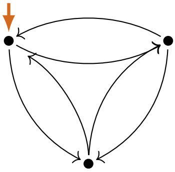

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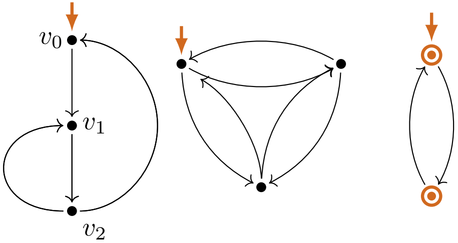

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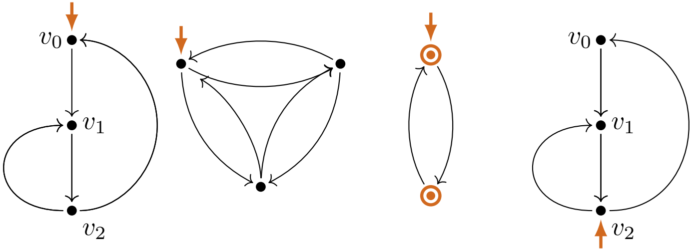

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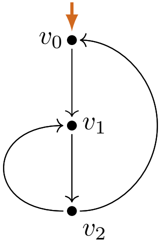

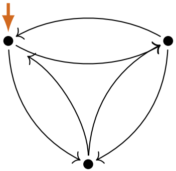

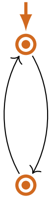

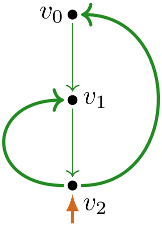

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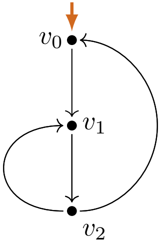

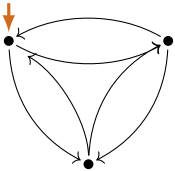

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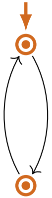

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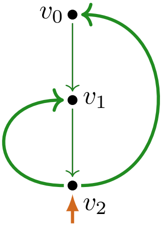

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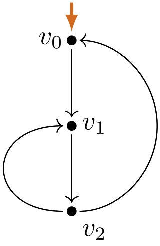

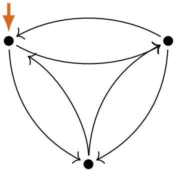

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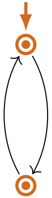

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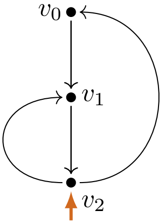

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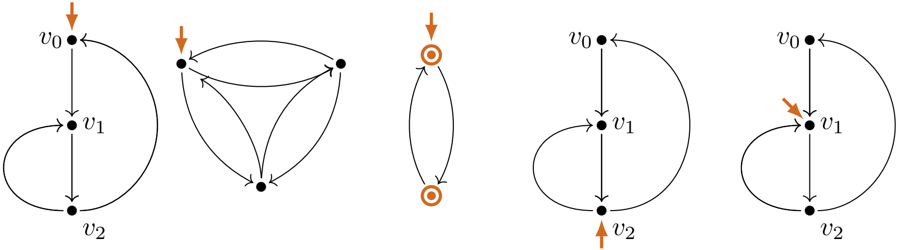

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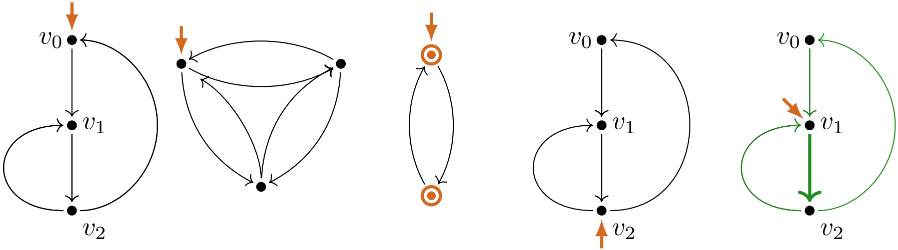

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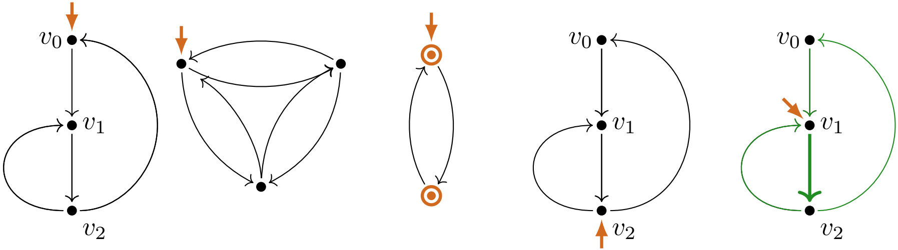

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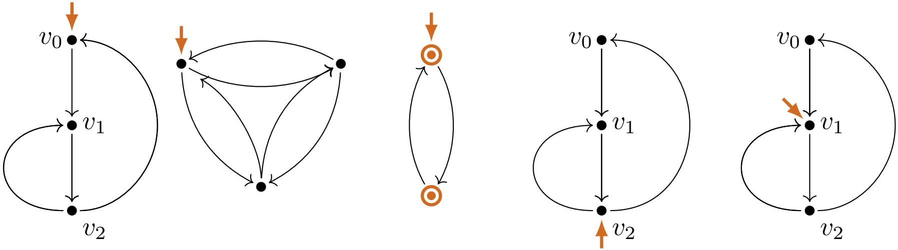

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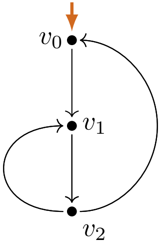

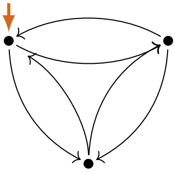

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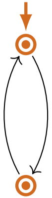

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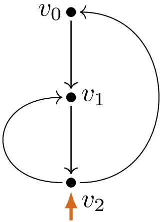

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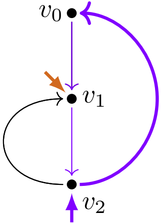
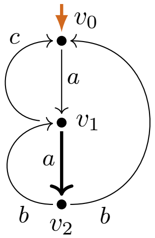


loop subchart

Loop existence and elimination



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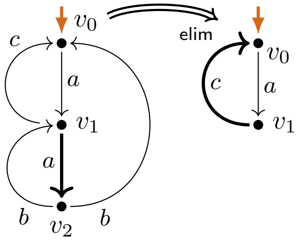

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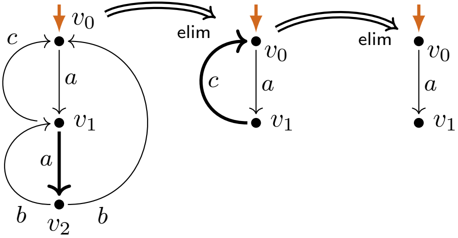

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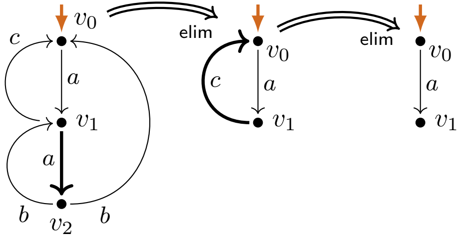

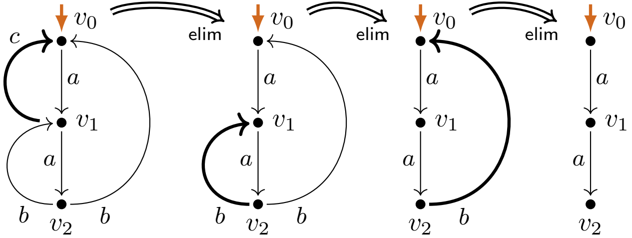

LEE

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A chart $\mathcal{C}$ satisfies LEE (*loop existence and elimination*) if:

$$\exists \mathcal{C}_0 \left(\mathcal{C} \longrightarrow^{*}_{\mathsf{elim}} \mathcal{C}_0 \not\longrightarrow_{\mathsf{elim}} \right.$$
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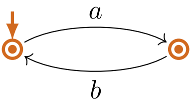

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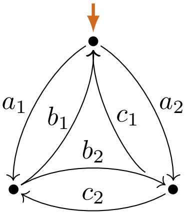

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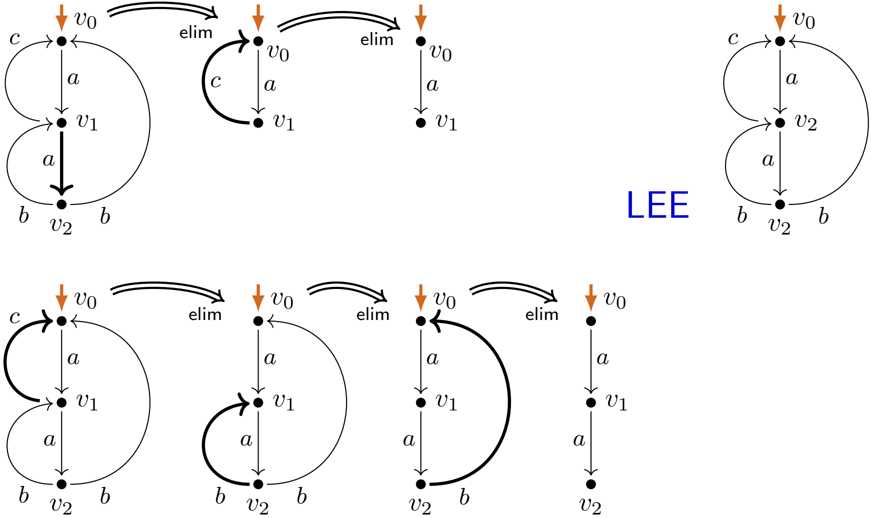

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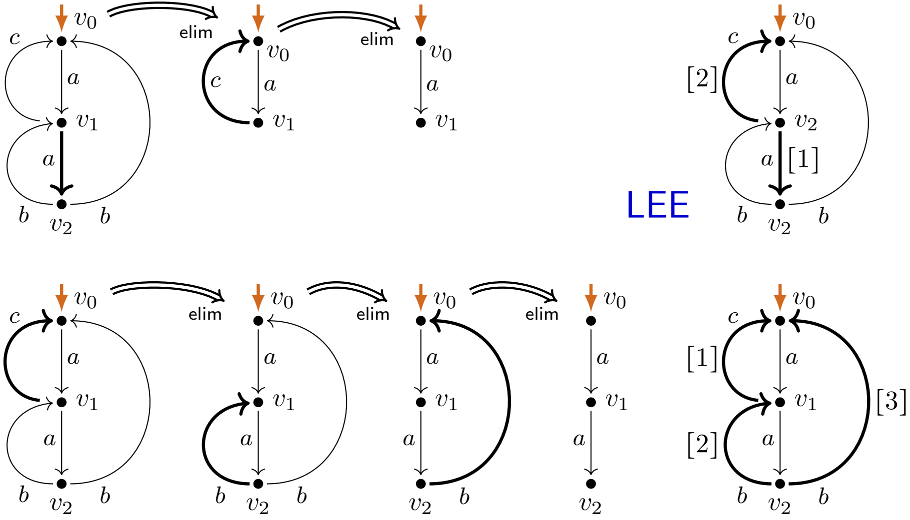
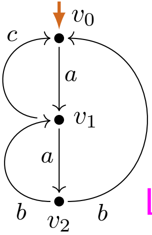
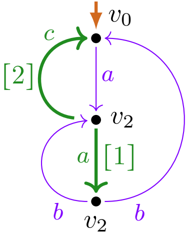


LEE witness and LEE-charts

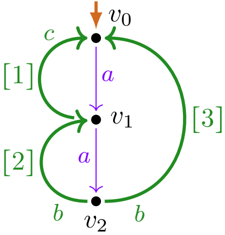


LEE-chart

LEE-witness



LEE-witness



Properties of LEE-charts

Theorem (⇐ G/Fokkink, 2020)

A process graph G

is $[\![\cdot]\!]_P$-expressible by a 1-free regular expression

(i.e. P-expressible modulo bisimilarity by a 1-free reg. expr.)

if and only if

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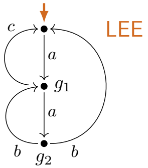
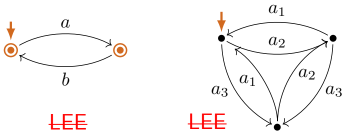
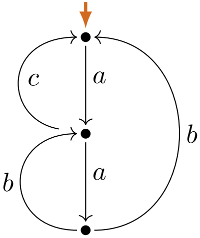
Hence $[\![\cdot]\!]_P$-expressible | **not** $[\![\cdot]\!]_P$-expressible by 1-free regular expressions:





Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

G_4



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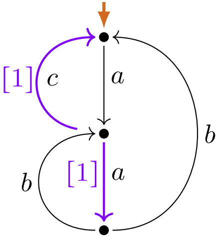

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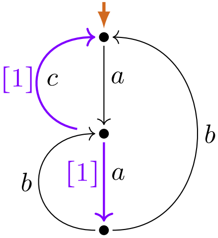

$$\Big(\qquad\qquad\qquad\qquad \Big)^{*} \cdot 0$$

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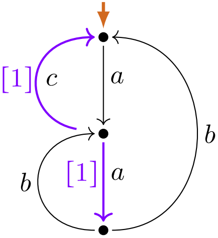

$$\Big(\quad \overset{\downarrow a}{} \quad \Big)^* \cdot 0$$

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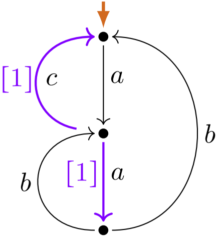


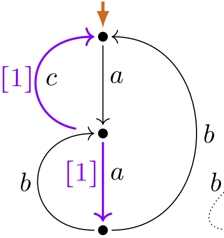
$$(a \cdot 1) \cdot ((\qquad\qquad\qquad\qquad)^* \cdot 0)$$

$$\Big\downarrow a$$

$$(\qquad\qquad\qquad\qquad)^* \cdot 0$$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$(a \cdot 1) \cdot ((\qquad)^* \cdot 0)$

a

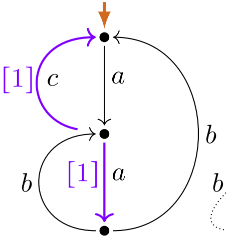
$(\qquad)^* \cdot 0$

b

b

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$$(a \cdot 1) \cdot ((\qquad)^* \cdot 0)$$

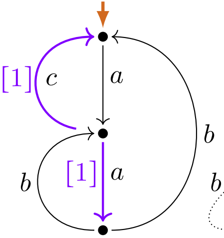
$$\Big\downarrow a$$

$$(\qquad)^* \cdot 0$$

$$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot ((\qquad)^*) \cdot 0)$$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$$(a \cdot 1) \cdot ((\qquad)^* \cdot 0)$$

$$\downarrow a$$

$$(\qquad)^* \cdot 0$$

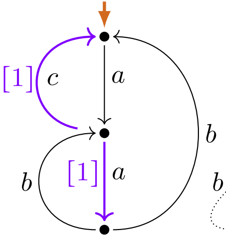
$$\downarrow a$$

$$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot ((\qquad)^*) \cdot 0)$$

c b b

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$$(a \cdot 1) \cdot \big(\big(\qquad \big)^{*} \cdot 0\big)$$

$$\big\downarrow a$$

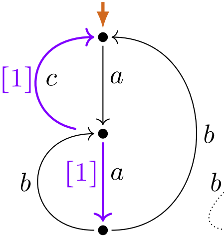
$$\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^{*} \cdot 0$$

$$\big\downarrow a$$

$$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot \big(\big(\qquad \big)^{*}) \cdot 0\big)$$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$$(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)$$

$$\Big\downarrow a$$

$$\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0$$

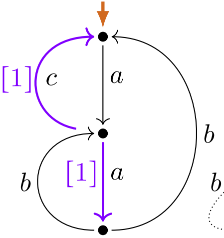
$$\Big\downarrow a$$

$$(b\cdot 1+b\cdot(a\cdot 1))\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*})\cdot 0\Big)$$

c b b

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$$\overbrace{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)}^{e}$$

$$\downarrow a$$

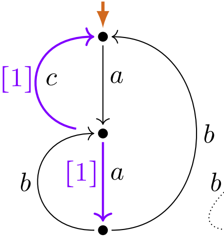
$$\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0 \quad \xrightarrow{c} e$$

$$\downarrow a$$

$$(b\cdot 1+b\cdot(a\cdot 1))\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0\Big) \quad \xrightarrow{b} \big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0, \quad \xrightarrow{b} e$$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$P(e) \;\underline{\rightarrow}\; G_4$

$$\overbrace{(a\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)}^{e}$$

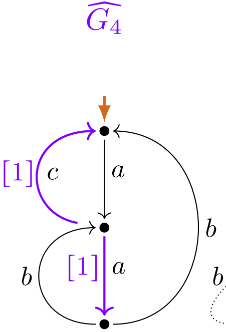
$$\downarrow a$$

$$\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0$$

$$\downarrow a$$

$$(b\cdot 1+b\cdot(a\cdot 1))\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0\Big)$$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)



$$P(e) \mathrel{\underline{\to}} G_4$$

$$\overbrace{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)}^{e}$$

$$\cancel{\downarrow}\, a$$

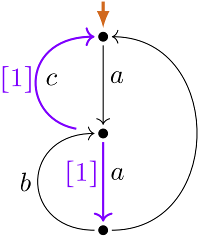
$$\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0$$

$$\downarrow a$$

$$(b\cdot 1+b\cdot(a\cdot 1))\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0\Big)$$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$P(e) \rightleftharpoons G_4$

$$\overbrace{(a \cdot 1) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^{*} \cdot 0\big)}^{e}$$

$$\downarrow a$$

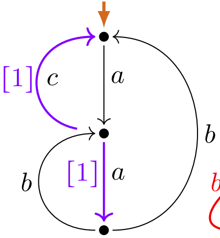
$$\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^{*} \cdot 0$$

$$\downarrow a$$

$$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^{*}\big) \cdot 0\big)$$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$P(e) \trianglerighteq G_4$

$$\overbrace{(a\cdot 1)\cdot\Big(\Big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\Big)^*\cdot 0\Big)}^{e}$$

$$\downarrow a$$

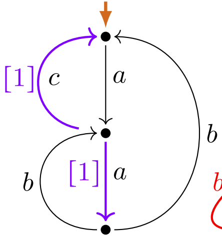
$$\Big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\Big)^*\cdot 0$$

$$\downarrow a$$

$$(b\cdot 1+b\cdot(a\cdot 1))\cdot\Big(\Big(\Big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\Big)^*\Big)\cdot 0\Big)$$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$$P(e) \mathrel{\underline{\to}} G_4 \mathrel{\not\underline{\leftarrow}} P(e)$$

$$\overbrace{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)}^{e}$$

$$\downarrow a$$

$$\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0$$

$$\downarrow a$$

$$(b\cdot 1+b\cdot(a\cdot 1))\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0\Big)$$

Interpretation of extracted expression

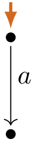
G_5

$$\textcolor{green}{P}(\textcolor{orange}{e}) = G_5$$

$$\overbrace{(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^{*} \cdot 0\Big)}^{e}$$

Interpretation of extracted expression

G_5



$P(e) = G_5$

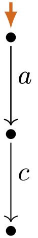
$$\overbrace{(a \cdot 1) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0 \big)}^{e}$$

$$\Big\downarrow a$$

$$(1 \cdot 1) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0 \big)$$

Interpretation of extracted expression

G_5



$$P(e) = G_5$$

$$\overbrace{(a\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)}^{e}$$

$$\downarrow a$$

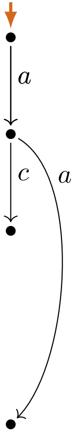
$$(1\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)$$

$$\downarrow c$$

$$\big((1\cdot(a\cdot 1))\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\big)\cdot 0$$

Interpretation of extracted expression

G_5



$P(e) = G_5$

$$\overbrace{(a\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^*\cdot 0\big)}^{e}$$

$$\downarrow a$$

$$(1\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^*\cdot 0\big)$$

$$\downarrow c$$

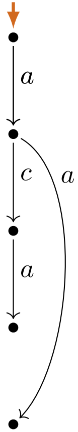
$$\big((1\cdot(a\cdot 1))\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^*\big)\cdot 0$$

$$a$$

$$\big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^*\big)\cdot 0$$

Interpretation of extracted expression

G_5



$P(e) = G_5$

$$\overbrace{(a\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)}^{e}$$

$$\downarrow a$$

$$(1\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)$$

$$\downarrow c$$

$$\big((1\cdot(a\cdot 1))\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\big)\cdot 0$$

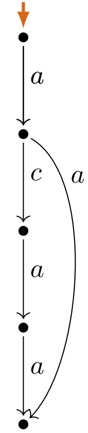
$$\downarrow a$$

$$\big((1\cdot 1)\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\big)\cdot 0\big)$$

$$\big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\big)\cdot 0$$

Interpretation of extracted expression

G_5



$\textsf{P}(e) = G_5$

$$\overbrace{(a\cdot 1)\cdot\big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\big)}^{e}$$

$$\downarrow a$$

$$(1\cdot 1)\cdot\big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\big)$$

$$\downarrow c$$

$$\big((1\cdot(a\cdot 1))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0$$

$$\downarrow a$$

$$\big((1\cdot 1)\cdot\big(c\cdot(a\cdot 1)+{\color{red}a}\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0\big)$$

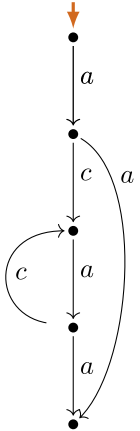
$$\downarrow a$$

$$\big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0$$

a

Interpretation of extracted expression

G_5



$$P(e) = G_5$$

$$\overbrace{(a \cdot 1) \cdot \big((c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)))^{*} \cdot 0 \big)}^{e}$$

$$\downarrow a$$

$$(1 \cdot 1) \cdot \big((c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)))^{*} \cdot 0 \big)$$

$$\downarrow c$$

$$\big((1 \cdot (a \cdot 1)) \cdot (c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)))^{*} \big) \cdot 0$$

$$\downarrow a$$

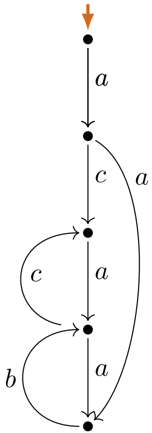
$$\big((1 \cdot 1) \cdot ({\color{red}c} \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)))^{*} \big) \cdot 0$$

$$\downarrow a$$

$$\big((1 \cdot ({\color{red}b} \cdot 1 + b \cdot (a \cdot 1))) \cdot (c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)))^{*} \big) \cdot 0$$

Interpretation of extracted expression

G_5



$P(e) = G_5$

$$\overbrace{(a\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)}^{e}$$

$$\Big\downarrow a$$

$$(1\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)$$

$$\Big\downarrow c$$

$$\big((1\cdot(a\cdot 1))\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\big)\cdot 0$$

$$\Big\downarrow a$$

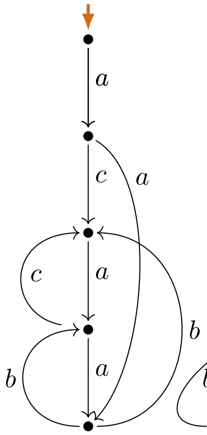
$$\big((1\cdot 1)\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\big)\cdot 0\big)$$

$$\Big\downarrow a$$

$$\big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\big)\cdot 0$$

Interpretation of extracted expression

G_5



$P(e) = G_5$

$$\overbrace{(a\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)}^{e}$$

$$\Big\downarrow a$$

$$(1\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)$$

$$\Big\downarrow c$$

$$\big((1\cdot(a\cdot 1))\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\big)\cdot 0$$

$$\Big\downarrow a$$

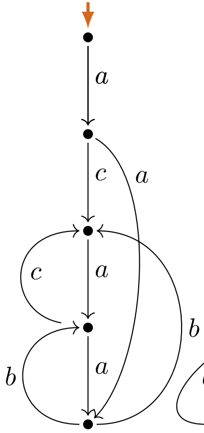
$$\big((1\cdot 1)\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\big)\cdot 0\big)$$

$$\Big\downarrow a$$

$$\big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\big)\cdot 0$$

Interpretation of extracted expression

G_5



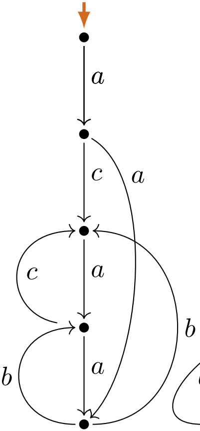
$$P(e) = G_5 \rightrightarrows G_4$$

$$\begin{tikzcd}
\overbrace{(a \cdot 1) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \cdot 0 \big)}^{e} \arrow[d, "a"] \\
(1 \cdot 1) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \cdot 0 \big) \arrow[d, "c"] \arrow[ddd, bend left, "a"] \\
\big((1 \cdot (a \cdot 1)) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \big) \cdot 0 \arrow[d, "a"] \\
\big((1 \cdot 1) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \big) \cdot 0 \big) \arrow[d, "a"] \arrow[u, bend left, "c"] \\
\big((1 \cdot (b \cdot 1 + b \cdot (a \cdot 1))) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \big) \cdot 0 \arrow[u, bend left, "b"] \arrow[uu, bend right, "b"]
\end{tikzcd}$$

Interpretation of extracted expression

G_5

$$P(e) = G_5 \rightrightarrows G_4 \not\simeq G_5$$



$$\begin{tikzcd}
\overbrace{(a\cdot 1)\cdot\big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\big)}^{e} \arrow[d, "a"] \\
(1\cdot 1)\cdot\big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\big) \arrow[d, "c"] \arrow[ddd, bend left, "a"] \\
\big((1\cdot(a\cdot 1))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0 \arrow[d, "a"] \\
\big((1\cdot 1)\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0 \arrow[d, "a"] \arrow[u, bend left, "c"] \\
\big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0 \arrow[u, bend left, "b"] \arrow[uu, bend right, "b"]
\end{tikzcd}$$

Image of P **not** closed under bisimulation collapse

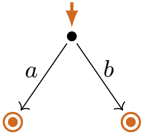
$P(uf)$

$P(uf)$

$$uf := a \cdot \overbrace{(a \cdot (a + a \cdot 0))^*}^{uf_a} + b \cdot \overbrace{(b \cdot (b + b \cdot 0))^*}^{uf_b}$$

Image of P **not** closed under bisimulation collapse

$P(uf)$



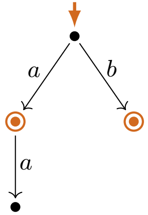
$P(uf)$

$$uf := a \cdot \overbrace{(a \cdot (a + a \cdot 0))^*}^{uf_a} + b \cdot \overbrace{(b \cdot (b + b \cdot 0))^*}^{uf_b}$$

$$\xrightarrow{a}\ 1 \cdot uf_a \Downarrow \qquad\qquad \xrightarrow{b}\ \Downarrow 1 \cdot uf_b$$

Image of P **not** closed under bisimulation collapse

$P(uf)$



$P(uf)$

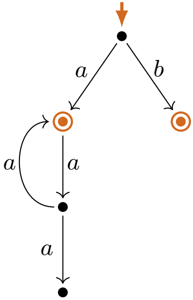
$$uf := a \cdot \overbrace{(a \cdot (a + a \cdot 0))^*}^{uf_a} + b \cdot \overbrace{(b \cdot (b + b \cdot 0))^*}^{uf_b}$$

$$uf \xrightarrow{a} 1 \cdot uf_a \Downarrow \qquad uf \xrightarrow{b} \Downarrow 1 \cdot uf_b$$

$$1 \cdot uf_a \xrightarrow{a} (1 \cdot (a + a \cdot 0)) \cdot uf_a$$

Image of P **not** closed under bisimulation collapse

$P(uf)$



$P(uf)$

$$uf := a \cdot \overbrace{(a \cdot (a + a \cdot 0))^*}^{uf_a} + b \cdot \overbrace{(b \cdot (b + b \cdot 0))^*}^{uf_b}$$

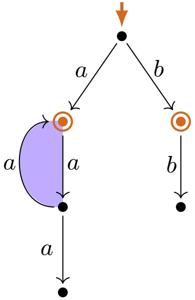
$$uf \xrightarrow{a} 1 \cdot uf_a \Downarrow \qquad uf \xrightarrow{b} \Downarrow 1 \cdot uf_b$$

$$1 \cdot uf_a \xrightarrow{a} (1 \cdot (a + a \cdot 0)) \cdot uf_a \xrightarrow{a} 1 \cdot uf_a$$

$$(1 \cdot (a + a \cdot 0)) \cdot uf_a \xrightarrow{a} (1 \cdot 0) \cdot uf_a$$

Image of *P* **not** closed under bisimulation collapse

$P(uf)$



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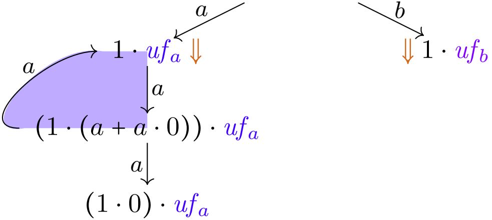
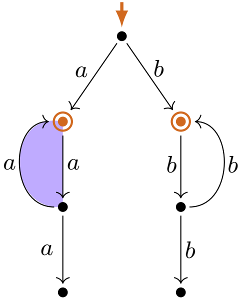


Image of P **not** closed under bisimulation collapse

$P(uf)$



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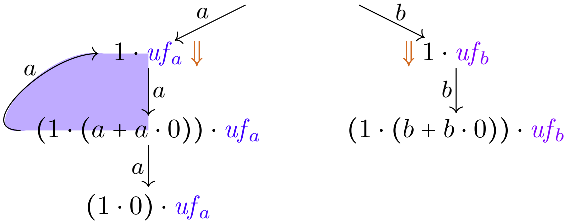

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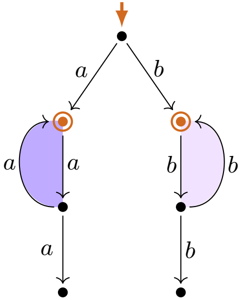

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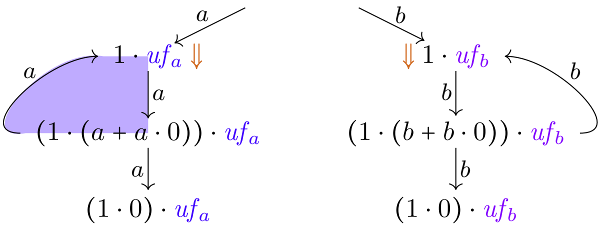

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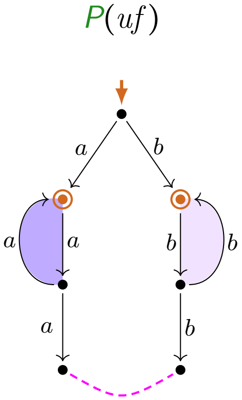

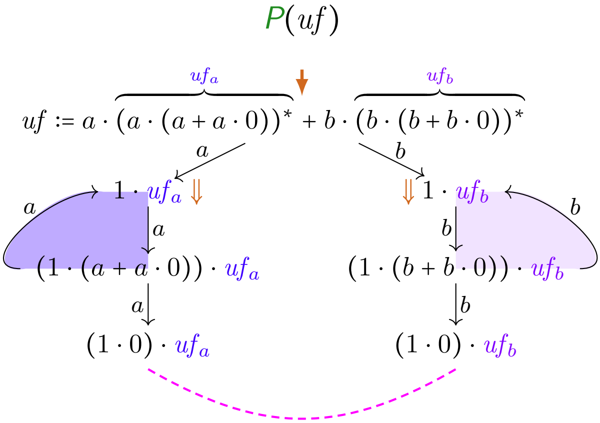

Compact process interpretation $P^{\bullet}$

Definition (Transition system specification $\mathcal{T}$)

$$\frac{}{\mathbf{1}{\Downarrow}} \qquad \frac{e_i{\Downarrow}}{(e_1 + e_2){\Downarrow}}\;(i \in \{1,2\}) \qquad \frac{e_1{\Downarrow} \quad e_2{\Downarrow}}{(e_1 \cdot e_2){\Downarrow}} \qquad \frac{}{(e^*){\Downarrow}}$$

$$\frac{}{a \xrightarrow{a} \mathbf{1}} \qquad \frac{e_i \xrightarrow{a} e_i'}{e_1 + e_2 \xrightarrow{a} e_i'}\;(i \in \{1,2\})$$

$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2} \qquad \frac{e_1{\Downarrow} \quad e_2 \xrightarrow{a} e_2'}{e_1 \cdot e_2 \xrightarrow{a} e_2'} \qquad \frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e' \cdot e^*}$$

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Definition (Transition system specification $\mathcal{T}^{\bullet}$, changed rules w.r.t. $\mathcal{T}$)

$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2} \text{ (if } e_1' \text{ is normed)}$$

$$\frac{e \xrightarrow{a} e'}{e^{*} \xrightarrow{a} e' \cdot e^{*}} \text{ (if } e' \text{ is normed)}$$

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$$\frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e' \cdot e^*} \text{ (if } e' \text{ is normed)} \qquad \frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e'} \text{ (if } e' \text{ is not normed)}$$

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Definition

The compact process (graph) interpretation $P^{\bullet}(e)$ of a reg. expr's e:

$P^{\bullet}(e) :=$ labeled transition graph generated by e by derivations in $\mathcal{T}^{\bullet}$.

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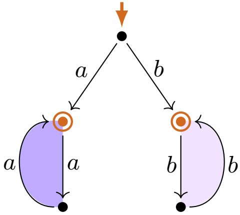
Lemma ($P^{\bullet}$ increases sharing; $P^{\bullet}$, P have same bisimulation semantics)

(i) $P(e) \rightarrow\!\!\!\!\!\_\; P^{\bullet}(e)$ *for all regular expressions* e.

(ii) (G *is* $[\![\cdot]\!]_{P^{\bullet}}$*-expressible* $\Longleftrightarrow$ G *is* $[\![\cdot]\!]_{P}$*-expressible) for all graphs* G.

Image of $P^{\bullet}$ under bisimulation collapse . . .

$P^{\bullet}(uf)$



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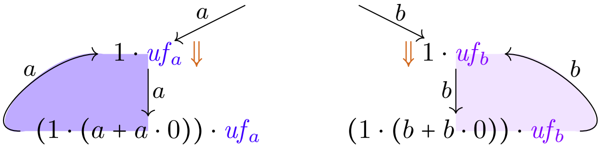

Image of $P^{\bullet}$ under bisimulation collapse ...

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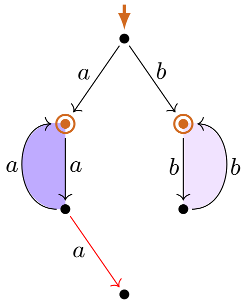

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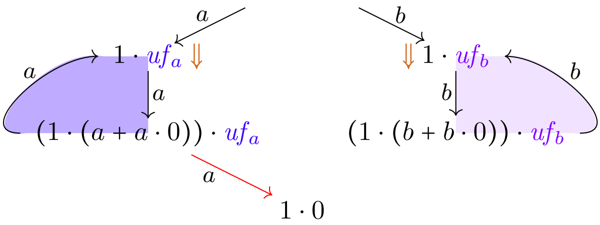
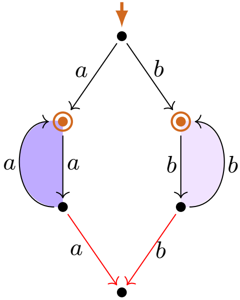
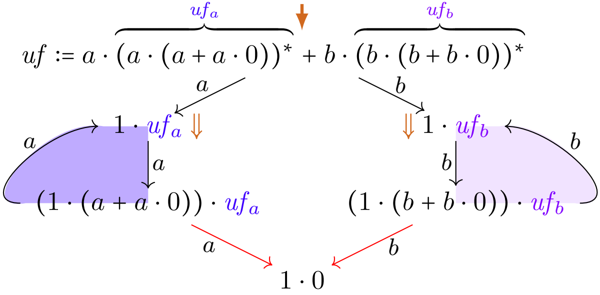


Image of $P^\bullet$ under bisimulation collapse ...

$P^\bullet(uf)$



$P^\bullet(uf)$



Under-Star-/1-Free regular expressions

Definition

The set $RExp^{(\cancel{1})}(A)$ of 1-free regular expressions over A is defined by:

$$f,\, f_1,\, f_2 \;::=\; 0 \mid a \mid f_1 + f_2 \mid f_1 \cdot f_2 \mid f_1{}^{\star} \cdot f_2 \qquad \text{(for } a \in A\text{)},$$

Under-Star-/1-Free regular expressions

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the set $RExp^{(\mathbb{1}\backslash *)}(A)$ of under-star-1-free regular expressions over A by:

$$uf, uf_1, uf_2 \;::=\; 0 \mid 1 \mid a \mid uf_1 + uf_2 \mid uf_1 \cdot uf_2 \mid f^{\star} \qquad (\text{for } a \in A).$$

Interpretation correspondence of $P^{\bullet}$ with LEE

$(\textbf{Int})^{(\cancel{1}\backslash *)}_{P^{\bullet}}$: *$P^{\bullet}$-expressible graphs satisfy* LEE:

Compact process **i**nterpretations $P^{\bullet}(uf)$
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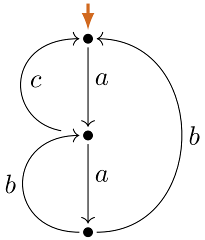
are finite process graphs that satisfy LEE.

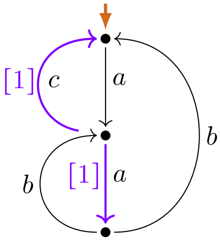
Refined extraction expression (example)

G_4



Refined extraction expression (example)

$\widehat{G_4}$



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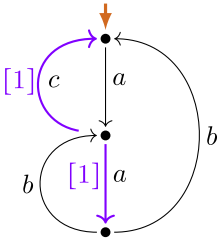
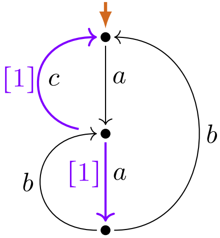


$$(1 \cdot (\qquad\qquad\qquad)^{*}) \cdot 0$$

Refined extraction expression (example)

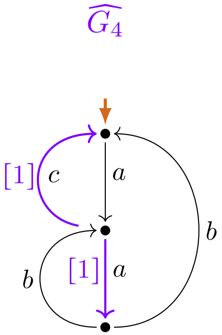
$\widehat{G_4}$



a

$(1 \cdot (\qquad\qquad)^*) \cdot 0$

Refined extraction expression (example)



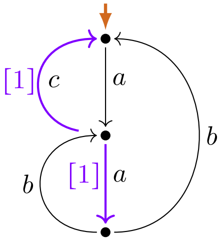
$$((1 \cdot a) \cdot (\qquad\qquad)^*) \cdot 0$$

$$\Big\downarrow a$$

$$(1 \cdot (\qquad\qquad)^*) \cdot 0$$

Refined extraction expression (example)

$\widehat{G_4}$



$((1 \cdot a) \cdot (\qquad)^*) \cdot 0$

$\downarrow a$

$(1 \cdot (\qquad)^*) \cdot 0$

$b \qquad b$

Refined extraction expression (example)

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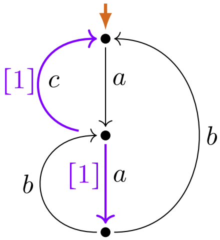


$((1 \cdot a) \cdot (\qquad)^*) \cdot 0$

a

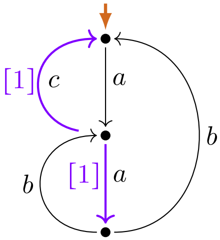
$(1 \cdot (\qquad)^*) \cdot 0$

b $\qquad$ b

$((1 \cdot (b + b \cdot a)) \cdot (\qquad)^*) \cdot 0$

Refined extraction expression (example)

$\widehat{G_4}$



$((1 \cdot a) \cdot (\qquad)^*) \cdot 0$

$(1 \cdot (\qquad)^*) \cdot 0$

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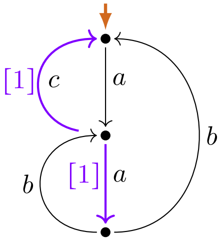
Refined extraction expression (example)

$\widehat{G_4}$



$$((1 \cdot a) \cdot (\qquad\qquad)^*) \cdot 0$$

$$\big\downarrow a$$

$$(1 \cdot (c \cdot a + a \cdot (b + b \cdot a))^*) \cdot 0$$

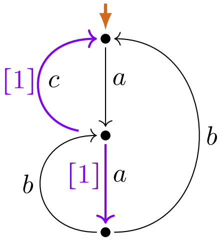
$$\big\downarrow a$$

$$((1 \cdot (b + b \cdot a)) \cdot (\qquad\qquad)^*) \cdot 0$$

c b b

Refined extraction expression (example)

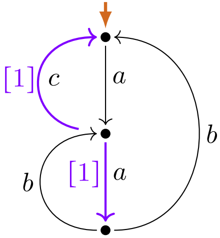
$\widehat{G_4}$

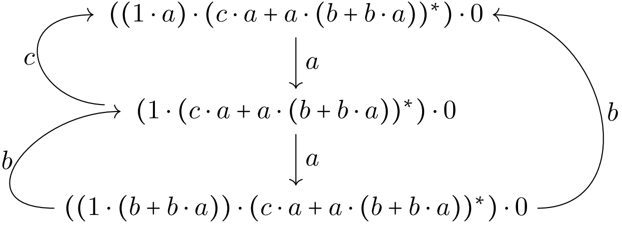


$$
\begin{tikzcd}
((1 \cdot a) \cdot (c \cdot a + a \cdot (b + b \cdot a))^{*}) \cdot 0 \arrow[d, dotted, "a"] \\
(1 \cdot (c \cdot a + a \cdot (b + b \cdot a))^{*}) \cdot 0 \arrow[d, dotted, "a"] \arrow[u, dotted, bend left, "c"] \\
((1 \cdot (b + b \cdot a)) \cdot (c \cdot a + a \cdot (b + b \cdot a))^{*}) \cdot 0 \arrow[u, dotted, bend left, "b"] \arrow[uu, dotted, bend right, "b"']
\end{tikzcd}
$$

Refined extraction expression (example)

$\widehat{G_4}$





Refined extraction expression (example)

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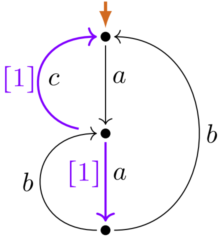

$$P^{\bullet}(uf) = P(uf) \simeq G_4$$

$$\begin{tikzcd}
\overbrace{((1\cdot a)\cdot(c\cdot a + a\cdot(b + b\cdot a))^{*})\cdot 0}^{uf} \arrow[d, "a"] \\
(1\cdot(c\cdot a + a\cdot(b + b\cdot a))^{*})\cdot 0 \arrow[d, "a"] \arrow[u, bend left=60, "c"] \\
((1\cdot(b + b\cdot a))\cdot(c\cdot a + a\cdot(b + b\cdot a))^{*})\cdot 0 \arrow[u, bend left=60, "b"] \arrow[uu, bend right=80, "b"']
\end{tikzcd}$$

Interpretation/extraction correspondences of $P^{\bullet}$ with LEE

$(\textbf{Int})^{(\cancel{1}\backslash *)}_{P^{\bullet}}$: *$P^{\bullet}$-expressible graphs satisfy LEE:*

Compact process **i**nterpretations $P^{\bullet}(uf)$
of under-star-1-free regular expressions uf
are finite process graphs that satisfy LEE.

$(\textbf{Extr})^{(\cancel{1}\backslash *)}_{P^{\bullet}}$: LEE *implies $[\![\cdot]\!]_P$-expressibility by under-star-1-free reg. expr's*:

From every finite process graph G with LEE
an under-star-1-free regular expression uf can be **e**xtracted
such that $G \rightarrow P(uf)$.

Interpretation/extraction correspondences of $P^{\bullet}$ with LEE

$\mathbf{(Int)}_{P^{\bullet}}^{(\cancel{1}\backslash *)}$: *$P^{\bullet}$-expressible graphs satisfy LEE*:

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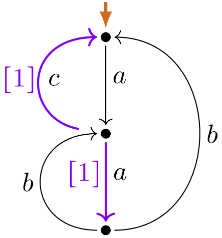
such that $G \underline{\simeq} P(uf)$.

(ImColl)$_{P^{\bullet}}^{(\mathbf{1}\backslash *)}$: The image of $P^{\bullet}$,
restricted to under-star-1-free regular expressions,
is closed under bisimulation collapse.

Refined extraction expression (example)

$\widehat{G_4}$



$$P^{\bullet}(uf) = P(uf) \simeq G_4$$

$$\begin{tikzcd}
\overbrace{((1\cdot a)\cdot(c\cdot a+a\cdot(b+b\cdot a))^{*})\cdot 0}^{uf} \arrow[d, "a"] \\
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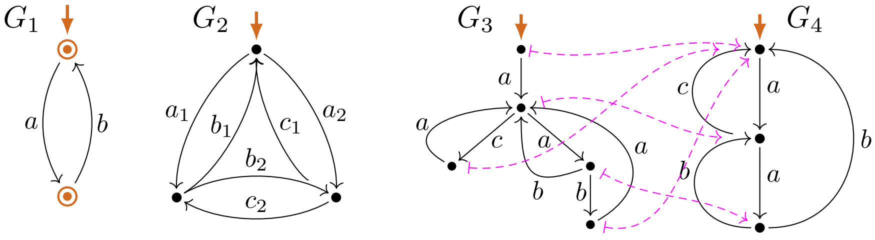
\end{tikzcd}$$

P-/$P^{\bullet}$-expressibility and $[\![\cdot]\!]_P$-expressibility (examples)



not P-expressible

not $[\![\cdot]\!]_P$-expressible

P-/$P^{\bullet}$-expressible

$[\![\cdot]\!]_P$-expressible

$P^{\bullet}$-expressible

$[\![\cdot]\!]_P$-expressible

Summary and outlook

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- image of 1-free expressions under the process interpretation P
  is **not** closed under bisimulation collapse

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Outlook on an extension:

- image of 1-free reg. expr's under $P^{\bullet}$ $=$ finite process graphs with LEE.

Summary and outlook

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- compact process interpretation $P^{\bullet}$
- refined expression extraction from LEE-process-graphs
- image of 1-free reg. expr's under $P^{\bullet}$ **is closed** under collapse
- A finite process graph G is $[\![\cdot]\!]_P$-expressible by a 1-free regular expression
  $\Longleftrightarrow$ the bisim. collapse of G is $P^{\bullet}$-expressible by a 1-free reg. expr..

Outlook on an extension:

- image of 1-free reg. expr's under $P^{\bullet}$ $=$ finite process graphs with LEE.

  A finite process graph G is $P^{\bullet}$-expressible by a 1-free regular expression
  $\Longleftrightarrow$ G satisfies LEE.

Summary and outlook

- 1-free regular expressions defined (also) with unary star
- image of 1-free expressions under the process interpretation P
  is **not** closed under bisimulation collapse
- compact process interpretation $P^{\bullet}$
- refined expression extraction from LEE-process-graphs
- image of 1-free reg. expr's under $P^{\bullet}$ **is closed** under collapse
- A finite process graph G is $[\![\cdot]\!]_P$-expressible by a 1-free regular expression
  $\Longleftrightarrow$ the bisimulation collapse of G satisfies LEE (G/Fokkink 2020).

Outlook on an extension:

- image of 1-free reg. expr's under $P^{\bullet}$ **=** finite process graphs with LEE.

  A finite process graph G is $P^{\bullet}$-expressible by a 1-free regular expression
  $\Longleftrightarrow$ G satisfies LEE.