Closing the Image of the Process Interpretation of 1-Free Regular Expressions Under Bisim. Collapse

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Overview

- 1-free regular expressions (with unary/binary star)
- process interpretation / semantics of regular expressions
  - expressible/not expressible process graphs
- loop existence and elimination property (LEE) (G/Fokkink, 2020)
  - interpretation/extraction correspondences with 1-free reg. expr's
  - LEE is preserved under bisimulation collapse
- Q: Image of process interpretation of 1-free regular expressions closed under bisimulation collapse?
  - A1: No. — But . . .
- compact process interpretation
- refined expression extraction
  - A2: compact process interpretation is image-closed under collapse
- outlook: consequences

Regular Expressions

Definition (~ *Copi–Elgot–Wright, 1958*)

Regular expressions over alphabet A with unary Kleene star:

$$e, e_1, e_2 ::= \mathbf{0} \mid \boldsymbol{a} \mid e_1 + e_2 \mid e_1 \cdot e_2 \mid e^{*} \qquad \text{(for } \boldsymbol{a} \in A\text{)}.$$

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- symbol $\mathbf{0}$ instead of $\varnothing$, symbol $\mathbf{1}$ instead of $\{\varnothing\}$ and ϵ
- with unary Kleene star *: $\mathbf{1}$ is definable as $\mathbf{0}^*$

Regular Expressions

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Definition

1-free regular expressions over alphabet A with binary Kleene star:

$$f, f_1, f_2 \;::=\; \mathbf{0} \mid \boldsymbol{a} \mid f_1 + f_2 \mid f_1 \cdot f_2 \mid f_1{}^{\circledast} f_2 \qquad (\text{for } \boldsymbol{a} \in A).$$

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Under-Star-/1-Free regular expressions

Definition

The set $\mathit{RExp}^{(\mathbf{1})}(A)$ of 1-free regular expressions over A is defined by:

$$f, f_1, f_2 \;::=\; 0 \mid a \mid f_1 + f_2 \mid f_1 \cdot f_2 \mid f_1{}^{\star} \cdot f_2 \qquad (\text{for } a \in A),$$

the set $\mathit{RExp}^{(\mathbf{1}\backslash *)}(A)$ of under-star-1-free regular expressions over A by:

$$uf, uf_1, uf_2 \;::=\; 0 \mid 1 \mid a \mid uf_1 + uf_2 \mid uf_1 \cdot uf_2 \mid f^{\star} \qquad (\text{for } a \in A).$$

Under the language interpretation, subclasses of minor relevance:

- 1-free regular expressions denote all regular languages without ϵ.
- Under-star-1-free regular expressions denote all regular languages.

Process interpretation $P(\cdot)$ of regular expressions *(Milner, 1984)*

$\mathbf{0} \xmapsto{P}$ deadlock $\boldsymbol{\delta}$, no termination

$\mathbf{1} \xmapsto{P}$ empty-step process $\boldsymbol{\epsilon}$, then terminate

$a \xmapsto{P}$ atomic action a, then terminate

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$e_1 \cdot e_2 \stackrel{P}{\longmapsto}$ (*sequentialization*) execute $P(e_1)$, then $P(e_2)$

$e^* \stackrel{P}{\longmapsto}$ (*iteration*) repeat (terminate or execute $P(e)$)

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Process semantics $[\![\cdot]\!]_P$ of regular expressions *(Milner, 1984)*

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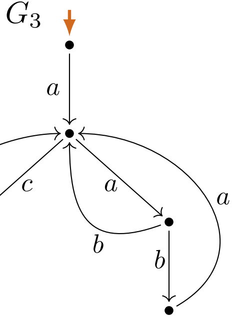
$[\![e]\!]_P := [P(e)]_{\underline{\leftrightarrow}}$ (bisimilarity equivalence class of process $P(e)$)

P-expressibility and $[\![\cdot]\!]_P$-expressibility (examples)



$$P\Big(\overbrace{(a\cdot(c\cdot a+a\cdot(b+b\cdot a))^*)\cdot 0}^{f}\Big)$$

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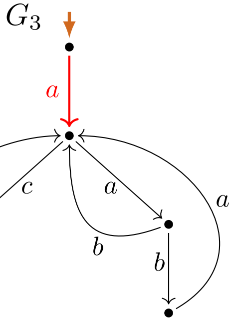

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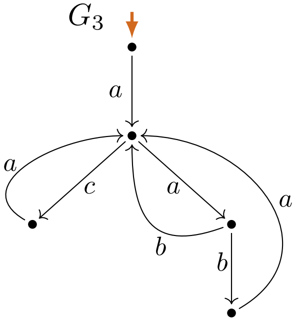

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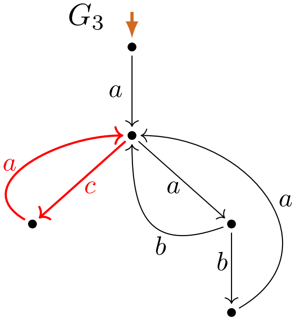

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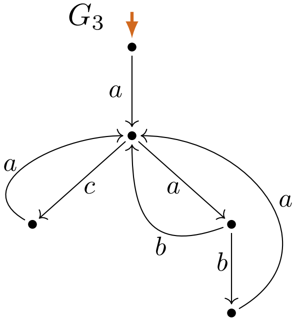

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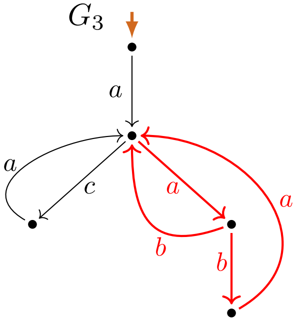

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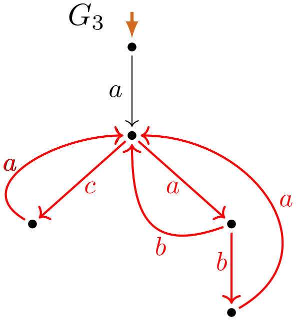

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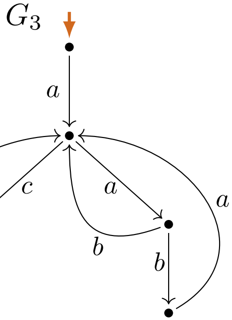

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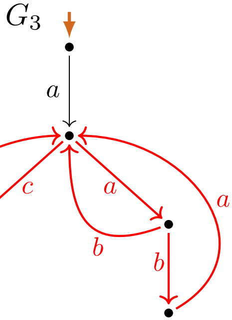

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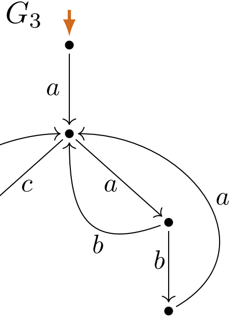

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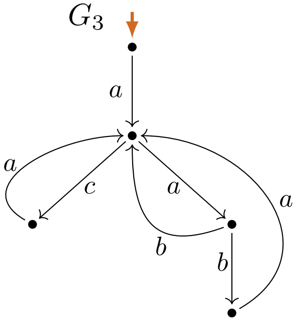

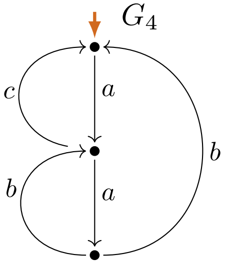

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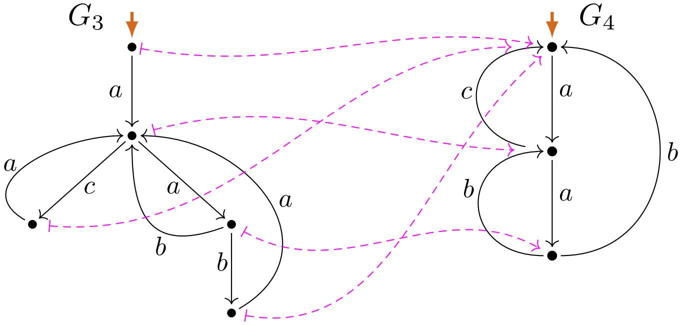

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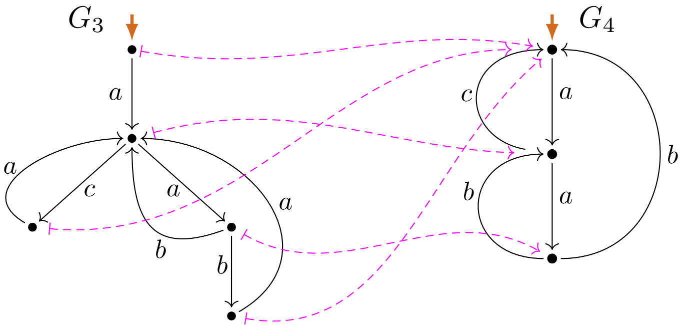

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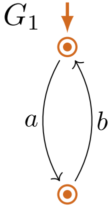

not P-expressible

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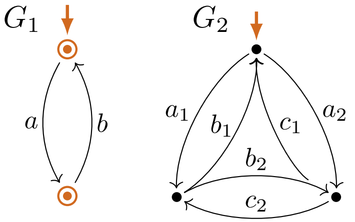

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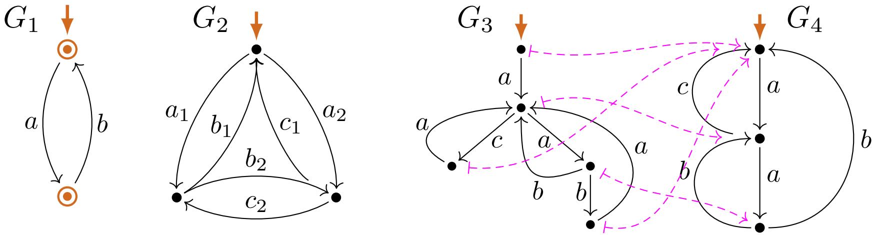

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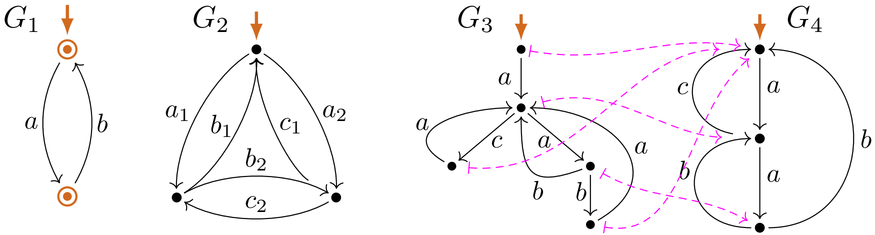

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not $[\![\cdot]\!]_P$-expressible

P-expressible
$[\![\cdot]\!]_P$-expressible

?
$[\![\cdot]\!]_P$-expressible

Process interpretation P (formal definition)

Definition (Transition system specification $\mathcal{T}$)

$$\frac{}{a \xrightarrow{a} 1} \qquad \frac{e_i \xrightarrow{a} e_i'}{e_1 + e_2 \xrightarrow{a} e_i'} \; (i \in \{1,2\})$$

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Definition

The process (graph) interpretation $P(e)$ of a regular expression e:

$P(e)$:= labeled transition graph generated by e by derivations in $\mathcal{T}$.

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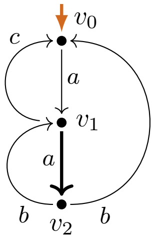
Loop existence and elimination



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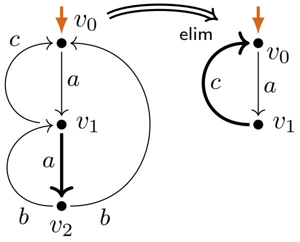

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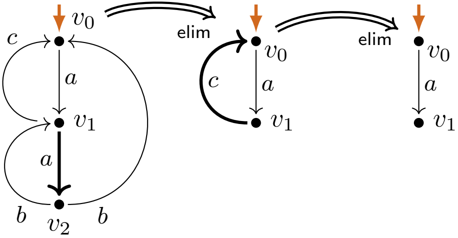

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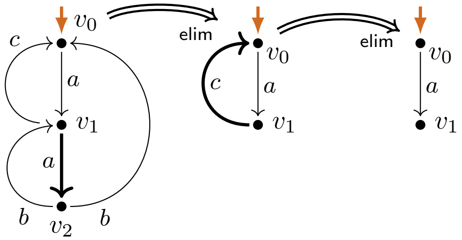

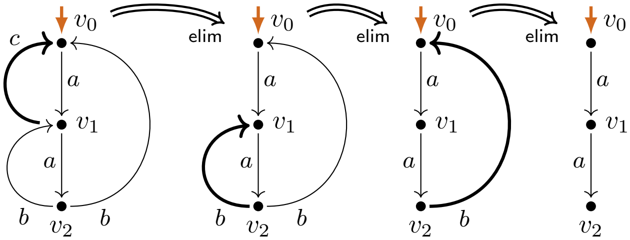

LEE

Definition

A chart $\mathcal{C}$ satisfies LEE (*loop existence and elimination*) if:

$$\exists \mathcal{C}_0 \left(\mathcal{C} \longrightarrow^{*}_{\text{elim}} \mathcal{C}_0 \not\longrightarrow_{\text{elim}} \right.$$
$$\left. \wedge \ \mathcal{C}_0 \text{ permits no infinite path} \right).$$

LEE

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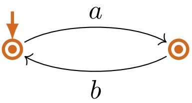
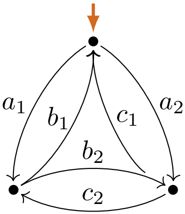
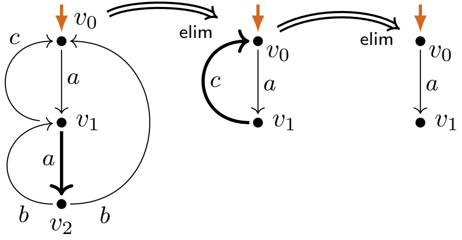


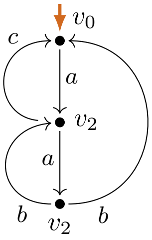


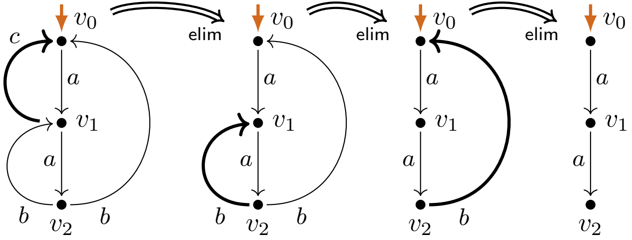
~~LEE~~ ~~LEE~~

LEE

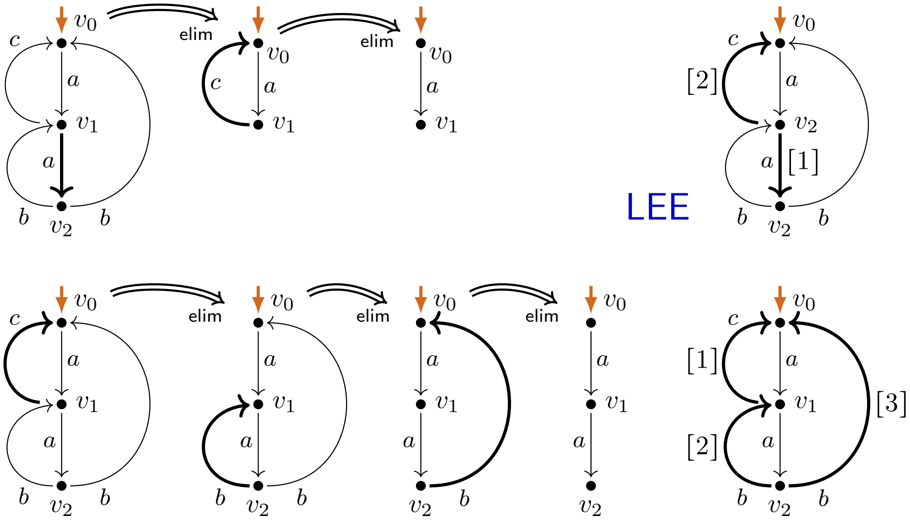


LEE

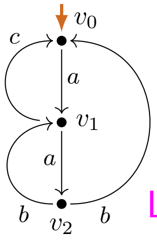




LEE

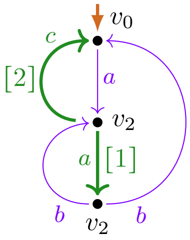


LEE witness and LEE-charts

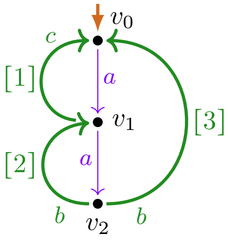


LEE-chart

LEE-witness



LEE-witness



Properties of LEE-charts

Theorem ($\Leftarrow$ G/Fokkink, 2020)

A process graph G

is $[\![\cdot]\!]_P$-expressible by an under-star-1-free regular expression

(i.e. P-expressible modulo bisimilarity by an $(\cancel{1}\backslash *)$ reg. expr.)

if and only if

the bisimulation collapse of G satisfies LEE.

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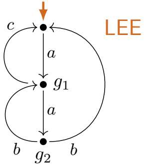
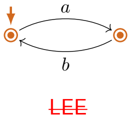
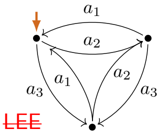
Hence $[\![\cdot]\!]_P$-expressible | **not** $[\![\cdot]\!]_P$-expressible by 1-free regular expressions:







Interpretation/extraction correspondences with LEE

($\Leftarrow$ G/Fokkink 2020, G 2021)

(Int)$_{P}^{(\cancel{1}\backslash *)}$: $P^{\bullet}$-($\cancel{1}\backslash *$)-*expressible graphs have structural property* LEE

Process **i**nterpretations $P(e)$
of under-star-1-free regular expressions e
are finite process graphs that satisfy LEE.

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(Extr)$_{P}$: LEE *implies* $[\![\cdot]\!]_{P}$*-expressibility*

From every finite process graph G with LEE
a regular expression e can be **e**xtracted
such that $G \underline{\leftrightarrow} P(e)$.

Interpretation/extraction correspondences with LEE

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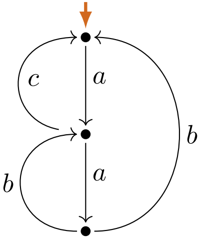
From every finite process graph G with LEE
a regular expression e can be **e**xtracted
such that $G \leftrightarrow P(e)$.

(Coll): LEE *is preserved under collapse*

The class of finite process graphs with LEE
is closed under bisimulation **c**ollapse.

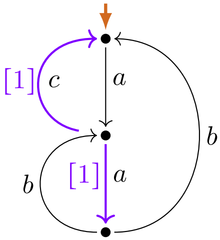
Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

G_4



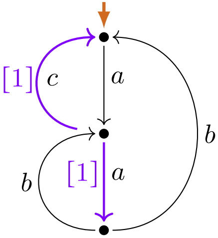
Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



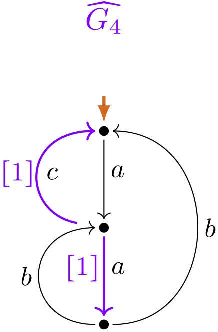
Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$$(\qquad\qquad\qquad)^* \cdot 0$$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

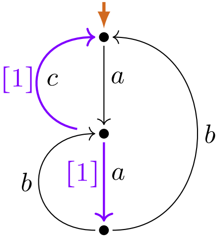


$\downarrow a$

$$(\qquad\qquad\qquad)^* \cdot 0$$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



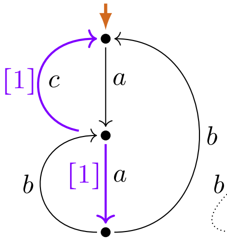
$$(a \cdot 1) \cdot ((\qquad\qquad\qquad\qquad)^* \cdot 0)$$

$$\downarrow a$$

$$(\qquad\qquad\qquad\qquad)^* \cdot 0$$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$(a \cdot 1) \cdot ((\qquad)^* \cdot 0)$

a

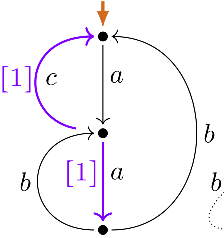
$(\qquad)^* \cdot 0$

b

b

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$$(a \cdot 1) \cdot \Big(\Big(\qquad\qquad\qquad \Big)^{*} \cdot 0\Big)$$

$$\downarrow a \qquad\qquad b$$

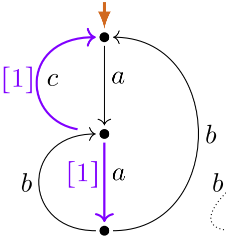
$$\Big(\qquad\qquad\qquad \Big)^{*} \cdot 0$$

$$b$$

$$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot \Big(\Big(\qquad\qquad\qquad \Big)^{*}\Big) \cdot 0\Big)$$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$(a \cdot 1) \cdot \big(\big(\qquad \big)^* \cdot 0\big)$

$\Big\downarrow a$

$\big(\qquad \big)^* \cdot 0$

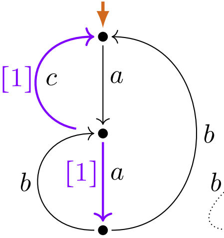
$\Big\downarrow a$

$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot \big(\big(\qquad \big)^*\big) \cdot 0\big)$

$c \qquad b \qquad b$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$$(a \cdot 1) \cdot \big(\big(\qquad \big)^* \cdot 0\big)$$

$$\downarrow a$$

$$\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0$$

$$\downarrow a$$

$$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot \big(\big(\qquad \big)^*) \cdot 0\big)$$

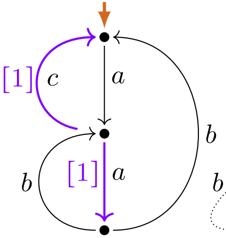
c

b

b

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$$(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \cdot 0 \Big)$$

$$\downarrow a$$

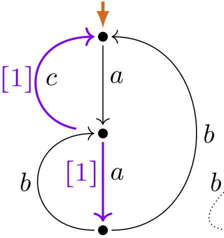
$$\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \cdot 0$$

$$\downarrow a$$

$$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \big) \cdot 0 \Big)$$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$$\overbrace{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)}^{e}$$

$$\Big\downarrow a$$

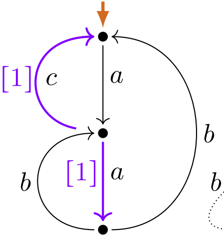
$$\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0$$

$$\Big\downarrow a$$

$$(b\cdot 1+b\cdot(a\cdot 1))\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0\Big)$$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$P(e) \rightleftharpoons G_4$

$$\overbrace{(a\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)}^{e}$$

$$\downarrow a$$

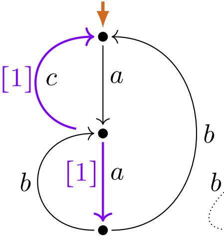
$$(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0$$

$$\downarrow a$$

$$(b\cdot 1+b\cdot(a\cdot 1))\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*})\cdot 0\big)$$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$P(e) \mathrel{\underline{\to}} G_4$

$$\overbrace{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)}^{e}$$

$$\downarrow\!\!\!\!\!-\, a$$

$$\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0$$

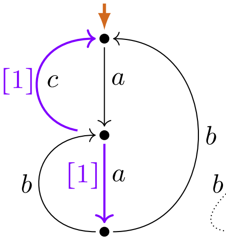
$$\downarrow a$$

$$(b\cdot 1+b\cdot(a\cdot 1))\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0\Big)$$

c b b

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$$P(e) \;\underline{\rightarrow}\; G_4$$

$$\overbrace{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)}^{e}$$

$$\downarrow a$$

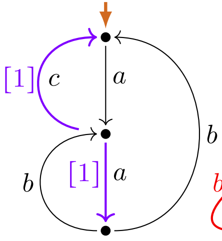
$$\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0$$

$$\downarrow a$$

$$(b\cdot 1+b\cdot(a\cdot 1))\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0\Big)$$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$P(e) \mathrel{\underline{\to}} G_4$

$$\overbrace{(a \cdot 1) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0 \big)}^{e}$$

$$\downarrow a$$

$$\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0 \quad \xrightarrow{c} \quad (a \cdot 1) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0 \big)$$

$$\downarrow a$$

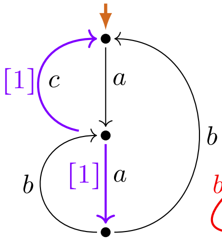
$$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \big) \cdot 0 \big)$$

$\xrightarrow{b}$ $\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0$

$\xrightarrow{b}$ $(a \cdot 1) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0 \big)$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$$P(e) \mathrel{\underline{\rightarrow}} G_4 \not\mathrel{\underline{\leftarrow}} P(e)$$

$$\overbrace{(a \cdot 1) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0 \big)}^{e}$$

$$\downarrow a$$

$$\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0$$

$$\downarrow a$$

$$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \big) \cdot 0 \big)$$

Interpretation of extracted expression

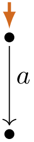
G_5

$P(e) = G_5$

$$\overbrace{(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0 \Big)}^{e}$$

Interpretation of extracted expression

G_5



$$P(e) = G_5$$

$$\overbrace{(a \cdot 1) \cdot \big((c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)))\big)^{*} \cdot 0\big)}^{e}$$

$$\Big\downarrow a$$

$$(1 \cdot 1) \cdot \big((c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)))\big)^{*} \cdot 0\big)$$

Interpretation of extracted expression

G_5

$$\begin{tikzcd} \arrow[d] \\ \bullet \arrow[d, "a"] \\ \bullet \arrow[d, "c"] \\ \bullet \end{tikzcd}$$

$P(e) = G_5$

$$\overbrace{(a \cdot 1) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0 \big)}^{e}$$

$$\downarrow a$$

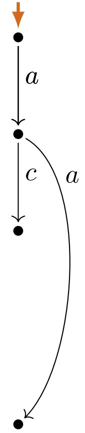
$$(1 \cdot 1) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0 \big)$$

$$\downarrow c$$

$$\big((1 \cdot (a \cdot 1)) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \big) \cdot 0$$

Interpretation of extracted expression

G_5



$P(e) = G_5$

$$\overbrace{(a \cdot 1) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0\big)}^{e}$$

$$\downarrow a$$

$$(1 \cdot 1) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0\big)$$

$$\downarrow c$$

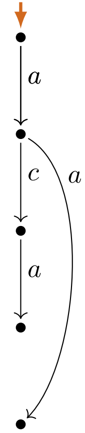
$$\big((1 \cdot (a \cdot 1)) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^*\big) \cdot 0$$

$$a$$

$$\big((1 \cdot (b \cdot 1 + b \cdot (a \cdot 1))) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^*\big) \cdot 0$$

Interpretation of extracted expression

G_5



$$P(e) = G_5$$

$$\overbrace{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^*\cdot 0\Big)}^{e}$$

$$\downarrow a$$

$$(1\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^*\cdot 0\Big)$$

$$\downarrow c$$

$$\Big((1\cdot(a\cdot 1))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^*\Big)\cdot 0$$

$$\downarrow a$$

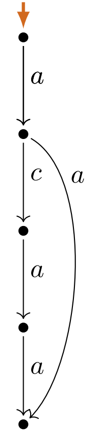
$$\Big((1\cdot 1)\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^*\Big)\cdot 0\Big)$$

$$a$$

$$\Big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^*\Big)\cdot 0$$

Interpretation of extracted expression

G_5



$P(e) = G_5$

$$\overbrace{(a \cdot 1) \cdot \big((c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)))^{*} \cdot 0\big)}^{e}$$

$$\downarrow a$$

$$(1 \cdot 1) \cdot \big((c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)))^{*} \cdot 0\big)$$

$$\downarrow c$$

$$\big((1 \cdot (a \cdot 1)) \cdot (c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)))^{*}\big) \cdot 0$$

$$\downarrow a$$

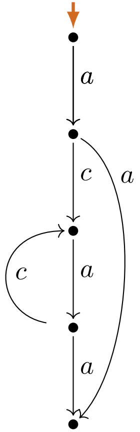
$$\big((1 \cdot 1) \cdot (c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)))^{*}\big) \cdot 0\big)$$

$$\downarrow a$$

$$\big((1 \cdot (b \cdot 1 + b \cdot (a \cdot 1))) \cdot (c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)))^{*}\big) \cdot 0$$

Interpretation of extracted expression

G_5

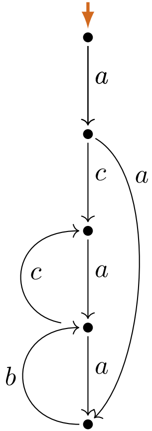


$P(e) = G_5$

$$
\begin{array}{c}
\overbrace{(a\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)}^{e} \\
\downarrow a \\
(1\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big) \\
\downarrow c \\
\big((1\cdot(a\cdot 1))\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\big)\cdot 0 \\
\downarrow a \\
\big((1\cdot 1)\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\big)\cdot 0\big) \\
\downarrow a \\
\big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\big)\cdot 0
\end{array}
$$

Interpretation of extracted expression

G_5



$P(e) = G_5$

$$\overbrace{(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0\Big)}^{e}$$

$$\downarrow a$$

$$(1 \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0\Big)$$

$$\downarrow c$$

$$\Big((1 \cdot (a \cdot 1)) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^*\Big) \cdot 0$$

$$\downarrow a$$

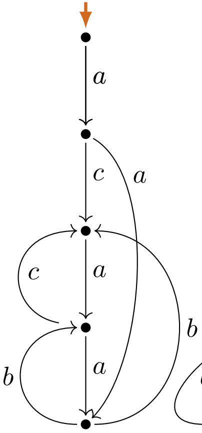
$$\Big((1 \cdot 1) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^*\Big) \cdot 0\Big)$$

$$\downarrow a$$

$$\Big((1 \cdot (b \cdot 1 + b \cdot (a \cdot 1))) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^*\Big) \cdot 0$$

Interpretation of extracted expression

G_5



$P(e) = G_5$

$$\overbrace{(a \cdot 1) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \cdot 0 \big)}^{e}$$

$$\downarrow a$$

$$(1 \cdot 1) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \cdot 0 \big)$$

$$\downarrow c$$

$$\big((1 \cdot (a \cdot 1)) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \big) \cdot 0$$

$$\downarrow a$$

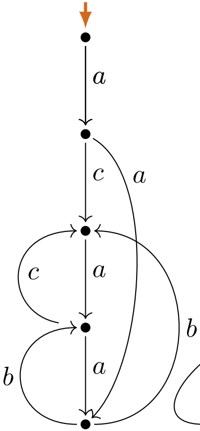
$$\big((1 \cdot 1) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \big) \cdot 0 \big)$$

$$\downarrow a$$

$$\big((1 \cdot (b \cdot 1 + b \cdot (a \cdot 1))) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \big) \cdot 0$$

Interpretation of extracted expression

G_5

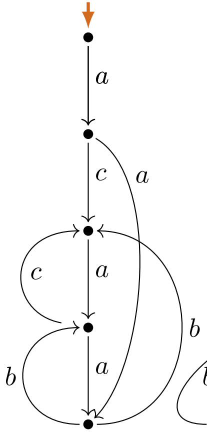


$$P(e) = G_5 \underline{\to} G_4$$

$$\begin{array}{c}
\overbrace{(a\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)}^{e} \\
\downarrow a \\
(1\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big) \\
\downarrow c \\
\big((1\cdot(a\cdot 1))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0 \\
\downarrow a \\
\big((1\cdot 1)\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0\big) \\
\downarrow a \\
\big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0
\end{array}$$

Interpretation of extracted expression

G_5



$$P(e) = G_5 \Rightarrow G_4 \not\simeq G_5$$

$$\overbrace{(a \cdot 1) \cdot \big((c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)))^{*} \cdot 0\big)}^{e}$$

$$\Big\downarrow a$$

$$(1 \cdot 1) \cdot \big((c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)))^{*} \cdot 0\big)$$

$$\Big\downarrow c$$

$$\big((1 \cdot (a \cdot 1)) \cdot (c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)))^{*}\big) \cdot 0$$

$$\Big\downarrow a$$

$$\big((1 \cdot 1) \cdot (c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)))^{*}\big) \cdot 0\big)$$

$$\Big\downarrow a$$

$$\big((1 \cdot (b \cdot 1 + b \cdot (a \cdot 1))) \cdot (c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)))^{*}\big) \cdot 0$$

Overview

- 1-free regular expressions (with unary/binary star)
- process interpretation/semantics of regular expressions
  - expressible/not expressible process graphs
- loop existence and elimination property (LEE) (G/Fokkink, 2020)
  - interpretation/extraction correspondences with 1-free reg. expr's
  - LEE is preserved under bisimulation collapse
- Q: Image of process interpretation of 1-free regular expressions closed under bisimulation collapse?
  - A1: No. — But ...
- compact process interpretation
- refined expression extraction
  - A2: compact process interpretation is image-closed under collapse
- outlook: consequences

Image of P is **not** closed under bisimulation collapse

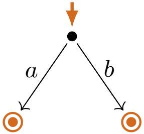
$P(uf)$

$P(uf)$

$$uf := a \cdot \overbrace{(a \cdot (a + a \cdot 0))^*}^{uf_a} + b \cdot \overbrace{(b \cdot (b + b \cdot 0))^*}^{uf_b}$$

Image of P is **not** closed under bisimulation collapse

$P(uf)$



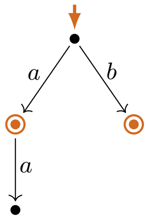
$P(uf)$

$$uf := a \cdot \overbrace{(a \cdot (a + a \cdot 0))^*}^{uf_a} + b \cdot \overbrace{(b \cdot (b + b \cdot 0))^*}^{uf_b}$$

$$\xrightarrow{a} 1 \cdot uf_a \Downarrow \qquad \qquad \xrightarrow{b} \Downarrow 1 \cdot uf_b$$

Image of P is **not** closed under bisimulation collapse

$P(uf)$



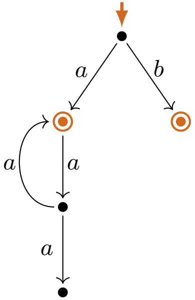
$P(uf)$

$$uf := a \cdot \overbrace{(a \cdot (a + a \cdot 0))^*}^{uf_a} + b \cdot \overbrace{(b \cdot (b + b \cdot 0))^*}^{uf_b}$$

$$\begin{tikzcd}
 & uf \arrow[dl, "a"'] \arrow[dr, "b"] & \\
1 \cdot uf_a \Downarrow \arrow[d, "a"] & & \Downarrow 1 \cdot uf_b \\
(1 \cdot (a + a \cdot 0)) \cdot uf_a & &
\end{tikzcd}$$

Image of *P* is **not** closed under bisimulation collapse

$P(uf)$



$P(uf)$

$$uf := a \cdot \overbrace{(a \cdot (a + a \cdot 0))^*}^{uf_a} + b \cdot \overbrace{(b \cdot (b + b \cdot 0))^*}^{uf_b}$$

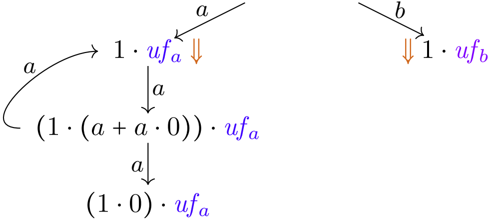
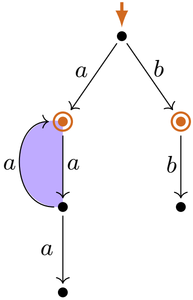


Image of P is **not** closed under bisimulation collapse

$P(uf)$



$P(uf)$

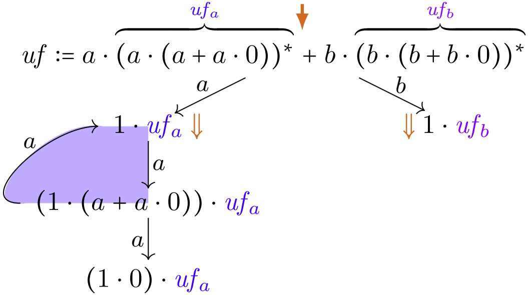
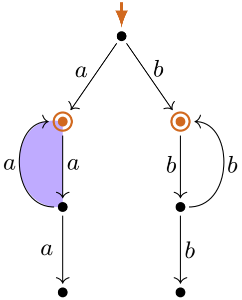


Image of *P* is **not** closed under bisimulation collapse

$P(uf)$



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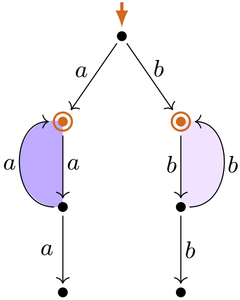
$$\begin{tikzcd}
 & uf \arrow[dl, "a"'] \arrow[dr, "b"] & \\
1 \cdot uf_a \arrow[d, "a"] & & 1 \cdot uf_b \arrow[d, "b"] \\
(1 \cdot (a + a \cdot 0)) \cdot uf_a \arrow[u, bend left, "a"] \arrow[d, "a"] & & (1 \cdot (b + b \cdot 0)) \cdot uf_b \\
(1 \cdot 0) \cdot uf_a & &
\end{tikzcd}$$

Image of *P* is **not** closed under bisimulation collapse

$P(uf)$



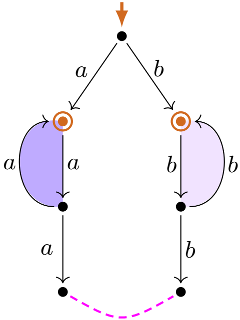
$P(uf)$

$$uf := a\cdot \overbrace{(a\cdot(a+a\cdot 0))^*}^{uf_a} + b\cdot \overbrace{(b\cdot(b+b\cdot 0))^*}^{uf_b}$$

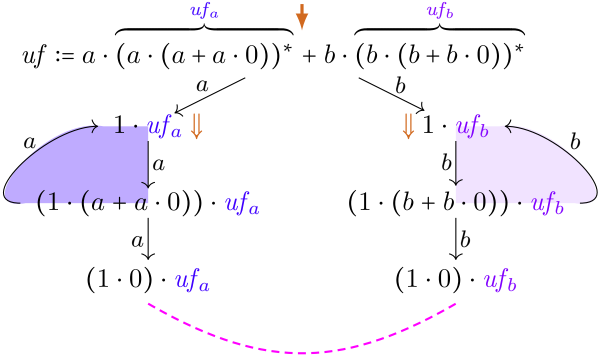
$$\begin{tikzcd}
 & uf \arrow[dl, "a"'] \arrow[dr, "b"] & \\
1\cdot uf_a \Downarrow \arrow[d, "a"] & & \Downarrow 1\cdot uf_b \arrow[d, "b"] \\
(1\cdot(a+a\cdot 0))\cdot uf_a \arrow[u, bend left, "a"] \arrow[d, "a"] & & (1\cdot(b+b\cdot 0))\cdot uf_b \arrow[u, bend right, "b"'] \arrow[d, "b"] \\
(1\cdot 0)\cdot uf_a & & (1\cdot 0)\cdot uf_b
\end{tikzcd}$$

Image of *P* is **not** closed under bisimulation collapse

$P(uf)$



$P(uf)$



Compact process interpretation $P^{\bullet}$

Definition (Transition system specification $\mathcal{T}$)

$$\frac{}{\mathbf{1}{\Downarrow}} \qquad \frac{e_i{\Downarrow}}{(e_1 + e_2){\Downarrow}}\,(i \in \{1,2\}) \qquad \frac{e_1{\Downarrow} \quad e_2{\Downarrow}}{(e_1 \cdot e_2){\Downarrow}} \qquad \frac{}{(e^*){\Downarrow}}$$

$$\frac{}{a \xrightarrow{a} \mathbf{1}} \qquad \frac{e_i \xrightarrow{a} e_i'}{e_1 + e_2 \xrightarrow{a} e_i'}\,(i \in \{1,2\})$$

$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2} \qquad \frac{e_1{\Downarrow} \quad e_2 \xrightarrow{a} e_2'}{e_1 \cdot e_2 \xrightarrow{a} e_2'} \qquad \frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e' \cdot e^*}$$

Compact process interpretation $P^{\bullet}$

Definition (Transition system specification $\mathcal{T}$)

$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2}$$

$$\frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e' \cdot e^*}$$

Compact process interpretation $P^{\bullet}$

Definition (Transition system specification $\mathcal{T}^{\bullet}$, changed rules w.r.t. $\mathcal{T}$)

$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2} \text{ (if } e_1' \text{ is normed)}$$

$$\frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e' \cdot e^*} \text{ (if } e' \text{ is normed)}$$

Compact process interpretation $P^{\bullet}$

Definition (Transition system specification $\mathcal{T}^{\bullet}$, changed rules w.r.t. $\mathcal{T}$)

$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2} \text{ (if } e_1' \text{ is normed)} \qquad \frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1'} \text{ (if } e_1' \text{ is not normed)}$$

$$\frac{e \xrightarrow{a} e'}{e^{*} \xrightarrow{a} e' \cdot e^{*}} \text{ (if } e' \text{ is normed)} \qquad \frac{e \xrightarrow{a} e'}{e^{*} \xrightarrow{a} e'} \text{ (if } e' \text{ is not normed)}$$

Compact process interpretation $P^{\bullet}$

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$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2} \text{ (if } e_1' \text{ is normed)} \qquad \frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1'} \text{ (if } e_1' \text{ is not normed)}$$

$$\frac{e \xrightarrow{a} e'}{e^{*} \xrightarrow{a} e' \cdot e^{*}} \text{ (if } e' \text{ is normed)} \qquad \frac{e \xrightarrow{a} e'}{e^{*} \xrightarrow{a} e'} \text{ (if } e' \text{ is not normed)}$$

Definition

The compact process (graph) interpretation $P^{\bullet}(e)$ of a reg. expr's e:

$P^{\bullet}(e) :=$ labeled transition graph generated by e by derivations in $\mathcal{T}^{\bullet}$.

Compact process interpretation $P^{\bullet}$

Definition (Transition system specification $\mathcal{T}^{\bullet}$, changed rules w.r.t. $\mathcal{T}$)

$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2} \text{ (if } e_1' \text{ is normed)} \qquad \frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1'} \text{ (if } e_1' \text{ is not normed)}$$

$$\frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e' \cdot e^*} \text{ (if } e' \text{ is normed)} \qquad \frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e'} \text{ (if } e' \text{ is not normed)}$$

Definition

The compact process (graph) interpretation $P^{\bullet}(e)$ of a reg. expr's e:

$P^{\bullet}(e)$:= labeled transition graph generated by e by derivations in $\mathcal{T}^{\bullet}$.

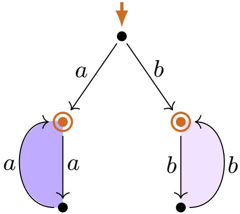
Lemma ($P^{\bullet}$ increases sharing; $P^{\bullet}$, P have same bisimulation semantics)

(i) $P(e) \mathrel{\underline{\rightarrow}} P^{\bullet}(e)$ *for all regular expressions* e.

(ii) (G *is* $[\![\cdot]\!]_{P^{\bullet}}$*-expressible* $\Longleftrightarrow$ G *is* $[\![\cdot]\!]_{P}$*-expressible) for all graphs* G.

Image of $P^{\bullet}$ under bisimulation collapse . . .

$P^{\bullet}(uf)$



$P^{\bullet}(uf)$

$$uf := a \cdot \overbrace{(a \cdot (a + a \cdot 0))^{*}}^{uf_a} + b \cdot \overbrace{(b \cdot (b + b \cdot 0))^{*}}^{uf_b}$$

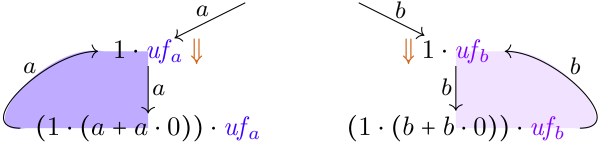
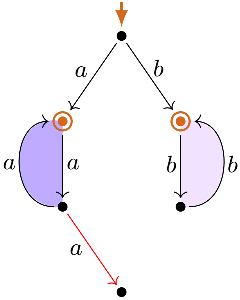


Image of $P^{\bullet}$ under bisimulation collapse . . .

$P^{\bullet}(uf)$



$P^{\bullet}(uf)$

$$uf := a \cdot \overbrace{(a \cdot (a + a \cdot 0))^{*}}^{uf_a} + b \cdot \overbrace{(b \cdot (b + b \cdot 0))^{*}}^{uf_b}$$

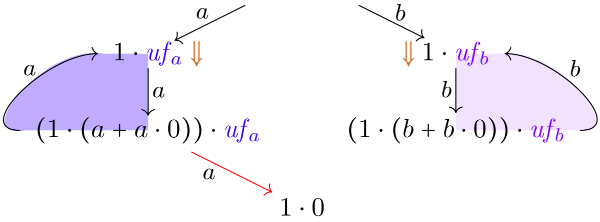
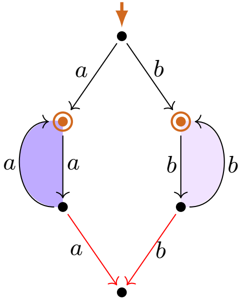


Image of $P^{\bullet}$ under bisimulation collapse ...

$P^{\bullet}(uf)$



$P^{\bullet}(uf)$

$$uf := a \cdot \overbrace{(a \cdot (a + a \cdot 0))^{*}}^{uf_a} + b \cdot \overbrace{(b \cdot (b + b \cdot 0))^{*}}^{uf_b}$$

$$uf \xrightarrow{a} 1 \cdot uf_a \qquad uf \xrightarrow{b} 1 \cdot uf_b$$

$$1 \cdot uf_a \xrightarrow{a} (1 \cdot (a + a \cdot 0)) \cdot uf_a \xrightarrow{a} 1 \cdot uf_a$$

$$1 \cdot uf_b \xrightarrow{b} (1 \cdot (b + b \cdot 0)) \cdot uf_b \xrightarrow{b} 1 \cdot uf_b$$

$$(1 \cdot (a + a \cdot 0)) \cdot uf_a \xrightarrow{a} 1 \cdot 0 \qquad (1 \cdot (b + b \cdot 0)) \cdot uf_b \xrightarrow{b} 1 \cdot 0$$

Interpretation correspondence of $P^\bullet$ with LEE

(Int)$_{P^\bullet}^{(\cancel{1}\backslash *)}$: *By under-star-1-free expressions $P^\bullet$-expressible graphs satisfy* LEE:

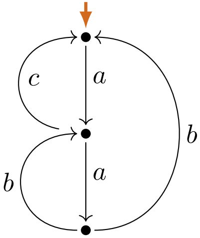
Compact process **i**nterpretations $P^\bullet(uf)$
of under-star-1-free regular expressions uf
are finite process graphs that satisfy LEE.

(Extr)$_{P^\bullet}^{(\cancel{1}\backslash *)}$: LEE *implies $[\![\cdot]\!]_P$-expressibility by under-star-1-free reg. expr's*:

From every finite process graph G with LEE
an under-star-1-free regular expression uf can be **e**xtracted
such that $G \rightharpoonup P(uf)$.

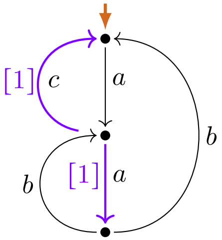
Refined extraction expression (example)

G_4



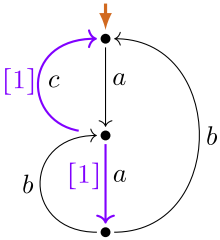
Refined extraction expression (example)

$\widehat{G_4}$



Refined extraction expression (example)

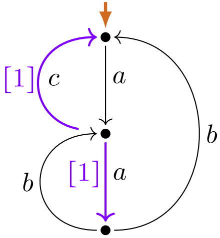
$\widehat{G_4}$



$$(1 \cdot (\qquad\qquad\qquad)^{*}) \cdot 0$$

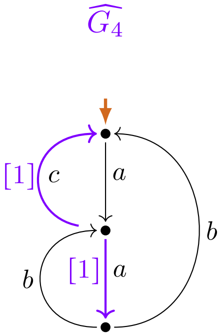
Refined extraction expression (example)

$\widehat{G_4}$



$$\left(1 \cdot \left(\quad \overset{a}{\downarrow} \quad \right)^{*}\right) \cdot 0$$

Refined extraction expression (example)



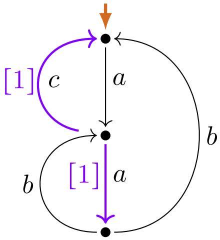
$((1 \cdot a) \cdot (\qquad)^*) \cdot 0$

$\downarrow a$

$(1 \cdot (\qquad)^*) \cdot 0$

Refined extraction expression (example)

$\widehat{G_4}$



$((1 \cdot a) \cdot (\qquad)^*) \cdot 0$

a

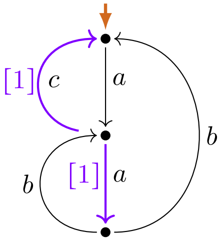
$(1 \cdot (\qquad)^*) \cdot 0$

b

b

Refined extraction expression (example)

$\widehat{G_4}$



$((1 \cdot a) \cdot (\qquad)^*) \cdot 0$

a

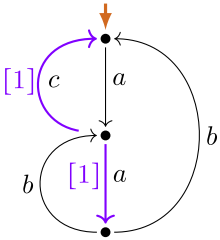
$(1 \cdot (\qquad)^*) \cdot 0$

b $\quad$ b

$((1 \cdot (b + b \cdot a)) \cdot (\qquad)^*) \cdot 0$

Refined extraction expression (example)

$\widehat{G_4}$



c $\quad ((1 \cdot a) \cdot (\qquad\qquad)^{*}) \cdot 0$

$\downarrow a$

$(1 \cdot (\qquad\qquad)^{*}) \cdot 0$ $\quad b$

b $\quad \downarrow a$

$((1 \cdot (b + b \cdot a)) \cdot (\qquad\qquad)^{*}) \cdot 0$

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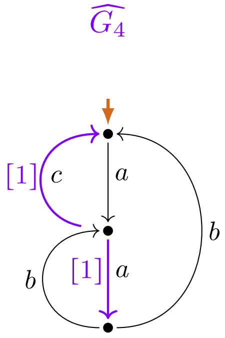

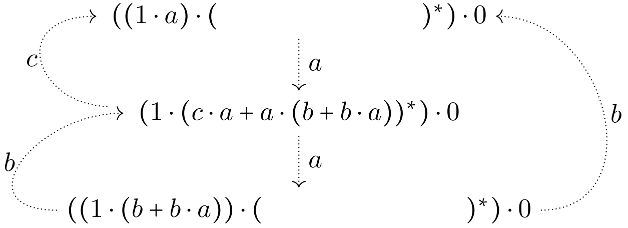

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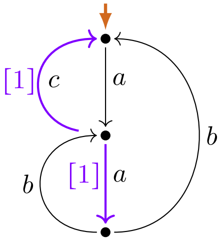

$$
\begin{tikzcd}
((1\cdot a)\cdot(c\cdot a+a\cdot(b+b\cdot a))^*)\cdot 0 \arrow[d, dotted, "a"] \\
(1\cdot(c\cdot a+a\cdot(b+b\cdot a))^*)\cdot 0 \arrow[d, dotted, "a"] \arrow[u, dotted, bend left, "c"] \\
((1\cdot(b+b\cdot a))\cdot(c\cdot a+a\cdot(b+b\cdot a))^*)\cdot 0 \arrow[u, dotted, bend left, "b"] \arrow[uu, dotted, bend right, "b"']
\end{tikzcd}
$$

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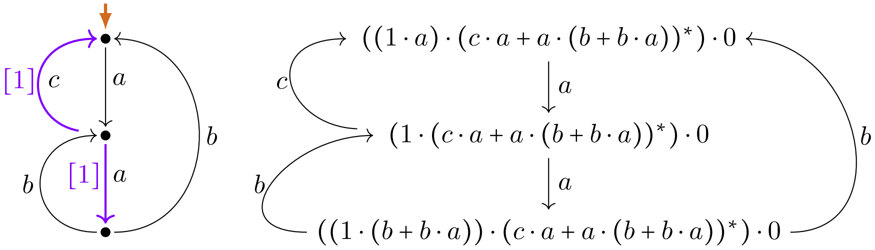

Refined extraction expression (example)

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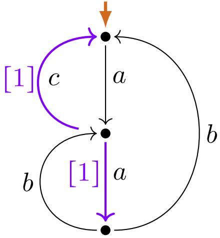
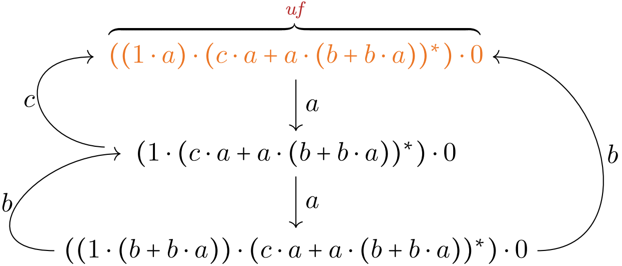
$P^{\bullet}(uf) = P(uf) \simeq G_4$





Interpretation/extraction correspondences of $P^{\bullet}$ with LEE

(Int)$^{(\cancel{1}\backslash *)}_{P^{\bullet}}$: *By under-star-1-free expressions $P^{\bullet}$-expressible graphs satisfy* LEE:

Compact process **i**nterpretations $P^{\bullet}(uf)$
of under-star-1-free regular expressions uf
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(Extr)$^{(\cancel{1}\backslash *)}_{P^{\bullet}}$: LEE *implies $[\![\cdot]\!]_P$-expressibility by under-star-1-free reg. expr's*:

From every finite process graph G with LEE
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(ImColl)$_{P^{\bullet}}^{(\cancel{1}\backslash *)}$: The image of $P^{\bullet}$,
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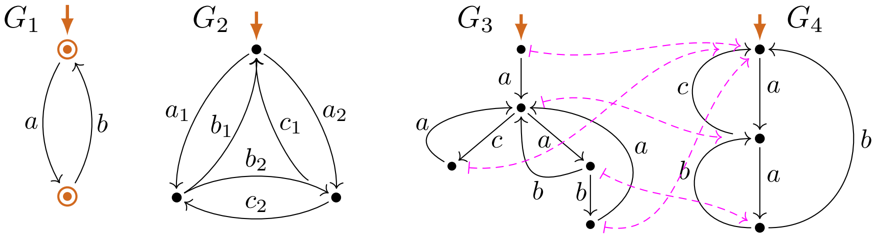
P-/$P^\bullet$-expressibility and $[\![\cdot]\!]_P$-expressibility (examples)



not P-expressible
not $[\![\cdot]\!]_P$-expressible

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Language semantics $[\![\cdot]\!]_L$ of reg. expr's *(Copi–Elgot–Wright, 1958)*

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$\mathbf{1} \xmapsto{L} \{\epsilon\}$ (ϵ the empty word)

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$[\![e]\!]_L := L(e)$ (language defined by e)

Loop charts (interpretations of innermost iterations)

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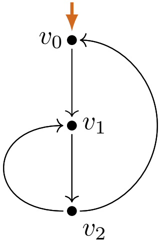

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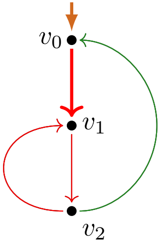

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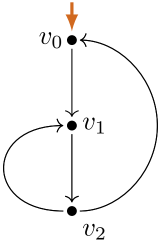

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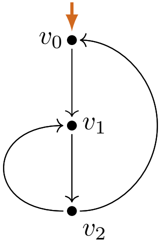

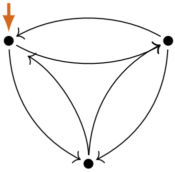

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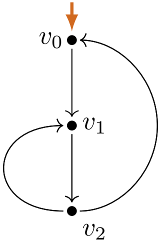

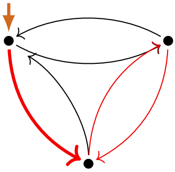

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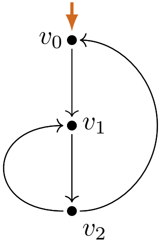

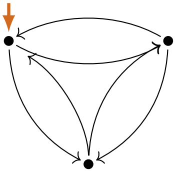

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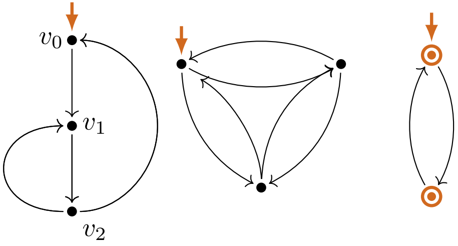

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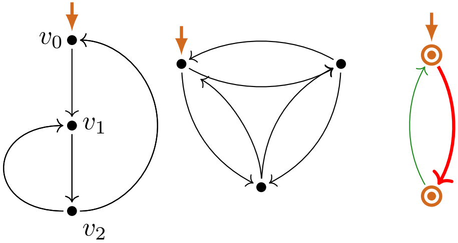

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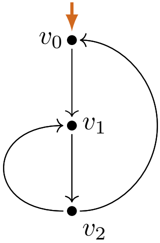

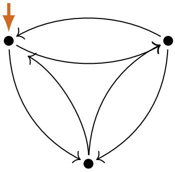

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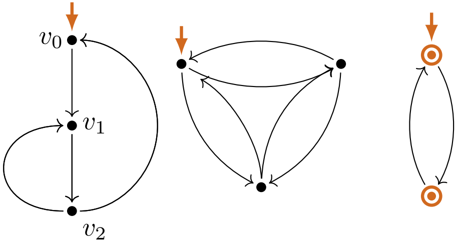

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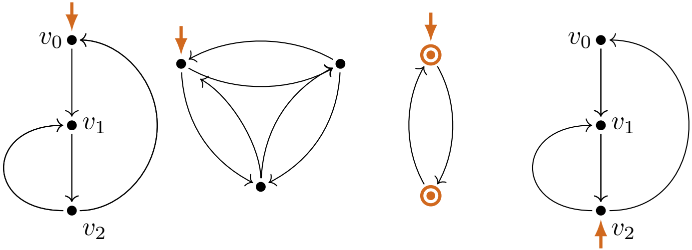

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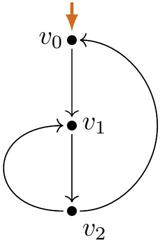

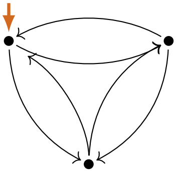

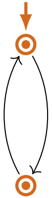

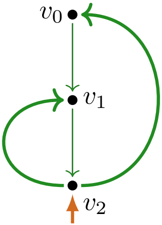

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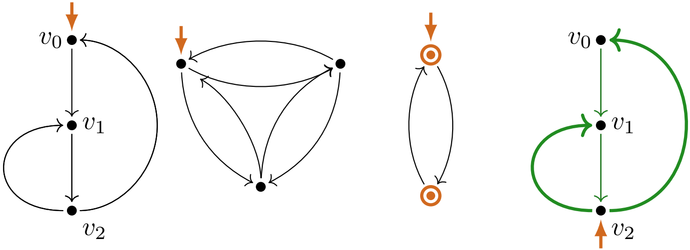

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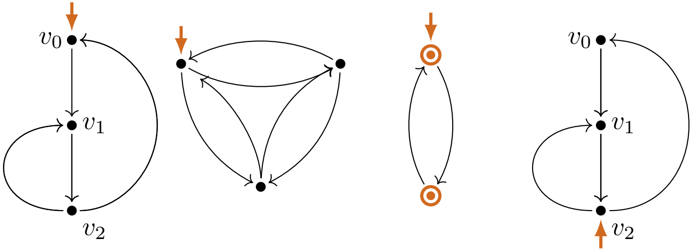

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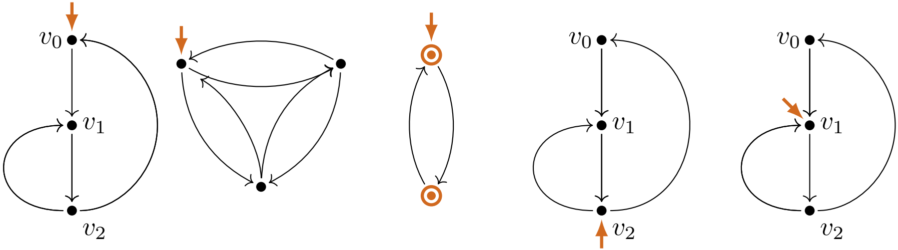

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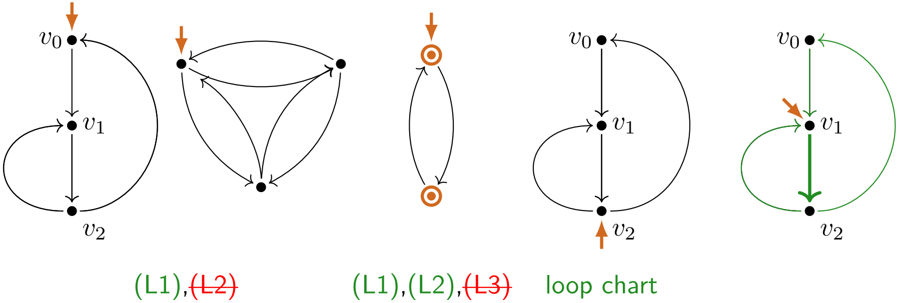

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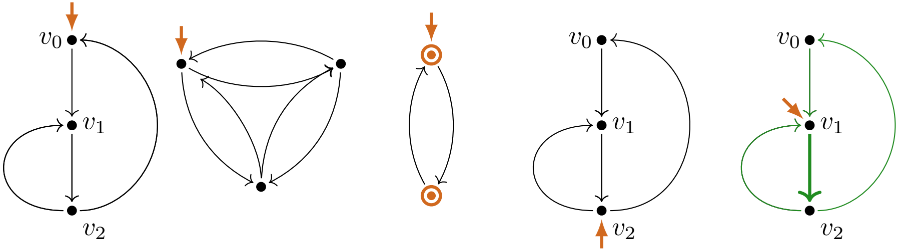

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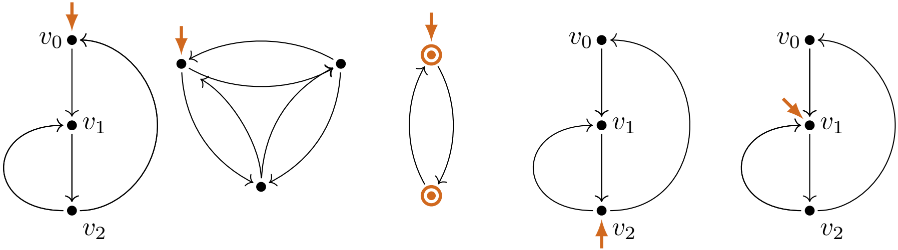

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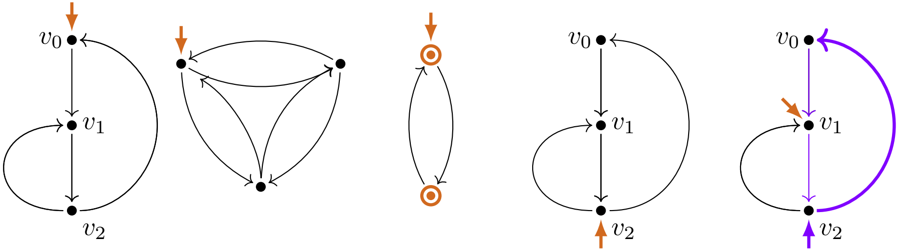

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