

# Introduction to Model Checking

<https://clegra.github.io/mc.html>

## Lecture 1

Clemens Grabmayer

<https://clegra.github.io>

Emilio Tuosto

<https://cs.gssi.it/emilio.tuosto/>

Department of Computer Science



January 26, 2026

# Organization

## Lectures (Clemens 4 / Emilio 3)

- ▶ this week:
  - ▶ Monday, 10.30–12.30, room B, Aurora
  - ▶ Tuesday, 14.30–16.30, P-2.6, Zenith
  - ▶ Thursday and Friday, 14.30–16.30, conference room -1, Zenith

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## Exam (more later)

- ▶ options:
  - ▶ small verification project (of an algorithm, e.g. in [Maude](#))
  - ▶ presentation about a paper
  - ▶ written exam?

# Topics of the course

- ▶ modeling systems by **labeled transition systems (LTSs)**
- ▶ **safety**, **liveness**, and **fairness** properties

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- ▶ Linear Temporal Logic (LTL)
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    - ▶ model check LTSs and properties via product automata
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- ▶ Maude examples model-checker
- ▶ (partial model checking)
  - ▶ (partially known systems (state properties/states/transitions))

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  - ▶ embedded systems, software engin., hardware design, **explainable AI**
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- ▶ provides **diagnostic information** for debugging
- ▶ has sound **mathematical underpinning** (**logic** and **process theory**)

# Model Checking

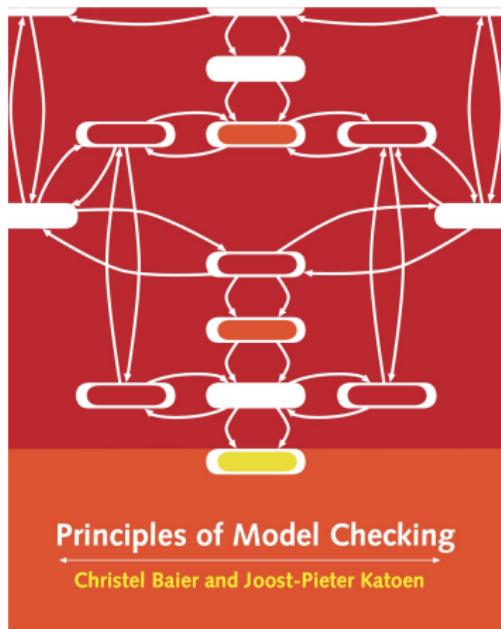
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**Course Goals** are introduction to:

- ▶ Theory:
  - ▶ **modeling** systems by **labeled transition systems**,
  - ▶ expressing **properties** by **temporal-logic formulas**
  - ▶ **model-checking algorithms**
- ▶ Practice related: see **Maude examples**

# Book



- ▶ pdf online available (see [1])
- ▶ we study chapters 1–3, 5, 6

# Lectures

1. Introduction (preview extended, LTSs)
2. Modeling by labeled transition systems
  - ▶ executions; traces; non-determinism; examples
3. Linear-Time Behaviour and Properties
  - ▶ invariant, safety, and liveness (and fairness) properties
4. Linear Temporal Logic (LTL)
  - ▶ syntax and semantics; interpretation of LTSs; examples
5. LTL (continued)
  - ▶ model checking of LTL formulas, and fairness in LTL
  - ▶ Maude examples
6. Computation Tree Logic (CTL)
  - ▶ syntax and semantics, examples
7. Extensions of CTL, and Outlook
  - ▶ expressibility differences with LTL
  - ▶ model checking formulas in CTL
  - ▶ ( $\mu$ -calculus | partial model-checking | Maude examples)

# Software correctness

- ▶ Software is ubiquitous  $\implies$  software is valuable

# Software correctness

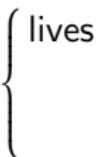
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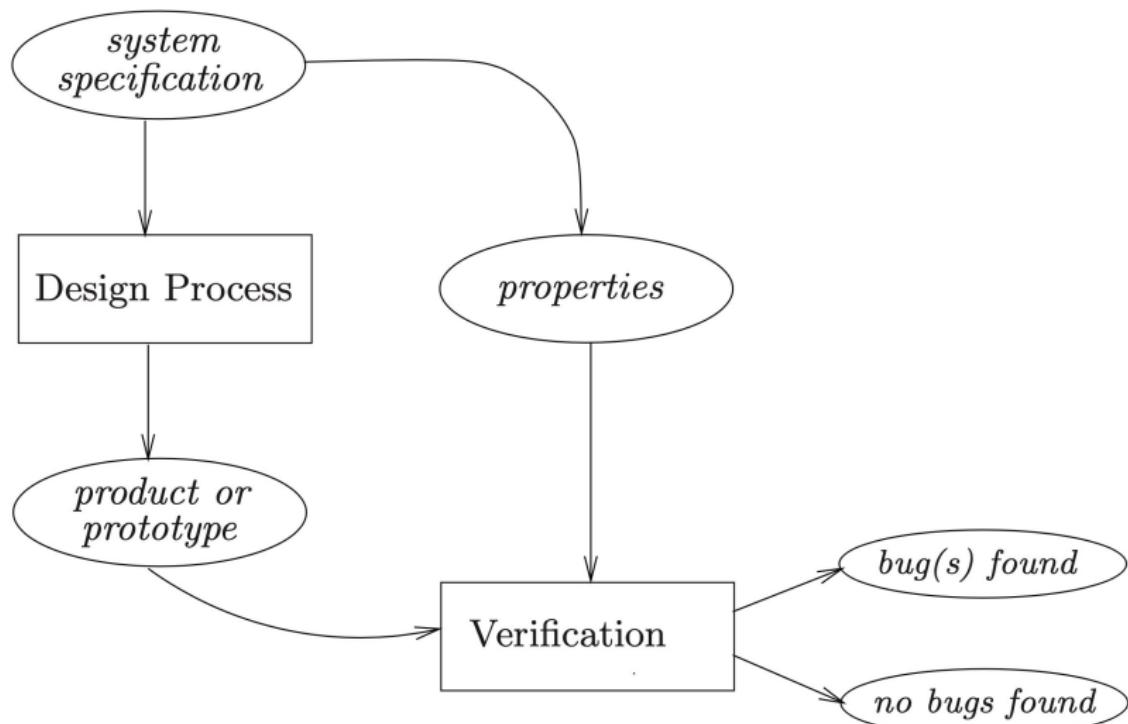
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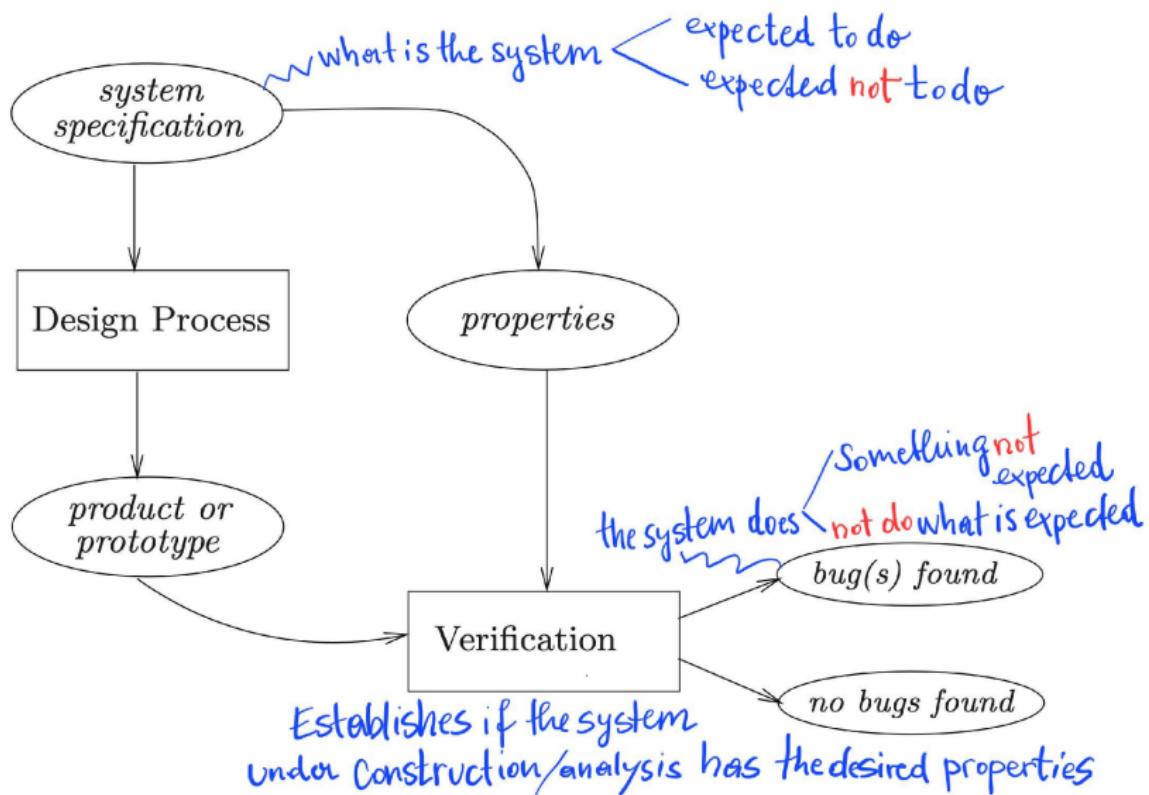
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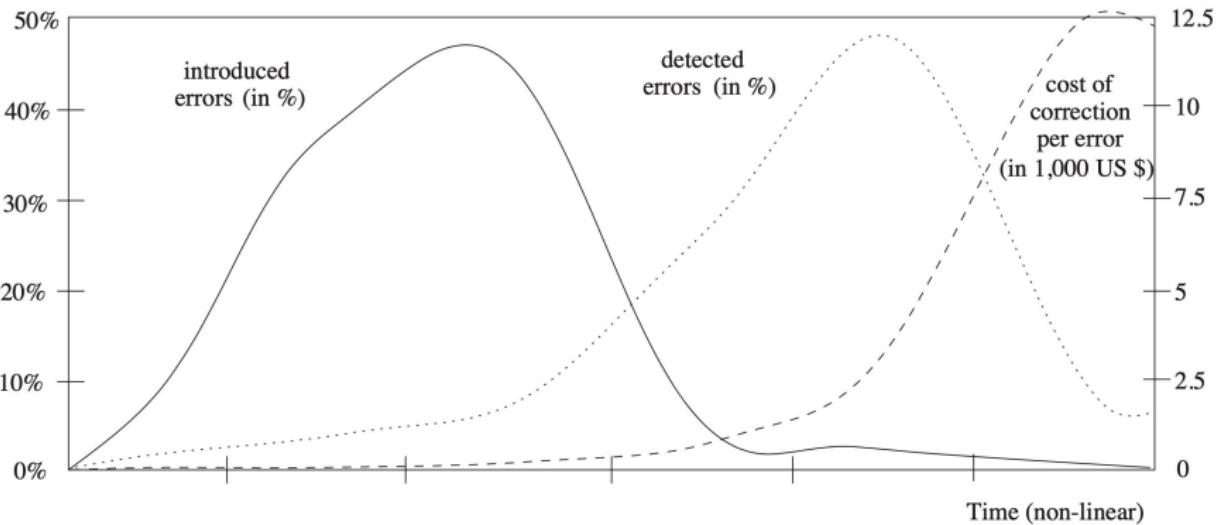
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  - exhaustive testing of all execution paths is infeasible
  - When do we stop?
- ▶ Catching software errors: **the sooner the better.**

# Software lifecycle, error introduction/detection, repair costs

Analysis	Conceptual Design	Programming	Unit Testing	System Testing	Operation
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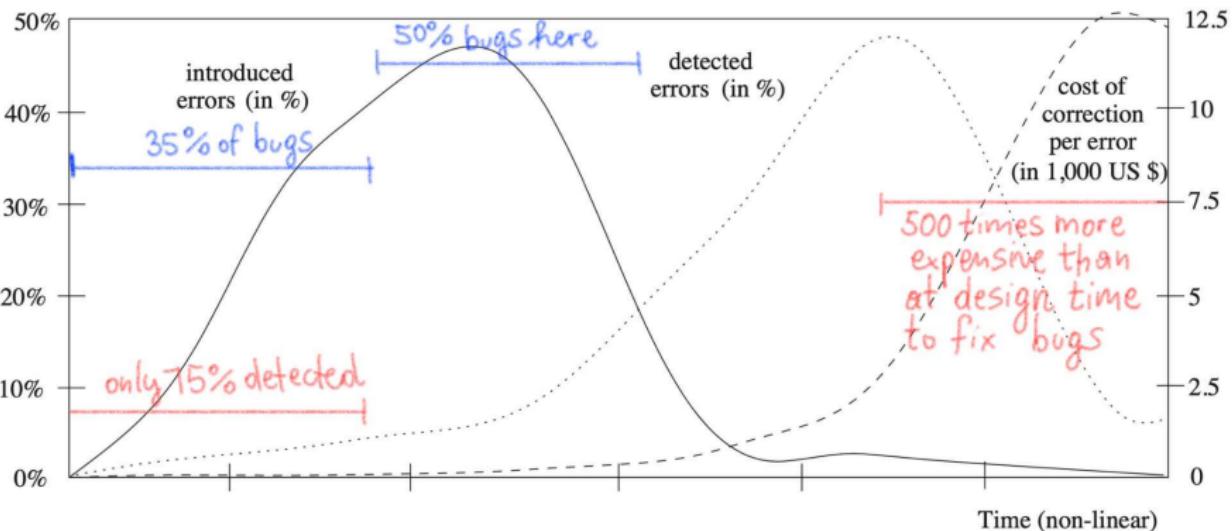


Liggesmeyer et al: "Qualitätssicherung technischer Systeme . . ." [4, 1998]

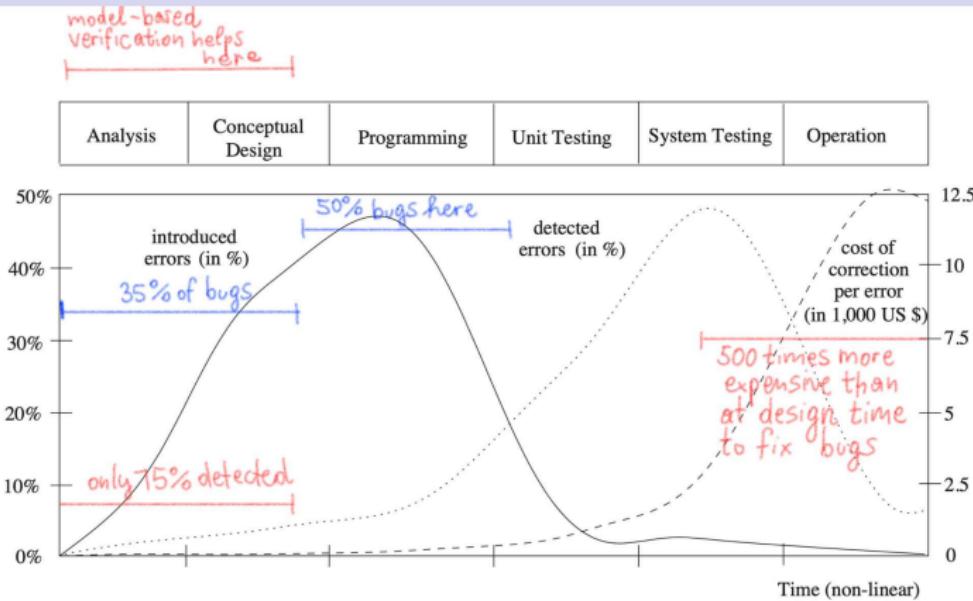
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model-based verification helps here

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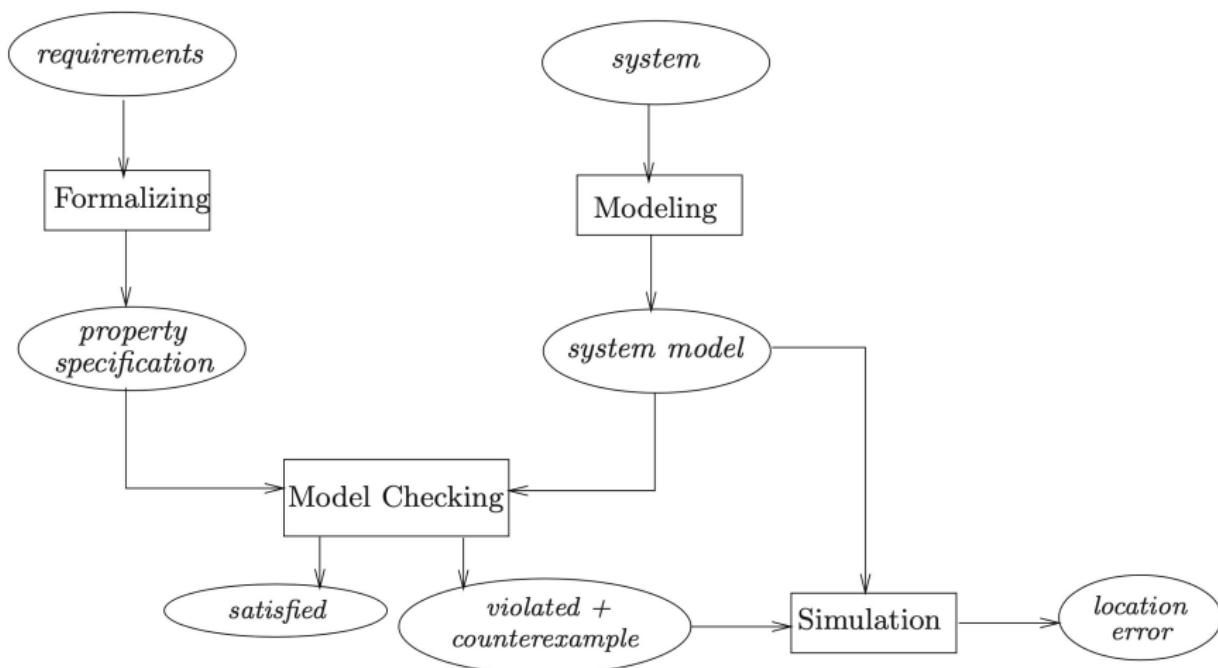


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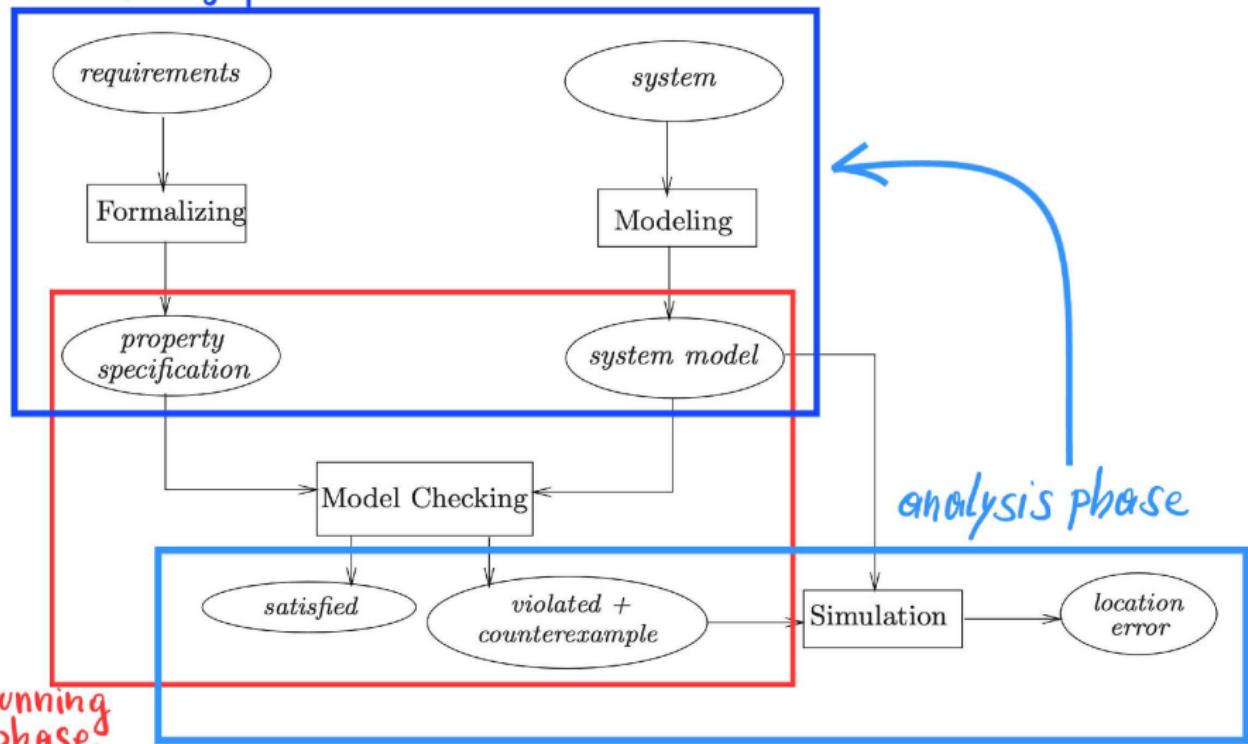
"In software and hardware design of complex systems, more time and effort are spent on verification than on construction. Techniques are sought to reduce and ease the verification efforts while increasing their coverage. Formal methods offer a large potential to obtain an early integration of verification in the design process, to provide more effective verification techniques, and to reduce the verification time."

# Model checking



# Model checking

modeling phase



running  
phase

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 $a$  and  $b$  do not occur at the same time,

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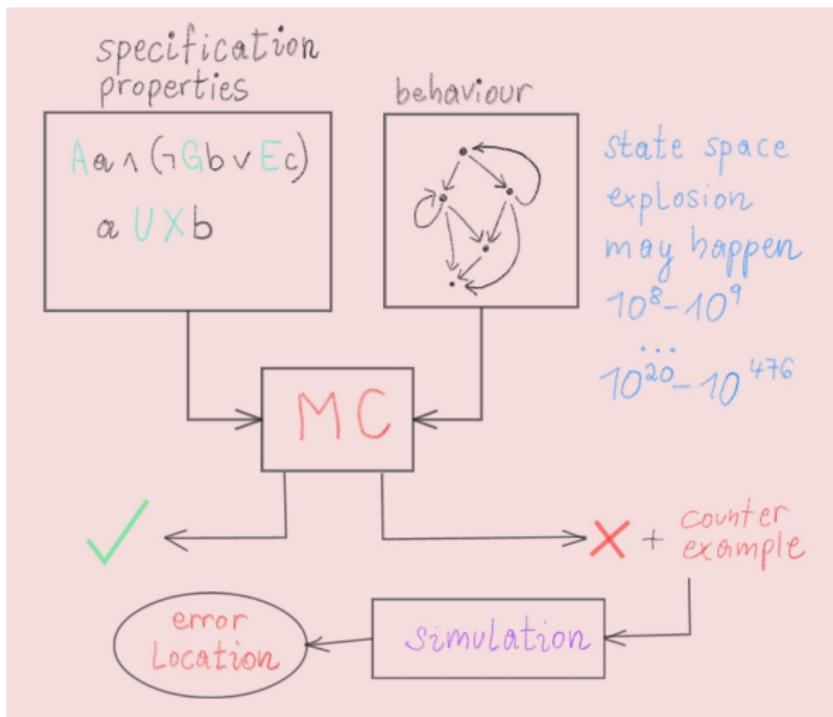
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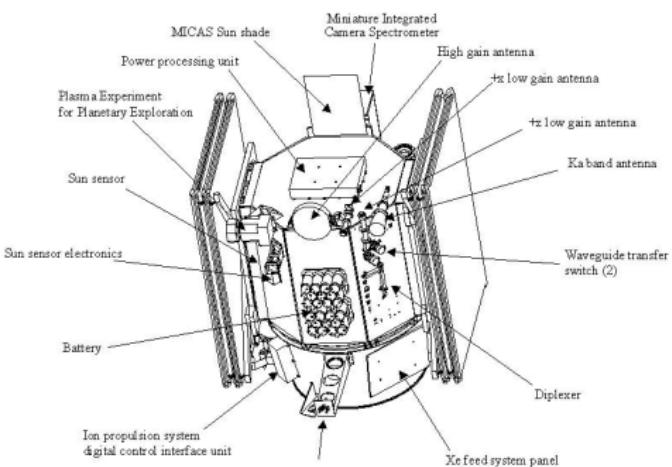
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 $= \text{it will never happen that}$   
 $\text{events } a \text{ and } b \text{ occur at the same time.}$

# Glancing at temporal logic (its use for model checking)



# Deep Space 1 (NASA)



- ▶ Flyby of asteroid 9969 Braille (1999)
- ▶ Entered the coma of Comet Borrelly (2001)
- ▶ Model checking discovered 5 concurrency errors

# Example (program concurrency/non-determinism)

Programs **Inc**, **Dec**, and **Reset** cooperate, and use a shared variable **x**:

```
proc Inc
  while 0 ≤ x ≤ 200
    do
      if x < 200
        then x := x + 1
      fi
    od
```

```
proc Dec
  while 0 ≤ x ≤ 200
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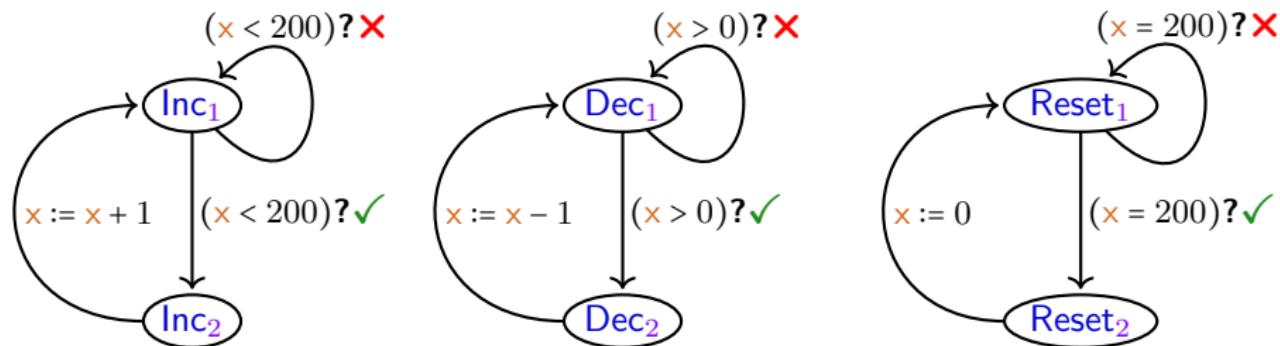
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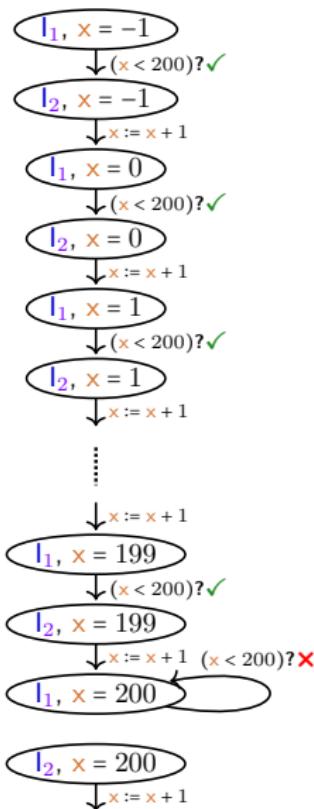
**od**

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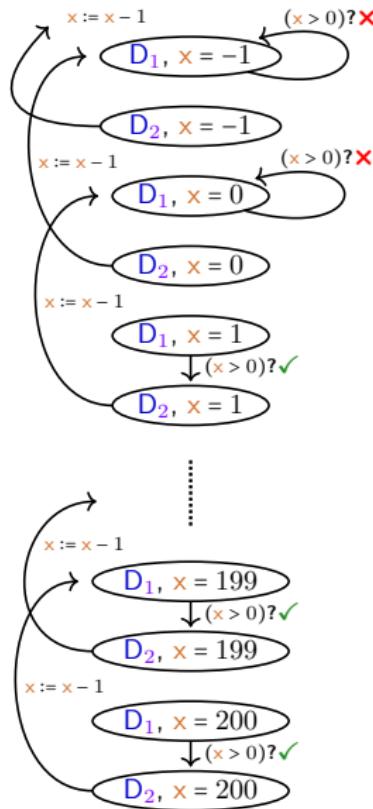
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## Program graphs (PG)

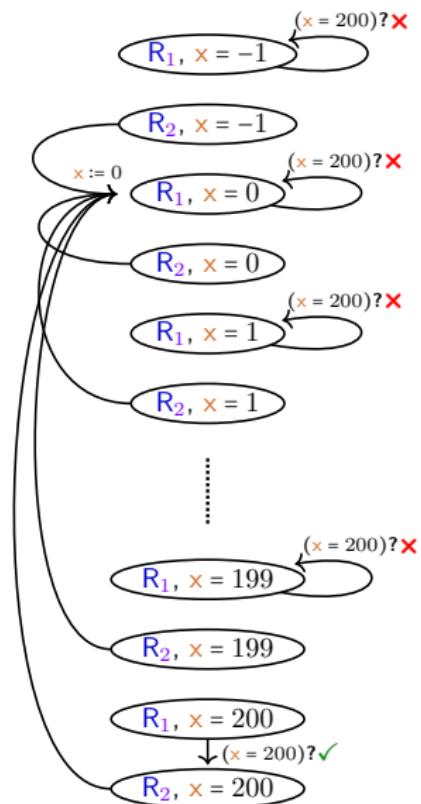
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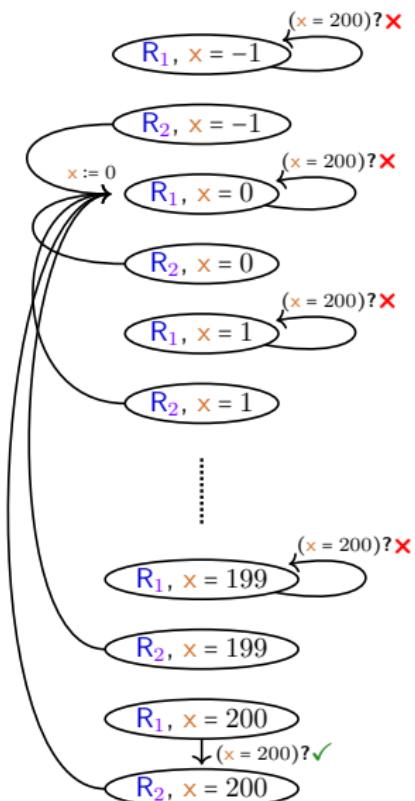
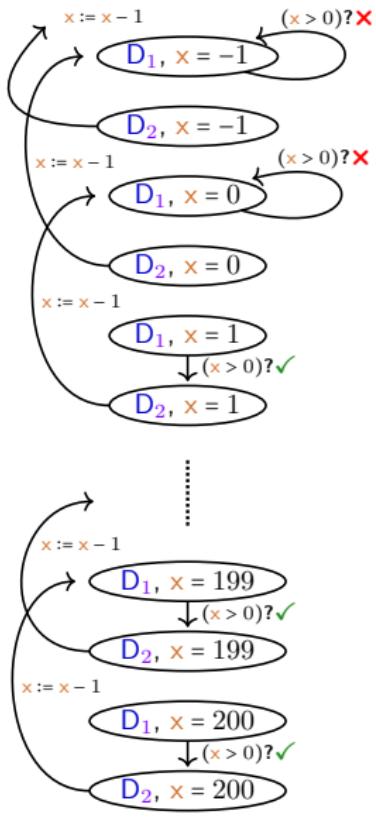
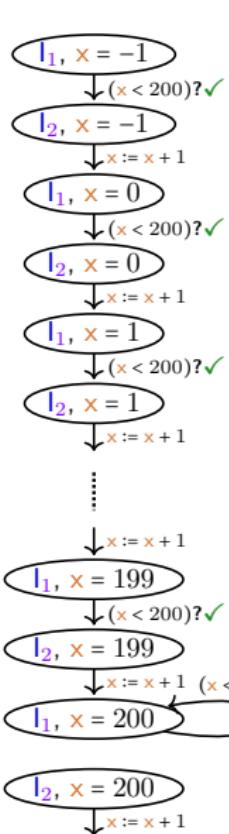
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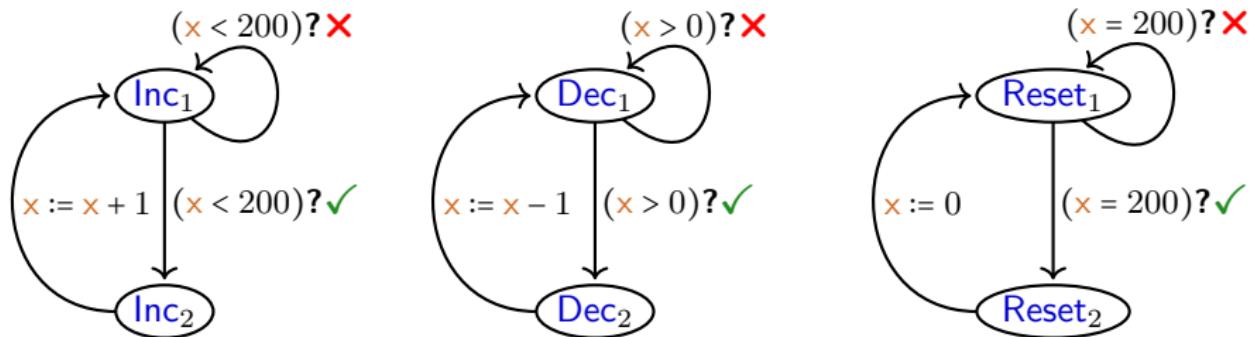
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# Formalizing properties (in temporal logic)



We assume  $x := 0$  initially.

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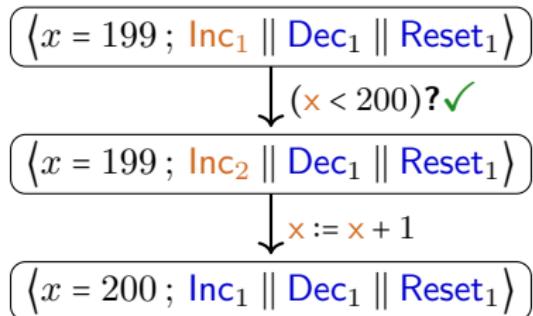
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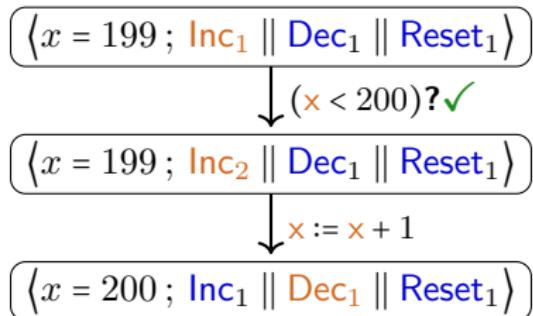
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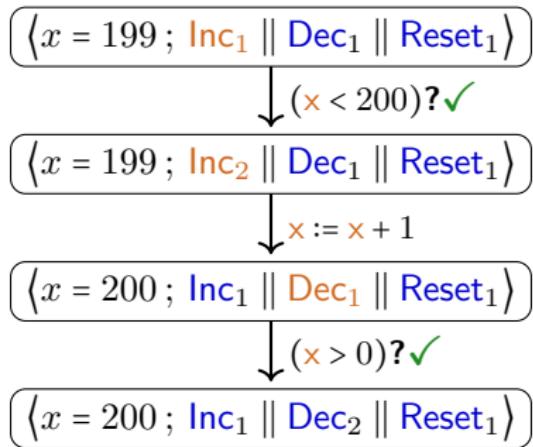
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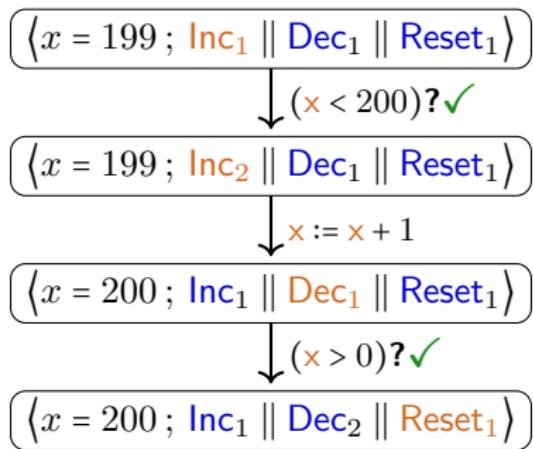
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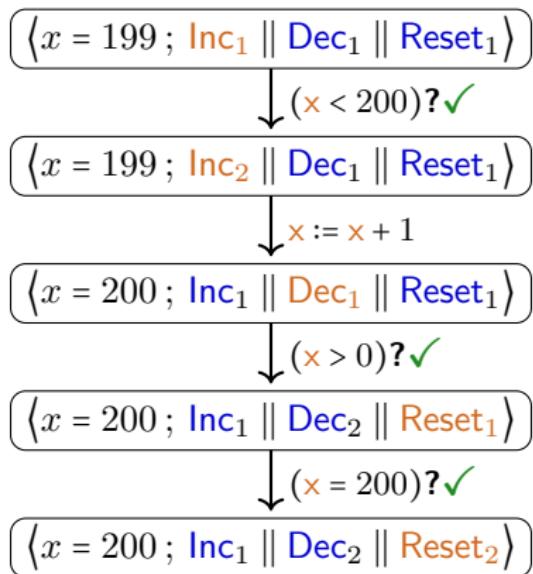
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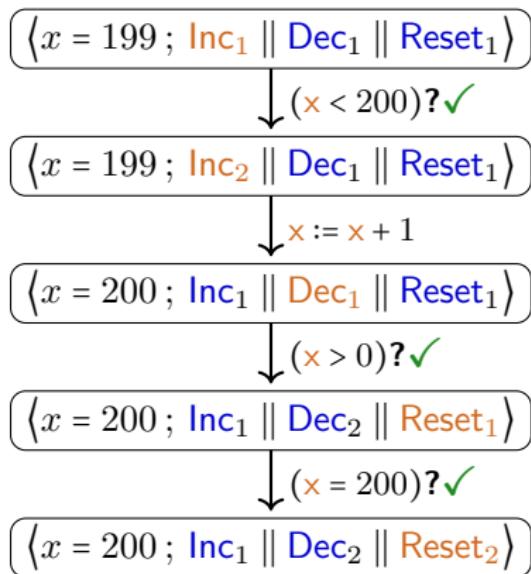
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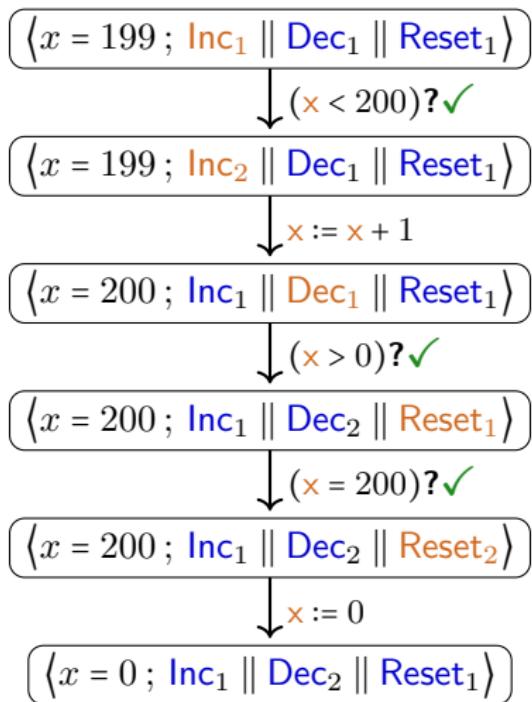
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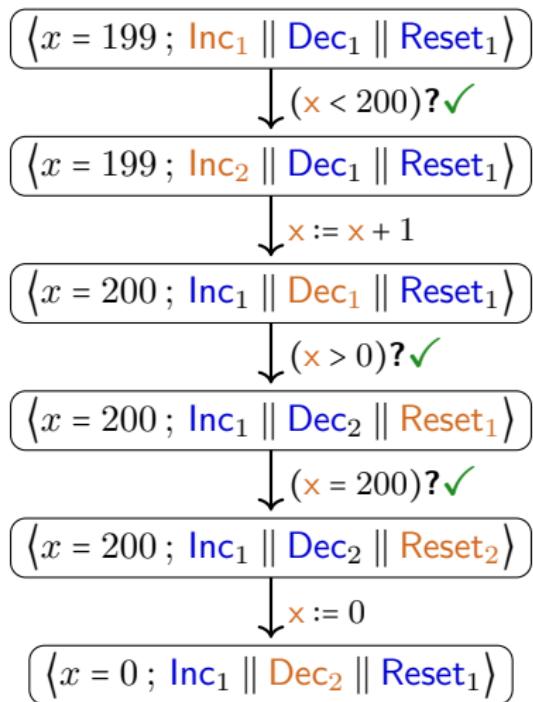
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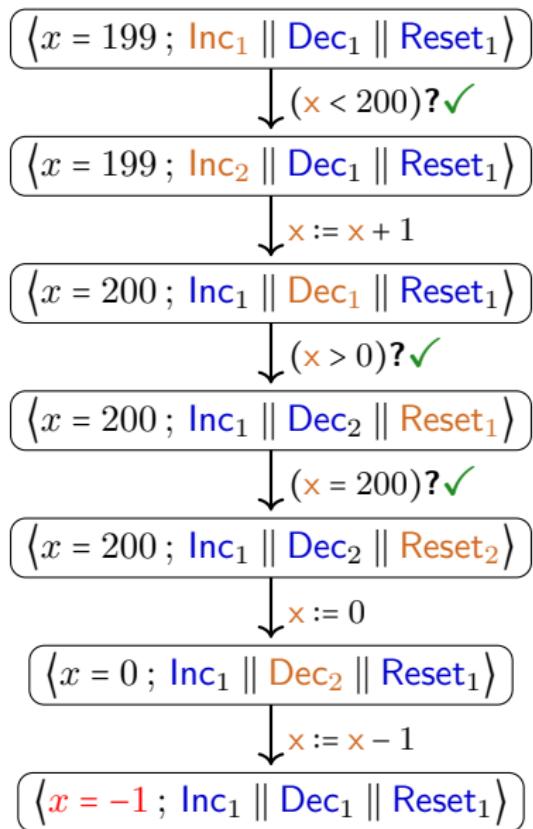
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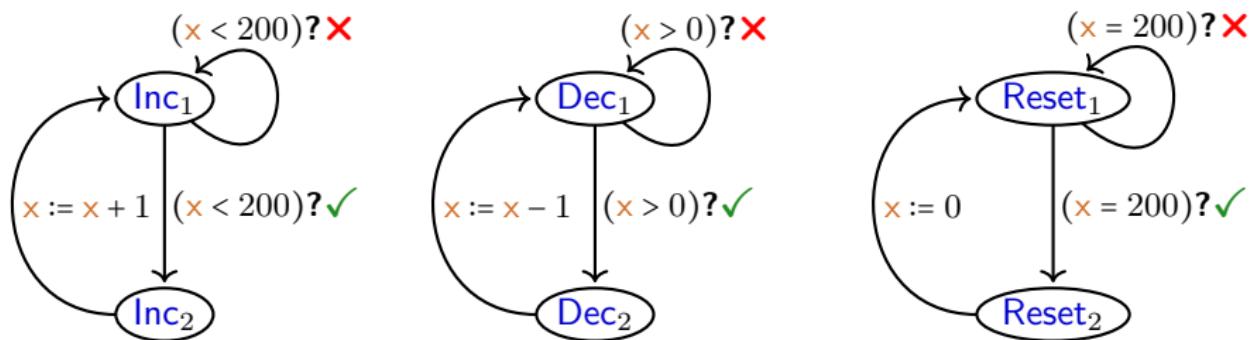
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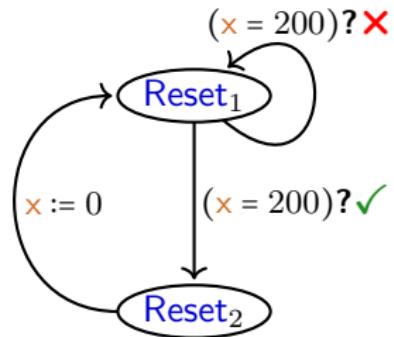
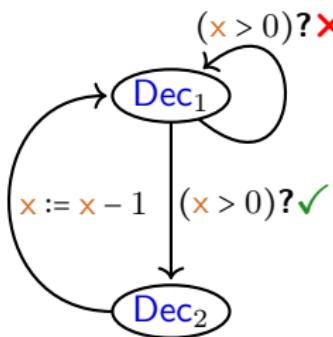
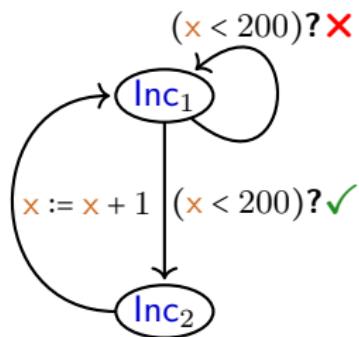
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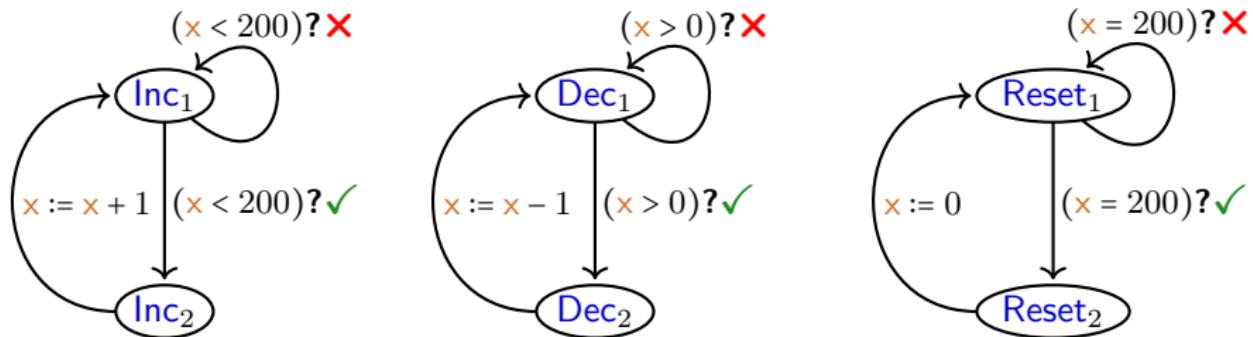
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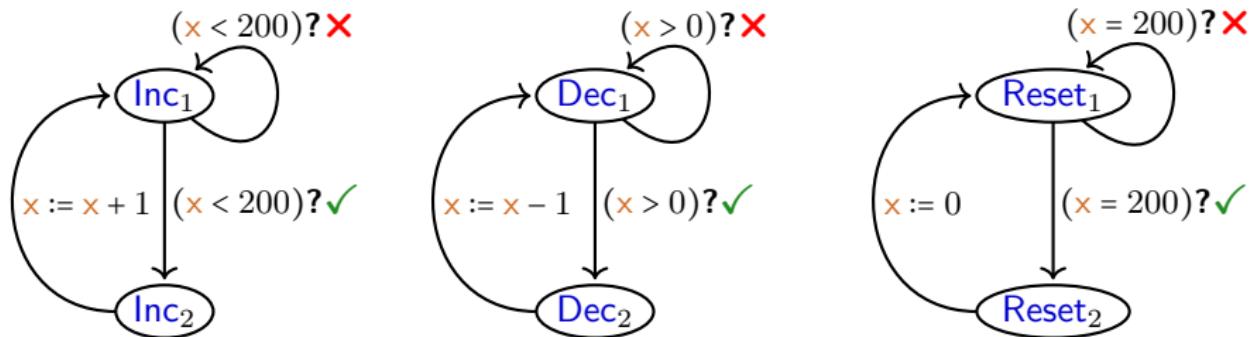


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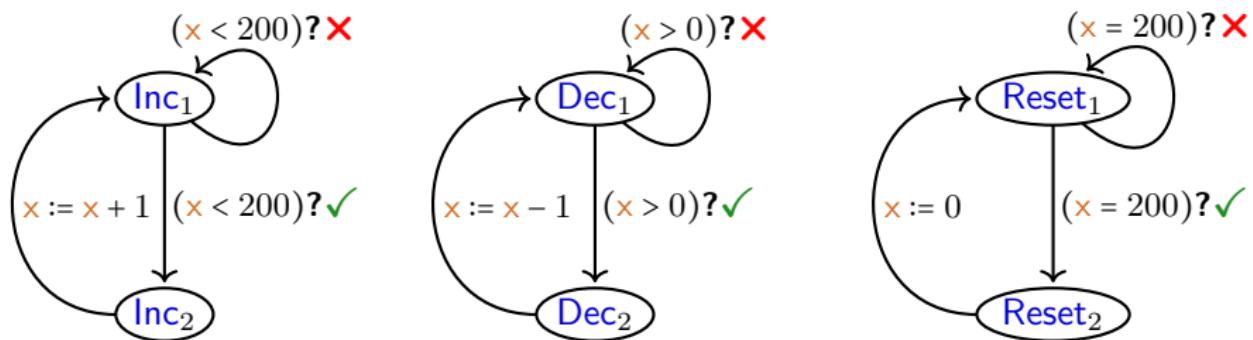


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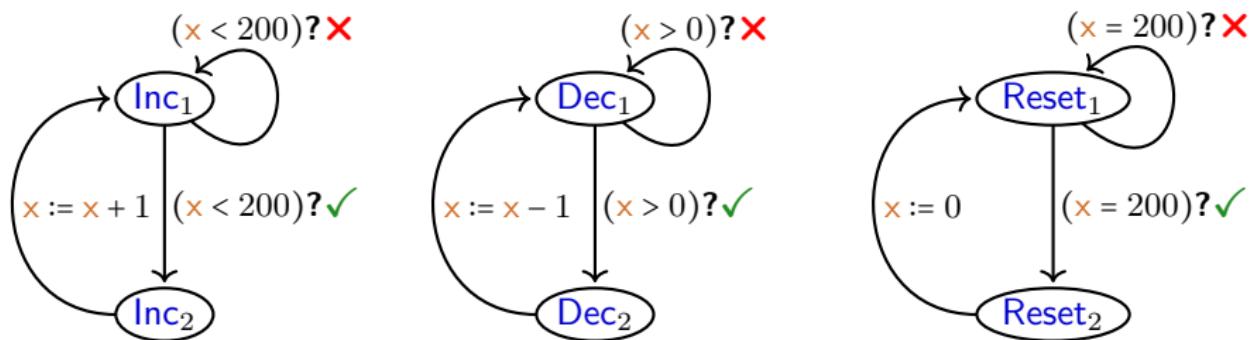
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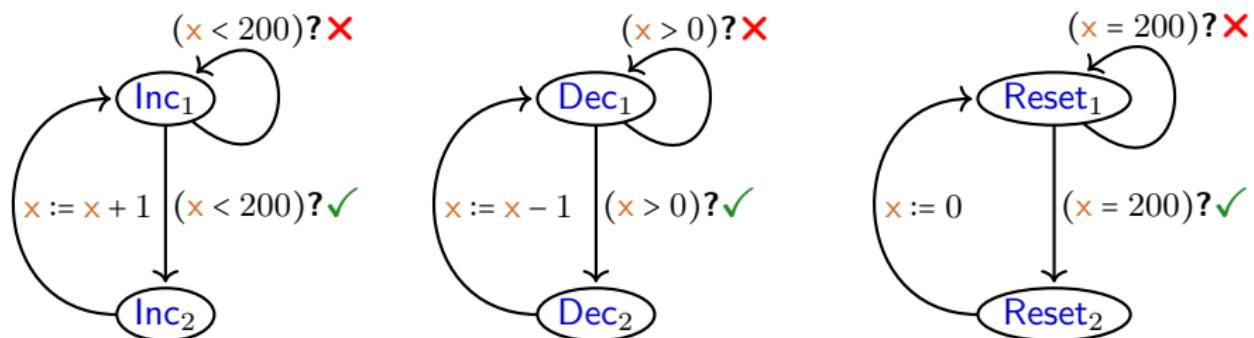
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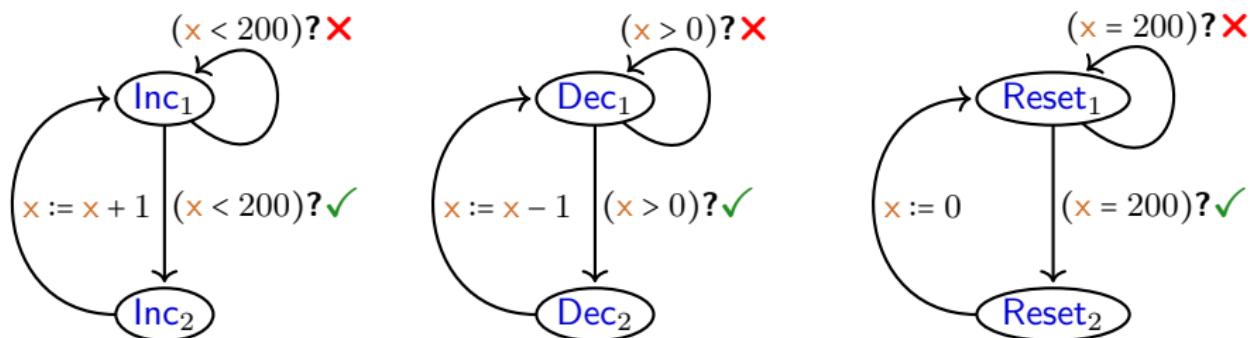
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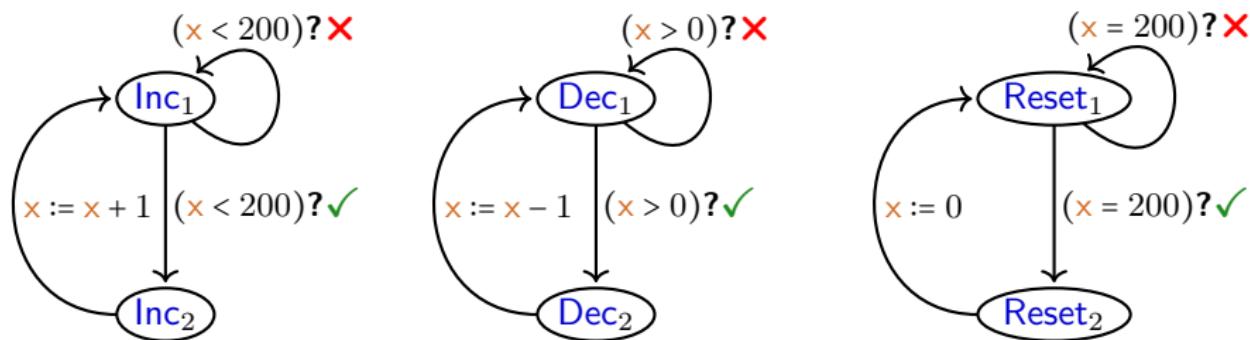
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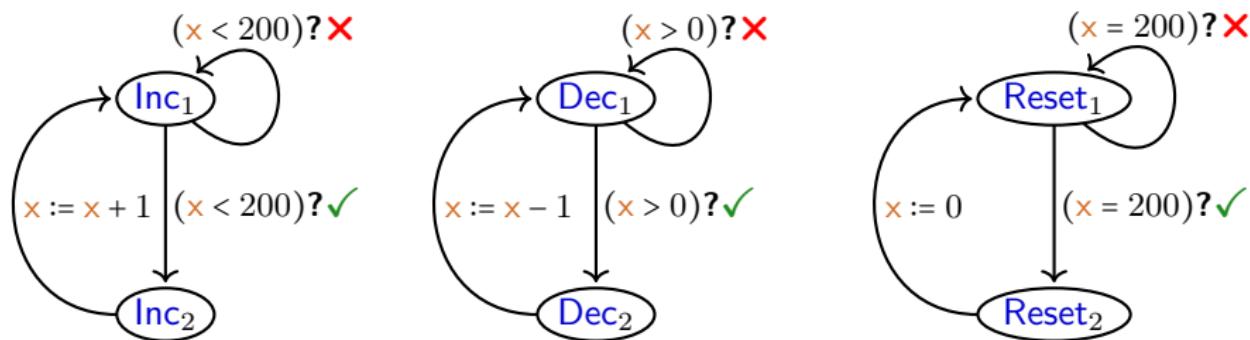
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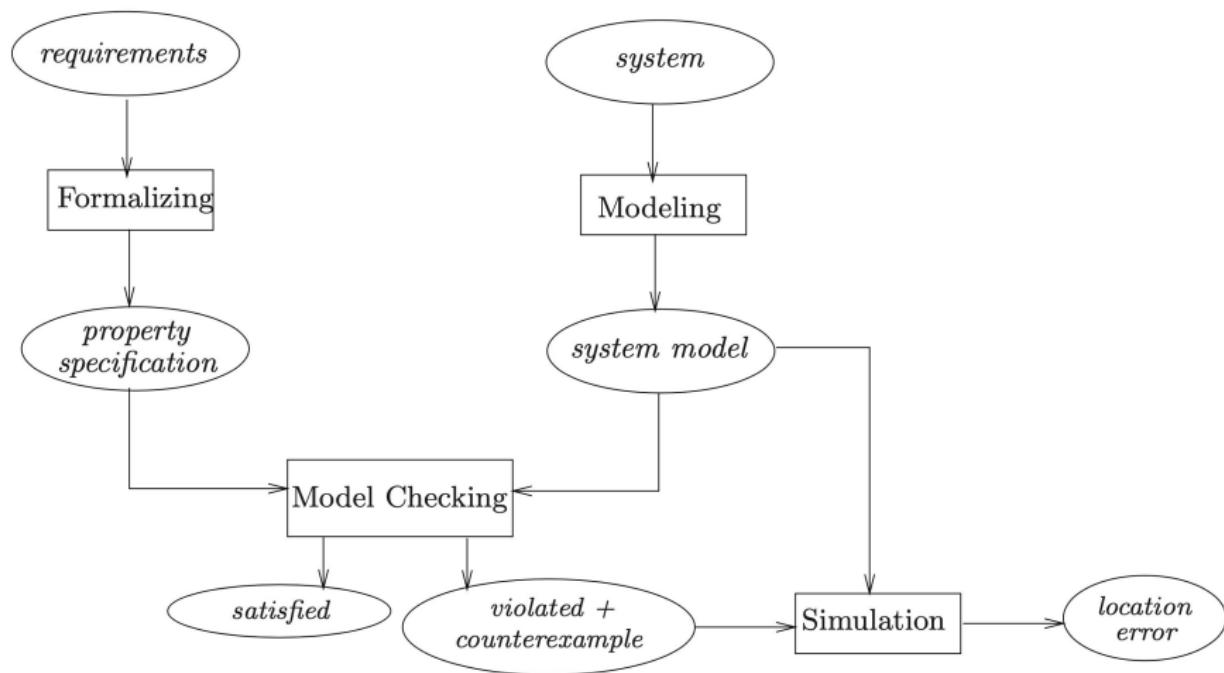
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# Model checking: validation



*Any [such] verification is only as good as the model of the system.*

# Modeling: Verification versus Validation

## VERIFICATION

Are we building the **thing right?**



Does the **design** satisfy the  
expected properties?

## VALIDATION

Are we building the **right thing?**



Is the **design** faithfully capturing  
the requirements?

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## Strengths:

- ▶ it is a **general** technique
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See Baier and Katoen [1] for more references.

# Lectures

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2. Modeling by labeled transition systems
  - ▶ executions; traces; non-determinism; examples
3. Linear-Time Behaviour and Properties
  - ▶ invariant, safety, and liveness (and fairness) properties
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5. LTL (continued)
  - ▶ model checking of LTL formulas, and fairness in LTL
  - ▶ Maude examples
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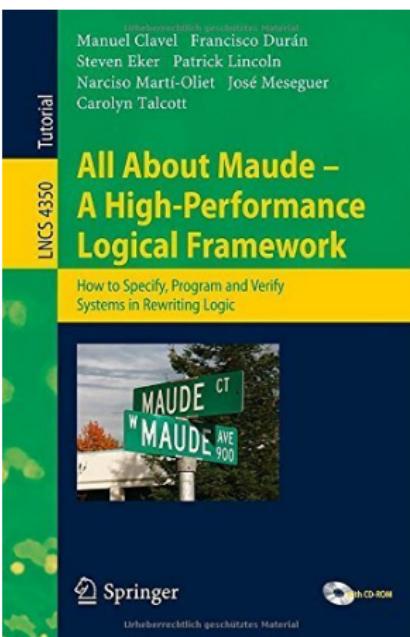
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# Book Maude



- ▶ Maude documentation page:  
<https://maude.cs.illinois.edu/documentation>

# Maude code (idea)

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ceq (S1 S2 S3 x |= counterge0) = true  if (0 = x \vee 0 < x)
ceq (S1 S2 S3 x |= counterlt0) = true  if (x < 0)
ceq (S1 S2 S3 x |= counterle200) = true  if (x < 200 \vee x = 200)

```

# Maude output (simplified)

```
Maude> red modelCheck(initial, <> counterlt0)
reduce in COUNTERS-CHECK : modelCheck(initial, <> counterlt0)
result ModelCheckResult:
result Bool : true
```

# Maude output (simplified)

```
Maude> red modelCheck(initial, <> counterlt0)
reduce in COUNTERS-CHECK : modelCheck(initial, <> counterlt0)
result ModelCheckResult:
result Bool : true
```

```
Maude> red modelCheck(initial, [](counterge0 /\ counterle200)
reduce in COUNTERS-CHECK :
           modelCheck(initial, [](counterge0 /\ counterle200)
result ModelCheckResult:
counterexample({Inc1 Dec1 Reset1 199}
               {Inc2 Dec1 Reset1 199}
               {Inc1 Dec1 Reset1 200}
               {Inc1 Dec2 Reset1 200}
               {Inc1 Dec2 Reset2 200}
               {Inc1 Dec2 Reset1 0}
               {Inc1 Dec1 Reset1 -1})
```

# Lectures (tentative)

1. Introduction (preview extended)
2. Modeling by labeled transition systems
  - ▶ executions; traces; non-determinism; examples
3. Linear-Time Behaviour and Properties
  - ▶ invariant, safety, and liveness (and fairness) properties
4. Linear Temporal Logic (LTL)
  - ▶ syntax and semantics; interpretation of LTSs; examples
5. LTL (continued)
  - ▶ fairness in LTL
  - ▶ model checking of LTL formulas
6. Computation Tree Logic (CTL)
  - ▶ syntax and semantics, examples
7. Extensions of CTL, and Outlook
  - ▶ expressibility differences with LTL
  - ▶ model checking formulas in CTL
  - ▶ Maude examples | partial model-checking |  $\mu$ -calculus

# Exam options

Together we decide on a topic.

Study an article, or a book chapter.

or

Dive deeper into a proof from the lecture.

or

Model a basic algorithm, and check basic properties.

or

Develop an idea that motivates you.

or

...

You give a 25-minute presentation about what you found.

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## Exam examples

Presentations about articles:

- ▶ *Progress, Justness, and Fairness* (2019) by R. van Glabbeek, P. Höfner
- ▶ *Comparison of Model Checking Tools for Information Systems* (2010) by M. Frappier et al. (focusing on SPIN and NuSMV)

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