Introduction to Model Checking

(Preview of Core Course)

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Model Checking

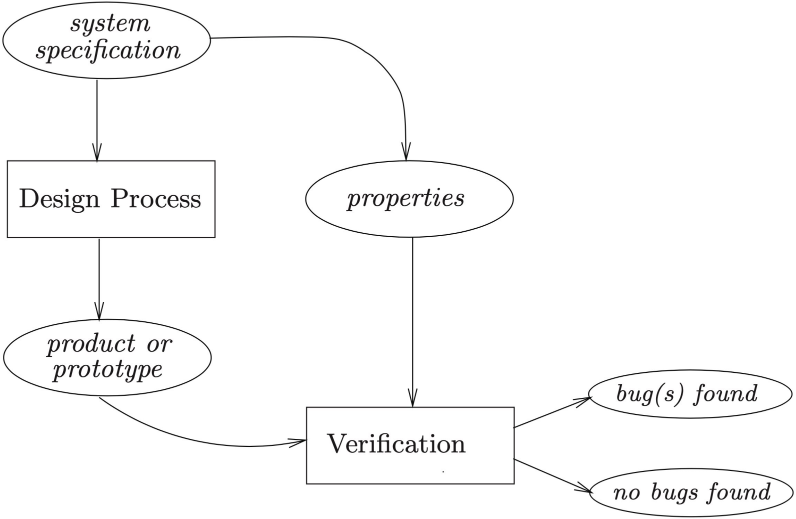
. . . is an effective automatable technique:

- *to expose potential software design errors;*
- *that, given a finite-state model of a system and a formal property, systematically checks whether this property holds for that model.*

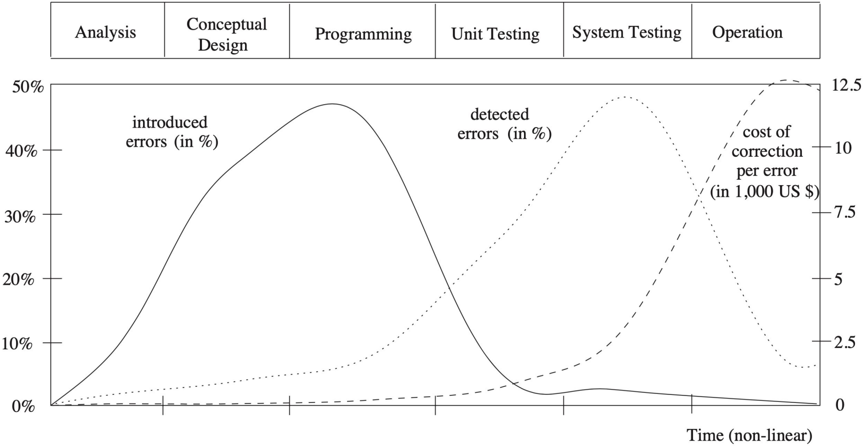
Strengths:

- widely applicable
  (embedded systems, software engineering, hardware design)
- supports partial verification (of modules)
- provides diagnostic information for debugging
- has sound mathematical underpinning (logic and process theory)

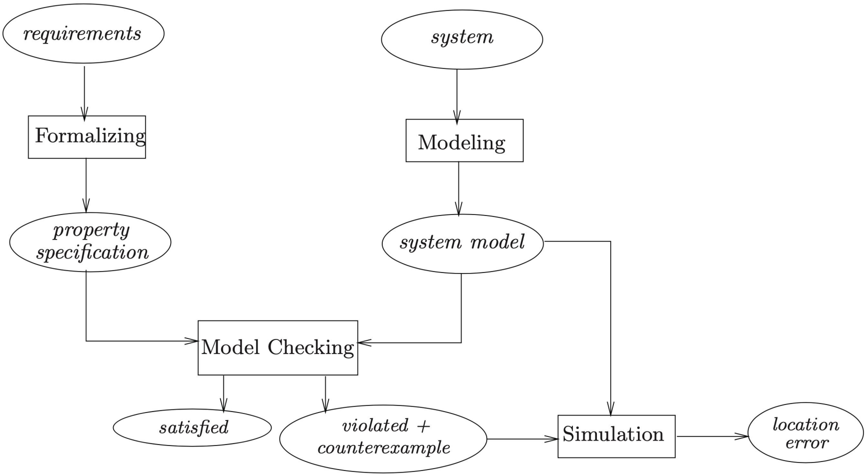
Hard-/Software Verification (traditionally)



Error introduction, detection, and repair costs



Model checking



Example: concurrency and non-determinism

Programs Inc, Dec, and Reset cooperate, and use a shared variable x:

```
proc Inc
  while true
    do
      if x < 200
        then x := x + 1
      fi
    od
```

```
proc Dec
  while true
    do
      if x > 0
        then x := x - 1
      fi
    od
```

```
proc Reset
  while true
    do
      if x = 200
        then x := 0
      fi
    od
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Example: concurrency and non-determinism

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  while true
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```

Question: Is $0 \leq x \leq 200$ always guaranteed?

Modeling (by labeled transition systems)

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  while true
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      if $x < 200$
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Modeling (by labeled transition systems)

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1:    if $x < 200$
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      fi
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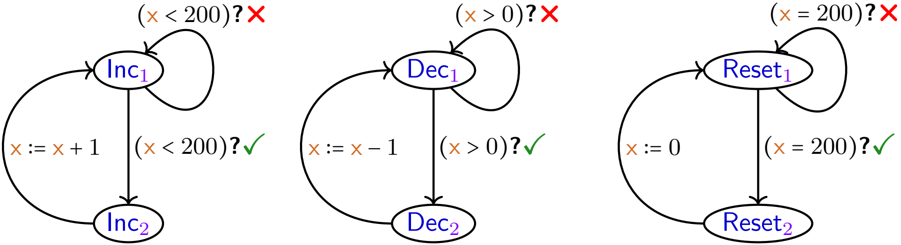
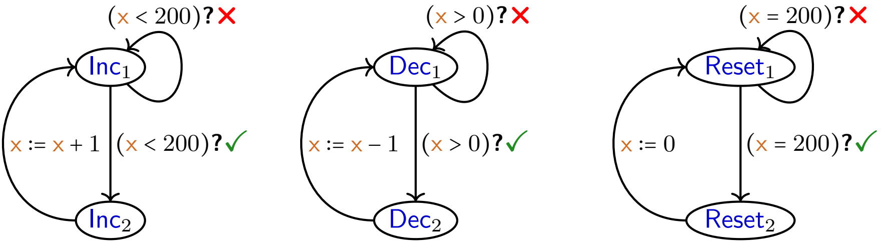


Labeled transition systems (LTSs)

Formalizing properties (in temporal logic)



$\mathsf{Inc}_1 \parallel \mathsf{Dec}_1 \parallel \mathsf{Reset}_1 \overset{?}{\models} \quad \Box(0 \leq x \wedge x \leq 200)$ (Linear-TL formula)

Counterexample (offending execution trace)

$$\boxed{\left\langle x = 199\,;\ \mathsf{Inc}_1 \parallel \mathsf{Dec}_1 \parallel \mathsf{Reset}_1 \right\rangle}$$

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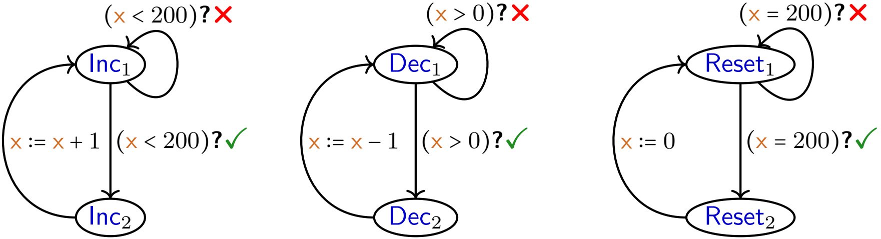
Formalizing properties (in temporal logic)



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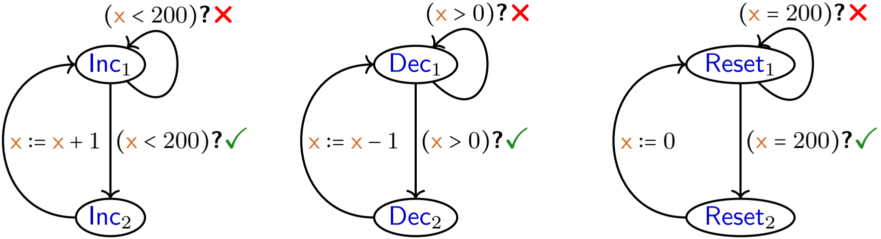

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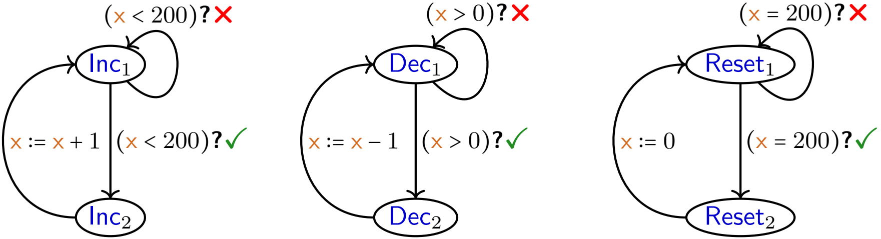

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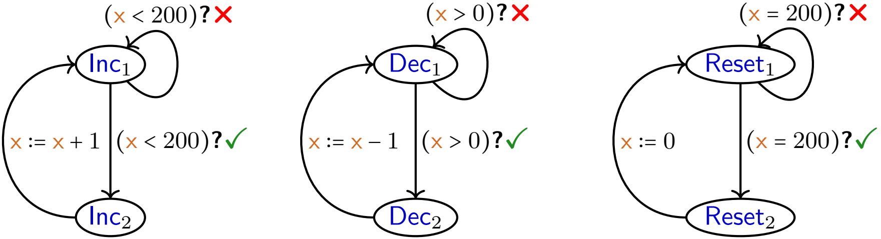

Inc$_1$ || Dec$_1$ || Reset$_1$ $\not\models$ $\Box(0 \leq x \ \wedge \ x \leq 200)$ (Linear-TL formula)

Inc$_1$ || Dec$_1$ || Reset$_1$ $\models$ $\Diamond(x < 0)$ (LTL formula)

Inc$_1$ || Dec$_1$ || Reset$_1$ $\forall\Box(0 \leq x \ \wedge \ x \leq 200)$ (Computation-Tree-L formula)

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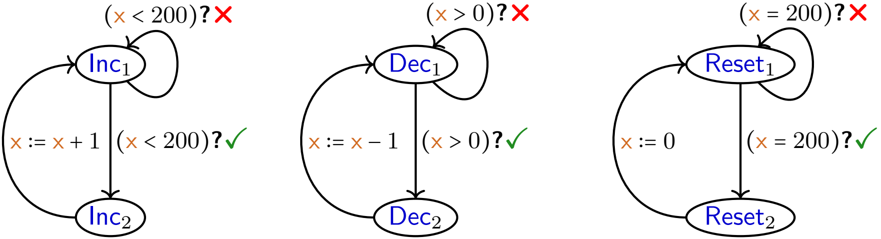

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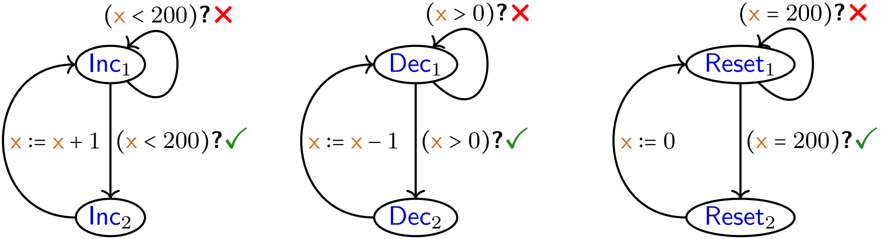

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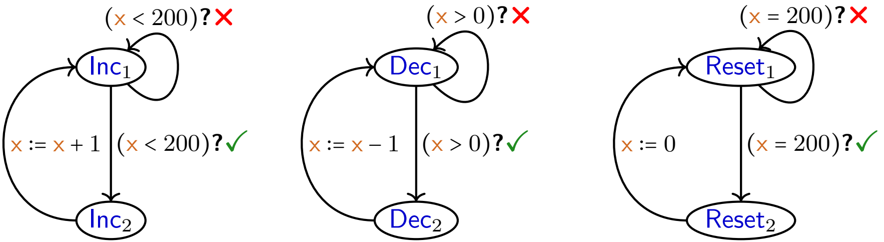
Formalizing properties (in temporal logic)



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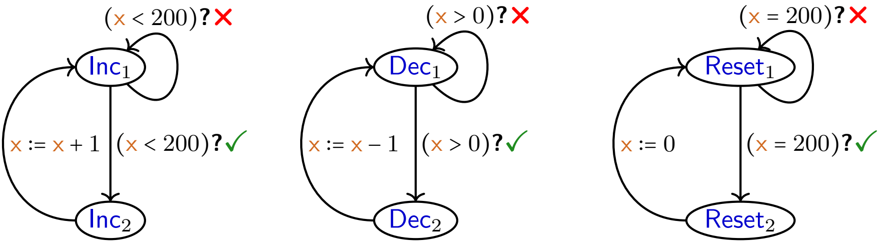

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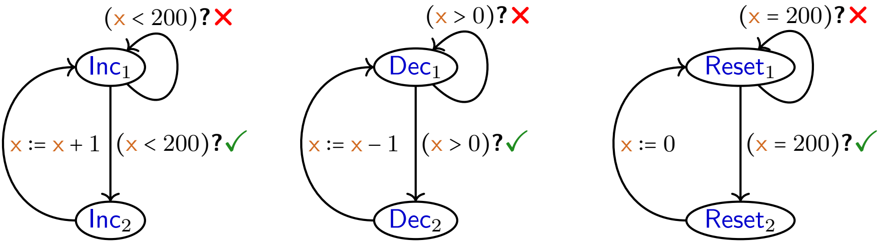

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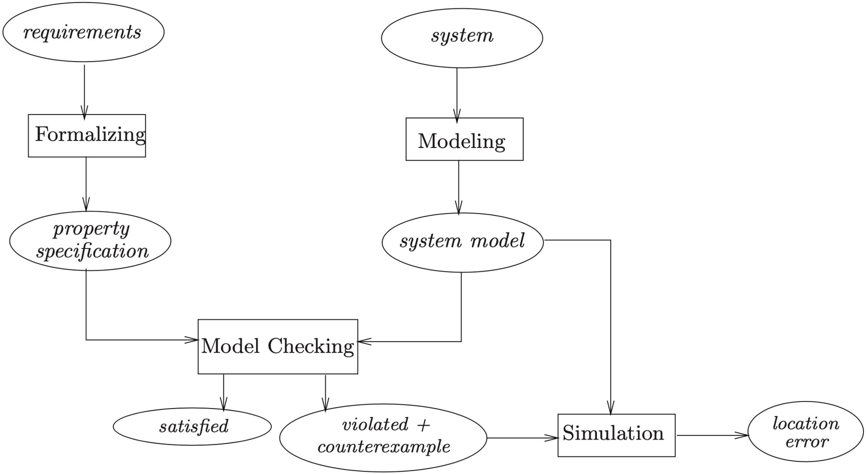
Model checking



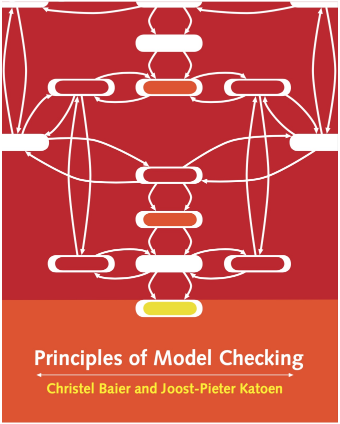
Any [such] verification is only as good as the model of the system.

Course topics

- modeling systems by labeled transition systems (LTSs)
- linear time behaviour properties (based on execution traces)
- concepts of fairness
- Linear Temporal Logic (LTL)
  - model checking
    - express properties by Büchi automata
    - model check LTSs and properties via product automata
- Computation Tree Logic (CTL) and variants (CTL^+, CTL^*)
- Partial model checking
  - for partially unknown systems (state properties/states/transitions)

- analysing system behavior with mCRL2

Book


Principles of Model Checking
Christel Baier and Joost-Pieter Katoen

- pdf available:
  https://is.ifmo.ru/books/_principles_of_model_checking.pdf

Course organization

Lectures (Emilio 2/Clemens 5)

- presentations on blackboard
- notes after the lecture (notes 2024/25 available)
- February (first/second week)

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  - presentation about a paper
  - written exam?

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Thank you – we are looking forward to the course!