The Graph Structure of Process Interpretations of Regular Expressions

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Nancy
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Overview

- regular expressions (unary/binary star/1-free-under-star $(*/\overline{1})$)
- Milner's process interpretation P/semantics $[\![\cdot]\!]_P$
  - P-/$[\![\cdot]\!]_P$-expressible graphs ($\rightsquigarrow$ expressibility question)
  - axioms for $[\![\cdot]\!]_P$-identity ($\rightsquigarrow$ completeness question)
- loop existence and elimination (LEE)
  - defined by loop elimination rewrite system, its completion
  - describes interpretations of $(*/\overline{1})$ reg. expr.s (extraction possible)
  - LEE-witnesses: labelings of process graphs with LEE
  - LEE is preserved under bisimulation collapse (stepwise collapse)
- 1-LEE = sharing via 1-transitions facilitates LEE
- LEE/1-LEE characterize image of $P^\bullet$ (restricted/unrestricted)
  - where $P^\bullet$ a compact (sharing-increased) refinement of P
- outlook on work-to-do

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  - LEE-witnesses: labelings of process graphs with LEE
  - LEE is preserved under bisimulation collapse (stepwise collapse)
- 1-LEE = sharing via 1-transitions facilitates LEE
  - describes interpretations of all reg. expr.s (extraction possible)
  - not preserved under bisimulation collapse (approximation possible)
- LEE/1-LEE characterize image of $P^\bullet$ (restricted/unrestricted)
  - where $P^\bullet$ a compact (sharing-increased) refinement of P
  - via refined extraction using LEE/1-LEE
- outlook on work-to-do

Regular Expressions

Definition (*~ Copi–Elgot–Wright, 1958*)

Regular expressions over alphabet A with unary Kleene star:

$$e, e_1, e_2 \ ::= \ \mathbf{0} \mid \boldsymbol{a} \mid e_1 + e_2 \mid e_1 \cdot e_2 \mid e^* \qquad (\text{for } \boldsymbol{a} \in A).$$

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Regular Expressions

Definition (~ *Kleene, 1951*, ~ *Copi–Elgot–Wright, 1958*)

Regular expressions over alphabet A with unary / binary Kleene star:

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$$e, e_1, e_2 ::= \mathbf{0} \mid \mathbf{1} \mid a \mid e_1 + e_2 \mid e_1 \cdot e_2 \mid e_1 \circledast e_2 \qquad (\text{for } a \in A).$$

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Regular Expressions (1-free)

Definition (~ *Kleene, 1951*, ~ *Copi–Elgot–Wright, 1958*)

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Definition (for process interpretation)

1-free regular expressions over alphabet A with binary Kleene star:

$$f, f_1, f_2 ::= \mathbf{0} \mid \boldsymbol{a} \mid f_1 + f_2 \mid f_1 \cdot f_2 \mid f_1 \circledast f_2 \qquad (\text{for } \boldsymbol{a} \in A).$$

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Regular Expressions (under-star-/1-free)

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Definition (for process interpretation)

The set $RExp^{(1)}(A)$ of 1-free regular expressions over A is defined by:

$$f, f_1, f_2 \ ::= \ \mathbf{0} \mid \boldsymbol{a} \mid f_1 + f_2 \mid f_1 \cdot f_2 \mid f_1{}^\star \cdot f_2 \qquad (\text{for } \boldsymbol{a} \in A),$$

the set $RExp^{(*/1)}(A)$ of under-star-1-free regular expressions over A by:

$$uf, uf_1, uf_2 \ ::= \ \mathbf{0} \mid \mathbf{1} \mid \boldsymbol{a} \mid uf_1 + uf_2 \mid uf_1 \cdot uf_2 \mid f^\star \qquad (\text{for } \boldsymbol{a} \in A).$$

Process interpretation P of regular expressions *(Milner, 1984)*

$\mathbf{0} \xmapsto{P}$ deadlock δ, no termination

$\mathbf{1} \xmapsto{P}$ empty-step process ϵ, then terminate

$a \xmapsto{P}$ atomic action a, then terminate

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$e_1 + e_2 \overset{P}{\longmapsto}$ *(choice)* execute $P(e_1)$ or $P(e_2)$

$e_1 \cdot e_2 \overset{P}{\longmapsto}$ *(sequentialization)* execute $P(e_1)$, then $P(e_2)$

$e^* \overset{P}{\longmapsto}$ *(iteration)* repeat (terminate or execute $P(e)$)

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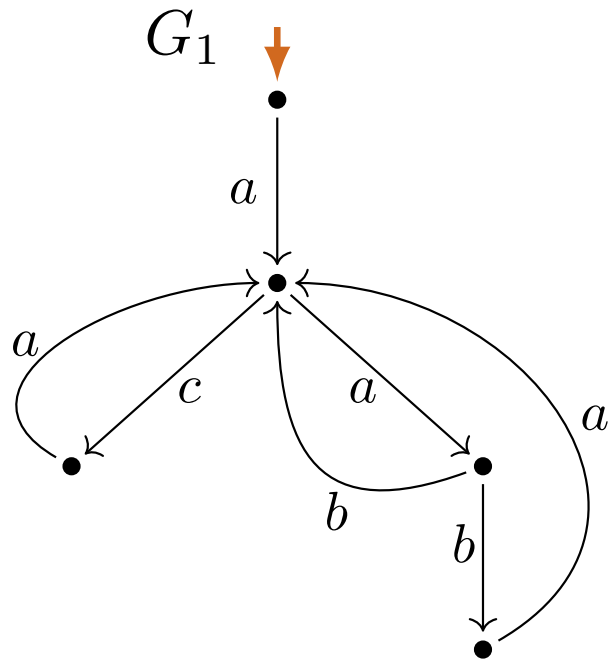
$$[\![e]\!]_P := [P(e)]_{\leftrightarrow} \quad \text{(bisimilarity equivalence class of process } P(e))$$

P-expressibility and $[\![\cdot]\!]_P$-expressibility (example, informally)



$$P\Big(\overbrace{(a\cdot(c\cdot a+a\cdot(b+b\cdot a))^*)\cdot 0}^{f}\Big)$$

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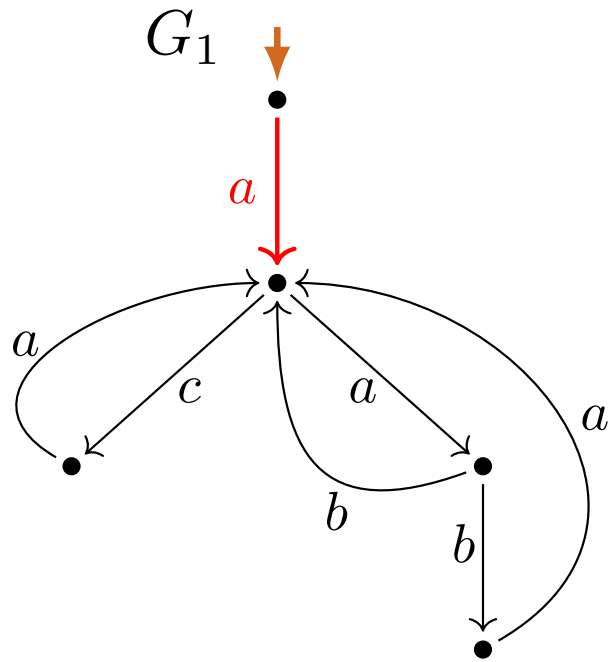

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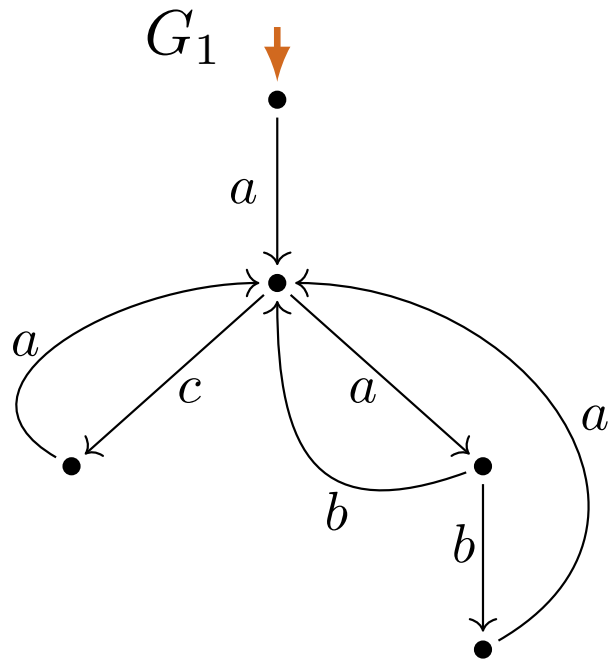

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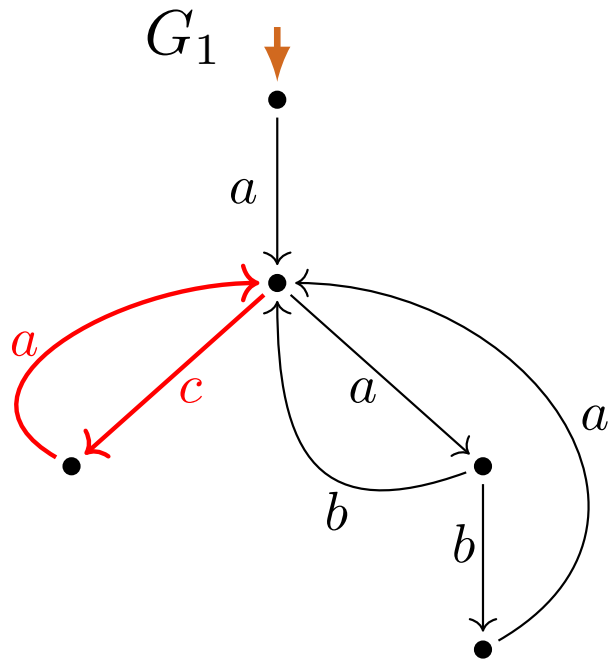

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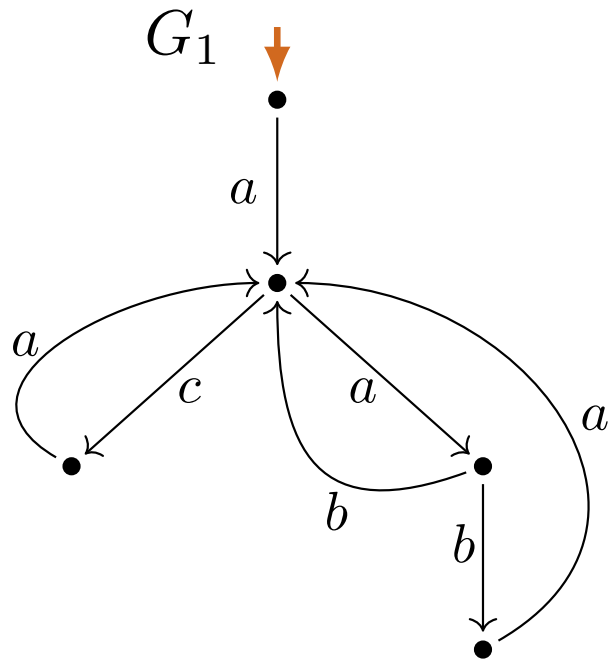

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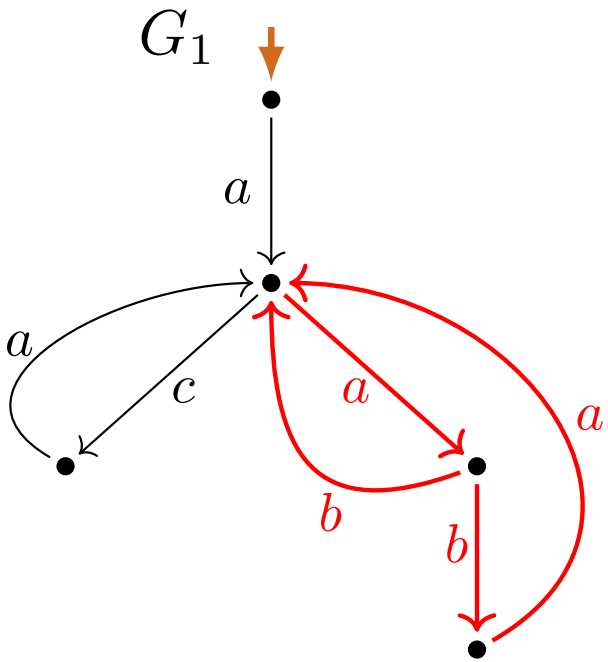

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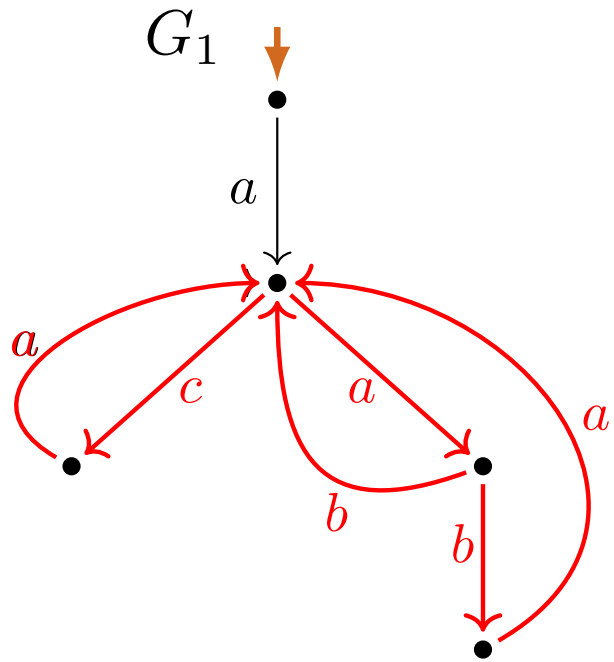

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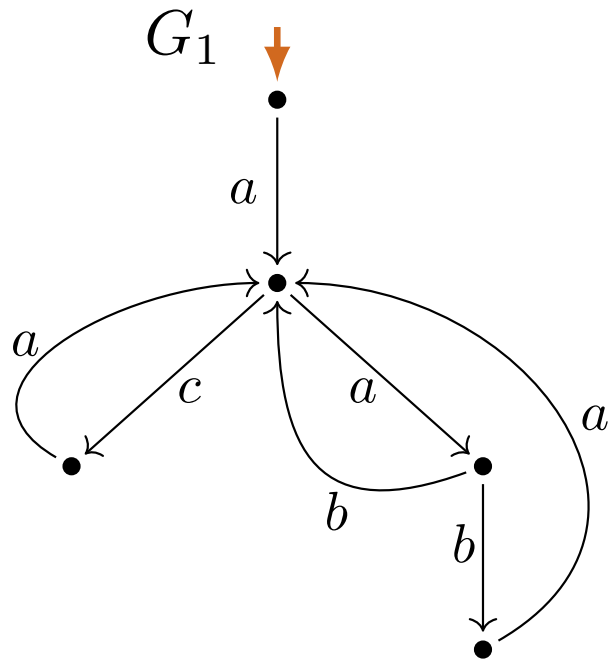

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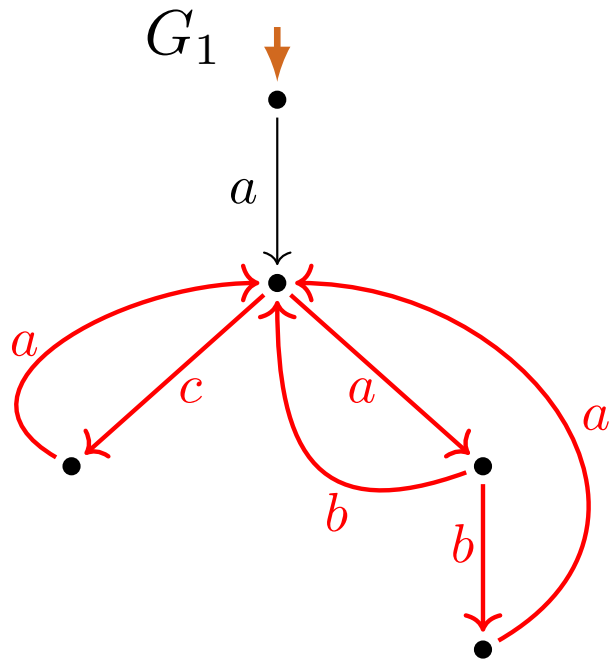

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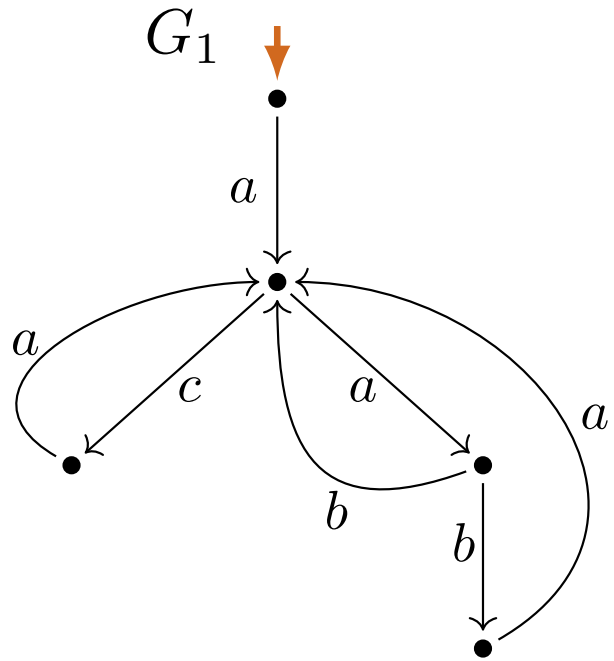

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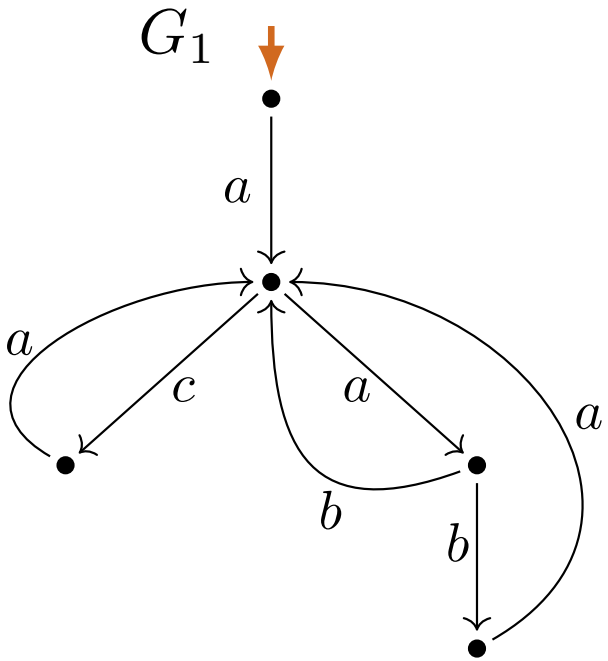

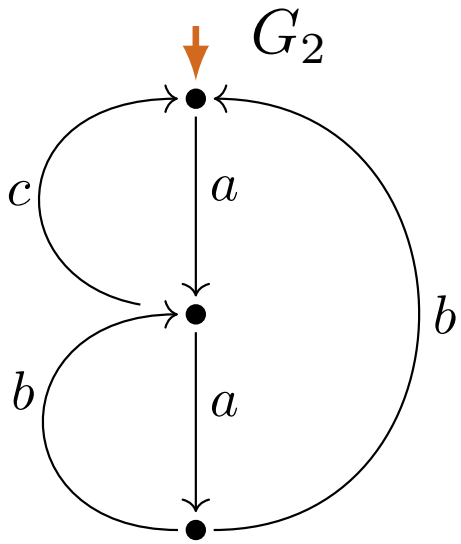

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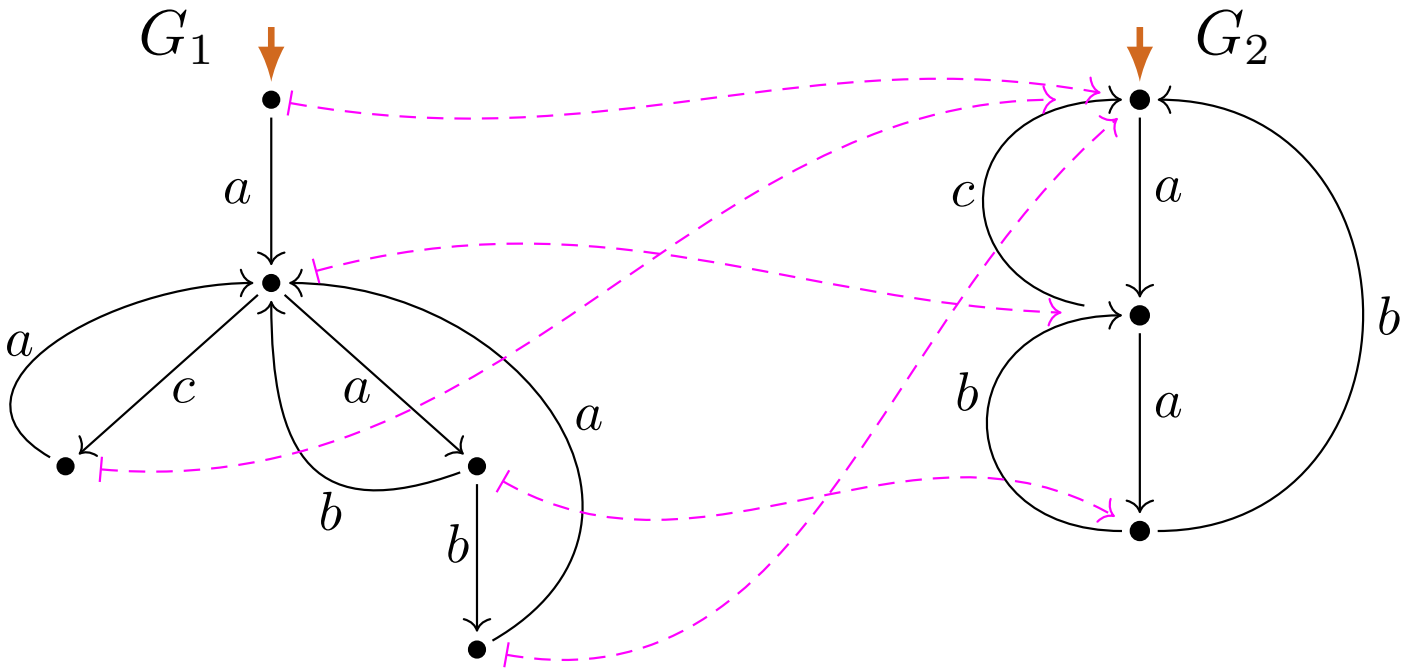

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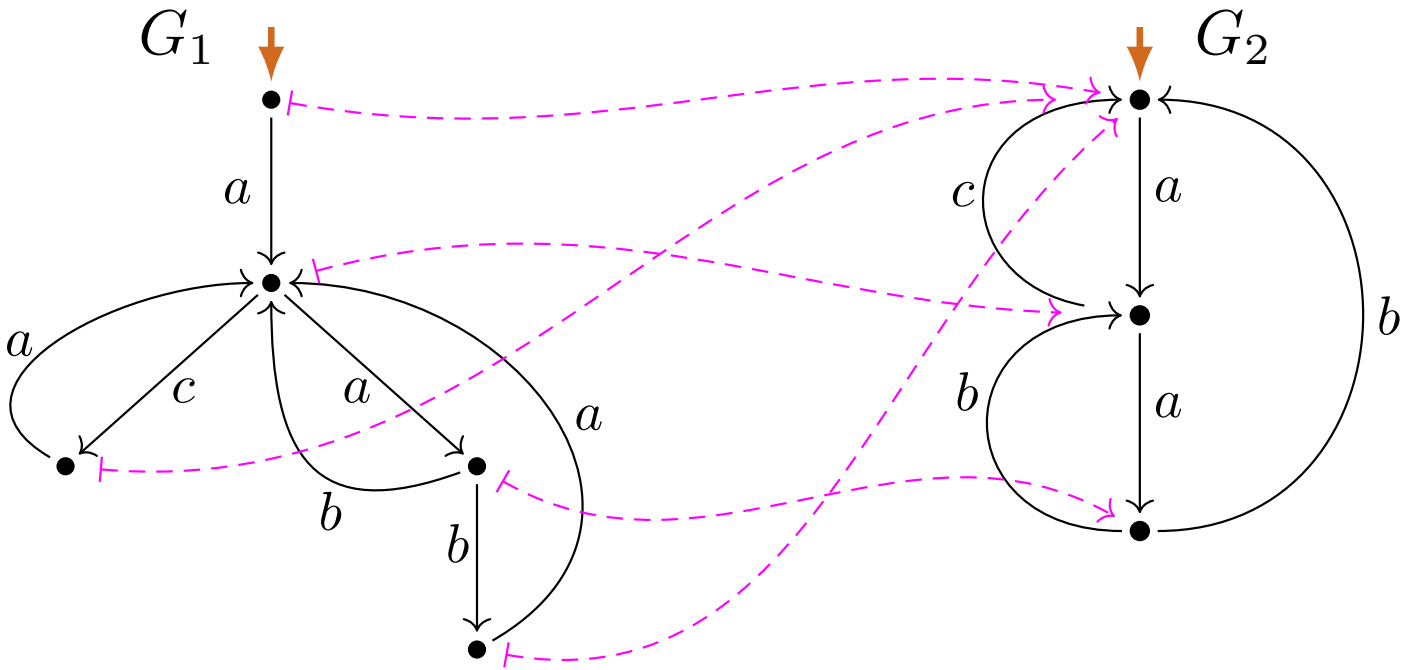

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Process interpretation P (formally)

Definition (Transition system specification $\mathcal{T}$)

$$\frac{}{a \xrightarrow{a} 1} \qquad \frac{e_i \xrightarrow{a} e_i'}{e_1 + e_2 \xrightarrow{a} e_i'} \ (i \in \{1, 2\})$$

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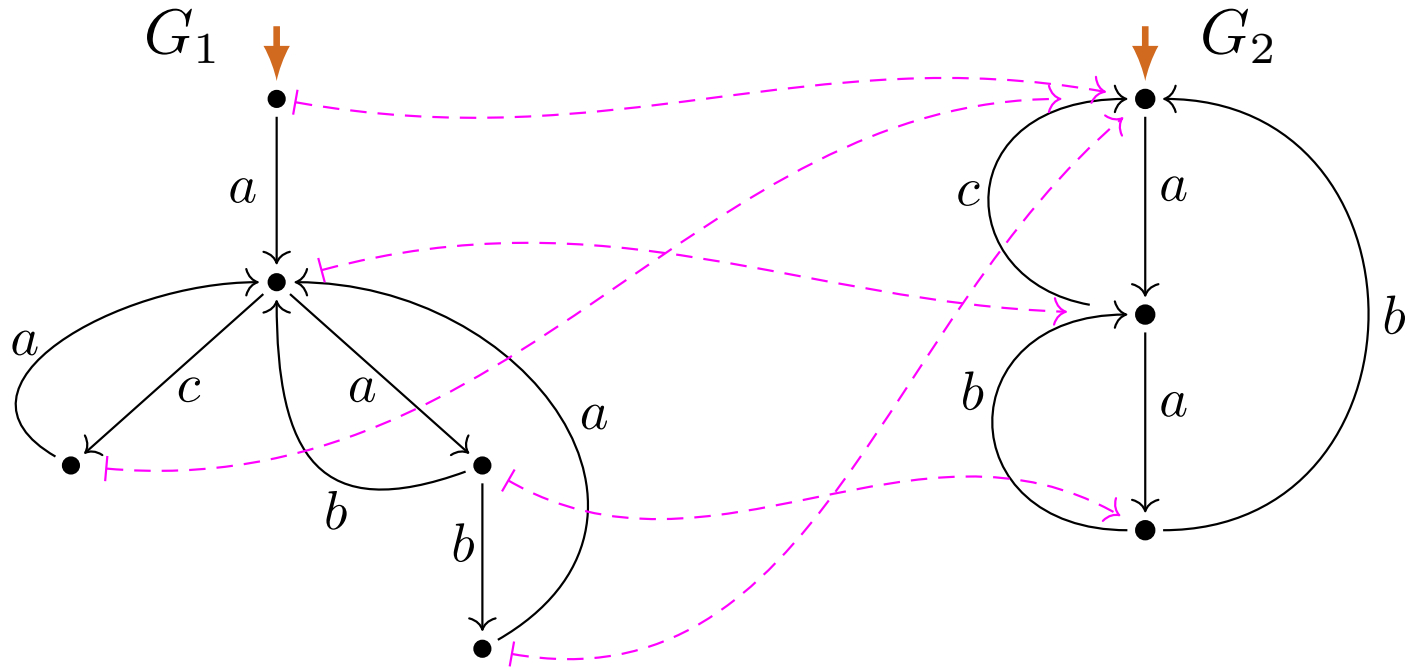
Definition

The process (graph) interpretation $P(e)$ of a regular expression e:

$P(e)$:= labeled transition graph generated by e by derivations in $\mathcal{T}$.

P-expressibility and $[\![\cdot]\!]_P$-expressibility (example, informally)



$$P\Big(\overbrace{a \cdot ((c \cdot a + a \cdot (b + b \cdot a)^*) \cdot 0)}^{f}\Big)$$

$G_1 \in [\![f]\!]_P$ $\qquad$ $G_2 \in [\![f]\!]_P$

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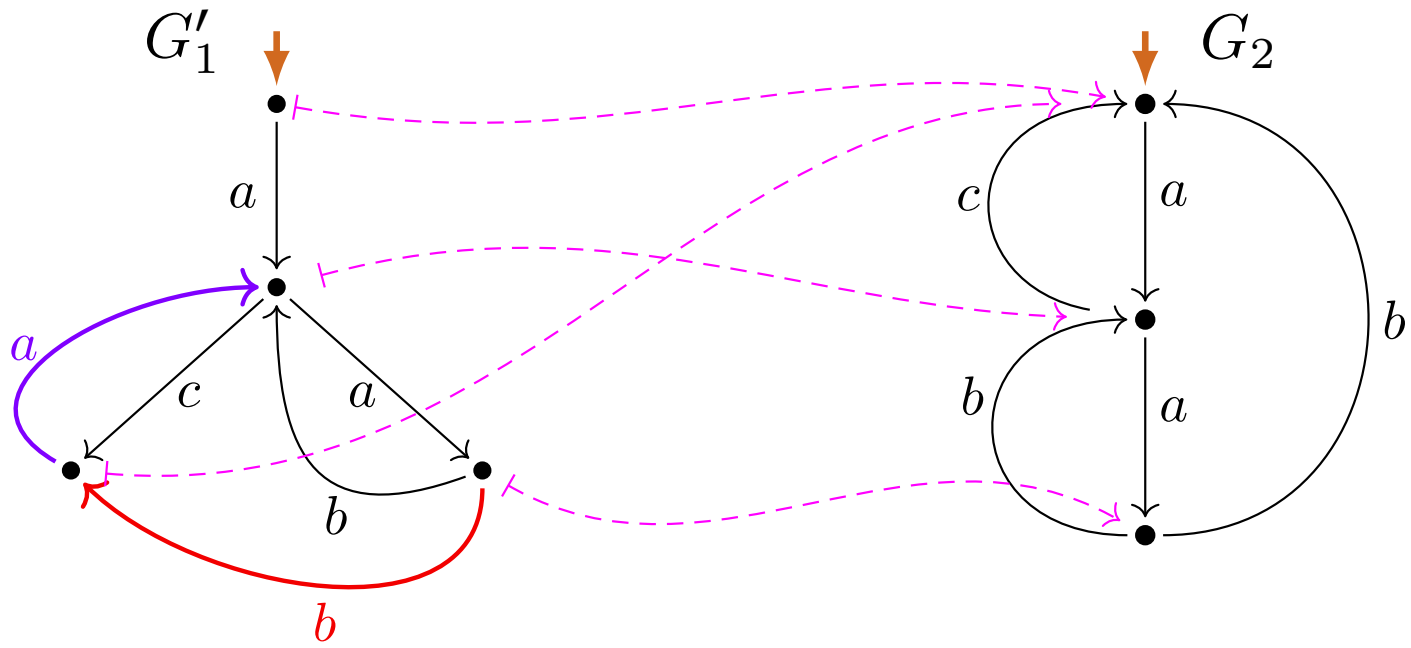

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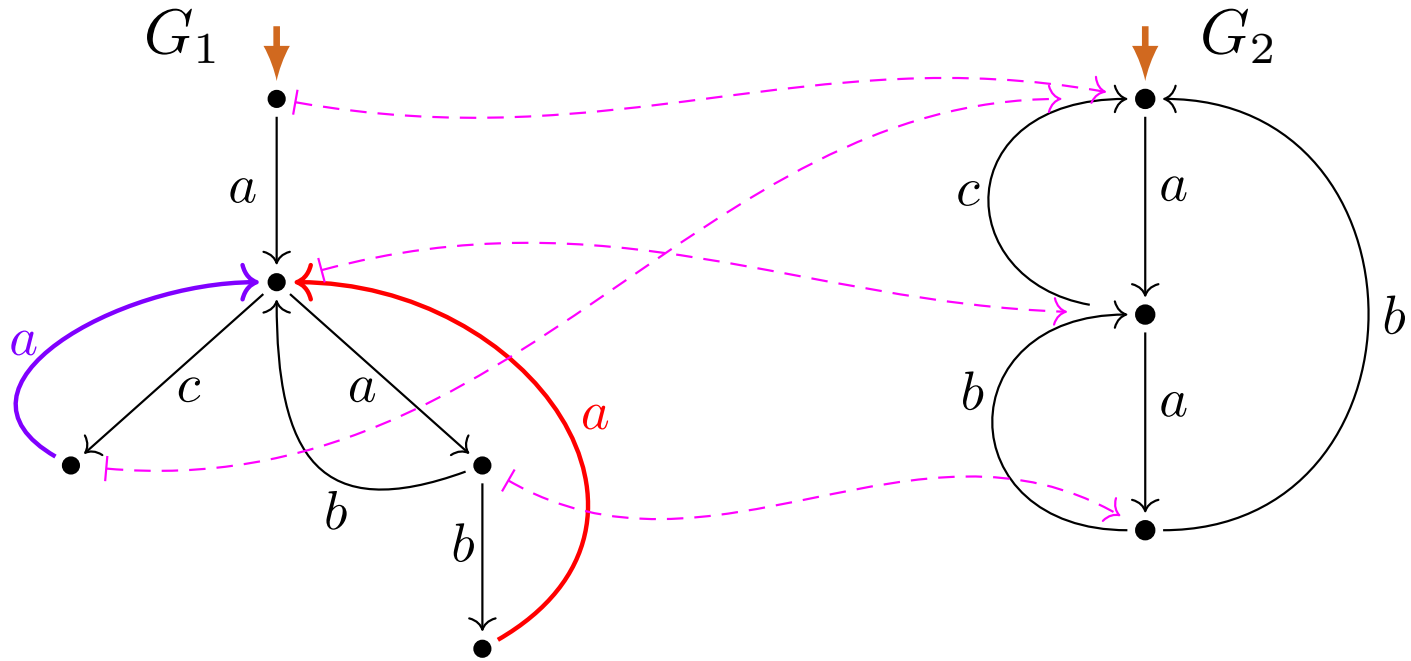

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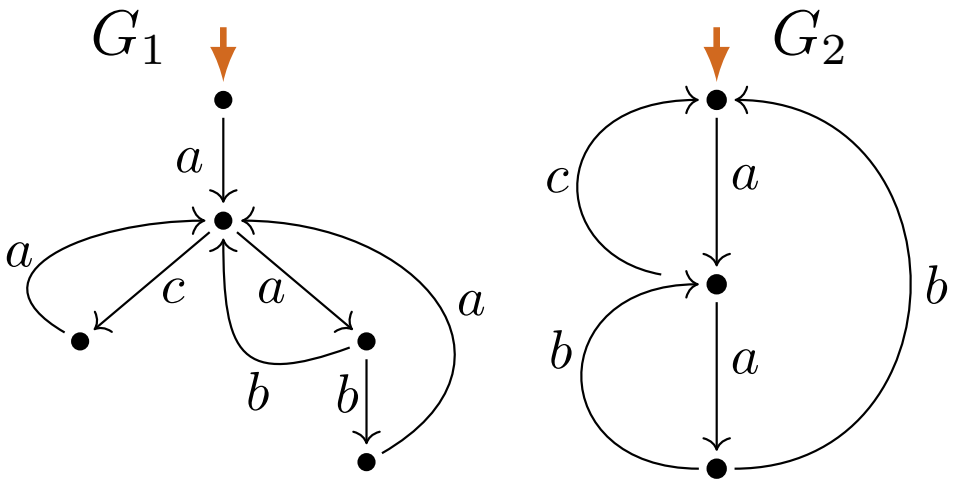
$G_1 \in [\![f]\!]_P$ $\qquad$ $G_2 \in [\![f]\!]_P$

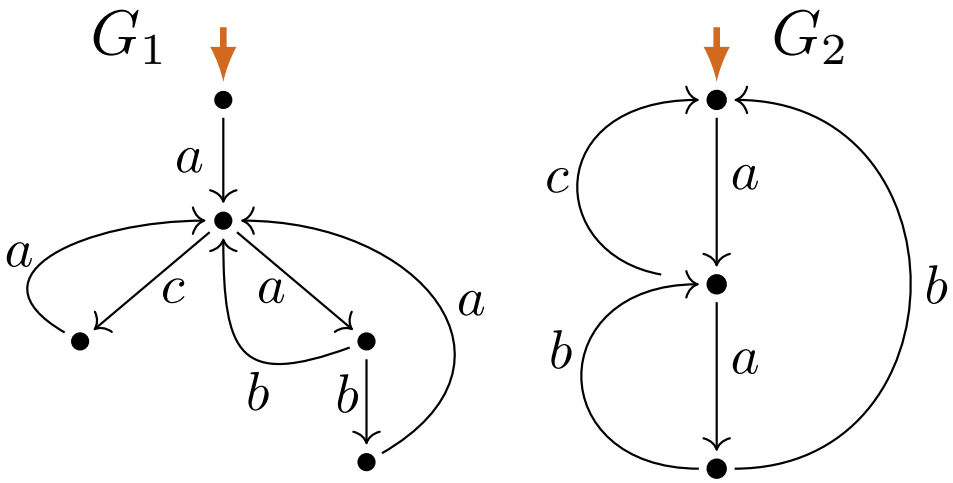
P-expressibility and $[\![\cdot]\!]_P$-expressibility (examples)



P-expressible

$[\![\cdot]\!]_P$-expressible $[\![\cdot]\!]_P$-expressible

P-expressibility and $[\![\cdot]\!]_P$-expressibility (examples)



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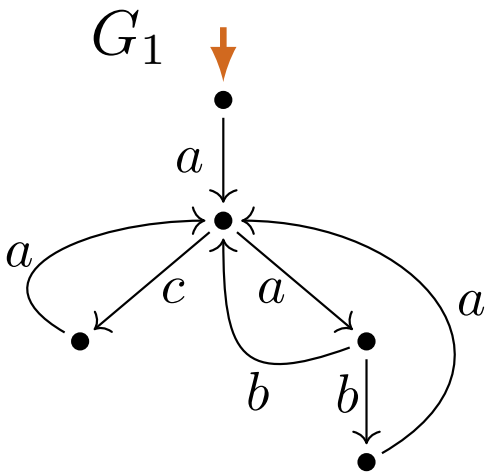

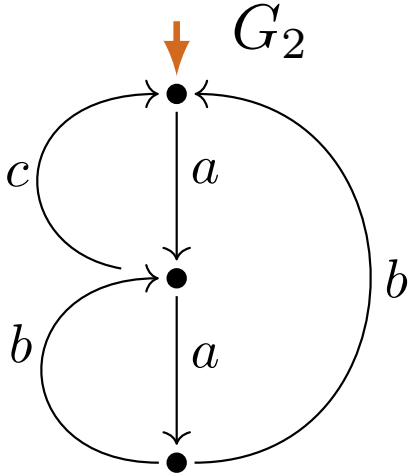

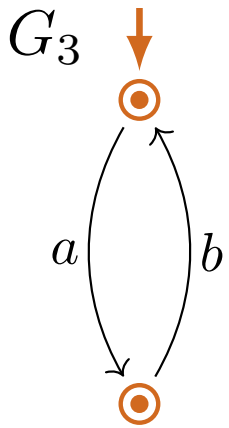

P-expressible

$[\![\cdot]\!]_P$-expressible

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$[\![\cdot]\!]_P$-expressible

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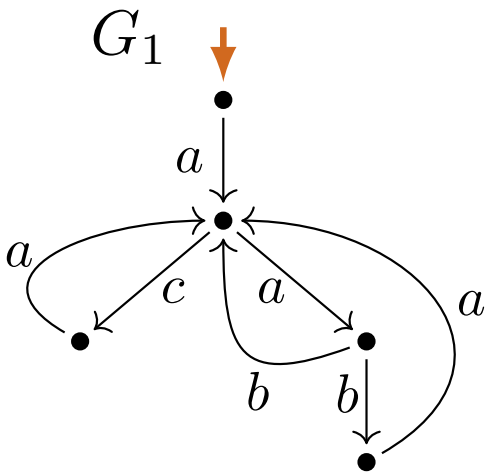

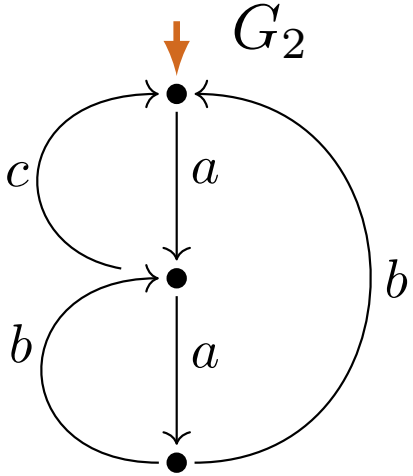

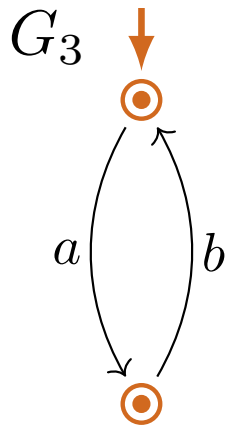

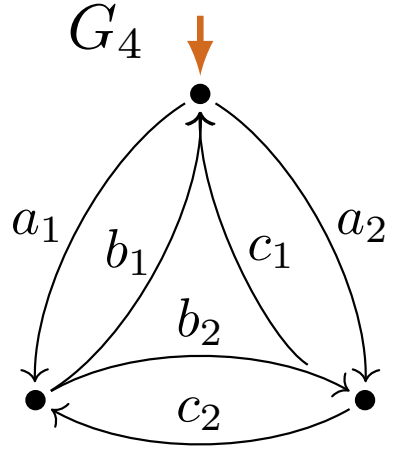

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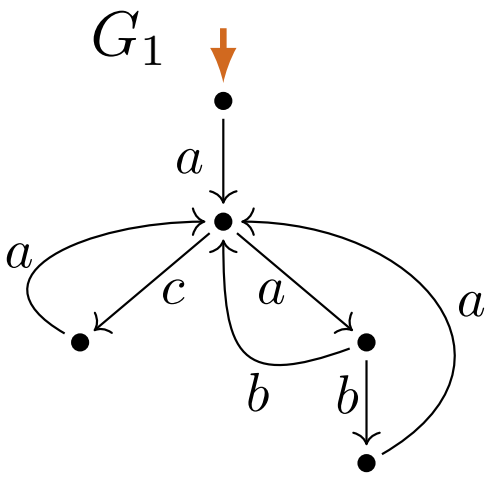
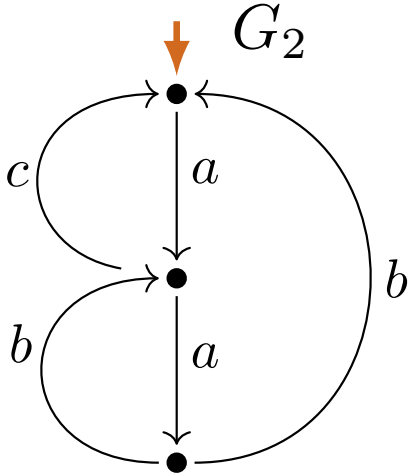
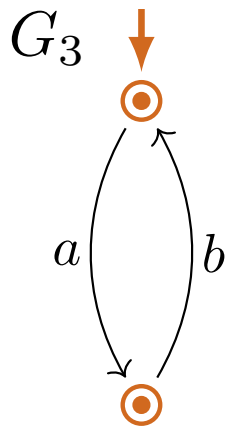
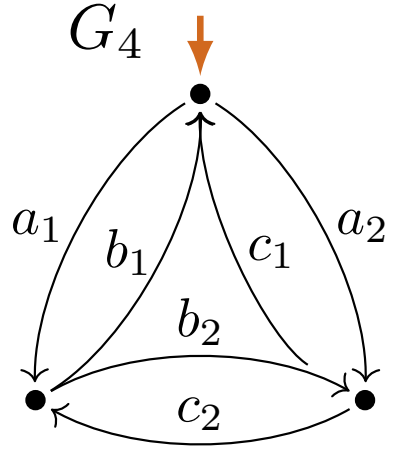








P-expressible

$[\![\cdot]\!]_P$-expressible

?

$[\![\cdot]\!]_P$-expressible

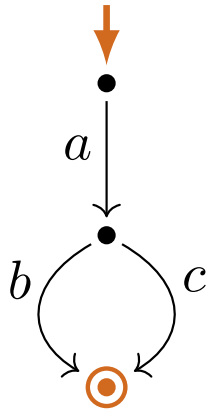
not P-expressible

not $[\![\cdot]\!]_P$-expressible

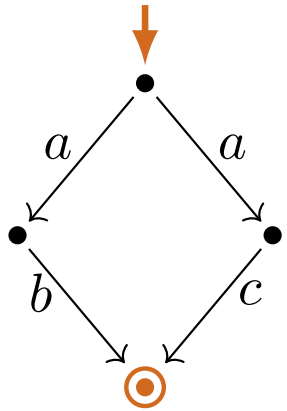
Q2: How can P-expressibility and $[\![\cdot]\!]_P$-expressibility be characterized?

Process semantics equality $=_{[\![\cdot]\!]_P}$

- Fewer identities hold for $=_{[\![\cdot]\!]_P}$ than for $=_{[\![\cdot]\!]_L}$: .



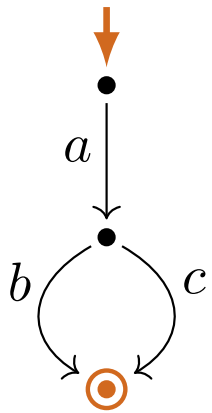
$P(a \cdot (b + c))$



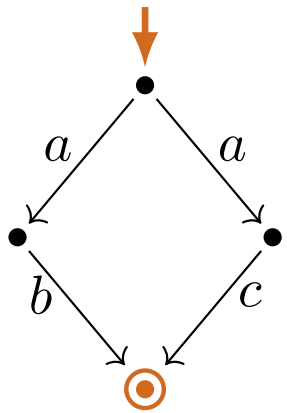
$P(a \cdot b + a \cdot c)$

Process semantics equality $=_{[\![\cdot]\!]_P}$

- Fewer identities hold for $=_{[\![\cdot]\!]_P}$ than for $=_{[\![\cdot]\!]_L}$: .



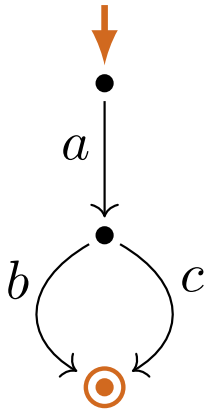
$\not\leftrightarrow$



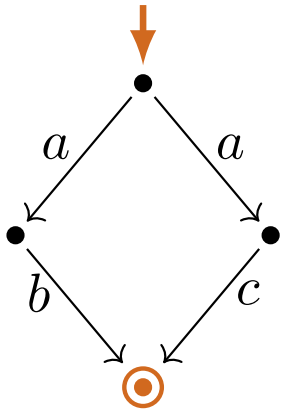
$P(a \cdot (b + c))$ $\not\leftrightarrow$ $P(a \cdot b + a \cdot c)$

Process semantics equality $=_{[\![\cdot]\!]_P}$

- Fewer identities hold for $=_{[\![\cdot]\!]_P}$ than for $=_{[\![\cdot]\!]_L}$: .



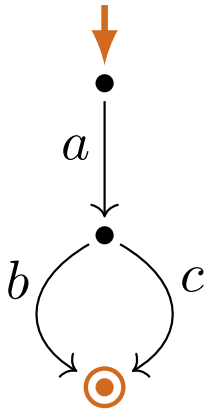
$\neq_{[\![\cdot]\!]_P}$



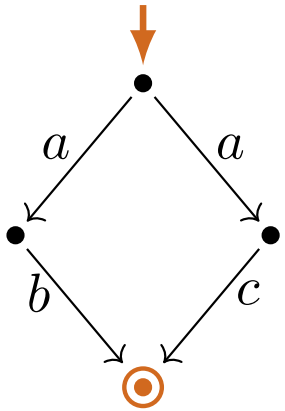
$a \cdot (b + c)$ $\neq_{[\![\cdot]\!]_P}$ $a \cdot b + a \cdot c$

Process semantics equality $=_{[\![\cdot]\!]_P}$

- Fewer identities hold for $=_{[\![\cdot]\!]_P}$ than for $=_{[\![\cdot]\!]_L}$: .



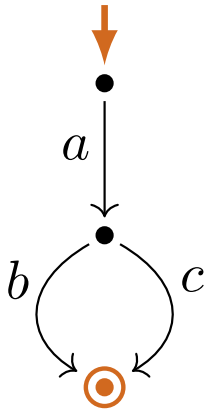
$=_{[\![\cdot]\!]_L}$



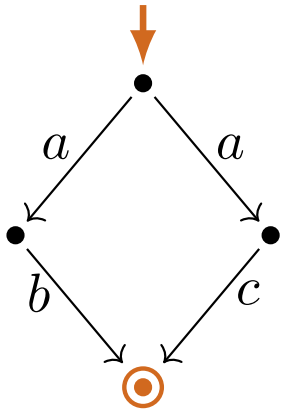
$a \cdot (b + c)$ $=_{[\![\cdot]\!]_L}$ $a \cdot b + a \cdot c$

Process semantics equality $=_{[\![\cdot]\!]_P}$

- Fewer identities hold for $=_{[\![\cdot]\!]_P}$ than for $=_{[\![\cdot]\!]_L}$: $=_{[\![\cdot]\!]_P} \subsetneq =_{[\![\cdot]\!]_L}$.



$\neq_{[\![\cdot]\!]_P}$



$a \cdot (b + c)$ $\neq_{[\![\cdot]\!]_P}$ $a \cdot b + a \cdot c$

Milner's proof system Mil

Axioms:

(A1)	$e + (f + g) = (e + f) + g$	(A7)	$e = 1 \cdot e$
(A2)	$e + 0 = e$	(A8)	$e = e \cdot 1$
(A3)	$e + f = f + e$	(A9)	$0 = 0 \cdot e$
(A4)	$e + e = e$	(A10)	$e^* = 1 + e \cdot e^*$
(A5)	$e \cdot (f \cdot g) = (e \cdot f) \cdot g$	(A11)	$e^* = (1 + e)^*$
(A6)	$(e + f) \cdot g = e \cdot g + f \cdot g$		

But: $e \cdot (f + g) \neq e \cdot f + e \cdot g$

But: $e \cdot 0 \neq 0$

Inference rules: rules of equational logic *plus*

$$\frac{e = f \cdot e + g}{e = f^* \cdot g}\ \mathsf{RSP}^*$$ (if f does not terminate immediately)

Milner's Question (Q1)

Is Mil complete with respect to $=_{[\![\cdot]\!]_P}$? (Does $e =_{[\![\cdot]\!]_P} f \Longrightarrow e =_{\mathsf{Mil}} f$ hold?)

Milner's questions

(Q1) Complete axiomatization:

Is the proof system Mil *complete for* $=_{[\![\cdot]\!]_P}$*?*

(Q2) $[\![\cdot]\!]_P$-Expressibility:

What structural property characterizes process graphs that are $[\![\cdot]\!]_P$*-expressible ?*

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(Q1) Complete axiomatization:

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- is decidable (Baeten/Corradini/G, 2007)

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- is decidable (Baeten/Corradini/G, 2007)
- partial new answer (G/Fokkink, 2020):
  - bisimulation collapse has loop existence & elimination property (LEE) if expressible by under-star-1-free regular expression

Milner's questions

(Q1) Complete axiomatization:

Is the proof system Mil *complete for* $=_{[\![\cdot]\!]_P}$*?*

- series of partial completeness results for:
  - exitless iterations (Fokkink, 1998)
  - with a stronger fixed-point rule (G, 2006)
  - under-star 1-free, and without 0 (Corradini/de Nicola/Labella, 2004)
  - with 0 but under-star-1-free (G/Fokkink, 2020)

(Q2) $[\![\cdot]\!]_P$-Expressibility:

What structural property characterizes process graphs that are $[\![\cdot]\!]_P$*-expressible ?*

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Milner's questions

(Q1) Complete axiomatization:

Is the proof system Mil *complete for* $=_{[\![\cdot]\!]_P}$*?*

- Yes! (G, 2022, proof summary, employing LEE and crystallization)
- series of partial completeness results for:
  - exitless iterations (Fokkink, 1998)
  - with a stronger fixed-point rule (G, 2006)
  - under-star 1-free, and without 0 (Corradini/de Nicola/Labella, 2004)
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Question (Q2) specialized

$(Q2)_0$ *P*-Expressibility and *P*-$(*/\not{1})$-Expressibility:

What structural property characterizes:

- *process graphs that are P-expressible ?*
  (. . . that are in the image of P?)
- *process graphs that are P-expressible by* $(*/\not{1})$ *regular expressions?*
  (. . . that are in the image of $(*/\not{1})$ *expressions under P?)*

Loop Existence and Elimination (LEE)

Loop graphs (interpretations of innermost iterations without 1)

Definition

A process graph is a loop graph if:

(L1) There is an infinite path from the start vertex.

(L2) Every infinite path from the start vertex returns to it.

(L3) Termination is only possible at the start vertex.

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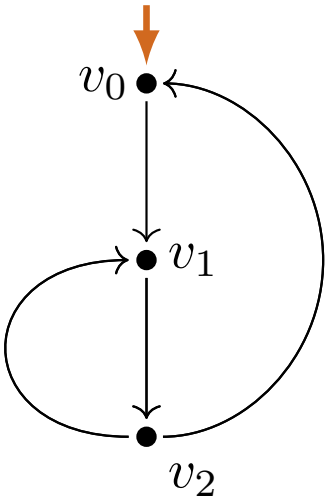

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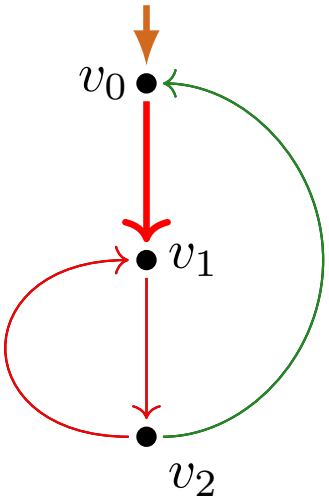

(L1),~~(L2)~~

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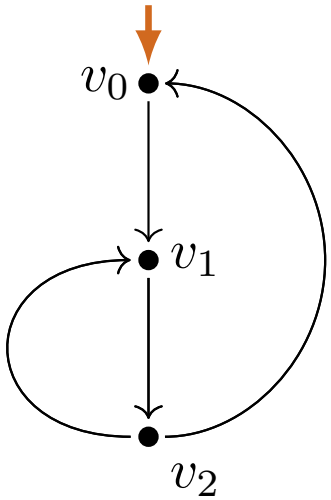

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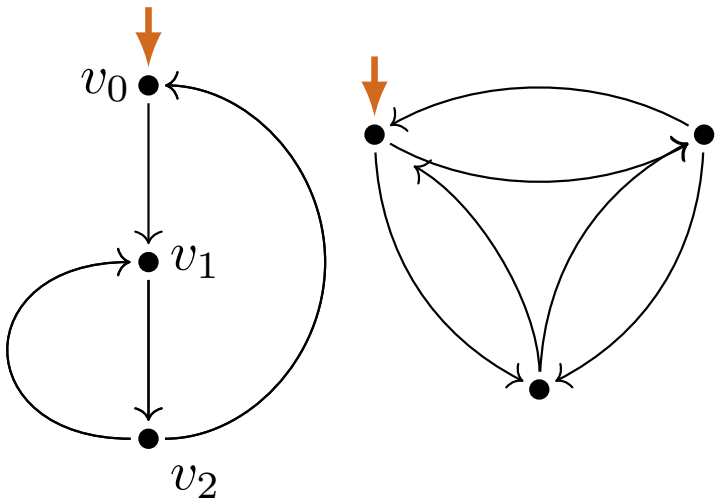

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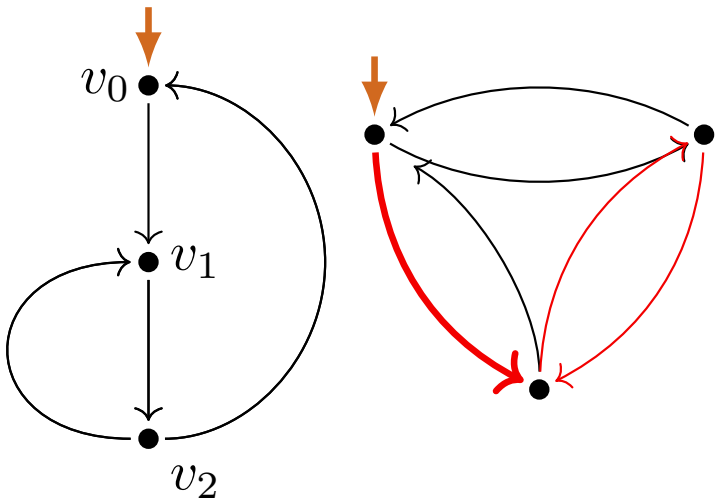

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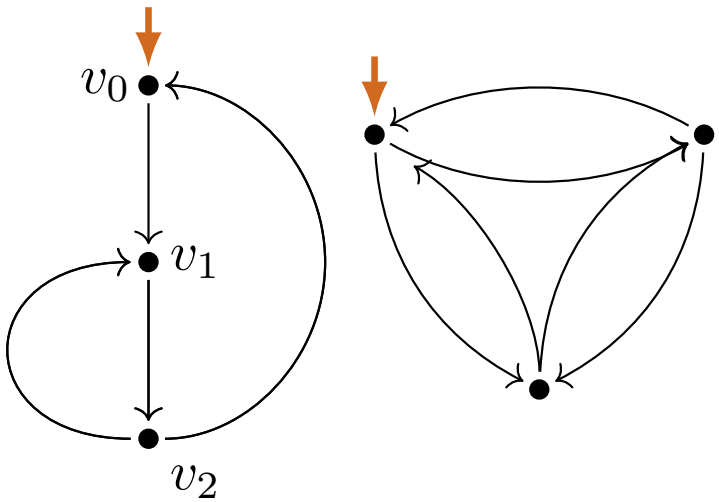

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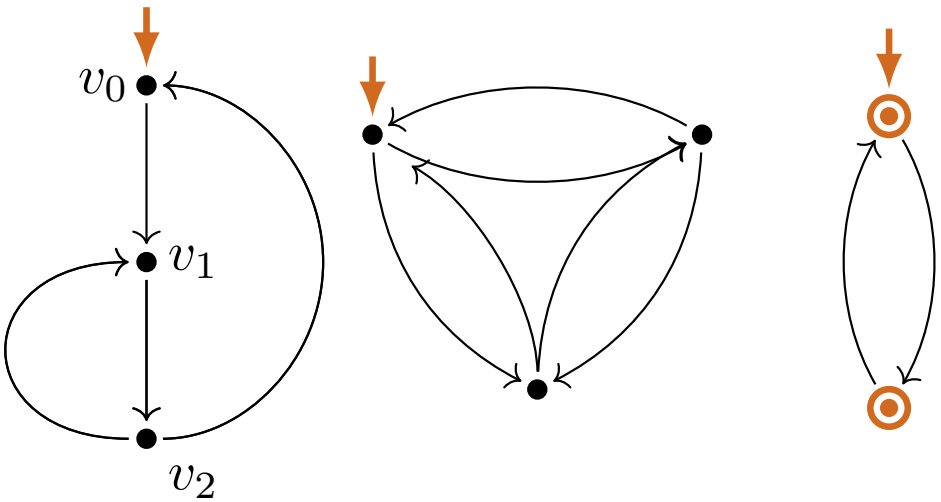

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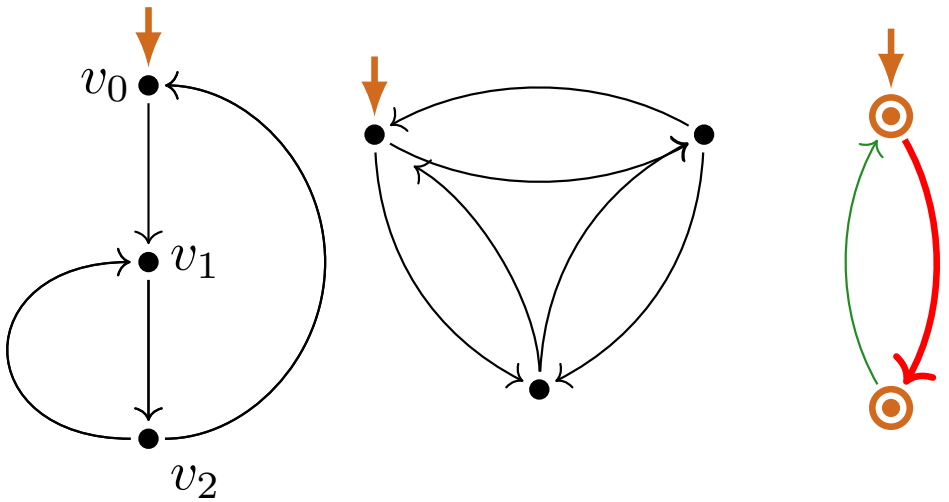

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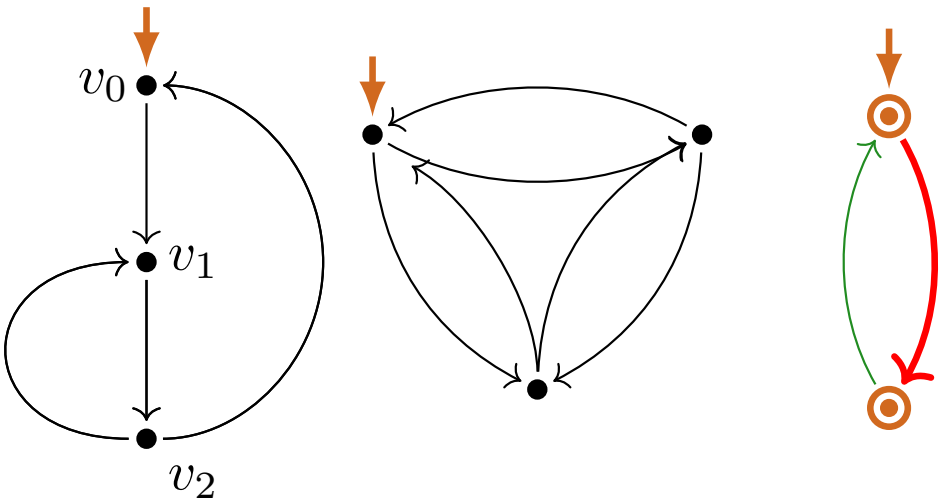

(L1),~~(L2)~~ (L1),(L2),~~(L3)~~

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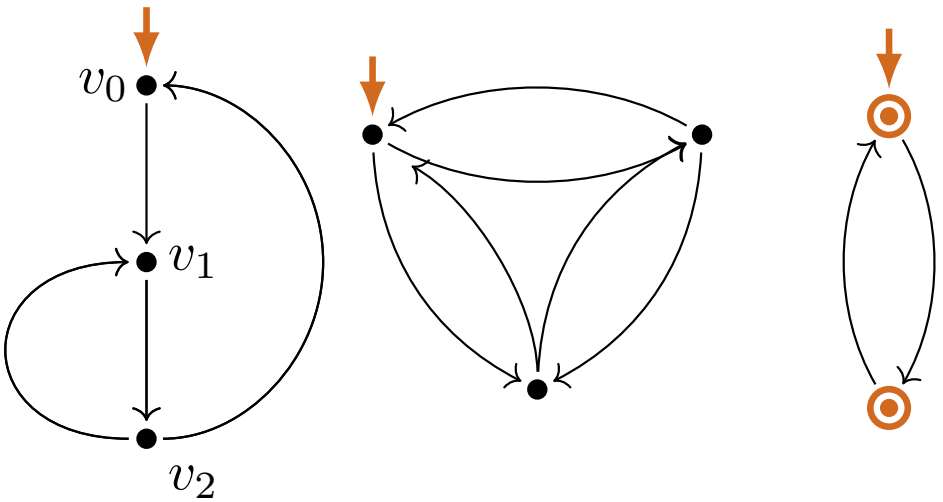

(L1),~~(L2)~~ (L1),(L2),~~(L3)~~

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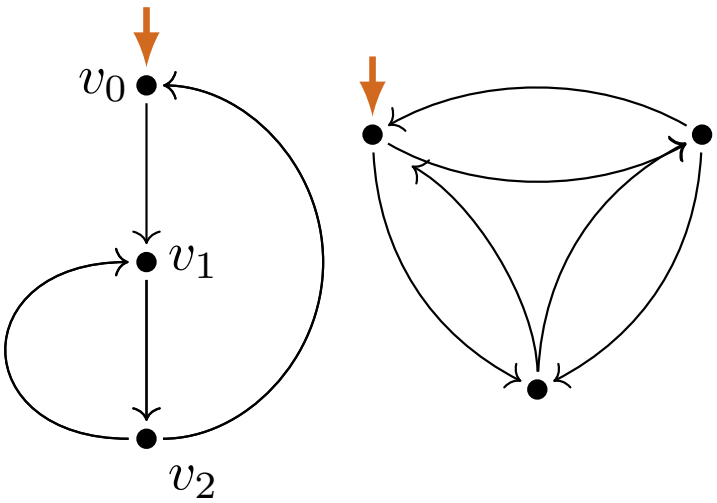
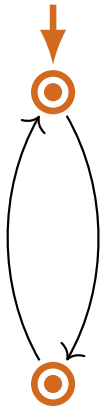
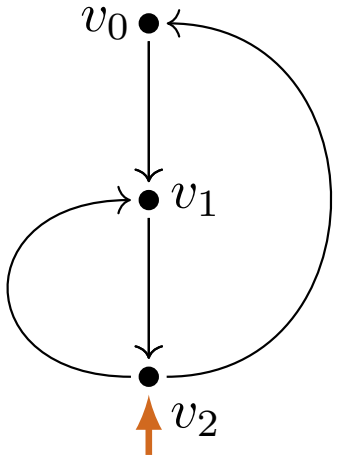


(L1),~~(L2)~~

(L1),(L2),~~(L3)~~





Loop graphs (interpretations of innermost iterations without 1)

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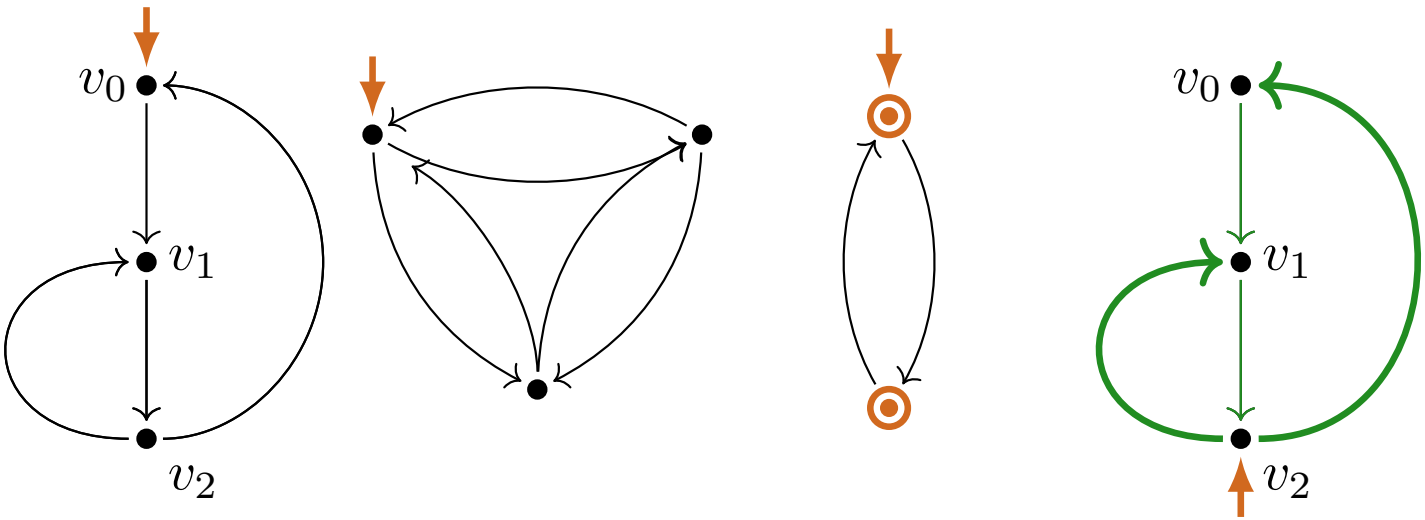

(L1),~~(L2)~~ (L1),(L2),~~(L3)~~

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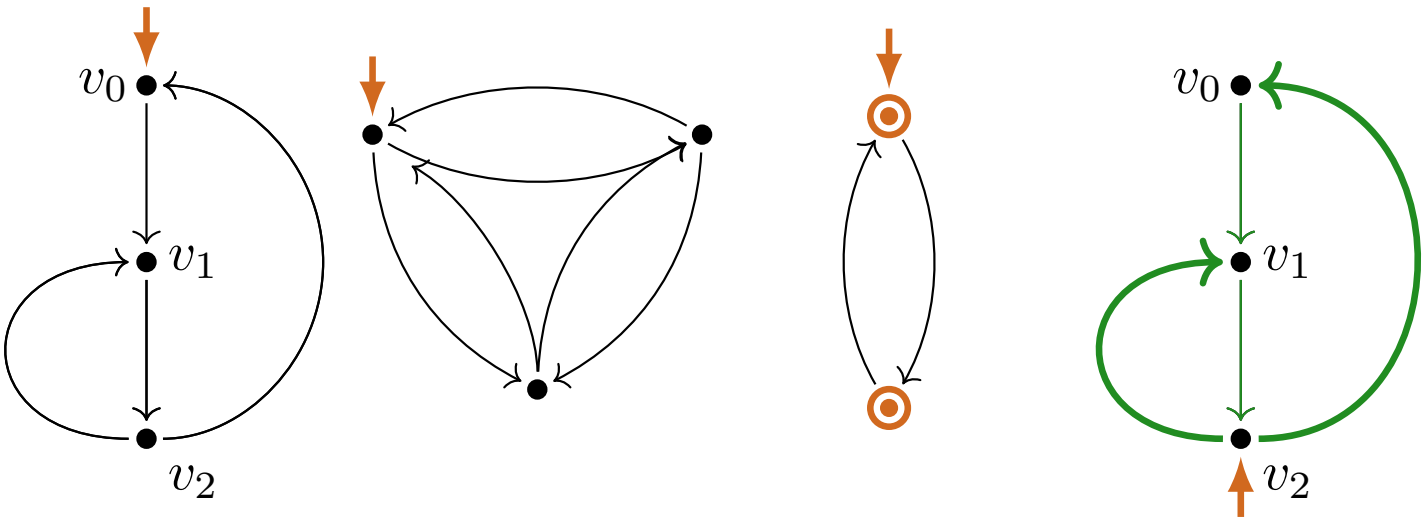

(L1),~~(L2)~~ (L1),(L2),~~(L3)~~ loop chart

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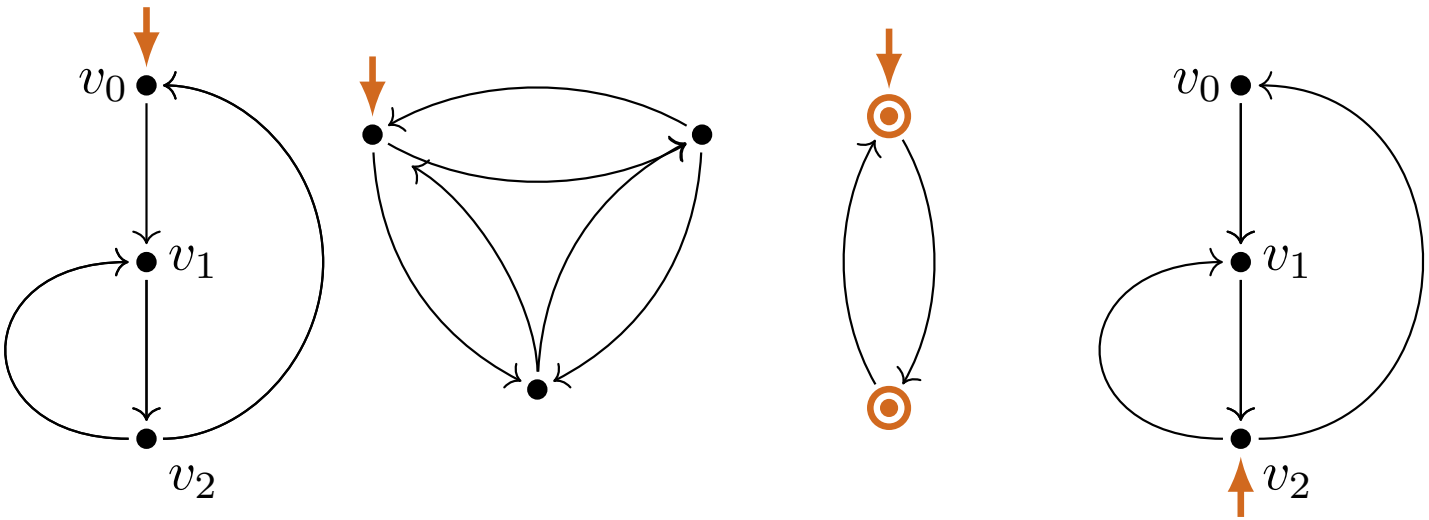

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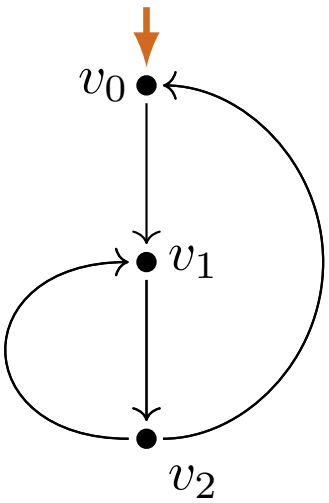
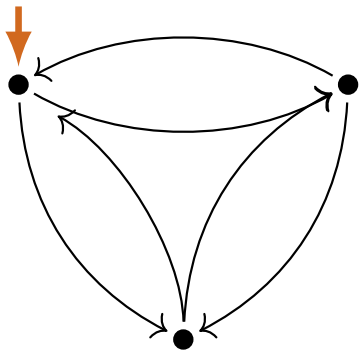
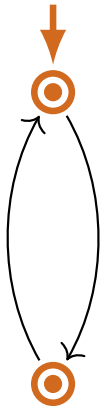
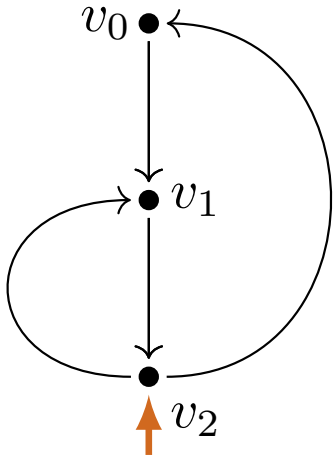


(L1),~~(L2)~~

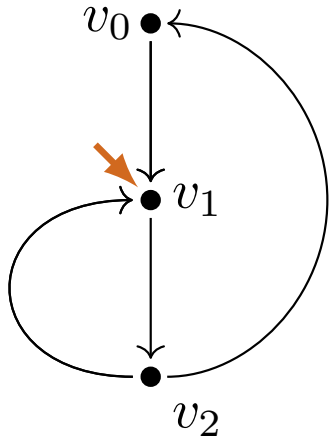




(L1),(L2),~~(L3)~~



loop chart



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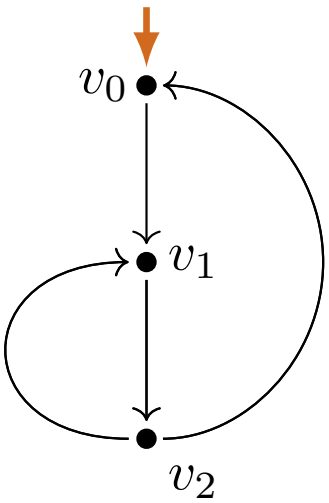

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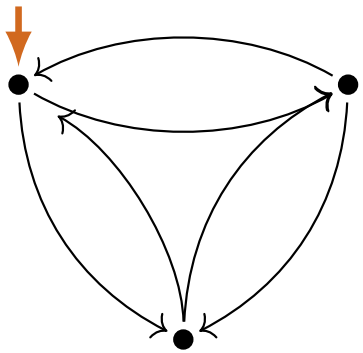

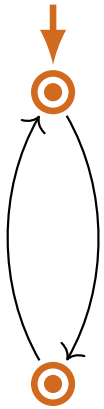

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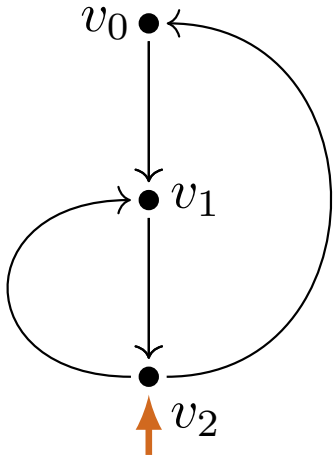
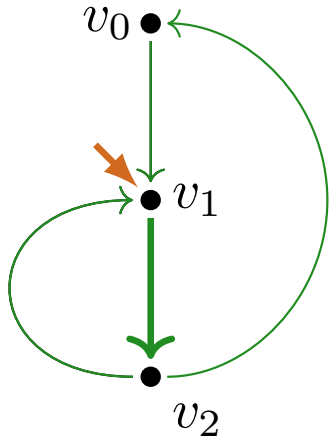


loop chart



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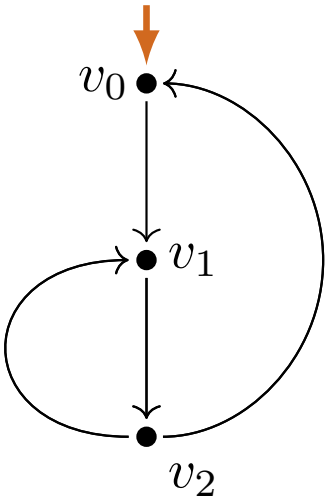

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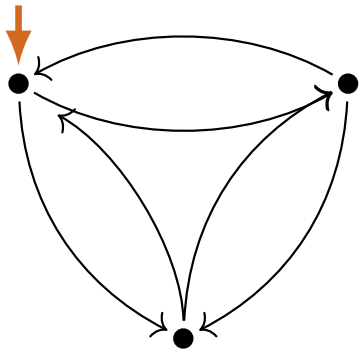

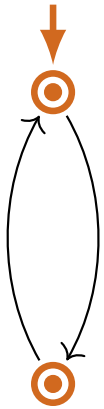

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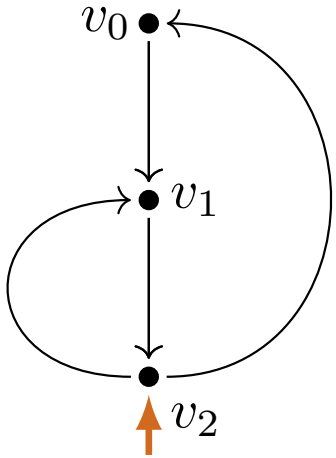
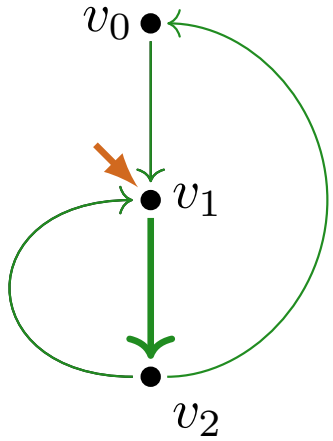


loop chart



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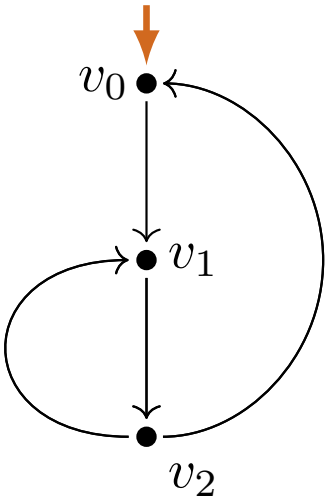

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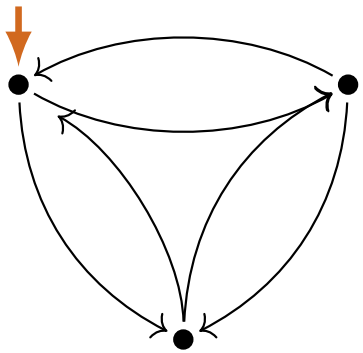

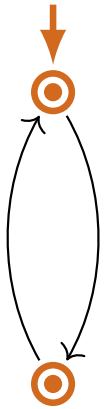

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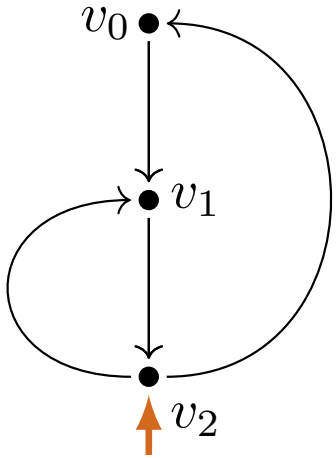

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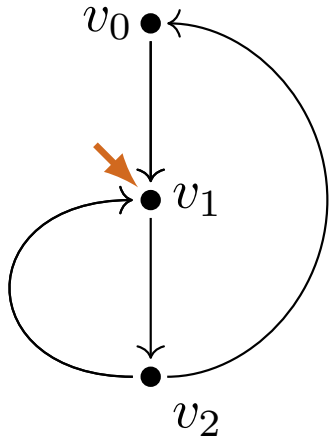

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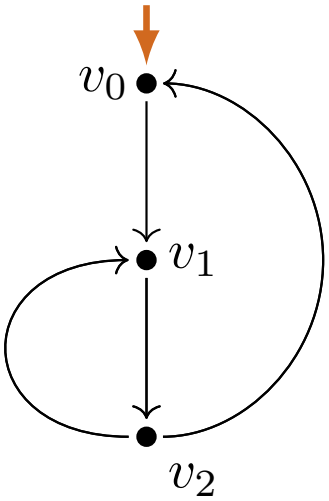

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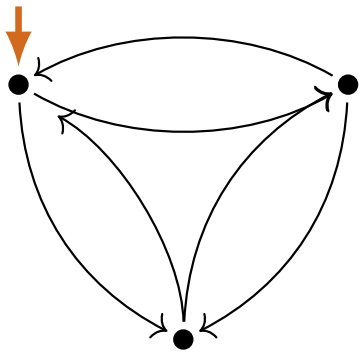

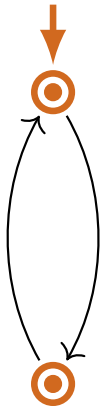

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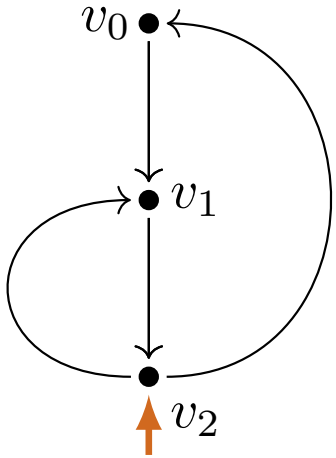
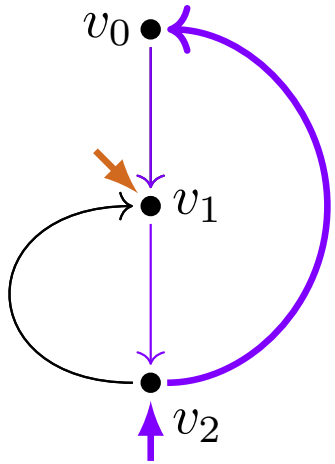
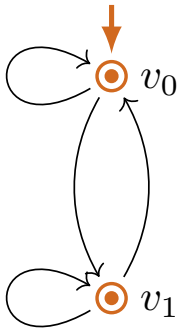


loop chart



loop subchart

Loop elimination



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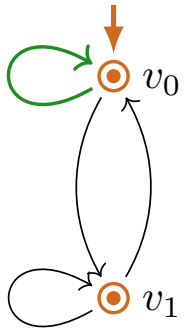

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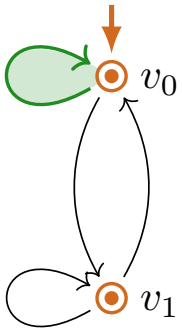

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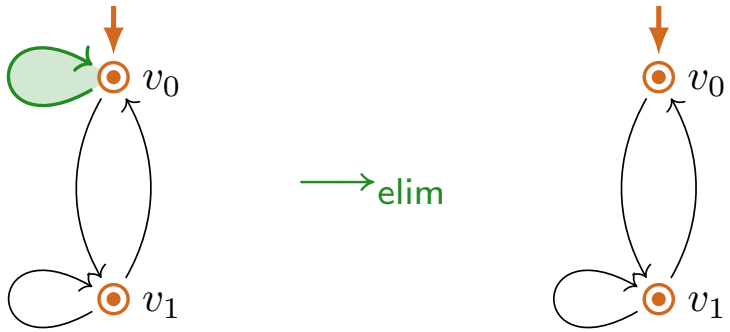

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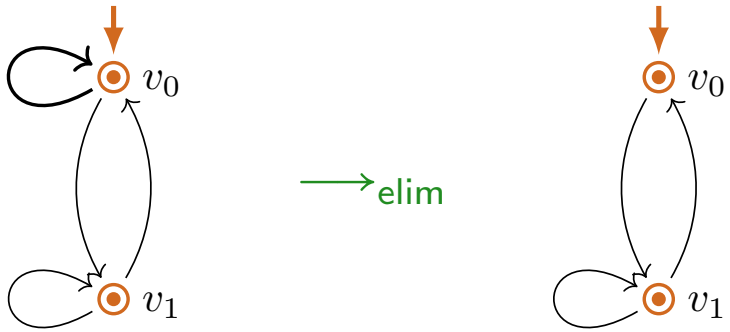

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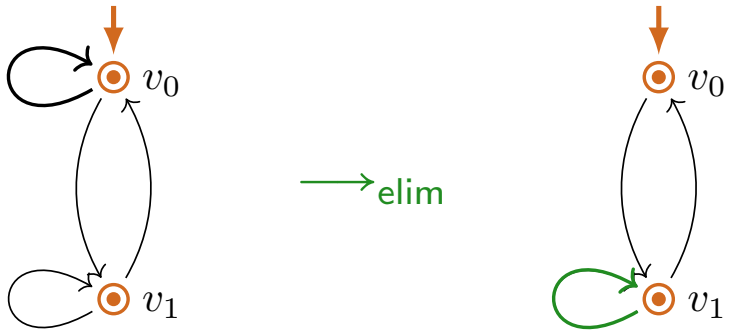

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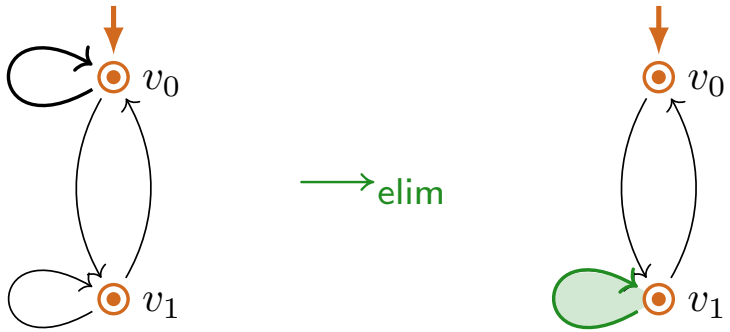

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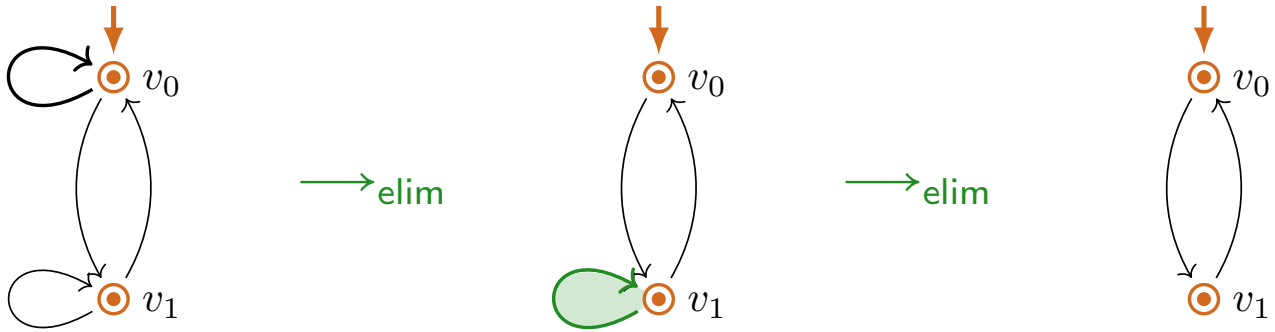

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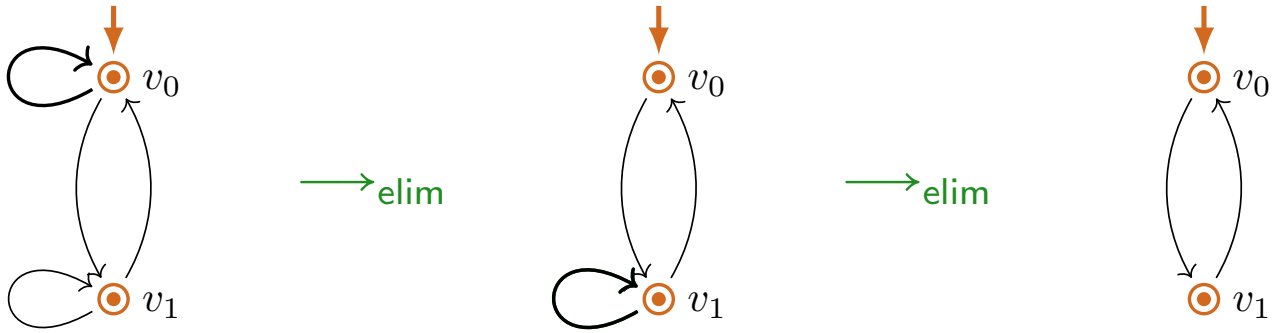

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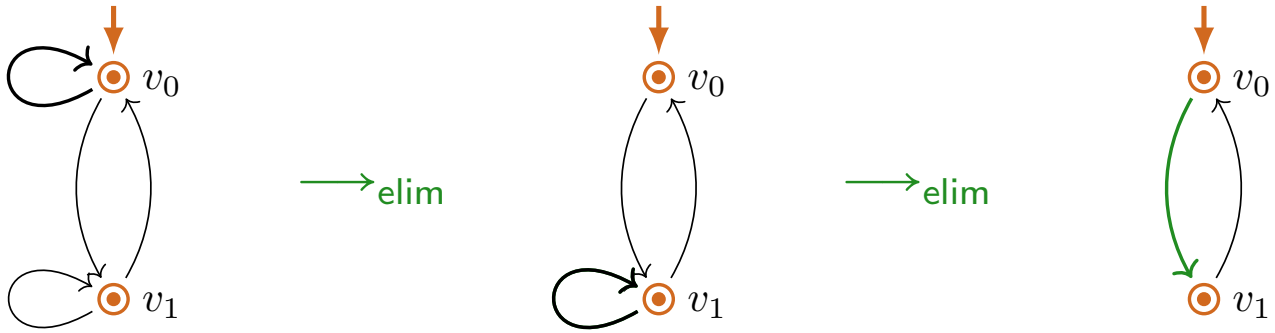

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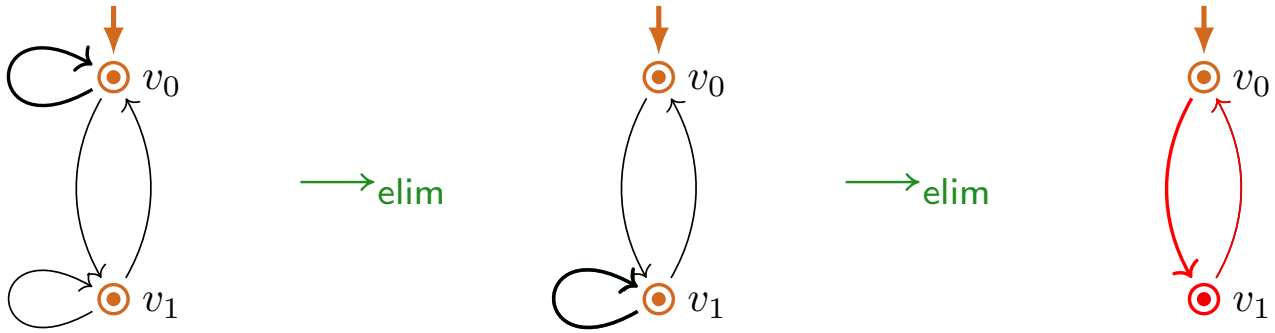

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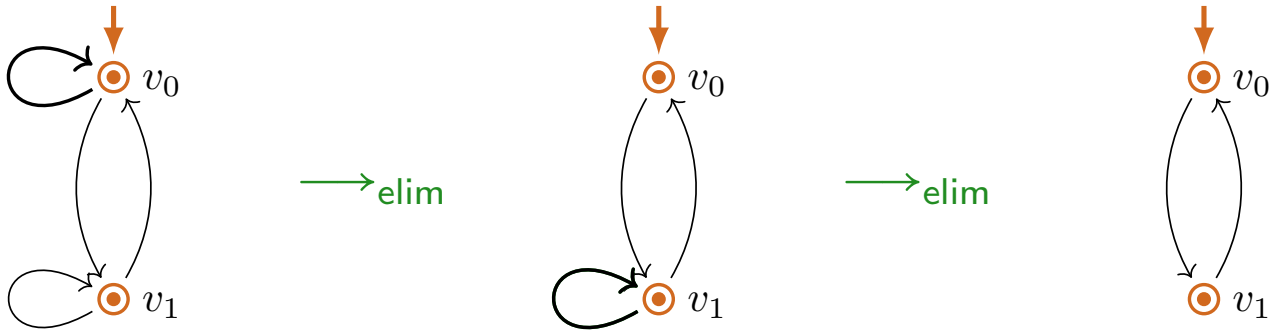

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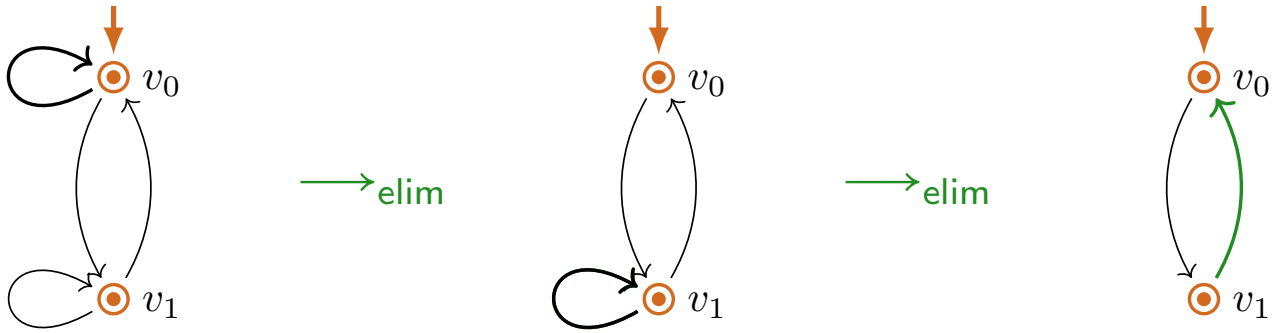

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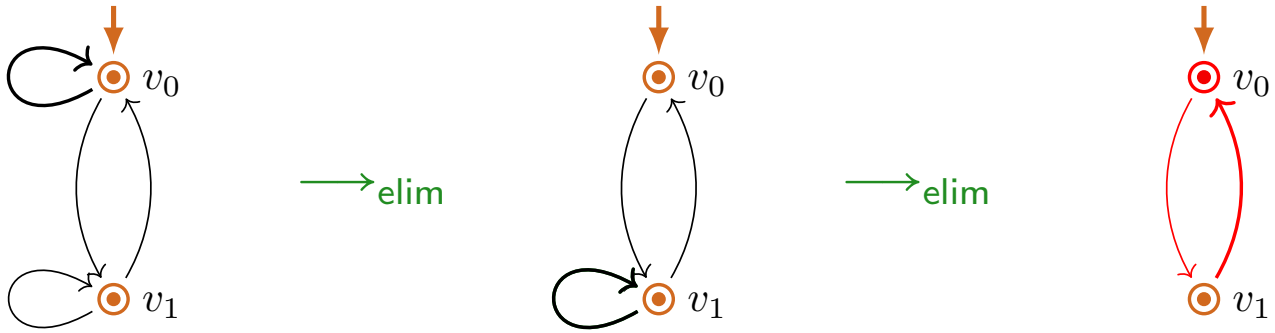

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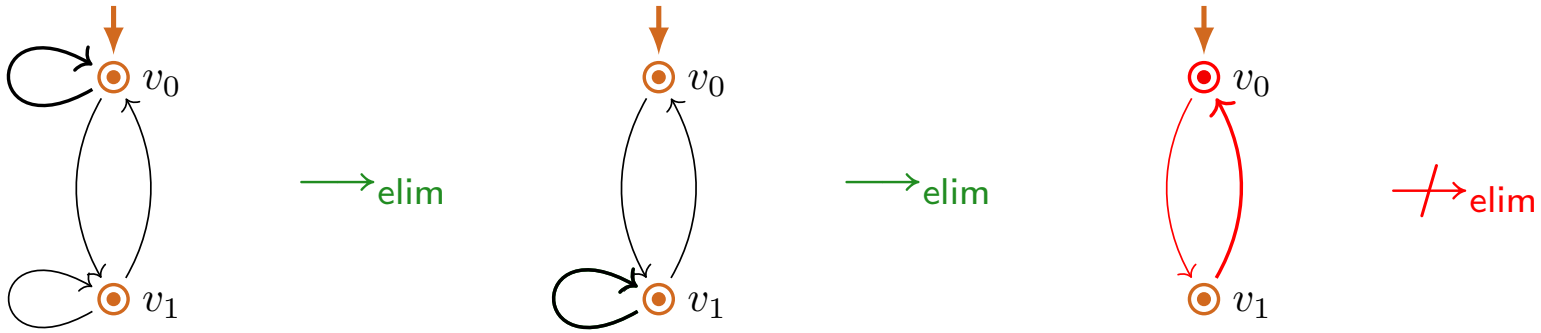

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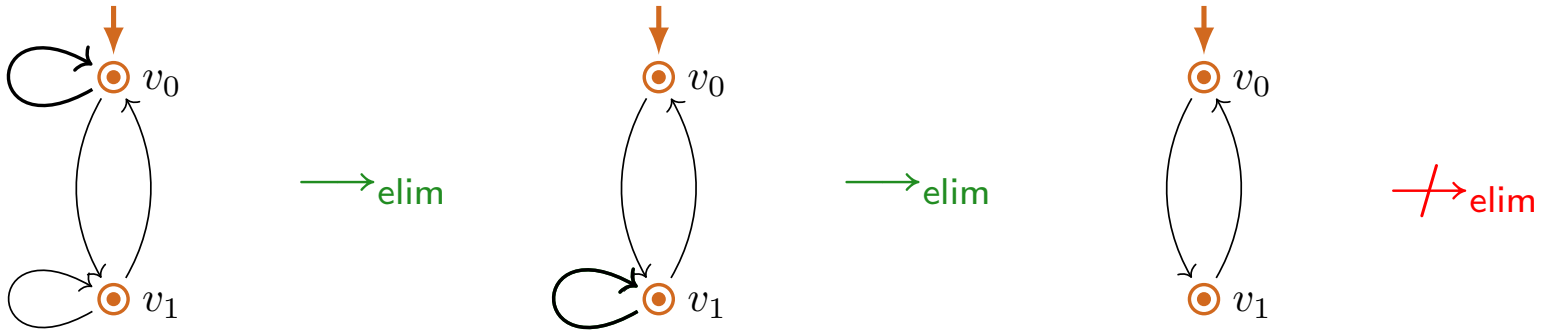

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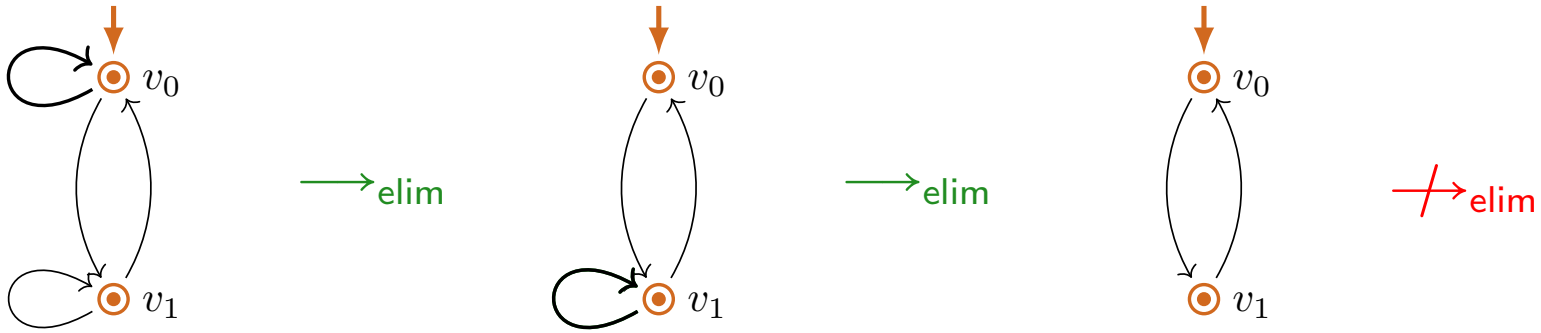
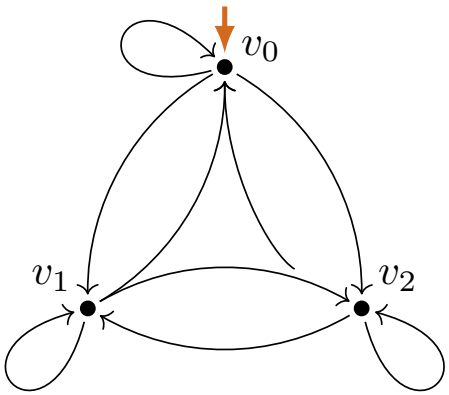
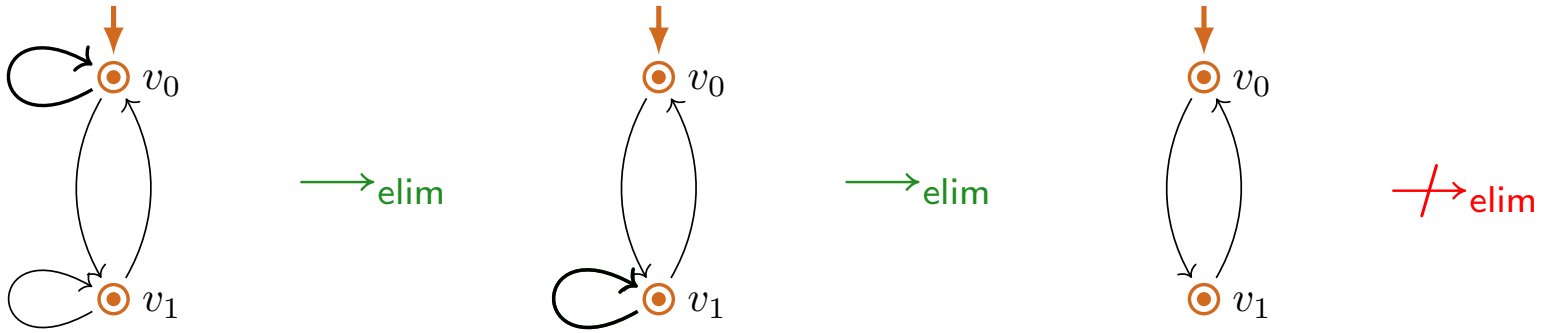
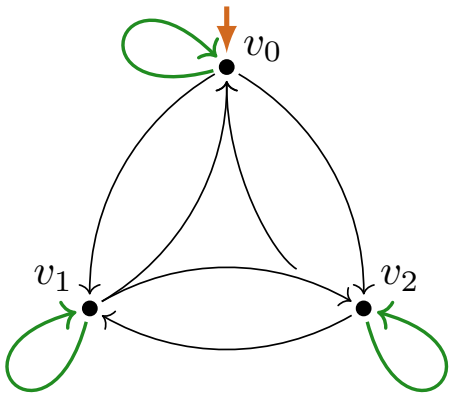


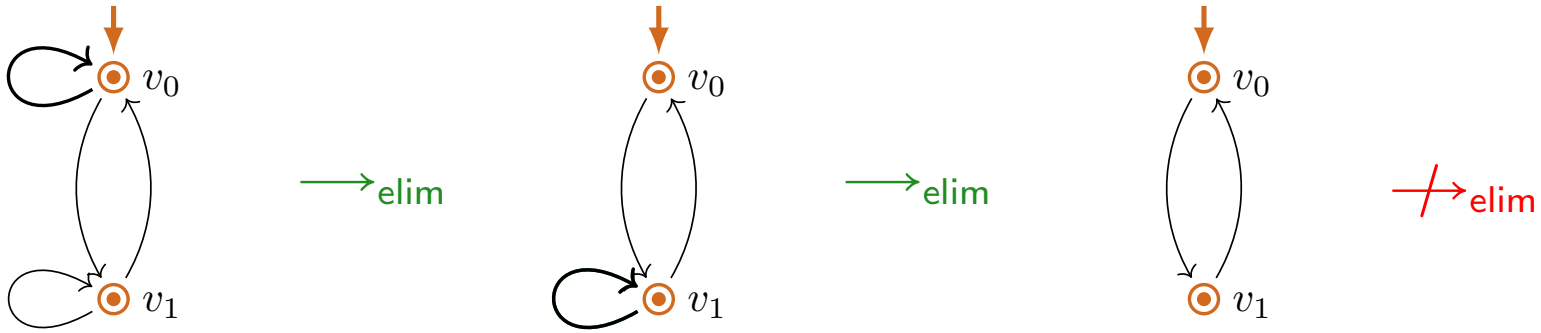


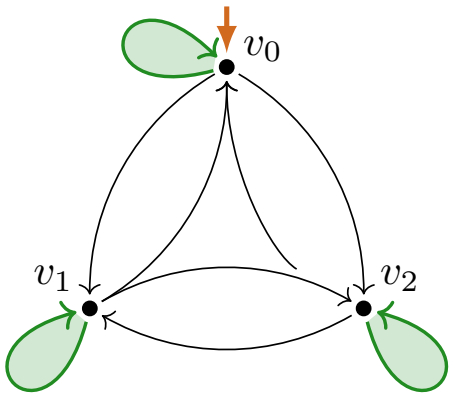
Loop elimination



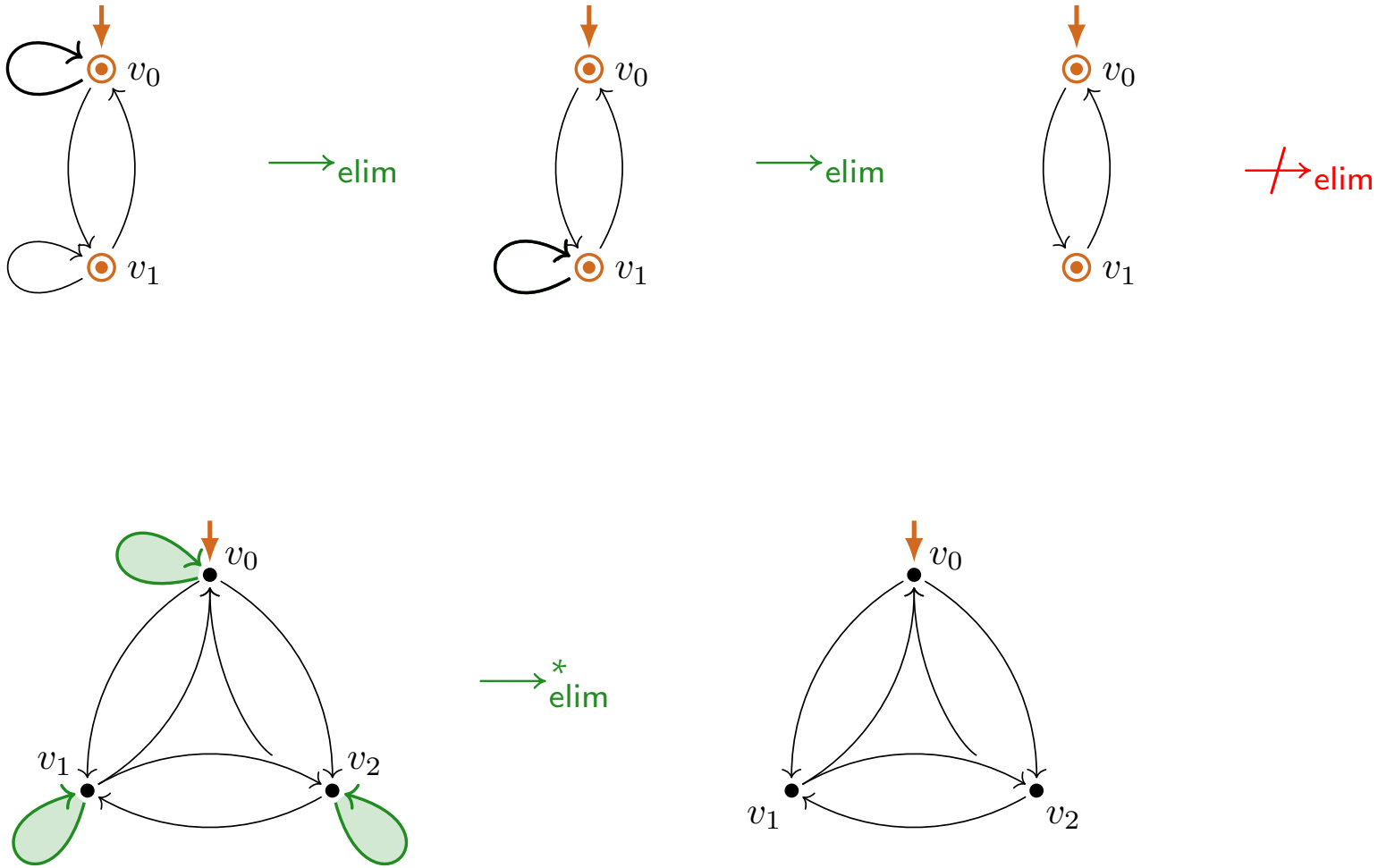


Loop elimination

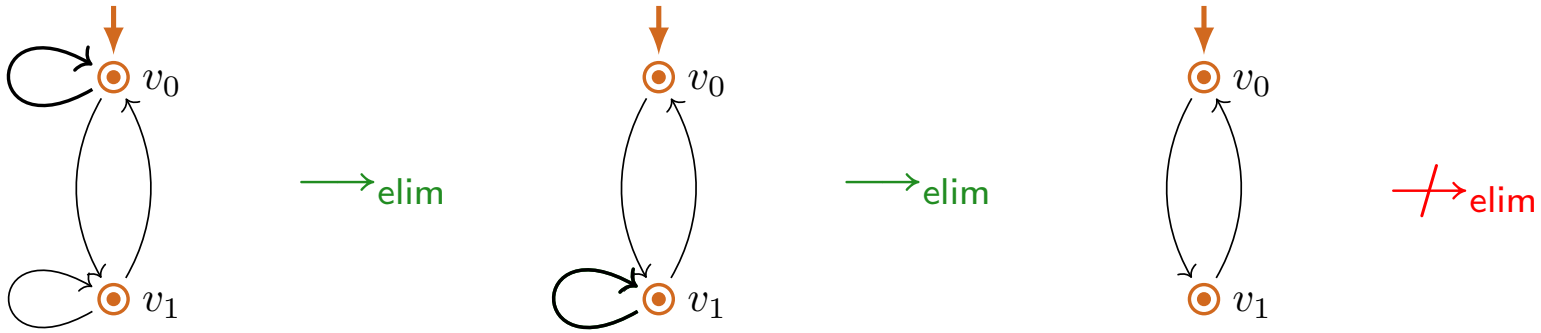


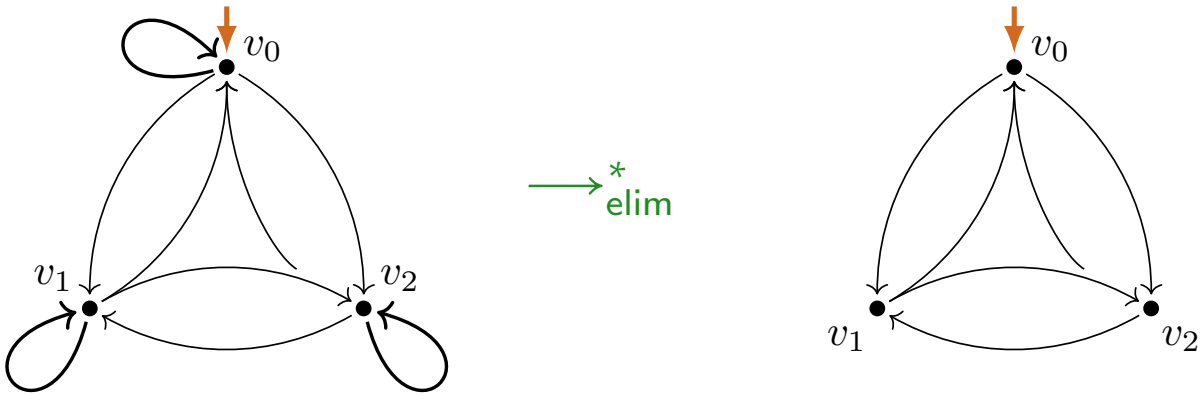


Loop elimination

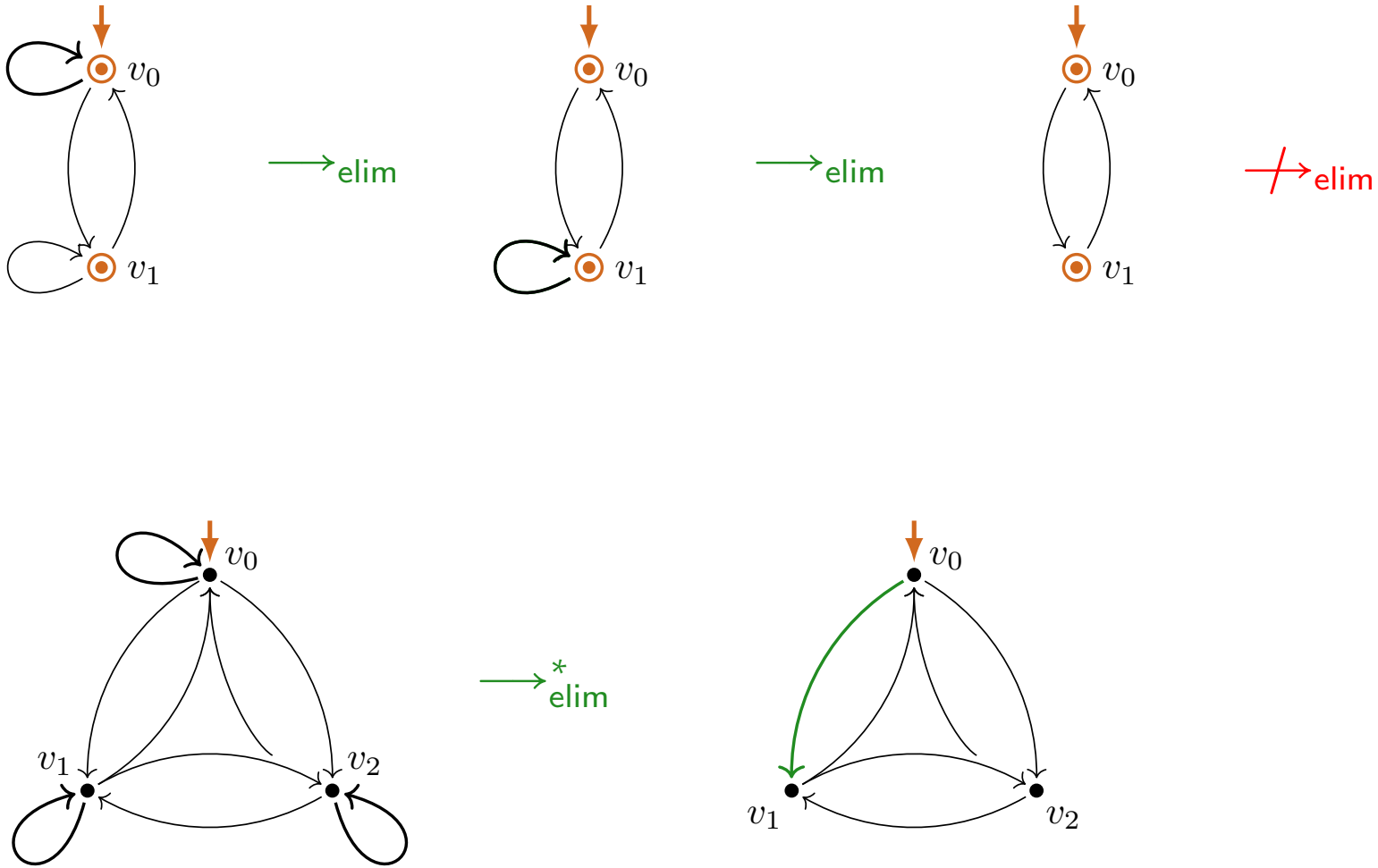


Loop elimination

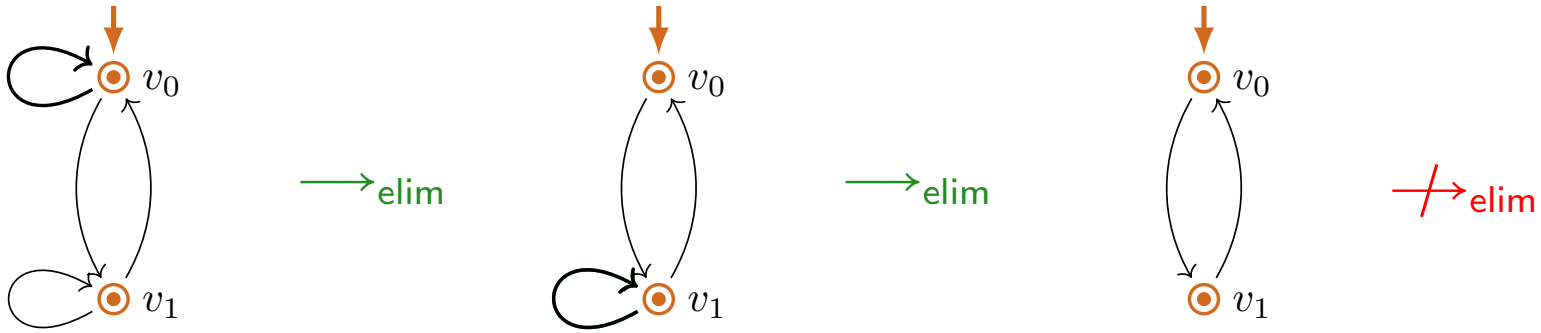


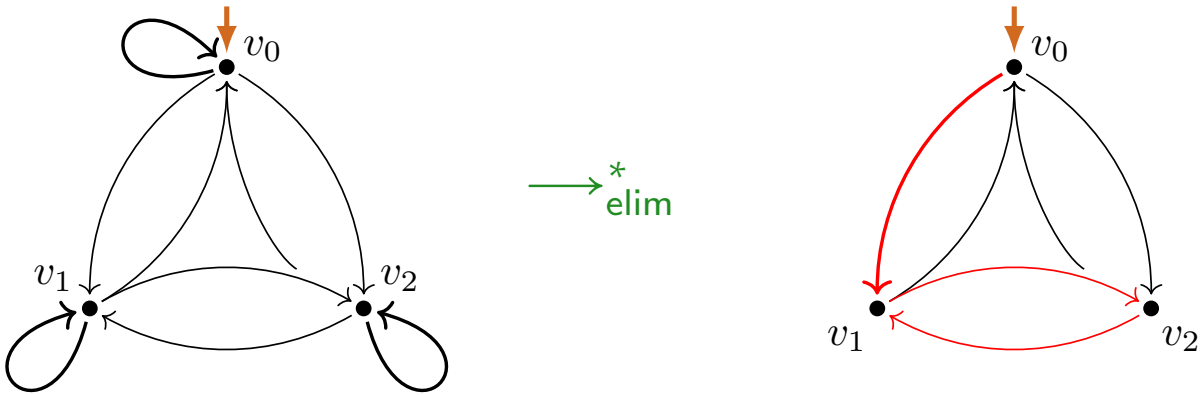


Loop elimination

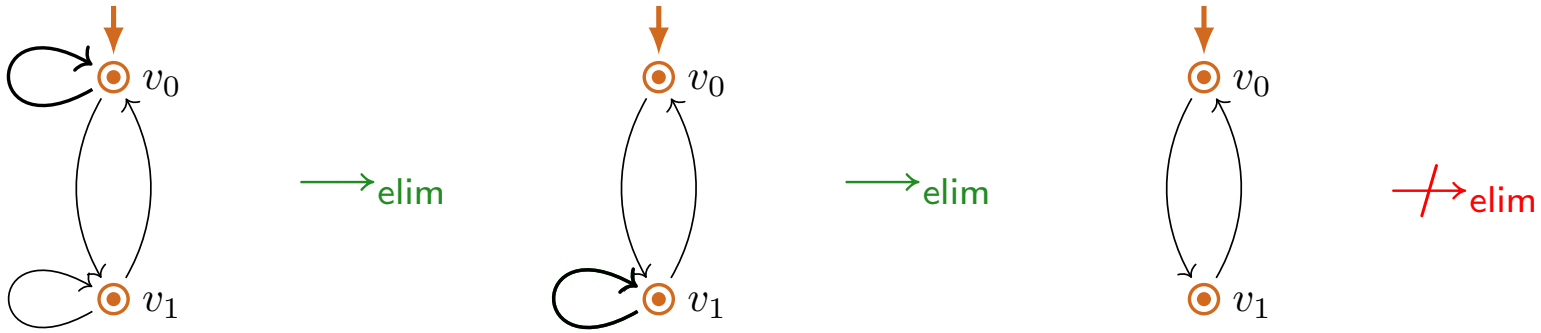


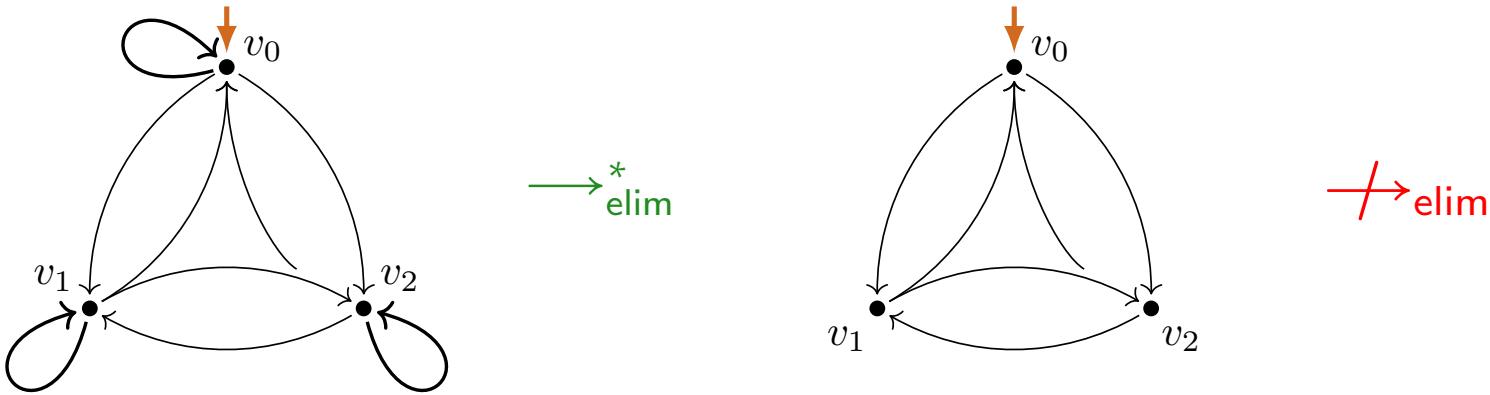
Loop elimination





Loop elimination





Loop elimination

$\longrightarrow_{\text{elim}}$: eliminate a transition-induced loop by:

- removing the loop-entry transition(s)
- garbage collection

$\longrightarrow_{\text{prune}}$: remove a transition to a deadlocking state

Lemma

(i) $\longrightarrow_{\text{elim}} \cup \longrightarrow_{\text{prune}}$ *is terminating.*

Loop elimination

$\longrightarrow_{\text{elim}}$: eliminate a transition-induced loop by:

- removing the loop-entry transition(s)
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Lemma

(i) $\longrightarrow_{\text{elim}} \cup \longrightarrow_{\text{prune}}$ *is terminating.*

Loop elimination, and properties

$\longrightarrow_{\text{elim}}$: eliminate a transition-induced loop by:

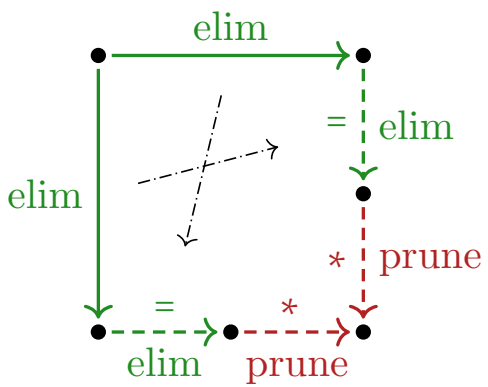
- removing the loop-entry transition(s)
- garbage collection

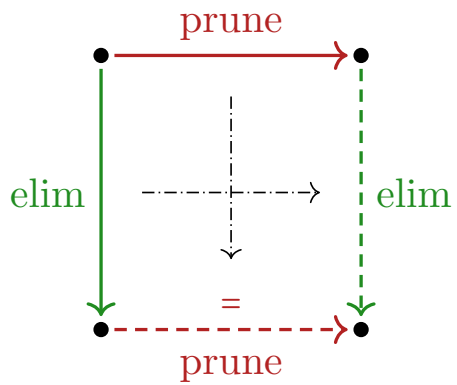
$\longrightarrow_{\text{prune}}$: remove a transition to a deadlocking state

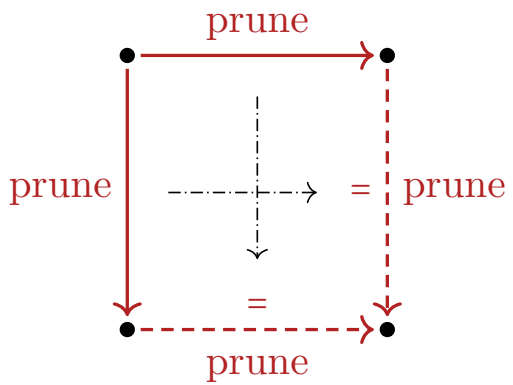
Lemma

(i) $\longrightarrow_{\text{elim}} \cup \longrightarrow_{\text{prune}}$ *is terminating.*

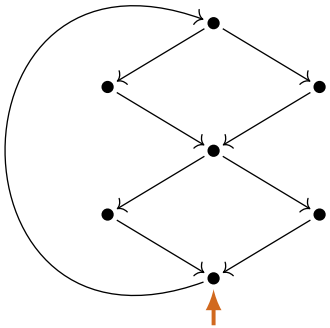
(ii) $\longrightarrow_{\text{elim}} \cup \longrightarrow_{\text{prune}}$ *is decreasing* [Van Oostrom, de Bruijn]



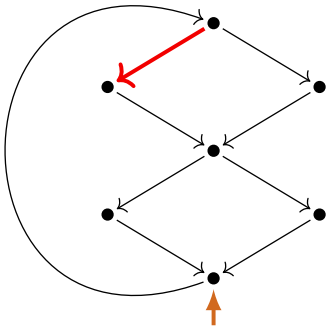




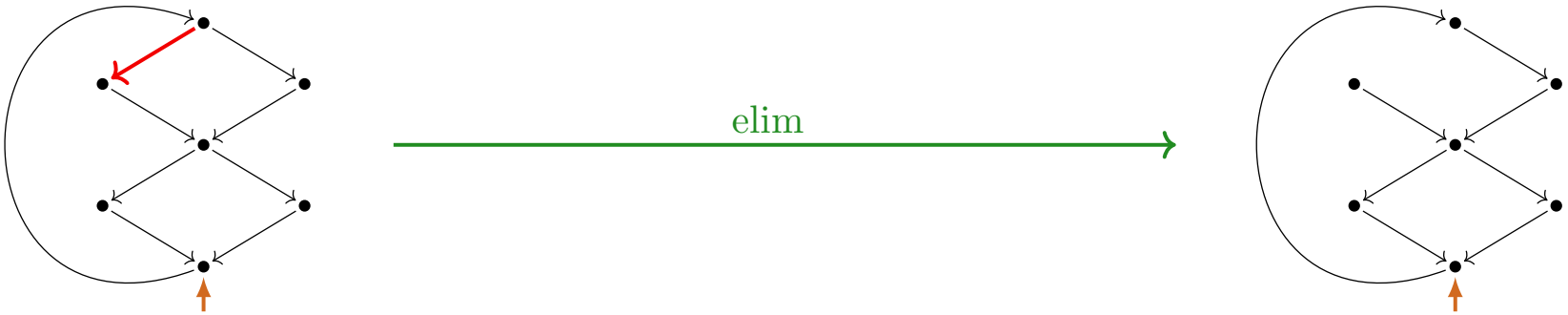
‘Critical pair’: bi-loop elimination



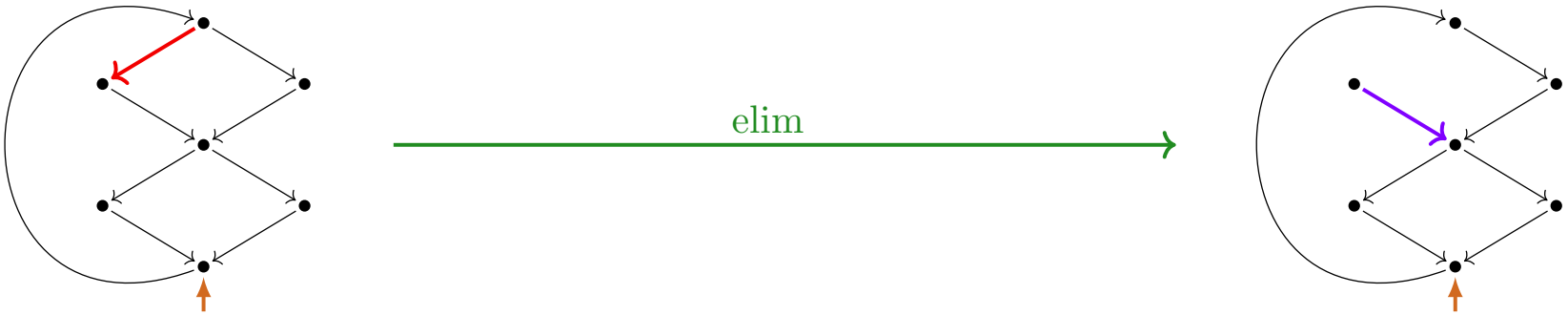
‘Critical pair’: bi-loop elimination



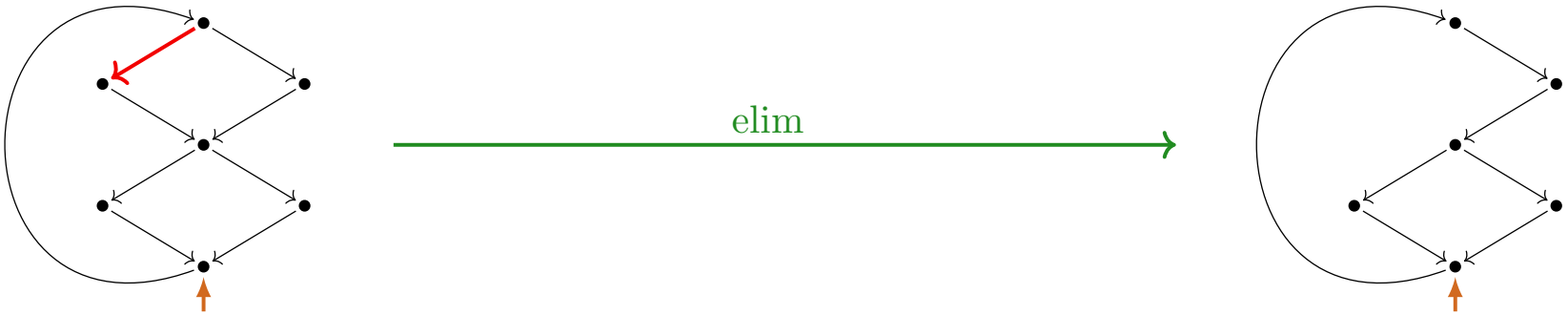
‘Critical pair’: bi-loop elimination



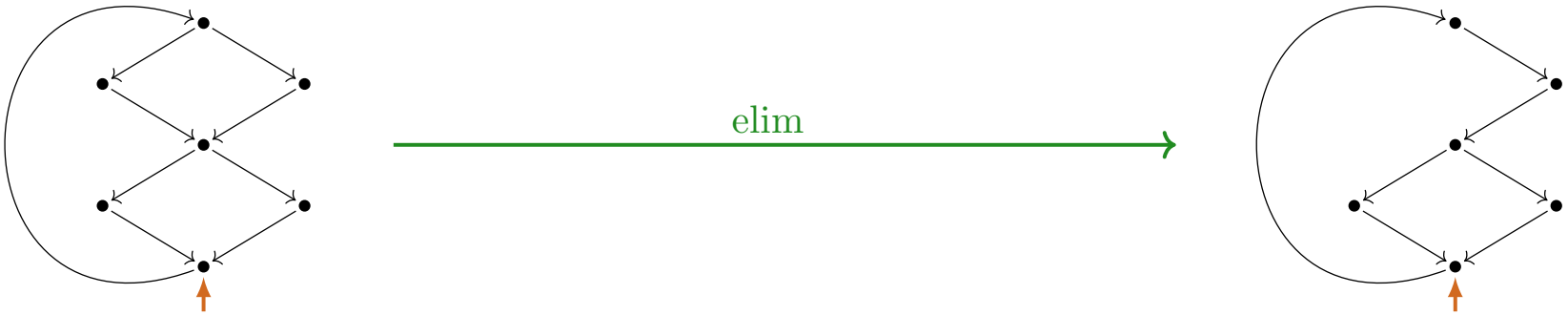
‘Critical pair’: bi-loop elimination



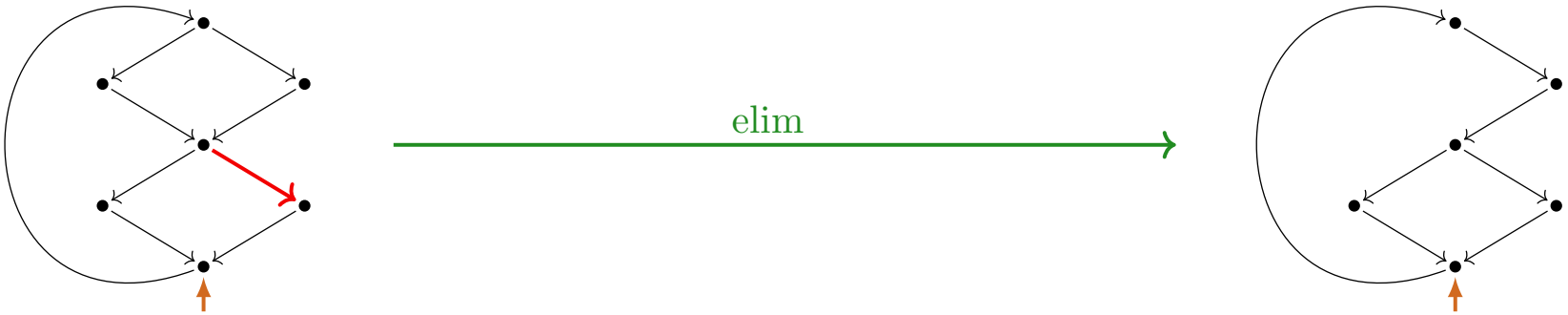
‘Critical pair’: bi-loop elimination



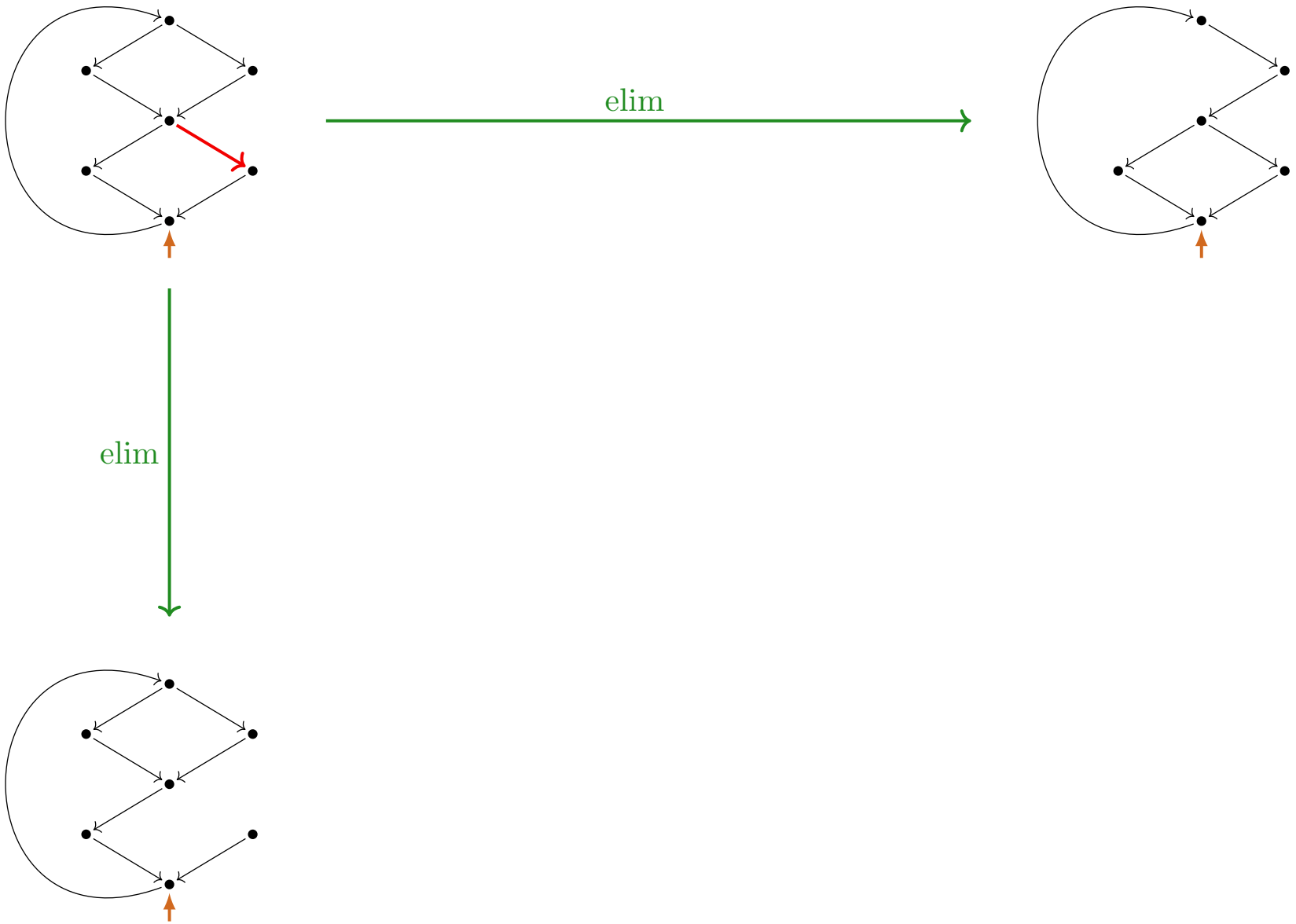
‘Critical pair’: bi-loop elimination



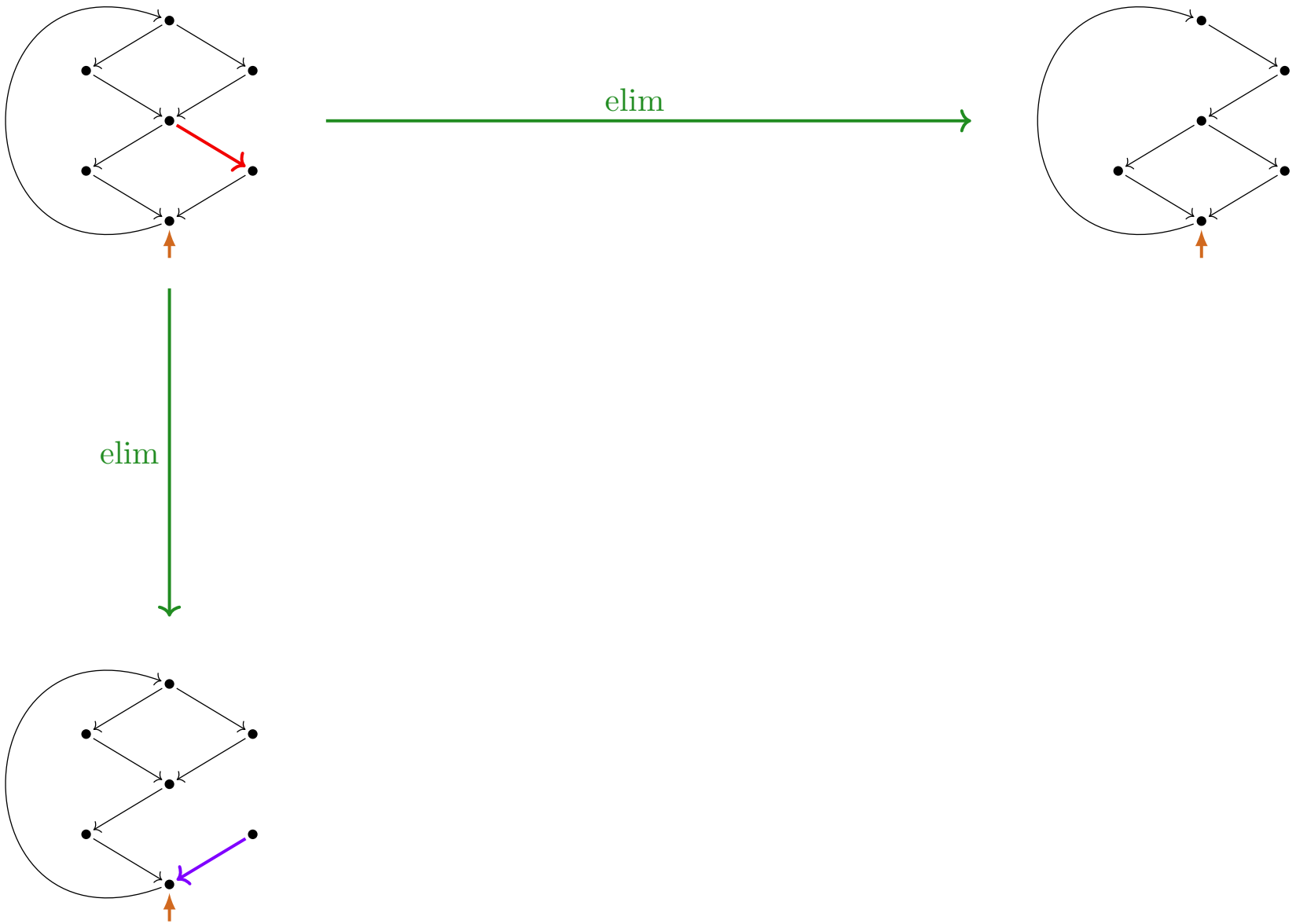
‘Critical pair’: bi-loop elimination



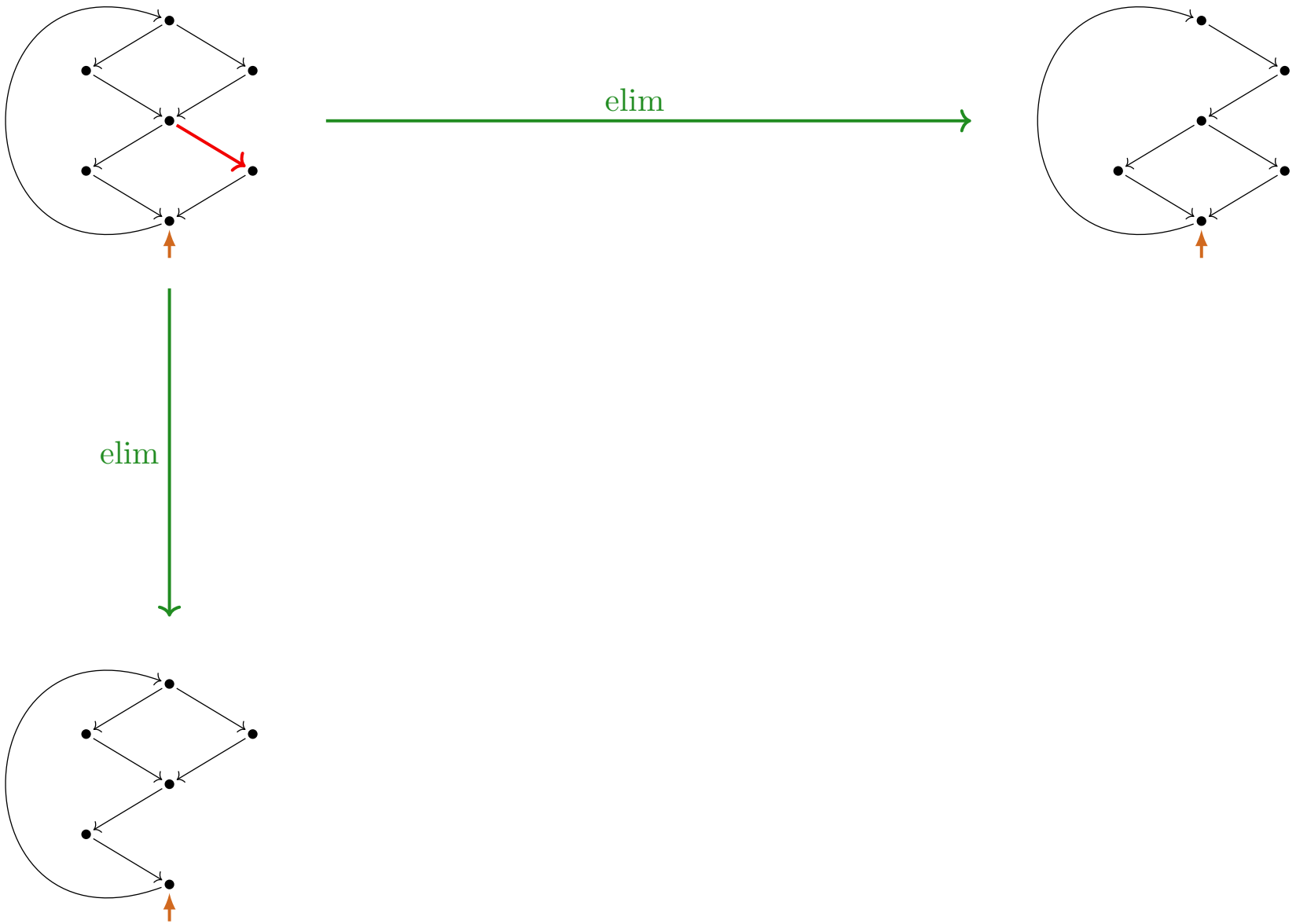
‘Critical pair’: bi-loop elimination



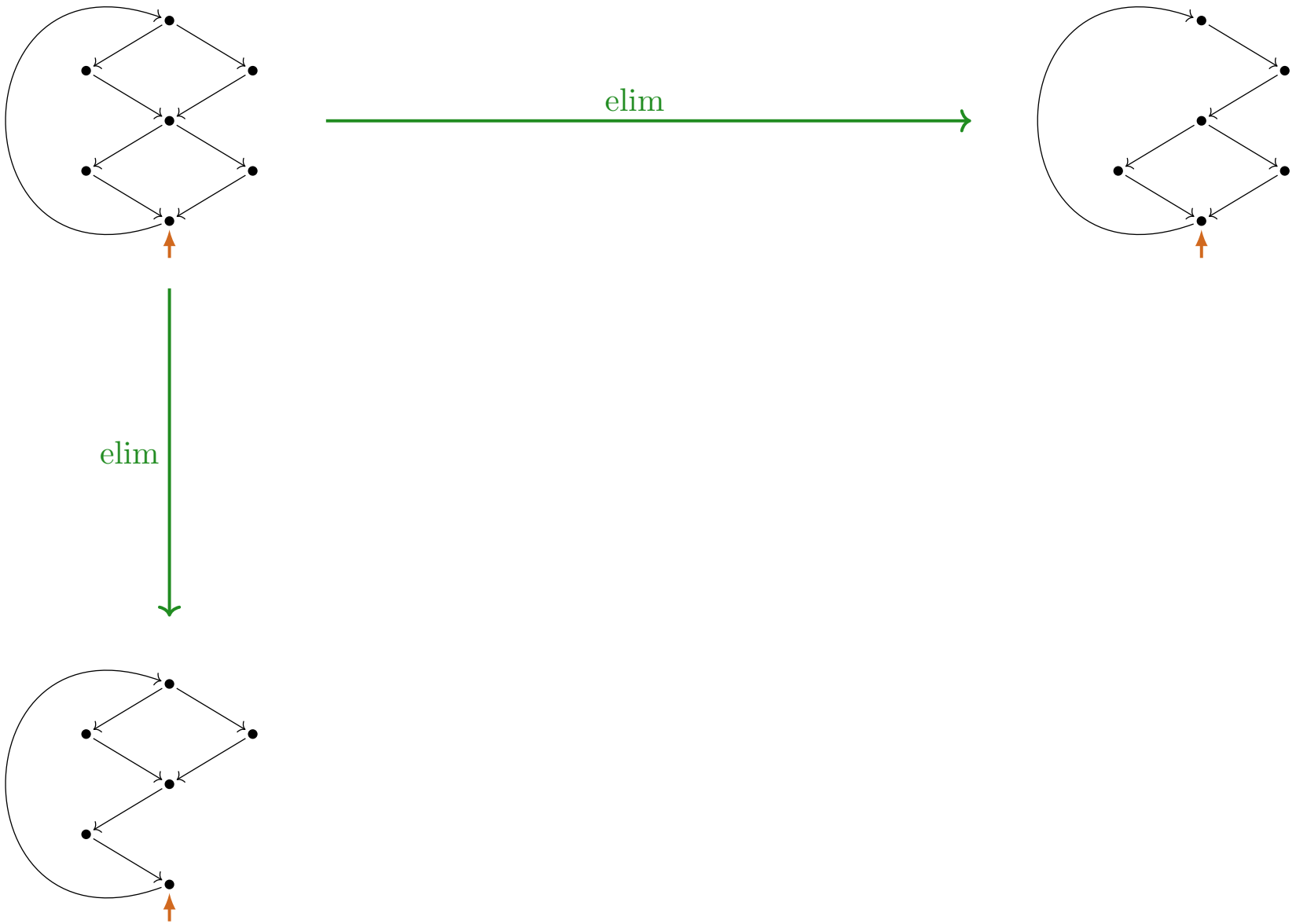
‘Critical pair’: bi-loop elimination



‘Critical pair’: bi-loop elimination



‘Critical pair’: bi-loop elimination



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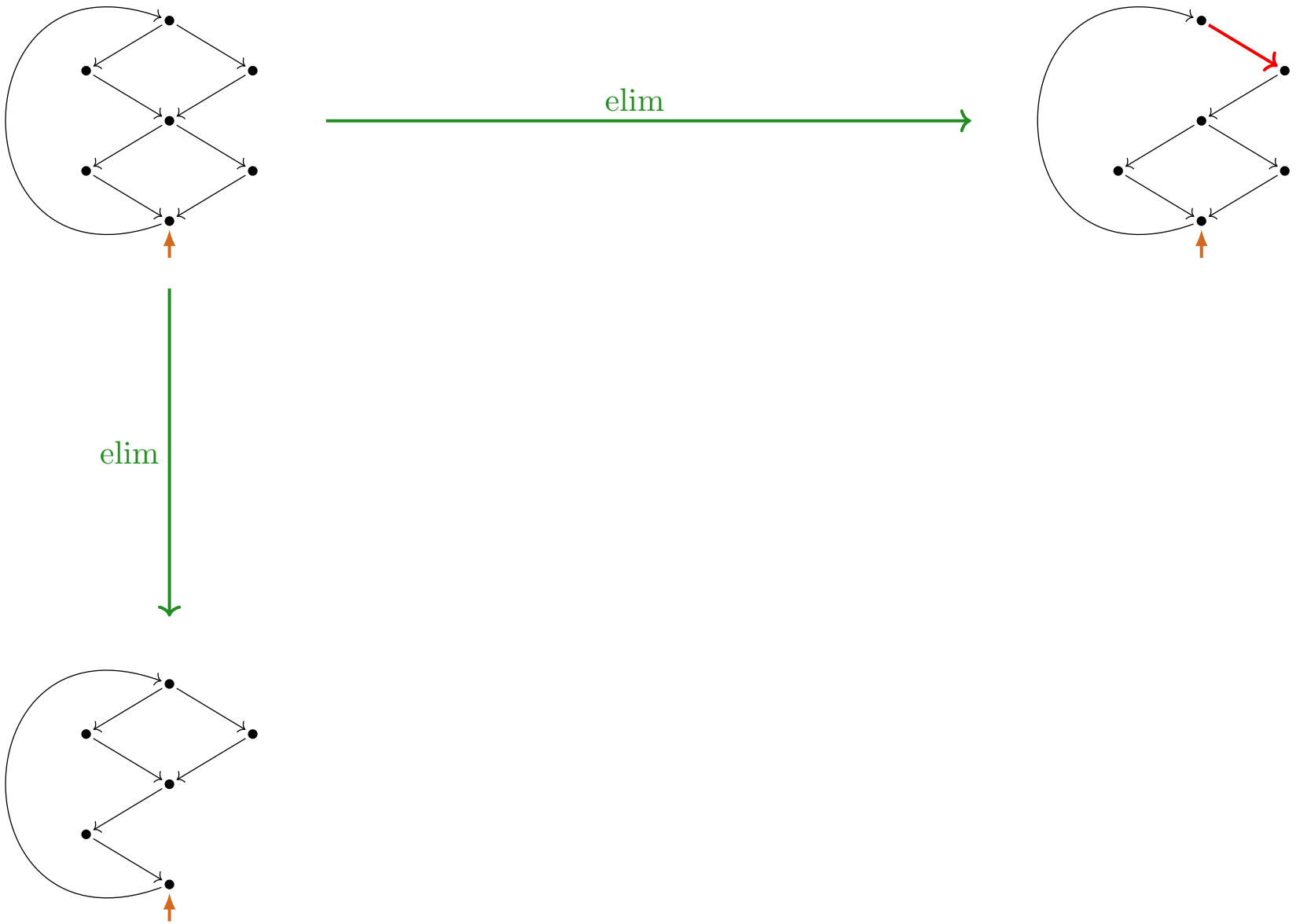

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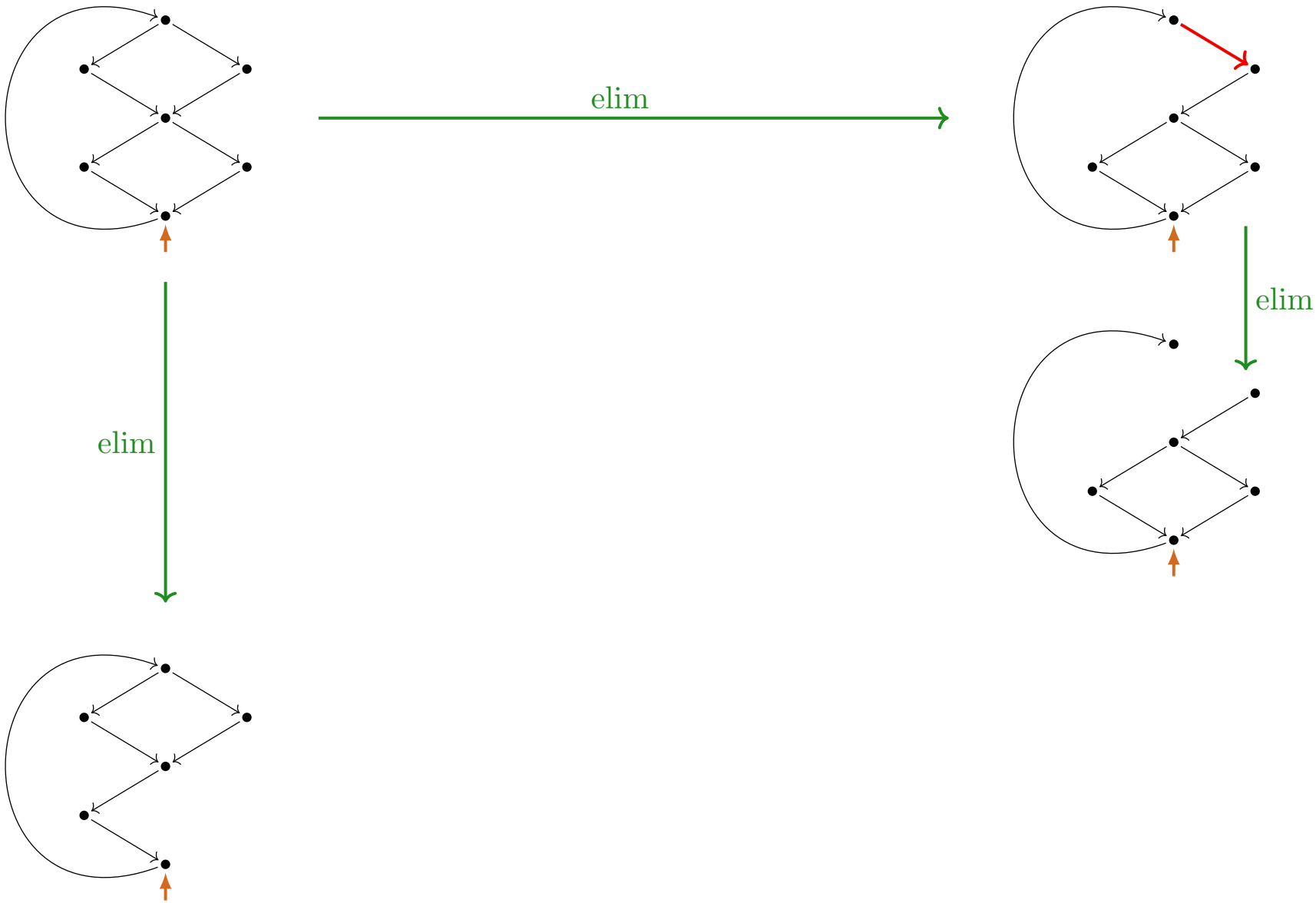

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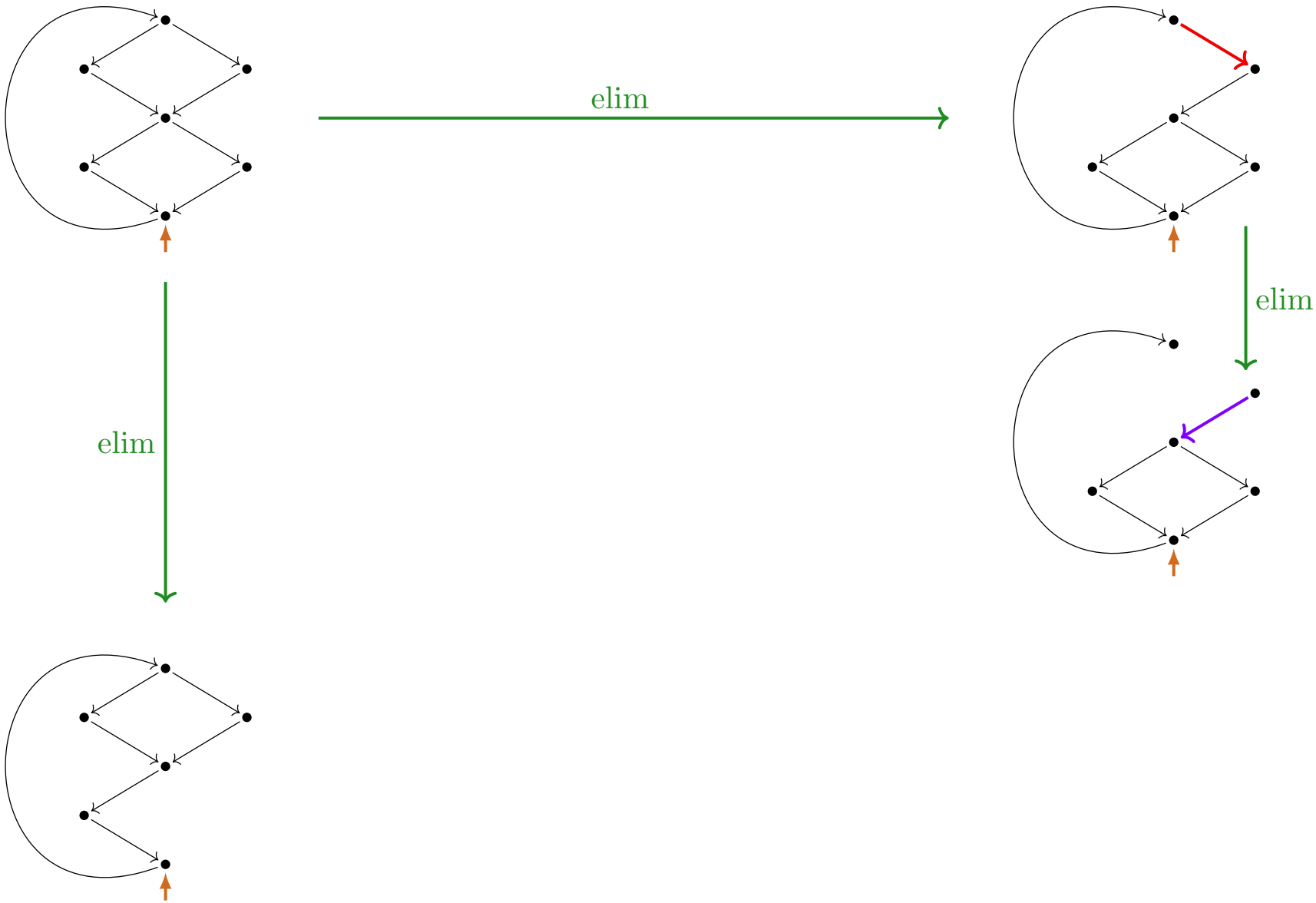
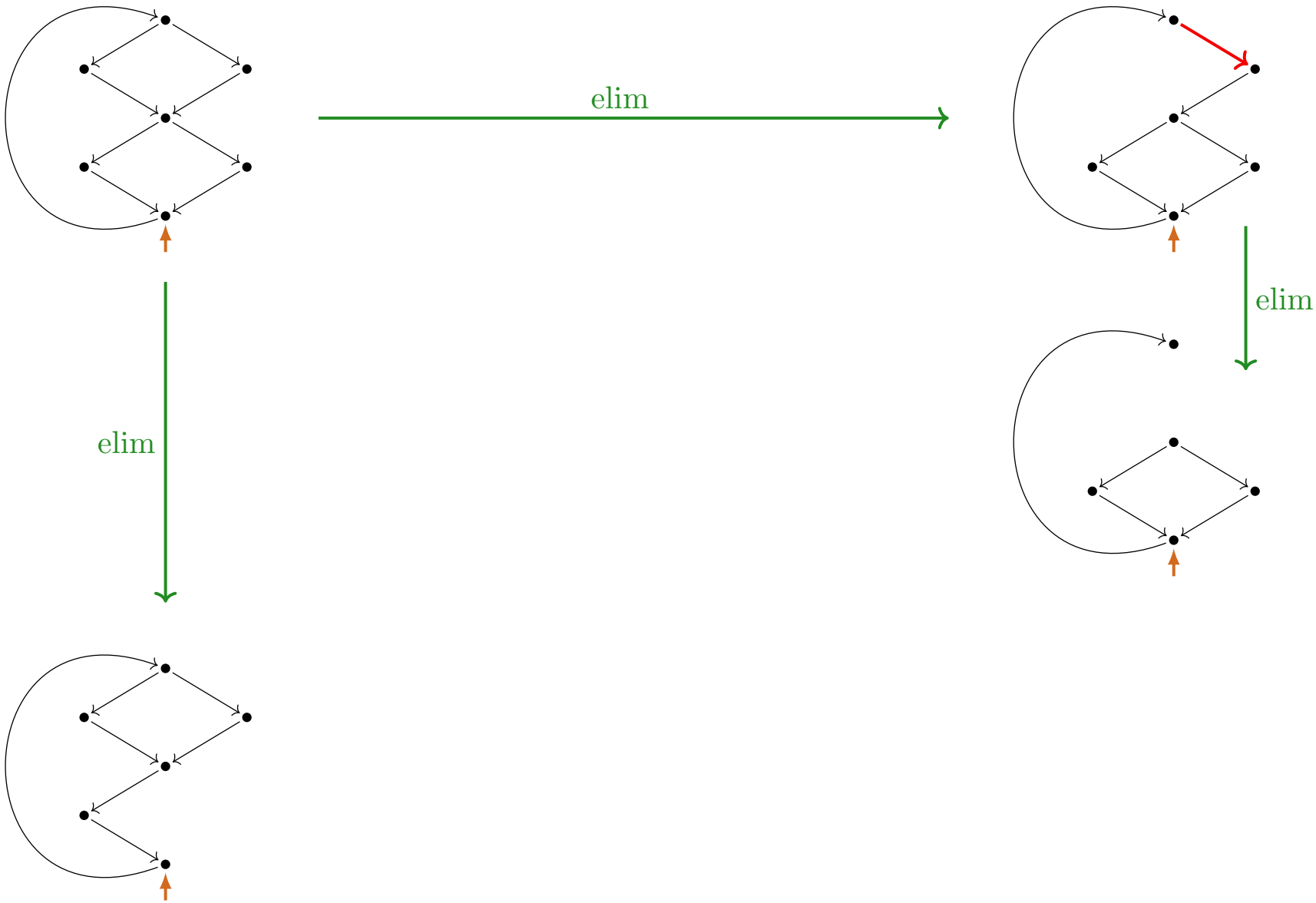


‘Critical pair’: bi-loop elimination



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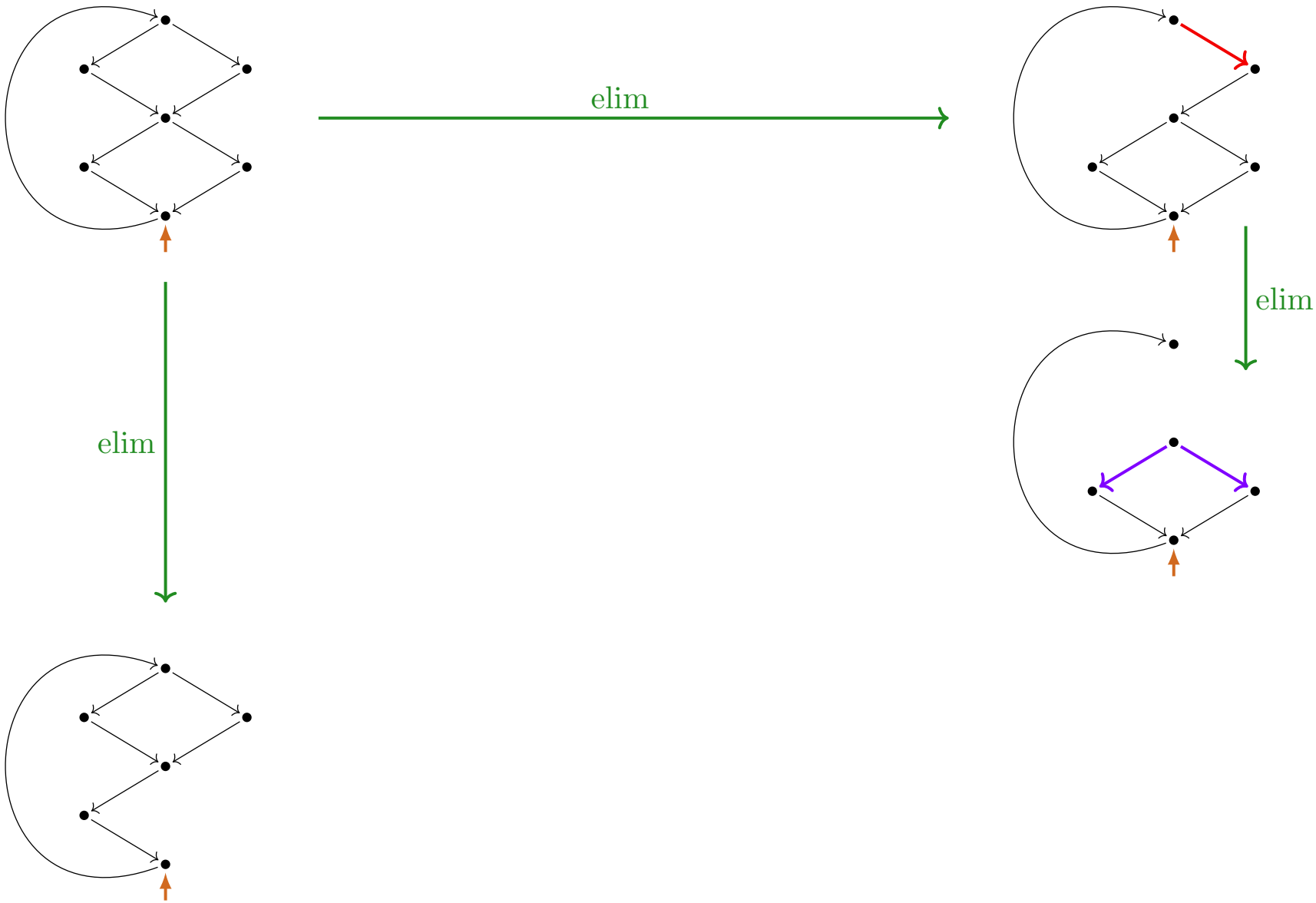
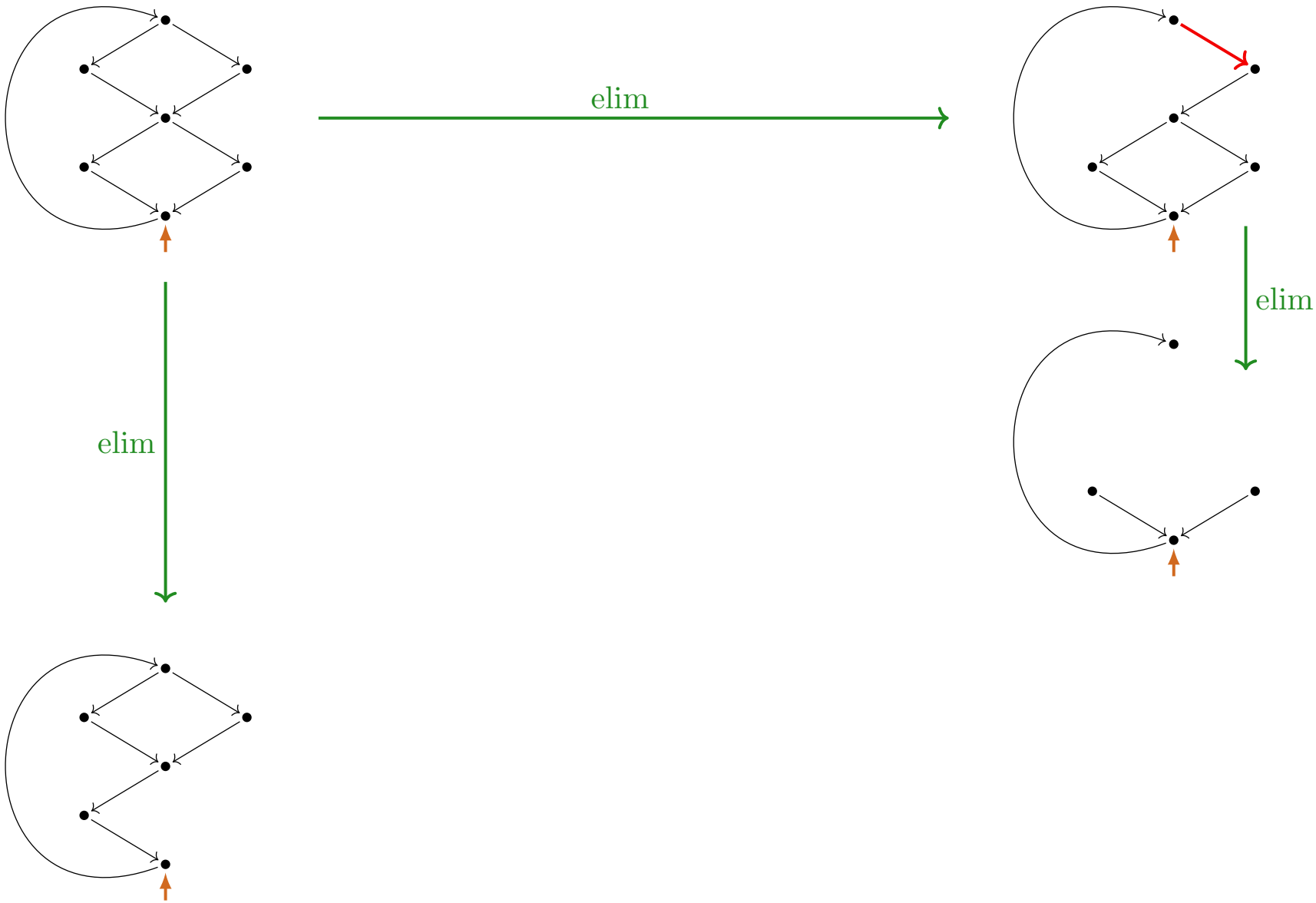
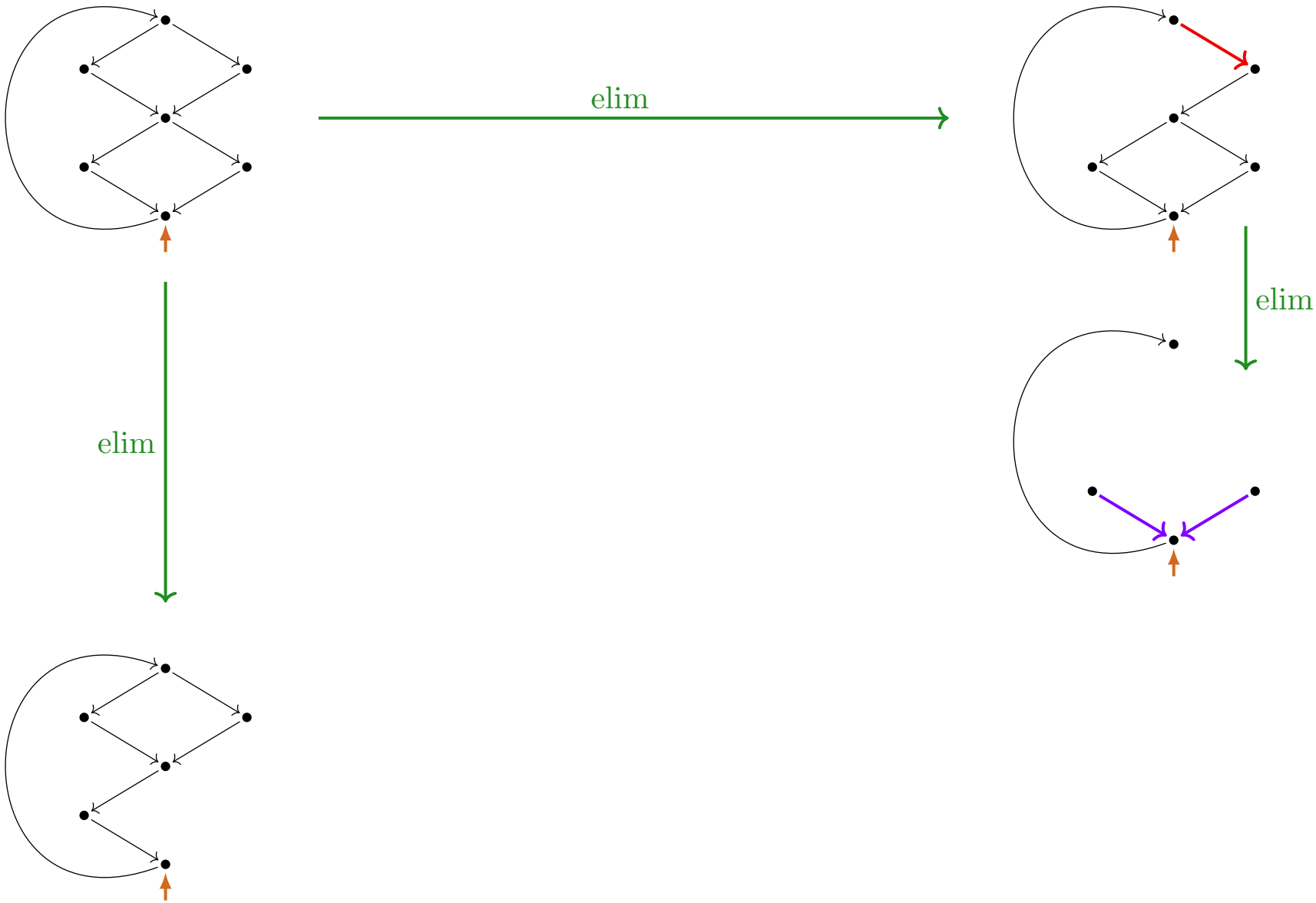


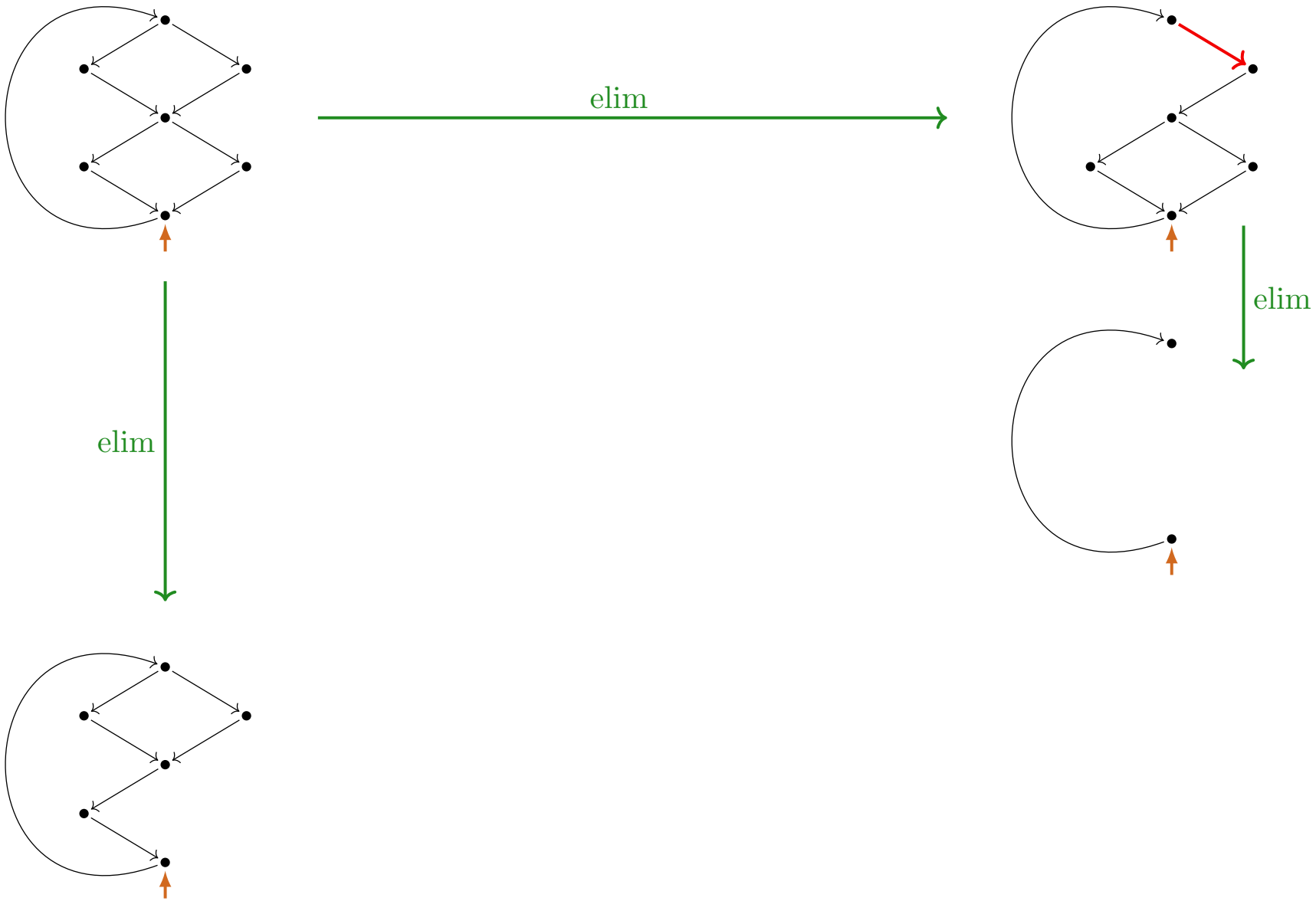
‘Critical pair’: bi-loop elimination



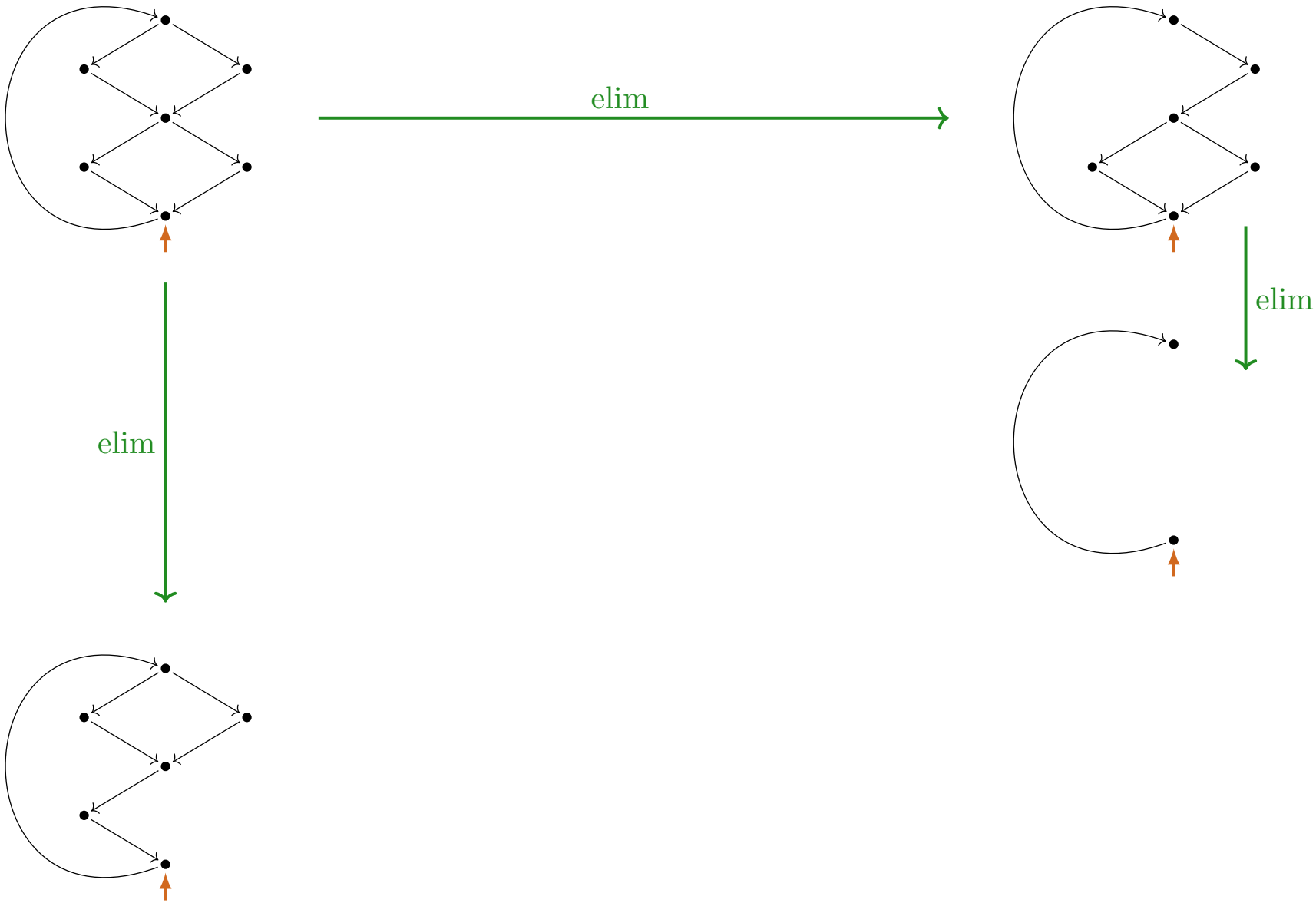
‘Critical pair’: bi-loop elimination



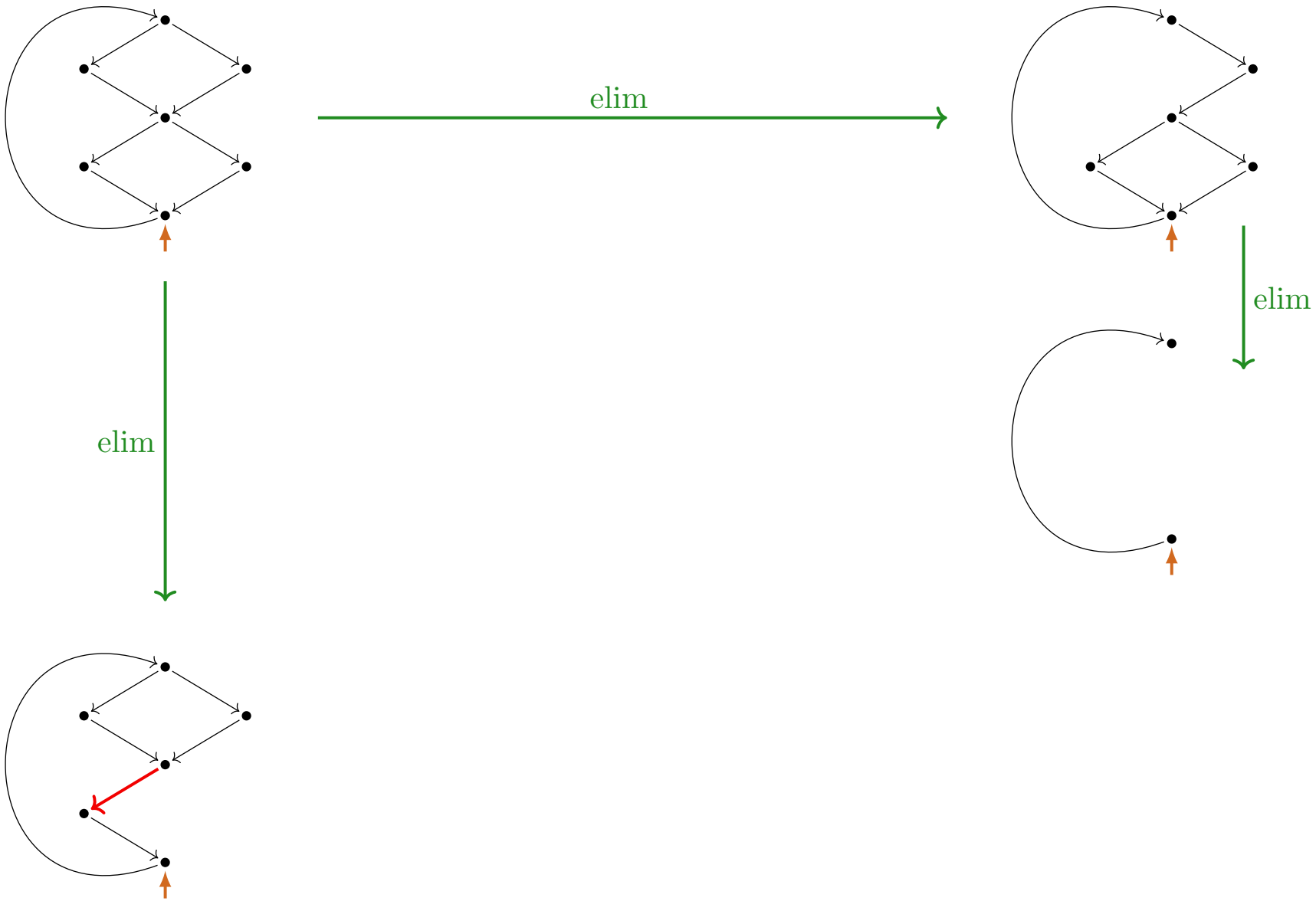
‘Critical pair’: bi-loop elimination



‘Critical pair’: bi-loop elimination



‘Critical pair’: bi-loop elimination



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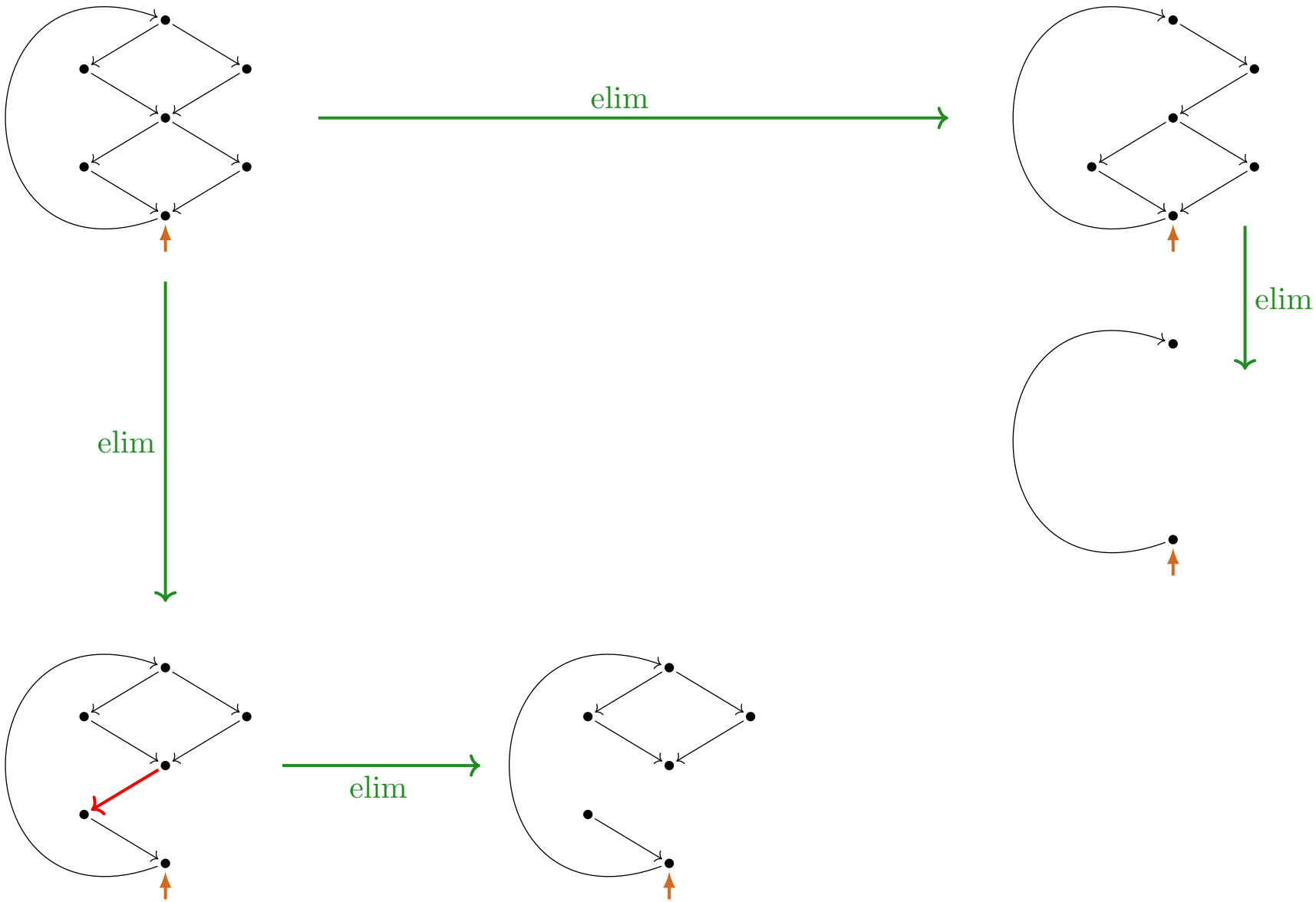
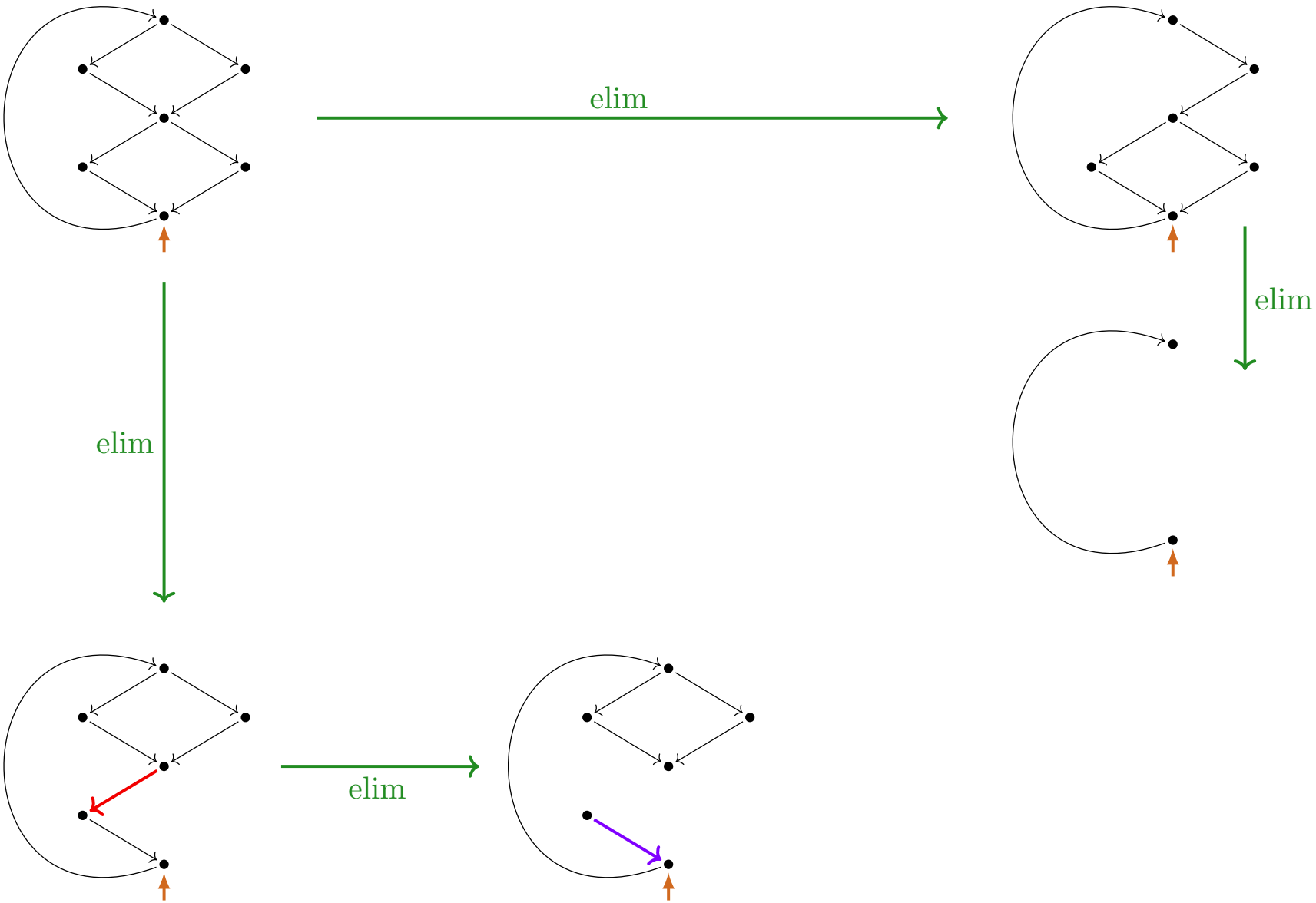
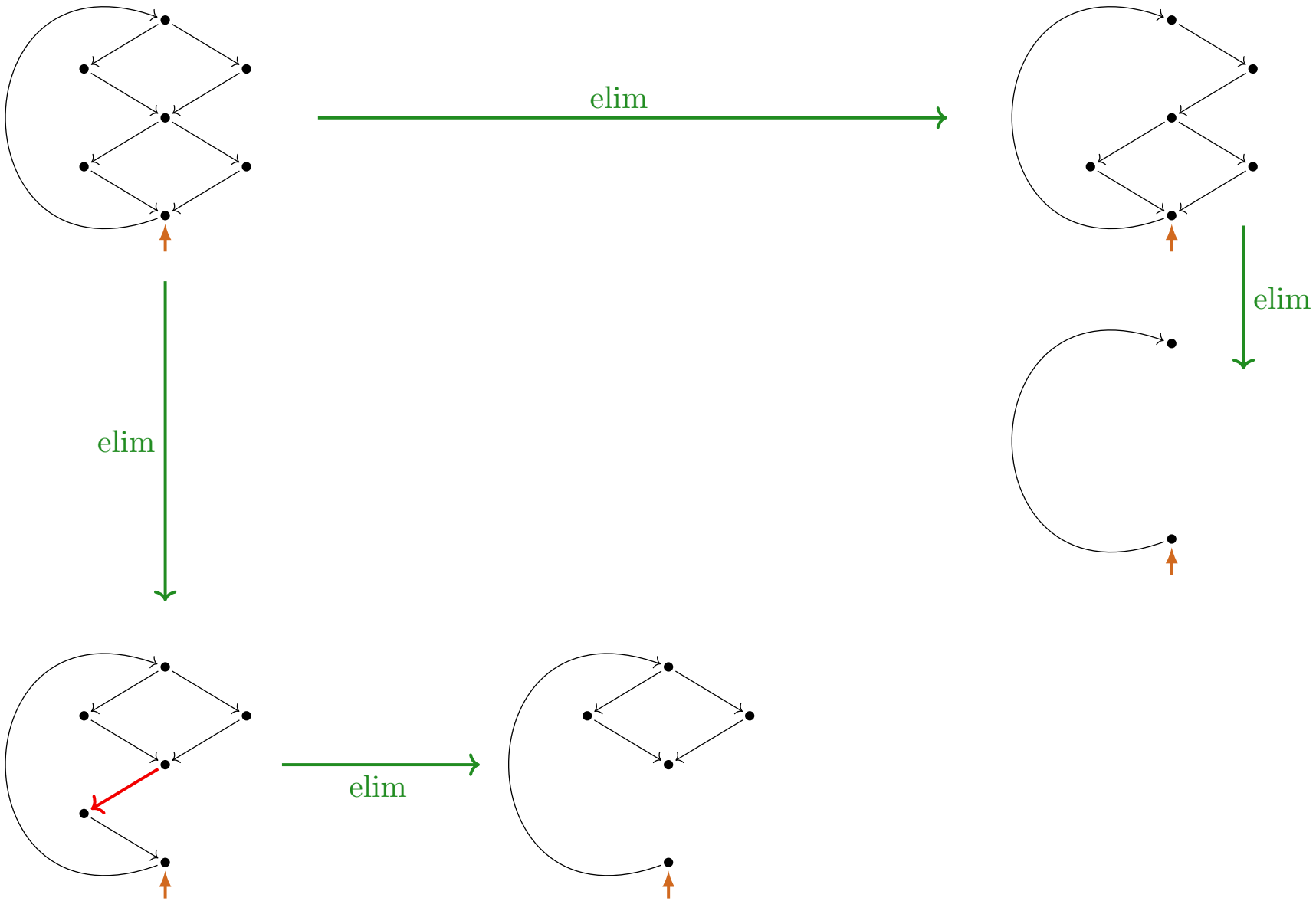


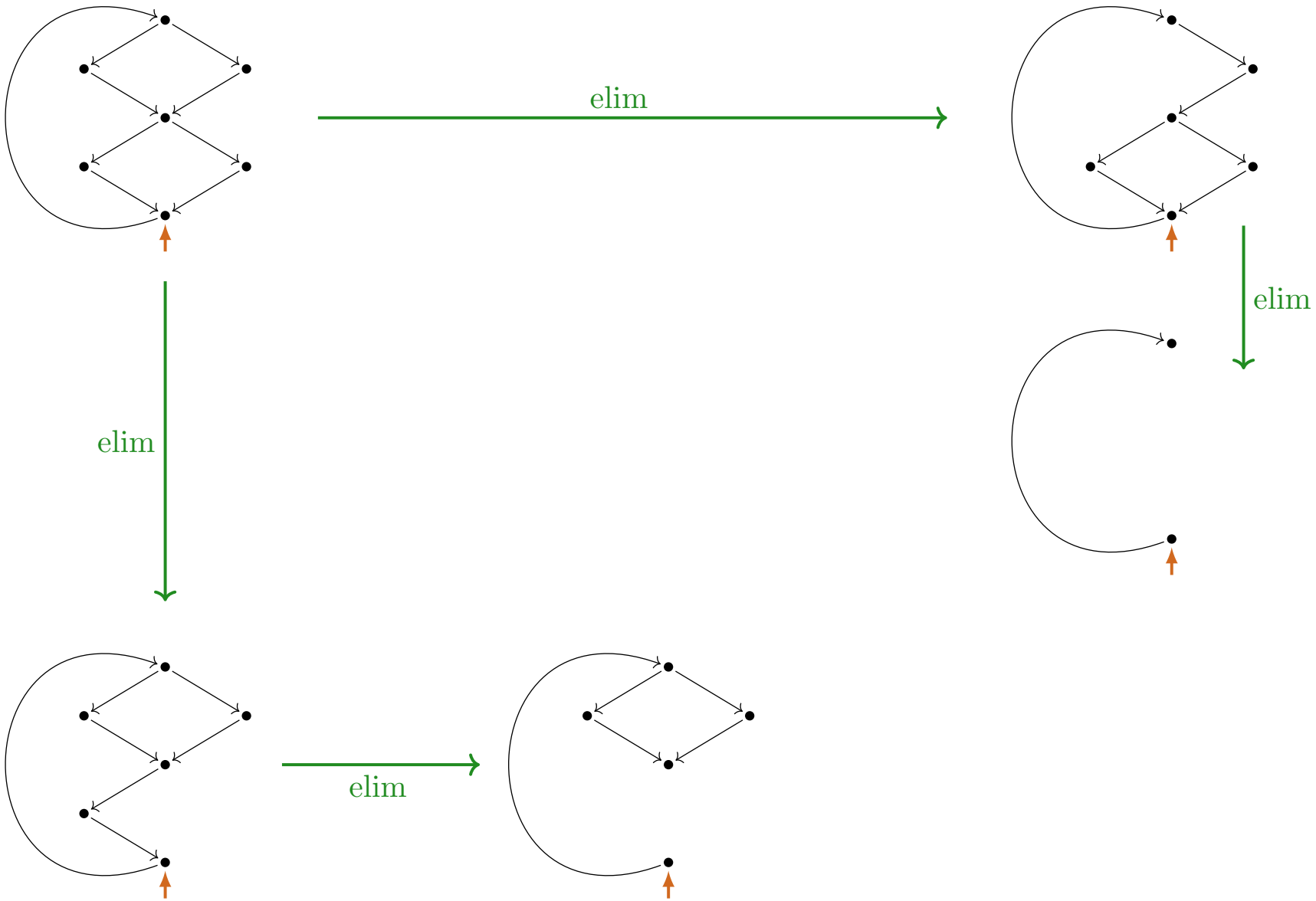
‘Critical pair’: bi-loop elimination



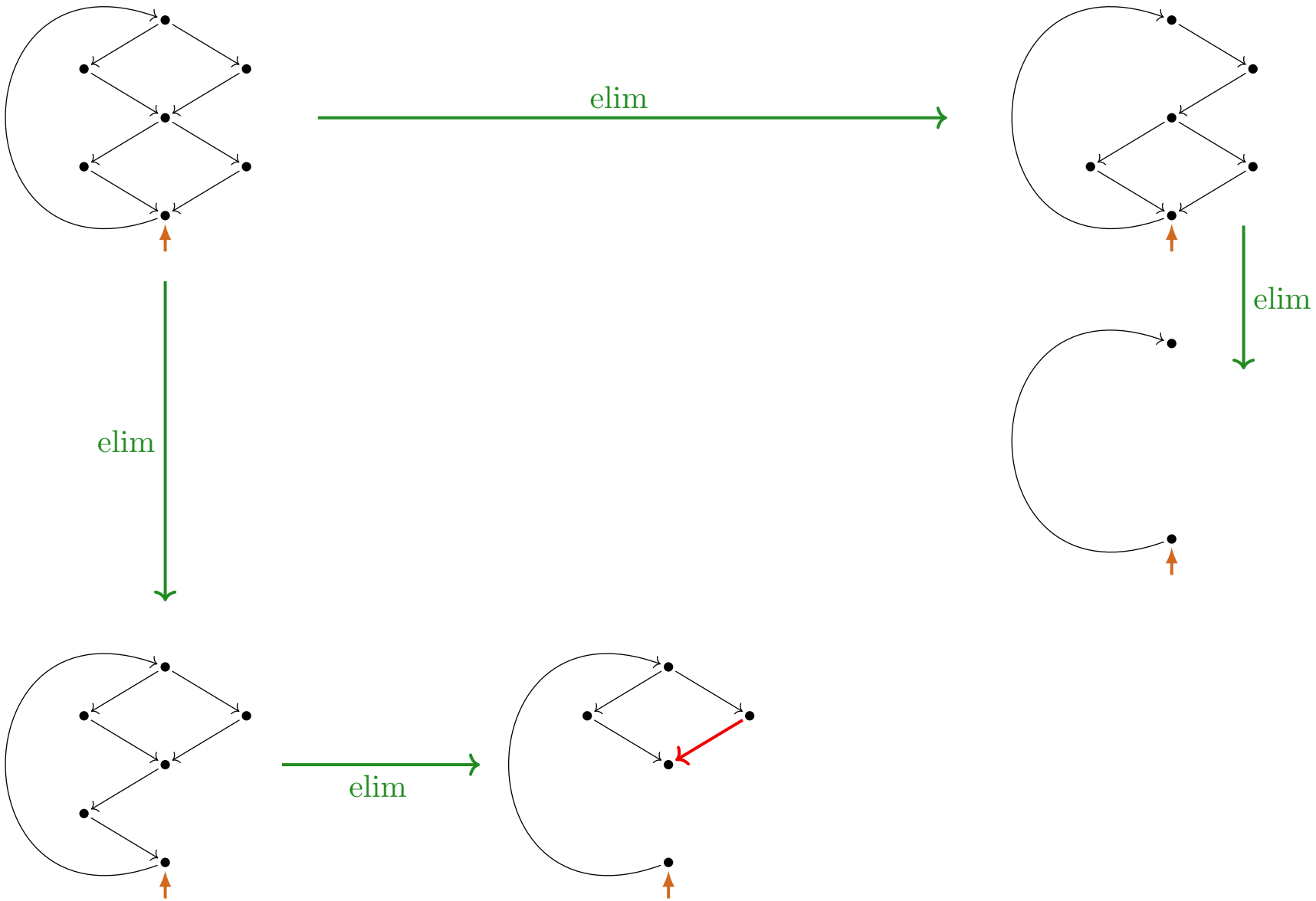
‘Critical pair’: bi-loop elimination



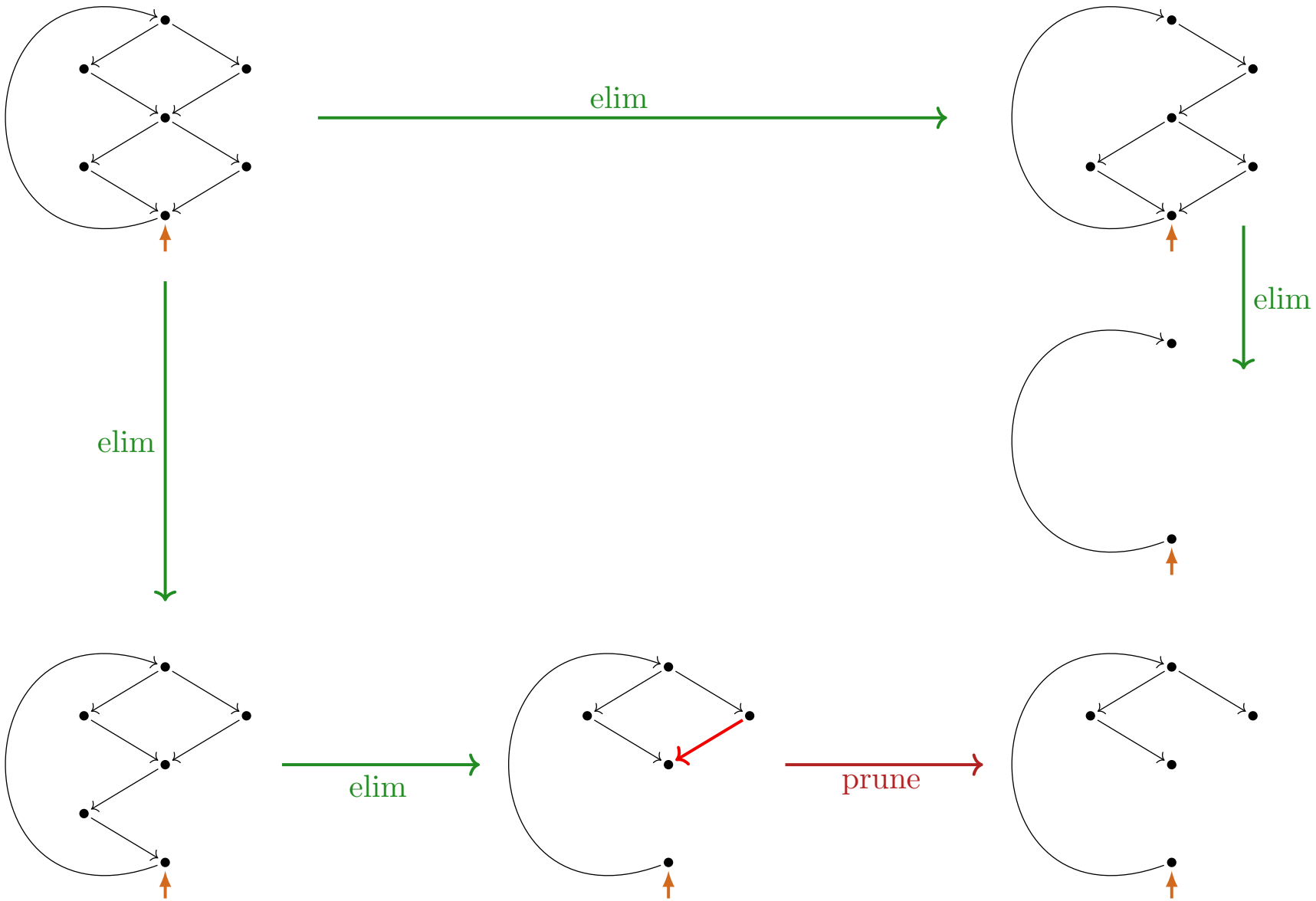
‘Critical pair’: bi-loop elimination



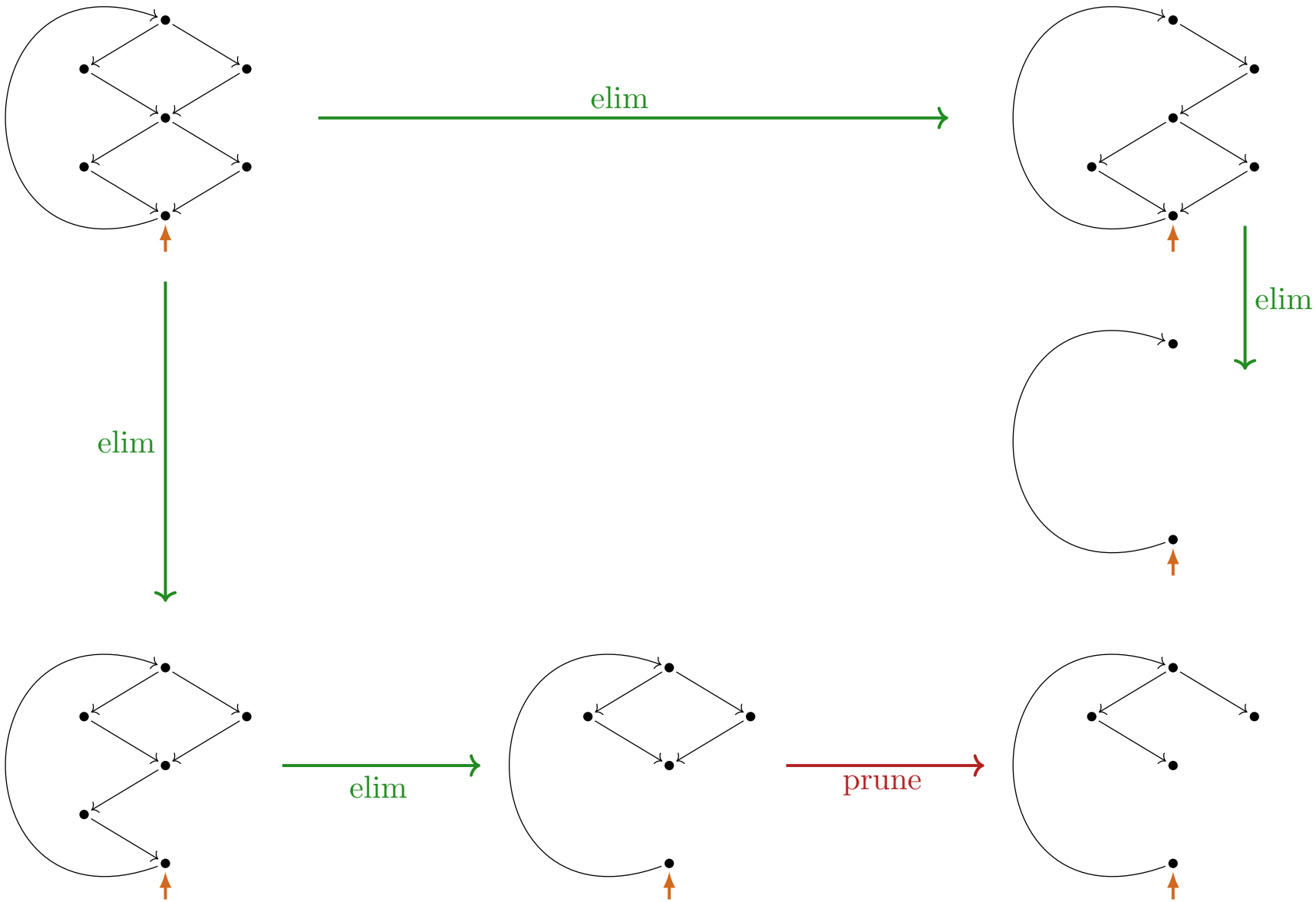
‘Critical pair’: bi-loop elimination



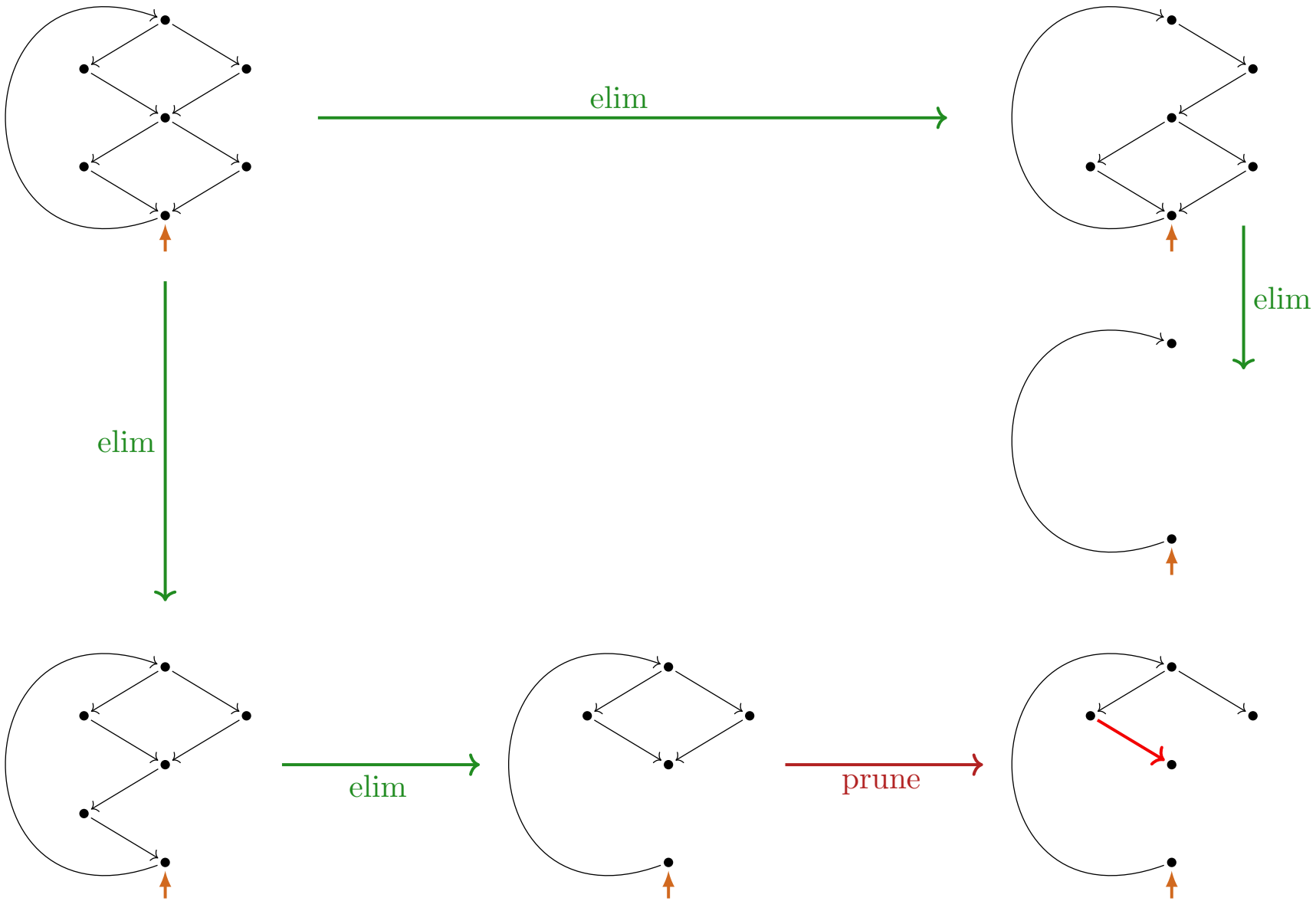
‘Critical pair’: bi-loop elimination



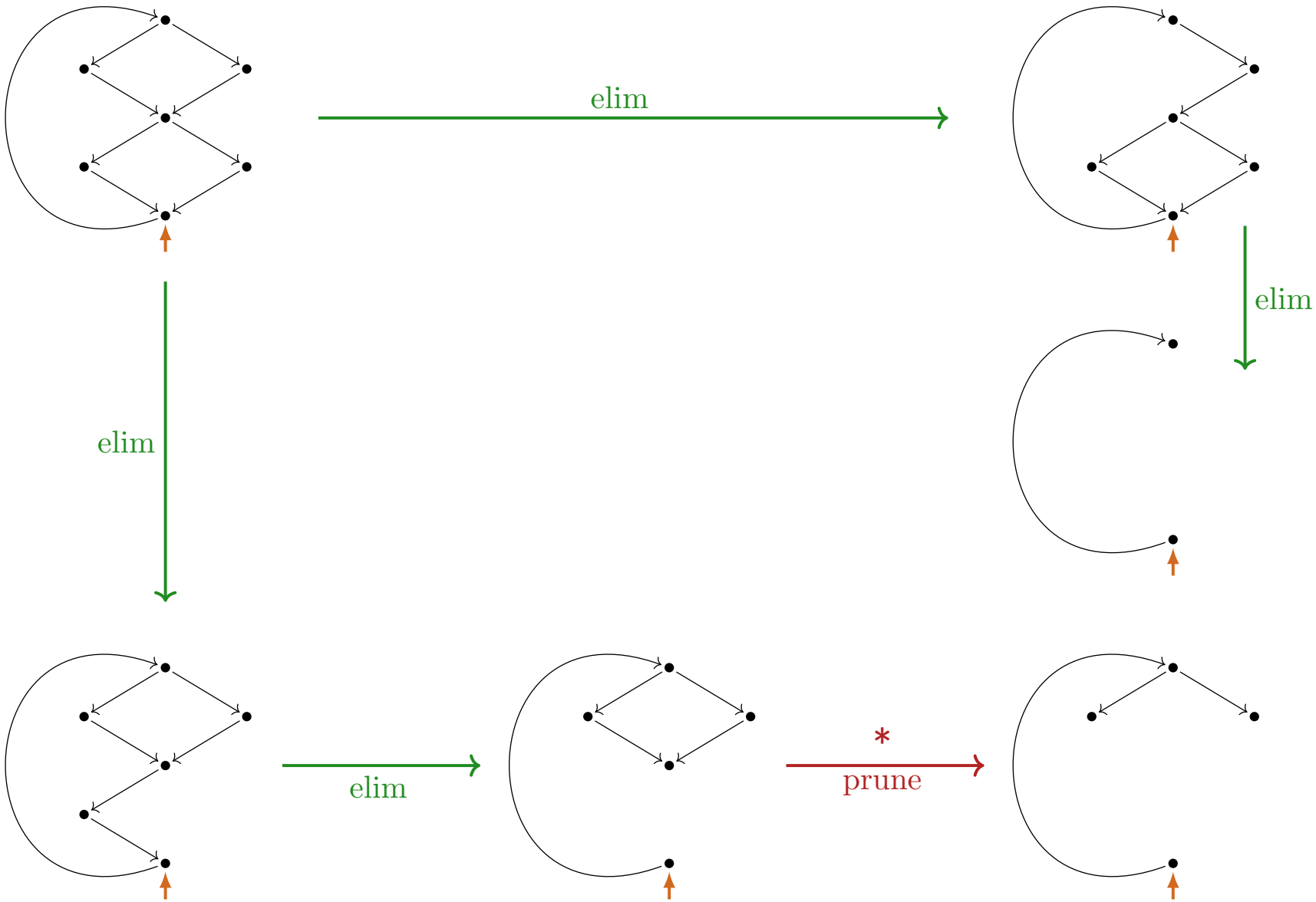
‘Critical pair’: bi-loop elimination



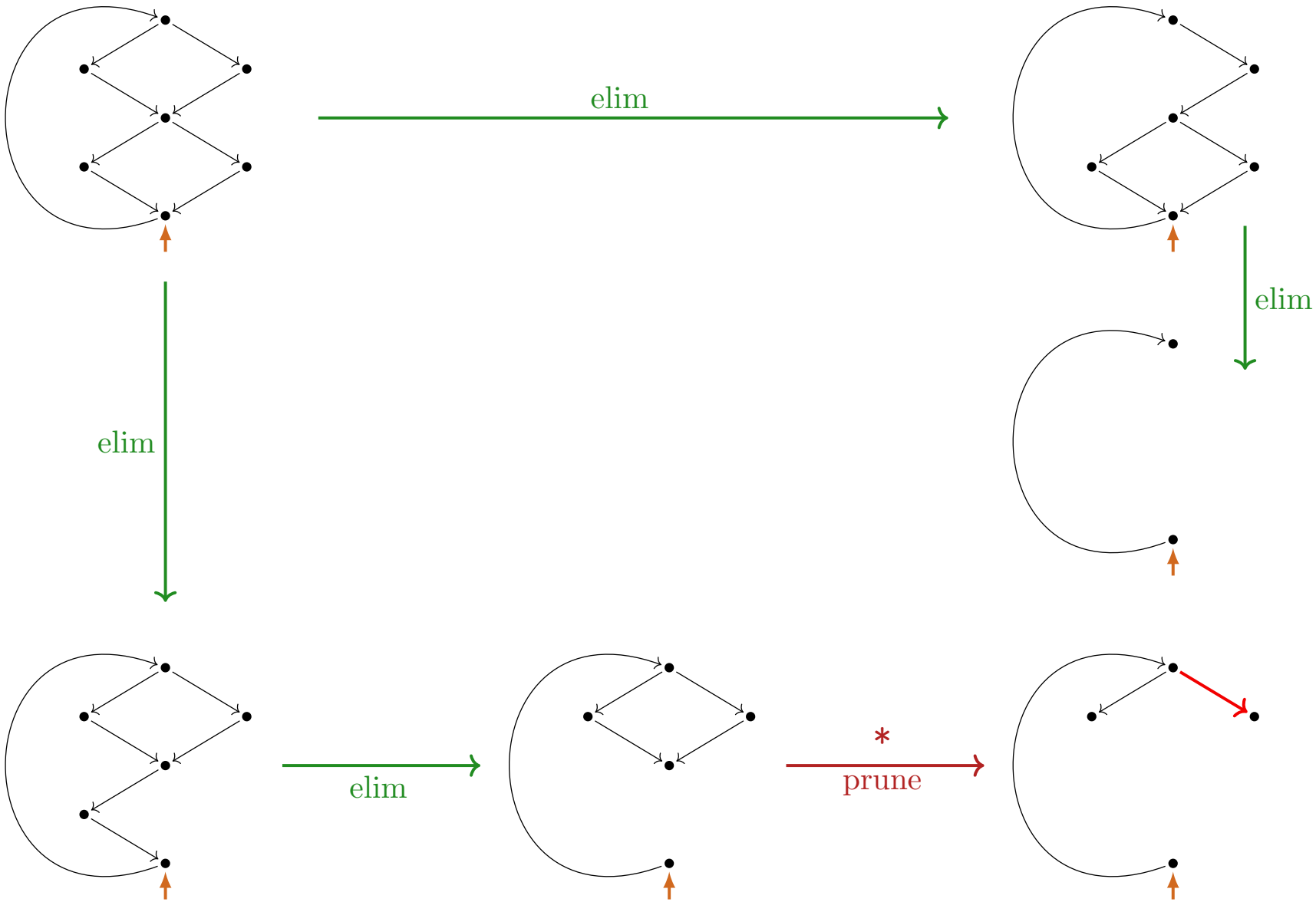
‘Critical pair’: bi-loop elimination



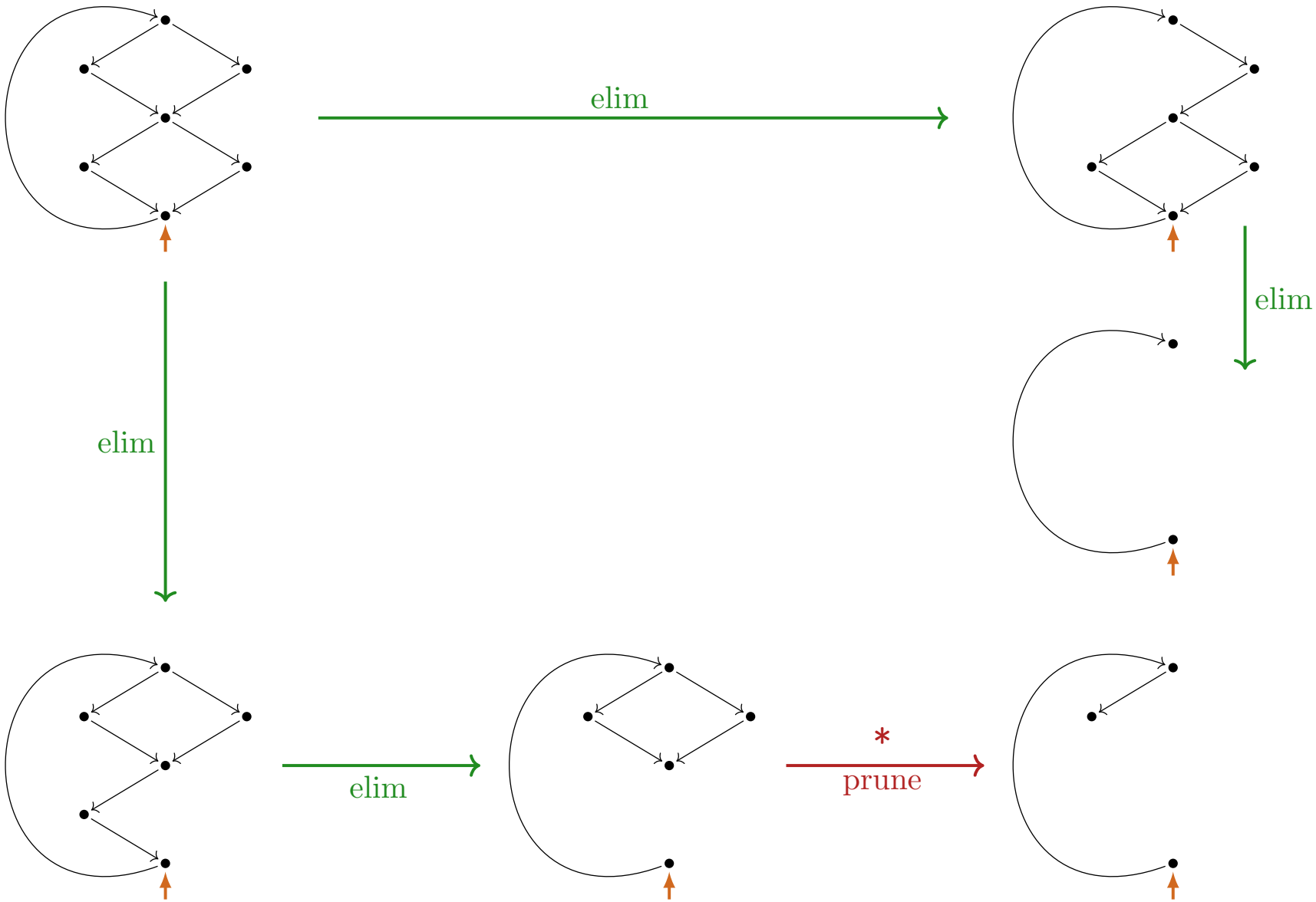
‘Critical pair’: bi-loop elimination



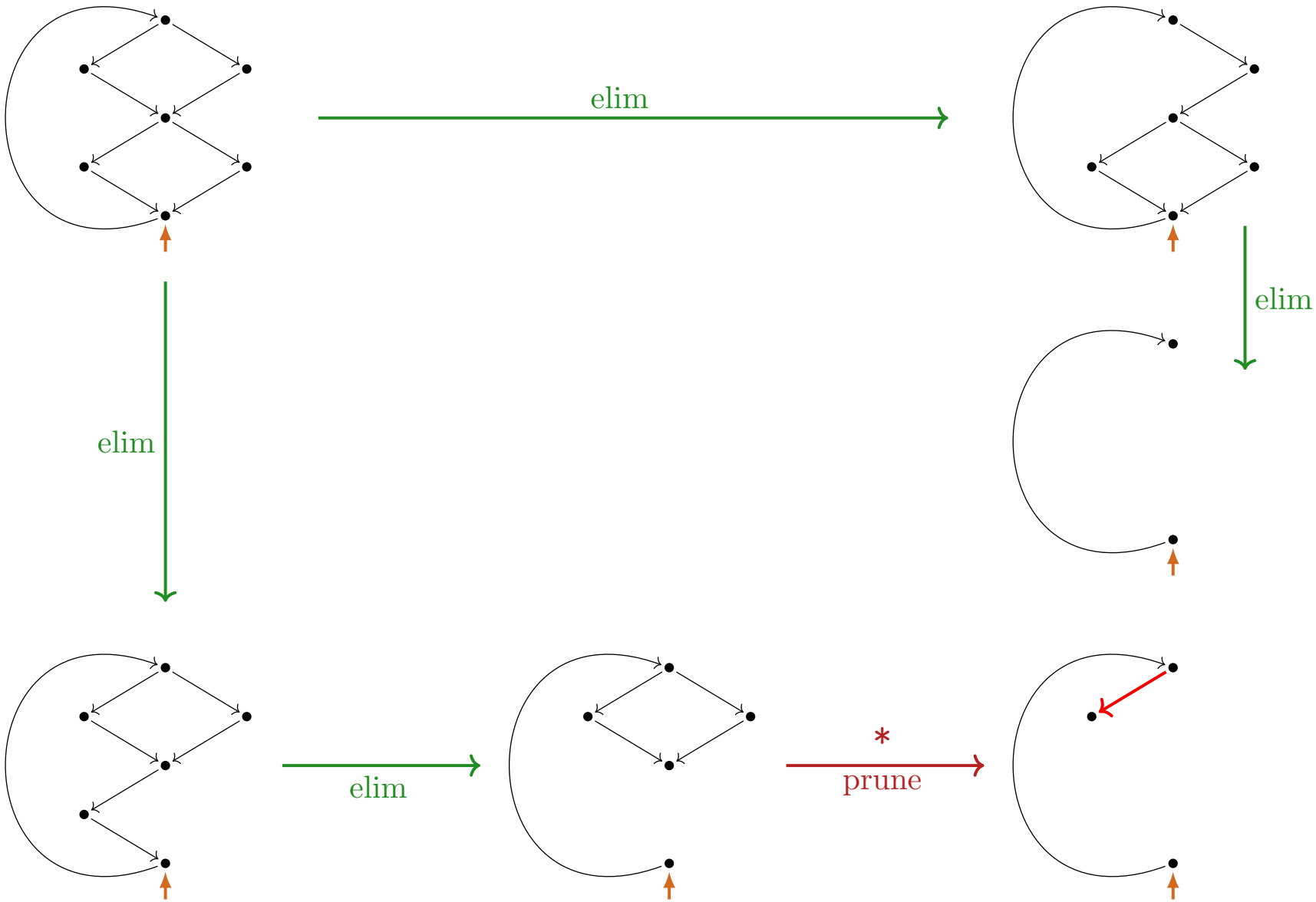
‘Critical pair’: bi-loop elimination



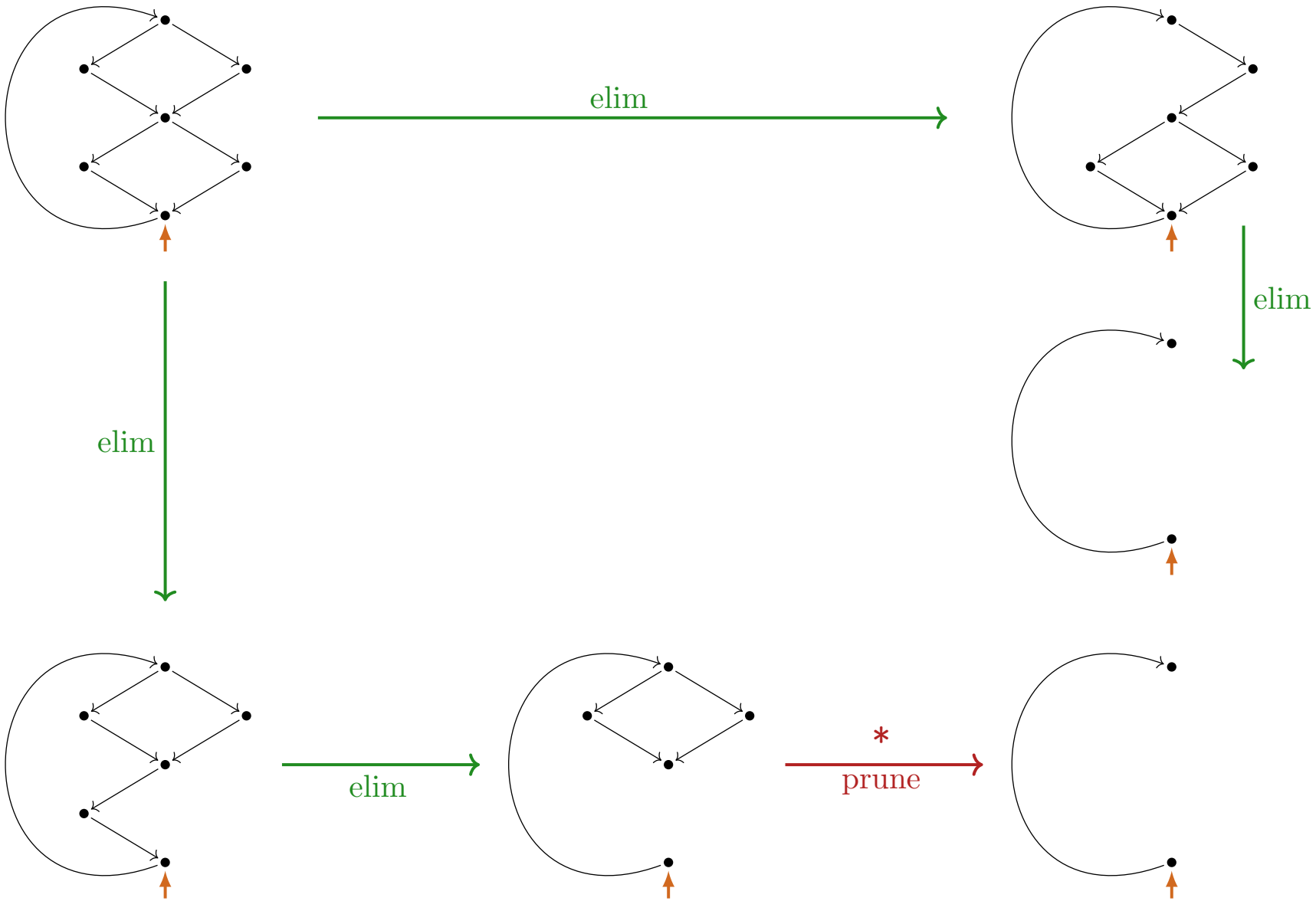
‘Critical pair’: bi-loop elimination



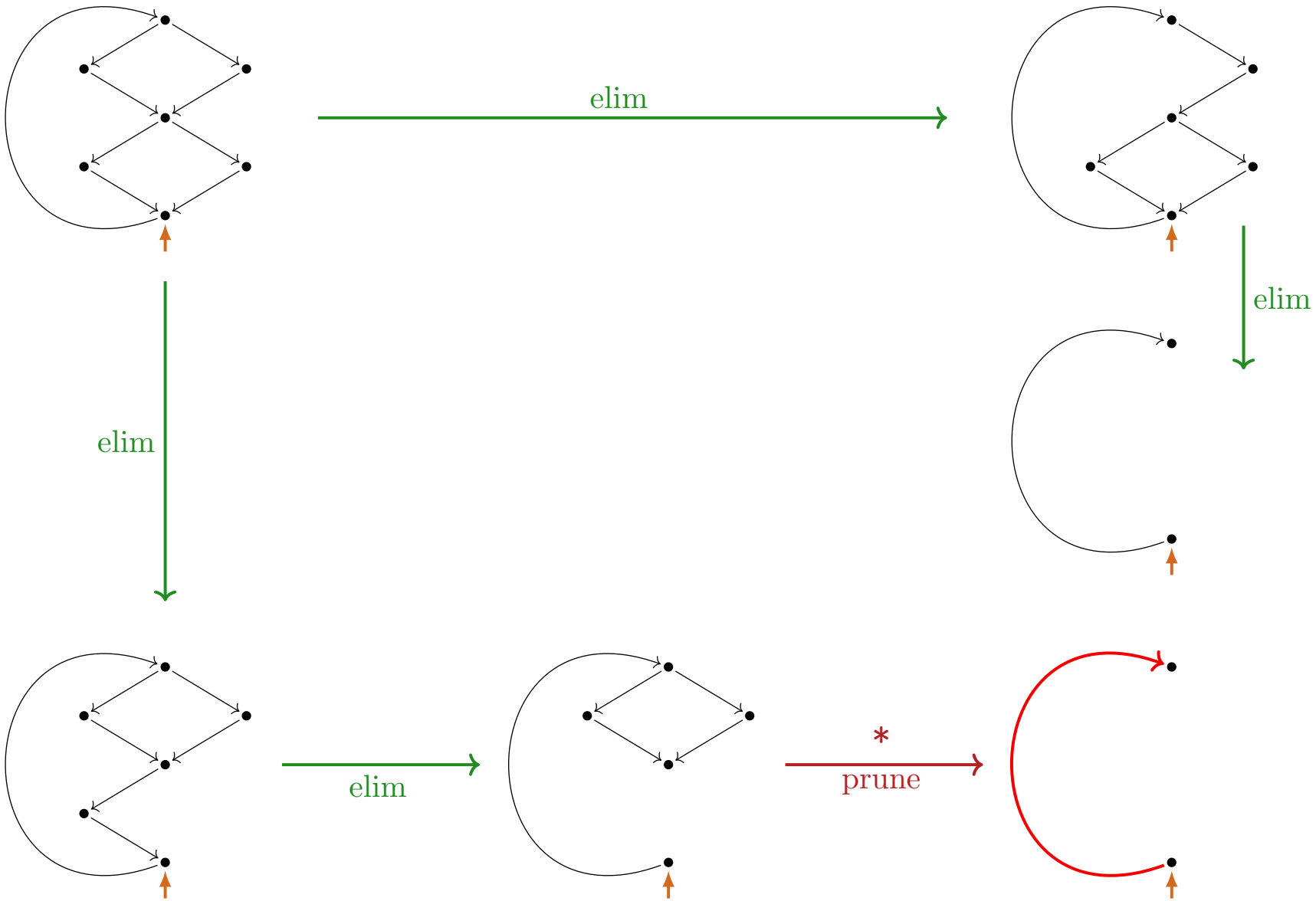
‘Critical pair’: bi-loop elimination



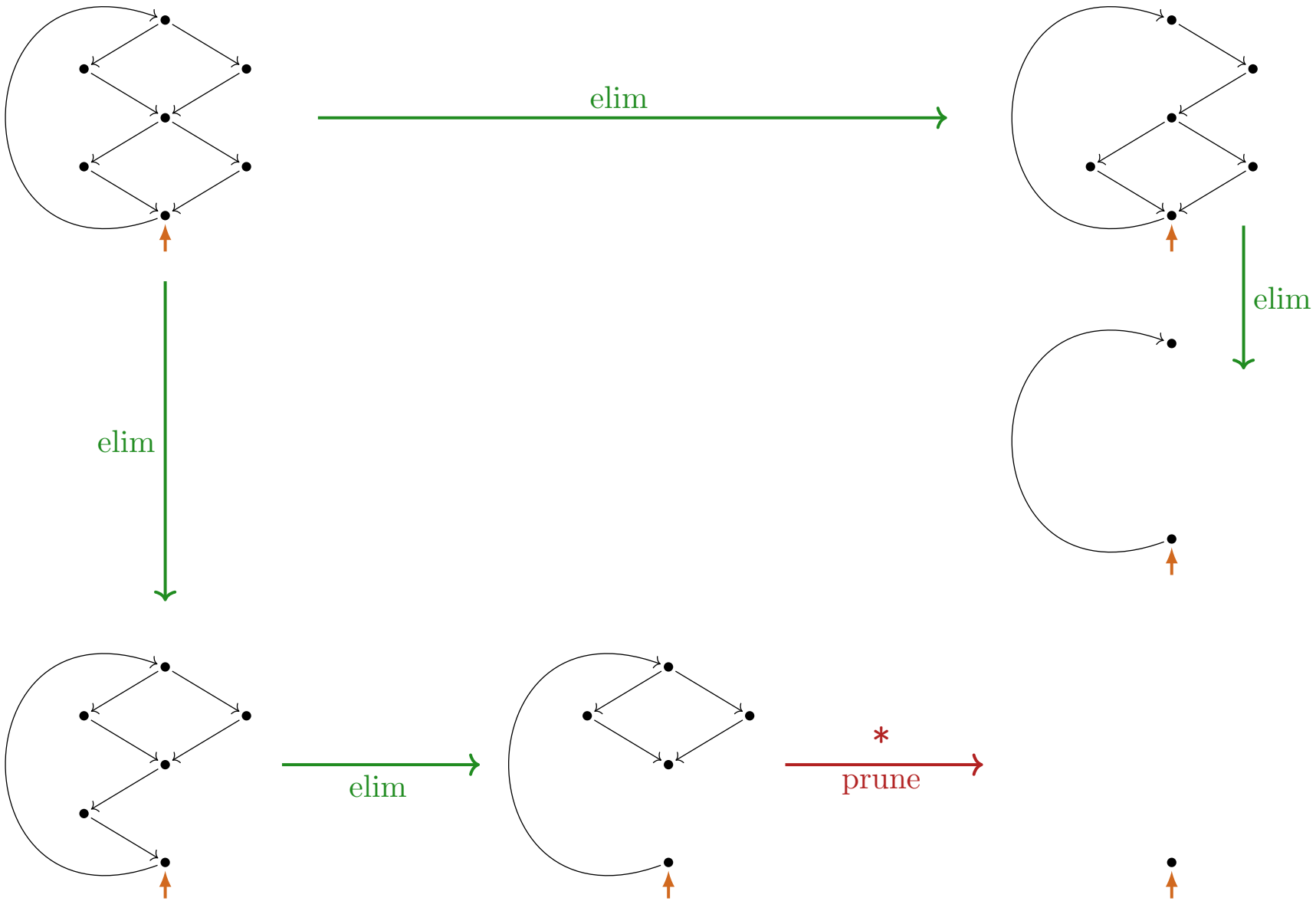
‘Critical pair’: bi-loop elimination



‘Critical pair’: bi-loop elimination



‘Critical pair’: bi-loop elimination



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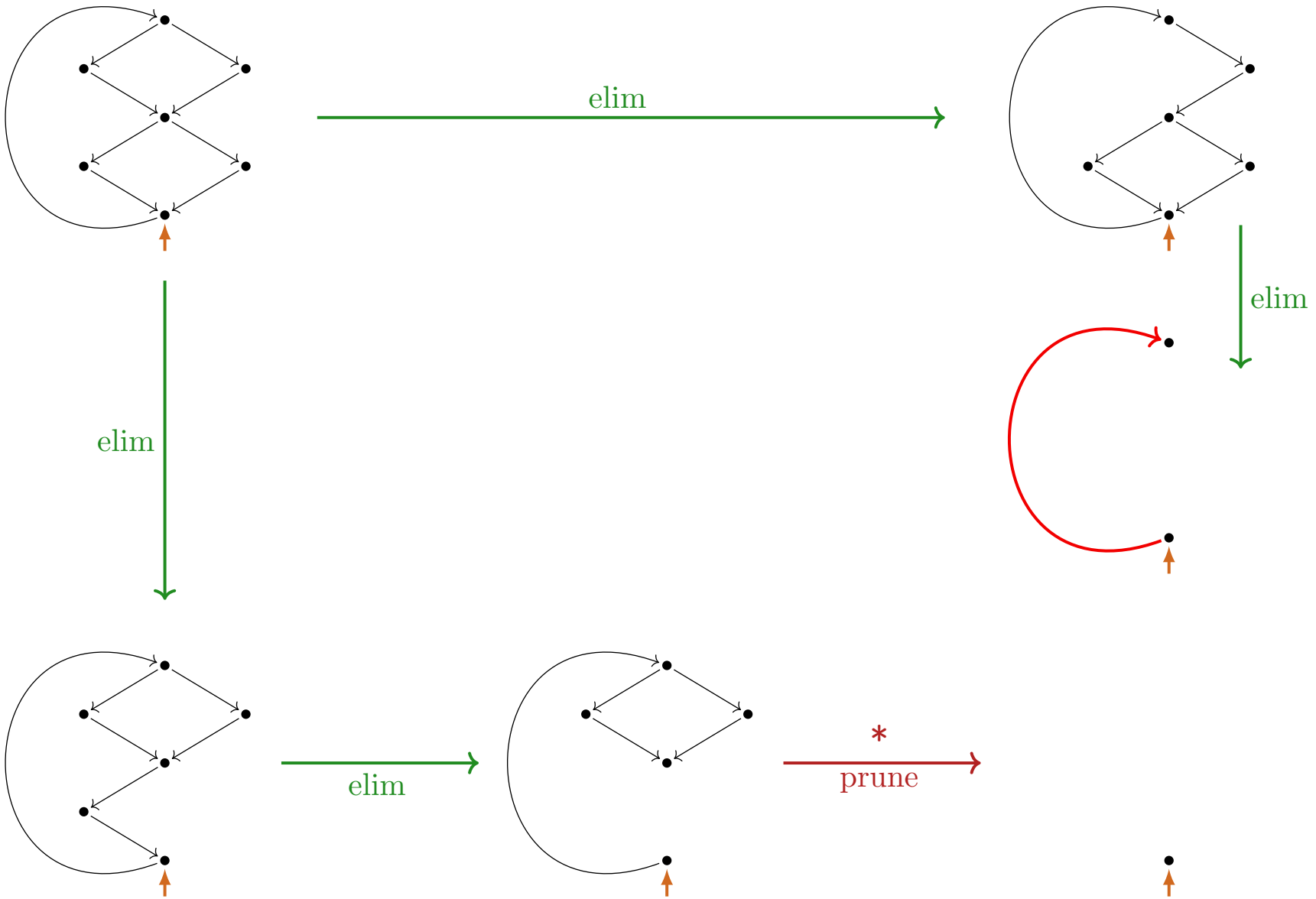

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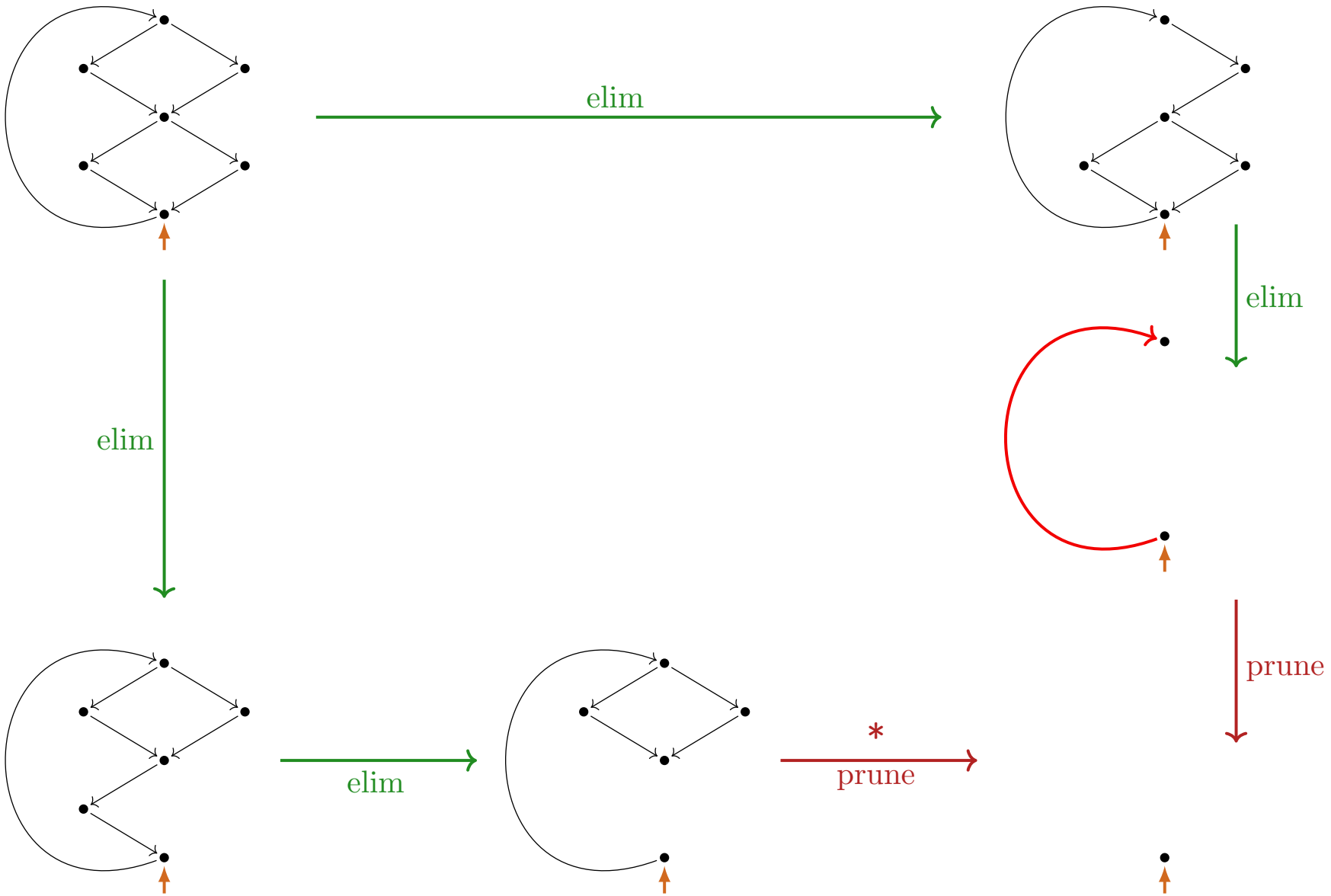

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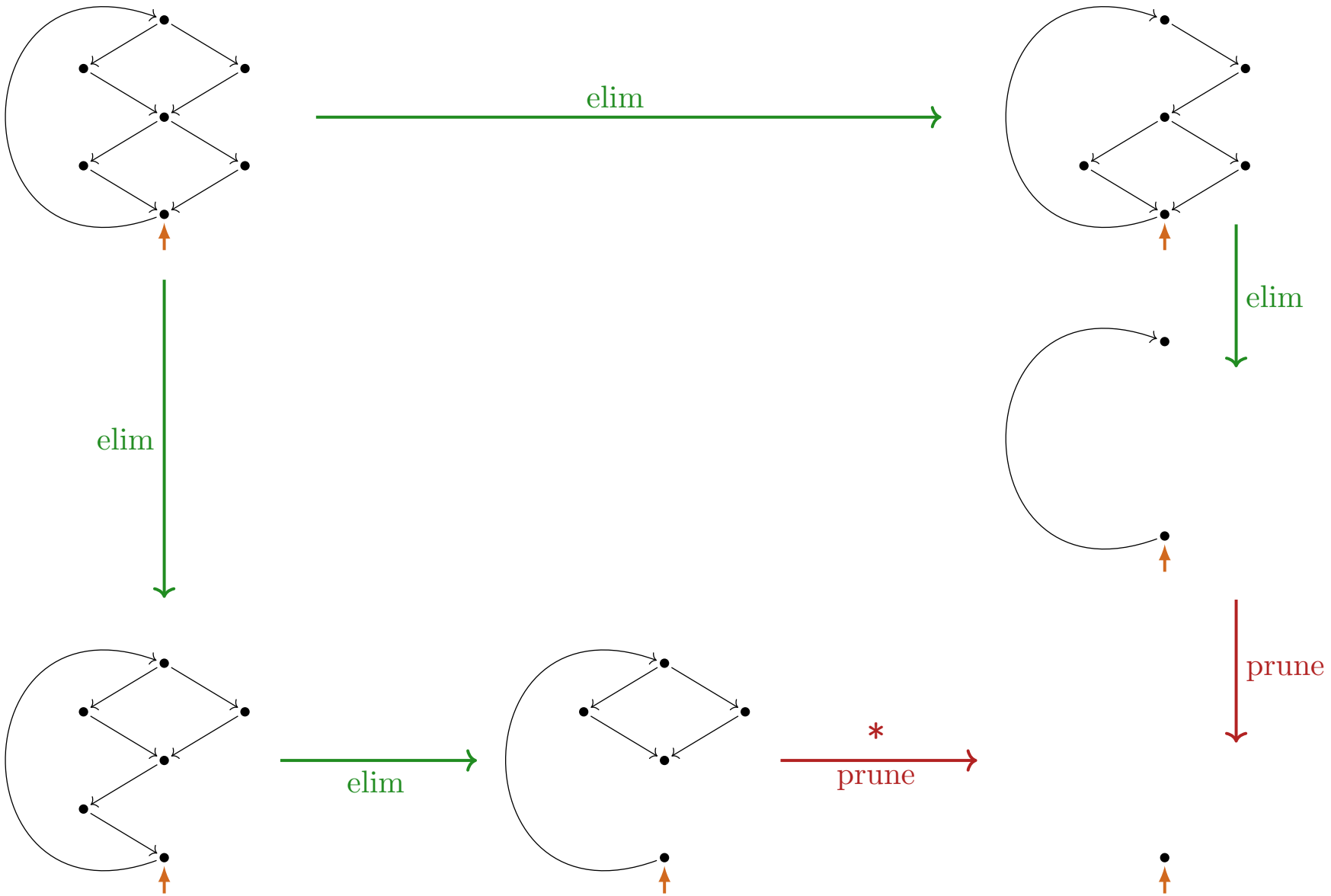

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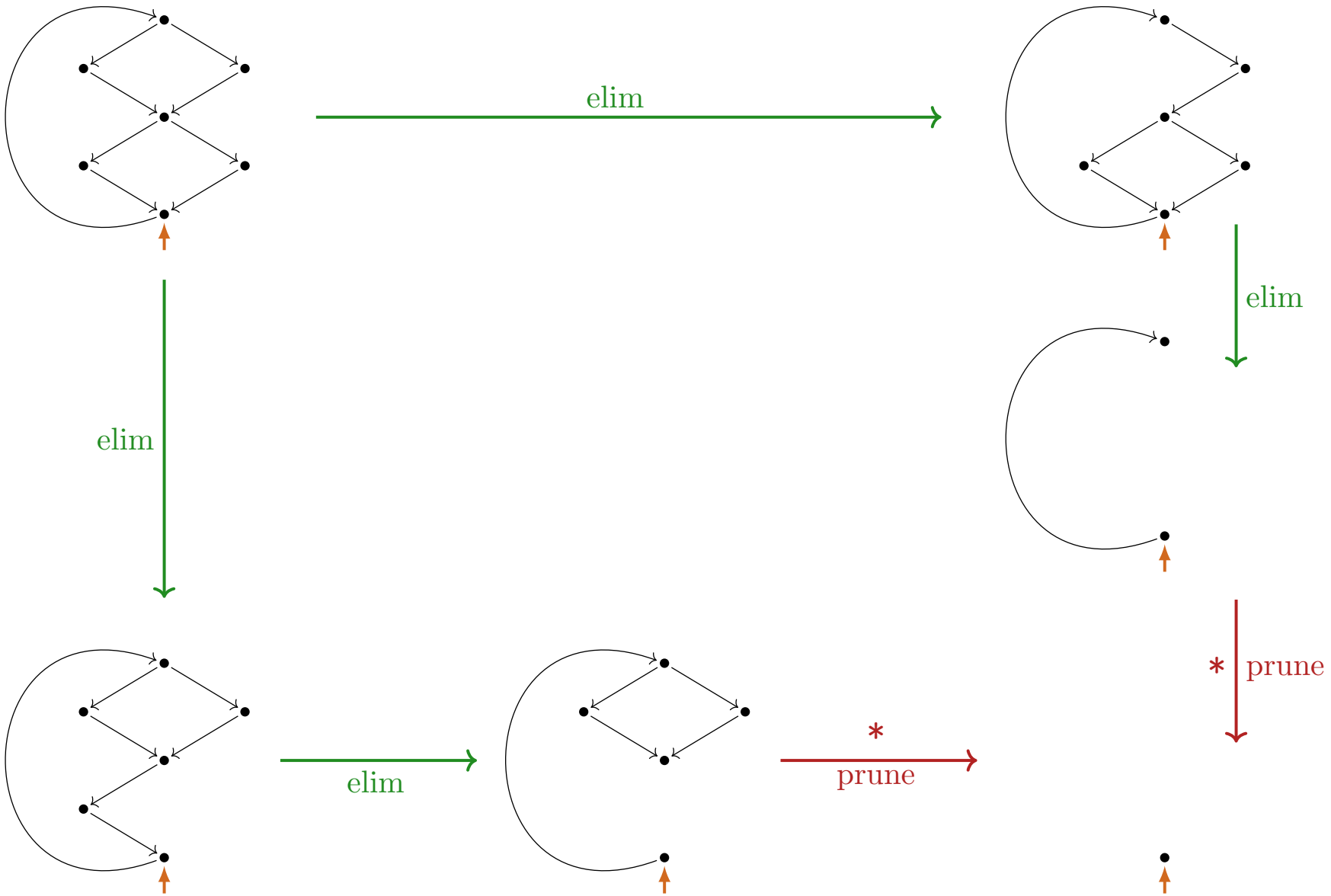

Loop elimination, and properties

$\longrightarrow_{\text{elim}}$: eliminate a transition-induced loop by:

- removing the loop-entry transition(s)
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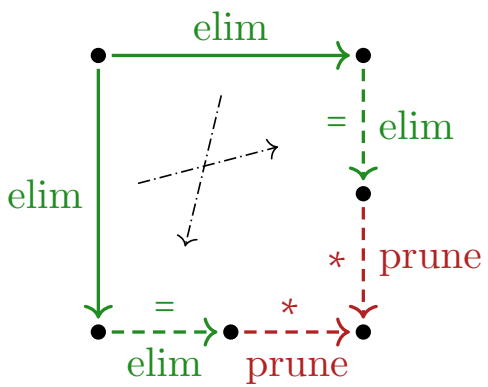
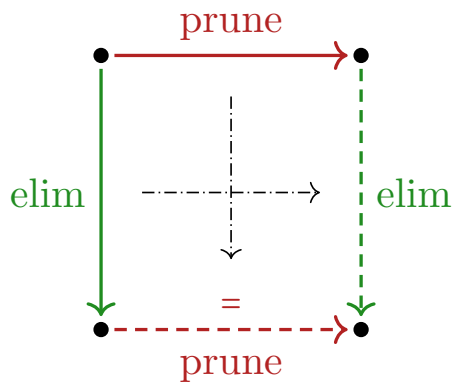
$\longrightarrow_{\text{prune}}$: remove a transition to a deadlocking state

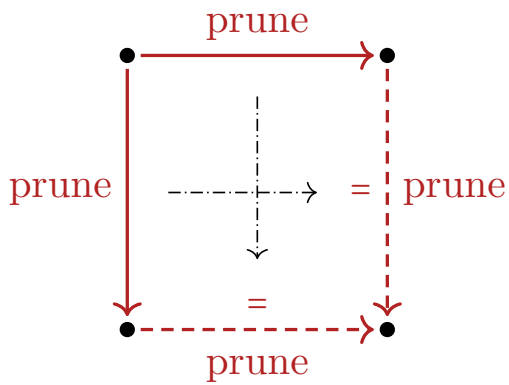
Lemma

(i) $\longrightarrow_{\text{elim}} \cup \longrightarrow_{\text{prune}}$ *is terminating.*

(ii) $\longrightarrow_{\text{elim}} \cup \longrightarrow_{\text{prune}}$ *is decreasing, and so due to (i) locally confluent.*







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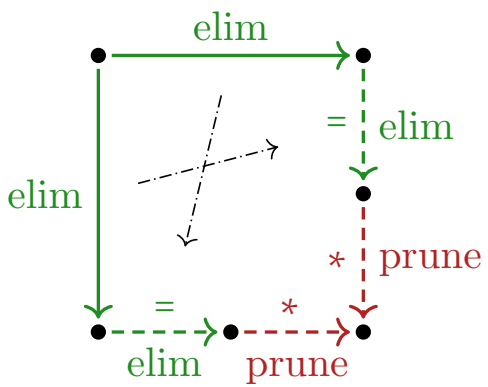

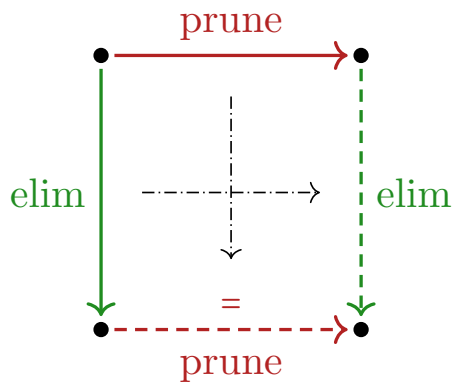

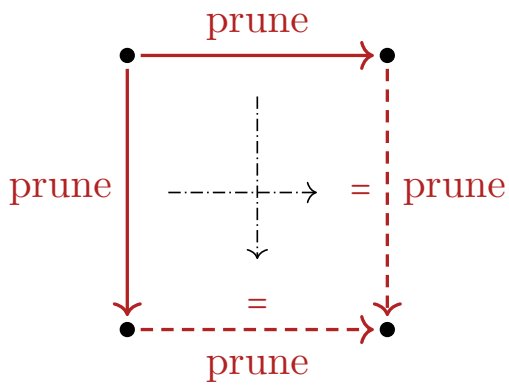

Structure property LEE

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A process graph G satisfies LEE (*loop existence and elimination*) if:

$$\exists G_0 \left(G \longrightarrow^*_{\text{elim}} G_0 \not\longrightarrow_{\text{elim}} \right.$$
$$\left. \wedge\ G_0 \text{ has no infinite trace} \right).$$

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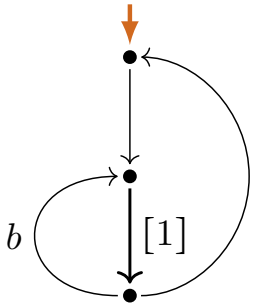
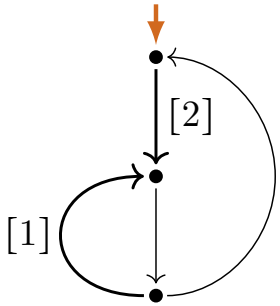
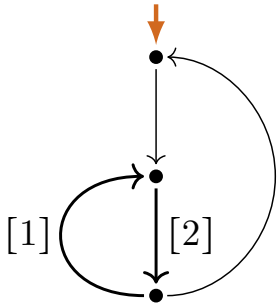
Theorem (efficient decidability)

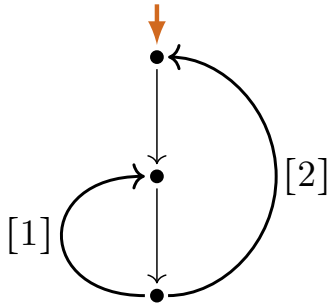
The problem of deciding LEE(G) *for process graphs G is in* PTIME.

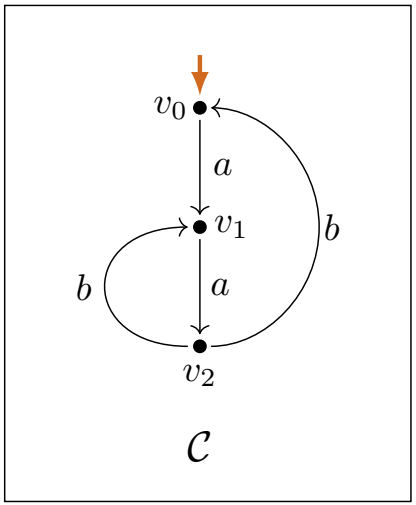
7 LEE-witnesses

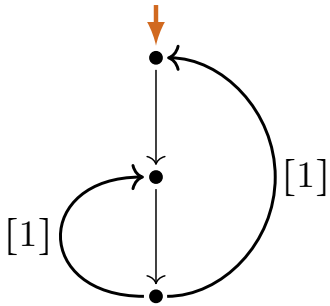


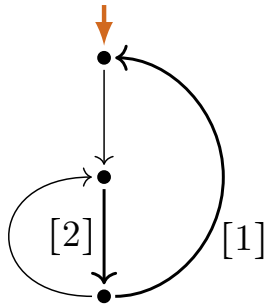


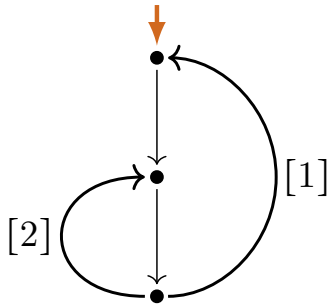












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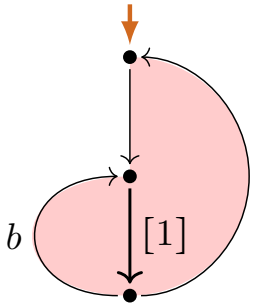

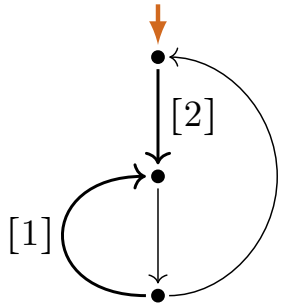

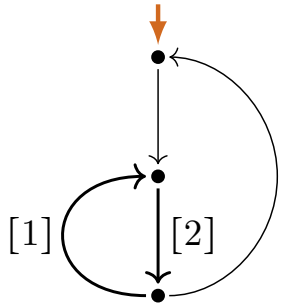

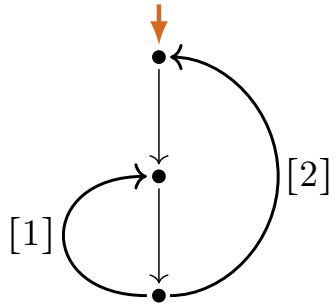

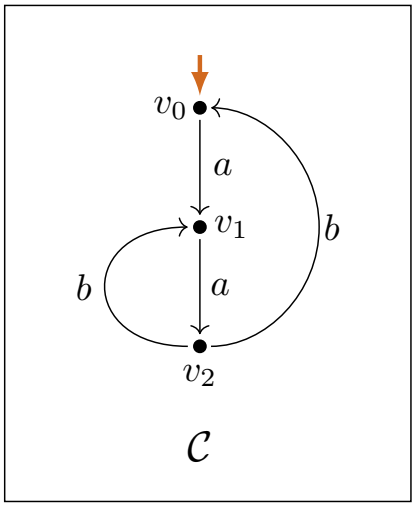

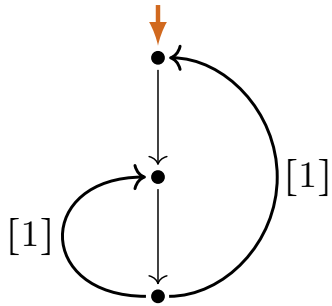

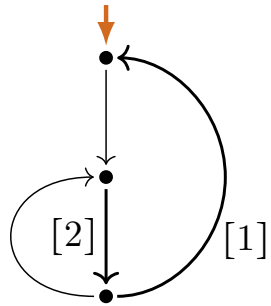

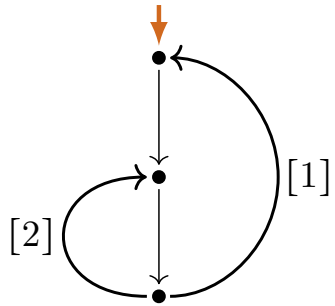

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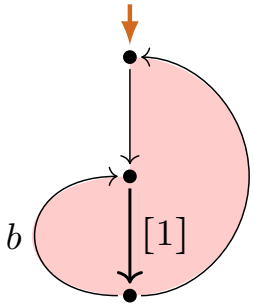

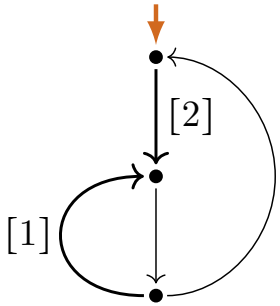

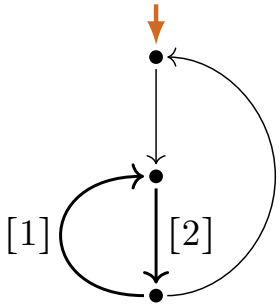

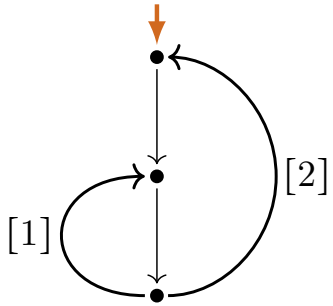

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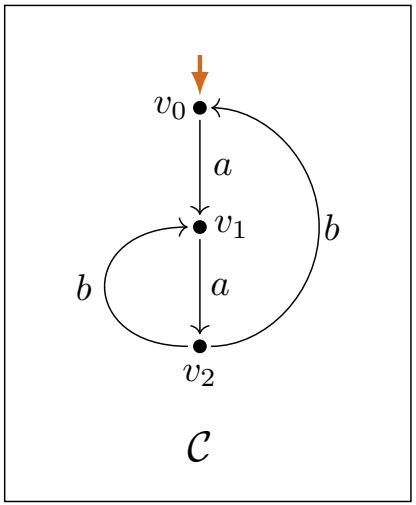

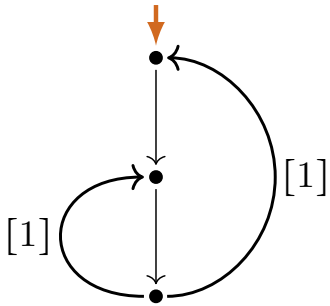

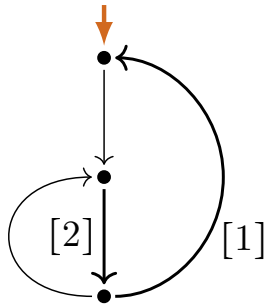

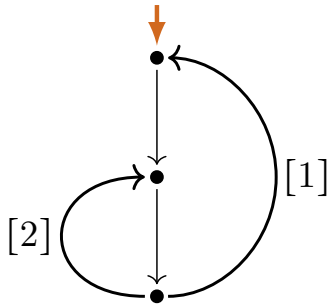

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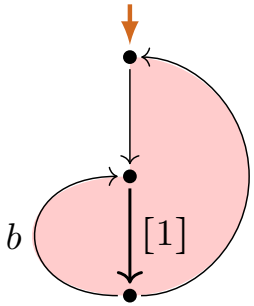

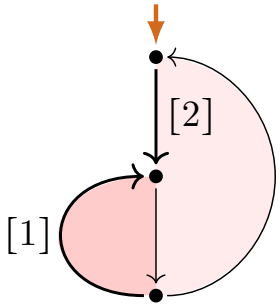

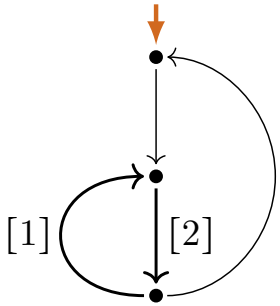

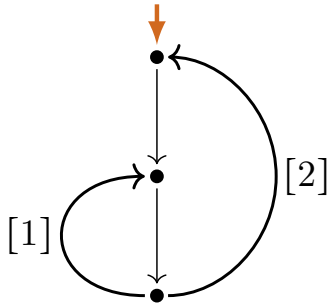

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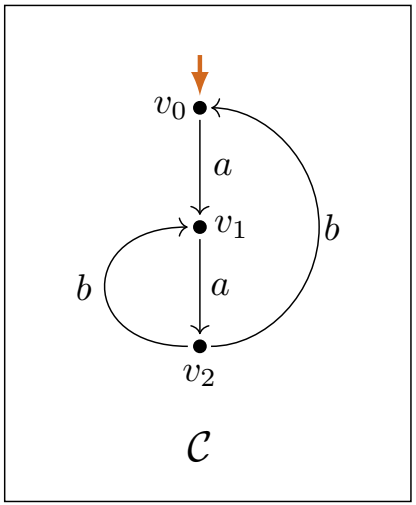

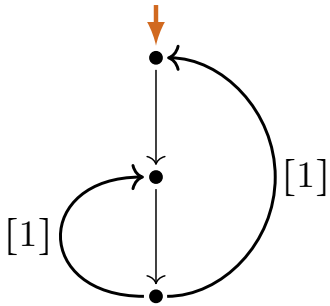

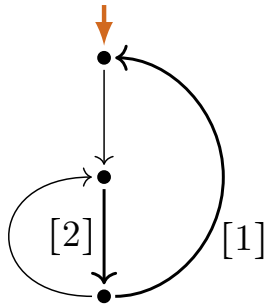

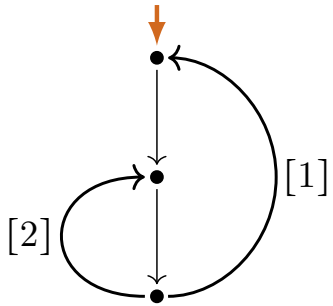

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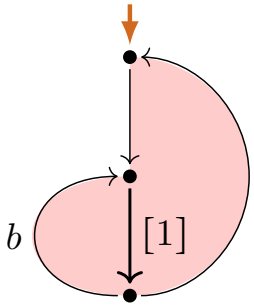

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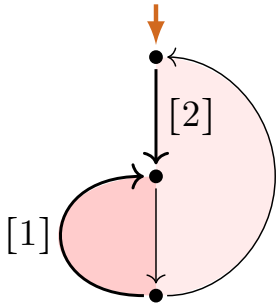

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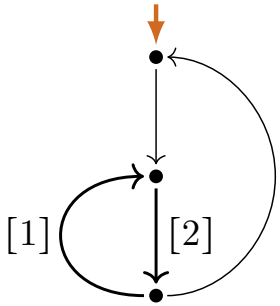
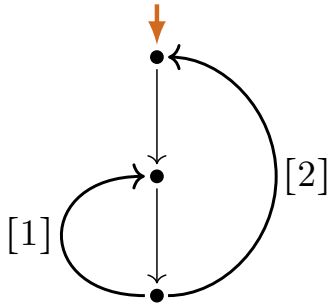
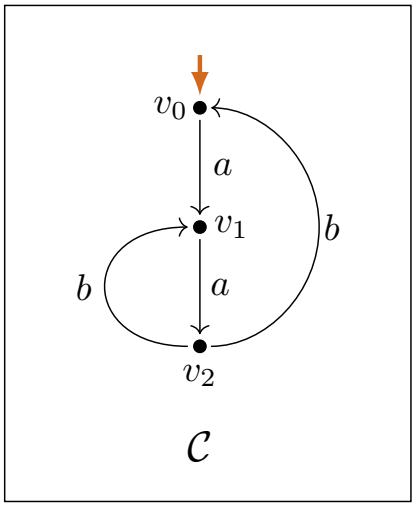
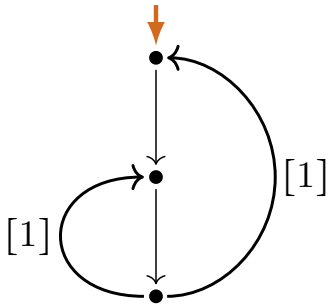
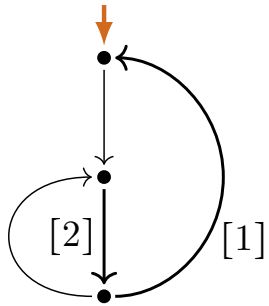
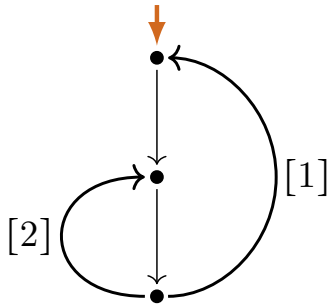




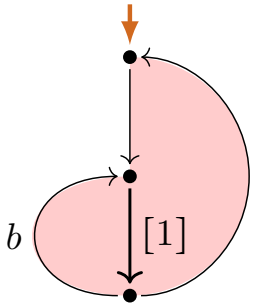








7 LEE-witnesses



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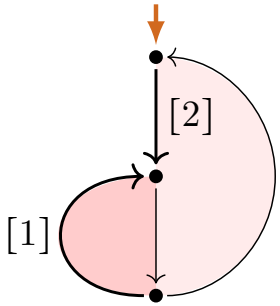

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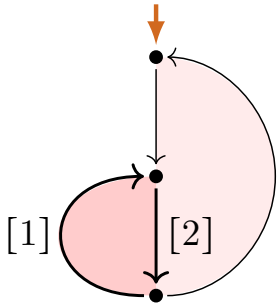
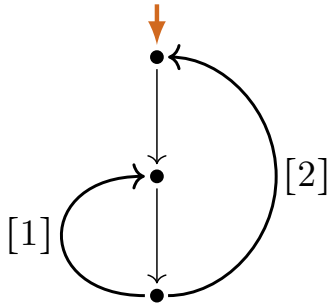
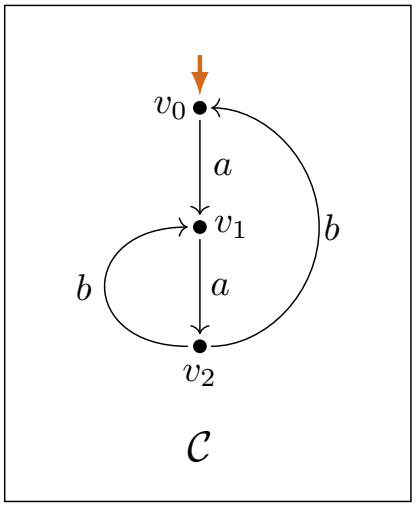
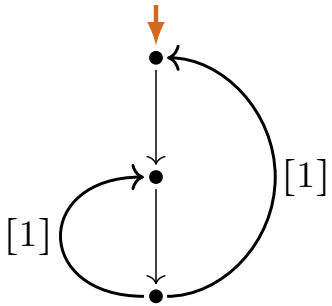
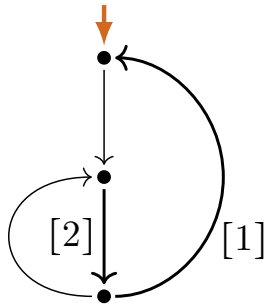
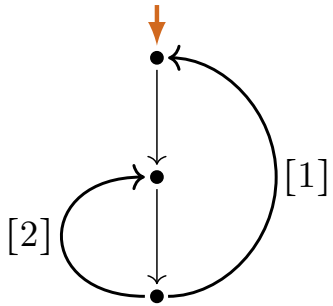




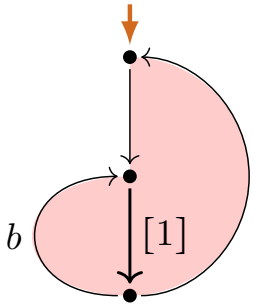








7 LEE-witnesses



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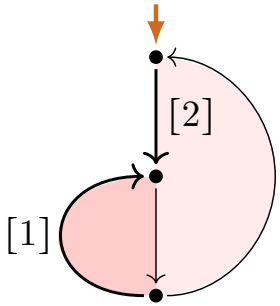

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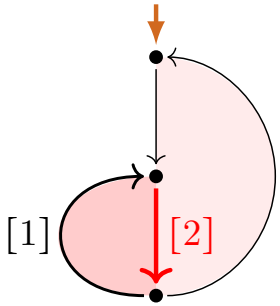
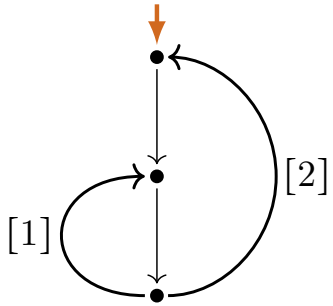
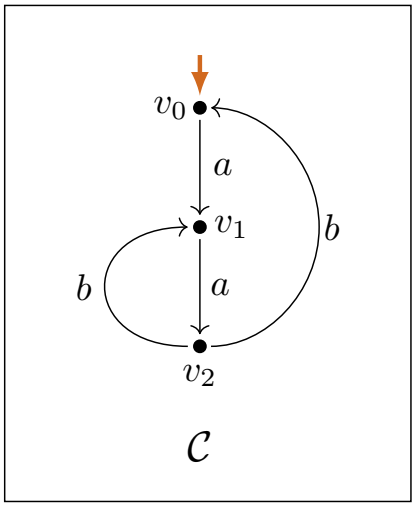
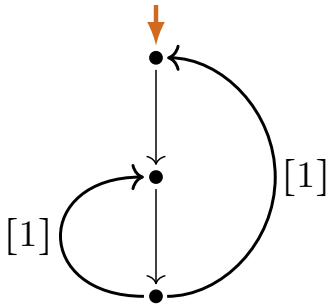
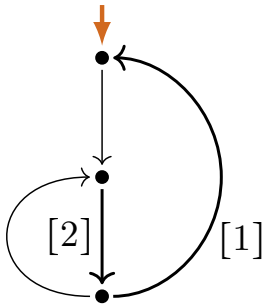
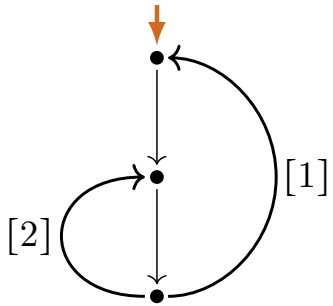




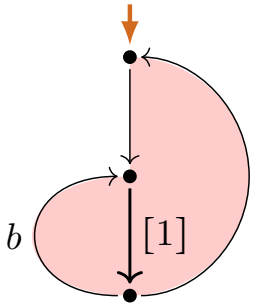








7 LEE-witnesses



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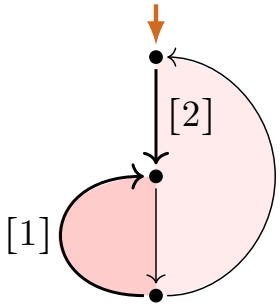

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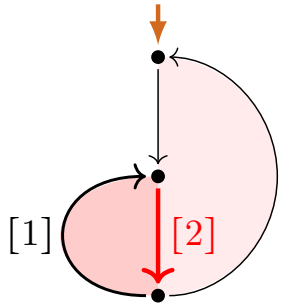

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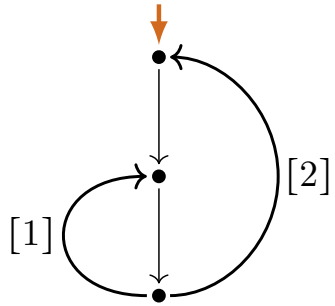
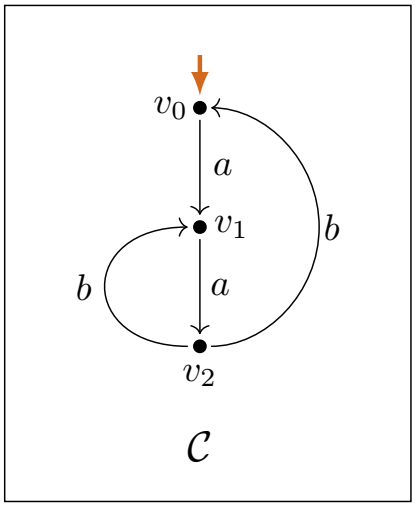
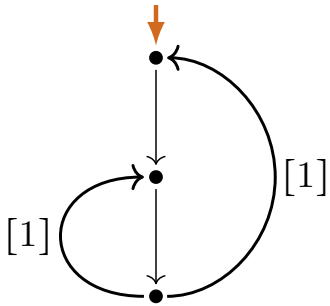
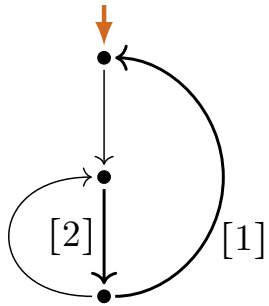
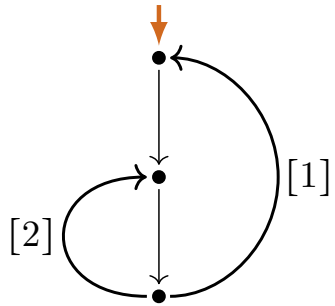


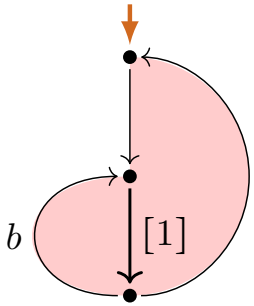








7 LEE-witnesses



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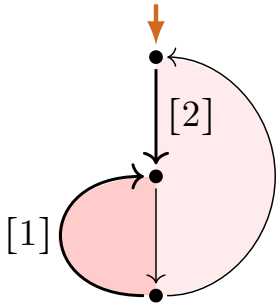

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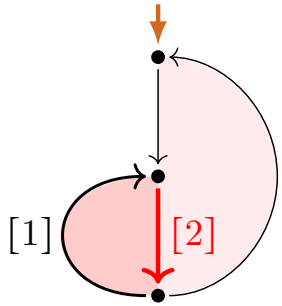

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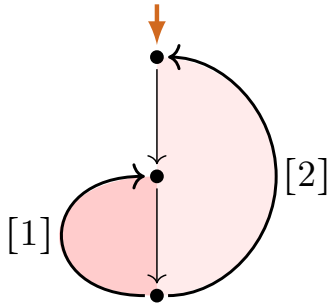
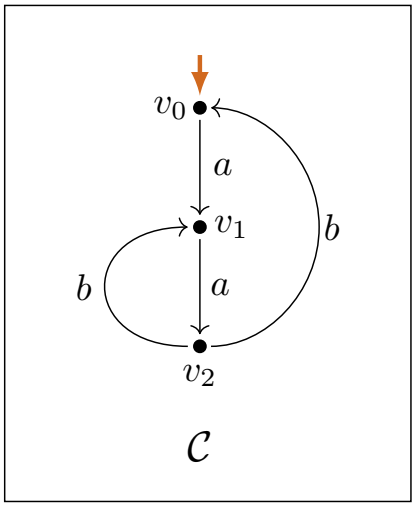
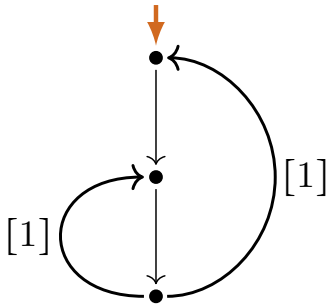
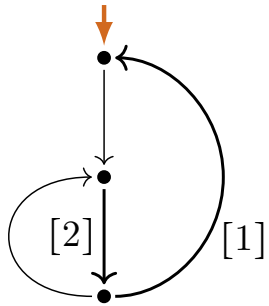
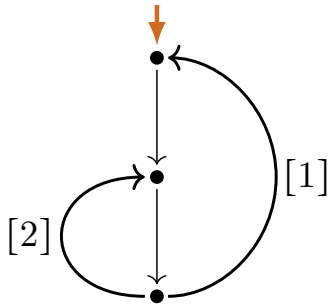


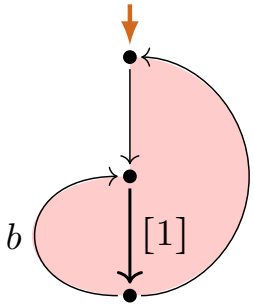








7 LEE-witnesses



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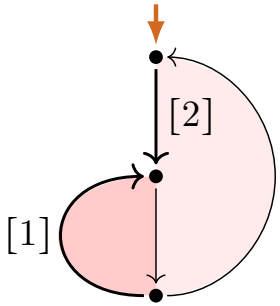

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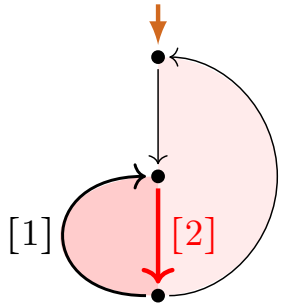

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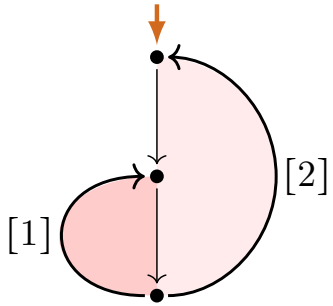

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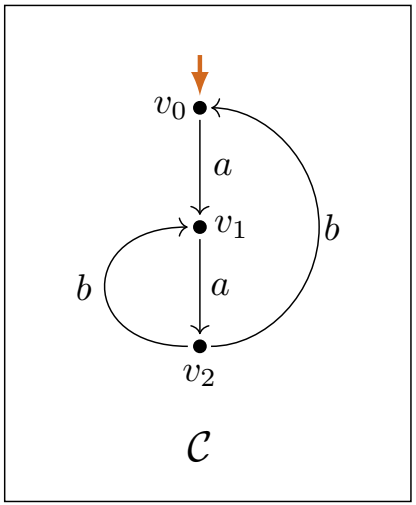
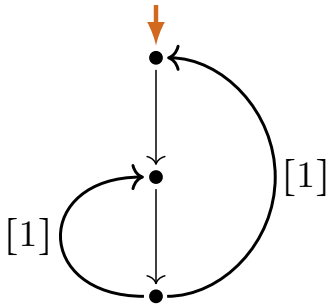
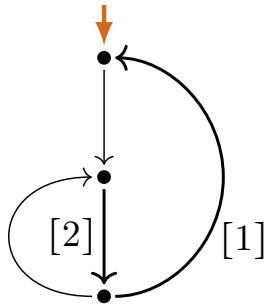
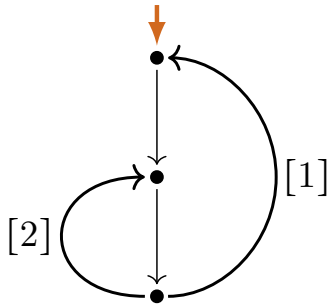
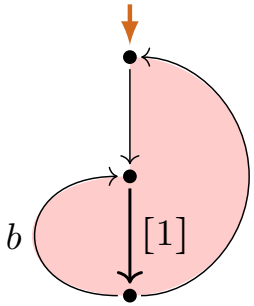








7 LEE-witnesses



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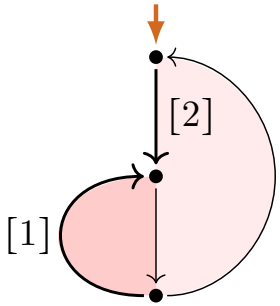

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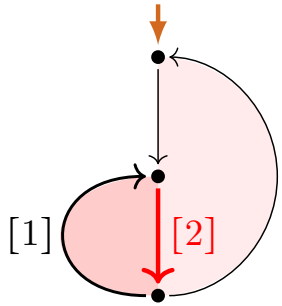

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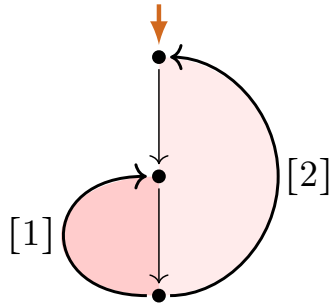

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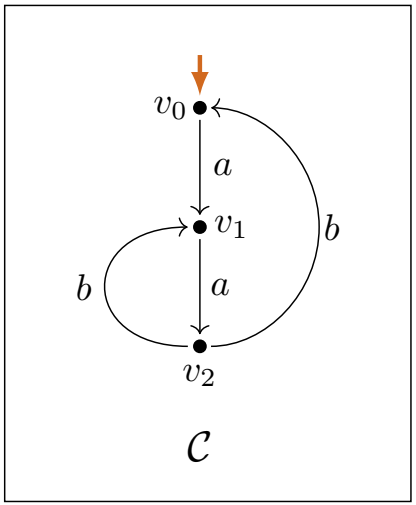
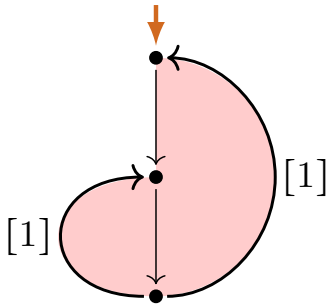
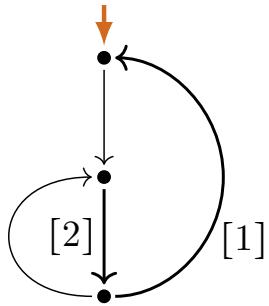
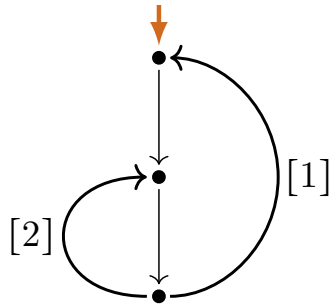
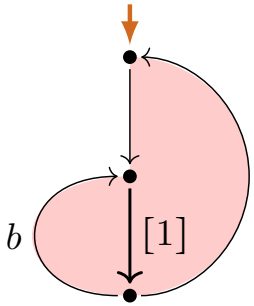








7 LEE-witnesses



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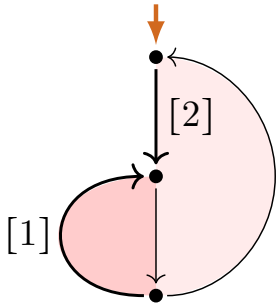

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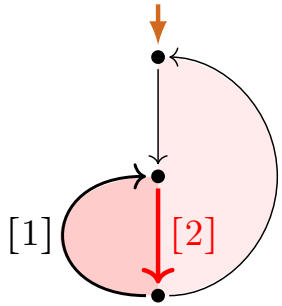

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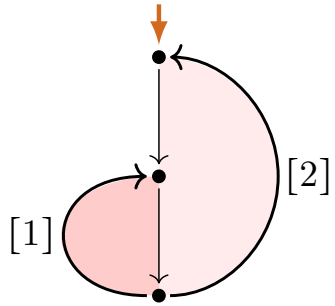

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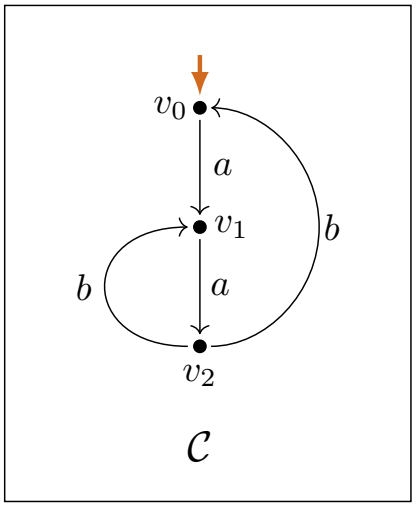

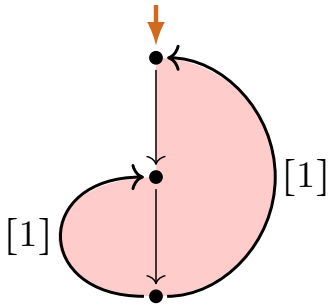

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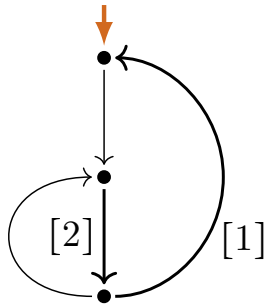
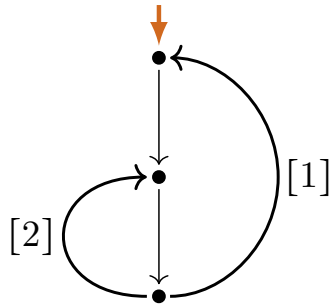
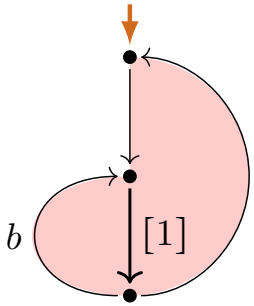




7 LEE-witnesses



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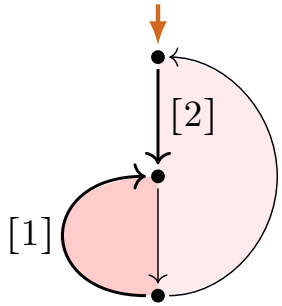

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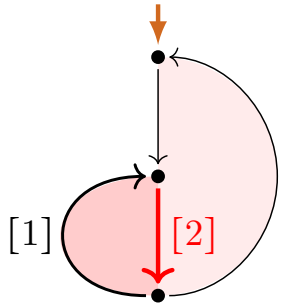

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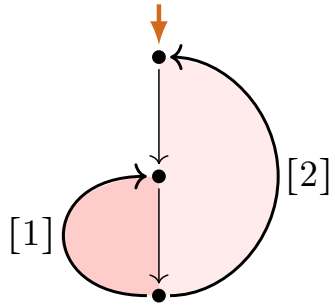

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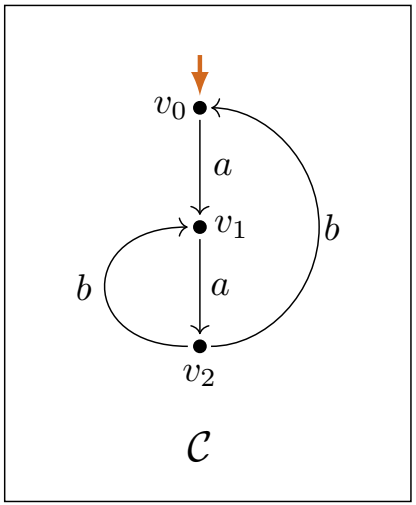

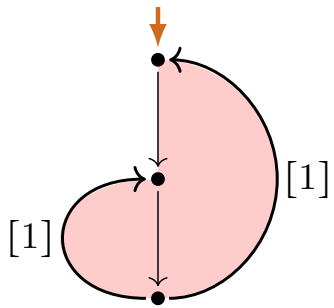

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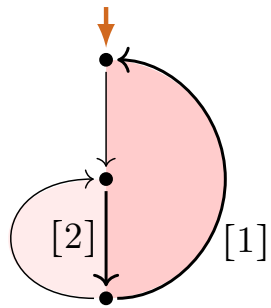
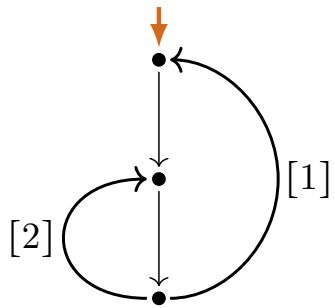
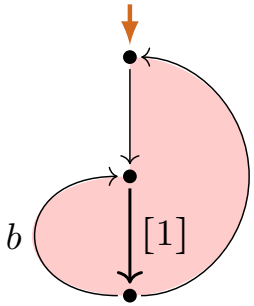




7 LEE-witnesses



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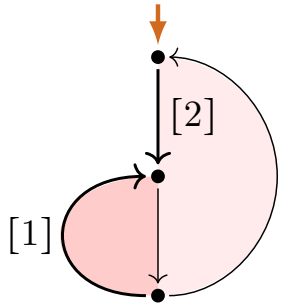

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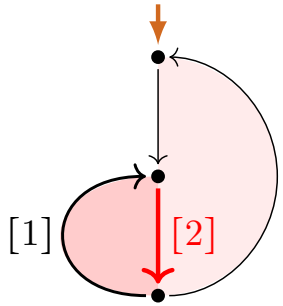

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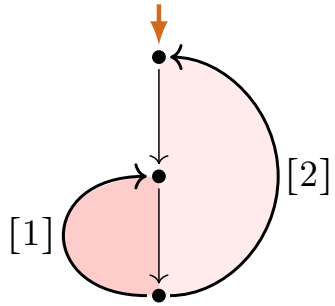

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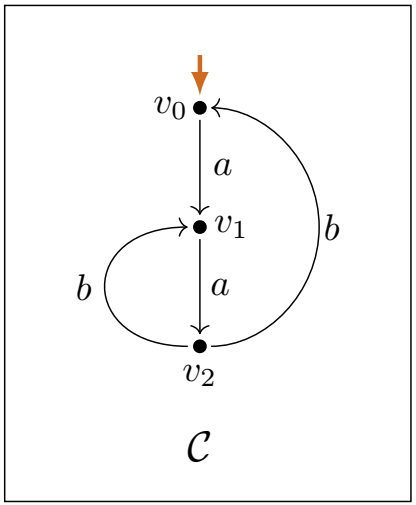

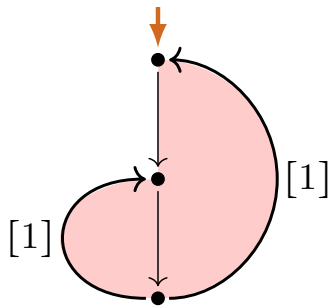

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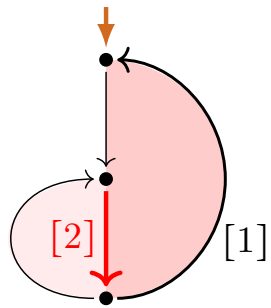
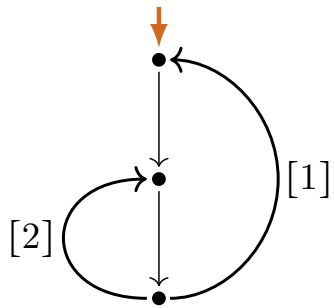
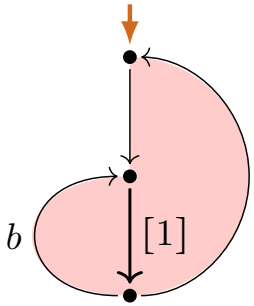




7 LEE-witnesses



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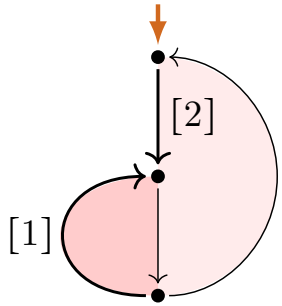

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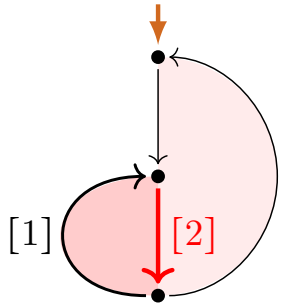

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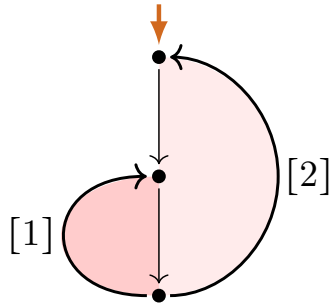

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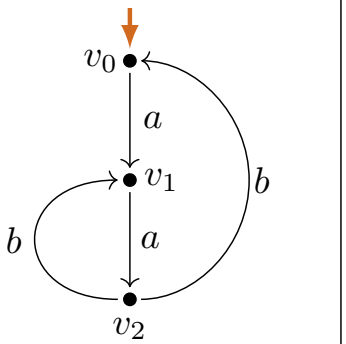

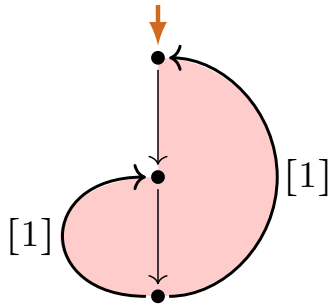

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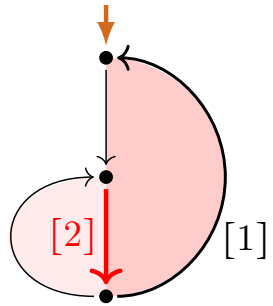

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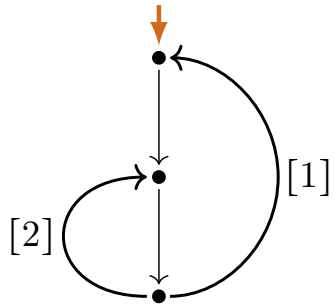

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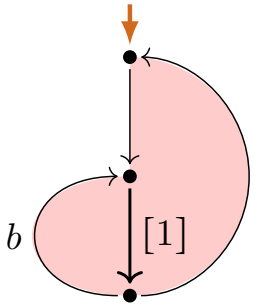

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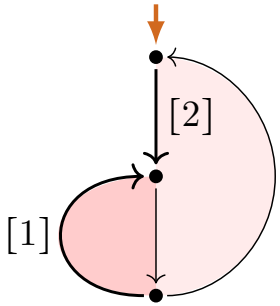

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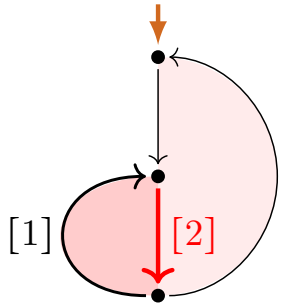

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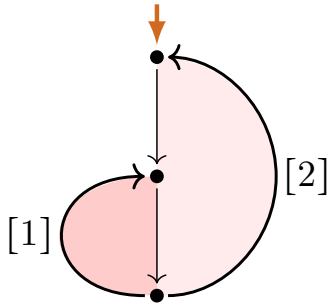

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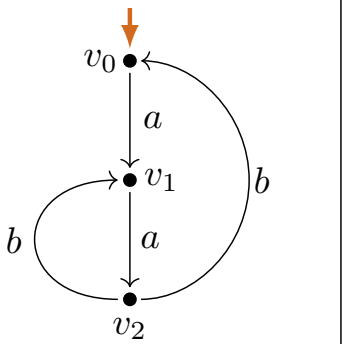

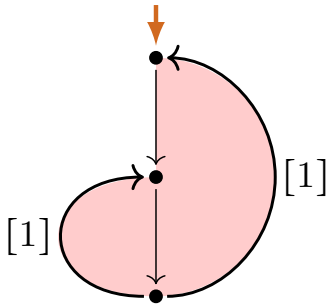

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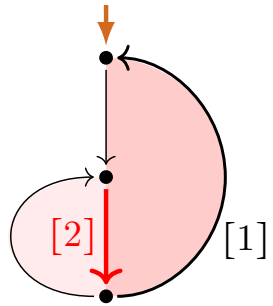

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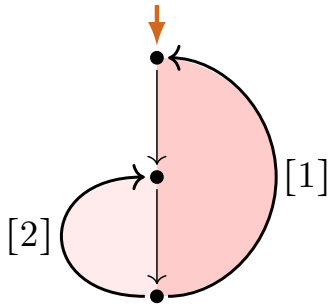

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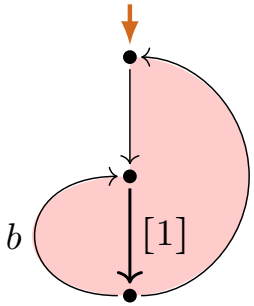

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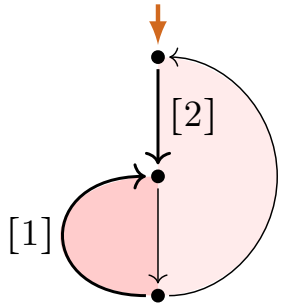

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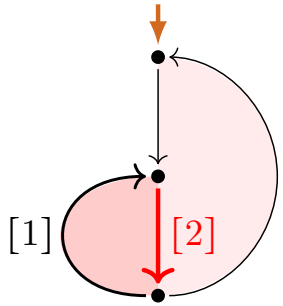

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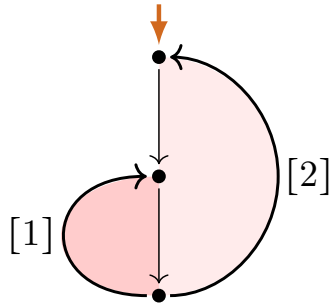

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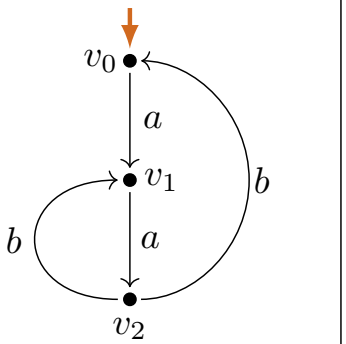

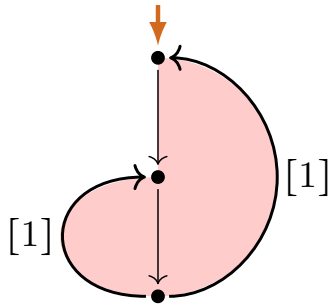

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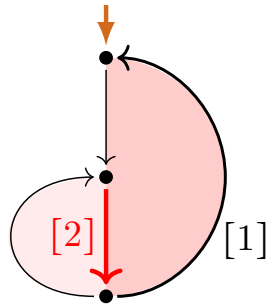

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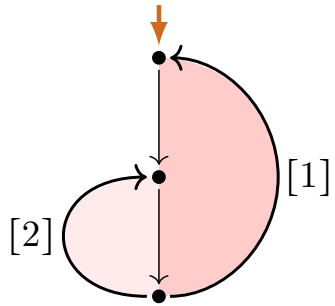

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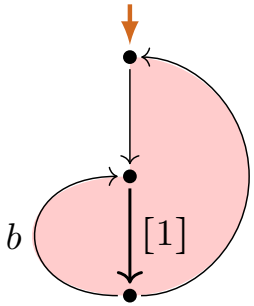
7 LEE-witnesses



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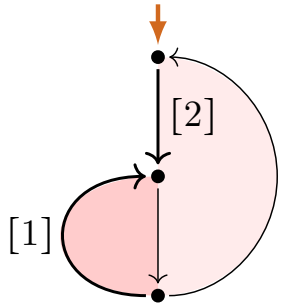

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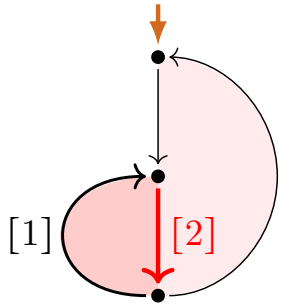

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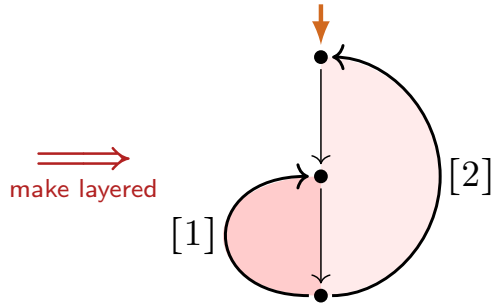

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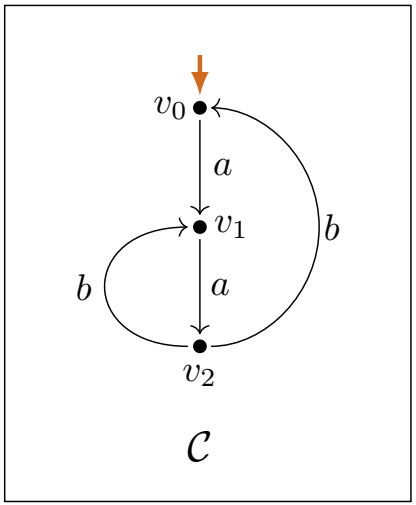

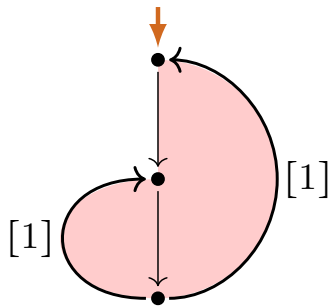

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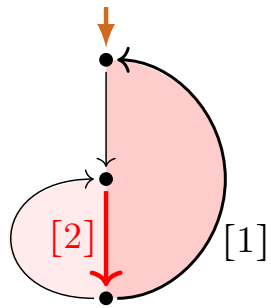

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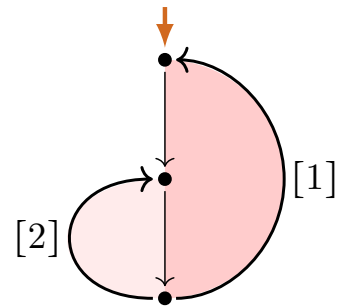
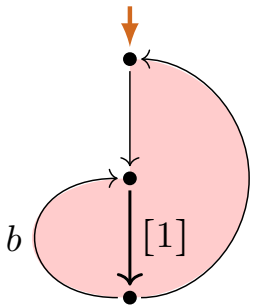


layered

7 LEE-witnesses



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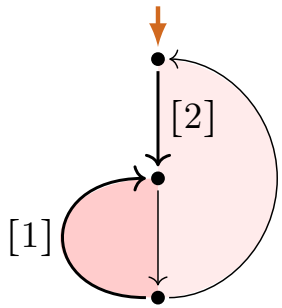

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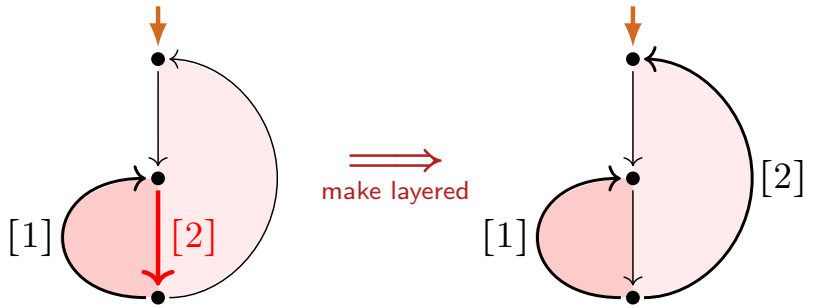

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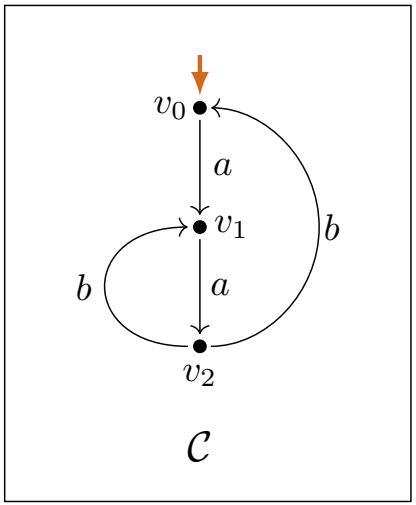

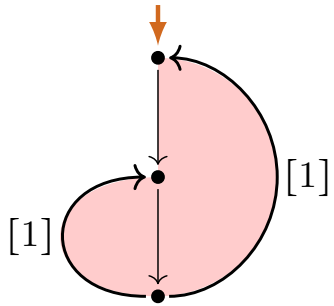

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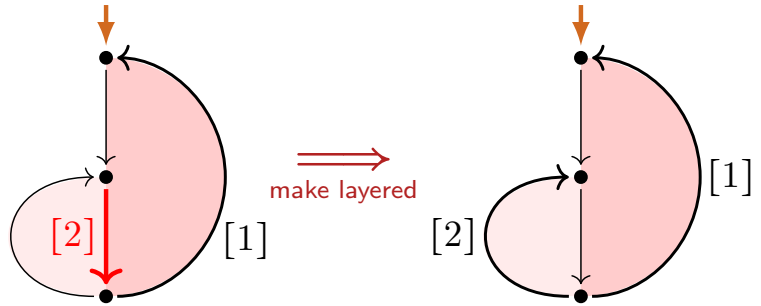
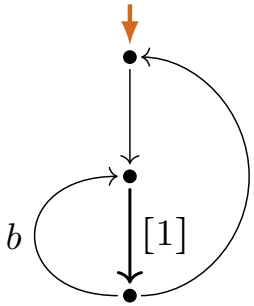


not layered layered

7 LEE-witnesses



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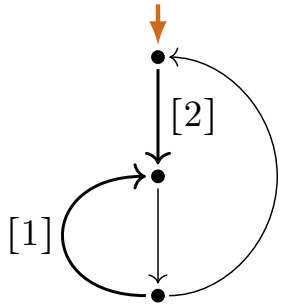

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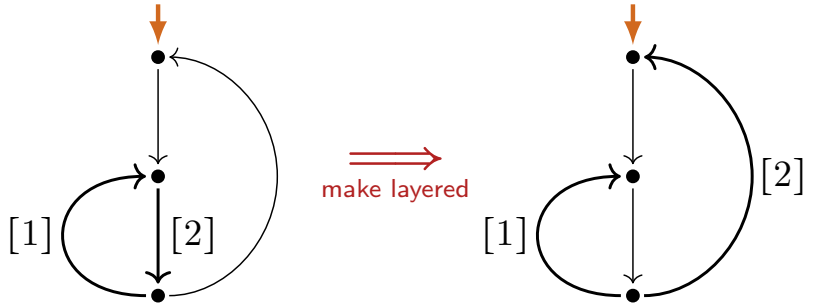

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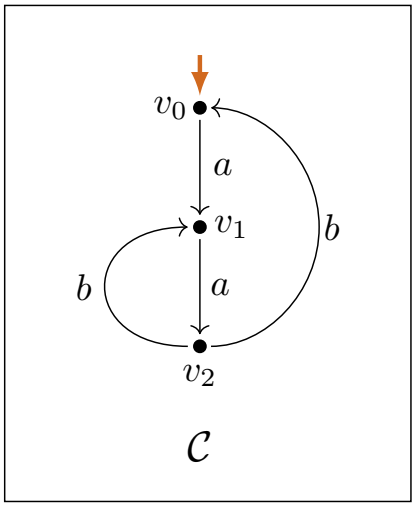

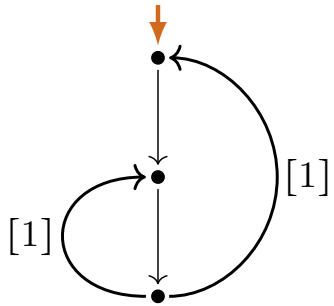

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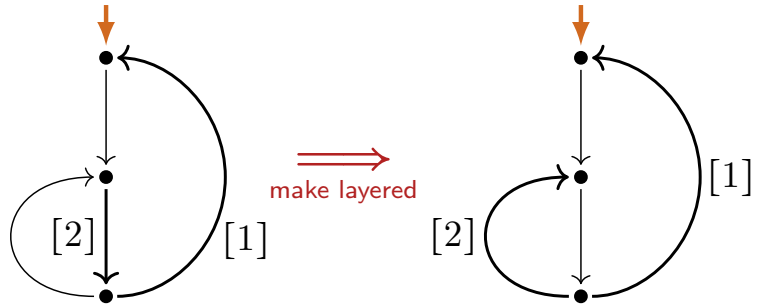

not layered layered

Interpretation/extraction correspondences with LEE

(⇐ G/Fokkink 2020, G 2021)

(Int)$_P^{(*/\not{1})}$: *P-$(*/\not{1})$-expressible graphs have the structural property* LEE.

Process **i**nterpretations $P(e)$ of $(*/\not{1})$ regular expressions e are finite process graphs that satisfy LEE.

(Extr)$_P$: LEE *implies* $[\![\cdot]\!]_P$*-expressibility*

From every finite process graph G with LEE a regular expression e can be **e**xtracted such that $G \leftrightarrows P(e)$.

Interpretation/extraction correspondences with LEE

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(Extr)$_P$: LEE *implies* $[\![\cdot]\!]_P$*-expressibility*

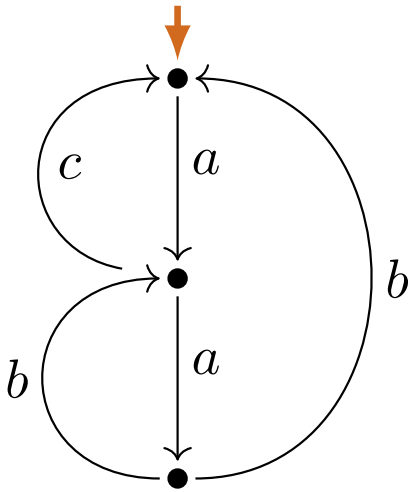
From every finite process graph G with LEE a regular expression e can be **e**xtracted such that $G \leftrightarrows P(e)$.

(Coll): LEE *is preserved under collapse*

The class of finite process graphs with LEE is closed under bisimulation **c**ollapse.

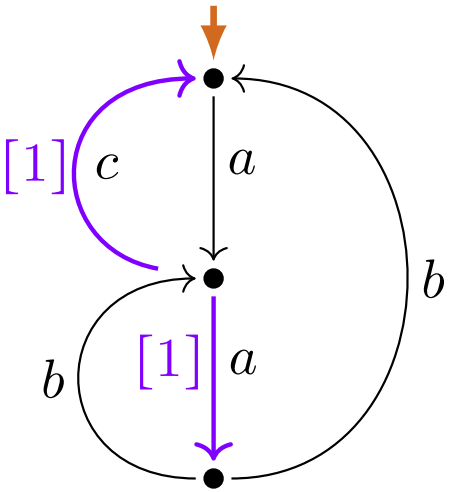
Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

G_2



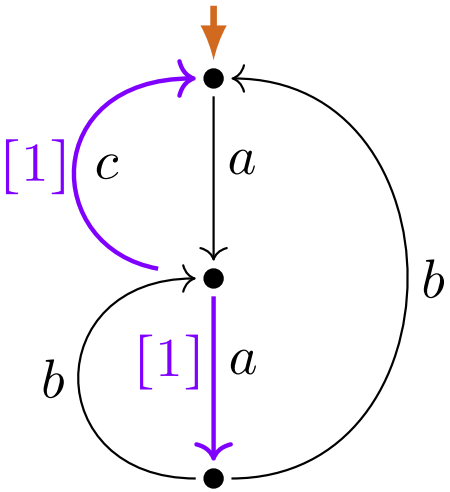
Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_2}$



Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_2}$

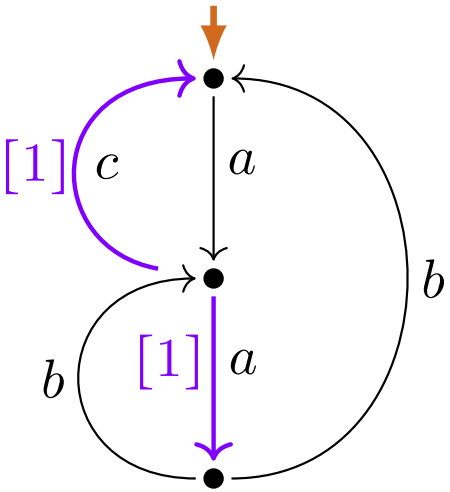


$$($$

$$)^* \cdot 0$$

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

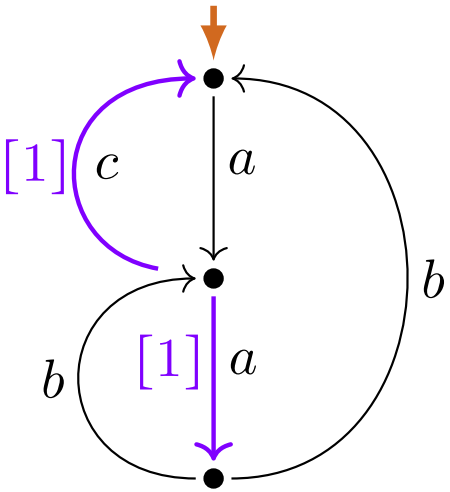
$\widehat{G_2}$



$($ a $)^* \cdot 0$

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_2}$

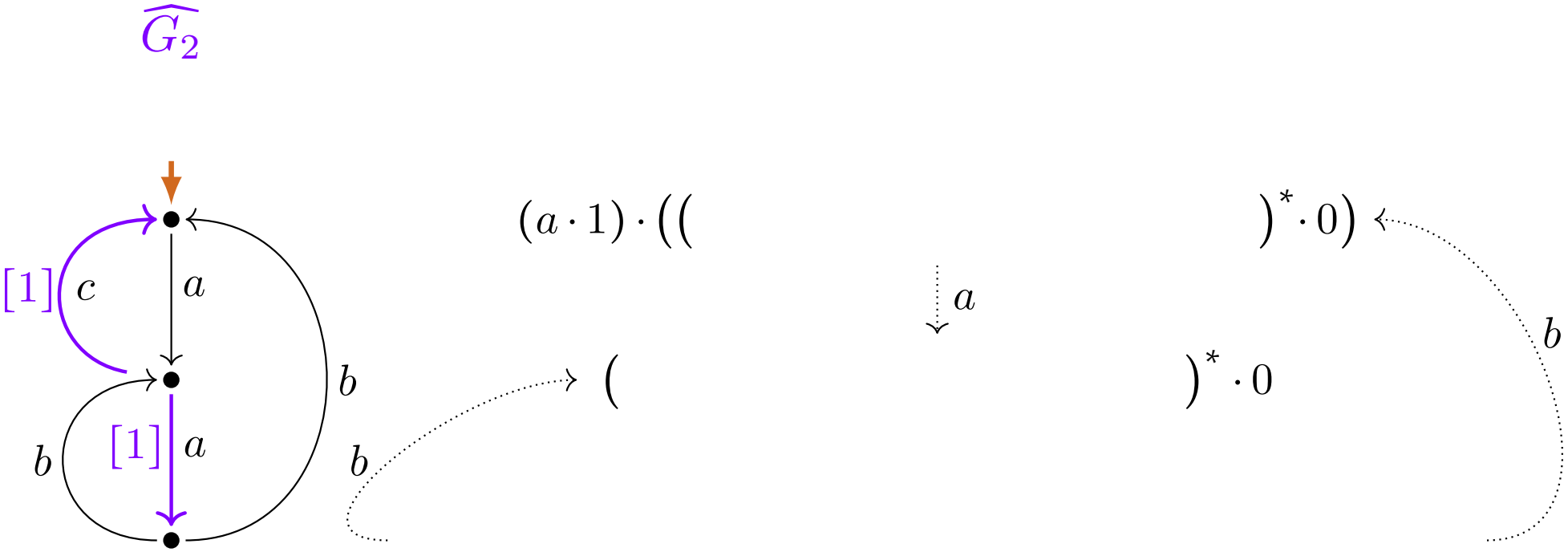


$(a \cdot 1) \cdot (($ $)^* \cdot 0)$

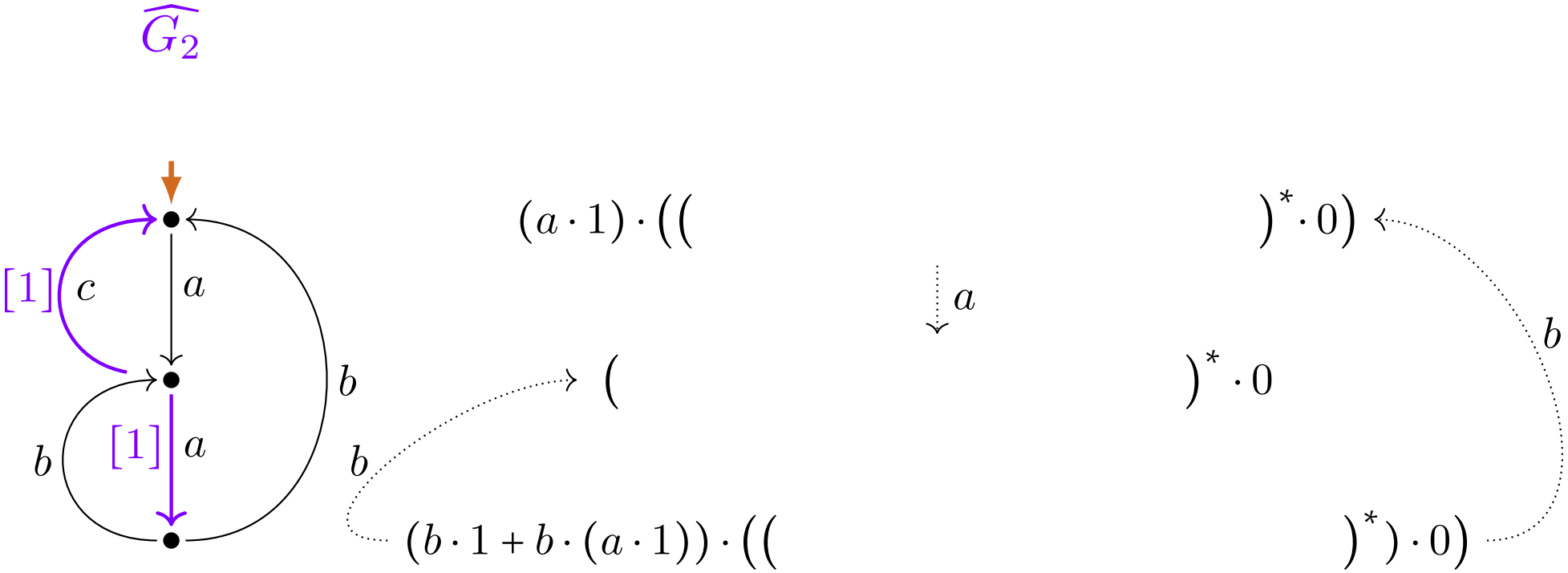
a

$($ $)^* \cdot 0$

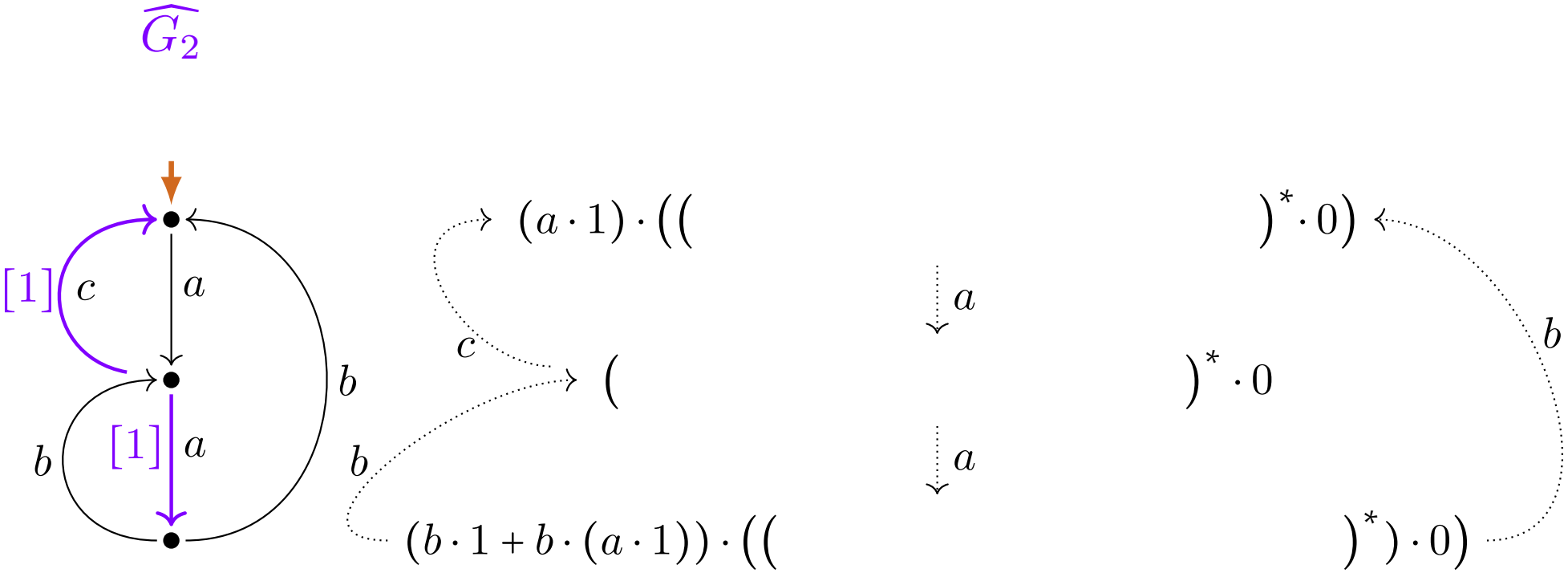
Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)



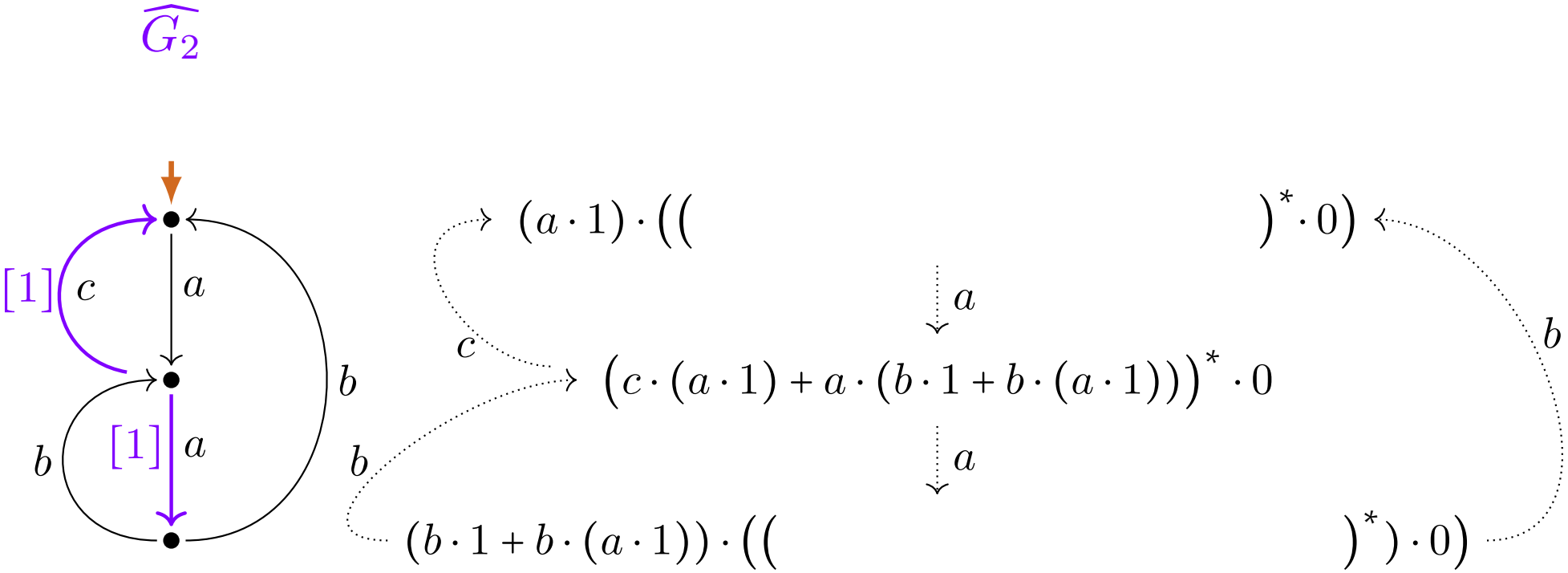
Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)



Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

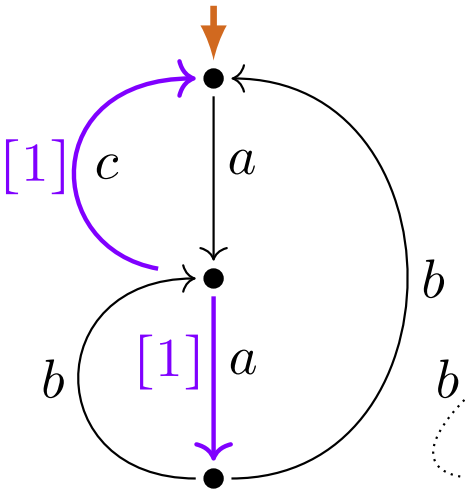


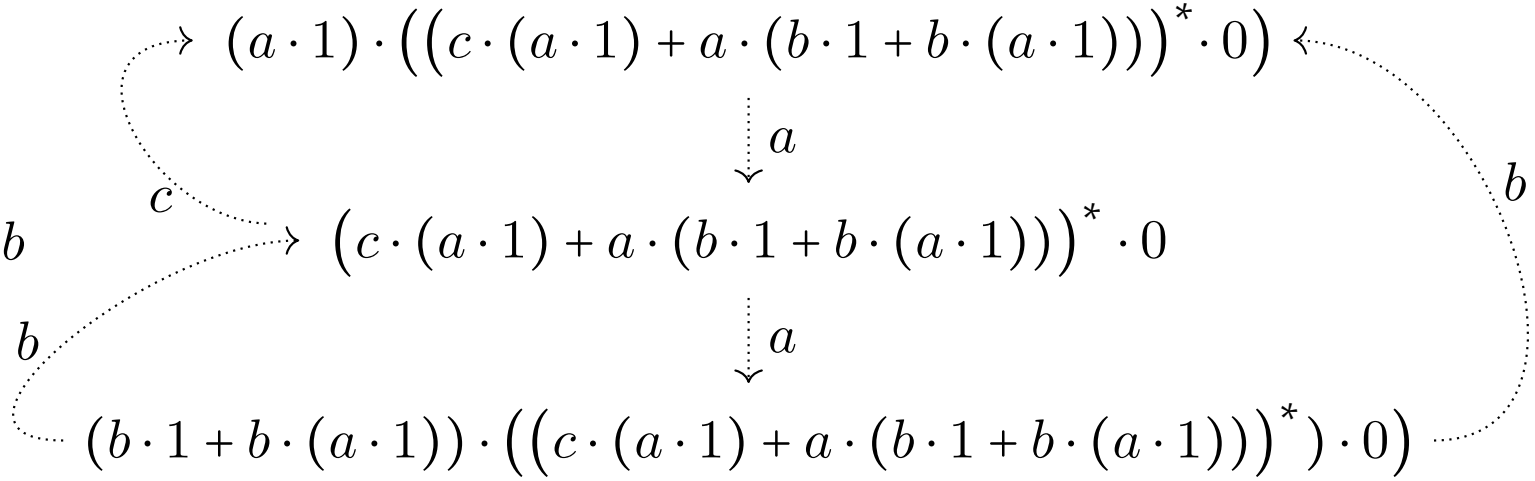
Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)



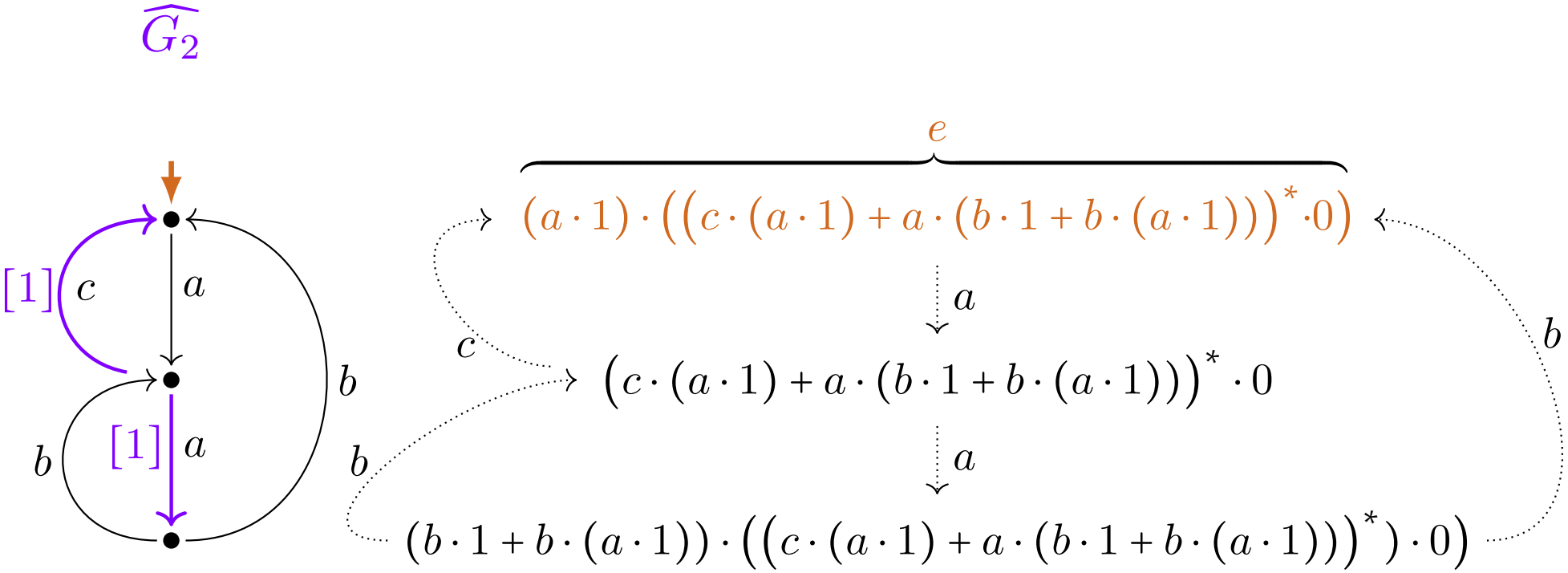
Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_2}$



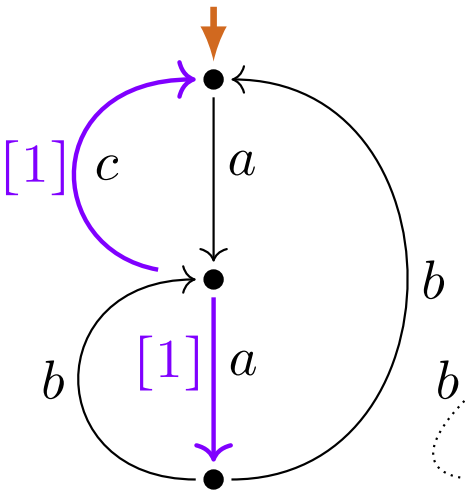


Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)



Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_2}$



$P(e) \rightrightarrows G_2$

e

$(a \cdot 1) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0\big)$

a

c

$\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0$

b

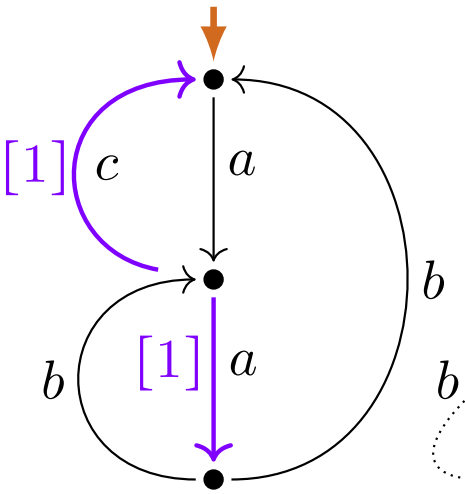
a

b

$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^*\big) \cdot 0\big)$

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_2}$



$P(e) \rightrightarrows G_2$

e

$(a \cdot 1) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0\big)$

$\downarrow a$

c b

$\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0$

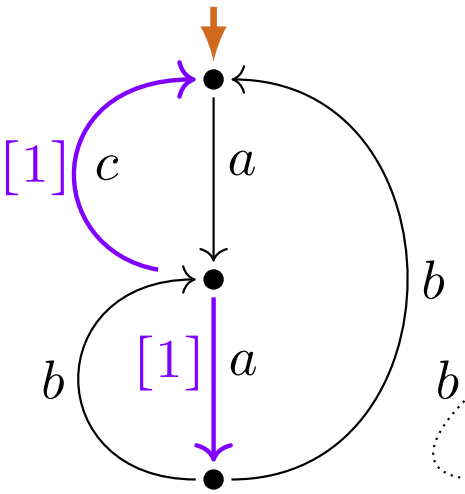
$\downarrow a$

b

$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^*\big) \cdot 0\big)$

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_2}$



$P(e) \rightrightarrows G_2$

$$\overbrace{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^*\cdot 0\Big)}^{e}$$

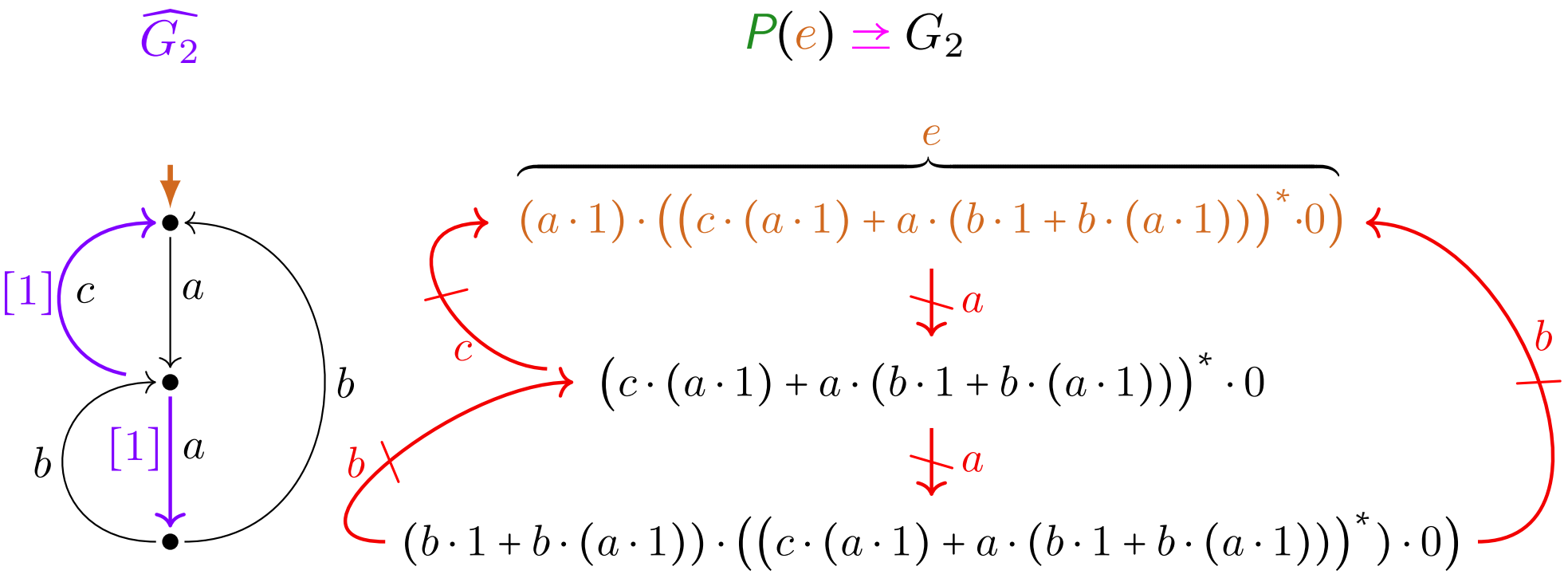
$\not\downarrow a$

$$\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^*\cdot 0$$

$\not\downarrow a$

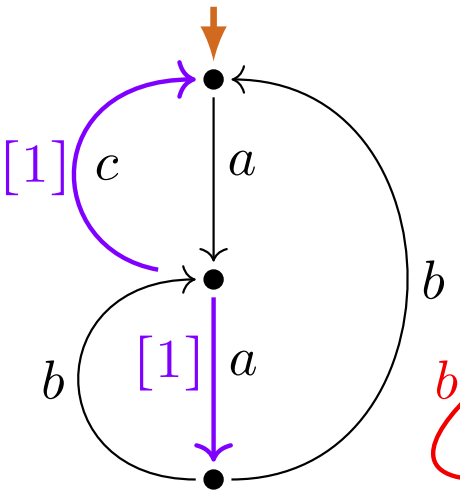
$$(b\cdot 1+b\cdot(a\cdot 1))\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^*\Big)\cdot 0\Big)$$

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)



Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_2}$



$P(e) \rightarrow G_2 \not\leftarrow P(e)$

$$\overbrace{(a\cdot 1)\cdot\big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^*\cdot 0\big)}^{e}$$

$\downarrow a$

$$\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^*\cdot 0$$

$\downarrow a$

$$(b\cdot 1+b\cdot(a\cdot 1))\cdot\big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^*\big)\cdot 0\big)$$

c b b b

Interpretation of extracted expression

G'_2

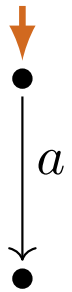
$$P(e) = G'_2$$

$$\overbrace{(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0\Big)}^{e}$$

Interpretation of extracted expression

G'_2

$$P(e) = G'_2$$



$$\overbrace{(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0\Big)}^{e}$$

$$\downarrow a$$

$$(1 \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0\Big)$$

Interpretation of extracted expression

G'_2

$$P(e) = G'_2$$

$$\overbrace{(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0\Big)}^{e}$$

$$\downarrow a$$

$$(1 \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0\Big)$$

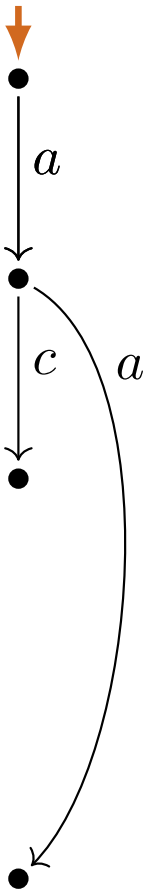
$$\downarrow c$$

$$\Big((1 \cdot (a \cdot 1)) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^*\Big) \cdot 0$$

Interpretation of extracted expression

G'_2

$P(e) = G'_2$



$$\overbrace{(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0\Big)}^{e}$$

$$\downarrow a$$

$$(1 \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0\Big)$$

$$\downarrow c$$

$$\Big((1 \cdot (a \cdot 1)) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^*\Big) \cdot 0$$

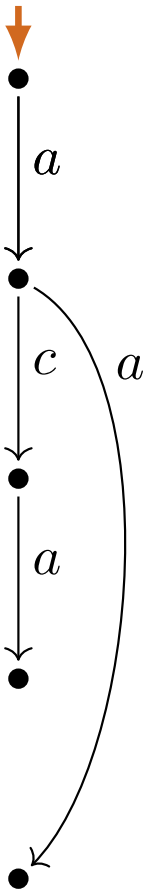
a

$$\Big((1 \cdot (b \cdot 1 + b \cdot (a \cdot 1))) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^*\Big) \cdot 0$$

Interpretation of extracted expression

G'_2

$P(e) = G'_2$



$$\overbrace{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^*\cdot 0\Big)}^{e}$$

$$\downarrow a$$

$$(1\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^*\cdot 0\Big)$$

$$\downarrow c$$

$$\Big((1\cdot(a\cdot 1))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^*\Big)\cdot 0$$

$$\downarrow a$$

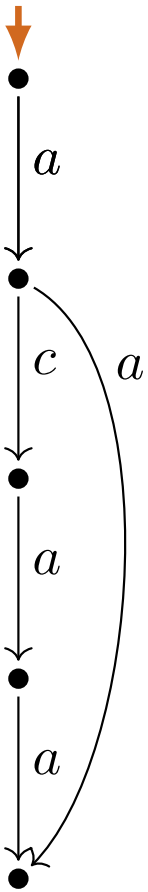
$$\Big((1\cdot 1)\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^*\Big)\cdot 0\Big)$$

$$\Big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^*\Big)\cdot 0$$

Interpretation of extracted expression

G'_2

$P(e) = G'_2$



$$\overbrace{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^*\cdot 0\Big)}^{e}$$

$$\downarrow a$$

$$(1\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^*\cdot 0\Big)$$

$$\downarrow c$$

$$\Big((1\cdot(a\cdot 1))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^*\Big)\cdot 0$$

$$\downarrow a$$

$$\Big((1\cdot 1)\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^*\Big)\cdot 0\Big)$$

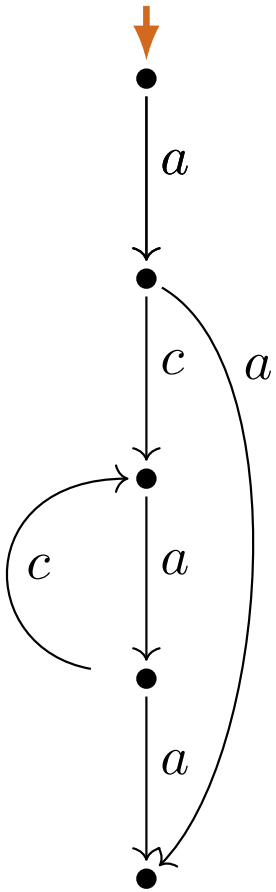
$$\downarrow a$$

$$\Big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^*\Big)\cdot 0$$

a

Interpretation of extracted expression

G'_2



$P(e) = G'_2$

$$\overbrace{(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0\Big)}^{e}$$

$$\downarrow a$$

$$(1 \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0\Big)$$

$$\downarrow c$$

$$\Big((1 \cdot (a \cdot 1)) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^*\Big) \cdot 0$$

$$\downarrow a$$

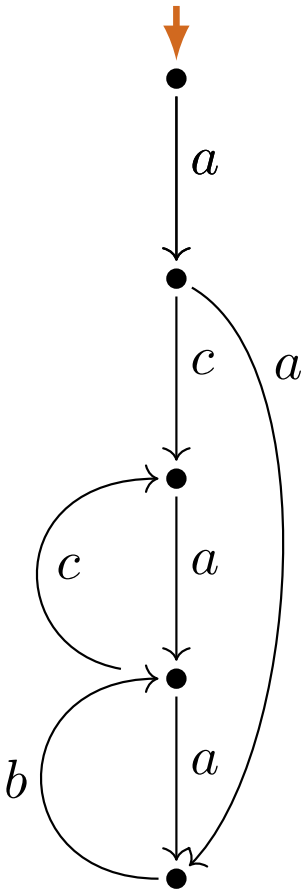
$$\Big((1 \cdot 1) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^*\Big) \cdot 0\Big)$$

$$\downarrow a$$

$$\Big((1 \cdot (b \cdot 1 + b \cdot (a \cdot 1))) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^*\Big) \cdot 0$$

Interpretation of extracted expression

G'_2



$P(e) = G'_2$

$$\overbrace{(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0\Big)}^{e}$$

$$\downarrow a$$

$$(1 \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0\Big)$$

$$\downarrow c$$

$$\Big((1 \cdot (a \cdot 1)) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^*\Big) \cdot 0$$

$$\downarrow a$$

$$\Big((1 \cdot 1) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^*\Big) \cdot 0\Big)$$

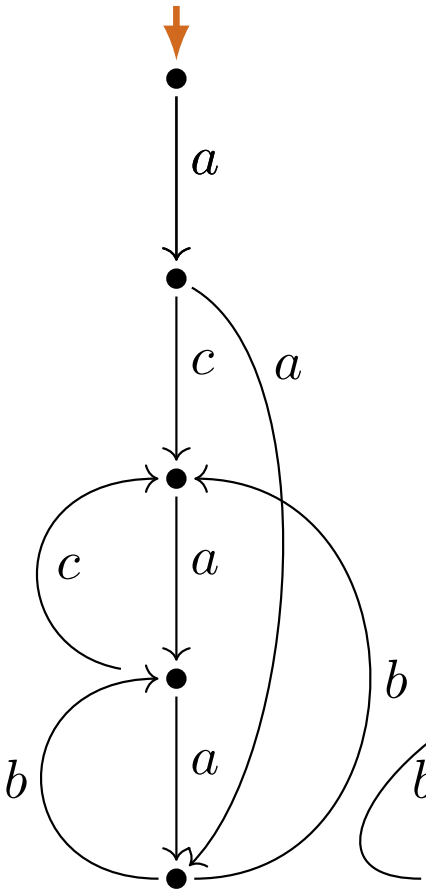
$$\downarrow a$$

$$\Big((1 \cdot (b \cdot 1 + b \cdot (a \cdot 1))) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^*\Big) \cdot 0$$

Interpretation of extracted expression

G'_2

$P(e) = G'_2$



$$\overbrace{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^*\cdot 0\Big)}^{e}$$

$$\downarrow a$$

$$(1\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^*\cdot 0\Big)$$

$$\downarrow c$$

$$\Big((1\cdot(a\cdot 1))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^*\Big)\cdot 0$$

$$\downarrow a$$

$$\Big((1\cdot 1)\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^*\Big)\cdot 0\Big)$$

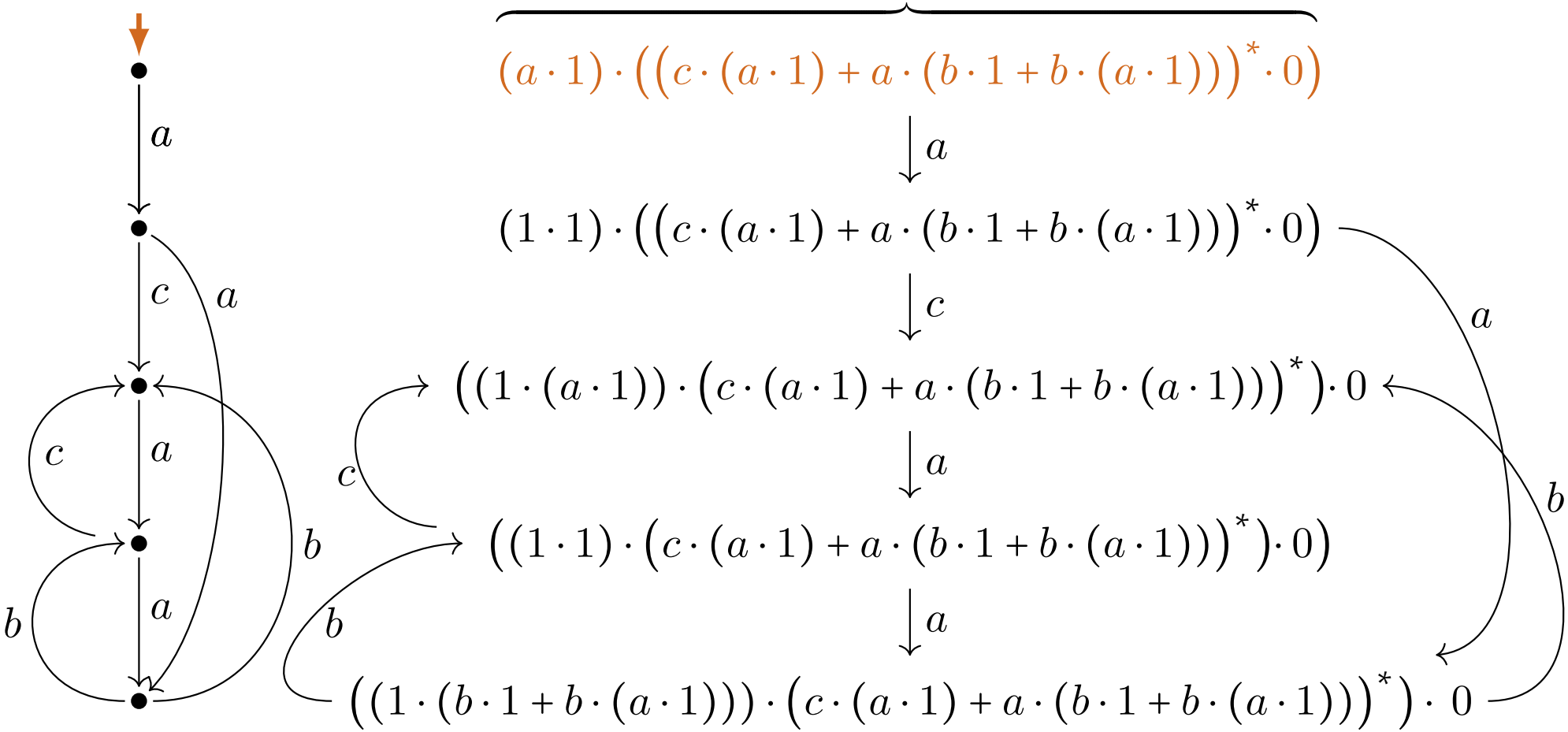
$$\downarrow a$$

$$\Big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^*\Big)\cdot 0$$

Interpretation of extracted expression

G_2'

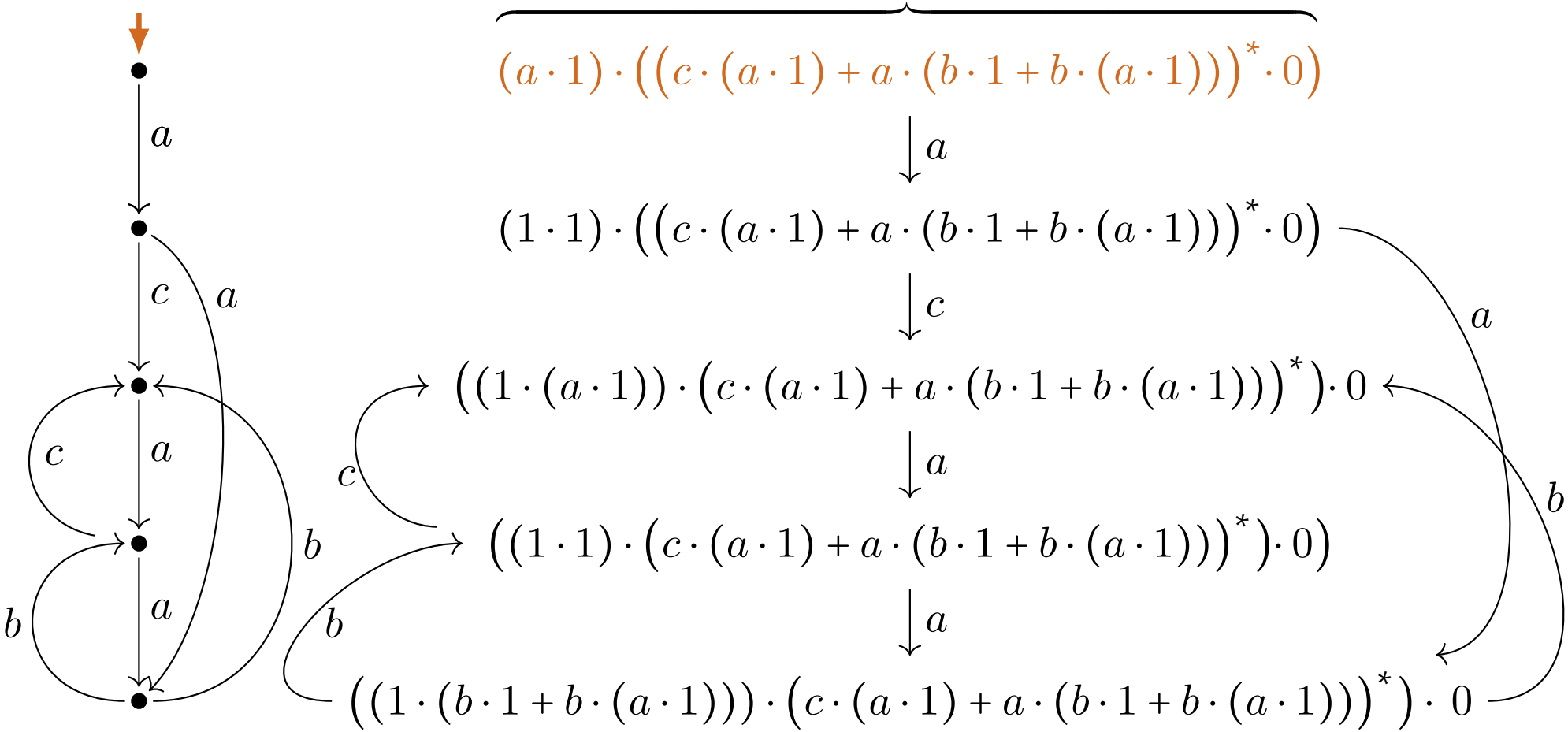
$$P(e) = G_2' \rightrightarrows G_2$$



Interpretation of extracted expression

G'_2

$$P(e) = G'_2 \rightarrow G_2 \not\rightarrow G'_2$$



LEE under bisimulation?

LEE under bisimulation

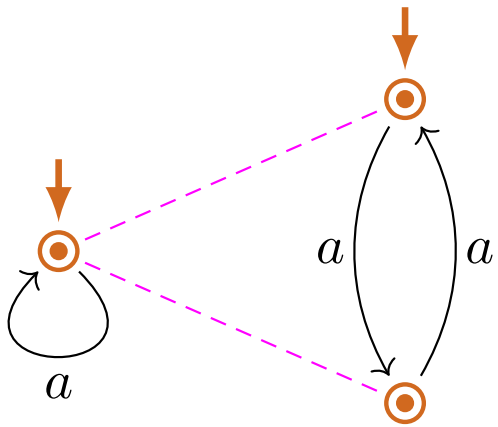
Observation

- LEE is not invariant under bisimulation.

LEE under bisimulation

Observation

- LEE is not invariant under bisimulation.

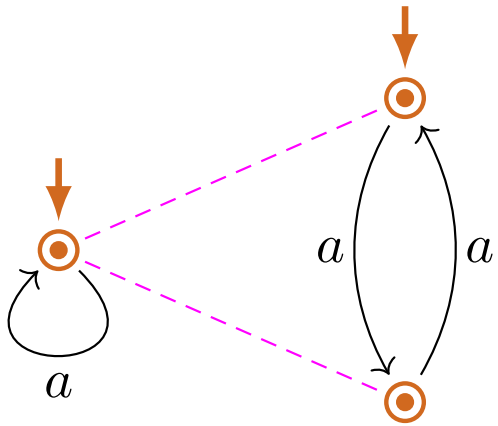


LEE ¬LEE

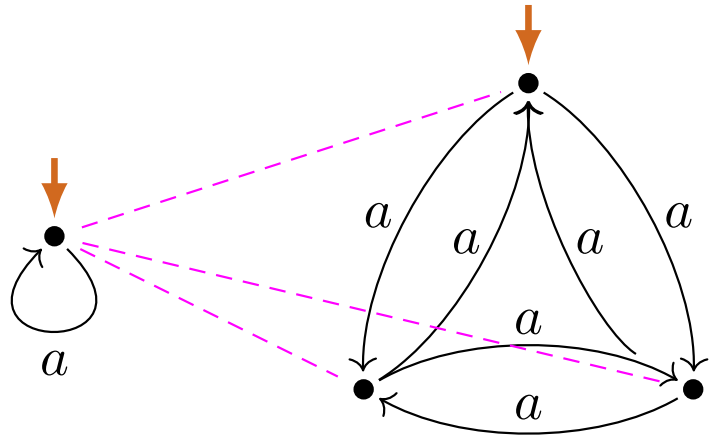
LEE under bisimulation

Observation

- LEE is not invariant under bisimulation.



LEE ¬LEE

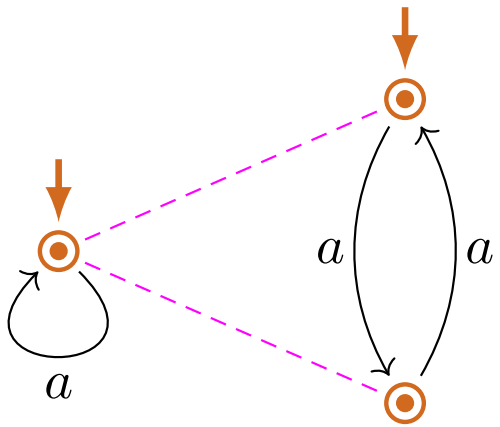


LEE ¬LEE

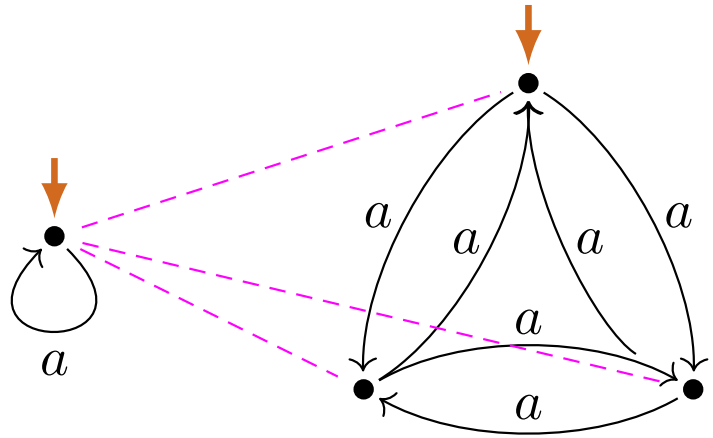
LEE under bisimulation

Observation

- LEE is not invariant under bisimulation.
- LEE is not preserved by converse functional bisimulation.



LEE ¬LEE



LEE ¬LEE

LEE under functional bisimulation

Lemma

(i) LEE *is preserved by* *functional bisimulations*:

$$\mathsf{LEE}(G_1) \wedge G_1 \rightarrow\!\!\!\!\!\!= \; G_2 \implies \mathsf{LEE}(G_2)\,.$$

LEE under functional bisimulation

Lemma

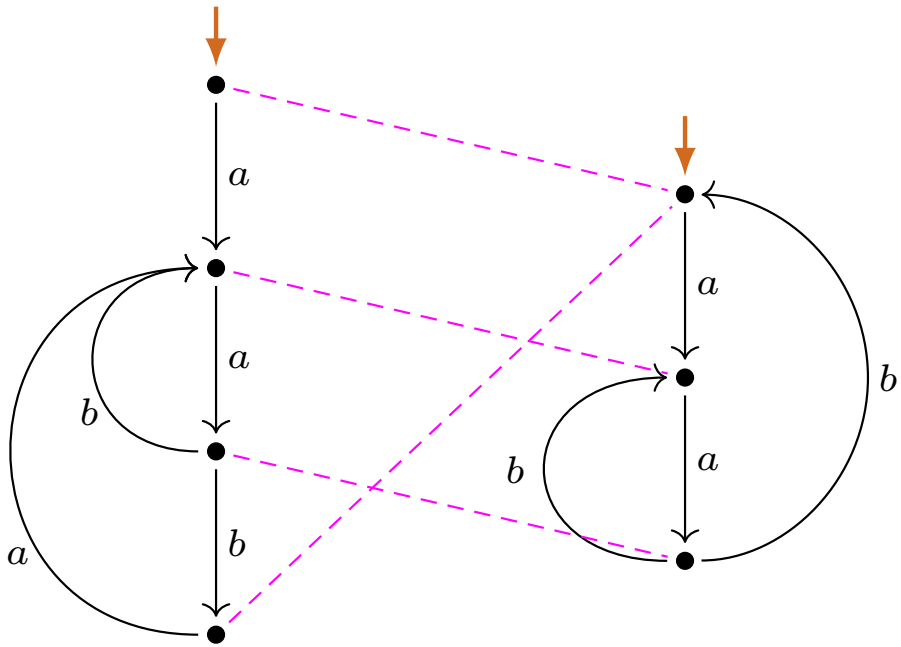
(i) LEE *is preserved by* *functional bisimulations*:

$$\mathsf{LEE}(G_1) \wedge G_1 \twoheadrightarrow G_2 \implies \mathsf{LEE}(G_2)\,.$$

Proof (Idea).

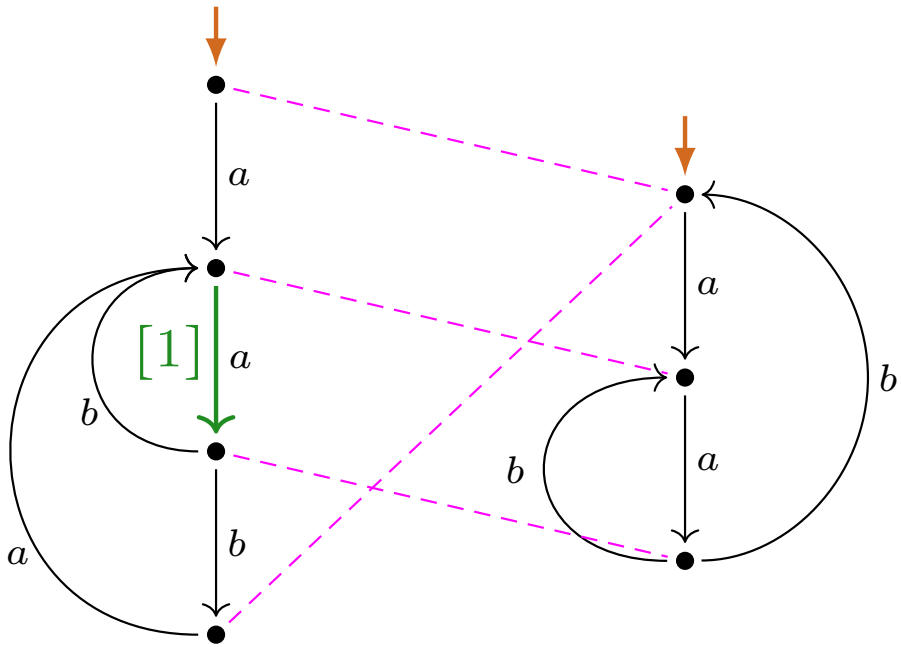
Use loop elimination in G_1 to carry out loop elimination in G_2.

Collapsing LEE-witnesses



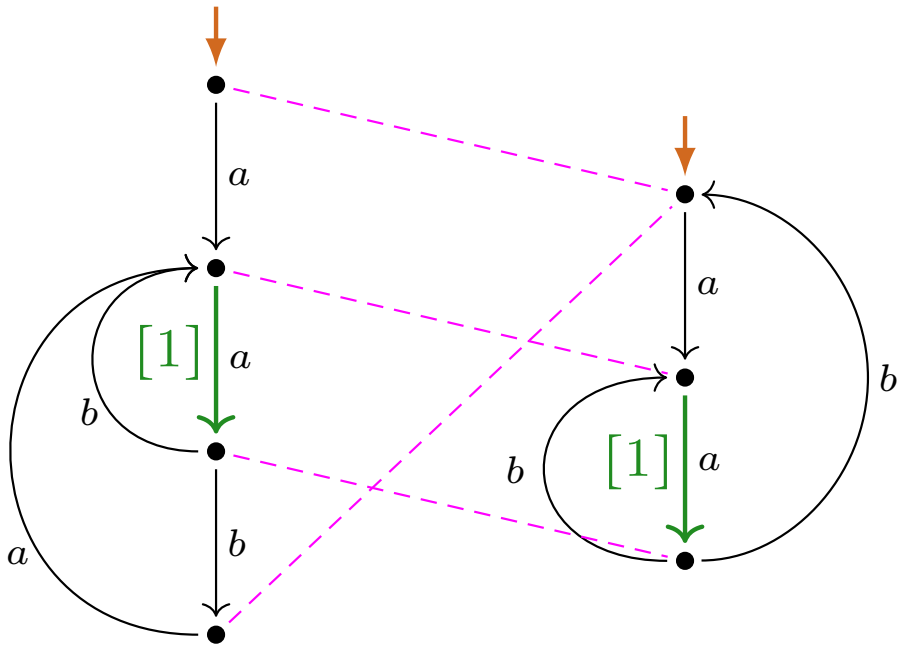
$P(a(a(b+ba))^* \cdot 0)$

Collapsing LEE-witnesses



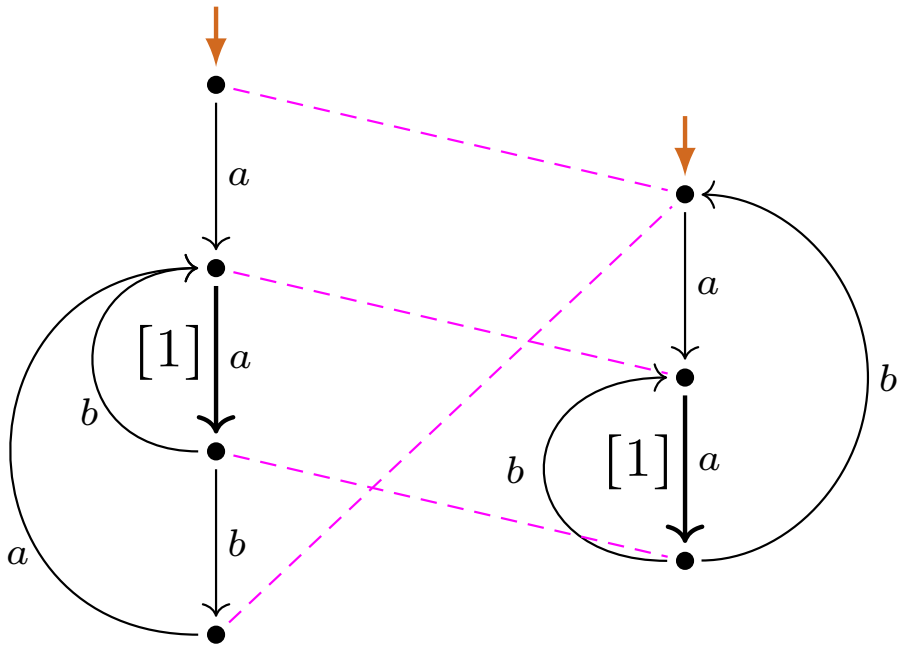
$P(a(a(b+ba))^* \cdot 0)$

Collapsing LEE-witnesses



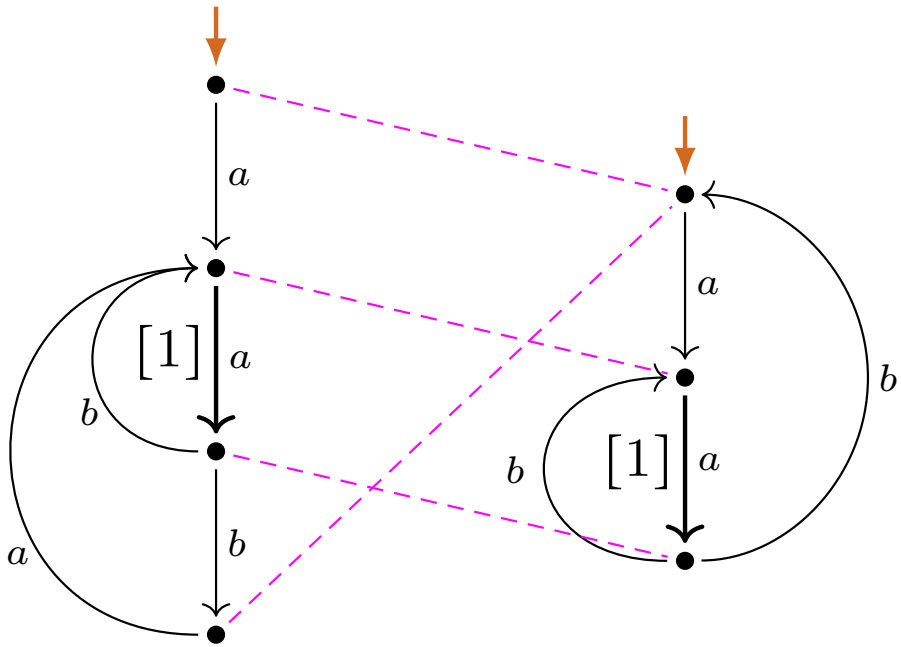
$P(a(a(b+ba))^* \cdot 0)$

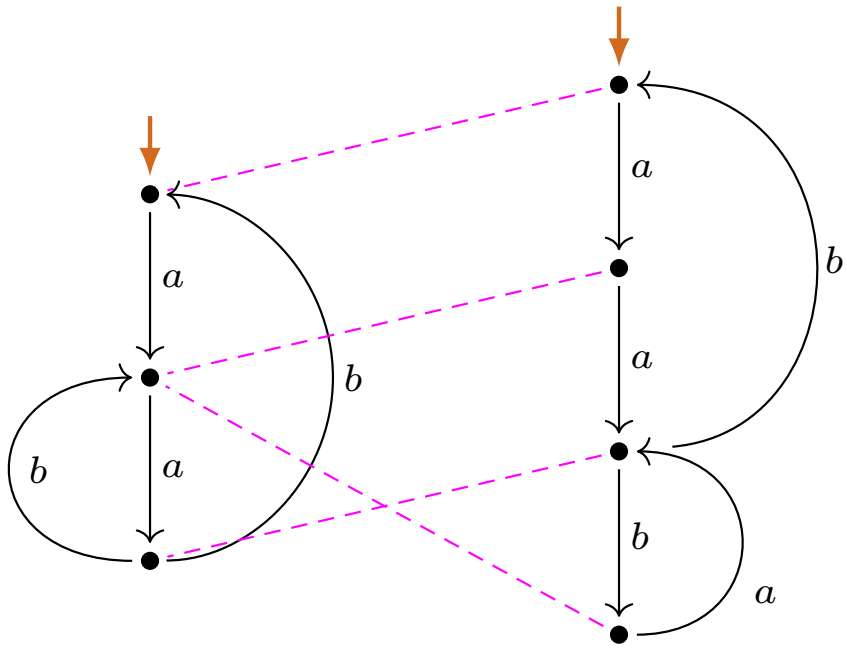
Collapsing LEE-witnesses



$P(a(a(b+ba))^* \cdot 0)$

Collapsing LEE-witnesses

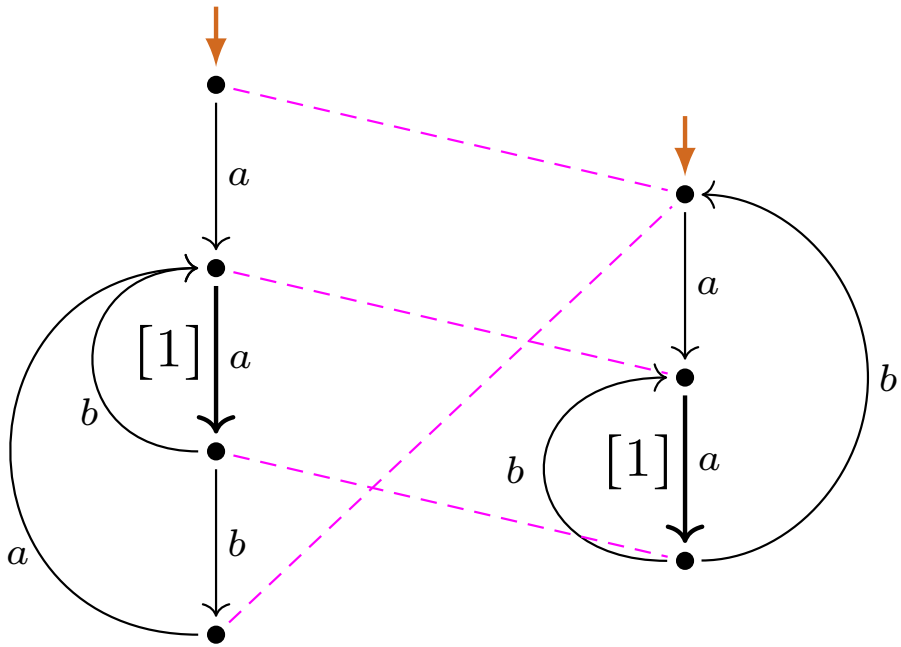




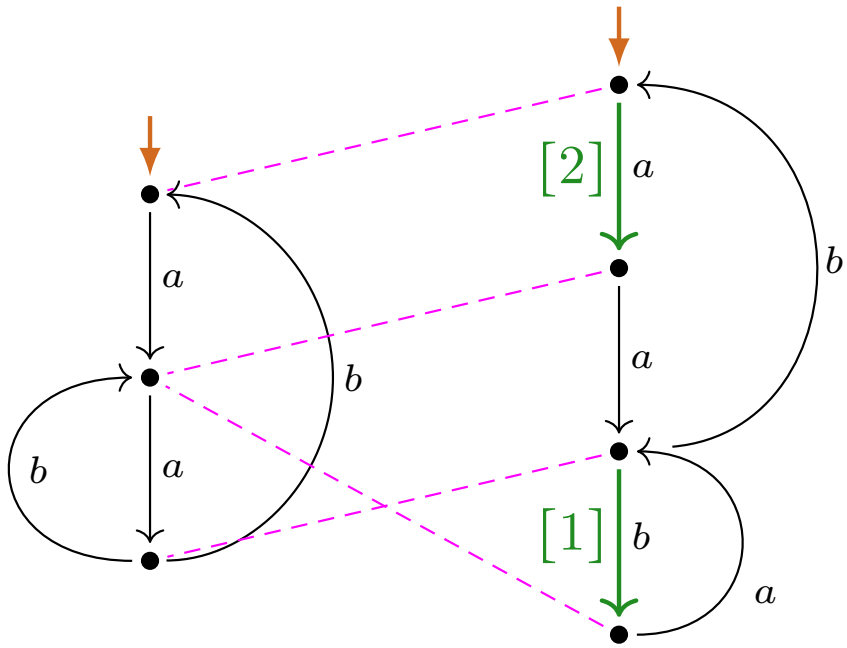
$P(a(a(b+ba))^* \cdot 0)$

$P((aa(ba)^* \cdot b)^* \cdot 0)$

Collapsing LEE-witnesses

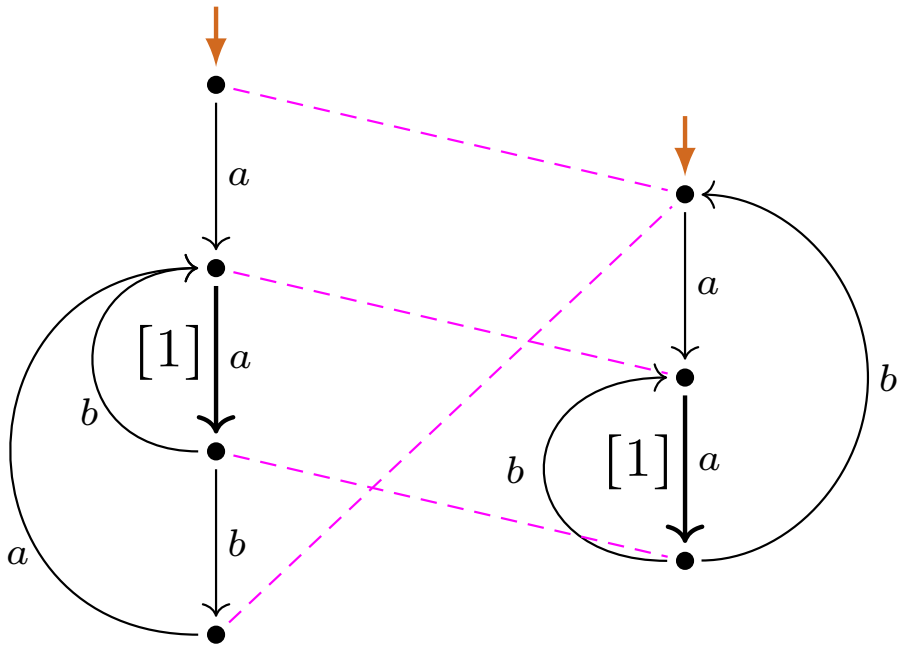


$P(a(a(b+ba))^* \cdot 0)$

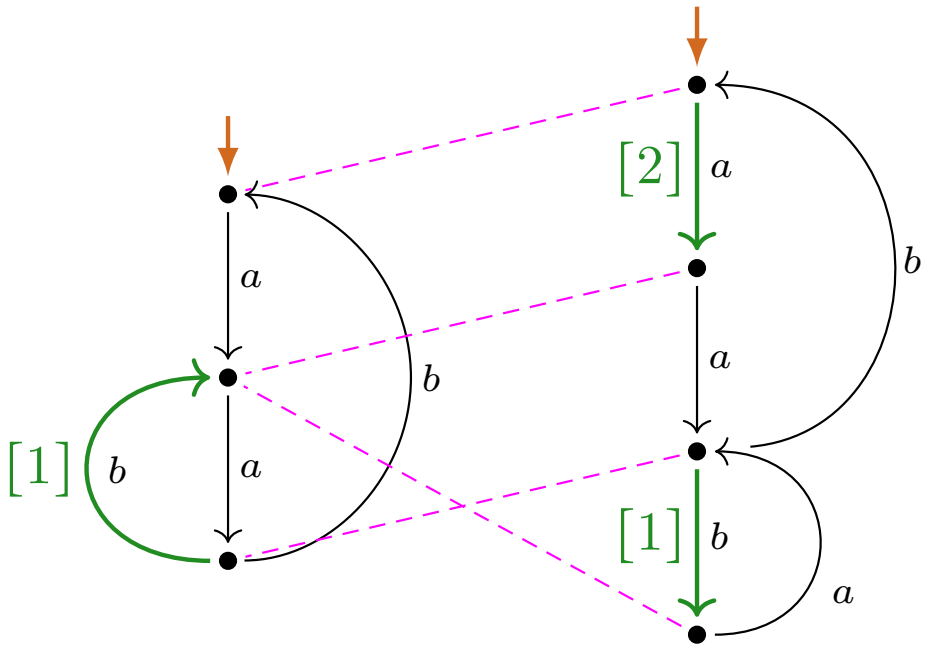


$P((aa(ba)^* \cdot b)^* \cdot 0)$

Collapsing LEE-witnesses

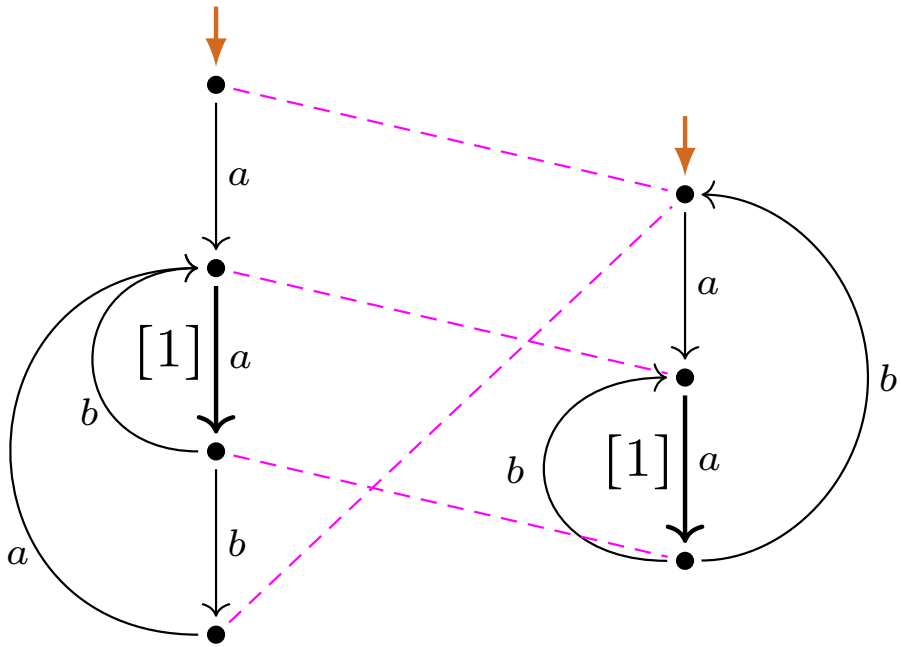


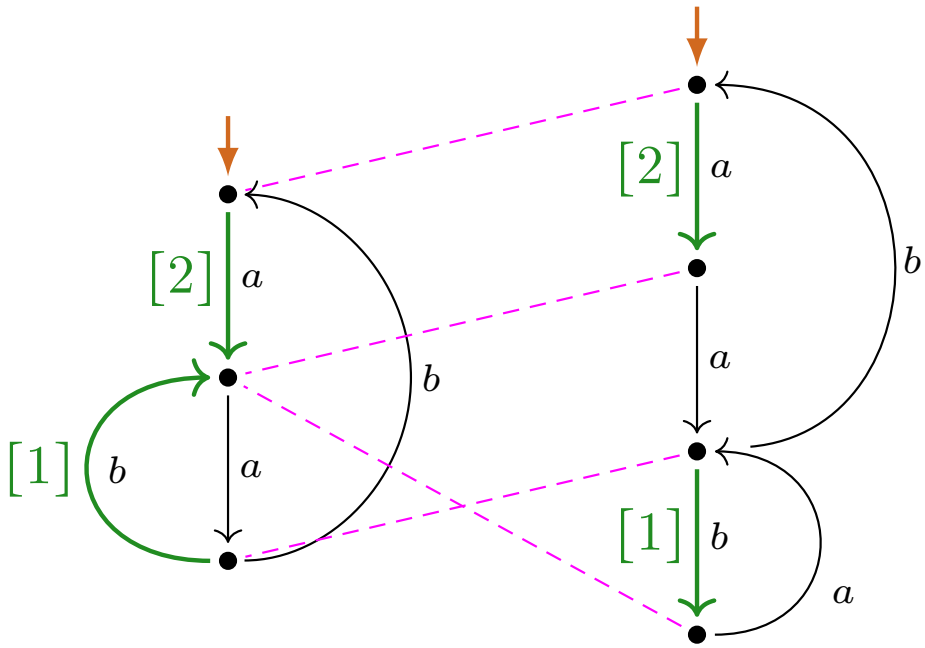
$P(a(a(b+ba))^* \cdot 0)$



$P((aa(ba)^* \cdot b)^* \cdot 0)$

Collapsing LEE-witnesses

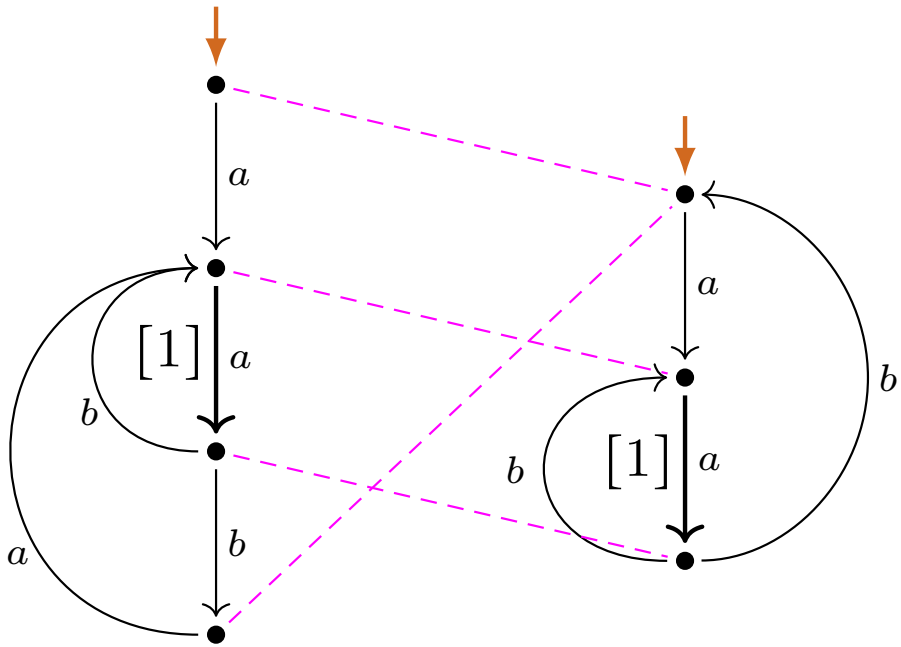


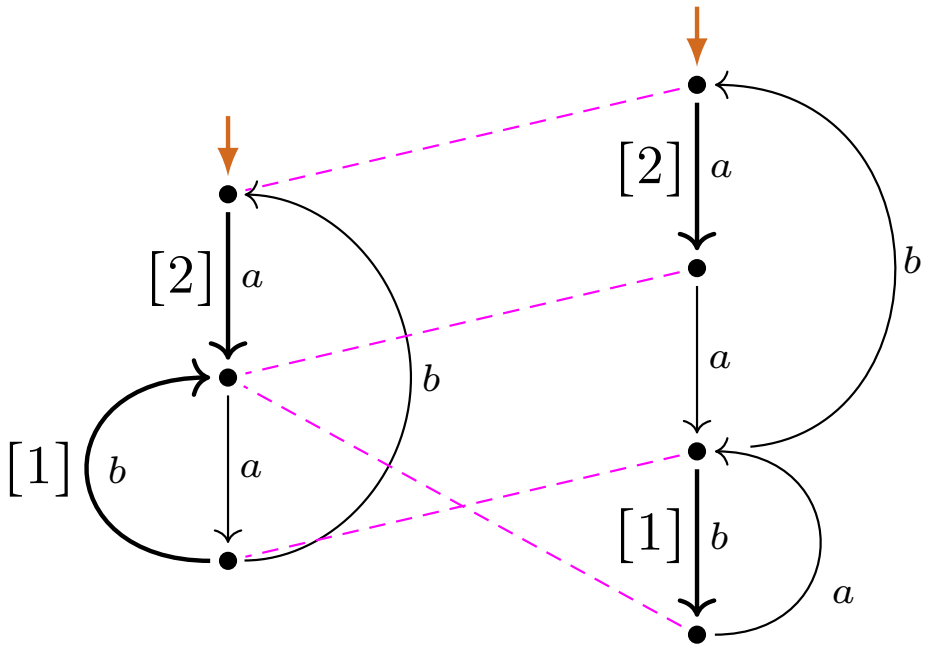


$P(a(a(b+ba))^* \cdot 0)$

$P((aa(ba)^* \cdot b)^* \cdot 0)$

Collapsing LEE-witnesses





$P(a(a(b+ba))^* \cdot 0)$

$P((aa(ba)^* \cdot b)^* \cdot 0)$

LEE under functional bisimulation / bisimulation collapse

Lemma

(i) LEE *is preserved by functional bisimulations:*

$$\mathsf{LEE}(G_1) \wedge G_1 \twoheadrightarrow G_2 \implies \mathsf{LEE}(G_2)\,.$$

(ii) LEE *is preserved from a process graph to its bisimulation collapse:*

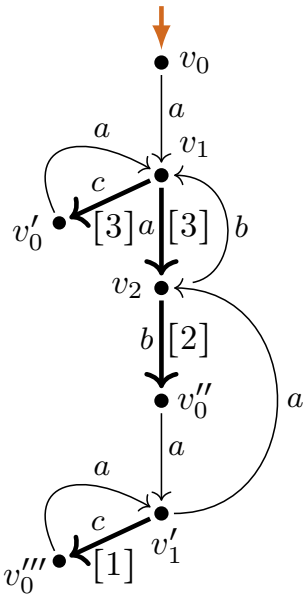
$$\mathsf{LEE}(G) \wedge G \textit{ has bisimulation collapse } C \implies \mathsf{LEE}(C)\,.$$

Idea of Proof for (i)

Use loop elimination in G_1 to carry out loop elimination in G_2.

LLEE-preserving collapse of LLEE-charts (G/Fokkink, LICS'20)

(no 1-transitions!)



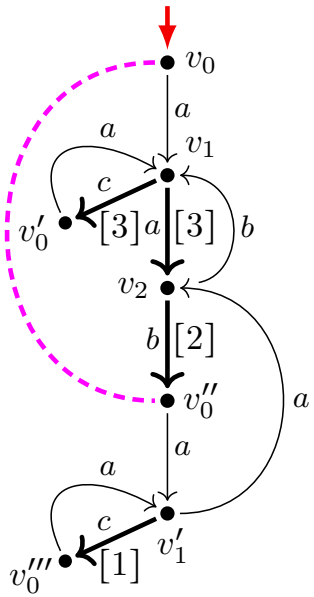
(C1.1)

Lemma

The bisimulation collapse of a LLEE-chart is again a LLEE-chart.

LLEE-preserving collapse of LLEE-charts (G/Fokkink, LICS'20)

(no 1-transitions!)



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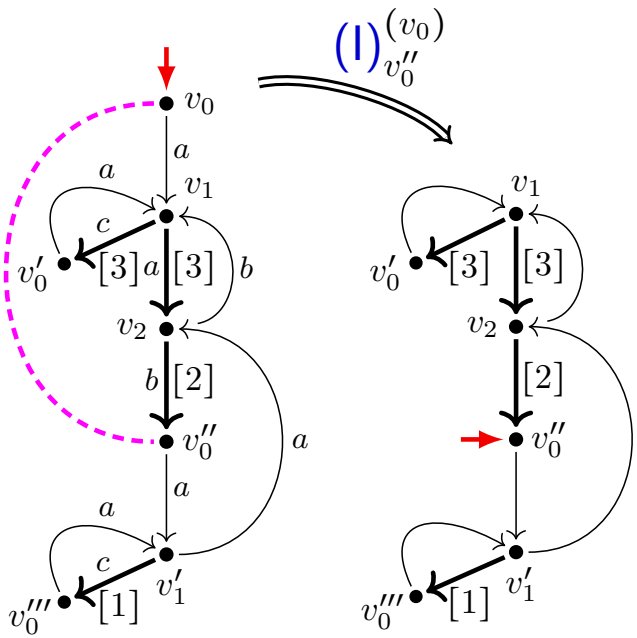

(C1.1) (C2)

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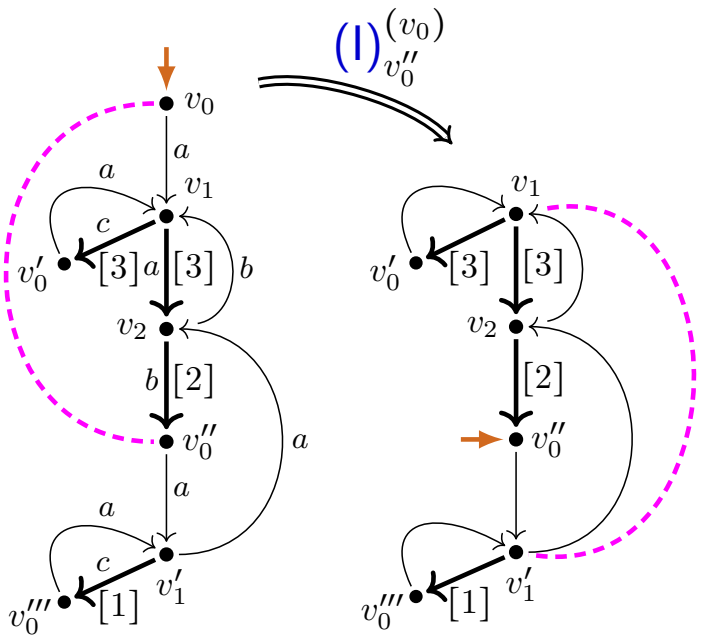
The bisimulation collapse of a LLEE-chart is again a LLEE-chart.

LLEE-preserving collapse of LLEE-charts (G/Fokkink, LICS'20)

(no 1-transitions!)



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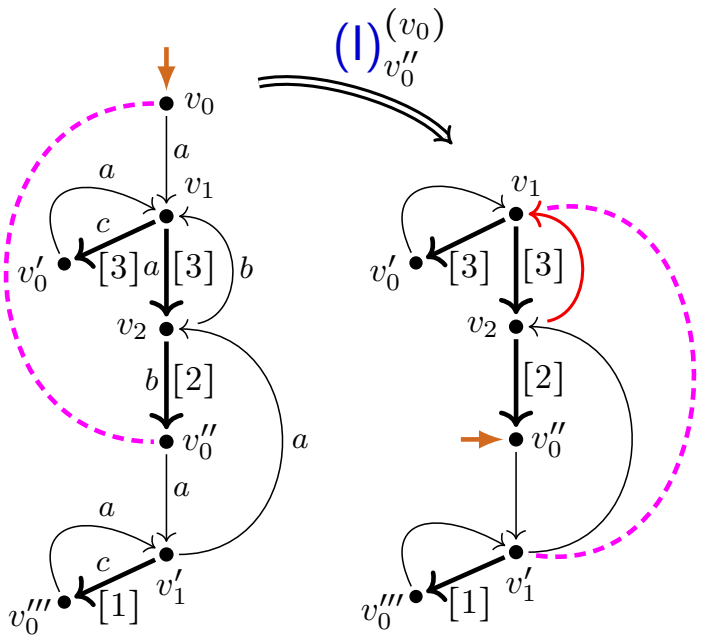
(no 1-transitions!)



(C1.1) (C2)

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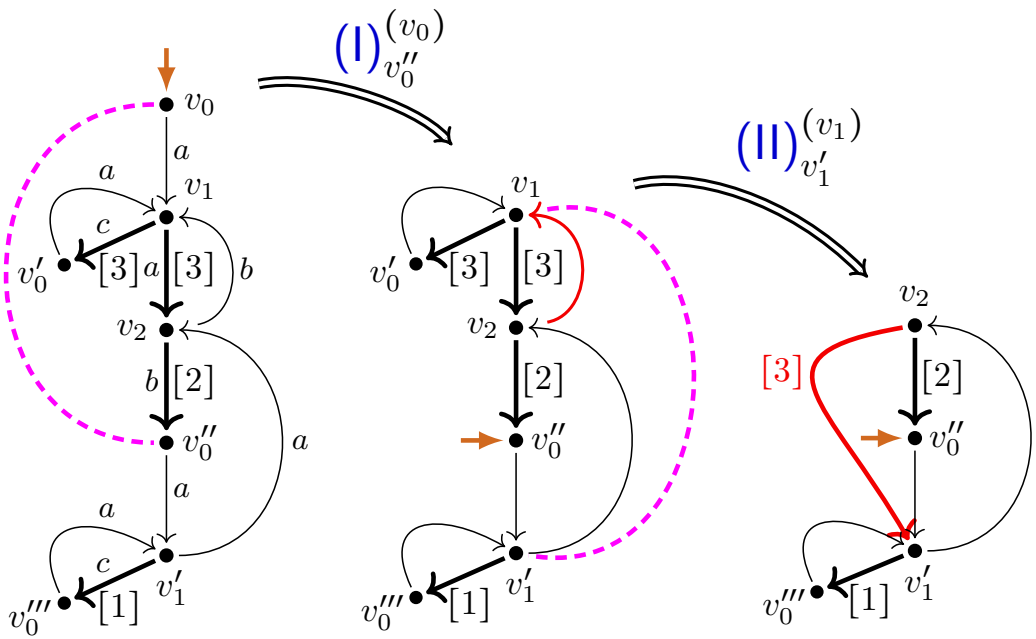
LLEE-preserving collapse of LLEE-charts (G/Fokkink, LICS'20)

(no 1-transitions!)



(C1.1) (C2)

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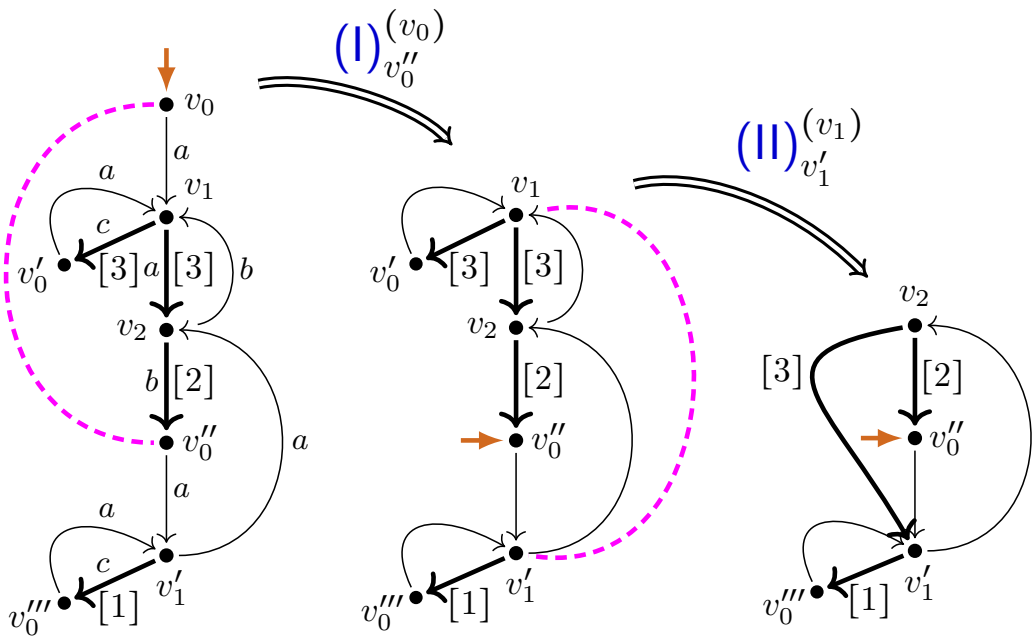
LLEE-preserving collapse of LLEE-charts (G/Fokkink, LICS'20)

(no 1-transitions!)



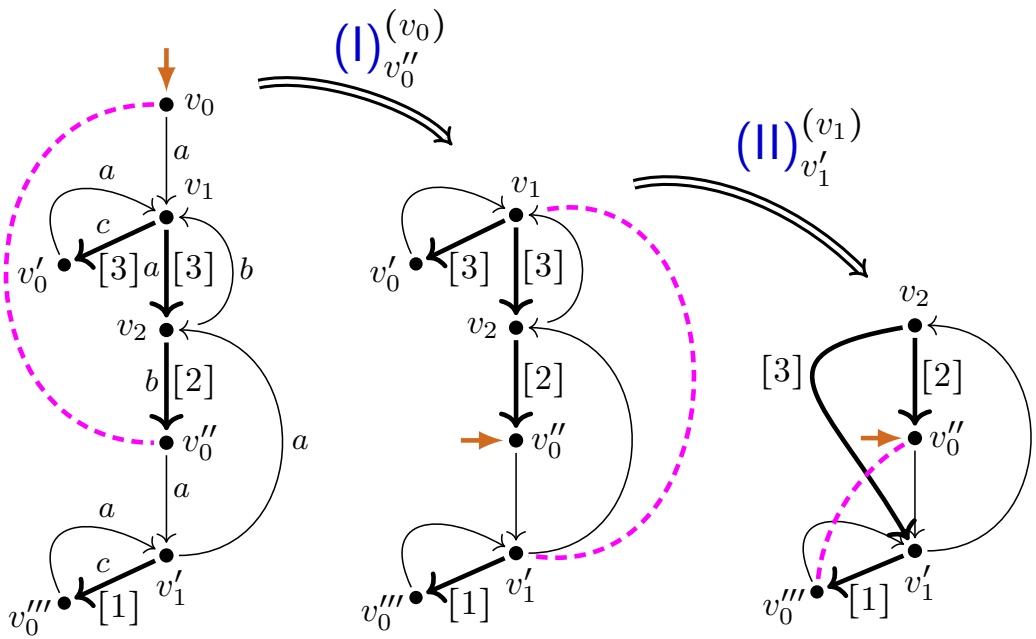
(C1.1) (C2)

Lemma

The bisimulation collapse of a LLEE-chart is again a LLEE-chart.

LLEE-preserving collapse of LLEE-charts (G/Fokkink, LICS'20)

(no 1-transitions!)



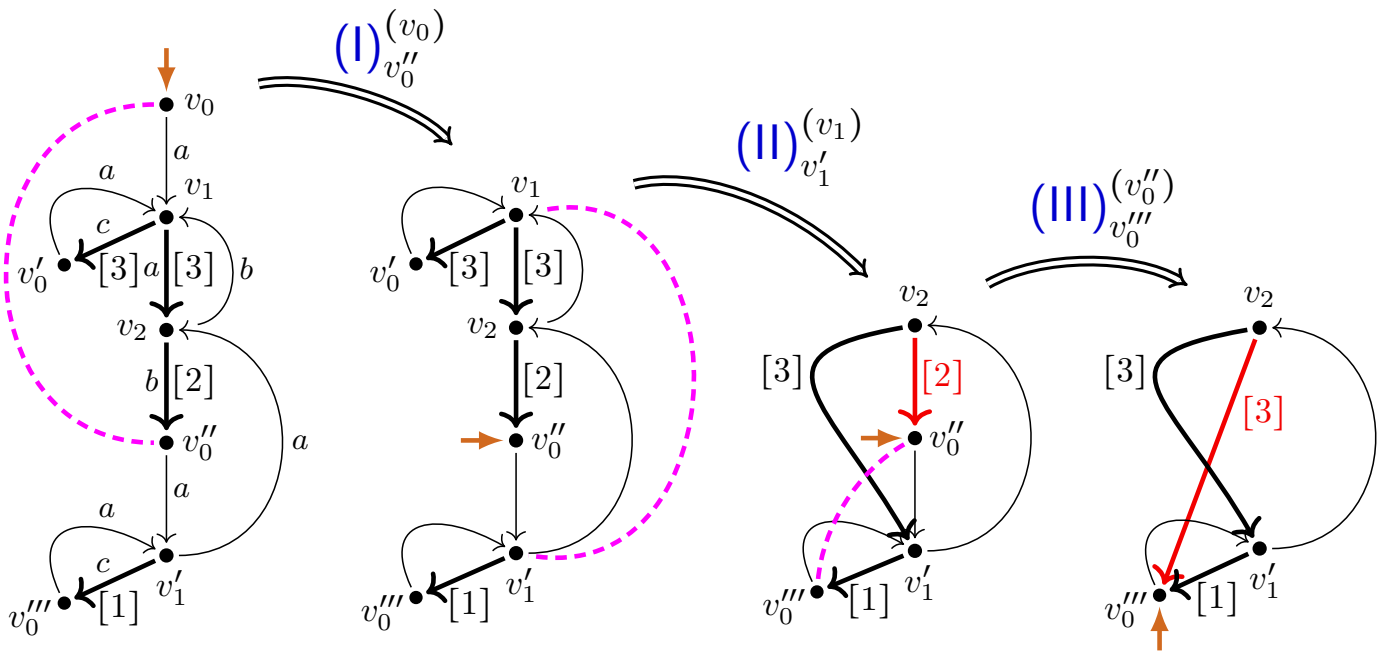
(C1.1) (C2)

Lemma

The bisimulation collapse of a LLEE-chart is again a LLEE-chart.

LLEE-preserving collapse of LLEE-charts (G/Fokkink, LICS'20)

(no 1-transitions!)

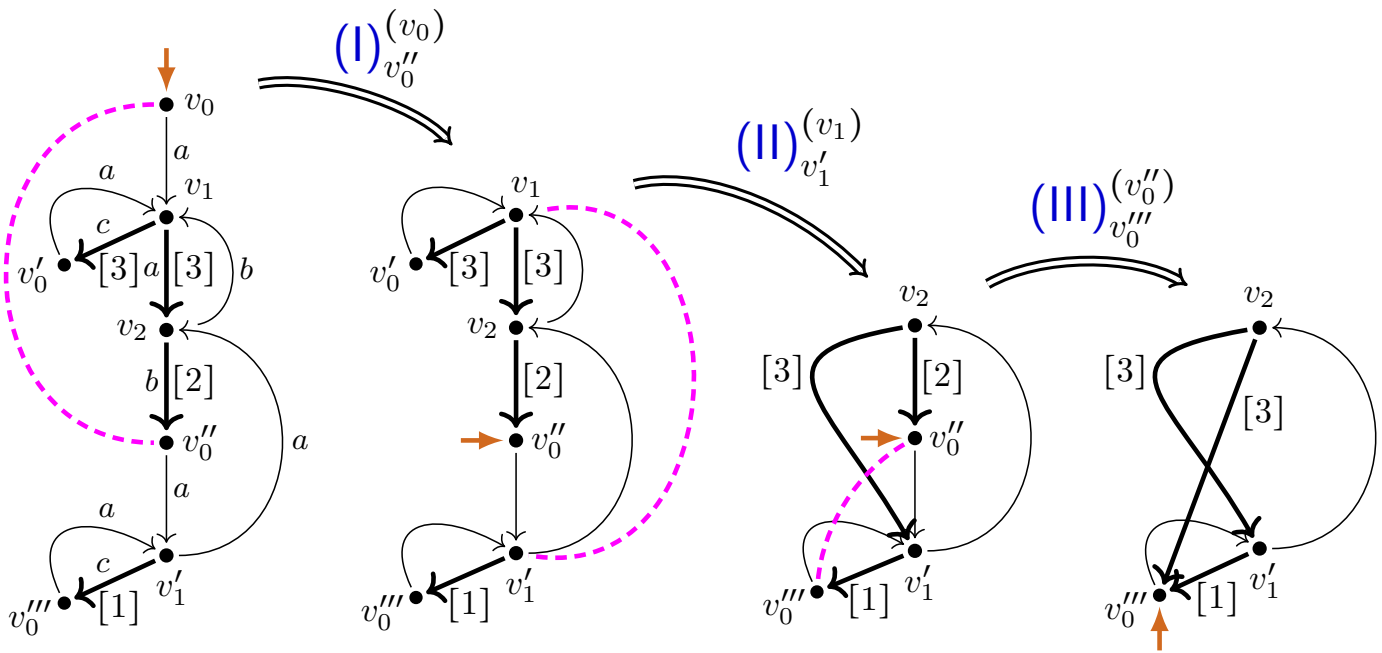


Lemma

The bisimulation collapse of a LLEE-chart is again a LLEE-chart.

LLEE-preserving collapse of LLEE-charts (G/Fokkink, LICS'20)

(no 1-transitions!)

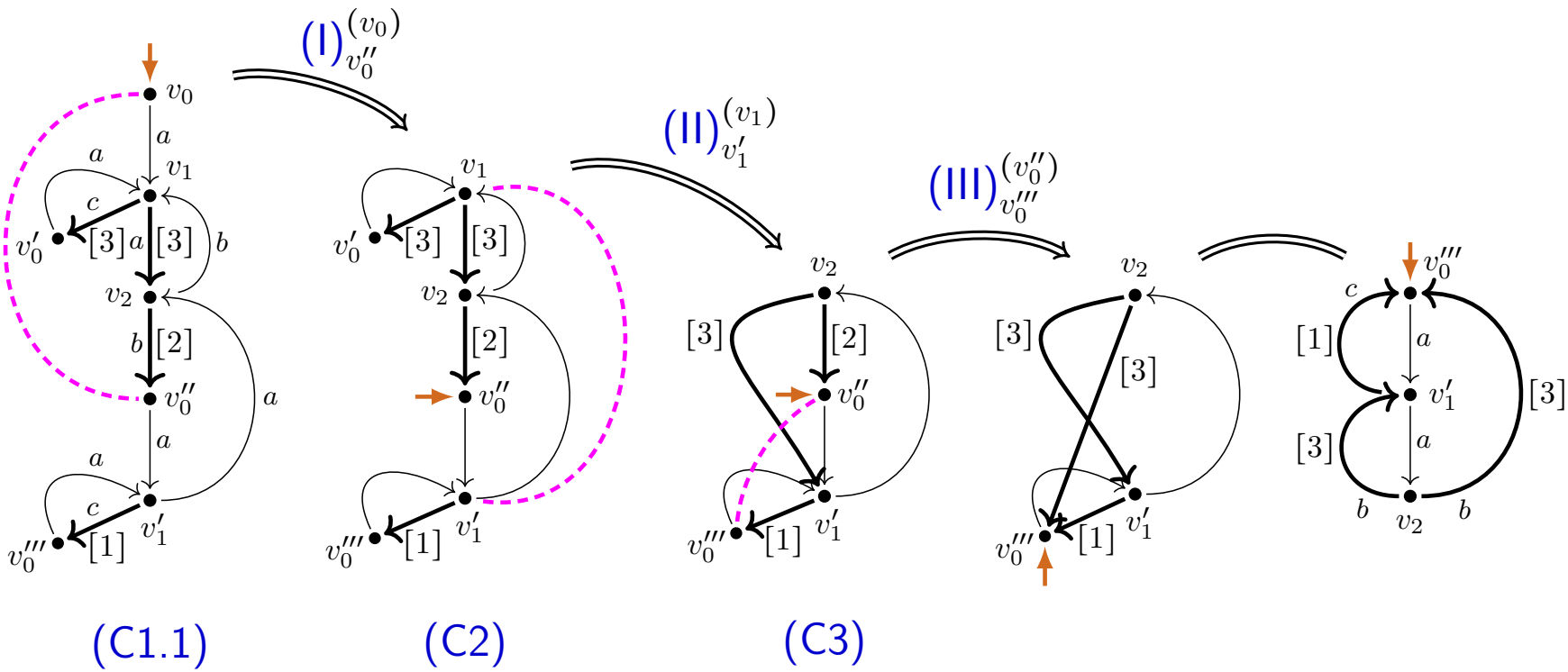


Lemma

The bisimulation collapse of a LLEE-chart is again a LLEE-chart.

LLEE-preserving collapse of LLEE-charts (G/Fokkink, LICS'20)

(no 1-transitions!)



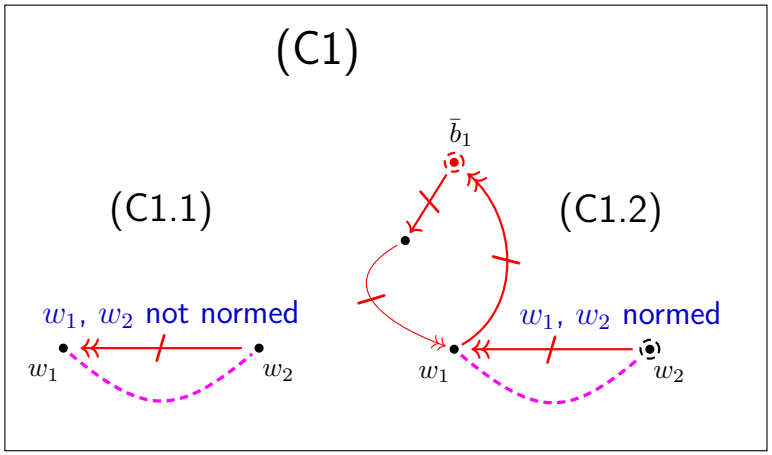
Lemma

The bisimulation collapse of a LLEE-chart is again a LLEE-chart.

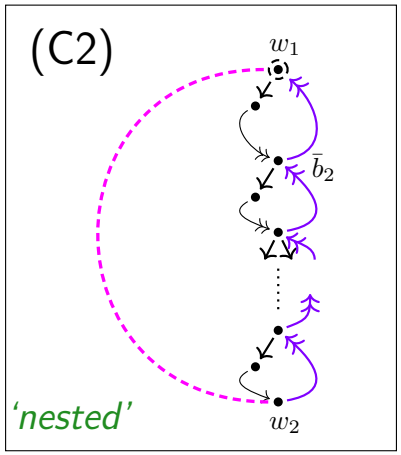
Reduced bisimilarity redundancies in LLEE-graphs (no 1-trans.!)

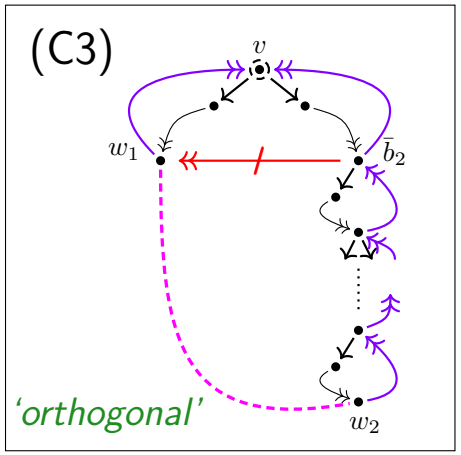
(G/Fokkink, LICS'20)

w_1, w_2 in different scc's



w_1, w_2 in the same scc





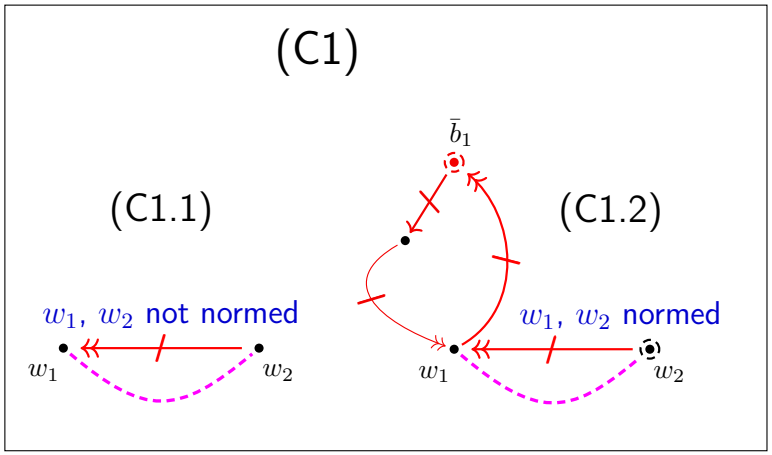
Lemma

Every not collapsed LLEE-graph contains bisimilar vertices $w_1 \neq w_2$ of kind (C1), (C2), or (C3) (a reduced bisimilarity redundancy $\langle w_1, w_2 \rangle$):

Reduced bisimilarity redundancies in LLEE-graphs (no 1-trans.!)

(G/Fokkink, LICS'20)

w_1, w_2 in different scc's



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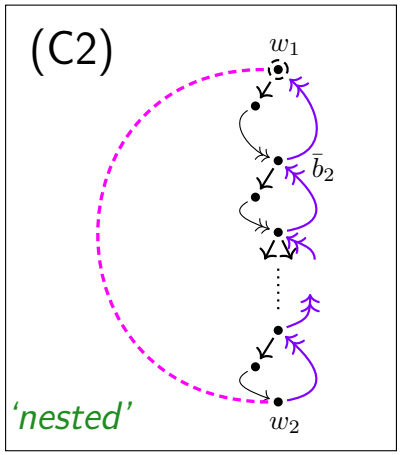

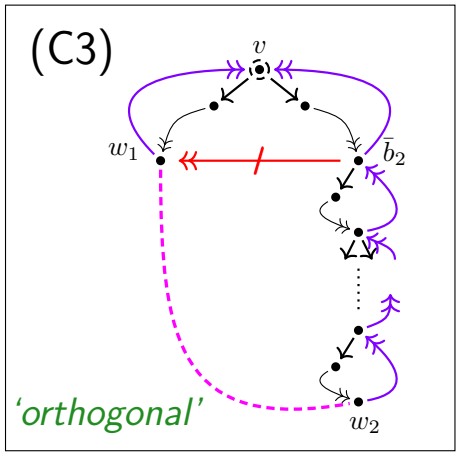

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Every reduced bisimilarity redundancy in a LLEE-graph can be eliminated LLEE-preservingly.

Properties of LEE-charts

Theorem ($\Leftarrow$ G/Fokkink, 2020)

A process graph G

is $[\![\cdot]\!]_P$-expressible by an under-star-1-free regular expression

(i.e. P-expressible modulo bisimilarity by an $(\not{1}\backslash *)$ reg. expr.)

if and only if

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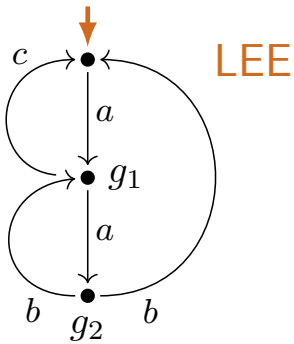
Hence $[\![\cdot]\!]_P$-expressible | **not** $[\![\cdot]\!]_P$-expressible by 1-free regular expressions:



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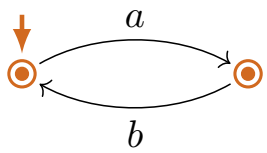

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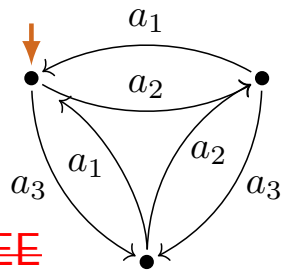

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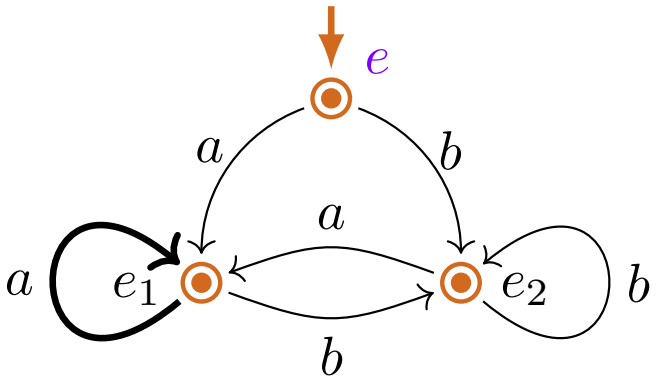
1-LEE

$\hat{=}$ sharing via 1-transitions facilitates LEE

Failure of LEE in general (example)

$P((a^* \cdot b^*)^*)$



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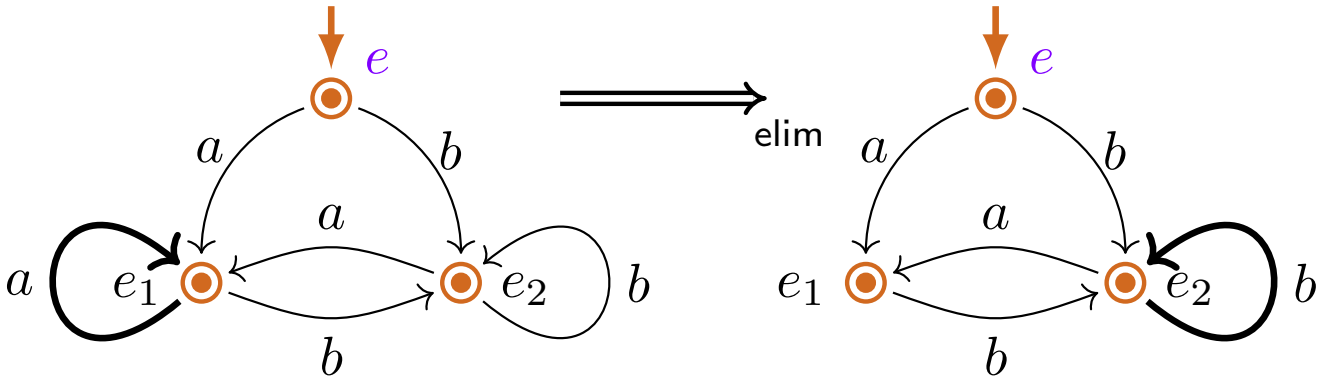

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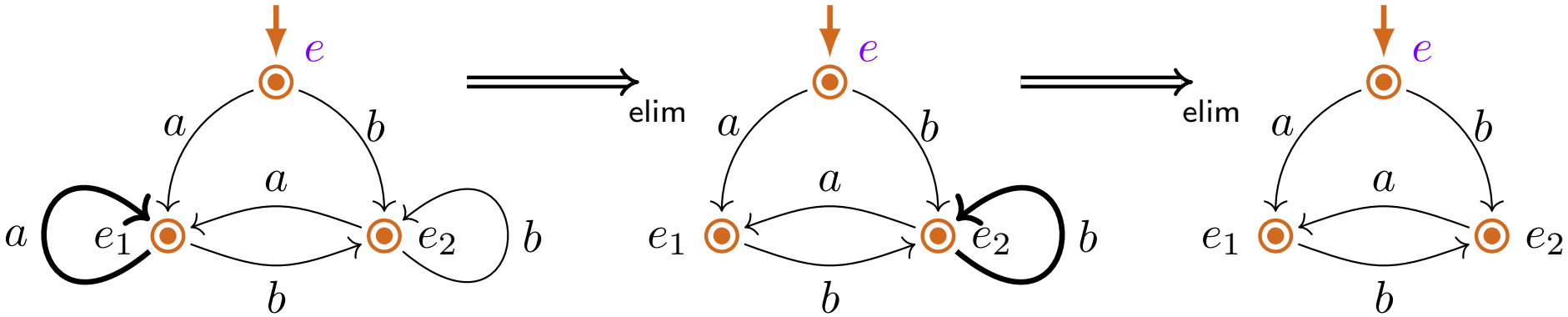

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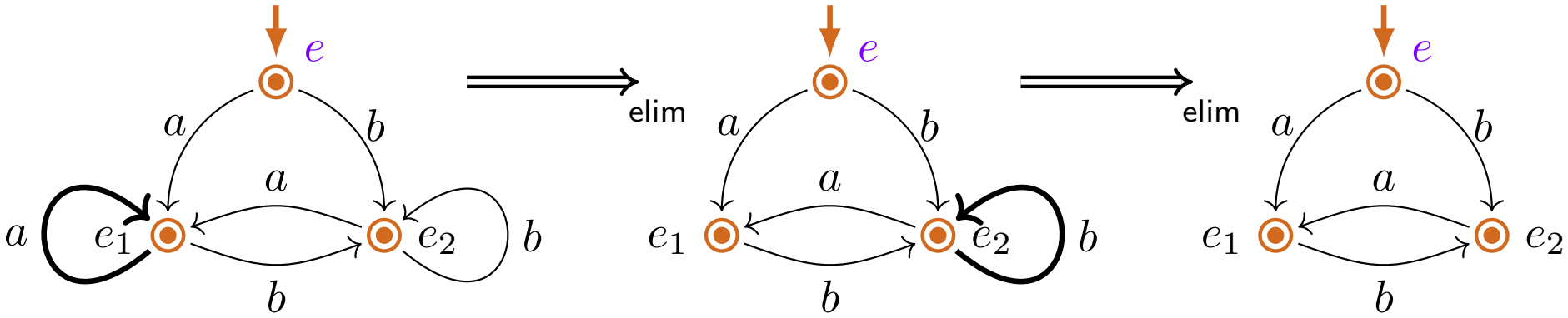

no loop subchart,
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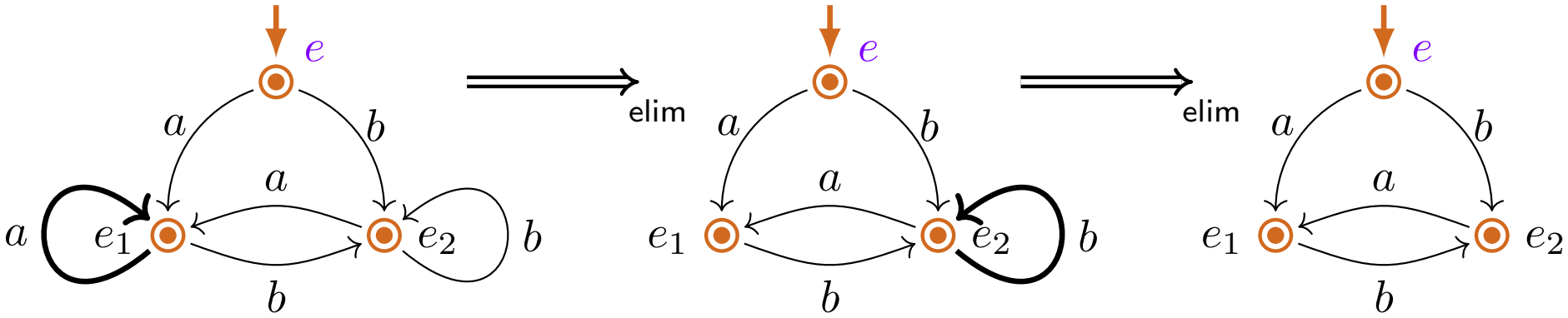

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1-Graphs and induced graphs

Definition

$\xrightarrow{1} \cdot \cdots \cdot \xrightarrow{1} \cdot \xrightarrow{a} \quad \widehat{\equiv} \quad \xrightarrow{(a]}$ induced a-transitions, for $a \in A$

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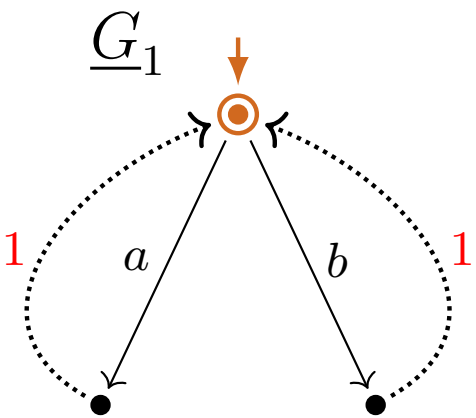

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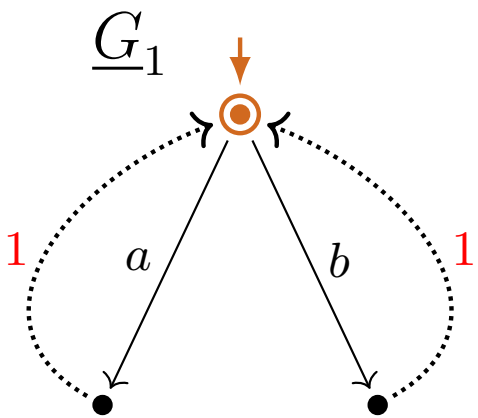

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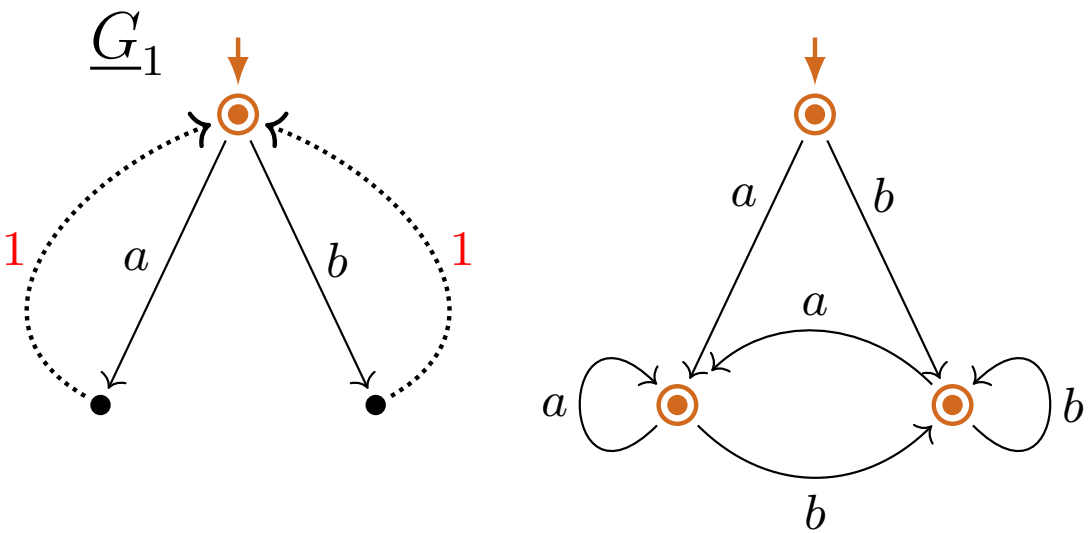

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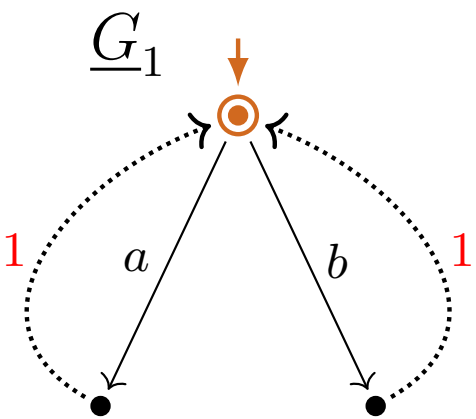

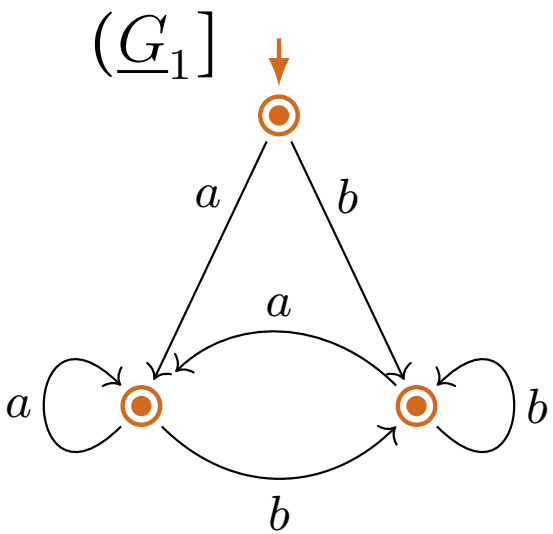

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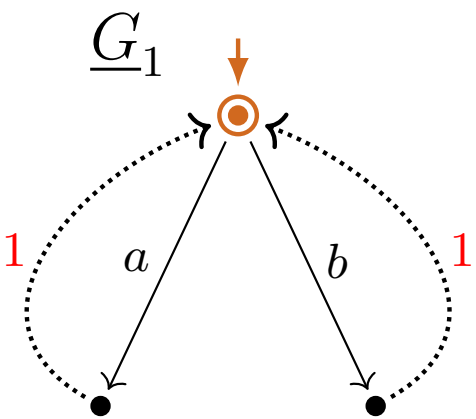

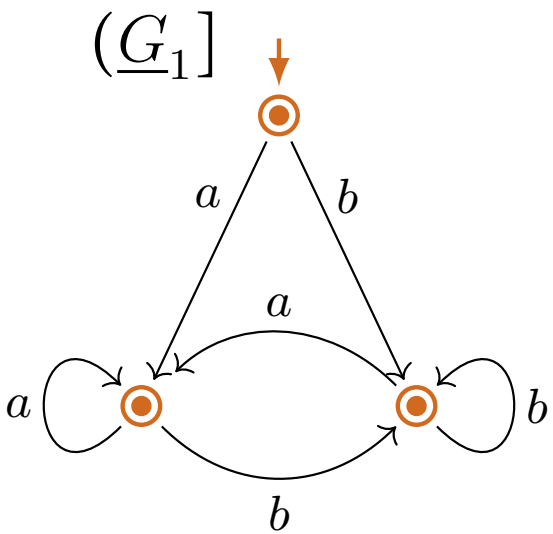

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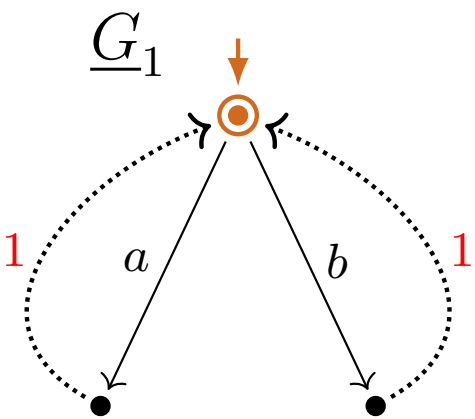

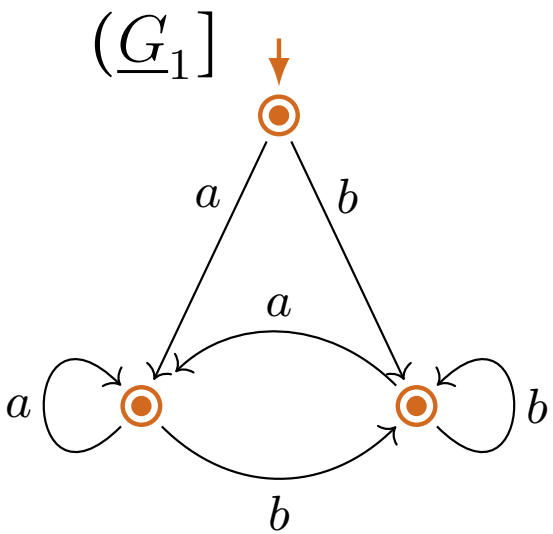

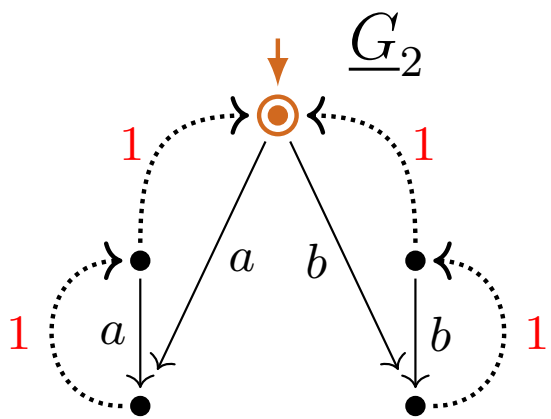

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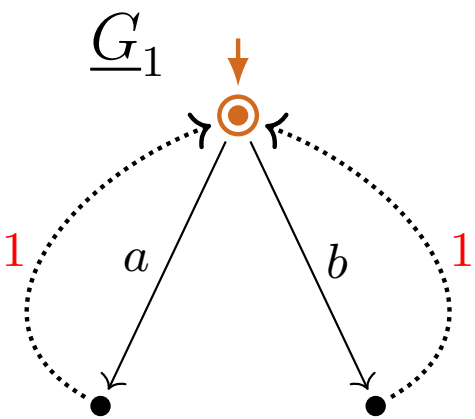

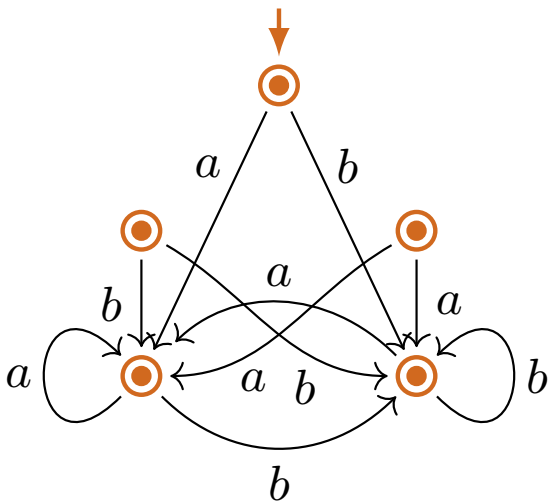

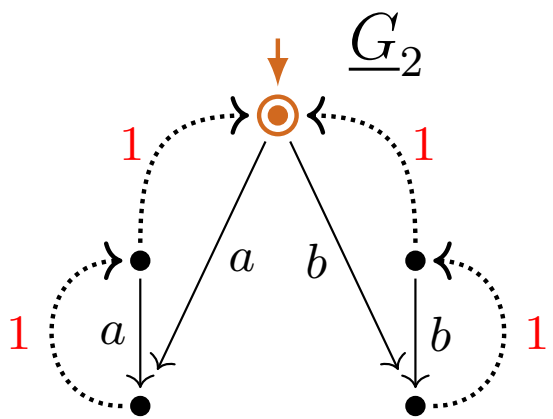

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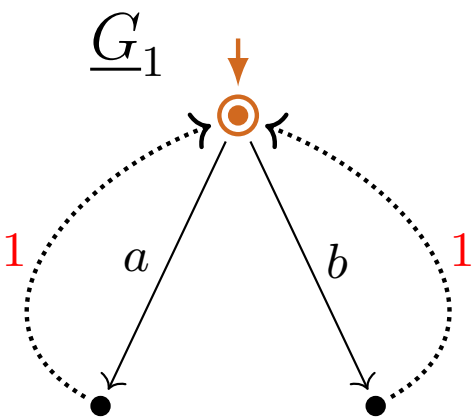

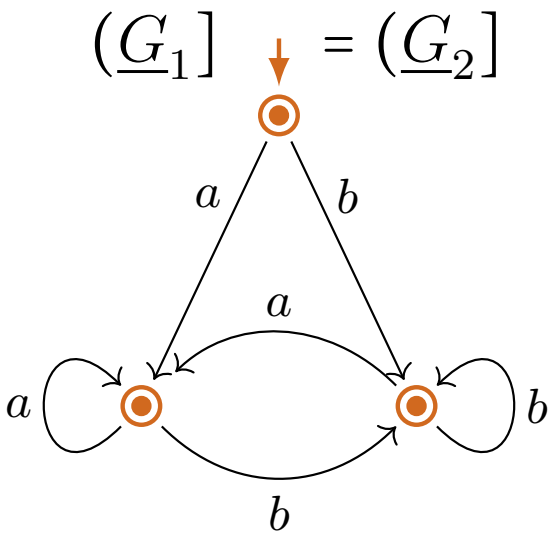

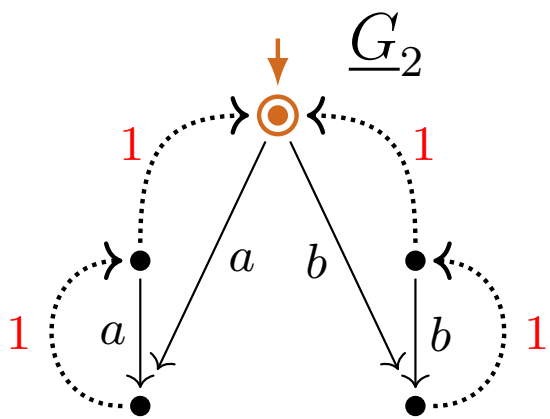

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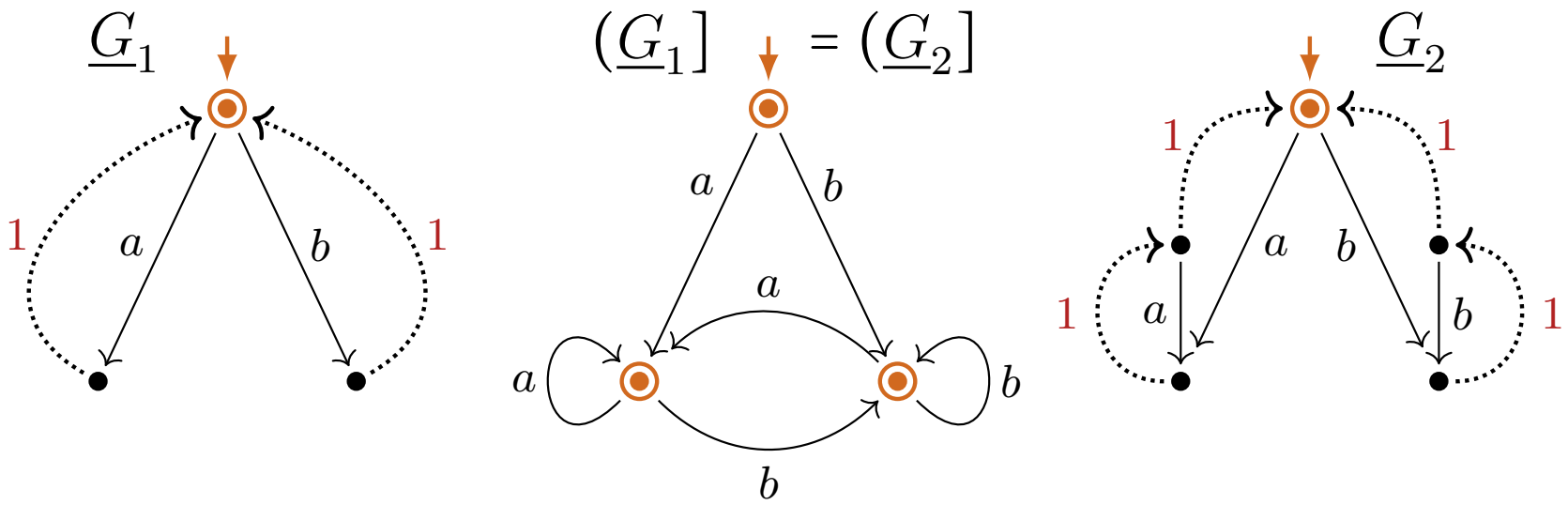

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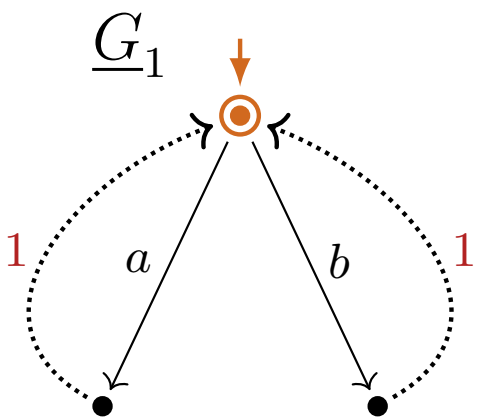

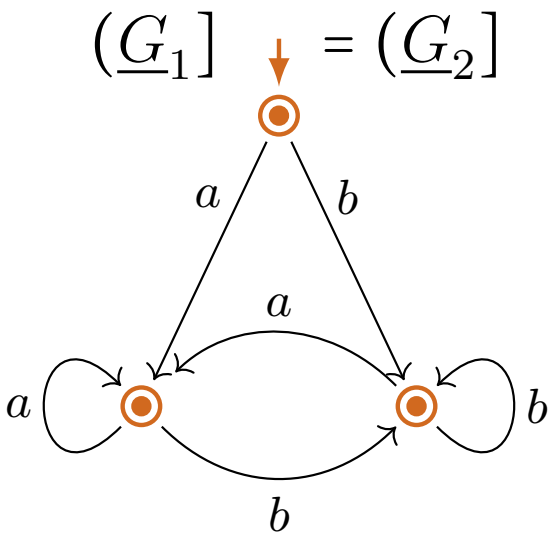

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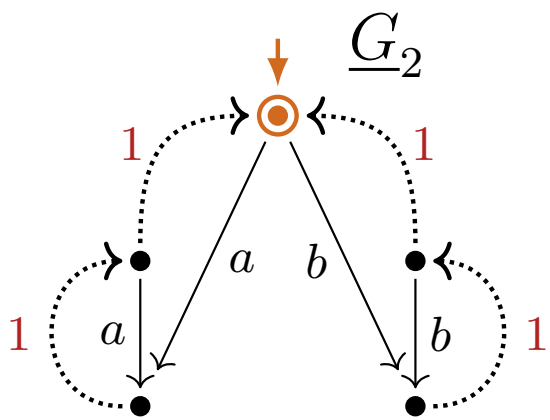

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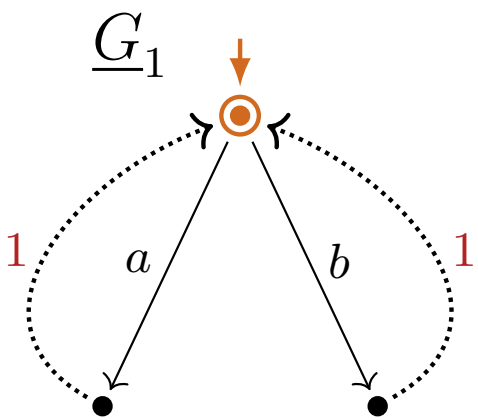

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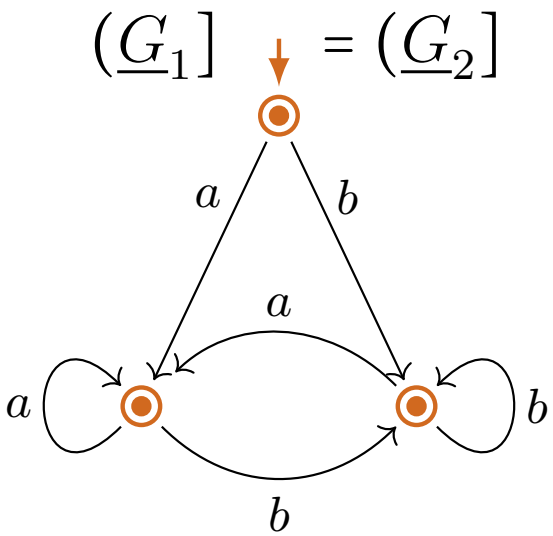

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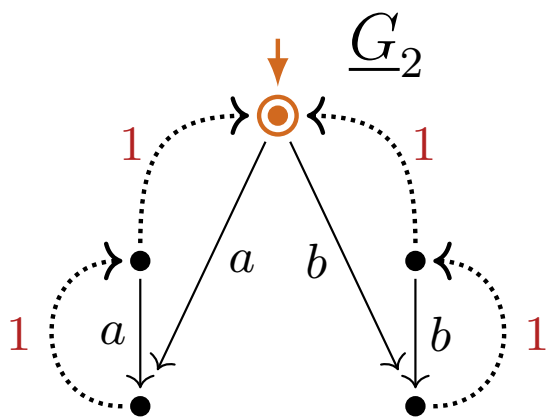

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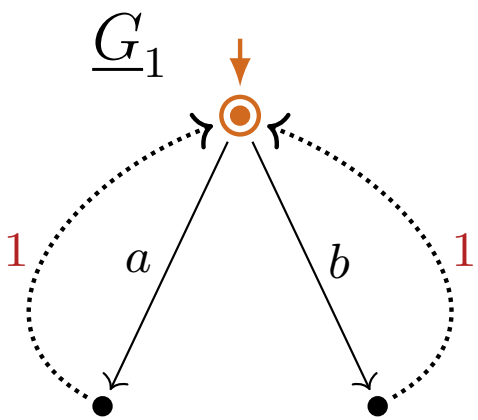

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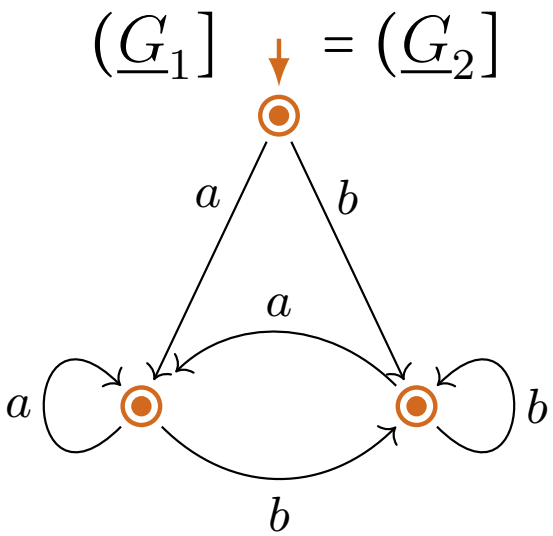

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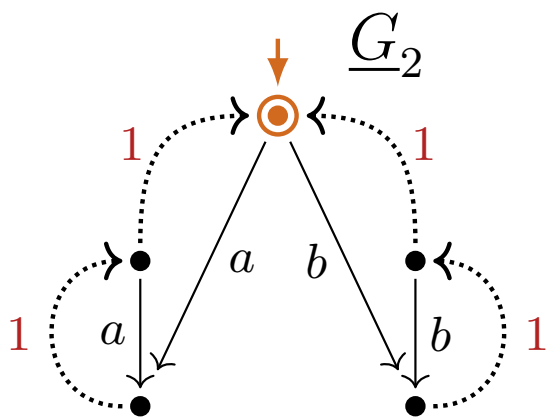

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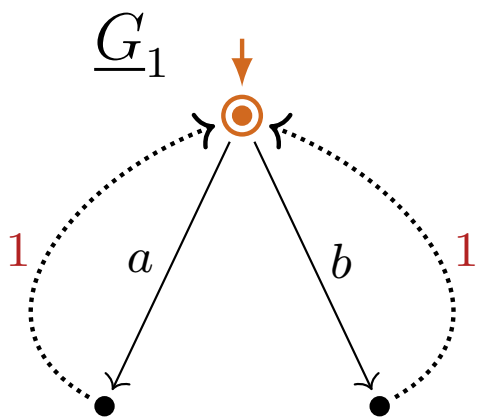
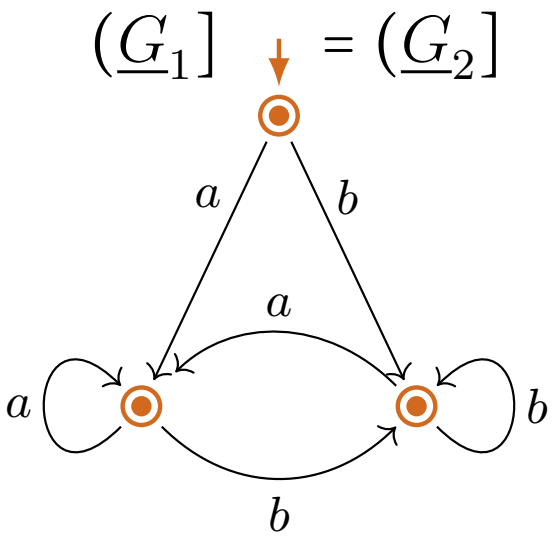
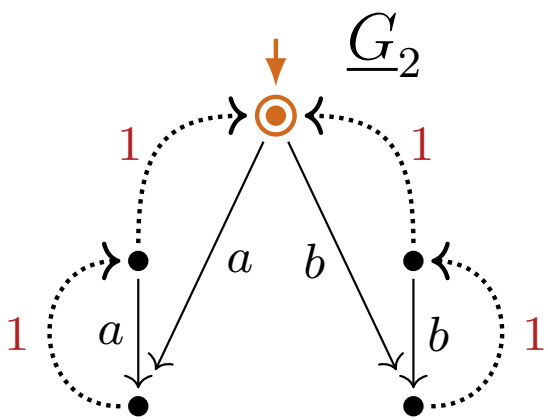


LEE



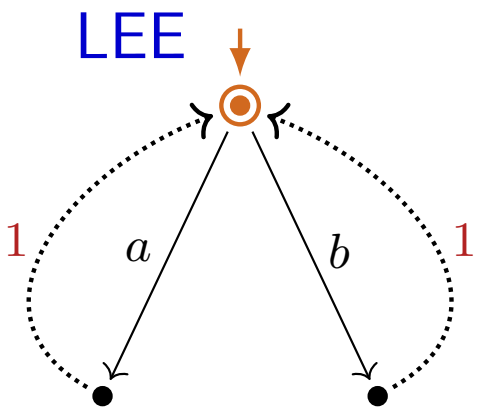
~~LEE~~

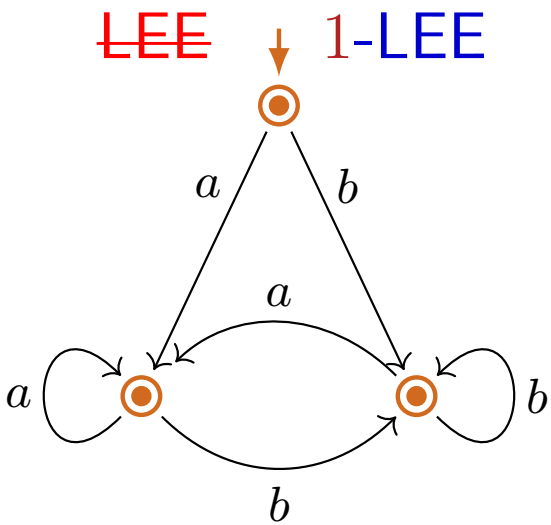


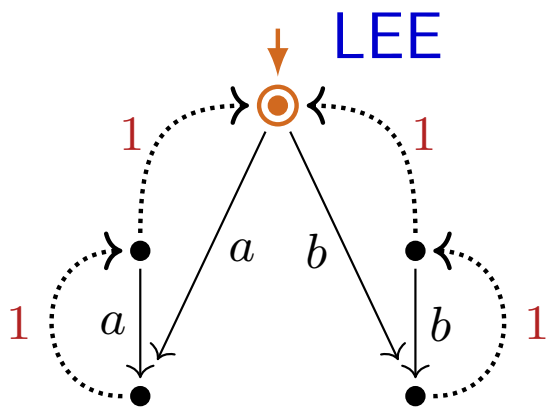
LEE

1-LEE

1-LEE holds for process interpretations







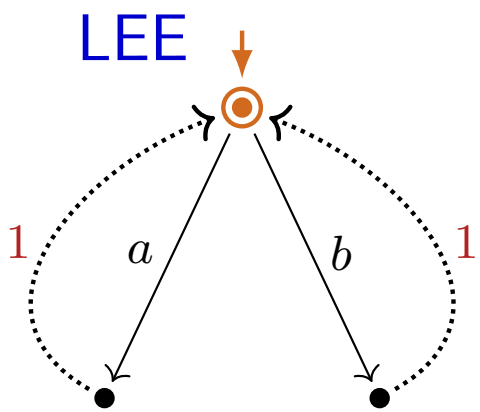
$P((a^* \cdot b^*)^*)$

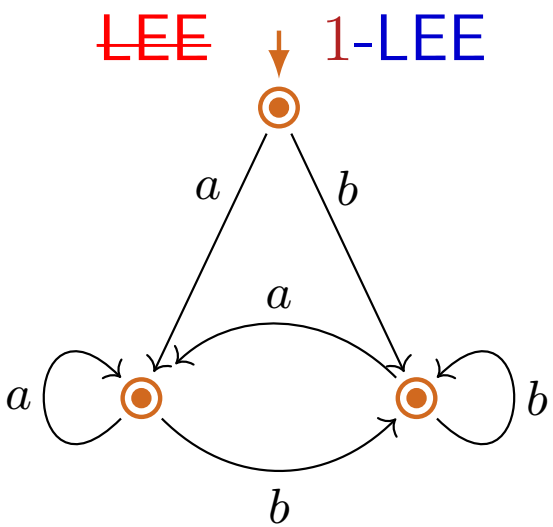
$P((b^* \cdot a^*)^*)$

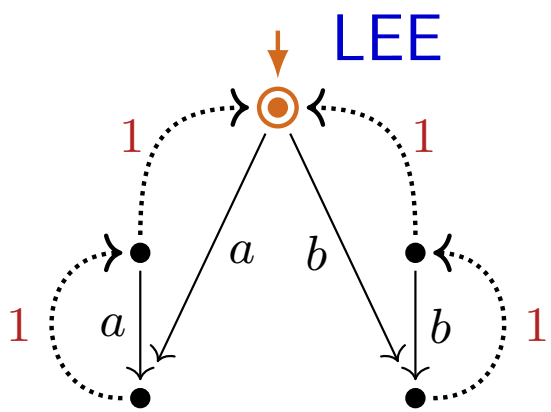
1-LEE holds for process interpretations

Lemma

There is a 1-graph interpretation $\underline{P}$ of reg. expression e as 1-graphs $\underline{P}(e)$ such that for all $e \in RExp$: (i): LEE($\underline{P}(e)$), (ii): $(\underline{P}(e)] = P(e)$.







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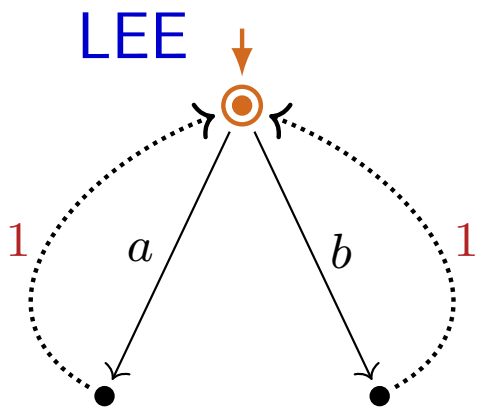

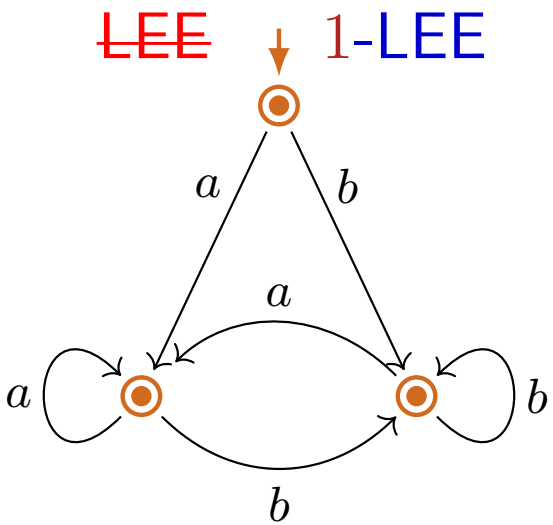

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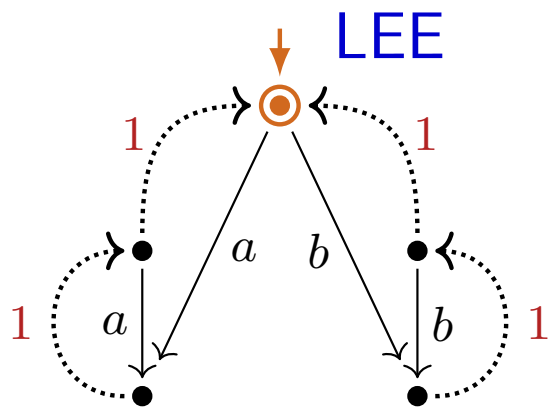

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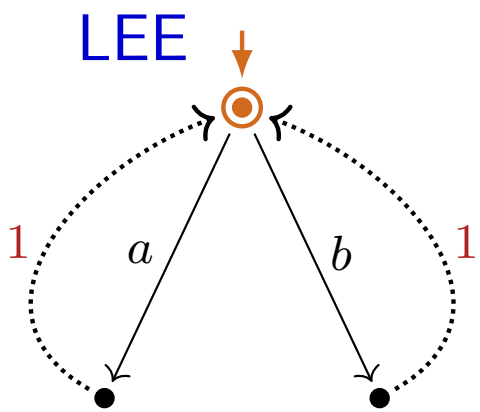
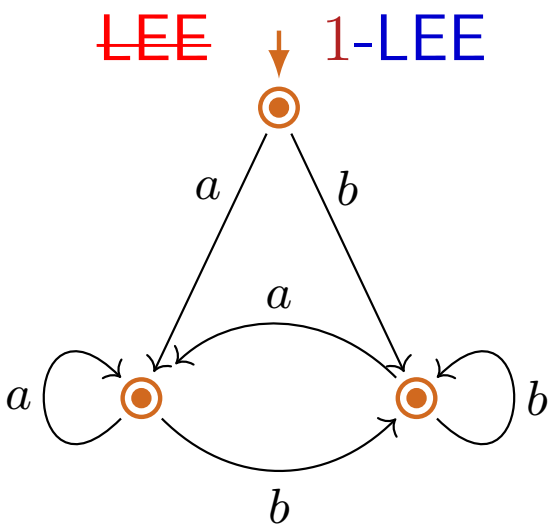
Theorem

1-LEE($P(e)$) holds for all regular expressions e.





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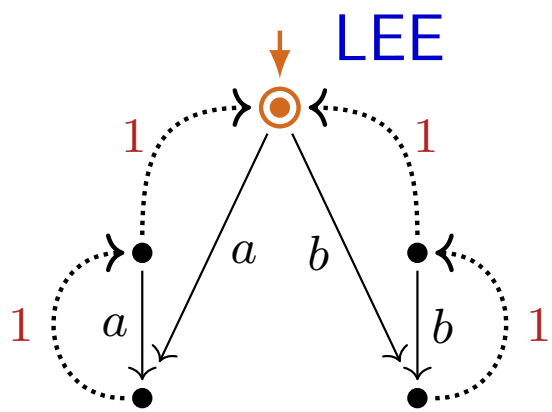

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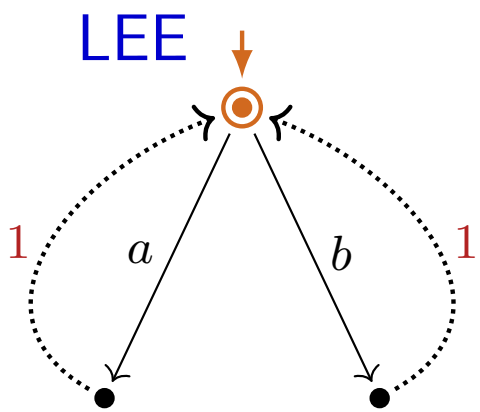

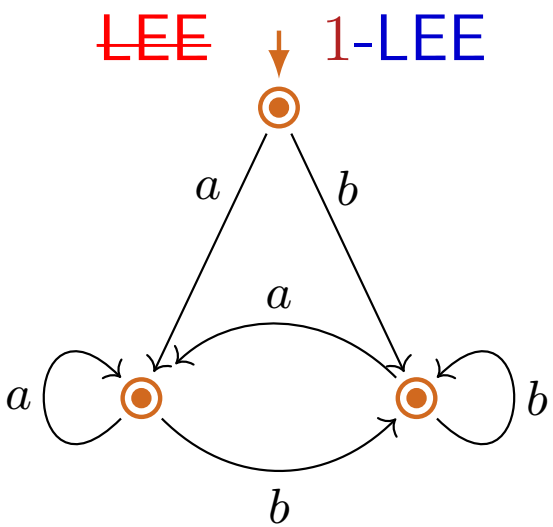

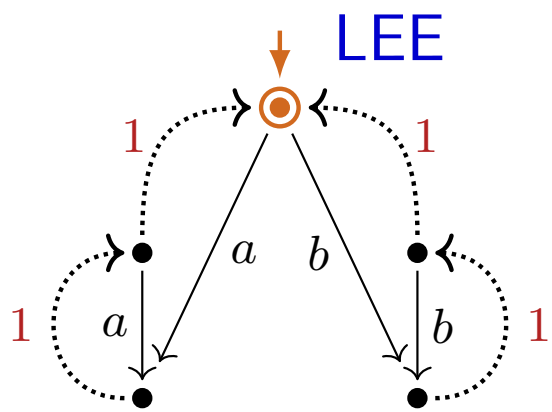

1-LEE($P((a^* \cdot b^*)^*)$) $\quad \underline{P}((a^* \cdot b^*)^*)$

1-LEE($P((b^* \cdot a^*)^*)$) $\quad \underline{P}((b^* \cdot a^*)^*)$

Interpretation/extraction correspondences with 1-LEE

(⇐ G 2021/22/23)

(Int)$_P$: *P-expressible graphs have the structural property* 1-LEE

Process **int**erpretations $P(e)$ of regular expressions e
are finite process graphs that satisfy 1-LEE.

(Extr)$_P$: 1-LEE *implies* $[\![\cdot]\!]_P$*-expressibility*

From every finite 1-process-graph $\underline{G}$ with 1-LEE
a regular expression e can be **extr**acted such that $\underline{G} \leftrightarrows P(e)$.

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~~**(Coll)**~~: 1-LEE *is not preserved under collapse*

The class of finite process graphs with 1-LEE
is not closed under bisimulation **coll**apse.

Interpretation/extraction correspondences of $P^\bullet$ with 1-LEE

(Int)$_{P^\bullet}$: *$P^\bullet$-expressible graphs satisfy* 1-LEE:

Compact process **i**nterpretations $P^\bullet(e)$ of regular expressions e are finite process graphs that satisfy 1-LEE.

(Extr)$_{P^\bullet}$: LEE *implies* $[\![\cdot]\!]_P$*-expressibility:*

From every finite process graph G with 1-LEE an regular expression e can be **e**xtracted such that $G \mathrel{\dot{\simeq}} P^\bullet(e)$.

From every finite collapsed process graph G with 1-LEE a regular expression e can be **e**xtracted such that $G \simeq P^\bullet(e)$.

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~~**(ImColl)**~~$_{P^\bullet}$: The image of $P^\bullet$ is not closed under bisimulation **c**ollapse .

1-LEE/ LEE characterize

the un-/restricted image of $P^\bullet$

Image of P is **not closed** under bisimulation collapse

not even for $(*/1)$ regular expressions

$P(uf)$

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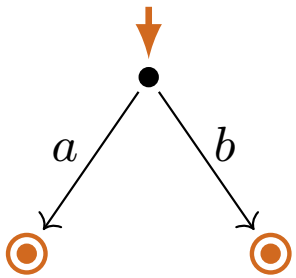
$$uf := a \cdot \overbrace{(a \cdot (a + a \cdot 0))^*}^{uf_a} + b \cdot \overbrace{(b \cdot (b + b \cdot 0))^*}^{uf_b}$$

Image of P is **not closed** under bisimulation collapse

not even for $(*/\perp)$ regular expressions

$P(uf)$



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a: $1 \cdot uf_a \Downarrow$

b: $\Downarrow 1 \cdot uf_b$

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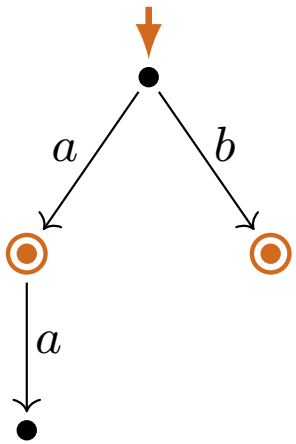

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$$\xrightarrow{a} 1 \cdot uf_a \Downarrow \qquad \Downarrow 1 \cdot uf_b \xleftarrow{b}$$

$$\downarrow a$$

$$(1 \cdot (a + a \cdot 0)) \cdot uf_a$$

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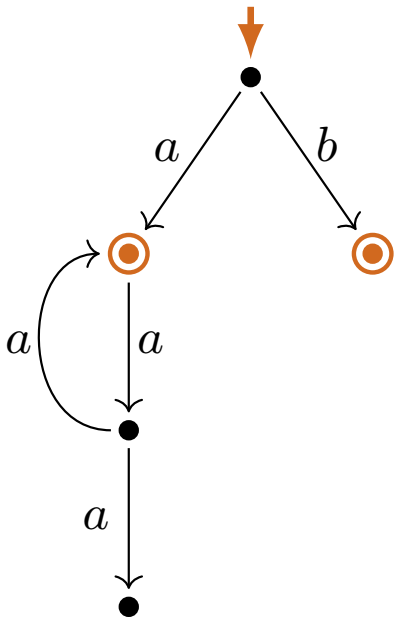

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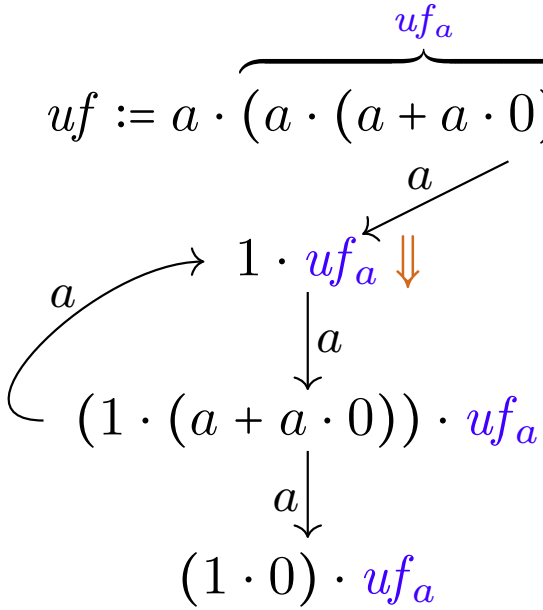

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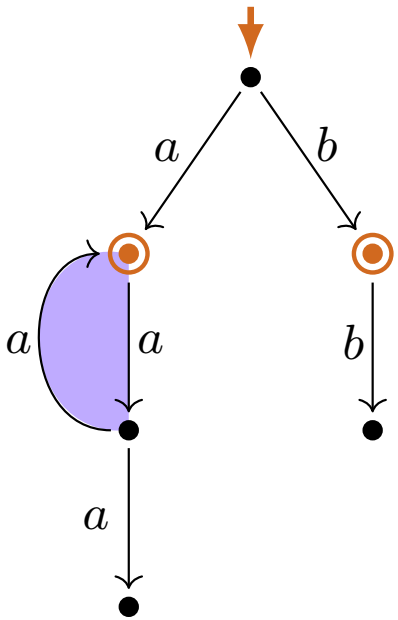

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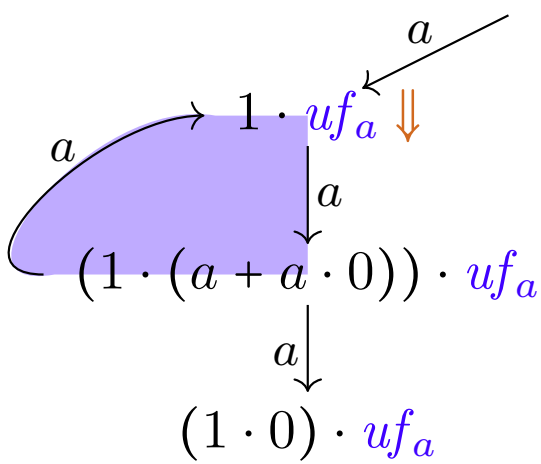

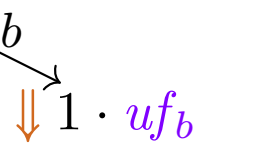

Image of P is **not closed** under bisimulation collapse

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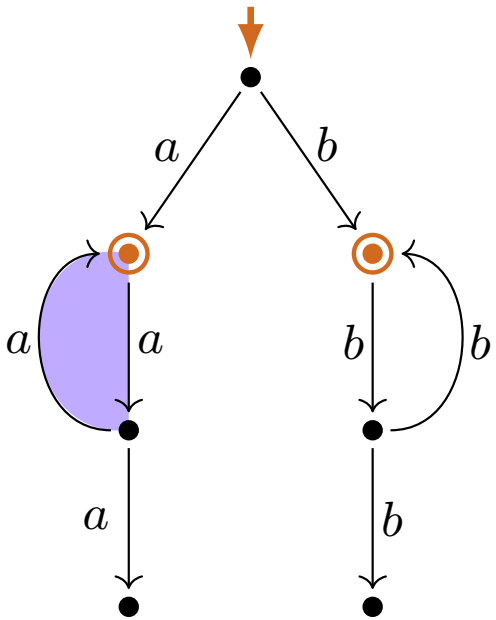

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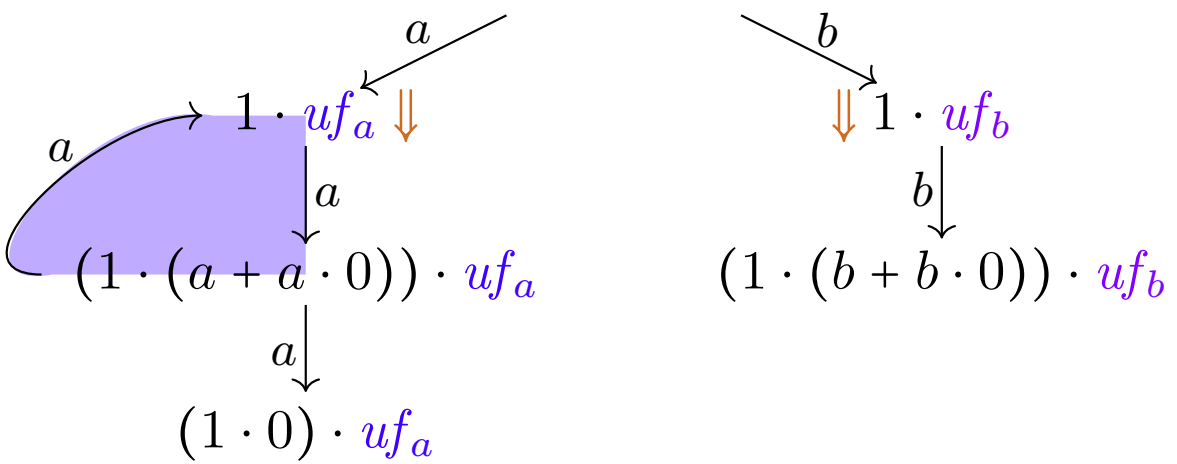

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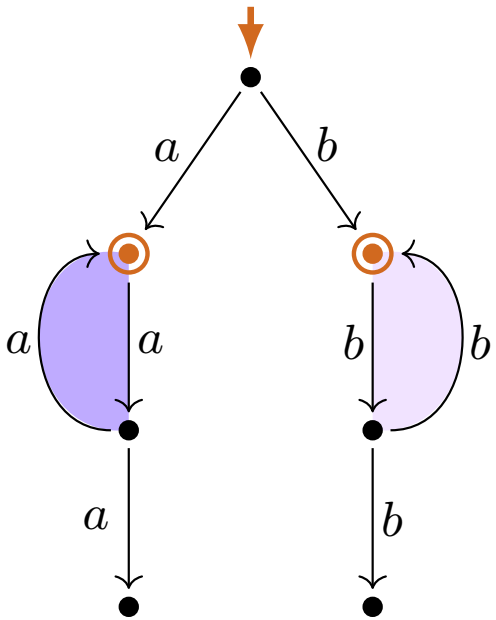

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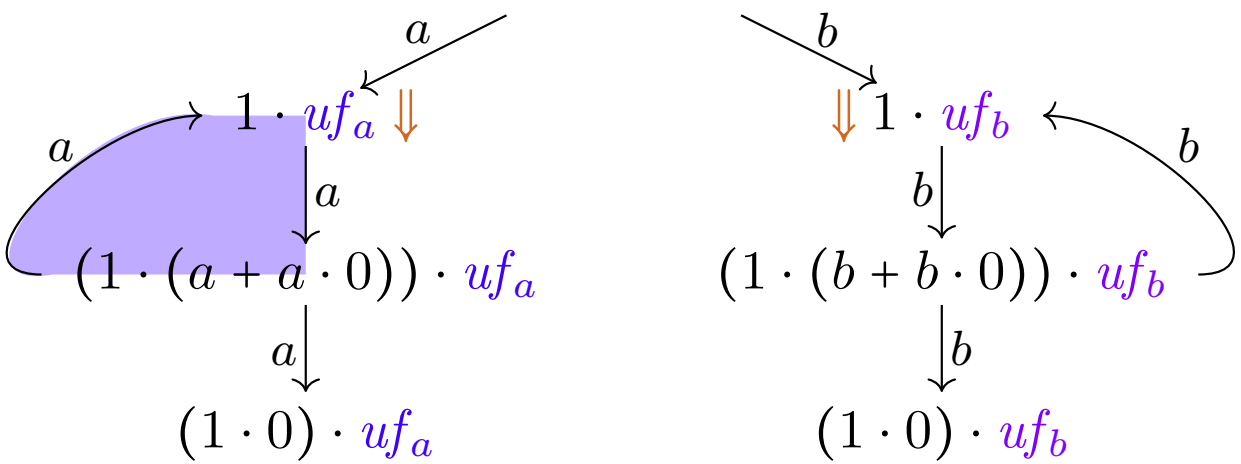

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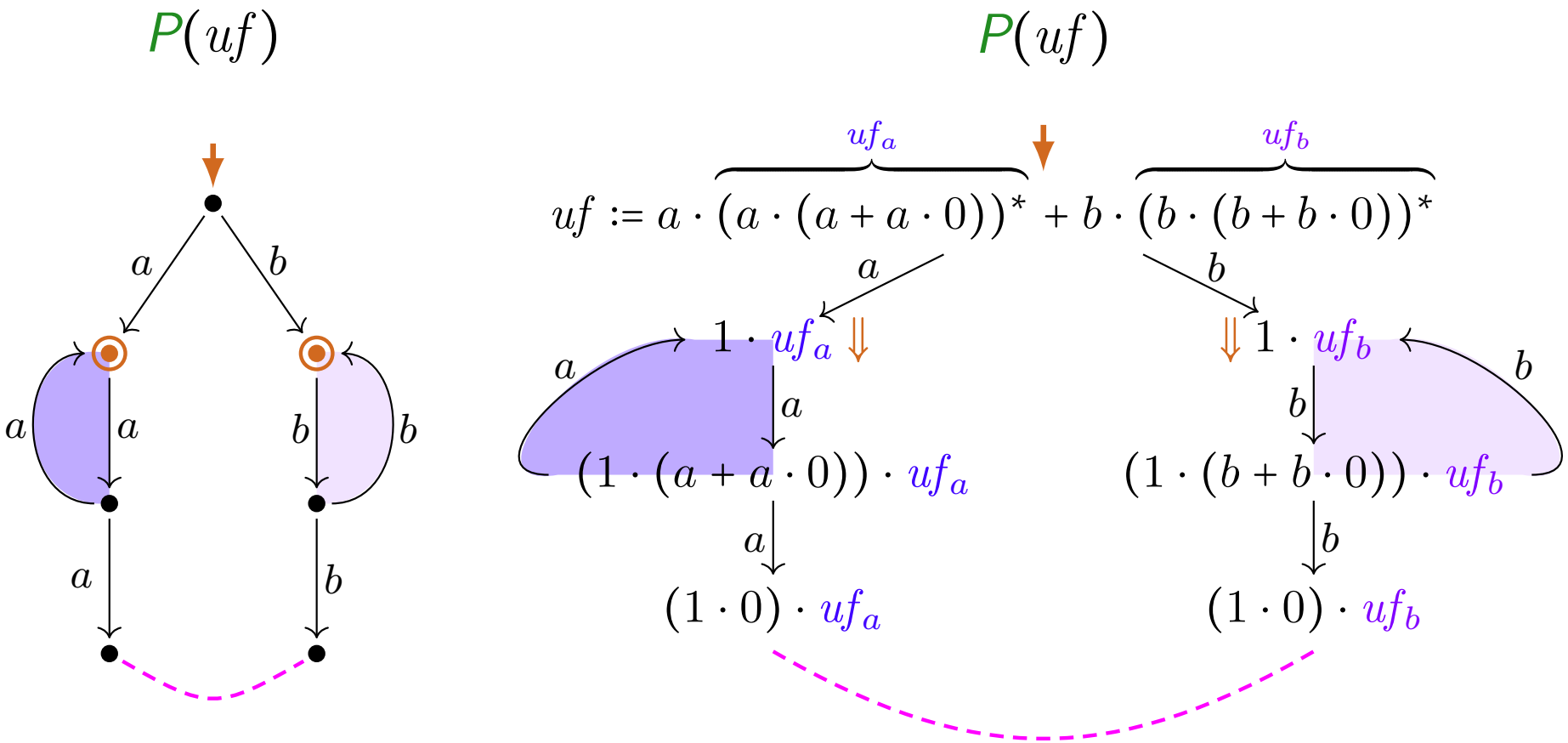

Compact process interpretation $P^\bullet$

Definition (Transition system specification $\mathcal{T}$)

$$\frac{}{\mathbf{1}\Downarrow} \qquad \frac{e_i\Downarrow}{(e_1 + e_2)\Downarrow}\ (i \in \{1,2\}) \qquad \frac{e_1\Downarrow \qquad e_2\Downarrow}{(e_1 \cdot e_2)\Downarrow} \qquad \frac{}{(e^*)\Downarrow}$$

$$\frac{}{a \xrightarrow{a} \mathbf{1}} \qquad \frac{e_i \xrightarrow{a} e_i'}{e_1 + e_2 \xrightarrow{a} e_i'}\ (i \in \{1,2\})$$

$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2} \qquad \frac{e_1\Downarrow \qquad e_2 \xrightarrow{a} e_2'}{e_1 \cdot e_2 \xrightarrow{a} e_2'} \qquad \frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e' \cdot e^*}$$

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Compact process interpretation $P^\bullet$

Definition (Transition system specification $\mathcal{T}^\bullet$, changed rules w.r.t. $\mathcal{T}$)

$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2} \text{ (if } e_1' \text{ is normed)}$$

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The compact process (graph) interpretation $P^\bullet(e)$ of a reg. expr's e:

$P^\bullet(e)$:= labeled transition graph generated by e by derivations in $\mathcal{T}^\bullet$.

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Lemma ($P^\bullet$ increases sharing; $P^\bullet$, P have same bisimulation semantics)

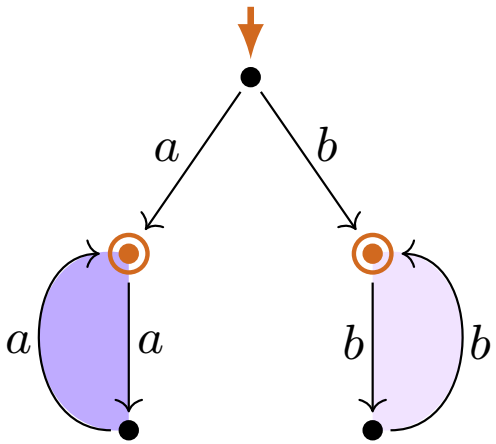
(i) $P(e) \mathrel{\rightarrow\!\!\!\!\!=} P^\bullet(e)$ *for all regular expressions* e.

(ii) (G *is* $[\![\cdot]\!]_{P^\bullet}$*-expressible* $\iff$ G *is* $[\![\cdot]\!]_P$*-expressible) for all graphs* G.

Image of P restricted to $(*/\not{1})$ regular expressions

. . . contains all of its bisimulation collapses

$P^\bullet(uf)$



$P^\bullet(uf)$

$$uf := a \cdot \overbrace{(a \cdot (a + a \cdot 0))^*}^{uf_a} + b \cdot \overbrace{(b \cdot (b + b \cdot 0))^*}^{uf_b}$$

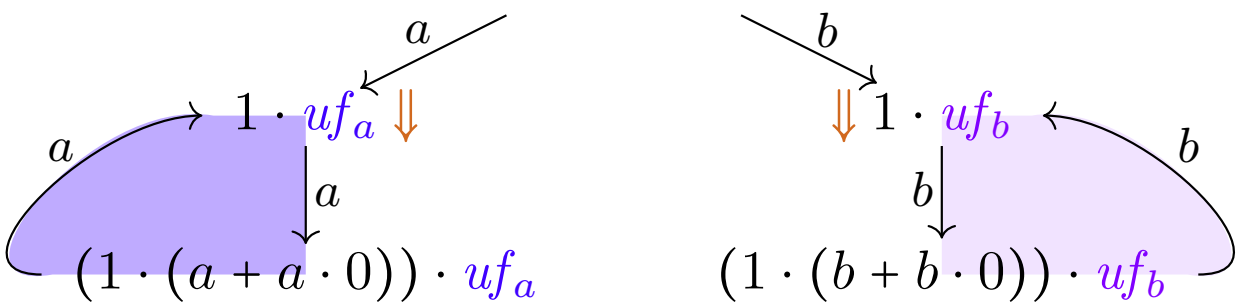
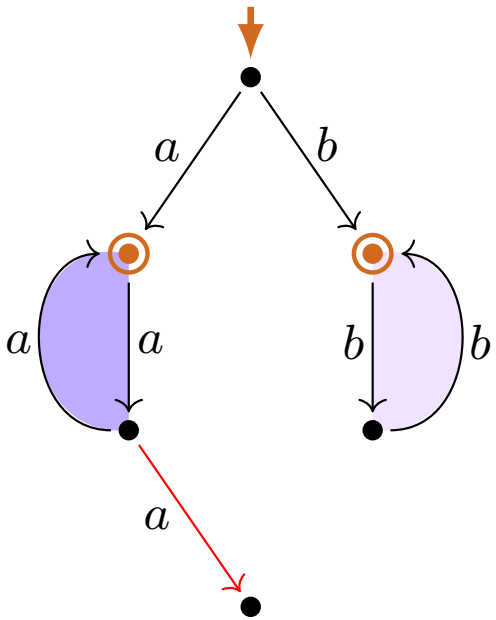


Image of P restricted to $(*/\not{1})$ regular expressions

. . . contains all of its bisimulation collapses

$P^\bullet(uf)$



$P^\bullet(uf)$

$$uf := \overbrace{a \cdot (a \cdot (a + a \cdot 0))^*}^{uf_a} + \overbrace{b \cdot (b \cdot (b + b \cdot 0))^*}^{uf_b}$$

a $\quad$ b

$1 \cdot uf_a \Downarrow$ $\quad$ $\Downarrow 1 \cdot uf_b$

a $\quad$ a $\quad$ b $\quad$ b

$(1 \cdot (a + a \cdot 0)) \cdot uf_a$ $\quad$ $(1 \cdot (b + b \cdot 0)) \cdot uf_b$

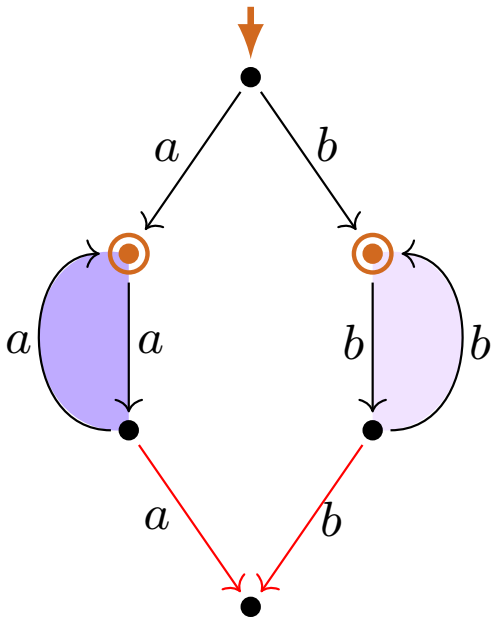
a

$1 \cdot 0$

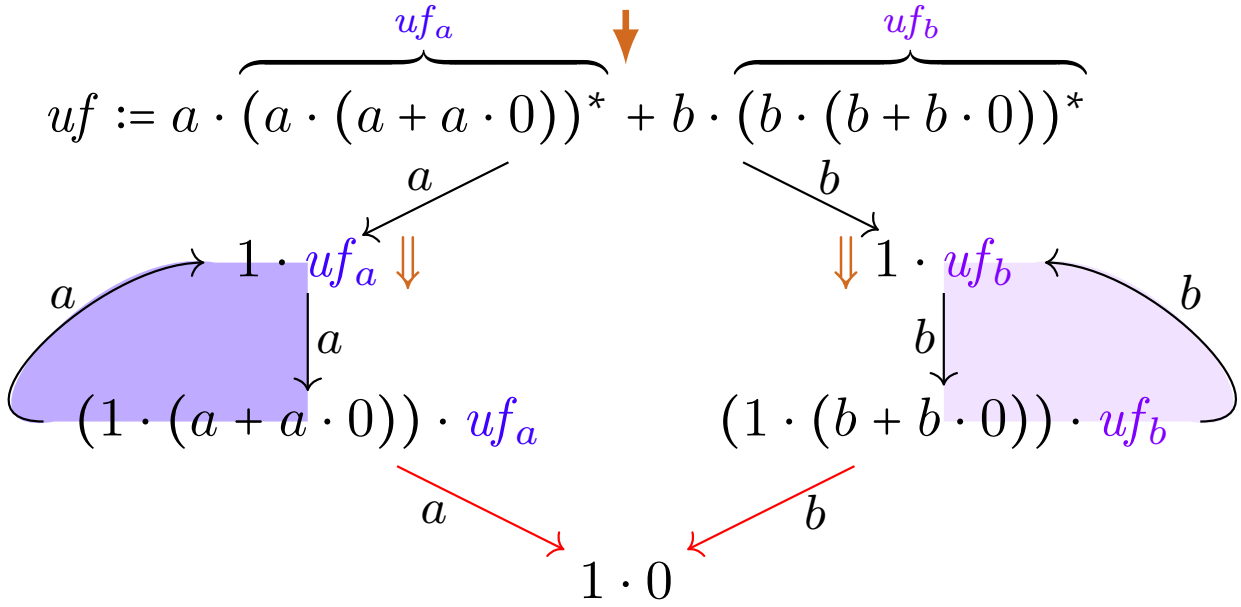
Image of P restricted to $(*/\not{1})$ regular expressions

. . . contains all of its bisimulation collapses

$P^\bullet(uf)$



$P^\bullet(uf)$



Interpretation correspondence of $P^\bullet$ with LEE

(Int)$^{(*/\not{1})}_{P^\bullet}$: *By under-star-1-free expressions $P^\bullet$-expressible graphs satisfy* LEE:

Compact process **i**nterpretations $P^\bullet(uf)$
of under-star-1-free regular expressions uf
are finite process graphs that satisfy LEE.

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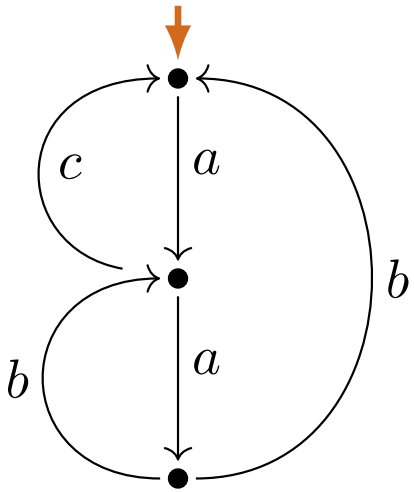
an under-star-1-free regular expression uf can be **e**xtracted
such that $G \twoheadrightarrow P(uf)$.

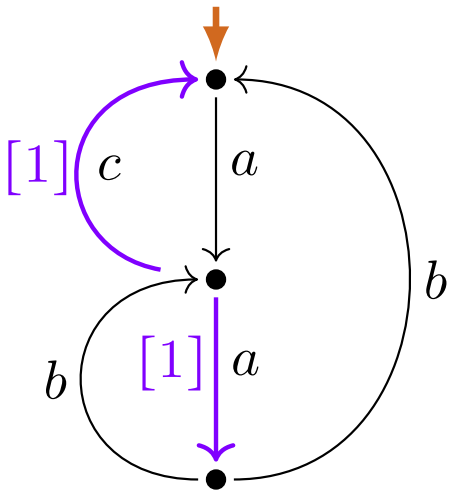
Refined extraction expression (example)

G_4



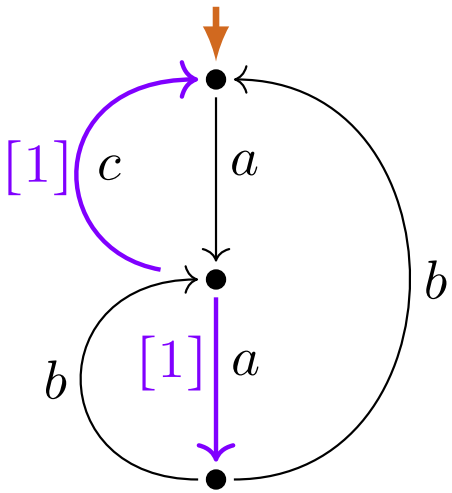
Refined extraction expression (example)

$\widehat{G_4}$



Refined extraction expression (example)

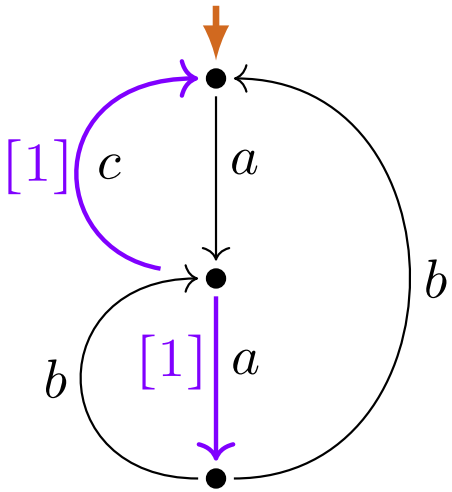
$\widehat{G_4}$



$(1 \cdot ($ $)^*) \cdot 0$

Refined extraction expression (example)

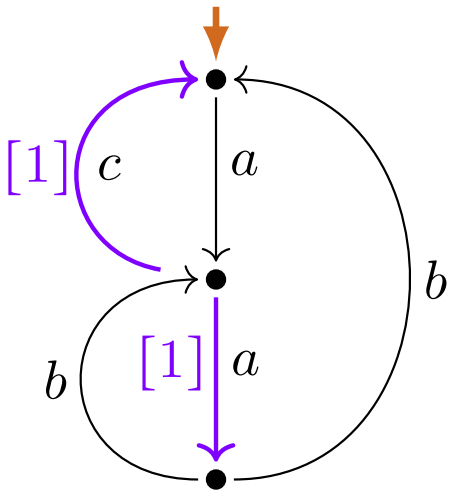
$\widehat{G_4}$



$$(1 \cdot (\ \overset{a}{\downarrow}\ )^*) \cdot 0$$

Refined extraction expression (example)

$\widehat{G_4}$

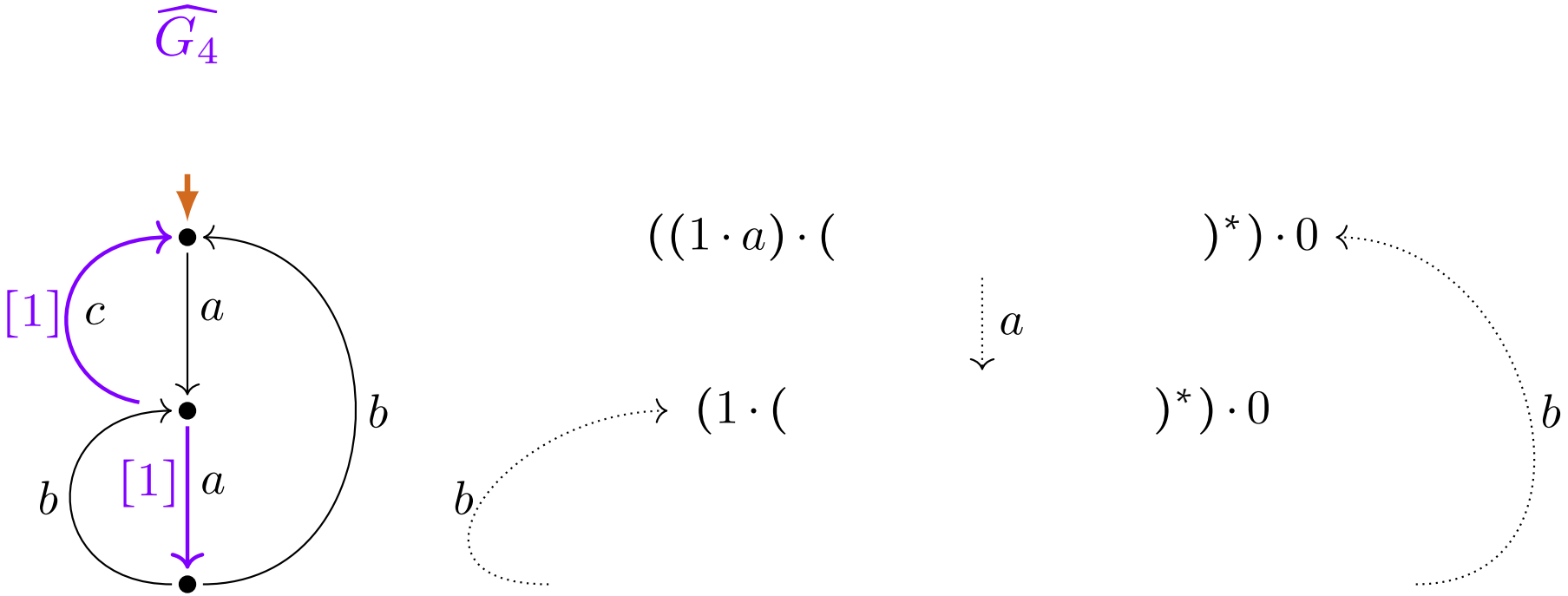


$$((1 \cdot a) \cdot (\quad)^*) \cdot 0$$

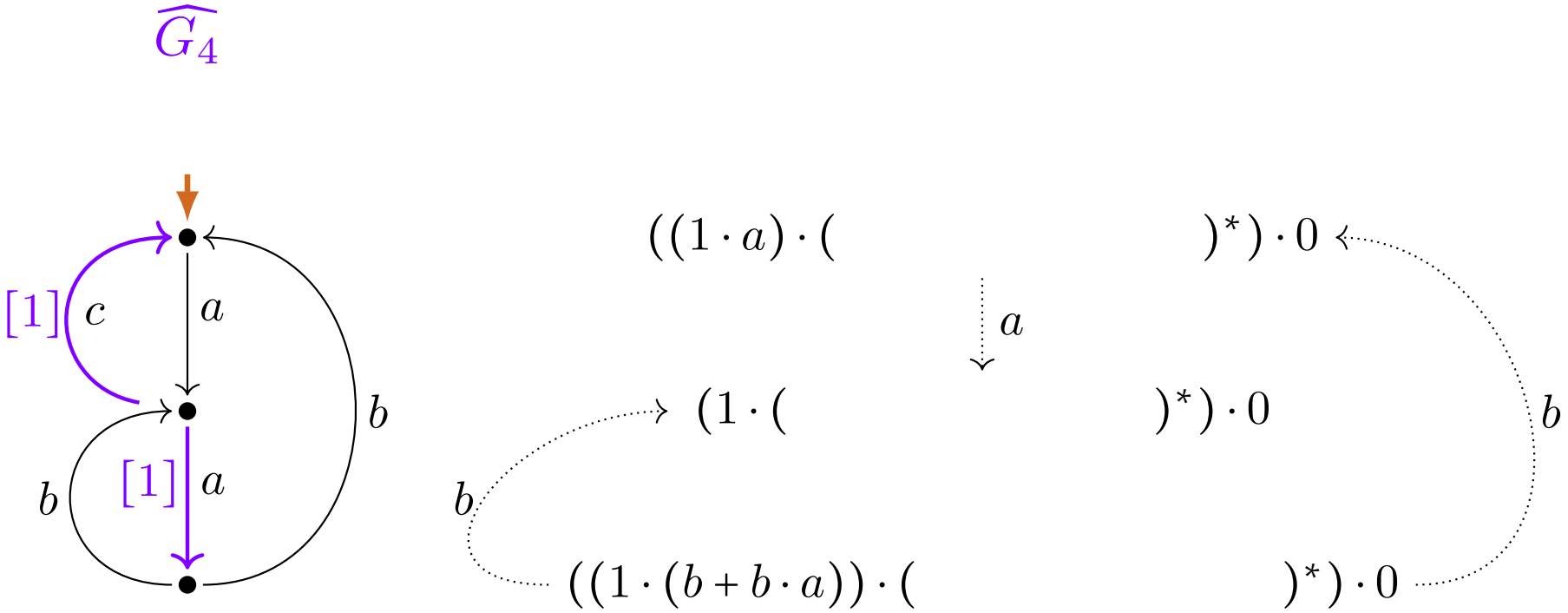
$$\downarrow a$$

$$(1 \cdot (\quad)^*) \cdot 0$$

Refined extraction expression (example)

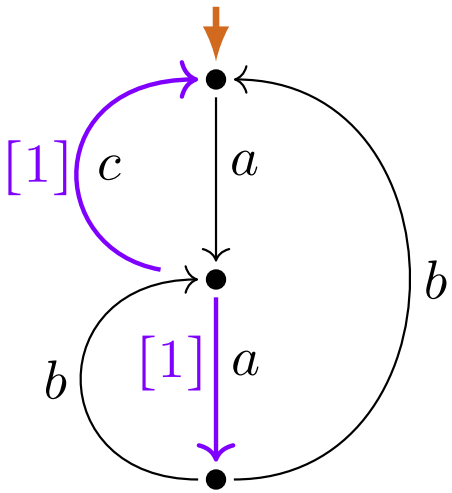


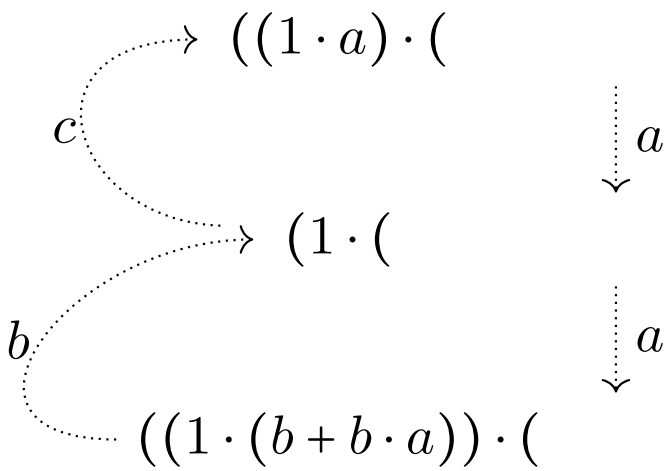
Refined extraction expression (example)

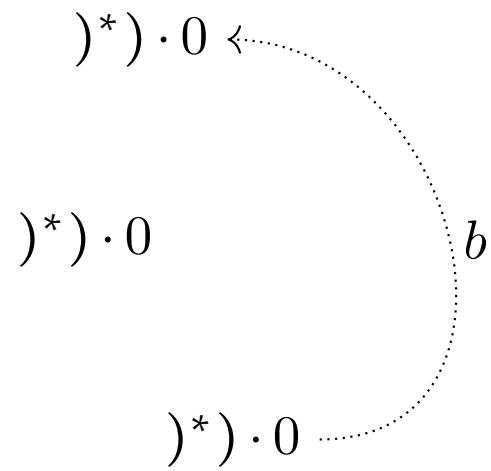


Refined extraction expression (example)

$\widehat{G_4}$

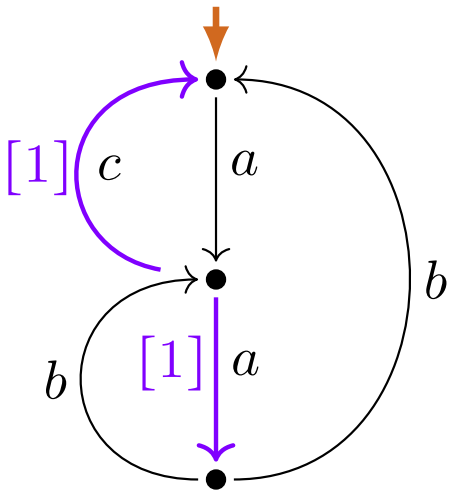


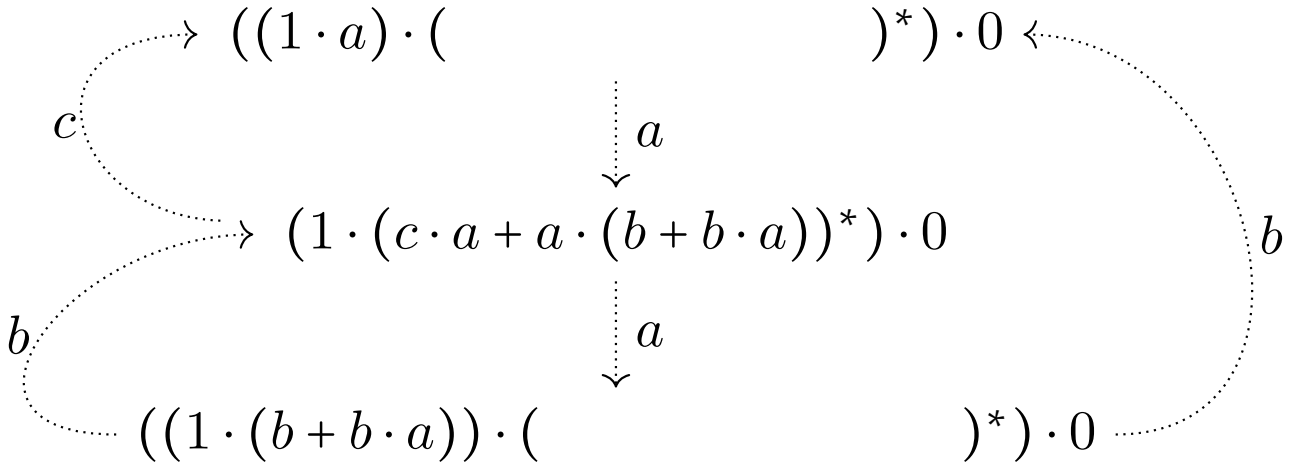




Refined extraction expression (example)

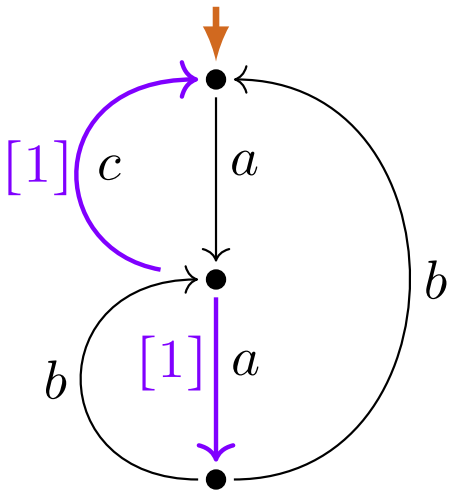
$\widehat{G_4}$

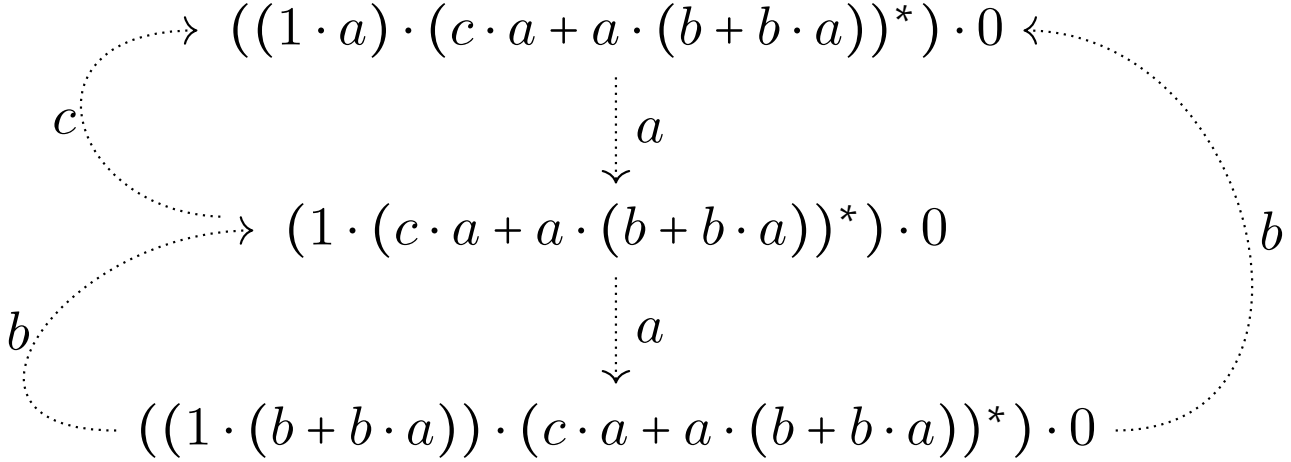




Refined extraction expression (example)

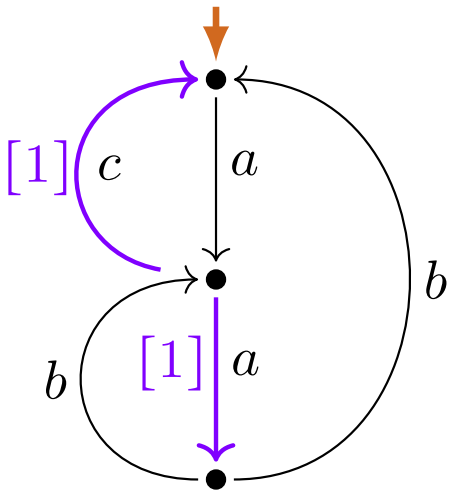
$\widehat{G_4}$

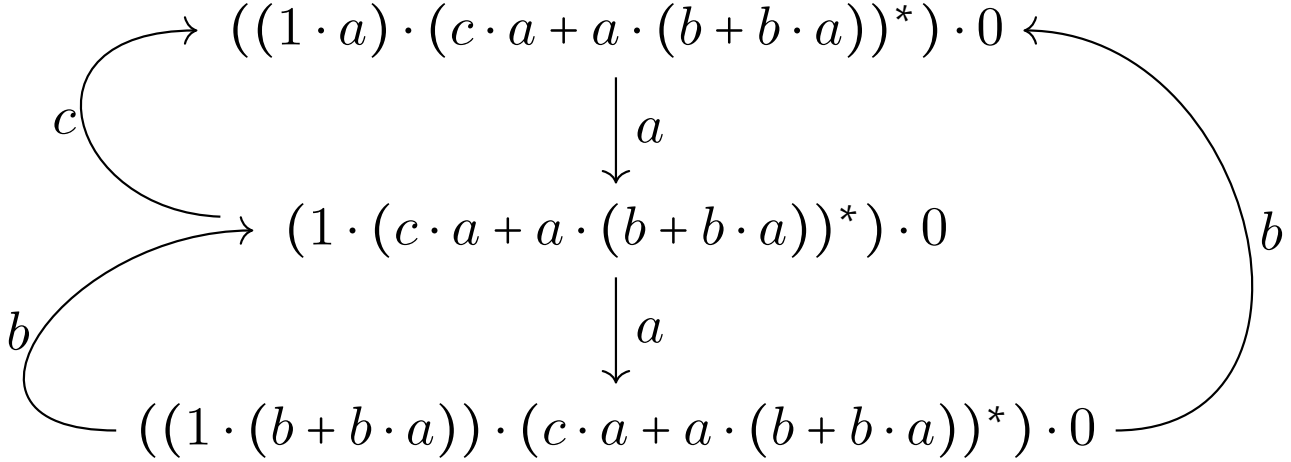




Refined extraction expression (example)

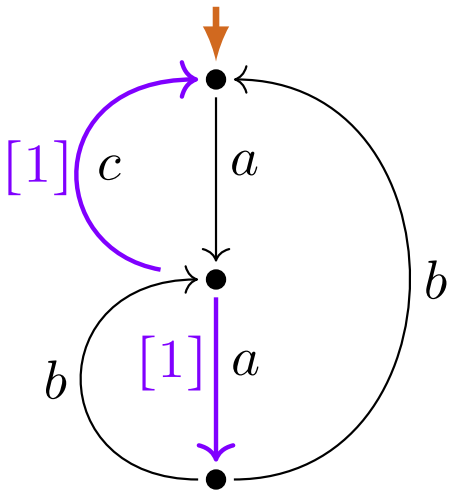
$\widehat{G_4}$



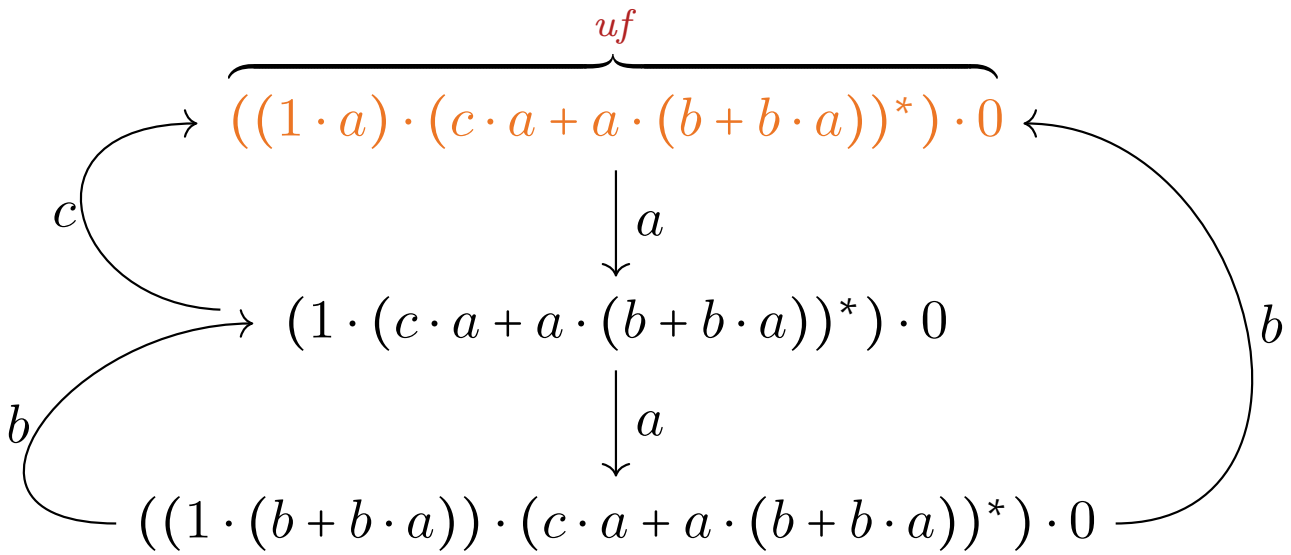


Refined extraction expression (example)

$\widehat{G_4}$



$P^\bullet(uf) = P(uf) \simeq G_4$



Interpretation/extraction correspondences of $P^\bullet$ with LEE

(Int)$^{(*/\not{1})}_{P^\bullet}$: *By under-star-1-free expressions $P^\bullet$-expressible graphs satisfy* LEE:

Compact process **i**nterpretations $P^\bullet(uf)$
of under-star-1-free regular expressions uf
are finite process graphs that satisfy LEE.

(Extr)$^{(*/\not{1})}_{P^\bullet}$: LEE *implies $[\![\cdot]\!]_P$-expressibility by under-star-1-free reg. expr's*:

From every finite process graph G with LEE
an under-star-1-free regular expression uf can be **e**xtracted
such that $G \mathrel{\dot{\simeq}} P^\bullet(uf)$.

From every finite collapsed process graph G with LEE
an under-star-1-free regular expression uf can be **e**xtracted
such that $G \simeq P^\bullet(uf)$.

Interpretation/extraction correspondences of $P^\bullet$ with LEE

(Int)$^{(*/\not{1})}_{P^\bullet}$: *By under-star-1-free expressions $P^\bullet$-expressible graphs satisfy* LEE:

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From every finite process graph G with LEE
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such that $G \rightarrow\!\!\!\!\!\cdot\;\; P^\bullet(uf)$.

From every finite collapsed process graph G with LEE
an under-star-1-free regular expression uf can be **e**xtracted
such that $G \simeq P^\bullet(uf)$.

(ImColl)$^{(*/\not{1})}_{P^\bullet}$: The image of $P^\bullet$,
restricted to under-star-1-free regular expressions,
is closed under bisimulation **c**ollapse.

LEE $\triangleq$ image of $P^\bullet|_{RExp^{(*/\not{1})}}$

Theorem

For every process graph G TFAE:

(i) LEE(G).

LEE $\triangleq$ image of $P^\bullet|_{RExp^{(*/\not{1})}}$

Theorem

For every process graph G TFAE:

(i) LEE(G).

(ii) *G is $P^\bullet$-expressible by an $(*/\not{1})$ regular expression (i.e. $G \simeq P^\bullet(e)$ for some $e \in RExp^{(*/\not{1})}$).*

LEE $\triangleq$ image of $P^\bullet|_{RExp^{(*/\not{1})}}$

Theorem

For every process graph G TFAE:

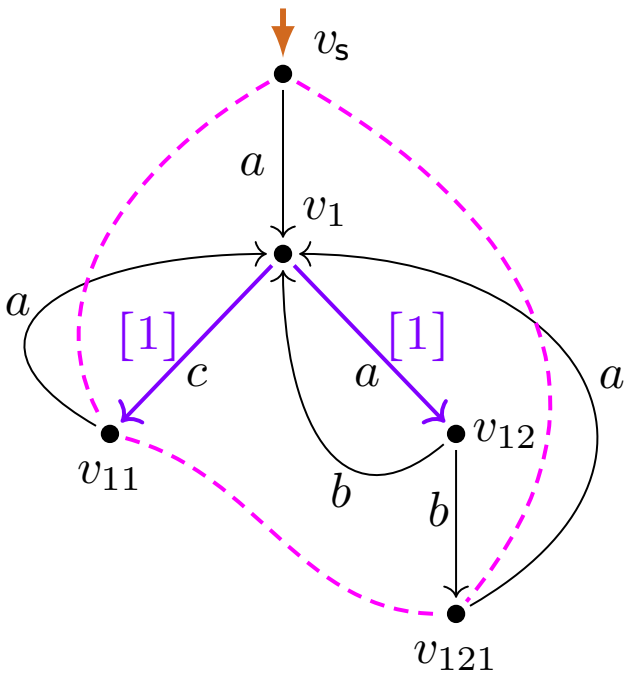
(i) LEE(G).

(ii) G *is* $P^\bullet$*-expressible by an* $(*/\not{1})$ *regular expression (i.e.* $G \simeq P^\bullet(e)$ *for some* $e \in RExp^{(*/\not{1})}$*).*

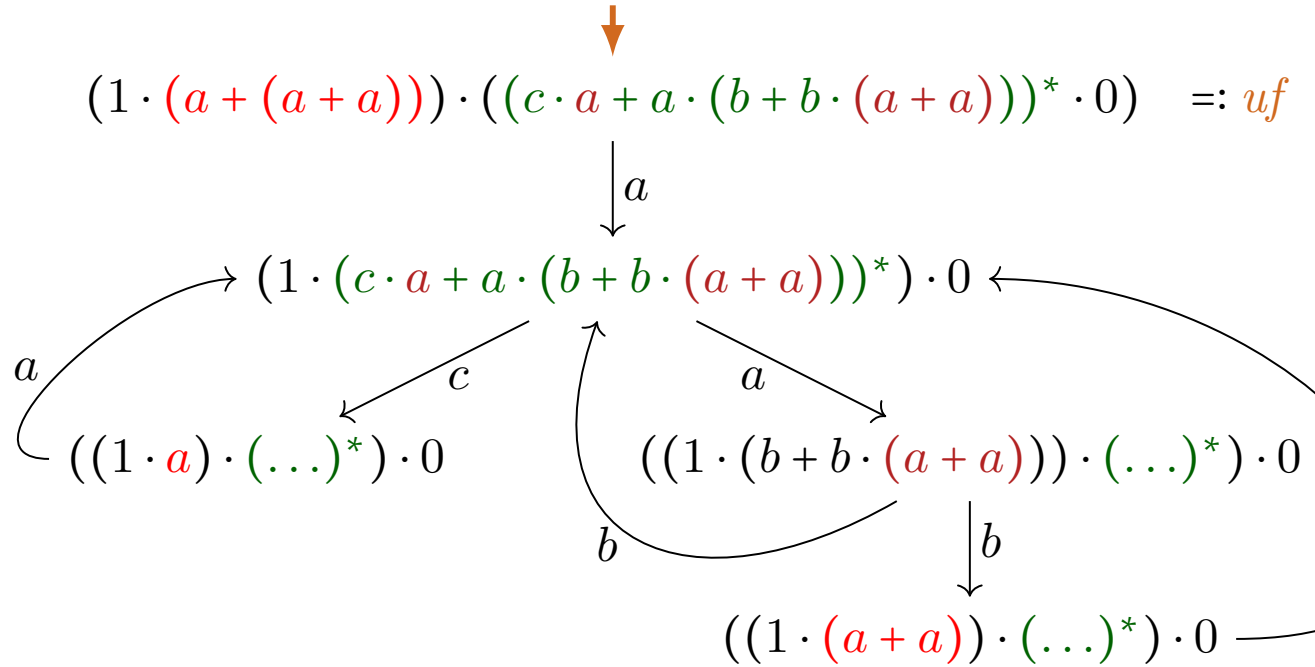
(iii) G *is isomorphic to a graph in the image of* $P^\bullet$ *on* $(*/\not{1})$ *reg. expr's (i.e.* $G \simeq G'$ *for some* $G' \in im(P^\bullet|_{RExp^{(*/\not{1})}})$*).*

Adapted (refined) extraction from LLEE-graph

$G_1 \,/\, \widehat{G_1}$



$P^\bullet(uf) = P(uf) \simeq G_1$



1-LEE $\hat{=}$ image of $P^\bullet$

Theorem

For every process graph G TFAE:

(i) 1-LEE(G)

(i.e. $G = (\underline{G}]$ for some 1-transition-process-graph $\underline{G}$ with LEE($\underline{G}$)).

1-LEE $\hat{=}$ image of $P^\bullet$

Theorem

For every process graph G TFAE:

(i) 1-LEE(G)
(i.e. $G = (\underline{G}]$ for some 1-transition-process-graph $\underline{G}$ with LEE($\underline{G}$)).

(ii) G *is $P^\bullet$-expressible by a regular expression*
(i.e. $G \simeq P^\bullet(e)$ for some $e \in RExp$).

1-LEE $\hat{=}$ image of $P^\bullet$

Theorem

For every process graph G TFAE:

(i) 1-LEE(G)
(i.e. $G = (\underline{G}]$ for some 1-transition-process-graph $\underline{G}$ with LEE($\underline{G}$)).

(ii) G *is* $P^\bullet$*-expressible by a regular expression*
(i.e. $G \simeq P^\bullet(e)$ for some $e \in RExp$).

(iii) G *is isomorphic to a graph in the image of* $P^\bullet$
(i.e. $G \simeq G'$ for some $G' \in im(P^\bullet)$).

Summary

- process interpretation P/semantics $[\![\cdot]\!]_P$ of regular expressions
  - expressibility and completeness questions
- loop existence and elimination (LEE)
  - loop elimination rewrite system can be completed
  - interpretation/extraction correspondences with $(*/\not{1})$ reg. expr.s
  - LEE-witnesses: labelings of graphs with LEE
  - stepwise LEE-preserving bisimulation collapse
- 1-LEE = sharing via 1-transitions facilitates LEE
  - interpretation/extraction correspondences with all regular expressions
  - not preserved under bisim. collapse (approximation possible)
- Characterizations of the image of $P^\bullet$ (refinement of P):
  - LEE $\hat{=}$ image of $P^\bullet|_{RExp(*/\not{1})}$ $\not\supseteq$ image of $P|_{RExp(*/\not{1})}$
  - 1-LEE $\hat{=}$ image of $P^\bullet$ $\not\supseteq$ image of P
- outlook on work-to-do

My next aims

Completeness problem, solution (journal articles):

A1: graph structure of regular expression processes (LEE/1-LEE)

A2: motivation of crystallization

A4: details of crystallization procedure,
and completeness of Milner's proof system

Expressibility problem

A3: LEE is decidable in polynomial time (conference article).

Q: Is 1-LEE decidable in polynomial time**?**

P: Is expressibility by a regular expression, for a finite process graph, decidable in polynomial time/fixed-parameter tractable time**?**

Resources

- Slides/abstract on clegra.github.io
  - slides: .../lf/IFIP-1_6-2024.pdf
  - abstract: .../lf/abstract-IFIP-1_6-2024.pdf
- CG: Closing the Image of the Process Interpretation of 1-Free Regular Expressions Under Bisimulation Collapse
  - TERMGRAPH 2024, extended abstract.
- CG: The Image of the Process Interpretation of Regular Expressions is Not Closed under Bisimulation Collapse,
  - arXiv:2303.08553, 2021/2023.
- CG: Milner's Proof System for Regular Expressions Modulo Bisimilarity is Complete,
  - LICS 2022, arXiv:2209.12188, poster.
- CG, Wan Fokkink: A Complete Proof System for 1-Free Regular Expressions Modulo Bisimilarity,
  - LICS 2020, arXiv:2004.12740, video on youtube.
- CG: Modeling Terms by Graphs with Structure Constraints,
  - TERMGRAPH 2018, EPTCS 288, arXiv:1902.02010.

Language semantics $[\![\cdot]\!]_L$ of reg. expr's *(Copi–Elgot–Wright, 1958)*

$$\mathbf{0} \xmapsto{L} \text{empty language } \varnothing$$

$$\mathbf{1} \xmapsto{L} \{\epsilon\} \qquad (\epsilon \text{ the empty word})$$

$$a \xmapsto{L} \{a\}$$

Language semantics $[\![\cdot]\!]_L$ of reg. expr's *(Copi–Elgot–Wright, 1958)*

$$\mathbf{0} \xmapsto{L} \text{empty language } \varnothing$$

$$\mathbf{1} \xmapsto{L} \{\epsilon\} \qquad (\epsilon \text{ the empty word})$$

$$a \xmapsto{L} \{a\}$$

$$e_1 + e_2 \xmapsto{L} \text{union of } L(e_1) \text{ and } L(e_2)$$

$$e_1 \cdot e_2 \xmapsto{L} \text{element-wise concatenation of } L(e_1) \text{ and } L(e_2)$$

$$e^* \xmapsto{L} \text{set of words formed by concatenating words in } L(e), \text{ and adding the empty word } \epsilon$$

Language semantics $[\![\cdot]\!]_L$ of reg. expr's *(Copi–Elgot–Wright, 1958)*

$\mathbf{0} \overset{L}{\longmapsto}$ empty language $\varnothing$

$\mathbf{1} \overset{L}{\longmapsto} \{\epsilon\}$ (ϵ the empty word)

$a \overset{L}{\longmapsto} \{a\}$

$e_1 + e_2 \overset{L}{\longmapsto}$ union of $L(e_1)$ and $L(e_2)$

$e_1 \cdot e_2 \overset{L}{\longmapsto}$ element-wise concatenation of $L(e_1)$ and $L(e_2)$

$e^* \overset{L}{\longmapsto}$ set of words formed by concatenating words in $L(e)$, and adding the empty word ϵ

$[\![e]\!]_L := L(e)$ (language defined by e)