Closing the Image of the Process Interpretation of 1-Free Regular Expressions under Bisim. Collapse

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Computer Science Seminar

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Overview

- 1-free regular expressions (with unary/binary star)
- process interpretation / semantics of regular expressions
  - expressible/not expressible process graphs
- loop existence and elimination property (LEE) (G/Fokkink, 2020)
  - interpretation/extraction correspondences with 1-free reg. expr's
  - LEE is preserved under bisimulation collapse
- Q: Image of process interpretation of 1-free regular expressions closed under bisimulation collapse?
  - A1: No. — But ...
- compact process interpretation
- refined expression extraction
  - A2: compact process interpretation is image-closed under collapse
- outlook: consequences

Regular Expressions

Definition (~ *Copi–Elgot–Wright, 1958*)

Regular expressions over alphabet A with unary Kleene star:

$$e, e_1, e_2 \ ::= \ \mathbf{0} \ \mid \ \boldsymbol{a} \ \mid \ e_1 + e_2 \ \mid \ e_1 \cdot e_2 \ \mid \ e^{*} \qquad \text{(for } \boldsymbol{a} \in A\text{).}$$

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- symbol $\mathbf{0}$ instead of $\varnothing$, symbol $\mathbf{1}$ instead of $\{\epsilon\}$
- with unary star $*$: $\mathbf{1}$ is definable as $\mathbf{0}^{*}$

Regular Expressions

Definition (~ *Kleene, 1951*, ~ *Copi–Elgot–Wright, 1958*)

Regular expressions over alphabet A with unary / binary Kleene star:

$$e, e_1, e_2 \;::=\; \mathbf{0} \mid \boldsymbol{a} \mid e_1 + e_2 \mid e_1 \cdot e_2 \mid e^{*} \qquad (\text{for } \boldsymbol{a} \in A).$$

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- with binary star $^{\circledast}$: $\mathbf{1}$ is not definable (in its absence)

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- symbol $\mathbf{0}$ instead of $\emptyset$, symbol $\mathbf{1}$ instead of $\{\epsilon\}$
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Definition

1-free regular expressions over alphabet A with binary Kleene star:

$$f, f_1, f_2 \ ::= \ \mathbf{0} \mid \boldsymbol{a} \mid f_1 + f_2 \mid f_1 \cdot f_2 \mid f_1{}^{\circledast} f_2 \qquad (\text{for } \boldsymbol{a} \in A).$$

Regular Expressions

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Regular expressions over alphabet A with unary / binary Kleene star:

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Under-Star-/1-Free regular expressions

Definition

The set $\mathit{RExp}^{(\cancel{1})}(A)$ of 1-free regular expressions over A is defined by:

$$f,\, f_1,\, f_2 \;::=\; 0 \mid a \mid f_1 + f_2 \mid f_1 \cdot f_2 \mid {f_1}^{\star} \cdot f_2 \qquad \text{(for } a \in A\text{)},$$

the set $\mathit{RExp}^{(\cancel{1}\backslash *)}(A)$ of under-star-1-free regular expressions over A by:

$$uf,\, uf_1,\, uf_2 \;::=\; 0 \mid 1 \mid a \mid uf_1 + uf_2 \mid uf_1 \cdot uf_2 \mid f^{\star} \qquad \text{(for } a \in A\text{)}.$$

Under-Star-/1-Free regular expressions

Definition

The set $RExp^{(\neq 1)}(A)$ of 1-free regular expressions over A is defined by:

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the set $RExp^{(\neq 1 \backslash *)}(A)$ of under-star-1-free regular expressions over A by:

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Under the language interpretation, subclasses of minor relevance:

- 1-free regular expressions denote all regular languages without ϵ.
- Under-star-1-free regular expressions denote all regular languages.

Process interpretation $P(\cdot)$ of regular expressions *(Milner, 1984)*

$\mathbf{0} \xrightarrow{P}$ deadlock $\boldsymbol{\delta}$, no termination

$\mathbf{1} \xrightarrow{P}$ empty-step process $\boldsymbol{\epsilon}$, then terminate

$a \xrightarrow{P}$ atomic action a, then terminate

Process interpretation $P(\cdot)$ of regular expressions *(Milner, 1984)*

$$\mathbf{0} \xmapsto{P} \text{deadlock } \boldsymbol{\delta}\text{, no termination}$$

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$$a \xmapsto{P} \text{atomic action } a\text{, then terminate}$$

$$e_1 + e_2 \xmapsto{P} (\textit{choice}) \text{ execute } P(e_1) \text{ or } P(e_2)$$

$$e_1 \cdot e_2 \xmapsto{P} (\textit{sequentialization}) \text{ execute } P(e_1)\text{, then } P(e_2)$$

$$e^* \xmapsto{P} (\textit{iteration}) \text{ repeat (terminate or execute } P(e)\text{)}$$

Process interpretation $P(\cdot)$ of regular expressions *(Milner, 1984)*

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$e_1 + e_2 \xmapsto{P}$ (*choice*) execute $P(e_1)$ or $P(e_2)$

$e_1 \cdot e_2 \xmapsto{P}$ (*sequentialization*) execute $P(e_1)$, then $P(e_2)$

$e^* \xmapsto{P}$ (*iteration*) repeat (terminate or execute $P(e)$)

$e_1{}^{\circledast} e_2 \xmapsto{P}$ (*iteration-exit*) repeat (terminate or execute $P(e_1)$), then $P(e_2)$

Process semantics $[\![\cdot]\!]_P$ of regular expressions *(Milner, 1984)*

$\mathbf{0} \overset{P}{\longmapsto}$ deadlock $\boldsymbol{\delta}$, no termination

$\mathbf{1} \overset{P}{\longmapsto}$ empty-step process ϵ, then terminate

$a \overset{P}{\longmapsto}$ atomic action a, then terminate

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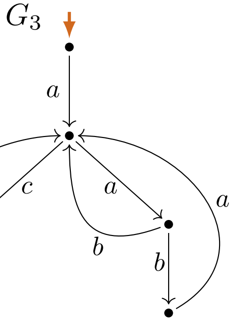
$e^* \overset{P}{\longmapsto}$ (*iteration*) repeat (terminate or execute $P(e)$)

$e_1{}^{\circledast} e_2 \overset{P}{\longmapsto}$ (*iteration-exit*) repeat (terminate or execute $P(e_1)$), then $P(e_2)$

$[\![e]\!]_P \;:=\; [P(e)]_{\underline{\leftrightarrow}}$ (bisimilarity equivalence class of process $P(e)$)

P-expressibility and $[\![\cdot]\!]_P$-expressibility (examples)



$$P\Big(\overbrace{(a\cdot(c\cdot a + a\cdot(b + b\cdot a))^*)\cdot 0}^{f}\Big)$$

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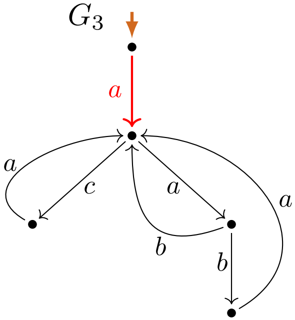

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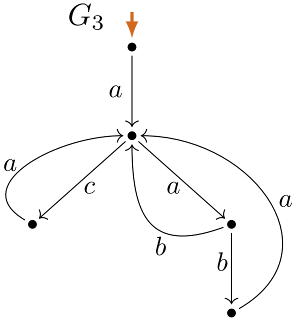

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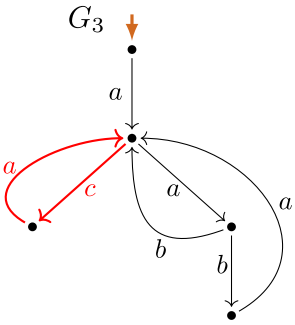

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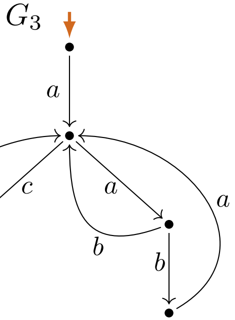

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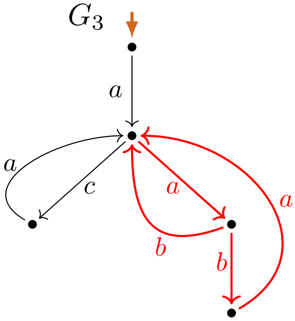

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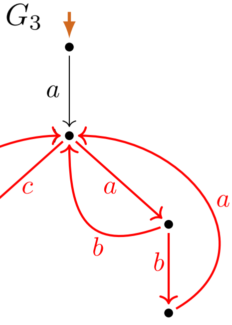

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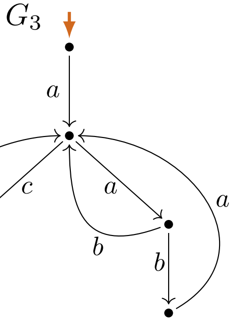

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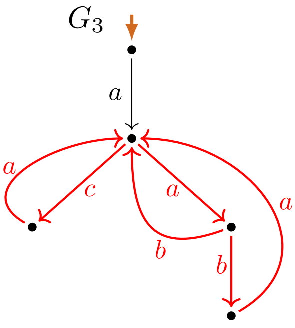

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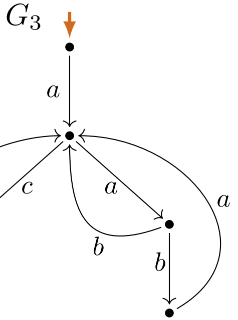

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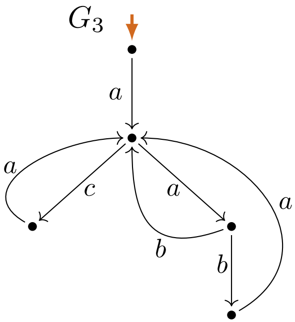
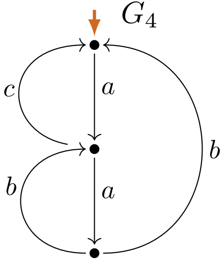
P-expressibility and $[\![\cdot]\!]_P$-expressibility (examples)





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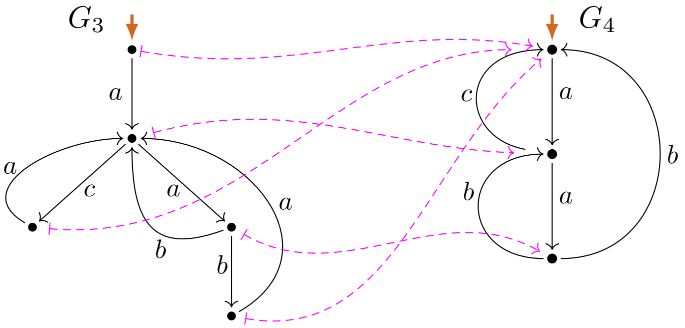

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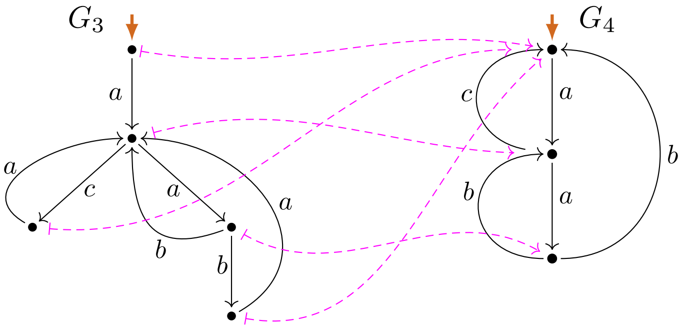

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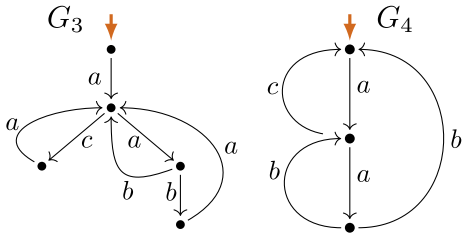
$$G_4 \in [\![f]\!]_P$$

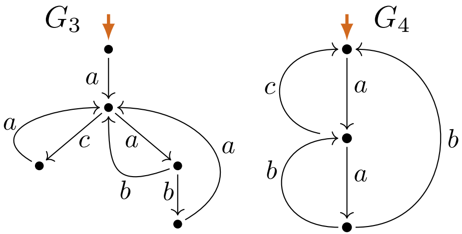
P-expressibility and $[\![\cdot]\!]_P$-expressibility (examples)



P-expressible

$[\![\cdot]\!]_P$-expressible $\quad$ $[\![\cdot]\!]_P$-expressible

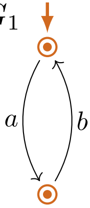
P-expressibility and $[\![\cdot]\!]_P$-expressibility (examples)



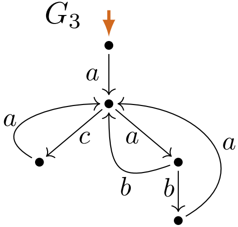
P-expressible	**?**
$[\![\cdot]\!]_P$-expressible	$[\![\cdot]\!]_P$-expressible

P-expressibility and $[\![\cdot]\!]_P$-expressibility (examples)

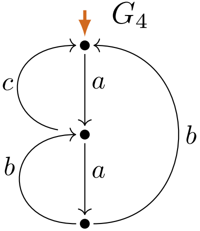
G_1



G_3



G_4



not P-expressible

not $[\![\cdot]\!]_P$-expressible

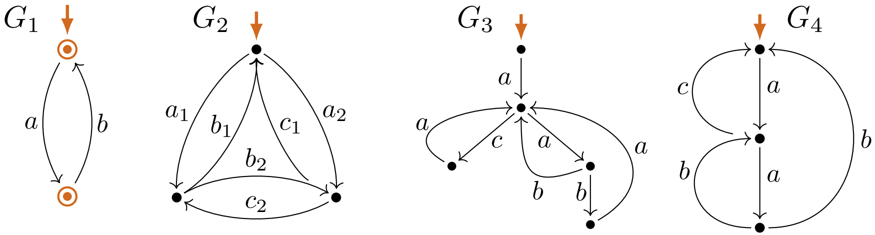
P-expressible

$[\![\cdot]\!]_P$-expressible

?

$[\![\cdot]\!]_P$-expressible

P-expressibility and $[\![\cdot]\!]_P$-expressibility (examples)



not P-expressible
not $[\![\cdot]\!]_P$-expressible

P-expressible
$[\![\cdot]\!]_P$-expressible

?
$[\![\cdot]\!]_P$-expressible

Process interpretation P (formal definition)

Definition (Transition system specification $\mathcal{T}$)

$$\frac{}{a \xrightarrow{a} 1} \qquad \frac{e_i \xrightarrow{a} e_i'}{e_1 + e_2 \xrightarrow{a} e_i'} \ (i \in \{1,2\})$$

Process interpretation P (formal definition)

Definition (Transition system specification $\mathcal{T}$)

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$$\frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e' \cdot e^*}$$

Process interpretation P (formal definition)

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$$\frac{}{\mathbf{1}{\Downarrow}} \qquad \frac{e_i{\Downarrow}}{(e_1 + e_2){\Downarrow}}\ (i \in \{1,2\}) \qquad \frac{e_1{\Downarrow} \qquad e_2{\Downarrow}}{(e_1 \cdot e_2){\Downarrow}} \qquad \frac{}{(e^*){\Downarrow}}$$

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Process interpretation P (formal definition)

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Definition

The process (graph) interpretation $P(e)$ of a regular expression e:

$P(e)$:= labeled transition graph generated by e by derivations in $\mathcal{T}$.

Overview

- 1-free regular expressions (with unary/binary star)
- process interpretation/semantics of regular expressions
  - expressible/not expressible process graphs
- loop existence and elimination property (LEE) (G/Fokkink, 2020)
  - interpretation/extraction correspondences with 1-free reg. expr's
  - LEE is preserved under bisimulation collapse
- Q: Image of process interpretation of 1-free regular expressions closed under bisimulation collapse?
  - A1: No. — But ...
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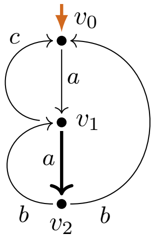
  - A2: compact process interpretation is image-closed under collapse
- outlook: consequences

Loop existence and elimination



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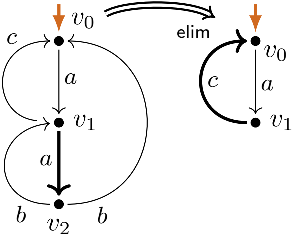

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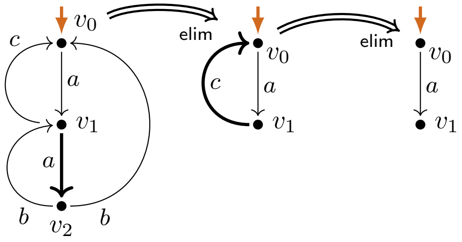
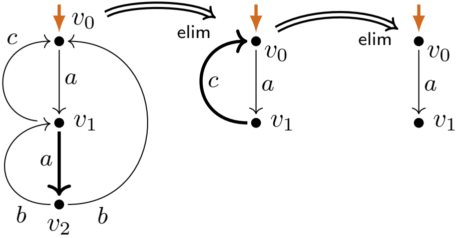
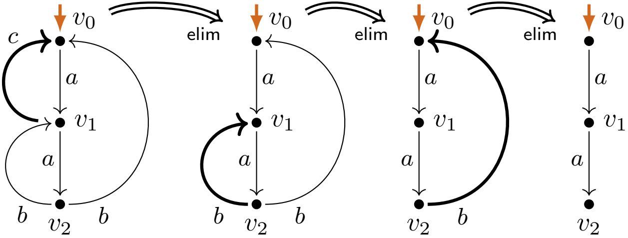


Loop existence and elimination





LEE

Definition

A chart $\mathcal{C}$ satisfies LEE (*loop existence and elimination*) if:

$$\exists \mathcal{C}_0 \left(\mathcal{C} \longrightarrow^{*}_{\mathsf{elim}} \mathcal{C}_0 \not\longrightarrow_{\mathsf{elim}} \right.$$
$$\left. \wedge\ \mathcal{C}_0 \text{ permits no infinite path} \right).$$

LEE

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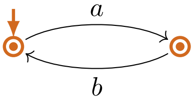
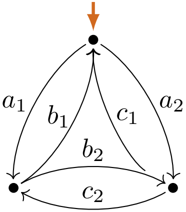
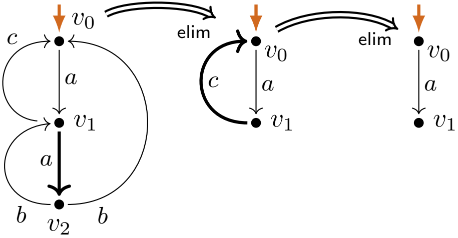


~~LEE~~



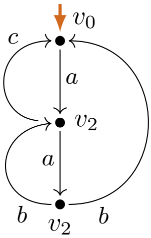
~~LEE~~

LEE

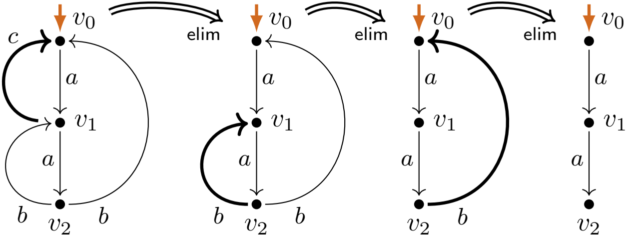


v_0
c
a
v_1
a
b
v_2
b
elim
v_0
c
a
v_1
elim
v_0
a
v_1

LEE

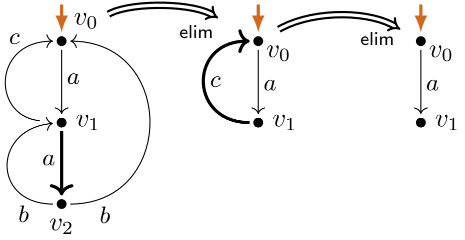


v_0
c
a
v_2
a
b
v_2
b

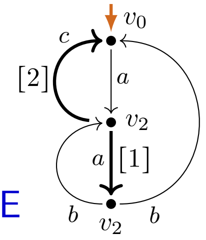


v_0
c
a
v_1
a
b
v_2
b
elim
v_0
a
v_1
a
b
v_2
b
elim
v_0
a
v_1
a
v_2
b
elim
v_0
a
v_1
a
v_2

LEE

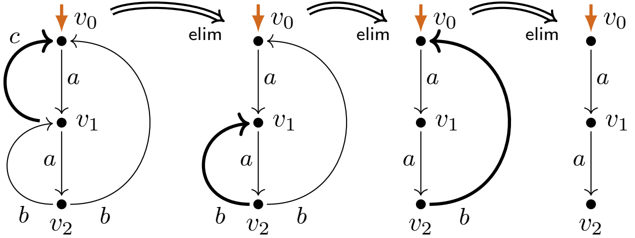


v_0
v_1
v_2
a
b
c
elim
elim

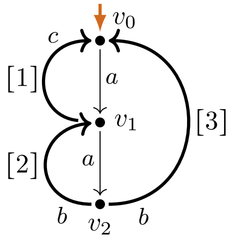


v_0
v_2
a
b
c
[1]
[2]

LEE

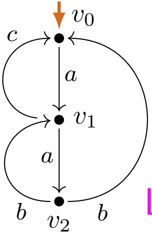


v_0
v_1
v_2
a
b
c
elim
elim
elim



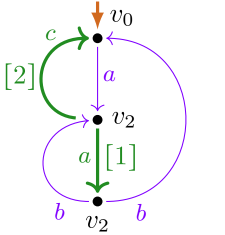
v_0
v_1
v_2
a
b
c
[1]
[2]
[3]

LEE witness and LEE-charts

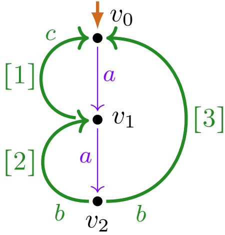


LEE-chart

LEE-witness



LEE-witness



Properties of LEE-charts

Theorem ($\Leftarrow$ G/Fokkink, 2020)

A process graph G
is $[\![\cdot]\!]_P$-expressible by an under-star-1-free regular expression
(i.e. P-expressible modulo bisimilarity by an $(\cancel{1}\backslash *)$ reg. expr.)
if and only if
the bisimulation collapse of G satisfies LEE.

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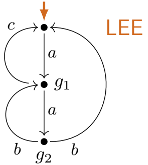
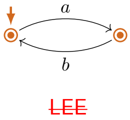
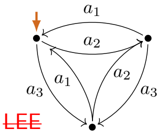
Hence $[\![\cdot]\!]_P$-expressible | **not** $[\![\cdot]\!]_P$-expressible by 1-free regular expressions:







Interpretation/extraction correspondences with LEE

($\Leftarrow$ G/Fokkink 2020, G 2021)

(Int)$_P^{(\cancel{1}\backslash *)}$: $P^{\bullet}$-$(\cancel{1}\backslash *)$*-expressible graphs have structural property* LEE

Process **i**nterpretations $P(e)$
of under-star-1-free regular expressions e
are finite process graphs that satisfy LEE.

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(Extr)$_{P}$: LEE *implies* $[\![\cdot]\!]_{P}$*-expressibility*

From every finite process graph G with LEE
a regular expression e can be **e**xtracted
such that $G \underline{\leftrightarrow} P(e)$.

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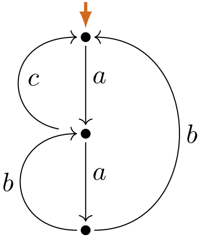
(Coll): LEE *is preserved under collapse*

The class of finite process graphs with LEE
is closed under bisimulation **c**ollapse.

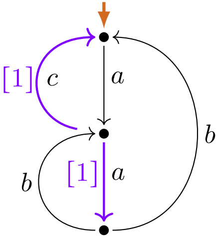
Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

G_4



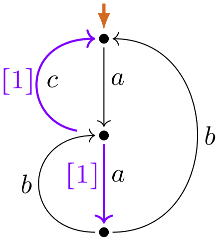
Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



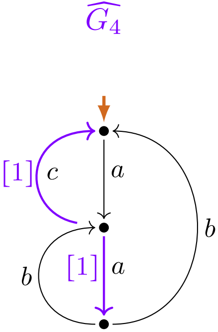
Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$$\Big(\qquad\qquad\qquad\qquad \Big)^{*} \cdot 0$$

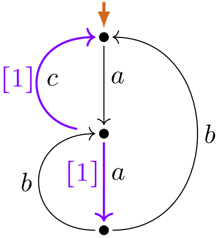
Expression extraction using LEE (G/Fokkink 2020, G 2021/22)



$$\Big(\quad \overset{\vdots\, a}{\downarrow} \quad \Big)^* \cdot 0$$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



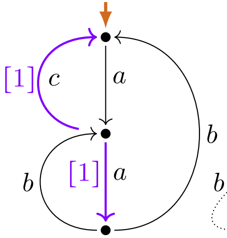
$(a \cdot 1) \cdot ((\qquad)^* \cdot 0)$

$\downarrow a$

$(\qquad)^* \cdot 0$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$(a \cdot 1) \cdot ((\qquad)^{*} \cdot 0)$

a

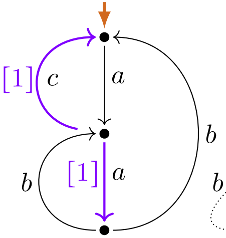
$(\qquad)^{*} \cdot 0$

b

b

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$(a \cdot 1) \cdot ((\qquad)^* \cdot 0)$

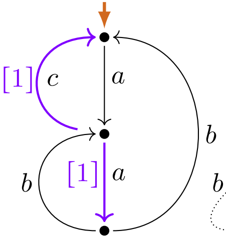
$\downarrow a$

$(\qquad)^* \cdot 0$

$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot ((\qquad)^*) \cdot 0)$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



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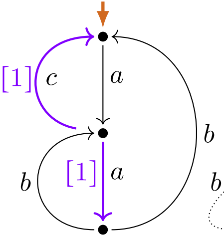
$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot ((\qquad)^*) \cdot 0)$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

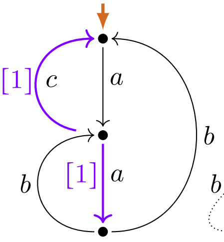
$\widehat{G_4}$



$$
\begin{tikzcd}
(a\cdot 1)\cdot\big(\big(\qquad \big)^*\cdot 0\big) \arrow[d, dotted, "a"] \\
\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^*\cdot 0 \arrow[u, dotted, bend left, "c"] \arrow[d, dotted, "a"] \\
(b\cdot 1+b\cdot(a\cdot 1))\cdot\big(\big(\qquad \big)^*\big)\cdot 0\big) \arrow[u, dotted, bend left, "b"] \arrow[uu, dotted, bend right, "b"']
\end{tikzcd}
$$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$$
\begin{array}{c}
(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0 \Big) \\
\Big\downarrow a \\
\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0 \\
\Big\downarrow a \\
(b \cdot 1 + b \cdot (a \cdot 1)) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*}) \cdot 0 \Big)
\end{array}
$$

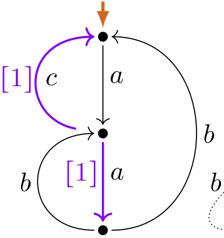
c

b

b

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$$\overbrace{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)}^{e}$$

$$\downarrow a$$

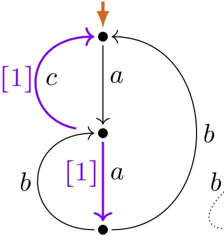
$$\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0$$

$$\downarrow a$$

$$(b\cdot 1+b\cdot(a\cdot 1))\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0\Big)$$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$$P(e) \rightarrow G_4$$

$$\overbrace{(a \cdot 1) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0 \big)}^{e}$$

$$\downarrow a$$

$$\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0$$

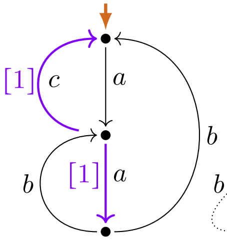
$$\downarrow a$$

$$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*}) \cdot 0 \big)$$

c b b

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$P(e) \underline{\to} G_4$

$$\overbrace{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)}^{e}$$

$$\cancel{\downarrow}\; a$$

$$\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0$$

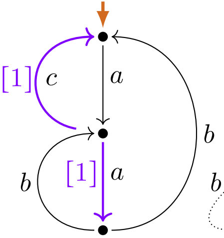
$$\downarrow a$$

$$(b\cdot 1+b\cdot(a\cdot 1))\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*})\cdot 0\Big)$$

$c \quad b \quad b$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$P(e) \;\underline{\to}\; G_4$

$$\overbrace{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)}^{e}$$

$$\downarrow a$$

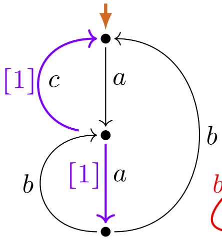
$$\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0$$

$$\downarrow a$$

$$(b\cdot 1+b\cdot(a\cdot 1))\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0\Big)$$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$\mathsf{P}(e) \mathrel{\underline{\rightarrow}} G_4$

$$\overbrace{(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0 \Big)}^{e}$$

$$\downarrow a$$

$$\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0$$

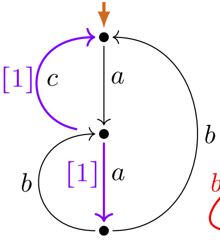
$$\downarrow a$$

$$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \Big) \cdot 0 \Big)$$

c b b

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$P(e) \;\underline{\rightarrow}\; G_4 \not\simeq P(e)$

$$\overbrace{(a\cdot 1)\cdot\big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\big)}^{e}$$

$$\Big\downarrow a$$

$$\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0$$

$$\Big\downarrow a$$

$$(b\cdot 1+b\cdot(a\cdot 1))\cdot\big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0\big)$$

Interpretation of extracted expression

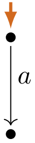
G_5

$P(e) = G_5$

$$\overbrace{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)}^{e}$$

Interpretation of extracted expression

G_5



$$P(e) = G_5$$

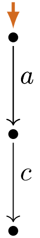
$$\overbrace{(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^{*} \cdot 0\Big)}^{e}$$

$$\Big\downarrow a$$

$$(1 \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^{*} \cdot 0\Big)$$

Interpretation of extracted expression

G_5



$P(e) = G_5$

$$\overbrace{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)}^{e}$$

$$\downarrow a$$

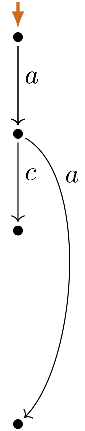
$$(1\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)$$

$$\downarrow c$$

$$\Big((1\cdot(a\cdot 1))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0$$

Interpretation of extracted expression

G_5



$P(e) = G_5$

$$\overbrace{(a \cdot 1) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0 \big)}^{e}$$

$$\downarrow a$$

$$(1 \cdot 1) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0 \big)$$

$$\downarrow c$$

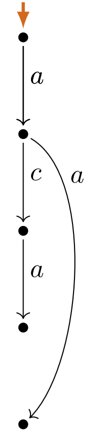
$$\big((1 \cdot (a \cdot 1)) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \big) \cdot 0$$

$$\xrightarrow{a}$$

$$\big((1 \cdot (b \cdot 1 + b \cdot (a \cdot 1))) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \big) \cdot 0$$

Interpretation of extracted expression

G_5



$P(e) = G_5$

$$\overbrace{(a\cdot 1)\cdot\big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\big)}^{e}$$

$$\downarrow a$$

$$(1\cdot 1)\cdot\big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\big)$$

$$\downarrow c$$

$$\big((1\cdot(a\cdot 1))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0$$

$$\downarrow a$$

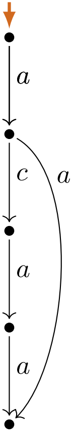
$$\big((1\cdot 1)\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0\big)$$

$$a$$

$$\big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0$$

Interpretation of extracted expression

G_5



$P(e) = G_5$

$$\overbrace{(a\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)}^{e}$$

$$\downarrow a$$

$$(1\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)$$

$$\downarrow c$$

$$\big((1\cdot(a\cdot 1))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0$$

$$\downarrow a$$

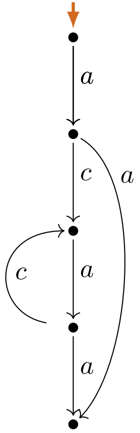
$$\big((1\cdot 1)\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0\big)$$

$$\downarrow a$$

$$\big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0$$

Interpretation of extracted expression

G_5



$P(e) = G_5$

$$\overbrace{(a\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)}^{e}$$

$$\downarrow a$$

$$(1\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)$$

$$\downarrow c$$

$$\big((1\cdot(a\cdot 1))\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\big)\cdot 0$$

$$\downarrow a$$

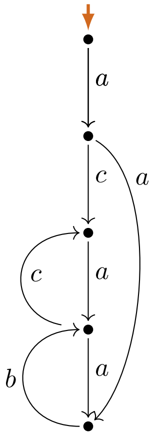
$$\big((1\cdot 1)\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\big)\cdot 0\big)$$

$$\downarrow a$$

$$\big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\big)\cdot 0$$

Interpretation of extracted expression

G_5



$P(e) = G_5$

$$\overbrace{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)}^{e}$$

$$\Big\downarrow a$$

$$(1\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)$$

$$\Big\downarrow c$$

$$\Big((1\cdot(a\cdot 1))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0$$

$$\Big\downarrow a$$

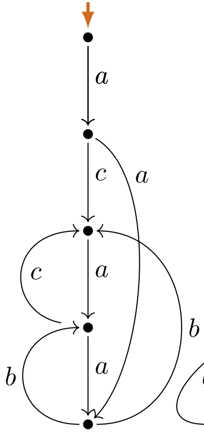
$$\Big((1\cdot 1)\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0\Big)$$

$$\Big\downarrow a$$

$$\Big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0$$

Interpretation of extracted expression

G_5



$P(e) = G_5$

$$\overbrace{(a \cdot 1) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0 \big)}^{e}$$

$$\downarrow a$$

$$(1 \cdot 1) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0 \big)$$

$$\downarrow c$$

$$\big((1 \cdot (a \cdot 1)) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \big) \cdot 0$$

$$\downarrow a$$

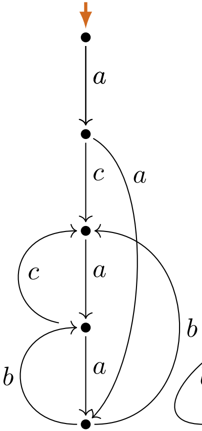
$$\big((1 \cdot 1) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \big) \cdot 0 \big)$$

$$\downarrow a$$

$$\big((1 \cdot (b \cdot 1 + b \cdot (a \cdot 1))) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \big) \cdot 0$$

Interpretation of extracted expression

G_5



$$P(e) = G_5 \rightarrow G_4$$

$$\overbrace{(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \cdot 0 \Big)}^{e}$$

$$\downarrow a$$

$$(1 \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \cdot 0 \Big)$$

$$\downarrow c$$

$$\Big((1 \cdot (a \cdot 1)) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \Big) \cdot 0$$

$$\downarrow a$$

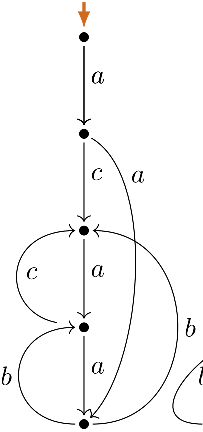
$$\Big((1 \cdot 1) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \Big) \cdot 0$$

$$\downarrow a$$

$$\Big((1 \cdot (b \cdot 1 + b \cdot (a \cdot 1))) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \Big) \cdot 0$$

Interpretation of extracted expression

G_5



$$P(e) = G_5 \Rightarrow G_4 \not\underline{\leftrightarrow} G_5$$

$$\overbrace{(a\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)}^{e}$$

$$\downarrow a$$

$$(1\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)$$

$$\downarrow c$$

$$\big((1\cdot(a\cdot 1))\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\big)\cdot 0$$

$$\downarrow a$$

$$\big((1\cdot 1)\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\big)\cdot 0\big)$$

$$\downarrow a$$

$$\big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\big)\cdot 0$$

Overview

- 1-free regular expressions (with unary/binary star)
- process interpretation/semantics of regular expressions
  - expressible/not expressible process graphs
- loop existence and elimination property (LEE) (G/Fokkink, 2020)
  - interpretation/extraction correspondences with 1-free reg. expr's
  - LEE is preserved under bisimulation collapse
- Q: Image of process interpretation of 1-free regular expressions closed under bisimulation collapse?
  - A1: No. — But ...
- compact process interpretation
- refined expression extraction
  - A2: compact process interpretation is image-closed under collapse
- outlook: consequences

Image of *P* is **not** closed under bisimulation collapse

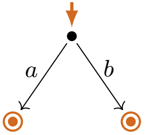
$P(uf)$

$P(uf)$

$$uf := a \cdot \overbrace{(a \cdot (a + a \cdot 0))^*}^{uf_a} + b \cdot \overbrace{(b \cdot (b + b \cdot 0))^*}^{uf_b}$$

Image of P is **not** closed under bisimulation collapse

$P(uf)$



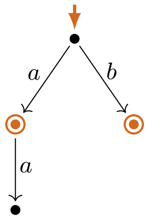
$P(uf)$

$$uf := a \cdot \overbrace{(a \cdot (a + a \cdot 0))^*}^{uf_a} + b \cdot \overbrace{(b \cdot (b + b \cdot 0))^*}^{uf_b}$$

$$1 \cdot uf_a \Downarrow \xleftarrow{a} \quad \xrightarrow{b} \Downarrow 1 \cdot uf_b$$

Image of P is **not** closed under bisimulation collapse

$P(uf)$



$P(uf)$

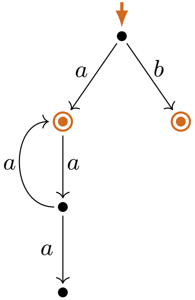
$$uf := a \cdot \overbrace{(a \cdot (a + a \cdot 0))^*}^{uf_a} + b \cdot \overbrace{(b \cdot (b + b \cdot 0))^*}^{uf_b}$$

$$uf \xrightarrow{a} 1 \cdot uf_a \Downarrow \qquad uf \xrightarrow{b} \Downarrow 1 \cdot uf_b$$

$$1 \cdot uf_a \xrightarrow{a} (1 \cdot (a + a \cdot 0)) \cdot uf_a$$

Image of P is **not** closed under bisimulation collapse

$P(uf)$



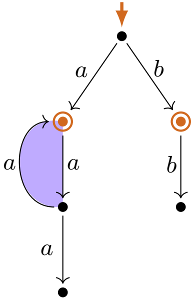
$P(uf)$

$$uf := a \cdot \overbrace{(a \cdot (a + a \cdot 0))^*}^{uf_a} + b \cdot \overbrace{(b \cdot (b + b \cdot 0))^*}^{uf_b}$$

$$\begin{tikzcd}
 & uf \arrow[dl, "a"'] \arrow[dr, "b"] & \\
1 \cdot uf_a \Downarrow \arrow[d, "a"] & & \Downarrow 1 \cdot uf_b \\
(1 \cdot (a + a \cdot 0)) \cdot uf_a \arrow[u, bend left, "a"] \arrow[d, "a"] & & \\
(1 \cdot 0) \cdot uf_a & &
\end{tikzcd}$$

Image of P is **not** closed under bisimulation collapse

$P(uf)$



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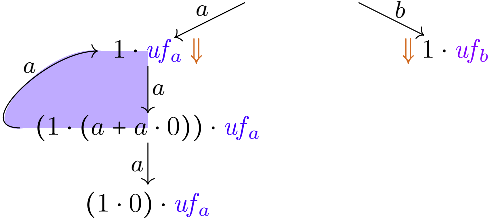
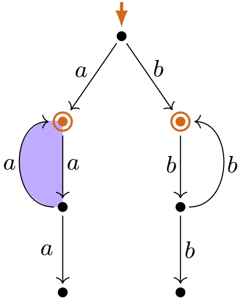


Image of *P* is **not** closed under bisimulation collapse

$P(uf)$



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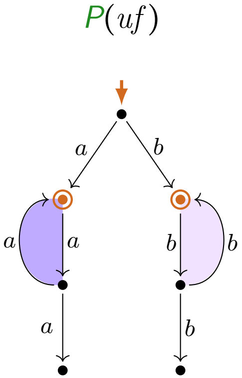
$$uf \xrightarrow{a} 1 \cdot uf_a \Downarrow \qquad uf \xrightarrow{b} \Downarrow 1 \cdot uf_b$$

$$1 \cdot uf_a \xrightarrow{a} (1 \cdot (a + a \cdot 0)) \cdot uf_a \xrightarrow{a} (1 \cdot 0) \cdot uf_a \qquad (1 \cdot (a + a \cdot 0)) \cdot uf_a \xrightarrow{a} 1 \cdot uf_a$$

$$1 \cdot uf_b \xrightarrow{b} (1 \cdot (b + b \cdot 0)) \cdot uf_b$$

Image of P is **not** closed under bisimulation collapse



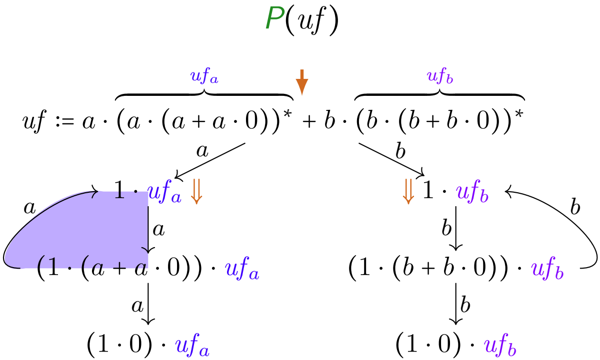
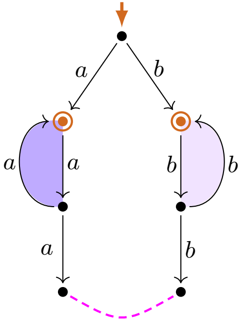


Image of P is **not** closed under bisimulation collapse

$P(uf)$



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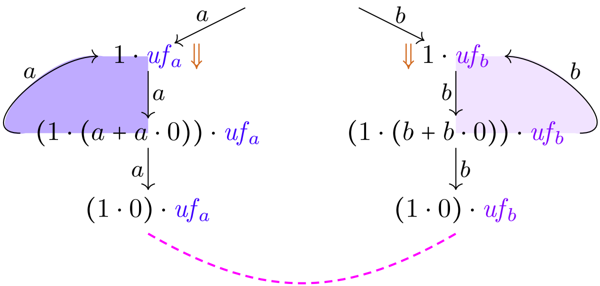

Compact process interpretation $P^{\bullet}$

Definition (Transition system specification $\mathcal{T}$)

$$\frac{}{\mathbf{1}{\Downarrow}} \qquad \frac{e_i{\Downarrow}}{(e_1 + e_2){\Downarrow}}\;(i \in \{1,2\}) \qquad \frac{e_1{\Downarrow} \quad e_2{\Downarrow}}{(e_1 \cdot e_2){\Downarrow}} \qquad \frac{}{(e^{*}){\Downarrow}}$$

$$\frac{}{a \xrightarrow{a} \mathbf{1}} \qquad \frac{e_i \xrightarrow{a} e_i'}{e_1 + e_2 \xrightarrow{a} e_i'}\;(i \in \{1,2\})$$

$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2} \qquad \frac{e_1{\Downarrow} \quad e_2 \xrightarrow{a} e_2'}{e_1 \cdot e_2 \xrightarrow{a} e_2'} \qquad \frac{e \xrightarrow{a} e'}{e^{*} \xrightarrow{a} e' \cdot e^{*}}$$

Compact process interpretation $P^{\bullet}$

Definition (Transition system specification $\mathcal{T}$)

$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2}$$

$$\frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e' \cdot e^*}$$

Compact process interpretation $P^{\bullet}$

Definition (Transition system specification $\mathcal{T}^{\bullet}$, changed rules w.r.t. $\mathcal{T}$)

$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2} \text{ (if } e_1' \text{ is normed)}$$

$$\frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e' \cdot e^*} \text{ (if } e' \text{ is normed)}$$

Compact process interpretation $P^\bullet$

Definition (Transition system specification $\mathcal{T}^\bullet$, changed rules w.r.t. $\mathcal{T}$)

$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2} \text{ (if } e_1' \text{ is normed)} \qquad \frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1'} \text{ (if } e_1' \text{ is not normed)}$$

$$\frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e' \cdot e^*} \text{ (if } e' \text{ is normed)} \qquad \frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e'} \text{ (if } e' \text{ is not normed)}$$

Compact process interpretation $P^{\bullet}$

Definition (Transition system specification $\mathcal{T}^{\bullet}$, changed rules w.r.t. $\mathcal{T}$)

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$$\frac{e \xrightarrow{a} e'}{e^{*} \xrightarrow{a} e' \cdot e^{*}} \text{ (if } e' \text{ is normed)} \qquad \frac{e \xrightarrow{a} e'}{e^{*} \xrightarrow{a} e'} \text{ (if } e' \text{ is not normed)}$$

Definition

The compact process (graph) interpretation $P^{\bullet}(e)$ of a reg. expr's e:

$P^{\bullet}(e) :=$ labeled transition graph generated by e by derivations in $\mathcal{T}^{\bullet}$.

Compact process interpretation $P^{\bullet}$

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Definition

The compact process (graph) interpretation $P^{\bullet}(e)$ of a reg. expr's e:

$P^{\bullet}(e)$:= labeled transition graph generated by e by derivations in $\mathcal{T}^{\bullet}$.

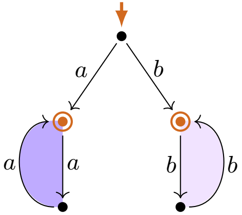
Lemma ($P^{\bullet}$ increases sharing; $P^{\bullet}$, P have same bisimulation semantics)

(i) $P(e) \mathrel{\underline{\to}} P^{\bullet}(e)$ *for all regular expressions* e.

(ii) (G *is* $[\![\cdot]\!]_{P^{\bullet}}$*-expressible* $\iff$ G *is* $[\![\cdot]\!]_{P}$*-expressible) for all graphs* G.

Image of $P^{\bullet}$ under bisimulation collapse ...

$P^{\bullet}(uf)$



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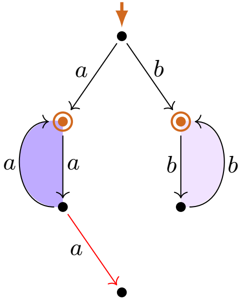
$$uf \xrightarrow{a} 1 \cdot uf_a \Downarrow \qquad uf \xrightarrow{b} \Downarrow 1 \cdot uf_b$$

$$1 \cdot uf_a \xrightarrow{a} (1 \cdot (a + a \cdot 0)) \cdot uf_a \xrightarrow{a} 1 \cdot uf_a$$

$$1 \cdot uf_b \xrightarrow{b} (1 \cdot (b + b \cdot 0)) \cdot uf_b \xrightarrow{b} 1 \cdot uf_b$$

Image of $P^{\bullet}$ under bisimulation collapse . . .

$P^{\bullet}(uf)$



$P^{\bullet}(uf)$

$$uf := a \cdot \overbrace{(a \cdot (a + a \cdot 0))^*}^{uf_a} + b \cdot \overbrace{(b \cdot (b + b \cdot 0))^*}^{uf_b}$$

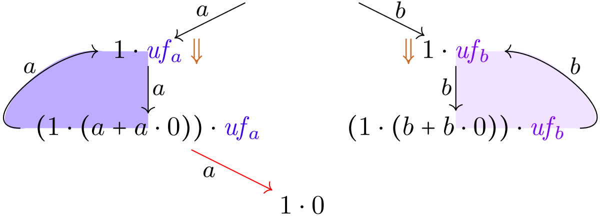
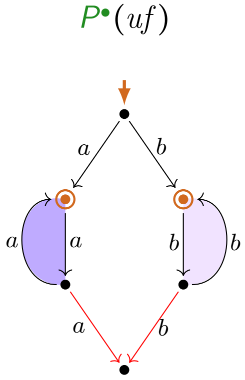
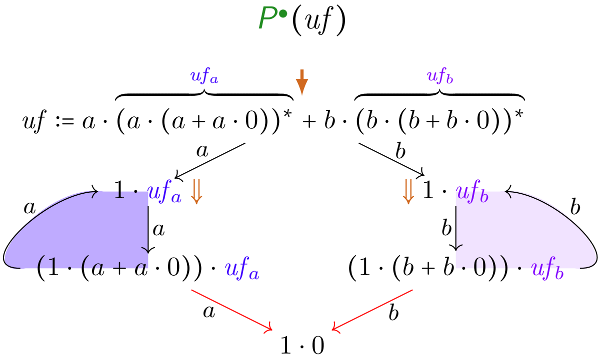


Image of $P^{\bullet}$ under bisimulation collapse . . .





Interpretation correspondence of $P^{\bullet}$ with LEE

(Int)$_{P^{\bullet}}^{(\cancel{1}\backslash *)}$: *By under-star-1-free expressions $P^{\bullet}$-expressible graphs satisfy* LEE:

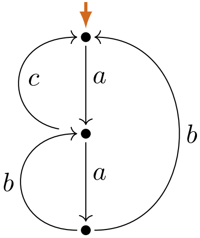
Compact process **i**nterpretations $P^{\bullet}(uf)$
of under-star-1-free regular expressions uf
are finite process graphs that satisfy LEE.

(Extr)$_{P^{\bullet}}^{(\cancel{1}\backslash *)}$: LEE *implies $[\![\cdot]\!]_P$-expressibility by under-star-1-free reg. expr's*:

From every finite process graph G with LEE
an under-star-1-free regular expression uf can be **e**xtracted
such that $G \mathrel{\underline{\rightarrow}} P(uf)$.

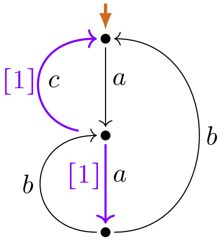
Refined extraction expression (example)

G_4

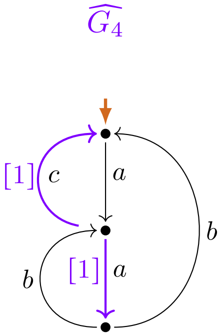


Refined extraction expression (example)

$\widehat{G_4}$



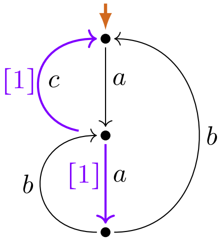
Refined extraction expression (example)



$$(1 \cdot (\qquad\qquad\qquad)^{*}) \cdot 0$$

Refined extraction expression (example)

$\widehat{G_4}$



$$(1 \cdot (\qquad \overset{a}{\downarrow} \qquad)^*) \cdot 0$$

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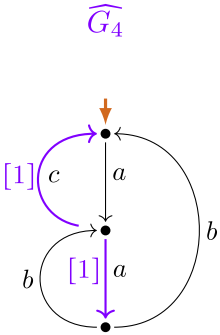

$$((1 \cdot a) \cdot (\qquad\qquad\qquad)^*) \cdot 0$$

$$\Big\downarrow a$$

$$(1 \cdot (\qquad\qquad\qquad)^*) \cdot 0$$

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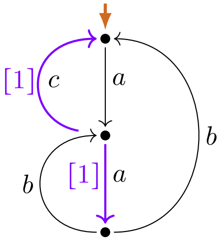

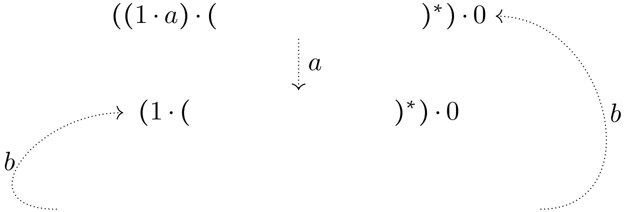

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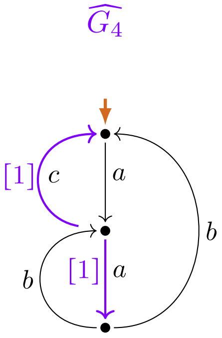

$((1 \cdot a) \cdot ($ $)^{*}) \cdot 0$

$\downarrow a$ $\quad b$

$(1 \cdot ($ $)^{*}) \cdot 0$

b

$((1 \cdot (b + b \cdot a)) \cdot ($ $)^{*}) \cdot 0$

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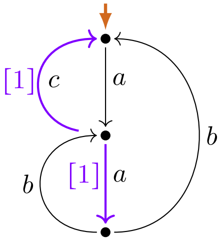

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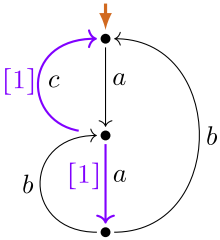


$$((1\cdot a)\cdot(\qquad\qquad\qquad\qquad)^{*})\cdot 0$$

$$\downarrow a$$

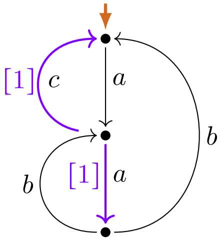
$$(1\cdot(c\cdot a + a\cdot(b + b\cdot a))^{*})\cdot 0$$

$$\downarrow a$$

$$((1\cdot(b + b\cdot a))\cdot(\qquad\qquad\qquad\qquad)^{*})\cdot 0$$

Refined extraction expression (example)

$\widehat{G_4}$



$$
\begin{tikzcd}
((1 \cdot a) \cdot (c \cdot a + a \cdot (b + b \cdot a))^{*}) \cdot 0 \arrow[d, dotted, "a"] \\
(1 \cdot (c \cdot a + a \cdot (b + b \cdot a))^{*}) \cdot 0 \arrow[d, dotted, "a"] \arrow[u, dotted, bend left, "c"] \\
((1 \cdot (b + b \cdot a)) \cdot (c \cdot a + a \cdot (b + b \cdot a))^{*}) \cdot 0 \arrow[u, dotted, bend left, "b"] \arrow[uu, dotted, bend right, "b"']
\end{tikzcd}
$$

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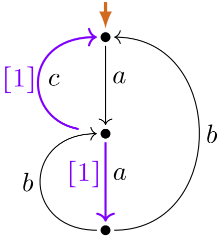

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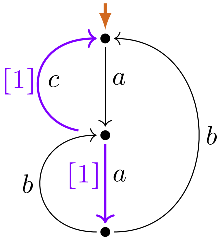

$$P^{\bullet}(uf) = P(uf) \simeq G_4$$

$$\begin{tikzcd}
\overbrace{((1\cdot a)\cdot(c\cdot a + a\cdot(b + b\cdot a))^{*})\cdot 0}^{uf} \arrow[d, "a"] \\
(1\cdot(c\cdot a + a\cdot(b + b\cdot a))^{*})\cdot 0 \arrow[u, bend left, "c"] \arrow[d, "a"] \\
((1\cdot(b + b\cdot a))\cdot(c\cdot a + a\cdot(b + b\cdot a))^{*})\cdot 0 \arrow[u, bend left, "b"] \arrow[uu, bend right, "b"']
\end{tikzcd}$$

Interpretation/extraction correspondences of $P^{\bullet}$ with LEE

(Int)$_{P^{\bullet}}^{(\cancel{1}\backslash *)}$: *By under-star-1-free expressions $P^{\bullet}$-expressible graphs satisfy* LEE:

Compact process **i**nterpretations $P^{\bullet}(uf)$
of under-star-1-free regular expressions uf
are finite process graphs that satisfy LEE.

(Extr)$_{P^{\bullet}}^{(\cancel{1}\backslash *)}$: LEE *implies $[\![\cdot]\!]_P$-expressibility by under-star-1-free reg. expr's*:

From every finite process graph G with LEE
an under-star-1-free regular expression uf can be **e**xtracted
such that $G \rightrightarrows P(uf)$.

From every finite collapsed process graph G with LEE
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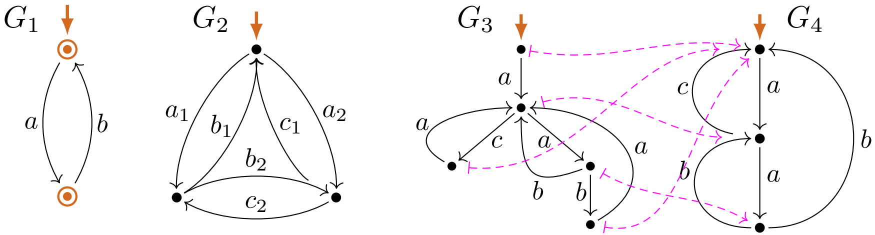
(ImColl)$^{(\cancel{1}\backslash *)}_{P^{\bullet}}$: The image of $P^{\bullet}$,
restricted to under-star-1-free regular expressions,
is closed under bisimulation **c**ollapse.

P-/$P^{\bullet}$-expressibility and $[\![\cdot]\!]_P$-expressibility (examples)



not P-expressible

not $[\![\cdot]\!]_P$-expressible

P-/$P^{\bullet}$-expressible $\quad$ $P^{\bullet}$-expressible

$[\![\cdot]\!]_P$-expressible $\quad$ $[\![\cdot]\!]_P$-expressible

Summary and outlook

- 1-free/under-star-1-free $(\cancel{1}\backslash *)$ reg. expr's defined (also) with unary star
- image of $(\cancel{1}\backslash *)$ regular expressions under the process interpretation P
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Outlook on an extension:

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Resources

- Slides/extended abstract on `clegra.github.io`
  - slides: `.../lf/TG-2024.pdf`
  - extended abstract: `.../lf/closing-bs-i-pi-us1f.pdf`

- CG, Wan Fokkink: A Complete Proof System for 1-Free Regular Expressions Modulo Bisimilarity,
  - LICS 2020, arXiv:2004.12740, video on youtube.
- CG: Modeling Terms by Graphs with Structure Constraints,
  - TERMGRAPH 2018, EPTCS 288, arXiv:1902.02010.
- CG: The Image of the Process Interpretation of Regular Expressions is Not Closed under Bisimulation Collapse,
  - arXiv:2303.08553.
- CG: Milner's Proof System for Regular Expressions Modulo Bisimilarity is Complete,
  - LICS 2022, arXiv:2209.12188, poster.

Language semantics $[\![\cdot]\!]_L$ of reg. expr's *(Copi–Elgot–Wright, 1958)*

$$\mathbf{0} \xmapsto{L} \text{empty language } \varnothing$$

$$\mathbf{1} \xmapsto{L} \{\epsilon\} \qquad (\epsilon \text{ the empty word})$$

$$a \xmapsto{L} \{a\}$$

Language semantics $[\![\cdot]\!]_L$ of reg. expr's *(Copi–Elgot–Wright, 1958)*

$\mathbf{0} \stackrel{L}{\longmapsto}$ empty language $\varnothing$

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$a \stackrel{L}{\longmapsto} \{a\}$

$e_1 + e_2 \stackrel{L}{\longmapsto}$ union of $L(e_1)$ and $L(e_2)$

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$e^* \stackrel{L}{\longmapsto}$ set of words formed by concatenating words in $L(e)$, and adding the empty word ϵ

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$[\![e]\!]_L := L(e)$ (language defined by e)

Loop charts (interpretations of innermost iterations)

Definition

A chart is a loop chart if:

(L1) There is an infinite path from the start vertex.

(L2) Every infinite path from the start vertex returns to it.

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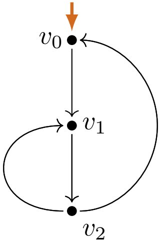

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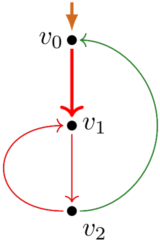

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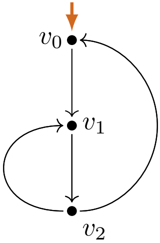

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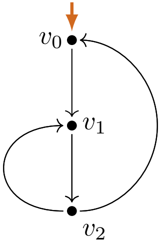

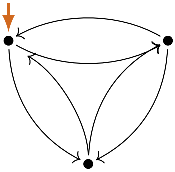

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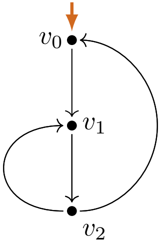

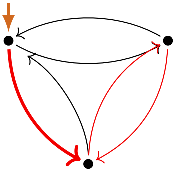

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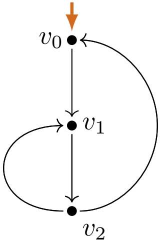

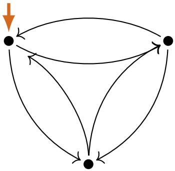

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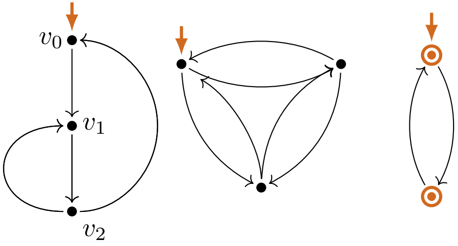

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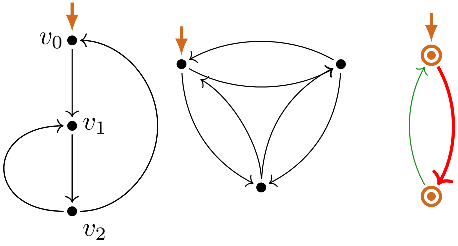

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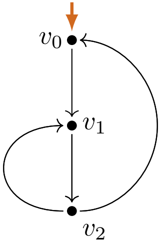

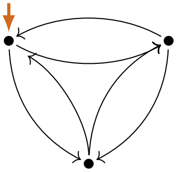

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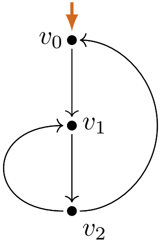

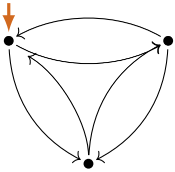

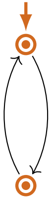

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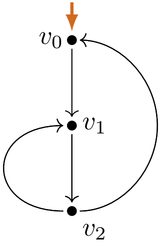

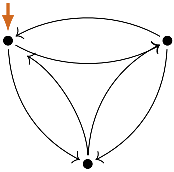

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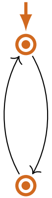

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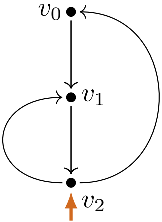

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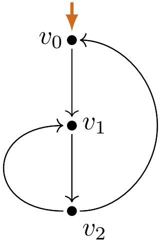
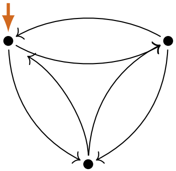
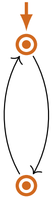
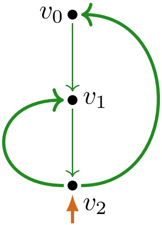




(L1),~~(L2)~~



(L1),(L2),~~(L3)~~



Loop charts (interpretations of innermost iterations)

Definition

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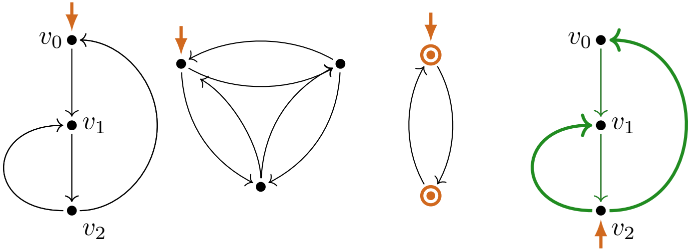

(L1),~~(L2)~~ (L1),(L2),~~(L3)~~ loop chart

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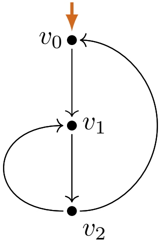

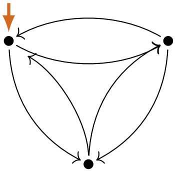

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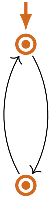
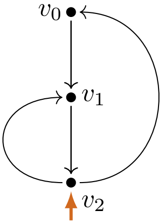


(L1),(L2),~~(L3)~~



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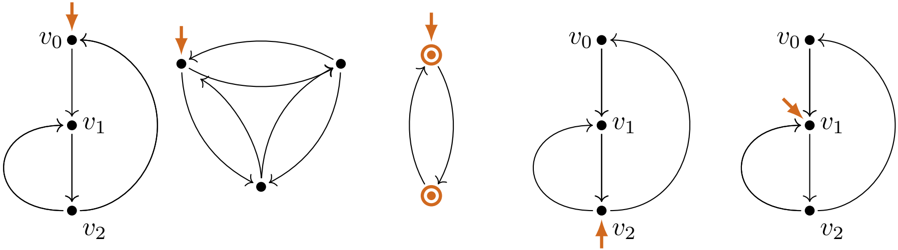

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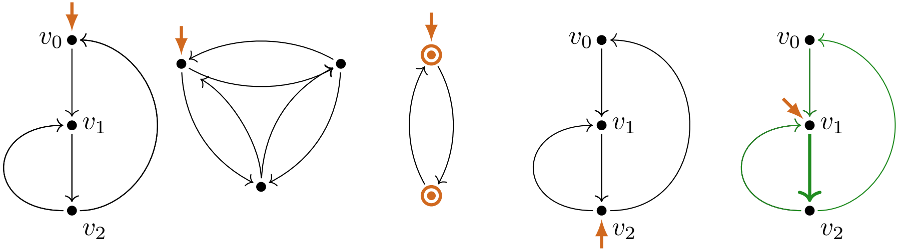

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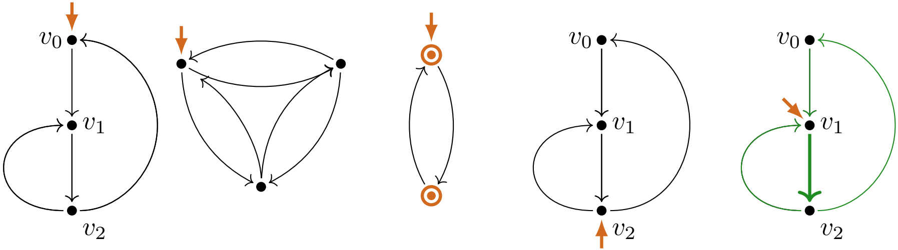

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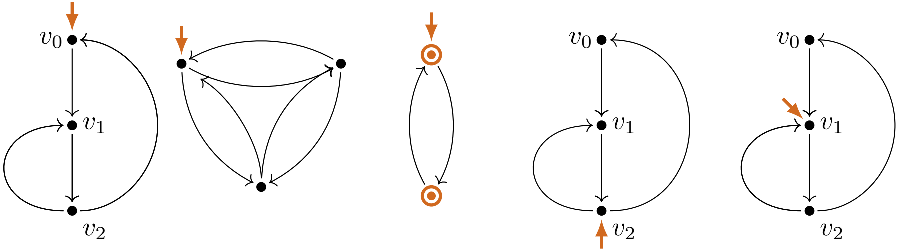

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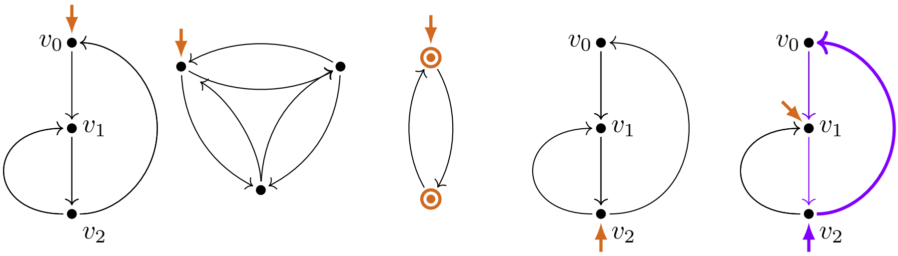

(L1),~~(L2)~~ (L1),(L2),~~(L3)~~ loop chart loop subchart