The Graph Structure of Process Interpretations of Regular Expressions

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https://clegra.github.io

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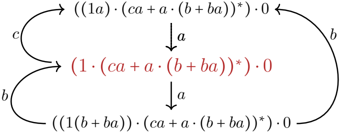

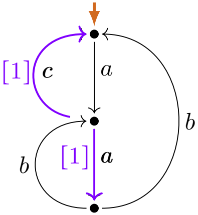

IFIP 1.6 Working Group Meeting

Nancy

July 1, 2024

Overview

- regular expressions (unary/binary star/1-free-under-star $(*/\cancel{1})$)
- Milner's process interpretation P/semantics $[\![\cdot]\!]_P$
  - P-/$[\![\cdot]\!]_P$-expressible graphs ($\rightsquigarrow$ expressibility question)
  - axioms for $[\![\cdot]\!]_P$-identity ($\rightsquigarrow$ completeness question)
- loop existence and elimination (LEE)
  - defined by loop elimination rewrite system, its completion
  - describes interpretations of $(*/\cancel{1})$ reg. expr.s (extraction possible)
  - LEE-witnesses: labelings of process graphs with LEE
  - LEE is preserved under bisimulation collapse (stepwise collapse)
- 1-LEE = sharing via 1-transitions facilitates LEE
- LEE/1-LEE characterize image of $P^{\bullet}$ (restricted/unrestricted)
  - where $P^{\bullet}$ a compact (sharing-increased) refinement of P
- outlook on work-to-do

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  - LEE-witnesses: labelings of process graphs with LEE
  - LEE is preserved under bisimulation collapse (stepwise collapse)
- 1-LEE = sharing via 1-transitions facilitates LEE
  - describes interpretations of all reg. expr.s (extraction possible)
  - not preserved under bisimulation collapse (approximation possible)
- LEE/1-LEE characterize image of $P^{\bullet}$ (restricted/unrestricted)
  - where $P^{\bullet}$ a compact (sharing-increased) refinement of P
  - via refined extraction using LEE/1-LEE
- outlook on work-to-do

Regular Expressions

Definition (~ *Copi–Elgot–Wright, 1958*)

Regular expressions over alphabet A with unary Kleene star:

$$e, e_1, e_2 \;::=\; \mathbf{0} \;\mid\; \boldsymbol{a} \;\mid\; e_1 + e_2 \;\mid\; e_1 \cdot e_2 \;\mid\; e^{*} \qquad \text{(for } \boldsymbol{a} \in A\text{)}.$$

- symbol $\mathbf{0}$ instead of $\varnothing$, symbol $\mathbf{1}$ instead of $\{\epsilon\}$

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- with unary star $*$: $\mathbf{1}$ is definable as $\mathbf{0}^{*}$

Regular Expressions

Definition (~ *Kleene, 1951*, ~ *Copi–Elgot–Wright, 1958*)

Regular expressions over alphabet A with unary / binary Kleene star:

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- with binary star ${}^{\circledast}$: $\mathbf{1}$ is not definable (in its absence)

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Regular Expressions (1-free)

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- symbol $\mathbf{0}$ instead of $\emptyset$, symbol $\mathbf{1}$ instead of $\{\epsilon\}$
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Definition (for process interpretation)

1-free regular expressions over alphabet A with binary Kleene star:

$$f, f_1, f_2 \;::=\; \mathbf{0} \mid a \mid f_1 + f_2 \mid f_1 \cdot f_2 \mid f_1{}^{\circledast} f_2 \qquad \text{(for } a \in A\text{).}$$

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Regular Expressions (under-star-/1-free)

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Definition (for process interpretation)

The set $\mathit{RExp}^{(\cancel{1})}(A)$ of 1-free regular expressions over A is defined by:

$$f, f_1, f_2 ::= \mathbf{0} \mid \boldsymbol{a} \mid f_1 + f_2 \mid f_1 \cdot f_2 \mid f_1{}^{\star} \cdot f_2 \qquad \text{(for } \boldsymbol{a} \in A\text{),}$$

the set $\mathit{RExp}^{(*/\cancel{1})}(A)$ of under-star-1-free regular expressions over A by:

$$uf, uf_1, uf_2 ::= \mathbf{0} \mid \mathbf{1} \mid \boldsymbol{a} \mid uf_1 + uf_2 \mid uf_1 \cdot uf_2 \mid f^{\star} \qquad \text{(for } \boldsymbol{a} \in A\text{).}$$

Process interpretation P of regular expressions *(Milner, 1984)*

$\mathbf{0} \xmapsto{P}$ deadlock δ, no termination

$\mathbf{1} \xmapsto{P}$ empty-step process ϵ, then terminate

$a \xmapsto{P}$ atomic action a, then terminate

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$e_1 + e_2 \xmapsto{P}$ (*choice*) execute $P(e_1)$ or $P(e_2)$

$e_1 \cdot e_2 \xmapsto{P}$ (*sequentialization*) execute $P(e_1)$, then $P(e_2)$

$e^* \xmapsto{P}$ (*iteration*) repeat (terminate or execute $P(e)$)

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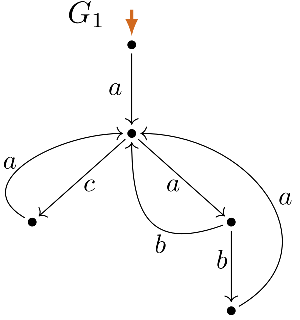
$[\![e]\!]_P \;:=\; [P(e)]_{\underline{\leftrightarrow}}$ (bisimilarity equivalence class of process $P(e)$)

P-expressibility and $[\![\cdot]\!]_P$-expressibility (example, informally)



$$P\Big(\overbrace{(a\cdot(c\cdot a+a\cdot(b+b\cdot a))^*)\cdot 0}^{f}\Big)$$

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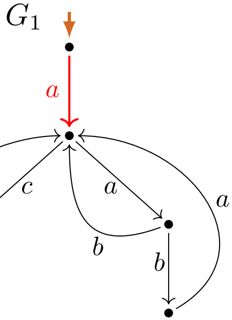

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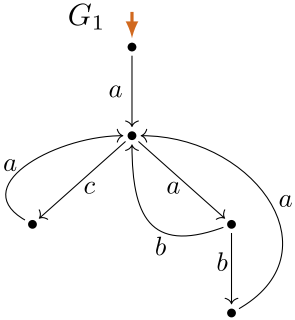

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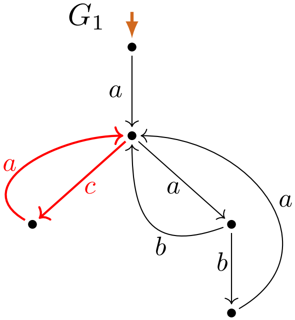

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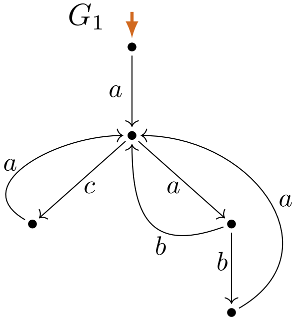

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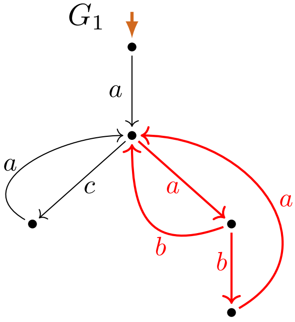

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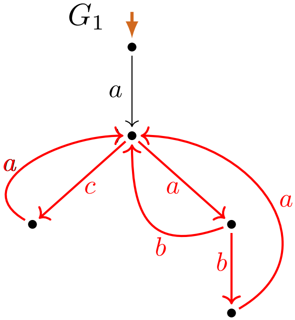

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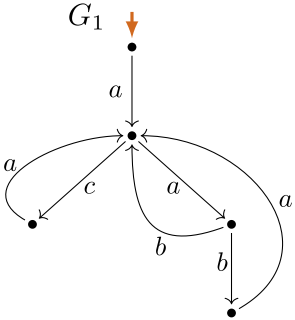

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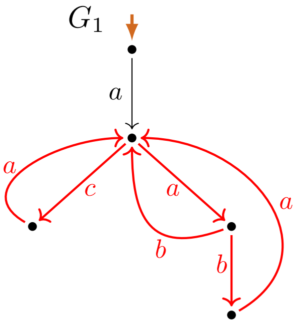

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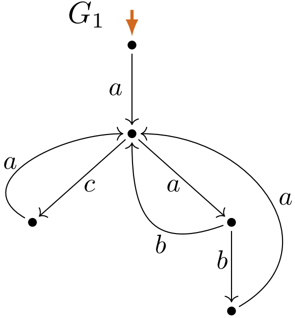

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$$G_1 \in [\![f]\!]_P$$

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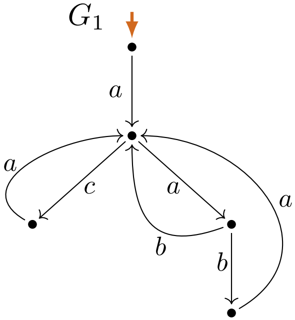

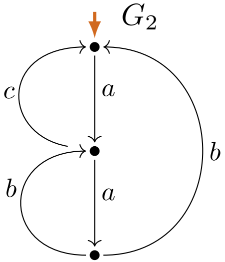

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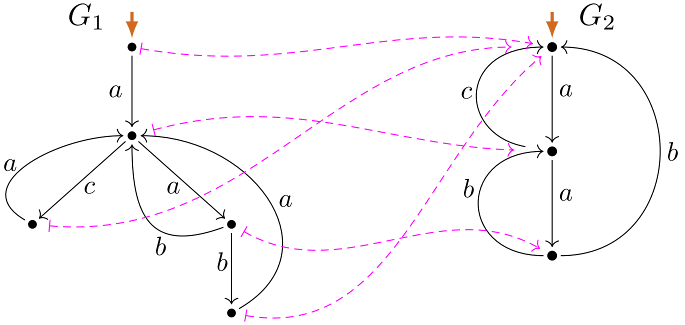

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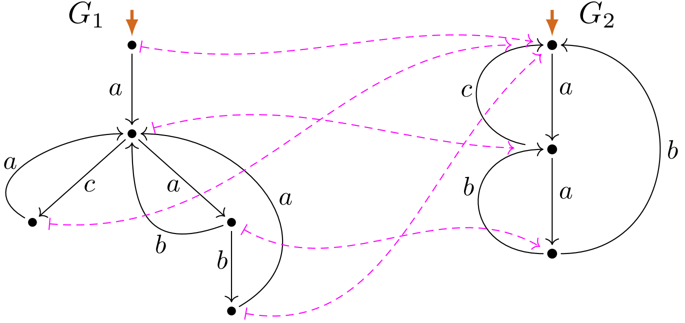

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$$G_1 \in [\![f]\!]_P \qquad G_2 \in [\![f]\!]_P$$

Process interpretation P (formally)

Definition (Transition system specification $\mathcal{T}$)

$$\frac{}{a \xrightarrow{a} 1} \qquad \frac{e_i \xrightarrow{a} e_i'}{e_1 + e_2 \xrightarrow{a} e_i'} \; (i \in \{1,2\})$$

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$$\frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e' \cdot e^*}$$

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$$\frac{}{\mathbf{1}{\Downarrow}} \qquad \frac{e_i{\Downarrow}}{(e_1 + e_2){\Downarrow}}\;(i \in \{1,2\}) \qquad \frac{e_1{\Downarrow} \qquad e_2{\Downarrow}}{(e_1 \cdot e_2){\Downarrow}} \qquad \frac{}{(e^*){\Downarrow}}$$

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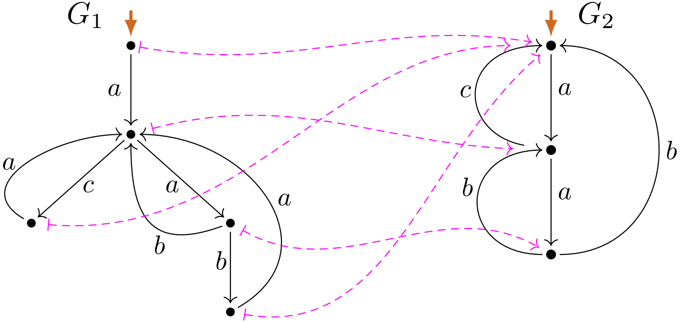
Definition

The process (graph) interpretation $P(e)$ of a regular expression e:

$P(e)$:= labeled transition graph generated by e by derivations in $\mathcal{T}$.

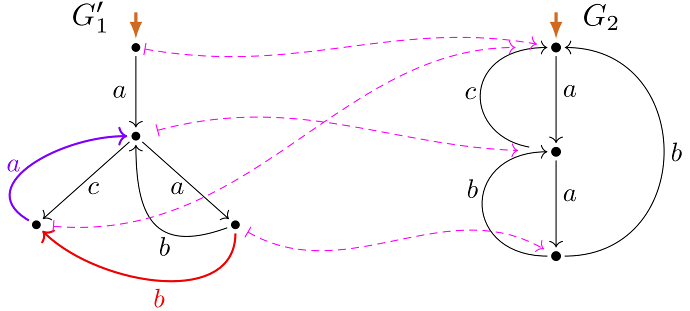
P-expressibility and $[\![\cdot]\!]_P$-expressibility (example, informally)



$$P\big(\overbrace{a\cdot((c\cdot a+a\cdot(b+b\cdot a)^{*})\cdot 0)}^{f}\big)$$

$G_1 \in [\![f]\!]_P$ $\qquad$ $G_2 \in [\![f]\!]_P$

P-expressibility and $[\![\cdot]\!]_P$-expressibility (example, formally)



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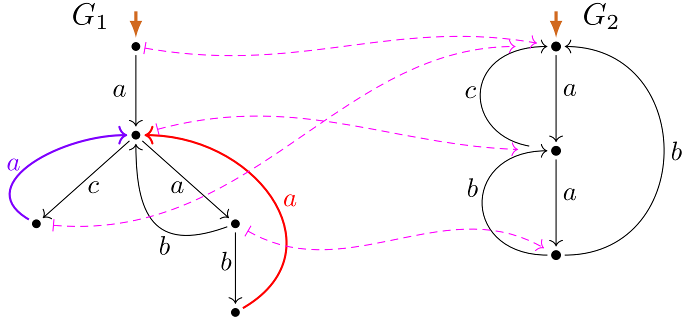

$$P\Big(\overbrace{a \cdot ((c \cdot a + a \cdot (b + b \cdot (a + a))^*) \cdot 0)}^{f}\Big)$$

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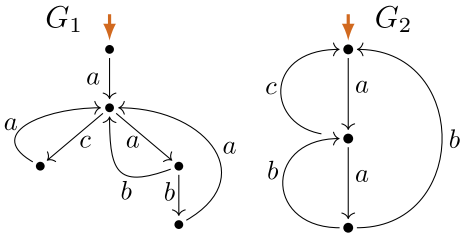
P-expressibility and $[\![\cdot]\!]_P$-expressibility (examples)



P-expressible

$[\![\cdot]\!]_P$-expressible $[\![\cdot]\!]_P$-expressible

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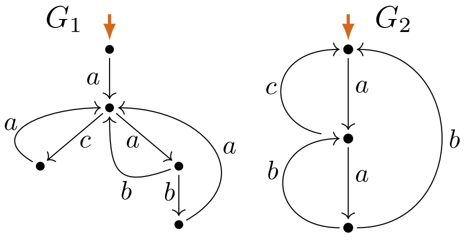

P-expressible ?

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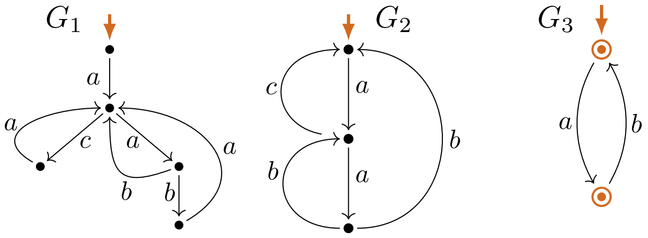
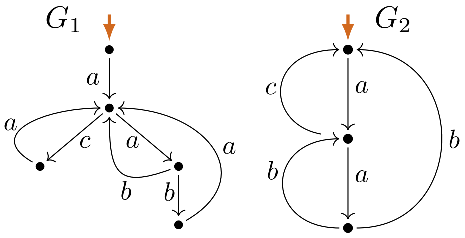
P-expressibility and $[\![\cdot]\!]_P$-expressibility (examples)



P-expressible	**?**	**not** P-expressible
$[\![\cdot]\!]_P$-expressible	$[\![\cdot]\!]_P$-expressible	**not** $[\![\cdot]\!]_P$-expressible

P-expressibility and $[\![\cdot]\!]_P$-expressibility (examples)

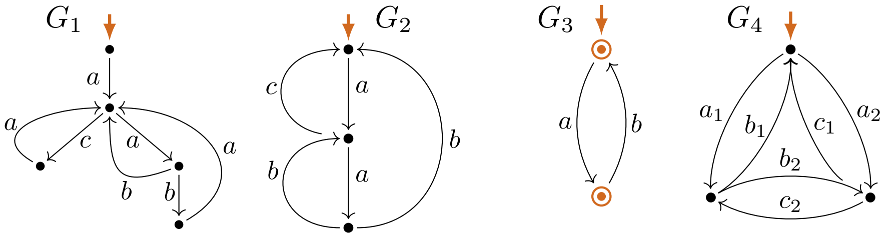


G_1: P-expressible, $[\![\cdot]\!]_P$-expressible

G_2: **?**, $[\![\cdot]\!]_P$-expressible

G_3, G_4: **not** P-expressible, **not** $[\![\cdot]\!]_P$-expressible

P-expressibility and $[\![\cdot]\!]_P$-expressibility (examples)

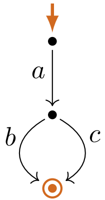


P-expressible	**?**	**not** P-expressible
$[\![\cdot]\!]_P$-expressible	$[\![\cdot]\!]_P$-expressible	**not** $[\![\cdot]\!]_P$-expressible

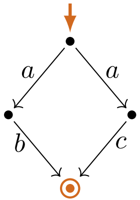
Q2: How can P-expressibility and $[\![\cdot]\!]_P$-expressibility be characterized?

Process semantics equality $=_{[\![\cdot]\!]_P}$

- Fewer identities hold for $=_{[\![\cdot]\!]_P}$ than for $=_{[\![\cdot]\!]_L}$: .



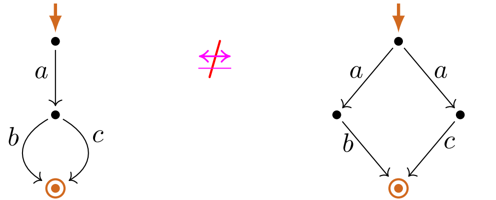
$P(a \cdot (b + c))$



$P(a \cdot b + a \cdot c)$

Process semantics equality $=_{[\![\cdot]\!]_P}$

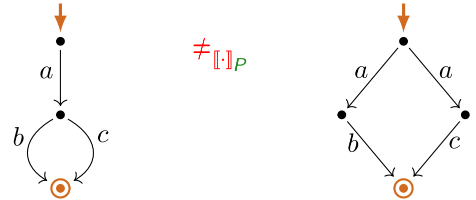
- Fewer identities hold for $=_{[\![\cdot]\!]_P}$ than for $=_{[\![\cdot]\!]_L}$: .



$P(a \cdot (b + c)) \not\leftrightarrow P(a \cdot b + a \cdot c)$

Process semantics equality $=_{[\![\cdot]\!]_P}$

- Fewer identities hold for $=_{[\![\cdot]\!]_P}$ than for $=_{[\![\cdot]\!]_L}$: .

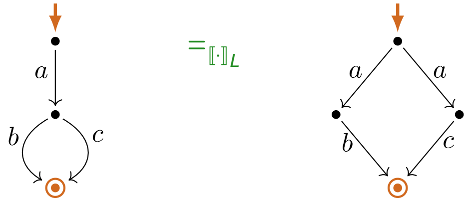


$\neq_{[\![\cdot]\!]_P}$

$$a \cdot (b + c) \quad \neq_{[\![\cdot]\!]_P} \quad a \cdot b + a \cdot c$$

Process semantics equality $=_{[\![\cdot]\!]_P}$

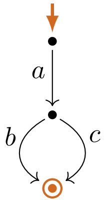
- Fewer identities hold for $=_{[\![\cdot]\!]_P}$ than for $=_{[\![\cdot]\!]_L}$: .



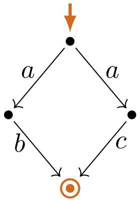
$$a \cdot (b + c) \quad =_{[\![\cdot]\!]_L} \quad a \cdot b + a \cdot c$$

Process semantics equality $=_{[\![\cdot]\!]_P}$

- Fewer identities hold for $=_{[\![\cdot]\!]_P}$ than for $=_{[\![\cdot]\!]_L}$: $=_{[\![\cdot]\!]_P} \subsetneqq =_{[\![\cdot]\!]_L}$.



$\neq_{[\![\cdot]\!]_P}$



$$a \cdot (b + c) \quad \neq_{[\![\cdot]\!]_P} \quad a \cdot b + a \cdot c$$

Milner's proof system Mil

Axioms :

(A1) $e + (f + g) = (e + f) + g$

(A2) $e + 0 = e$

(A3) $e + f = f + e$

(A4) $e + e = e$

(A5) $e \cdot (f \cdot g) = (e \cdot f) \cdot g$

(A6) $(e + f) \cdot g = e \cdot g + f \cdot g$

(A7) $e = 1 \cdot e$

(A8) $e = e \cdot 1$

(A9) $0 = 0 \cdot e$

(A10) $e^* = 1 + e \cdot e^*$

(A11) $e^* = (1 + e)^*$

But: $e \cdot (f + g) \neq e \cdot f + e \cdot g$

But: $e \cdot 0 \neq 0$

Inference rules : rules of equational logic *plus*

$$\frac{e = f \cdot e + g}{e = f^* \cdot g} \text{ RSP}^*$$ (if f does not terminate immediately)

Milner's Question (Q1)

Is Mil complete with respect to $=_{[\![\cdot]\!]_P}$? (Does $e =_{[\![\cdot]\!]_P} f \Longrightarrow e =_{\mathsf{Mil}} f$ hold?)

Milner's questions

(Q1) Complete axiomatization:

Is the proof system Mil *complete for* $=_{[\![\cdot]\!]_P}$*?*

(Q2) $[\![\cdot]\!]_P$-Expressibility:

What structural property characterizes
process graphs that are $[\![\cdot]\!]_P$*-expressible ?*

Milner's questions

(Q1) Complete axiomatization:
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What structural property characterizes process graphs that are $[\![\cdot]\!]_P$*-expressible ?*

- is decidable (Baeten/Corradini/G, 2007)

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What structural property characterizes process graphs that are $[\![\cdot]\!]_P$*-expressible ?*

- is decidable (Baeten/Corradini/G, 2007)
- partial new answer (G/Fokkink, 2020):
  - bisimulation collapse has loop existence & elimination property (LEE) if expressible by under-star-1-free regular expression

Milner's questions

(Q1) Complete axiomatization:

Is the proof system Mil *complete for* $=_{[\![\cdot]\!]_P}$*?*

- series of partial completeness results for:
  - exitless iterations (Fokkink, 1998)
  - with a stronger fixed-point rule (G, 2006)
  - under-star 1-free, and without 0 (Corradini/de Nicola/Labella, 2004)
  - with 0 but under-star-1-free (G/Fokkink, 2020)

(Q2) $[\![\cdot]\!]_P$-Expressibility:

What structural property characterizes
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- is decidable (Baeten/Corradini/G, 2007)
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Milner's questions

(Q1) Complete axiomatization:

Is the proof system Mil *complete for* $=_{[\![\cdot]\!]_P}$*?*

- Yes! (G, 2022, proof summary, employing LEE and crystallization)
- series of partial completeness results for:
  - exitless iterations (Fokkink, 1998)
  - with a stronger fixed-point rule (G, 2006)
  - under-star 1-free, and without 0 (Corradini/de Nicola/Labella, 2004)
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Question (Q2) specialized

$(\mathrm{Q2})_{\mathbf{0}}$ P-Expressibility and P-$(*/\mathbb{1})$-Expressibility:

What structural property characterizes:

- *process graphs that are P-expressible ?*
  (... that are in the image of P?)
- *process graphs that are P-expressible by $(*/\mathbb{1})$ regular expressions?*
  (... that are in the image of $(*/\mathbb{1})$ expressions under P?)

Loop Existence and Elimination (LEE)

Loop graphs (interpretations of innermost iterations without 1)

Definition

A process graph is a loop graph if:

(L1) There is an infinite path from the start vertex.

(L2) Every infinite path from the start vertex returns to it.

(L3) Termination is only possible at the start vertex.

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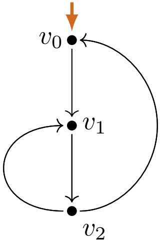

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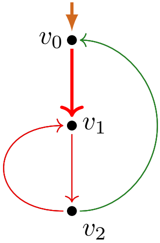

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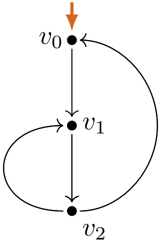

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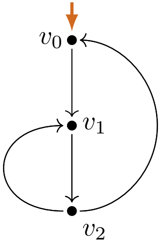

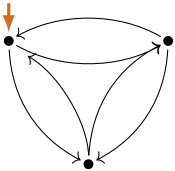

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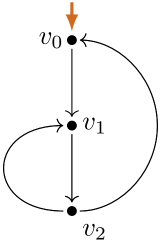

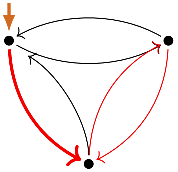

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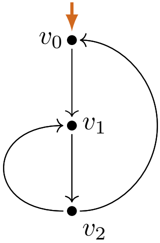

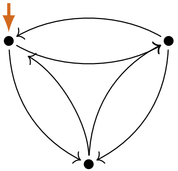

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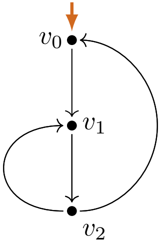

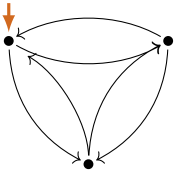

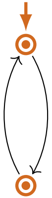

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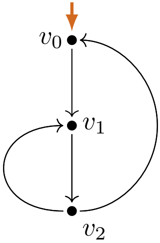

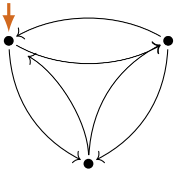

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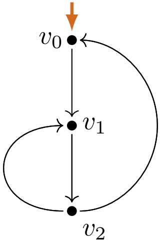

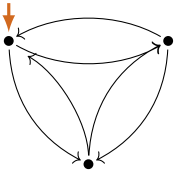

(L1),~~(L2)~~

(L1),(L2),~~(L3)~~

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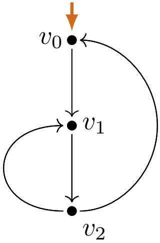
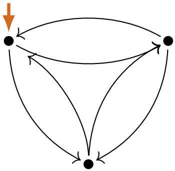
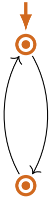




(L1),~~(L2)~~



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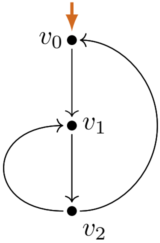

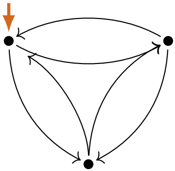

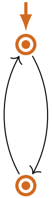

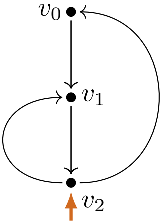

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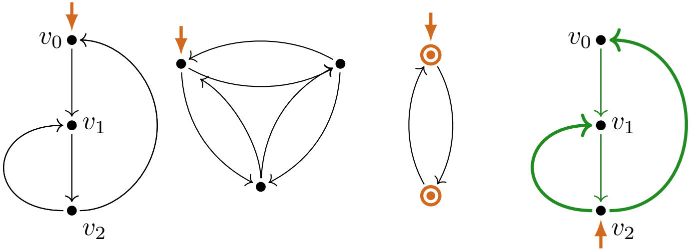

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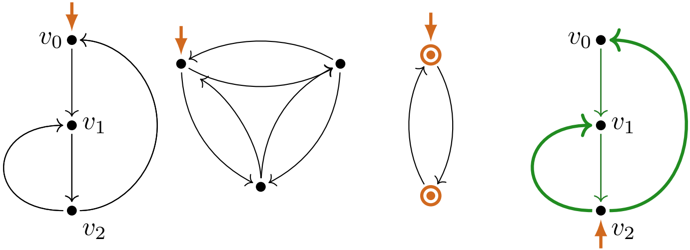

(L1),~~(L2)~~ (L1),(L2),~~(L3)~~ loop chart

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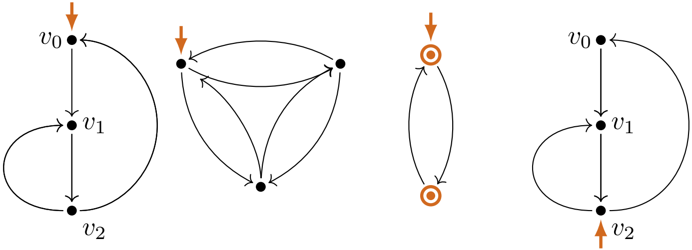

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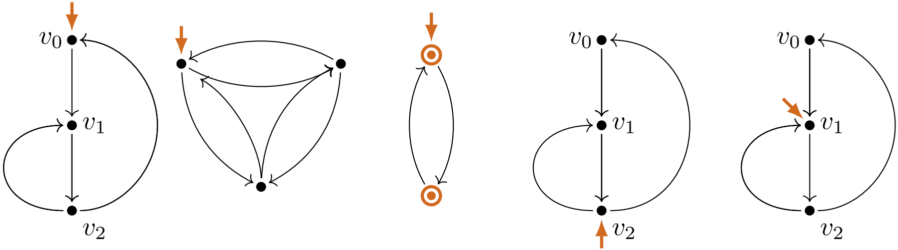

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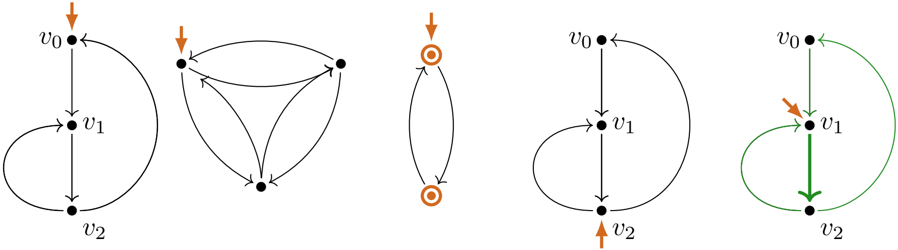

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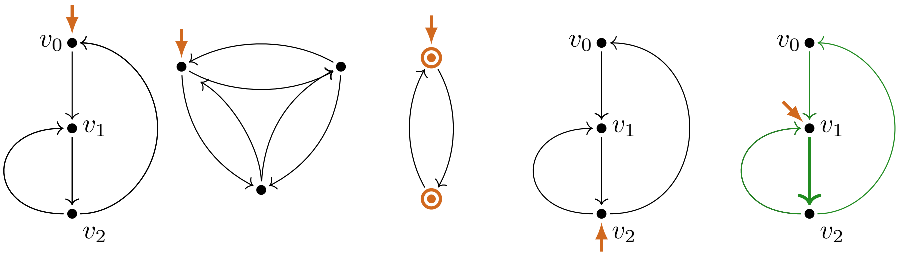

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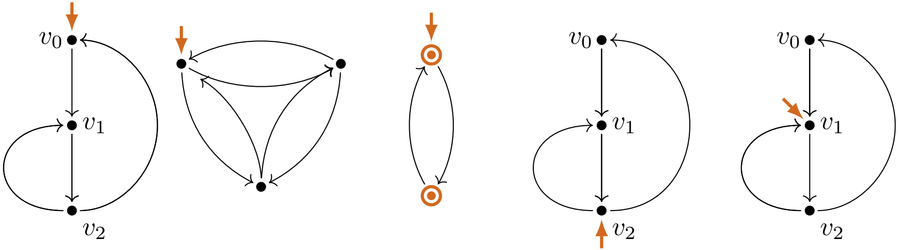

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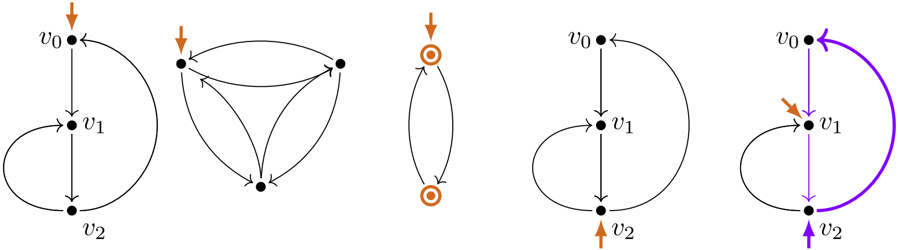
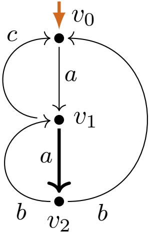
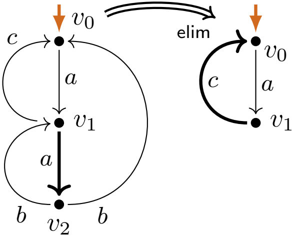


(L1),~~(L2)~~ (L1),(L2),~~(L3)~~ loop chart loop subchart

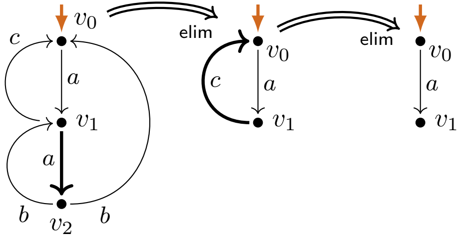
Loop existence and elimination



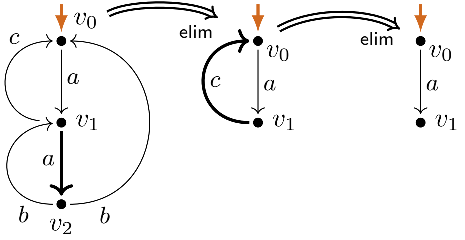
Loop existence and elimination

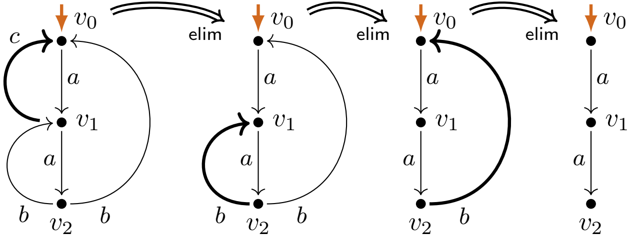


Loop existence and elimination



Loop existence and elimination





LEE

Definition

A chart $\mathcal{C}$ satisfies LEE (*loop existence and elimination*) if:

$$\exists \mathcal{C}_0 \left(\mathcal{C} \longrightarrow^{*}_{\text{elim}} \mathcal{C}_0 \not\longrightarrow_{\text{elim}} \right.$$
$$\left. \wedge\ \mathcal{C}_0 \text{ permits no infinite path} \right).$$

LEE

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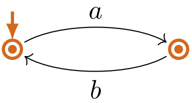
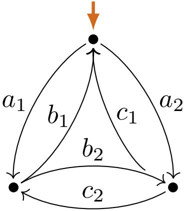
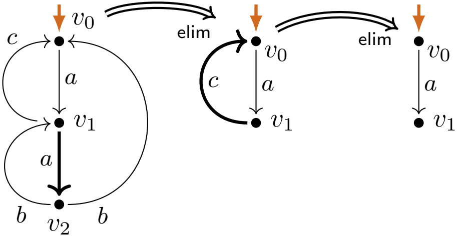


~~LEE~~



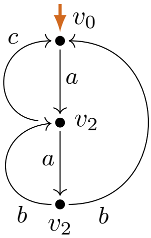
~~LEE~~

LEE

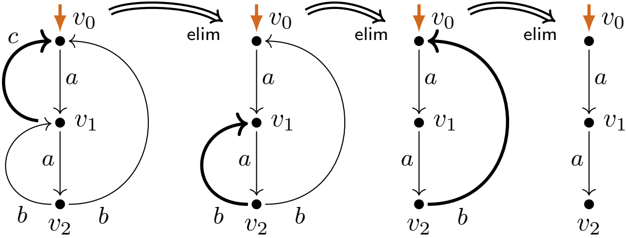


v_0
c
a
v_1
a
b
v_2
b
elim
v_0
c
a
v_1
elim
v_0
a
v_1

LEE

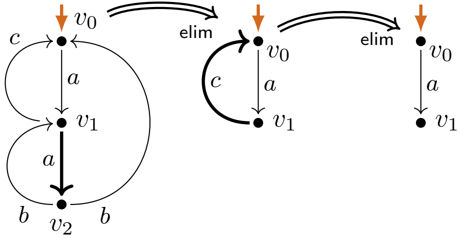


v_0
c
a
v_2
a
b
v_2
b

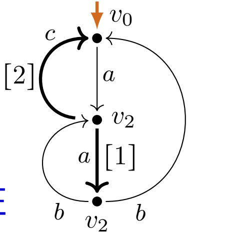


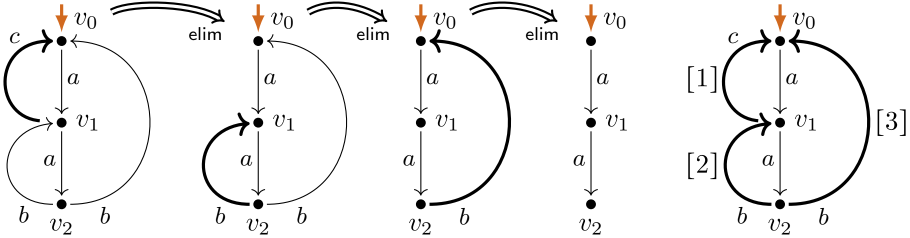
v_0
c
a
v_1
a
b
v_2
b
elim
v_0
a
v_1
a
b
v_2
b
elim
v_0
a
v_1
a
v_2
b
elim
v_0
a
v_1
a
v_2

LEE



LEE





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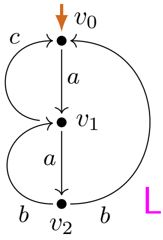
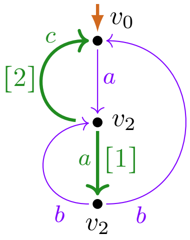
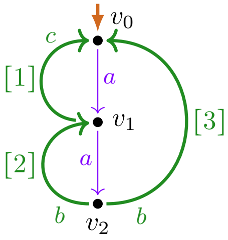


LEE-graph

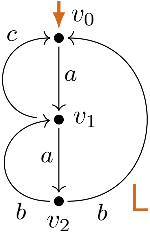
LEE-witness



LEE-witness

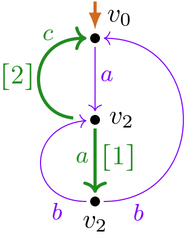


Layered LEE

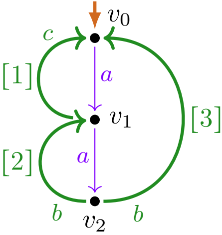


LLEE-graph

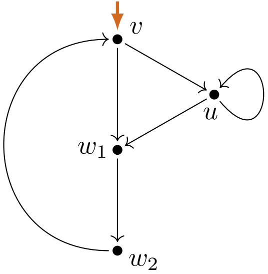
layered
LLEE-witness

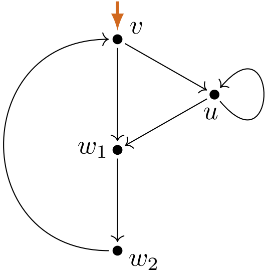


layered
LLEE-witness

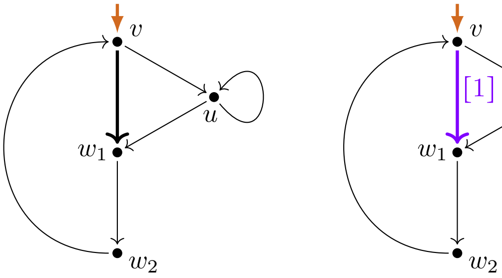


Layered LEE-witness (LLEE-witness)

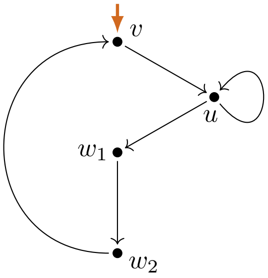


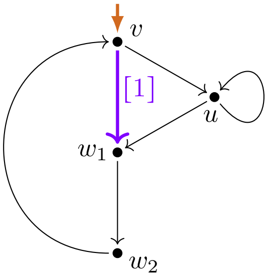


Layered LEE-witness (LLEE-witness)

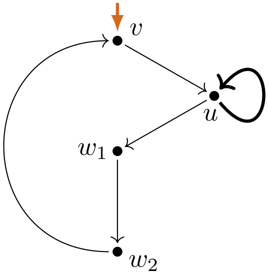


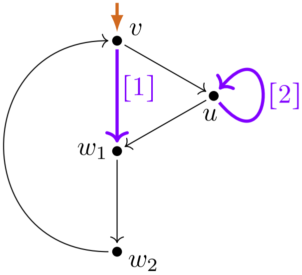
Layered LEE-witness (LLEE-witness)



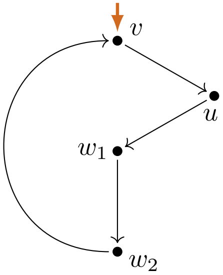


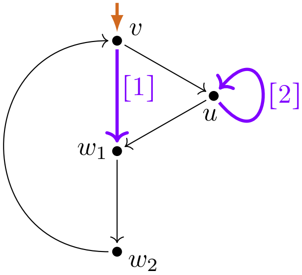
Layered LEE-witness (LLEE-witness)



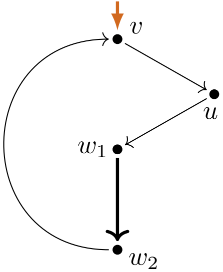


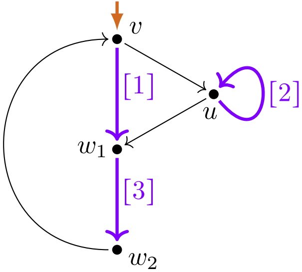
Layered LEE-witness (LLEE-witness)



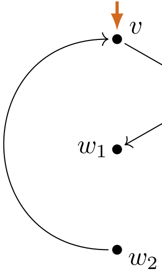


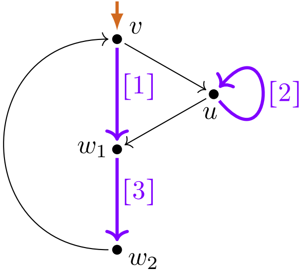
Layered LEE-witness (LLEE-witness)





Layered LEE-witness (LLEE-witness)





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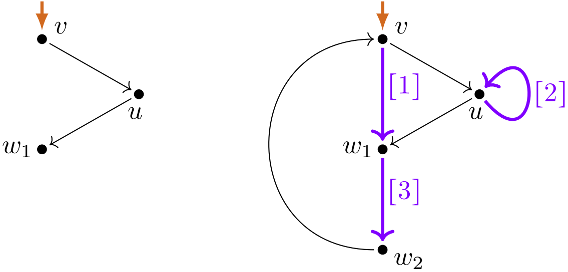
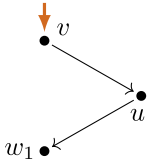
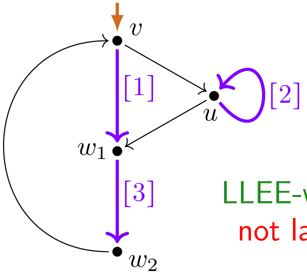


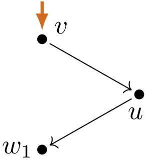
Layered LEE-witness (LLEE-witness)

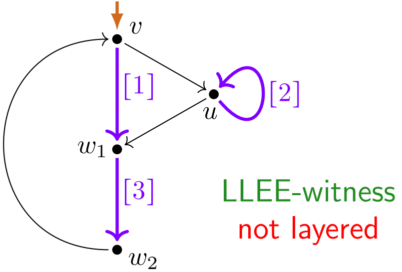


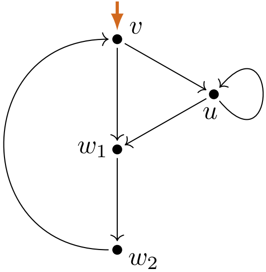


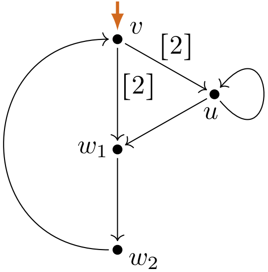
LLEE-witness
not layered

Layered LEE-witness (LLEE-witness)

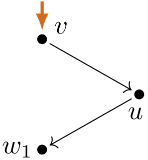


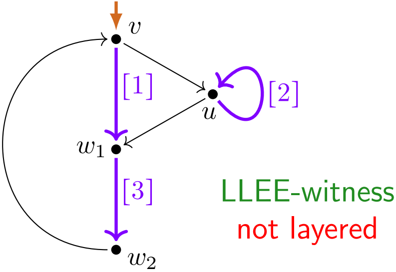


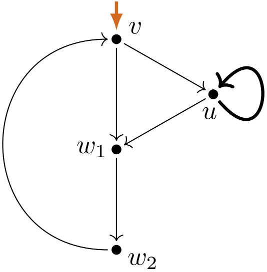


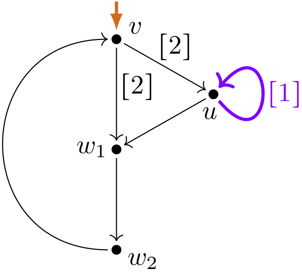


Layered LEE-witness (LLEE-witness)

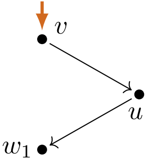


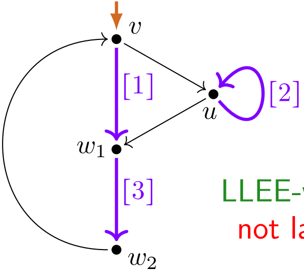




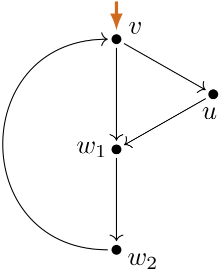


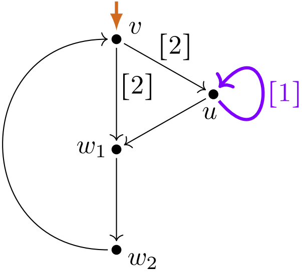
Layered LEE-witness (LLEE-witness)



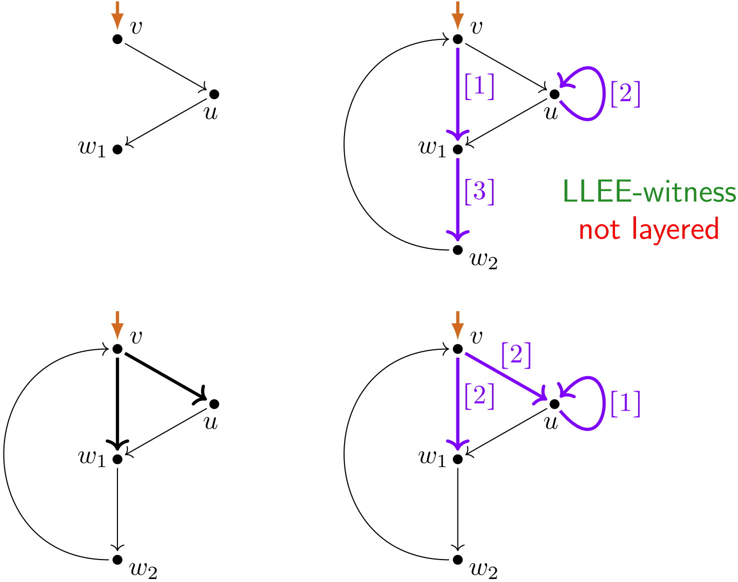


LLEE-witness
not layered

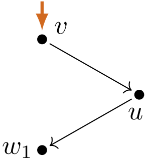


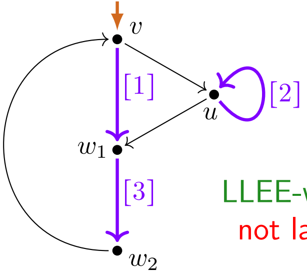


Layered LEE-witness (LLEE-witness)

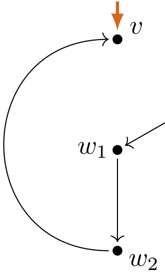


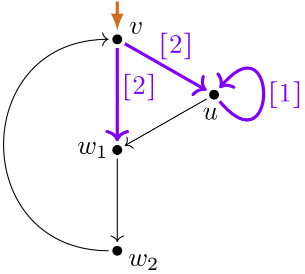
Layered LEE-witness (LLEE-witness)



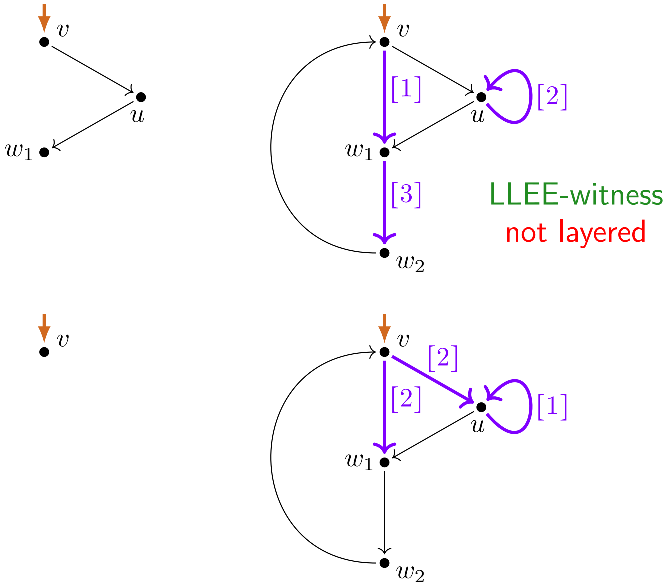


LLEE-witness
not layered

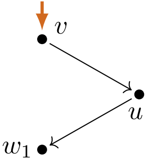


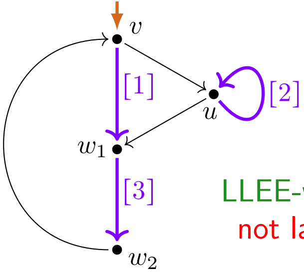


Layered LEE-witness (LLEE-witness)

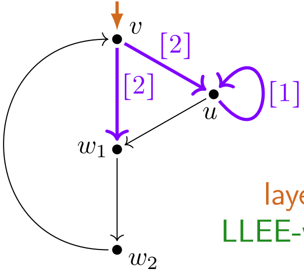


Layered LEE-witness (LLEE-witness)



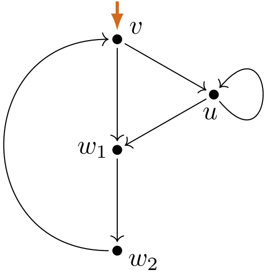


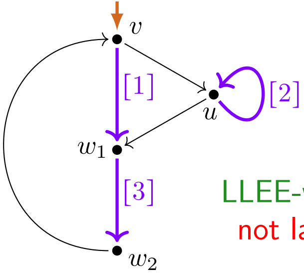
LLEE-witness
not layered



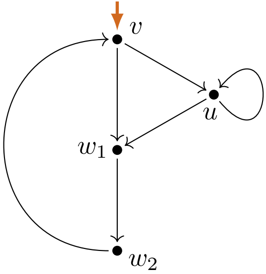
layered
LLEE-witness

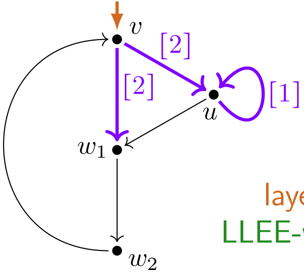
Layered LEE-witness (LLEE-witness)





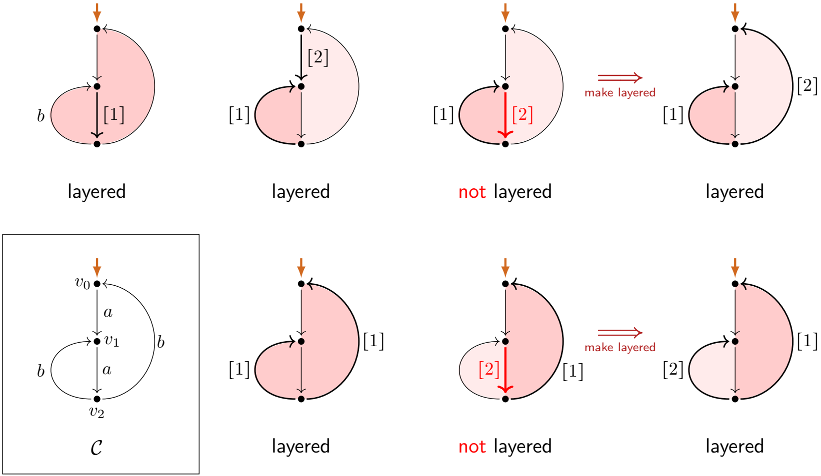
LLEE-witness
not layered





layered
LLEE-witness

7 LEE-witnesses



Loop elimination

$\longrightarrow_{\text{elim}}$: eliminate a transition-induced loop by:

- removing the loop-entry transition(s)
- garbage collection

$\longrightarrow_{\text{prune}}$: remove a transition to a deadlocking state

Lemma

(i) $\longrightarrow_{\text{elim}} \cup \longrightarrow_{\text{prune}}$ *is terminating.*

Loop elimination

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- garbage collection

$\longrightarrow_{\text{prune}}$: remove a transition to a deadlocking state

Lemma

(i) $\longrightarrow_{\text{elim}} \cup \longrightarrow_{\text{prune}}$ *is terminating.*

Loop elimination, and properties

$\longrightarrow_{\text{elim}}$: eliminate a transition-induced loop by:

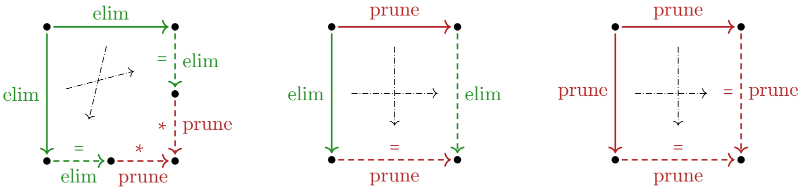
- removing the loop-entry transition(s)
- garbage collection

$\longrightarrow_{\text{prune}}$: remove a transition to a deadlocking state

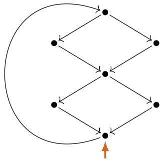
Lemma

(i) $\longrightarrow_{\text{elim}} \cup \longrightarrow_{\text{prune}}$ *is terminating.*

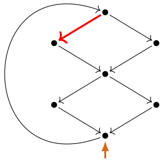
(ii) $\longrightarrow_{\text{elim}} \cup \longrightarrow_{\text{prune}}$ *is decreasing* [Van Oostrom, de Bruijn]



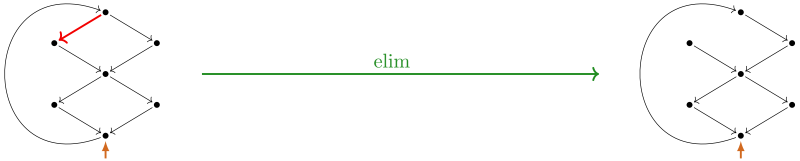
‘Critical pair’: bi-loop elimination



‘Critical pair’: bi-loop elimination



‘Critical pair’: bi-loop elimination



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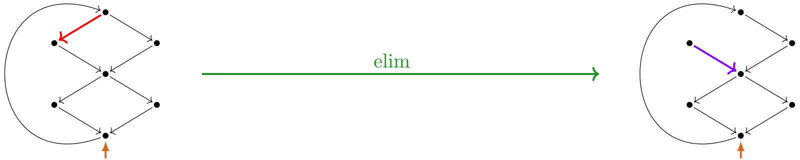

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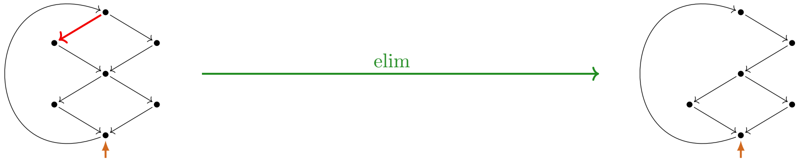

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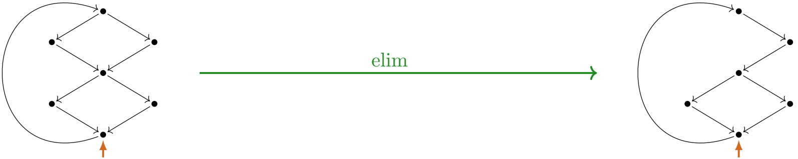

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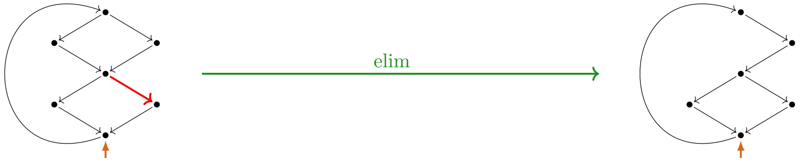

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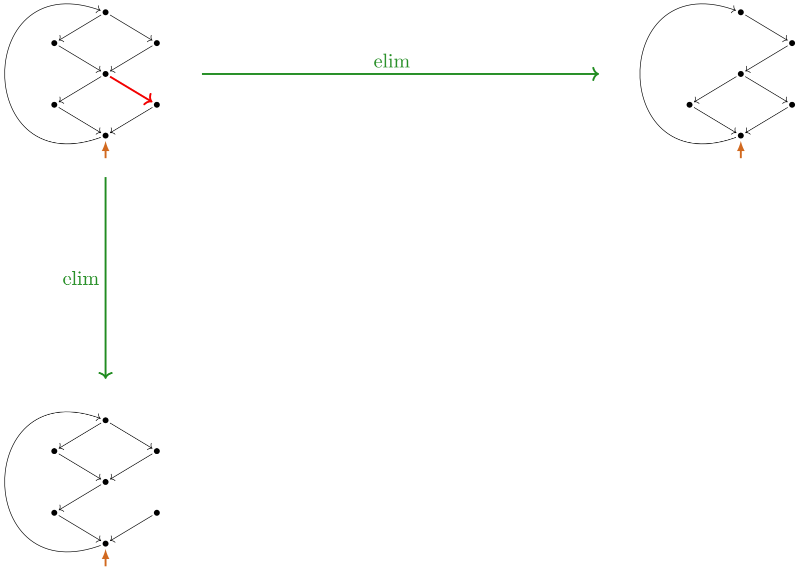

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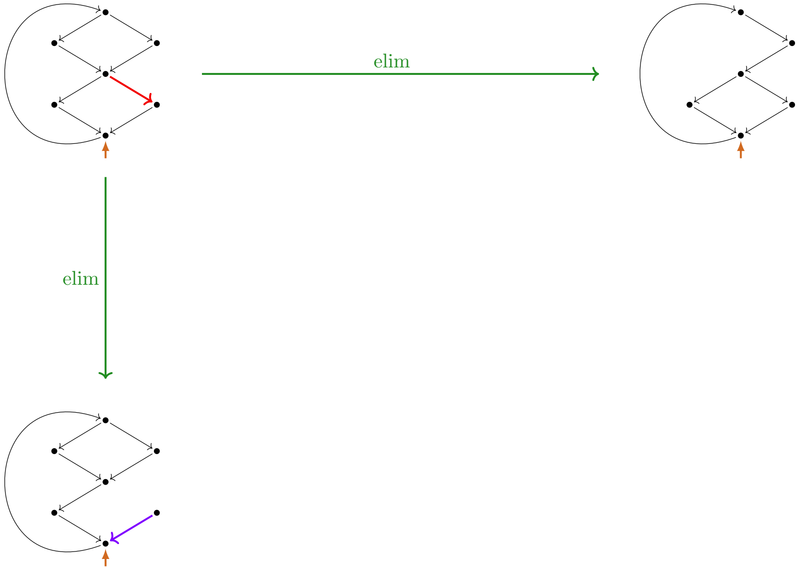

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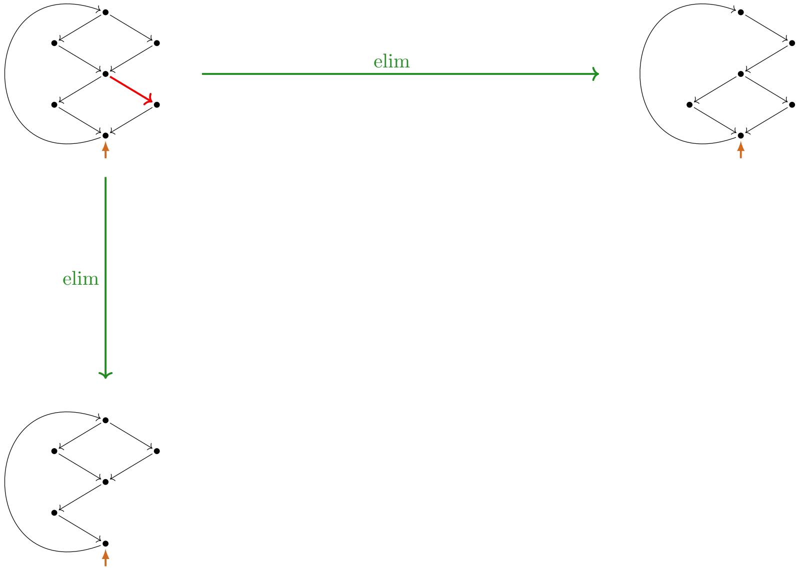

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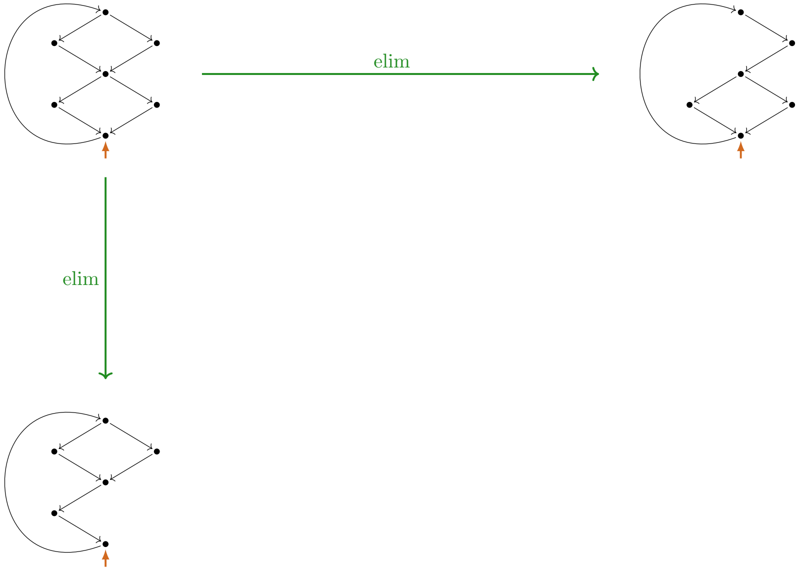

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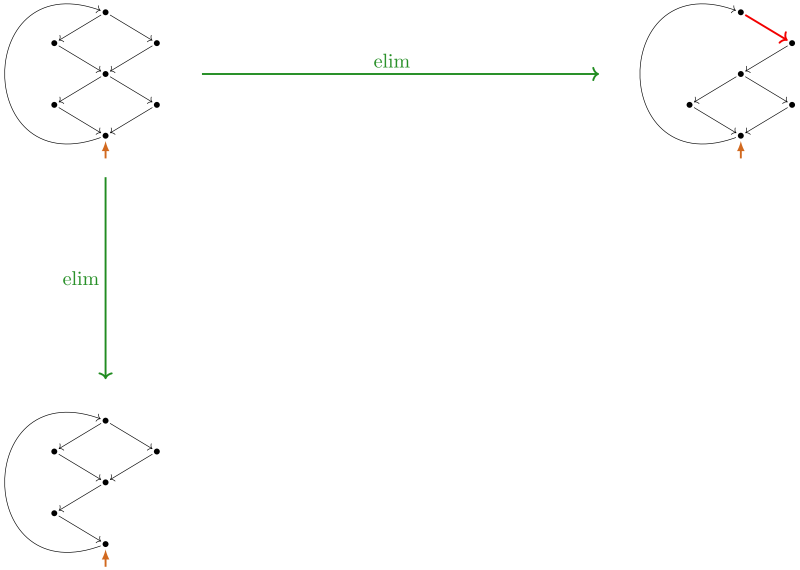

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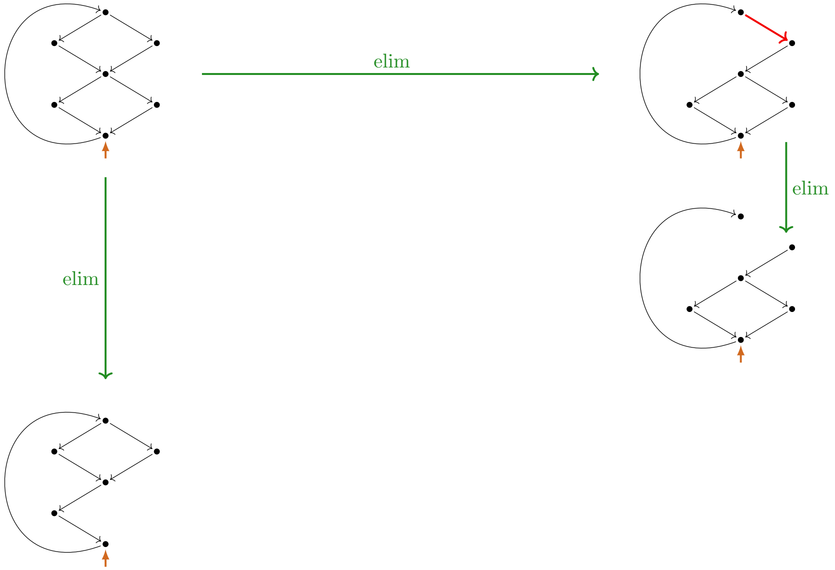

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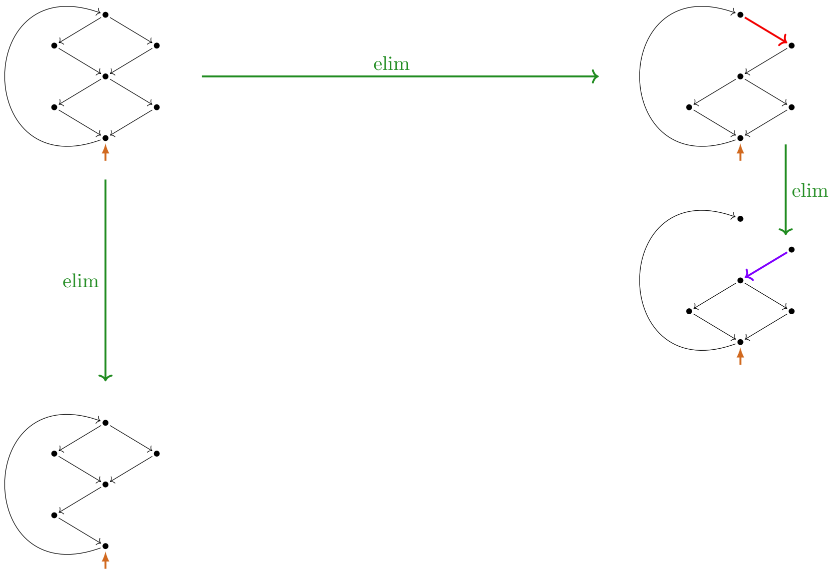

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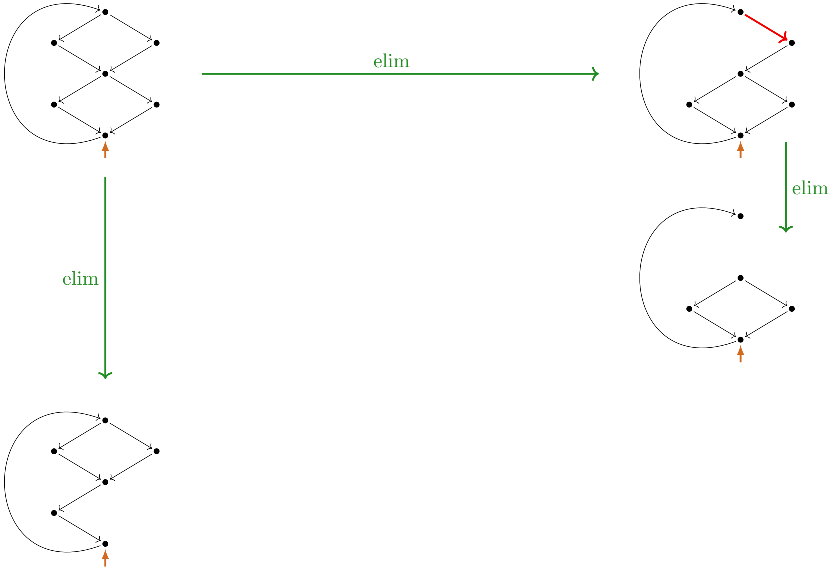

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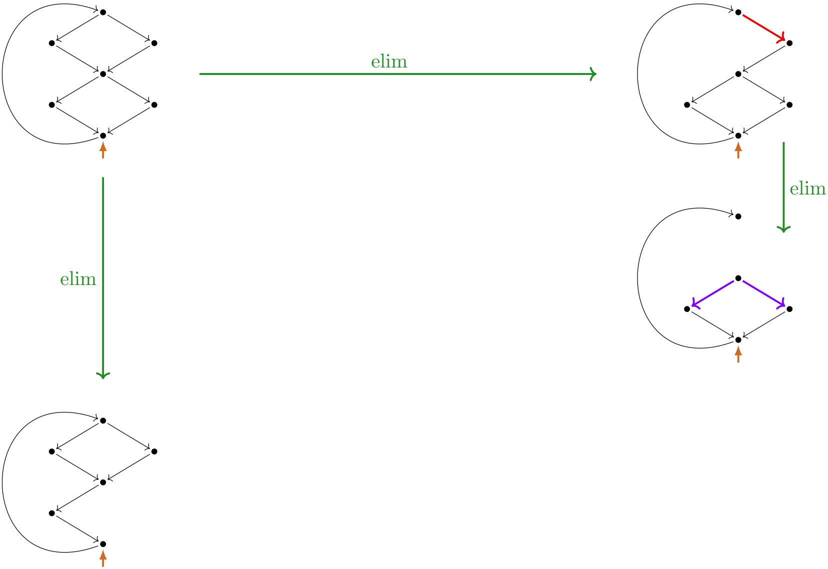
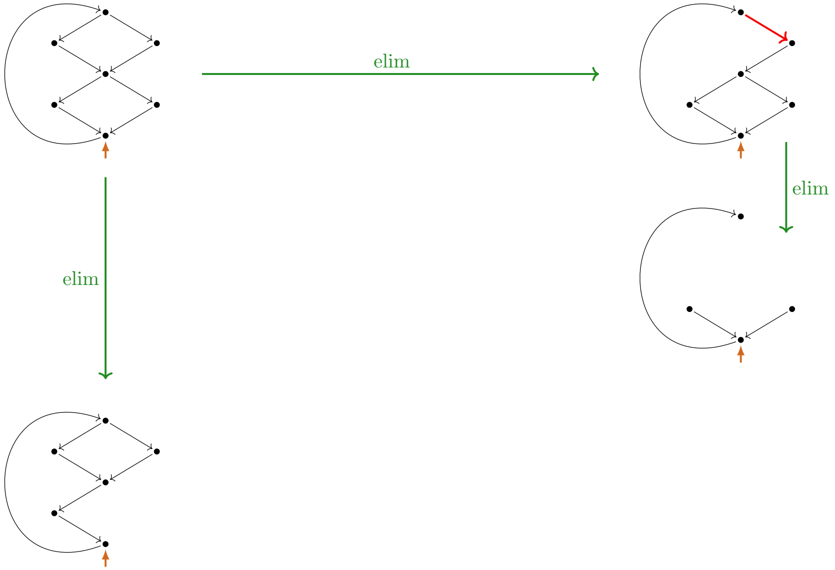


‘Critical pair’: bi-loop elimination



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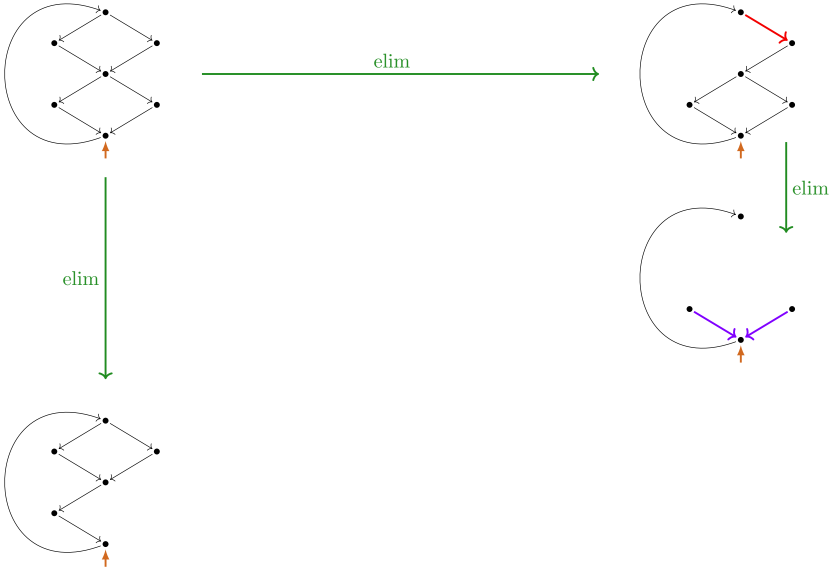

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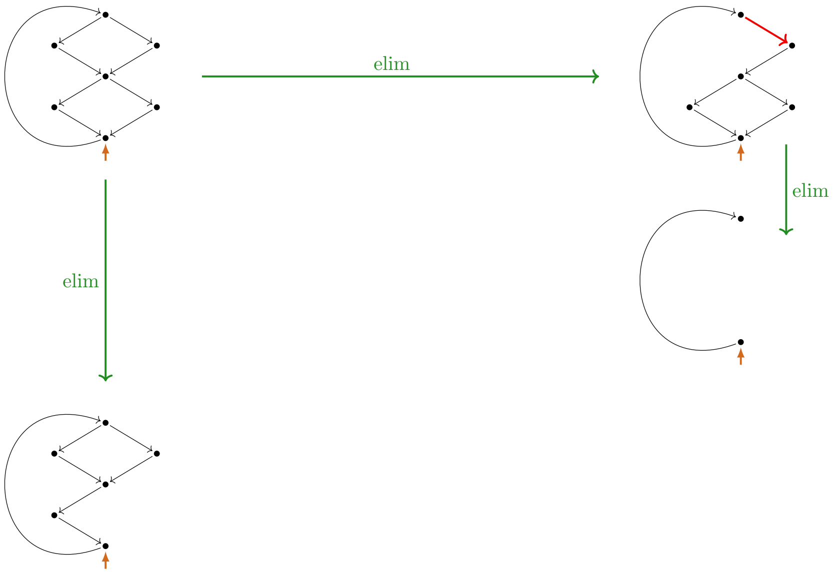

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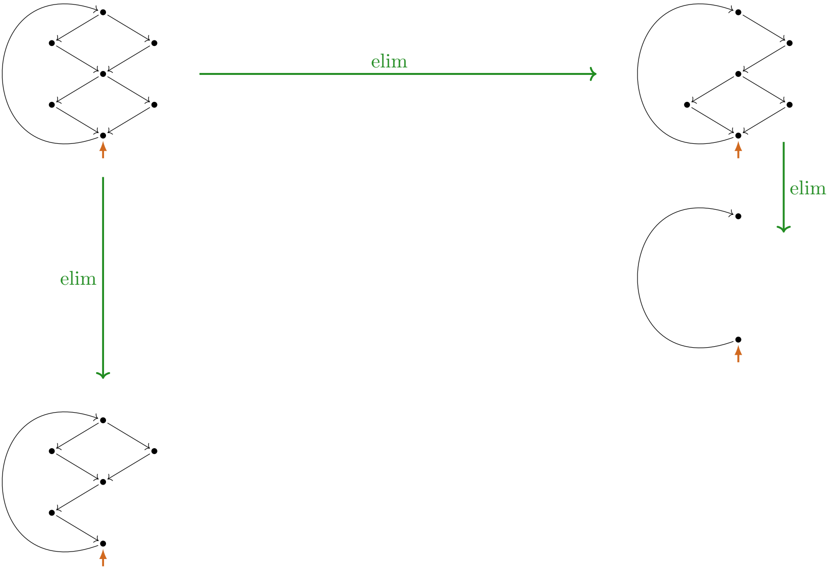

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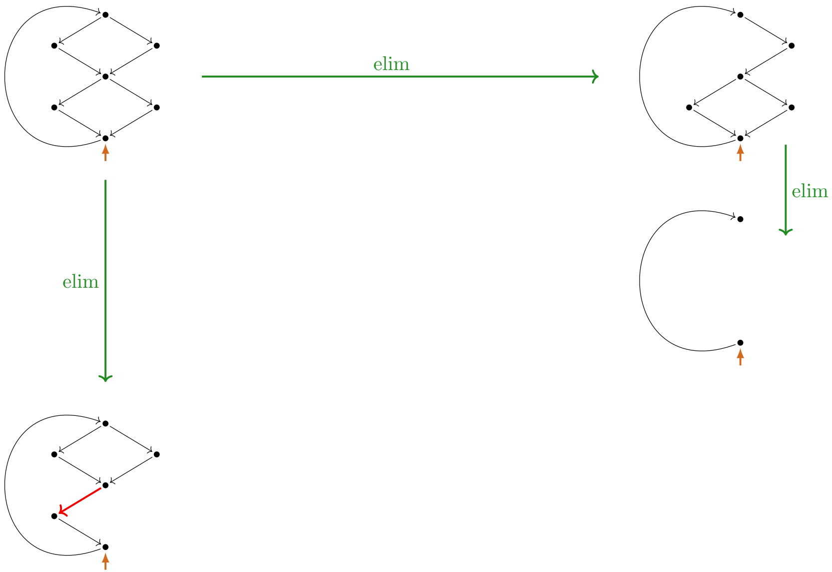

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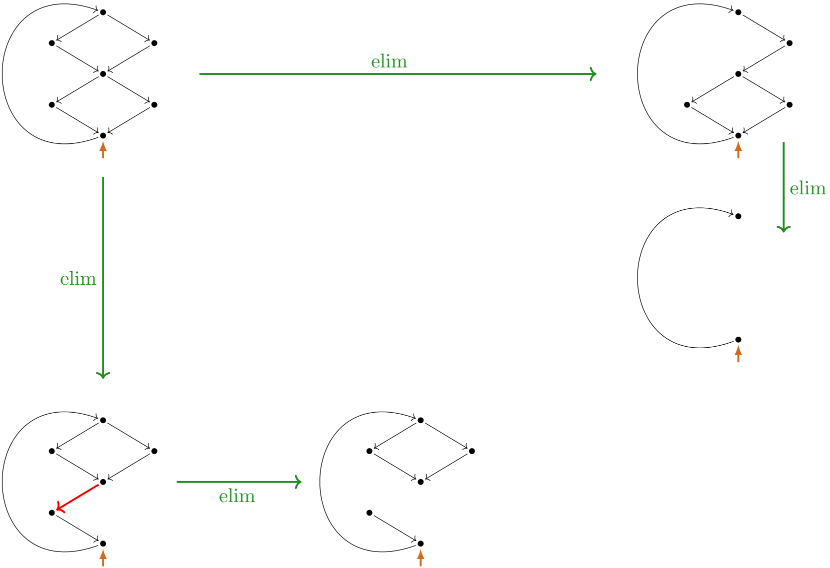

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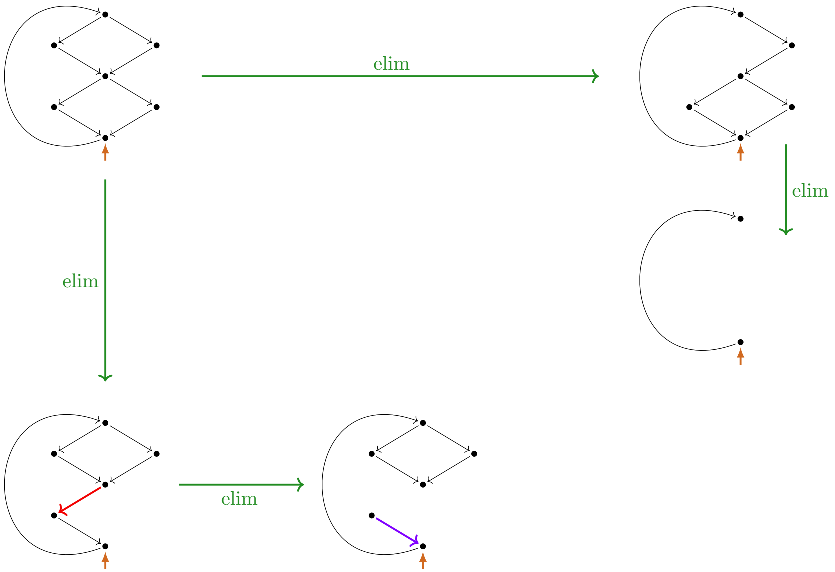

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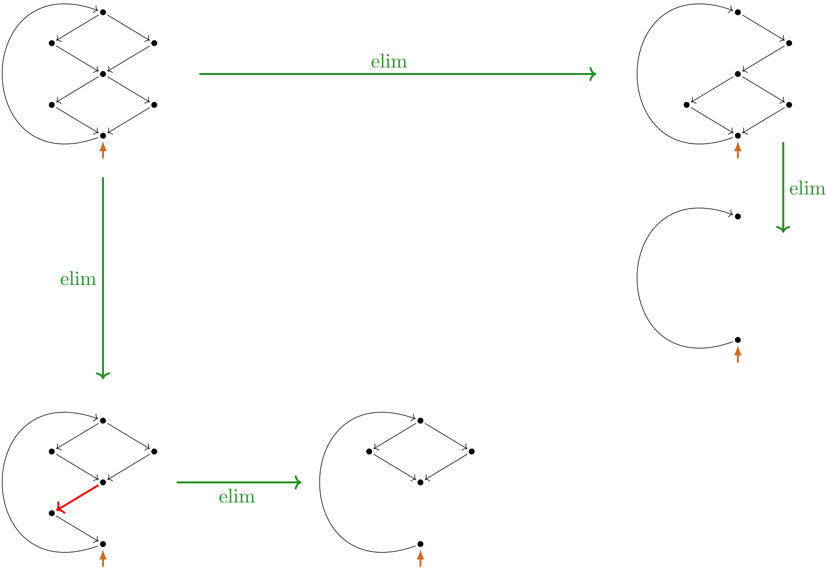

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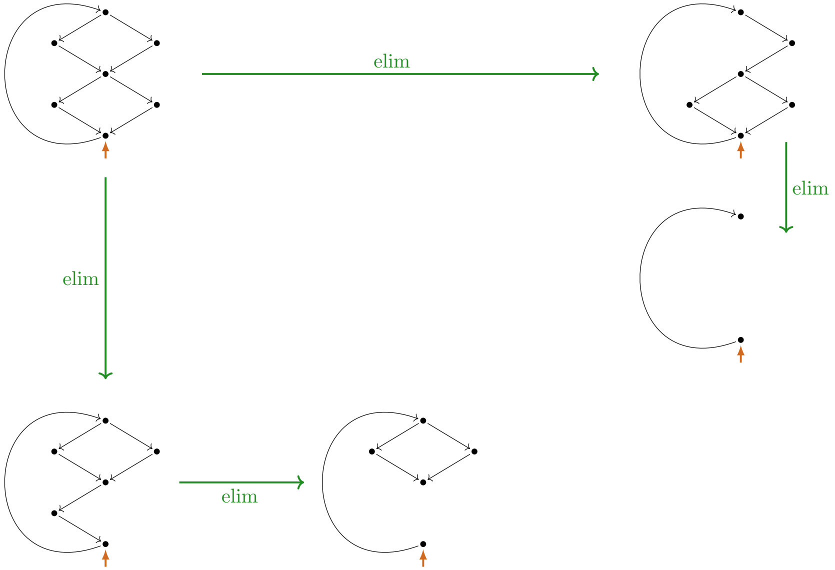

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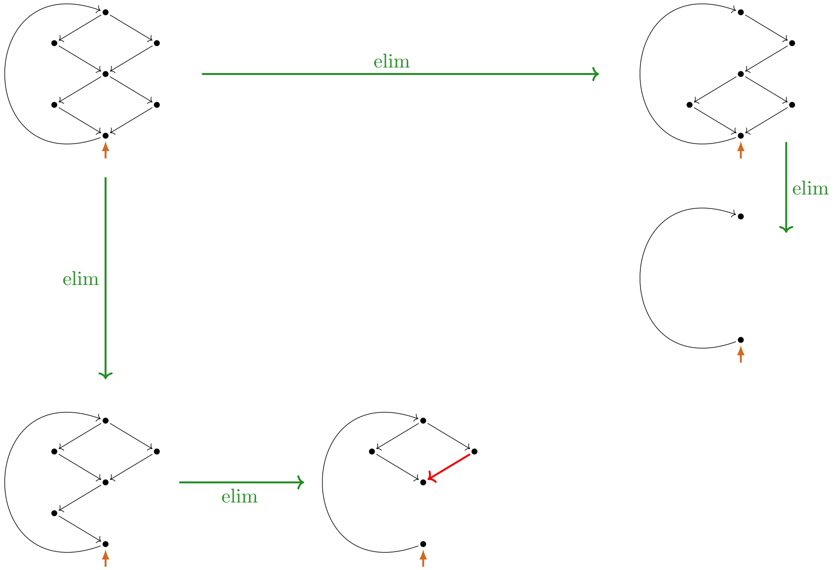

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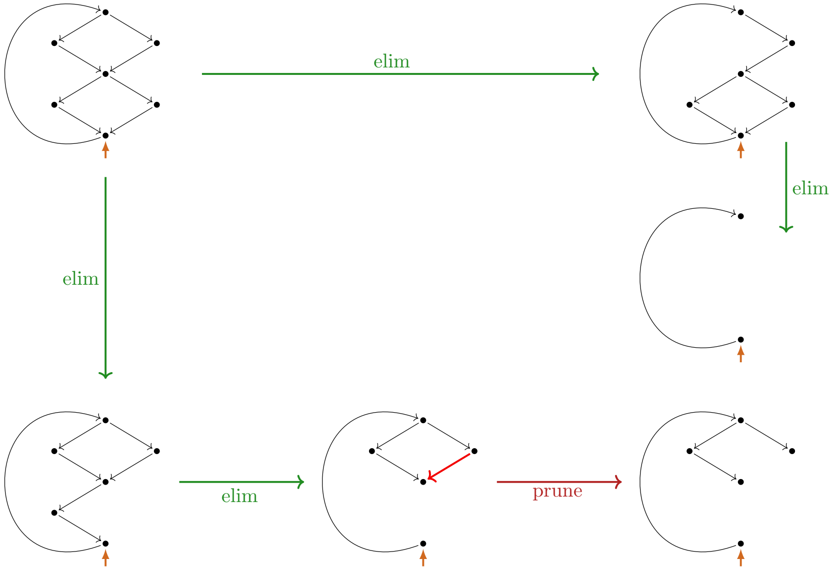

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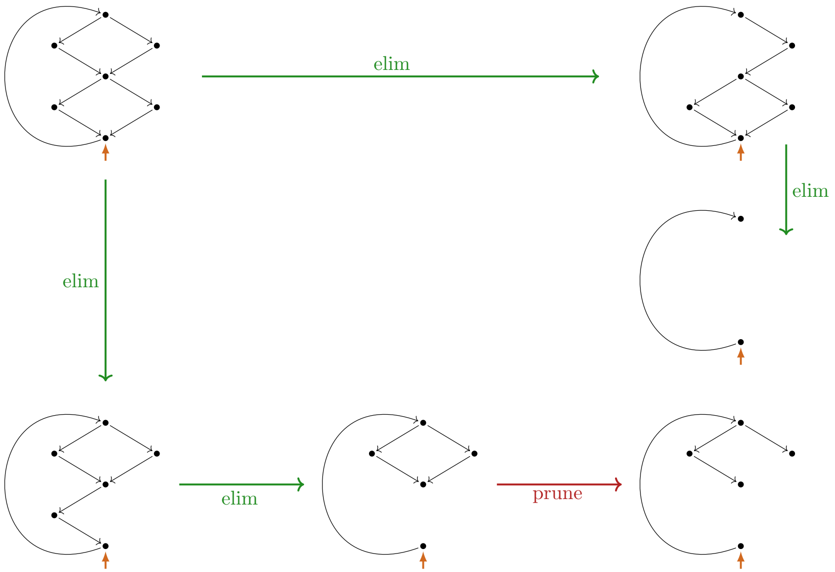

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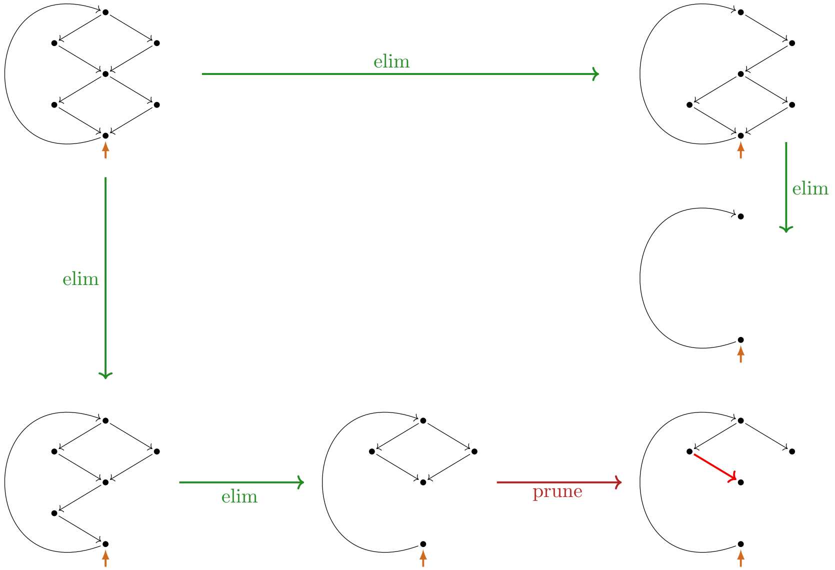

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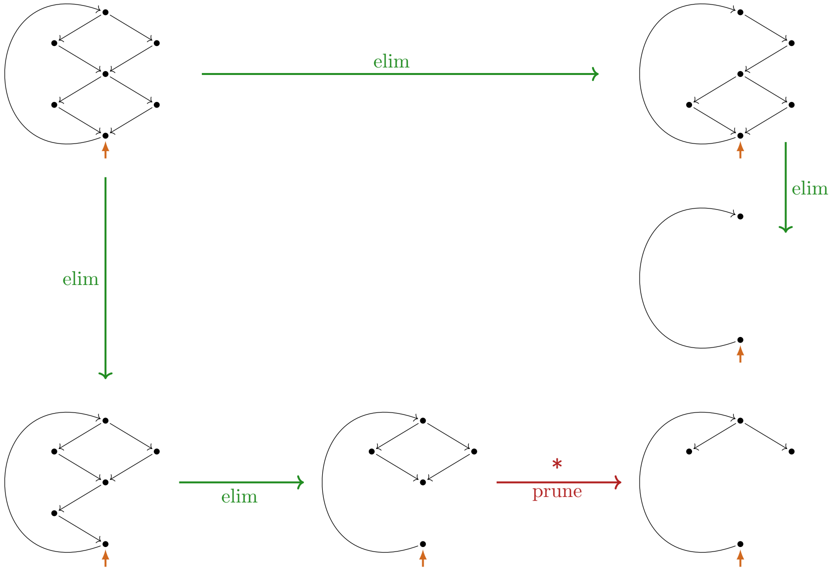

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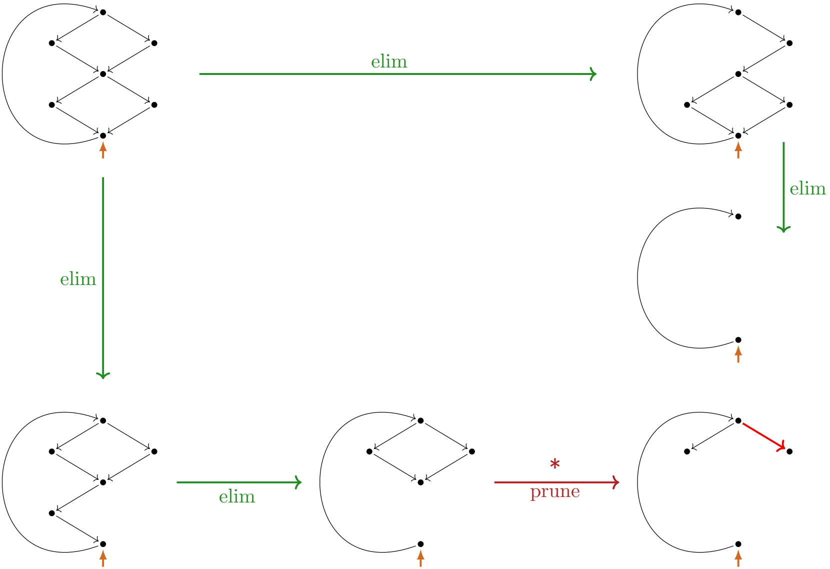

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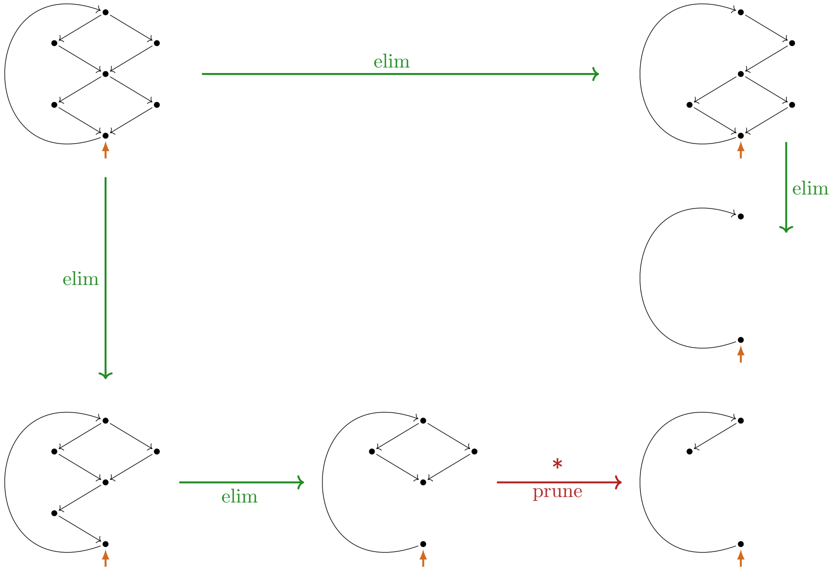

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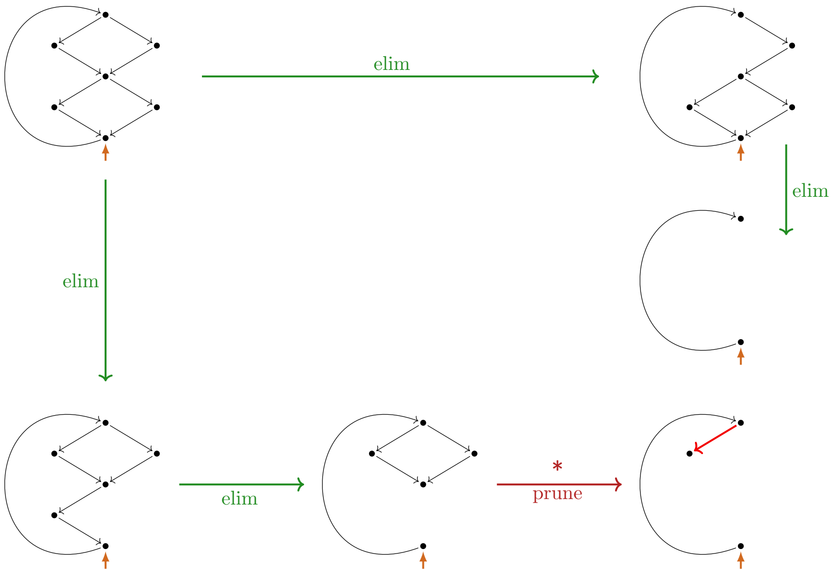

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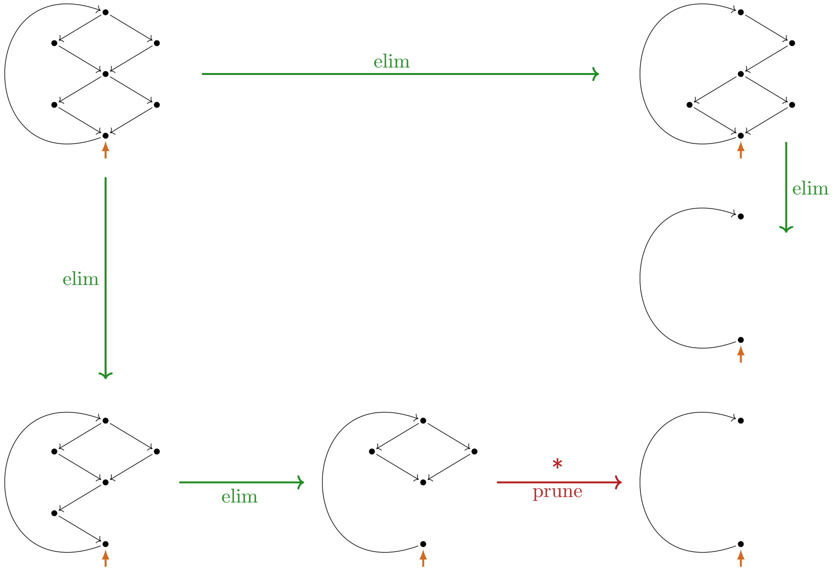

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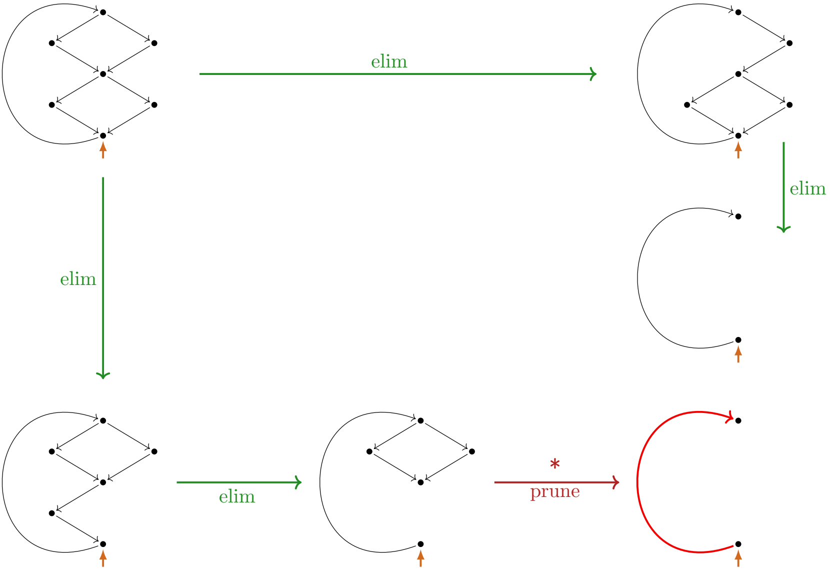

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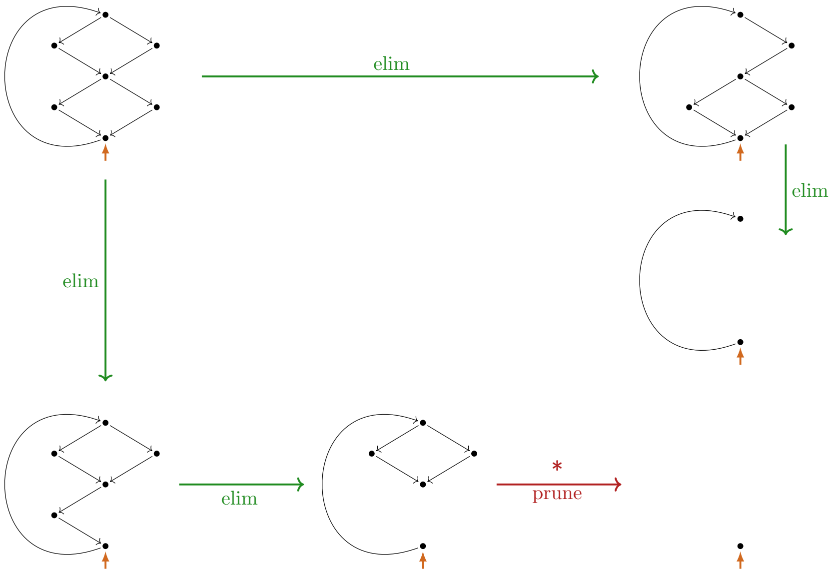

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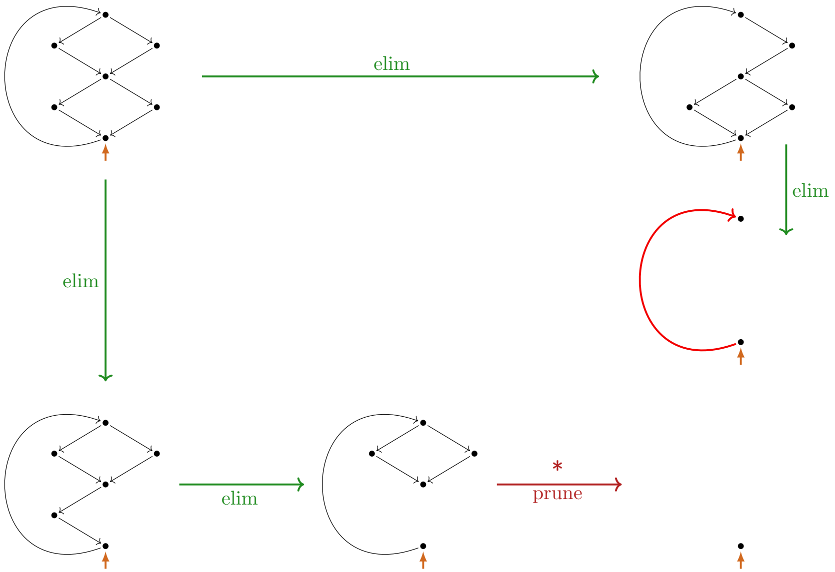

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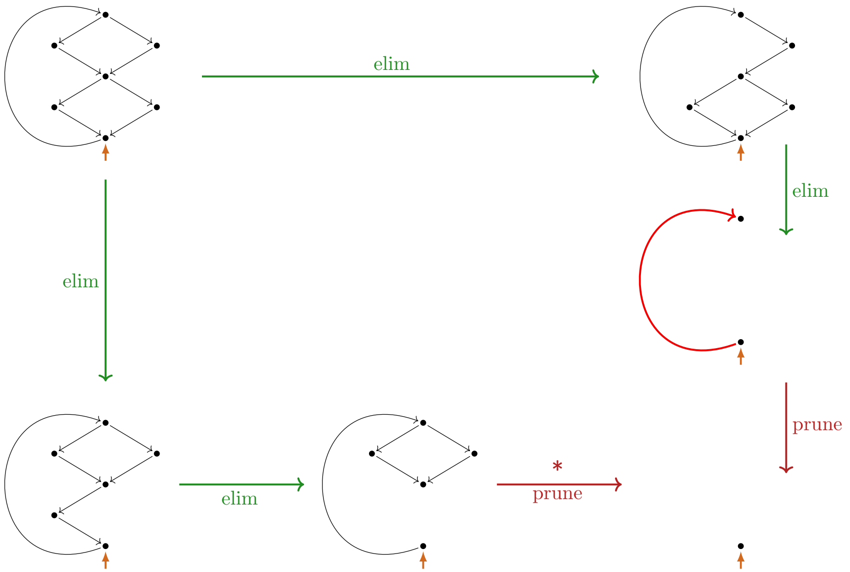

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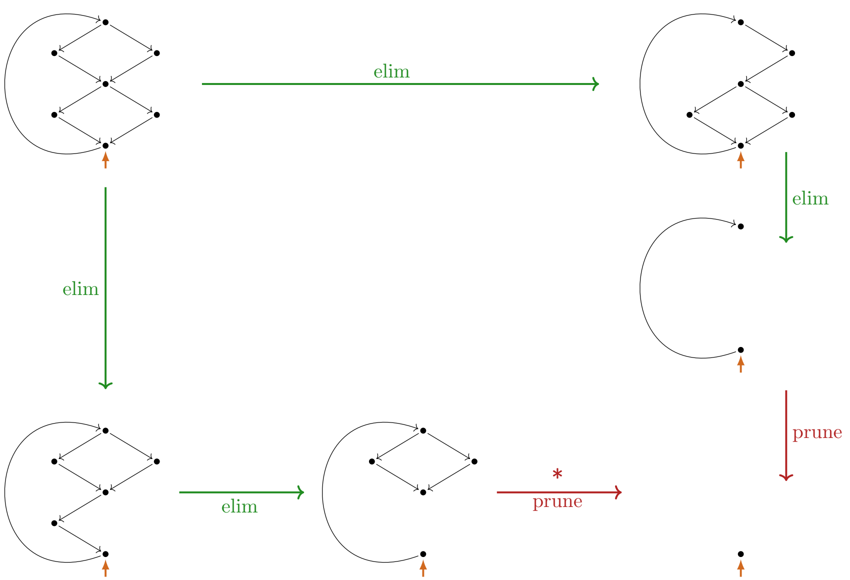

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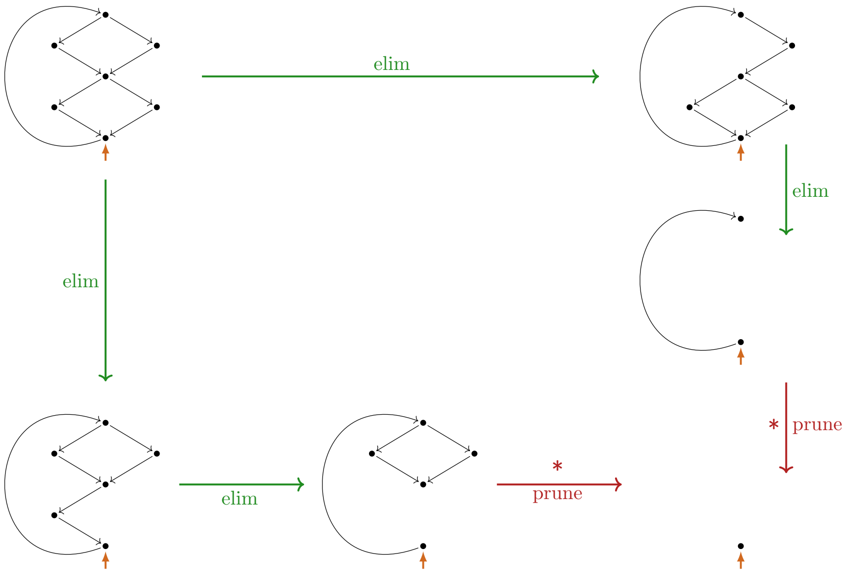


Loop elimination, and properties

$\longrightarrow_{\text{elim}}$: eliminate a transition-induced loop by:

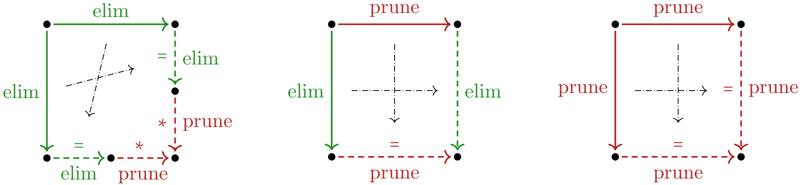
- removing the loop-entry transition(s)
- garbage collection

$\longrightarrow_{\text{prune}}$: remove a transition to a deadlocking state

Lemma

(i) $\longrightarrow_{\text{elim}} \cup \longrightarrow_{\text{prune}}$ *is terminating.*

(ii) $\longrightarrow_{\text{elim}} \cup \longrightarrow_{\text{prune}}$ *is decreasing, and so due to (i) locally confluent.*



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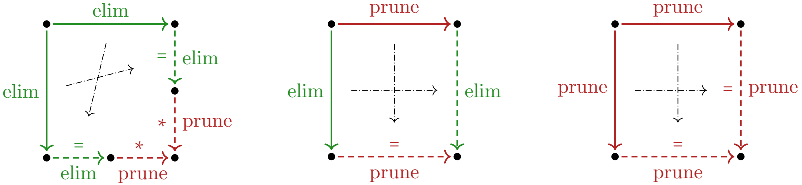
(iii) $\longrightarrow_{\text{elim}} \cup \longrightarrow_{\text{prune}}$ *is confluent.*



Structure property LEE

$$\mathsf{LEE}(G) :\Longleftrightarrow \exists G_0 \big(G \longrightarrow^{*}_{\mathsf{elim}} G_0 \not\longrightarrow_{\mathsf{elim}} \wedge G_0 \text{ has no infinite trace} \big).$$

Lemma (by using termination and confluence)

For every process graph G the following are equivalent:

(i) $\mathsf{LEE}(G)$.

(ii) *There is an $\longrightarrow_{\mathsf{elim}}$ normal form without an infinite trace.*

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Theorem (efficient decidability)

The problem of deciding $\mathsf{LEE}(G)$ for process graphs G is in PTIME.

Interpretation/extraction correspondences with LEE

($\Leftarrow$ G/Fokkink 2020, G 2021)

(Int)$_{P}^{(*/\mathbb{1})}$: *P-$(*/\mathbb{1})$-expressible graphs have the structural property LEE.*

Process **i**nterpretations $P(e)$ of $(*/\mathbb{1})$ regular expressions e are finite process graphs that satisfy LEE.

(Extr)$_{P}$: LEE *implies $[\![\cdot]\!]_{P}$-expressibility*

From every finite process graph G with LEE a regular expression e can be **e**xtracted such that $G \underline{\leftrightarrow} P(e)$.

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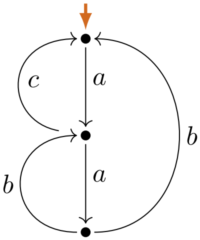
(Coll): LEE *is preserved under collapse*

The class of finite process graphs with LEE is closed under bisimulation **c**ollapse.

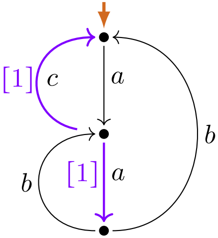
Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

G_2



Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_2}$



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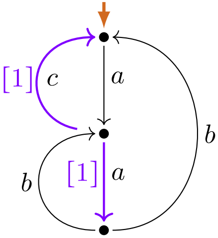
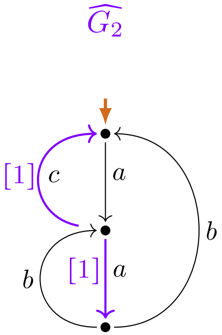


$$(\qquad\qquad\qquad)^{*} \cdot 0$$

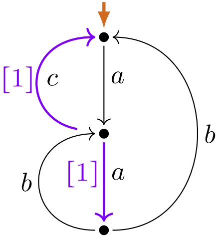
Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)



$$\left(\quad \overset{\vdots\, a}{} \quad \right)^* \cdot 0$$

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_2}$



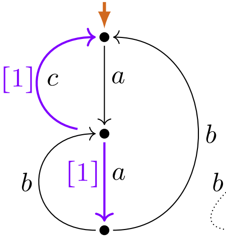
$$(a \cdot 1) \cdot ((\qquad\qquad\qquad)^* \cdot 0)$$

$$\downarrow a$$

$$(\qquad\qquad\qquad)^* \cdot 0$$

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_2}$



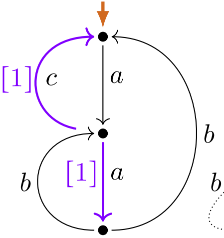
$(a \cdot 1) \cdot (($ $)^* \cdot 0)$

a

b $($ $)^* \cdot 0$ b

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_2}$



$(a \cdot 1) \cdot ((\qquad)^* \cdot 0)$

$\downarrow a$

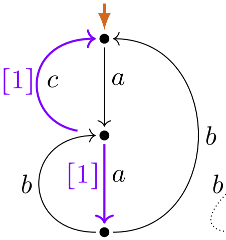
$(\qquad)^* \cdot 0$

b $\qquad$ b

$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot ((\qquad)^*) \cdot 0)$

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_2}$



$(a \cdot 1) \cdot ((\qquad)^* \cdot 0)$

$\downarrow a$

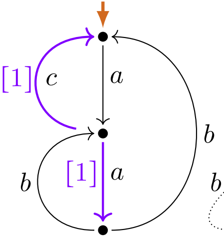
$(\qquad)^* \cdot 0$

$\downarrow a$

$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot ((\qquad)^*) \cdot 0)$

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_2}$



$$(a \cdot 1) \cdot ((\qquad\qquad\qquad\qquad)^* \cdot 0)$$

$$\downarrow a$$

$$\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0$$

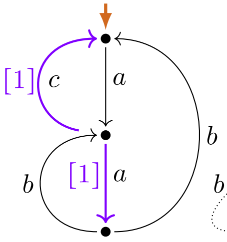
$$\downarrow a$$

$$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot ((\qquad\qquad\qquad\qquad)^*) \cdot 0)$$

c b b

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_2}$



$$(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^*\cdot 0\Big)$$

$$\downarrow a$$

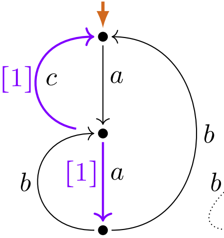
$$\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^*\cdot 0$$

$$\downarrow a$$

$$(b\cdot 1+b\cdot(a\cdot 1))\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^*)\cdot 0\Big)$$

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_2}$



$$\overbrace{(a \cdot 1) \cdot \left(\left(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\right)^{*} \cdot 0\right)}^{e}$$

$$\downarrow a$$

$$\left(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\right)^{*} \cdot 0$$

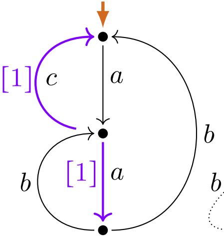
$$\downarrow a$$

$$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot \left(\left(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\right)^{*}\right) \cdot 0\Big)$$

c b b

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_2}$



$P(e) \rightleftharpoons G_2$

$$\overbrace{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)}^{e}$$

$$\downarrow a$$

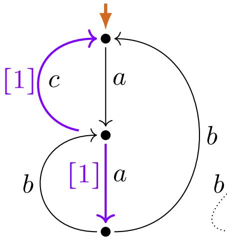
$$\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0$$

$$\downarrow a$$

$$(b\cdot 1+b\cdot(a\cdot 1))\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0\Big)$$

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_2}$



$P(e) \Rightarrow G_2$

$$\overbrace{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)}^{e}$$

$$\downarrow a$$

$$\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0$$

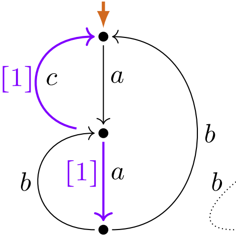
$$\downarrow a$$

$$(b\cdot 1+b\cdot(a\cdot 1))\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0\Big)$$

c b b

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_2}$



$P(e) \underline{\rightarrow} G_2$

$$\overbrace{(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0 \Big)}^{e}$$

$$\downarrow a$$

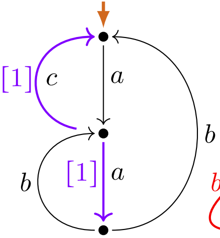
$$\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0$$

$$\downarrow a$$

$$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*}) \cdot 0 \Big)$$

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_2}$



$P(e) \rightrightarrows G_2$

$$\overbrace{(a \cdot 1) \cdot \big((c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)))^* \cdot 0\big)}^{e}$$

$$\downarrow a$$

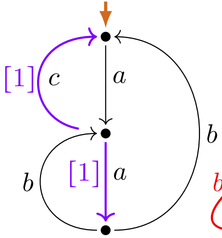
$$\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0$$

$$\downarrow a$$

$$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^*) \cdot 0\big)$$

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_2}$



$P(e) \trianglerighteq G_2 \not\simeq P(e)$

$$\overbrace{(a\cdot 1)\cdot\big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\big)}^{e}$$

$$\downarrow a$$

$$\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0$$

$$\downarrow a$$

$$(b\cdot 1+b\cdot(a\cdot 1))\cdot\big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0\big)$$

Interpretation of extracted expression

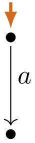
G_2'

$$P(e) = G_2'$$

$$\overbrace{(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^{*} \cdot 0\Big)}^{e}$$

Interpretation of extracted expression

G'_2



$$P(e) = G'_2$$

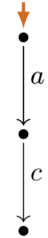
$$\overbrace{(a \cdot 1) \cdot \left(\left(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\right)^{*} \cdot 0\right)}^{e}$$

$$\Big\downarrow a$$

$$(1 \cdot 1) \cdot \left(\left(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\right)^{*} \cdot 0\right)$$

Interpretation of extracted expression

G_2'



$$P(e) = G_2'$$

$$\overbrace{(a\cdot 1)\cdot\left(\left(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\right)^{*}\cdot 0\right)}^{e}$$

$$\downarrow a$$

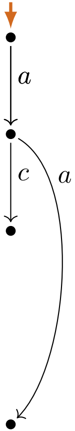
$$(1\cdot 1)\cdot\left(\left(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\right)^{*}\cdot 0\right)$$

$$\downarrow c$$

$$\left((1\cdot(a\cdot 1))\cdot\left(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\right)^{*}\right)\cdot 0$$

Interpretation of extracted expression

G'_2



$P(e) = G'_2$

$$\overbrace{(a\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)}^{e}$$

$$\downarrow a$$

$$(1\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)$$

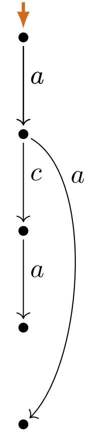
$$\downarrow c$$

$$\big((1\cdot(a\cdot 1))\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\big)\cdot 0$$

$$\big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\big)\cdot 0$$

Interpretation of extracted expression

G'_2



$P(e) = G'_2$

$$\overbrace{(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \cdot 0 \Big)}^{e}$$

$$\downarrow a$$

$$(1 \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \cdot 0 \Big)$$

$$\downarrow c$$

$$\Big((1 \cdot (a \cdot 1)) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \Big) \cdot 0$$

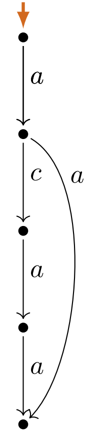
$$\downarrow a$$

$$\Big((1 \cdot 1) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \Big) \cdot 0 \Big)$$

$$\Big((1 \cdot (b \cdot 1 + b \cdot (a \cdot 1))) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \Big) \cdot 0$$

Interpretation of extracted expression

G_2'



$P(e) = G_2'$

$$\overbrace{(a\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)}^{e}$$

$$\downarrow a$$

$$(1\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)$$

$$\downarrow c$$

$$\big((1\cdot(a\cdot 1))\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\big)\cdot 0$$

$$\downarrow a$$

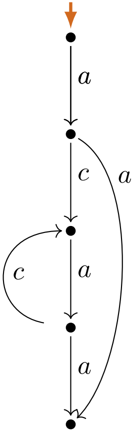
$$\big((1\cdot 1)\cdot(c\cdot(a\cdot 1)+{\color{red}a}\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\big)\cdot 0\big)$$

$$\downarrow a$$

$$\big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\big)\cdot 0$$

Interpretation of extracted expression

G_2'



$$P(e) = G_2'$$

$$\overbrace{(a \cdot 1) \cdot \big((c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)))^{*} \cdot 0\big)}^{e}$$

$$\downarrow a$$

$$(1 \cdot 1) \cdot \big((c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)))^{*} \cdot 0\big)$$

$$\downarrow c$$

$$\big((1 \cdot (a \cdot 1)) \cdot (c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)))^{*}\big) \cdot 0$$

$$\downarrow a$$

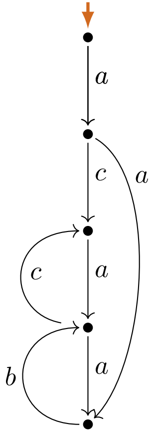
$$\big((1 \cdot 1) \cdot (c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)))^{*}\big) \cdot 0\big)$$

$$\downarrow a$$

$$\big((1 \cdot (b \cdot 1 + b \cdot (a \cdot 1))) \cdot (c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)))^{*}\big) \cdot 0$$

Interpretation of extracted expression

G_2'



$P(e) = G_2'$

$$\overbrace{(a\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)}^{e}$$

$$\downarrow a$$

$$(1\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)$$

$$\downarrow c$$

$$\big((1\cdot(a\cdot 1))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0$$

$$\downarrow a$$

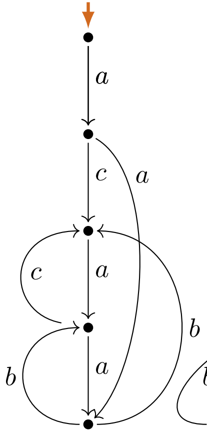
$$\big((1\cdot 1)\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0\big)$$

$$\downarrow a$$

$$\big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0$$

Interpretation of extracted expression

G_2'



$P(e) = G_2'$

$$\overbrace{(a\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)}^{e}$$

$$\downarrow a$$

$$(1\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)$$

$$\downarrow c$$

$$\big((1\cdot(a\cdot 1))\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\big)\cdot 0$$

$$\downarrow a$$

$$\big((1\cdot 1)\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\big)\cdot 0\big)$$

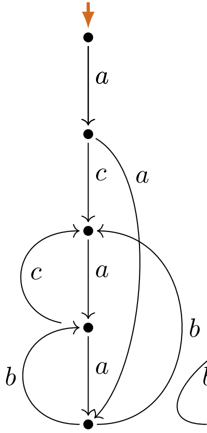
$$\downarrow a$$

$$\big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\big)\cdot 0$$

a c b b

Interpretation of extracted expression

G_2'



$$P(e) = G_2' \mathrel{\underline{\to}} G_2$$

$$\overbrace{(a \cdot 1) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \cdot 0 \big)}^{e}$$

$$\downarrow a$$

$$(1 \cdot 1) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \cdot 0 \big)$$

$$\downarrow c$$

$$\big((1 \cdot (a \cdot 1)) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \big) \cdot 0$$

$$\downarrow a$$

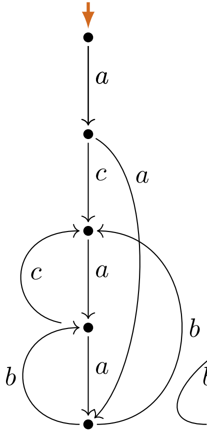
$$\big((1 \cdot 1) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \big) \cdot 0 \big)$$

$$\downarrow a$$

$$\big((1 \cdot (b \cdot 1 + b \cdot (a \cdot 1))) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \big) \cdot 0$$

Interpretation of extracted expression

G'_2



$$P(e) = G'_2 \Rightarrow G_2 \not\simeq G'_2$$

$$\overbrace{(a\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^*\cdot 0\big)}^{e}$$

$$\downarrow a$$

$$(1\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^*\cdot 0\big)$$

$$\downarrow c$$

$$\big((1\cdot(a\cdot 1))\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^*\big)\cdot 0$$

$$\downarrow a$$

$$\big((1\cdot 1)\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^*\big)\cdot 0\big)$$

$$\downarrow a$$

$$\big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^*\big)\cdot 0$$

LEE under bisimulation?

LEE under bisimulation

Observation

- LEE is not invariant under bisimulation.

LEE under bisimulation

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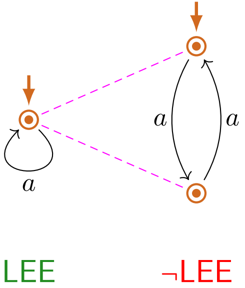
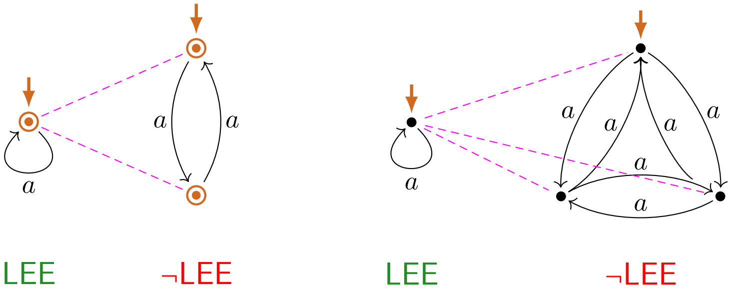
- LEE is not invariant under bisimulation.



LEE under bisimulation

Observation

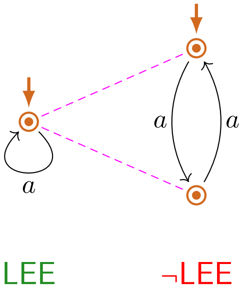
- LEE is not invariant under bisimulation.

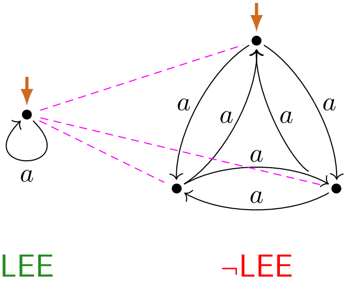


LEE under bisimulation

Observation

- LEE is not invariant under bisimulation.
- LEE is not preserved by converse functional bisimulation.





LEE under functional bisimulation

Lemma

(i) LEE *is preserved by functional bisimulations:*

$$\mathsf{LEE}(G_1) \wedge G_1 \mathrel{\underline{\rightarrow}} G_2 \implies \mathsf{LEE}(G_2)\,.$$

LEE under functional bisimulation

Lemma

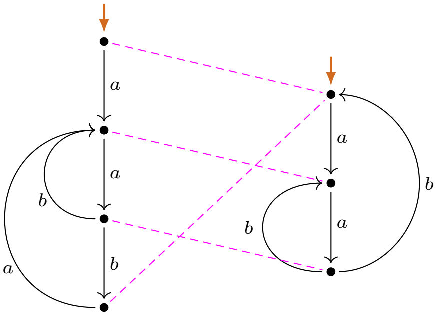
(i) LEE *is preserved by functional bisimulations:*

$$\mathsf{LEE}(G_1) \wedge G_1 \mathrel{\underline{\to}} G_2 \implies \mathsf{LEE}(G_2)\,.$$

Proof (Idea).

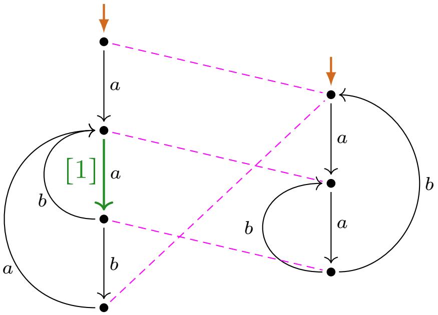
Use loop elimination in G_1 to carry out loop elimination in G_2.

Collapsing LEE-witnesses



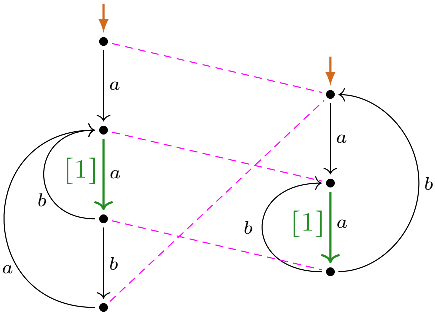
$P(a(a(b+ba))^* \cdot 0)$

Collapsing LEE-witnesses



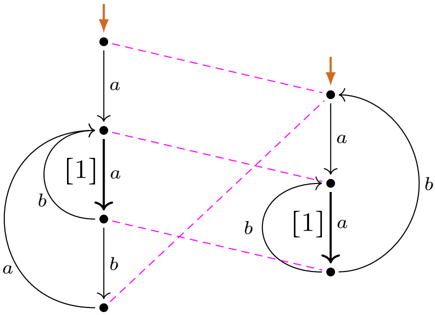
$P(a(a(b+ba))^* \cdot 0)$

Collapsing LEE-witnesses



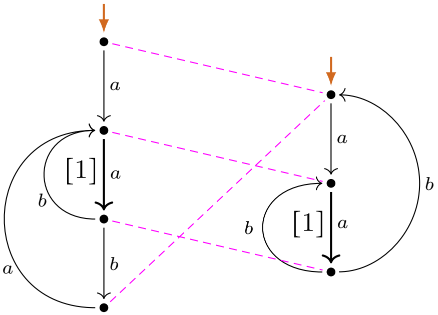
$P(a(a(b+ba))^* \cdot 0)$

Collapsing LEE-witnesses

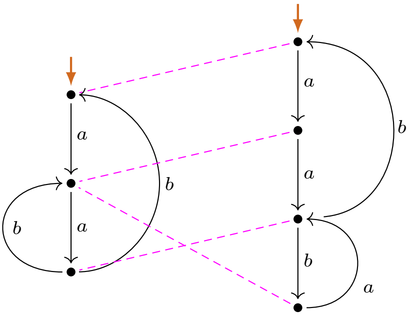


$P(a(a(b+ba))^* \cdot 0)$

Collapsing LEE-witnesses

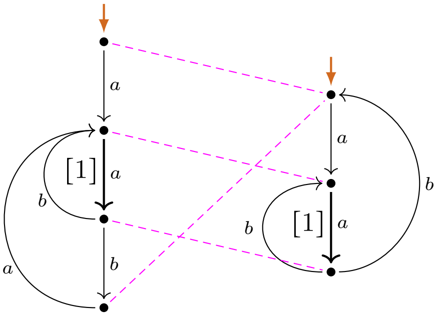


$P(a(a(b+ba))^{*} \cdot 0)$

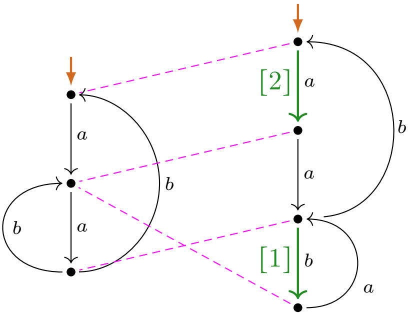


$P((aa(ba)^{*} \cdot b)^{*} \cdot 0)$

Collapsing LEE-witnesses

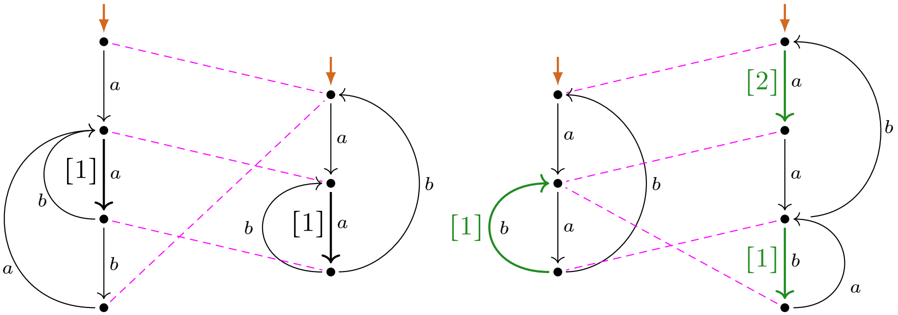


$P(a(a(b+ba))^* \cdot 0)$



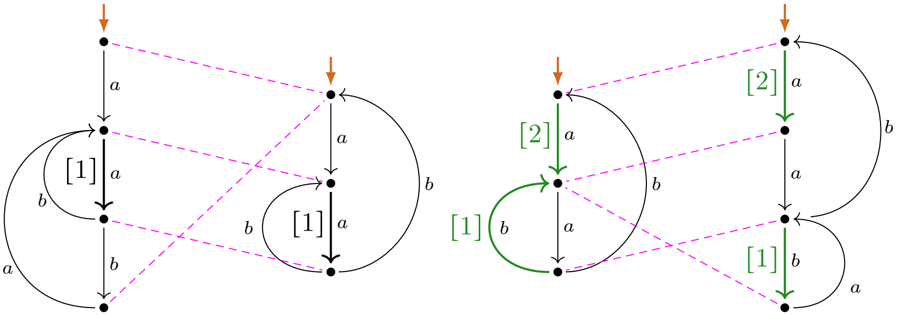
$P((aa(ba)^* \cdot b)^* \cdot 0)$

Collapsing LEE-witnesses



$P(a(a(b+ba))^* \cdot 0)$ $P((aa(ba)^* \cdot b)^* \cdot 0)$

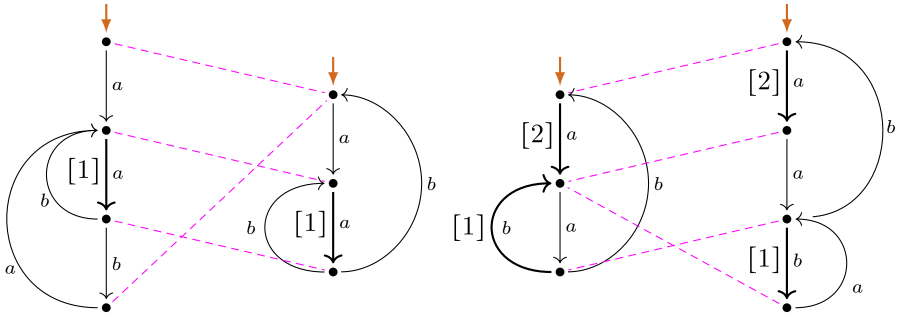
Collapsing LEE-witnesses



$P(a(a(b+ba))^* \cdot 0)$

$P((aa(ba)^* \cdot b)^* \cdot 0)$

Collapsing LEE-witnesses



$P(a(a(b+ba))^* \cdot 0)$

$P((aa(ba)^* \cdot b)^* \cdot 0)$

LEE under functional bisimulation / bisimulation collapse

Lemma

(i) LEE *is preserved by functional bisimulations:*

$$\mathsf{LEE}(G_1) \wedge G_1 \mathrel{\underline{\rightarrow}} G_2 \implies \mathsf{LEE}(G_2)\,.$$

(ii) LEE *is preserved from a process graph to its bisimulation collapse:*

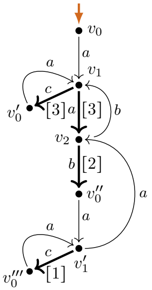
$$\mathsf{LEE}(G) \wedge G \textit{ has bisimulation collapse } C \implies \mathsf{LEE}(C)\,.$$

Idea of Proof for (i)

Use loop elimination in G_1 to carry out loop elimination in G_2.

LLEE-preserving collapse of LLEE-charts (G/Fokkink, LICS'20)

(no 1-transitions!)



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The bisimulation collapse of a LLEE-chart is again a LLEE-chart.

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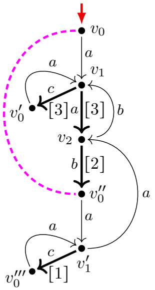

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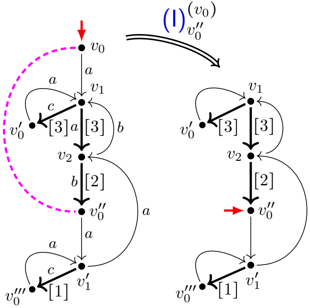

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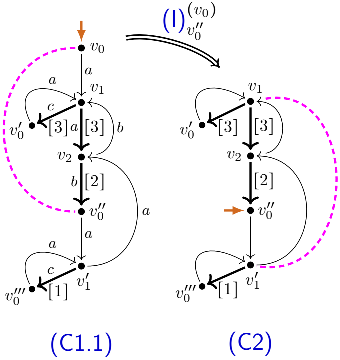

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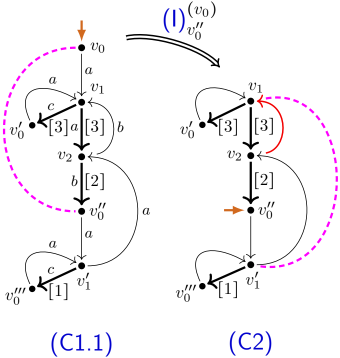

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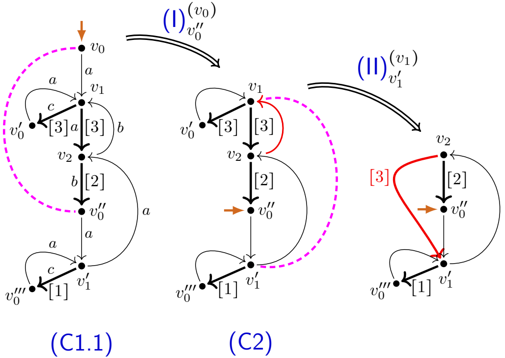

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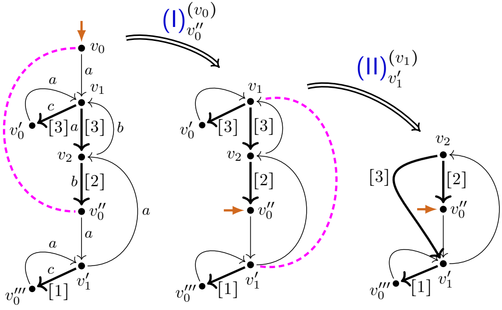

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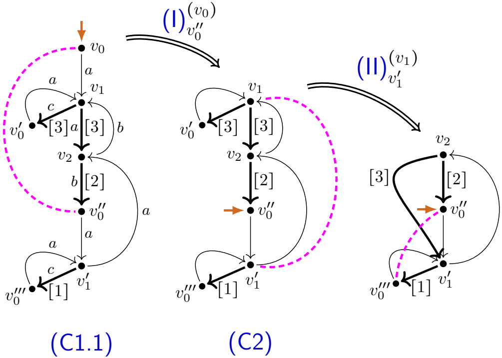

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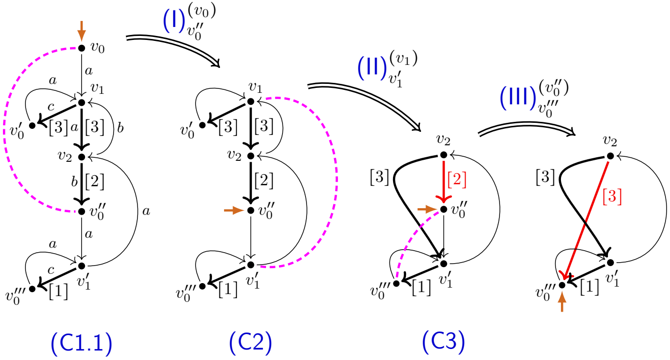

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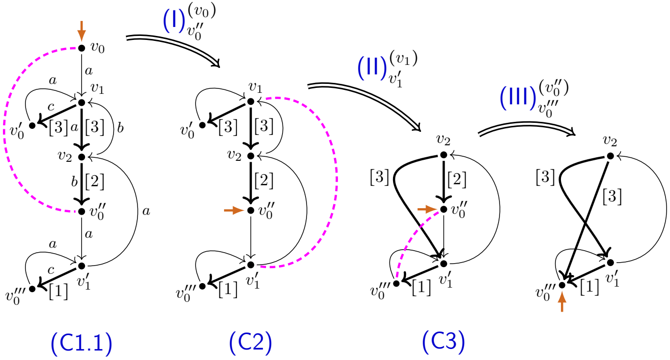

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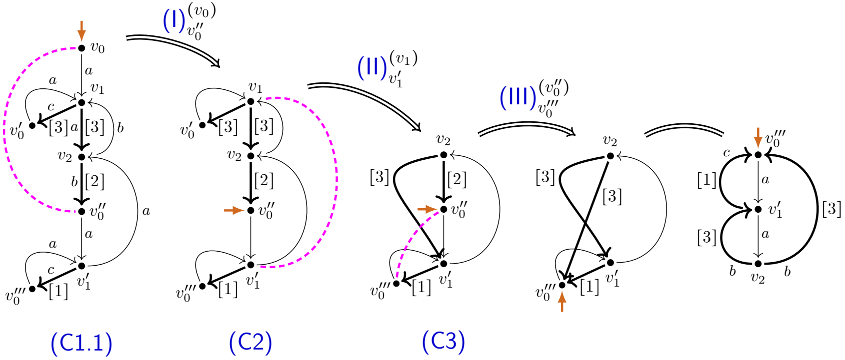

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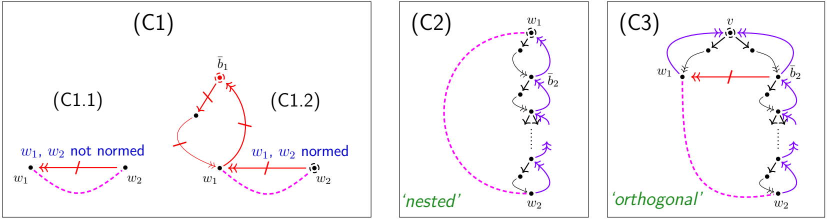
Reduced bisimilarity redundancies in LLEE-graphs (no 1-trans.!)

(G/Fokkink, LICS'20)

w_1, w_2 in different scc's

w_1, w_2 in the same scc



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Every not collapsed LLEE-graph contains bisimilar vertices $w_1 \neq w_2$ of kind (C1), (C2), or (C3) (a reduced bisimilarity redundancy $\langle w_1, w_2 \rangle$):

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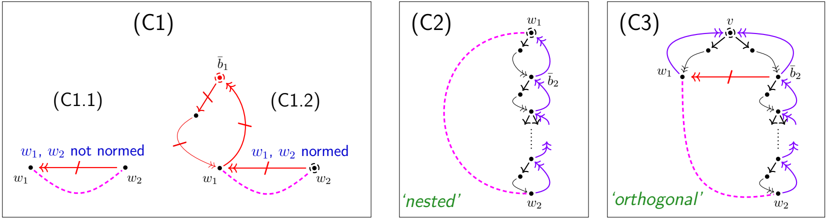

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Every reduced bisimilarity redundancy in a LLEE-graph can be eliminated LLEE-preservingly.

Properties of LEE-charts

Theorem ($\Leftarrow$ G/Fokkink, 2020)

A process graph G
is $[\![\cdot]\!]_P$-expressible by an under-star-1-free regular expression
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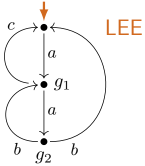
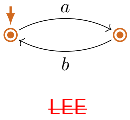
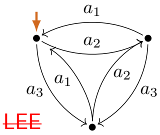
Hence $[\![\cdot]\!]_P$-expressible | **not** $[\![\cdot]\!]_P$-expressible by 1-free regular expressions:





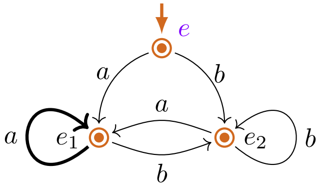


1-LEE

$\hat{=}$ sharing via 1-transitions facilitates LEE

Failure of LEE in general (example)

$P((a^* \cdot b^*)^*)$



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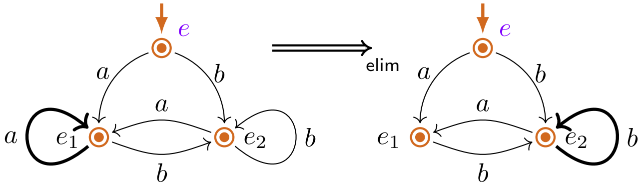

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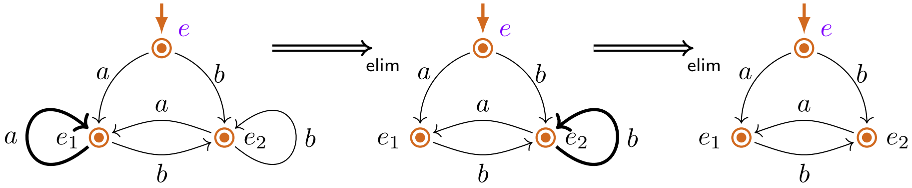

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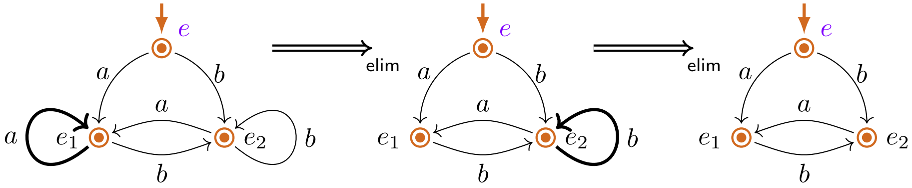

no loop subchart,
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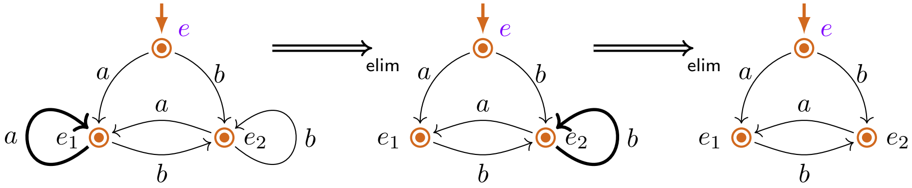

~~LEE~~

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1-Graphs and induced graphs

Definition

$\xrightarrow{1} \cdot \cdots \cdot \xrightarrow{1} \cdot \xrightarrow{a} \quad \widehat{=} \quad \xrightarrow{(a]}$ induced a-transitions, for $a \in A$

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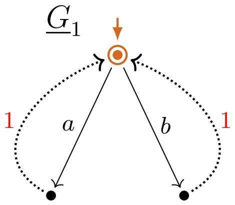

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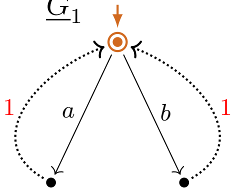
$\underline{G}_1$



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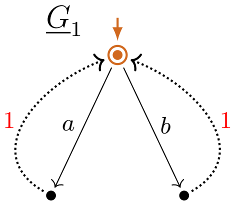

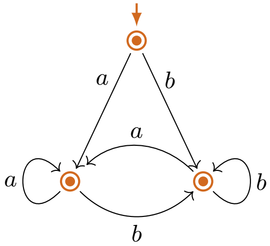

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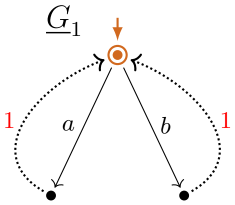

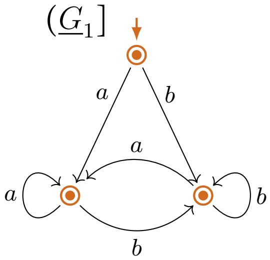

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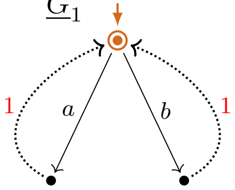

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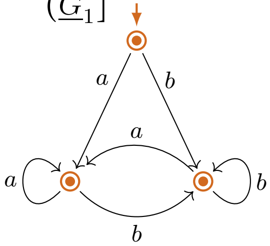

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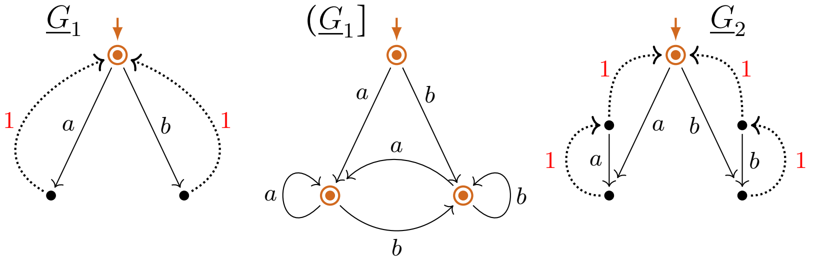

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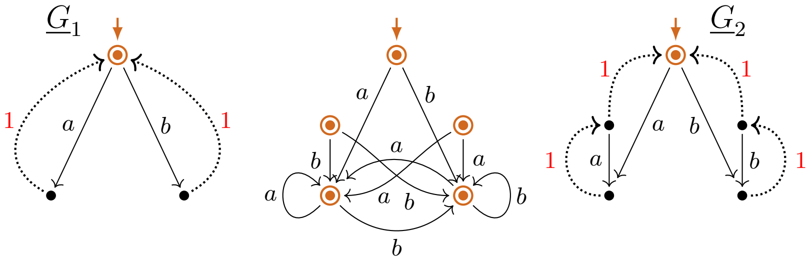

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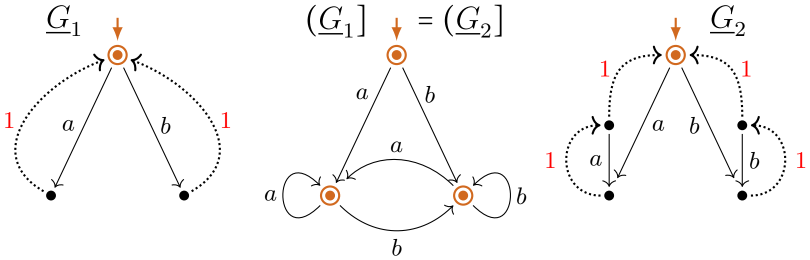

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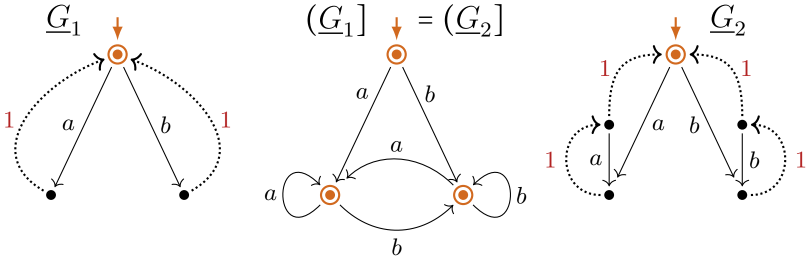

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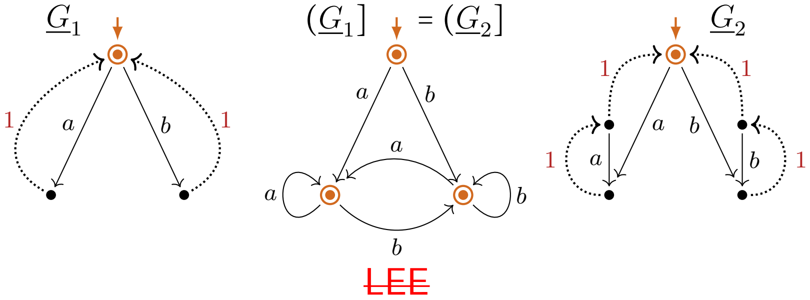

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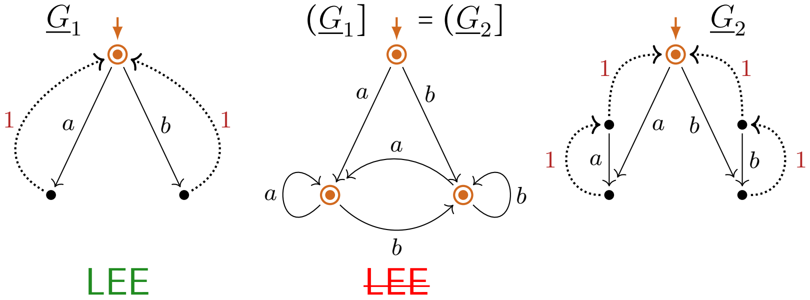

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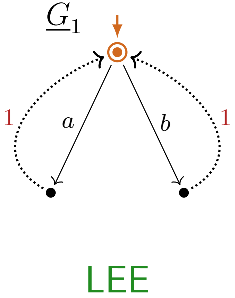

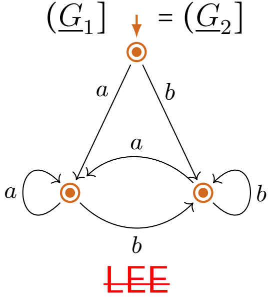

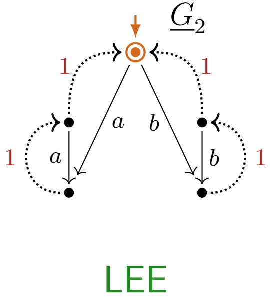

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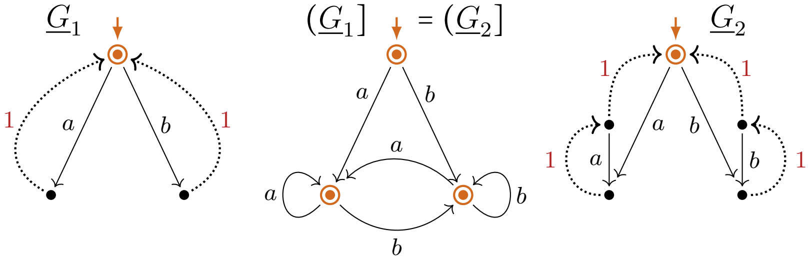
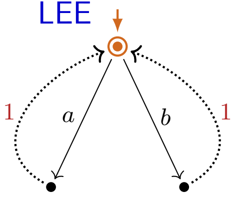
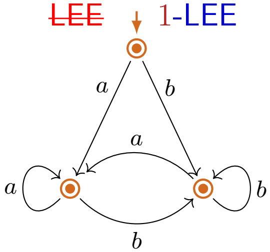


LEE ~~LEE~~ LEE

1-LEE

1-LEE holds for process interpretations





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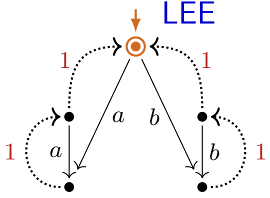

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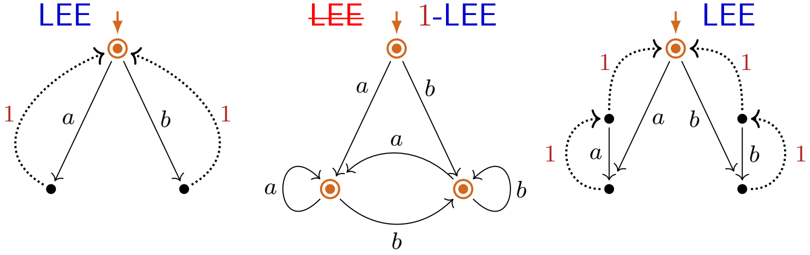

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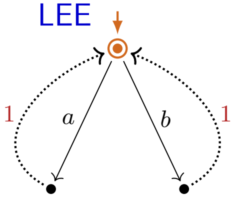
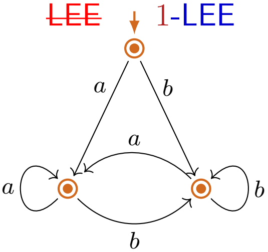
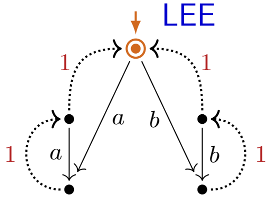






$P((a^* \cdot b^*)^*)$ $\underline{P}((a^* \cdot b^*)^*)$

$P((b^* \cdot a^*)^*)$ $\underline{P}((b^* \cdot a^*)^*)$

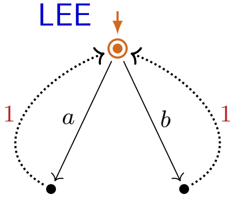
1-LEE holds for process interpretations

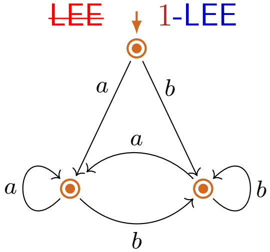
Lemma

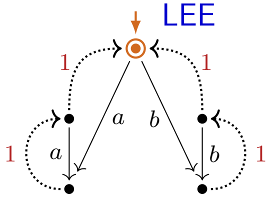
There is a 1-graph interpretation $\underline{P}$ of reg. expression e as 1-graphs $\underline{P}(e)$ such that for all $e \in RExp$: (i): LEE($\underline{P}(e)$), (ii): $(\underline{P}(e)] = P(e)$.

Theorem

1-LEE($P(e)$) holds for all regular expressions e.







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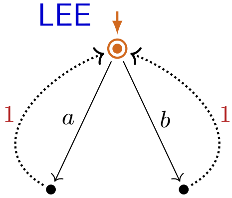

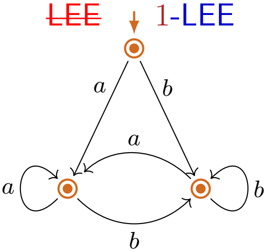

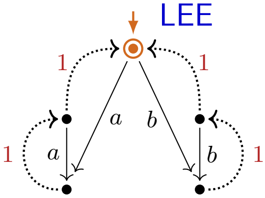

1-LEE($P((a^* \cdot b^*)^*)$) $\underline{P}((a^* \cdot b^*)^*)$

1-LEE($P((b^* \cdot a^*)^*)$) $\underline{P}((b^* \cdot a^*)^*)$

Interpretation/extraction correspondences with 1-LEE

($\Leftarrow$ G 2021/22/23)

(Int)$_P$: *P-expressible graphs have the structural property* 1-LEE

Process **int**erpretations $P(e)$ of regular expressions e are finite process graphs that satisfy 1-LEE.

(Extr)$_P$: 1-LEE *implies* $[\![\cdot]\!]_P$*-expressibility*

From every finite 1-process-graph $\underline{G}$ with 1-LEE a regular expression e can be **extr**acted such that $\underline{G} \leftrightarrow P(e)$.

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~~**(Coll)**~~: 1-LEE *is not preserved under collapse*

The class of finite process graphs with 1-LEE
is not closed under bisimulation **coll**apse.

Interpretation/extraction correspondences of $P^{\bullet}$ with 1-LEE

(Int)$_{P^{\bullet}}$: *$P^{\bullet}$-expressible graphs satisfy* 1-LEE:

Compact process **i**nterpretations $P^{\bullet}(e)$ of regular expressions e
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(Extr)$_{P^{\bullet}}$: LEE *implies* $[\![\cdot]\!]_P$*-expressibility:*

From every finite process graph G with 1-LEE
an regular expression e can be **e**xtracted
such that $G \rightrightarrows P^{\bullet}(e)$.

From every finite collapsed process graph G with 1-LEE
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such that $G \simeq P^{\bullet}(e)$.

Interpretation/extraction correspondences of $P^\bullet$ with 1-LEE

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~~**(ImColl)**$_{P^\bullet}$~~: The image of $P^\bullet$ is not closed under bisimulation **c**ollapse .

1-LEE/ LEE characterize

the un-/restricted image of $P^{\bullet}$

Image of P is **not closed** under bisimulation collapse

not even for $(*/\cancel{1})$ regular expressions

$P(uf)$

$P(uf)$

$$uf := a \cdot \overbrace{(a \cdot (a + a \cdot 0))^*}^{uf_a} + b \cdot \overbrace{(b \cdot (b + b \cdot 0))^*}^{uf_b}$$

Image of P is **not closed** under bisimulation collapse

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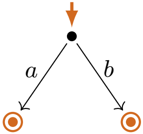

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$$uf \xrightarrow{a} 1 \cdot uf_a \Downarrow \qquad uf \xrightarrow{b} {\Downarrow}\, 1 \cdot uf_b$$

Image of *P* is **not closed** under bisimulation collapse
not even for $(*/\cancel{1})$ regular expressions

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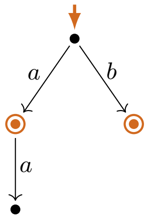

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$$uf \xrightarrow{a} 1 \cdot uf_a \Downarrow \qquad uf \xrightarrow{b} \Downarrow 1 \cdot uf_b$$

$$1 \cdot uf_a \xrightarrow{a} (1 \cdot (a + a \cdot 0)) \cdot uf_a$$

Image of P is **not closed** under bisimulation collapse

not even for $(*/\cancel{1})$ regular expressions

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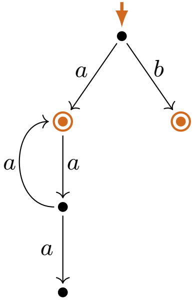

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$$\begin{tikzcd}
 & uf \arrow[dl, "a"'] \arrow[dr, "b"] & \\
1\cdot uf_a \Downarrow \arrow[d, "a"] & & \Downarrow 1\cdot uf_b \\
(1\cdot(a+a\cdot 0))\cdot uf_a \arrow[u, bend left, "a"] \arrow[d, "a"] & & \\
(1\cdot 0)\cdot uf_a & &
\end{tikzcd}$$

Image of P is **not closed** under bisimulation collapse

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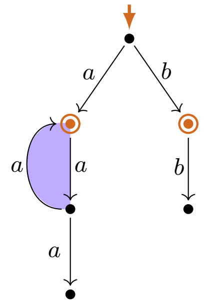

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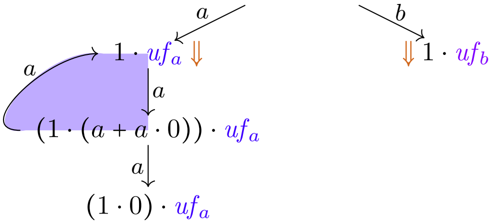

Image of P is **not closed** under bisimulation collapse

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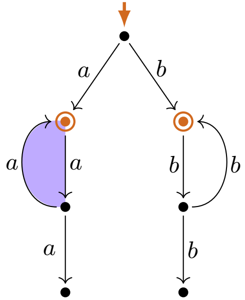

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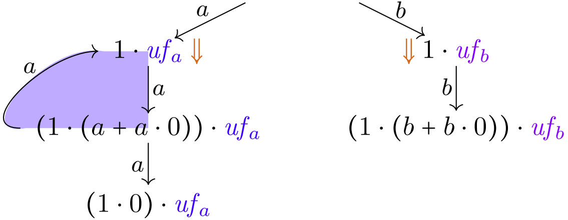

Image of P is **not closed** under bisimulation collapse

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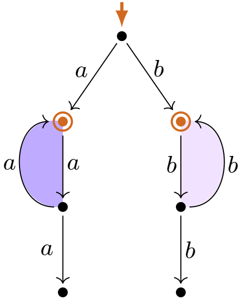
$P(uf)$



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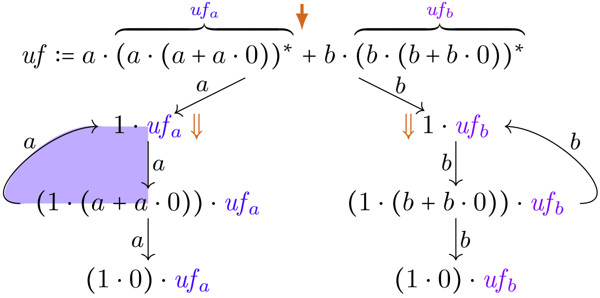

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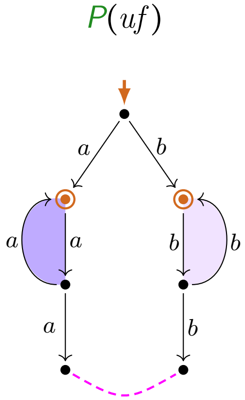

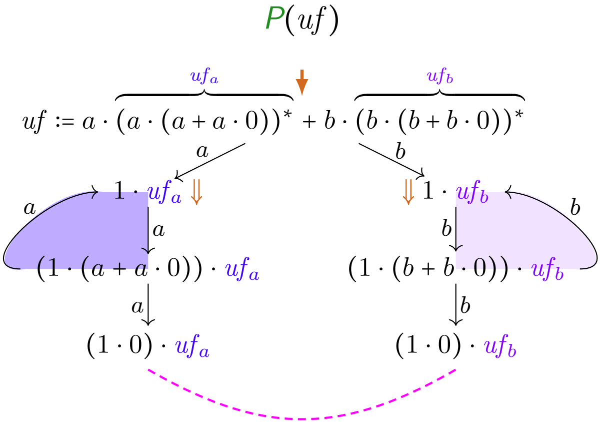

Compact process interpretation $P^{\bullet}$

Definition (Transition system specification $\mathcal{T}$)

$$\frac{}{\mathbf{1}{\Downarrow}} \qquad \frac{e_i{\Downarrow}}{(e_1 + e_2){\Downarrow}}\ (i \in \{1,2\}) \qquad \frac{e_1{\Downarrow} \qquad e_2{\Downarrow}}{(e_1 \cdot e_2){\Downarrow}} \qquad \frac{}{(e^*){\Downarrow}}$$

$$\frac{}{a \xrightarrow{a} \mathbf{1}} \qquad \frac{e_i \xrightarrow{a} e_i'}{e_1 + e_2 \xrightarrow{a} e_i'}\ (i \in \{1,2\})$$

$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2} \qquad \frac{e_1{\Downarrow} \qquad e_2 \xrightarrow{a} e_2'}{e_1 \cdot e_2 \xrightarrow{a} e_2'} \qquad \frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e' \cdot e^*}$$

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Compact process interpretation $P^{\bullet}$

Definition (Transition system specification $\mathcal{T}^{\bullet}$, changed rules w.r.t. $\mathcal{T}$)

$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2} \text{ (if } e_1' \text{ is normed)}$$

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$$\frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e' \cdot e^*} \text{ (if } e' \text{ is normed)} \qquad \frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e'} \text{ (if } e' \text{ is not normed)}$$

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Definition

The compact process (graph) interpretation $P^{\bullet}(e)$ of a reg. expr's e:

$P^{\bullet}(e)$:= labeled transition graph generated by e by derivations in $\mathcal{T}^{\bullet}$.

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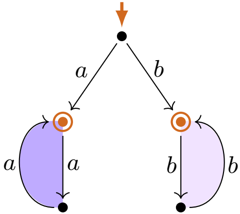
Lemma ($P^{\bullet}$ increases sharing; $P^{\bullet}$, P have same bisimulation semantics)

(i) $P(e) \mathrel{\underline{\to}} P^{\bullet}(e)$ *for all regular expressions* e.

(ii) *(*G *is* $[\![\cdot]\!]_{P^{\bullet}}$*-expressible* $\Longleftrightarrow$ G *is* $[\![\cdot]\!]_{P}$*-expressible) for all graphs* G.

Image of P restricted to $(*/\cancel{1})$ regular expressions ... contains all of its bisimulation collapses

$P^{\bullet}(uf)$



$P^{\bullet}(uf)$

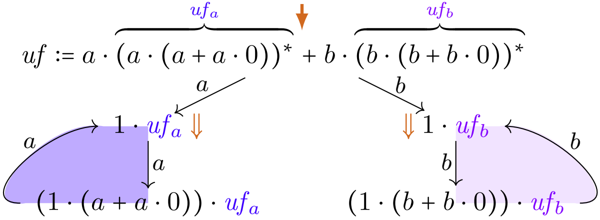


Image of P restricted to $(*/\cancel{1})$ regular expressions
. . . contains all of its bisimulation collapses

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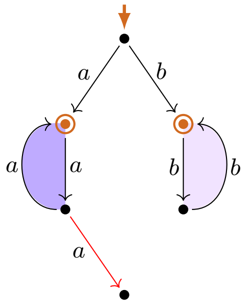

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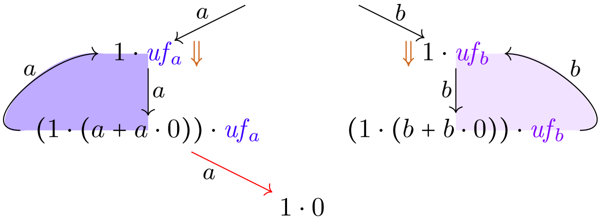
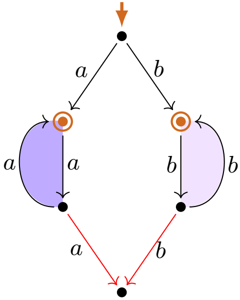
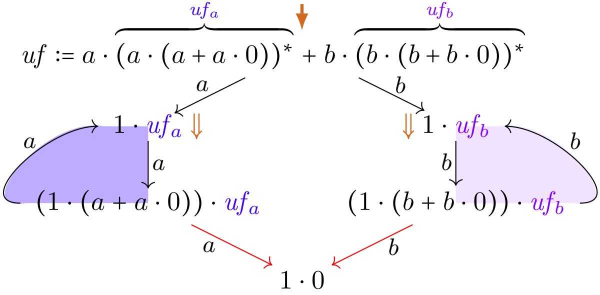


Image of P restricted to $(*/\!\!\!1)$ regular expressions … contains all of its bisimulation collapses

$P^{\bullet}(uf)$



$P^{\bullet}(uf)$



Interpretation correspondence of $P^{\bullet}$ with LEE

(Int)$^{(*/\cancel{1})}_{P^{\bullet}}$: *By under-star-1-free expressions $P^{\bullet}$-expressible graphs satisfy* LEE:

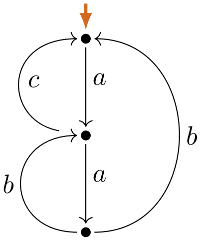
Compact process **i**nterpretations $P^{\bullet}(uf)$
of under-star-1-free regular expressions uf
are finite process graphs that satisfy LEE.

(Extr)$^{(*/\cancel{1})}_{P^{\bullet}}$: LEE *implies* $[\![\cdot]\!]_P$*-expressibility by under-star-1-free reg. expr's*:

From every finite process graph G with LEE
an under-star-1-free regular expression uf can be **e**xtracted
such that $G \mathrel{\underline{\rightarrow}} P(uf)$.

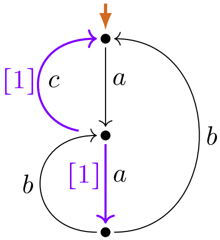
Refined extraction expression (example)

G_4



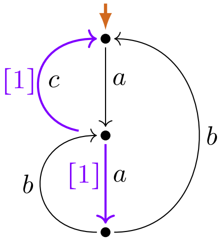
Refined extraction expression (example)

$\widehat{G_4}$



Refined extraction expression (example)

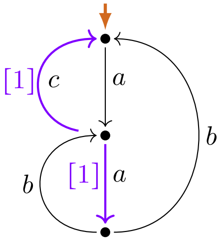
$\widehat{G_4}$



$$(1 \cdot (\qquad\qquad\qquad)^{*}) \cdot 0$$

Refined extraction expression (example)

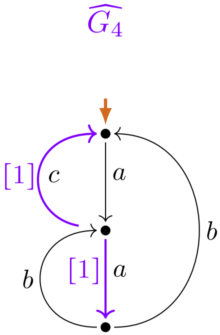
$\widehat{G_4}$



$\overset{a}{\downarrow}$

$(1 \cdot (\qquad\qquad\qquad)^{*}) \cdot 0$

Refined extraction expression (example)



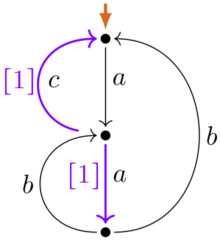
$$((1 \cdot a) \cdot (\qquad\qquad\qquad)^*) \cdot 0$$

$$\downarrow a$$

$$(1 \cdot (\qquad\qquad\qquad)^*) \cdot 0$$

Refined extraction expression (example)

$\widehat{G_4}$



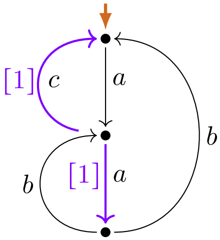
$((1 \cdot a) \cdot (\qquad)^*) \cdot 0$

$\downarrow a$

$(1 \cdot (\qquad)^*) \cdot 0$

Refined extraction expression (example)

$\widehat{G_4}$



$((1 \cdot a) \cdot ($ $\quad\quad\quad\quad$ $)^*) \cdot 0$

a

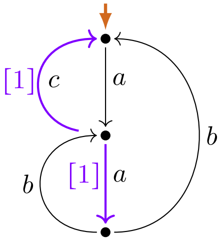
$(1 \cdot ($ $\quad\quad\quad\quad$ $)^*) \cdot 0$

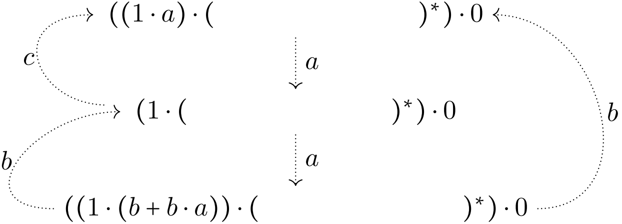
b $\quad\quad\quad\quad$ b

$((1 \cdot (b + b \cdot a)) \cdot ($ $\quad\quad\quad\quad$ $)^*) \cdot 0$

Refined extraction expression (example)

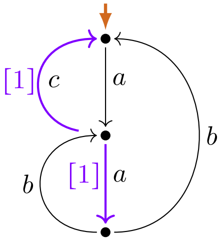
$\widehat{G_4}$





Refined extraction expression (example)

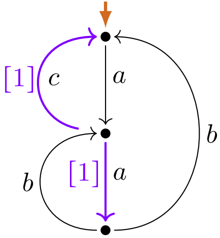
$\widehat{G_4}$



$$
\begin{tikzcd}[column sep=small]
((1\cdot a)\cdot(\qquad\qquad\qquad)^*)\cdot 0 \arrow[d, dotted, "a"] \\
(1\cdot(c\cdot a + a\cdot(b+b\cdot a))^*)\cdot 0 \arrow[d, dotted, "a"] \arrow[u, dotted, bend left, "c"] \\
((1\cdot(b+b\cdot a))\cdot(\qquad\qquad\qquad)^*)\cdot 0 \arrow[u, dotted, bend left, "b"] \arrow[uu, dotted, bend right, "b"']
\end{tikzcd}
$$

Refined extraction expression (example)

$\widehat{G_4}$



$$\begin{tikzcd}
((1 \cdot a) \cdot (c \cdot a + a \cdot (b + b \cdot a))^*) \cdot 0 \arrow[d, dotted, "a"] \\
(1 \cdot (c \cdot a + a \cdot (b + b \cdot a))^*) \cdot 0 \arrow[u, dotted, bend left, "c"] \arrow[d, dotted, "a"] \\
((1 \cdot (b + b \cdot a)) \cdot (c \cdot a + a \cdot (b + b \cdot a))^*) \cdot 0 \arrow[u, dotted, bend left, "b"] \arrow[uu, dotted, bend right, "b"']
\end{tikzcd}$$

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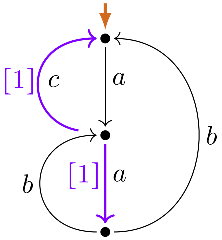

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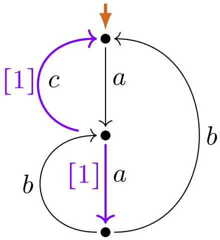

$$\mathsf{P}^{\bullet}(uf) = \mathsf{P}(uf) \simeq G_4$$

$$\begin{tikzcd}
\overbrace{((1 \cdot a) \cdot (c \cdot a + a \cdot (b + b \cdot a))^{*}) \cdot 0}^{uf} \arrow[d, "a"] \\
(1 \cdot (c \cdot a + a \cdot (b + b \cdot a))^{*}) \cdot 0 \arrow[d, "a"] \arrow[u, bend left, "c"] \\
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\end{tikzcd}$$

Interpretation/extraction correspondences of $P^{\bullet}$ with LEE

(Int)$_{P^{\bullet}}^{(*/\cancel{1})}$: *By under-star-1-free expressions $P^{\bullet}$-expressible graphs satisfy LEE:*
Compact process **i**nterpretations $P^{\bullet}(uf)$
of under-star-1-free regular expressions uf
are finite process graphs that satisfy LEE.

(Extr)$_{P^{\bullet}}^{(*/\cancel{1})}$: LEE *implies $[\![\cdot]\!]_P$-expressibility by under-star-1-free reg. expr's:*
From every finite process graph G with LEE
an under-star-1-free regular expression uf can be **e**xtracted
such that $G \rightarrow\!\!\!\!\!\! = P^{\bullet}(uf)$.
From every finite collapsed process graph G with LEE
an under-star-1-free regular expression uf can be **e**xtracted
such that $G \simeq P^{\bullet}(uf)$.

Interpretation/extraction correspondences of $P^{\bullet}$ with LEE

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(Extr)$_{P^{\bullet}}^{(*/\cancel{1})}$: LEE *implies $[\![\cdot]\!]_P$-expressibility by under-star-1-free reg. expr's*:

From every finite process graph G with LEE
an under-star-1-free regular expression uf can be extracted
such that $G \Rightarrow P^{\bullet}(uf)$.

From every finite collapsed process graph G with LEE
an under-star-1-free regular expression uf can be extracted
such that $G \simeq P^{\bullet}(uf)$.

(ImColl)$_{P^{\bullet}}^{(*/\cancel{1})}$: The image of $P^{\bullet}$,
restricted to under-star-1-free regular expressions,
is closed under bisimulation collapse.

LEE $\triangleq$ image of $P^{\bullet}|_{REsp^{(*/\cancel{1})}}$

Theorem

For every process graph G TFAE:

(i) LEE(G).

LEE $\triangleq$ image of $P^{\bullet}|_{RExp^{(*/\cancel{1})}}$

Theorem

For every process graph G TFAE:

- *(i)* LEE(G).
- *(ii)* G *is* $P^{\bullet}$*-expressible by an* $(*/\cancel{1})$ *regular expression* *(i.e.* $G \simeq P^{\bullet}(e)$ *for some* $e \in RExp^{(*/\cancel{1})}$*).*

LEE $\triangleq$ image of $P^{\bullet}|_{RExp^{(*/\cancel{1})}}$

Theorem

For every process graph G TFAE:

(i) LEE(G).

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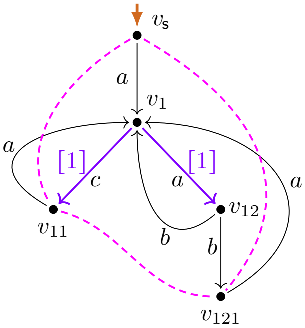
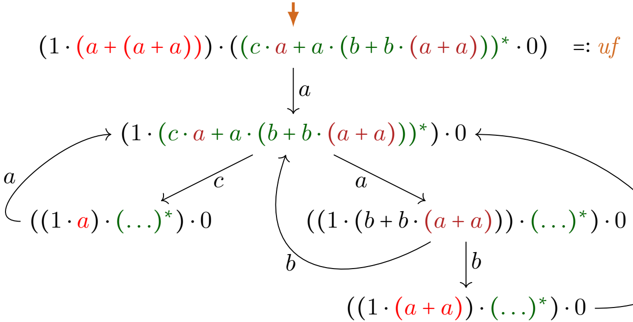
(iii) G *is isomorphic to a graph in the image of* $P^{\bullet}$ *on* $(*/\cancel{1})$ *reg. expr's* *(i.e.* $G \simeq G'$ *for some* $G' \in im(P^{\bullet}|_{RExp^{(*/\cancel{1})}})$*).*

Adapted (refined) extraction from LLEE-graph

$G_1 \,/\, \widehat{G_1}$



$P^{\bullet}(uf) = P(uf) \simeq G_1$



1-LEE $\hat{=}$ image of $P^{\bullet}$

Theorem

For every process graph G TFAE:

(i) 1-LEE(G)
(i.e. $G = (\underline{G}]$ for some 1-transition-process-graph $\underline{G}$ with LEE($\underline{G}$)*).*

1-LEE $\hat{=}$ image of $P^{\bullet}$

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For every process graph G TFAE:

(i) 1-LEE(G)
(i.e. $G = (\underline{G}]$ for some 1-transition-process-graph $\underline{G}$ with LEE($\underline{G}$)).

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1-LEE $\triangleq$ image of $P^{\bullet}$

Theorem

For every process graph G TFAE:

- *(i)* 1-LEE(G)
  (i.e. $G = (\underline{G}]$ for some 1-transition-process-graph $\underline{G}$ with LEE($\underline{G}$)*).*
- *(ii) G is $P^{\bullet}$-expressible by a regular expression*
  (i.e. $G \simeq P^{\bullet}(e)$ for some $e \in RExp$).
- *(iii) G is isomorphic to a graph in the image of $P^{\bullet}$*
  (i.e. $G \simeq G'$ for some $G' \in im(P^{\bullet})$).

Summary

- process interpretation P / semantics $[\![\cdot]\!]_P$ of regular expressions
  - expressibility and completeness questions
- loop existence and elimination (LEE)
  - loop elimination rewrite system can be completed
  - interpretation/extraction correspondences with $(*/\cancel{1})$ reg. expr.s
  - LEE-witnesses: labelings of graphs with LEE
  - stepwise LEE-preserving bisimulation collapse
- 1-LEE = sharing via 1-transitions facilitates LEE
  - interpretation/extraction correspondences with all regular expressions
  - not preserved under bisim. collapse (approximation possible)
- Characterizations of the image of $P^{\bullet}$ (refinement of P):
  - LEE $\triangleq$ image of $P^{\bullet}|_{RExp^{(*/\cancel{1})}} \supsetneqq$ image of $P|_{RExp^{(*/\cancel{1})}}$
  - 1-LEE $\triangleq$ image of $P^{\bullet} \supsetneqq$ image of P
- outlook on work-to-do

My next aims

Completeness problem, solution (journal articles):

A1: graph structure of regular expression processes (LEE/1-LEE)

A2: motivation of crystallization

A4: details of crystallization procedure,
and completeness of Milner's proof system

Expressibility problem

A3: LEE is decidable in polynomial time (conference article).

Q: Is 1-LEE decidable in polynomial time**?**

P: Is expressibility by a regular expression, for a finite process graph,
decidable in polynomial time/fixed-parameter tractable time**?**

Resources

- Slides/abstract on `clegra.github.io`
  - slides: `.../lf/IFIP-1_6-2024.pdf`
  - abstract: `.../lf/abstract-IFIP-1_6-2024.pdf`
- CG: Closing the Image of the Process Interpretation of 1-Free Regular Expressions Under Bisimulation Collapse
  - TERMGRAPH 2024, extended abstract.
- CG: The Image of the Process Interpretation of Regular Expressions is Not Closed under Bisimulation Collapse,
  - arXiv:2303.08553, 2021/2023.
- CG: Milner's Proof System for Regular Expressions Modulo Bisimilarity is Complete,
  - LICS 2022, arXiv:2209.12188, poster.
- CG, Wan Fokkink: A Complete Proof System for 1-Free Regular Expressions Modulo Bisimilarity,
  - LICS 2020, arXiv:2004.12740, video on youtube.
- CG: Modeling Terms by Graphs with Structure Constraints,
  - TERMGRAPH 2018, EPTCS 288, arXiv:1902.02010.

Language semantics $[\![\cdot]\!]_L$ of reg. expr's *(Copi–Elgot–Wright, 1958)*

$\mathbf{0} \xmapsto{L}$ empty language $\varnothing$

$\mathbf{1} \xmapsto{L} \{\epsilon\}$ (ϵ the empty word)

$a \xmapsto{L} \{a\}$

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$e_1 \cdot e_2 \xmapsto{L}$ element-wise concatenation of $L(e_1)$ and $L(e_2)$

$e^* \xmapsto{L}$ set of words formed by concatenating words in $L(e)$, and adding the empty word ϵ

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$[\![e]\!]_L := L(e)$ (language defined by e)