Closing the Image of the Process Interpretation of 1-Free Regular Expressions Under Bisim. Collapse

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TERMGRAPH 2024

Luxembourg

April 7, 2024

Overview

- 1-free regular expressions (with unary/binary star)
- process interpretation / semantics of regular expressions
  - expressible/not expressible process graphs
- loop existence and elimination property (LEE) (G/Fokkink, 2020)
  - interpretation/extraction correspondences with 1-free reg. expr's
  - LEE is preserved under bisimulation collapse
- Q: Image of process interpretation of 1-free regular expressions closed under bisimulation collapse?
  - A1: No. — But . . .
- compact process interpretation
- refined expression extraction
  - A2: compact process interpretation is image-closed under collapse
- outlook: consequences

Regular Expressions

Definition (~ *Copi–Elgot–Wright, 1958*)

Regular expressions over alphabet A with unary Kleene star:

$$e, e_1, e_2 \;::=\; \mathbf{0} \;\mid\; \boldsymbol{a} \;\mid\; e_1 + e_2 \;\mid\; e_1 \cdot e_2 \;\mid\; e^{*} \qquad (\text{for } \boldsymbol{a} \in A).$$

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- symbol $\mathbf{0}$ instead of $\varnothing$, symbol $\mathbf{1}$ instead of $\{\epsilon\}$
- with unary star $*$: $\mathbf{1}$ is definable as $\mathbf{0}^*$

Regular Expressions

Definition (~ *Kleene, 1951*, ~ *Copi–Elgot–Wright, 1958*)

Regular expressions over alphabet A with unary / binary Kleene star:

$$e, e_1, e_2 \;::=\; \mathbf{0} \mid \phantom{\mathbf{1} \mid} \boldsymbol{a} \mid e_1 + e_2 \mid e_1 \cdot e_2 \mid e^{*} \qquad (\text{for } \boldsymbol{a} \in A).$$

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- with binary star $^{\circledast}$: $\mathbf{1}$ is not definable (in its absence)

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Definition

1-free regular expressions over alphabet A with binary Kleene star:

$$f, f_1, f_2 ::= \mathbf{0} \mid \boldsymbol{a} \mid f_1 + f_2 \mid f_1 \cdot f_2 \mid f_1{}^{\circledast} f_2 \qquad (\text{for } \boldsymbol{a} \in A).$$

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Under-Star-/1-Free regular expressions

Definition

The set $\mathit{RExp}^{(\not{1})}(A)$ of 1-free regular expressions over A is defined by:

$$f, f_1, f_2 ::= 0 \mid a \mid f_1 + f_2 \mid f_1 \cdot f_2 \mid {f_1}^{\star} \cdot f_2 \qquad \text{(for } a \in A\text{)},$$

the set $\mathit{RExp}^{(\not{1}\backslash *)}(A)$ of under-star-1-free regular expressions over A by:

$$uf, uf_1, uf_2 ::= 0 \mid 1 \mid a \mid uf_1 + uf_2 \mid uf_1 \cdot uf_2 \mid f^{\star} \qquad \text{(for } a \in A\text{)}.$$

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Under the language interpretation, subclasses of minor relevance:

- 1-free regular expressions denote all regular languages without ϵ.
- Under-star-1-free regular expressions denote all regular languages.

Process interpretation $P(\cdot)$ of regular expressions *(Milner, 1984)*

$\mathbf{0} \xmapsto{P}$ deadlock $\boldsymbol{\delta}$, no termination

$\mathbf{1} \xmapsto{P}$ empty-step process $\boldsymbol{\epsilon}$, then terminate

$a \xmapsto{P}$ atomic action a, then terminate

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$e_1 + e_2 \xmapsto{P}$ (*choice*) execute $P(e_1)$ or $P(e_2)$

$e_1 \cdot e_2 \xmapsto{P}$ (*sequentialization*) execute $P(e_1)$, then $P(e_2)$

$e^{*} \xmapsto{P}$ (*iteration*) repeat (terminate or execute $P(e)$)

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$e^* \xrightarrow{P}$ (*iteration*) repeat (terminate or execute $P(e)$)

$e_1 {}^{\circledast} e_2 \xrightarrow{P}$ (*iteration-exit*) repeat (terminate or execute $P(e_1)$), then $P(e_2)$

Process semantics $[\![\cdot]\!]_P$ of regular expressions *(Milner, 1984)*

$$\mathbf{0} \stackrel{P}{\longmapsto} \text{deadlock } \boldsymbol{\delta}\text{, no termination}$$

$$\mathbf{1} \stackrel{P}{\longmapsto} \text{empty-step process } \boldsymbol{\epsilon}\text{, then terminate}$$

$$a \stackrel{P}{\longmapsto} \text{atomic action } a\text{, then terminate}$$

$$e_1 + e_2 \stackrel{P}{\longmapsto} (\textit{choice}) \text{ execute } P(e_1) \text{ or } P(e_2)$$

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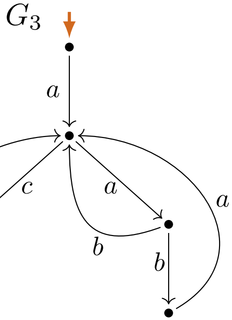
$$e^* \stackrel{P}{\longmapsto} (\textit{iteration}) \text{ repeat (terminate or execute } P(e))$$

$$e_1{}^{\circledast} e_2 \stackrel{P}{\longmapsto} (\textit{iteration-exit}) \text{ repeat (terminate or execute } P(e_1)\text{)}, \text{ then } P(e_2)$$

$$[\![e]\!]_P := [P(e)]_{\underline{\leftrightarrow}} \quad (\text{bisimilarity equivalence class of process } P(e))$$

P-expressibility and $[\![\cdot]\!]_P$-expressibility (examples)



$$P\big(\overbrace{(a \cdot (c \cdot a + a \cdot (b + b \cdot a))^*) \cdot 0}^{f}\big)$$

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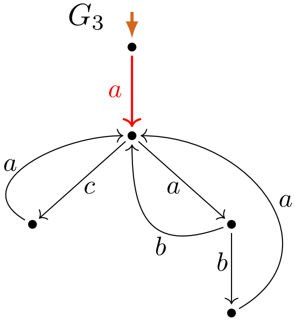

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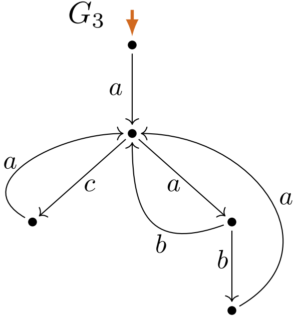

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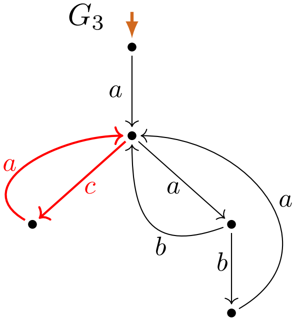

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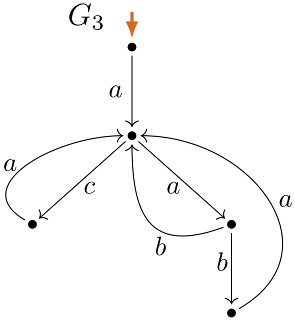

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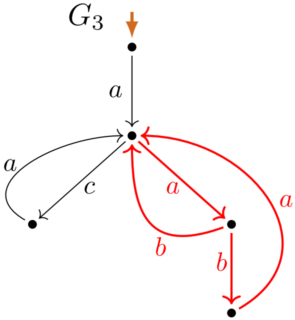

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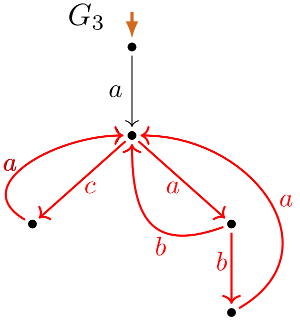

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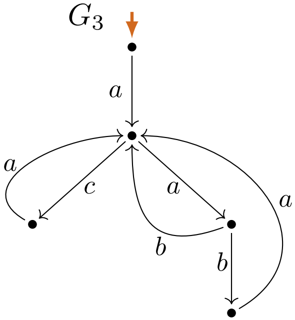

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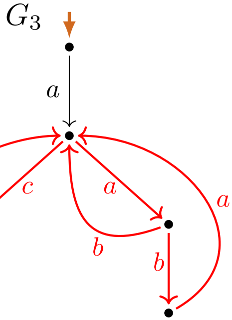

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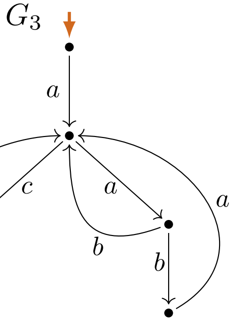

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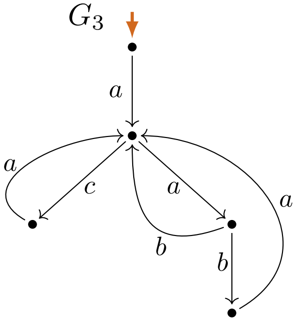

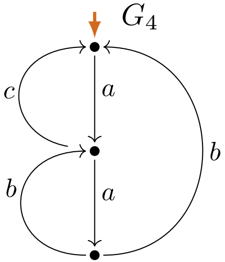

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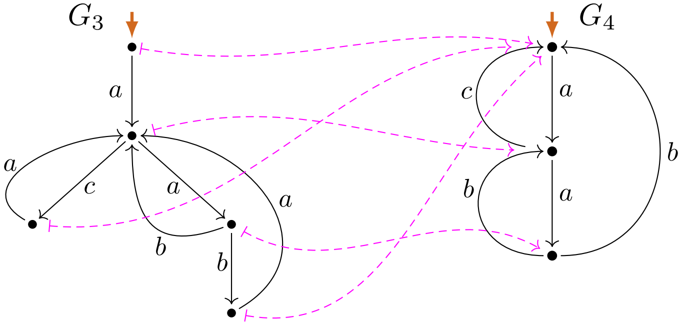

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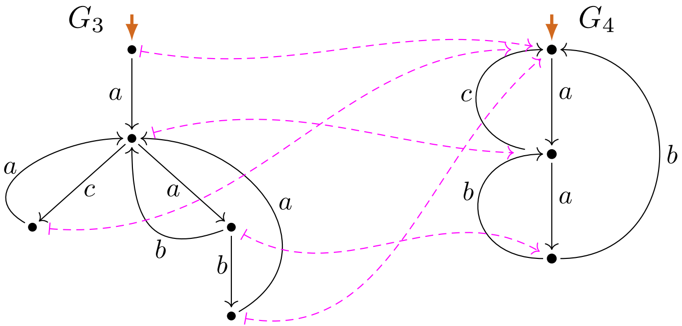

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$$G_3 \in [\![f]\!]_P \qquad G_4 \in [\![f]\!]_P$$

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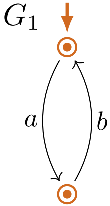

not P-expressible

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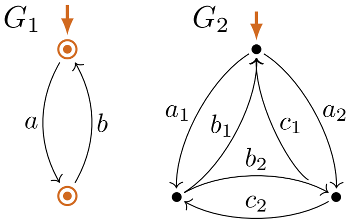

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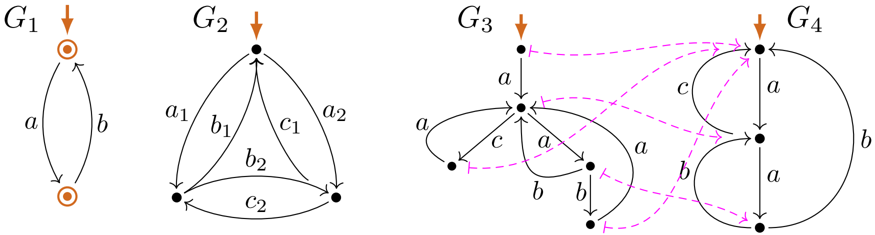

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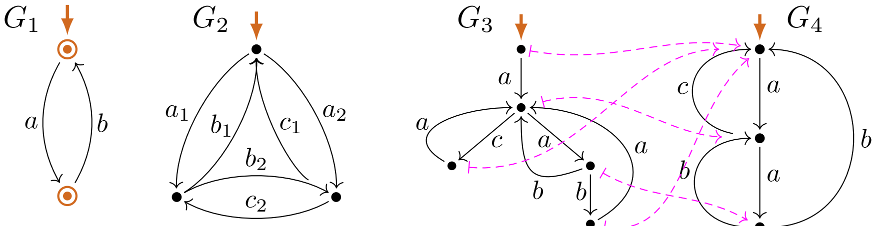

not P-expressible | P-expressible | **?**

not $[\![\cdot]\!]_P$-expressible | $[\![\cdot]\!]_P$-expressible | $[\![\cdot]\!]_P$-expressible

Process interpretation P (formal definition)

Definition (Transition system specification $\mathcal{T}$)

$$\frac{}{a \xrightarrow{a} 1} \qquad \frac{e_i \xrightarrow{a} e_i'}{e_1 + e_2 \xrightarrow{a} e_i'} \; (i \in \{1,2\})$$

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Definition

The process (graph) interpretation $P(e)$ of a regular expression e:

$P(e)$:= labeled transition graph generated by e by derivations in $\mathcal{T}$.

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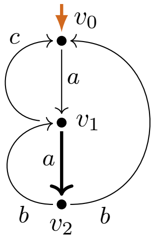
Loop existence and elimination



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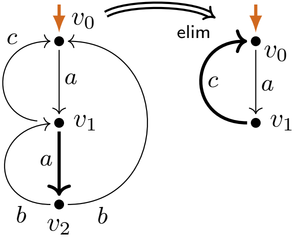

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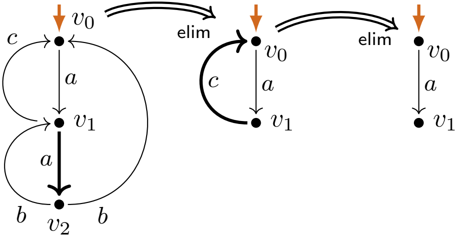

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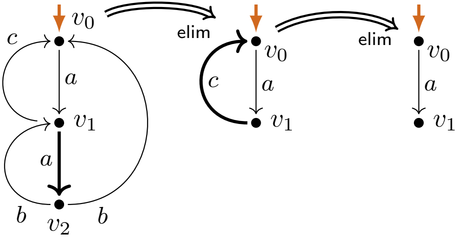

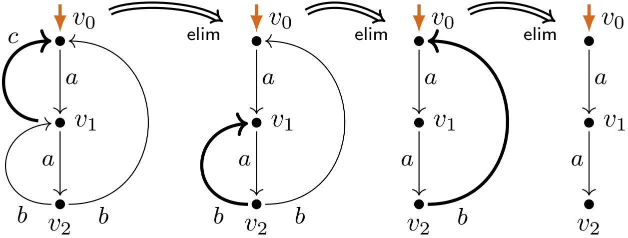

LEE

Definition

A chart $\mathcal{C}$ satisfies LEE (*loop existence and elimination*) if:

$$\exists \mathcal{C}_0 \left(\mathcal{C} \longrightarrow^{*}_{\text{elim}} \mathcal{C}_0 \not\longrightarrow_{\text{elim}} \right.$$
$$\left. \wedge\ \mathcal{C}_0 \text{ permits no infinite path} \right).$$

LEE

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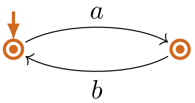
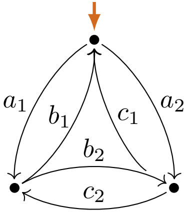
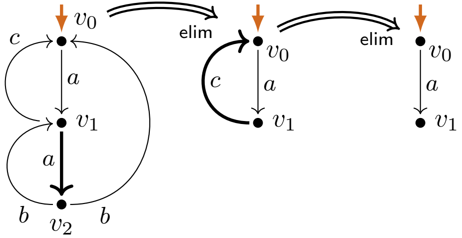


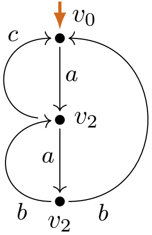


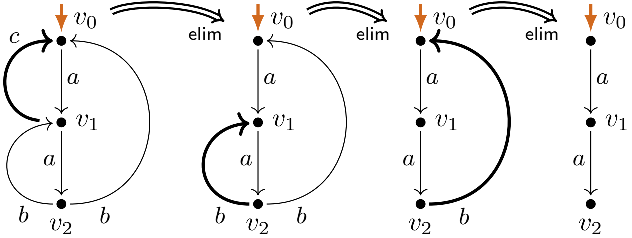
~~LEE~~ ~~LEE~~

LEE

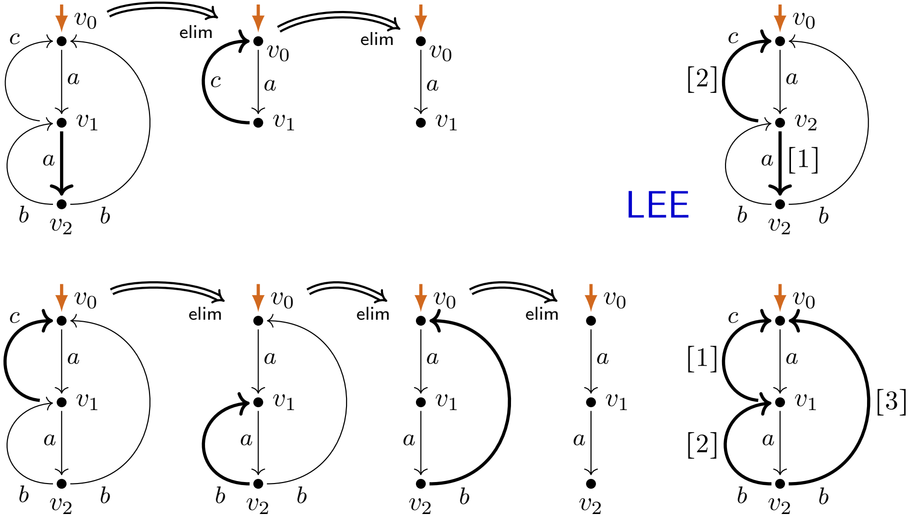


LEE

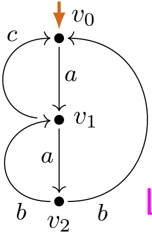




LEE

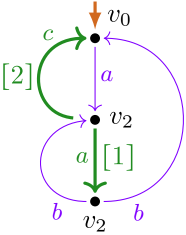


LEE witness and LEE-charts

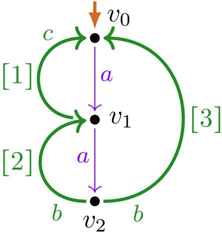


LEE-chart

LEE-witness



LEE-witness



Properties of LEE-charts

Theorem ($\Leftarrow$ G/Fokkink, 2020)

A process graph G

is $[\![\cdot]\!]_P$-expressible by an under-star-1-free regular expression

(i.e. P-expressible modulo bisimilarity by an $(\cancel{1}\backslash *)$ reg. expr.)

if and only if

the bisimulation collapse of G satisfies LEE.

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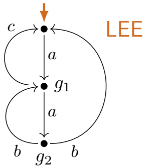
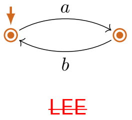
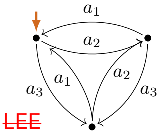
Hence $[\![\cdot]\!]_P$-expressible | **not** $[\![\cdot]\!]_P$-expressible by 1-free regular expressions:







Interpretation/extraction correspondences with LEE

($\Leftarrow$ G/Fokkink 2020, G 2021)

$\textbf{(Int)}_{P}^{(\cancel{1}\backslash *)}$: $P^{\bullet}$-$(\cancel{1}\backslash *)$-*expressible graphs have structural property* LEE

Process **i**nterpretations $P(e)$
of under-star-1-free regular expressions e
are finite process graphs that satisfy LEE.

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(Extr)$_{P}$: LEE *implies* $[\![\cdot]\!]_{P}$*-expressibility*

From every finite process graph G with LEE
a regular expression e can be **e**xtracted
such that $G \mathrel{\underline{\leftrightarrow}} P(e)$.

Interpretation/extraction correspondences with LEE

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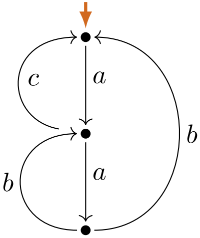
(Coll): LEE *is preserved under collapse*

The class of finite process graphs with LEE
is closed under bisimulation **c**ollapse.

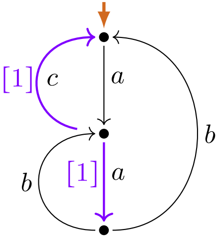
Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

G_4



Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

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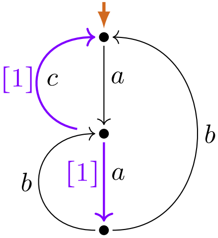
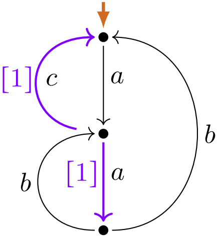


$$\left(\qquad\qquad\qquad\qquad \right)^* \cdot 0$$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

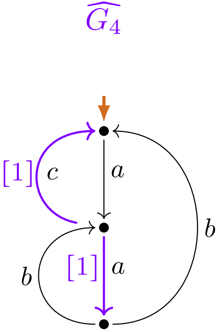
$\widehat{G_4}$



a

$$(\qquad)^* \cdot 0$$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)



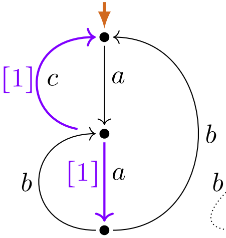
$(a \cdot 1) \cdot ((\qquad)^* \cdot 0)$

$\downarrow a$

$(\qquad)^* \cdot 0$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$(a \cdot 1) \cdot ((\qquad)^* \cdot 0)$

a

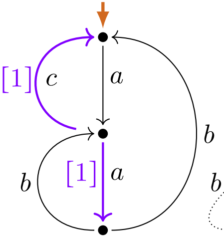
$(\qquad)^* \cdot 0$

b

b

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$(a \cdot 1) \cdot ((\qquad)^* \cdot 0)$

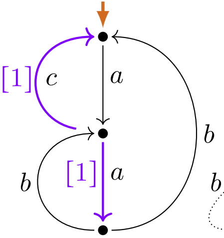
$\downarrow a$

$b \qquad (\qquad)^* \cdot 0 \qquad b$

$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot ((\qquad)^*) \cdot 0)$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$(a \cdot 1) \cdot ((\qquad)^* \cdot 0)$

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Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

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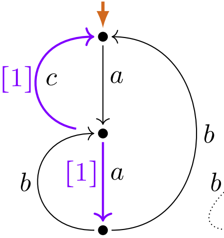


$$(a\cdot 1)\cdot\big(\big(\qquad\qquad\big)^{*}\cdot 0\big)$$

$$\big\downarrow a$$

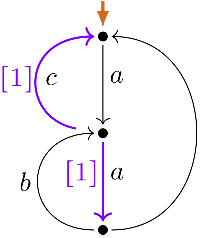
$$\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0$$

$$\big\downarrow a$$

$$(b\cdot 1+b\cdot(a\cdot 1))\cdot\big(\big(\qquad\qquad\big)^{*}\big)\cdot 0\big)$$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$$(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \cdot 0 \Big)$$

$$\Big\downarrow a$$

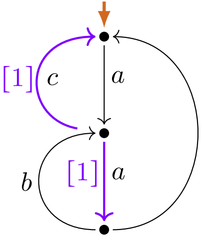
$$\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \cdot 0$$

$$\Big\downarrow a$$

$$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^*) \cdot 0 \Big)$$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$$\overbrace{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)}^{e}$$

$$\Big\downarrow a$$

$$\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0 \xrightarrow{c} e$$

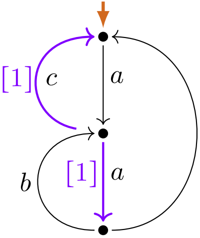
$$\Big\downarrow a$$

$$(b\cdot 1+b\cdot(a\cdot 1))\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0\Big)$$

$$\xrightarrow{b} \big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0 \qquad \xrightarrow{b} e$$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$

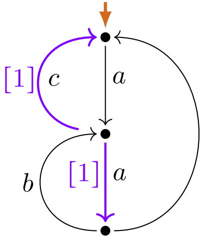


$P(e) \;\underline{\to}\; G_4$

$$\begin{tikzcd}
\overbrace{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)}^{e} \arrow[d, dotted, "a"] \\
\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0 \arrow[u, dotted, bend left, "c"] \arrow[d, dotted, "a"] \\
(b\cdot 1+b\cdot(a\cdot 1))\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*})\cdot 0\Big) \arrow[u, dotted, bend left, "b"] \arrow[uu, dotted, bend right, "b"']
\end{tikzcd}$$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$P(e) \succeq G_4$

$$\overbrace{(a\cdot 1)\cdot\big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\big)}^{e}$$

$$\downarrow a$$

$$\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0$$

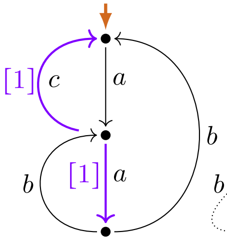
$$\downarrow a$$

$$(b\cdot 1+b\cdot(a\cdot 1))\cdot\big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0\big)$$

c b b

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$P(e) \mathrel{\underline{\to}} G_4$

$$\overbrace{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)}^{e}$$

$$\Big\downarrow a$$

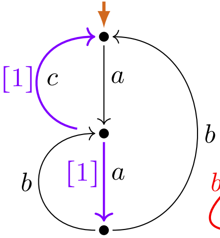
$$\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0$$

$$\Big\downarrow a$$

$$(b\cdot 1+b\cdot(a\cdot 1))\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0\Big)$$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$$P(e) \mathrel{\underline{\to}} G_4$$

$$\overbrace{(a\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)}^{e}$$

$$\downarrow a$$

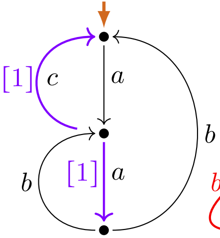
$$\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0$$

$$\downarrow a$$

$$(b\cdot 1+b\cdot(a\cdot 1))\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*})\cdot 0\big)$$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$$P(e) \;\underline{\rightarrow}\; G_4 \;\not\underline{\leftarrow}\; P(e)$$

$$\overbrace{(a\cdot 1)\cdot\big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\big)}^{e}$$

$$\downarrow a$$

$$\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0$$

$$\downarrow a$$

$$(b\cdot 1+b\cdot(a\cdot 1))\cdot\big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*})\cdot 0\big)$$

Interpretation of extracted expression

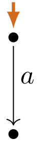
G_5

$$P(e) = G_5$$

$$\overbrace{(a \cdot 1) \cdot \left(\left(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \right)^{*} \cdot 0 \right)}^{e}$$

Interpretation of extracted expression

G_5 $\qquad\qquad$ $P(e) = G_5$



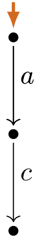
$$\overbrace{(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0 \Big)}^{e}$$

$$\big\downarrow a$$

$$(1 \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0 \Big)$$

Interpretation of extracted expression

G_5



$$P(e) = G_5$$

$$\overbrace{(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0 \Big)}^{e}$$

$$\Big\downarrow a$$

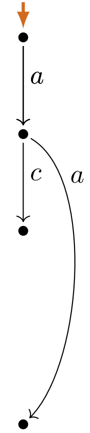
$$(1 \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0 \Big)$$

$$\Big\downarrow c$$

$$\Big((1 \cdot (a \cdot 1)) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \Big) \cdot 0$$

Interpretation of extracted expression

G_5



$P(e) = G_5$

$$\overbrace{(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0\Big)}^{e}$$

$$\downarrow a$$

$$(1 \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0\Big) \xrightarrow{a}$$

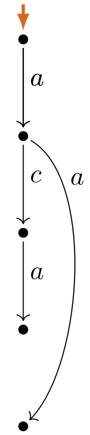
$$\downarrow c$$

$$\Big((1 \cdot (a \cdot 1)) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^*\Big) \cdot 0$$

$$\Big((1 \cdot (b \cdot 1 + b \cdot (a \cdot 1))) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^*\Big) \cdot 0$$

Interpretation of extracted expression

G_5



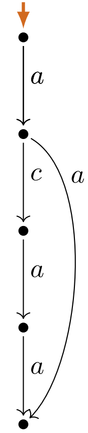
$P(e) = G_5$

$$
\begin{array}{c}
\overbrace{(a\cdot 1)\cdot\big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\big)}^{e} \\
\big\downarrow a \\
(1\cdot 1)\cdot\big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\big) \\
\big\downarrow c \\
\big((1\cdot(a\cdot 1))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0 \\
\big\downarrow a \\
\big((1\cdot 1)\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0\big)
\end{array}
$$

$$
(1\cdot 1)\cdot\big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\big) \xrightarrow{a} \big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0
$$

Interpretation of extracted expression

G_5



$P(e) = G_5$

$$\overbrace{(a\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)}^{e}$$

$$\downarrow a$$

$$(1\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)$$

$$\downarrow c$$

$$\big((1\cdot(a\cdot 1))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0$$

$$\downarrow a$$

$$\big((1\cdot 1)\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0\big)$$

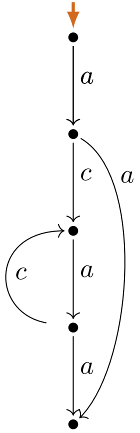
$$\downarrow a$$

$$\big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0$$

a

Interpretation of extracted expression

G_5



$P(e) = G_5$

$$\overbrace{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)}^{e}$$

$$\downarrow a$$

$$(1\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)$$

$$\downarrow c$$

$$\Big((1\cdot(a\cdot 1))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0$$

$$\downarrow a$$

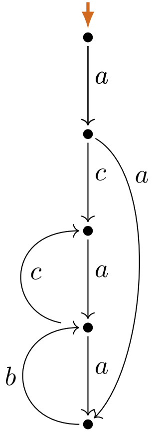
$$\Big((1\cdot 1)\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0)$$

$$\downarrow a$$

$$\Big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0$$

Interpretation of extracted expression

G_5



$P(e) = G_5$

$$\overbrace{(a \cdot 1) \cdot \left(\left(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \right)^{*} \cdot 0 \right)}^{e}$$

$$\downarrow a$$

$$(1 \cdot 1) \cdot \left(\left(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \right)^{*} \cdot 0 \right)$$

$$\downarrow c$$

$$\left((1 \cdot (a \cdot 1)) \cdot \left(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \right)^{*} \right) \cdot 0$$

$$\downarrow a$$

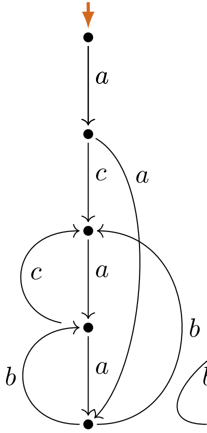
$$\left((1 \cdot 1) \cdot \left(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \right)^{*} \right) \cdot 0)$$

$$\downarrow a$$

$$\left((1 \cdot (b \cdot 1 + b \cdot (a \cdot 1))) \cdot \left(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \right)^{*} \right) \cdot 0$$

Interpretation of extracted expression

G_5 $\qquad\qquad$ $P(e) = G_5$



$$\overbrace{(a\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)}^{e}$$

$$\downarrow a$$

$$(1\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)$$

$$\downarrow c$$

$$\big((1\cdot(a\cdot 1))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0$$

$$\downarrow a$$

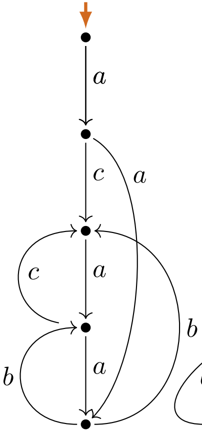
$$\big((1\cdot 1)\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0\big)$$

$$\downarrow a$$

$$\big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0$$

Interpretation of extracted expression

G_5



$$P(e) = G_5 \rightleftharpoons G_4$$

$$\overbrace{(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0\Big)}^{e}$$

$$\downarrow a$$

$$(1 \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0\Big)$$

$$\downarrow c$$

$$\Big((1 \cdot (a \cdot 1)) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^*\Big) \cdot 0$$

$$\downarrow a$$

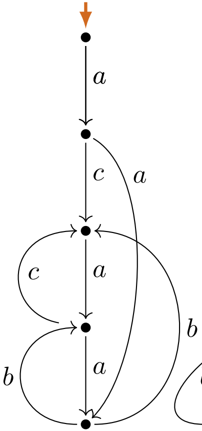
$$\Big((1 \cdot 1) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^*\Big) \cdot 0\Big)$$

$$\downarrow a$$

$$\Big((1 \cdot (b \cdot 1 + b \cdot (a \cdot 1))) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^*\Big) \cdot 0$$

Interpretation of extracted expression

G_5



$$P(e) = G_5 \Rightarrow G_4 \not\simeq G_5$$

$$\overbrace{(a\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)}^{e}$$

$$\big\downarrow a$$

$$(1\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)$$

$$\big\downarrow c$$

$$\big((1\cdot(a\cdot 1))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0$$

$$\big\downarrow a$$

$$\big((1\cdot 1)\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0\big)$$

$$\big\downarrow a$$

$$\big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0$$

Overview

- 1-free regular expressions (with unary/binary star)
- process interpretation/semantics of regular expressions
  - expressible/not expressible process graphs
- loop existence and elimination property (LEE) (G/Fokkink, 2020)
  - interpretation/extraction correspondences with 1-free reg. expr's
  - LEE is preserved under bisimulation collapse
- Q: Image of process interpretation of 1-free regular expressions closed under bisimulation collapse?
  - A1: No. — But ...
- compact process interpretation
- refined expression extraction
  - A2: compact process interpretation is image-closed under collapse
- outlook: consequences

Image of P is **not** closed under bisimulation collapse

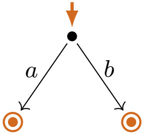
$P(uf)$

$P(uf)$

$$uf := a \cdot \overbrace{(a \cdot (a + a \cdot 0))^*}^{uf_a} + b \cdot \overbrace{(b \cdot (b + b \cdot 0))^*}^{uf_b}$$

Image of *P* is **not** closed under bisimulation collapse

$P(uf)$



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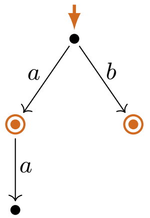

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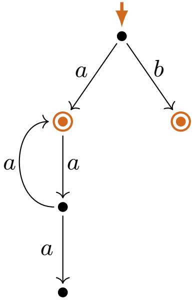
$$uf := a \cdot \overbrace{(a \cdot (a + a \cdot 0))^*}^{uf_a} + b \cdot \overbrace{(b \cdot (b + b \cdot 0))^*}^{uf_b}$$

$$uf \xrightarrow{a} 1 \cdot uf_a \Downarrow \qquad uf \xrightarrow{b} {\Downarrow} 1 \cdot uf_b$$

$$1 \cdot uf_a \xrightarrow{a} (1 \cdot (a + a \cdot 0)) \cdot uf_a$$

Image of P is **not** closed under bisimulation collapse

$P(uf)$



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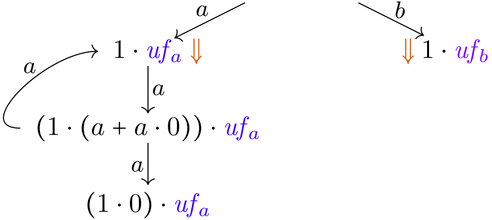
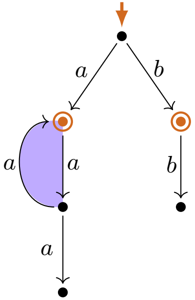


Image of *P* is **not** closed under bisimulation collapse

$P(uf)$



$P(uf)$

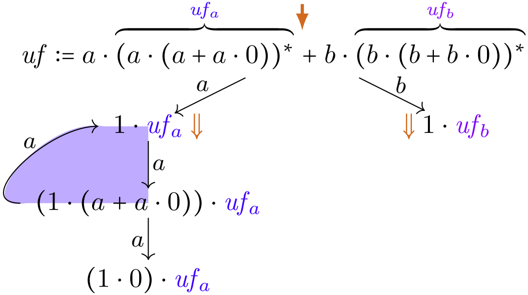
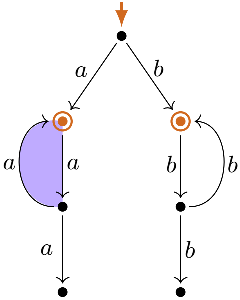


Image of *P* is **not** closed under bisimulation collapse

$P(uf)$



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$$uf \xrightarrow{a} 1 \cdot uf_a \Downarrow \qquad uf \xrightarrow{b} \Downarrow 1 \cdot uf_b$$

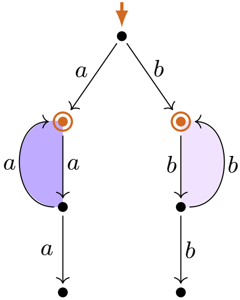
$$1 \cdot uf_a \xrightarrow{a} (1 \cdot (a + a \cdot 0)) \cdot uf_a \qquad 1 \cdot uf_b \xrightarrow{b} (1 \cdot (b + b \cdot 0)) \cdot uf_b$$

$$(1 \cdot (a + a \cdot 0)) \cdot uf_a \xrightarrow{a} 1 \cdot uf_a$$

$$(1 \cdot (a + a \cdot 0)) \cdot uf_a \xrightarrow{a} (1 \cdot 0) \cdot uf_a$$

Image of P is **not** closed under bisimulation collapse

$P(uf)$



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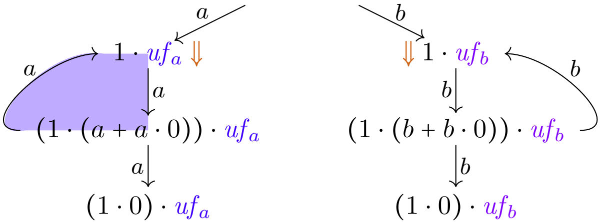
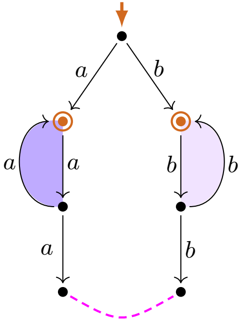


Image of *P* is **not** closed under bisimulation collapse

$P(uf)$



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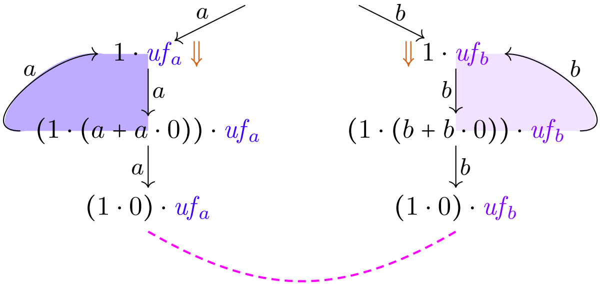

Compact process interpretation $P^{\bullet}$

Definition (Transition system specification $\mathcal{T}$)

$$\frac{}{\mathbf{1}\Downarrow} \qquad \frac{e_i\Downarrow}{(e_1 + e_2)\Downarrow}\,(i \in \{1,2\}) \qquad \frac{e_1\Downarrow \quad e_2\Downarrow}{(e_1 \cdot e_2)\Downarrow} \qquad \frac{}{(e^*)\Downarrow}$$

$$\frac{}{a \xrightarrow{a} \mathbf{1}} \qquad \frac{e_i \xrightarrow{a} e_i'}{e_1 + e_2 \xrightarrow{a} e_i'}\,(i \in \{1,2\})$$

$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2} \qquad \frac{e_1\Downarrow \quad e_2 \xrightarrow{a} e_2'}{e_1 \cdot e_2 \xrightarrow{a} e_2'} \qquad \frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e' \cdot e^*}$$

Compact process interpretation $P^{\bullet}$

Definition (Transition system specification $\mathcal{T}$)

$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2}$$

$$\frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e' \cdot e^*}$$

Compact process interpretation $P^{\bullet}$

Definition (Transition system specification $\mathcal{T}^{\bullet}$, changed rules w.r.t. $\mathcal{T}$)

$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2} \text{ (if } e_1' \text{ is normed)}$$

$$\frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e' \cdot e^*} \text{ (if } e' \text{ is normed)}$$

Compact process interpretation $P^{\bullet}$

Definition (Transition system specification $\mathcal{T}^{\bullet}$, changed rules w.r.t. $\mathcal{T}$)

$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2} \text{ (if } e_1' \text{ is normed)} \qquad \frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1'} \text{ (if } e_1' \text{ is not normed)}$$

$$\frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e' \cdot e^*} \text{ (if } e' \text{ is normed)} \qquad \frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e'} \text{ (if } e' \text{ is not normed)}$$

Compact process interpretation $P^{\bullet}$

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$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2} \text{ (if } e_1' \text{ is normed)} \qquad \frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1'} \text{ (if } e_1' \text{ is not normed)}$$

$$\frac{e \xrightarrow{a} e'}{e^{*} \xrightarrow{a} e' \cdot e^{*}} \text{ (if } e' \text{ is normed)} \qquad \frac{e \xrightarrow{a} e'}{e^{*} \xrightarrow{a} e'} \text{ (if } e' \text{ is not normed)}$$

Definition

The compact process (graph) interpretation $P^{\bullet}(e)$ of a reg. expr's e:

$P^{\bullet}(e) :=$ labeled transition graph generated by e by derivations in $\mathcal{T}^{\bullet}$.

Compact process interpretation $P^\bullet$

Definition (Transition system specification $\mathcal{T}^\bullet$, changed rules w.r.t. $\mathcal{T}$)

$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2} \text{ (if } e_1' \text{ is normed)} \qquad \frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1'} \text{ (if } e_1' \text{ is not normed)}$$

$$\frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e' \cdot e^*} \text{ (if } e' \text{ is normed)} \qquad \frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e'} \text{ (if } e' \text{ is not normed)}$$

Definition

The compact process (graph) interpretation $P^\bullet(e)$ of a reg. expr's e:

$P^\bullet(e)$:= labeled transition graph generated by e by derivations in $\mathcal{T}^\bullet$.

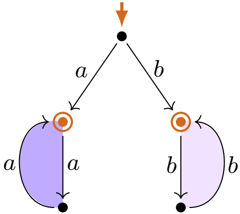
Lemma ($P^\bullet$ increases sharing; $P^\bullet$, P have same bisimulation semantics)

(i) $P(e) \Rightarrow P^\bullet(e)$ *for all regular expressions* e.

(ii) *(*G *is* $[\![\cdot]\!]_{P^\bullet}$*-expressible* $\Longleftrightarrow$ G *is* $[\![\cdot]\!]_P$*-expressible) for all graphs* G.

Image of $P^{\bullet}$ under bisimulation collapse ...

$P^{\bullet}(uf)$



$P^{\bullet}(uf)$

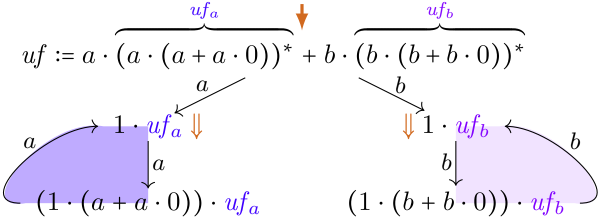


Image of $P^{\bullet}$ under bisimulation collapse ...

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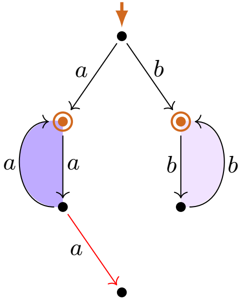


$P^{\bullet}(uf)$

$$uf := a \cdot \overbrace{(a \cdot (a + a \cdot 0))^*}^{uf_a} + b \cdot \overbrace{(b \cdot (b + b \cdot 0))^*}^{uf_b}$$

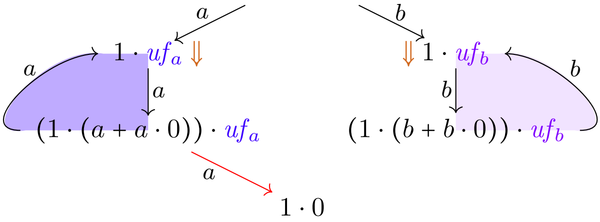
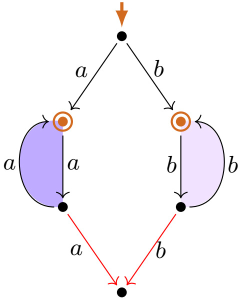


Image of $P^\bullet$ under bisimulation collapse ...

$P^\bullet(uf)$



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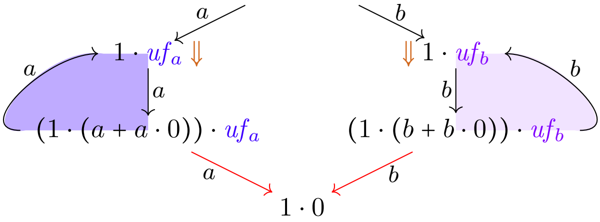


Interpretation correspondence of $P^{\bullet}$ with LEE

(Int)$^{(\cancel{1}\backslash *)}_{P^{\bullet}}$: *By under-star-1-free expressions $P^{\bullet}$-expressible graphs satisfy* LEE:

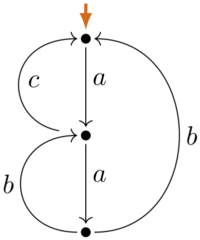
Compact process **i**nterpretations $P^{\bullet}(uf)$
of under-star-1-free regular expressions uf
are finite process graphs that satisfy LEE.

(Extr)$^{(\cancel{1}\backslash *)}_{P^{\bullet}}$: LEE *implies* $[\![\cdot]\!]_P$*-expressibility by under-star-1-free reg. expr's*:

From every finite process graph G with LEE
an under-star-1-free regular expression uf can be **e**xtracted
such that $G \rightrightarrows P(uf)$.

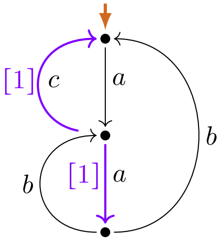
Refined extraction expression (example)

G_4



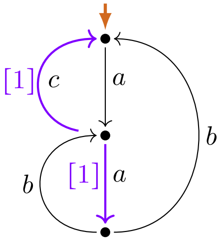
Refined extraction expression (example)

$\widehat{G_4}$



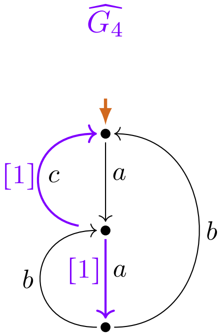
Refined extraction expression (example)

$\widehat{G_4}$



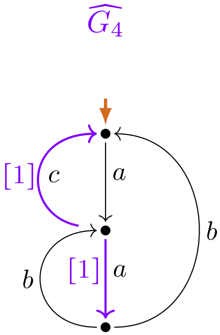
$$(1 \cdot (\qquad\qquad\qquad)^{*}) \cdot 0$$

Refined extraction expression (example)



$$(1 \cdot (\qquad \overset{a}{\downarrow} \qquad)^{*}) \cdot 0$$

Refined extraction expression (example)



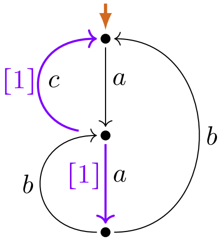
$$((1 \cdot a) \cdot (\qquad)^{*}) \cdot 0$$

$$\Big\downarrow a$$

$$(1 \cdot (\qquad)^{*}) \cdot 0$$

Refined extraction expression (example)

$\widehat{G_4}$



$$((1 \cdot a) \cdot (\qquad)^*) \cdot 0$$

a

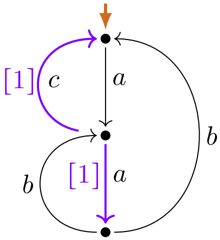
$$(1 \cdot (\qquad)^*) \cdot 0$$

b

b

Refined extraction expression (example)

$\widehat{G_4}$



$$((1 \cdot a) \cdot (\qquad)^*) \cdot 0$$

$$a$$

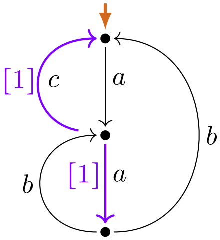
$$(1 \cdot (\qquad)^*) \cdot 0$$

$$b \qquad b$$

$$((1 \cdot (b + b \cdot a)) \cdot (\qquad)^*) \cdot 0$$

Refined extraction expression (example)

$\widehat{G_4}$



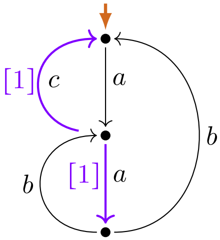
$((1 \cdot a) \cdot (\qquad)^*) \cdot 0$

$(1 \cdot (\qquad)^*) \cdot 0$

$((1 \cdot (b + b \cdot a)) \cdot (\qquad)^*) \cdot 0$

Refined extraction expression (example)

$\widehat{G_4}$



$$((1 \cdot a) \cdot (\qquad\qquad\qquad)^*) \cdot 0$$

$$\downarrow a \qquad c,\ b$$

$$(1 \cdot (c \cdot a + a \cdot (b + b \cdot a))^*) \cdot 0$$

$$\downarrow a$$

$$((1 \cdot (b + b \cdot a)) \cdot (\qquad\qquad\qquad)^*) \cdot 0$$

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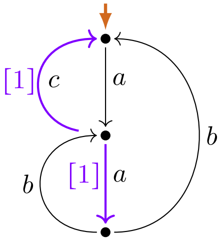

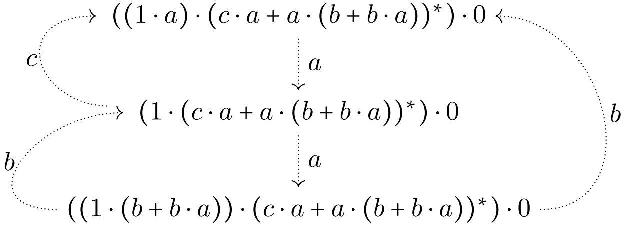

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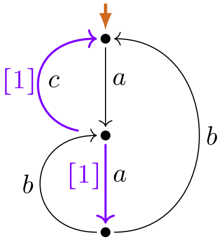

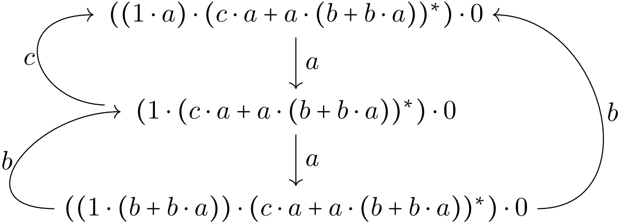

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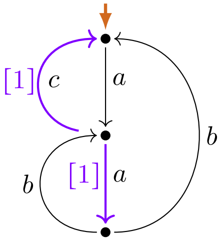

$P^{\bullet}(uf) = P(uf) \simeq G_4$

$$\begin{tikzcd}
\overbrace{((1 \cdot a) \cdot (c \cdot a + a \cdot (b + b \cdot a))^*) \cdot 0}^{uf} \arrow[d, "a"] \\
(1 \cdot (c \cdot a + a \cdot (b + b \cdot a))^*) \cdot 0 \arrow[u, bend left, "c"] \arrow[d, "a"] \\
((1 \cdot (b + b \cdot a)) \cdot (c \cdot a + a \cdot (b + b \cdot a))^*) \cdot 0 \arrow[u, bend left, "b"] \arrow[uu, bend right, "b"']
\end{tikzcd}$$

Interpretation/extraction correspondences of $P^{\bullet}$ with LEE

(Int)$_{P^{\bullet}}^{(\cancel{1}\backslash *)}$: *By under-star-1-free expressions $P^{\bullet}$-expressible graphs satisfy* LEE:

Compact process **i**nterpretations $P^{\bullet}(uf)$
of under-star-1-free regular expressions uf
are finite process graphs that satisfy LEE.

(Extr)$_{P^{\bullet}}^{(\cancel{1}\backslash *)}$: LEE *implies* $[\![\cdot]\!]_P$*-expressibility by under-star-1-free reg. expr's*:

From every finite process graph G with LEE
an under-star-1-free regular expression uf can be **e**xtracted
such that $G \Rightarrow P(uf)$.

From every finite collapsed process graph G with LEE
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(ImColl)$^{(\cancel{1}\backslash *)}_{P^{\bullet}}$: The image of $P^{\bullet}$,
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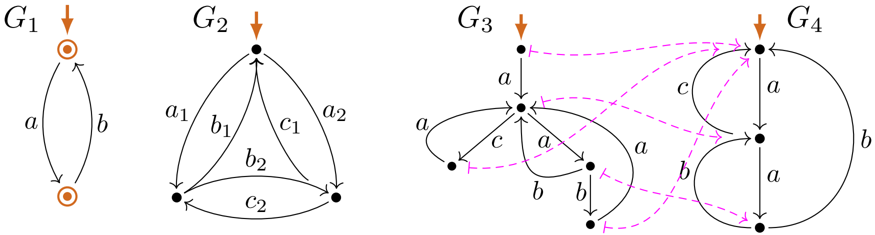
P-/$P^\bullet$-expressibility and $[\![\cdot]\!]_P$-expressibility (examples)



not P-expressible

not $[\![\cdot]\!]_P$-expressible

P-/$P^\bullet$-expressible

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$P^\bullet$-expressible

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Resources

- Slides/extended abstract on `clegra.github.io`
  - slides: `.../lf/TG-2024.pdf`
  - extended abstract: `.../lf/closing-bs-i-pi-us1f.pdf`

- CG, Wan Fokkink: A Complete Proof System for 1-Free Regular Expressions Modulo Bisimilarity,
  - LICS 2020, arXiv:2004.12740, video on youtube.
- CG: Modeling Terms by Graphs with Structure Constraints,
  - TERMGRAPH 2018, EPTCS 288, arXiv:1902.02010.
- CG: The Image of the Process Interpretation of Regular Expressions is Not Closed under Bisimulation Collapse,
  - arXiv:2303.08553.
- CG: Milner's Proof System for Regular Expressions Modulo Bisimilarity is Complete,
  - LICS 2022, arXiv:2209.12188, poster.

Language semantics $[\![\cdot]\!]_L$ of reg. expr's *(Copi–Elgot–Wright, 1958)*

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$[\![e]\!]_L := L(e)$ (language defined by e)

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A chart is a loop chart if:

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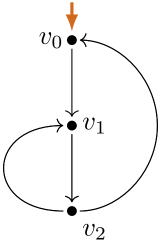

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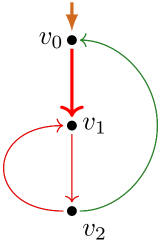

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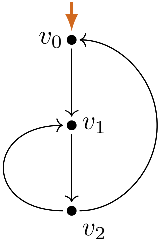

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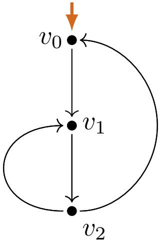

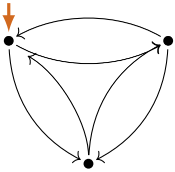

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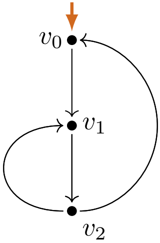

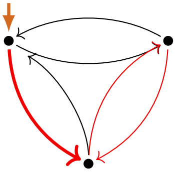

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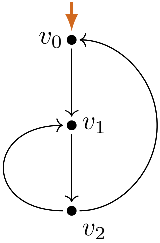

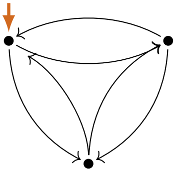

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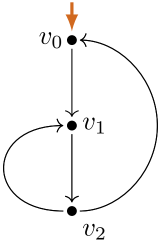

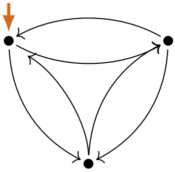

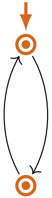

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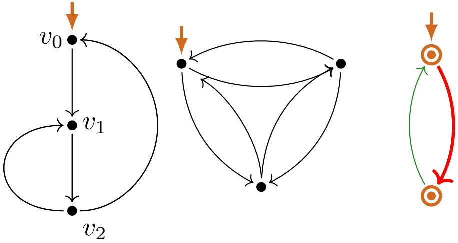

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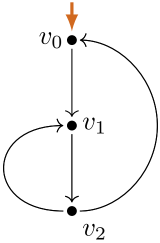

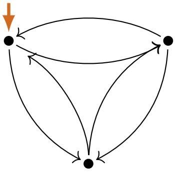

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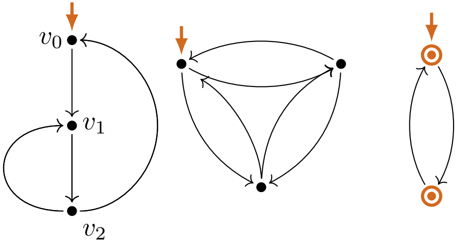

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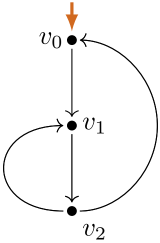

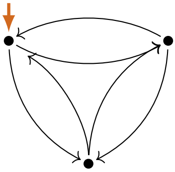

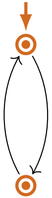

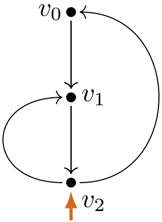

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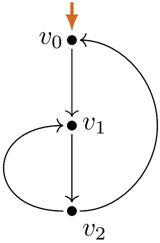

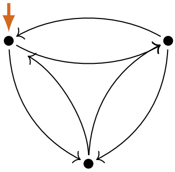

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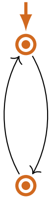

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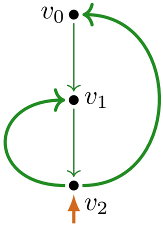

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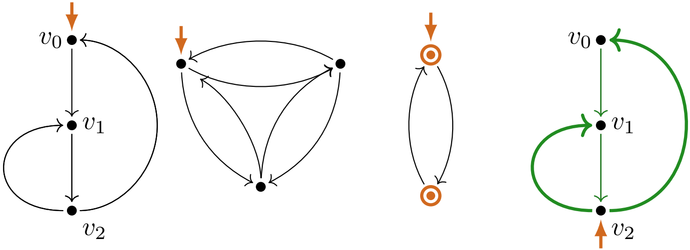

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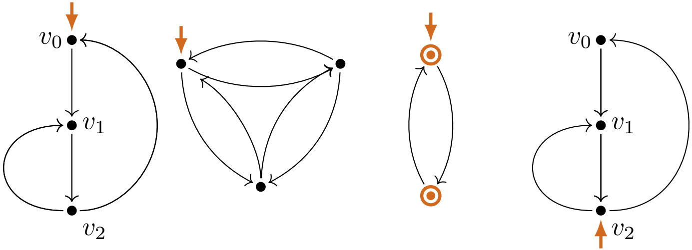

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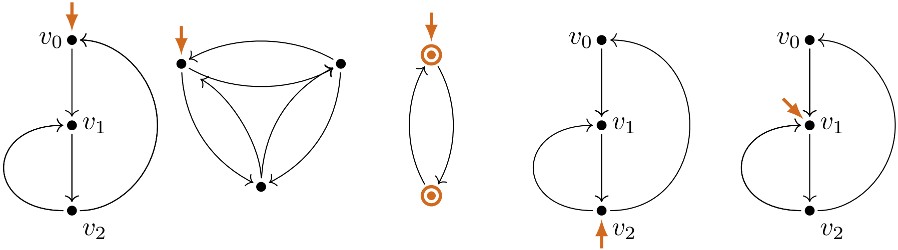

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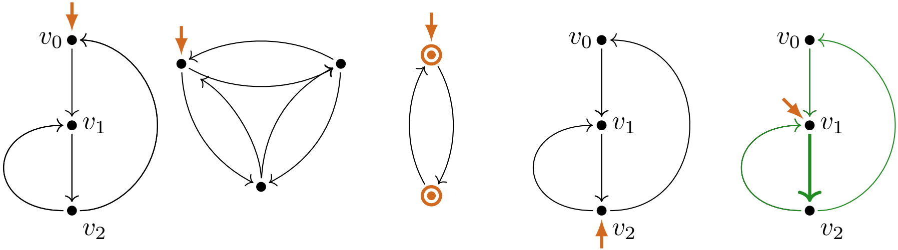

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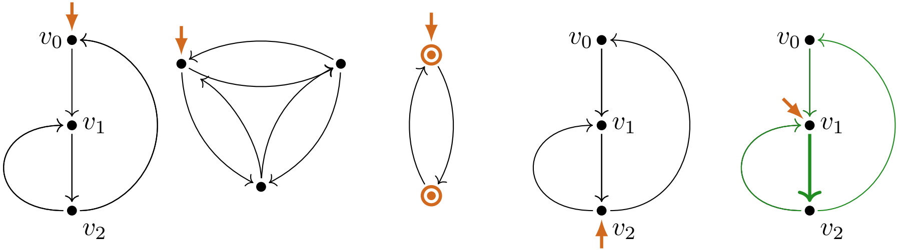

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Loop charts (interpretations of innermost iterations)

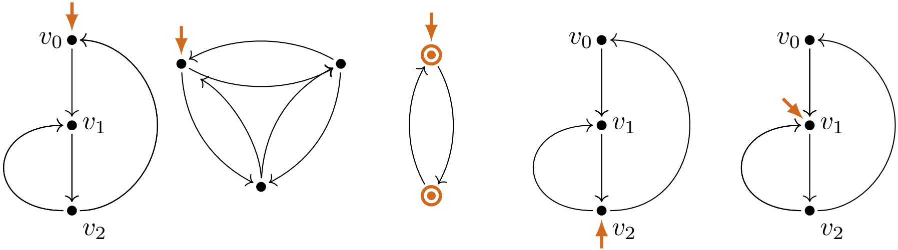
Definition

A chart is a loop chart if:

(L1) There is an infinite path from the start vertex.

(L2) Every infinite path from the start vertex returns to it.

(L3) Termination is only possible at the start vertex.



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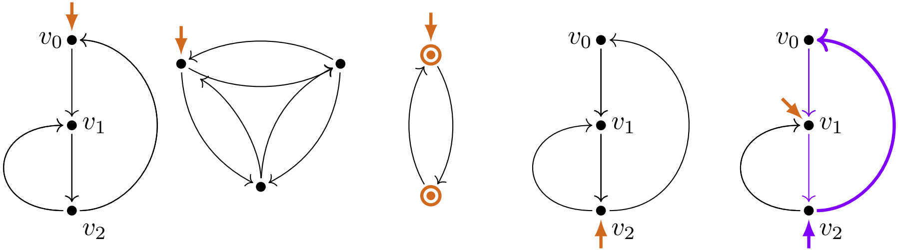

(L1),~~(L2)~~ (L1),(L2),~~(L3)~~ loop chart loop subchart