Core Course Preview

Introduction to Model Checking

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Model Checking

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- *to expose potential software design errors;*
- *that, given a finite-state model of a system and a formal property, systematically checks whether this property holds for that model.*

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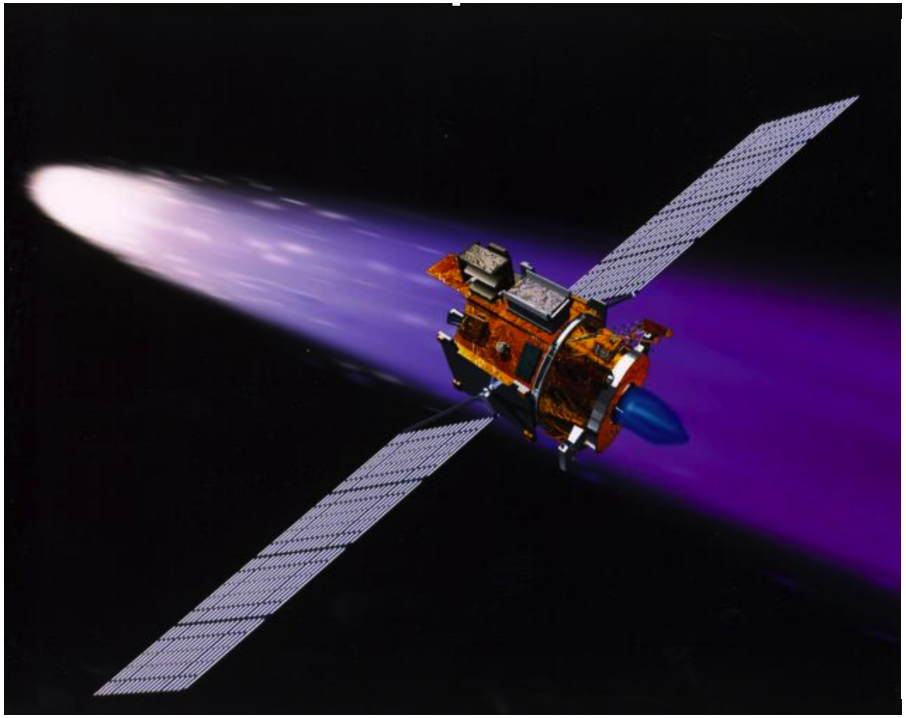
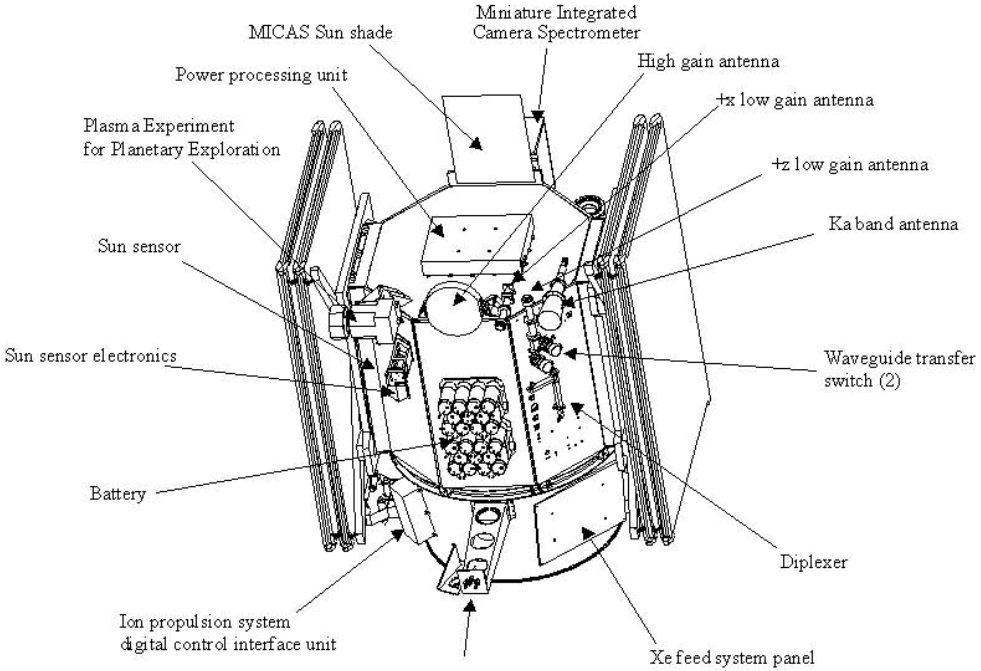
Course Goals are introduction to:

- Theory:
  - modeling systems by labeled transition systems,
  - expressing properties by temporal-logic formulas
  - model-checking algorithms
- Practice: use the Maude system for examples

Deep Space 1 (NASA)





- Flyby of asteroid 9969 Braille (1999)
- Entered the coma of Comet Borrelly (2001)
- Model checking discovered 5 concurrency errors

Example (program concurrency/non-determinism)

Programs Inc, Dec, and Reset cooperate, and use a shared variable x:

```
proc Inc
  while true
    do
      if x < 200
        then x := x + 1
      fi
    od
```

```
proc Dec
  while true
    do
      if x > 0
        then x := x - 1
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proc Reset
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      if x = 200
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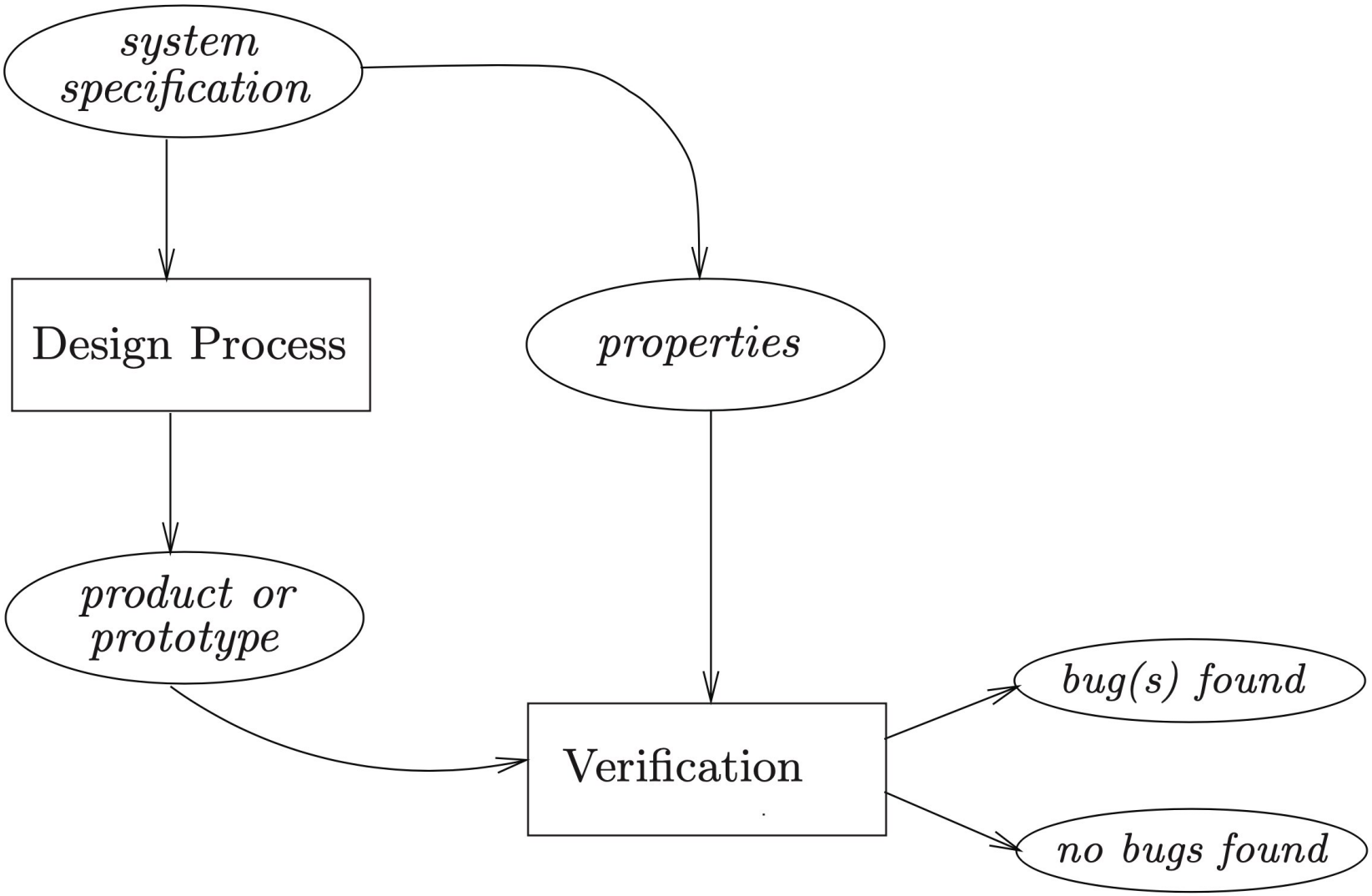
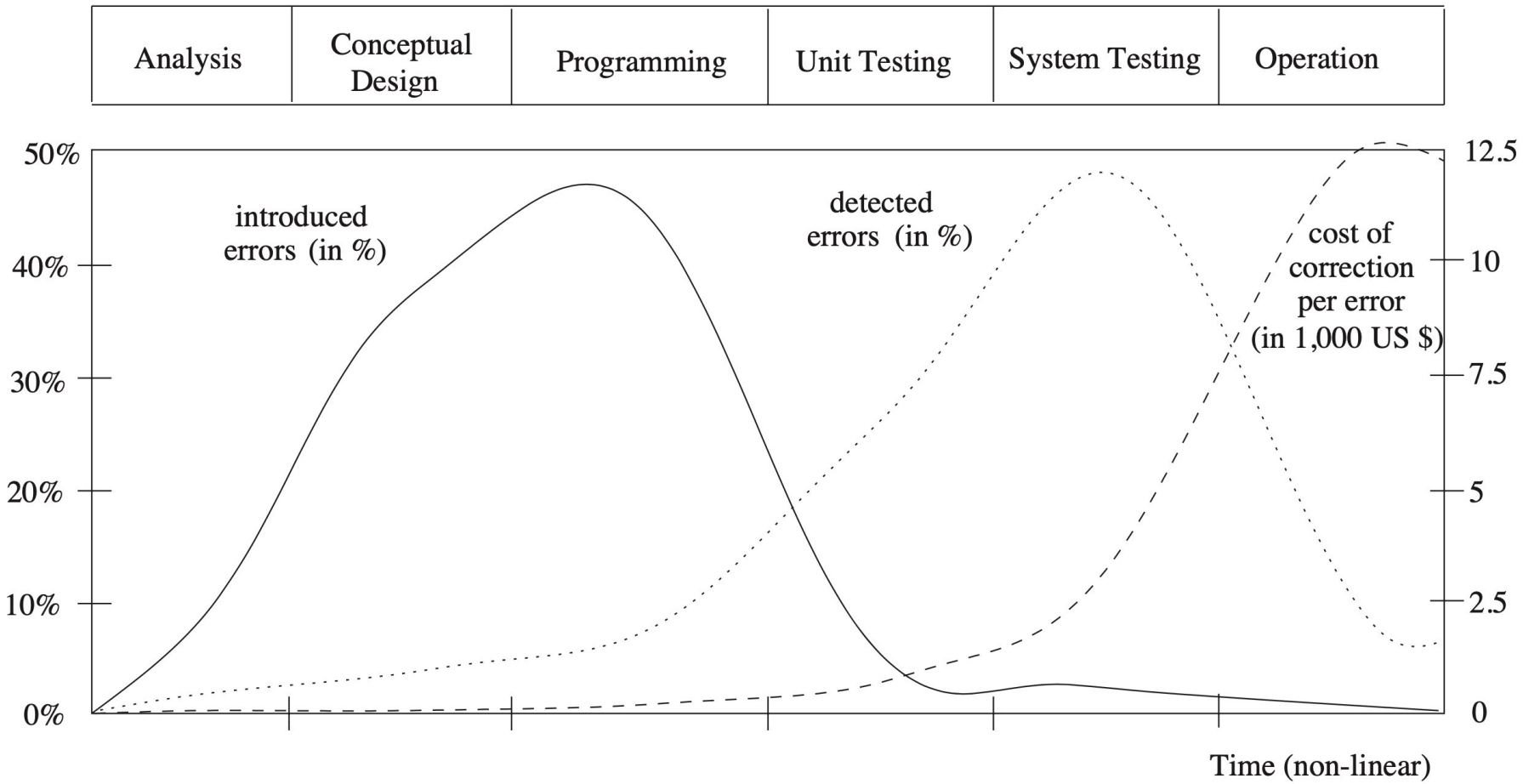
Question: Is $0 \leq x \leq 200$ always guaranteed?

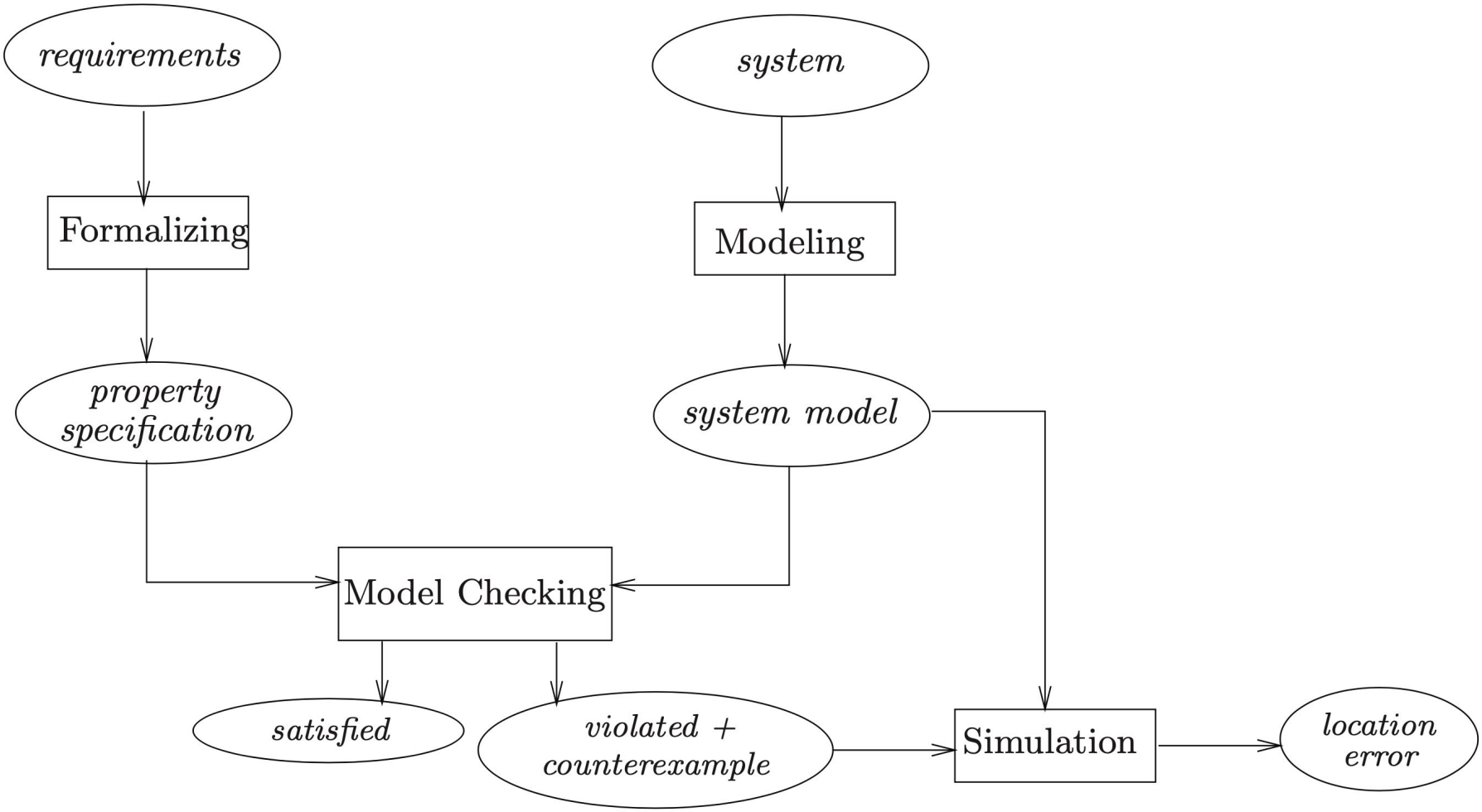
Hard-/Software Verification (traditionally)



Error introduction, detection, and repair costs



Model checking



Modeling (by program graphs)

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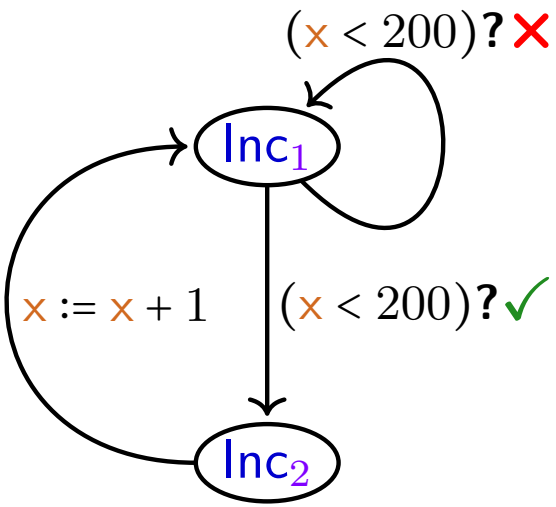
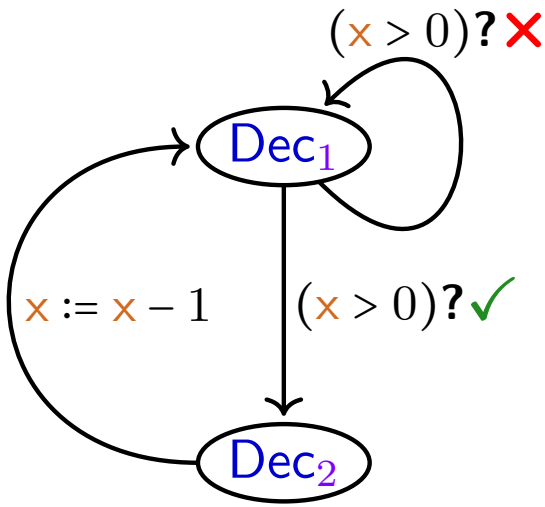
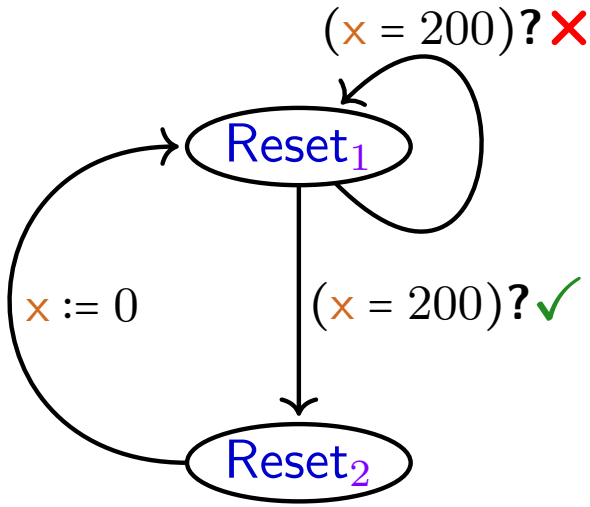
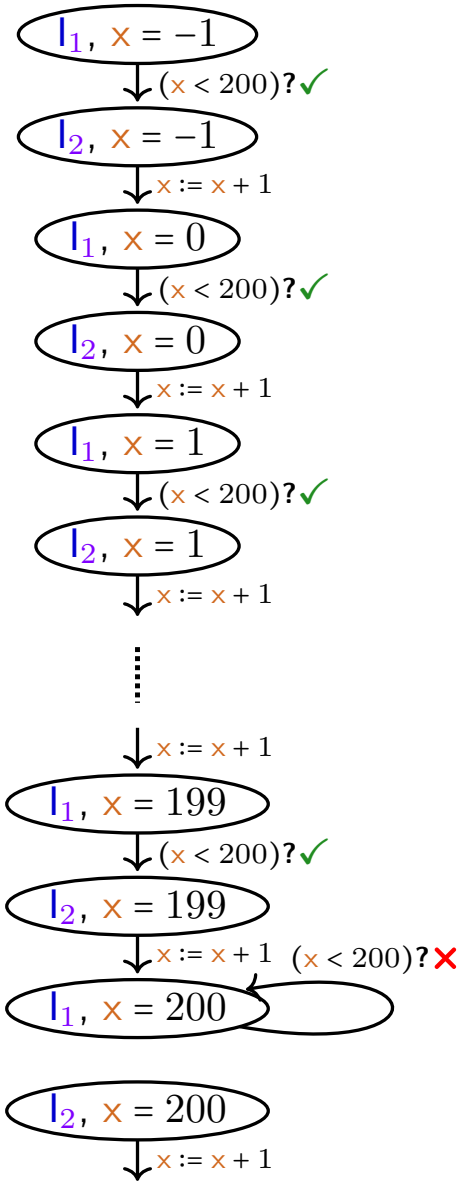






Program graphs (PG)

Modeling (by labeled transition systems, with state space explosion)



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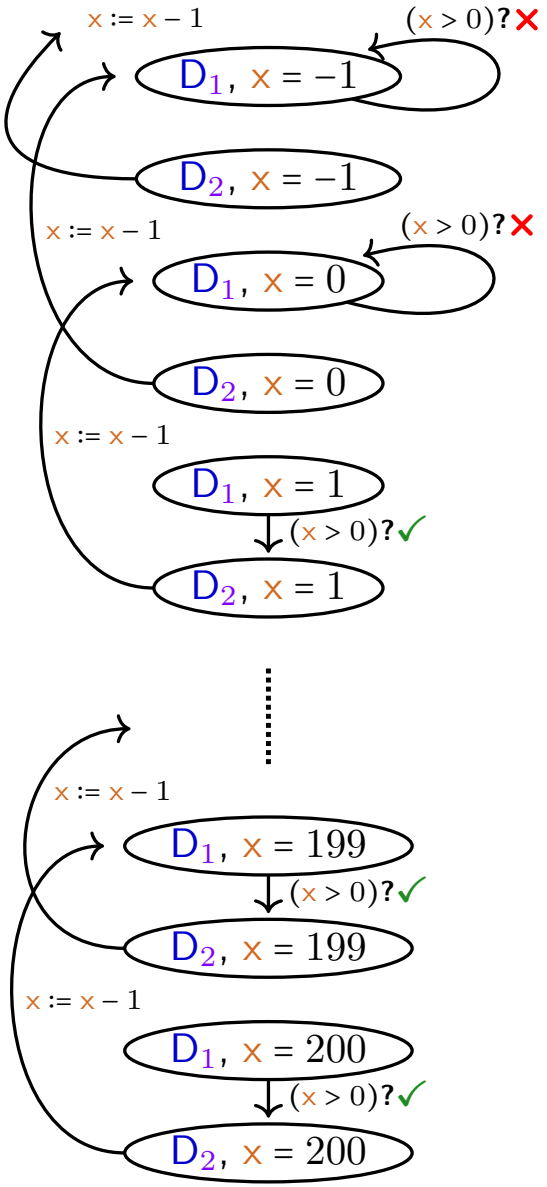

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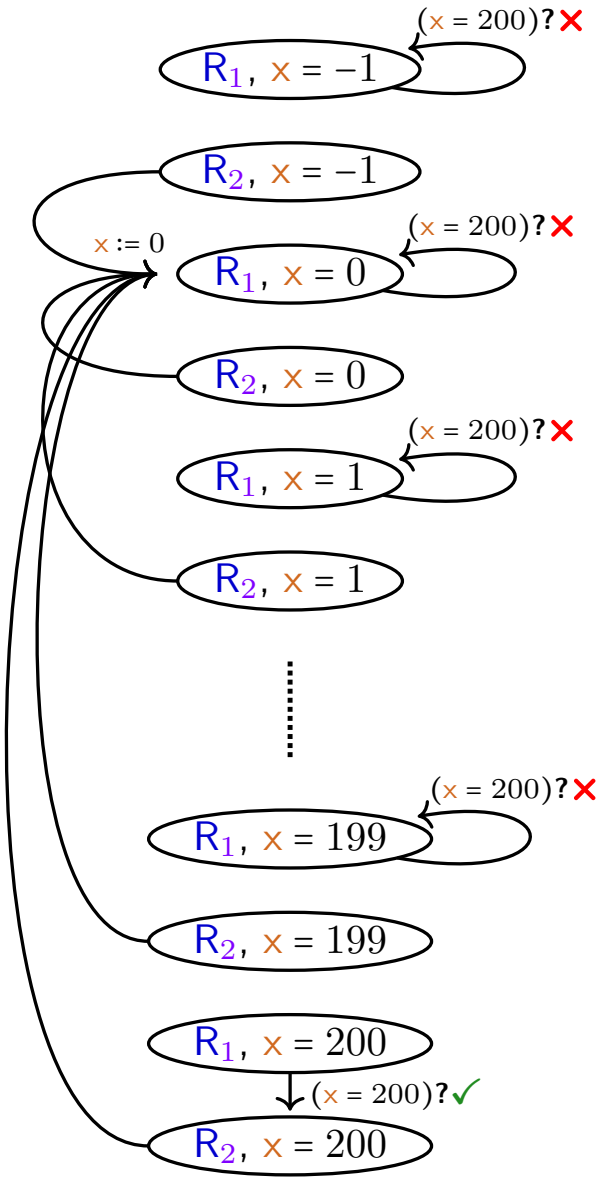

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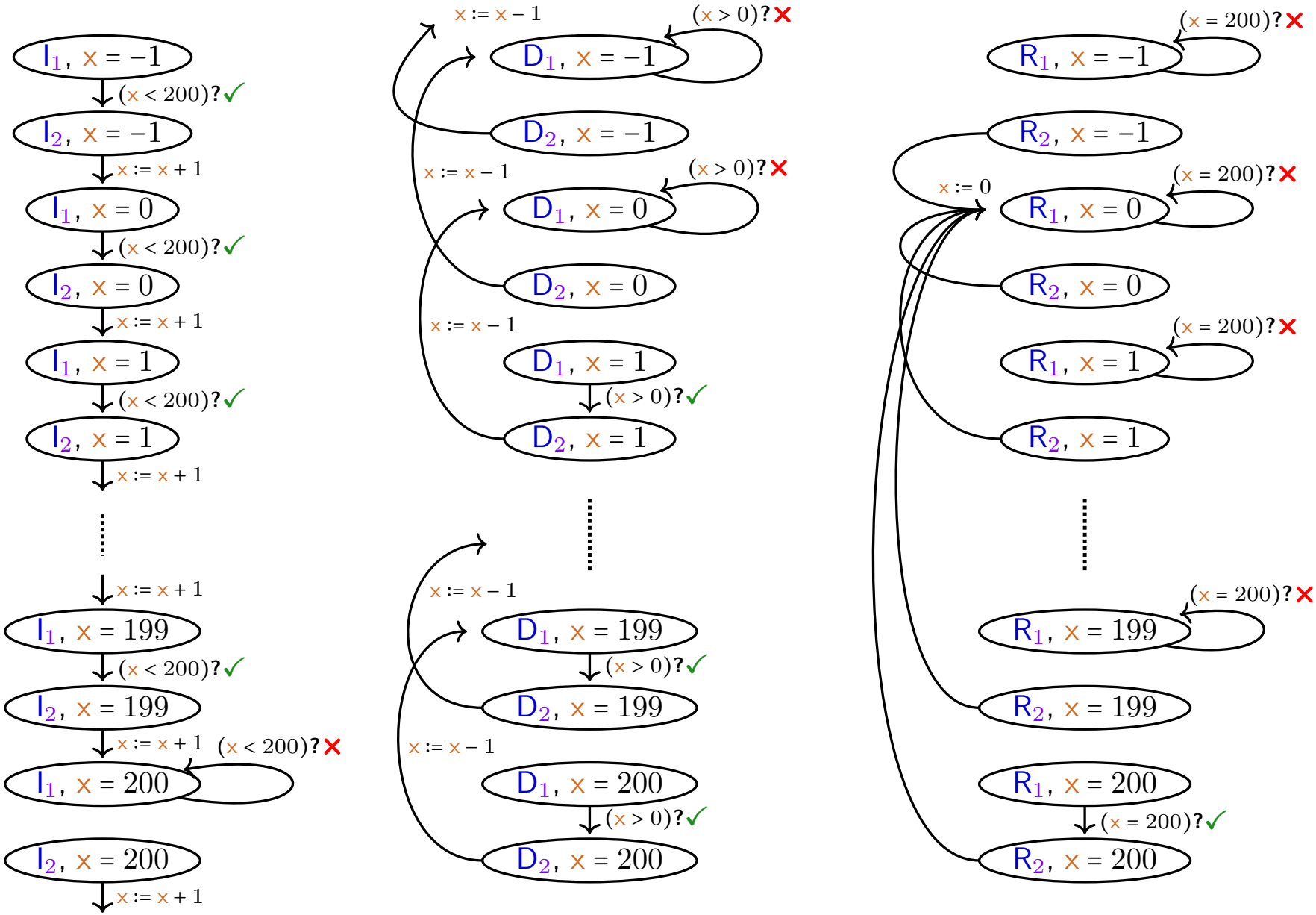
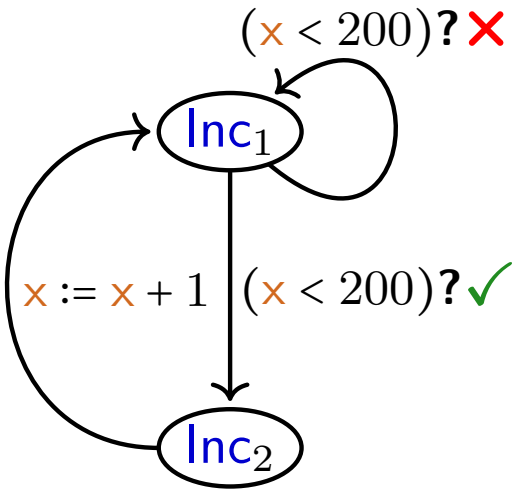
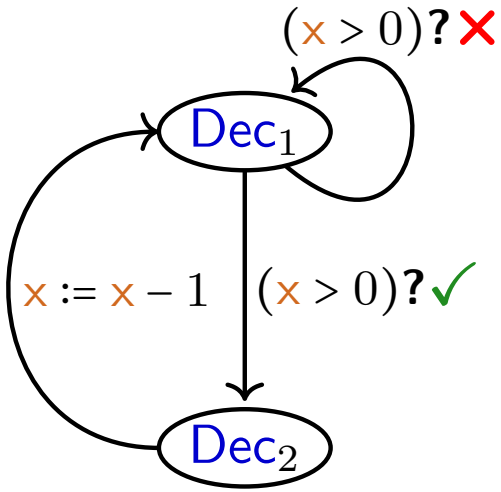
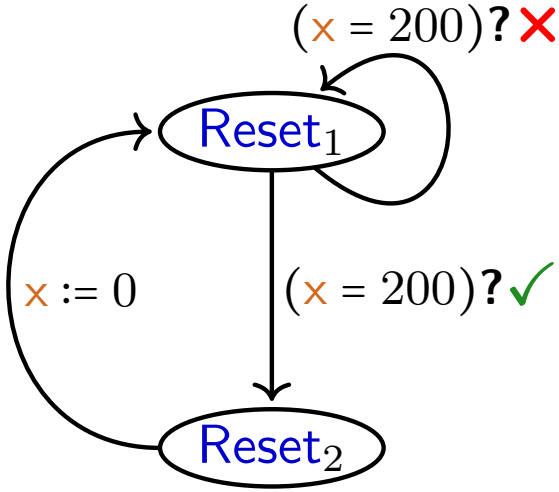


Formalizing properties (in temporal logic)







$\mathsf{Inc}_1 \parallel \mathsf{Dec}_1 \parallel \mathsf{Reset}_1 \stackrel{?}{\models} \Box(0 \leq x \wedge x \leq 200)$ (Linear-TL formula)

Counterexample (offending execution trace)

$$\langle x = 199 \,;\, \mathsf{Inc}_1 \parallel \mathsf{Dec}_1 \parallel \mathsf{Reset}_1 \rangle$$

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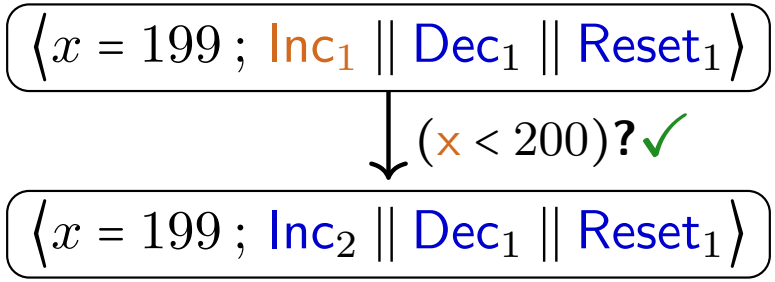

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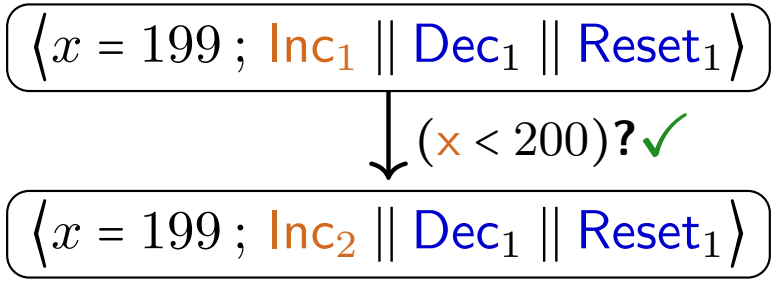

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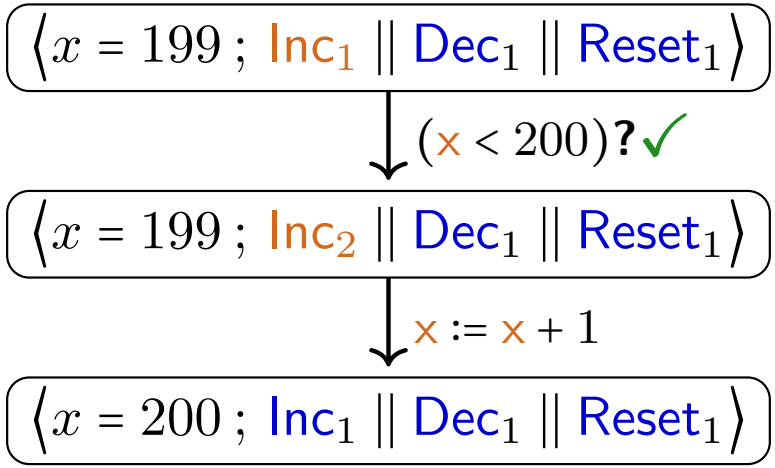

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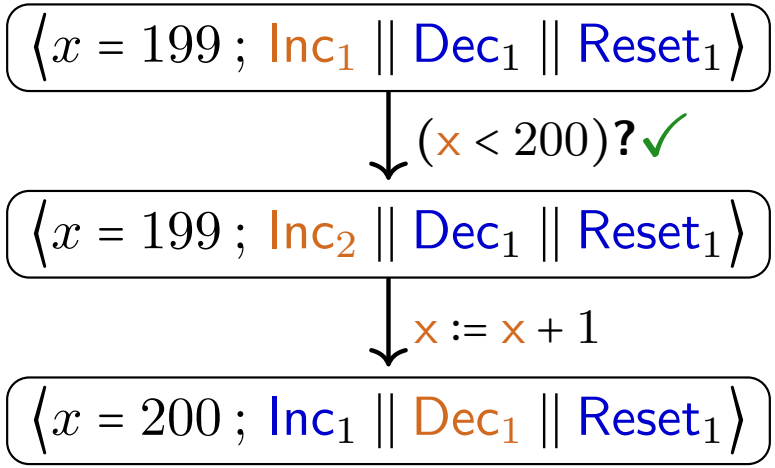

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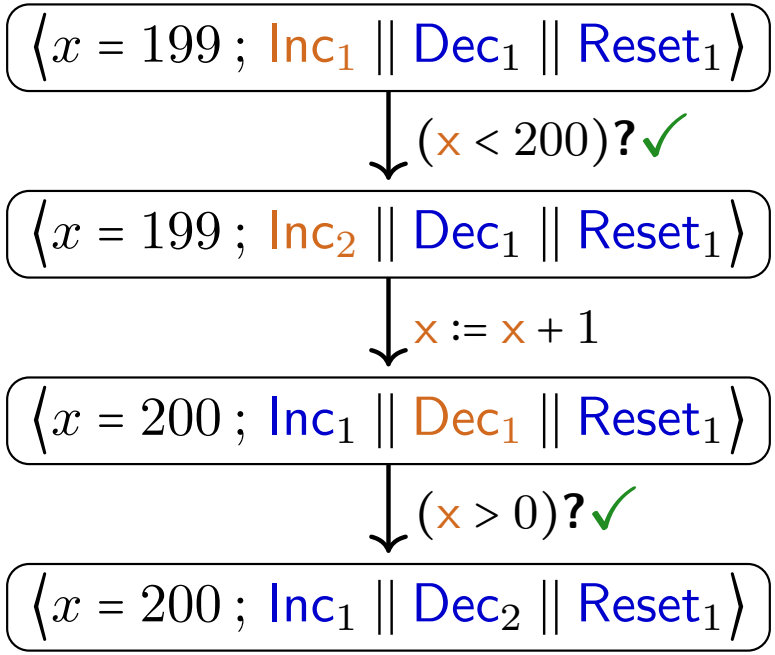

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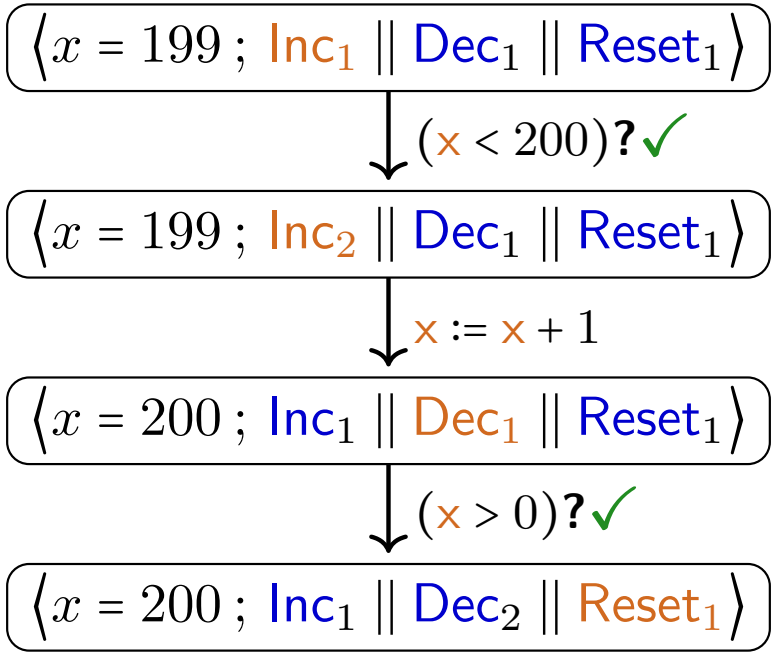

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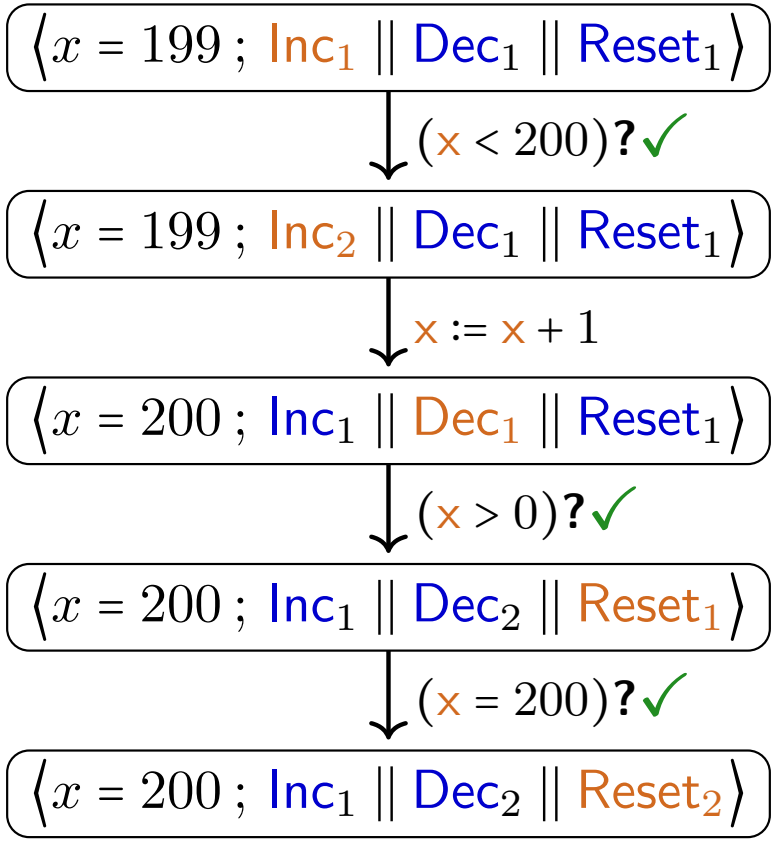

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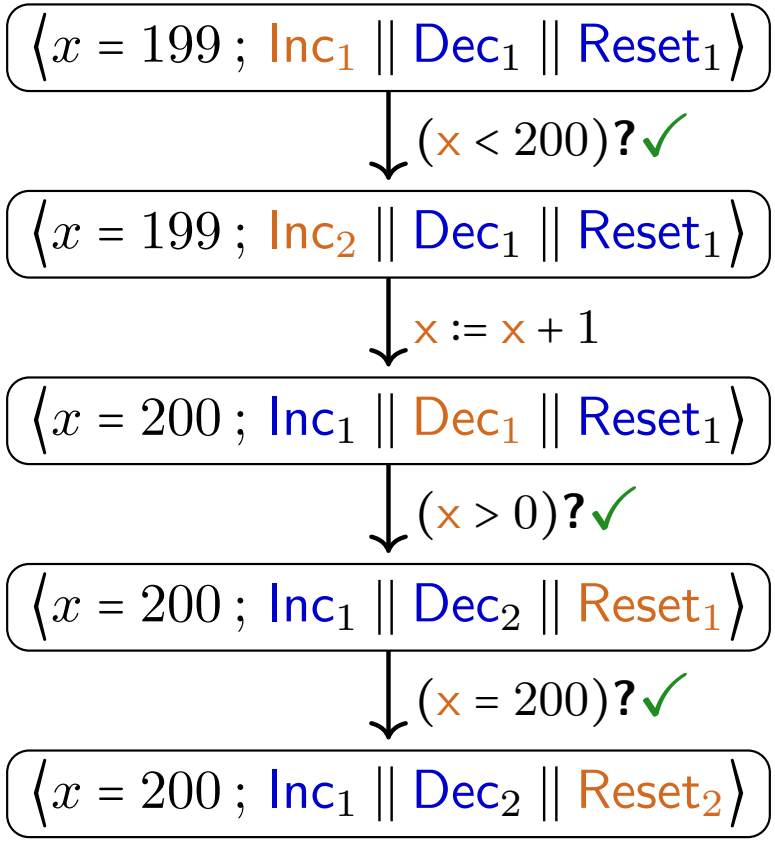

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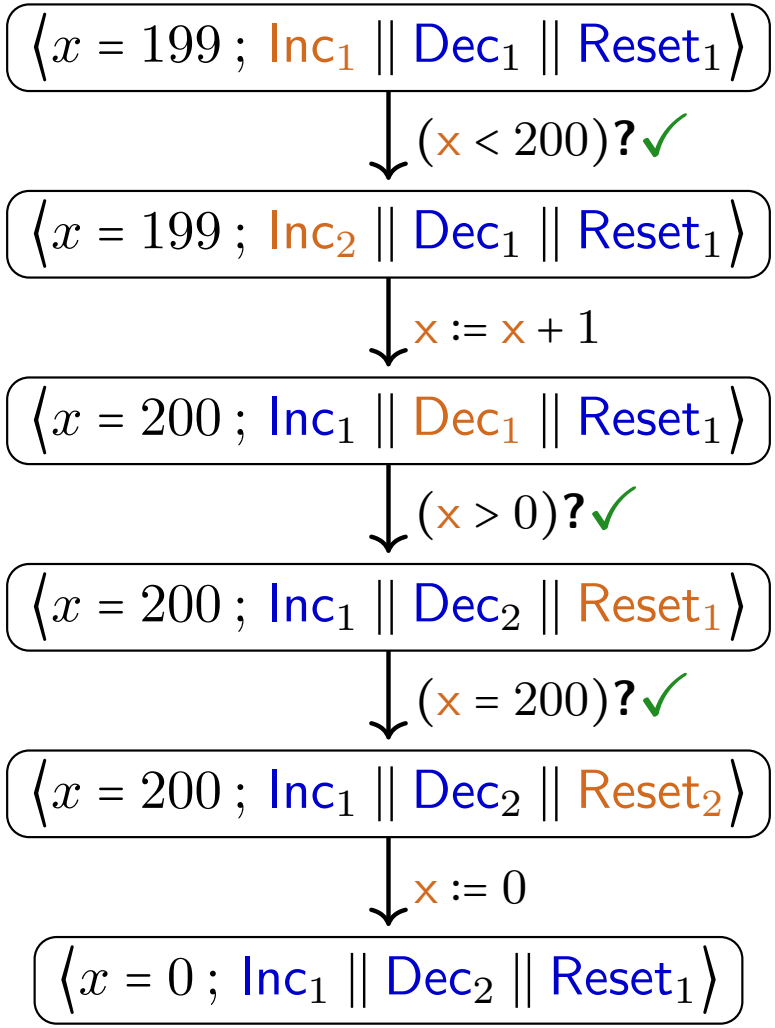

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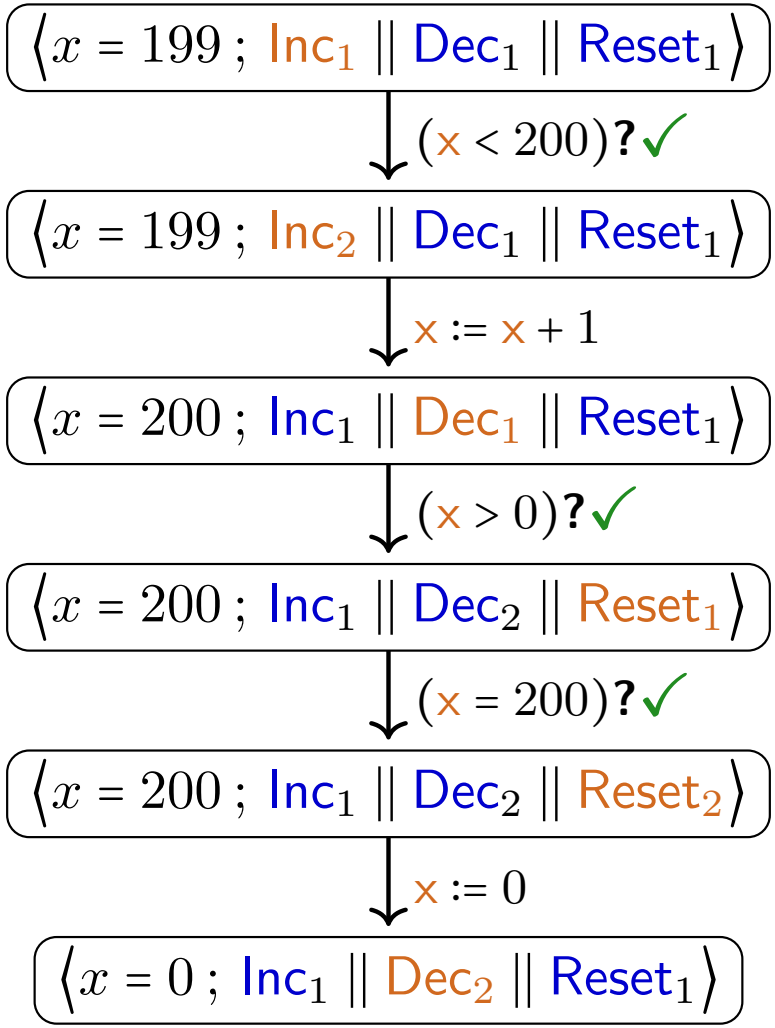

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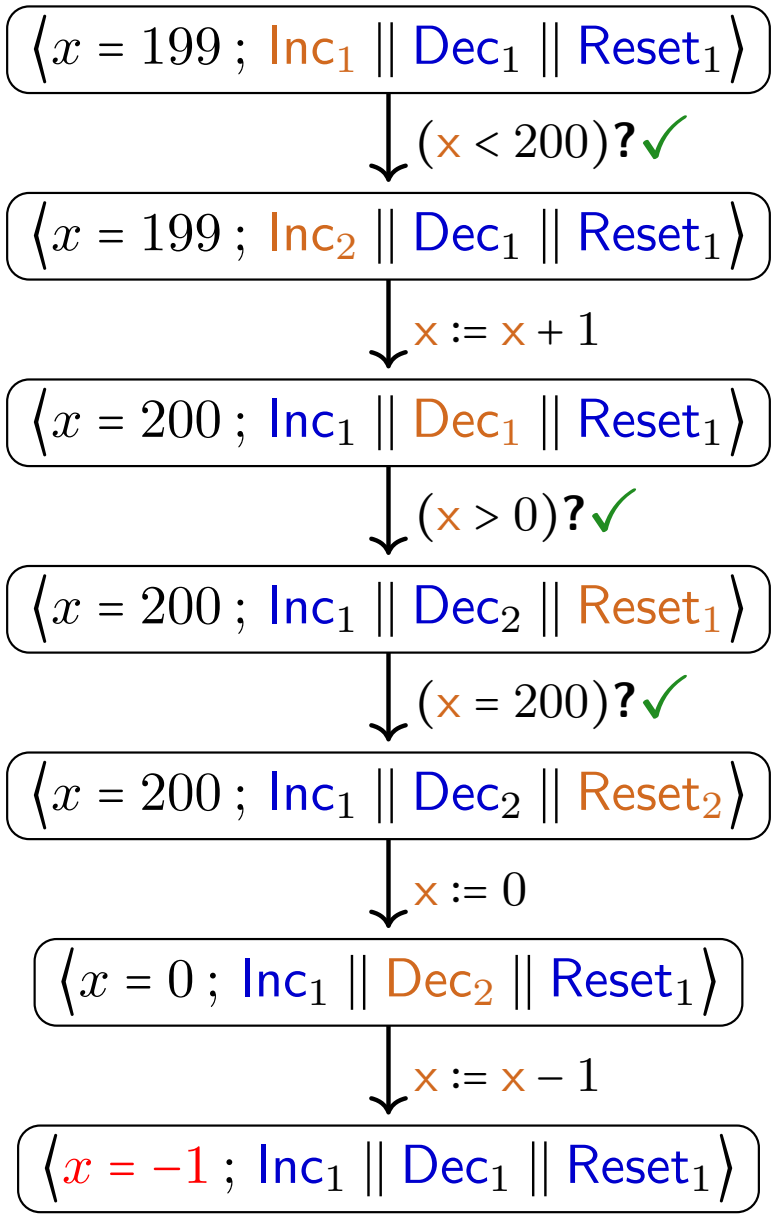
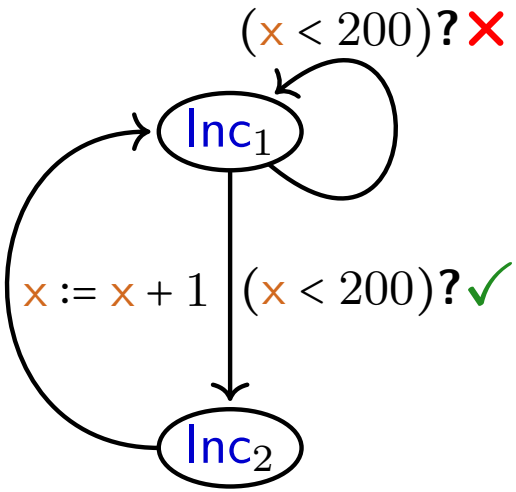
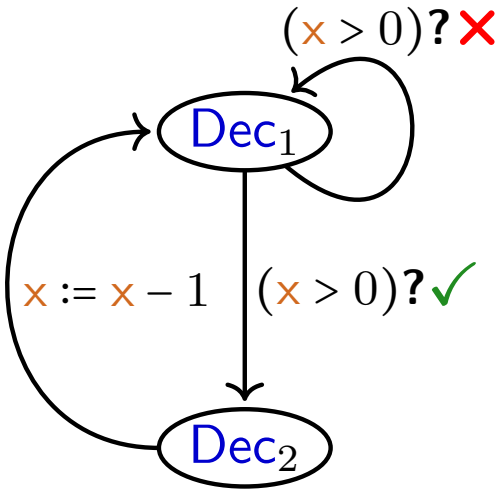
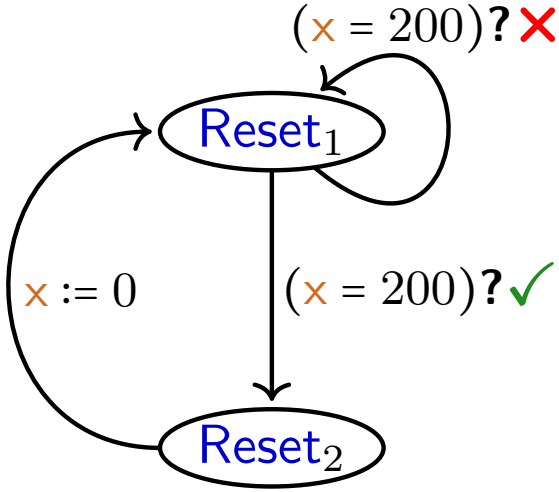


Formalizing properties (in temporal logic)







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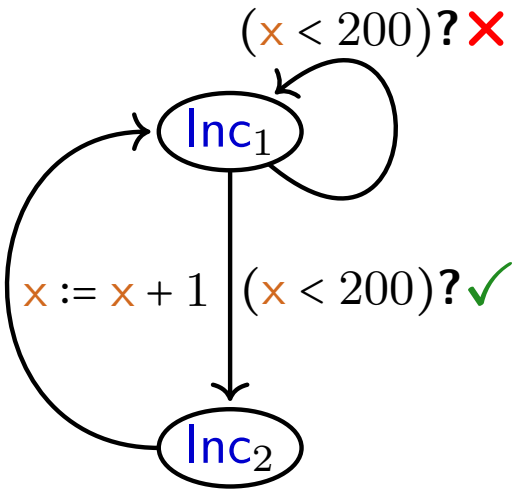
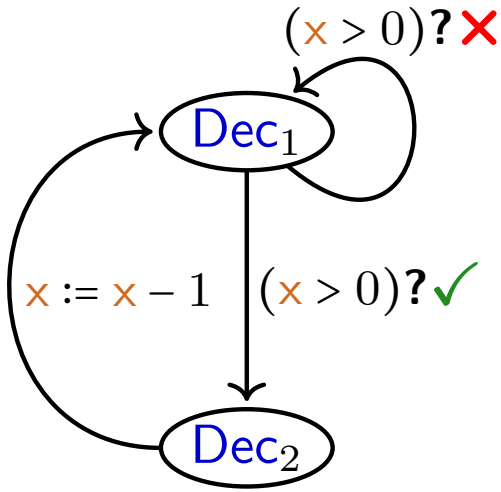
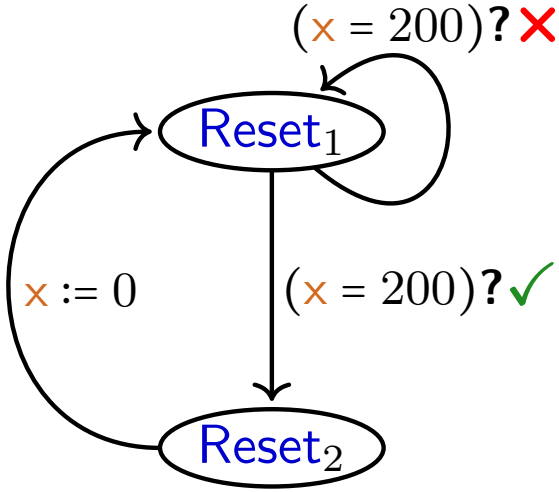
Formalizing properties (in temporal logic)







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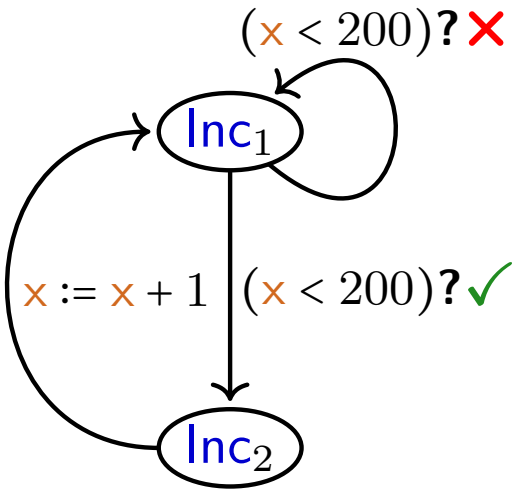

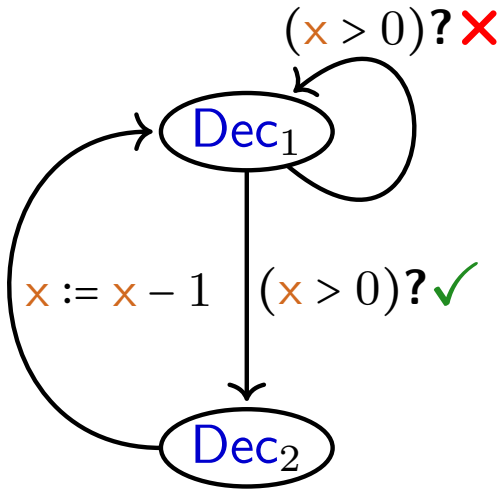

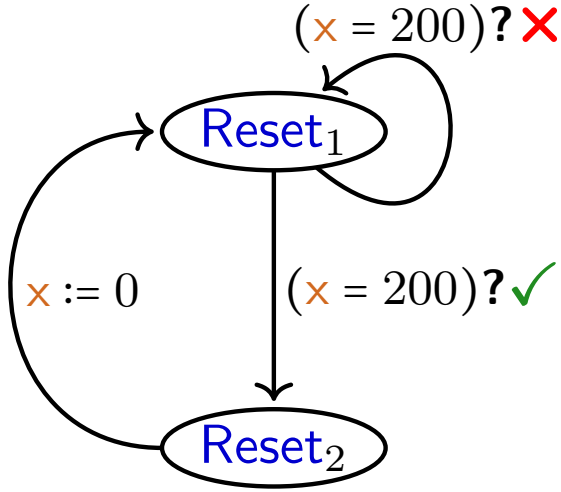

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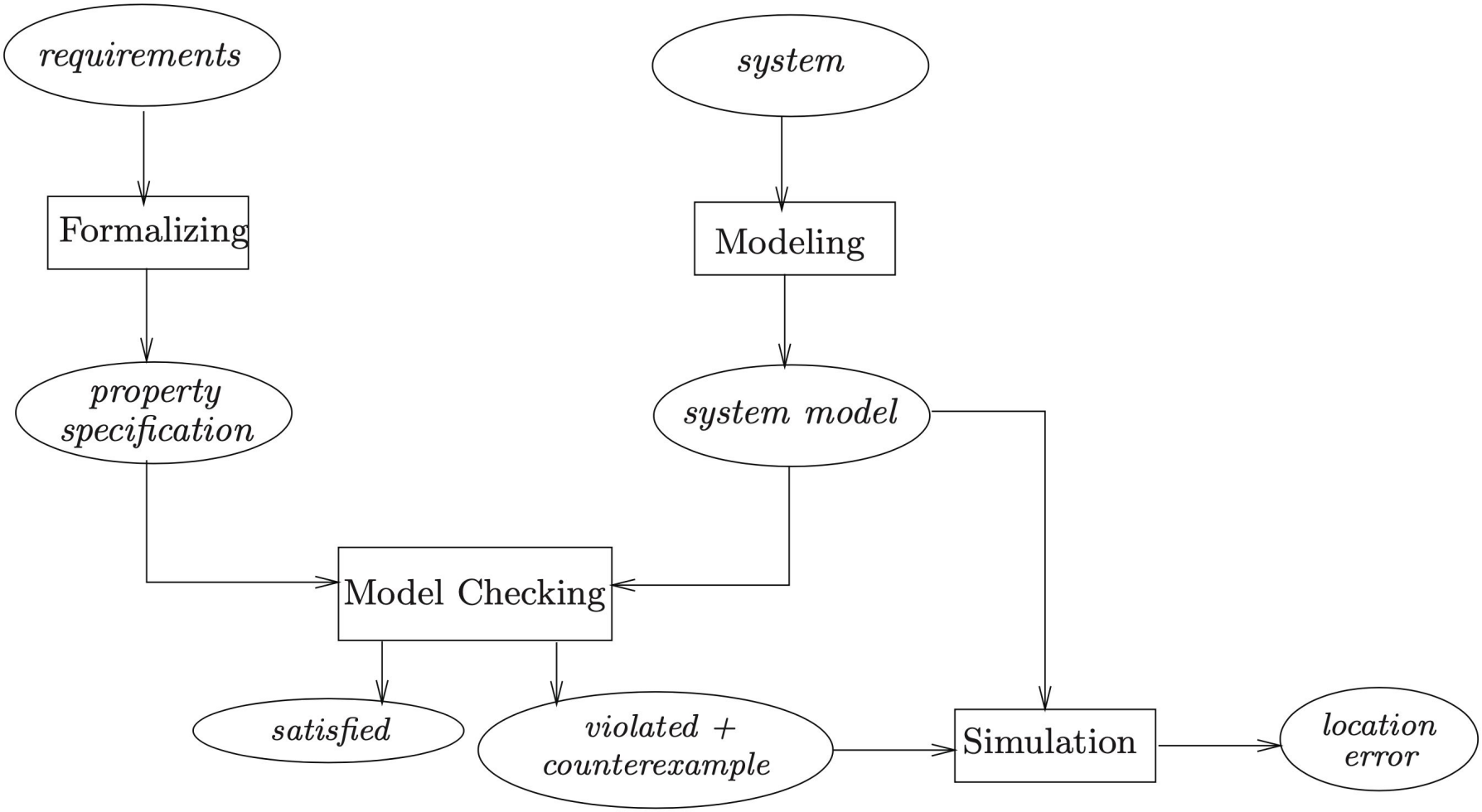
$\mathsf{Inc}_1 \parallel \mathsf{Dec}_1 \parallel \mathsf{Reset}_1 \not\models \forall\Box(0 \leq x \wedge x \leq 200)$ (Computation-Tree-L formula)

$\mathsf{Inc}_1 \parallel \mathsf{Dec}_1 \parallel \mathsf{Reset}_1 \models \exists\Box(0 \leq x \wedge x \leq 200)$ (CTL formula)

$\mathsf{Inc}_1 \parallel \mathsf{Dec}_1 \parallel \mathsf{Reset}_1 \models \forall\Box\exists\Diamond(x < 0)$ (CTL formula)

Model checking



Any [such] verification is only as good as the model of the system.

Maude code (idea)

```
crl [Inc1a]   :  Inc1 x => Inc2 x  if x < 200
rl  [Inc2]    :  Inc2 x => Inc1 (x + 1)
crl [Inc1b]   :  Inc1 x => Inc1 x  if not(x < 200)
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crl [Reset1a]  :  Reset1 x => Reset2 x  if x = 200
rl  [Reset2]   :  Reset2 x => Reset1 0
crl [Reset1b]  :  Reset1 x => Reset1 x  if not(x = 200)
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eq initial = { Dec1 Inc1 Reset1 199 }
```

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eq initial = { Dec1 Inc1 Reset1 199 }


ceq (S1 S2 S3 x |= counterge0) = true  if (0 = x \/ 0 < x)
ceq (S1 S2 S3 x |= counterlt0) = true  if (x < 0)
ceq (S1 S2 S3 x |= counterle200) = true  if (x < 200 \/ x = 200)
```

Maude output (simplified)

```
Maude> red modelCheck(initial, <> counterlt0)
reduce in COUNTERS-CHECK : modelCheck(initial, <> counterlt0)
result ModelCheckResult:
result Bool :  true
```

Maude output (simplified)

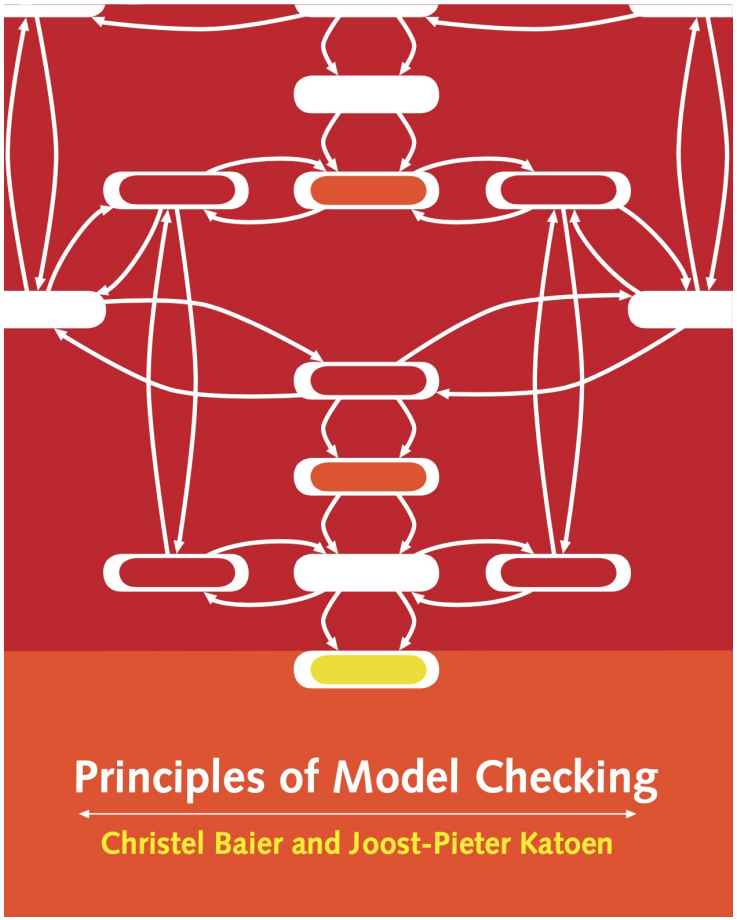
```
Maude> red modelCheck(initial, <> counterlt0)
reduce in COUNTERS-CHECK : modelCheck(initial, <> counterlt0)
result ModelCheckResult:
result Bool :  true


Maude> red modelCheck(initial, [](counterge0 /\counterle200)
reduce in COUNTERS-CHECK :
              modelCheck(initial, [](counterge0 /\counterle200)
result ModelCheckResult:
counterexample({Inc1 Dec1 Reset1 199}
               {Inc2 Dec1 Reset1 199}
               {Inc1 Dec1 Reset1 200}
               {Inc1 Dec2 Reset1 200}
               {Inc1 Dec2 Reset2 200}
               {Inc1 Dec2 Reset1 0}
               {Inc1 Dec1 Reset1 -1})
```

Topics of the course

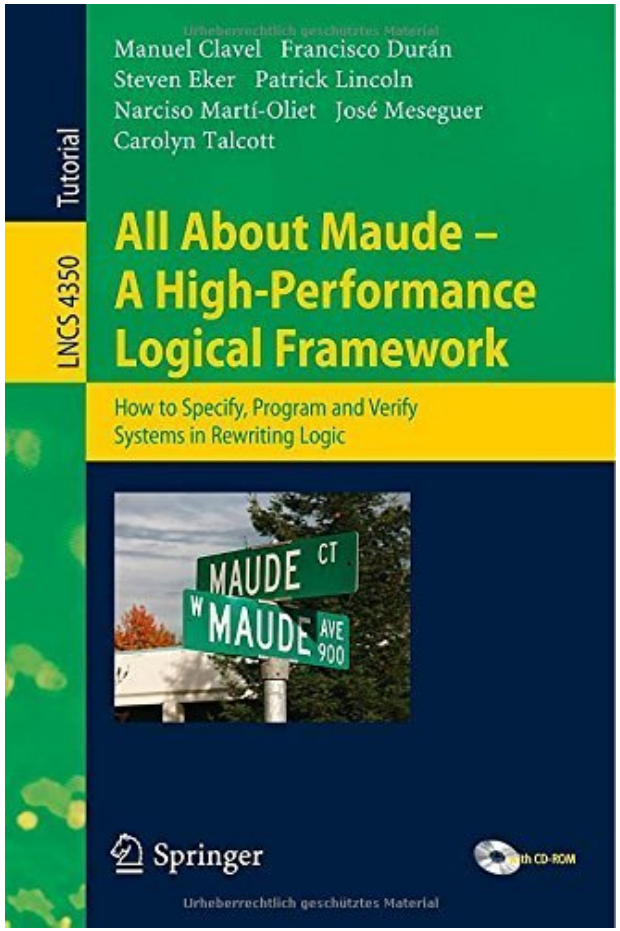
- modeling systems by labeled transition systems (LTSs)
- safety, liveness, and fairness properties
- Linear Temporal Logic (LTL)
  - model checking formulas
    - express properties by Büchi automata
    - model check LTSs and properties via product automata
- Computation Tree Logic (CTL)
- partial model checking
  - partially known systems (state properties/states/transitions)
- learning Maude and its model-checker

Book



- pdf available:
  https://is.ifmo.ru/books/_principles_of_model_checking.pdf

Book Maude



- pdf available:
  https://maude.cs.illinois.edu/w/images/0/0d/Maude-book.pdf

Organization

Lectures (Emilio 3 / Clemens 4)

- January 19 – January 28 (7 meetings)
- blackboard presentations
- notes after the lecture (last year's notes available)
- Maude examples at the end of each lecture

Webpage

- https://clegra.github.io/mc/mc.html

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Exam

- options:
  - verification project (of an algorithm, etc.) in Maude
  - presentation about a paper
  - written exam

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Thank you – we are looking forward to the course!