Introduction to Model Checking

https://clegra.github.io/mc.html

Lecture 1

Clemens Grabmayer
https://clegra.github.io

Emilio Tuosto
https://cs.gssi.it/emilio.tuosto/

Department of Computer Science

GRAN SASSO SCIENCE INSTITUTE
SCHOOL OF ADVANCED STUDIES
Scuola Universitaria Superiore

January 26, 2026

Organization

Lectures (Clemens 4 / Emilio 3)

- this week:
  - Monday, 10.30–12.30, room B, Aurora
  - Tuesday, Thursday, Friday, 14.30–16.30, conference room -1, Zenith

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Exam (more later)

- options:
  - small verification project (of an algorithm, e.g. in Maude)
  - presentation about a paper
  - written exam?

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- (partial model checking)
  - (partially known systems (state properties/states/transitions))

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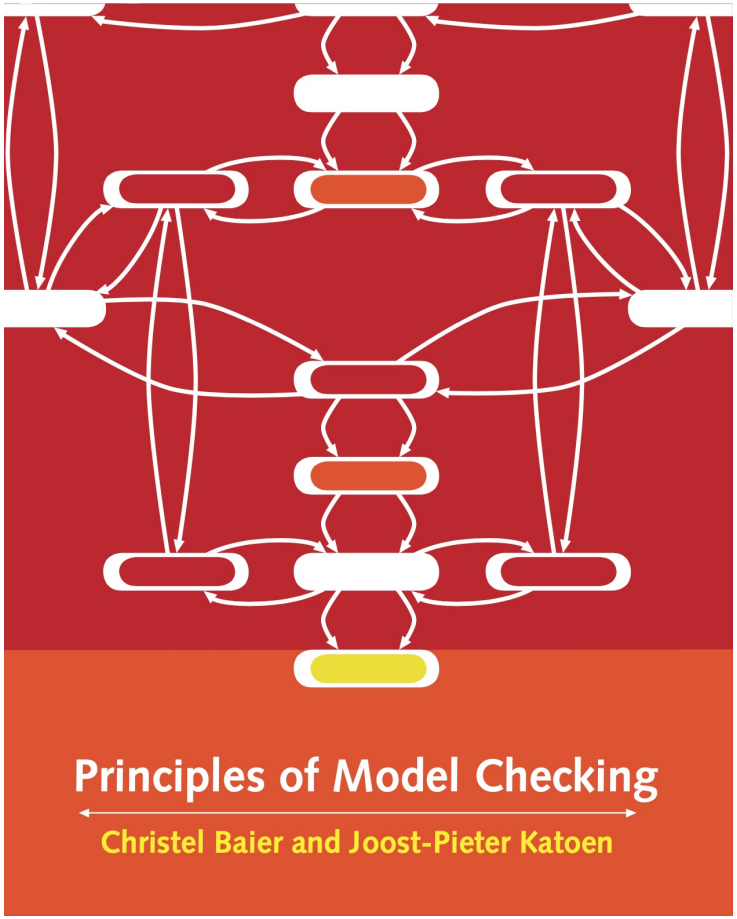
Course Goals are introduction to:

- Theory:
  - modeling systems by labeled transition systems,
  - expressing properties by temporal-logic formulas
  - model-checking algorithms
- Practice related: see Maude examples

Book



- pdf online available (see [1])
- we study chapters 1–3, 5, 6

Lectures

1. Introduction (preview extended, LTSs)
2. Modeling by labeled transition systems
   - executions; traces; non-determinism; examples
3. Linear-Time Behaviour and Properties
   - invariant, safety, and liveness (and fairness) properties
4. Linear Temporal Logic (LTL)
   - syntax and semantics; interpretation of LTSs; examples
5. LTL (continued)
   - model checking of LTL formulas, and fairness in LTL
   - Maude examples
6. Computation Tree Logic (CTL)
   - syntax and semantics, examples
7. Extensions of CTL, and Outlook
   - expressibility differences with LTL
   - model checking formulas in CTL
   - (μ-calculus | partial model-checking | Maude examples)

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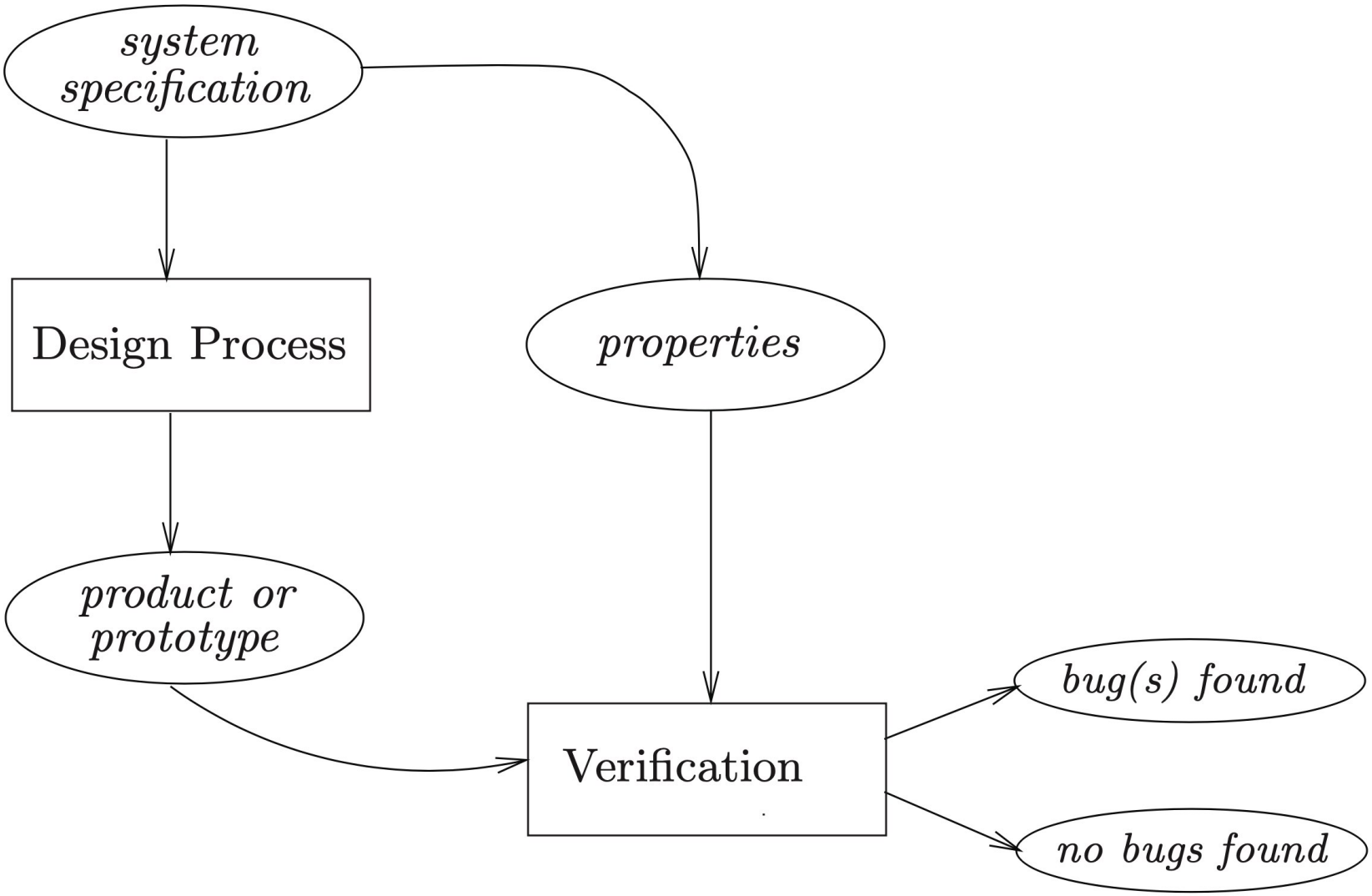
Hard-/Software Verification (traditionally)



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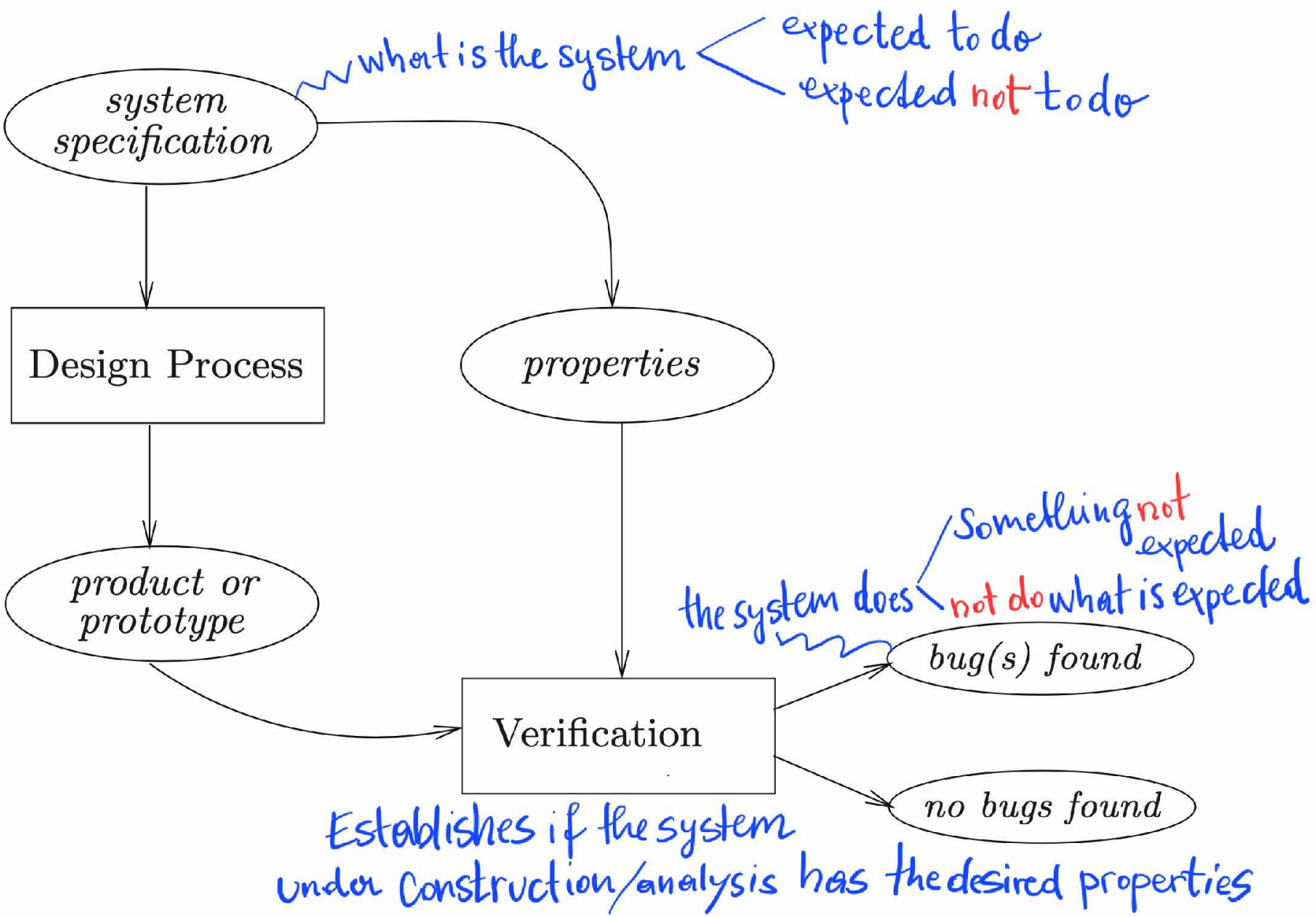

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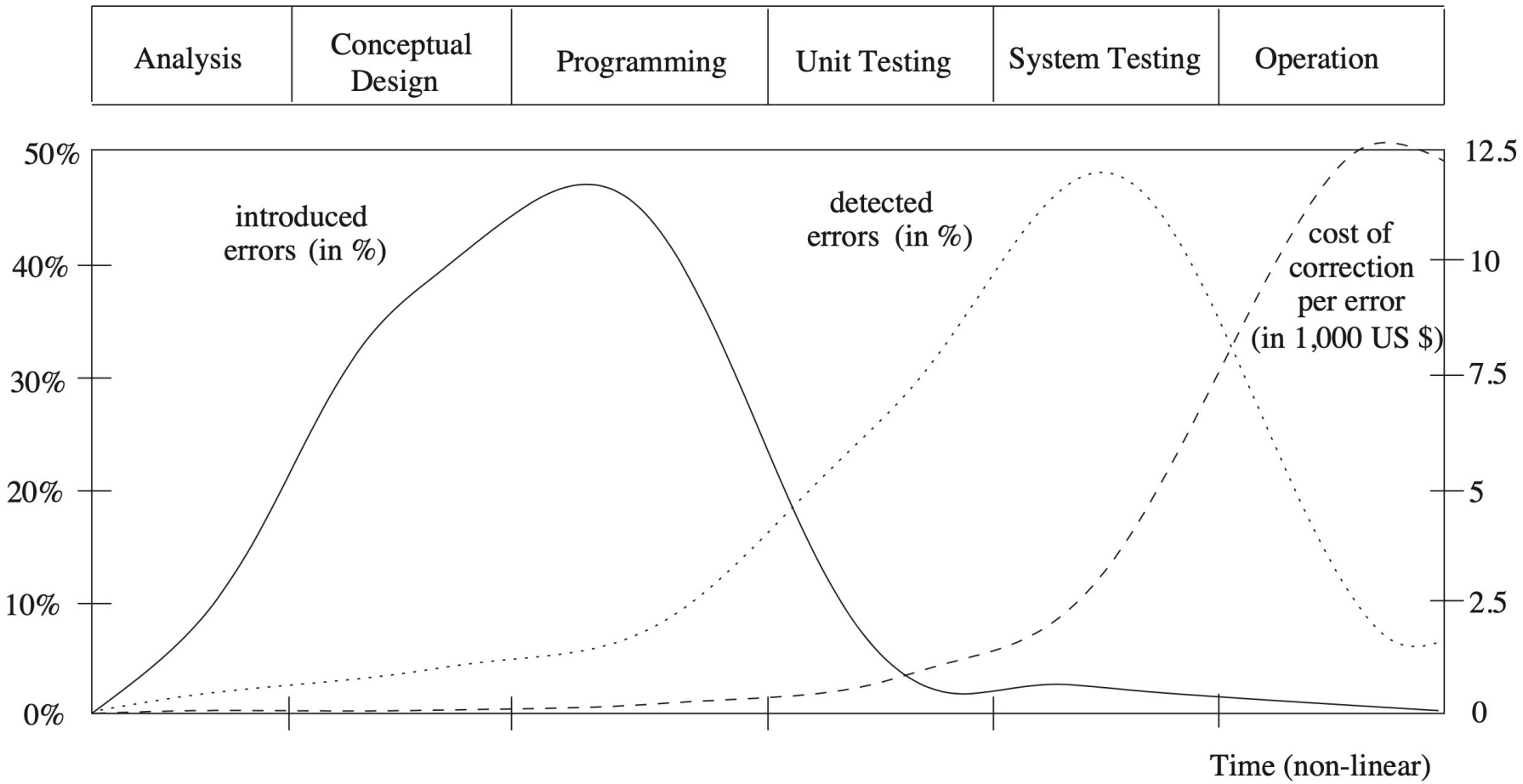
- Catching software errors: the sooner the better.

Software lifecycle, error introduction/detection, repair costs



Liggesmeyer et al: "Qualitätssicherung technischer Systeme . . ." [4, 1998]

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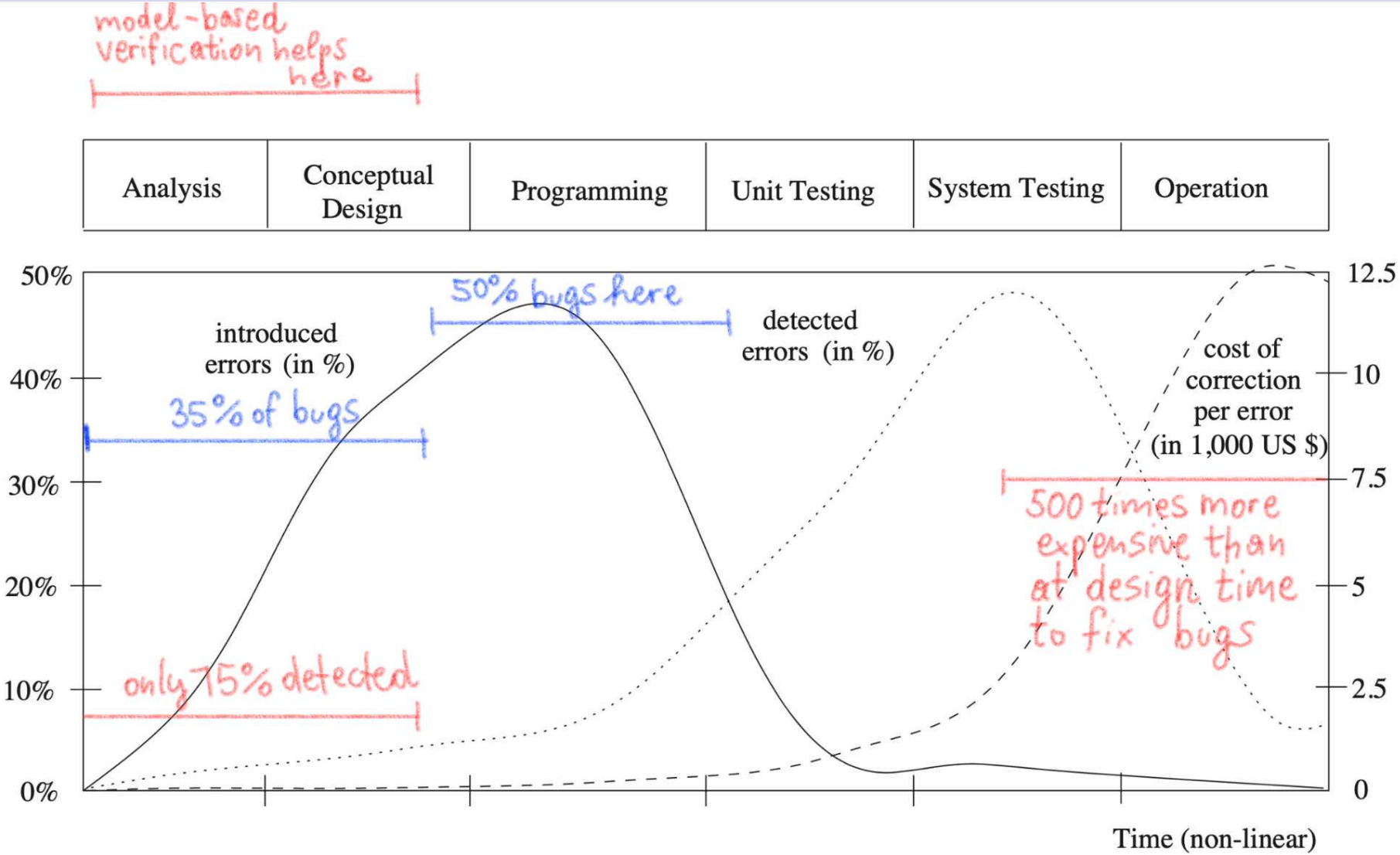

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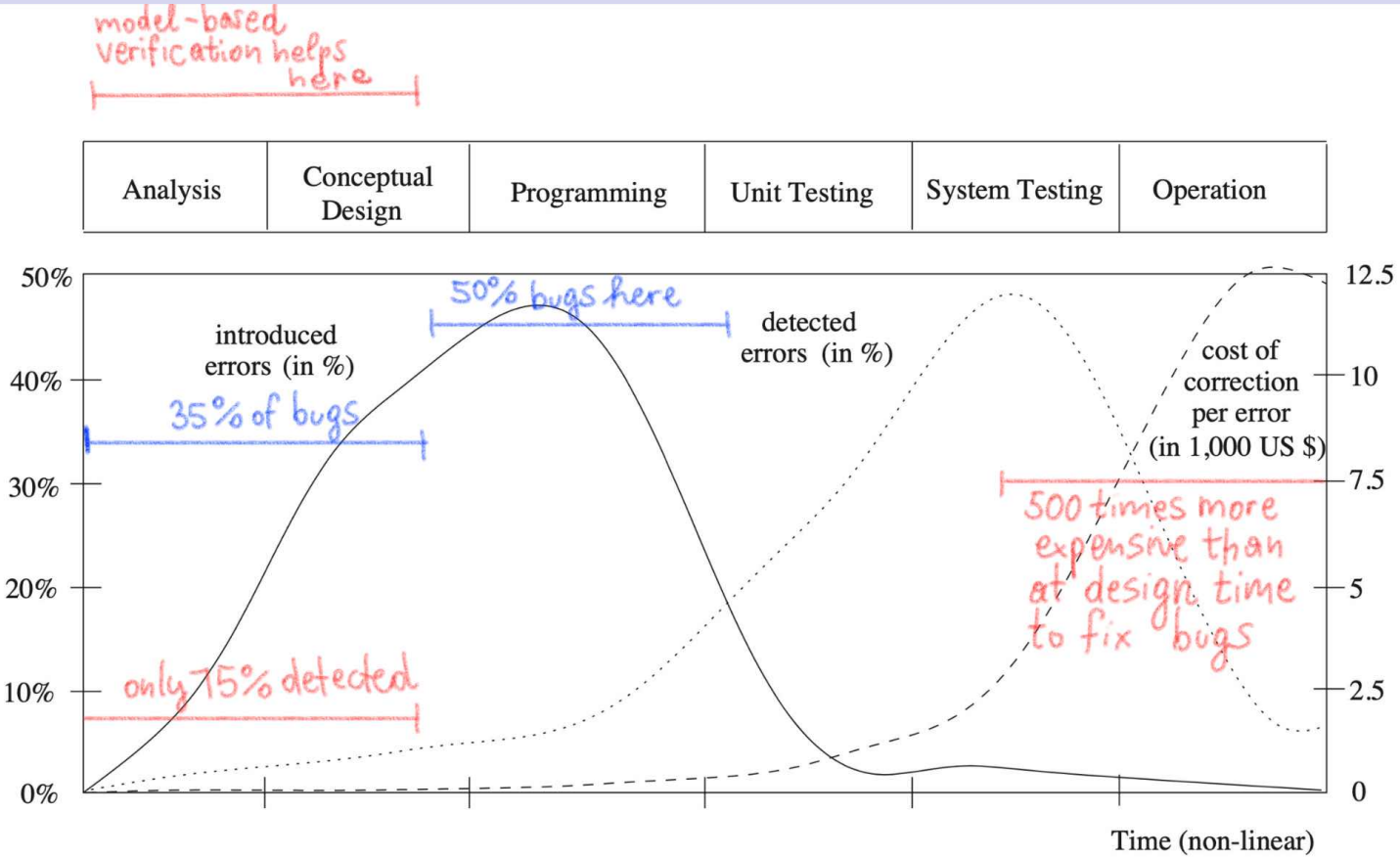
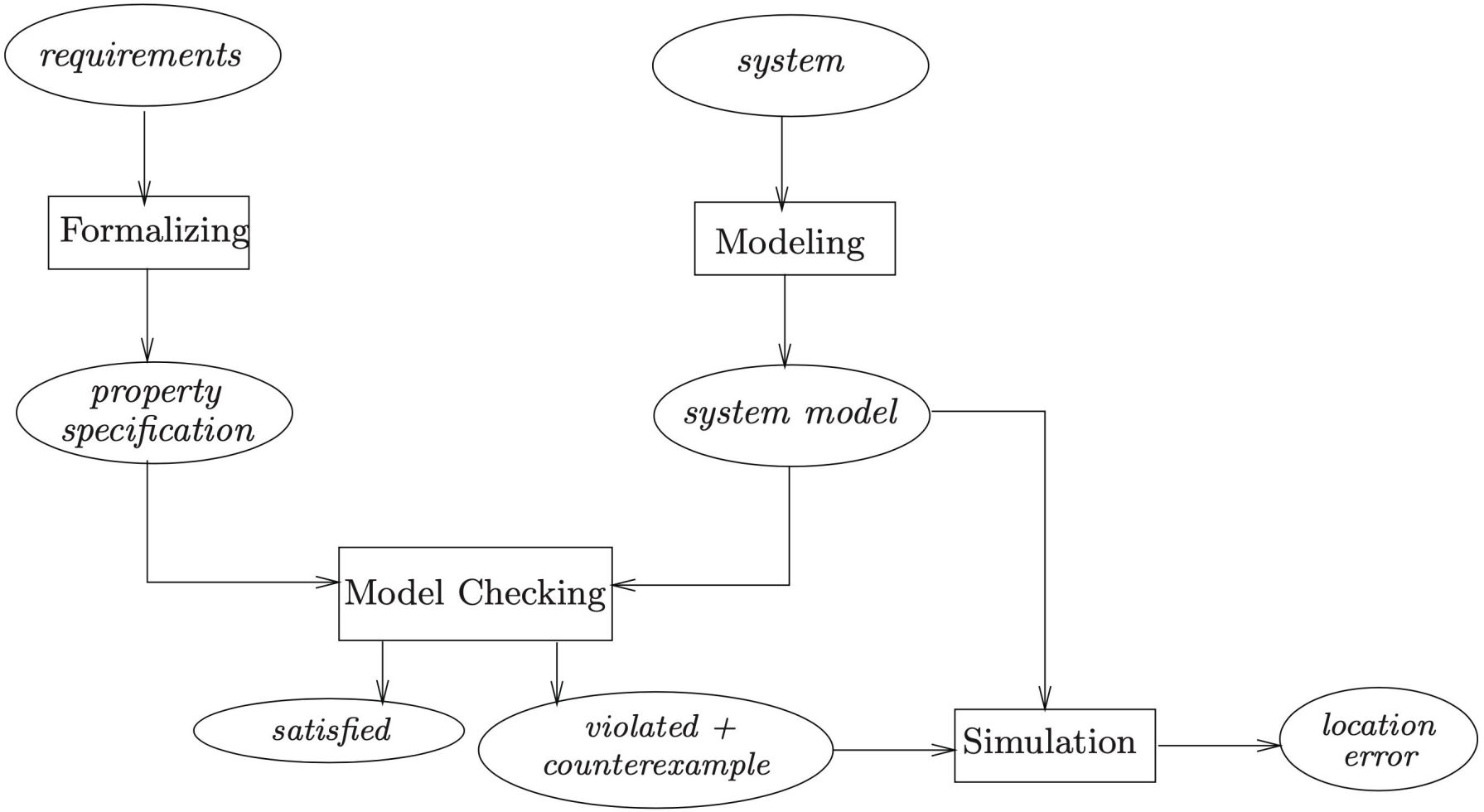


"In software and hardware design of complex systems, more time and effort are spent on verification than on construction. Techniques are sought to reduce and ease the verification efforts while increasing their coverage. Formal methods offer a large potential to obtain an early integration of verification in the design process, to provide more effective verification techniques, and to reduce the verification time."

Model checking



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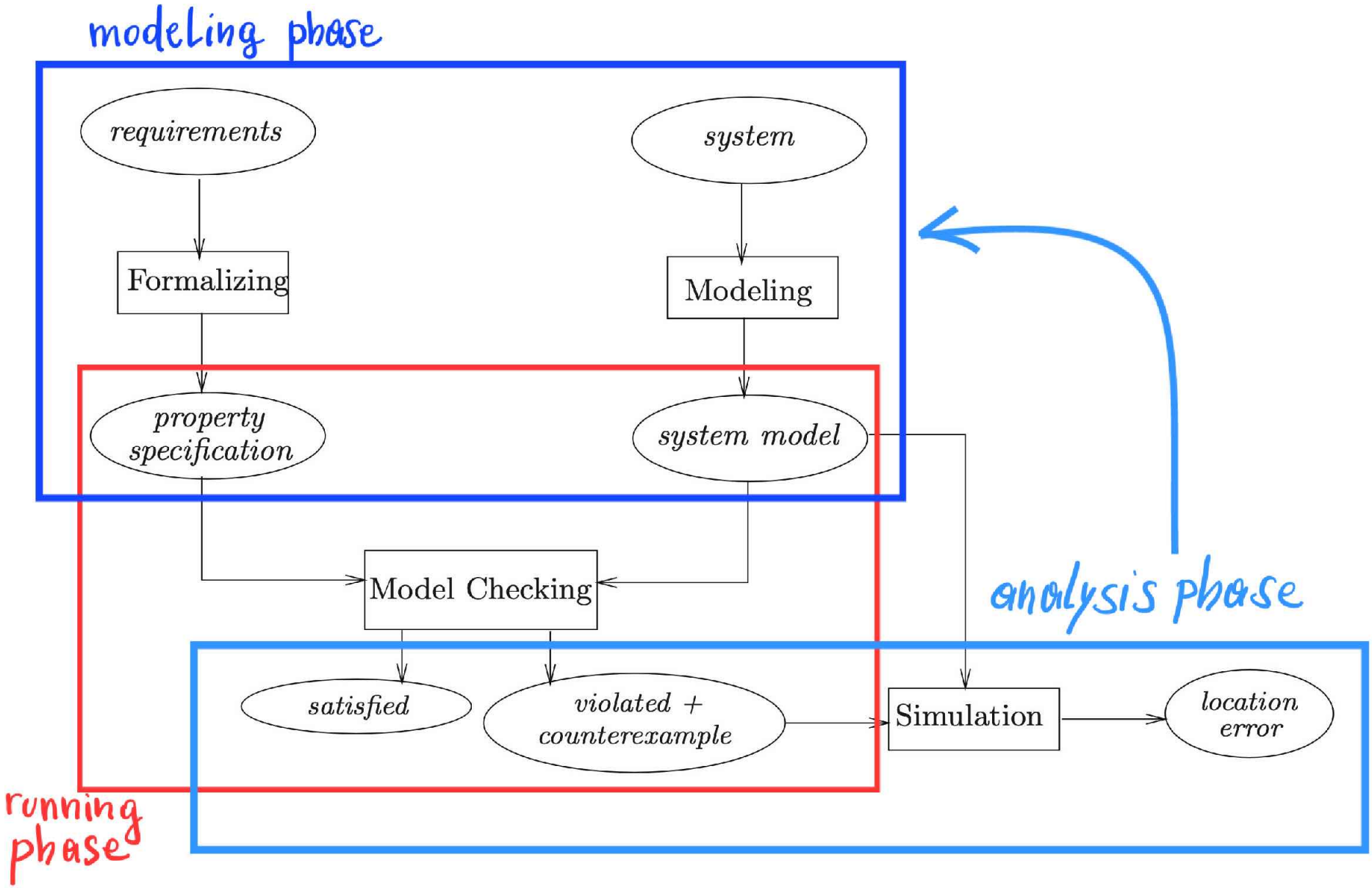

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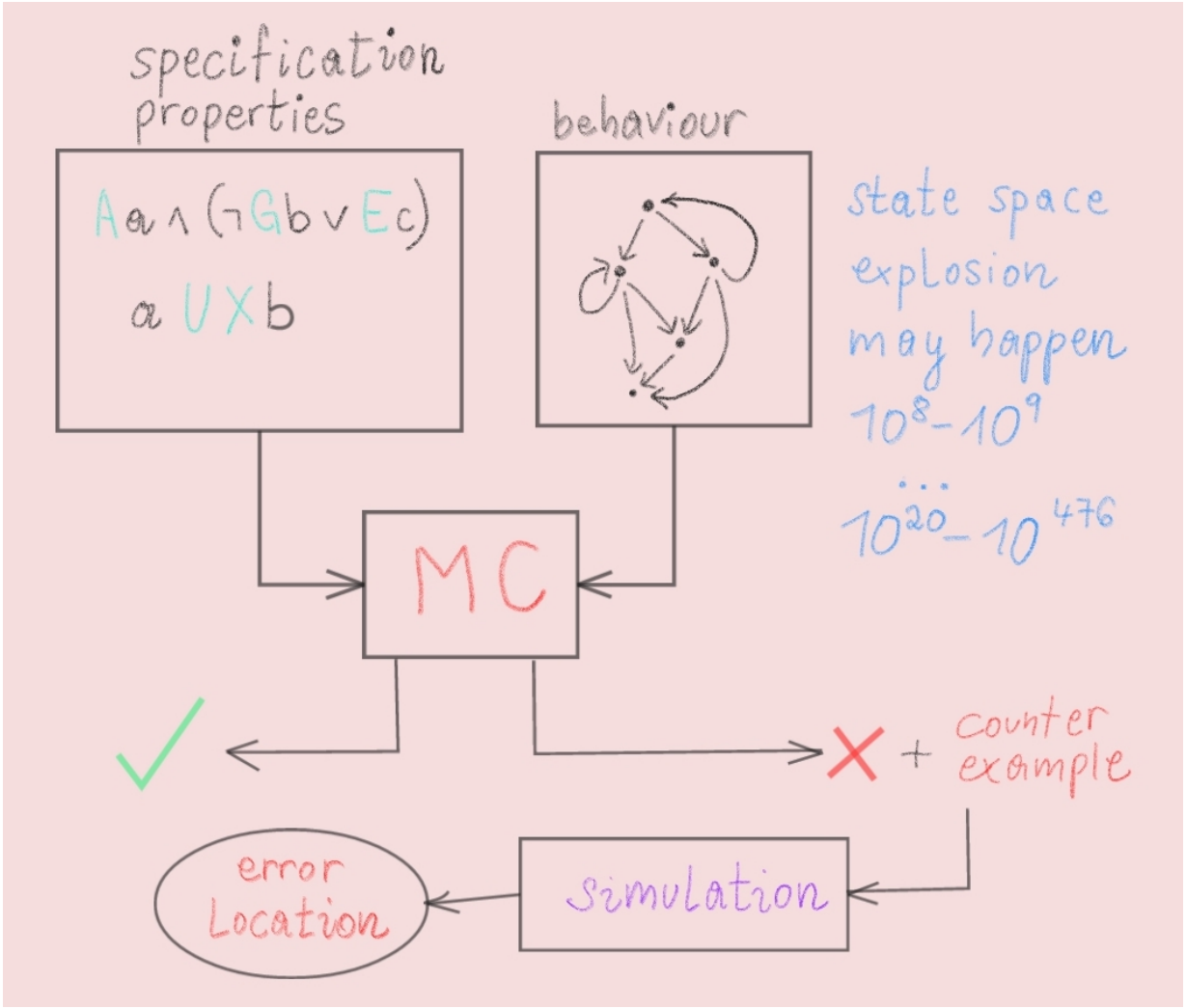
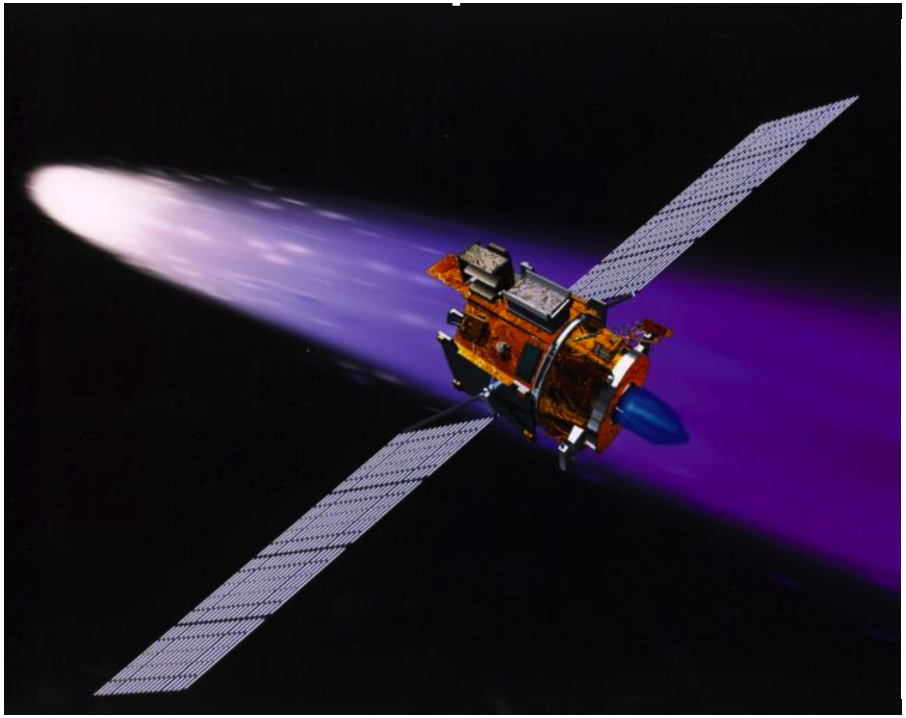
$\Box(\neg(a \wedge b)) \quad \hat{=} \quad$ for all time moments, events a and b do not occur at the same time,

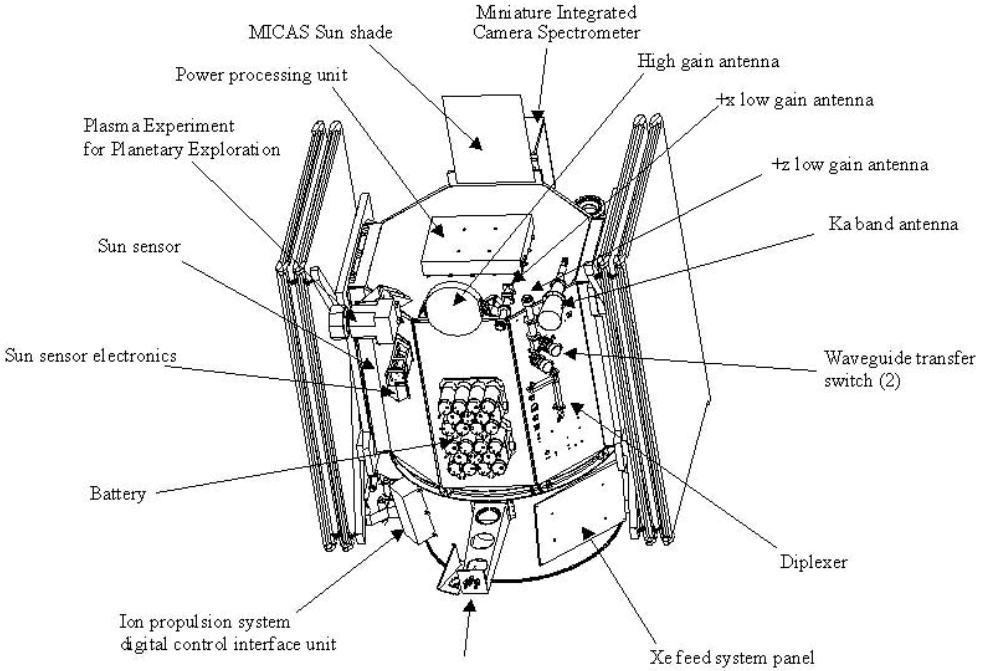
$= \quad$ it will never happen that events a and b occur at the same time.

Glancing at temporal logic (its use for model checking)



Deep Space 1 (NASA)





- Flyby of asteroid 9969 Braille (1999)
- Entered the coma of Comet Borrelly (2001)
- Model checking discovered 5 concurrency errors

Example (program concurrency/non-determinism)

Programs Inc, Dec, and Reset cooperate, and use a shared variable x:

proc Inc
  while $0 \leq x \leq 200$
    do
      if $x < 200$
        then $x := x + 1$
      fi
    od

proc Dec
  while $0 \leq x \leq 200$
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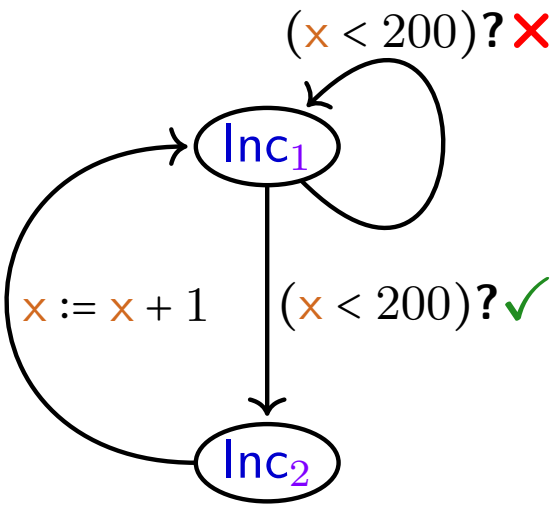
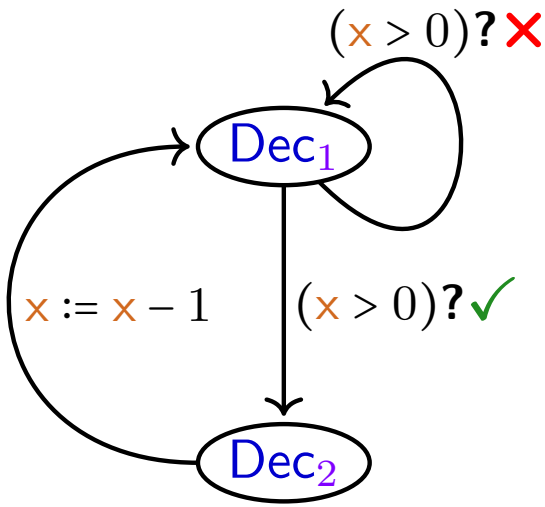
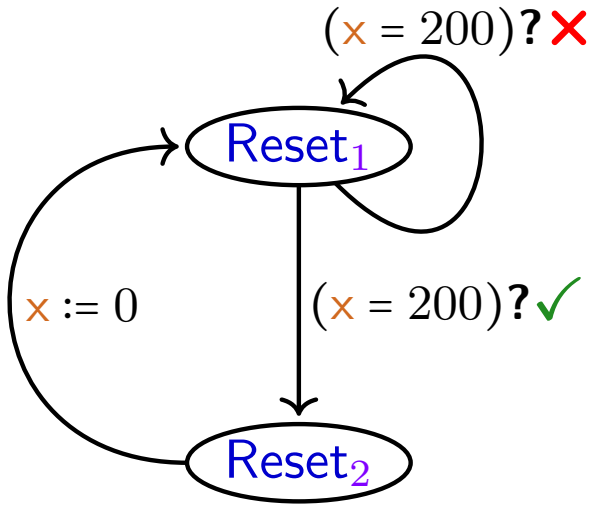
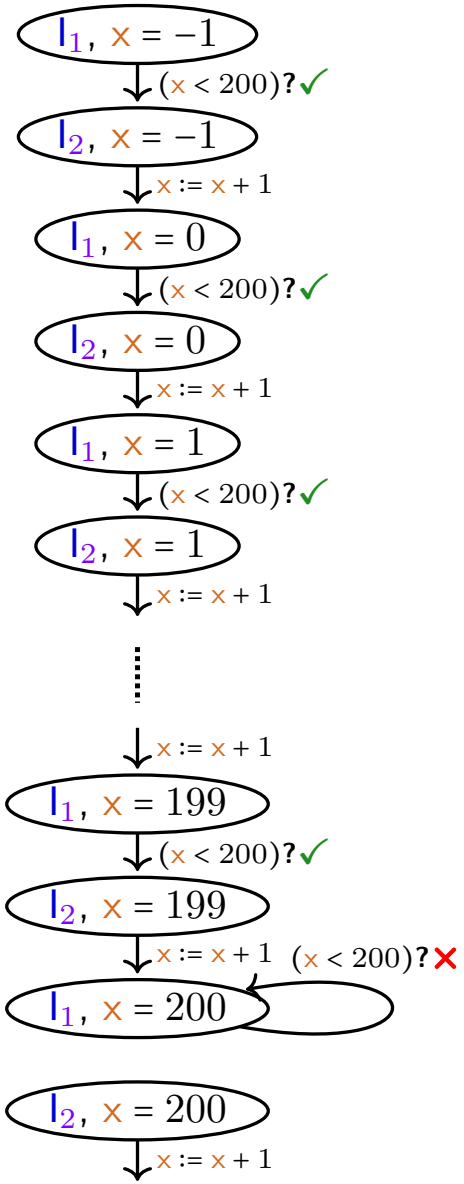






Program graphs (PG)

Modeling (by labeled transition systems, with state space explosion)



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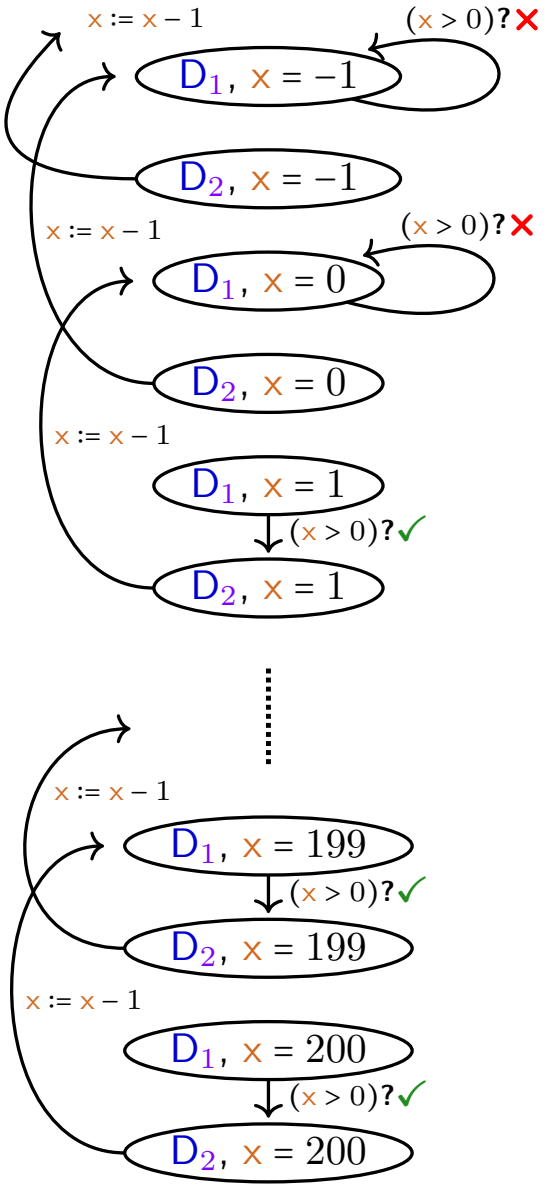

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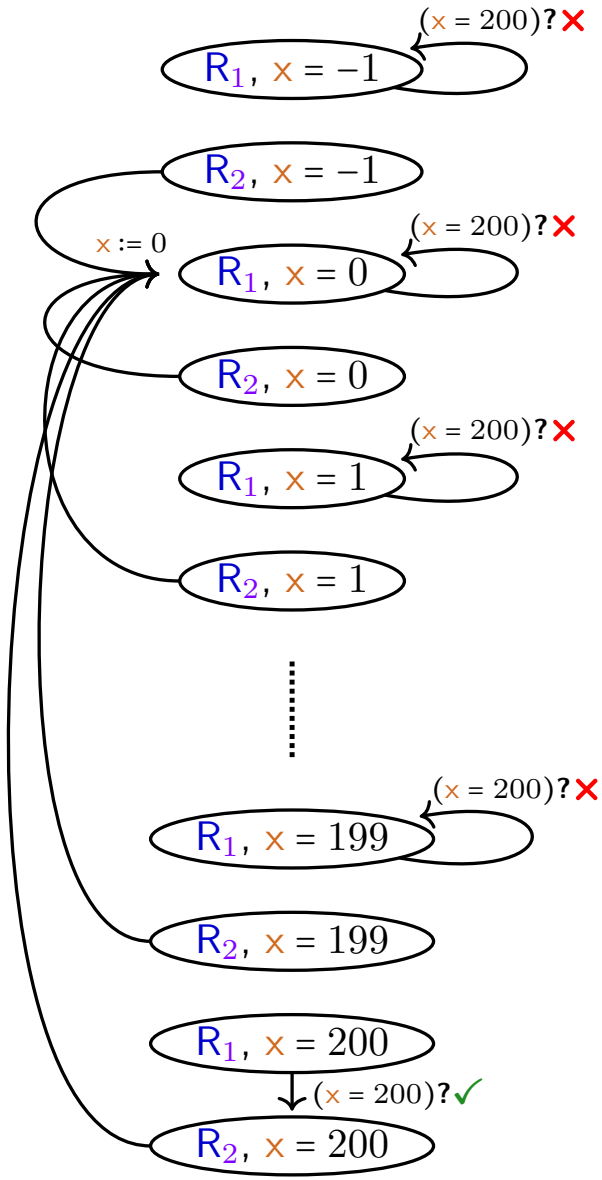

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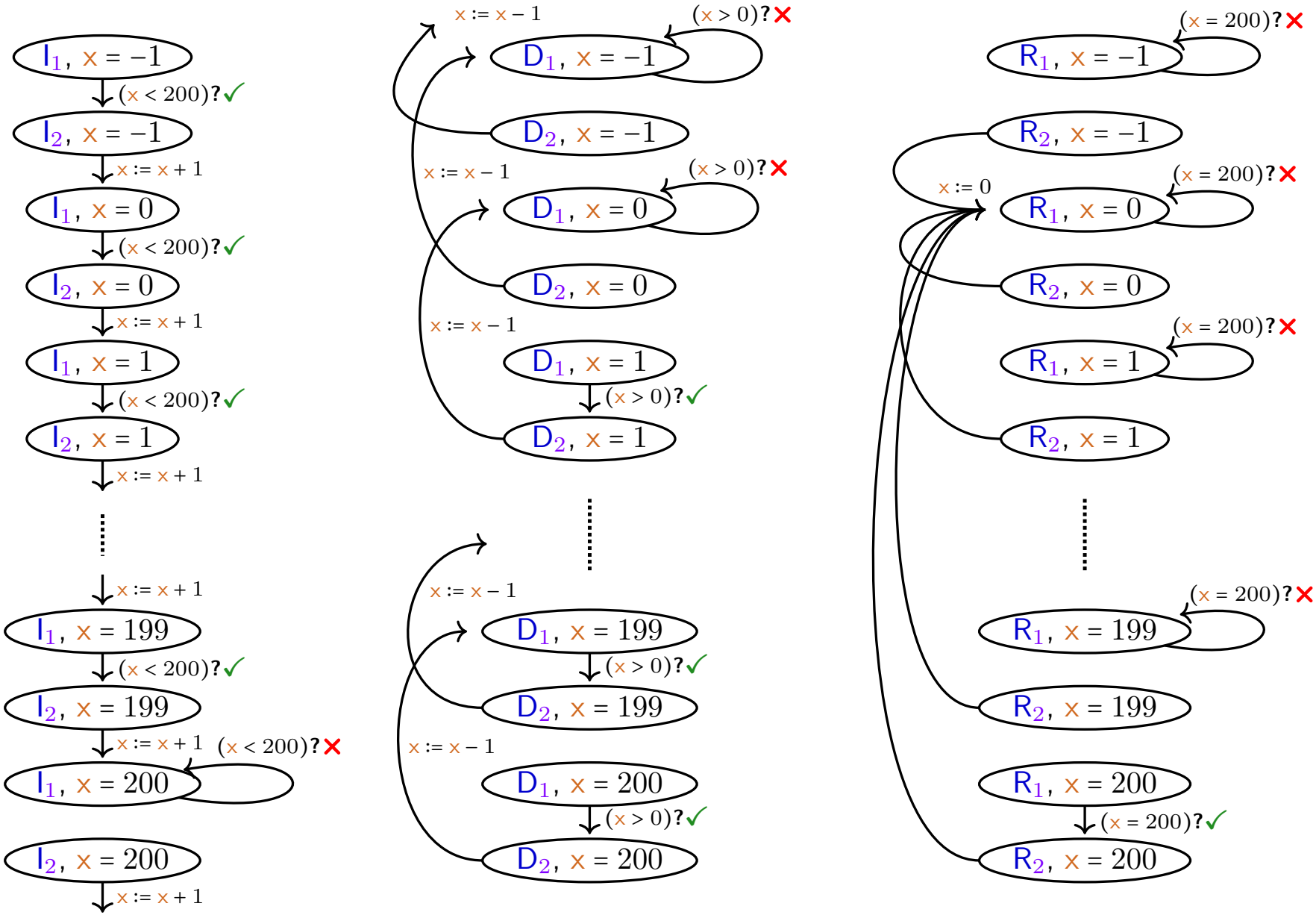
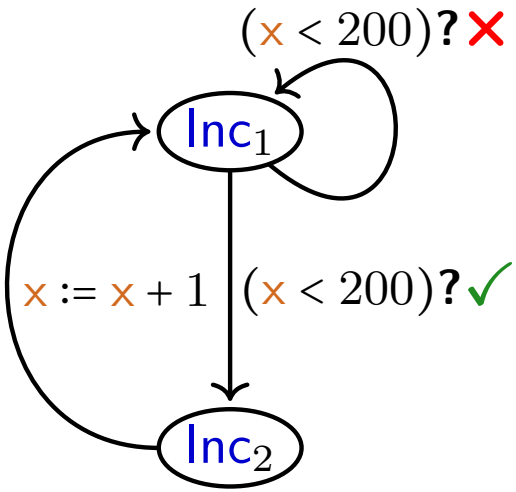
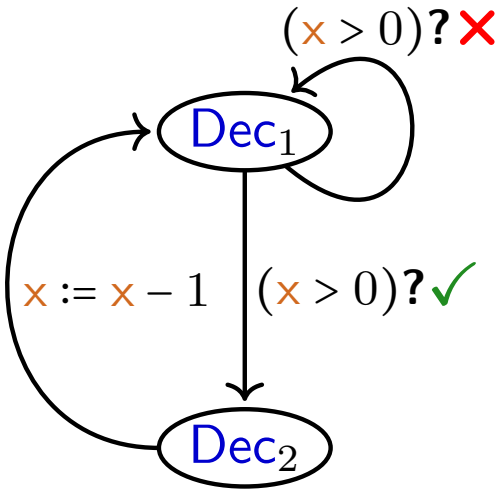
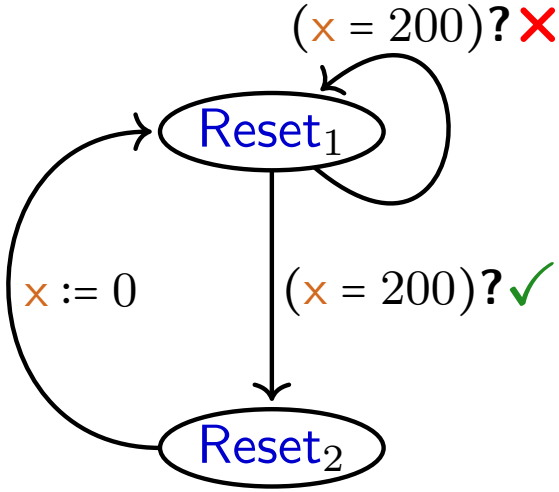


Formalizing properties (in temporal logic)







We assume $x := 0$ initially.

$\mathsf{Inc}_1 \parallel \mathsf{Dec}_1 \parallel \mathsf{Reset}_1 \stackrel{?}{\models} \Box(0 \leq x \wedge x \leq 200)$ (Linear-TL formula)

Counterexample (offending execution trace)

$$\langle x = 199 \,;\, \mathsf{Inc}_1 \parallel \mathsf{Dec}_1 \parallel \mathsf{Reset}_1 \rangle$$

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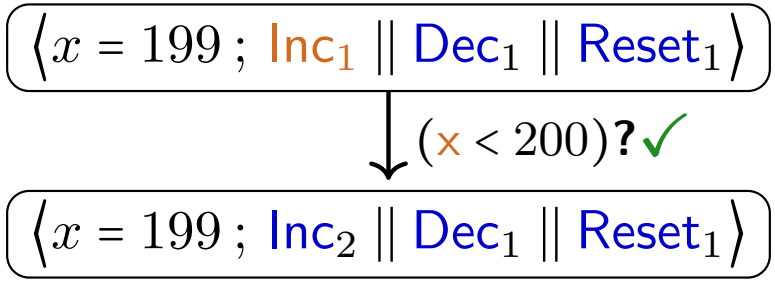

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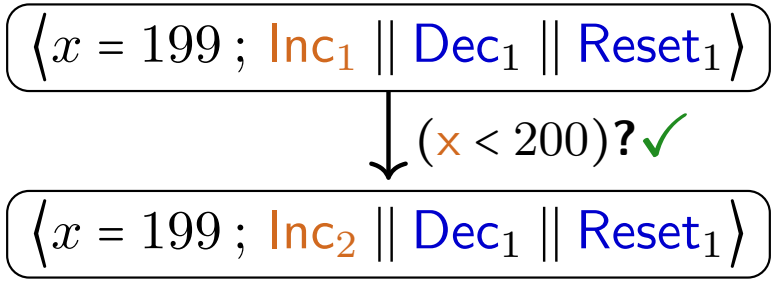

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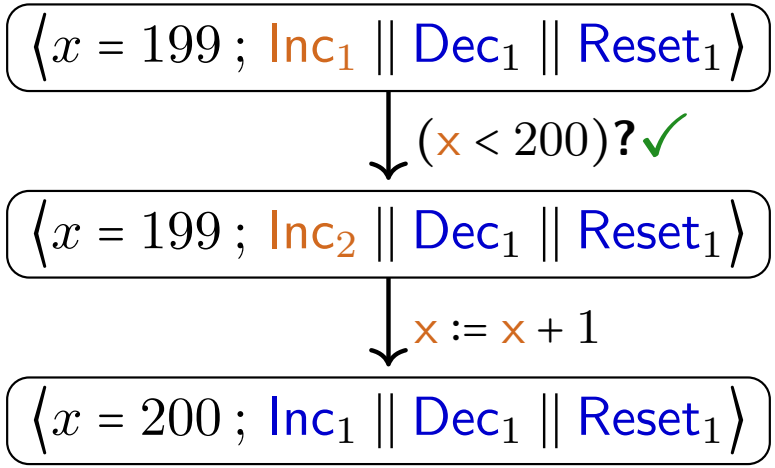

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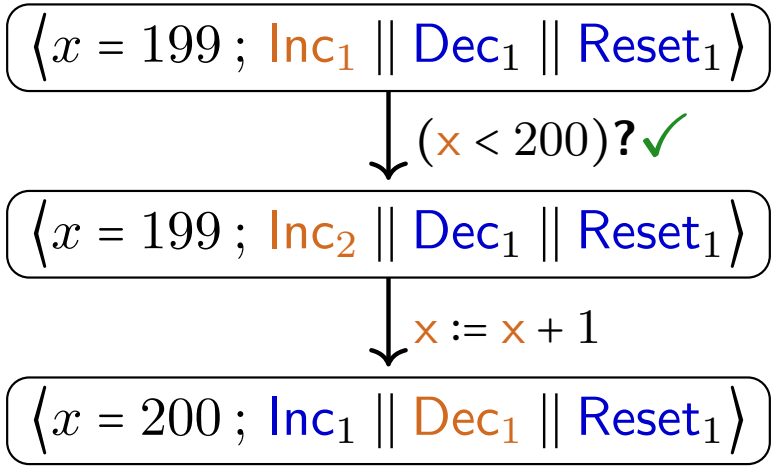

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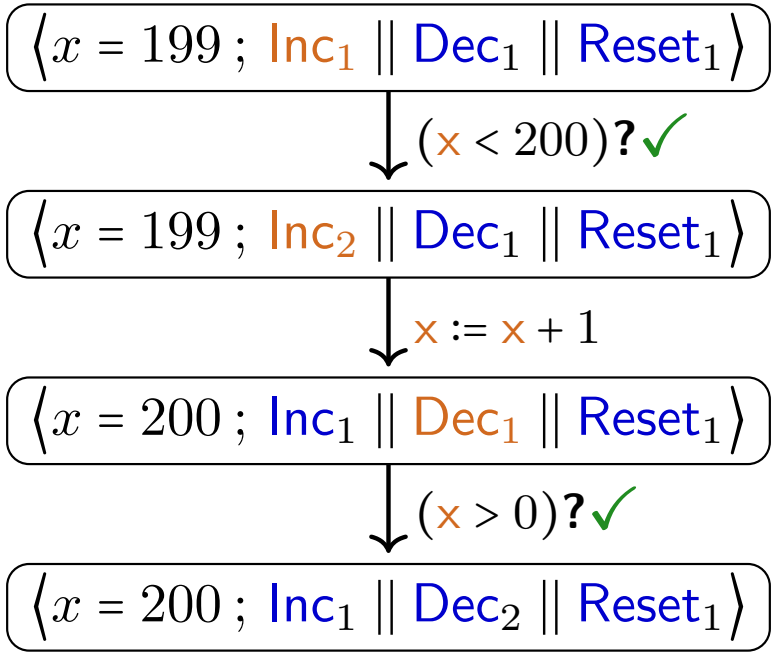

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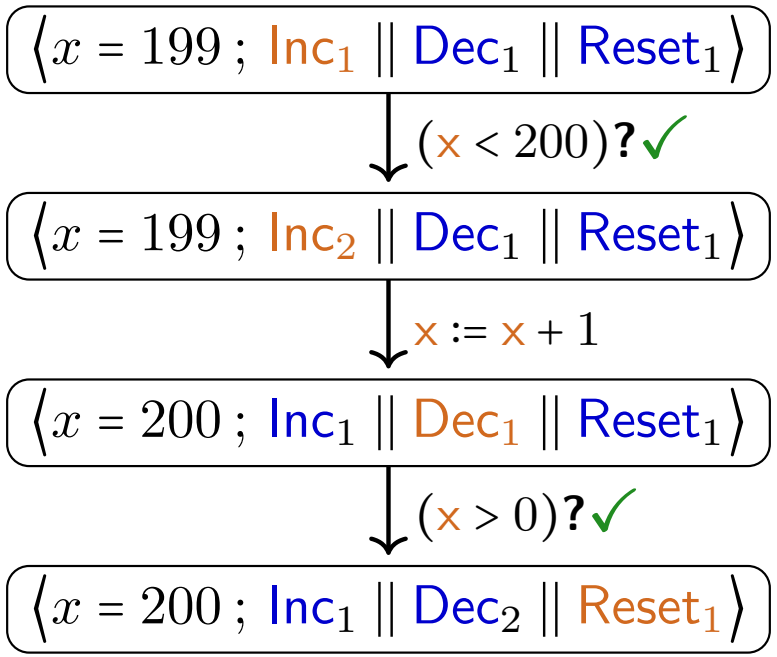
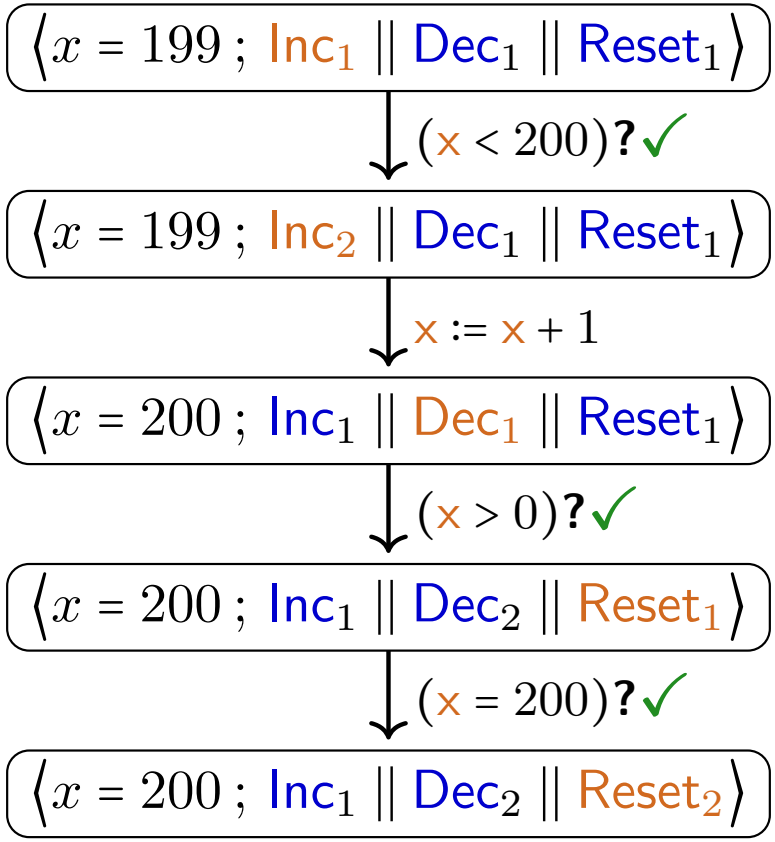


Counterexample (offending execution trace)



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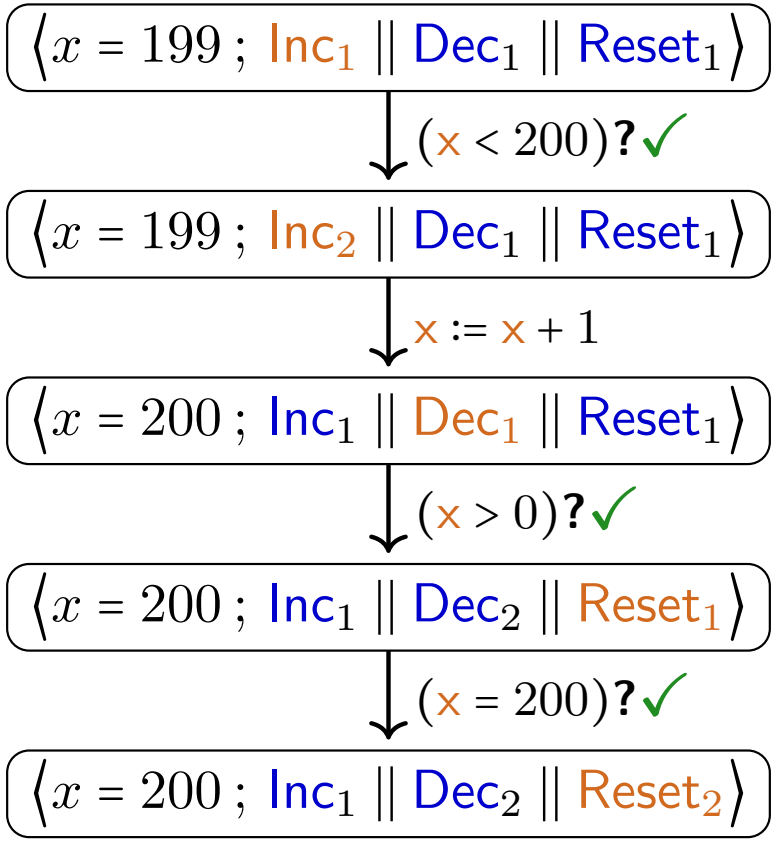

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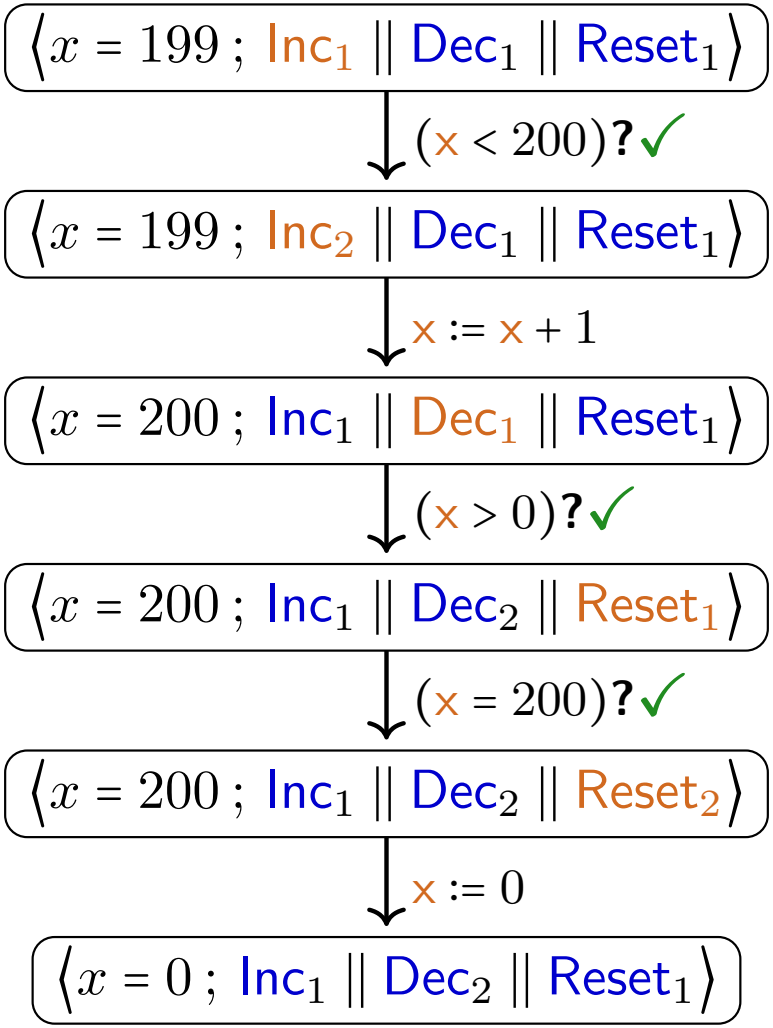

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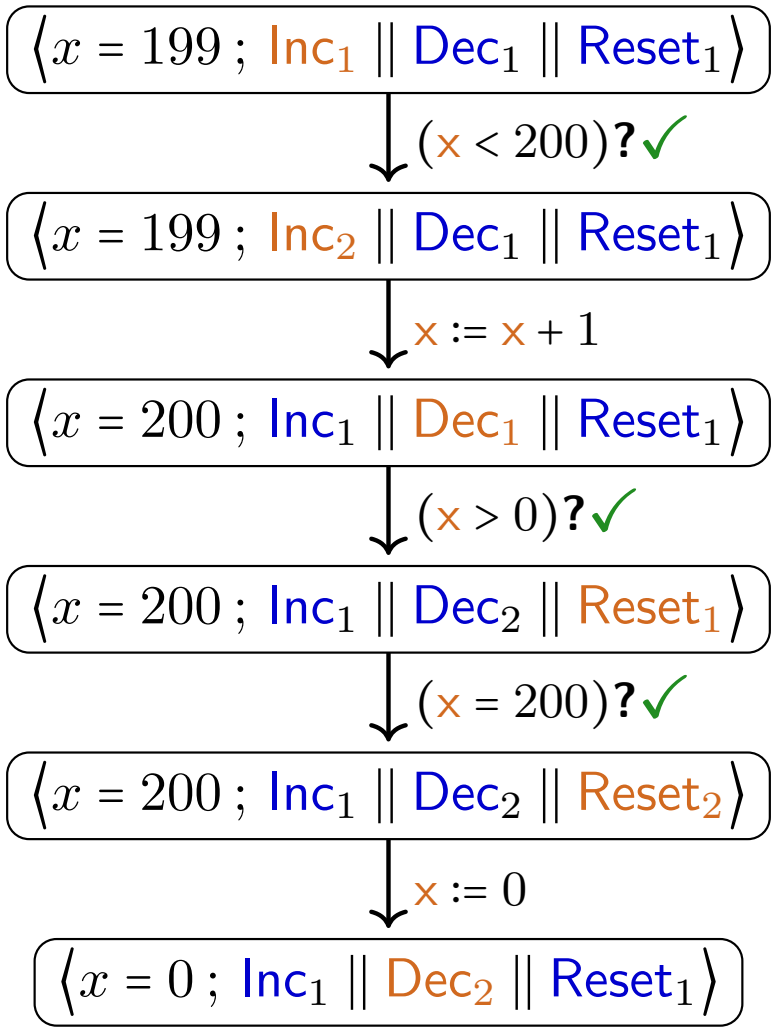

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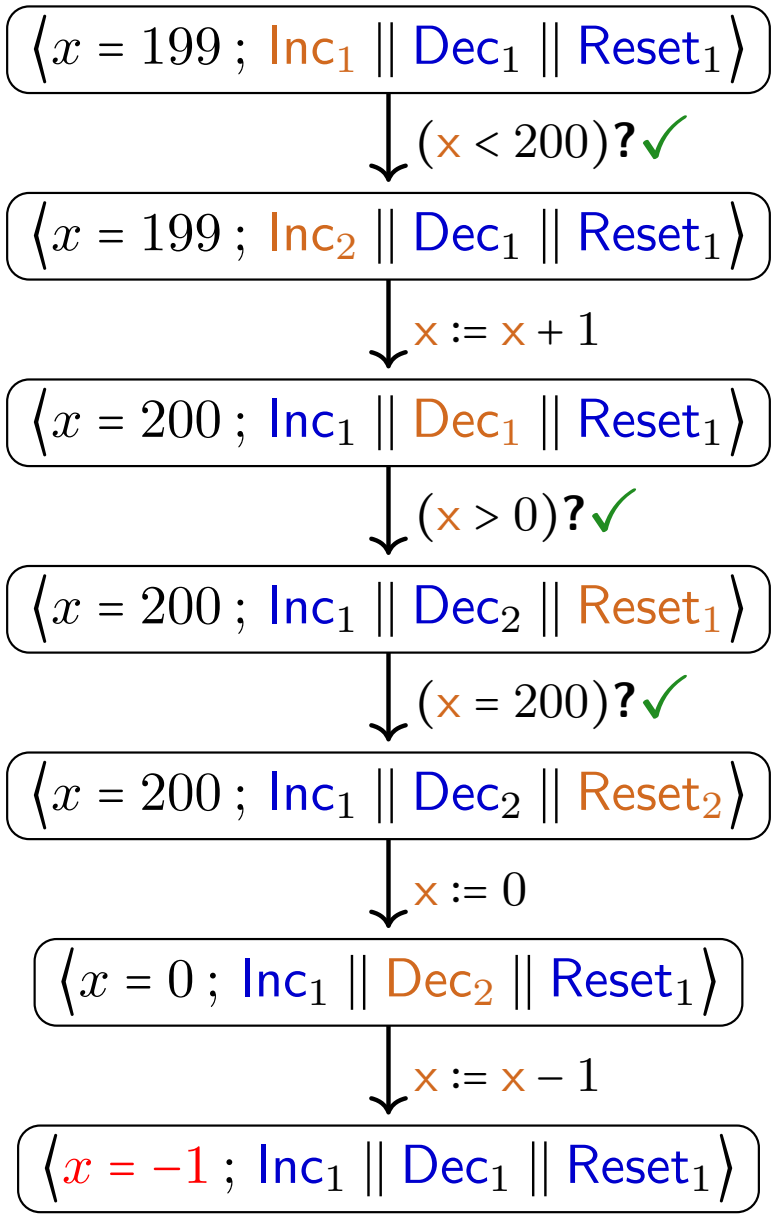
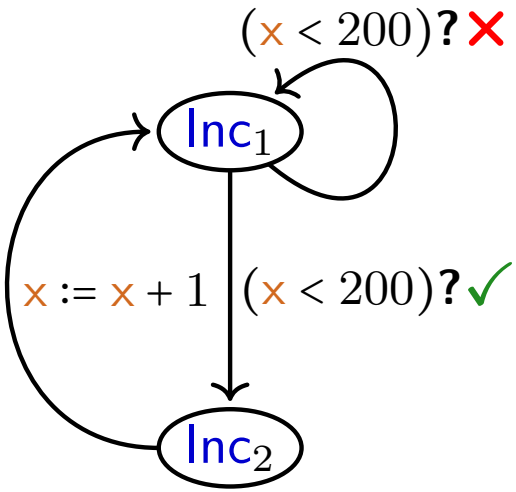
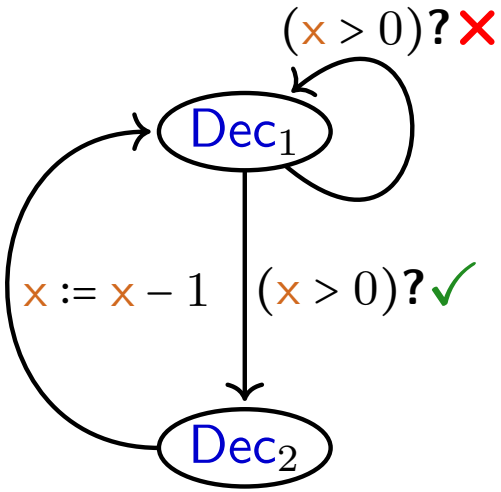
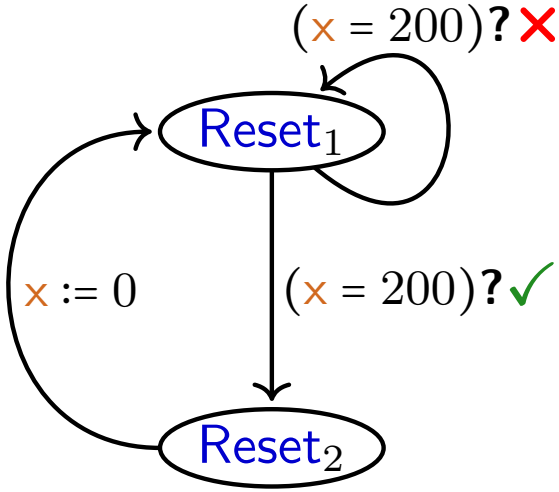


Formalizing properties (in temporal logic)







We assume $x := 0$ initially.

$\mathsf{Inc}_1 \parallel \mathsf{Dec}_1 \parallel \mathsf{Reset}_1 \stackrel{?}{\models} \Box(0 \leq x \wedge x \leq 200)$ (Linear-TL formula)

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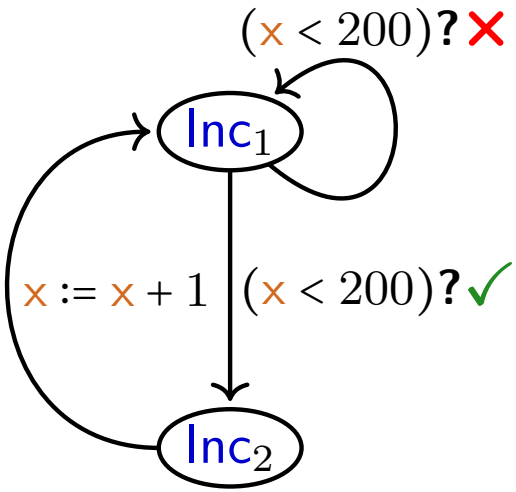

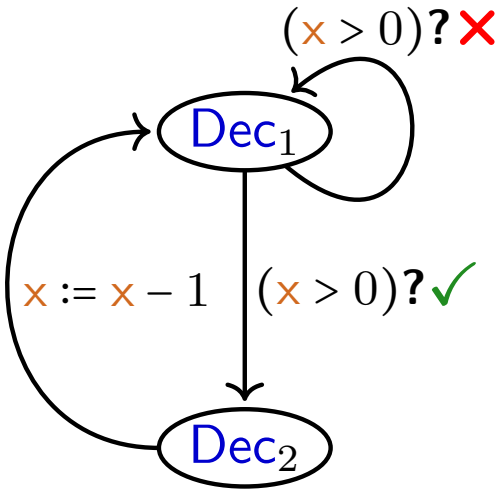

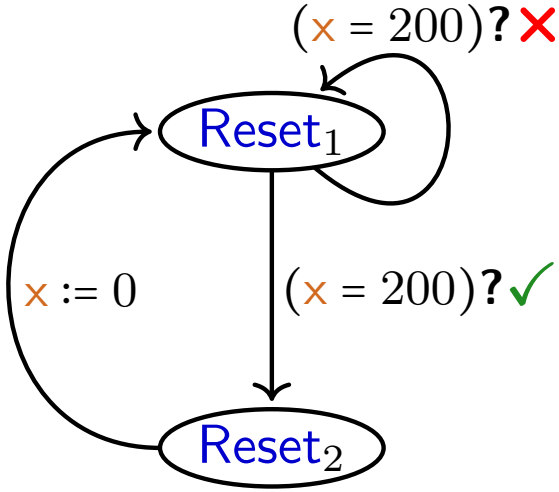

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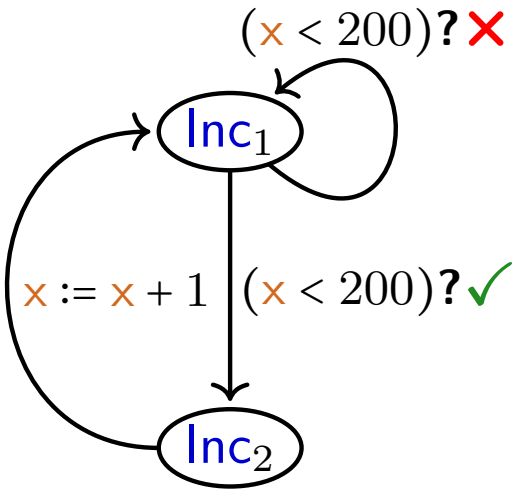
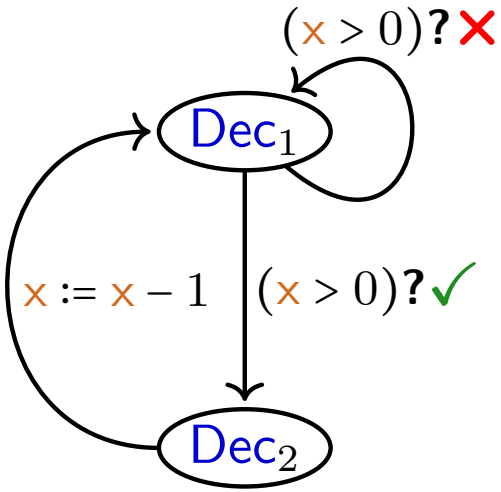
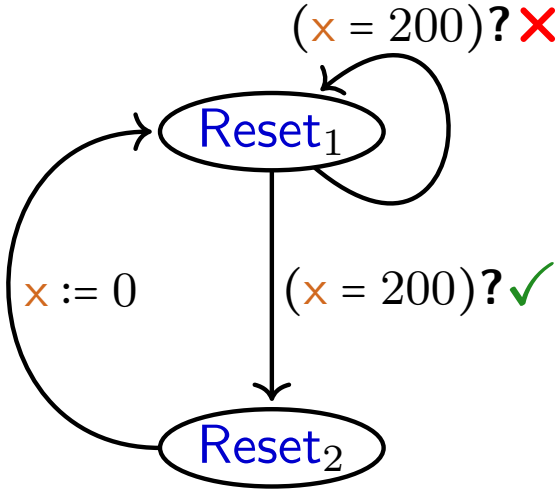
Formalizing properties (in temporal logic)







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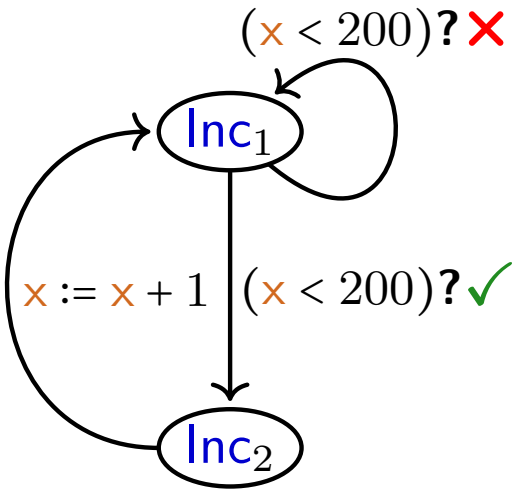
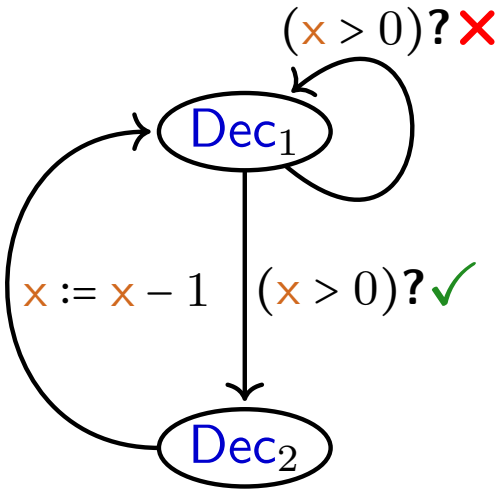
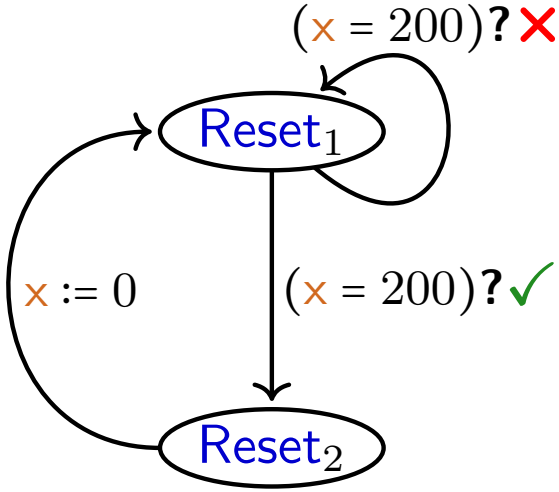
Formalizing properties (in temporal logic)







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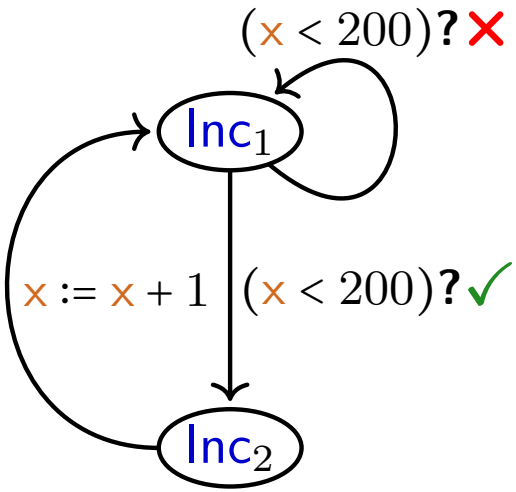

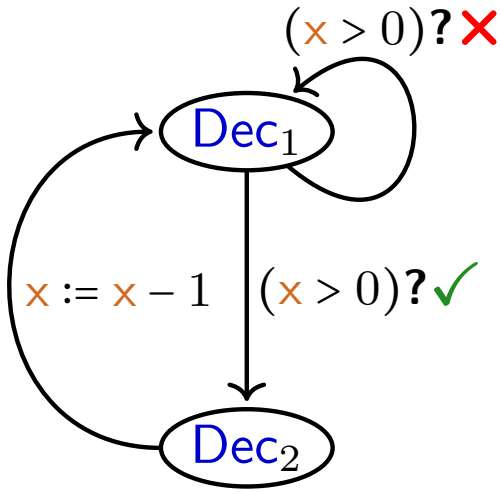

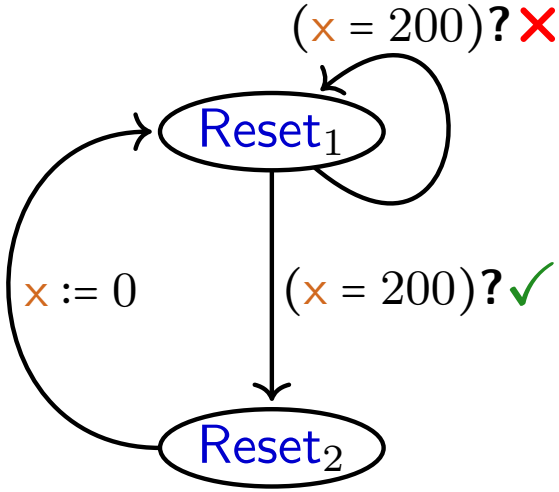

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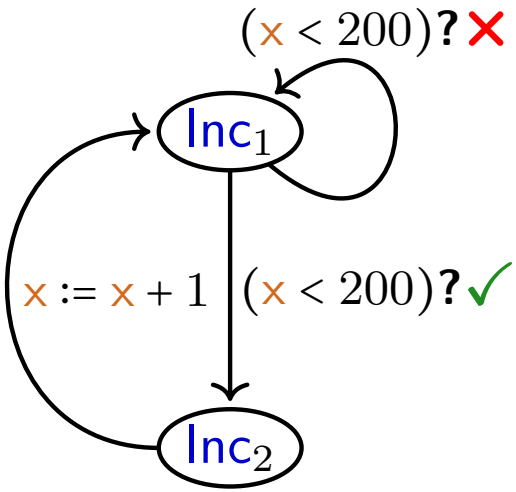
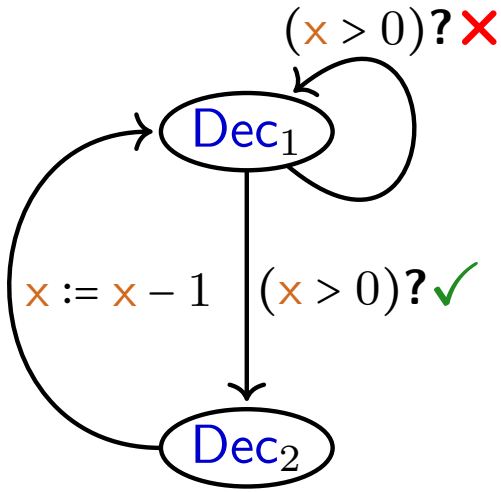
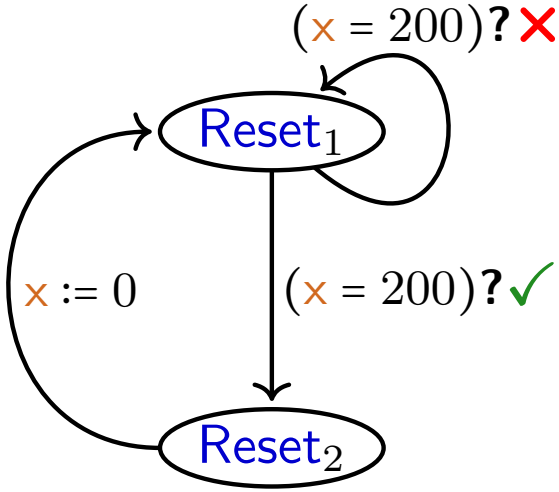
$\mathsf{Inc}_1 \parallel \mathsf{Dec}_1 \parallel \mathsf{Reset}_1 \quad \forall\Box(0 \leq x \wedge x \leq 200)$ (Computation-Tree-L formula)

Formalizing properties (in temporal logic)







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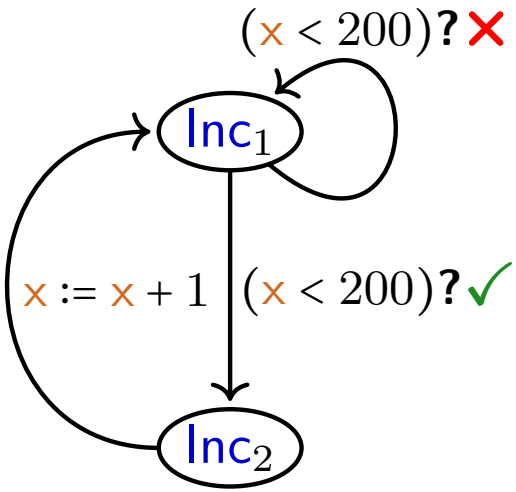
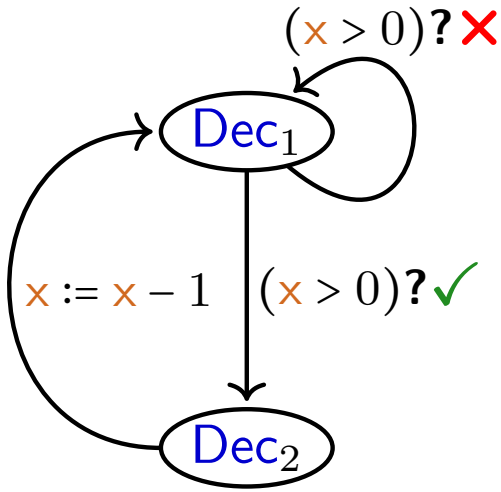
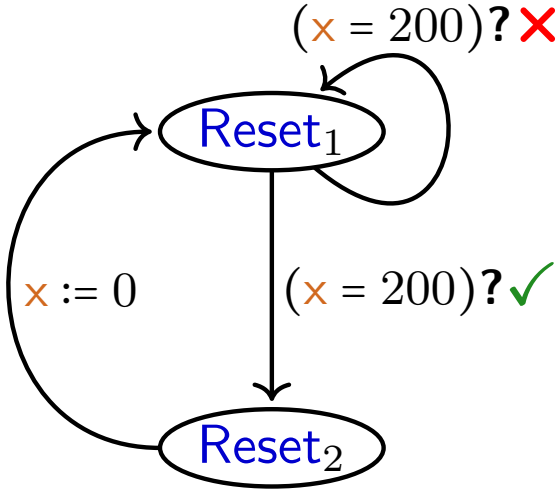
Formalizing properties (in temporal logic)







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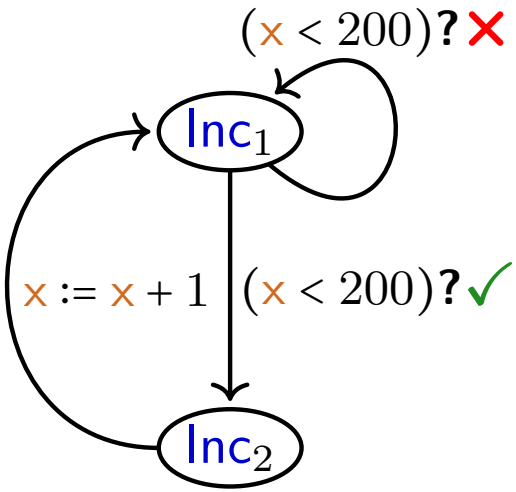
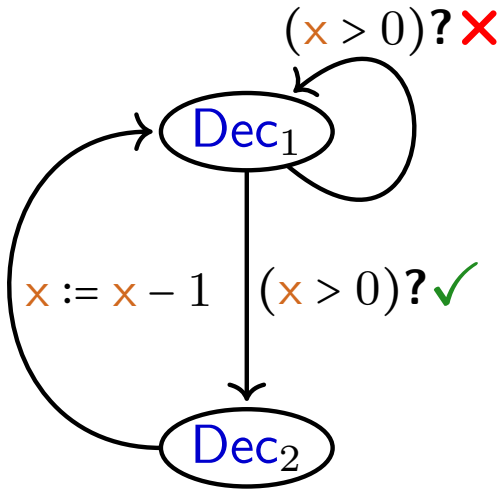
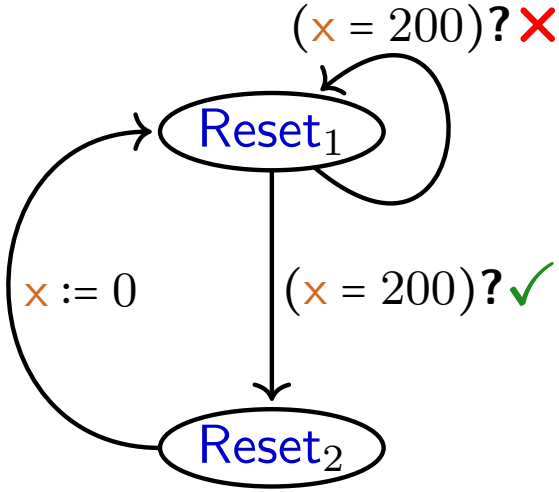
Formalizing properties (in temporal logic)







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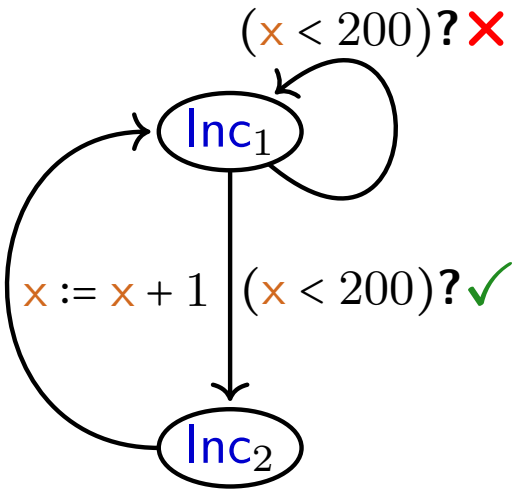
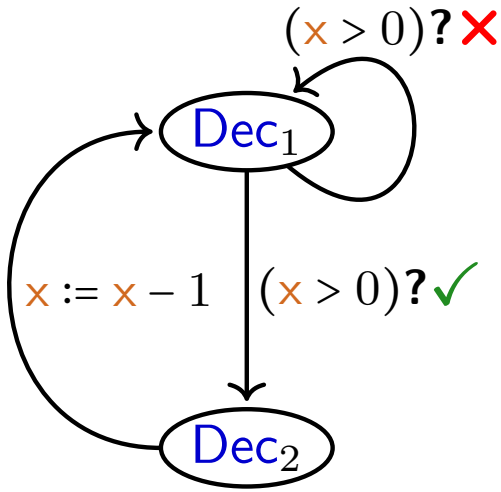
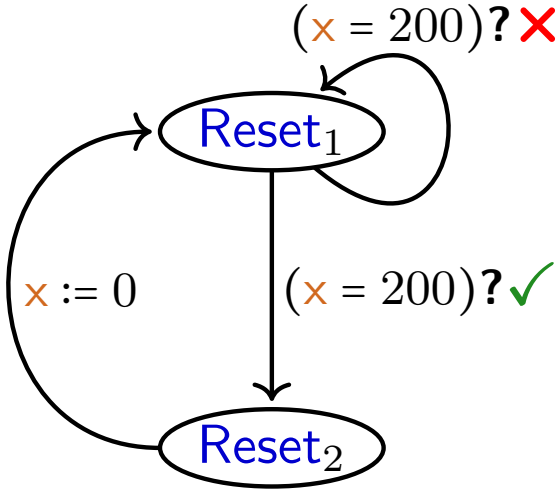
Formalizing properties (in temporal logic)







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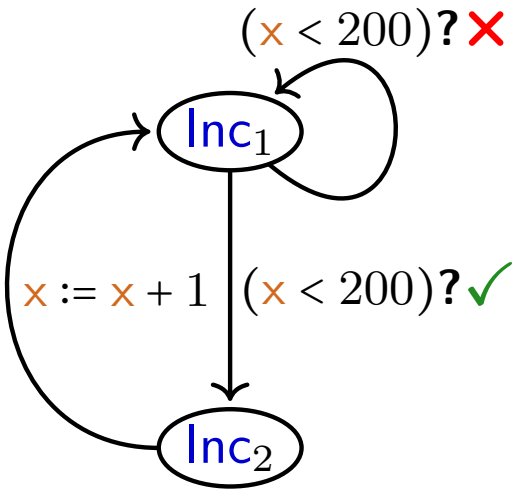
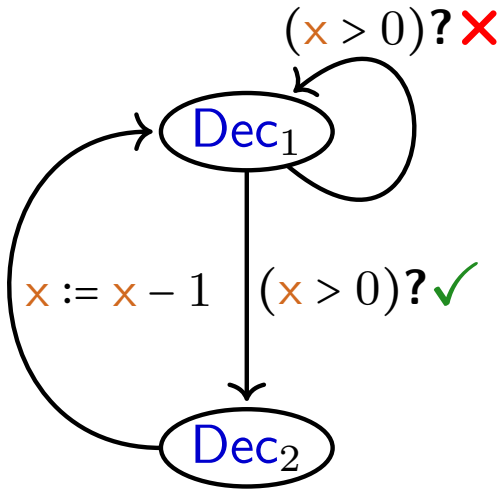
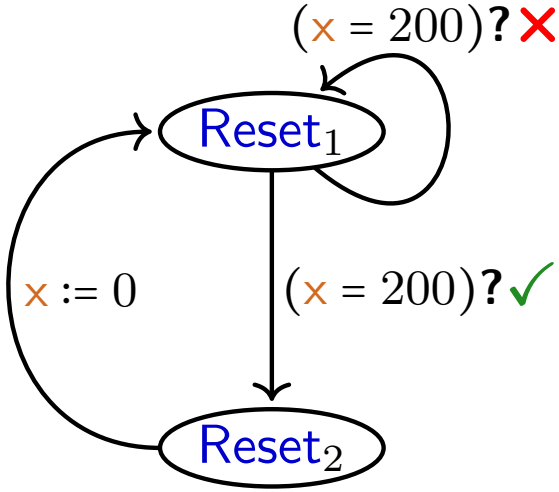
Formalizing properties (in temporal logic)







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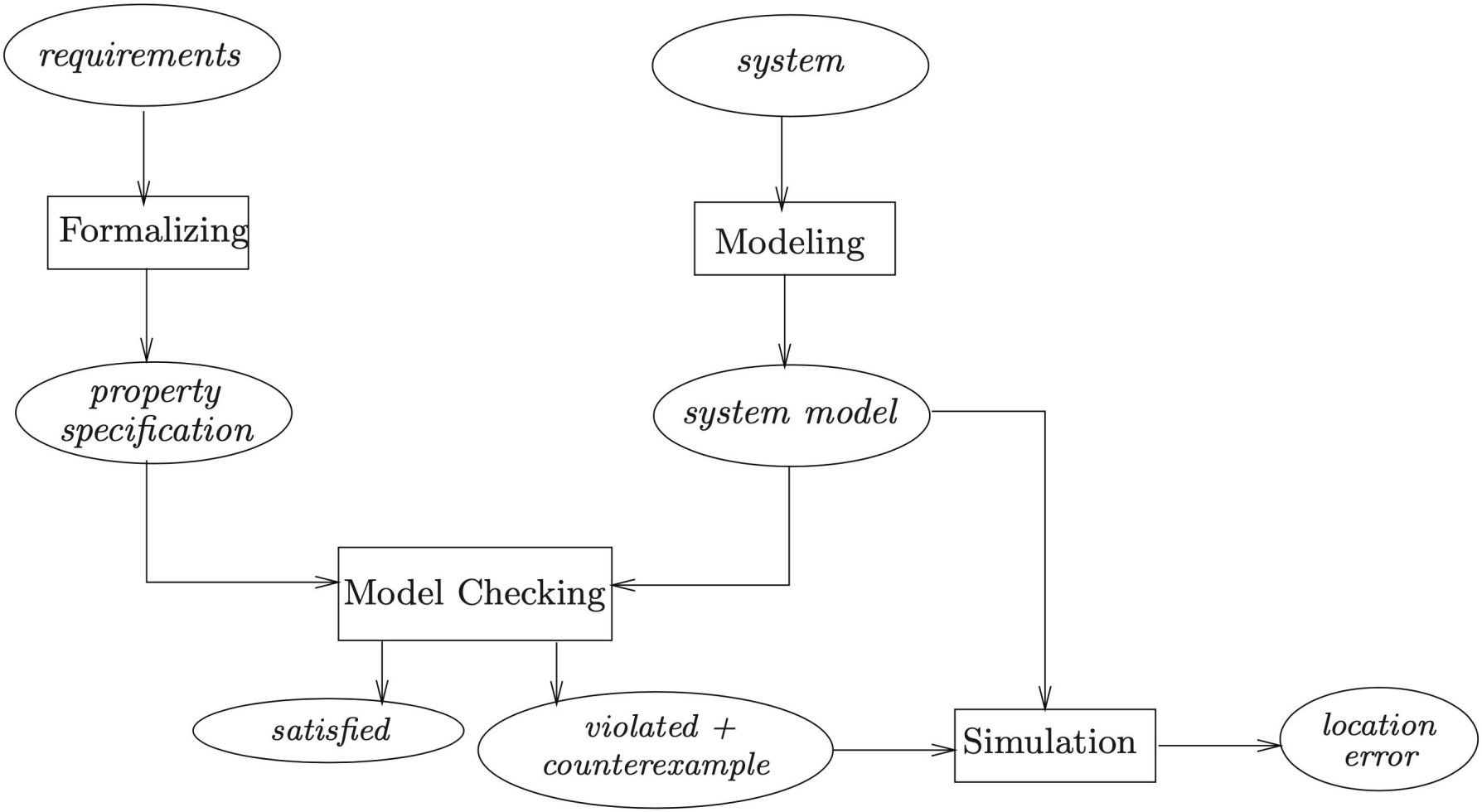
Model checking: validation



Any [such] verification is only as good as the model of the system.

Modeling: Verification versus Validation

VERIFICATION

Are we building the thing right?

Does the design satisfy the expected properties?

VALIDATION

Are we building the right thing?

Is the design faithfully capturing the requirements?

Model-checking: strengths and weaknesses

Strengths:

- it is a general technique
- supports partial verification, checking properties individually

Weaknesses:

Model-checking: strengths and weaknesses

Strengths:

- it is a general technique
- supports partial verification, checking properties individually
- bug-detection is not dependent on chance
- provides diagnostic information for debugging

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- potential push-button technology
- widely used in industry

Weaknesses:

Model-checking: strengths and weaknesses

Strengths:

- it is a general technique
- supports partial verification, checking properties individually
- bug-detection is not dependent on chance
- provides diagnostic information for debugging
- potential push-button technology
- widely used in industry
- can be integrated in existing development cycles
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Model-checking: strengths and weaknesses

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- it is a general technique
- supports partial verification, checking properties individually
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- modeling requires expertise (finding smaller models, state properties as formulas)
- model-checker may have defects as well
- does not permit to directly check generalizations (infinitely many components, parameterized systems)

Origins of model checking

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See Baier and Katoen [1] for more references.

Lectures

1. Introduction (preview extended, LTSs)
2. Modeling by labeled transition systems
   - executions; traces; non-determinism; examples
3. Linear-Time Behaviour and Properties
   - invariant, safety, and liveness (and fairness) properties
4. Linear Temporal Logic (LTL)
   - syntax and semantics; interpretation of LTSs; examples
5. LTL (continued)
   - model checking of LTL formulas, and fairness in LTL
   - Maude examples
6. Computation Tree Logic (CTL)
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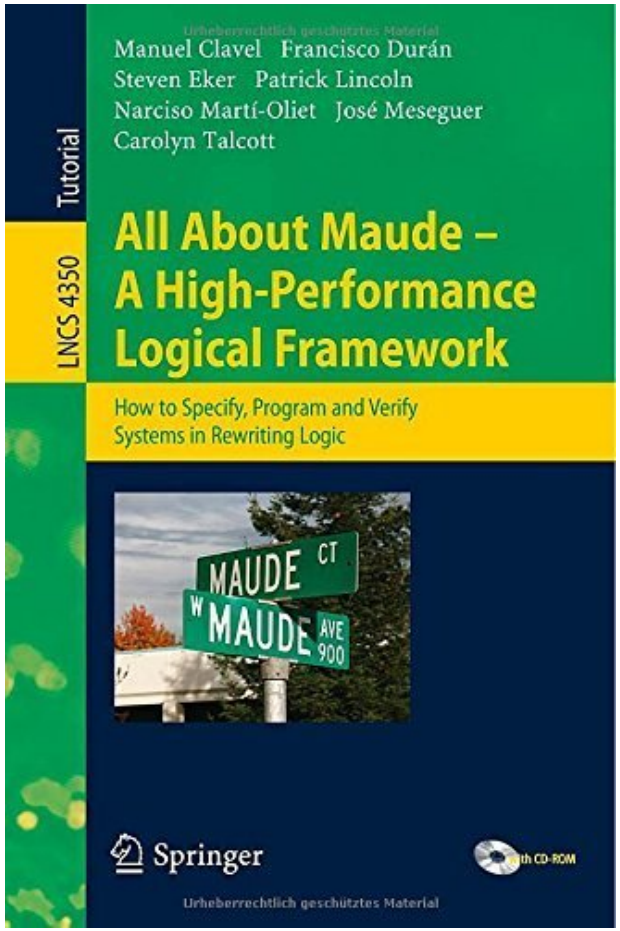
Book Maude



- pdf available:
  https://maude.cs.illinois.edu/w/images/0/0d/Maude-book.pdf

Maude code (idea)

```
crl [Inc1a]   :  Inc1 x => Inc2 x  if x < 200
rl  [Inc2]    :  Inc2 x => Inc1 (x + 1)
crl [Inc1b]   :  Inc1 x => Inc1 x  if not(x < 200)
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eq initial = { Dec1 Inc1 Reset1 0 }
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eq initial = { Dec1 Inc1 Reset1 0 }


ceq (S1 S2 S3 x |= counterge0) = true  if (0 = x \/ 0 < x)
ceq (S1 S2 S3 x |= counterlt0) = true  if (x < 0)
ceq (S1 S2 S3 x |= counterle200) = true  if (x < 200 \/ x = 200)
```

Maude output (simplified)

```
Maude> red modelCheck(initial, <> counterlt0)
reduce in COUNTERS-CHECK : modelCheck(initial, <> counterlt0)
result ModelCheckResult:
result Bool :  true
```

Maude output (simplified)

```
Maude> red modelCheck(initial, <> counterlt0)
reduce in COUNTERS-CHECK : modelCheck(initial, <> counterlt0)
result ModelCheckResult:
result Bool :  true


Maude> red modelCheck(initial, [](counterge0 /\counterle200)
reduce in COUNTERS-CHECK :
                modelCheck(initial, [](counterge0 /\counterle200)
result ModelCheckResult:
counterexample({Inc1 Dec1 Reset1 199}
               {Inc2 Dec1 Reset1 199}
               {Inc1 Dec1 Reset1 200}
               {Inc1 Dec2 Reset1 200}
               {Inc1 Dec2 Reset2 200}
               {Inc1 Dec2 Reset1 0}
               {Inc1 Dec1 Reset1 -1})
```

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Exam options

Together we decide on a topic.

Study an article, or a book chapter.
or
Dive deeper into a proof from the lecture.
or
Model a basic algorithm, and check basic properties.
or
Develop an idea that motivates you.
or
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You give a 25-minute presentation about what you found.

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Exam examples

Presentations about articles:

- *Progress, Justness, and Fairness* (2019) by R. van Glabbeek, P. Höfner
- *Comparison of Model Checking Tools for Information Systems* (2010) by M. Frappier et al. (focusing on SPIN and NuSMV)

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