

Core Course Preview

Introduction to Model Checking

<https://clegra.github.io/mc/mc.html>

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Model Checking

... is an effective automatable technique:

- ▶ *to expose potential software design errors;*
- ▶ *that, given a finite-state model of a system and a formal property, systematically checks whether this property holds for that model.*

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- ▶ widely applied in industry
 - ▶ embedded systems, software engineering, hardware design, symbolic AI
- ▶ supports partial verification (of system parts)
- ▶ provides diagnostic information for debugging
- ▶ has sound mathematical underpinning (logic and process theory)

Model Checking

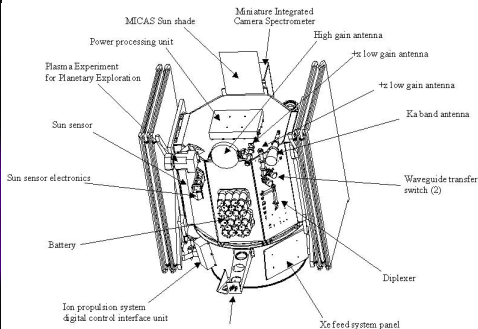
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Course Goals are introduction to:

- ▶ **Theory:**
 - ▶ modeling systems by labeled transition systems,
 - ▶ expressing properties by temporal-logic formulas
 - ▶ model-checking algorithms
- ▶ **Practice:** use the Maude system for examples

Deep Space 1 (NASA)



- ▶ Flyby of asteroid 9969 Braille (1999)
- ▶ Entered the coma of Comet Borrelly (2001)
- ▶ Model checking discovered **5 concurrency errors**

Example (program concurrency/non-determinism)

Programs [Inc](#), [Dec](#), and [Reset](#) cooperate, and use a shared variable x :

```
proc Inc
  while true
  do
    if  $x < 200$ 
    then  $x := x + 1$ 
    fi
  od
```

```
proc Dec
  while true
  do
    if  $x > 0$ 
    then  $x := x - 1$ 
    fi
  od
```

```
proc Reset
  while true
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    if  $x = 200$ 
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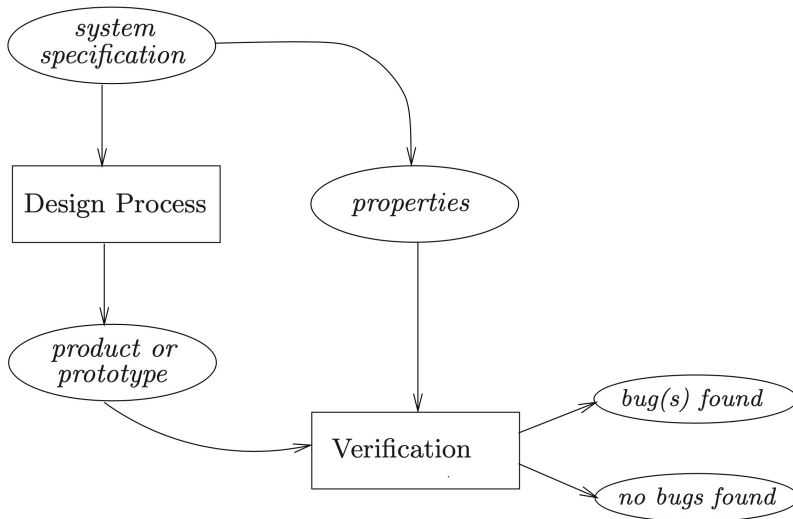
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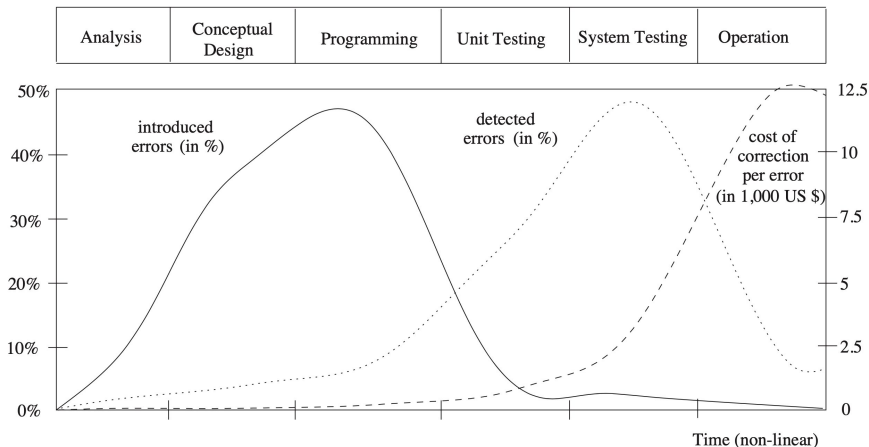
```
proc Reset
  while true
  do
    if  $x = 200$ 
    then  $x := 0$ 
    fi
  od
```

Question: Is $0 \leq x \leq 200$ always guaranteed?

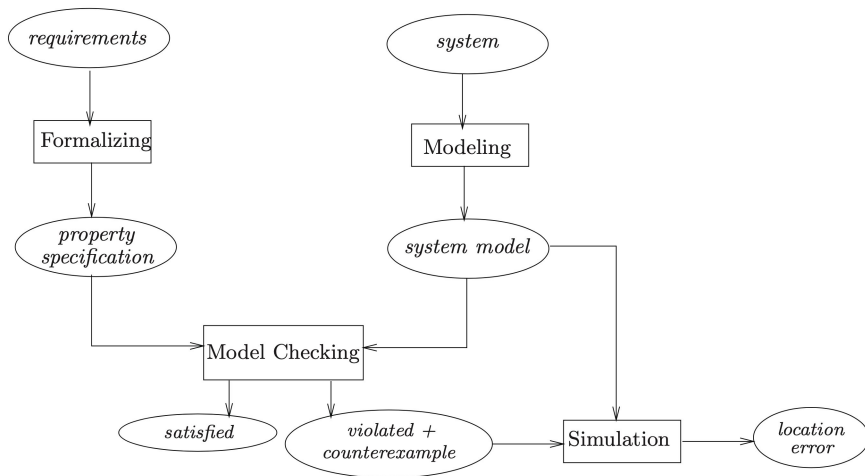
Hard-/Software Verification (traditionally)



Error introduction, detection, and repair costs



Model checking



Modeling (by program graphs)

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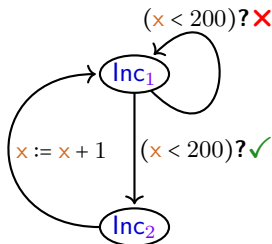
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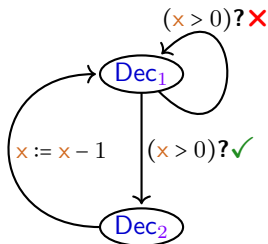


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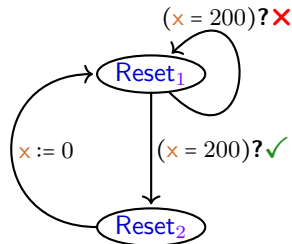


proc Reset

while true

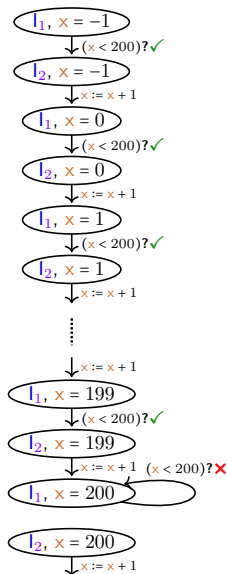
do

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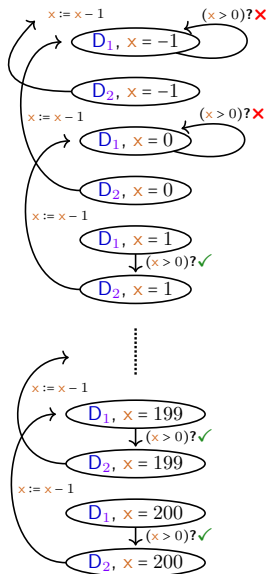


Program graphs (PG)

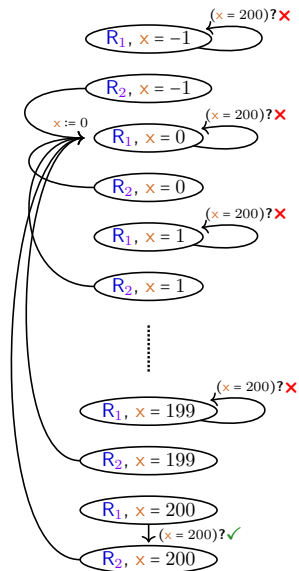
Modeling (by **labeled transition systems**, with state space explosion)



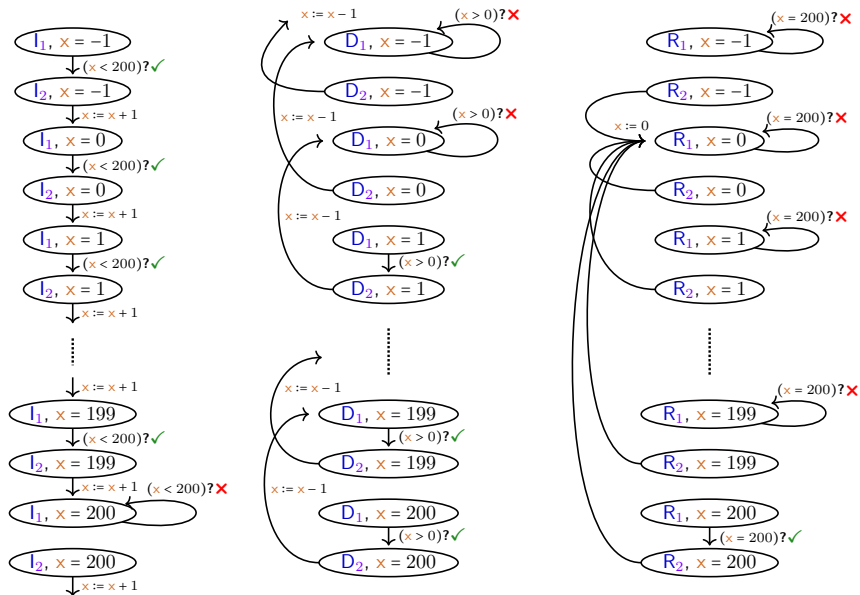
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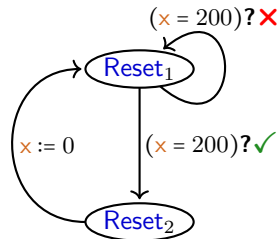
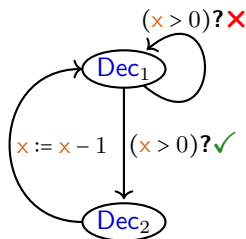
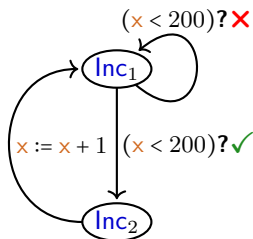
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Formalizing properties (in temporal logic)



$\text{Inc}_1 \parallel \text{Dec}_1 \parallel \text{Reset}_1 \stackrel{?}{\models} \Box(0 \leq x \wedge x \leq 200) \quad (\text{Linear-TL formula})$

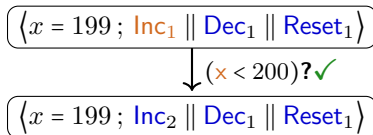
Counterexample (offending execution trace)

$$\langle x = 199 ; \text{Inc}_1 \parallel \text{Dec}_1 \parallel \text{Reset}_1 \rangle$$

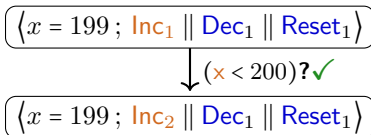
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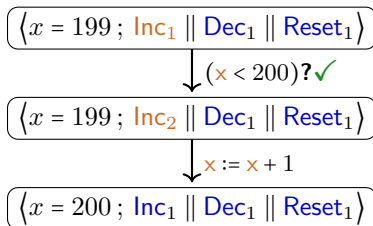
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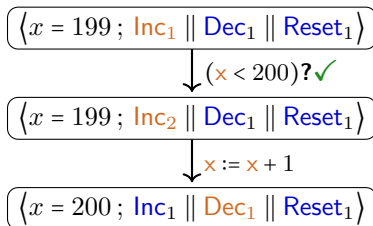
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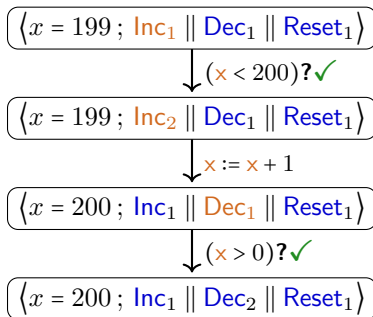
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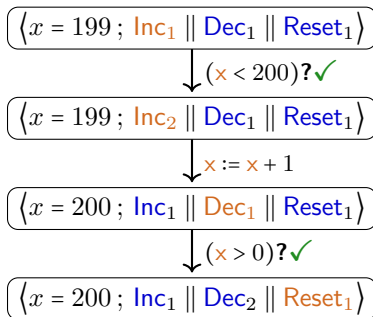
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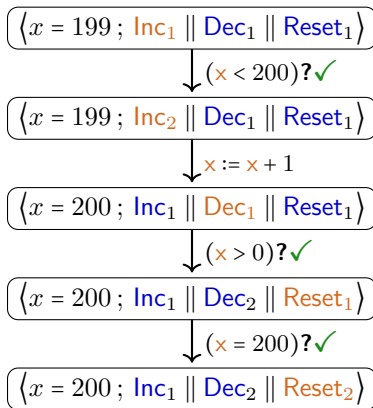
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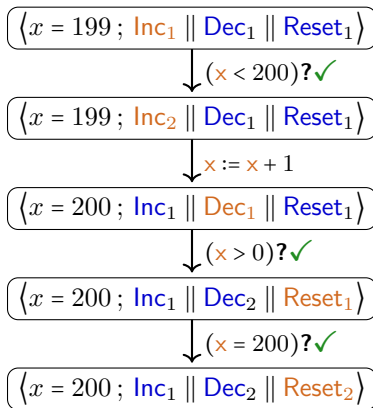
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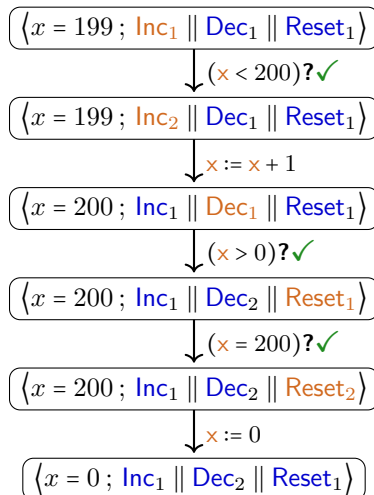
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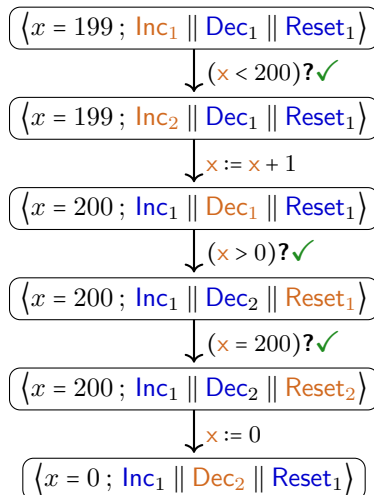
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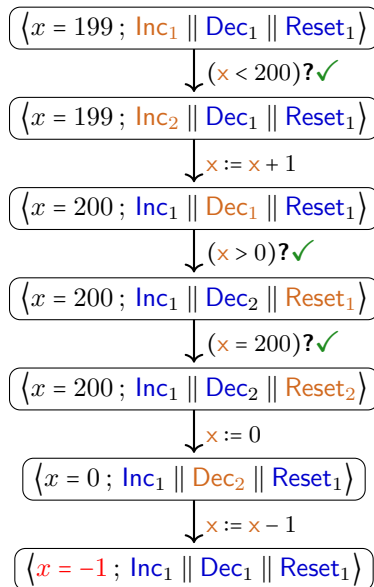
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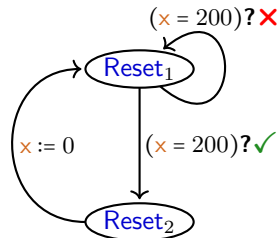
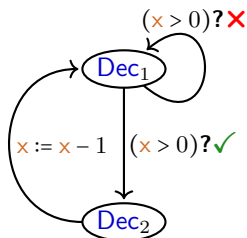
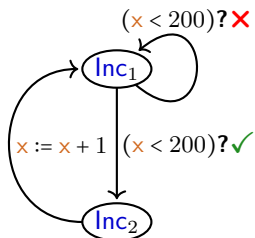
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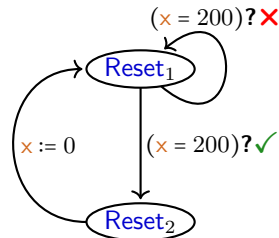
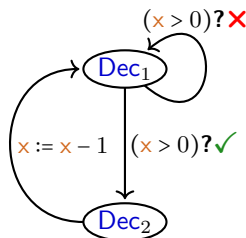
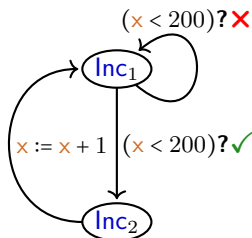


Formalizing properties (in temporal logic)



$\text{Inc}_1 \parallel \text{Dec}_1 \parallel \text{Reset}_1 \not\models \Box(0 \leq x \wedge x \leq 200)$ (Linear-TL formula)

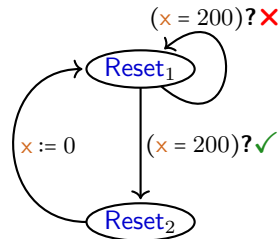
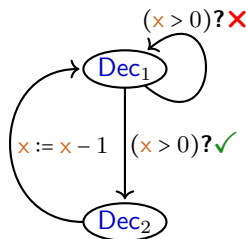
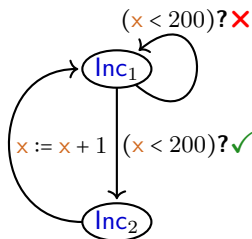
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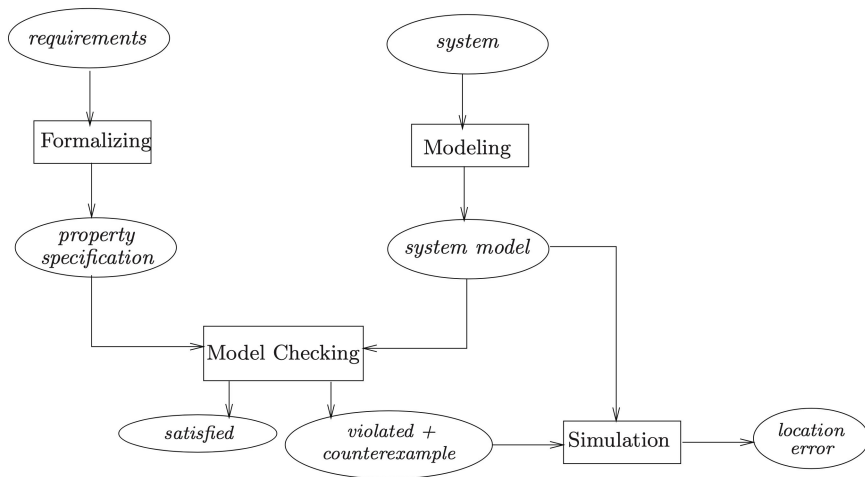
$\text{Inc}_1 \parallel \text{Dec}_1 \parallel \text{Reset}_1 \models \Diamond(x < 0)$ (LTL formula)

$\text{Inc}_1 \parallel \text{Dec}_1 \parallel \text{Reset}_1 \not\models \forall \Box(0 \leq x \wedge x \leq 200)$ (Computation-Tree-L formula)

$\text{Inc}_1 \parallel \text{Dec}_1 \parallel \text{Reset}_1 \models \exists \Box(0 \leq x \wedge x \leq 200)$ (CTL formula)

$\text{Inc}_1 \parallel \text{Dec}_1 \parallel \text{Reset}_1 \models \forall \Box \exists \Diamond(x < 0)$ (CTL formula)

Model checking



Any [such] verification is only as good as the model of the system.

Possible Maude code (idea)

```
cr1 [Inc1a]      :  Inc1 x => Inc2 x   if x < 200
r1  [Inc2]       :  Inc2 x => Inc1 x + 1
cr1 [Inc1b]      :  Inc1 x => Inc1 x   if not(x < 200)
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eq initial = { Dec1 Inc1 Reset1 199 }
```

Possible Maude code (idea)

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eq initial = { Dec1 Inc1 Reset1 199 }

ceq (S1 S2 S3 x |= counterge0) = true   if (0 < x \ / x = 0)
ceq (S1 S2 S3 x |= counterlt0) = true   if (x < 0)
ceq (S1 S2 S3 x |= counterle200) = true if (x < 200 \ / x = 200)

```


Maude output

```
Maude> red modelCheck(initial, <> counterlt0)
reduce in COUNTERS-CHECK : modelCheck(initial, <> counterlt0)
result ModelCheckResult:
result Bool : true
```

Maude output

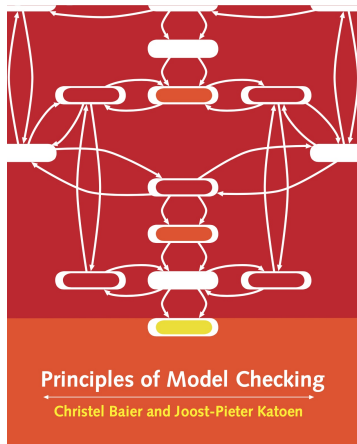
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```
Maude> red modelCheck(initial, [](counterge0 /\counterle200)
reduce in COUNTERS-CHECK :
      modelCheck(initial, [](counterge0 /\counterle200)
result ModelCheckResult:
counterexample({Inc1 Dec1 Reset1 199}
               {Inc2 Dec1 Reset1 199}
               {Inc1 Dec1 Reset1 200}
               {Inc1 Dec2 Reset1 200}
               {Inc1 Dec2 Reset2 200}
               {Inc1 Dec2 Reset1 0}
               {Inc1 Dec1 Reset1 -1})
```

Topics of the course

- ▶ modeling systems by [labeled transition systems \(LTSs\)](#)
- ▶ [safety](#), [liveness](#), and [fairness](#) properties
- ▶ [Linear Temporal Logic \(LTL\)](#)
 - ▶ model checking formulas
 - ▶ express properties by [Büchi automata](#)
 - ▶ model check LTSs and properties via [product automata](#)
- ▶ [Computation Tree Logic \(CTL\)](#)
- ▶ [partial](#) model checking
 - ▶ partially known systems (state properties/states/transitions)
- ▶ learning [Maude](#) and its [model-checker](#)

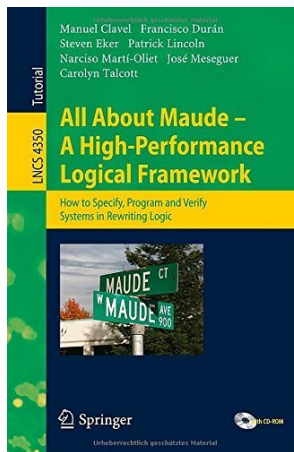
Book



- pdf available:

https://is.ifmo.ru/books/_principles_of_model_checking.pdf

Book Maude



- pdf available:

<https://maude.cs.illinois.edu/w/images/0/0d/Maude-book.pdf>

Organization

Lectures (Emilio 3 / Clemens 4)

- ▶ January 19–January 28 (7 meetings)
- ▶ **blackboard presentations**
- ▶ **notes** after the lecture (last year's **notes** available)
- ▶ **Maude examples** at the end of each lecture

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- ▶ options:
 - ▶ verification project (of an algorithm, etc.) in **Maude**
 - ▶ presentation about a paper
 - ▶ written exam



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