Closing the Image of the Process Interpretation of 1-Free Regular Expressions Under Bisim. Collapse

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Overview

- 1-free regular expressions (with unary/binary star)
- process interpretation / semantics of regular expressions
  - expressible/not expressible process graphs
- loop existence and elimination property (LEE) (G/Fokkink, 2020)
  - interpretation/extraction correspondences with 1-free reg. expr's
  - LEE is preserved under bisimulation collapse
- Q: Image of process interpretation of 1-free regular expressions closed under bisimulation collapse?
  - A1: No. — But . . .
- compact process interpretation
- refined expression extraction
  - A2: compact process interpretation is image-closed under collapse
- outlook: consequences

Regular Expressions

Definition (~ *Copi–Elgot–Wright, 1958*)

Regular expressions over alphabet A with unary Kleene star:

$$e, e_1, e_2 \;::=\; \mathbf{0} \;\mid\; \boldsymbol{a} \;\mid\; e_1 + e_2 \;\mid\; e_1 \cdot e_2 \;\mid\; e^{*} \qquad \text{(for } \boldsymbol{a} \in A\text{)}.$$

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- symbol $\mathbf{0}$ instead of $\varnothing$, symbol $\mathbf{1}$ instead of $\{\varnothing\}$ and ϵ
- with unary Kleene star *: $\mathbf{1}$ is definable as $\mathbf{0}^{*}$

Regular Expressions

Definition (~ *Kleene, 1951*, ~ *Copi–Elgot–Wright, 1958*)

Regular expressions over alphabet A with unary / binary Kleene star:

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Definition

1-free regular expressions over alphabet A with binary Kleene star:

$$f, f_1, f_2 \; ::= \; \mathbf{0} \mid \boldsymbol{a} \mid f_1 + f_2 \mid f_1 \cdot f_2 \mid f_1{}^{\circledast} f_2 \qquad \text{(for } \boldsymbol{a} \in A\text{).}$$

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Under-Star-/1-Free regular expressions

Definition

The set $\mathit{RExp}^{(\cancel{1})}(A)$ of 1-free regular expressions over A is defined by:

$$f,\, f_1,\, f_2 \;::=\; 0 \mid a \mid f_1 + f_2 \mid f_1 \cdot f_2 \mid {f_1}^{\star} \cdot f_2 \qquad \text{(for } a \in A\text{)},$$

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the set $\mathit{RExp}^{(\cancel{1}\backslash *)}(A)$ of under-star-1-free regular expressions over A by:

$$uf, uf_1, uf_2 \;::=\; 0 \mid 1 \mid a \mid uf_1 + uf_2 \mid uf_1 \cdot uf_2 \mid f^{\star} \qquad \text{(for } a \in A\text{)}.$$

Process interpretation $P(\cdot)$ of regular expressions *(Milner, 1984)*

$\mathbf{0} \xrightarrow{P}$ deadlock $\boldsymbol{\delta}$, no termination

$\mathbf{1} \xrightarrow{P}$ empty-step process $\boldsymbol{\epsilon}$, then terminate

$a \xrightarrow{P}$ atomic action a, then terminate

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$e_1 + e_2 \overset{P}{\longmapsto}$ (*choice*) execute $P(e_1)$ or $P(e_2)$

$e_1 \cdot e_2 \overset{P}{\longmapsto}$ (*sequentialization*) execute $P(e_1)$, then $P(e_2)$

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$e_1{}^{\circledast} e_2 \overset{P}{\longmapsto}$ *(iteration-exit)* repeat (terminate or execute $P(e_1)$), then $P(e_2)$

Process semantics $[\![\cdot]\!]_P$ of regular expressions *(Milner, 1984)*

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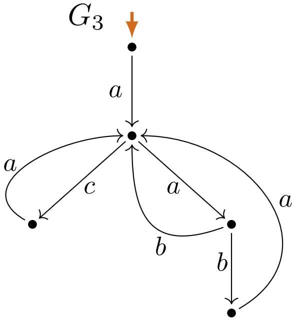
$[\![e]\!]_P \;:=\; [P(e)]_{\underline{\leftrightarrow}}$ (bisimilarity equivalence class of process $P(e)$)

P-expressibility and $[\![\cdot]\!]_P$-expressibility (examples)



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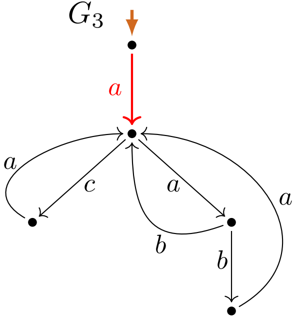

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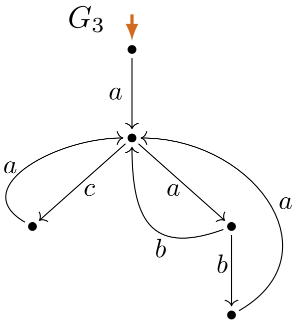

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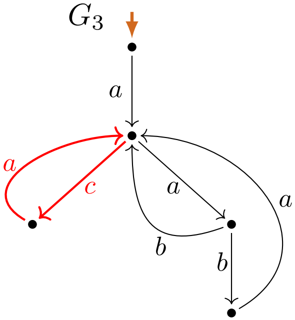

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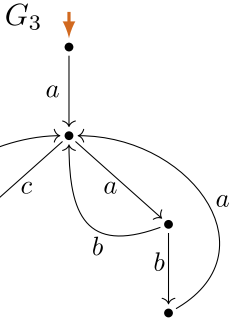

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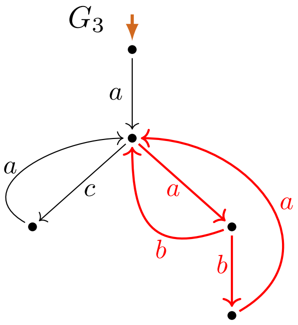

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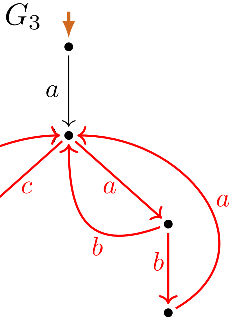

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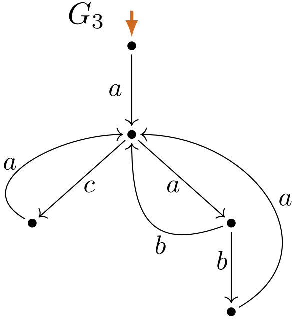

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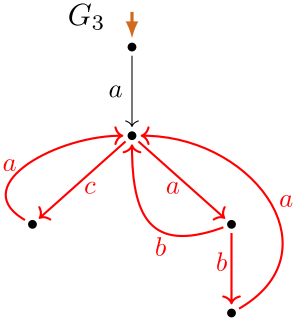

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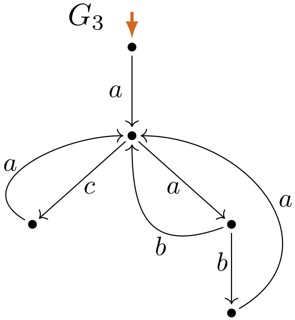

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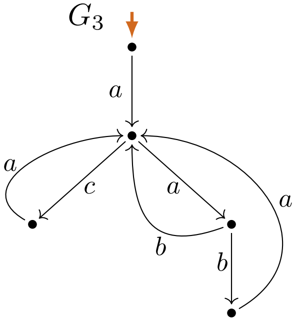

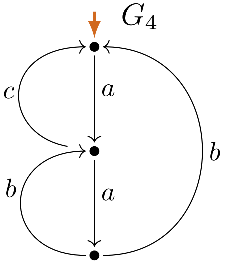

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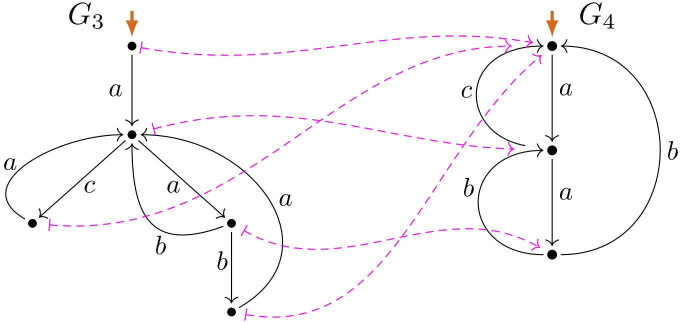

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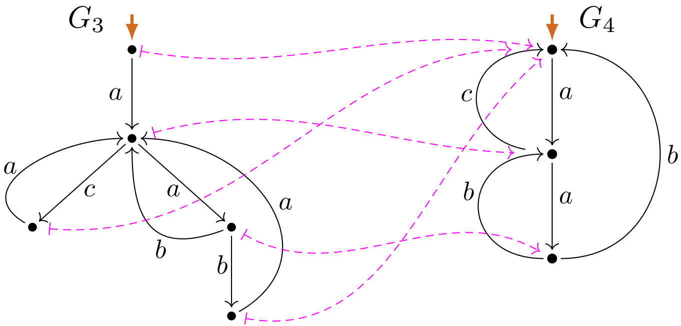

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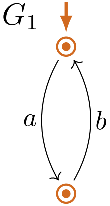

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not $[\![\cdot]\!]_P$-expressible

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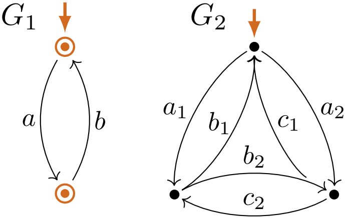

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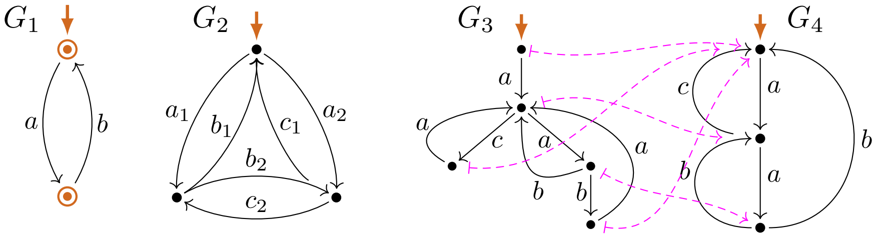

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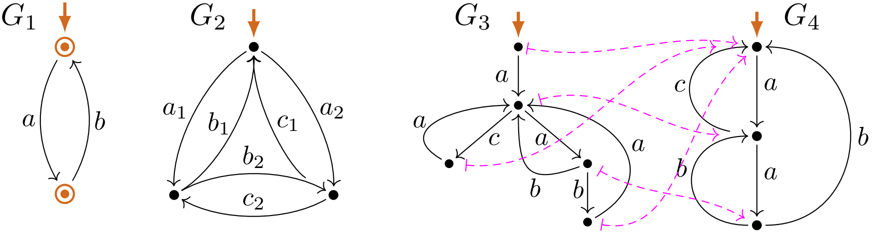

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not $[\![\cdot]\!]_P$-expressible

P-expressible
$[\![\cdot]\!]_P$-expressible

?
$[\![\cdot]\!]_P$-expressible

Process interpretation P (formal definition)

Definition (Transition system specification $\mathcal{T}$)

$$\frac{}{a \xrightarrow{a} 1} \qquad \frac{e_i \xrightarrow{a} e_i'}{e_1 + e_2 \xrightarrow{a} e_i'} \, (i \in \{1,2\})$$

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Definition

The process (graph) interpretation $P(e)$ of a regular expression e:

$P(e)$:= labeled transition graph generated by e by derivations in $\mathcal{T}$.

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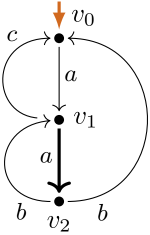
Loop existence and elimination



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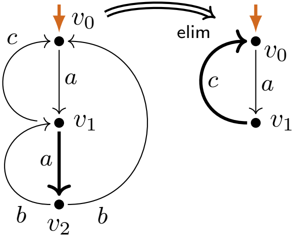

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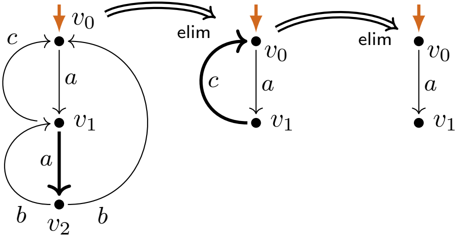
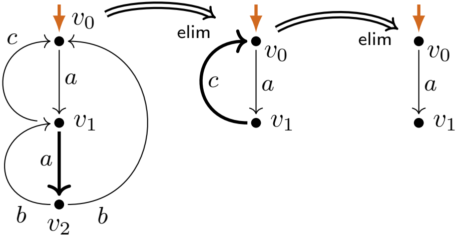
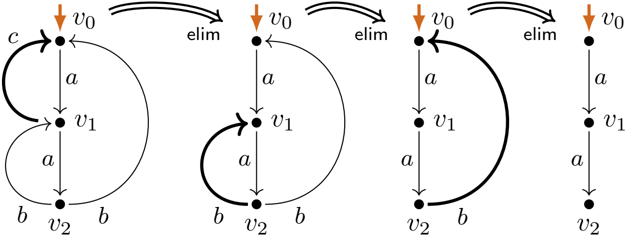


Loop existence and elimination





LEE

Definition

A chart $\mathcal{C}$ satisfies LEE (*loop existence and elimination*) if:

$$\exists \mathcal{C}_0 \left(\mathcal{C} \longrightarrow^{*}_{\text{elim}} \mathcal{C}_0 \not\longrightarrow_{\text{elim}} \right.$$
$$\left. \wedge\ \mathcal{C}_0 \text{ permits no infinite path} \right).$$

LEE

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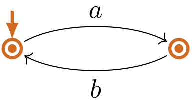
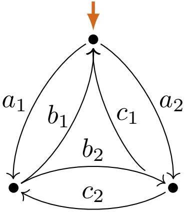
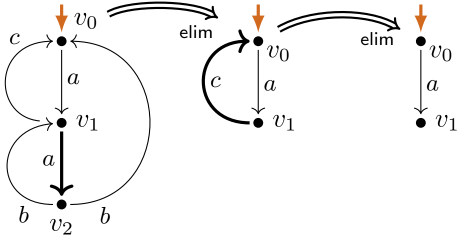


~~LEE~~

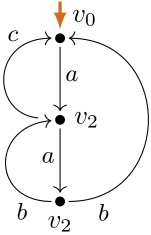


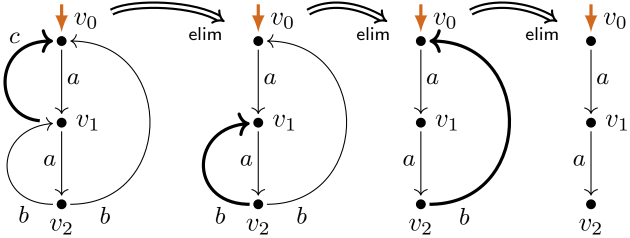
~~LEE~~

LEE

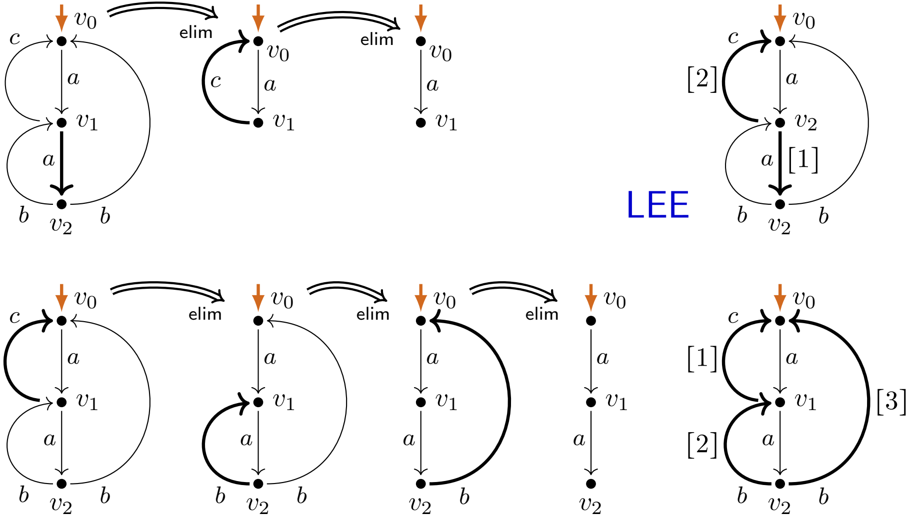


LEE



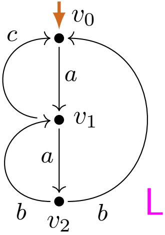


LEE



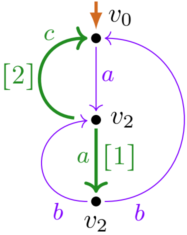
LEE

LEE witness and LEE-charts

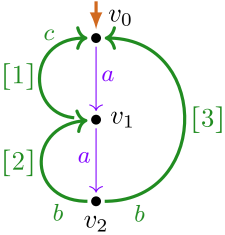


LEE-chart

LEE-witness



LEE-witness



Properties of LEE-charts

Theorem ($\Leftarrow$ G/Fokkink, 2020)

A process graph G

is $[\![\cdot]\!]_P$-expressible by an under-star-1-free regular expression

(i.e. P-expressible modulo bisimilarity by an $(\cancel{1}\backslash *)$ reg. expr.)

if and only if

the bisimulation collapse of G satisfies LEE.

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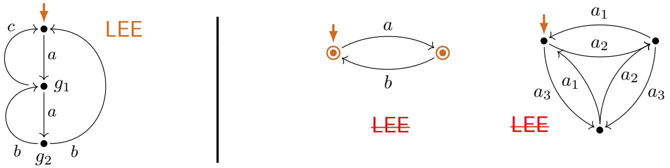
Hence $[\![\cdot]\!]_P$-expressible | **not** $[\![\cdot]\!]_P$-expressible by 1-free regular expressions:



Interpretation/extraction correspondences with LEE

($\Leftarrow$ G/Fokkink 2020, G 2021)

(Int)$^{(\cancel{1}\backslash *)}_{P}$: $P^{\bullet}$-$(\cancel{1}\backslash *)$-*expressible graphs have structural property* LEE

Process **i**nterpretations $P(e)$
of under-star-1-free regular expressions e
are finite process graphs that satisfy LEE.

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(Extr)$_{P}$: LEE *implies* $[\![\cdot]\!]_{P}$*-expressibility*

From every finite process graph G with LEE
a regular expression e can be **e**xtracted
such that $G \underline{\leftrightarrow} P(e)$.

Interpretation/extraction correspondences with LEE

(⇐ G/Fokkink 2020, G 2021)

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From every finite process graph G with LEE
a regular expression e can be **e**xtracted
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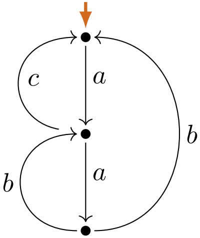
(Coll): LEE *is preserved under collapse*

The class of finite process graphs with LEE
is closed under bisimulation **c**ollapse.

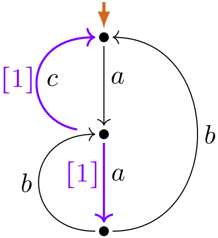
Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

G_4



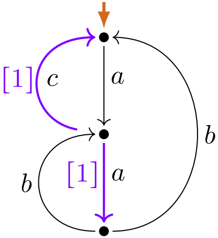
Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



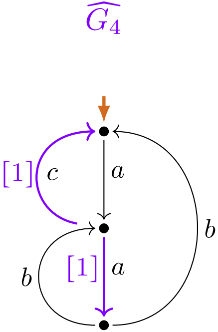
Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$$(\qquad\qquad\qquad\qquad)^* \cdot 0$$

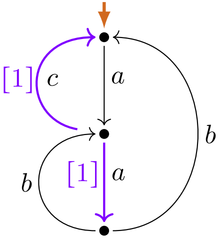
Expression extraction using LEE (G/Fokkink 2020, G 2021/22)



$$\Big(\quad \overset{\vdots\, a}{} \quad \Big)^{*} \cdot 0$$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



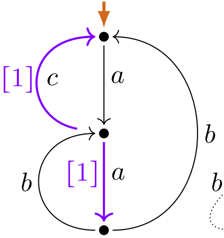
$(a \cdot 1) \cdot ((\qquad)^{*} \cdot 0)$

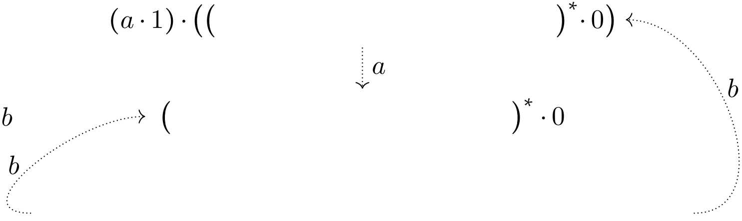
$\downarrow a$

$(\qquad)^{*} \cdot 0$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$





$(a \cdot 1) \cdot ((\qquad)^* \cdot 0)$

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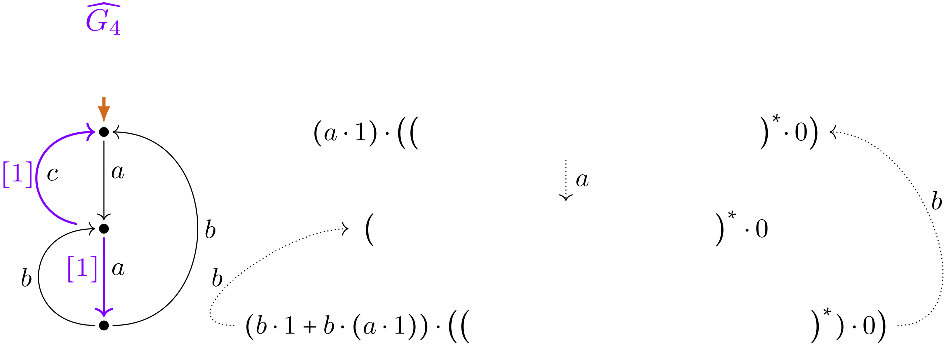
Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

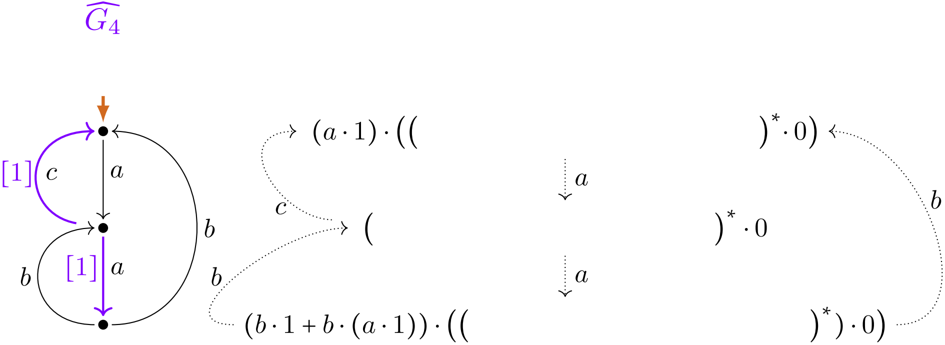


$(a \cdot 1) \cdot ((\qquad)^* \cdot 0)$

$(\qquad)^* \cdot 0$

$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot ((\qquad)^*) \cdot 0)$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)



$(a \cdot 1) \cdot ((\qquad)^* \cdot 0)$

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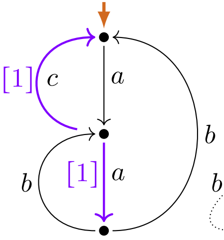
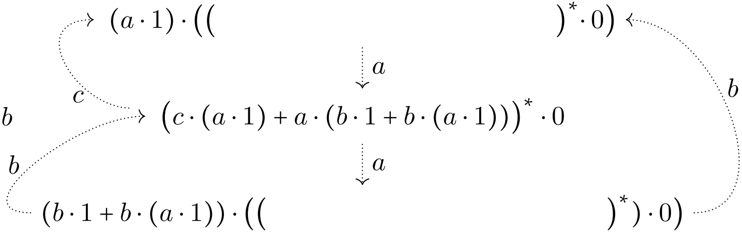
$\widehat{G_4}$



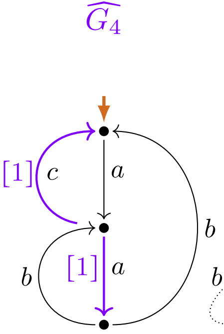


$$(a\cdot 1)\cdot\big(\big(\qquad\qquad\qquad\qquad\big)^*\cdot 0\big)$$

$$\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^*\cdot 0$$

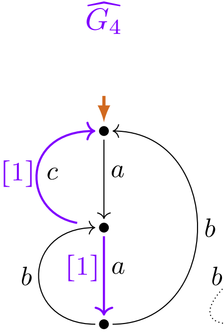
$$(b\cdot 1+b\cdot(a\cdot 1))\cdot\big(\big(\qquad\qquad\qquad\qquad\big)^*\big)\cdot 0\big)$$

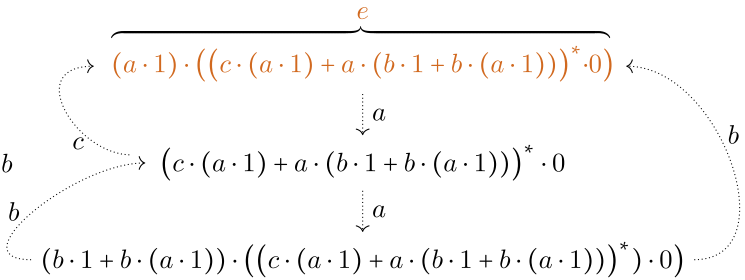
Expression extraction using LEE (G/Fokkink 2020, G 2021/22)



$$\begin{tikzcd}
(a \cdot 1) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0 \big) \arrow[d, dotted, "a"] \\
\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0 \arrow[u, bend left, dotted, "c"] \arrow[d, dotted, "a"] \\
(b \cdot 1 + b \cdot (a \cdot 1)) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*}) \cdot 0 \big) \arrow[u, bend left, dotted, "b"] \arrow[uu, bend right, dotted, "b"']
\end{tikzcd}$$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)





$$\overbrace{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)}^{e}$$

$$\Big\downarrow a$$

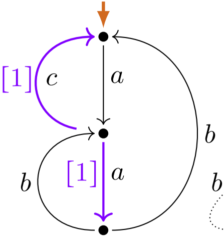
$$\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0$$

$$\Big\downarrow a$$

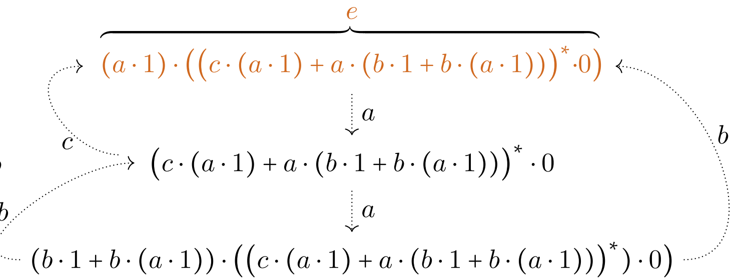
$$(b\cdot 1+b\cdot(a\cdot 1))\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*})\cdot 0\Big)$$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$P(e) \mathrel{\underline{\to}} G_4$



$$\overbrace{(a\cdot 1)\cdot\big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\big)}^{e}$$

$$\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0$$

$$(b\cdot 1+b\cdot(a\cdot 1))\cdot\big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0\big)$$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$

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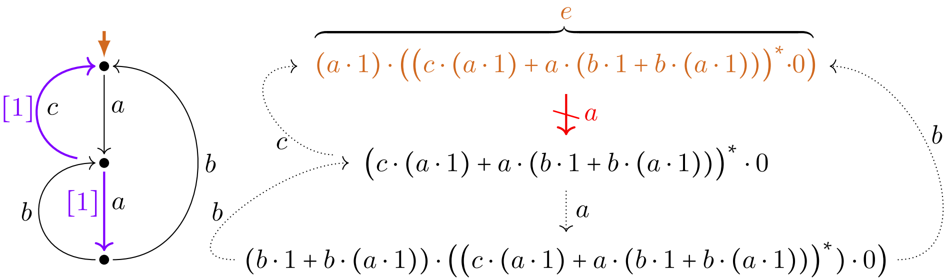


$$\overbrace{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)}^{e}$$

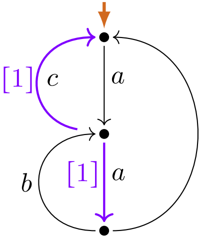
$$\not\downarrow a$$

$$\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0$$

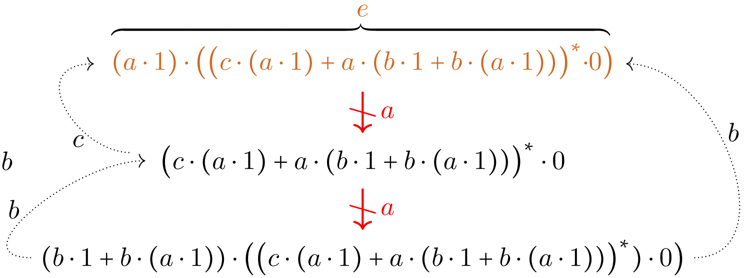
$$(b\cdot 1+b\cdot(a\cdot 1))\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0\Big)$$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$P(e) \rightarrowtail G_4$



$$\overbrace{(a\cdot 1)\cdot\big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\big)}^{e}$$

$$\downarrow a$$

$$\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0$$

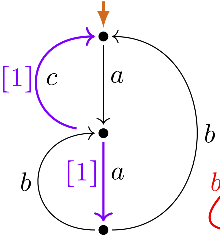
$$\downarrow a$$

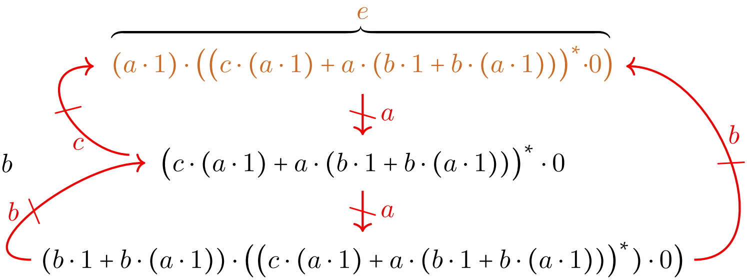
$$(b\cdot 1+b\cdot(a\cdot 1))\cdot\big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*})\cdot 0\big)$$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$

$P(e) \mathrel{\underline{\rightarrow}} G_4$



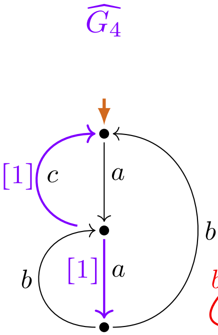


$$\overbrace{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)}^{e}$$

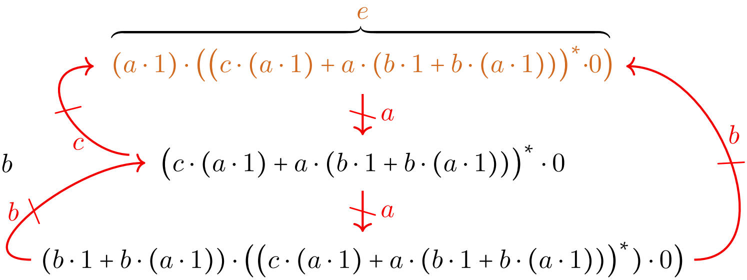
$$\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0$$

$$(b\cdot 1+b\cdot(a\cdot 1))\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*})\cdot 0\Big)$$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)



$$P(e) \mathrel{\underline{\rightarrow}} G_4 \not\mathrel{\underline{\leftarrow}} P(e)$$



$$\begin{tikzcd}
\overbrace{(a\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)}^{e} \arrow[d, "a"] \\
\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0 \arrow[d, "a"] \arrow[u, bend left, "c"] \\
(b\cdot 1+b\cdot(a\cdot 1))\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*})\cdot 0\Big) \arrow[u, bend left, "b"] \arrow[uu, bend right, "b"']
\end{tikzcd}$$

Interpretation of extracted expression

G_5

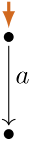
$$P(e) = G_5$$

$$\overbrace{(a \cdot 1) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0 \big)}^{e}$$

Interpretation of extracted expression

G_5

$P(e) = G_5$



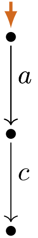
$$\underbrace{(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^{*} \cdot 0\Big)}^{e}$$

$$\downarrow a$$

$$(1 \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^{*} \cdot 0\Big)$$

Interpretation of extracted expression

G_5



$P(e) = G_5$

$$\overbrace{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)}^{e}$$

$$\Big\downarrow a$$

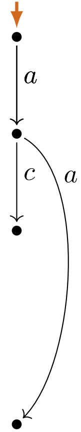
$$(1\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)$$

$$\Big\downarrow c$$

$$\Big((1\cdot(a\cdot 1))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0$$

Interpretation of extracted expression

G_5

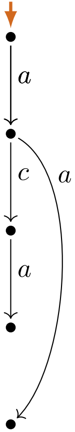


$P(e) = G_5$

$$\begin{tikzcd}
\overbrace{(a\cdot 1)\cdot\big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\big)}^{e} \arrow[d, "a"] \\
(1\cdot 1)\cdot\big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\big) \arrow[d, "c"] \arrow[dd, bend left, "a"] \\
\big((1\cdot(a\cdot 1))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0 \\
\big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0
\end{tikzcd}$$

Interpretation of extracted expression

G_5

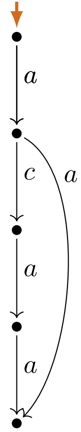


$P(e) = G_5$

$$\begin{tikzcd}[row sep=large]
\overbrace{(a\cdot 1)\cdot\Big(\Big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))\Big)^{*}\cdot 0\Big)}^{e} \arrow[d, "a"] \\
(1\cdot 1)\cdot\Big(\Big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))\Big)^{*}\cdot 0\Big) \arrow[d, "c"] \arrow[ddd, bend left, "a"] \\
\Big((1\cdot(a\cdot 1))\cdot\Big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))\Big)^{*}\Big)\cdot 0 \arrow[d, "a"] \\
\Big((1\cdot 1)\cdot\Big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))\Big)^{*}\Big)\cdot 0\Big) \\
\Big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot\Big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))\Big)^{*}\Big)\cdot 0
\end{tikzcd}$$

Interpretation of extracted expression

G_5

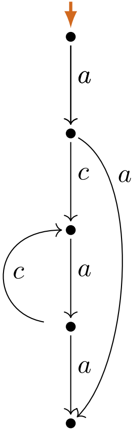


$P(e) = G_5$

$$\begin{tikzcd}
\overbrace{(a\cdot 1)\cdot\big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\big)}^{e} \arrow[d, "a"] \\
(1\cdot 1)\cdot\big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\big) \arrow[d, "c"] \arrow[dddd, bend left, "a"] \\
\big((1\cdot(a\cdot 1))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0 \arrow[d, "a"] \\
\big((1\cdot 1)\cdot\big(c\cdot(a\cdot 1)+{\color{red}a}\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0\big) \arrow[d, "a"] \\
\big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0
\end{tikzcd}$$

Interpretation of extracted expression

G_5

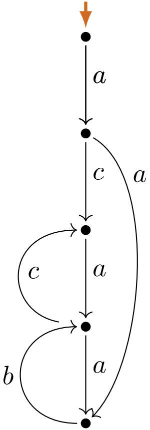


$P(e) = G_5$

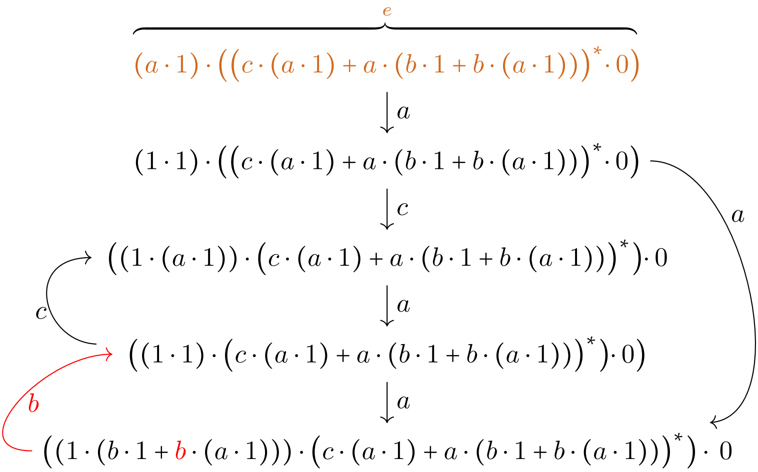
$$\begin{tikzcd}
\overbrace{(a\cdot 1)\cdot\big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\big)}^{e} \arrow[d, "a"] \\
(1\cdot 1)\cdot\big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\big) \arrow[d, "c"] \arrow[dddd, bend left, "a"] \\
\big((1\cdot(a\cdot 1))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0 \arrow[d, "a"] \\
\big((1\cdot 1)\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0\big) \arrow[d, "a"] \arrow[u, bend left, "c"] \\
\big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0
\end{tikzcd}$$

Interpretation of extracted expression

G_5



$P(e) = G_5$



$$\overbrace{(a\cdot 1)\cdot\big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^*\cdot 0\big)}^{e}$$

$$\downarrow a$$

$$(1\cdot 1)\cdot\big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^*\cdot 0\big)$$

$$\downarrow c$$

$$\big((1\cdot(a\cdot 1))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^*\big)\cdot 0$$

$$\downarrow a$$

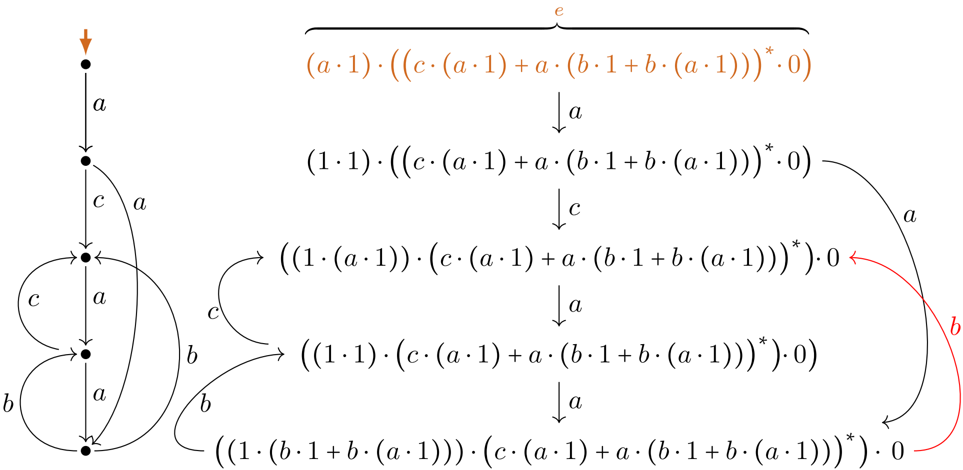
$$\big((1\cdot 1)\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^*\big)\cdot 0\big)$$

$$\downarrow a$$

$$\big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^*\big)\cdot 0$$

Interpretation of extracted expression

G_5 $\quad\quad$ $P(e) = G_5$



$$\overbrace{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)}^{e}$$

$$\big\downarrow a$$

$$(1\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)$$

$$\big\downarrow c$$

$$\Big((1\cdot(a\cdot 1))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0$$

$$\big\downarrow a$$

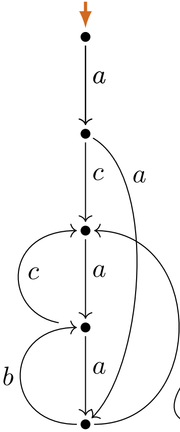
$$\Big((1\cdot 1)\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0\Big)$$

$$\big\downarrow a$$

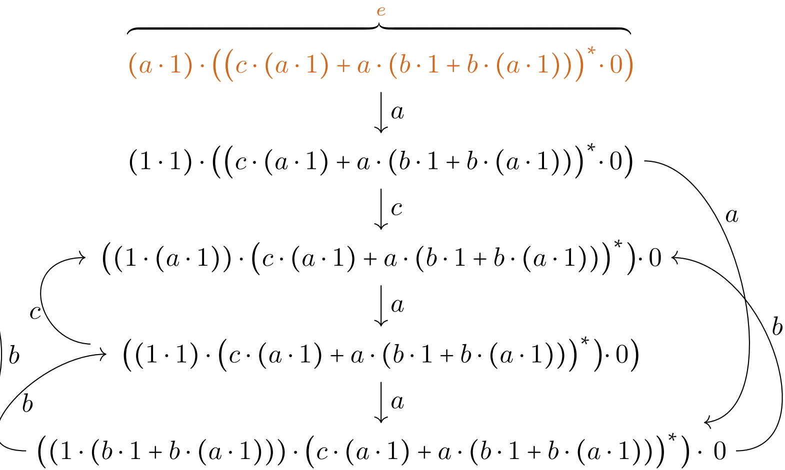
$$\Big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0$$

Interpretation of extracted expression

G_5



$P(e) = G_5 \underline{\rightarrow} G_4$



$$\overbrace{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)}^{e}$$

$$\downarrow a$$

$$(1\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)$$

$$\downarrow c$$

$$\Big((1\cdot(a\cdot 1))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0$$

$$\downarrow a$$

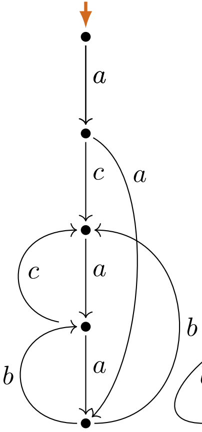
$$\Big((1\cdot 1)\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0\Big)$$

$$\downarrow a$$

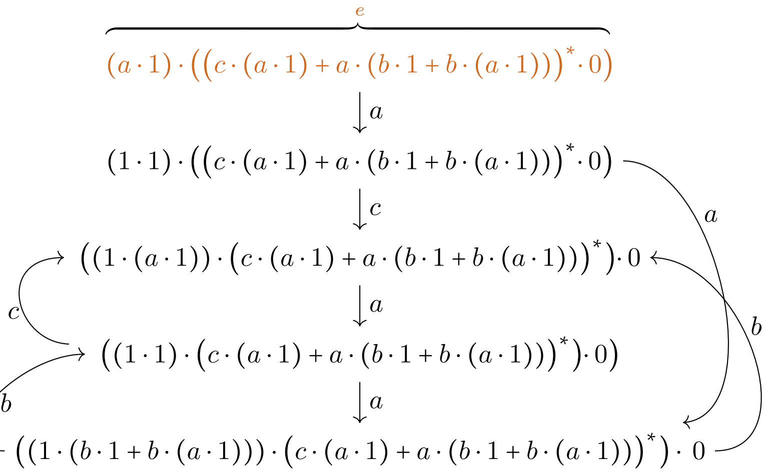
$$\Big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0$$

Interpretation of extracted expression

G_5



$$P(e) = G_5 \underline{\rightarrow} G_4 \not\simeq G_5$$



$$\overbrace{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)}^{e}$$

$$(1\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)$$

$$\Big((1\cdot(a\cdot 1))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0$$

$$\Big((1\cdot 1)\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0\Big)$$

$$\Big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0$$

Overview

- 1-free regular expressions (with unary/binary star)
- process interpretation / semantics of regular expressions
  - expressible/not expressible process graphs
- loop existence and elimination property (LEE) (G/Fokkink, 2020)
  - interpretation/extraction correspondences with 1-free reg. expr's
  - LEE is preserved under bisimulation collapse
- Q: Image of process interpretation of 1-free regular expressions closed under bisimulation collapse?
  - A1: No. — But . . .
- compact process interpretation
- refined expression extraction
  - A2: compact process interpretation is image-closed under collapse
- outlook: consequences

Image of *P* is **not** closed under bisimulation collapse

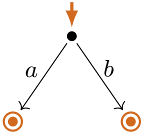
$P(uf)$

$P(uf)$

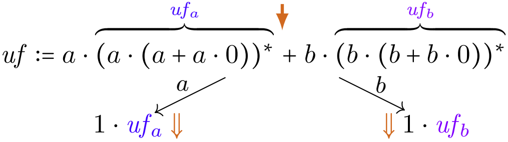
$$uf := a \cdot \overbrace{(a \cdot (a + a \cdot 0))^*}^{uf_a} \overset{\downarrow}{+} b \cdot \overbrace{(b \cdot (b + b \cdot 0))^*}^{uf_b}$$

Image of *P* is **not** closed under bisimulation collapse

$P(uf)$



$P(uf)$

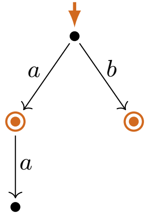


$$uf := a \cdot \overbrace{(a \cdot (a + a \cdot 0))^*}^{uf_a} + b \cdot \overbrace{(b \cdot (b + b \cdot 0))^*}^{uf_b}$$

$$1 \cdot uf_a \Downarrow \qquad \Downarrow 1 \cdot uf_b$$

Image of *P* is **not** closed under bisimulation collapse

P(uf)

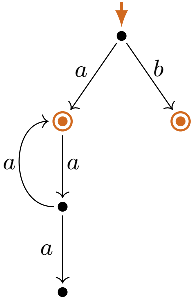


P(uf)

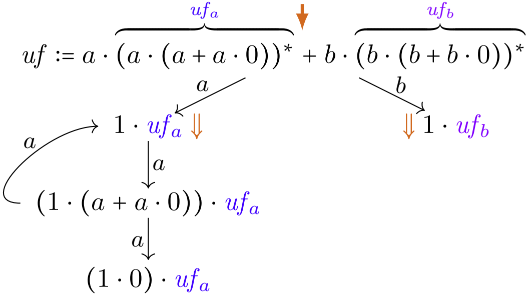
$$\begin{tikzcd}
& uf := a\cdot\overbrace{(a\cdot(a+a\cdot 0))^*}^{uf_a} + b\cdot\overbrace{(b\cdot(b+b\cdot 0))^*}^{uf_b} \arrow[dl, "a"'] \arrow[dr, "b"] & \\
1\cdot uf_a \Downarrow \arrow[d, "a"] & & \Downarrow 1\cdot uf_b \\
(1\cdot(a+a\cdot 0))\cdot uf_a & &
\end{tikzcd}$$

Image of *P* is **not** closed under bisimulation collapse

P(uf)



P(uf)



$$uf := a \cdot \overbrace{(a \cdot (a + a \cdot 0))^*}^{uf_a} \downarrow + b \cdot \overbrace{(b \cdot (b + b \cdot 0))^*}^{uf_b}$$

$1 \cdot uf_a \Downarrow$

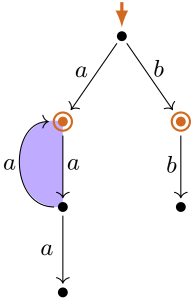
$(1 \cdot (a + a \cdot 0)) \cdot uf_a$

$(1 \cdot 0) \cdot uf_a$

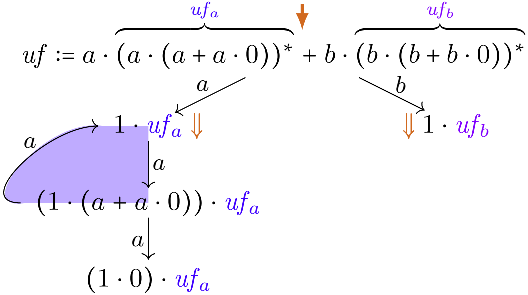
$\Downarrow 1 \cdot uf_b$

Image of *P* is **not** closed under bisimulation collapse

$P(uf)$



$P(uf)$



$$uf := a \cdot \overbrace{(a \cdot (a + a \cdot 0))^*}^{uf_a} + b \cdot \overbrace{(b \cdot (b + b \cdot 0))^*}^{uf_b}$$

$1 \cdot uf_a \Downarrow$

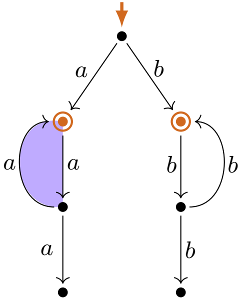
$\Downarrow 1 \cdot uf_b$

$(1 \cdot (a + a \cdot 0)) \cdot uf_a$

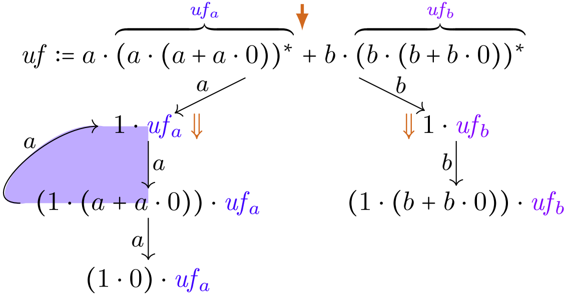
$(1 \cdot 0) \cdot uf_a$

Image of *P* is **not** closed under bisimulation collapse

$P(uf)$



$P(uf)$



$$uf := a \cdot \overbrace{(a \cdot (a + a \cdot 0))^*}^{uf_a} + b \cdot \overbrace{(b \cdot (b + b \cdot 0))^*}^{uf_b}$$

$1 \cdot uf_a \Downarrow$

$(1 \cdot (a + a \cdot 0)) \cdot uf_a$

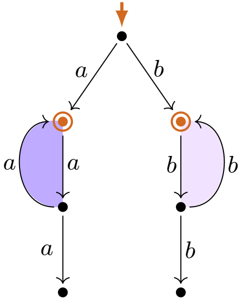
$(1 \cdot 0) \cdot uf_a$

$\Downarrow 1 \cdot uf_b$

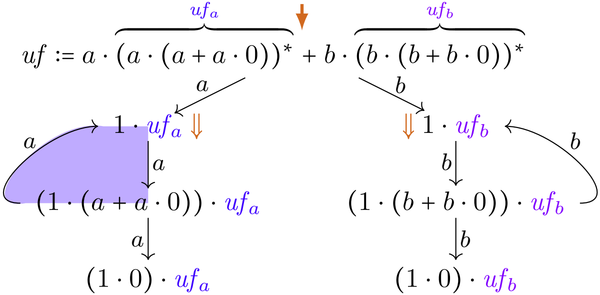
$(1 \cdot (b + b \cdot 0)) \cdot uf_b$

Image of P is **not** closed under bisimulation collapse

$P(uf)$



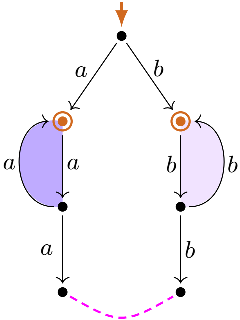
$P(uf)$



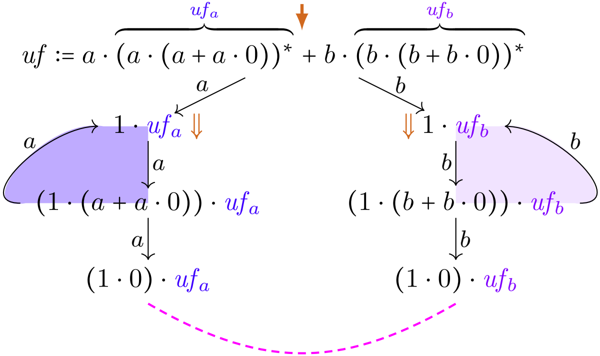
$$uf := a\cdot \overbrace{(a\cdot(a+a\cdot 0))^*}^{uf_a} + b\cdot \overbrace{(b\cdot(b+b\cdot 0))^*}^{uf_b}$$

Image of *P* is **not** closed under bisimulation collapse

$P(uf)$



$P(uf)$



$$uf := a \cdot \overbrace{(a \cdot (a + a \cdot 0))^*}^{uf_a} + b \cdot \overbrace{(b \cdot (b + b \cdot 0))^*}^{uf_b}$$

$1 \cdot uf_a \Downarrow$

$(1 \cdot (a + a \cdot 0)) \cdot uf_a$

$(1 \cdot 0) \cdot uf_a$

$\Downarrow 1 \cdot uf_b$

$(1 \cdot (b + b \cdot 0)) \cdot uf_b$

$(1 \cdot 0) \cdot uf_b$

Compact process interpretation $P^{\bullet}$

Definition (Transition system specification $\mathcal{T}$)

$$\frac{}{\mathbf{1}\Downarrow} \qquad \frac{e_i\Downarrow}{(e_1 + e_2)\Downarrow}\ (i \in \{1,2\}) \qquad \frac{e_1\Downarrow \qquad e_2\Downarrow}{(e_1 \cdot e_2)\Downarrow} \qquad \frac{}{(e^{*})\Downarrow}$$

$$\frac{}{a \xrightarrow{a} \mathbf{1}} \qquad \frac{e_i \xrightarrow{a} e_i'}{e_1 + e_2 \xrightarrow{a} e_i'}\ (i \in \{1,2\})$$

$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2} \qquad \frac{e_1\Downarrow \qquad e_2 \xrightarrow{a} e_2'}{e_1 \cdot e_2 \xrightarrow{a} e_2'} \qquad \frac{e \xrightarrow{a} e'}{e^{*} \xrightarrow{a} e' \cdot e^{*}}$$

Compact process interpretation $P^{\bullet}$

Definition (Transition system specification $\mathcal{T}$)

$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2}$$

$$\frac{e \xrightarrow{a} e'}{e^{*} \xrightarrow{a} e' \cdot e^{*}}$$

Compact process interpretation $P^{\bullet}$

Definition (Transition system specification $\mathcal{T}^{\bullet}$, changed rules w.r.t. $\mathcal{T}$)

$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2} \text{ (if } e_1' \text{ is normed)}$$

$$\frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e' \cdot e^*} \text{ (if } e' \text{ is normed)}$$

Compact process interpretation $P^{\bullet}$

Definition (Transition system specification $\mathcal{T}^{\bullet}$, changed rules w.r.t. $\mathcal{T}$)

$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2} \text{ (if } e_1' \text{ is normed)} \qquad \frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1'} \text{ (if } e_1' \text{ is not normed)}$$

$$\frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e' \cdot e^*} \text{ (if } e' \text{ is normed)} \qquad \frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e'} \text{ (if } e' \text{ is not normed)}$$

Compact process interpretation $P^{\bullet}$

Definition (Transition system specification $\mathcal{T}^{\bullet}$, changed rules w.r.t. $\mathcal{T}$)

$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2} \text{ (if } e_1' \text{ is normed)} \qquad \frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1'} \text{ (if } e_1' \text{ is not normed)}$$

$$\frac{e \xrightarrow{a} e'}{e^{*} \xrightarrow{a} e' \cdot e^{*}} \text{ (if } e' \text{ is normed)} \qquad \frac{e \xrightarrow{a} e'}{e^{*} \xrightarrow{a} e'} \text{ (if } e' \text{ is not normed)}$$

Definition

The compact process (graph) interpretation $P^{\bullet}(e)$ of a reg. expr's e:

$P^{\bullet}(e)$:= labeled transition graph generated by e by derivations in $\mathcal{T}^{\bullet}$.

Compact process interpretation $P^{\bullet}$

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$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2} \text{ (if } e_1' \text{ is normed)} \qquad \frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1'} \text{ (if } e_1' \text{ is not normed)}$$

$$\frac{e \xrightarrow{a} e'}{e^{*} \xrightarrow{a} e' \cdot e^{*}} \text{ (if } e' \text{ is normed)} \qquad \frac{e \xrightarrow{a} e'}{e^{*} \xrightarrow{a} e'} \text{ (if } e' \text{ is not normed)}$$

Definition

The compact process (graph) interpretation $P^{\bullet}(e)$ of a reg. expr's e:

$P^{\bullet}(e)$:= labeled transition graph generated by e by derivations in $\mathcal{T}^{\bullet}$.

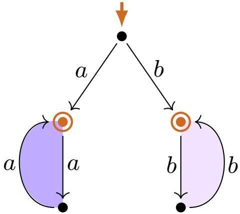
Lemma ($P^{\bullet}$ increases sharing; $P^{\bullet}$, P have same bisimulation semantics)

(i) $P(e) \mathrel{\underline{\rightarrow}} P^{\bullet}(e)$ *for all regular expressions* e.

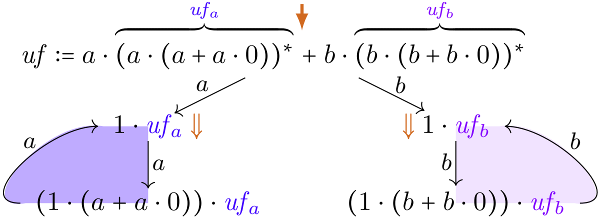
(ii) (G *is* $[\![\cdot]\!]_{P^{\bullet}}$*-expressible* $\Longleftrightarrow$ G *is* $[\![\cdot]\!]_{P}$*-expressible) for all graphs* G.

Image of $P^{\bullet}$ under bisimulation collapse ...

$P^{\bullet}(uf)$



$P^{\bullet}(uf)$



$$uf := a \cdot \overbrace{(a \cdot (a + a \cdot 0))^{*}}^{uf_a} + b \cdot \overbrace{(b \cdot (b + b \cdot 0))^{*}}^{uf_b}$$

$1 \cdot uf_a$ ⇓

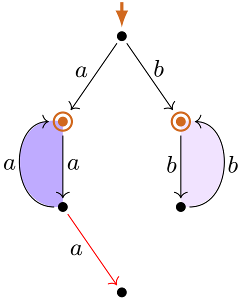
⇓ $1 \cdot uf_b$

$(1 \cdot (a + a \cdot 0)) \cdot uf_a$

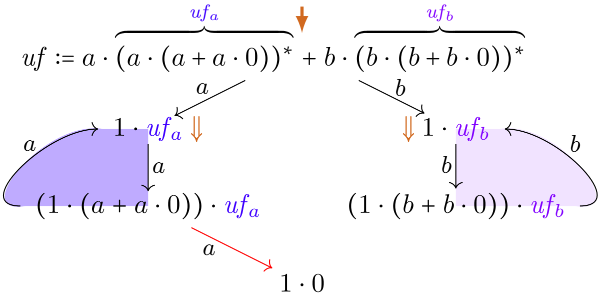
$(1 \cdot (b + b \cdot 0)) \cdot uf_b$

Image of $P^{\bullet}$ under bisimulation collapse . . .

$P^{\bullet}(uf)$



$P^{\bullet}(uf)$



$$uf := a \cdot \overbrace{(a \cdot (a + a \cdot 0))^{*}}^{uf_a} + b \cdot \overbrace{(b \cdot (b + b \cdot 0))^{*}}^{uf_b}$$

$1 \cdot uf_a \Downarrow$

$(1 \cdot (a + a \cdot 0)) \cdot uf_a$

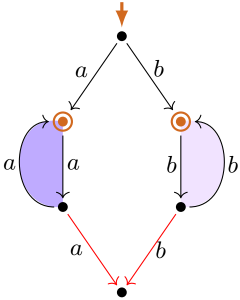
$\Downarrow 1 \cdot uf_b$

$(1 \cdot (b + b \cdot 0)) \cdot uf_b$

$1 \cdot 0$

Image of $P^{\bullet}$ under bisimulation collapse . . .

$P^{\bullet}(uf)$



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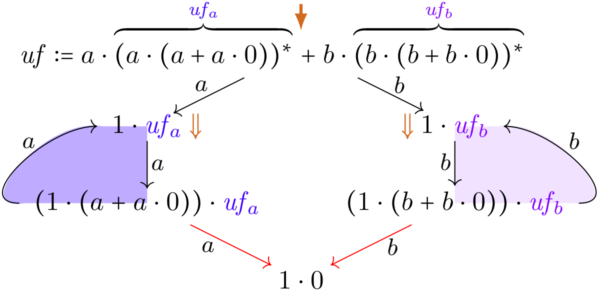

$uf := a \cdot \overbrace{(a \cdot (a + a \cdot 0))^{*}}^{uf_a} + b \cdot \overbrace{(b \cdot (b + b \cdot 0))^{*}}^{uf_b}$

$1 \cdot uf_a$ ⇓

⇓ $1 \cdot uf_b$

$(1 \cdot (a + a \cdot 0)) \cdot uf_a$

$(1 \cdot (b + b \cdot 0)) \cdot uf_b$

$1 \cdot 0$

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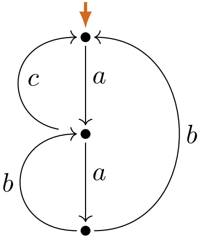
Refined extraction expression (example)

G_4



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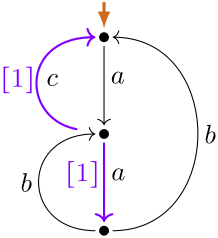

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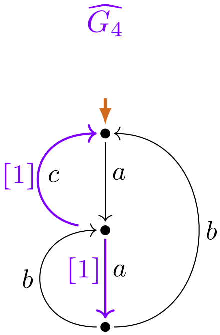

$$(1 \cdot (\qquad\qquad\qquad\qquad)^{*}) \cdot 0$$

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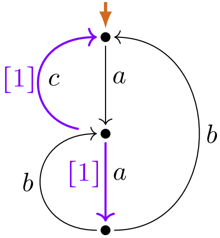

$$(1 \cdot (\qquad \overset{a}{\downarrow} \qquad)^{*}) \cdot 0$$

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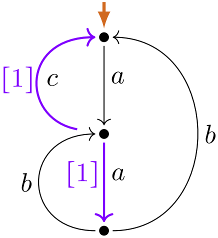

$$((1 \cdot a) \cdot (\qquad\qquad\qquad)^*) \cdot 0$$

$$\downarrow a$$

$$(1 \cdot (\qquad\qquad\qquad)^*) \cdot 0$$

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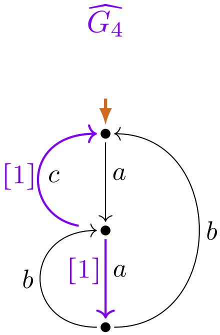
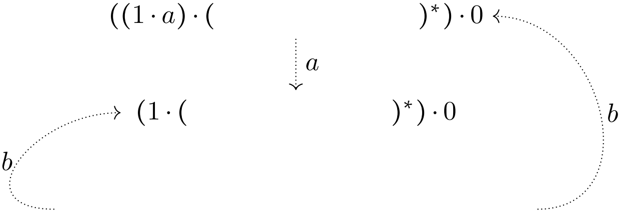



$((1 \cdot a) \cdot ($ $)^*) \cdot 0$

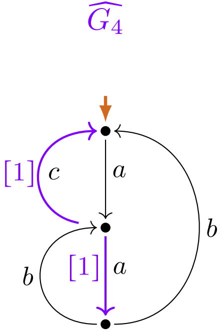
a

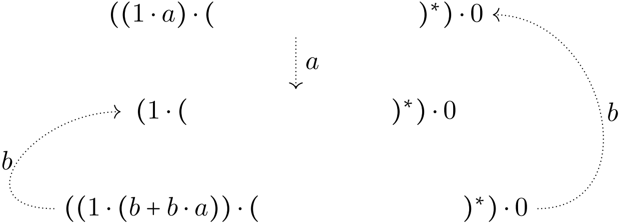
$(1 \cdot ($ $)^*) \cdot 0$

b

b

Refined extraction expression (example)





$((1 \cdot a) \cdot ($ $)^{*}) \cdot 0$

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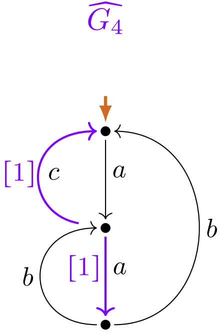

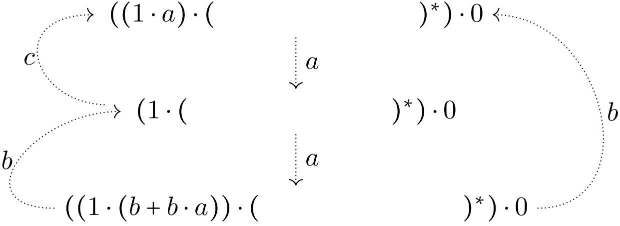

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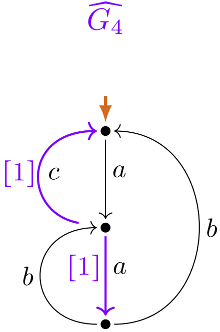

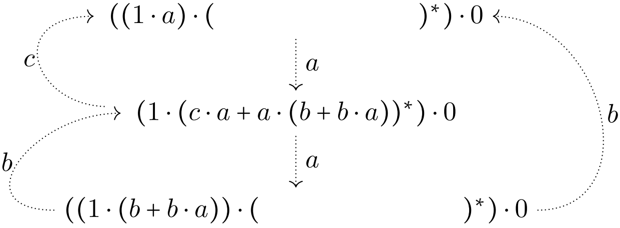

$((1 \cdot a) \cdot (\qquad\qquad)^{*}) \cdot 0$

$(1 \cdot (c \cdot a + a \cdot (b + b \cdot a))^{*}) \cdot 0$

$((1 \cdot (b + b \cdot a)) \cdot (\qquad\qquad)^{*}) \cdot 0$

Refined extraction expression (example)

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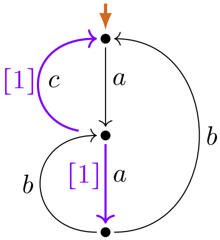

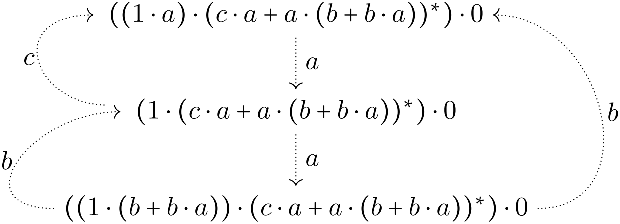
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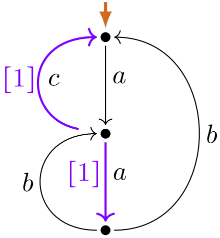

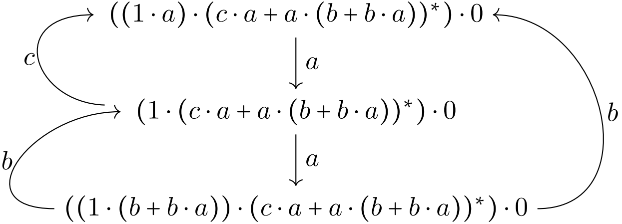

$((1 \cdot a) \cdot (c \cdot a + a \cdot (b + b \cdot a))^{*}) \cdot 0$

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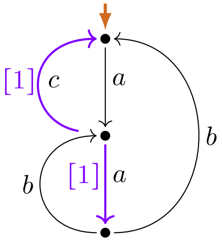
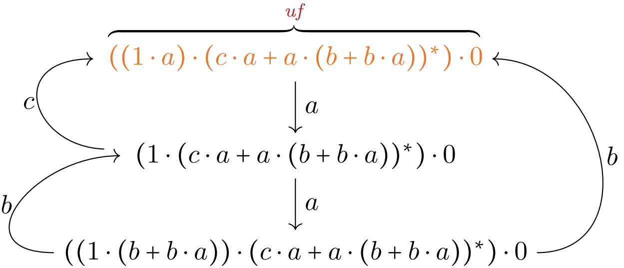


$\mathsf{P}^{\bullet}(uf) = \mathsf{P}(uf) \simeq G_4$



$\overbrace{((1\cdot a)\cdot(c\cdot a+a\cdot(b+b\cdot a))^{*})\cdot 0}^{uf}$

$(1\cdot(c\cdot a+a\cdot(b+b\cdot a))^{*})\cdot 0$

$((1\cdot(b+b\cdot a))\cdot(c\cdot a+a\cdot(b+b\cdot a))^{*})\cdot 0$

Interpretation/extraction correspondences of $P^{\bullet}$ with LEE

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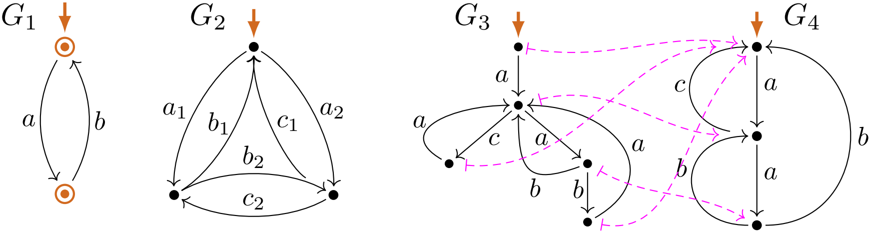
P-/$P^{\bullet}$-expressibility and $[\![\cdot]\!]_P$-expressibility (examples)



not P-expressible

not $[\![\cdot]\!]_P$-expressible

P-/$P^{\bullet}$-expressible

$[\![\cdot]\!]_P$-expressible

$P^{\bullet}$-expressible

$[\![\cdot]\!]_P$-expressible

Summary and outlook

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Loop charts (interpretations of innermost iterations)

Definition

A chart is a loop chart if:

(L1) There is an infinite path from the start vertex.

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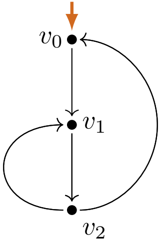

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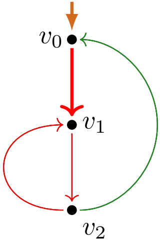

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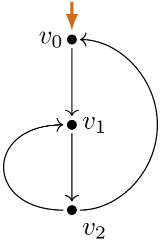

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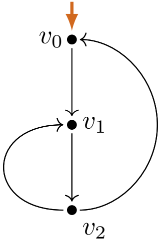

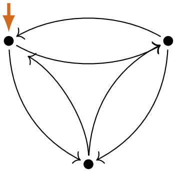

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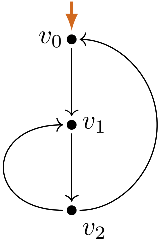

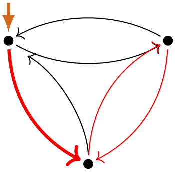

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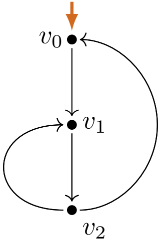

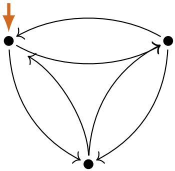

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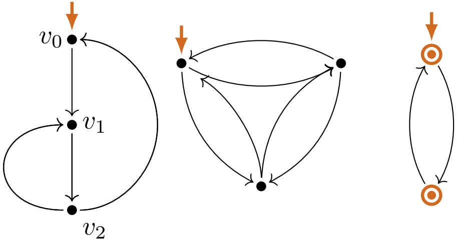

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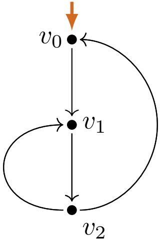

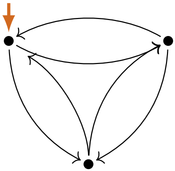

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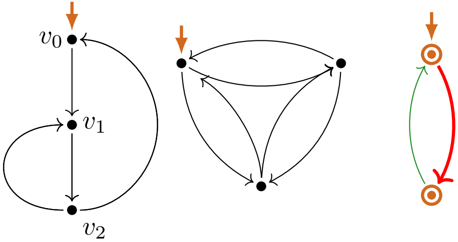

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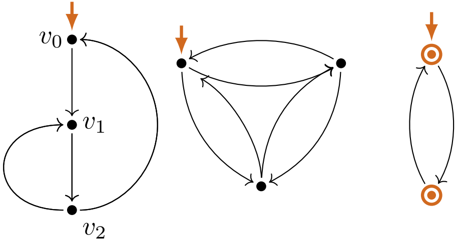

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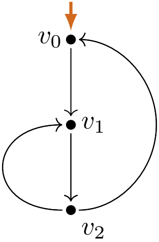

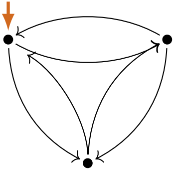

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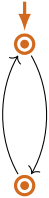
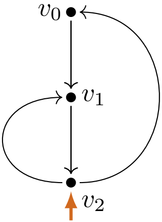


(L1),(L2),~~(L3)~~



Loop charts (interpretations of innermost iterations)

Definition

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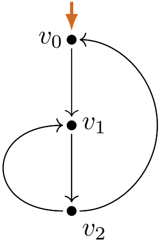

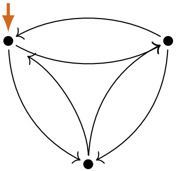

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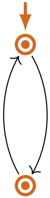

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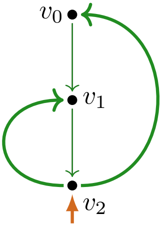

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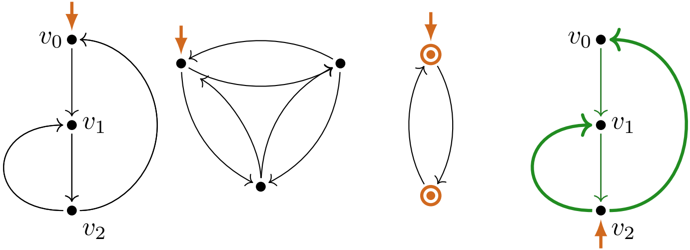

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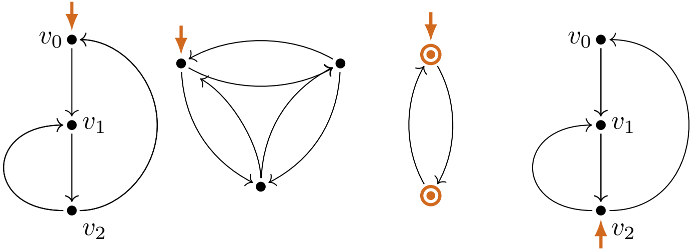

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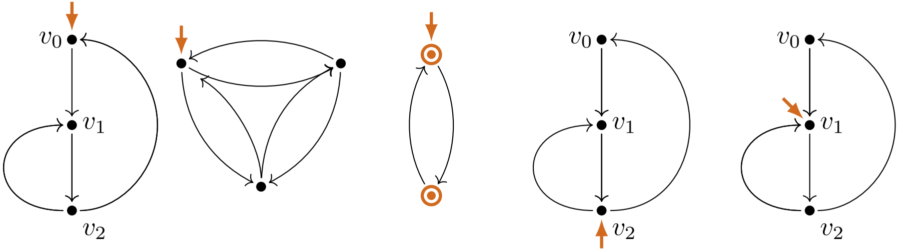

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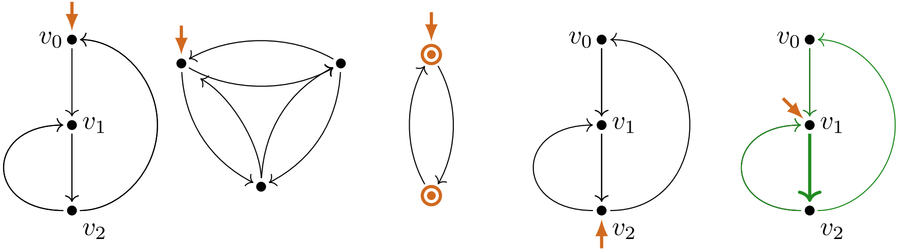

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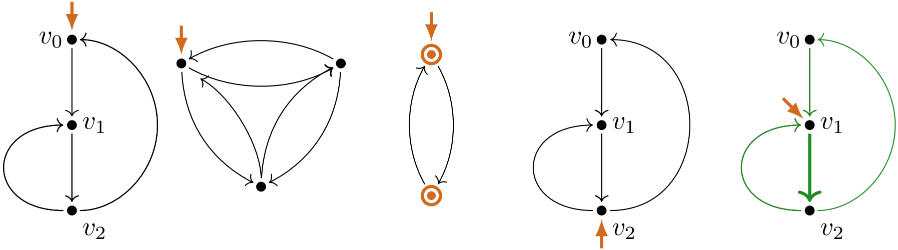

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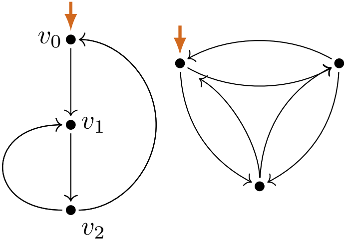

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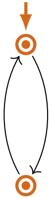

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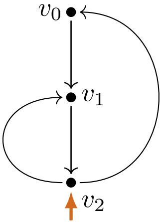
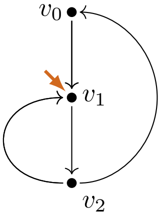


loop chart



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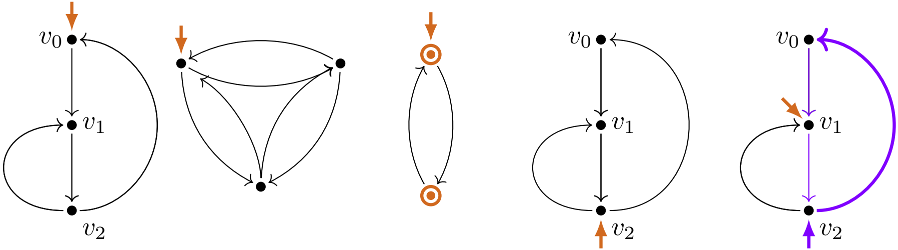

(L1),~~(L2)~~

(L1),(L2),~~(L3)~~

loop chart

loop subchart