Closing the Image of the Process Interpretation of 1-Free Regular Expressions Under Bisim. Collapse

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https://clegra.github.io

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Regular Expressions

Definition (~ *Copi–Elgot–Wright, 1958*)

Regular expressions over alphabet A with unary Kleene star:

$$e, e_1, e_2 \; ::= \; \mathbf{0} \;\mid\; \boldsymbol{a} \;\mid\; e_1 + e_2 \;\mid\; e_1 \cdot e_2 \;\mid\; e^{*} \qquad \text{(for } \boldsymbol{a} \in A\text{)}.$$

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- symbol $\mathbf{0}$ instead of $\varnothing$, symbol $\mathbf{1}$ instead of $\{\varnothing\}$ and ϵ
- with unary Kleene star *: $\mathbf{1}$ is definable as $\mathbf{0}^*$

Regular Expressions

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Regular expressions over alphabet A with unary / binary Kleene star:

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Definition

1-free regular expressions over alphabet A with binary Kleene star:

$$f, f_1, f_2 \ ::= \ \mathbf{0} \mid a \mid f_1 + f_2 \mid f_1 \cdot f_2 \mid f_1{}^{\circledast} f_2 \qquad (\text{for } a \in A).$$

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Under-Star-/1-Free regular expressions

Definition

The set $\mathit{RExp}^{(\cancel{1})}(A)$ of 1-free regular expressions over A is defined by:

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the set $RExp^{(\not{1}\backslash *)}(A)$ of under-star-1-free regular expressions over A by:

$$uf,\, uf_1,\, uf_2 \;::=\; 0 \mid 1 \mid a \mid uf_1 + uf_2 \mid uf_1 \cdot uf_2 \mid f^{\star} \qquad \text{(for } a \in A\text{)}.$$

Process interpretation $P(\cdot)$ of regular expressions *(Milner, 1984)*

$\mathbf{0} \xrightarrow{P}$ deadlock $\boldsymbol{\delta}$, no termination

$\mathbf{1} \xrightarrow{P}$ empty-step process $\boldsymbol{\epsilon}$, then terminate

$a \xrightarrow{P}$ atomic action a, then terminate

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$e_1 + e_2 \xrightarrow{P}$ (*choice*) execute $P(e_1)$ or $P(e_2)$

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$e^* \xrightarrow{P}$ (*iteration*) repeat (terminate or execute $P(e)$)

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$e_1{}^{\circledast} e_2 \xmapsto{P}$ *(iteration-exit)* repeat (terminate or execute $P(e_1)$), then $P(e_2)$

Process semantics $[\![\cdot]\!]_P$ of regular expressions *(Milner, 1984)*

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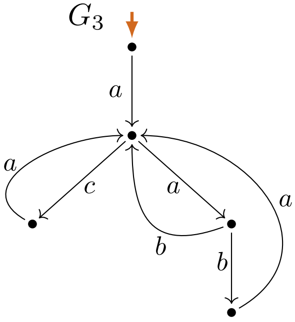
$[\![e]\!]_P := [P(e)]_{\underline{\leftrightarrow}}$ (bisimilarity equivalence class of process $P(e)$)

P-expressibility and $[\![\cdot]\!]_P$-expressibility (examples)



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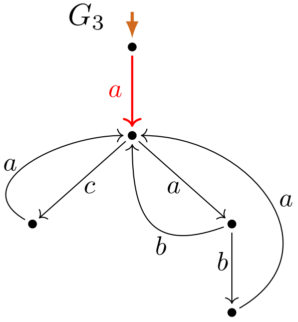

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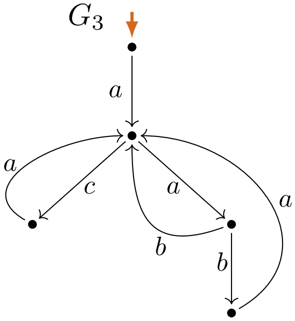

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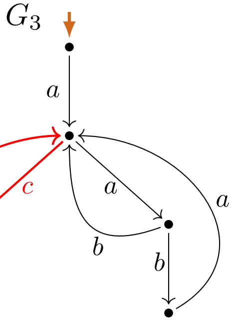

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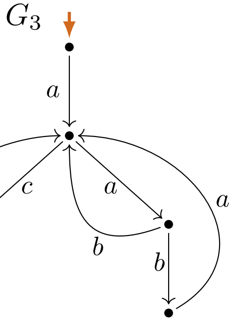

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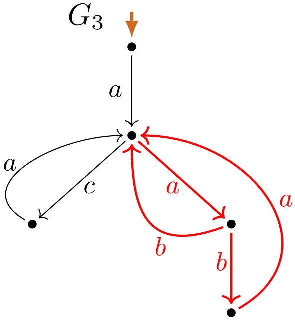

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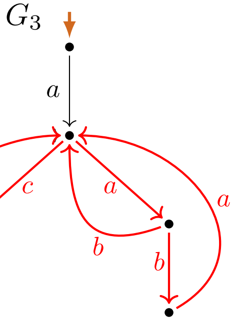

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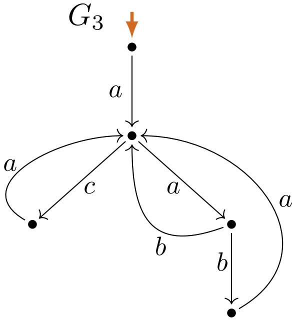

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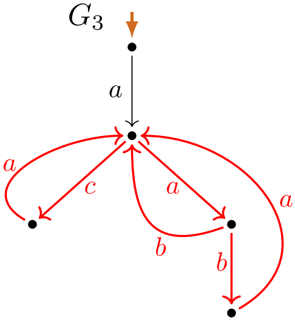

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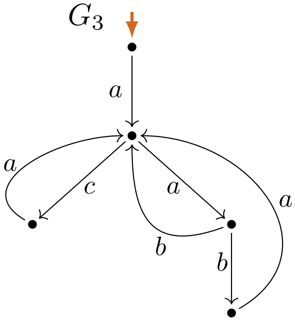

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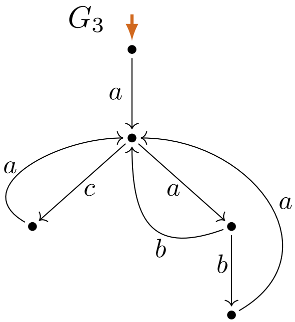

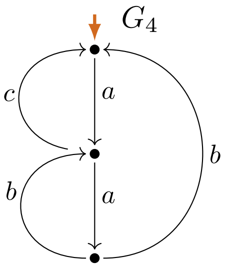

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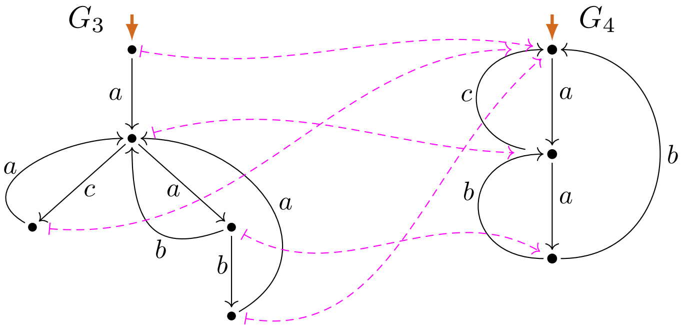

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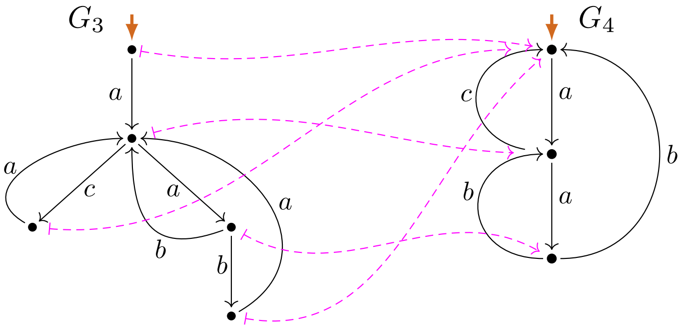

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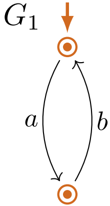

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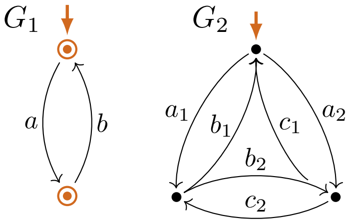

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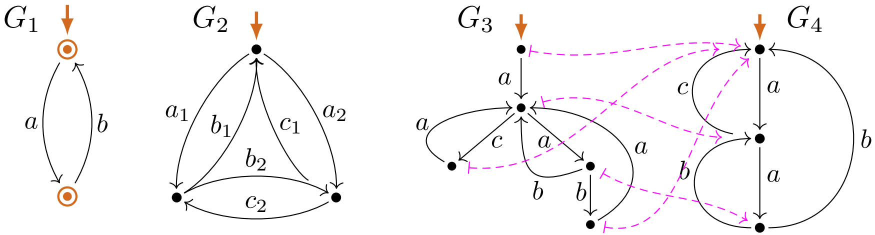

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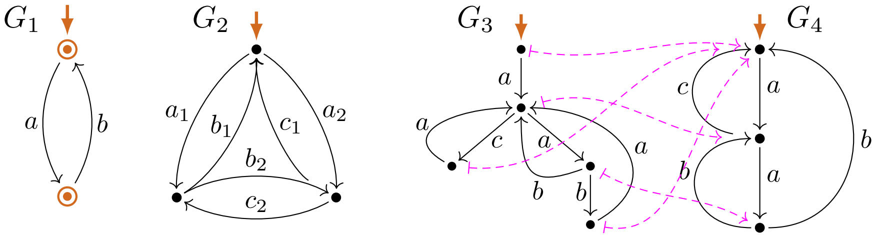

not P-expressible — P-expressible — **?**

not $[\![\cdot]\!]_P$-expressible — $[\![\cdot]\!]_P$-expressible — $[\![\cdot]\!]_P$-expressible

Process interpretation P (formal definition)

Definition (Transition system specification $\mathcal{T}$)

$$\frac{}{\boldsymbol{a} \xrightarrow{\boldsymbol{a}} \mathbf{1}} \qquad \frac{e_i \xrightarrow{\boldsymbol{a}} e_i'}{e_1 + e_2 \xrightarrow{\boldsymbol{a}} e_i'} \, (i \in \{1,2\})$$

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Definition

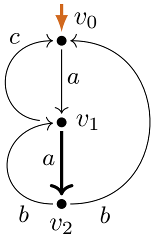
The process (graph) interpretation $P(e)$ of a regular expression e:

$P(e)$:= labeled transition graph generated by e by derivations in $\mathcal{T}$.

Overview

- 1-free regular expressions (with unary/binary star)
- process interpretation / semantics of regular expressions
  - expressible/not expressible process graphs
- loop existence and elimination property (LEE) (G/Fokkink, 2020)
  - interpretation/extraction correspondences with 1-free reg. expr's
  - LEE is preserved under bisimulation collapse
- Q: Image of process interpretation of 1-free regular expressions closed under bisimulation collapse?
  - A1: No. — But . . .
- compact process interpretation
- refined expression extraction
  - A2: compact process interpretation is image-closed under collapse
- outlook: consequences

Loop existence and elimination



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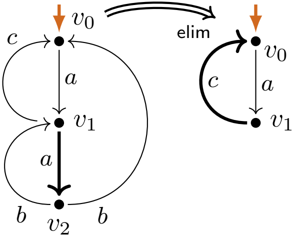

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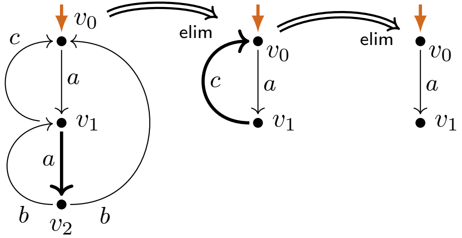

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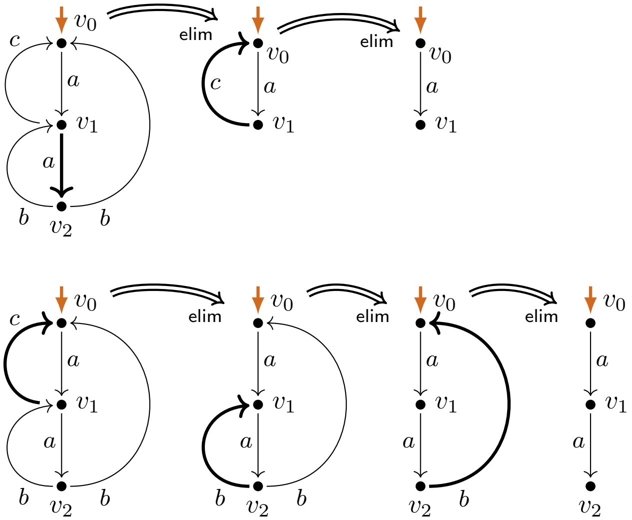

LEE

Definition

A chart $\mathcal{C}$ satisfies LEE (*loop existence and elimination*) if:

$$\exists \mathcal{C}_0 \left(\mathcal{C} \longrightarrow^{*}_{\text{elim}} \mathcal{C}_0 \not\longrightarrow_{\text{elim}} \right.$$
$$\left. \wedge \ \mathcal{C}_0 \text{ permits no infinite path} \right).$$

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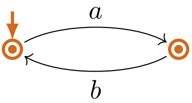
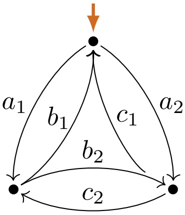
$$\exists \mathcal{C}_0 \left(\mathcal{C} \longrightarrow^{*}_{\text{elim}} \mathcal{C}_0 \not\longrightarrow_{\text{elim}} \right.$$
$$\left. \wedge\ \mathcal{C}_0 \text{ permits no infinite path} \right).$$

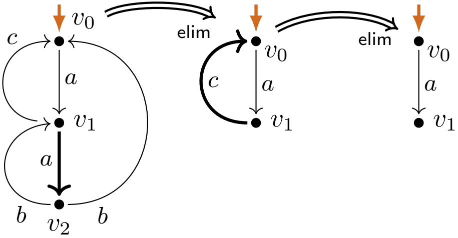


~~LEE~~

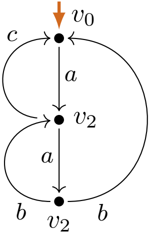


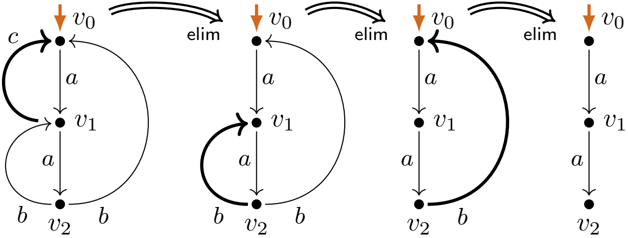
~~LEE~~

LEE

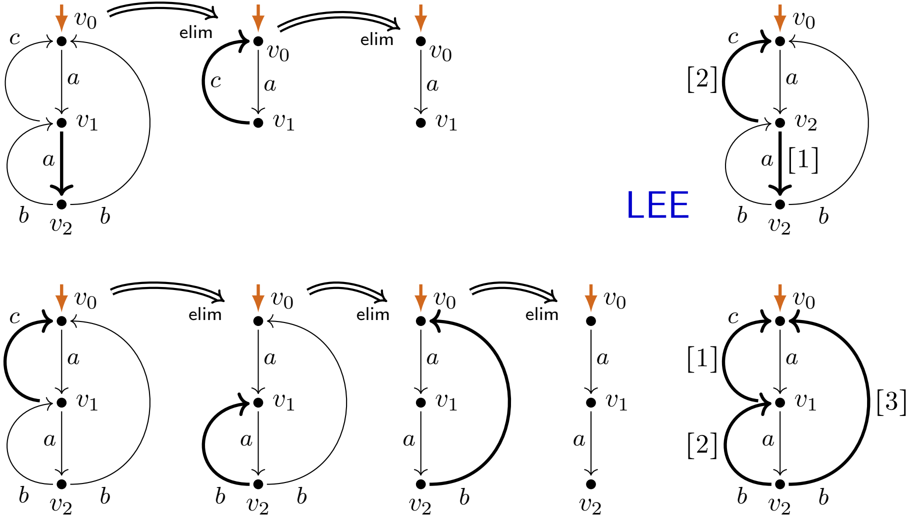


LEE

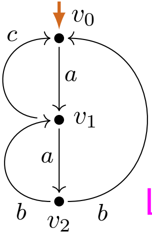




LEE

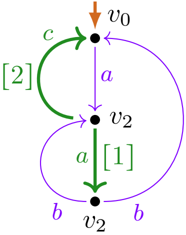


LEE witness and LEE-charts

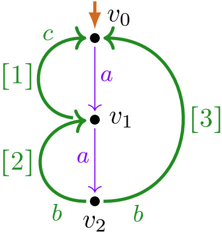


LEE-chart

LEE-witness



LEE-witness



Properties of LEE-charts

Theorem (⇐ G/Fokkink, 2020)

A process graph G

is $[\![\cdot]\!]_P$-expressible by an under-star-1-free regular expression

(i.e. P-expressible modulo bisimilarity by a 1-free reg. expr.)

if and only if

the bisimulation collapse of G satisfies LEE.

Properties of LEE-charts

Theorem (⇐ G/Fokkink, 2020)

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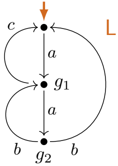
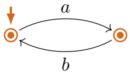
if and only if
the bisimulation collapse of G satisfies LEE.

Hence $[\![\cdot]\!]_P$-expressible | **not** $[\![\cdot]\!]_P$-expressible by 1-free regular expressions:

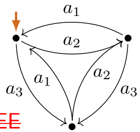


LEE



~~LEE~~

~~LEE~~



Interpretation/extraction correspondences with LEE

($\Leftarrow$ G/Fokkink 2020, G 2021)

(Int)$_{P}^{(\not{1})}$: *P-expressible graphs have the structural property* LEE

Process **i**nterpretations $P(e)$ of 1-free regular expressions e are finite process graphs that satisfy LEE.

Interpretation/extraction correspondences with LEE

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(Extr)$_{P}$: LEE *implies* $[\![\cdot]\!]_{P}$*-expressibility*

From every finite process graph G with LEE a regular expression e can be **e**xtracted such that $G \leftrightarrow P(e)$.

Interpretation/extraction correspondences with LEE

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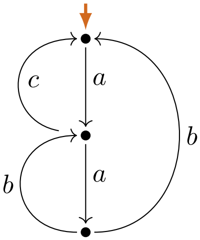
From every finite process graph G with LEE a regular expression e can be **e**xtracted such that $G \underline{\leftrightarrow} P(e)$.

(Coll): LEE *is preserved under collapse*

The class of finite process graphs with LEE is closed under bisimulation **c**ollapse.

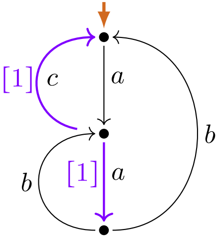
Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

G_4



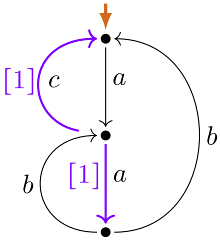
Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

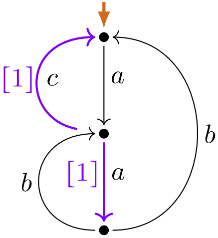
$\widehat{G_4}$



$$(\qquad\qquad\qquad)^* \cdot 0$$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

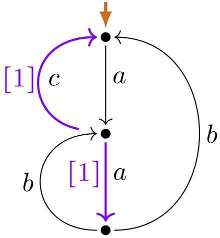
$\widehat{G_4}$



$\Big(\quad \overset{\vdots\, a}{\downarrow} \quad \Big)^* \cdot 0$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



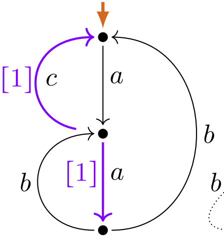
$$(a \cdot 1) \cdot ((\qquad\qquad)^* \cdot 0)$$

$$\Big\downarrow a$$

$$(\qquad\qquad)^* \cdot 0$$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$(a \cdot 1) \cdot ((\qquad)^* \cdot 0)$

a

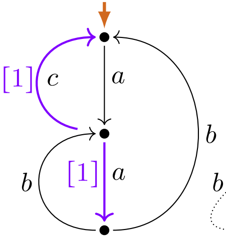
$(\qquad)^* \cdot 0$

b

b

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$(a \cdot 1) \cdot ((\qquad)^* \cdot 0)$

a

$(\qquad)^* \cdot 0$

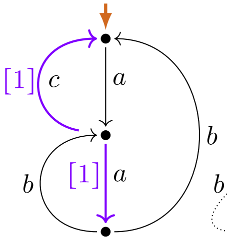
b

b

$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot ((\qquad)^*) \cdot 0)$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$(a \cdot 1) \cdot ((\qquad)^* \cdot 0)$

$c \qquad \downarrow a \qquad b$

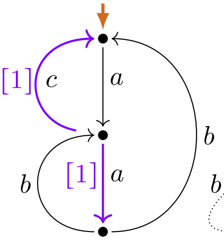
$(\qquad)^* \cdot 0$

$b \qquad \downarrow a$

$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot ((\qquad)^*) \cdot 0)$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$$(a \cdot 1) \cdot \big(\big(\qquad\qquad\qquad\qquad \big)^* \cdot 0\big)$$

$$\downarrow a$$

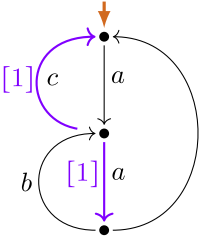
$$\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0$$

$$\downarrow a$$

$$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot \big(\big(\qquad\qquad\qquad\qquad \big)^*) \cdot 0\big)$$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

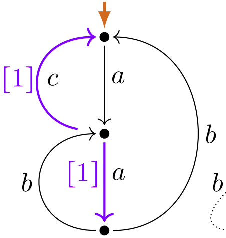
$\widehat{G_4}$



$$
\begin{tikzcd}
(a\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big) \arrow[d, dotted, "a"] \\
\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0 \arrow[d, dotted, "a"] \arrow[u, dotted, bend left, "c"] \\
(b\cdot 1+b\cdot(a\cdot 1))\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*})\cdot 0\big) \arrow[u, dotted, bend left, "b"] \arrow[uu, dotted, bend right, "b"']
\end{tikzcd}
$$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$$\overbrace{(a \cdot 1) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0 \big)}^{e}$$

$$\Big\downarrow a$$

$$\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0$$

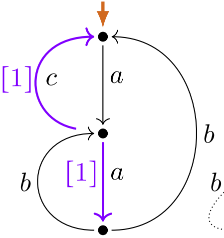
$$\Big\downarrow a$$

$$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*}) \cdot 0 \big)$$

c b b

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$P(e) \mathrel{\underline{\rightarrow}} G_4$

$$\overbrace{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)}^{e}$$

$$\Big\downarrow a$$

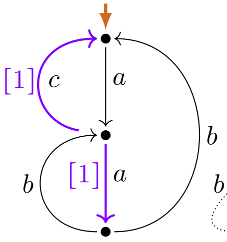
$$\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0$$

$$\Big\downarrow a$$

$$(b\cdot 1+b\cdot(a\cdot 1))\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0\Big)$$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$P(e) \mathrel{\underline{\to}} G_4$

$$\overbrace{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)}^{e}$$

$$\cancel{\downarrow}\; a$$

$$\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0$$

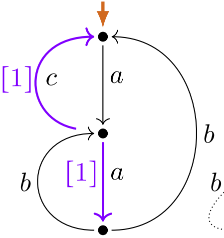
$$\downarrow a$$

$$(b\cdot 1+b\cdot(a\cdot 1))\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0\Big)$$

c b b

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$P(e) \mathrel{\underline{\to}} G_4$

$$\overbrace{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)}^{e}$$

$$\downarrow a$$

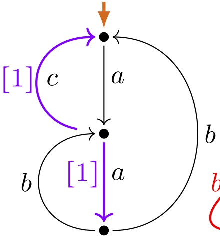
$$\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0$$

$$\downarrow a$$

$$(b\cdot 1+b\cdot(a\cdot 1))\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0\Big)$$

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$P(e) \;\underline{\to}\; G_4$

$$\overbrace{(a \cdot 1) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \cdot 0 \big)}^{e}$$

$$\downarrow a$$

$$\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \cdot 0$$

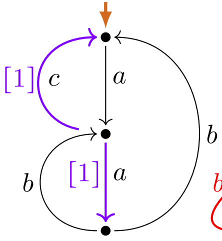
$$\downarrow a$$

$$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^*) \cdot 0 \big)$$

c b b

Expression extraction using LEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$$P(e) \,\underline{\rightarrow}\, G_4 \not\underline{\leftarrow} P(e)$$

$$\overbrace{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^*\cdot 0\Big)}^{e}$$

$$\downarrow a$$

$$\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^*\cdot 0$$

$$\downarrow a$$

$$(b\cdot 1+b\cdot(a\cdot 1))\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^*)\cdot 0\Big)$$

Interpretation of extracted expression

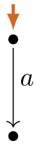
G_5

$P(e) = G_5$

$$\overbrace{(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^{*} \cdot 0\Big)}^{e}$$

Interpretation of extracted expression

G_5



$P(e) = G_5$

$$\overbrace{(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0 \Big)}^{e}$$

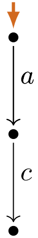
$$\Big\downarrow a$$

$$(1 \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0 \Big)$$

Interpretation of extracted expression

G_5

$P(e) = G_5$



$$\overbrace{(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^{*} \cdot 0\Big)}^{e}$$

$$\downarrow a$$

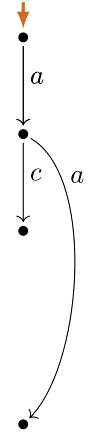
$$(1 \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^{*} \cdot 0\Big)$$

$$\downarrow c$$

$$\Big((1 \cdot (a \cdot 1)) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^{*}\Big) \cdot 0$$

Interpretation of extracted expression

G_5



$P(e) = G_5$

$$\overbrace{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)}^{e}$$

$$\Big\downarrow a$$

$$(1\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)$$

$$\Big\downarrow c$$

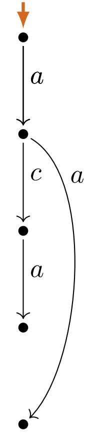
$$\Big((1\cdot(a\cdot 1))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0$$

$$\xrightarrow{a}$$

$$\Big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0$$

Interpretation of extracted expression

G_5



$\textcolor{green}{P}(\textcolor{orange}{e}) = G_5$

$$\overbrace{(a\cdot 1)\cdot\big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\big)}^{e}$$

$$\downarrow a$$

$$(1\cdot 1)\cdot\big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\big)$$

$$\downarrow c$$

$$\big((1\cdot(\textcolor{red}{a}\cdot 1))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0$$

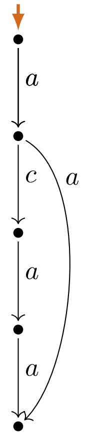
$$\textcolor{red}{\downarrow a}$$

$$\big((1\cdot 1)\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0\big)$$

$$\big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0$$

Interpretation of extracted expression

G_5



$P(e) = G_5$

$$\overbrace{(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0 \Big)}^{e}$$

$$\downarrow a$$

$$(1 \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0 \Big)$$

$$\downarrow c$$

$$\Big((1 \cdot (a \cdot 1)) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \Big) \cdot 0$$

$$\downarrow a$$

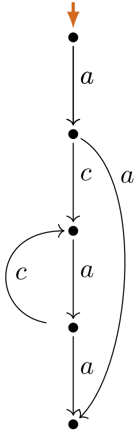
$$\Big((1 \cdot 1) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \Big) \cdot 0 \Big)$$

$$\downarrow a$$

$$\Big((1 \cdot (b \cdot 1 + b \cdot (a \cdot 1))) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \Big) \cdot 0$$

Interpretation of extracted expression

G_5 $\qquad\qquad\qquad$ $P(e) = G_5$



$$\overbrace{(a\cdot 1)\cdot\big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\big)}^{e}$$

$$\downarrow a$$

$$(1\cdot 1)\cdot\big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\big)$$

$$\downarrow c$$

$$\big((1\cdot(a\cdot 1))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0$$

$$\downarrow a$$

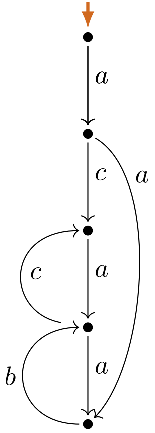
$$\big((1\cdot 1)\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0\big)$$

$$\downarrow a$$

$$\big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0$$

Interpretation of extracted expression

G_5



$\textcolor{green}{P}(\textcolor{orange}{e}) = G_5$

$$\overbrace{\textcolor{orange}{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)}}^{\textcolor{orange}{e}}$$

$$\downarrow a$$

$$(1\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)$$

$$\downarrow c$$

$$\Big((1\cdot(a\cdot 1))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0$$

$$\downarrow a$$

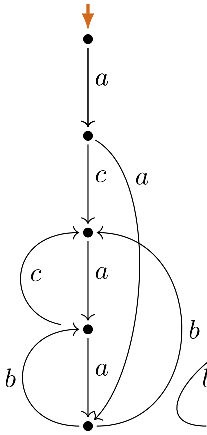
$$\Big(\big((1\cdot 1)\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0\Big)$$

$$\downarrow a$$

$$\Big((1\cdot(b\cdot 1+\textcolor{red}{b}\cdot(a\cdot 1)))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0$$

Interpretation of extracted expression

G_5



$P(e) = G_5$

$$\overbrace{(a \cdot 1) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0\big)}^{e}$$

$$\Big\downarrow a$$

$$(1 \cdot 1) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0\big)$$

$$\Big\downarrow c$$

$$\big((1 \cdot (a \cdot 1)) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^*\big) \cdot 0$$

$$\Big\downarrow a$$

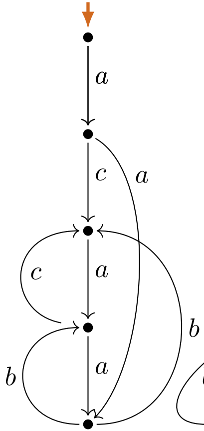
$$\big((1 \cdot 1) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^*\big) \cdot 0\big)$$

$$\Big\downarrow a$$

$$\big((1 \cdot (b \cdot 1 + b \cdot (a \cdot 1))) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^*\big) \cdot 0$$

Interpretation of extracted expression

G_5



$P(e) = G_5 \xrightarrow{} G_4$

$$\overbrace{(a\cdot 1)\cdot\big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\big)}^{e}$$

$$\downarrow a$$

$$(1\cdot 1)\cdot\big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\big)$$

$$\downarrow c$$

$$\big((1\cdot(a\cdot 1))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0$$

$$\downarrow a$$

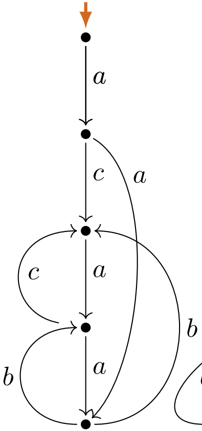
$$\big((1\cdot 1)\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0\big)$$

$$\downarrow a$$

$$\big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0$$

Interpretation of extracted expression

G_5



$$P(e) = G_5 \Rightarrow G_4 \not\simeq G_5$$

$$\overbrace{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)}^{e}$$

$$\downarrow a$$

$$(1\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)$$

$$\downarrow c$$

$$\Big((1\cdot(a\cdot 1))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0$$

$$\downarrow a$$

$$\Big((1\cdot 1)\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0\Big)$$

$$\downarrow a$$

$$\Big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0$$

Overview

- 1-free regular expressions (with unary/binary star)
- process interpretation/semantics of regular expressions
  - expressible/not expressible process graphs
- loop existence and elimination property (LEE) (G/Fokkink, 2020)
  - interpretation/extraction correspondences with 1-free reg. expr's
  - LEE is preserved under bisimulation collapse
- Q: Image of process interpretation of 1-free regular expressions closed under bisimulation collapse?
  - A1: No. — But ...
- compact process interpretation
- refined expression extraction
  - A2: compact process interpretation is image-closed under collapse
- outlook: consequences

Image of P is **not** closed under bisimulation collapse

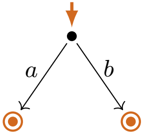
$P(uf)$

$P(uf)$

$$uf := a \cdot \overbrace{(a \cdot (a + a \cdot 0))^*}^{uf_a} + b \cdot \overbrace{(b \cdot (b + b \cdot 0))^*}^{uf_b}$$

Image of P is **not** closed under bisimulation collapse

$P(uf)$



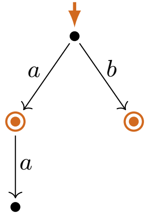
$P(uf)$

$$uf := a \cdot \overbrace{(a \cdot (a + a \cdot 0))^*}^{uf_a} + b \cdot \overbrace{(b \cdot (b + b \cdot 0))^*}^{uf_b}$$

$$1 \cdot uf_a \Downarrow \xleftarrow{a} \qquad \xrightarrow{b} \Downarrow 1 \cdot uf_b$$

Image of P is **not** closed under bisimulation collapse

$P(uf)$



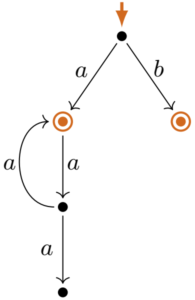
$P(uf)$

$$uf := a \cdot \overbrace{(a \cdot (a + a \cdot 0))^*}^{uf_a} + b \cdot \overbrace{(b \cdot (b + b \cdot 0))^*}^{uf_b}$$

$$\begin{tikzcd} & uf \arrow[dl, "a"'] \arrow[dr, "b"] & \\ 1 \cdot uf_a \Downarrow \arrow[d, "a"] & & \Downarrow 1 \cdot uf_b \\ (1 \cdot (a + a \cdot 0)) \cdot uf_a & & \end{tikzcd}$$

Image of P is **not** closed under bisimulation collapse

$P(uf)$



$P(uf)$

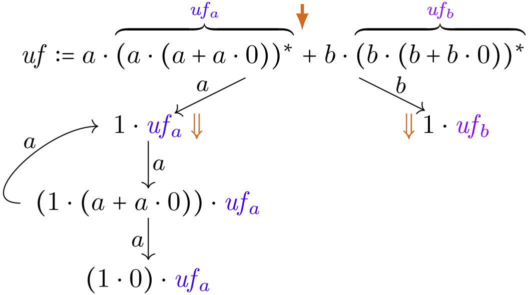
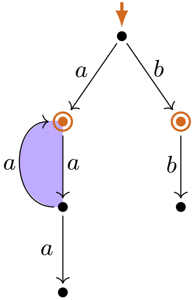


Image of *P* is **not** closed under bisimulation collapse

$P(uf)$



$P(uf)$

$$uf := a \cdot \overbrace{(a \cdot (a + a \cdot 0))^*}^{uf_a} + b \cdot \overbrace{(b \cdot (b + b \cdot 0))^*}^{uf_b}$$

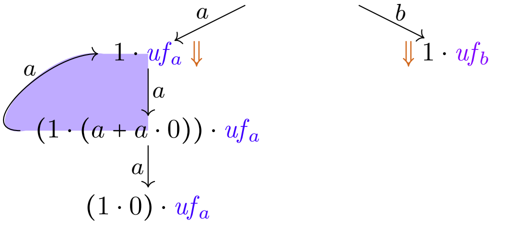
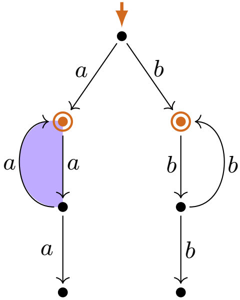


Image of *P* is **not** closed under bisimulation collapse

$P(uf)$



$P(uf)$

$$uf := a \cdot \overbrace{(a \cdot (a + a \cdot 0))^*}^{uf_a} + b \cdot \overbrace{(b \cdot (b + b \cdot 0))^*}^{uf_b}$$

$$uf \xrightarrow{a} 1 \cdot uf_a \Downarrow \qquad uf \xrightarrow{b} \Downarrow 1 \cdot uf_b$$

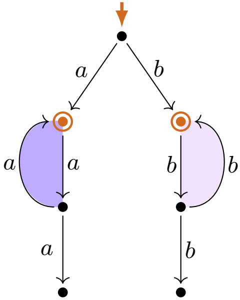
$$1 \cdot uf_a \xrightarrow{a} (1 \cdot (a + a \cdot 0)) \cdot uf_a \xrightarrow{a} (1 \cdot 0) \cdot uf_a$$

$$(1 \cdot (a + a \cdot 0)) \cdot uf_a \xrightarrow{a} 1 \cdot uf_a$$

$$1 \cdot uf_b \xrightarrow{b} (1 \cdot (b + b \cdot 0)) \cdot uf_b$$

Image of P is **not** closed under bisimulation collapse

$P(uf)$



$P(uf)$

$$uf := a \cdot \overbrace{(a \cdot (a + a \cdot 0))^*}^{uf_a} + b \cdot \overbrace{(b \cdot (b + b \cdot 0))^*}^{uf_b}$$

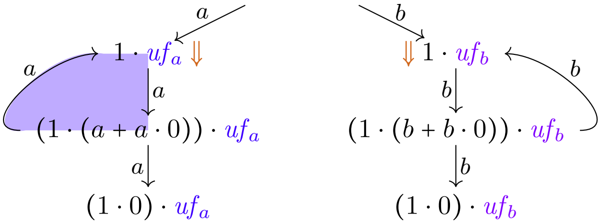
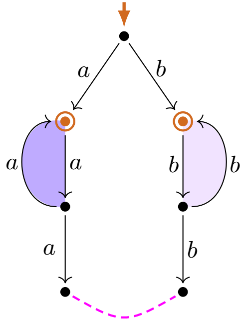
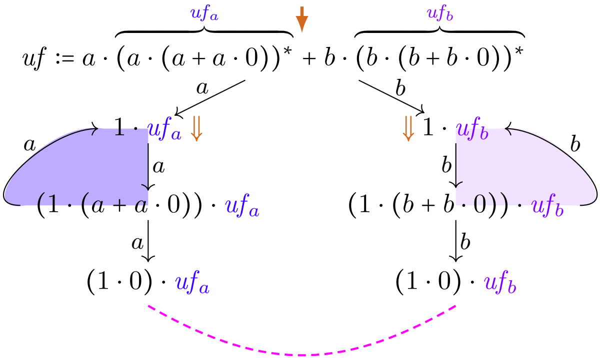


Image of P is **not** closed under bisimulation collapse

$P(uf)$



$P(uf)$



Compact process interpretation $P^{\bullet}$

Definition (Transition system specification $\mathcal{T}$)

$$\frac{}{\mathbf{1}{\Downarrow}} \qquad \frac{e_i{\Downarrow}}{(e_1 + e_2){\Downarrow}}\,(i \in \{1,2\}) \qquad \frac{e_1{\Downarrow} \qquad e_2{\Downarrow}}{(e_1 \cdot e_2){\Downarrow}} \qquad \frac{}{(e^{*}){\Downarrow}}$$

$$\frac{}{a \xrightarrow{a} \mathbf{1}} \qquad \frac{e_i \xrightarrow{a} e_i'}{e_1 + e_2 \xrightarrow{a} e_i'}\,(i \in \{1,2\})$$

$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2} \qquad \frac{e_1{\Downarrow} \qquad e_2 \xrightarrow{a} e_2'}{e_1 \cdot e_2 \xrightarrow{a} e_2'} \qquad \frac{e \xrightarrow{a} e'}{e^{*} \xrightarrow{a} e' \cdot e^{*}}$$

Compact process interpretation $P^{\bullet}$

Definition (Transition system specification $\mathcal{T}$)

$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2}$$

$$\frac{e \xrightarrow{a} e'}{e^{*} \xrightarrow{a} e' \cdot e^{*}}$$

Compact process interpretation $P^{\bullet}$

Definition (Transition system specification $\mathcal{T}^{\bullet}$, changed rules w.r.t. $\mathcal{T}$)

$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2} \text{ (if } e_1' \text{ is normed)}$$

$$\frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e' \cdot e^*} \text{ (if } e' \text{ is normed)}$$

Compact process interpretation $P^{\bullet}$

Definition (Transition system specification $\mathcal{T}^{\bullet}$, changed rules w.r.t. $\mathcal{T}$)

$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2} \text{ (if } e_1' \text{ is normed)} \qquad \frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1'} \text{ (if } e_1' \text{ is not normed)}$$

$$\frac{e \xrightarrow{a} e'}{e^{*} \xrightarrow{a} e' \cdot e^{*}} \text{ (if } e' \text{ is normed)} \qquad \frac{e \xrightarrow{a} e'}{e^{*} \xrightarrow{a} e'} \text{ (if } e' \text{ is not normed)}$$

Compact process interpretation $P^{\bullet}$

Definition (Transition system specification $\mathcal{T}^{\bullet}$, changed rules w.r.t. $\mathcal{T}$)

$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2} \text{ (if } e_1' \text{ is normed)} \qquad \frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1'} \text{ (if } e_1' \text{ is not normed)}$$

$$\frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e' \cdot e^*} \text{ (if } e' \text{ is normed)} \qquad \frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e'} \text{ (if } e' \text{ is not normed)}$$

Definition

The compact process (graph) interpretation $P^{\bullet}(e)$ of a reg. expr's e:

$P^{\bullet}(e)$:= labeled transition graph generated by e by derivations in $\mathcal{T}^{\bullet}$.

Compact process interpretation $P^{\bullet}$

Definition (Transition system specification $\mathcal{T}^{\bullet}$, changed rules w.r.t. $\mathcal{T}$)

$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2} \text{ (if } e_1' \text{ is normed)} \qquad \frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1'} \text{ (if } e_1' \text{ is not normed)}$$

$$\frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e' \cdot e^*} \text{ (if } e' \text{ is normed)} \qquad \frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e'} \text{ (if } e' \text{ is not normed)}$$

Definition

The compact process (graph) interpretation $P^{\bullet}(e)$ of a reg. expr's e:

$P^{\bullet}(e)$:= labeled transition graph generated by e by derivations in $\mathcal{T}^{\bullet}$.

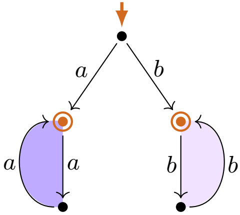
Lemma ($P^{\bullet}$ increases sharing; $P^{\bullet}$, P have same bisimulation semantics)

(i) $P(e) \mathrel{\underline{\rightarrow}} P^{\bullet}(e)$ *for all regular expressions* e.

(ii) (G *is* $[\![\cdot]\!]_{P^{\bullet}}$*-expressible* $\Longleftrightarrow$ G *is* $[\![\cdot]\!]_{P}$*-expressible) for all graphs* G.

Image of $P^{\bullet}$ under bisimulation collapse . . .

$P^{\bullet}(uf)$



$P^{\bullet}(uf)$

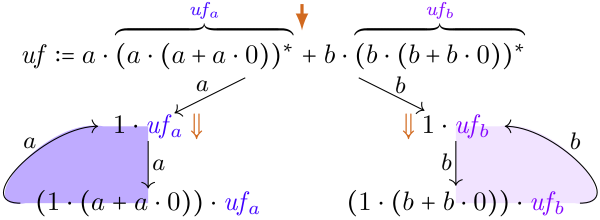
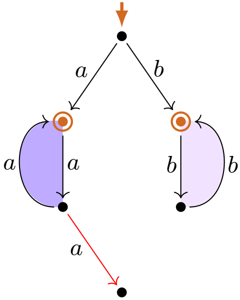


Image of $P^{\bullet}$ under bisimulation collapse . . .

$P^{\bullet}(uf)$



$P^{\bullet}(uf)$

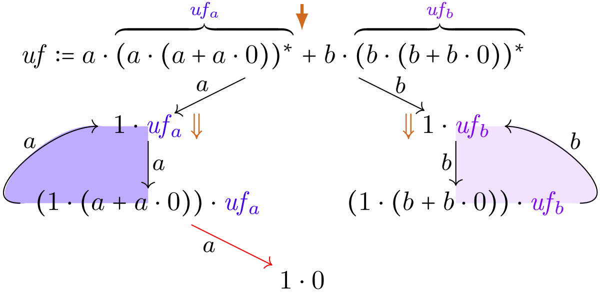
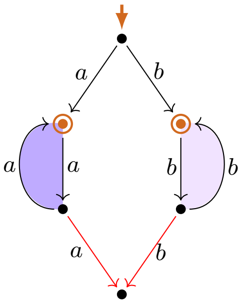
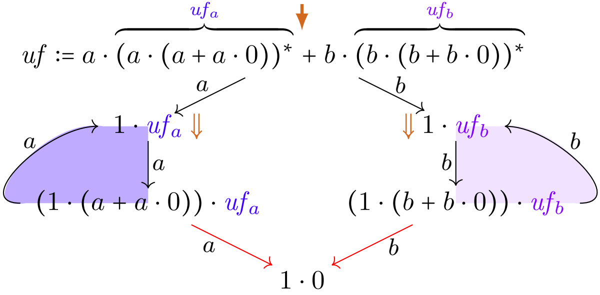


Image of $P^\bullet$ under bisimulation collapse . . .

$P^\bullet(uf)$



$P^\bullet(uf)$



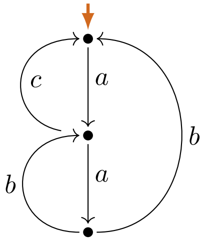
Interpretation correspondence of $P^{\bullet}$ with LEE

(Int)$^{(\cancel{1}\backslash *)}_{P^{\bullet}}$: *By under-star-1-free expressions $P^{\bullet}$-expressible graphs satisfy* LEE:
Compact process **i**nterpretations $P^{\bullet}(uf)$
of under-star-1-free regular expressions uf
are finite process graphs that satisfy LEE.

(Extr)$^{(\cancel{1}\backslash *)}_{P^{\bullet}}$: LEE *implies $[\![\cdot]\!]_P$-expressibility by under-star-1-free reg. expr's*:
From every finite process graph G with LEE
an under-star-1-free regular expression uf can be **e**xtracted
such that $G \rightrightarrows P(uf)$.

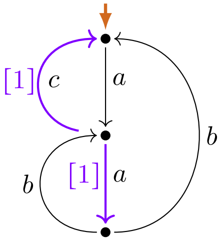
Refined extraction expression (example)

G_4



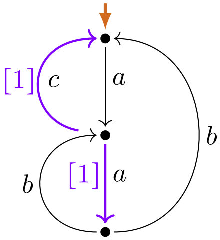
Refined extraction expression (example)

$\widehat{G_4}$



Refined extraction expression (example)

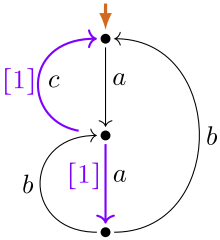
$\widehat{G_4}$



$$(1 \cdot (\qquad\qquad\qquad)^{*}) \cdot 0$$

Refined extraction expression (example)

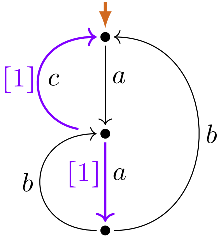
$\widehat{G_4}$



$$(1 \cdot (\qquad \overset{a}{\downarrow} \qquad)^*) \cdot 0$$

Refined extraction expression (example)

$\widehat{G_4}$



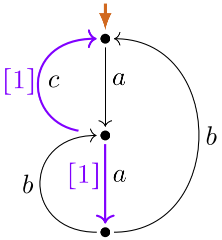
$$((1 \cdot a) \cdot (\qquad\qquad\qquad)^*) \cdot 0$$

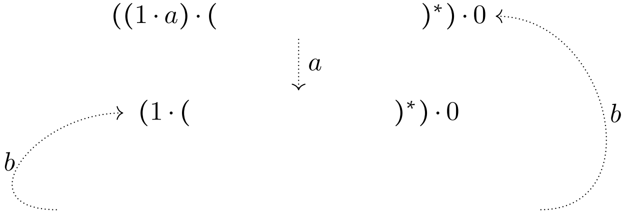
$$\Big\downarrow a$$

$$(1 \cdot (\qquad\qquad\qquad)^*) \cdot 0$$

Refined extraction expression (example)

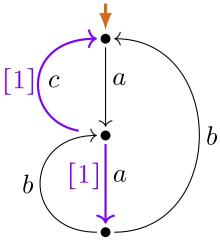
$\widehat{G_4}$





Refined extraction expression (example)

$\widehat{G_4}$



$((1 \cdot a) \cdot (\qquad)^*) \cdot 0$

a

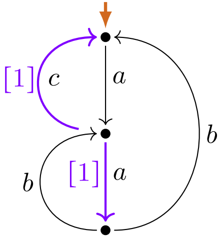
$(1 \cdot (\qquad)^*) \cdot 0$

b $\qquad$ b

$((1 \cdot (b + b \cdot a)) \cdot (\qquad)^*) \cdot 0$

Refined extraction expression (example)

$\widehat{G_4}$



$$((1 \cdot a) \cdot (\qquad\qquad)^*) \cdot 0$$

$$\downarrow a$$

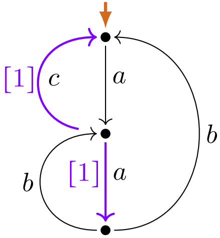
$$(1 \cdot (\qquad\qquad)^*) \cdot 0$$

$$\downarrow a$$

$$((1 \cdot (b + b \cdot a)) \cdot (\qquad\qquad)^*) \cdot 0$$

Refined extraction expression (example)

$\widehat{G_4}$



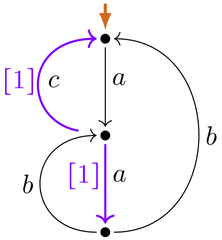
$$((1 \cdot a) \cdot (\qquad\qquad)^*) \cdot 0$$

$$(1 \cdot (c \cdot a + a \cdot (b + b \cdot a))^*) \cdot 0$$

$$((1 \cdot (b + b \cdot a)) \cdot (\qquad\qquad)^*) \cdot 0$$

Refined extraction expression (example)

$\widehat{G_4}$



$$((1 \cdot a) \cdot (c \cdot a + a \cdot (b + b \cdot a))^*) \cdot 0$$

$$\Big\downarrow a$$

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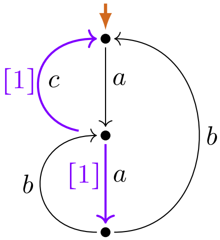

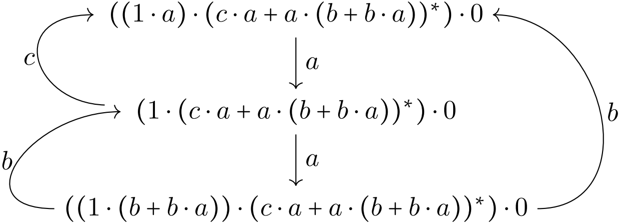

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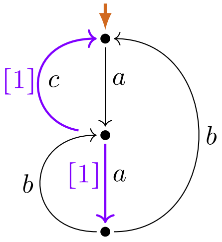
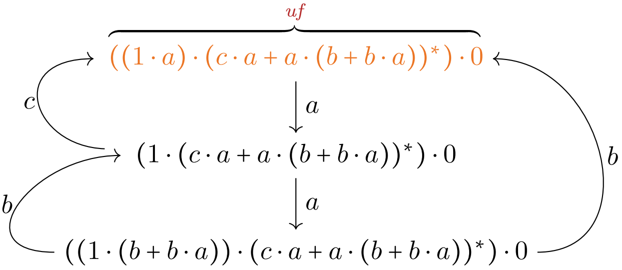


$P^{\bullet}(uf) = P(uf) \simeq G_4$



Interpretation/extraction correspondences of $P^{\bullet}$ with LEE

(Int)$^{(\cancel{1}\backslash *)}_{P^{\bullet}}$: *By under-star-1-free expressions $P^{\bullet}$-expressible graphs satisfy* LEE:

Compact process **i**nterpretations $P^{\bullet}(uf)$
of under-star-1-free regular expressions uf
are finite process graphs that satisfy LEE.

(Extr)$^{(\cancel{1}\backslash *)}_{P^{\bullet}}$: LEE *implies $[\![\cdot]\!]_P$-expressibility by under-star-1-free reg. expr's*:

From every finite process graph G with LEE
an under-star-1-free regular expression uf can be **e**xtracted
such that $G \mathrel{\underline{\to}} P(uf)$.

From every finite collapsed process graph G with LEE
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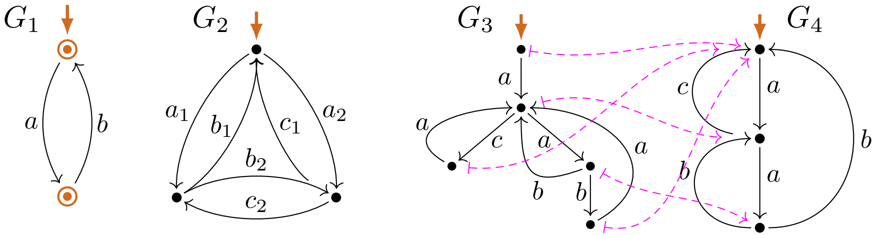
P-/$P^{\bullet}$-expressibility and $[\![\cdot]\!]_P$-expressibility (examples)



not P-expressible
not $[\![\cdot]\!]_P$-expressible

P-/$P^{\bullet}$-expressible
$[\![\cdot]\!]_P$-expressible

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Summary and outlook

- 1-free regular expressions defined (also) with unary star
- image of 1-free regular expressions under the process interpretation P
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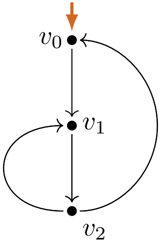

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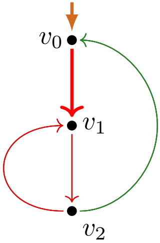

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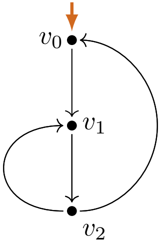

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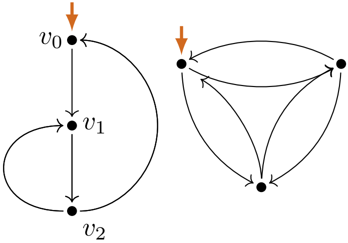

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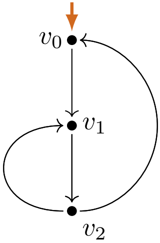

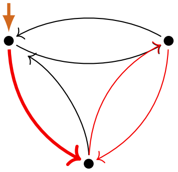

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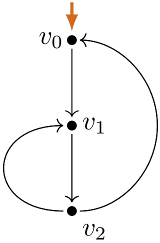

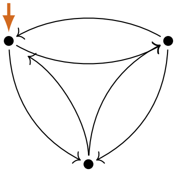

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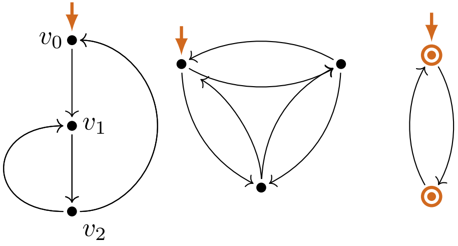

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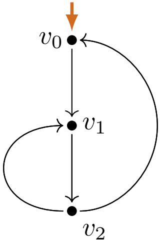

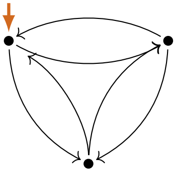

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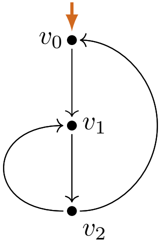

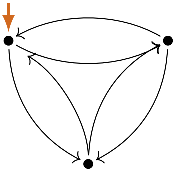

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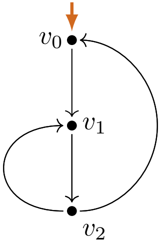

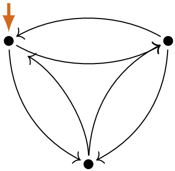

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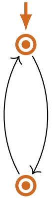

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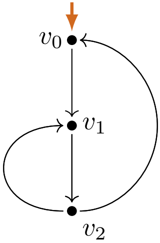

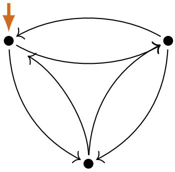

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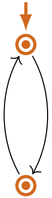

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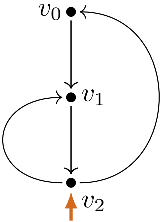

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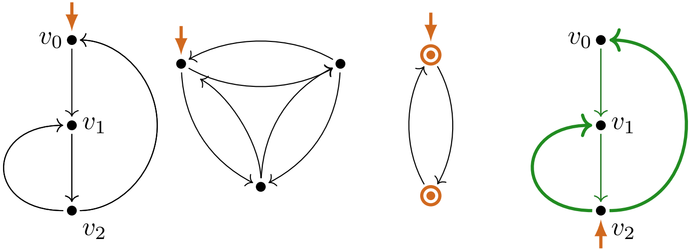

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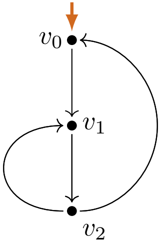

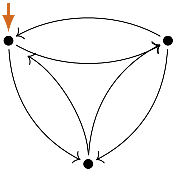

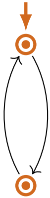

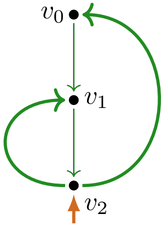

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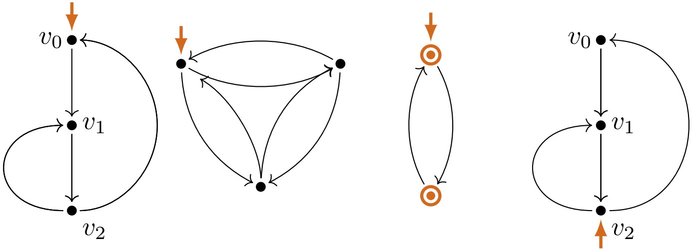

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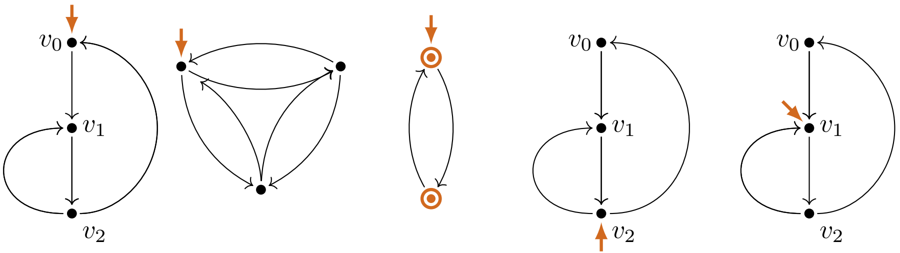

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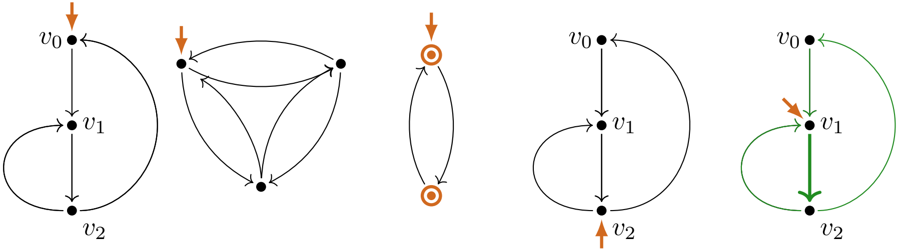

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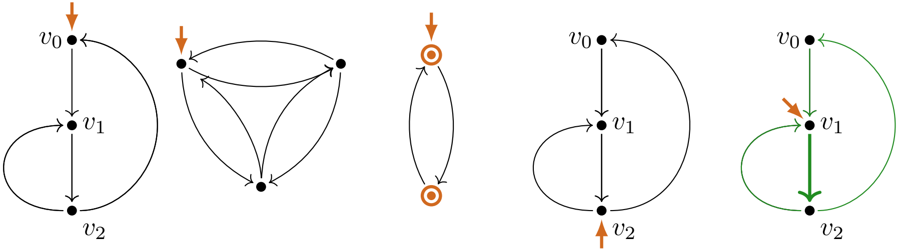

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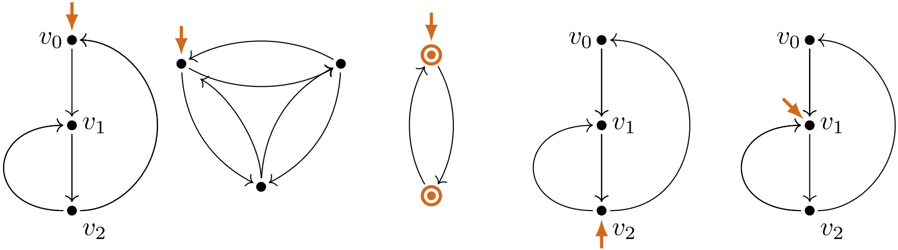

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