Lecture 5: Three More Models

Models of Computation

https://clegra.github.io/moc/moc.html

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Ph.D. Program, Advanced Courses Period
Gran Sasso Science Institute
L'Aquila, Italy

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Course overview

Monday, July 7 10.30 – 12.30	Tuesday, July 8 10.30 – 12.30	Wednesday, July 9 10.30 – 12.30	Thursday, July 10 10.30 – 12.30	Friday, July 11
intro	*classic models*			*additional models*
Introduction to Computability	**Machine Models**	**Recursive Functions**	**Lambda Calculus**	
computation and decision problems, from logic to computability, overview of models of computation relevance of MoCs	Post Machines, typical features, Turing's analysis of human computers, Turing machines, basic recursion theory	primitive recursive functions, Gödel–Herbrand recursive functions, partial recursive funct's, partial recursive = = Turing-computable, Church's Thesis	λ-terms, β-reduction, λ-definable functions, partial recursive = λ-definable = Turing computable	
	imperative programming	*algebraic programming*	*functional programming*	
				14.30 – 16.30
				Three more Models of Computation
				Post's Correspondence Problem, Interaction-Nets, Fractran
				comparing computational power

Some Models of Computation

machine model	mathematical model	sort
Turing machine Post machine register machine	Combinatory Logic λ-calculus Herbrand–Gödel recursive functions partial-recursive/μ-recursive functions Post canonical system (tag system) Post's Correspondence Problem Markov algorithms Lindenmayer systems	*classical*
	Fractran	*less well known*
cellular automata neural networks	term rewrite systems interaction nets logic-based models of computation concurrency and process algebra ς-calculus evolutionary programming/genetic algorithms	*modern*
abstract state machines		
hypercomputation		*speculative*
quantum computing bio-computing reversible computing		*physics-/biology-inspired*

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- Compare computational power of models of computation
- Fractran (by John Horton Conway, 1987, [2])

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$I = \{\langle g_1, g_1' \rangle, \ldots, \langle g_k, g_k' \rangle\}$, where $k \geq 1$, $g_i, g_i' \in \Sigma^+$ for $i \in \{1, \ldots, k\}$.

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Question: Is I solvable?

Do there exist $n \geq 1$, and $i_1, \ldots, i_n \in \{1, \ldots, k\}$ such that:

$$g_{i_1} g_{i_2} \ldots g_{i_n} = g'_{i_1} g'_{i_2} \ldots g'_{i_n} \quad ?$$

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Theorem

Codings of solvable instances of PCP:

$$\Big\{\Big\langle \overbrace{\{\langle g_1, g'_1\rangle, \ldots, \langle g_k, g'_k\rangle \mid k \geq 1, g_i, g'_i \in \Sigma^+\}}^{\text{PCP instance } I} \Big\rangle \mid I \text{ is solvable}\Big\}$$

form a set that is recursively enumerable, but not recursive.

Interaction Nets

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- a programming language with a simple graph rewriting semantics

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- electric circuits:
  - agents $\hat{=}$ gates,
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Analogy with:

- electric circuits:
  - agents $\hat{=}$ gates,
  - edges $\hat{=}$ wires
- agents as computation entities:
  - interaction rules specify behavior

Comparing computational power via encodings

- Simulation of models of computation $\mathcal{M}_1 = \langle D_1, \mathcal{F}_1 \rangle$, $\mathcal{M}_2 = \langle D_2, \mathcal{F}_2 \rangle$:

Comparing computational power via encodings

- Simulation of functions:
  function f_2 *simulates* function f_1 via *encoding* ρ if:

$\mathcal{M}_1$

$$\begin{array}{ccc} D_1 & \xrightarrow{\rho} & D_2 \\ {\scriptstyle f_1}\downarrow & & \downarrow{\scriptstyle f_2} \\ D_1 & \xrightarrow[\rho]{} & D_2 \end{array}$$

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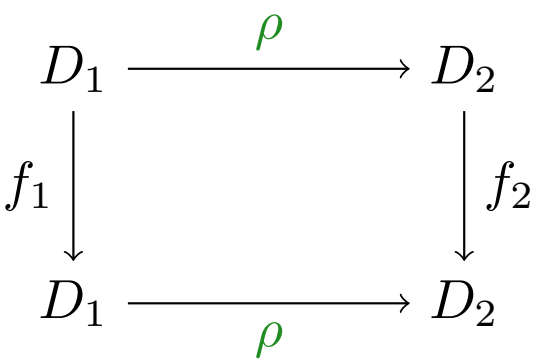

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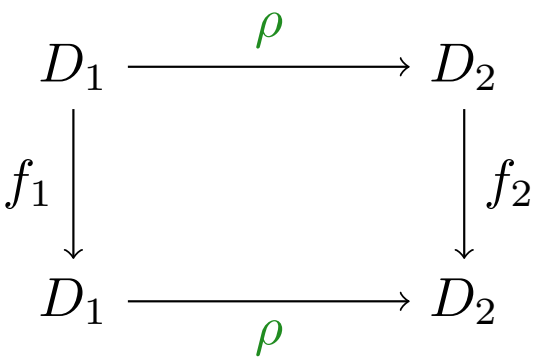

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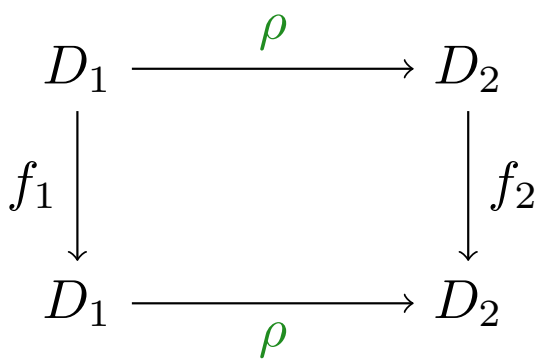

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$$\forall f_1 \in \mathcal{F}_1 \; \exists f_2 \in \mathcal{F}_2 \left(f_2 \text{ simulates } f_1 \text{ via } \rho \right)$$

Weak requirements on encodings (Boker/Dershowitz)

Traditional requirements on encodings are:

- *informally computable*/*effective*/*mechanizable in principle*
- *computable* with respect to a specific model (Turing machine, . . .)

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Definition (power subsumption pre-order [Boker/Dershowitz 2006 [1]])

(i) $\mathcal{M}_1 \lesssim \mathcal{M}_2$ if: there is an injective ρ such that $\mathcal{M}_1 \lesssim_\rho \mathcal{M}_2$

(ii) $\mathcal{M}_1 \lesssim_{\text{bijective}} \mathcal{M}_2$ if: there is a bijective ρ such that $\mathcal{M}_1 \lesssim_\rho \mathcal{M}_2$

Anomalies for decision models

However, we found anomalies of these definitions.

$\mathcal{M} = \langle D, \mathcal{F} \rangle$ is a *decision model* if $\{0,1\} \subseteq D$, $\forall f \in \mathcal{F}\,(f[D] \subseteq \{0,1\})$.

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Theorem (Endrullis/G/Hendriks, [3])

Let Σ and Γ with $\{0,1\} \subseteq \Sigma, \Gamma$ be alphabets.

Then for every countable decision model $\mathcal{M} = \langle \Sigma^*, \mathcal{F} \rangle$, it holds:

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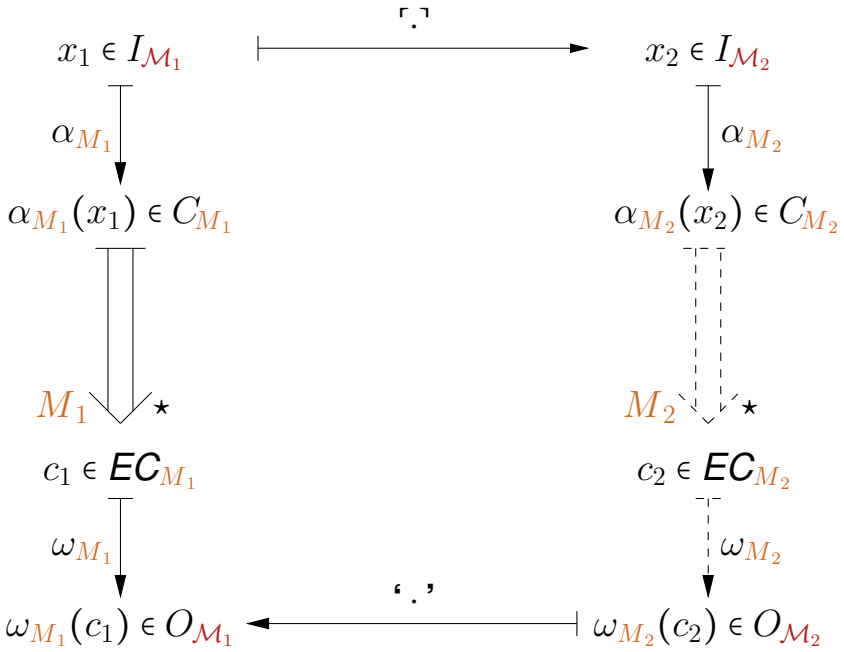
- but do not extend to all moc's

Simulations between models of computation

models $M_1 \in \mathcal{M}_1$ and $M_2 \in \mathcal{M}_2$ simulate each other with respect to computable coding $\ulcorner \cdot \urcorner : I_{\mathcal{M}_1} \to I_{\mathcal{M}_2}$ and decoding $\lq \cdot \rq : O_{\mathcal{M}_2} \to O_{\mathcal{M}_1}$ if:



Models of computation, viewed abstractly

A(n abstractly viewed) model of computation (MoC) is a class $\mathcal{M}$ of machines/systems/... such that every $M \in \mathcal{M}$ it holds:

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- M has a partial output function $\omega_M : EC_M \rightharpoonup O_{\mathcal{M}}$, which maps some end-configurations of M to output objects of M; ω_M is computable, and membership of end-configurations in $dom(\omega_M)$ is decidable.

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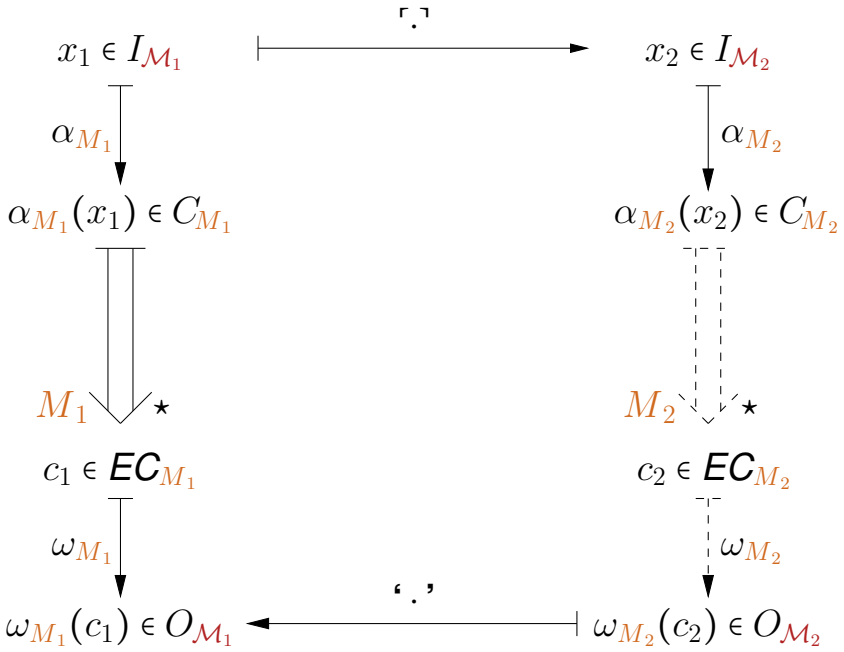

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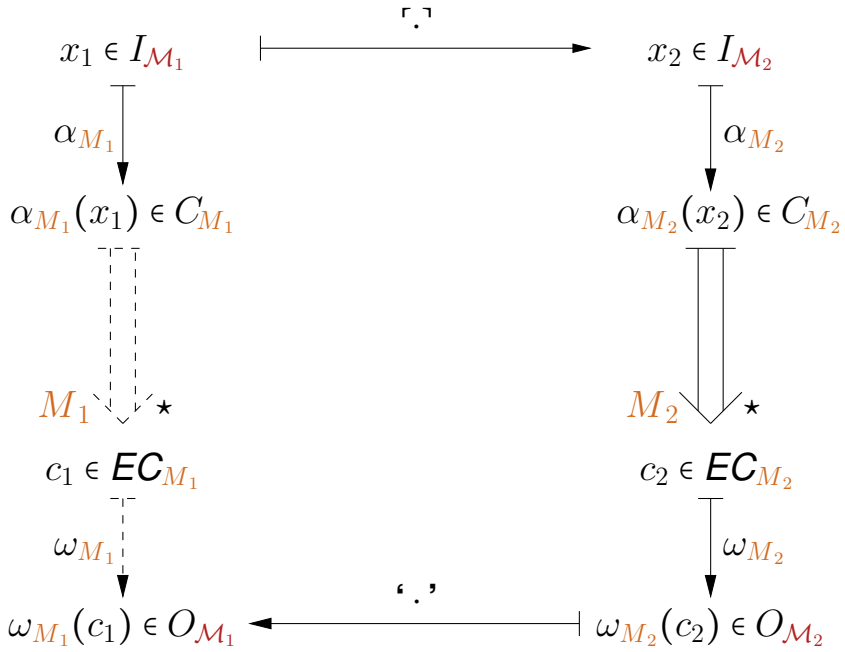

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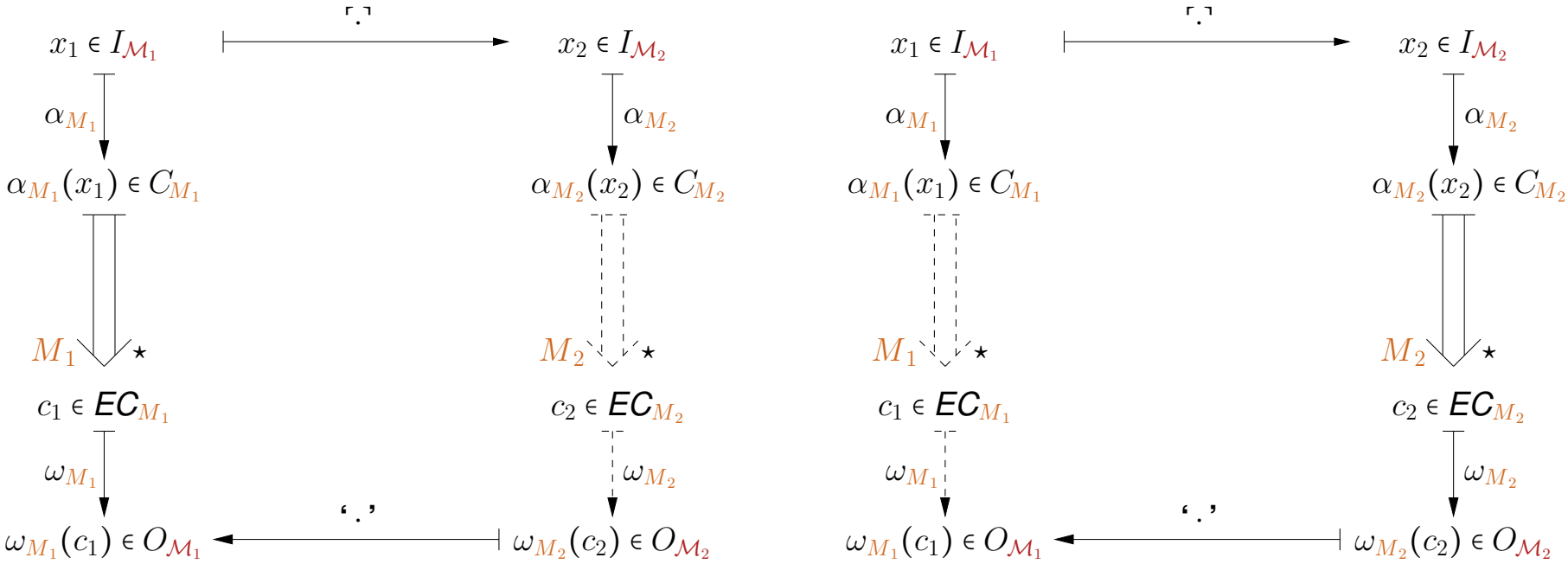

(defines a Galois connection)

Comparing Computational Power of MoC's

Definition

Let $\mathcal{M}_1$ and $\mathcal{M}_2$ be MoC's.

1. The computational power of $\mathcal{M}_1$ is subsumed by that of $\mathcal{M}_2$, denoted symbolically by $\mathcal{M}_1 \leq \mathcal{M}_2$, if:

$$(\exists \text{ a pair } \langle \ulcorner\cdot\urcorner, \text{`}\cdot\text{'} \rangle \text{ of computable encoding and decoding functions } \ulcorner\cdot\urcorner : I_{\mathcal{M}_1} \to I_{\mathcal{M}_2} \text{ and } \text{`}\cdot\text{'} : O_{\mathcal{M}_2} \to O_{\mathcal{M}_1}$$
$$(\forall M_1 \in \mathcal{M}_1)\,(\exists M_2 \in \mathcal{M}_2)$$
$$\big[\, M_1 \text{ and } M_2 \text{ simulate each other w.r.t. } \langle \ulcorner\cdot\urcorner, \text{`}\cdot\text{'} \rangle \,\big].$$

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   $(\forall M_1 \in \mathcal{M}_1)\,(\exists M_2 \in \mathcal{M}_2)$

   $\big[\, M_1$ and M_2 simulate each other w.r.t. $\langle \ulcorner\cdot\urcorner, \text{`}\cdot\text{'} \rangle \,\big]$.

2. The computational power of $\mathcal{M}_1$ is equivalent to that of $\mathcal{M}_2$, denoted by $\mathcal{M}_1 \sim \mathcal{M}_2$, if both $\mathcal{M}_1 \leq \mathcal{M}_2$ and $\mathcal{M}_2 \leq \mathcal{M}_1$ hold.

Comparing Computational Power of MoC's

Theorem

For all models $\mathcal{M}_1$ and $\mathcal{M}_2$, and encoding and decoding functions $\ulcorner\cdot\urcorner : I_{\mathcal{M}_1} \to I_{\mathcal{M}_2}$ and $\text{`}\cdot\text{'} : O_{\mathcal{M}_2} \to O_{\mathcal{M}_1}$ it holds:

$$\mathcal{M}_1 \leq_{\langle \ulcorner\cdot\urcorner, \text{`}\cdot\text{'} \rangle} \mathcal{M}_2 \implies \mathcal{F}(\mathcal{M}_1) \subseteq \left\{ \text{`}\cdot\text{'} \circ f \circ \ulcorner\cdot\urcorner \;\middle|\; f \in \mathcal{F}(\mathcal{M}_2) \right\}.$$

Turing completeness and equivalence

By $\mathcal{TM}(\Sigma)$ we mean the model of Turing machines over input alphabet Σ.

Definition

Let $\mathcal{M}$ a model of computation.

$\mathcal{M}$ is Turing-complete if $\mathcal{TM}(\Sigma) \leq \mathcal{M}$ for some alphabet Σ with $\Sigma \neq \varnothing$.

$\mathcal{M}$ is Turing-equivalent if $\mathcal{M} \sim \mathcal{TM}(\Sigma)$ for some alphabet $\Sigma \neq \varnothing$.

Fractran

John Horton Conway:

- article:
  - FRACTRAN:
    A Simple Universal Programming Language for Arithmetic
- talk video:
  - ”Fractran: A Ridiculous Logical Language”

Summary

- Post's Correspondence Problem (by Emil Post, 1946, [6])
- Interaction Nets (by Yves Lafont, 1990, [4])
- Compare computational power of models of computation
- Fractran (by John Horton Conway, 1987, [2])

Some Models of Computation

machine model	mathematical model	sort
Turing machine Post machine register machine	Combinatory Logic λ-calculus Herbrand–Gödel recursive functions partial-recursive/μ-recursive functions Post canonical system (tag system) Post's Correspondence Problem Markov algorithms Lindenmayer systems	*classical*
	Fractran	*less well known*
cellular automata neural networks	term rewrite systems interaction nets logic-based models of computation concurrency and process algebra ς-calculus evolutionary programming/genetic algorithms	*modern*
abstract state machines		
hypercomputation		*speculative*
quantum computing bio-computing reversible computing		*physics-/biology-inspired*

Course overview

Monday, July 7 10.30 – 12.30	Tuesday, July 8 10.30 – 12.30	Wednesday, July 9 10.30 – 12.30	Thursday, July 10 10.30 – 12.30	Friday, July 11
intro	*classic models*			*additional models*
Introduction to Computability	**Machine Models**	**Recursive Functions**	**Lambda Calculus**	
computation and decision problems, from logic to computability, overview of models of computation relevance of MoCs	Post Machines, typical features, Turing's analysis of human computers, Turing machines, basic recursion theory	primitive recursive functions, Gödel–Herbrand recursive functions, partial recursive funct's, partial recursive = = Turing-computable, Church's Thesis	λ-terms, β-reduction, λ-definable functions, partial recursive = λ-definable = Turing computable	
	imperative programming	*algebraic programming*	*functional programming*	
				14.30 – 16.30
				Three more Models of Computation
				Post's Correspondence Problem, Interaction-Nets, Fractran
				comparing computational power

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Haskell package on Hackage, `https://hackage.haskell.org/package/graph-rewriting-lambdascope`, 2010.
Lambdascope interaction-net animation tool.