The Graph Structure of Process Interpretations of Regular Expressions

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Nancy
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Overview

- regular expressions (unary/binary star/1-free-under-star $(*/\cancel{1})$)
- Milner's process interpretation P/semantics $[\![\cdot]\!]_P$
  - P-/$[\![\cdot]\!]_P$-expressible graphs ($\rightsquigarrow$ expressibility question)
  - axioms for $[\![\cdot]\!]_P$-identity ($\rightsquigarrow$ completeness question)
- loop existence and elimination (LEE)
  - defined by loop elimination rewrite system, its completion
  - describes interpretations of $(*/\cancel{1})$ reg. expr.s (extraction possible)
  - LEE-witnesses: labelings of process graphs with LEE
  - LEE is preserved under bisimulation collapse (stepwise collapse)
- 1-LEE = sharing via 1-transitions facilitates LEE
- LEE/1-LEE characterize image of $P^{\bullet}$ (restricted/unrestricted)
  - where $P^{\bullet}$ a compact (sharing-increased) refinement of P
- outlook on work-to-do

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  - describes interpretations of $(*/\cancel{1})$ reg. expr.s (extraction possible)
  - LEE-witnesses: labelings of process graphs with LEE
  - LEE is preserved under bisimulation collapse (stepwise collapse)
- 1-LEE = sharing via 1-transitions facilitates LEE
  - describes interpretations of all reg. expr.s (extraction possible)
  - not preserved under bisimulation collapse (approximation possible)
- LEE/1-LEE characterize image of $P^\bullet$ (restricted/unrestricted)
  - where $P^\bullet$ a compact (sharing-increased) refinement of P
  - via refined extraction using LEE/1-LEE
- outlook on work-to-do

Regular Expressions

Definition (~ *Copi–Elgot–Wright, 1958*)

Regular expressions over alphabet A with unary Kleene star:

$$e, e_1, e_2 \;::=\; \mathbf{0} \;\mid\; \boldsymbol{a} \;\mid\; e_1 + e_2 \;\mid\; e_1 \cdot e_2 \;\mid\; e^{*} \qquad \text{(for } \boldsymbol{a} \in A\text{)}.$$

- symbol $\mathbf{0}$ instead of $\varnothing$, symbol $\mathbf{1}$ instead of $\{\epsilon\}$

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- with unary star $*$: $\mathbf{1}$ is definable as $\mathbf{0}^{*}$

Regular Expressions

Definition (~ *Kleene, 1951*, ~ *Copi–Elgot–Wright, 1958*)

Regular expressions over alphabet A with unary / binary Kleene star:

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$$e, e_1, e_2 ::= \mathbf{0} \mid \mathbf{1} \mid \boldsymbol{a} \mid e_1 + e_2 \mid e_1 \cdot e_2 \mid e_1{}^{\circledast} e_2 \qquad \text{(for } \boldsymbol{a} \in A\text{)}.$$

- symbol $\mathbf{0}$ instead of $\varnothing$, symbol $\mathbf{1}$ instead of $\{\epsilon\}$
- with unary star *: $\mathbf{1}$ is definable as $\mathbf{0}^{*}$
- with binary star $^{\circledast}$: $\mathbf{1}$ is not definable (in its absence)

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Regular Expressions (1-free)

Definition (~ *Kleene, 1951*, ~ *Copi–Elgot–Wright, 1958*)

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Definition (for process interpretation)

1-free regular expressions over alphabet A with binary Kleene star:

$$f, f_1, f_2 ::= \mathbf{0} \mid \boldsymbol{a} \mid f_1 + f_2 \mid f_1 \cdot f_2 \mid f_1{}^{\circledast} f_2 \qquad (\text{for } \boldsymbol{a} \in A).$$

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Definition (for process interpretation)

1-free regular expressions over alphabet A with unary/binary Kleene star:

$$f, f_1, f_2 ::= \mathbf{0} \mid a \mid f_1 + f_2 \mid f_1 \cdot f_2 \mid (f_1{}^{*}) \cdot f_2 \qquad \text{(for } a \in A\text{)},$$

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Regular Expressions (under-star-/1-free)

Definition (~ *Kleene, 1951*, ~ *Copi–Elgot–Wright, 1958*)

Regular expressions over alphabet A with unary / binary Kleene star:

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$$e, e_1, e_2 \;::=\; \mathbf{0} \mid \mathbf{1} \mid \boldsymbol{a} \mid e_1 + e_2 \mid e_1 \cdot e_2 \mid e_1{}^{\circledast} e_2 \qquad (\text{for } \boldsymbol{a} \in A).$$

- symbol $\mathbf{0}$ instead of $\varnothing$, symbol $\mathbf{1}$ instead of $\{\epsilon\}$
- with unary star *: $\mathbf{1}$ is definable as $\mathbf{0}^{*}$
- with binary star $^{\circledast}$: $\mathbf{1}$ is not definable (in its absence)

Definition (for process interpretation)

The set $\mathit{RExp}^{(\cancel{1})}(A)$ of 1-free regular expressions over A is defined by:

$$f, f_1, f_2 \;::=\; \mathbf{0} \mid \boldsymbol{a} \mid f_1 + f_2 \mid f_1 \cdot f_2 \mid f_1{}^{\star} \cdot f_2 \qquad (\text{for } \boldsymbol{a} \in A),$$

the set $\mathit{RExp}^{(*/\cancel{1})}(A)$ of under-star-1-free regular expressions over A by:

$$uf, uf_1, uf_2 \;::=\; \mathbf{0} \mid \mathbf{1} \mid \boldsymbol{a} \mid uf_1 + uf_2 \mid uf_1 \cdot uf_2 \mid f^{\star} \qquad (\text{for } \boldsymbol{a} \in A).$$

Process interpretation P of regular expressions *(Milner, 1984)*

$\mathbf{0} \overset{P}{\longmapsto}$ deadlock $\boldsymbol{\delta}$, no termination

$\mathbf{1} \overset{P}{\longmapsto}$ empty-step process $\boldsymbol{\epsilon}$, then terminate

$a \overset{P}{\longmapsto}$ atomic action a, then terminate

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$e_1 + e_2 \stackrel{P}{\longmapsto}$ (*choice*) execute $P(e_1)$ or $P(e_2)$

$e_1 \cdot e_2 \stackrel{P}{\longmapsto}$ (*sequentialization*) execute $P(e_1)$, then $P(e_2)$

$e^* \stackrel{P}{\longmapsto}$ (*iteration*) repeat (terminate or execute $P(e)$)

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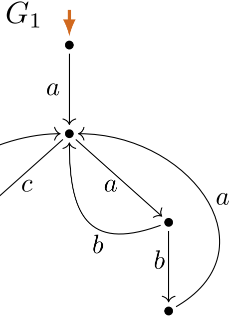
$e^* \xmapsto{P}$ (*iteration*) repeat (terminate or execute $P(e)$)

$[\![e]\!]_P := [P(e)]_{\underline{\leftrightarrow}}$ (bisimilarity equivalence class of process $P(e)$)

P-expressibility and $[\![\cdot]\!]_P$-expressibility (example, informally)



$$P\Big(\overbrace{(a \cdot (c \cdot a + a \cdot (b + b \cdot a))^*) \cdot 0}^{f} \Big)$$

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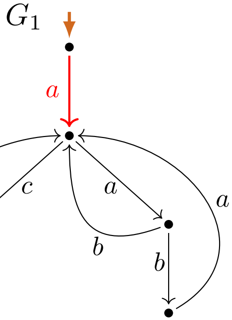

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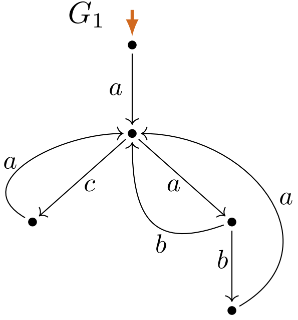

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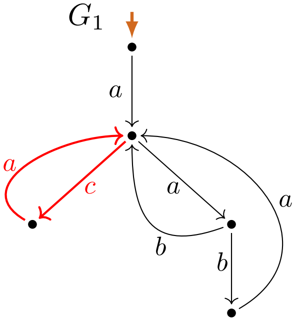

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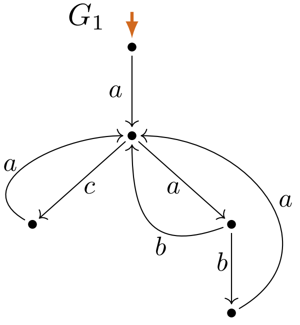

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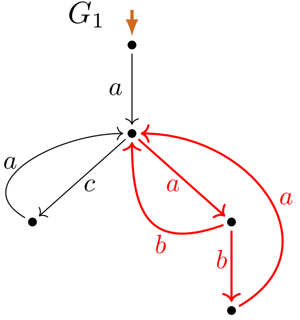

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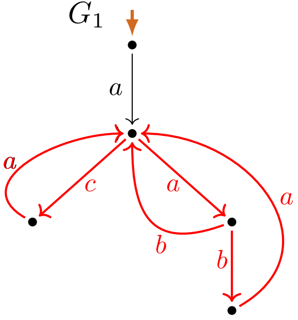

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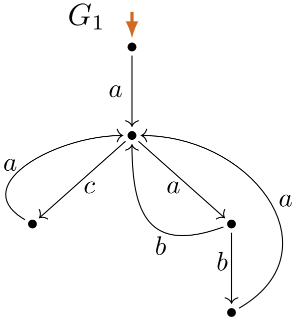

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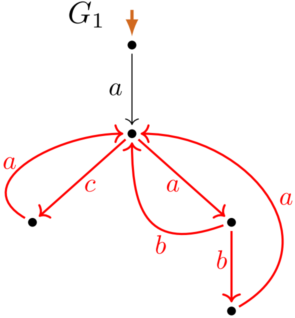

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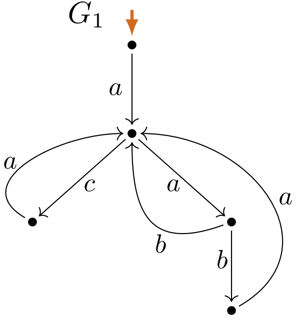

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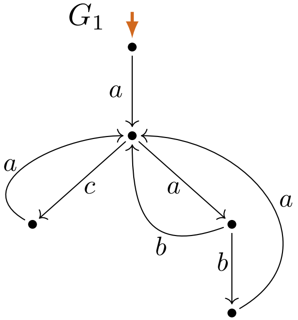

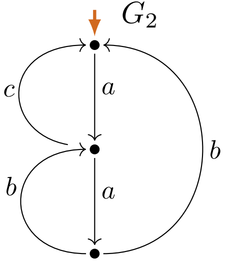

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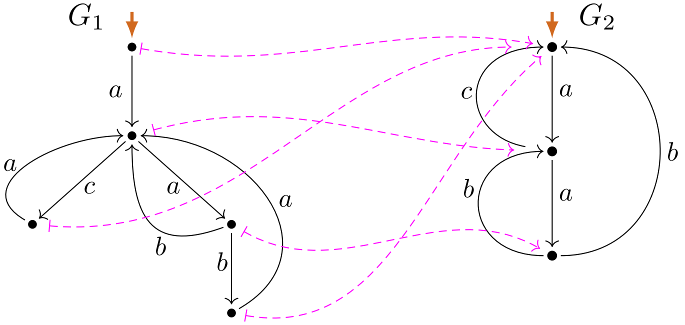

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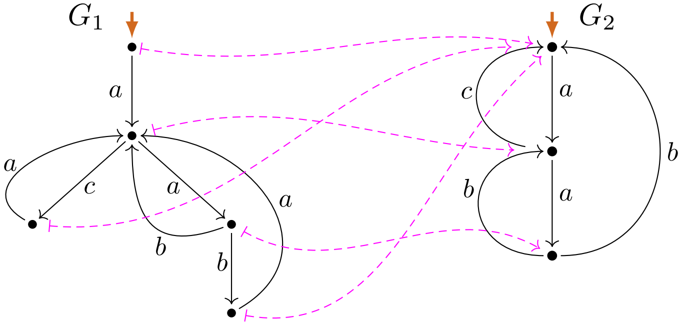

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Process interpretation P (formally)

Definition (Transition system specification $\mathcal{T}$)

$$\frac{}{\boldsymbol{a} \xrightarrow{\boldsymbol{a}} \mathbf{1}} \qquad \frac{e_i \xrightarrow{\boldsymbol{a}} e_i'}{e_1 + e_2 \xrightarrow{\boldsymbol{a}} e_i'} \; (i \in \{1, 2\})$$

Process interpretation P (formally)

Definition (Transition system specification $\mathcal{T}$)

$$\frac{}{a \xrightarrow{a} 1} \qquad \frac{e_i \xrightarrow{a} e_i'}{e_1 + e_2 \xrightarrow{a} e_i'} \,(i \in \{1,2\})$$

$$\frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e' \cdot e^*}$$

Process interpretation P (formally)

Definition (Transition system specification $\mathcal{T}$)

$$\frac{}{\mathbf{1}\Downarrow} \qquad \frac{e_i\Downarrow}{(e_1 + e_2)\Downarrow}\,(i \in \{1,2\}) \qquad \frac{e_1\Downarrow \quad e_2\Downarrow}{(e_1 \cdot e_2)\Downarrow} \qquad \frac{}{(e^*)\Downarrow}$$

$$\frac{}{a \xrightarrow{a} \mathbf{1}} \qquad \frac{e_i \xrightarrow{a} e_i'}{e_1 + e_2 \xrightarrow{a} e_i'}\,(i \in \{1,2\})$$

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Process interpretation P (formally)

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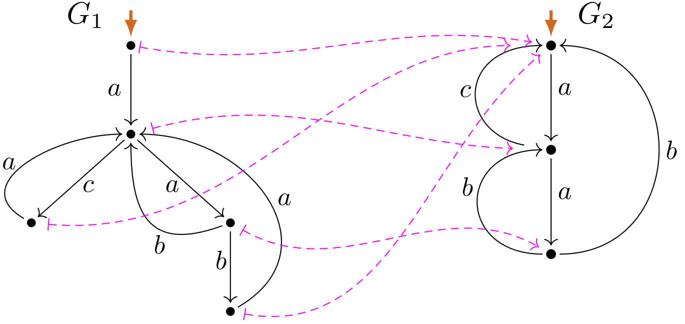
Definition

The process (graph) interpretation $P(e)$ of a regular expression e:

$P(e) :=$ labeled transition graph generated by e by derivations in $\mathcal{T}$.

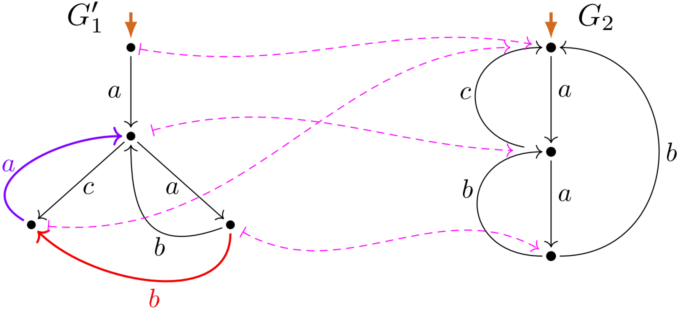
P-expressibility and $[\![\cdot]\!]_P$-expressibility (example, informally)



$$P\Big(\overbrace{a\cdot((c\cdot a + a\cdot(b + b\cdot a)^*)\cdot 0)}^{f}\Big)$$

$G_1 \in [\![f]\!]_P$ $\qquad$ $G_2 \in [\![f]\!]_P$

P-expressibility and $[\![\cdot]\!]_P$-expressibility (example, formally)



$$P\Big(\overbrace{a \cdot ((c \cdot a + a \cdot (b + b \cdot a)^*) \cdot 0)}^{f}\Big)$$

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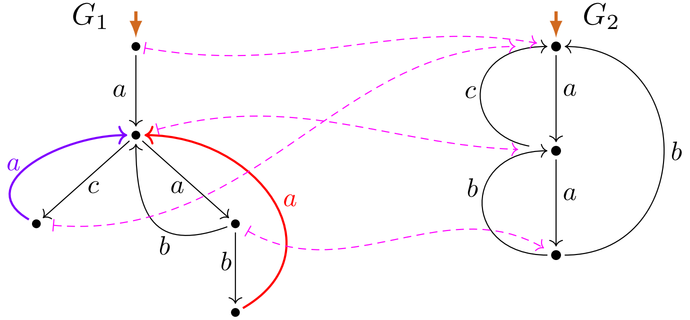
P-expressibility and $[\![\cdot]\!]_P$-expressibility (example, formally)



$$P\big(\overbrace{a \cdot ((c \cdot a + a \cdot (b + b \cdot (a + a))^*) \cdot 0)}^{f}\big)$$

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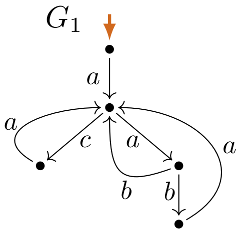
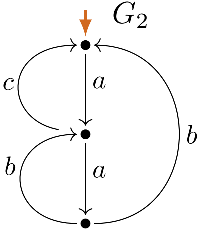
P-expressibility and $[\![\cdot]\!]_P$-expressibility (examples)

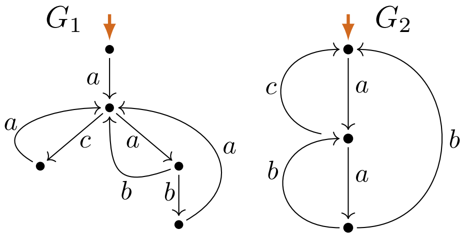




P-expressible

$[\![\cdot]\!]_P$-expressible $[\![\cdot]\!]_P$-expressible

P-expressibility and $[\![\cdot]\!]_P$-expressibility (examples)



P-expressible

?

$[\![\cdot]\!]_P$-expressible

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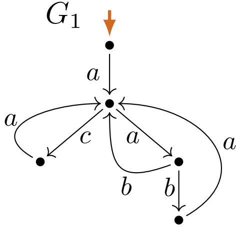
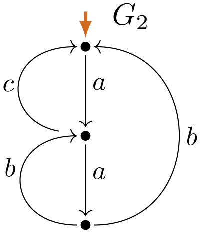
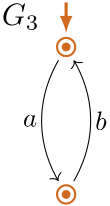
P-expressibility and $[\![\cdot]\!]_P$-expressibility (examples)





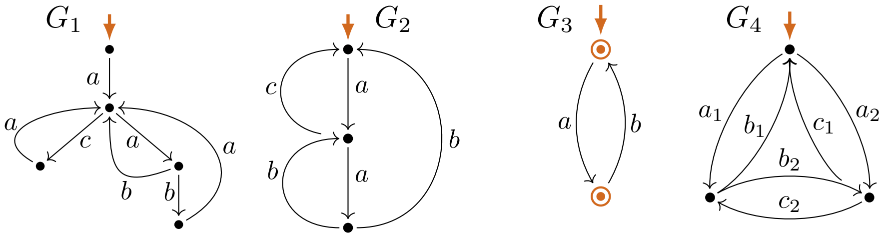


G_1: P-expressible, $[\![\cdot]\!]_P$-expressible

G_2: **?**, $[\![\cdot]\!]_P$-expressible

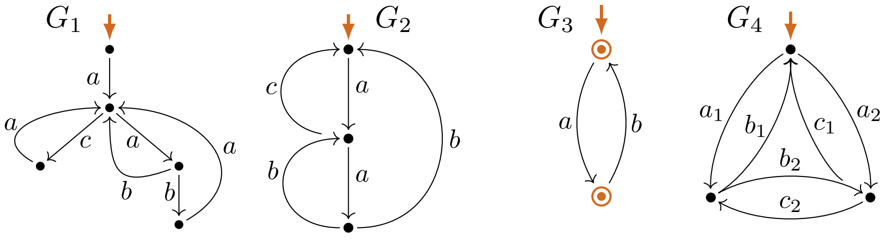
G_3: **not** P-expressible, **not** $[\![\cdot]\!]_P$-expressible

P-expressibility and $[\![\cdot]\!]_P$-expressibility (examples)



P-expressible	**?**	**not** P-expressible
$[\![\cdot]\!]_P$-expressible	$[\![\cdot]\!]_P$-expressible	**not** $[\![\cdot]\!]_P$-expressible

P-expressibility and $[\![\cdot]\!]_P$-expressibility (examples)

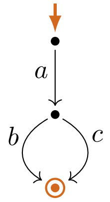


P-expressible	**?**	**not** P-expressible
$[\![\cdot]\!]_P$-expressible	$[\![\cdot]\!]_P$-expressible	**not** $[\![\cdot]\!]_P$-expressible

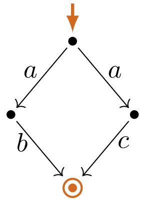
Q2: How can P-expressibility and $[\![\cdot]\!]_P$-expressibility be characterized?

Process semantics equality $=_{[\![\cdot]\!]_P}$

- Fewer identities hold for $=_{[\![\cdot]\!]_P}$ than for $=_{[\![\cdot]\!]_L}$: .



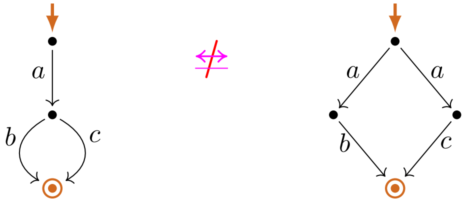
$P(a \cdot (b + c))$



$P(a \cdot b + a \cdot c)$

Process semantics equality $=_{[\![\cdot]\!]_P}$

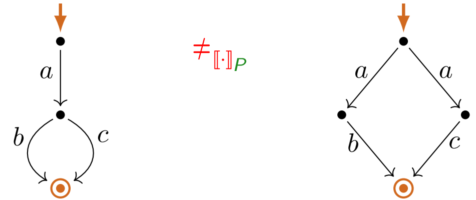
- Fewer identities hold for $=_{[\![\cdot]\!]_P}$ than for $=_{[\![\cdot]\!]_L}$: .



$$P(a \cdot (b + c)) \quad \not\leftrightarrow \quad P(a \cdot b + a \cdot c)$$

Process semantics equality $=_{[\![\cdot]\!]_P}$

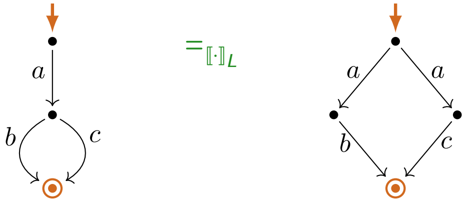
- Fewer identities hold for $=_{[\![\cdot]\!]_P}$ than for $=_{[\![\cdot]\!]_L}$: .



$$a \cdot (b + c) \quad \neq_{[\![\cdot]\!]_P} \quad a \cdot b + a \cdot c$$

Process semantics equality $=_{[\![\cdot]\!]_P}$

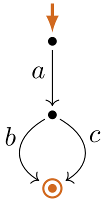
- Fewer identities hold for $=_{[\![\cdot]\!]_P}$ than for $=_{[\![\cdot]\!]_L}$: .



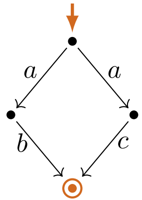
$$a \cdot (b + c) \quad =_{[\![\cdot]\!]_L} \quad a \cdot b + a \cdot c$$

Process semantics equality $=_{[\![\cdot]\!]_P}$

- Fewer identities hold for $=_{[\![\cdot]\!]_P}$ than for $=_{[\![\cdot]\!]_L}$: $\quad =_{[\![\cdot]\!]_P} \subsetneqq =_{[\![\cdot]\!]_L}$.



$\neq_{[\![\cdot]\!]_P}$



$$a \cdot (b + c) \quad \neq_{[\![\cdot]\!]_P} \quad a \cdot b + a \cdot c$$

Milner's proof system Mil

Axioms :

(A1)	$e + (f + g) = (e + f) + g$	(A7)	$e = 1 \cdot e$
(A2)	$e + 0 = e$	(A8)	$e = e \cdot 1$
(A3)	$e + f = f + e$	(A9)	$0 = 0 \cdot e$
(A4)	$e + e = e$	(A10)	$e^* = 1 + e \cdot e^*$
(A5)	$e \cdot (f \cdot g) = (e \cdot f) \cdot g$	(A11)	$e^* = (1 + e)^*$
(A6)	$(e + f) \cdot g = e \cdot g + f \cdot g$		

But: $e \cdot (f + g) \neq e \cdot f + e \cdot g$ But: $e \cdot 0 \neq 0$

Inference rules : rules of equational logic *plus*

$$\frac{e = f \cdot e + g}{e = f^* \cdot g}\ \mathsf{RSP}^* \quad \text{(if } f \text{ does not terminate immediately)}$$

Milner's Question (Q1)

Is Mil complete with respect to $=_{[\![\cdot]\!]_P}$? (Does $e =_{[\![\cdot]\!]_P} f \implies e =_{\mathsf{Mil}} f$ hold?)

Milner's questions

(Q1) Complete axiomatization:

Is the proof system Mil complete for $=_{[\![\cdot]\!]_P}$?

(Q2) $[\![\cdot]\!]_P$-Expressibility:

What structural property characterizes process graphs that are $[\![\cdot]\!]_P$-expressible ?

Milner's questions

(Q1) Complete axiomatization:
Is the proof system Mil *complete for* $=_{[\![\cdot]\!]_P}$*?*

(Q2) $[\![\cdot]\!]_P$-Expressibility:
What structural property characterizes process graphs that are $[\![\cdot]\!]_P$*-expressible ?*

- is decidable (Baeten/Corradini/G, 2007)

Milner's questions

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What structural property characterizes process graphs that are $[\![\cdot]\!]_P$-expressible ?

- is decidable (Baeten/Corradini/G, 2007)
- partial new answer (G/Fokkink, 2020):
  - bisimulation collapse has loop existence & elimination property (LEE) if expressible by under-star-1-free regular expression

Milner's questions

(Q1) Complete axiomatization:

Is the proof system Mil *complete for* $=_{[\![\cdot]\!]_P}$*?*

- series of partial completeness results for:
  - exitless iterations (Fokkink, 1998)
  - with a stronger fixed-point rule (G, 2006)
  - under-star 1-free, and without 0 (Corradini/de Nicola/Labella, 2004)
  - with 0 but under-star-1-free (G/Fokkink, 2020)

(Q2) $[\![\cdot]\!]_P$-Expressibility:

What structural property characterizes process graphs that are $[\![\cdot]\!]_P$*-expressible ?*

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Milner's questions

(Q1) Complete axiomatization:

Is the proof system Mil *complete for* $=_{[\![\cdot]\!]_P}$*?*

- Yes! (G, 2022, proof summary, employing LEE and crystallization)
- series of partial completeness results for:
  - exitless iterations (Fokkink, 1998)
  - with a stronger fixed-point rule (G, 2006)
  - under-star 1-free, and without 0 (Corradini/de Nicola/Labella, 2004)
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Question (Q2) specialized

$(\mathrm{Q2})_{\mathbf{0}}$ P-Expressibility and P-$(*/\mathit{1})$-Expressibility:

What structural property characterizes:

- *process graphs that are P-expressible ?*
  (... that are in the image of P?)
- *process graphs that are P-expressible by $(*/\mathit{1})$ regular expressions?*
  (... that are in the image of $(*/\mathit{1})$ expressions under P?)

Loop Existence and Elimination (LEE)

Loop graphs (interpretations of innermost iterations without 1)

Definition

A process graph is a loop graph if:

(L1) There is an infinite path from the start vertex.

(L2) Every infinite path from the start vertex returns to it.

(L3) Termination is only possible at the start vertex.

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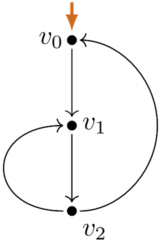

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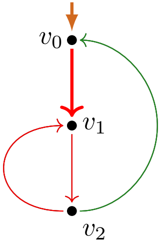

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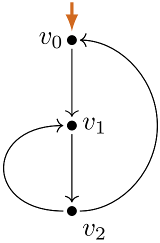

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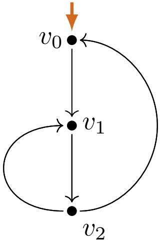

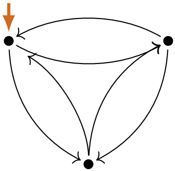

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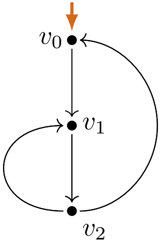

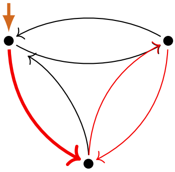

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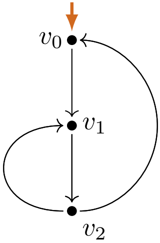

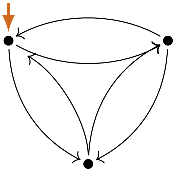

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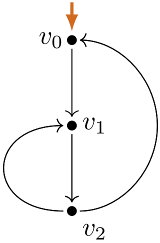

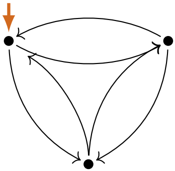

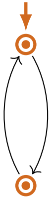

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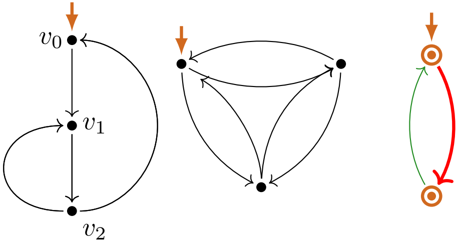

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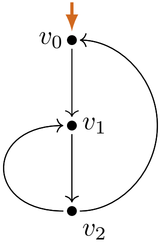

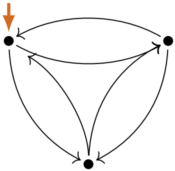

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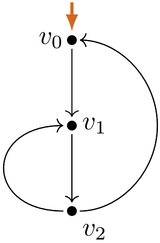

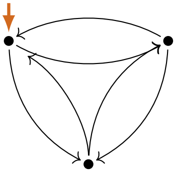

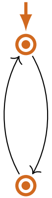

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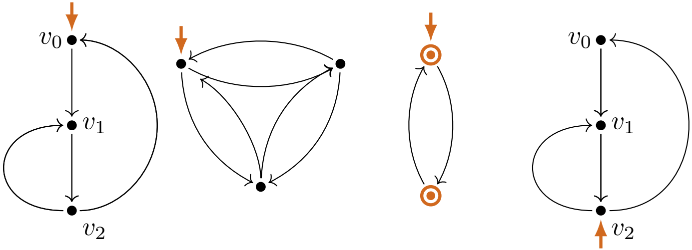

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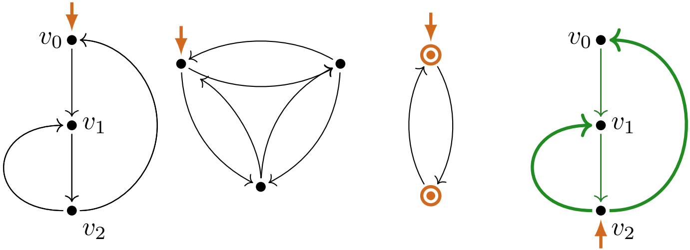

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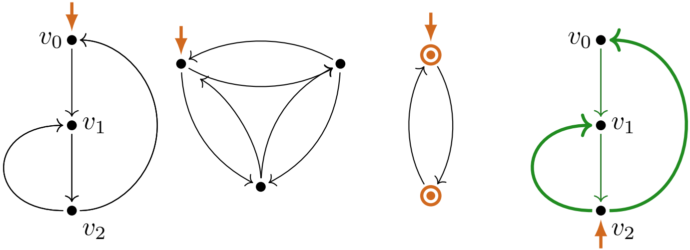

(L1),~~(L2)~~ (L1),(L2),~~(L3)~~ loop chart

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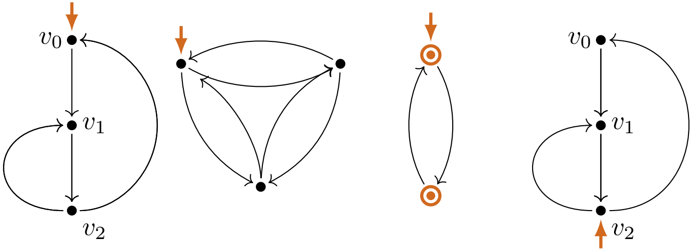

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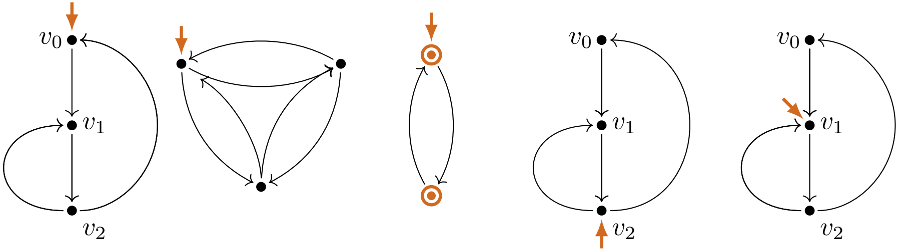

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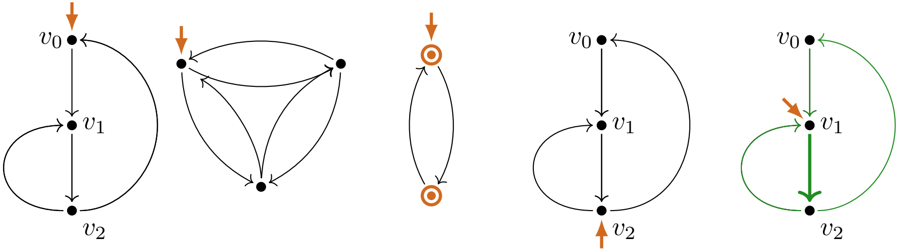

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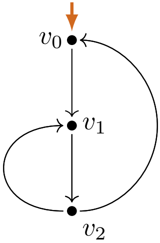

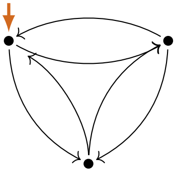

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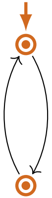
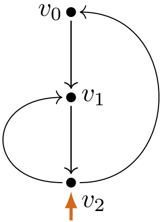
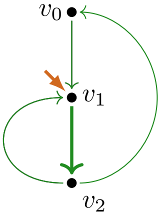


(L1),(L2),~~(L3)~~



loop chart



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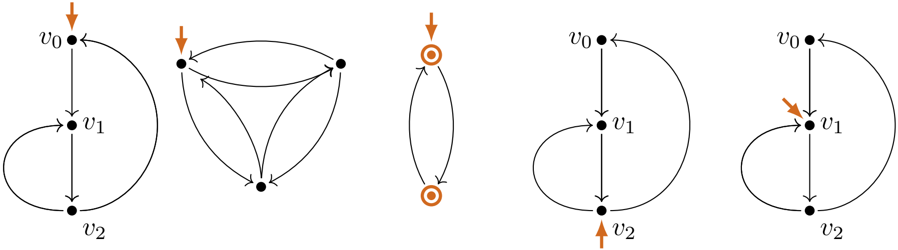


(L1),~~(L2)~~ (L1),(L2),~~(L3)~~ loop chart loop chart

Loop graphs (interpretations of innermost iterations without 1)

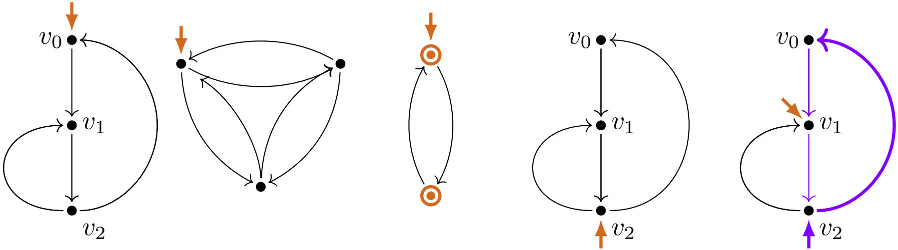
Definition

A process graph is a loop graph if:

(L1) There is an infinite path from the start vertex.

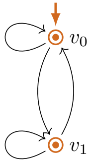
(L2) Every infinite path from the start vertex returns to it.

(L3) Termination is only possible at the start vertex.

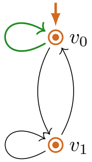


(L1),~~(L2)~~ (L1),(L2),~~(L3)~~ loop chart loop subchart

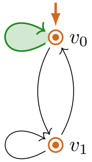
Loop elimination



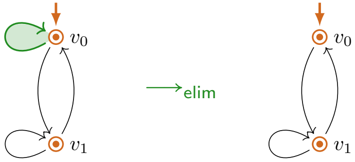
Loop elimination



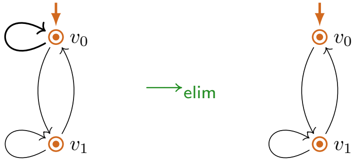
Loop elimination



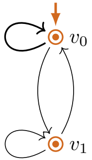
Loop elimination



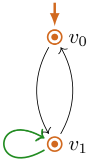
Loop elimination



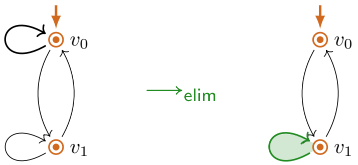
Loop elimination



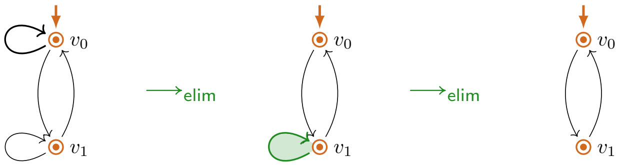
$\longrightarrow_{\text{elim}}$



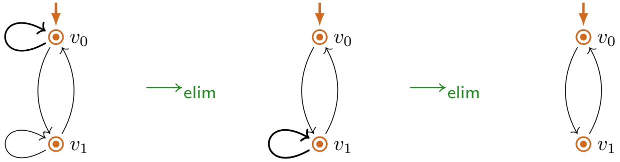
Loop elimination



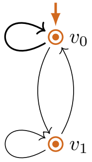
Loop elimination



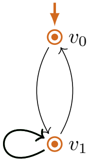
Loop elimination



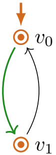
Loop elimination



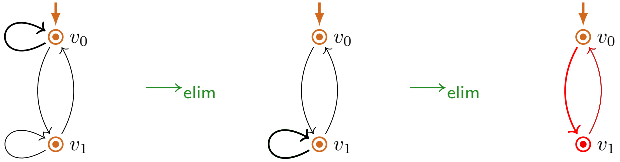
$\longrightarrow_{\text{elim}}$



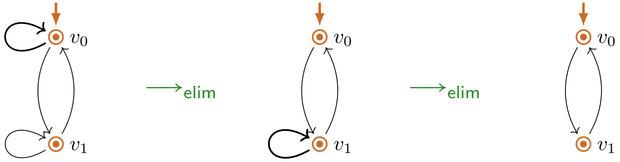
$\longrightarrow_{\text{elim}}$



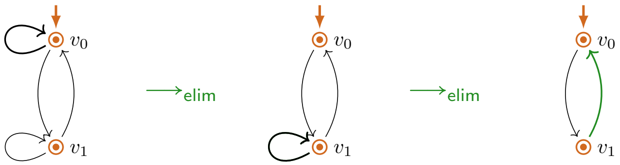
Loop elimination



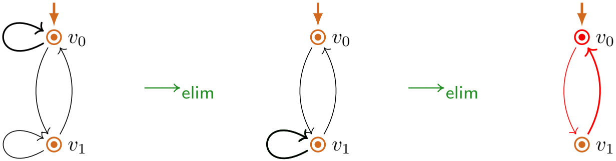
Loop elimination



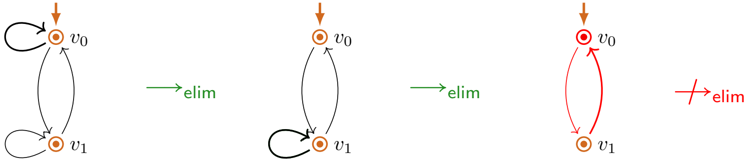
Loop elimination



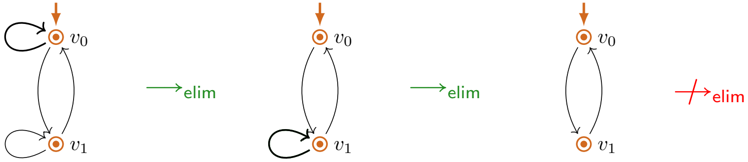
Loop elimination



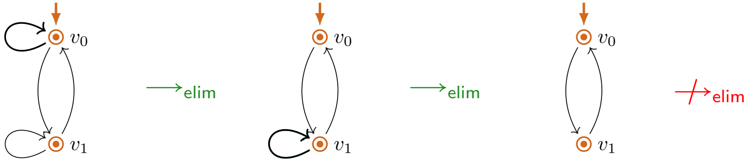
Loop elimination

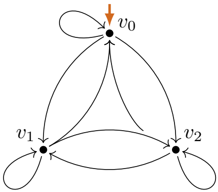


Loop elimination

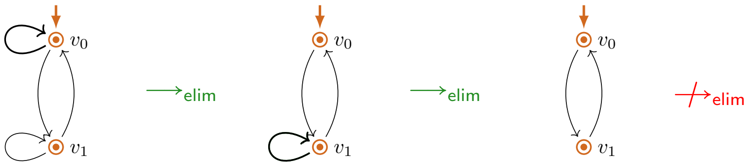


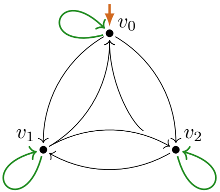
Loop elimination



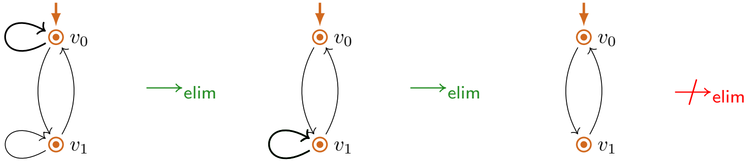


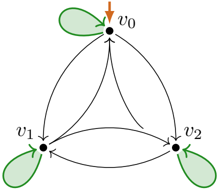
Loop elimination



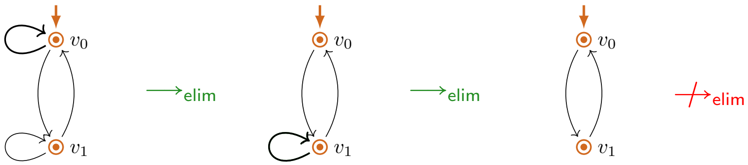


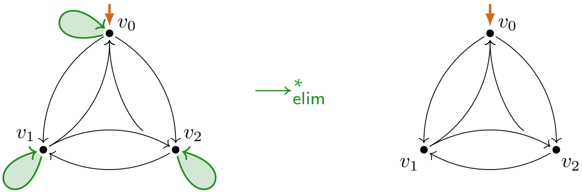
Loop elimination



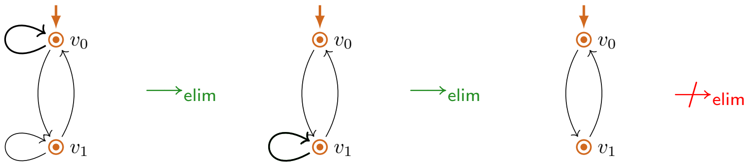


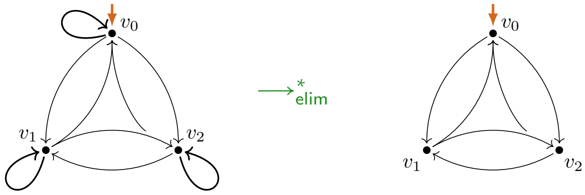
Loop elimination



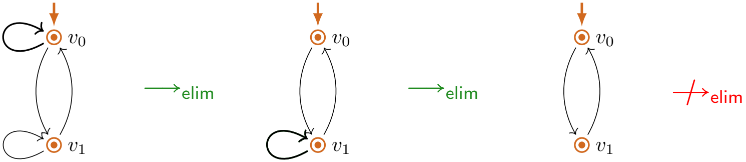


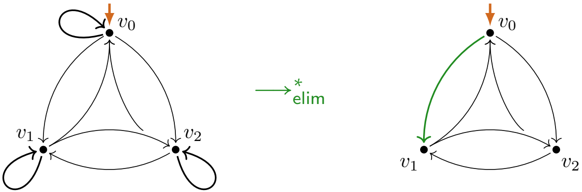
Loop elimination



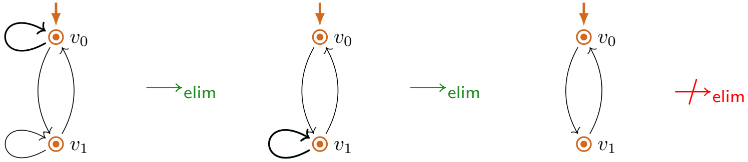


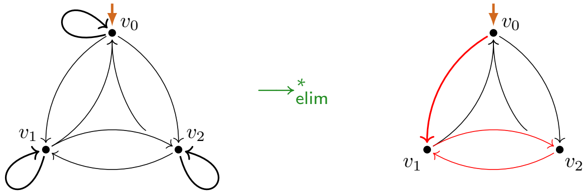
Loop elimination



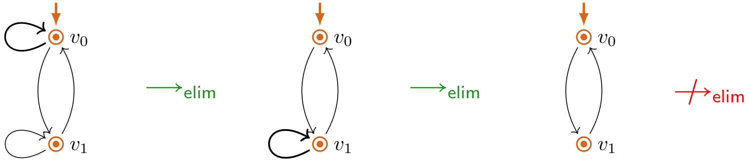


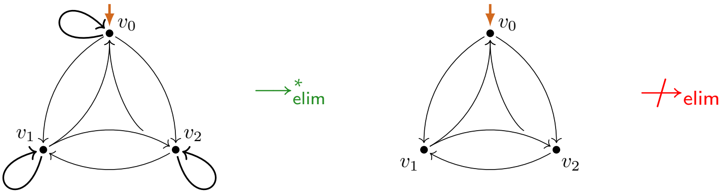
Loop elimination





Loop elimination





Loop elimination

$\longrightarrow_{\text{elim}}$: eliminate a transition-induced loop by:

- removing the loop-entry transition(s)
- garbage collection

$\longrightarrow_{\text{prune}}$: remove a transition to a deadlocking state

Lemma

(i) $\longrightarrow_{\text{elim}} \cup \longrightarrow_{\text{prune}}$ *is terminating.*

Loop elimination

$\longrightarrow_{\text{elim}}$: eliminate a transition-induced loop by:

- removing the loop-entry transition(s)
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Lemma

(i) $\longrightarrow_{\text{elim}} \cup \longrightarrow_{\text{prune}}$ *is terminating.*

Loop elimination, and properties

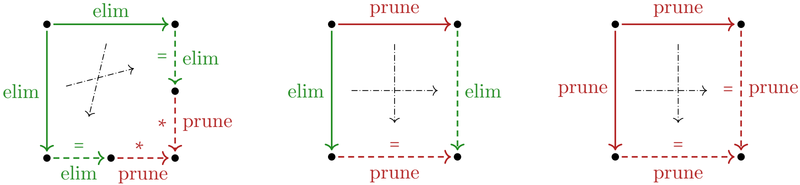
$\longrightarrow_{\text{elim}}$: eliminate a transition-induced loop by:
- removing the loop-entry transition(s)
- garbage collection

$\longrightarrow_{\text{prune}}$: remove a transition to a deadlocking state

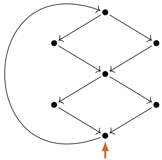
Lemma

(i) $\longrightarrow_{\text{elim}} \cup \longrightarrow_{\text{prune}}$ *is terminating.*

(ii) $\longrightarrow_{\text{elim}} \cup \longrightarrow_{\text{prune}}$ *is decreasing* [Van Oostrom, de Bruijn]



‘Critical pair’: bi-loop elimination



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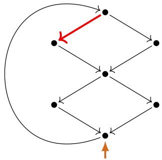

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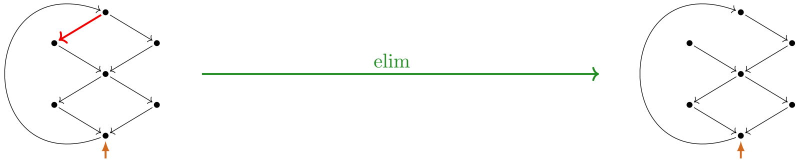

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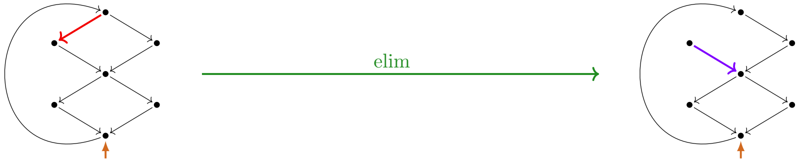

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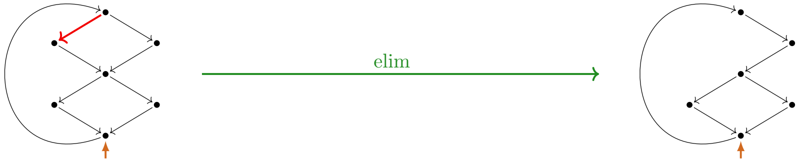

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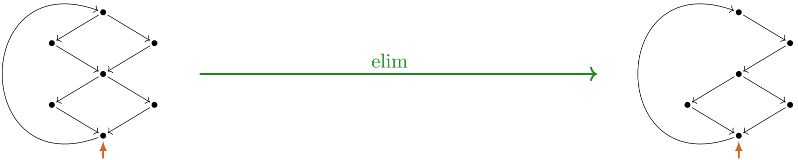

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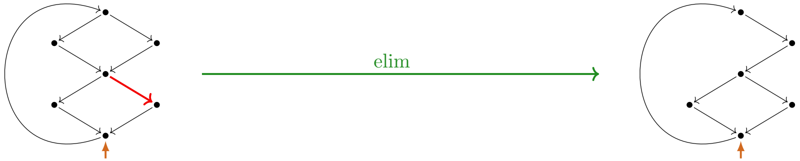

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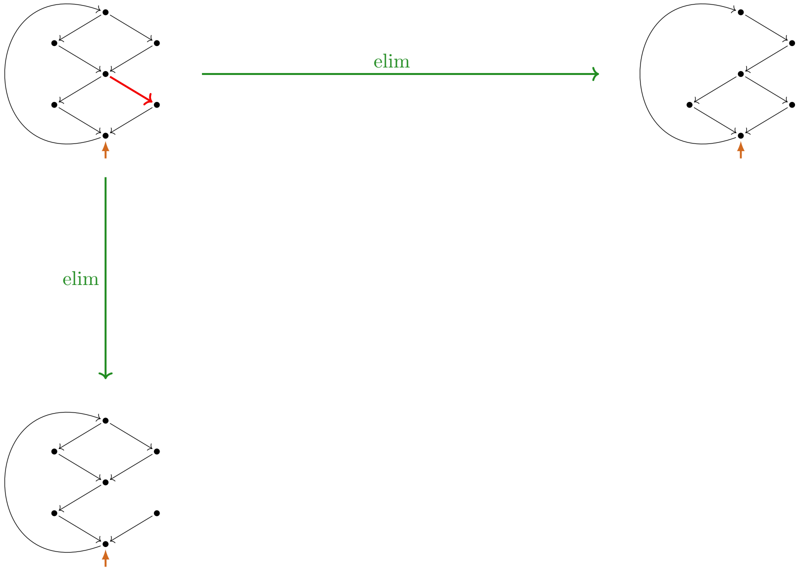

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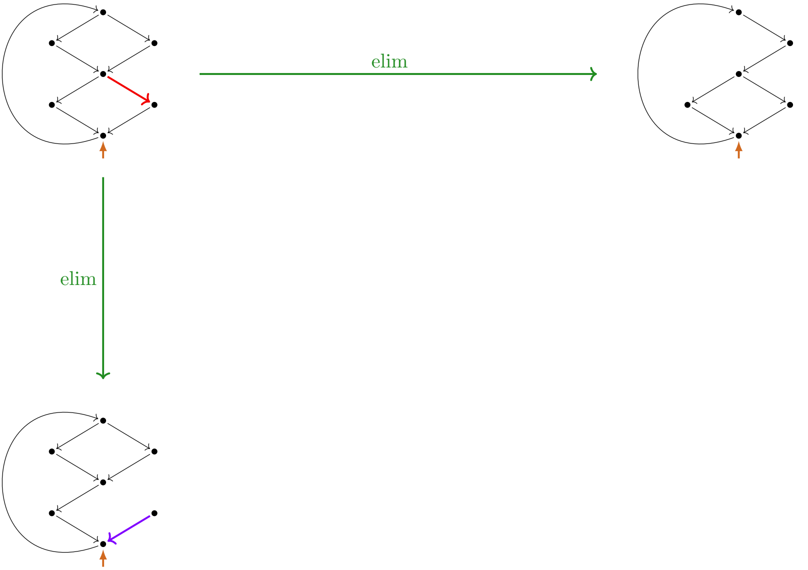

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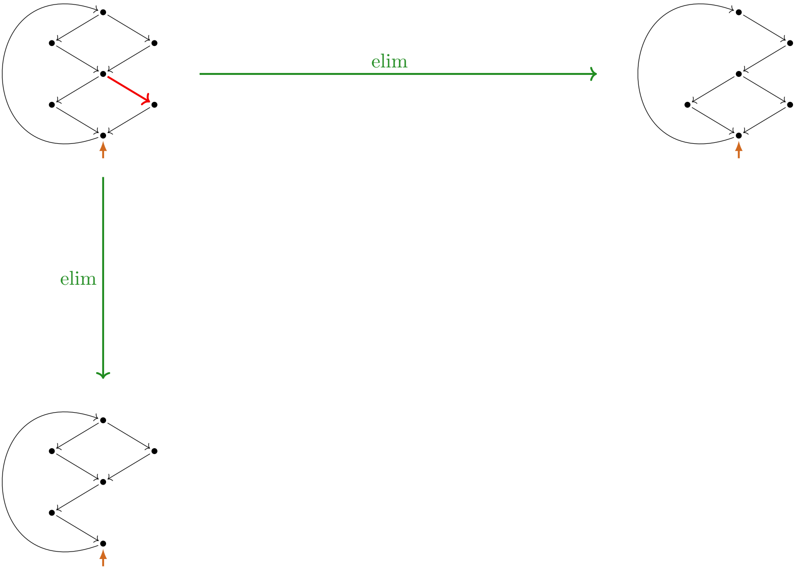

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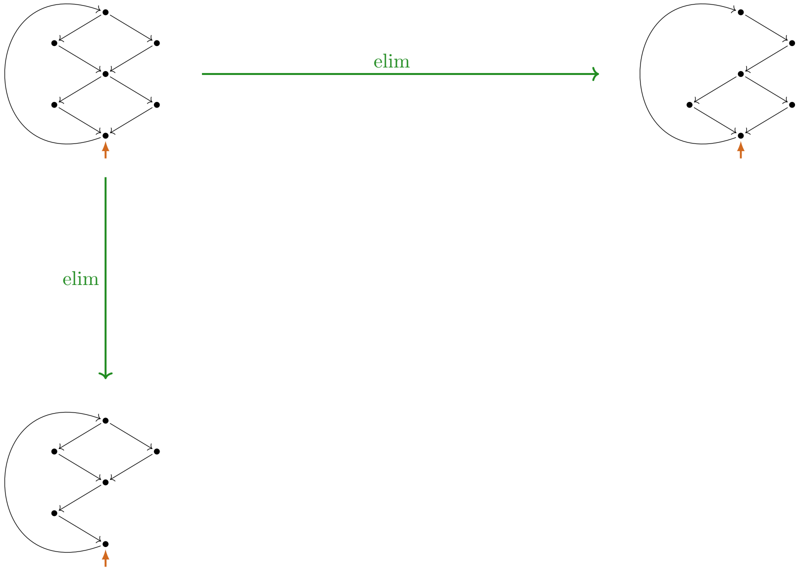

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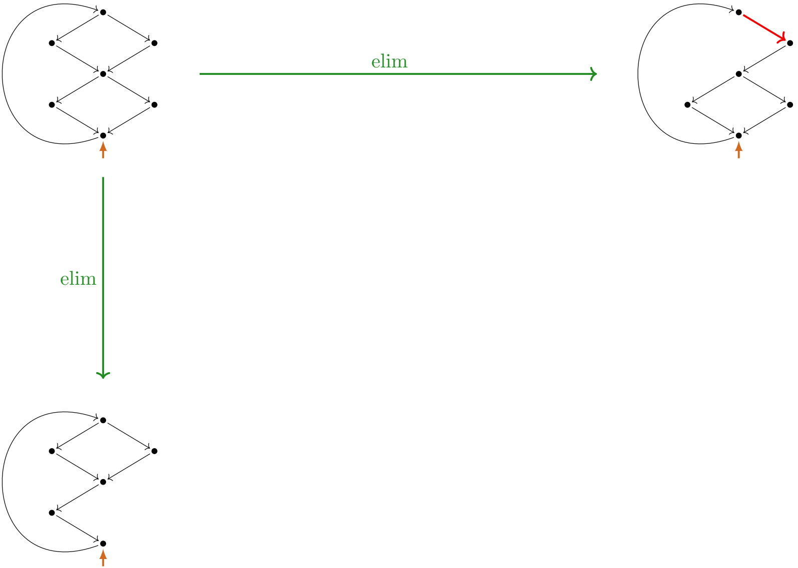

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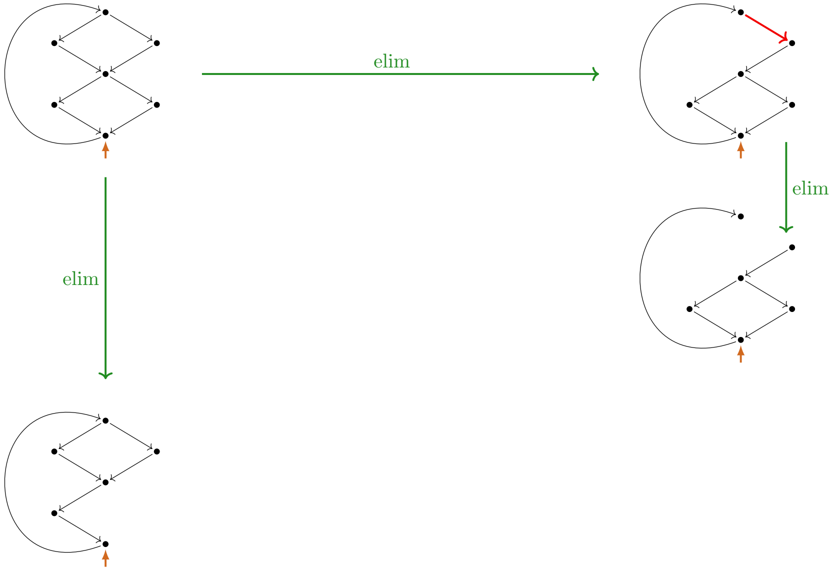

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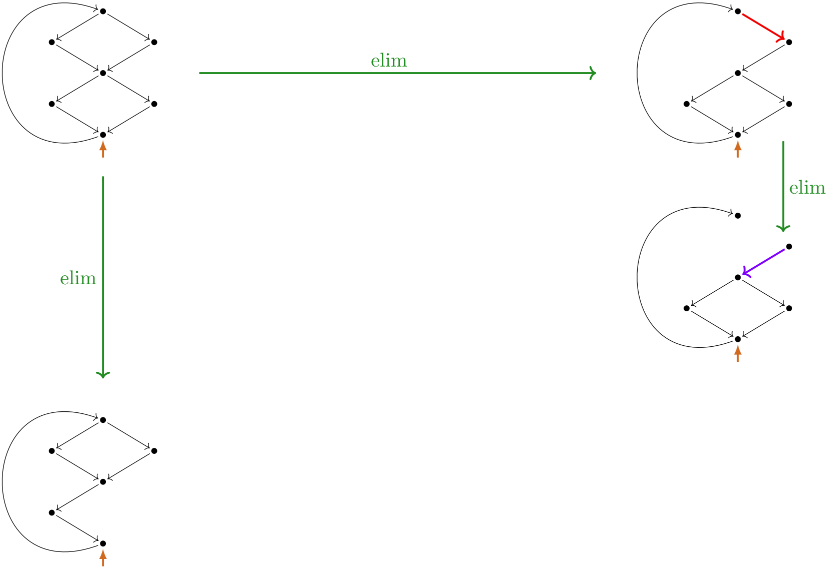

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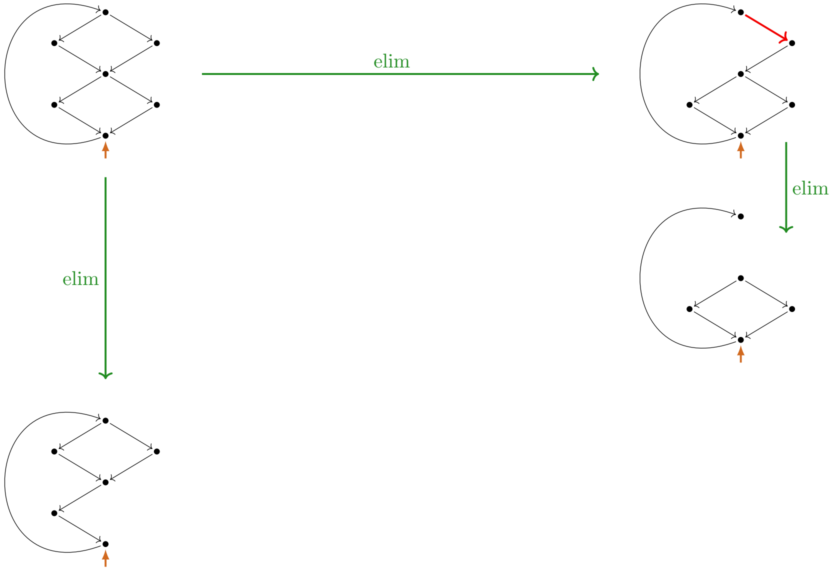

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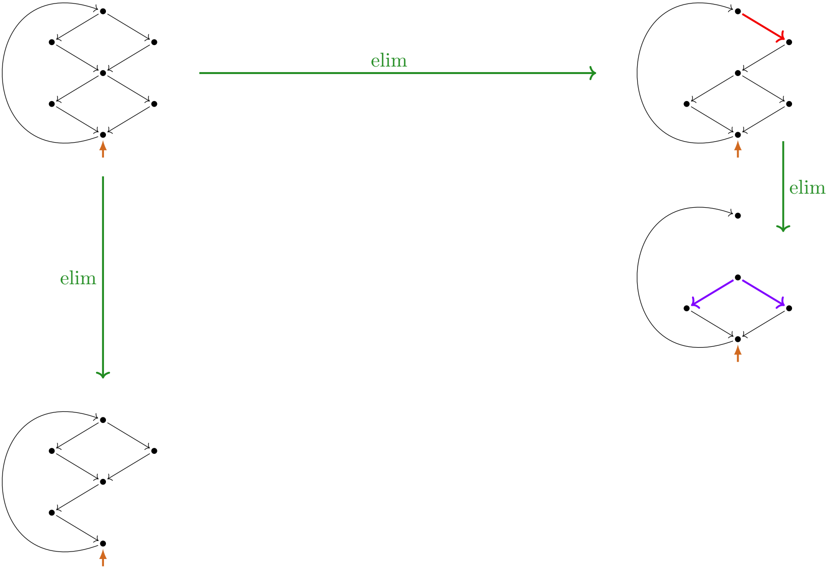

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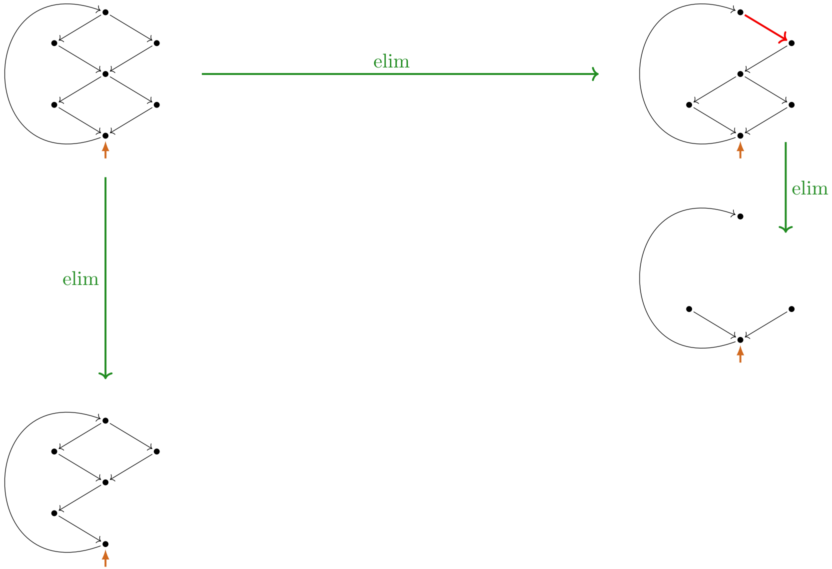

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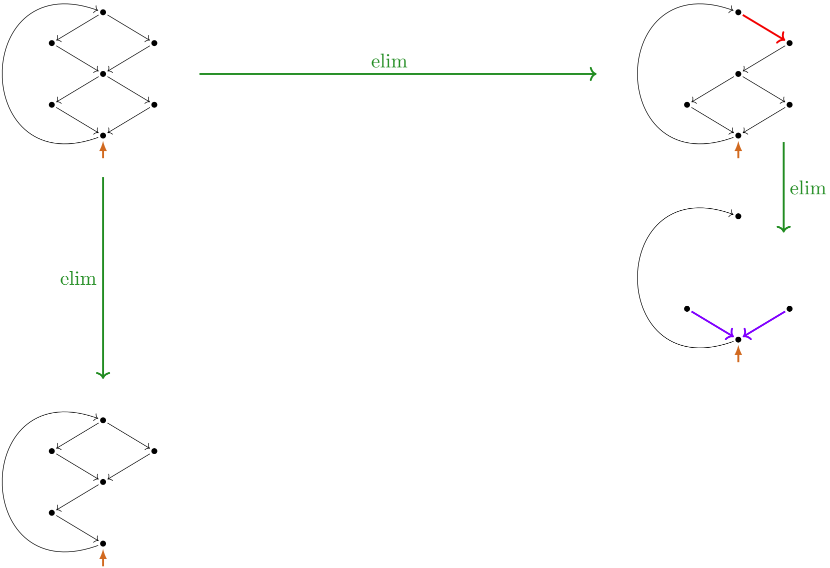

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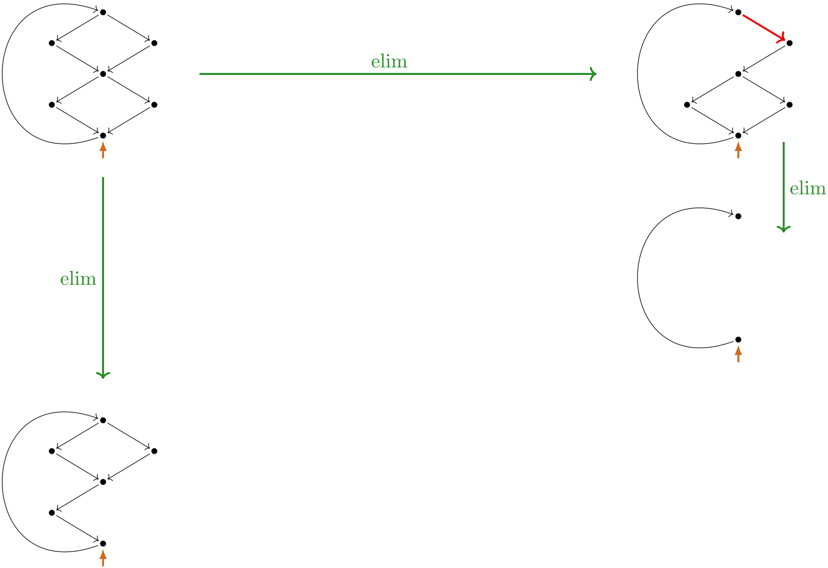

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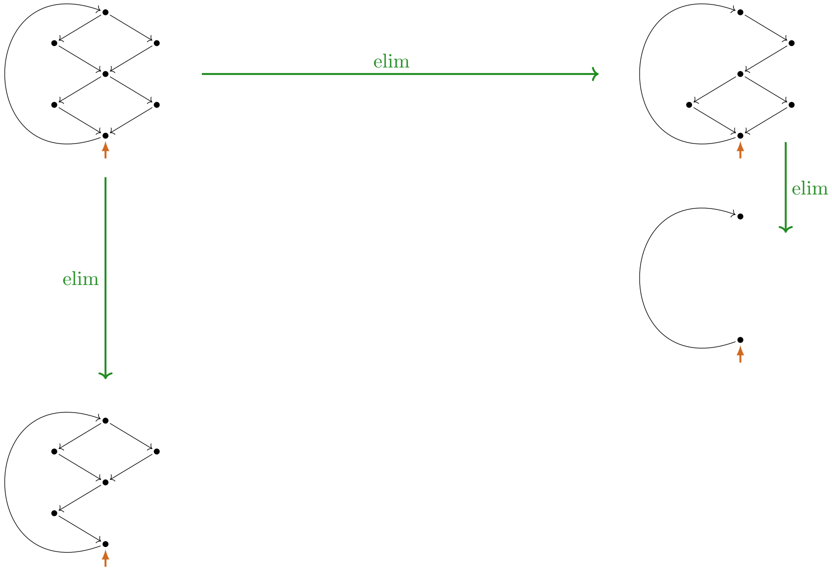

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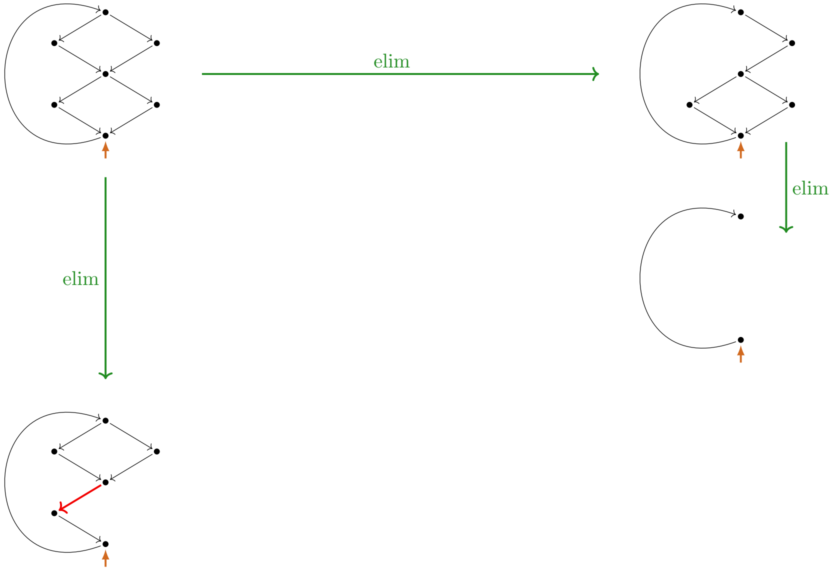

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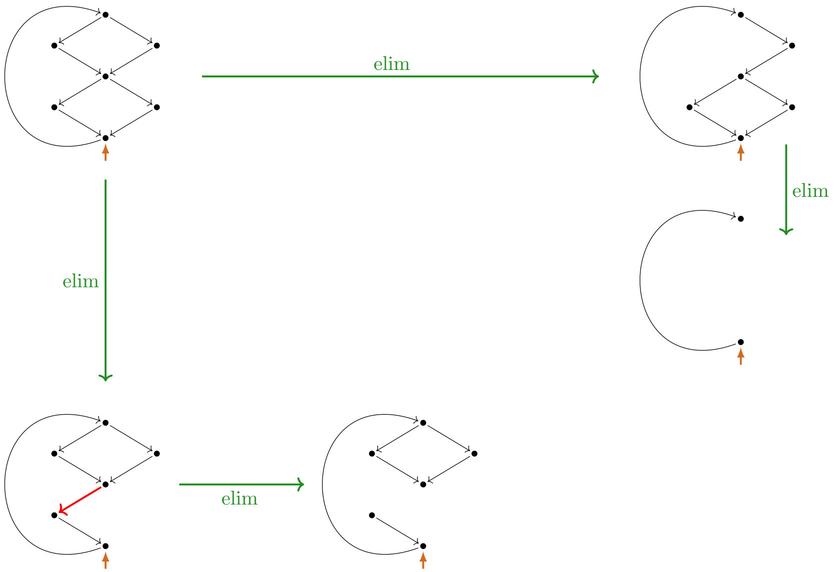

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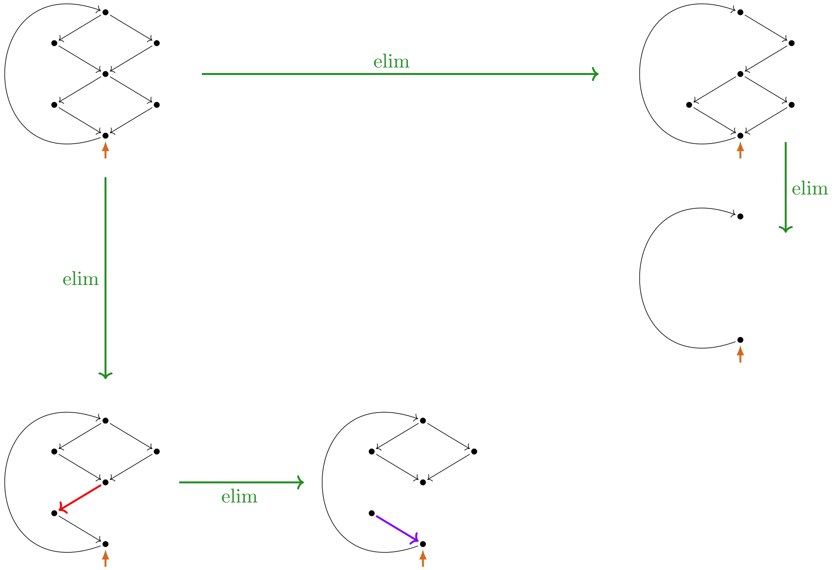

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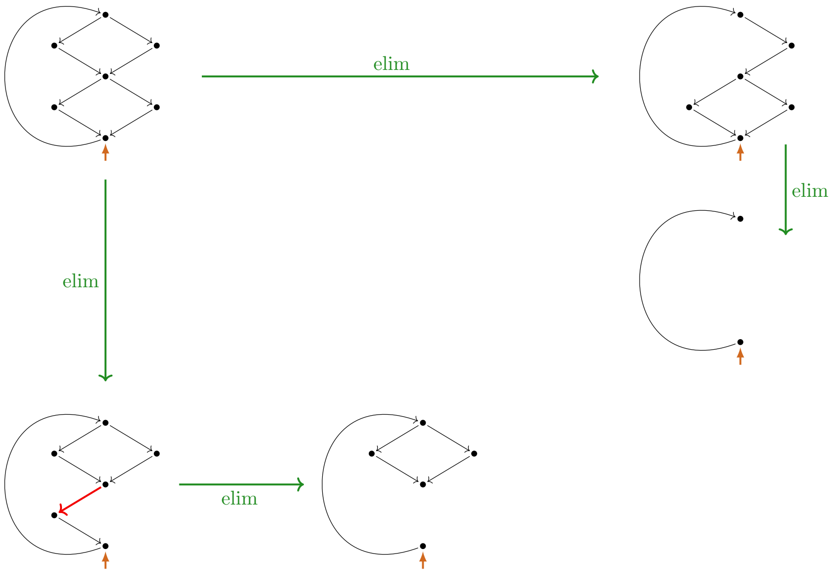

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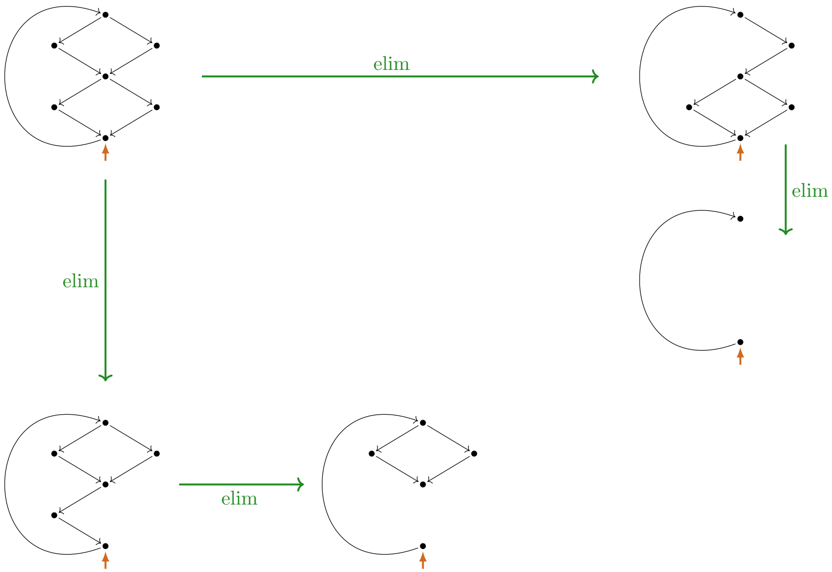

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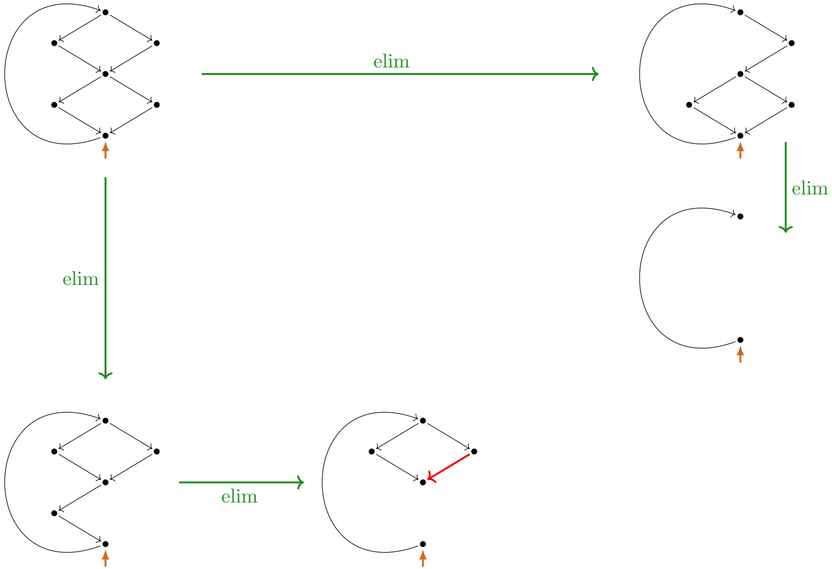

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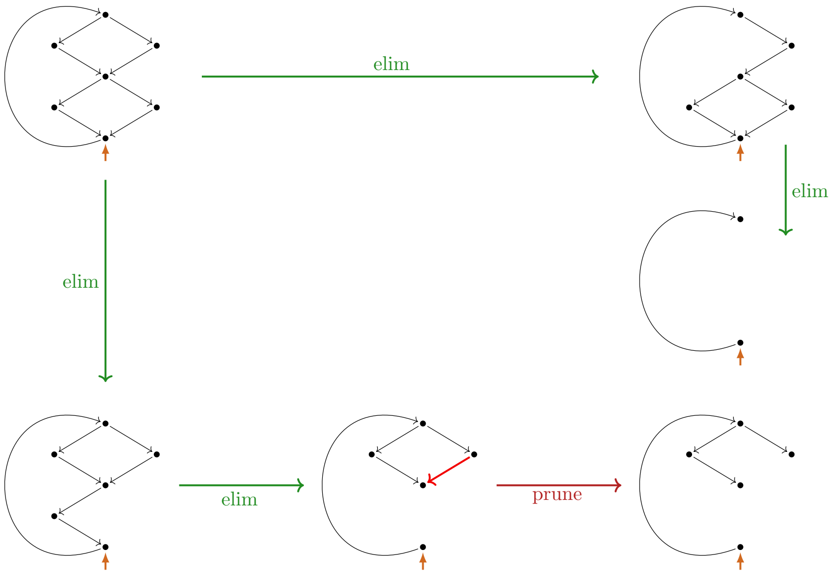

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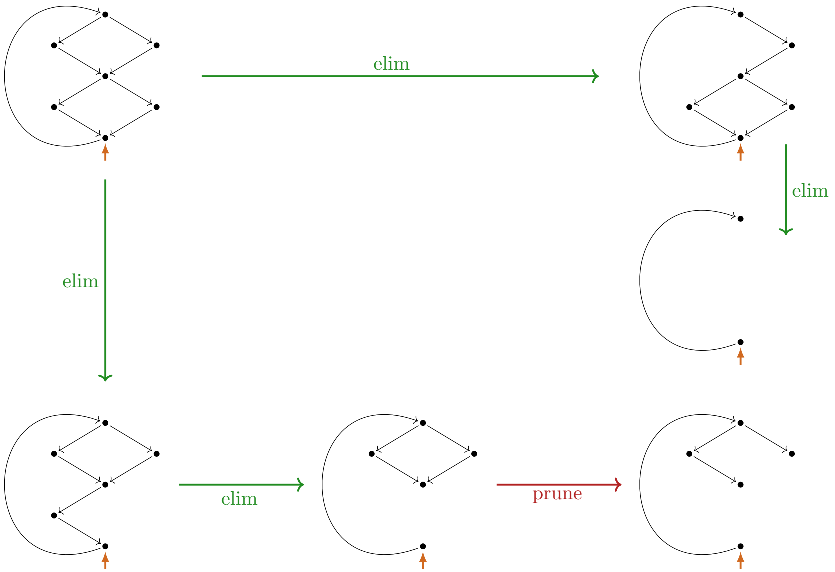

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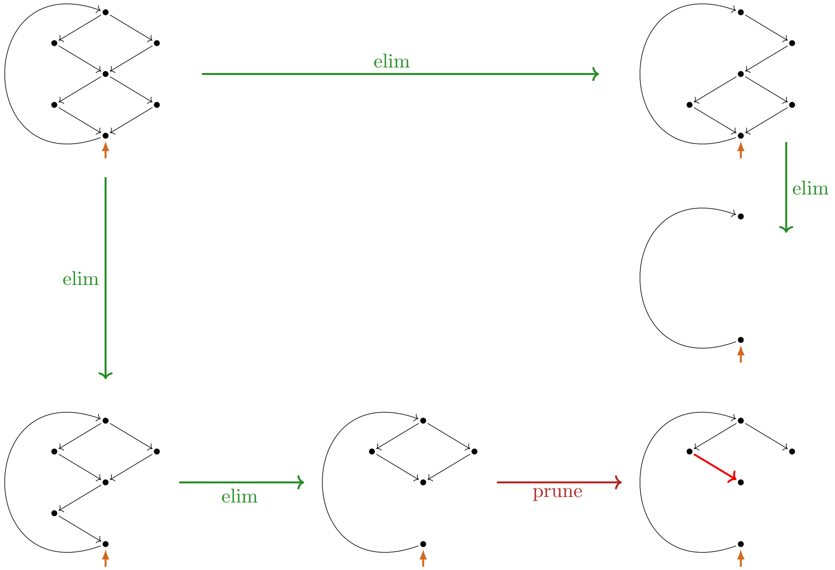

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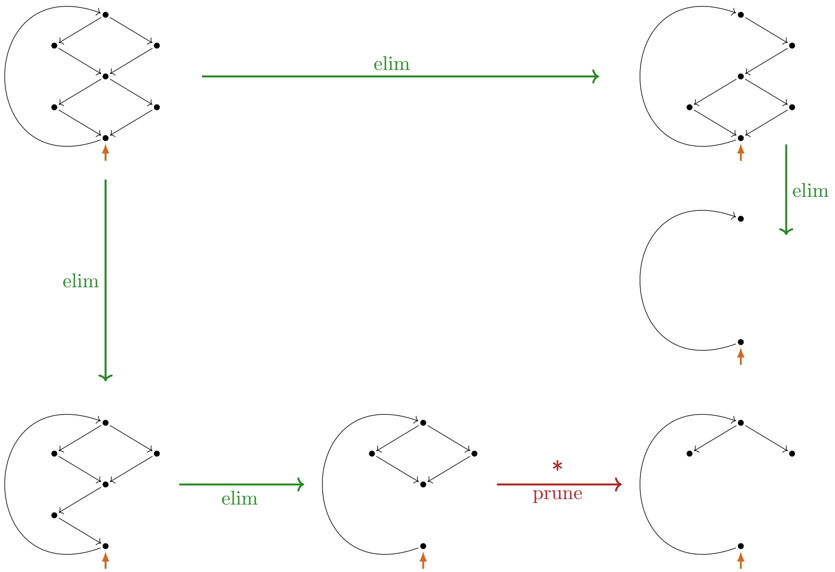

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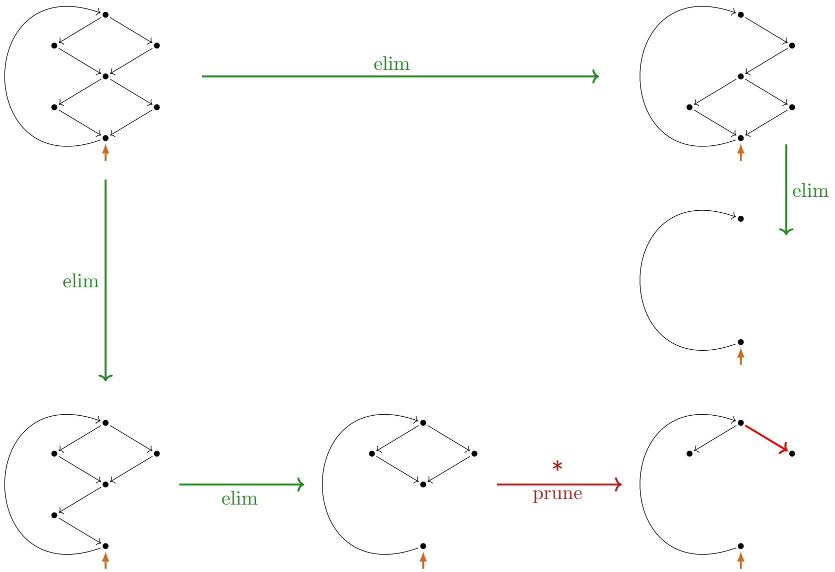
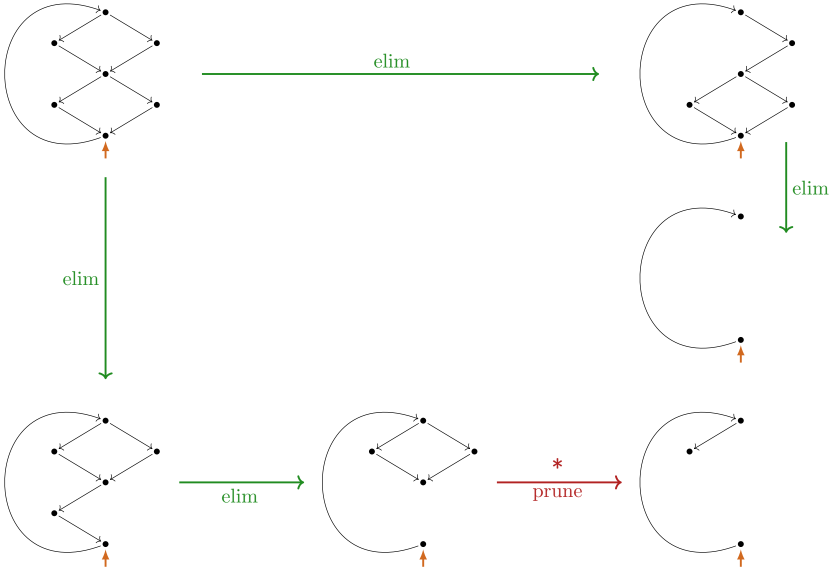


'Critical pair': bi-loop elimination



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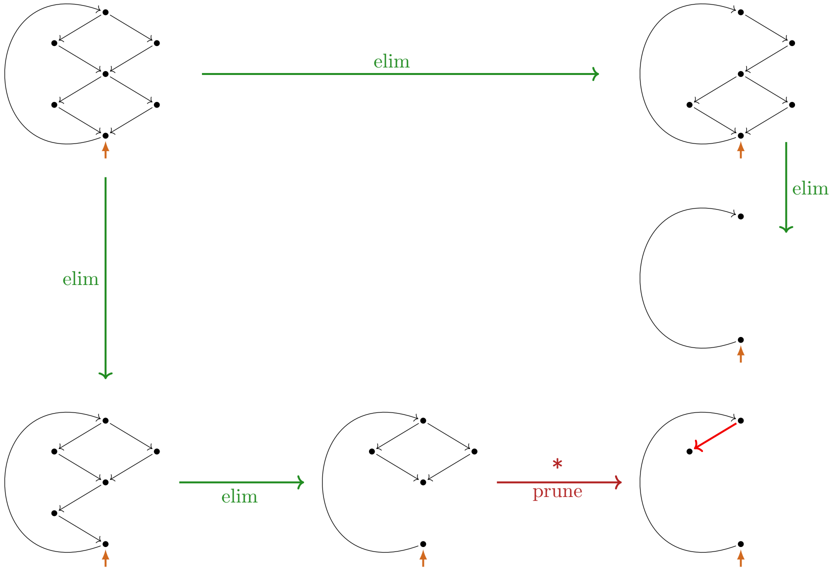

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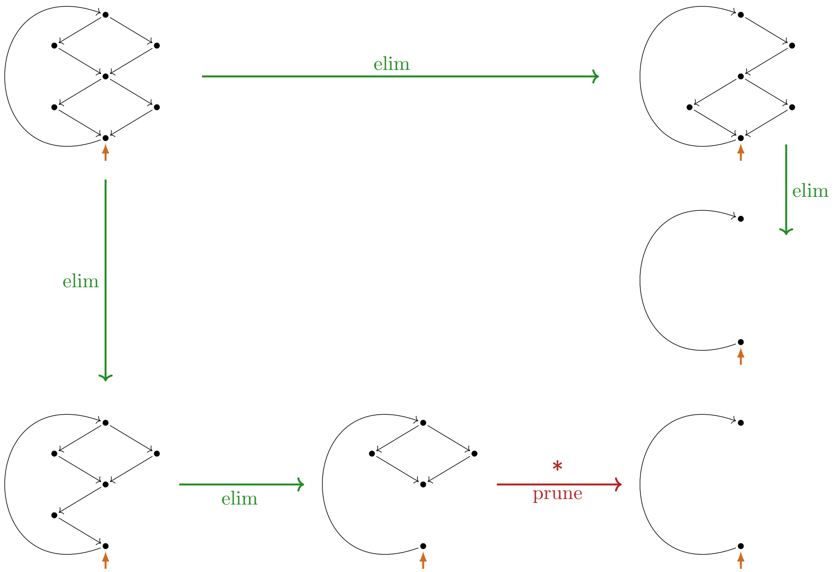

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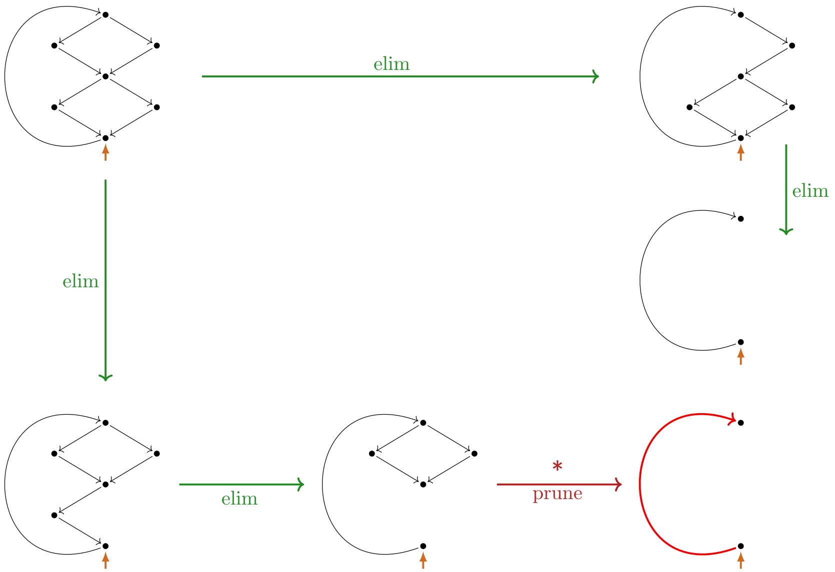

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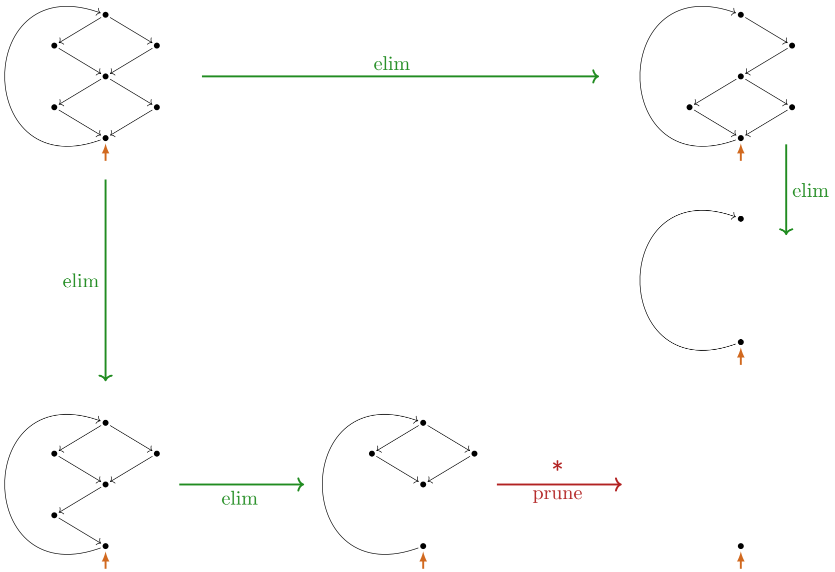

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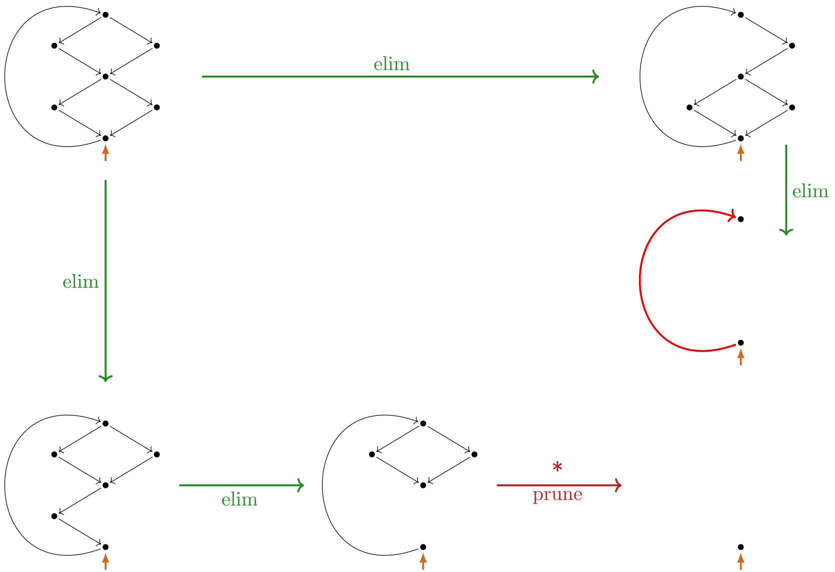

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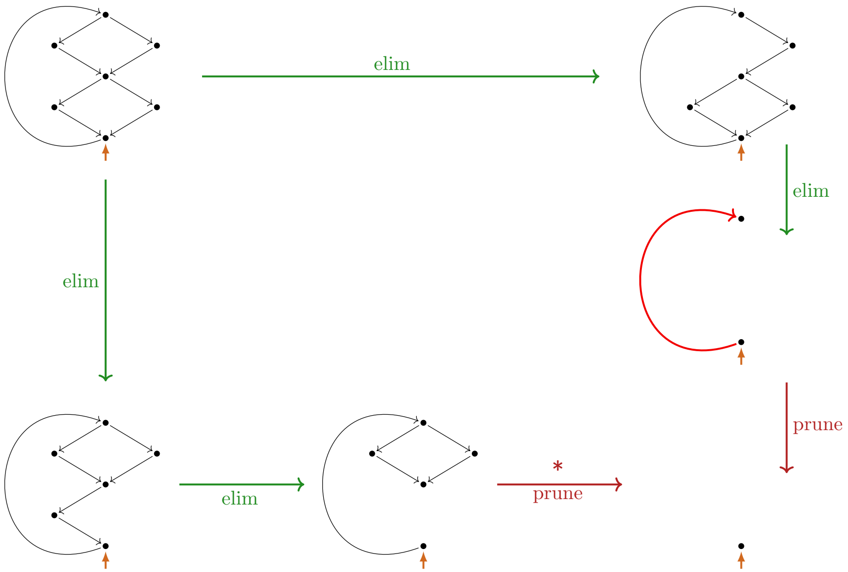

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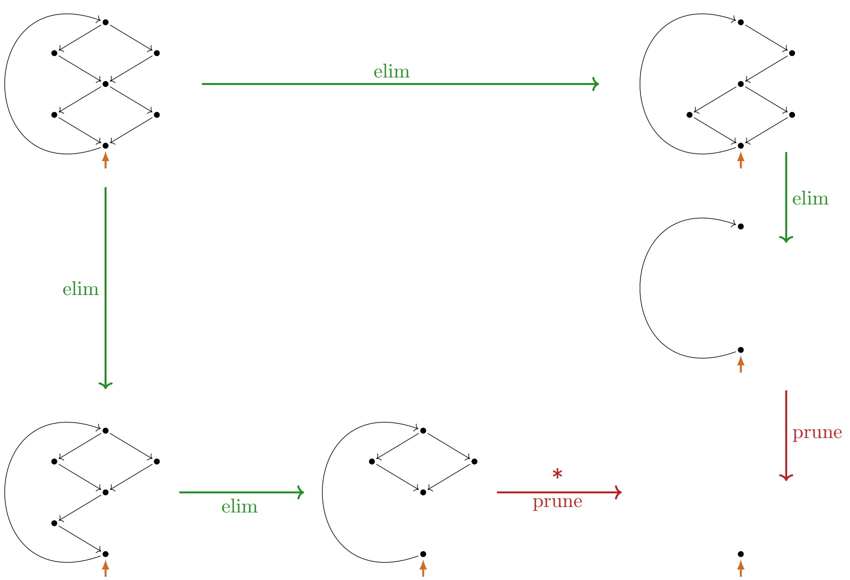

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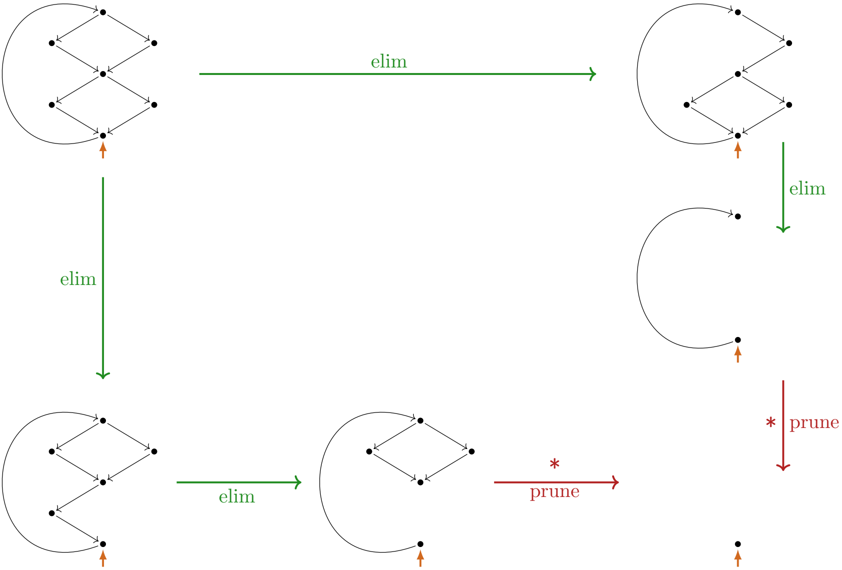


Loop elimination, and properties

$\longrightarrow_{\text{elim}}$: eliminate a transition-induced loop by:

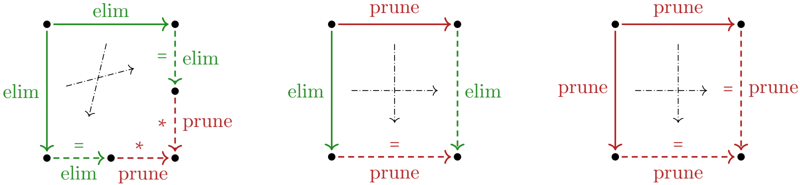
- removing the loop-entry transition(s)
- garbage collection

$\longrightarrow_{\text{prune}}$: remove a transition to a deadlocking state

Lemma

(i) $\longrightarrow_{\text{elim}} \cup \longrightarrow_{\text{prune}}$ *is terminating.*

(ii) $\longrightarrow_{\text{elim}} \cup \longrightarrow_{\text{prune}}$ *is decreasing, and so due to (i) locally confluent.*



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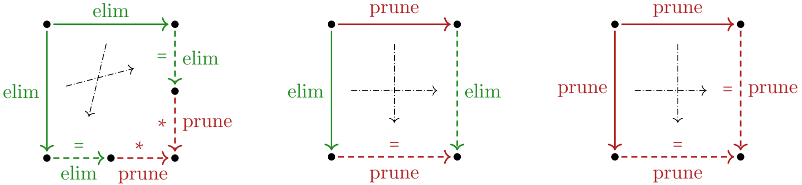

Structure property LEE

Definition

A process graph G satisfies LEE (*loop existence and elimination*) if:

$$\exists G_0 \left(G \longrightarrow^{*}_{\text{elim}} G_0 \not\longrightarrow_{\text{elim}} \right.$$
$$\left. \wedge \; G_0 \text{ has no infinite trace} \right).$$

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For every process graph G the following are equivalent:

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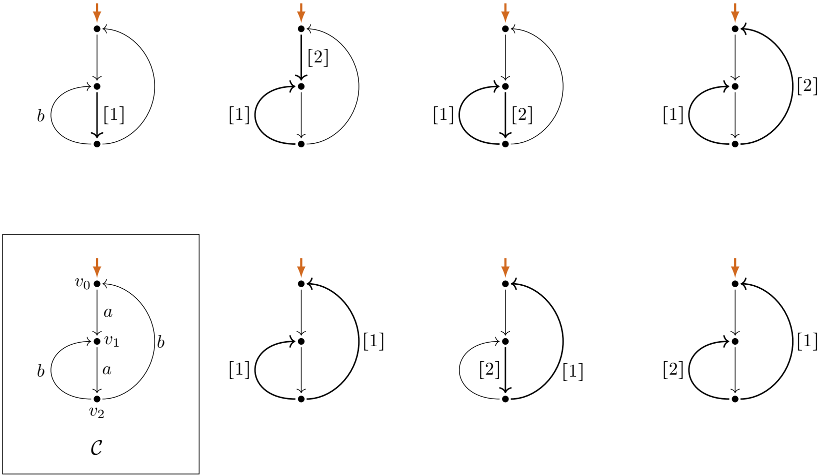
Theorem (efficient decidability)

The problem of deciding LEE(G) *for process graphs G is in* PTIME.

7 LEE-witnesses



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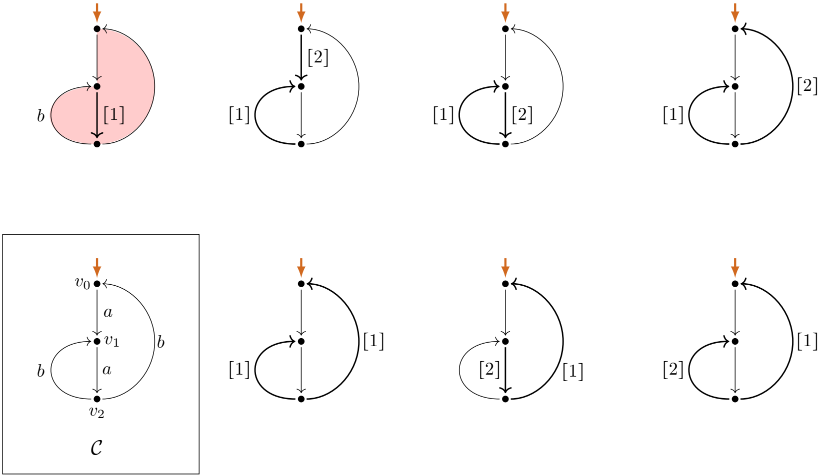

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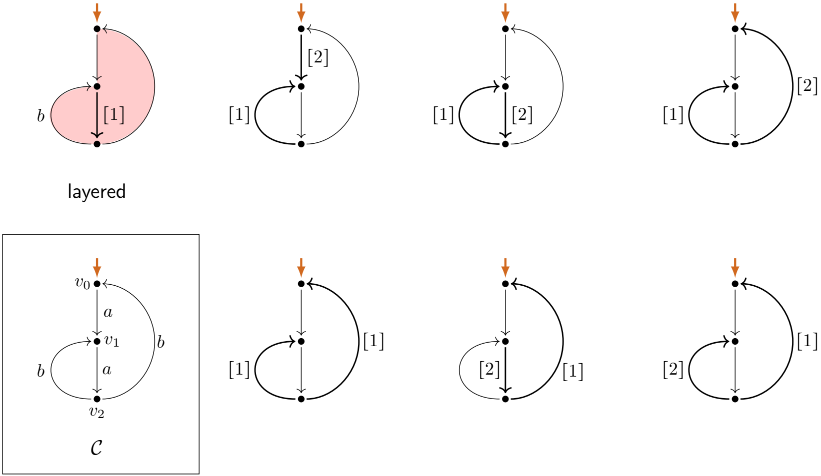

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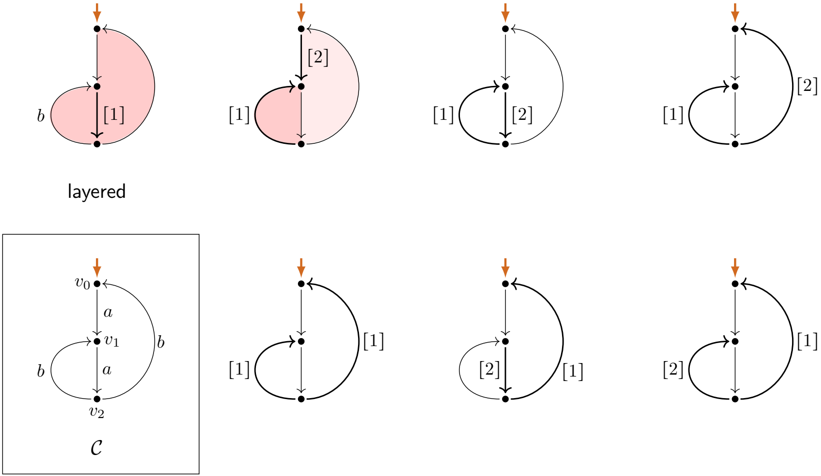

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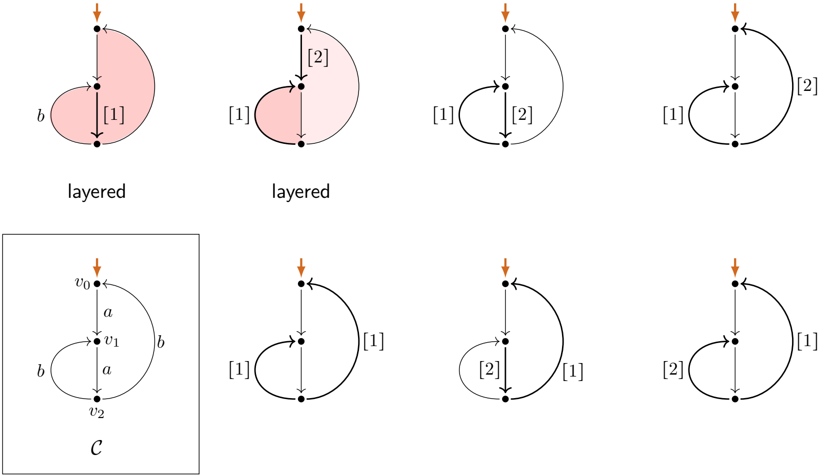

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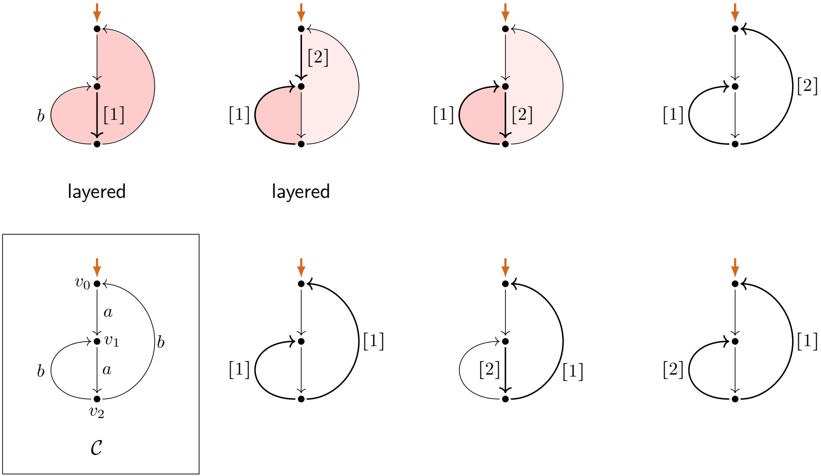

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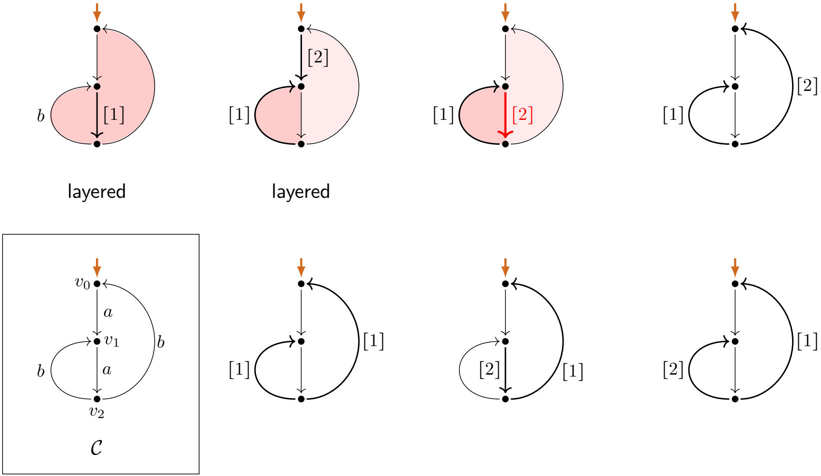

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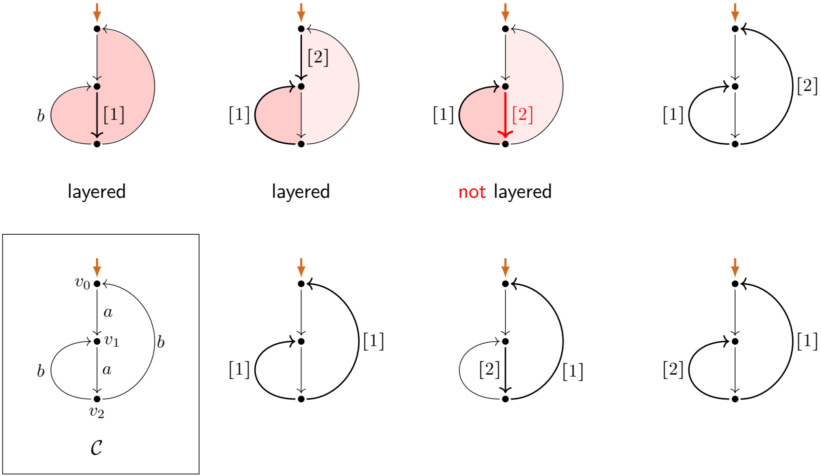

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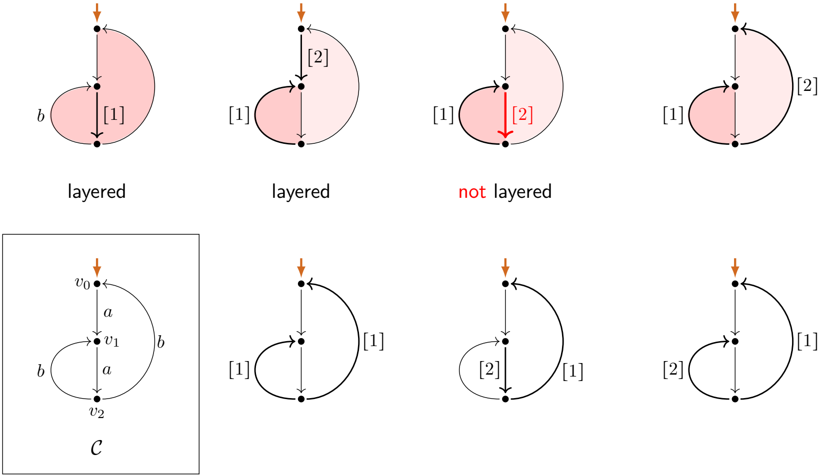

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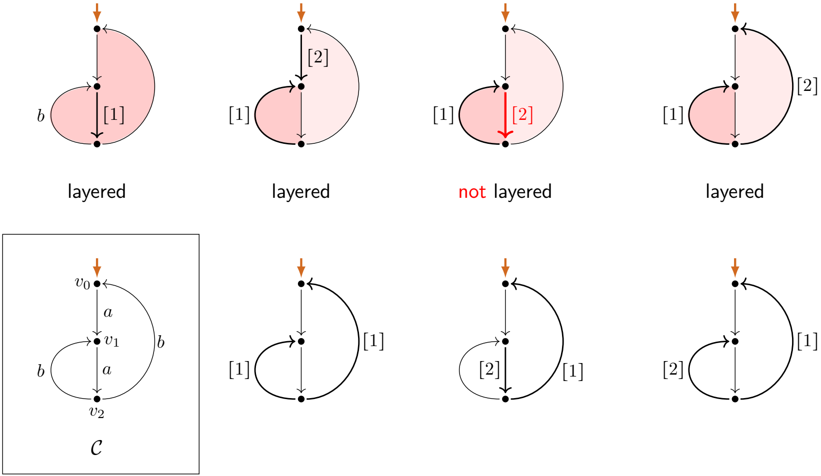

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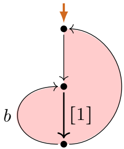

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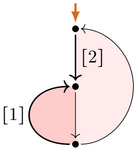

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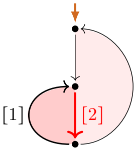

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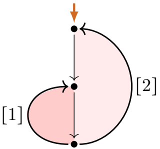

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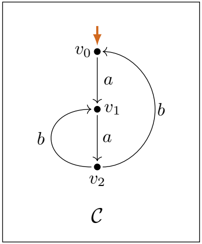

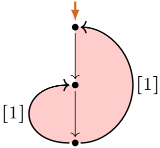

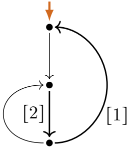

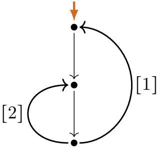

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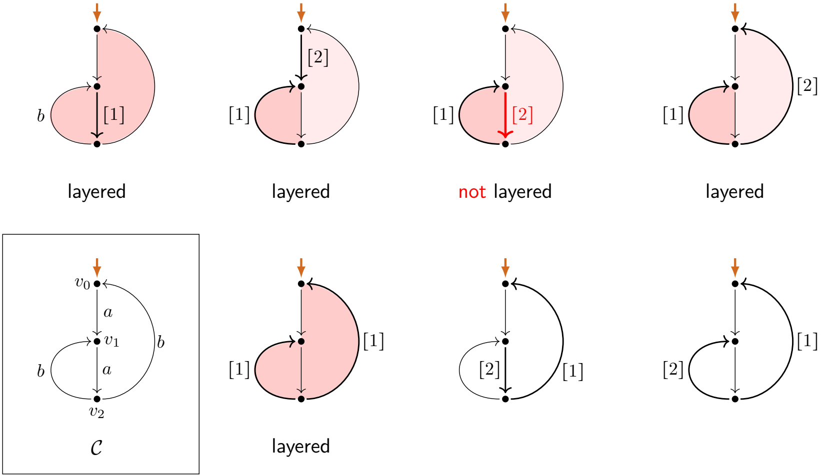

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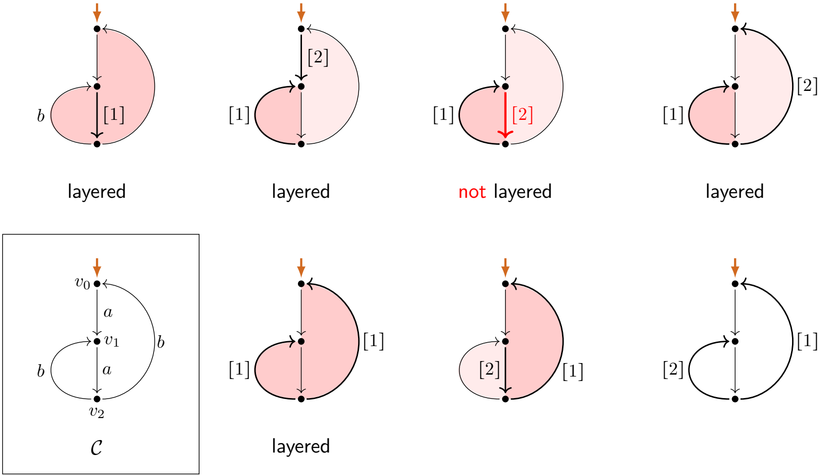

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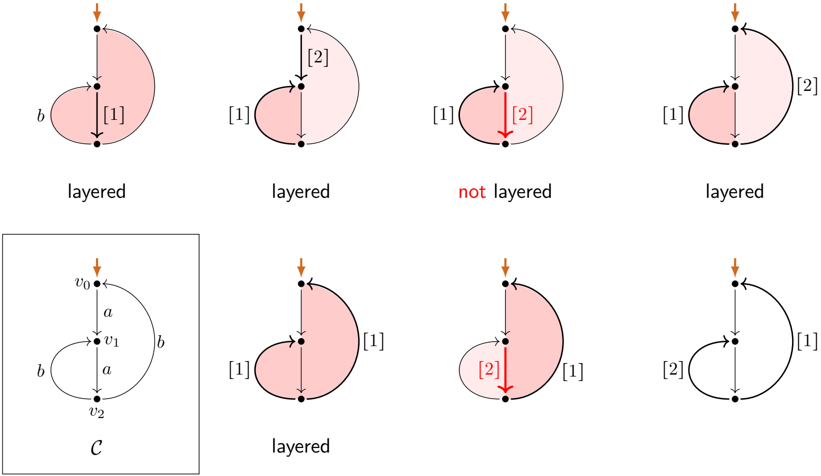

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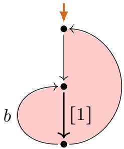

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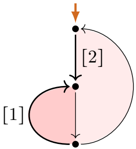

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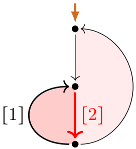

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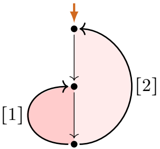

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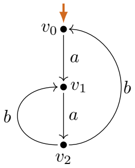

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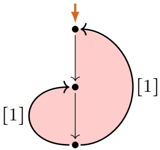

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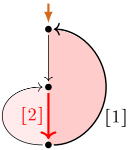

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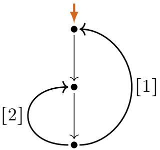

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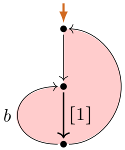

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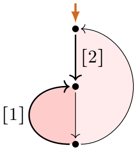

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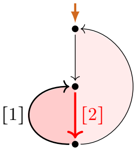

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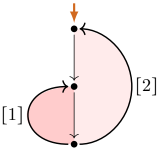

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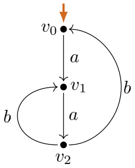

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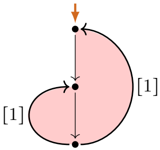

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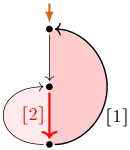

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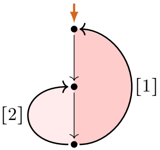

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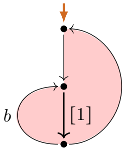

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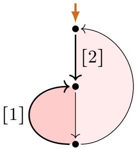

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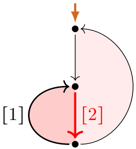

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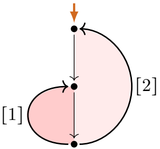

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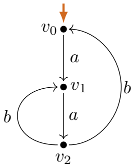

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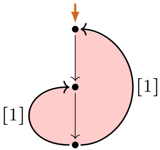

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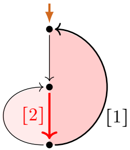

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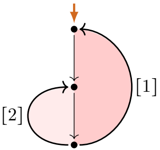

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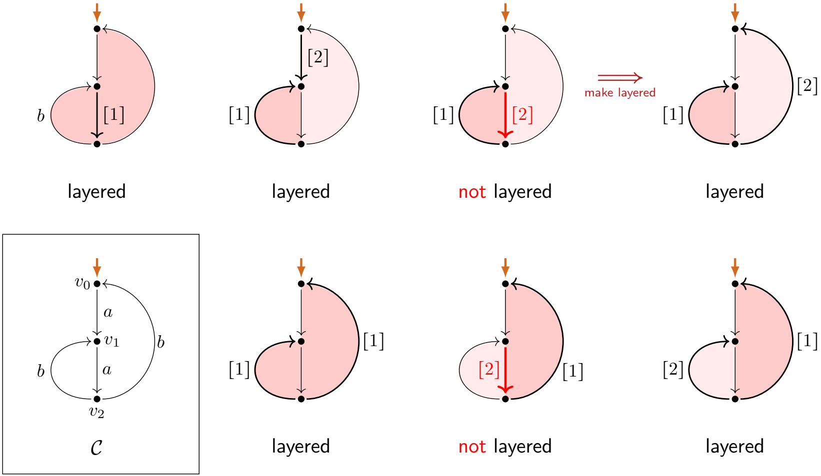

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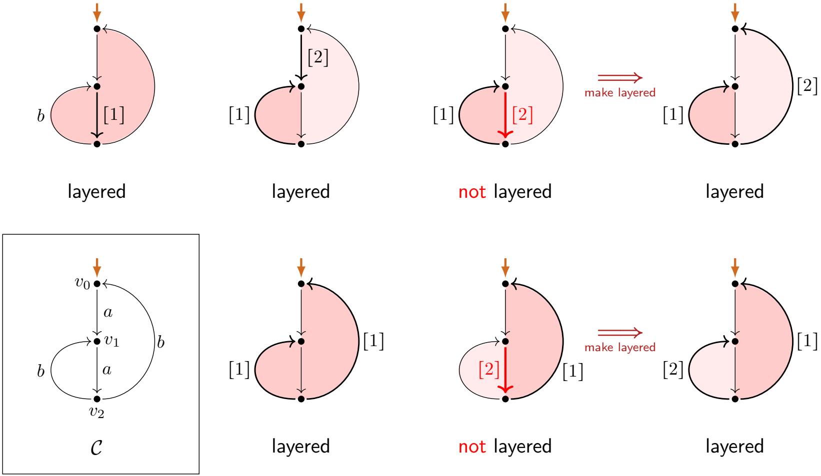

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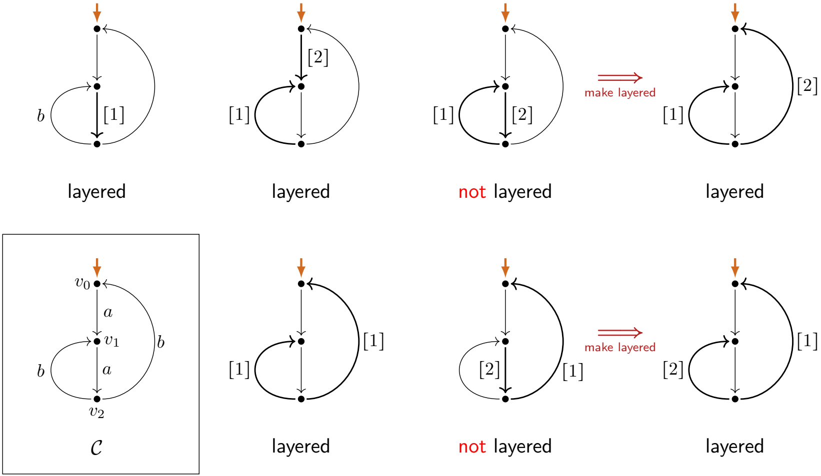

Interpretation/extraction correspondences with LEE

($\Leftarrow$ G/Fokkink 2020, G 2021)

(Int)$_P^{(*/\mathbf{1})}$: *P-$(*/\mathbf{1})$-expressible graphs have the structural property LEE.*

Process **i**nterpretations $P(e)$ of $(*/\mathbf{1})$ regular expressions e are finite process graphs that satisfy LEE.

(Extr)$_P$: LEE *implies $[\![\cdot]\!]_P$-expressibility*

From every finite process graph G with LEE a regular expression e can be **e**xtracted such that $G \,\underline{\leftrightarrow}\, P(e)$.

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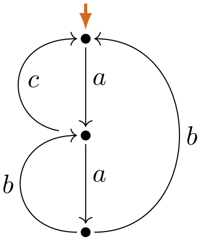
(Coll): LEE *is preserved under collapse*

The class of finite process graphs with LEE is closed under bisimulation **c**ollapse.

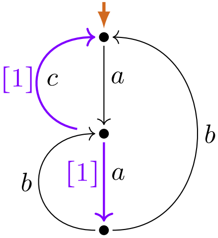
Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

G_2



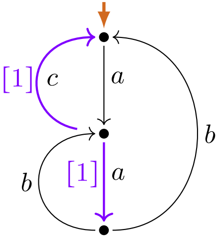
Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_2}$



Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

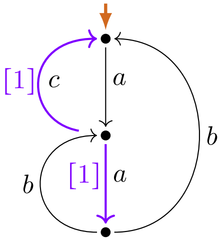
$\widehat{G_2}$



$$(\qquad\qquad\qquad\qquad)^* \cdot 0$$

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

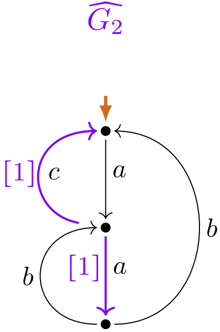
$\widehat{G_2}$



a

$\Big(\qquad\qquad\qquad\qquad \Big)^* \cdot 0$

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)



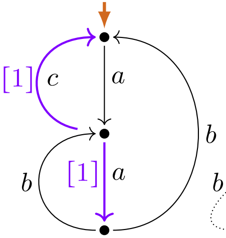
$$(a \cdot 1) \cdot ((\qquad)^{*} \cdot 0)$$

$$\downarrow a$$

$$(\qquad)^{*} \cdot 0$$

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_2}$



$(a \cdot 1) \cdot ((\qquad)^* \cdot 0)$

a

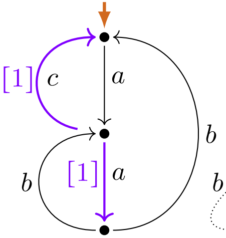
$(\qquad)^* \cdot 0$

b

b

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_2}$



$(a \cdot 1) \cdot ((\qquad)^* \cdot 0)$

$\downarrow a$

$(\qquad)^* \cdot 0$

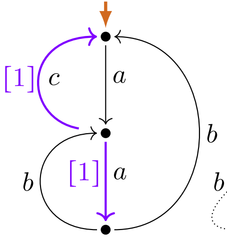
b

b

$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot ((\qquad)^*) \cdot 0)$

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_2}$



$(a \cdot 1) \cdot ((\qquad)^* \cdot 0)$

$\downarrow a$

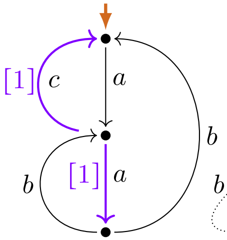
$(\qquad)^* \cdot 0$

$\downarrow a$

$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot ((\qquad)^*) \cdot 0)$

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_2}$



$$(a \cdot 1) \cdot \Big(\big(\qquad \qquad \qquad \big)^{*} \cdot 0 \Big)$$

$$\Big\downarrow a$$

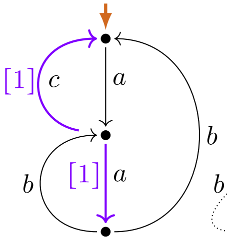
$$\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0$$

$$\Big\downarrow a$$

$$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot \Big(\big(\qquad \qquad \qquad \big)^{*} \big) \cdot 0 \Big)$$

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

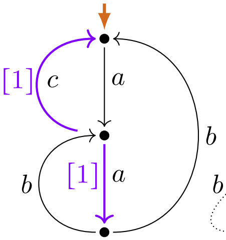
$\widehat{G_2}$



$$
\begin{tikzcd}
(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big) \arrow[d, dotted, "a"] \\
\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0 \arrow[d, dotted, "a"] \arrow[u, bend left, dotted, "c"] \\
(b\cdot 1+b\cdot(a\cdot 1))\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*})\cdot 0\Big) \arrow[u, bend left, dotted, "b"] \arrow[uu, bend right, dotted, "b"']
\end{tikzcd}
$$

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_2}$



$$\overbrace{(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0 \Big)}^{e}$$

$$\Big\downarrow a$$

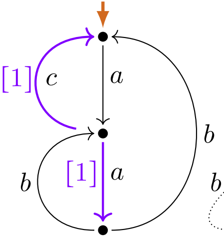
$$\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0$$

$$\Big\downarrow a$$

$$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*}) \cdot 0 \Big)$$

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_2}$



$P(e) \rightharpoonup G_2$

$$\overbrace{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)}^{e}$$

$$\downarrow a$$

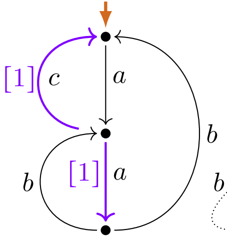
$$\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0$$

$$\downarrow a$$

$$(b\cdot 1+b\cdot(a\cdot 1))\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0\Big)$$

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_2}$



$P(e) \mathrel{\underline{\to}} G_2$

$$\overbrace{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)}^{e}$$

$$\Big\downarrow a$$

$$\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0$$

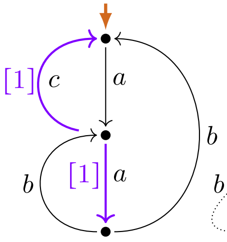
$$\downarrow a$$

$$(b\cdot 1+b\cdot(a\cdot 1))\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0\Big)$$

c b b

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_2}$



$P(e) \mathrel{\underline{\rightarrow}} G_2$

$$\overbrace{(a\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)}^{e}$$

$$\downarrow a$$

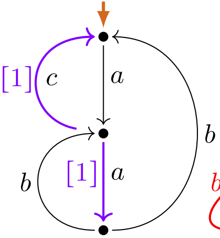
$$\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0$$

$$\downarrow a$$

$$(b\cdot 1+b\cdot(a\cdot 1))\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0\Big)$$

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_2}$



$P(e) \mathrel{\underline{\to}} G_2$

$$\overbrace{(a\cdot 1)\cdot\big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\big)}^{e}$$

$$\downarrow a$$

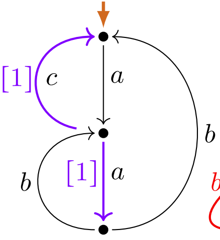
$$\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0$$

$$\downarrow a$$

$$(b\cdot 1+b\cdot(a\cdot 1))\cdot\big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*})\cdot 0\big)$$

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_2}$



$$P(e) \mathrel{\underline{\rightarrow}} G_2 \not\mathrel{\underline{\leftrightarrow}} P(e)$$

$$\overbrace{(a \cdot 1) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0 \big)}^{e}$$

$$\downarrow a$$

$$\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0$$

$$\downarrow a$$

$$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \big) \cdot 0 \big)$$

$$\not\xrightarrow{c} \qquad \not\xrightarrow{b} \qquad \not\xrightarrow{b}$$

Interpretation of extracted expression

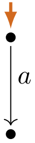
G_2'

$P(e) = G_2'$

$$\overbrace{(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \cdot 0 \Big)}^{e}$$

Interpretation of extracted expression

G_2'



$$P(e) = G_2'$$

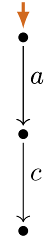
$$\overbrace{(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \cdot 0 \Big)}^{e}$$

$$\big\downarrow a$$

$$(1 \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \cdot 0 \Big)$$

Interpretation of extracted expression

G'_2 $\qquad\qquad$ $P(e) = G'_2$



$$\overbrace{(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0\Big)}^{e}$$

$$\Big\downarrow a$$

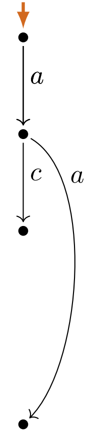
$$(1 \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0\Big)$$

$$\Big\downarrow c$$

$$\Big((1 \cdot (a \cdot 1)) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^*\Big) \cdot 0$$

Interpretation of extracted expression

G'_2



$P(e) = G'_2$

$$\overbrace{(a\cdot 1)\cdot\big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^*\cdot 0\big)}^{e}$$

$$\downarrow a$$

$$(1\cdot 1)\cdot\big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^*\cdot 0\big)$$

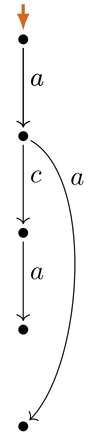
$$\downarrow c \qquad\qquad\qquad \xrightarrow{a}$$

$$\big((1\cdot(a\cdot 1))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^*\big)\cdot 0$$

$$\big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^*\big)\cdot 0$$

Interpretation of extracted expression

G_2'



$\color{green}{P}(\color{orange}{e}) = G_2'$

$$\overbrace{(a \cdot 1) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \cdot 0 \big)}^{e}$$

$$\downarrow a$$

$$(1 \cdot 1) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \cdot 0 \big)$$

$$\downarrow c$$

$$\big((1 \cdot (\color{red}{a} \cdot 1)) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \big) \cdot 0$$

$$\color{red}{\downarrow a}$$

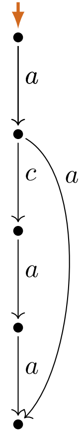
$$\big((1 \cdot 1) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \big) \cdot 0 \big)$$

$$a$$

$$\big((1 \cdot (b \cdot 1 + b \cdot (a \cdot 1))) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \big) \cdot 0$$

Interpretation of extracted expression

G_2'



$P(e) = G_2'$

$$\overbrace{(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0 \Big)}^{e}$$

$$\downarrow a$$

$$(1 \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0 \Big)$$

$$\downarrow c$$

$$\Big((1 \cdot (a \cdot 1)) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \Big) \cdot 0$$

$$\downarrow a$$

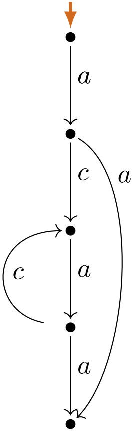
$$\Big((1 \cdot 1) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \Big) \cdot 0 \Big)$$

$$\downarrow a$$

$$\Big((1 \cdot (b \cdot 1 + b \cdot (a \cdot 1))) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \Big) \cdot 0$$

Interpretation of extracted expression

G_2'



$P(e) = G_2'$

$$\overbrace{(a \cdot 1) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0 \big)}^{e}$$

$$\downarrow a$$

$$(1 \cdot 1) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0 \big)$$

$$\downarrow c$$

$$\big((1 \cdot (a \cdot 1)) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \big) \cdot 0$$

$$\downarrow a$$

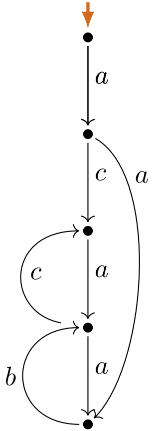
$$\big((1 \cdot 1) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \big) \cdot 0 \big)$$

$$\downarrow a$$

$$\big((1 \cdot (b \cdot 1 + b \cdot (a \cdot 1))) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \big) \cdot 0$$

Interpretation of extracted expression

G_2'



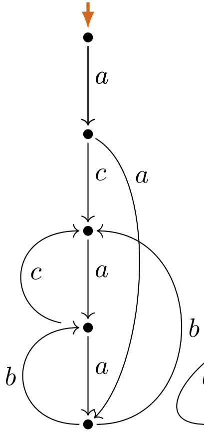
$$P(e) = G_2'$$

$$\begin{array}{c}
\overbrace{(a\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)}^{e} \\
\downarrow a \\
(1\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big) \\
\downarrow c \\
\big((1\cdot(a\cdot 1))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0 \\
\downarrow a \\
\big((1\cdot 1)\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0\big) \\
\downarrow a \\
\big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0
\end{array}$$

c b a

Interpretation of extracted expression

G'_2

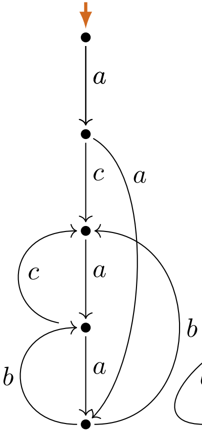


$$P(e) = G'_2$$

$$\begin{tikzcd}
\overbrace{(a\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)}^{e} \arrow[d, "a"] \\
(1\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big) \arrow[d, "c"] \arrow[ddd, bend left, "a"] \\
\big((1\cdot(a\cdot 1))\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\big)\cdot 0 \arrow[d, "a"] \\
\big((1\cdot 1)\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\big)\cdot 0 \arrow[d, "a"] \arrow[u, bend left, "c"] \\
\big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\big)\cdot 0 \arrow[u, bend left, "b"] \arrow[uu, bend right, "b"]
\end{tikzcd}$$

Interpretation of extracted expression

G_2'



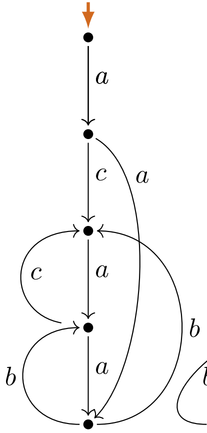
$$P(e) = G_2' \mathrel{\underline{\rightarrow}} G_2$$

$$\begin{tikzcd}
\overbrace{(a\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)}^{e} \arrow[d, "a"] \\
(1\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big) \arrow[d, "c"] \arrow[ddd, bend left, "a"] \\
\big((1\cdot(a\cdot 1))\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\big)\cdot 0 \arrow[d, "a"] \\
\big((1\cdot 1)\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\big)\cdot 0\big) \arrow[d, "a"] \arrow[u, bend left, "c"] \\
\big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\big)\cdot 0 \arrow[uu, bend right, "b"] \arrow[u, bend left, "b"]
\end{tikzcd}$$

Interpretation of extracted expression

G_2'

$$P(e) = G_2' \underline{\rightarrow} G_2 \not\simeq G_2'$$



$$\begin{array}{c}
\overbrace{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)}^{e}\\
\downarrow a\\
(1\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)\\
\downarrow c\\
\Big((1\cdot(a\cdot 1))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0\\
\downarrow a\\
\Big((1\cdot 1)\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0\Big)\\
\downarrow a\\
\Big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0
\end{array}$$

LEE under bisimulation?

LEE under bisimulation

Observation

- LEE is **not** invariant under bisimulation.

LEE under bisimulation

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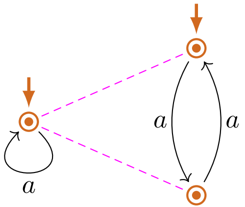
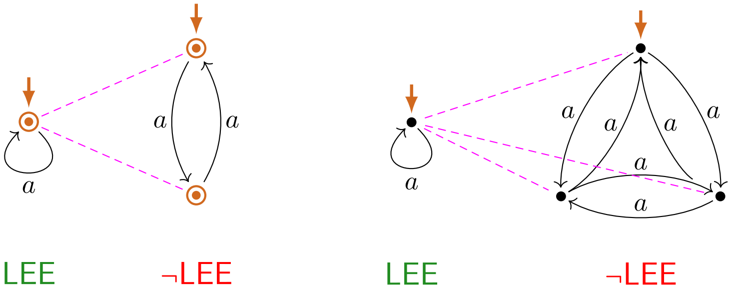


LEE ¬LEE

LEE under bisimulation

Observation

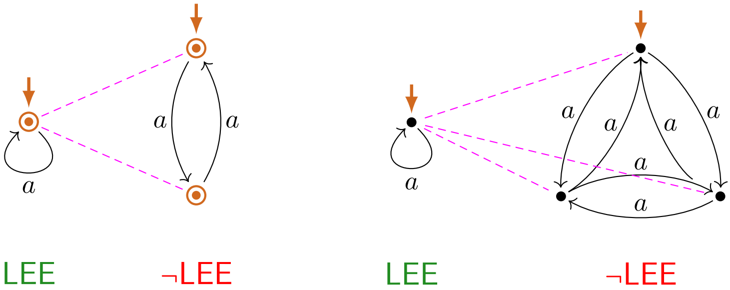
- LEE is not invariant under bisimulation.



LEE under bisimulation

Observation

- LEE is not invariant under bisimulation.
- LEE is not preserved by converse functional bisimulation.



LEE under functional bisimulation

Lemma

(i) LEE *is preserved by functional bisimulations:*

$$\mathsf{LEE}(G_1) \wedge G_1 \mathrel{\underline{\rightarrow}} G_2 \implies \mathsf{LEE}(G_2)\,.$$

LEE under functional bisimulation

Lemma

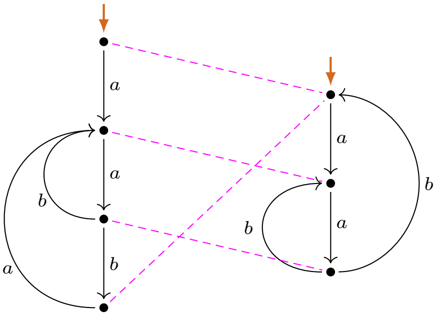
(i) LEE *is preserved by functional bisimulations:*

$$\mathsf{LEE}(G_1) \wedge G_1 \mathrel{\underline{\to}} G_2 \implies \mathsf{LEE}(G_2)\,.$$

Proof (Idea).

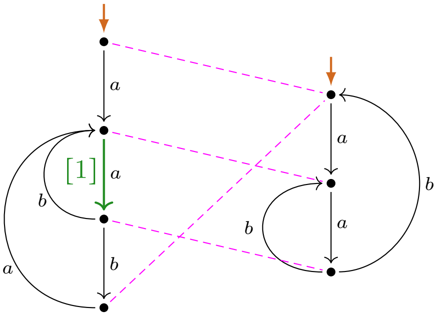
Use loop elimination in G_1 to carry out loop elimination in G_2.

Collapsing LEE-witnesses



$P(a(a(b+ba))^* \cdot 0)$

Collapsing LEE-witnesses



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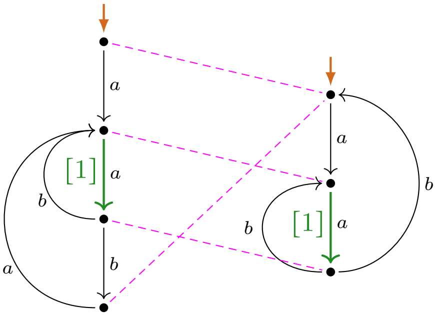

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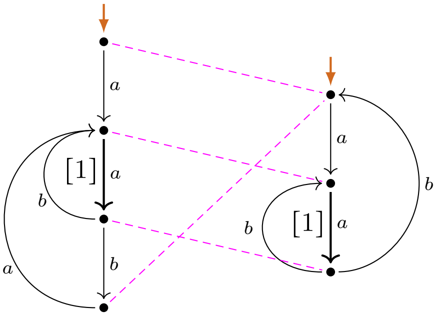

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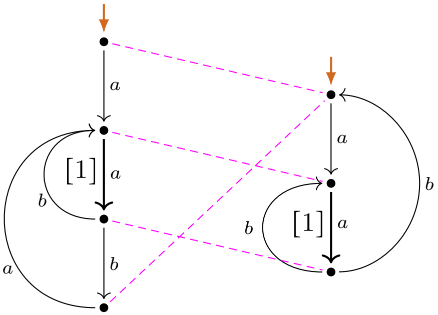

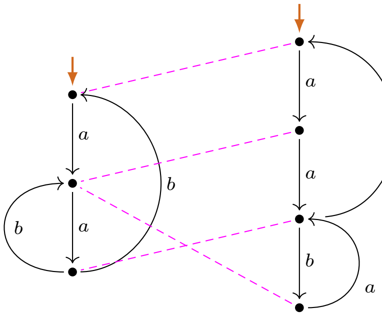

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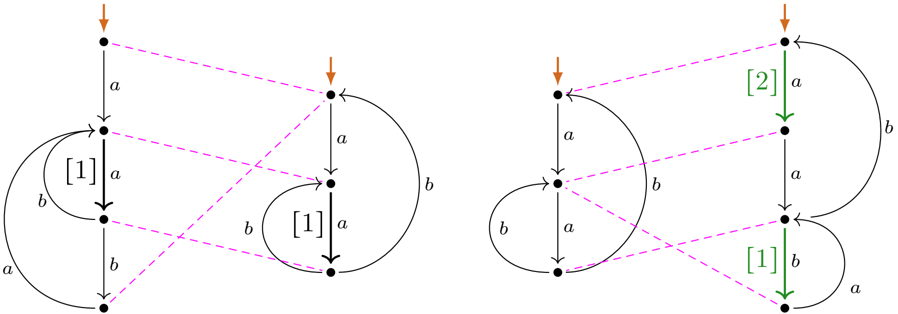

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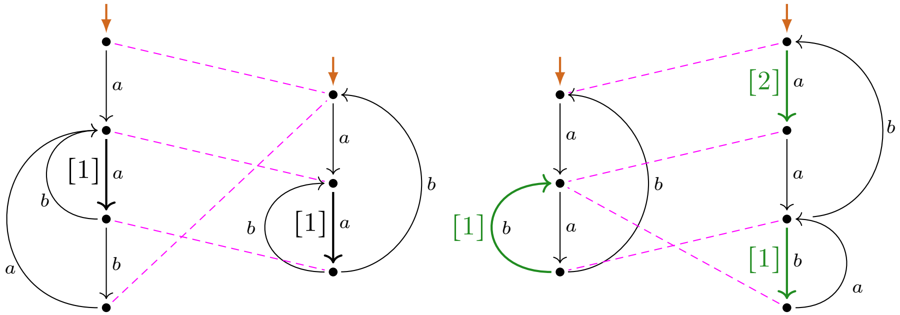

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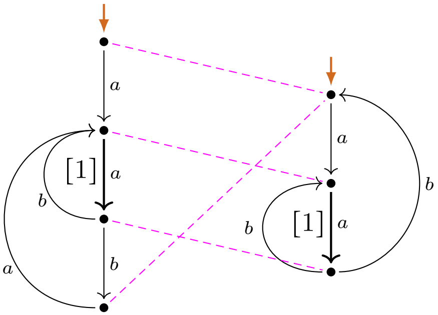

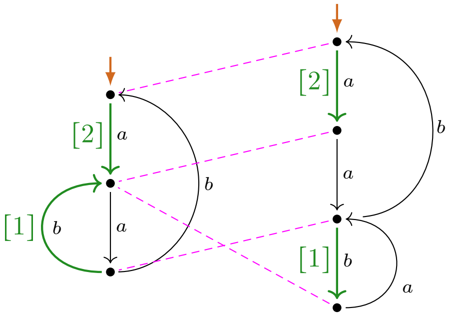

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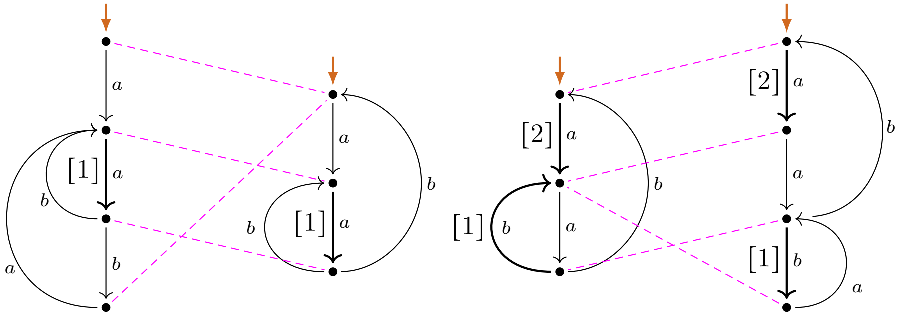

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LEE under functional bisimulation / bisimulation collapse

Lemma

(i) LEE *is preserved by functional bisimulations:*

$$\mathsf{LEE}(G_1) \wedge G_1 \mathrel{\underline{\to}} G_2 \implies \mathsf{LEE}(G_2)\,.$$

(ii) LEE *is preserved from a process graph to its bisimulation collapse:*

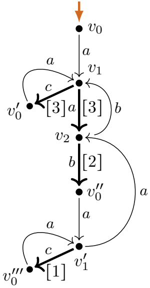
$$\mathsf{LEE}(G) \wedge G \textit{ has bisimulation collapse } C \implies \mathsf{LEE}(C)\,.$$

Idea of Proof for (i)

Use loop elimination in G_1 to carry out loop elimination in G_2.

LLEE-preserving collapse of LLEE-charts (G/Fokkink, LICS'20)

(no 1-transitions!)



(C1.1)

Lemma

The bisimulation collapse of a LLEE-chart is again a LLEE-chart.

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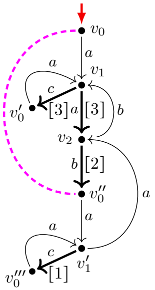

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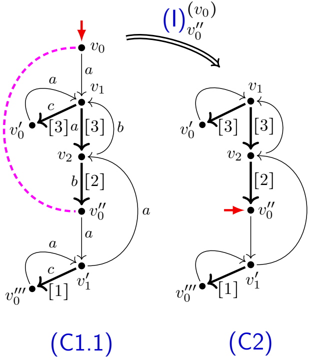

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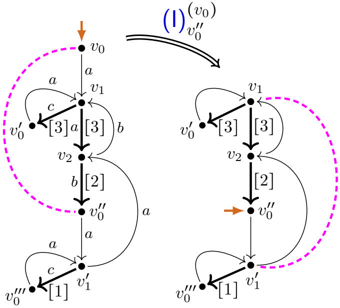

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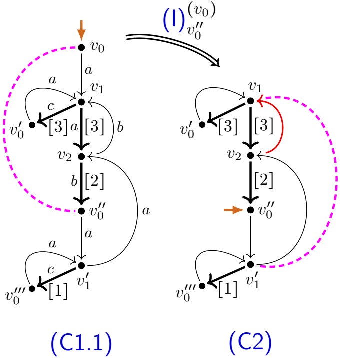

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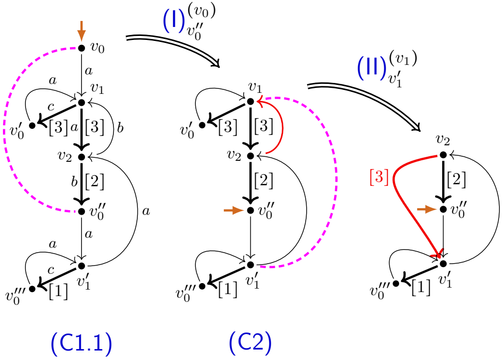

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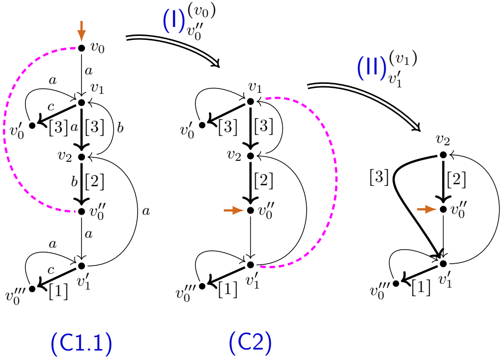

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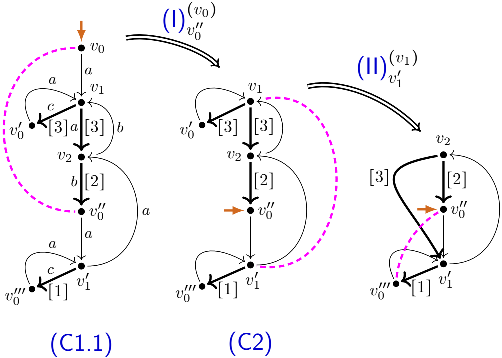

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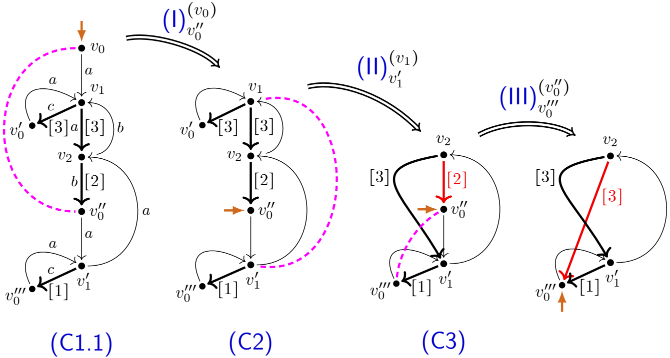

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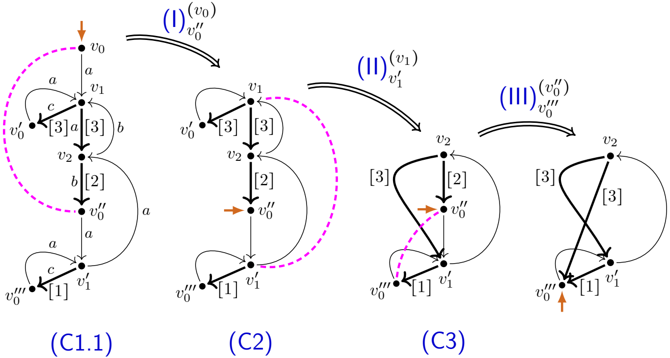

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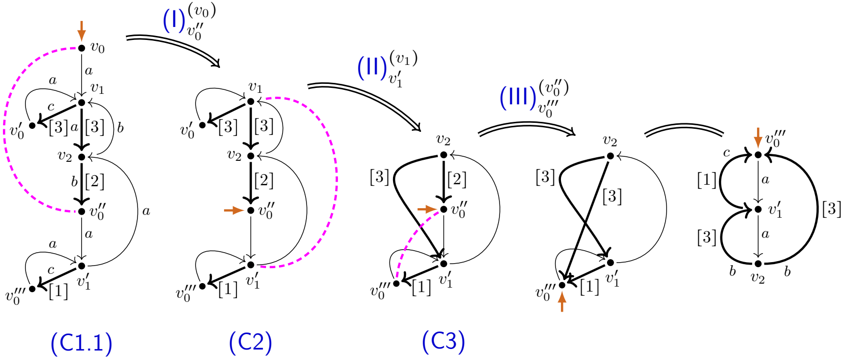


Lemma

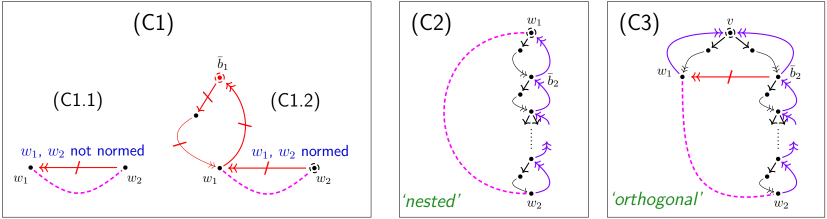
The bisimulation collapse of a LLEE-chart is again a LLEE-chart.

Reduced bisimilarity redundancies in LLEE-graphs (no 1-trans.!)

(G/Fokkink, LICS'20)

w_1, w_2 in different scc's

w_1, w_2 in the same scc



Lemma

Every not collapsed LLEE-graph contains bisimilar vertices $w_1 \neq w_2$ of kind (C1), (C2), or (C3) (a reduced bisimilarity redundancy $\langle w_1, w_2 \rangle$):

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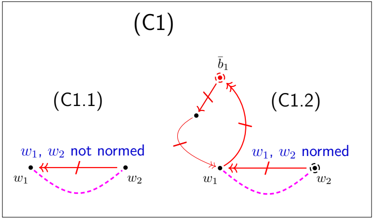

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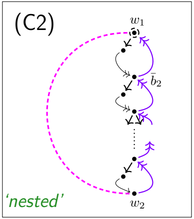

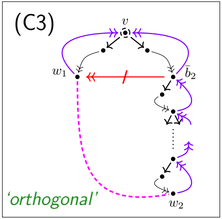

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Lemma

Every reduced bisimilarity redundancy in a LLEE-graph can be eliminated LLEE-preservingly.

Properties of LEE-charts

Theorem ($\Leftarrow$ G/Fokkink, 2020)

A process graph G
is $[\![\cdot]\!]_P$-expressible by an under-star-1-free regular expression
(i.e. P-expressible modulo bisimilarity by an $(\cancel{1}\backslash *)$ reg. expr.)
if and only if
the bisimulation collapse of G satisfies LEE.

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if and only if
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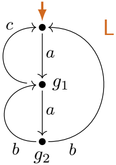
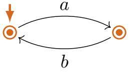
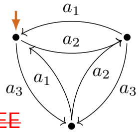
Hence $[\![\cdot]\!]_P$-expressible | **not** $[\![\cdot]\!]_P$-expressible by 1-free regular expressions:



LEE



~~LEE~~



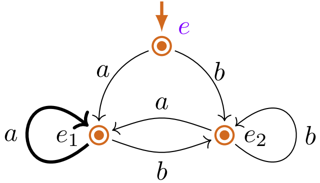
~~LEE~~

1-LEE

$\hat{=}$ sharing via 1-transitions facilitates LEE

Failure of LEE in general (example)

$P((a^* \cdot b^*)^*)$



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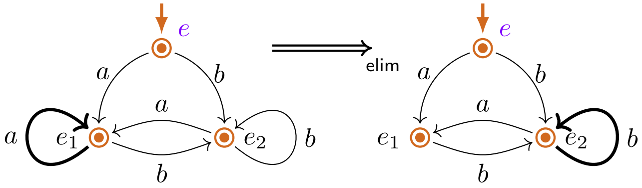

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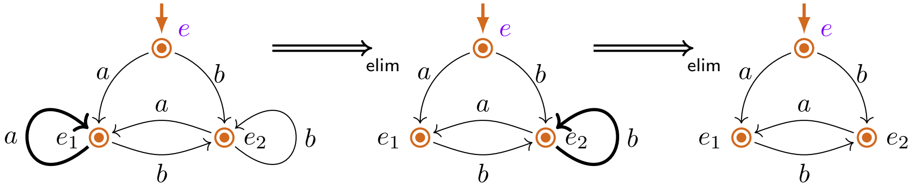

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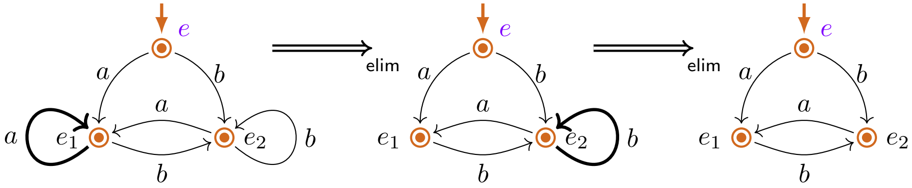

no loop subchart,
but infinite paths

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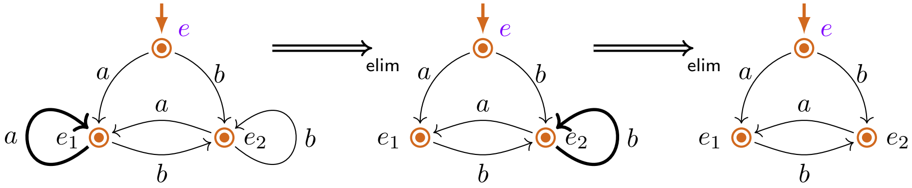

~~LEE~~

no loop subchart,
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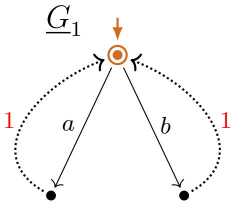
1-Graphs and induced graphs

Definition

$\xrightarrow{1} \cdot \cdots \cdot \xrightarrow{1} \cdot \xrightarrow{a} \quad \widehat{=} \quad \xrightarrow{(a]}$ induced a-transitions, for $a \in A$

$\xrightarrow{1} \cdot \cdots \cdot \xrightarrow{1} \cdot \Downarrow \quad \widehat{=} \quad \Downarrow^{(1)}$ induced termination.



1-Graphs and induced graphs

Definition

$v_1 \xrightarrow{1} \cdot \ \cdots \ \cdot \xrightarrow{1} \cdot \xrightarrow{a} v_2 \quad \widehat{=} \quad v_1 \xrightarrow{(a]} v_2$ induced a-transitions, for $a \in A$

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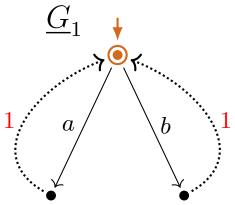

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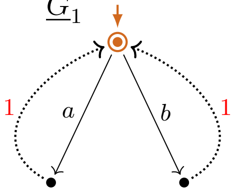
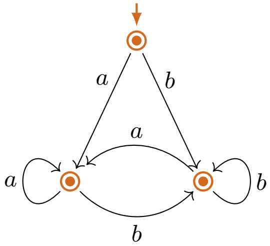
$\underline{G}_1$





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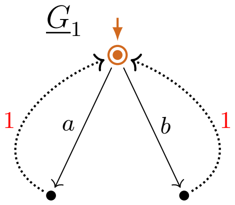

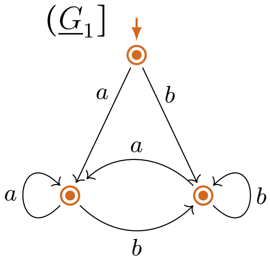

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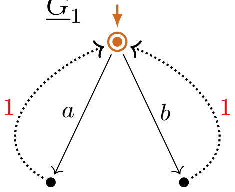
Definition

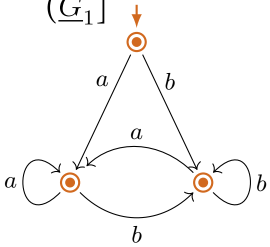
The induced (process) graph of a 1-graph $\underline{G} = \langle V, A, 1, v_s, \rightarrow, \Downarrow \rangle$ is:

$$(\underline{G}] = \langle V, A, v_s, \xrightarrow{(\cdot]}, \Downarrow^{(1)} \rangle .$$

$\underline{G}_1$



$(\underline{G}_1]$



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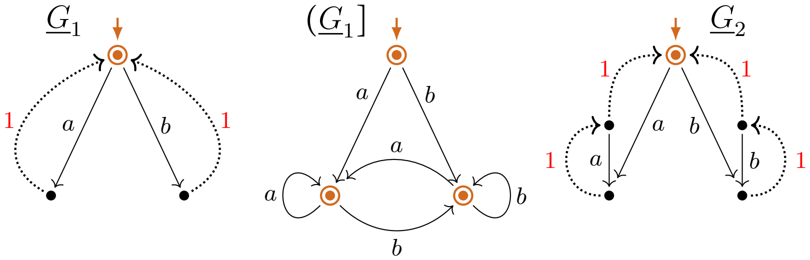

1-Graphs and induced graphs

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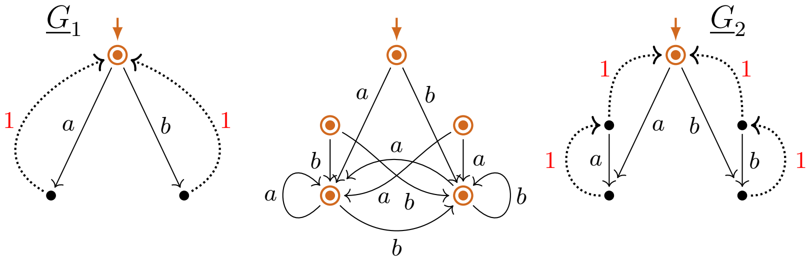
$$v_1 \xrightarrow{1} \cdot \;\cdots\; \cdot \xrightarrow{1} \cdot \xrightarrow{a} v_2 \;\;\widehat{=}\;\; v_1 \xrightarrow{(a]} v_2$$ induced a-transitions, for $a \in A$

$$v \xrightarrow{1} \cdot \;\cdots\; \cdot \xrightarrow{1} \cdot \Downarrow \;\;\widehat{=}\;\; v{\Downarrow}^{(1)}$$ induced termination.

Definition

The induced (process) graph of a 1-graph $\underline{G} = \langle V, A, 1, v_s, \rightarrow, \Downarrow \rangle$ is:

$$(\underline{G}] = \langle V, A, v_s, \xrightarrow{(\cdot]}, {\Downarrow}^{(1)} \rangle \,.$$



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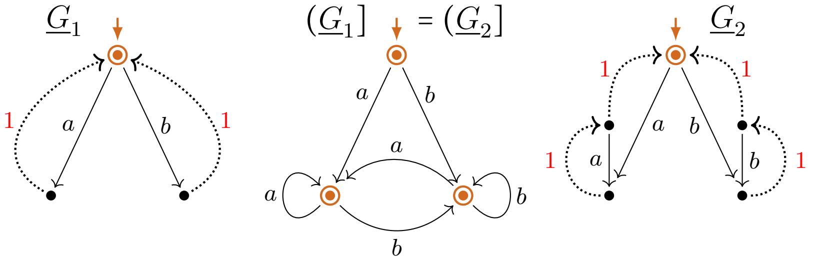

1-LEE

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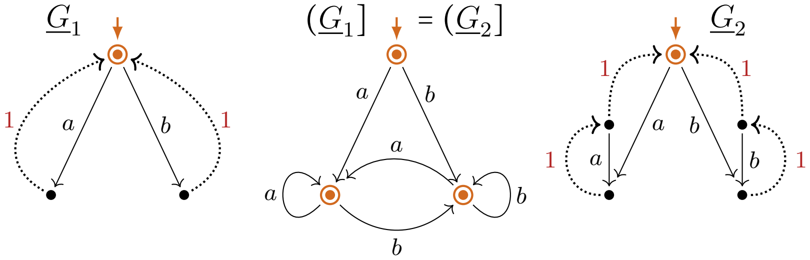

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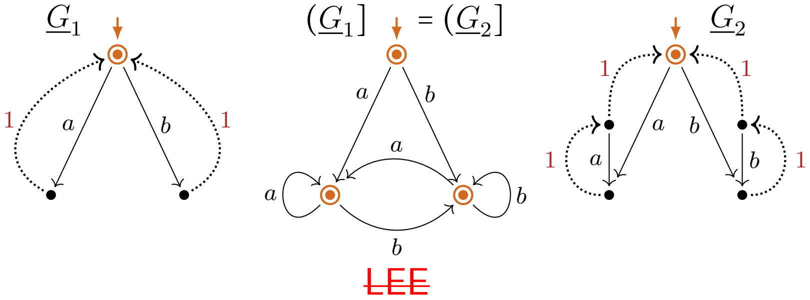

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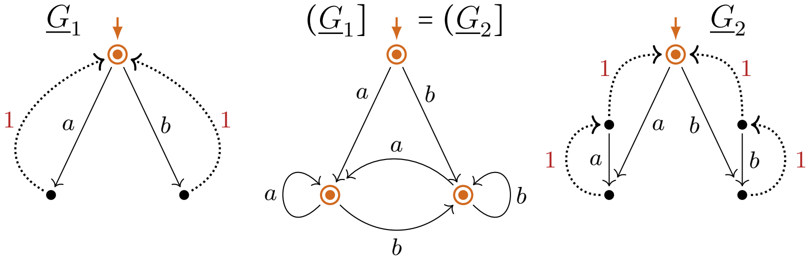

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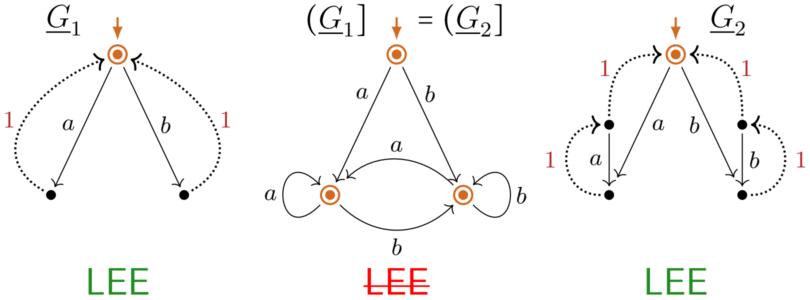

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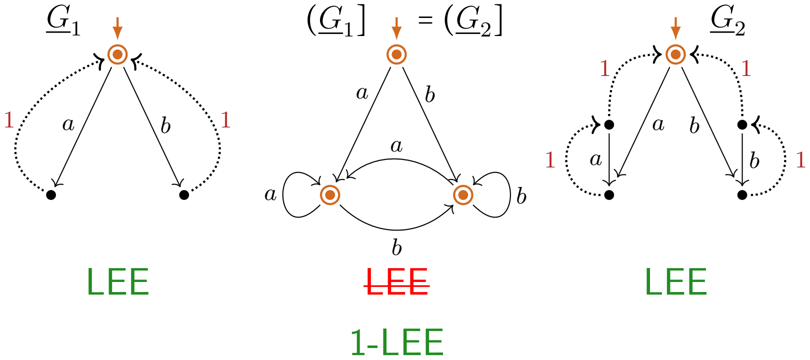
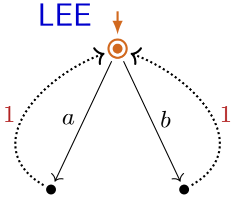
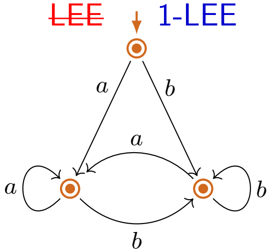


1-LEE holds for process interpretations





$$P((a^* \cdot b^*)^*)$$

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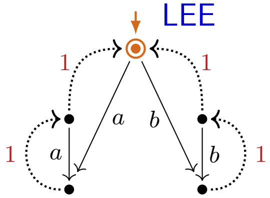
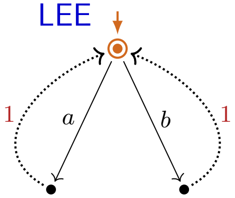
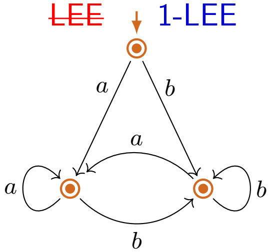


1-LEE holds for process interpretations

Lemma

There is a 1-graph interpretation $\underline{P}$ of reg. expression e as 1-graphs $\underline{P}(e)$ such that for all $e \in RExp$: (i): LEE($\underline{P}(e)$), (ii): $(\underline{P}(e)] = P(e)$.





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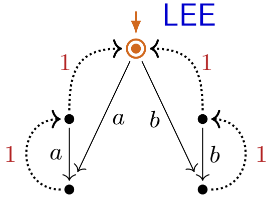

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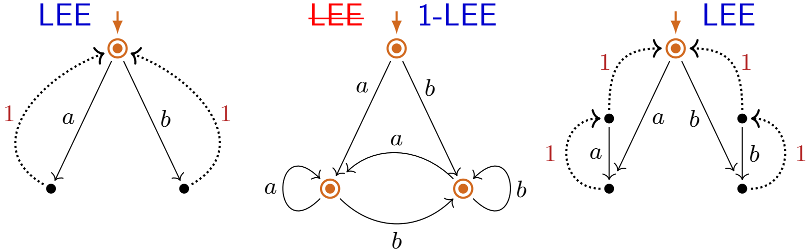

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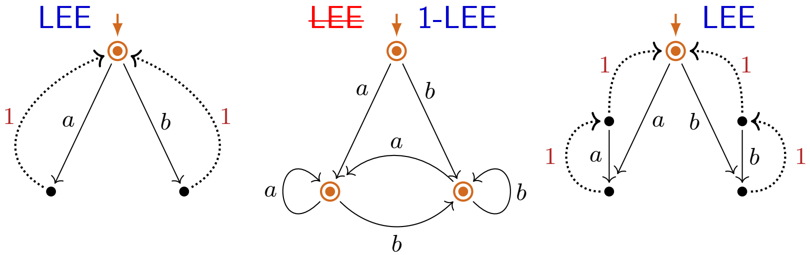
Theorem

$\text{1-LEE}(P(e))$ holds for all regular expressions e.



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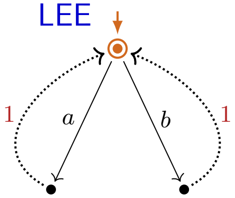
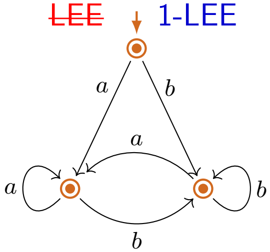
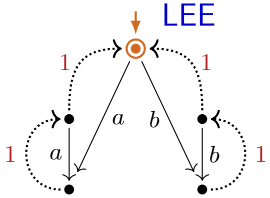
Theorem

1-LEE($P(e)$) holds for all regular expressions e.







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Interpretation/extraction correspondences with 1-LEE

($\Leftarrow$ G 2021/22/23)

(Int)$_P$: *P-expressible graphs have the structural property* 1-LEE

Process **int**erpretations $P(e)$ of regular expressions e
are finite process graphs that satisfy 1-LEE.

(Extr)$_P$: 1-LEE *implies* $[\![\cdot]\!]_P$*-expressibility*

From every finite 1-process-graph $\underline{G}$ with 1-LEE
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~~**(Coll)**~~: 1-LEE *is not preserved under collapse*

The class of finite process graphs with 1-LEE
is not closed under bisimulation **coll**apse.

Interpretation/extraction correspondences of $P^{\bullet}$ with 1-LEE

(Int)$_{P^{\bullet}}$: *$P^{\bullet}$-expressible graphs satisfy* 1-LEE:

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From every finite process graph G with 1-LEE
an regular expression uf can be **e**xtracted
such that $G \rightrightarrows P^{\bullet}(uf)$.

From every finite collapsed process graph G with 1-LEE
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~~**(ImColl)**$_{P^{\bullet}}$~~: The image of $P^{\bullet}$ is not closed under bisimulation **c**ollapse .

1-LEE/ LEE characterize

the un-/restricted image of $P^{\bullet}$

Image of P is **not** closed under bisimulation collapse

$P(uf)$

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$$uf := a \cdot \overbrace{(a \cdot (a + a \cdot 0))^*}^{uf_a} + b \cdot \overbrace{(b \cdot (b + b \cdot 0))^*}^{uf_b}$$

Image of P is **not** closed under bisimulation collapse

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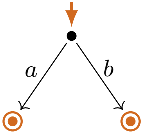

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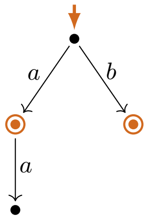

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$$1 \cdot uf_a \xrightarrow{a} (1 \cdot (a + a \cdot 0)) \cdot uf_a$$

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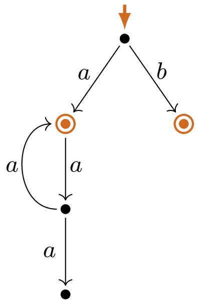

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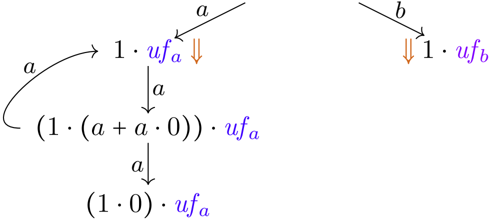

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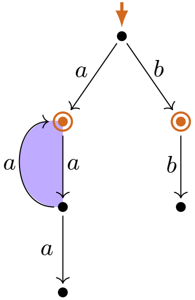

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$$\begin{tikzcd}
 & uf \arrow[dl, "a"'] \arrow[dr, "b"] & \\
1 \cdot uf_a \Downarrow \arrow[d, "a"] & & \Downarrow 1 \cdot uf_b \\
(1 \cdot (a + a \cdot 0)) \cdot uf_a \arrow[u, bend left, "a"] \arrow[d, "a"] & & \\
(1 \cdot 0) \cdot uf_a & &
\end{tikzcd}$$

Image of P is **not** closed under bisimulation collapse

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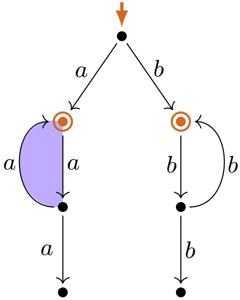

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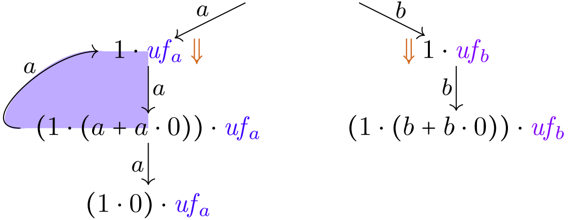

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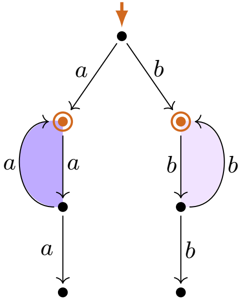

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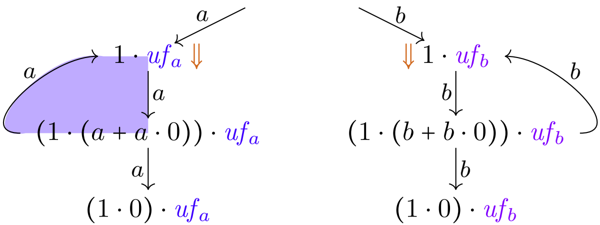

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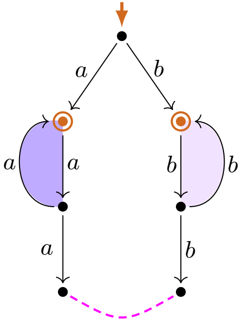

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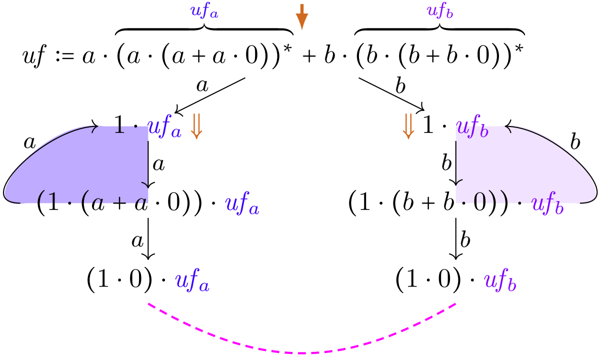

Compact process interpretation $P^{\bullet}$

Definition (Transition system specification $\mathcal{T}$)

$$\frac{}{\mathbf{1}{\Downarrow}} \qquad \frac{e_i{\Downarrow}}{(e_1 + e_2){\Downarrow}}\,(i \in \{1,2\}) \qquad \frac{e_1{\Downarrow} \qquad e_2{\Downarrow}}{(e_1 \cdot e_2){\Downarrow}} \qquad \frac{}{(e^*){\Downarrow}}$$

$$\frac{}{a \xrightarrow{a} \mathbf{1}} \qquad \frac{e_i \xrightarrow{a} e_i'}{e_1 + e_2 \xrightarrow{a} e_i'}\,(i \in \{1,2\})$$

$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2} \qquad \frac{e_1{\Downarrow} \qquad e_2 \xrightarrow{a} e_2'}{e_1 \cdot e_2 \xrightarrow{a} e_2'} \qquad \frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e' \cdot e^*}$$

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Definition (Transition system specification $\mathcal{T}^{\bullet}$, changed rules w.r.t. $\mathcal{T}$)

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The compact process (graph) interpretation $P^{\bullet}(e)$ of a reg. expr's e:

$P^{\bullet}(e)$:= labeled transition graph generated by e by derivations in $\mathcal{T}^{\bullet}$.

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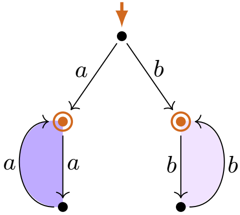
Lemma ($P^{\bullet}$ increases sharing; $P^{\bullet}$, P have same bisimulation semantics)

(i) $P(e) \mathrel{\underline{\to}} P^{\bullet}(e)$ *for all regular expressions* e.

(ii) *(*G *is* $[\![\cdot]\!]_{P^{\bullet}}$*-expressible* $\iff$ G *is* $[\![\cdot]\!]_{P}$*-expressible) for all graphs* G.

Image of $P^{\bullet}$ under bisimulation collapse ...

$P^{\bullet}(uf)$



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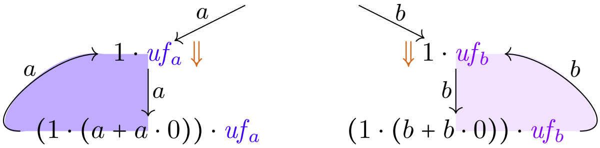

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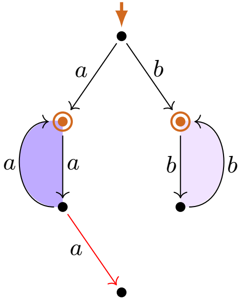

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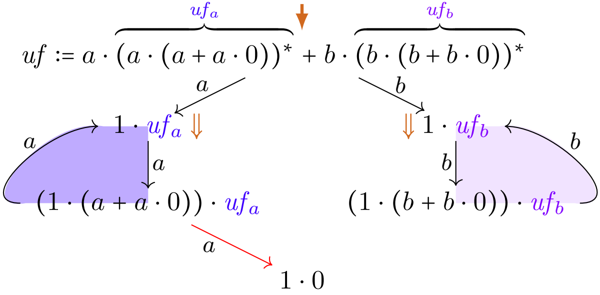

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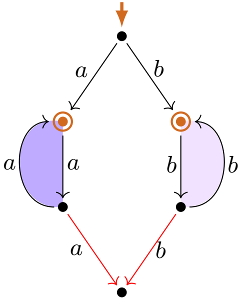
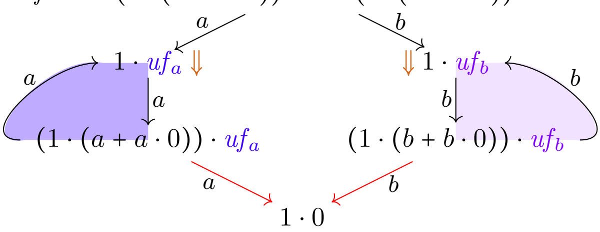


$P^\bullet(uf)$

$$uf := a \cdot \overbrace{(a \cdot (a + a \cdot 0))^*}^{uf_a} + b \cdot \overbrace{(b \cdot (b + b \cdot 0))^*}^{uf_b}$$



Interpretation correspondence of $P^{\bullet}$ with LEE

$(\textbf{Int})^{(*/\cancel{1})}_{P^{\bullet}}$: *By under-star-1-free expressions $P^{\bullet}$-expressible graphs satisfy* LEE:

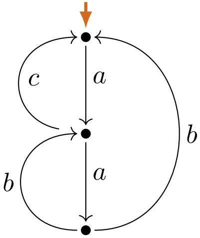
Compact process **i**nterpretations $P^{\bullet}(uf)$
of under-star-1-free regular expressions uf
are finite process graphs that satisfy LEE.

$(\textbf{Extr})^{(*/\cancel{1})}_{P^{\bullet}}$: LEE *implies $[\![\cdot]\!]_P$-expressibility by under-star-1-free reg. expr's*:

From every finite process graph G with LEE
an under-star-1-free regular expression uf can be **e**xtracted
such that $G \Rightarrow P(uf)$.

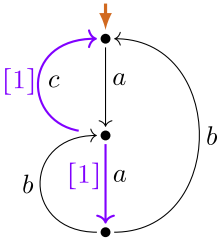
Refined extraction expression (example)

G_4



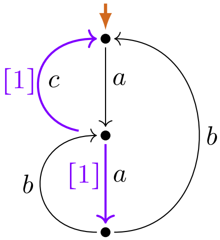
Refined extraction expression (example)

$\widehat{G_4}$



Refined extraction expression (example)

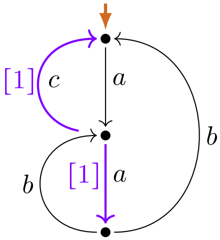
$\widehat{G_4}$



$$(1 \cdot (\qquad\qquad\qquad)^*) \cdot 0$$

Refined extraction expression (example)

$\widehat{G_4}$

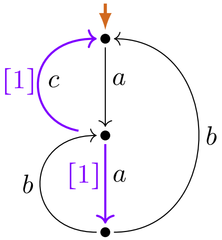


a

$(1 \cdot (\qquad)^*) \cdot 0$

Refined extraction expression (example)

$\widehat{G_4}$



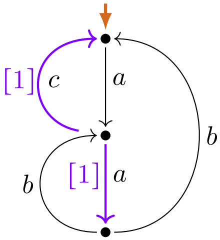
$$((1 \cdot a) \cdot (\qquad\qquad)^{*}) \cdot 0$$

$$\downarrow a$$

$$(1 \cdot (\qquad\qquad)^{*}) \cdot 0$$

Refined extraction expression (example)

$\widehat{G_4}$



$$((1 \cdot a) \cdot (\qquad\qquad)^{*}) \cdot 0$$

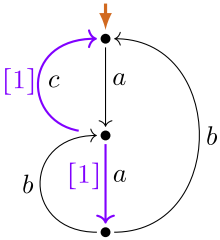
$$\downarrow a$$

$$(1 \cdot (\qquad\qquad)^{*}) \cdot 0$$

b $\qquad$ b

Refined extraction expression (example)

$\widehat{G_4}$



$$((1 \cdot a) \cdot (\qquad)^*) \cdot 0$$

$$\downarrow a$$

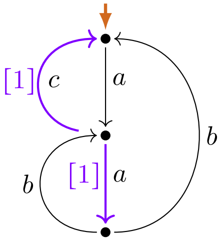
$$(1 \cdot (\qquad)^*) \cdot 0$$

$$b \qquad b$$

$$((1 \cdot (b + b \cdot a)) \cdot (\qquad)^*) \cdot 0$$

Refined extraction expression (example)

$\widehat{G_4}$



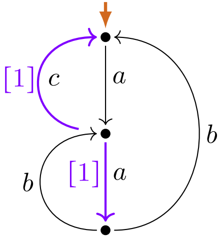
$$((1 \cdot a) \cdot (\qquad)^*) \cdot 0$$

$$(1 \cdot (\qquad)^*) \cdot 0$$

$$((1 \cdot (b + b \cdot a)) \cdot (\qquad)^*) \cdot 0$$

Refined extraction expression (example)

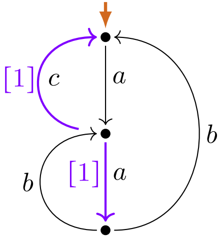
$\widehat{G_4}$



$$\begin{tikzcd}
((1 \cdot a) \cdot (\qquad\qquad)^*) \cdot 0 \arrow[d, dotted, "a"] \\
(1 \cdot (c \cdot a + a \cdot (b + b \cdot a))^*) \cdot 0 \arrow[u, dotted, bend left, "c"] \arrow[d, dotted, "a"] \\
((1 \cdot (b + b \cdot a)) \cdot (\qquad\qquad)^*) \cdot 0 \arrow[u, dotted, bend left, "b"] \arrow[uu, dotted, bend right, "b"']
\end{tikzcd}$$

Refined extraction expression (example)

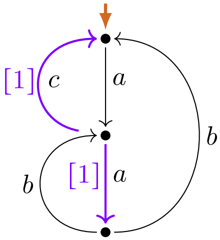
$\widehat{G_4}$



$$
\begin{tikzcd}
((1\cdot a)\cdot(c\cdot a+a\cdot(b+b\cdot a))^*)\cdot 0 \arrow[d, dotted, "a"] \\
(1\cdot(c\cdot a+a\cdot(b+b\cdot a))^*)\cdot 0 \arrow[d, dotted, "a"] \arrow[u, dotted, bend left, "c"] \\
((1\cdot(b+b\cdot a))\cdot(c\cdot a+a\cdot(b+b\cdot a))^*)\cdot 0 \arrow[u, dotted, bend left, "b"] \arrow[uu, dotted, bend right, "b"']
\end{tikzcd}
$$

Refined extraction expression (example)

$\widehat{G_4}$

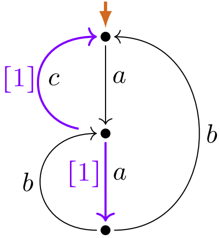


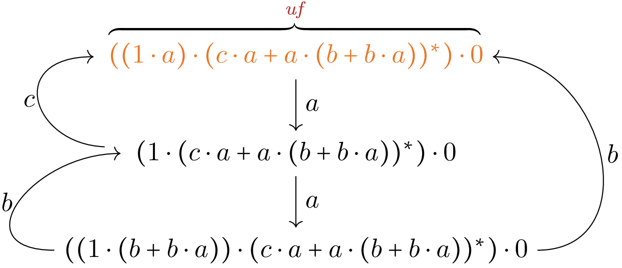
$$\begin{tikzcd}
((1\cdot a)\cdot(c\cdot a+a\cdot(b+b\cdot a))^{*})\cdot 0 \arrow[d, "a"] \\
(1\cdot(c\cdot a+a\cdot(b+b\cdot a))^{*})\cdot 0 \arrow[d, "a"] \arrow[u, bend left=60, "c"] \\
((1\cdot(b+b\cdot a))\cdot(c\cdot a+a\cdot(b+b\cdot a))^{*})\cdot 0 \arrow[u, bend left=60, "b"] \arrow[uu, bend right=60, "b"']
\end{tikzcd}$$

Refined extraction expression (example)

$\widehat{G_4}$

$P^\bullet(uf) = P(uf) \simeq G_4$





Interpretation/extraction correspondences of $P^{\bullet}$ with LEE

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From every finite process graph G with LEE
an under-star-1-free regular expression uf can be **e**xtracted
such that $G \mathrel{\underline{\to}} P^{\bullet}(uf)$.

From every finite collapsed process graph G with LEE
an under-star-1-free regular expression uf can be **e**xtracted
such that $G \simeq P^{\bullet}(uf)$.

Interpretation/extraction correspondences of $P^{\bullet}$ with LEE

(Int)$_{P^{\bullet}}^{(*/\cancel{1})}$: *By under-star-1-free expressions $P^{\bullet}$-expressible graphs satisfy* LEE:

Compact process **i**nterpretations $P^{\bullet}(uf)$
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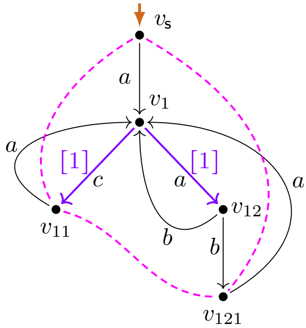
such that $G \Rightarrow P^{\bullet}(uf)$.

From every finite collapsed process graph G with LEE
an under-star-1-free regular expression uf can be **e**xtracted
such that $G \simeq P^{\bullet}(uf)$.

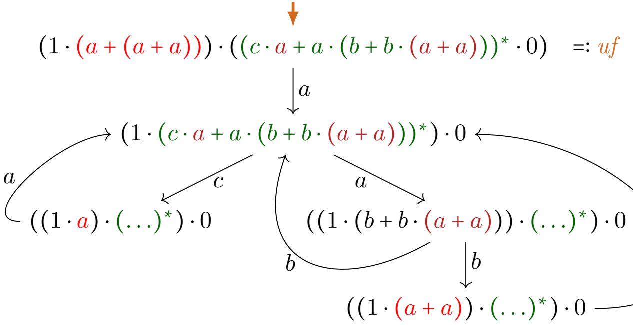
(ImColl)$_{P^{\bullet}}^{(*/\cancel{1})}$: The image of $P^{\bullet}$,
restricted to under-star-1-free regular expressions,
is closed under bisimulation **c**ollapse.

Adapted (refined) extraction from LLEE-graph

$G_1 \,/\, \widehat{G_1}$



$P^\bullet(uf) = P(uf) \simeq G_1$



LEE $\triangleq$ image of $P^{\bullet}|_{RExp^{(*/\cancel{1})}}$

Theorem

For every process graph G TFAE:

(i) LEE(G).

LEE $\triangleq$ image of $P^{\bullet}|_{RExp^{(*/\cancel{1})}}$

Theorem

For every process graph G TFAE:

(i) $\mathsf{LEE}(G)$.

(ii) G *is* $P^{\bullet}$*-expressible by an* $(*/\cancel{1})$ *regular expression*
(i.e. $G \simeq P^{\bullet}(e)$ *for some* $e \in RExp^{(*/\cancel{1})}$*).*

LEE $\triangleq$ image of $P^{\bullet}|_{RExp^{(*/\cancel{1})}}$

Theorem

For every process graph G TFAE:

(i) LEE(G).

(ii) G is $P^{\bullet}$-expressible by an $(*/\cancel{1})$ regular expression (i.e. $G \simeq P^{\bullet}(e)$ for some $e \in RExp^{(*/\cancel{1})}$).

(iii) G is isomorphic to a graph in the image of $P^{\bullet}$ on $(*/\cancel{1})$ reg. expr's (i.e. $G \simeq G'$ for some $G' \in im(P^{\bullet}|_{RExp^{(*/\cancel{1})}})$).

1-LEE $\hat{=}$ image of $P^{\bullet}$

Theorem

For every process graph G TFAE:

(i) 1-LEE(G)

(i.e. $G = (\underline{G}]$ for some 1-transition-process-graph $\underline{G}$ with LEE($\underline{G}$)*).*

1-LEE $\triangleq$ image of $P^{\bullet}$

Theorem

For every process graph G TFAE:

(i) 1-LEE(G)
(i.e. $G = (\underline{G}]$ for some 1-transition-process-graph $\underline{G}$ with LEE($\underline{G}$)*).*

(ii) G *is* $P^{\bullet}$*-expressible by a regular expression*
(i.e. $G \simeq P^{\bullet}(e)$ for some $e \in \mathit{RExp}$).

1-LEE $\triangleq$ image of $P^{\bullet}$

Theorem

For every process graph G TFAE:

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(i.e. $G = (\underline{G}]$ for some 1-transition-process-graph $\underline{G}$ with LEE($\underline{G}$)*).*

(ii) G is $P^{\bullet}$-expressible by a regular expression
(i.e. $G \simeq P^{\bullet}(e)$ for some $e \in \mathit{RExp}$).

(iii) G is isomorphic to a graph in the image of $P^{\bullet}$
(i.e. $G \simeq G'$ for some $G' \in \mathit{im}(P^{\bullet})$).

Summary

- process interpretation P/semantics $[\![\cdot]\!]_P$ of regular expressions
  - expressibility and completeness questions
- loop existence and elimination (LEE)
  - loop elimination rewrite system can be completed
  - interpretation/extraction correspondences with $(*/\cancel{1})$ reg. expr.s
  - LEE-witnesses: labelings of graphs with LEE
  - stepwise LEE-preserving bisimulation collapse
- 1-LEE = sharing via 1-transitions facilitates LEE
  - interpretation/extraction correspondences with all regular expressions
  - not preserved under bisim. collapse (approximation possible)
- Characterizations of the image of $P^{\bullet}$ (refinement of P):
  - LEE $\triangleq$ image of $P^{\bullet}|_{RExp^{(*/\cancel{1})}}$ $\supsetneqq$ image of $P|_{RExp^{(*/\cancel{1})}}$
  - 1-LEE $\triangleq$ image of $P^{\bullet}$ $\supsetneqq$ image of P
- outlook on work-to-do

My next aims

Completeness problem, solution (journal articles):

A1: graph structure of regular expression processes (LEE/1-LEE)

A2: motivation of crystallization

A4: details of crystallization procedure,
and completeness of Milner's proof system

Expressibility problem

A3: LEE is decidable in polynomial time (conference article).

Q: Is 1-LEE decidable in polynomial time**?**

P: Is expressibility by a regular expression, for a finite process graph,
decidable in polynomial time/fixed-parameter tractable time**?**

Resources

- Slides/abstract on `clegra.github.io`
  - slides: `.../lf/IFIP-1_6-2024.pdf`
  - abstract: `.../lf/abstract-IFIP-1_6-2024.pdf`
- CG: Closing the Image of the Process Interpretation of 1-Free Regular Expressions Under Bisimulation Collapse
  - TERMGRAPH 2024, extended abstract.
- CG: The Image of the Process Interpretation of Regular Expressions is Not Closed under Bisimulation Collapse,
  - arXiv:2303.08553, 2021/2023.
- CG: Milner's Proof System for Regular Expressions Modulo Bisimilarity is Complete,
  - LICS 2022, arXiv:2209.12188, poster.
- CG, Wan Fokkink: A Complete Proof System for 1-Free Regular Expressions Modulo Bisimilarity,
  - LICS 2020, arXiv:2004.12740, video on youtube.
- CG: Modeling Terms by Graphs with Structure Constraints,
  - TERMGRAPH 2018, EPTCS 288, arXiv:1902.02010.

Language semantics $[\![\cdot]\!]_L$ of reg. expr's *(Copi–Elgot–Wright, 1958)*

$\mathbf{0} \xmapsto{L}$ empty language $\varnothing$

$\mathbf{1} \xmapsto{L} \{\epsilon\}$ (ϵ the empty word)

$a \xmapsto{L} \{a\}$

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$\mathbf{0} \xmapsto{L}$ empty language $\varnothing$

$\mathbf{1} \xmapsto{L} \{\epsilon\}$ (ϵ the empty word)

$a \xmapsto{L} \{a\}$

$e_1 + e_2 \xmapsto{L}$ union of $L(e_1)$ and $L(e_2)$

$e_1 \cdot e_2 \xmapsto{L}$ element-wise concatenation of $L(e_1)$ and $L(e_2)$

$e^* \xmapsto{L}$ set of words formed by concatenating words in $L(e)$, and adding the empty word ϵ

Language semantics $[\![\cdot]\!]_L$ of reg. expr's *(Copi–Elgot–Wright, 1958)*

$$\mathbf{0} \xmapsto{L} \text{empty language } \varnothing$$

$$\mathbf{1} \xmapsto{L} \{\epsilon\} \qquad (\epsilon \text{ the empty word})$$

$$a \xmapsto{L} \{a\}$$

$$e_1 + e_2 \xmapsto{L} \text{union of } L(e_1) \text{ and } L(e_2)$$

$$e_1 \cdot e_2 \xmapsto{L} \text{element-wise concatenation of } L(e_1) \text{ and } L(e_2)$$

$$e^* \xmapsto{L} \text{set of words formed by concatenating words in } L(e), \text{ and adding the empty word } \epsilon$$

$$[\![e]\!]_L := L(e) \quad (\text{language defined by } e)$$