Core Course
Introduction to Model Checking

https://clegra.github.io/mc/mc.html

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GS
SI
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SCIENCE INSTITUTE
SCHOOL OF ADVANCED STUDIES
Scuola Universitaria Superiore

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Model Checking

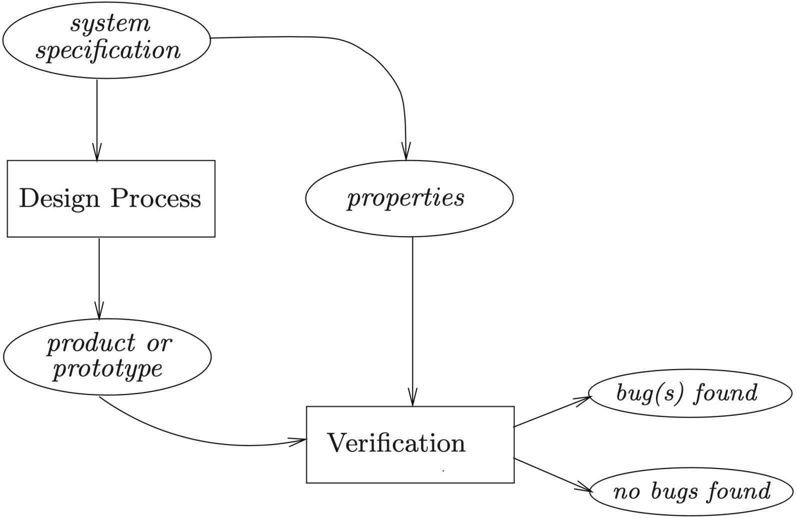
. . . is an effective automatable technique:

- *to expose potential software design errors;*
- *that, given a finite-state model of a system and a formal property, systematically checks whether this property holds for that model.*

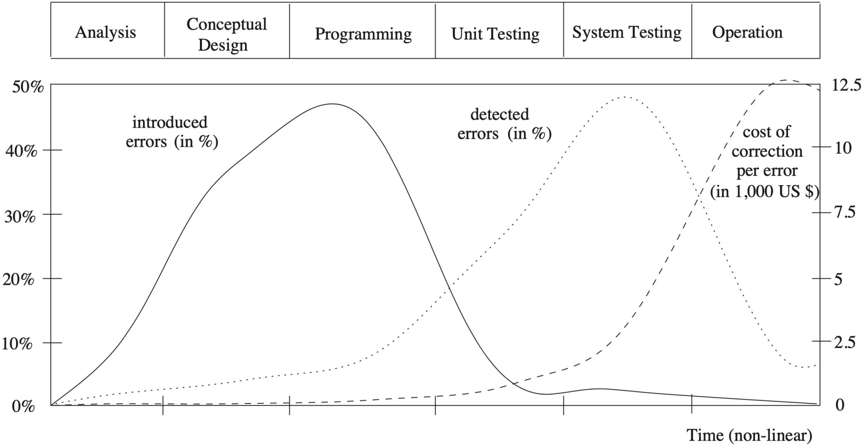
Strengths:

- widely applied in industry
  for: embedded systems, software engineering, hardware design, explainable AI
- supports partial verification (of system parts)
- provides diagnostic information for debugging
- has sound mathematical underpinning (logic and process theory)

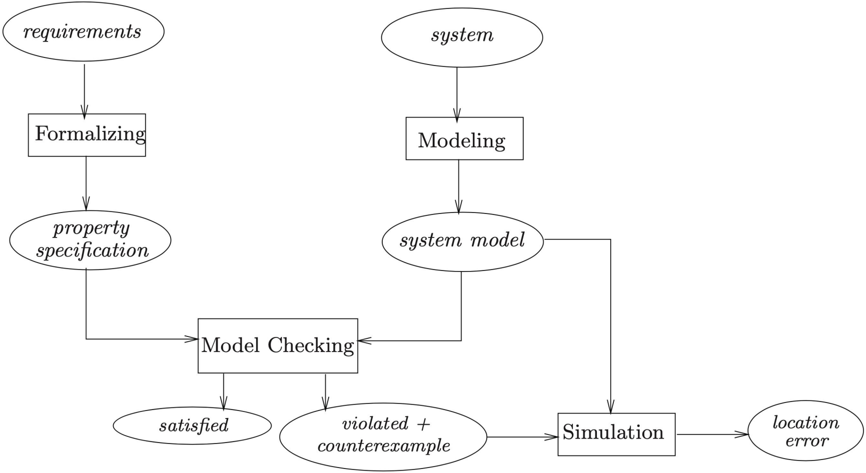
Hard-/Software Verification (traditionally)



Error introduction, detection, and repair costs



Model checking



Example (program concurrency/non-determinism)

Programs Inc, Dec, and Reset cooperate, and use a shared variable x:

```
proc Inc
  while true
    do
      if $x < 200$
        then $x := x + 1$
      fi
    od
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```
proc Dec
  while true
    do
      if $x > 0$
        then $x := x - 1$
      fi
    od
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proc Reset
  while true
    do
      if $x = 200$
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Question: Is $0 \leq x \leq 200$ always guaranteed?

Modeling (by labeled transition systems)

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Modeling (by labeled transition systems)

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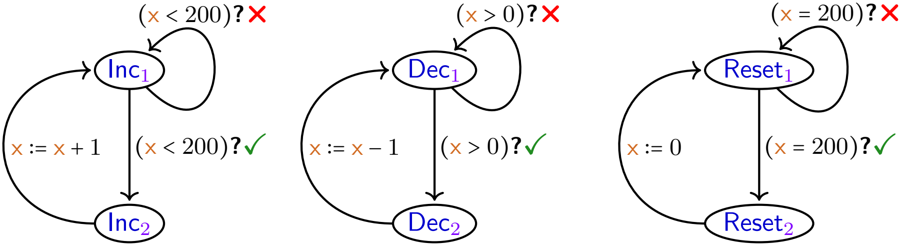
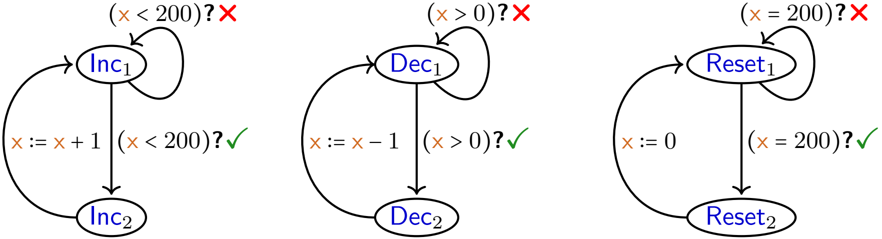


Labeled transition systems (LTSs)

Formalizing properties (in temporal logic)



$\mathsf{Inc}_1 \parallel \mathsf{Dec}_1 \parallel \mathsf{Reset}_1 \overset{?}{\vDash} \Box(0 \leq x \wedge x \leq 200)$ (Linear-TL formula)

Counterexample (offending execution trace)

$$\boxed{\left\langle x = 199\,;\ \mathsf{Inc}_1 \parallel \mathsf{Dec}_1 \parallel \mathsf{Reset}_1 \right\rangle}$$

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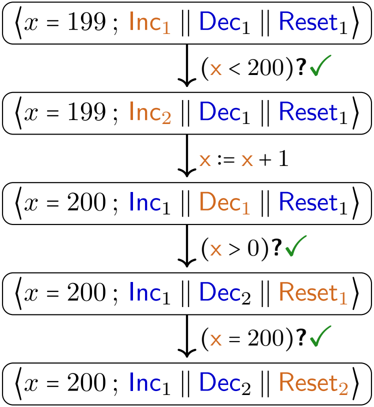

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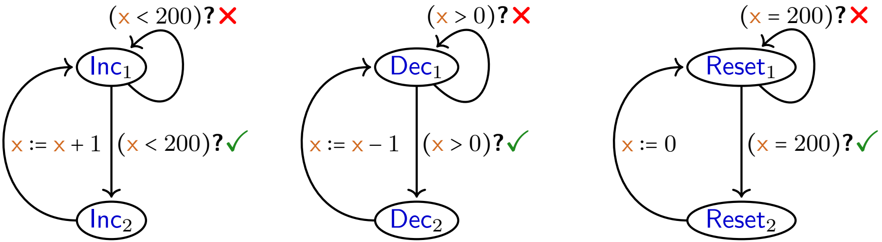
Formalizing properties (in temporal logic)



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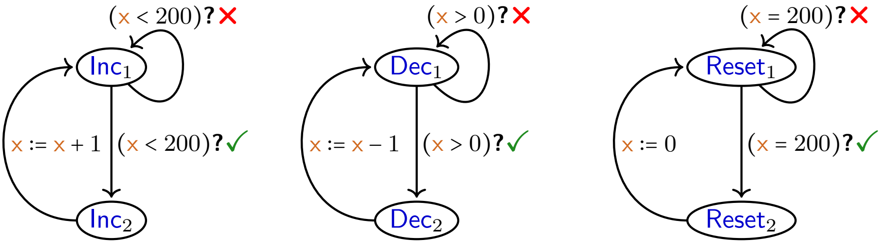

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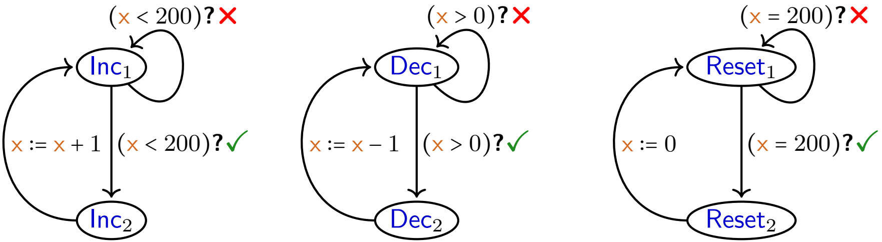

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$\mathsf{Inc}_1 \parallel \mathsf{Dec}_1 \parallel \mathsf{Reset}_1 \;\not\models\; \forall\Box(0 \leq x \,\wedge\, x \leq 200)$ (Computation-Tree-L formula)

$\mathsf{Inc}_1 \parallel \mathsf{Dec}_1 \parallel \mathsf{Reset}_1 \;\models\; \exists\Box(0 \leq x \,\wedge\, x \leq 200)$ (CTL formula)

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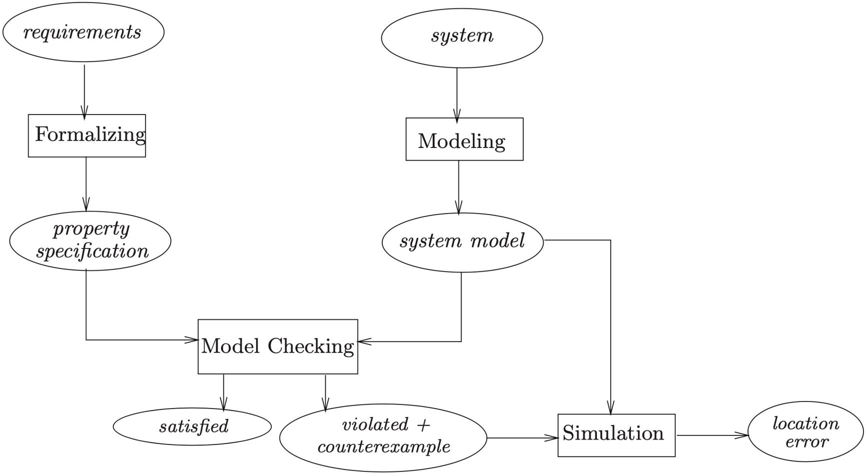
Model checking

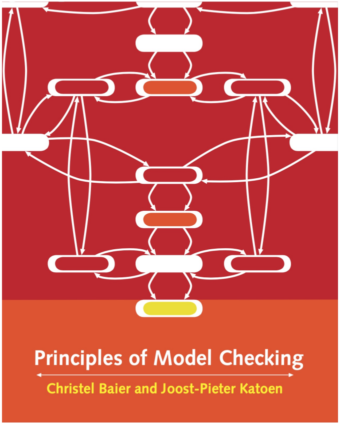


Any [such] verification is only as good as the model of the system.

Topics of the module

- modeling systems by labeled transition systems (LTSs)
- fairness
- Linear Temporal Logic (LTL)
  - model checking formulas
    - express properties by Büchi automata
    - model check LTSs and properties via product automata
- Computation Tree Logic (CTL)
- partial model checking
  - partially known systems (state properties/states/transitions)
- analysing system behavior with the mCRL2 model-checker toolbox

Book


Principles of Model Checking
Christel Baier and Joost-Pieter Katoen

- pdf available:
  https://is.ifmo.ru/books/_principles_of_model_checking.pdf

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Lectures (Emilio 2/Clemens 5)

- presentations on blackboard
- notes after the lecture (notes 2025/26 available)
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