The Graph Structure of Process Interpretations of Regular Expressions

Clemens Grabmayer

https://clegra.github.io

Department of Computer Science

GRAN SASSO SCIENCE INSTITUTE

SCHOOL OF ADVANCED STUDIES

Scuola Universitaria Superiore

L'Aquila, Italy

IFIP 1.6 Working Group Meeting

Nancy

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Overview

Process interpretation $P(\cdot)$ of regular expressions *(Milner, 1984)*

$\mathbf{0} \xmapsto{P}$ deadlock δ, no termination

$\mathbf{1} \xmapsto{P}$ empty-step process ϵ, then terminate

$a \xmapsto{P}$ atomic action a, then terminate

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$e_1 + e_2 \xmapsto{P}$ (*choice*) execute $P(e_1)$ or $P(e_2)$

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$e^* \xmapsto{P}$ (*iteration*) repeat (terminate or execute $P(e)$)

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$[\![e]\!]_P \;:=\; [P(e)]_{\underline{\leftrightarrow}}$ (bisimilarity equivalence class of process $P(e)$)

Process interpretation P (formal definition)

Definition (Transition system specification $\mathcal{T}$)

$$\frac{}{a \xrightarrow{a} 1} \qquad \frac{e_i \xrightarrow{a} e_i'}{e_1 + e_2 \xrightarrow{a} e_i'}\ (i \in \{1, 2\})$$

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Definition

The process (graph) interpretation $P(e)$ of a regular expression e:

$P(e)$:= labeled transition graph generated by e by derivations in $\mathcal{T}$.

Loop graphs (interpretations of innermost iterations without 1)

Definition

A process graph is a loop graph if:

(L1) There is an infinite path from the start vertex.

(L2) Every infinite path from the start vertex returns to it.

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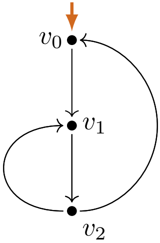

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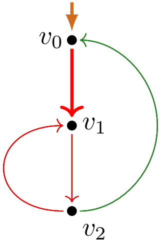

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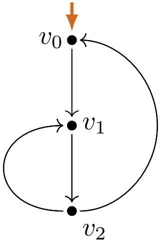

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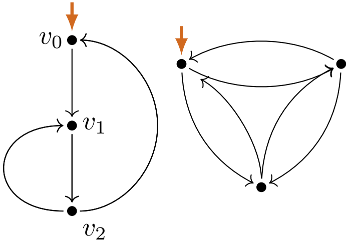

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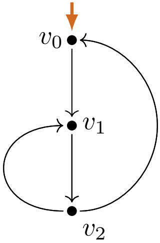

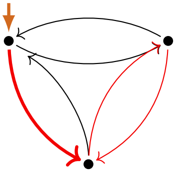

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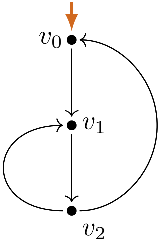

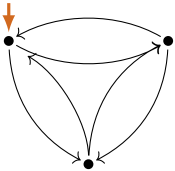

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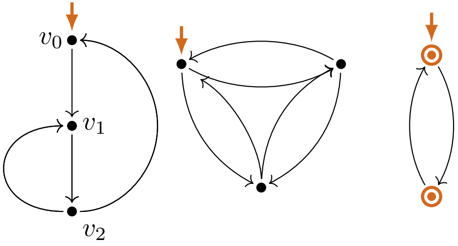

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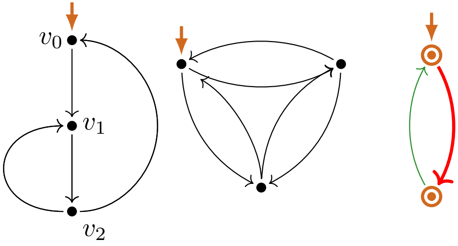

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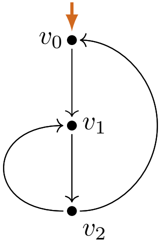

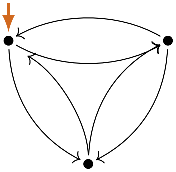

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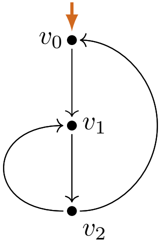

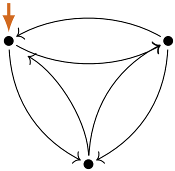

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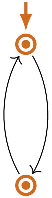

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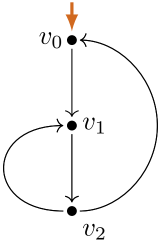

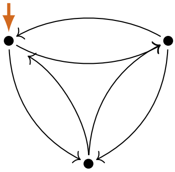

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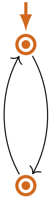

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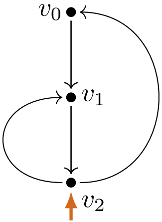

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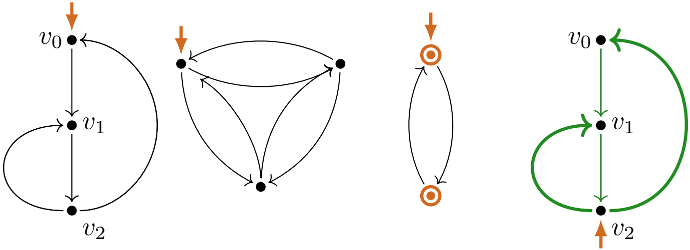

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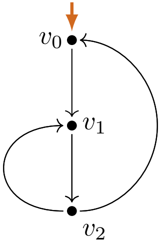

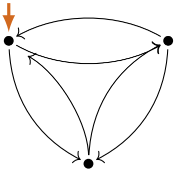

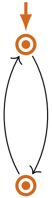

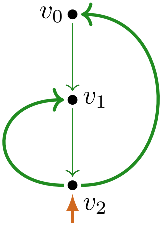

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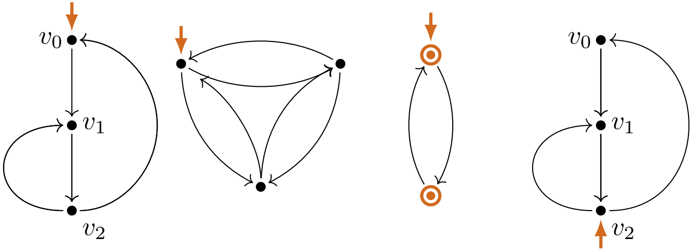

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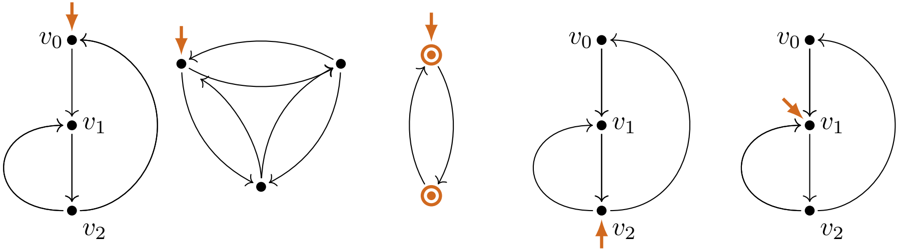

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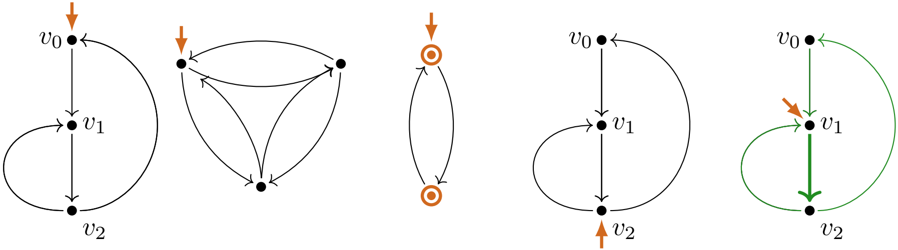

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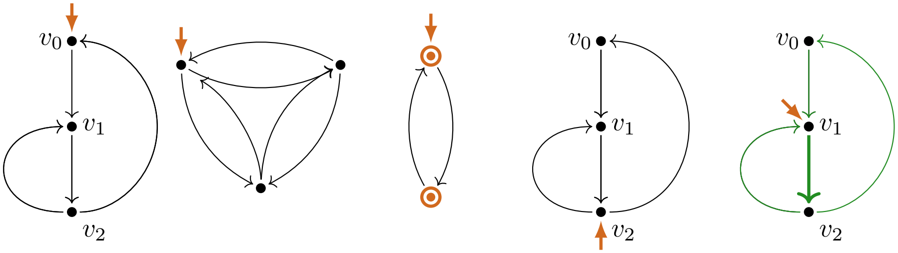

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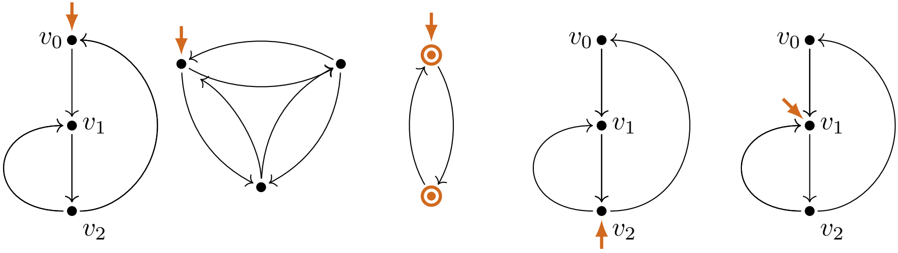

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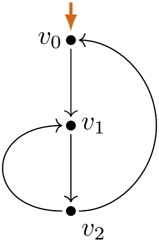

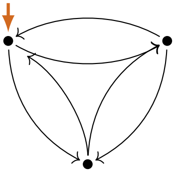

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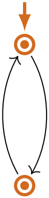

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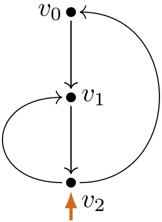

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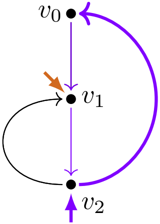

loop subchart

Loop elimination, and properties

$\longrightarrow_{\text{elim}}$: eliminate a transition-induced loop by:

- removing the loop-entry transition(s)
- garbage collection

$\longrightarrow_{\text{prune}}$: remove a transition to a deadlocking state

Lemma

(i) $\longrightarrow_{\text{elim}} \cup \longrightarrow_{\text{prune}}$ *is terminating.*

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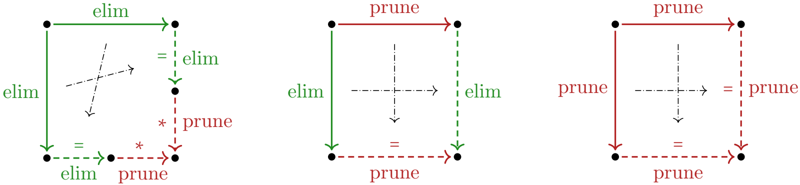
$\longrightarrow_{\text{prune}}$: remove a transition to a deadlocking state

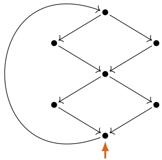
Lemma

(i) $\longrightarrow_{\text{elim}} \cup \longrightarrow_{\text{prune}}$ *is terminating.*

(ii) $\longrightarrow_{\text{elim}} \cup \longrightarrow_{\text{prune}}$ *is decreasing* [Van Oostrom, de Bruijn]



'Critical pair': bi-loop elimination



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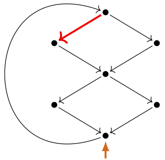

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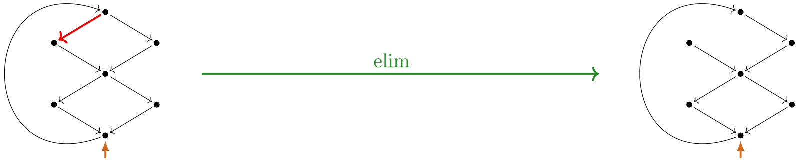

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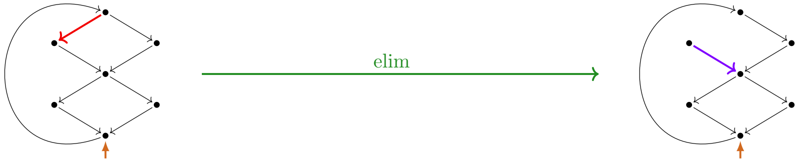

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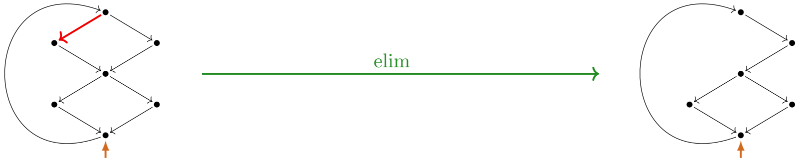

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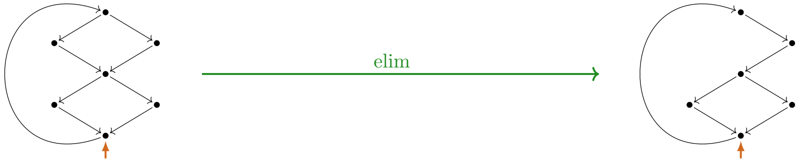

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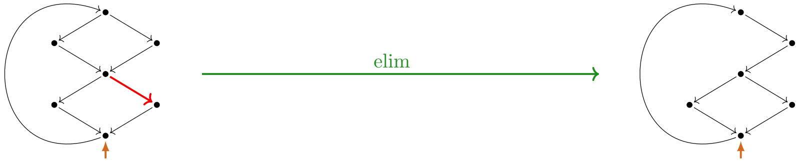

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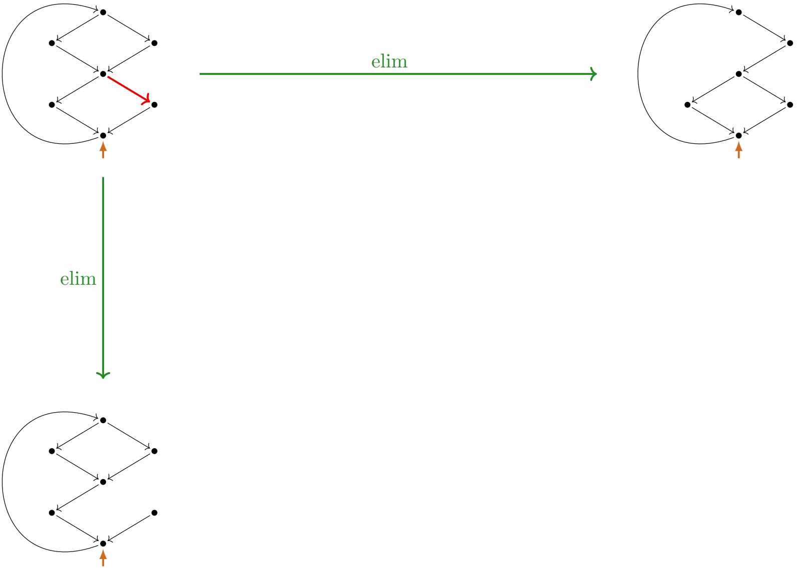

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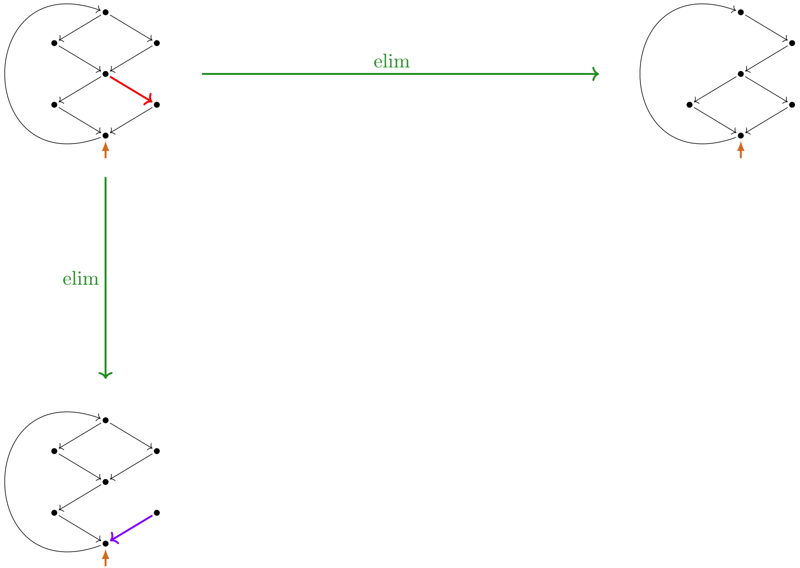

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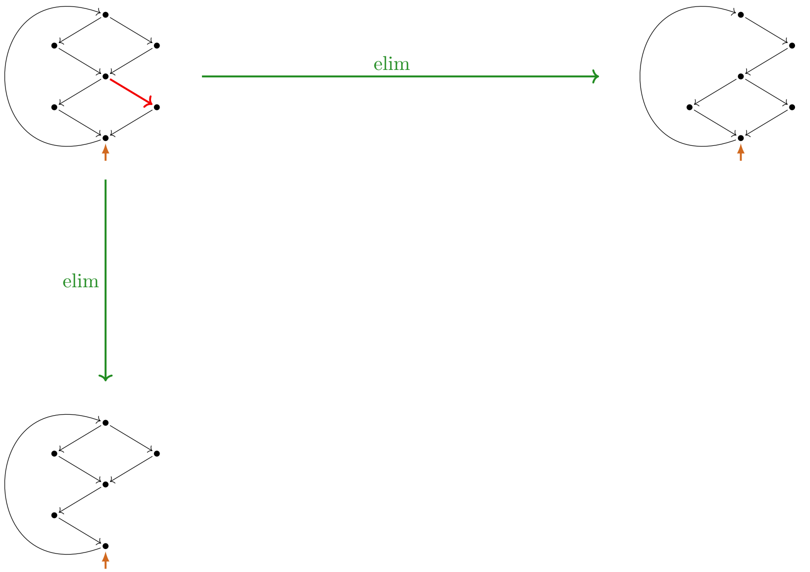

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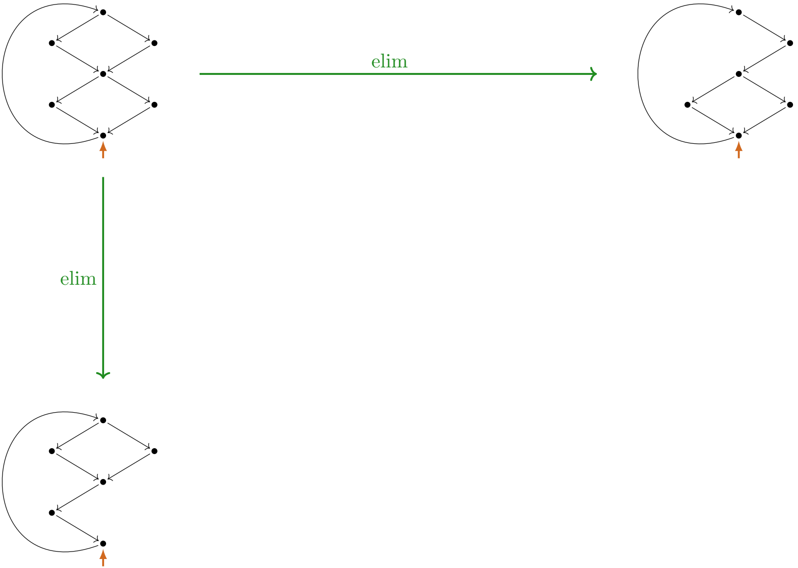

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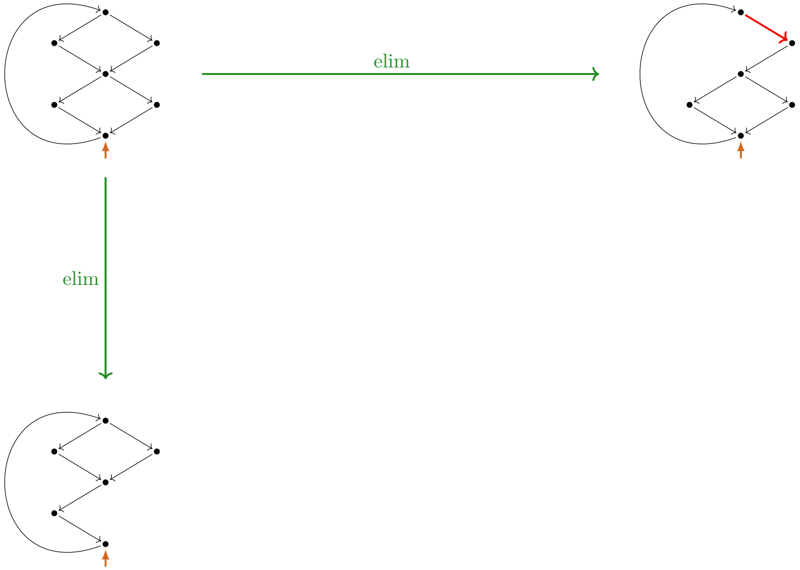

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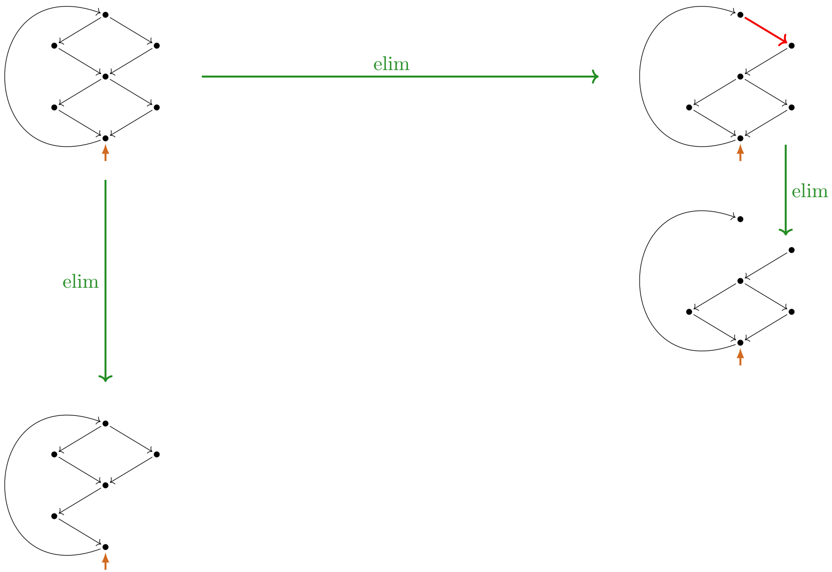

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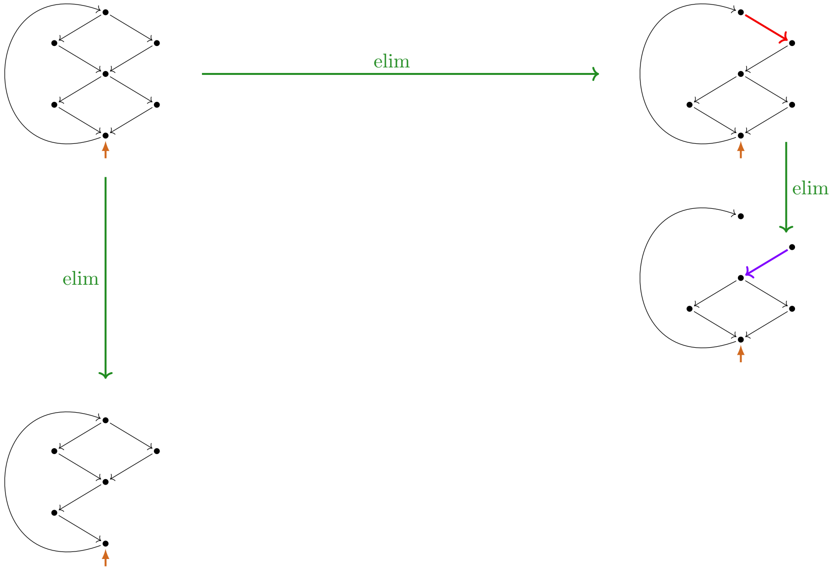

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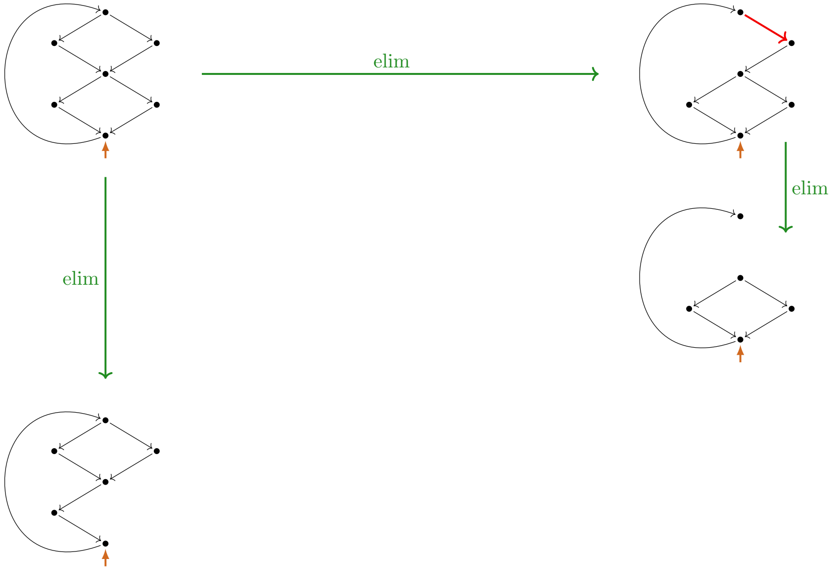

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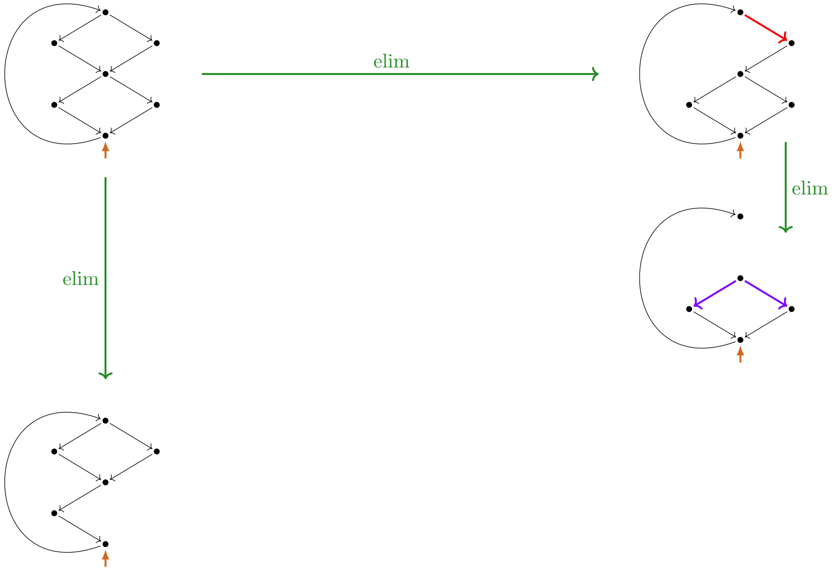

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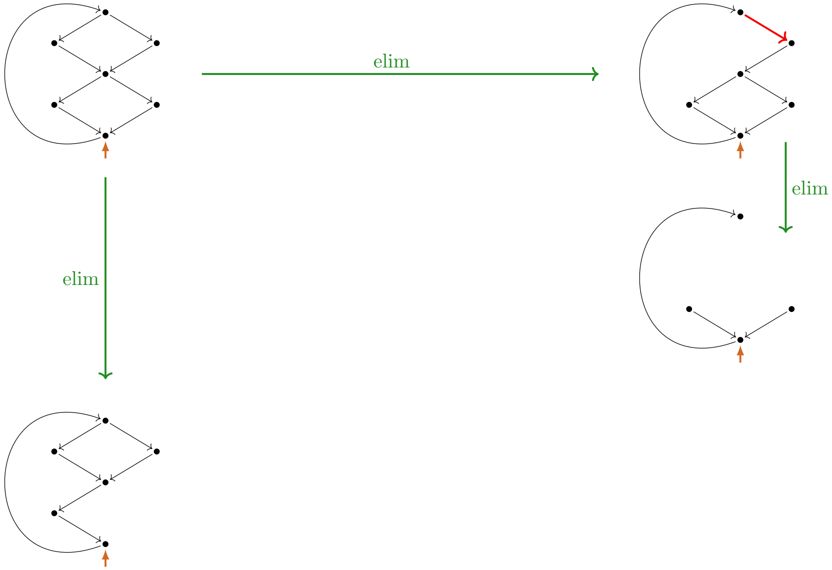

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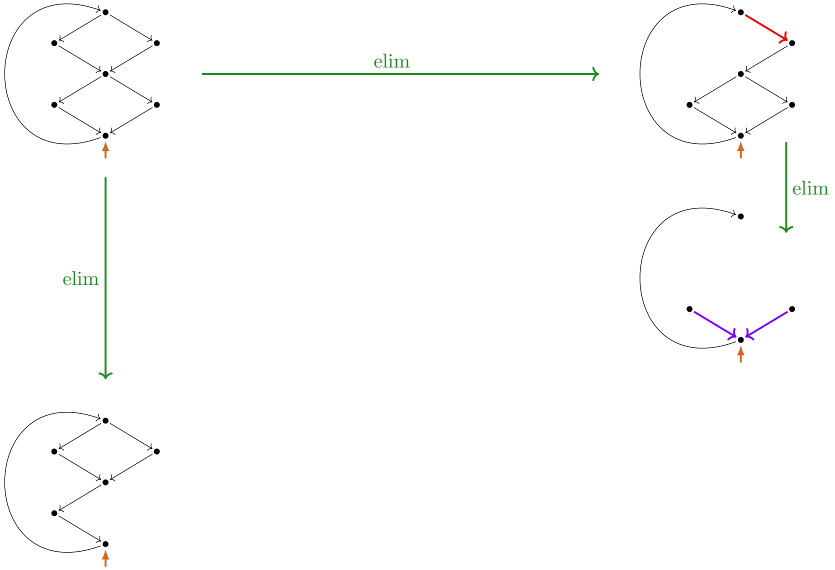

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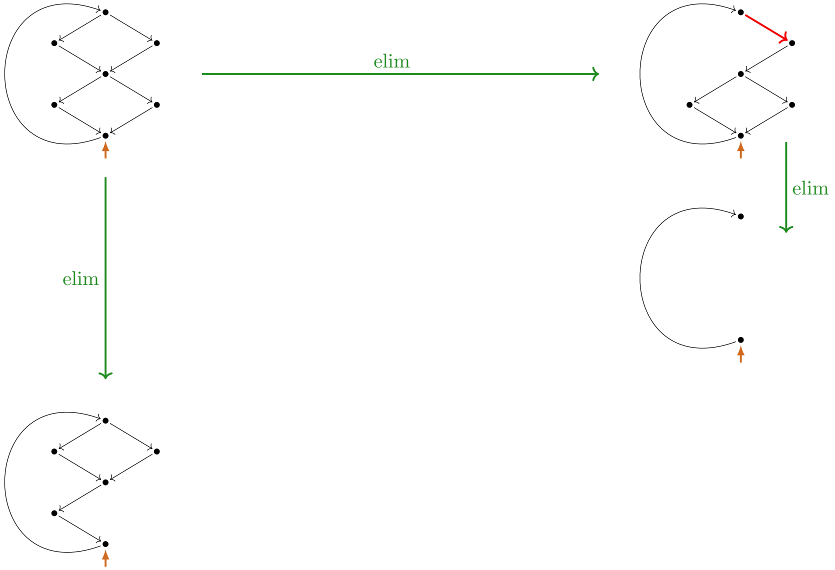

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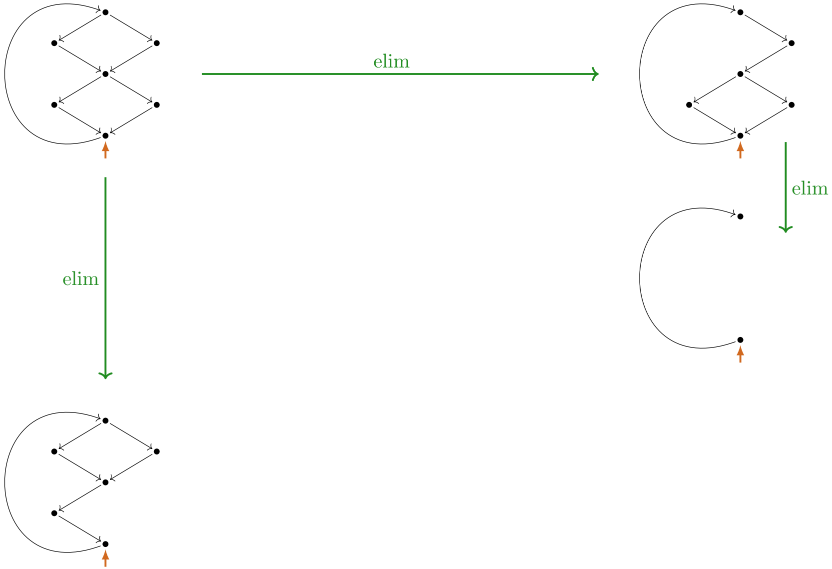

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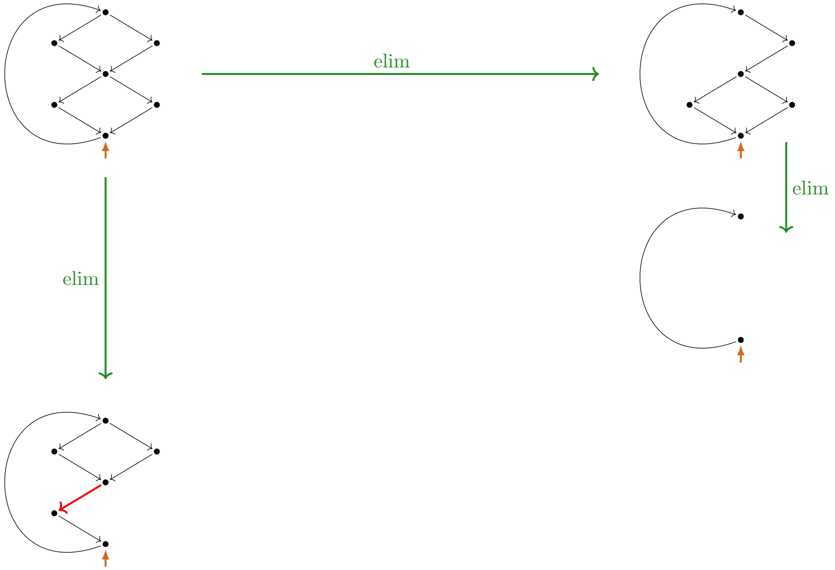

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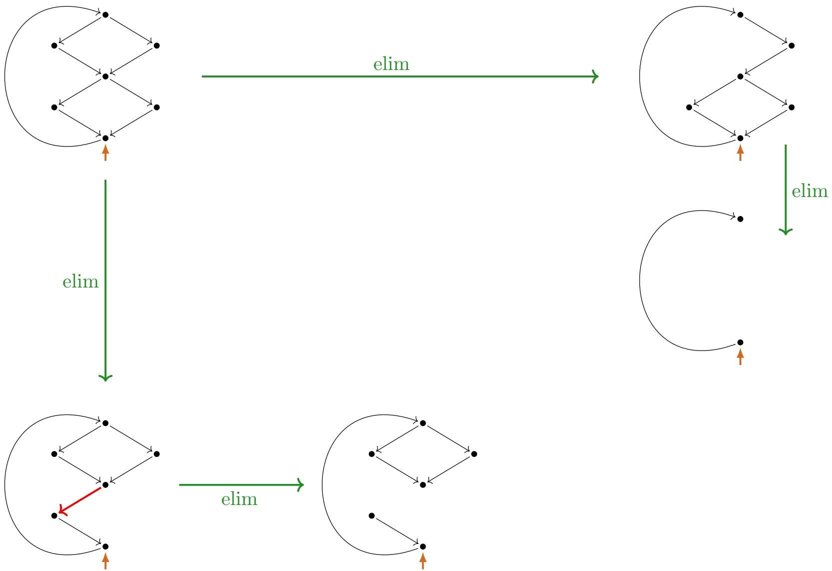

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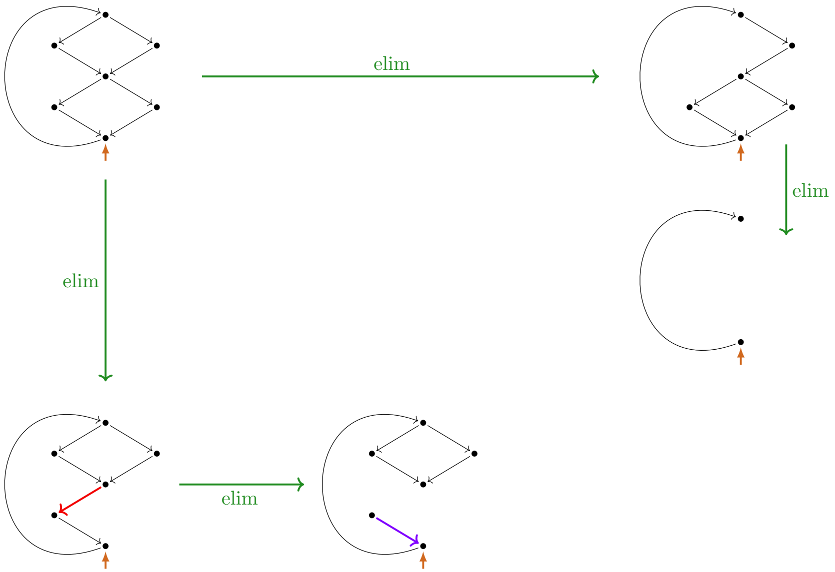

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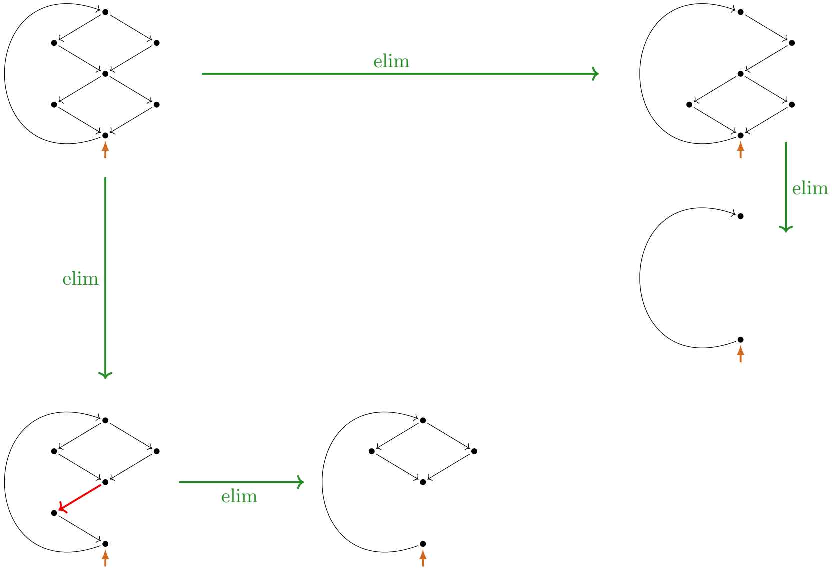

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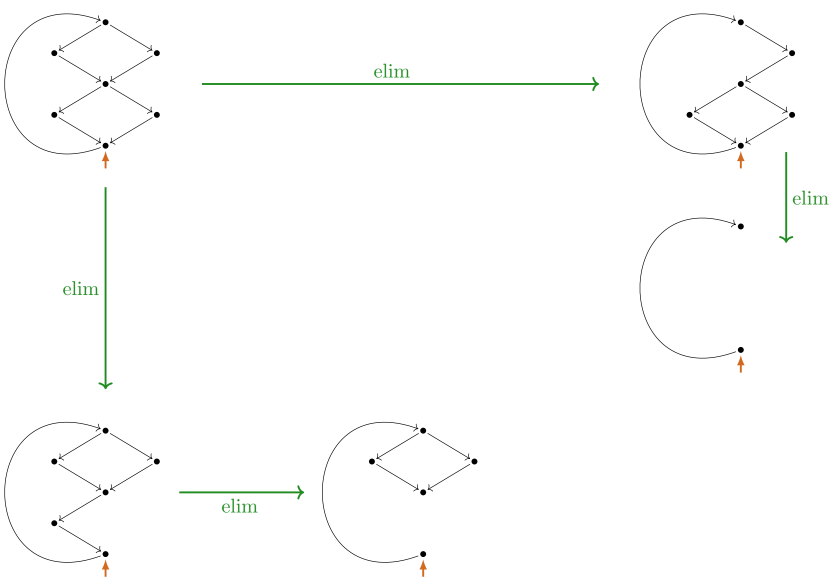

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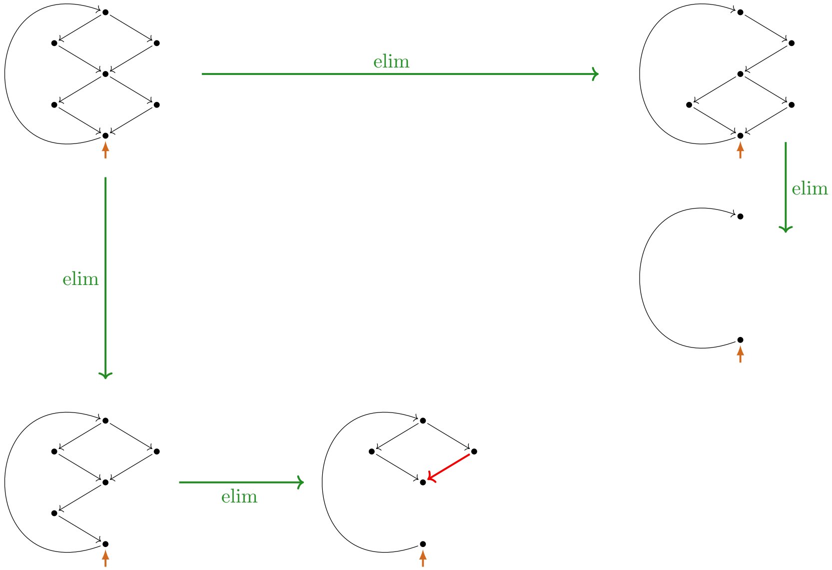

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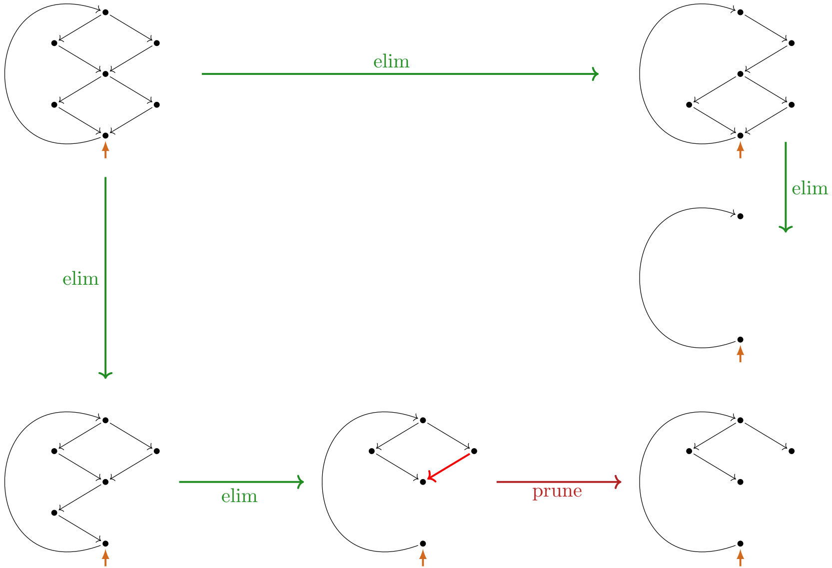

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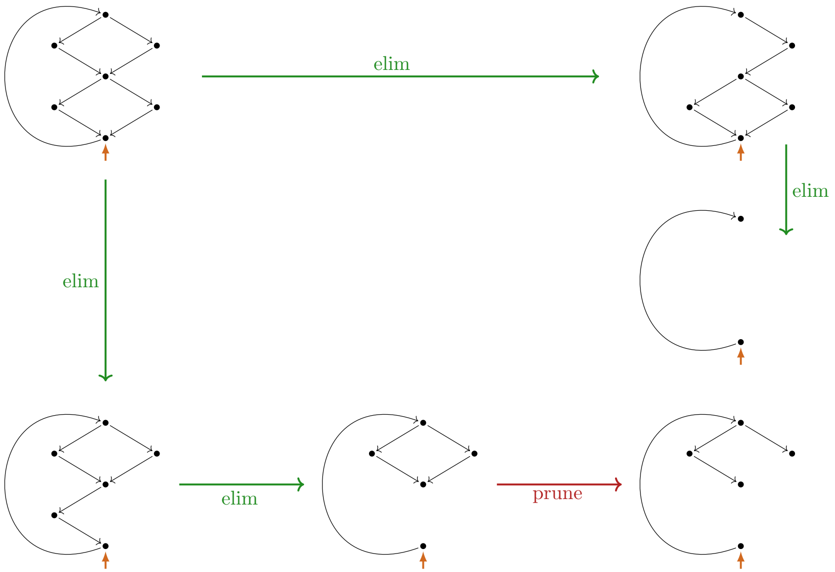

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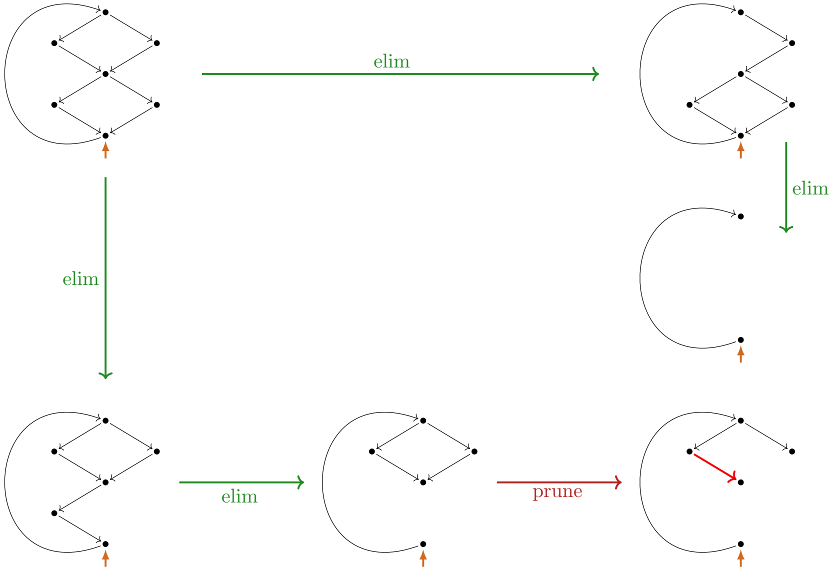

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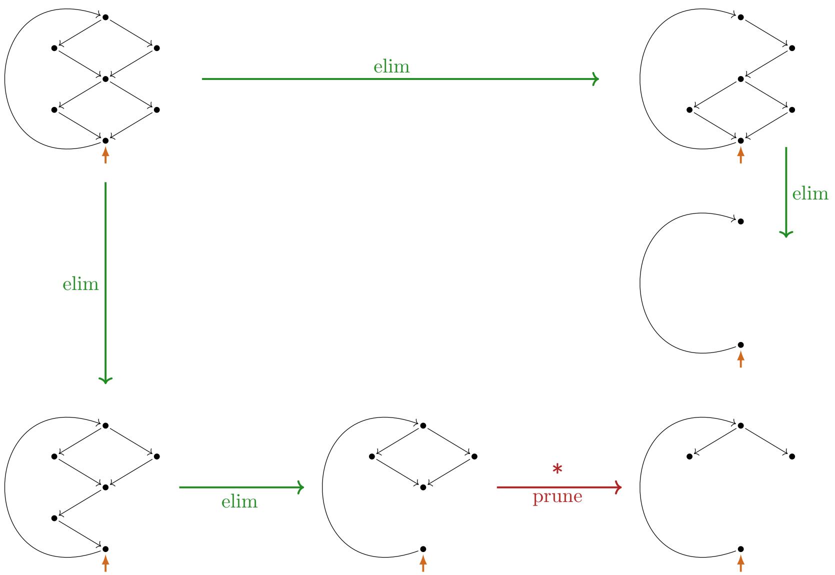

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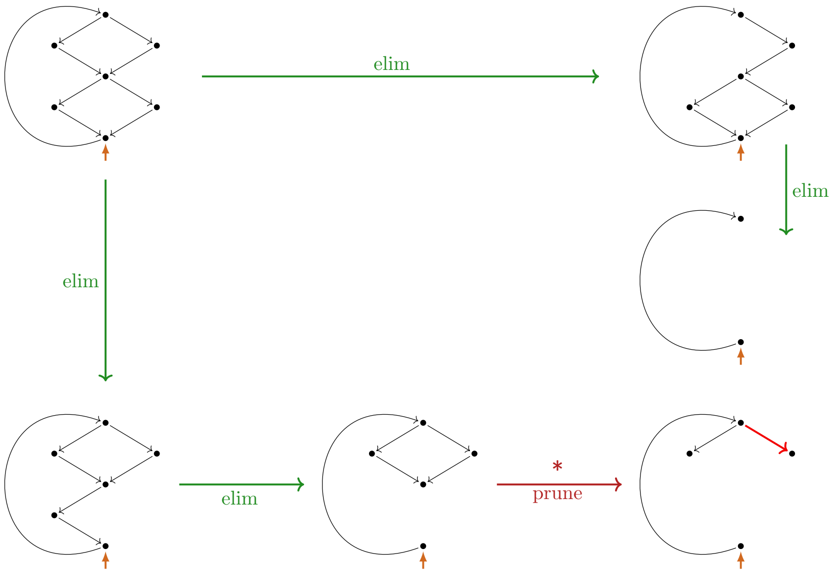

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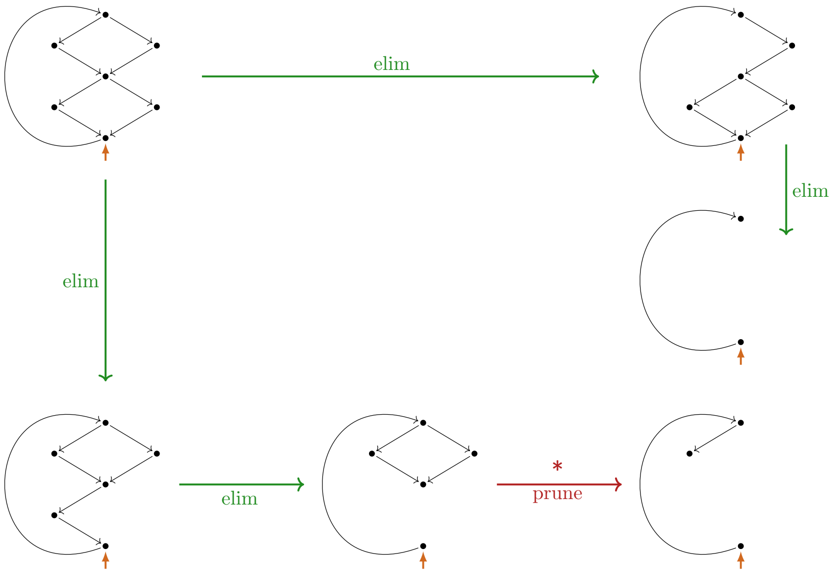

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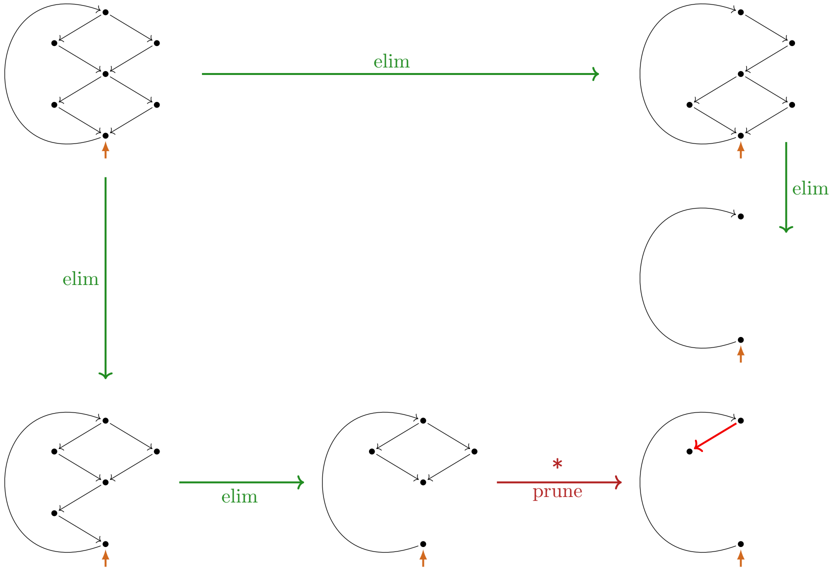

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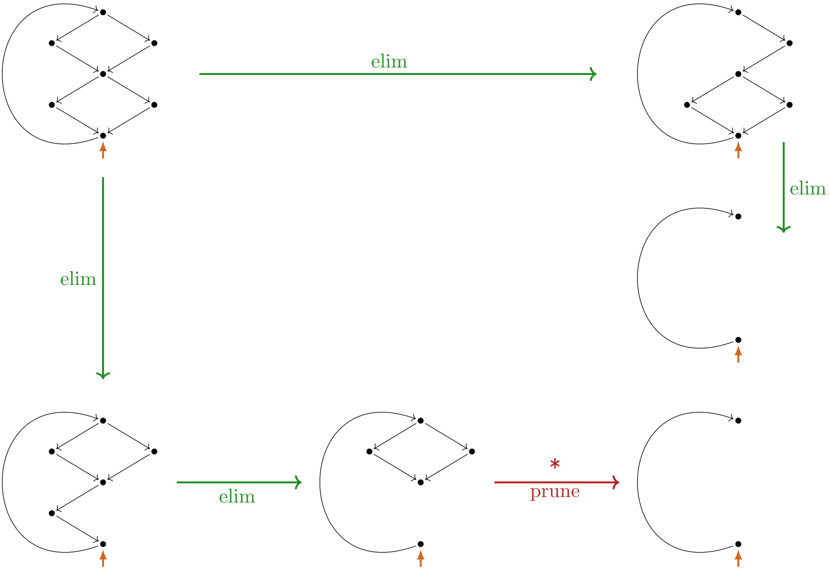

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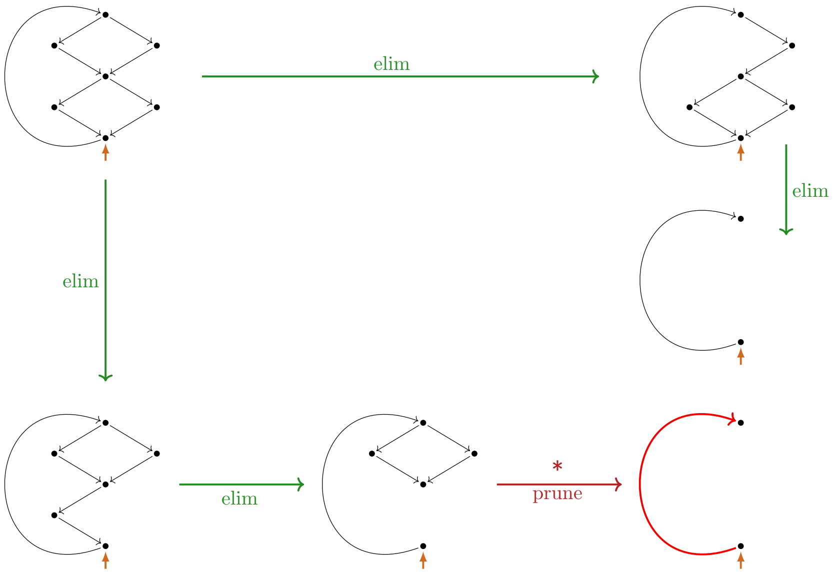

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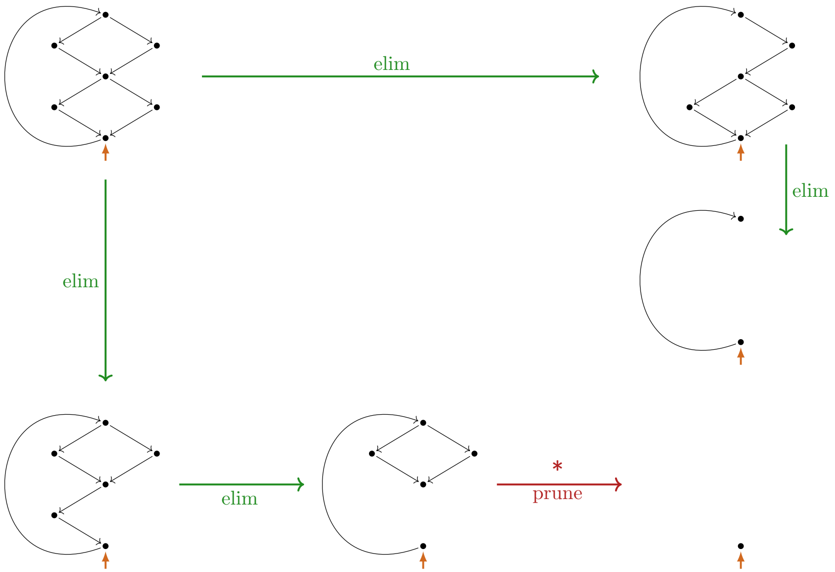

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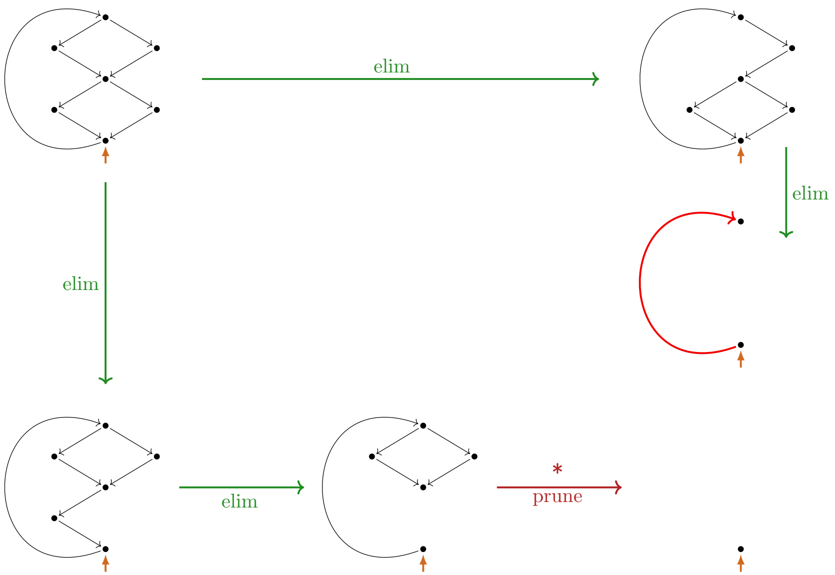

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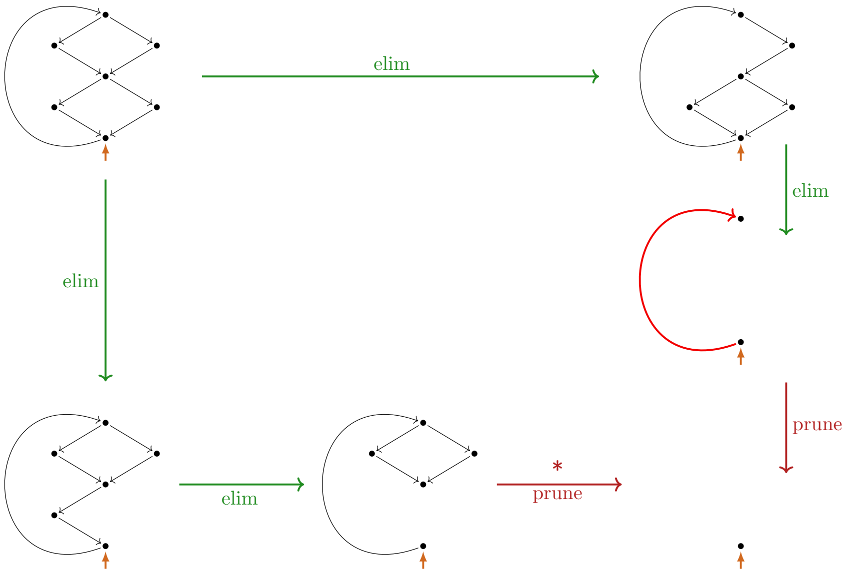

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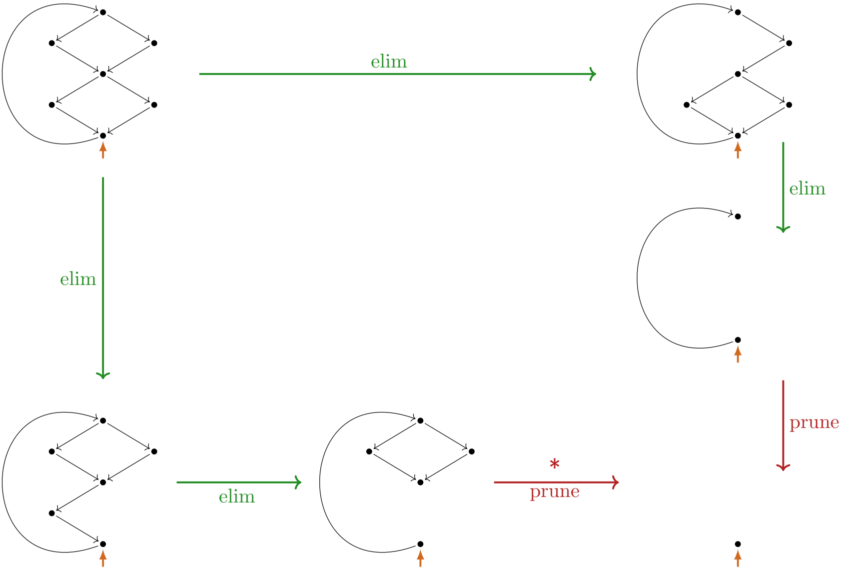

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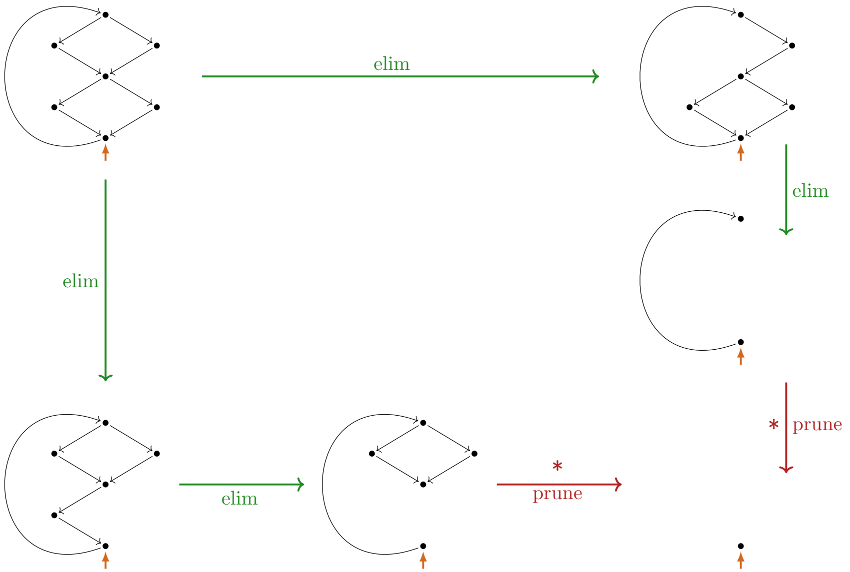

Loop elimination, and properties

$\longrightarrow_{\mathrm{elim}}$: eliminate a transition-induced loop by:

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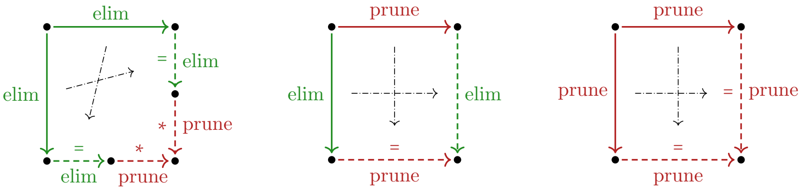
$\longrightarrow_{\mathrm{prune}}$: remove a transition to a deadlocking state

Lemma

(i) $\longrightarrow_{\mathrm{elim}} \cup \longrightarrow_{\mathrm{prune}}$ *is terminating.*

(ii) $\longrightarrow_{\mathrm{elim}} \cup \longrightarrow_{\mathrm{prune}}$ *is decreasing, and hence locally confluent.*



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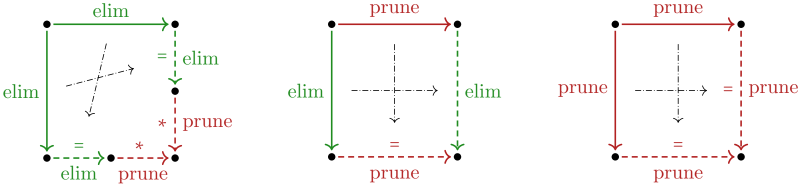

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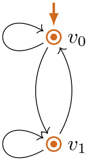

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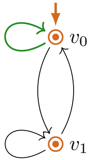

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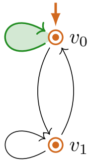

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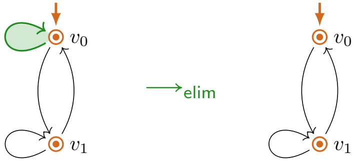

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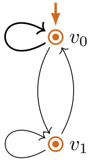
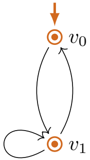


$\longrightarrow_{\text{elim}}$



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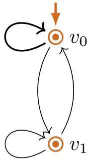

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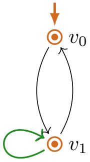

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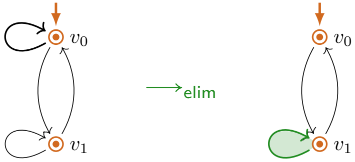

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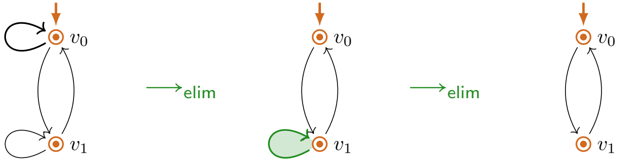

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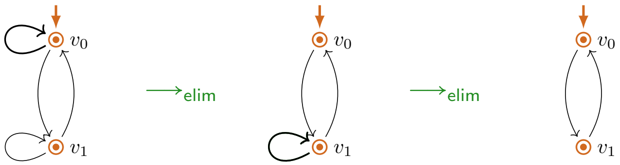

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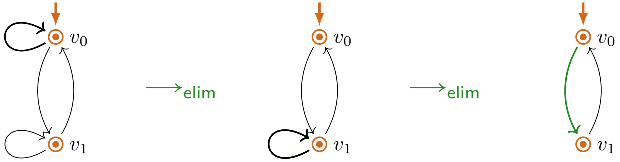

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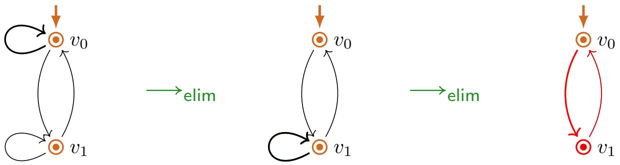

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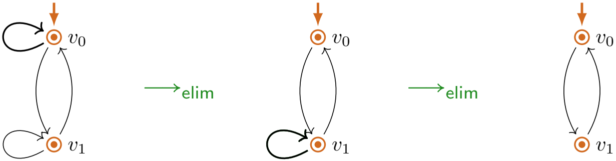

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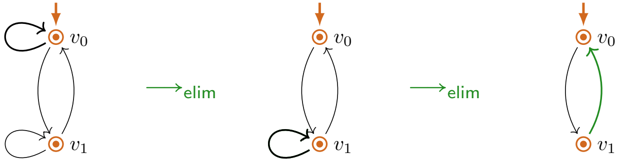

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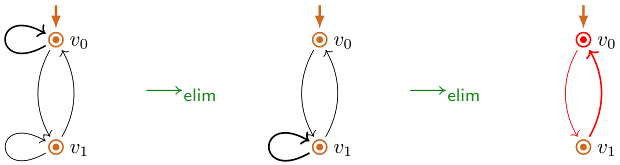

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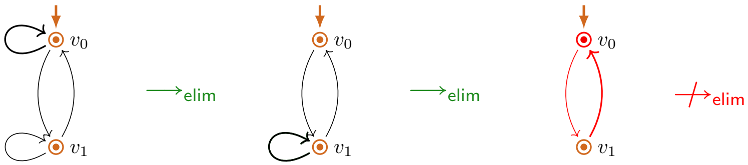

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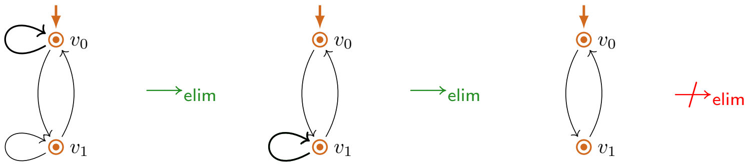

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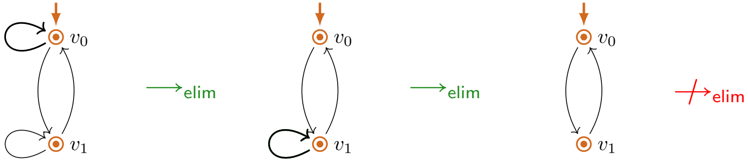

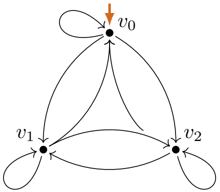

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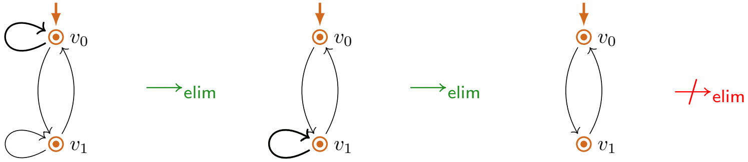

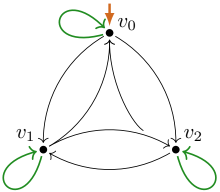

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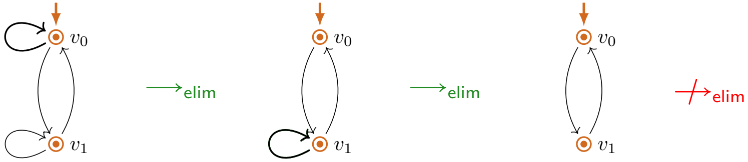

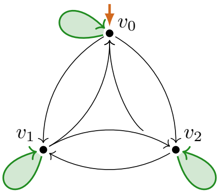

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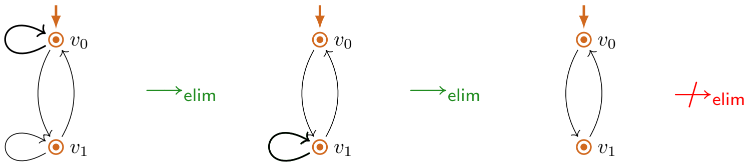

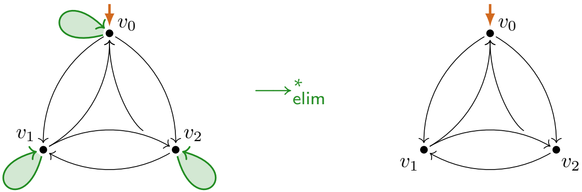

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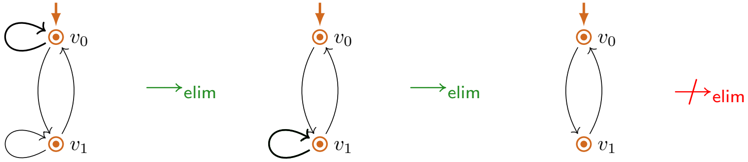

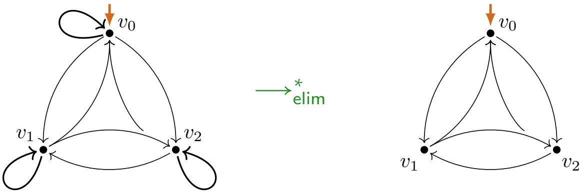

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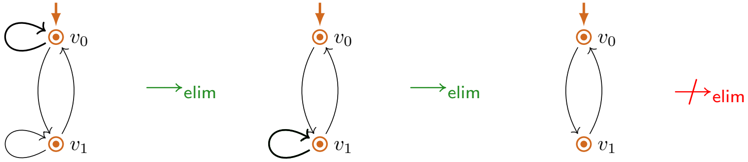

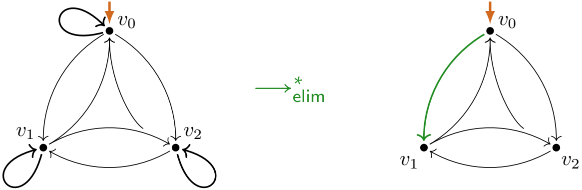

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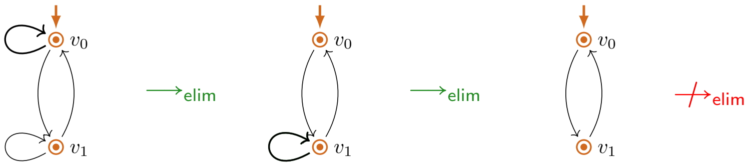

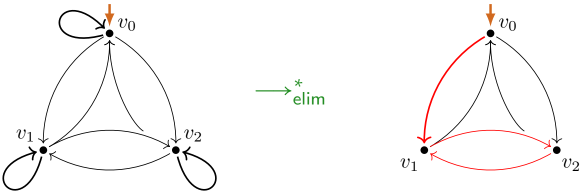

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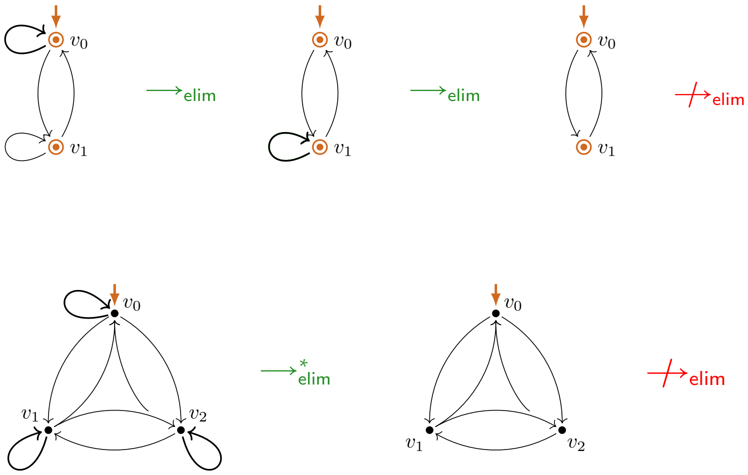

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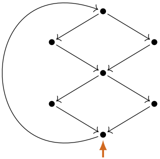

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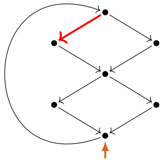

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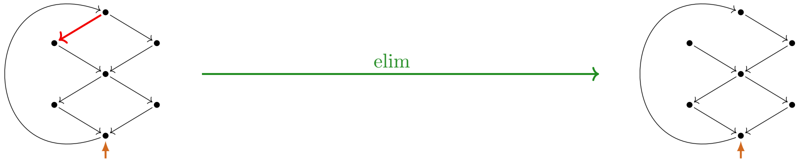

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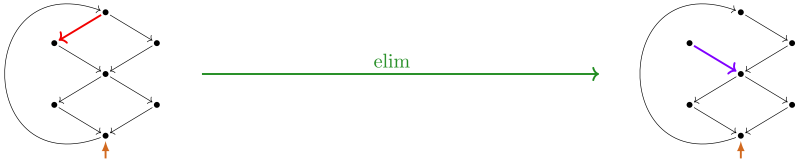

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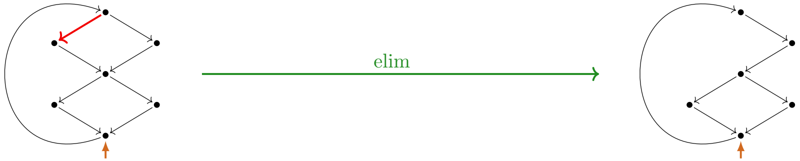

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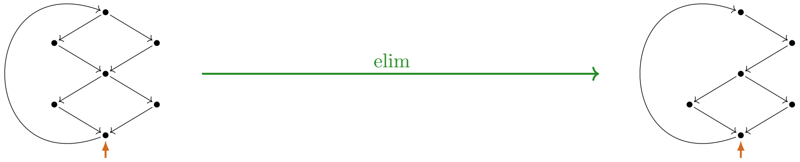

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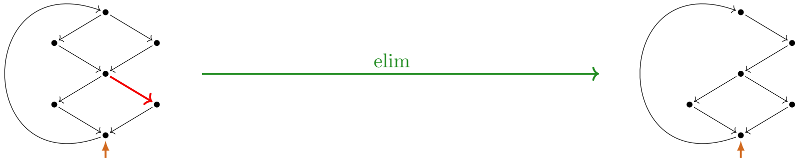

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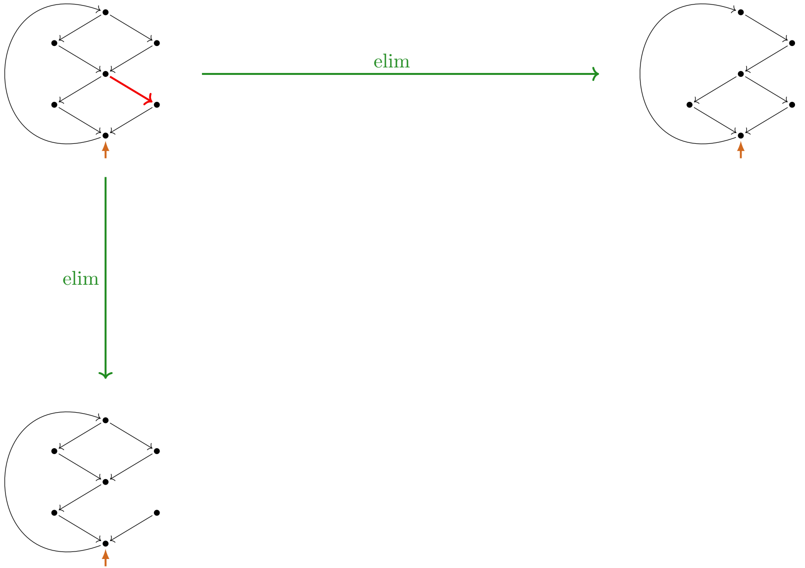

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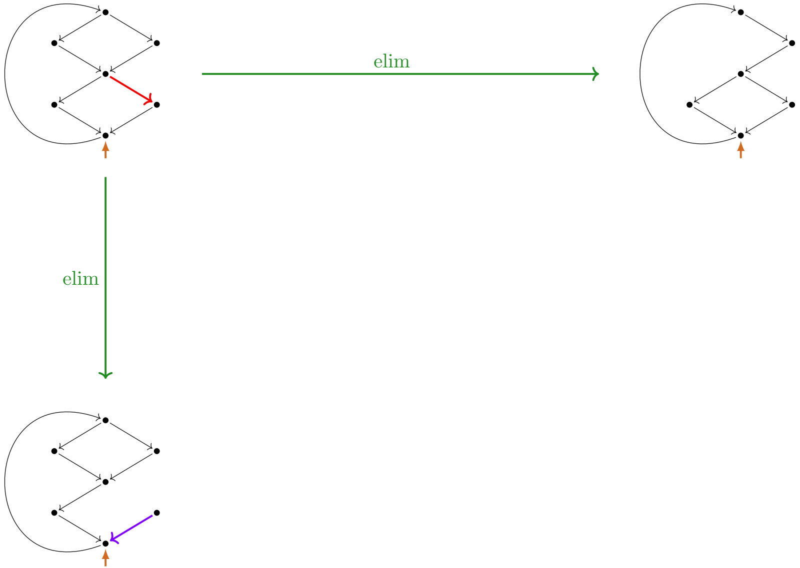

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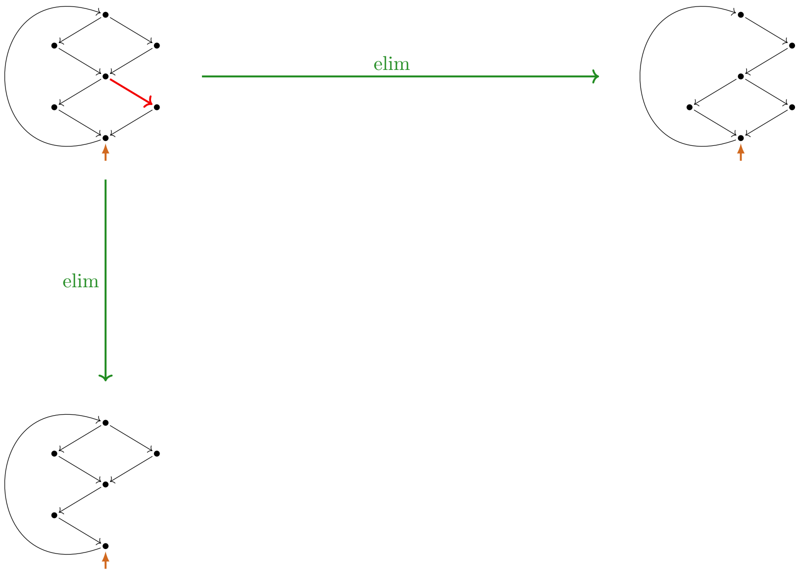

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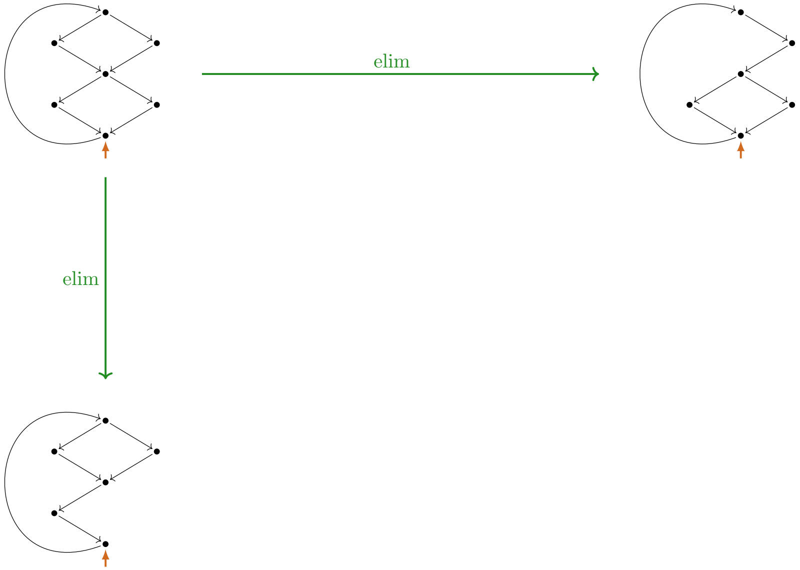

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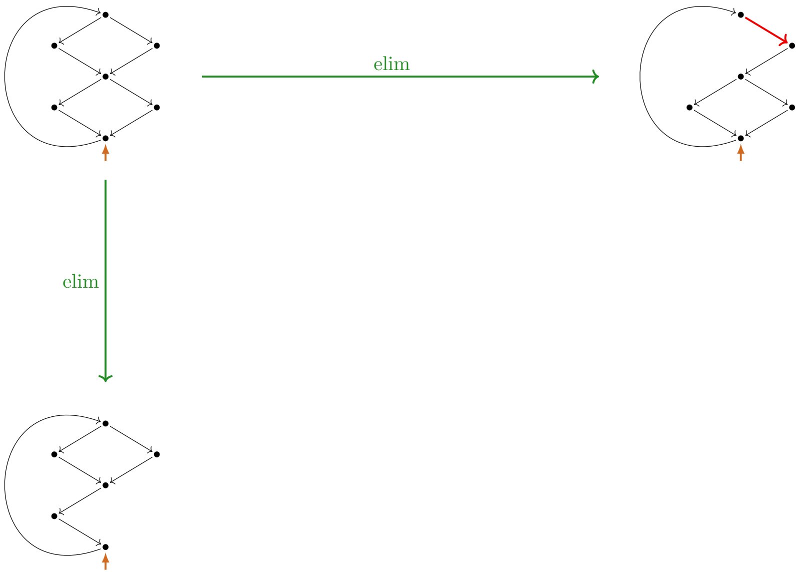

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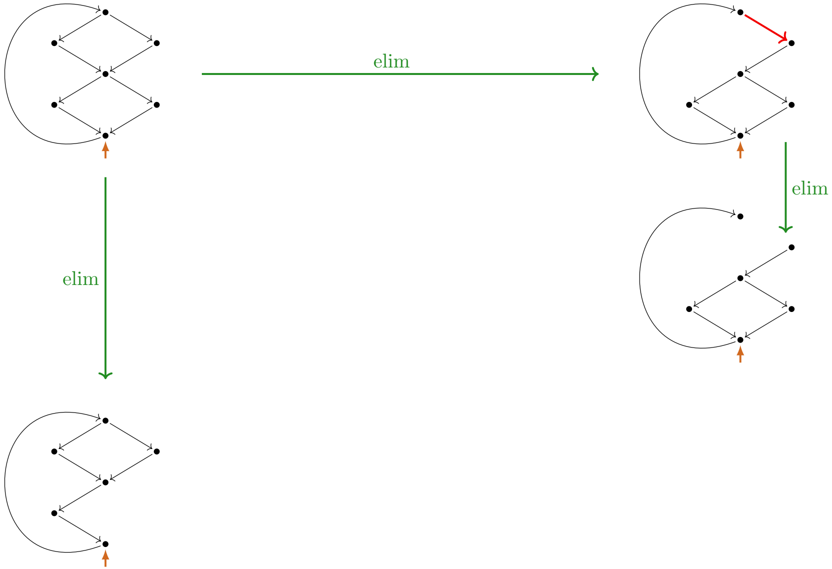

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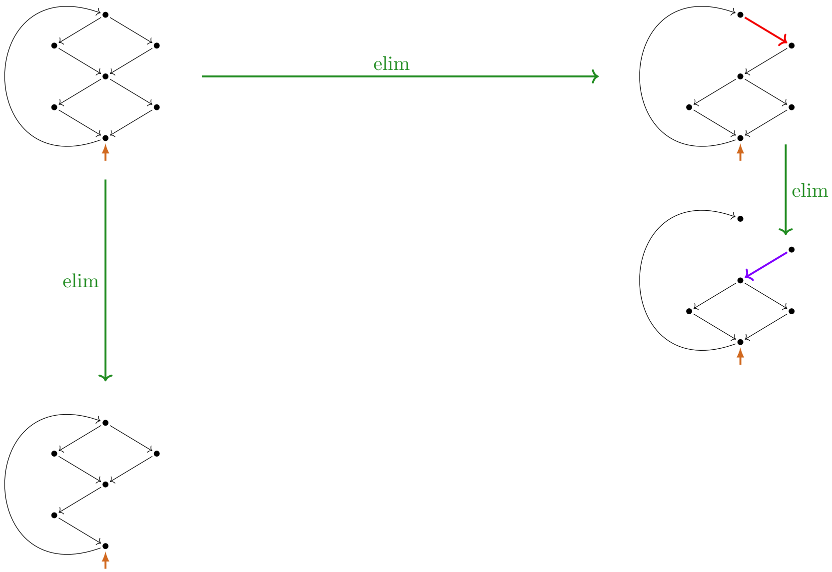

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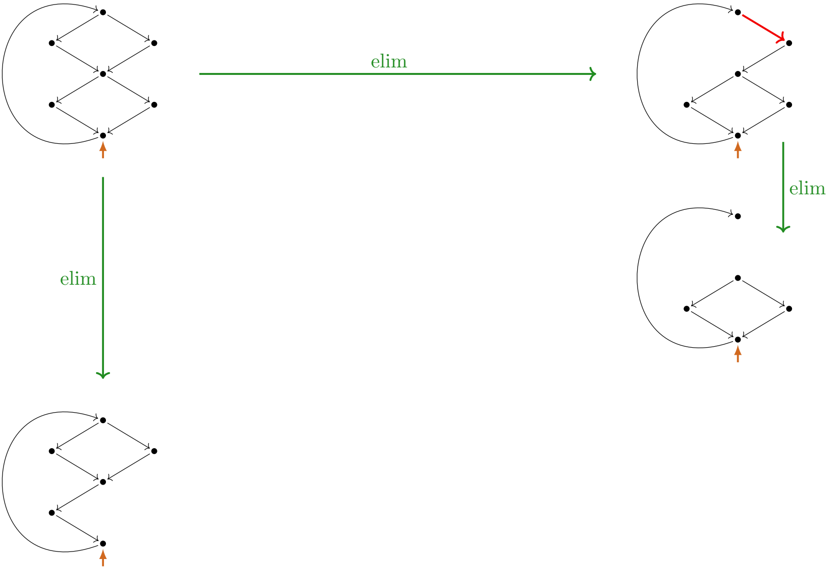

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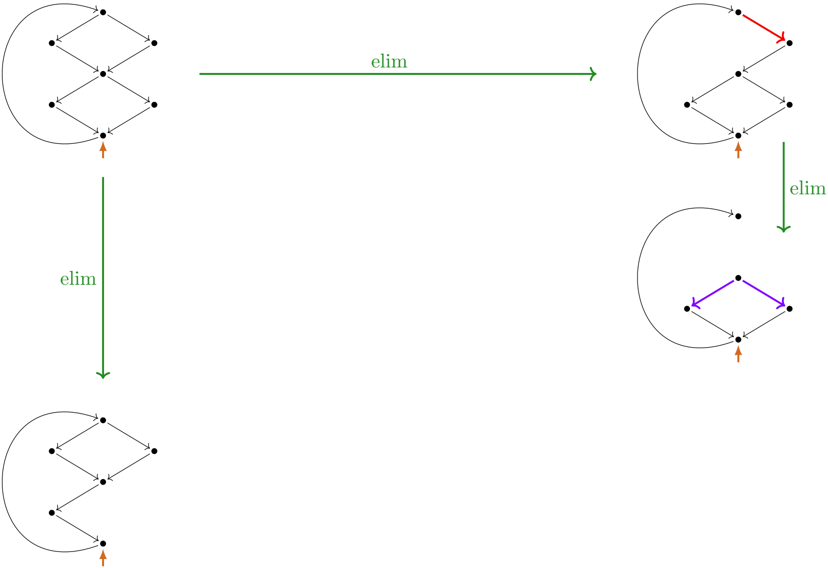

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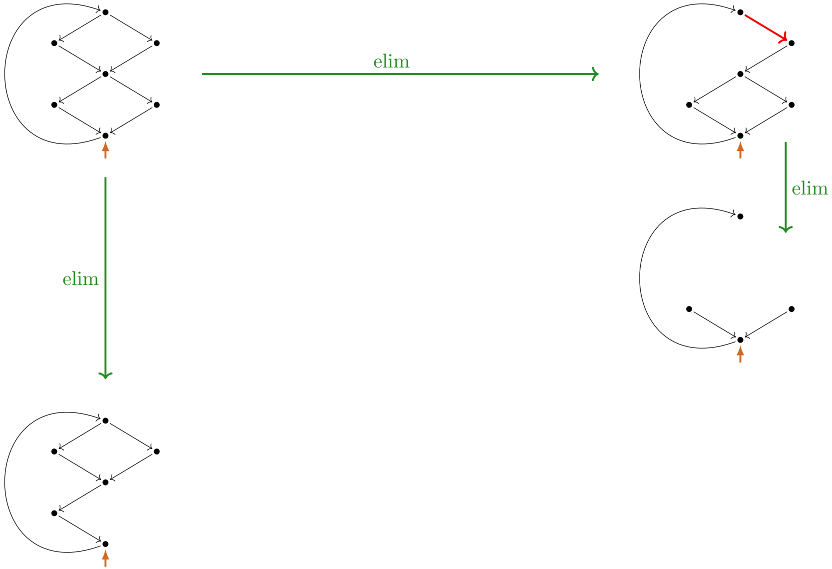

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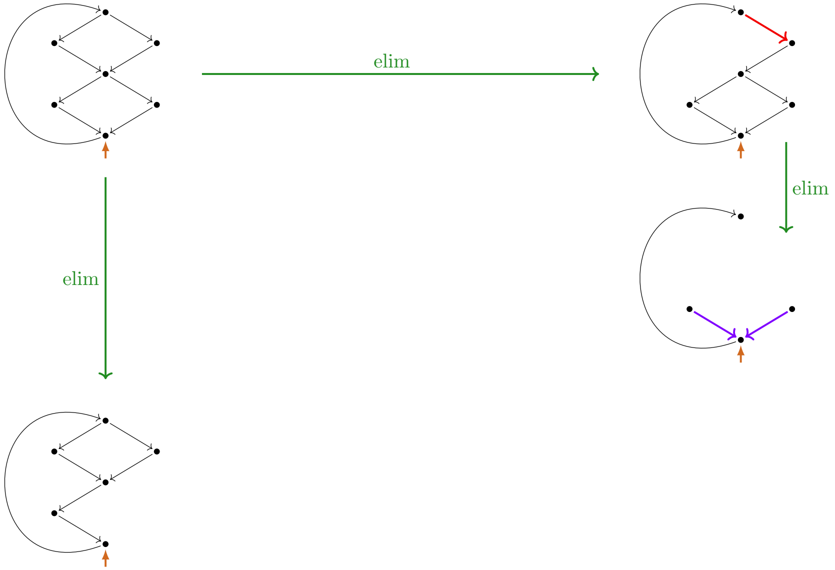

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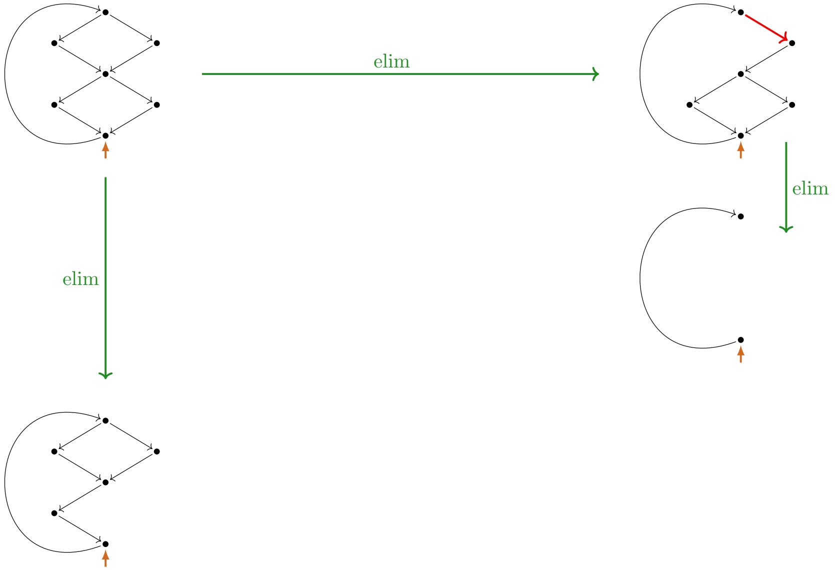

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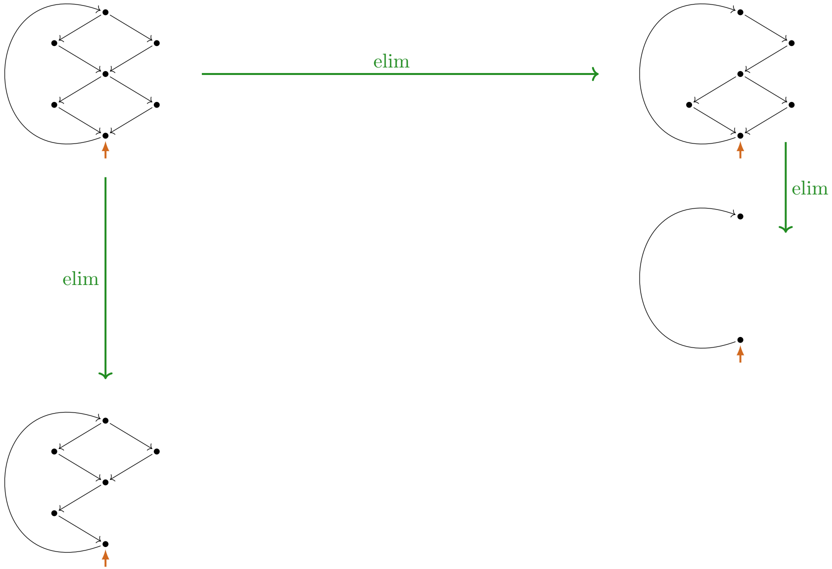

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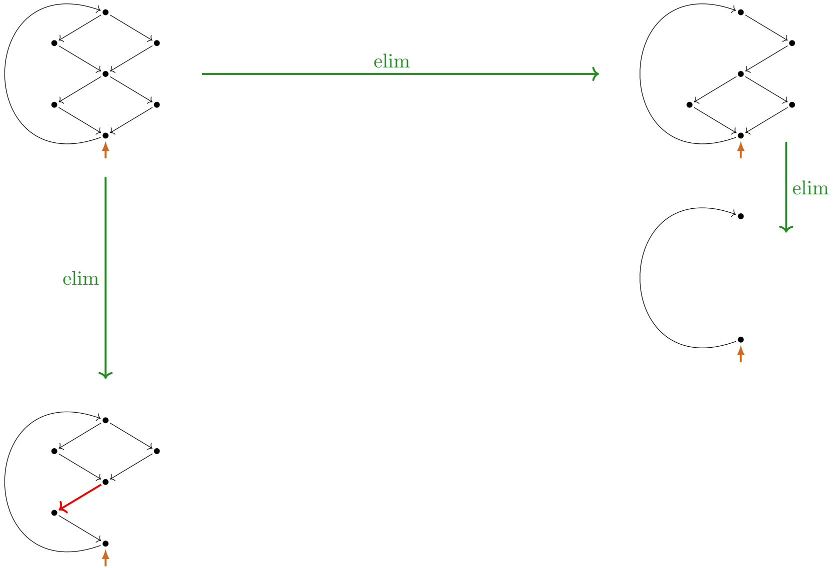

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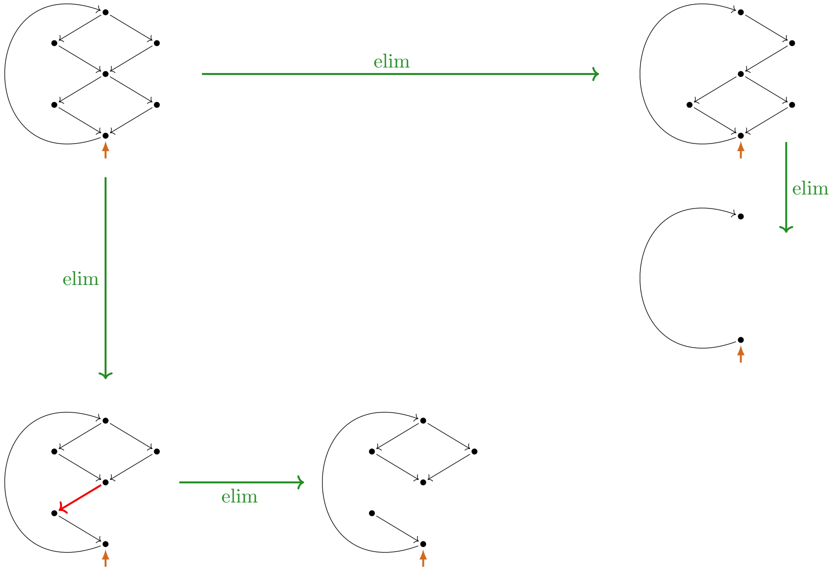

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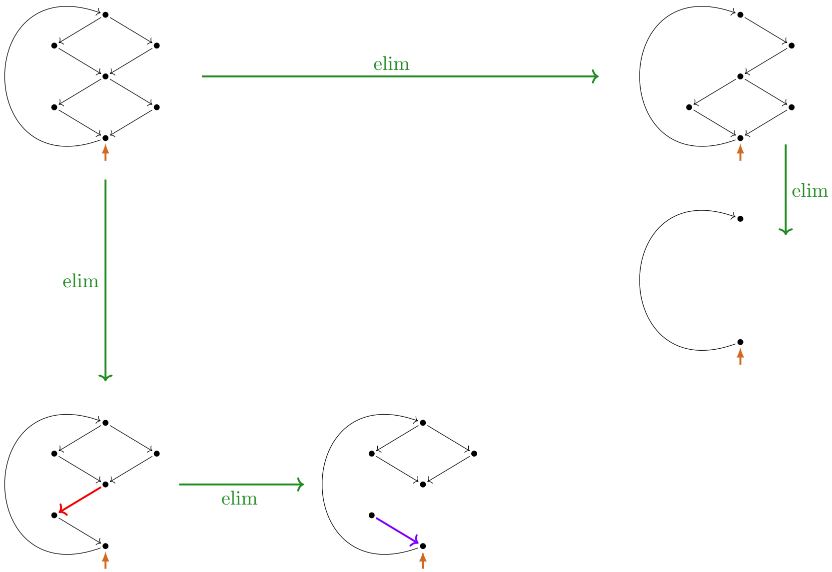

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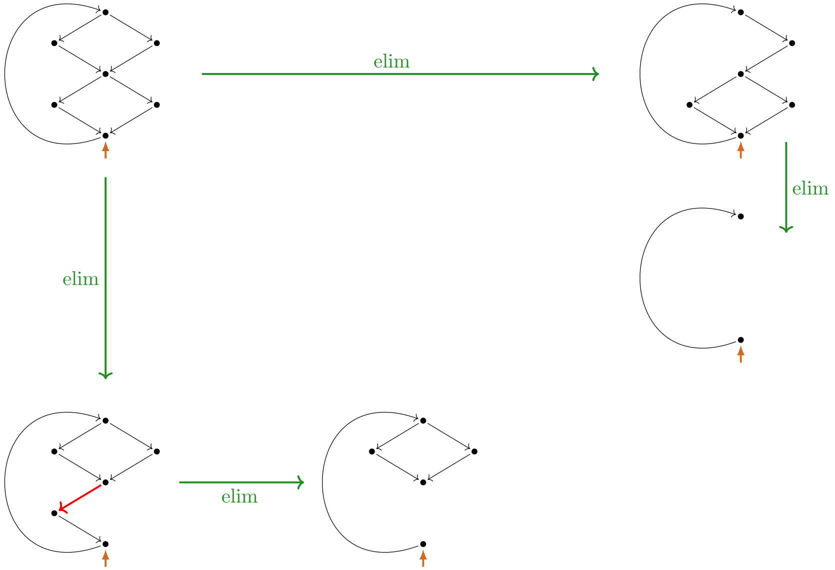

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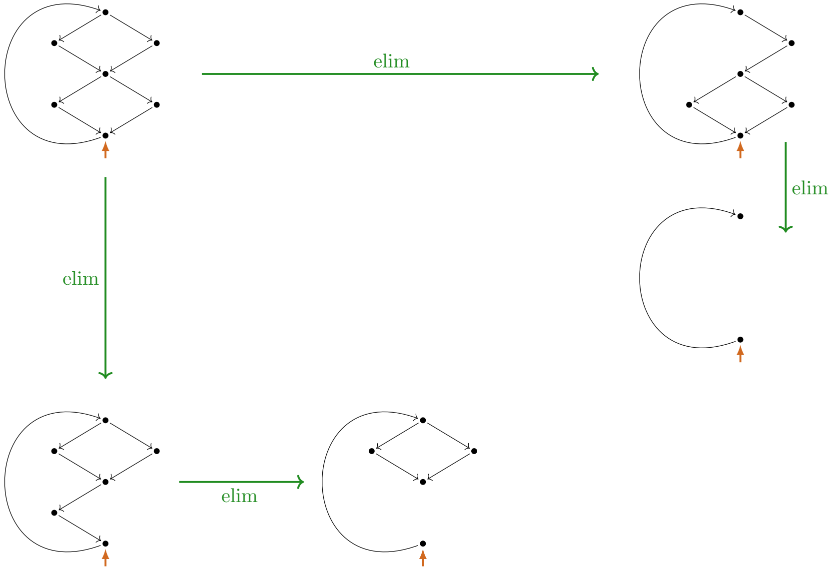

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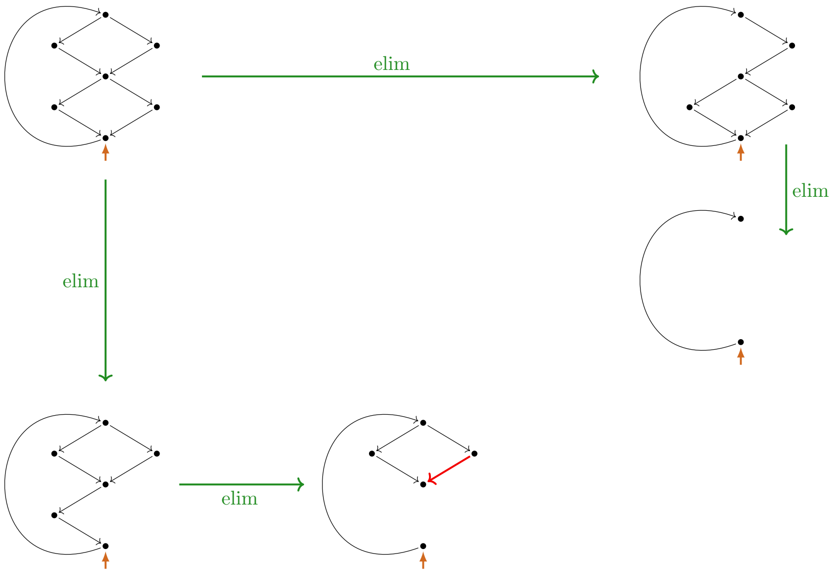

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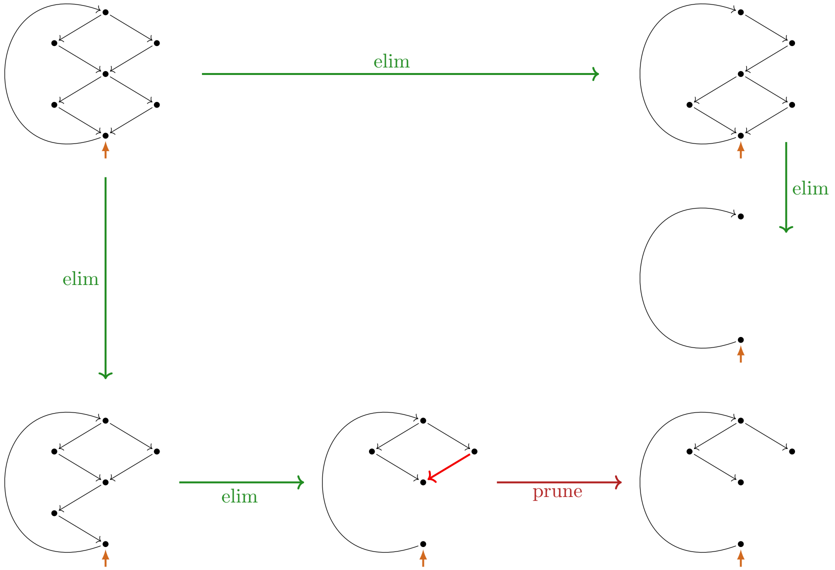

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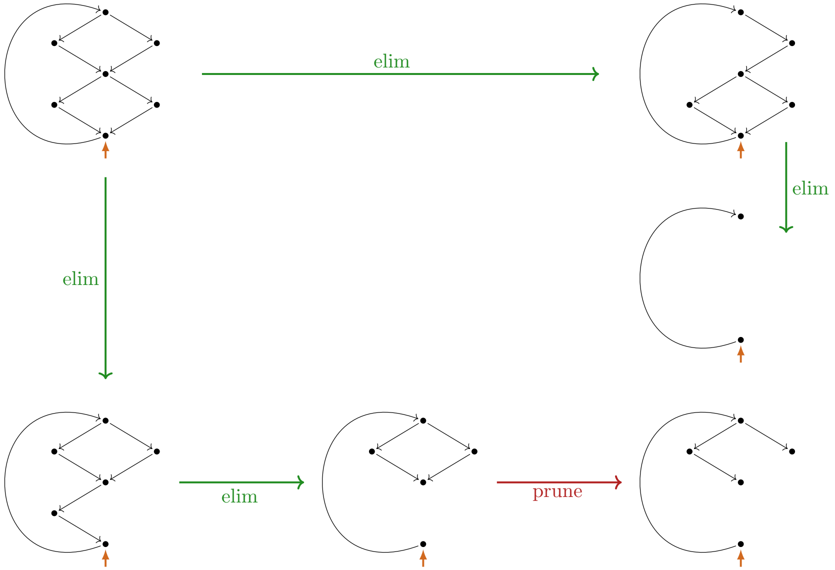

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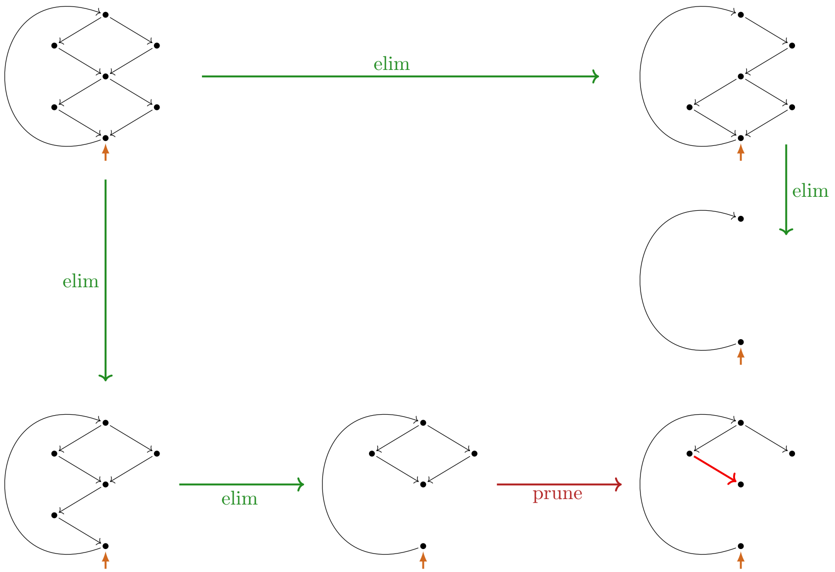

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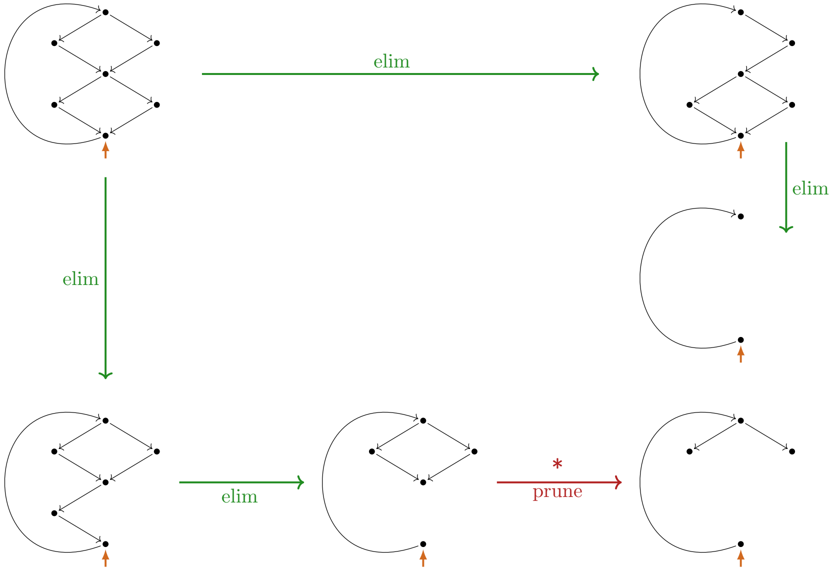

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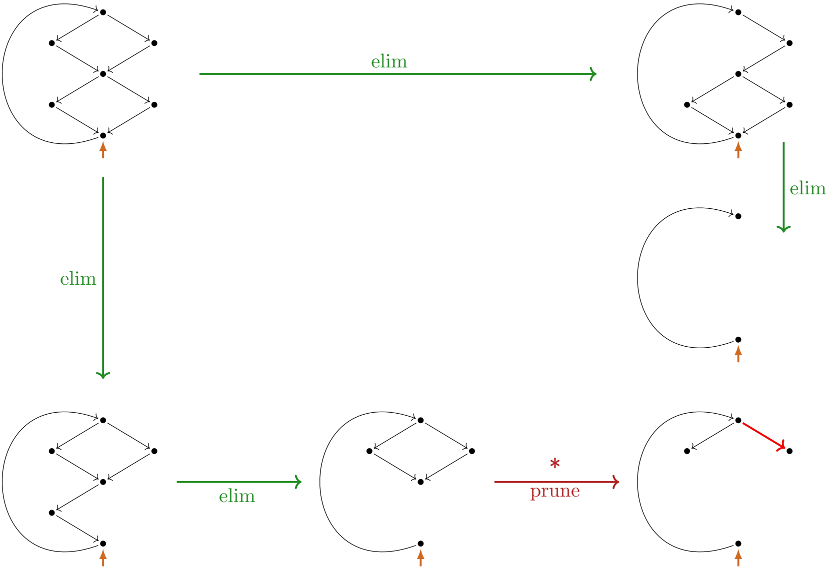

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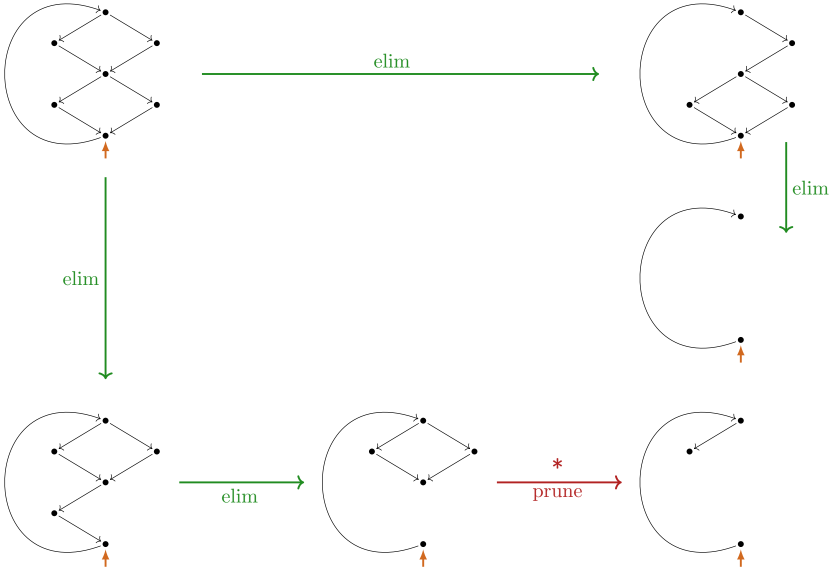

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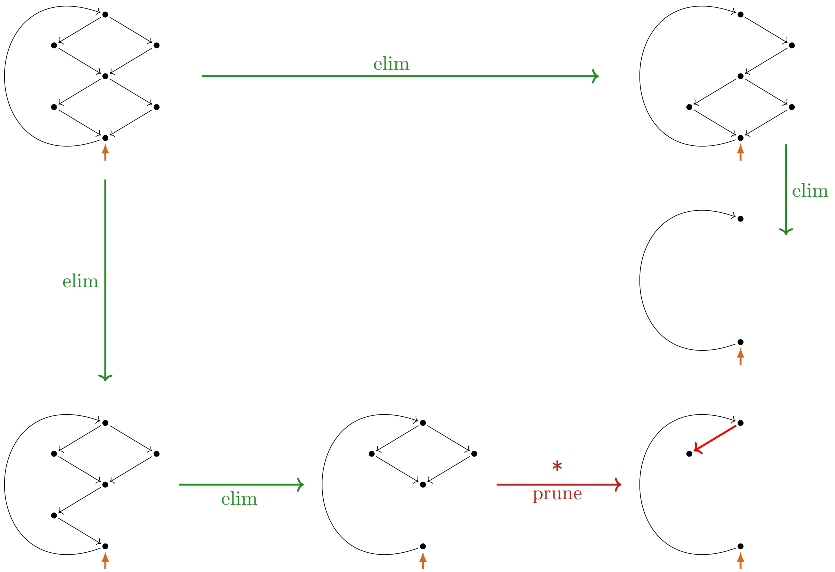

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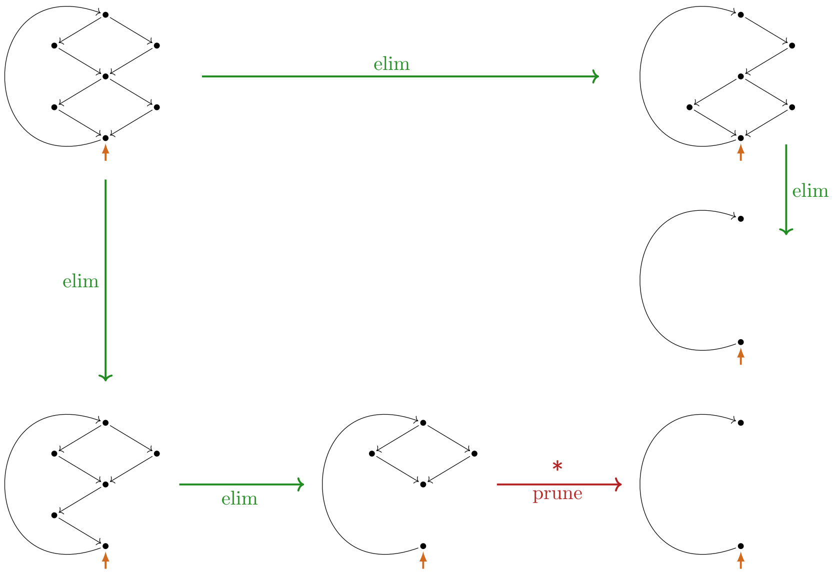

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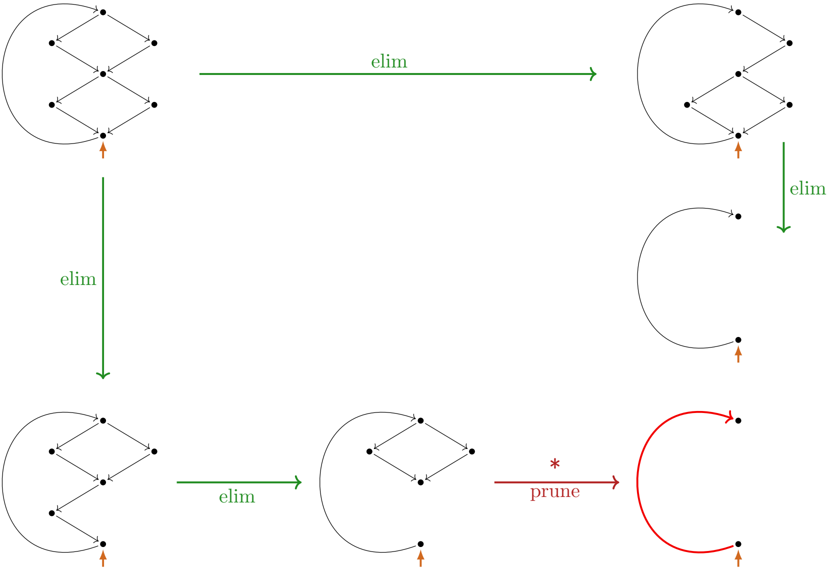

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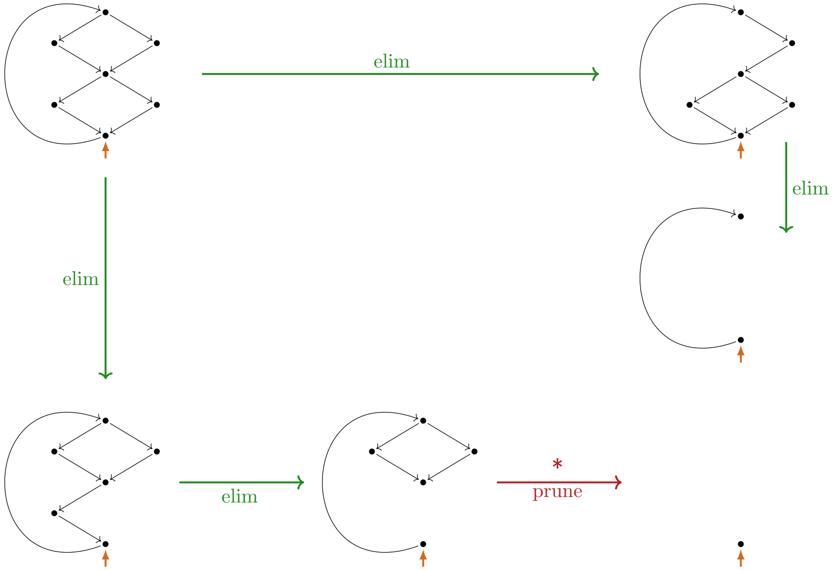

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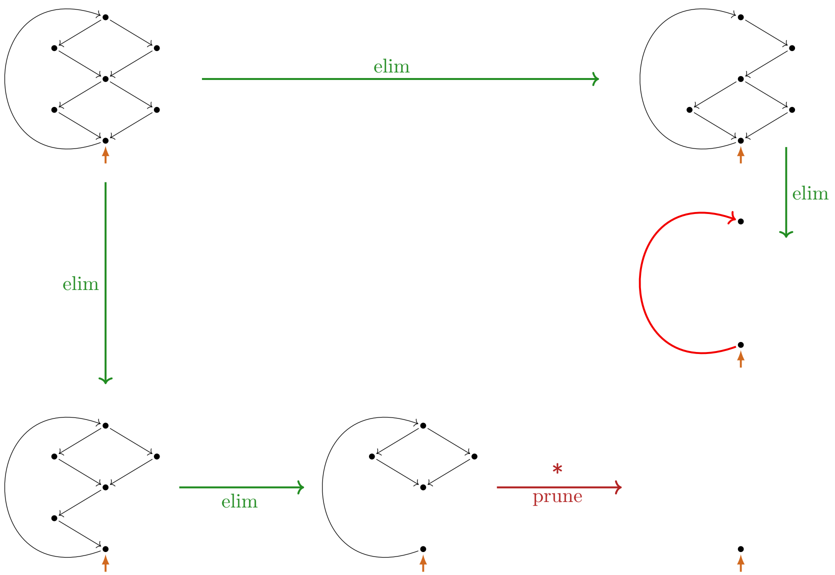

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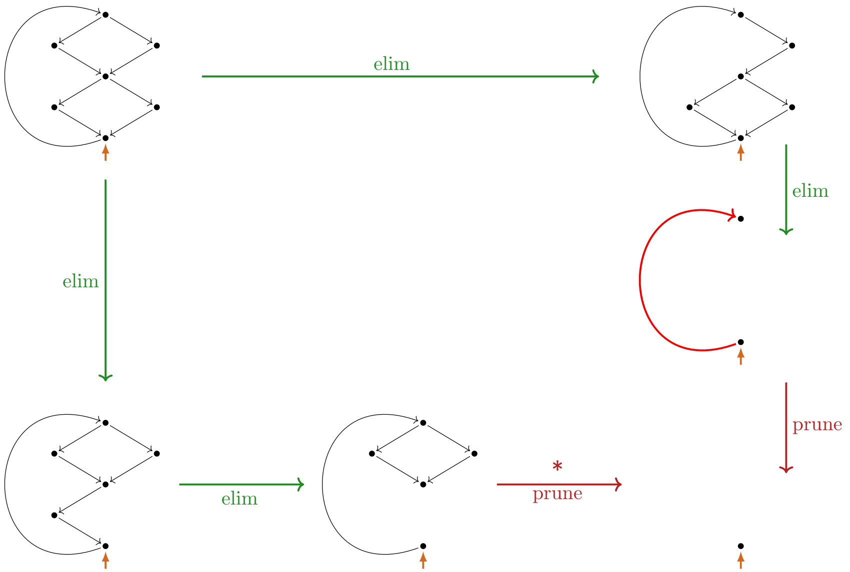

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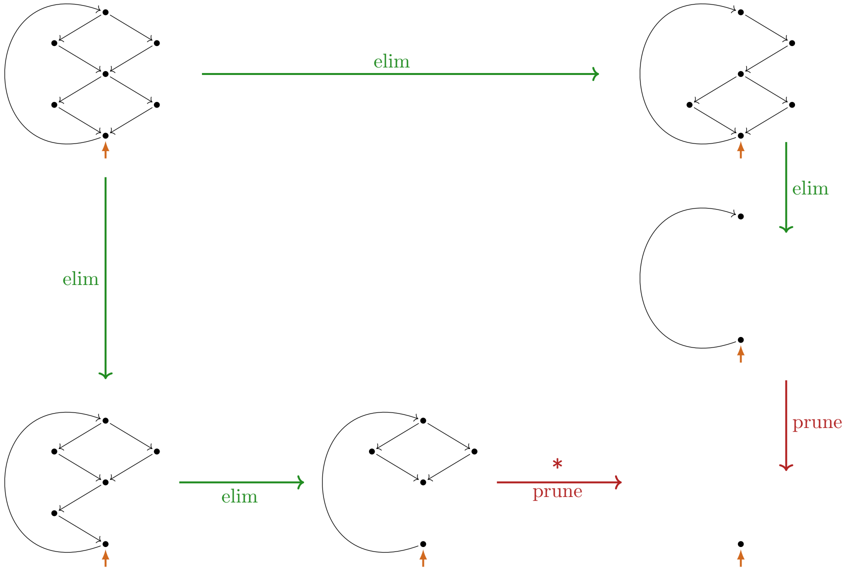

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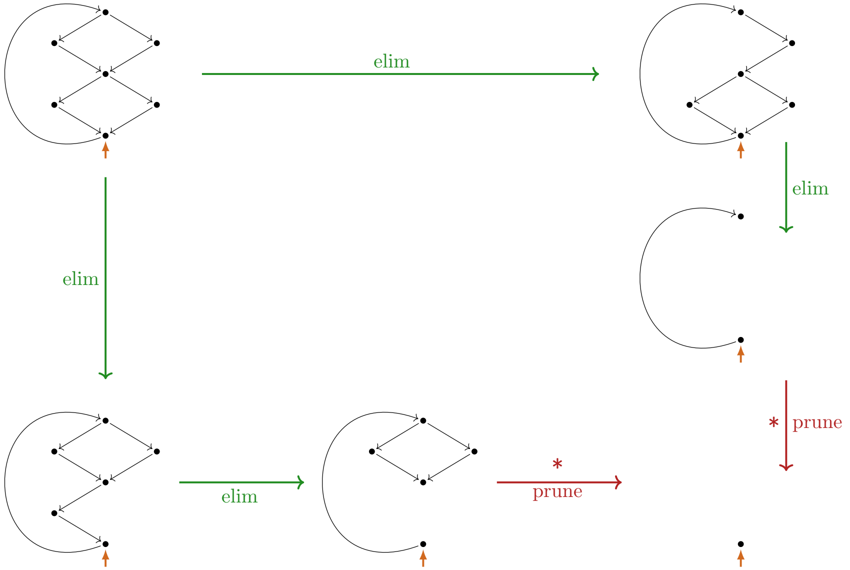

Loop elimination, and properties

$\longrightarrow_{\text{elim}}$: eliminate a transition-induced loop by:

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Lemma

(i) $\longrightarrow_{\text{elim}} \cup \longrightarrow_{\text{prune}}$ *is terminating.*

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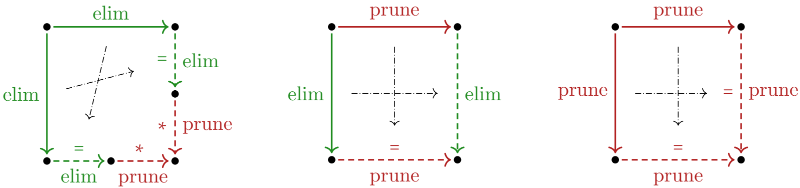

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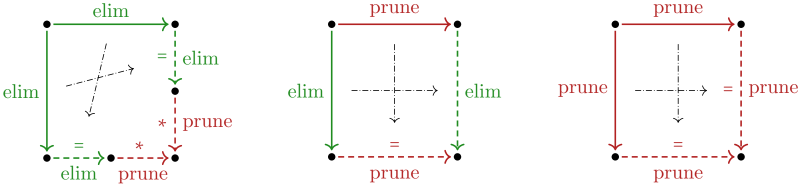

Structure property LEE

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A process graph G satisfies LEE (*loop existence and elimination*) if:

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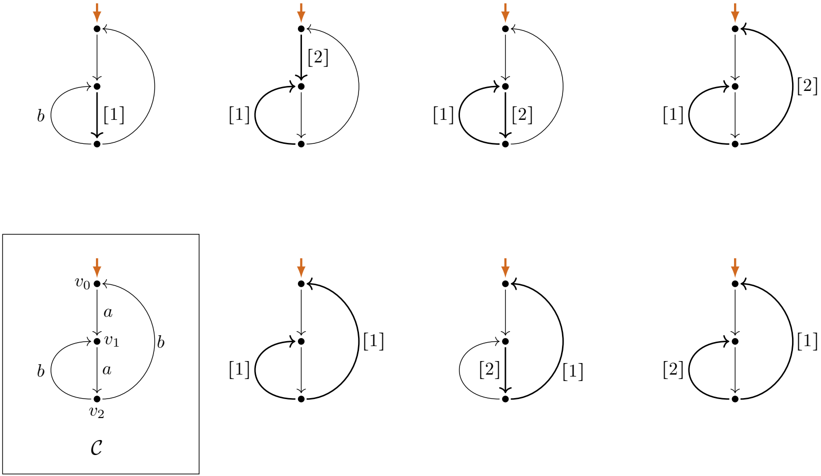
Theorem (efficient decidability)

The problem of deciding LEE(G) for process graphs G is in PTIME.

7 LEE-witnesses



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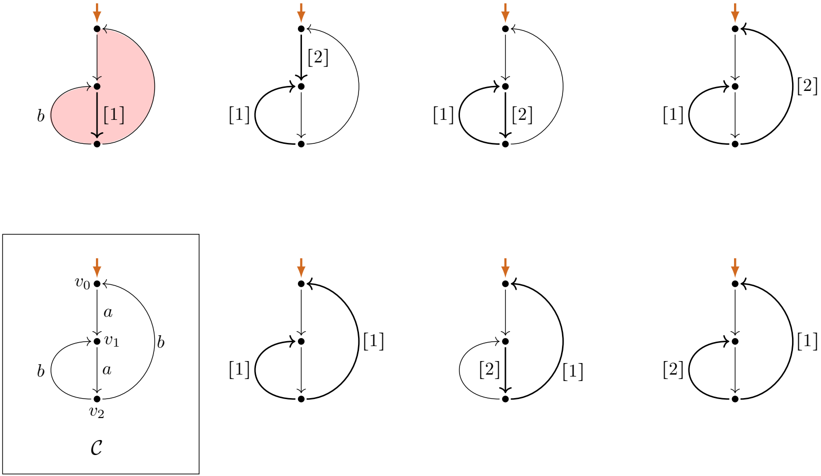

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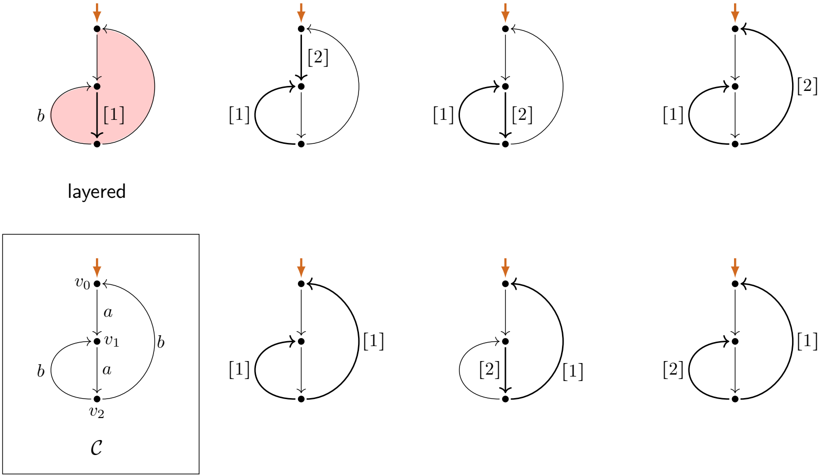

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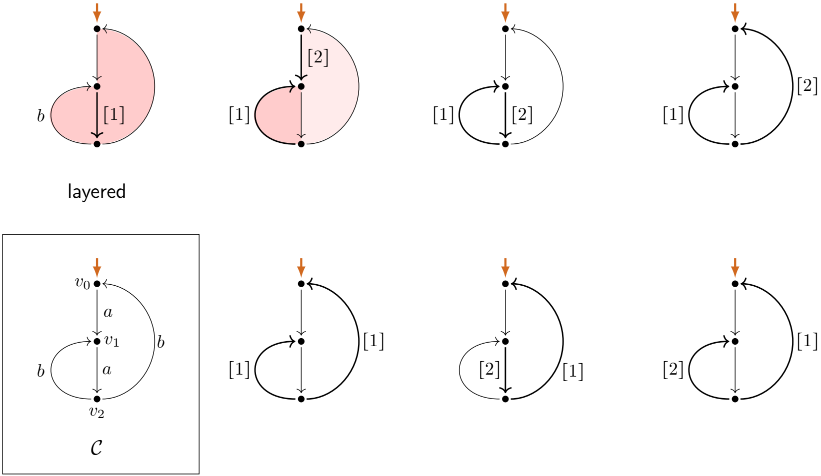

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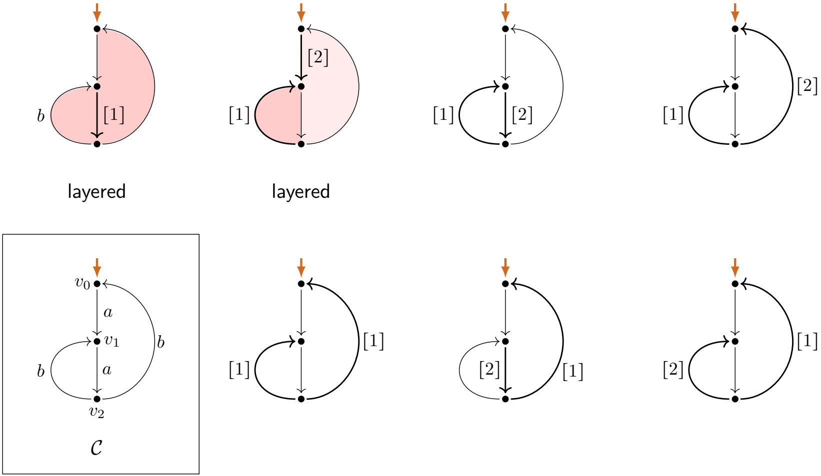

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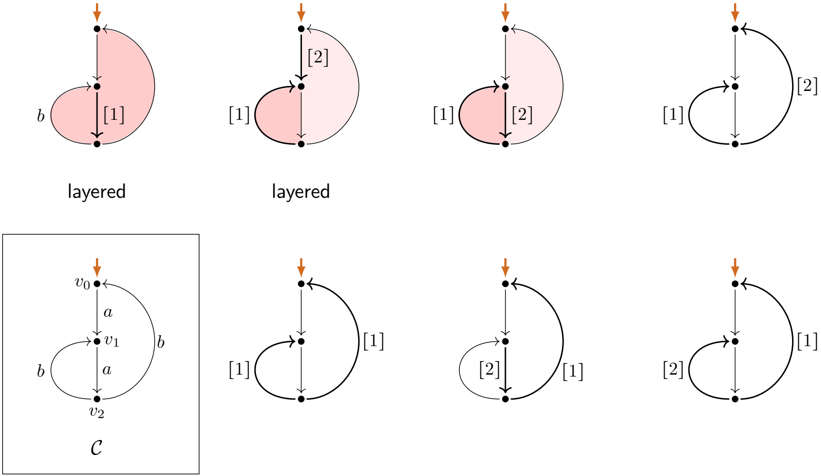

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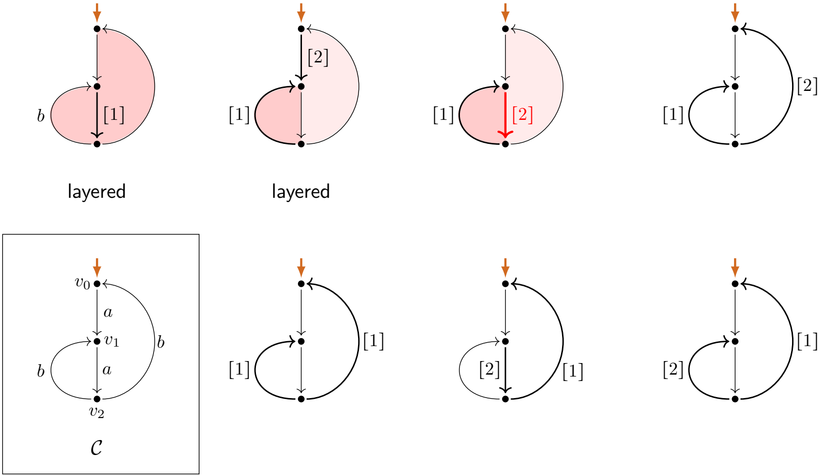

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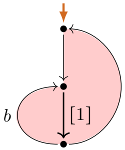

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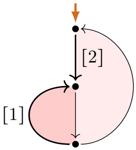

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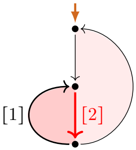

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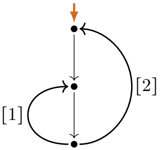

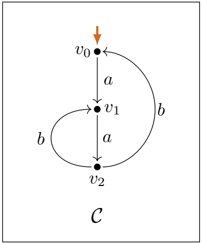

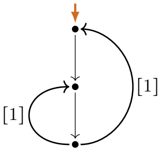

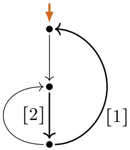

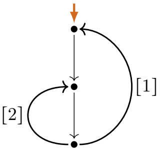

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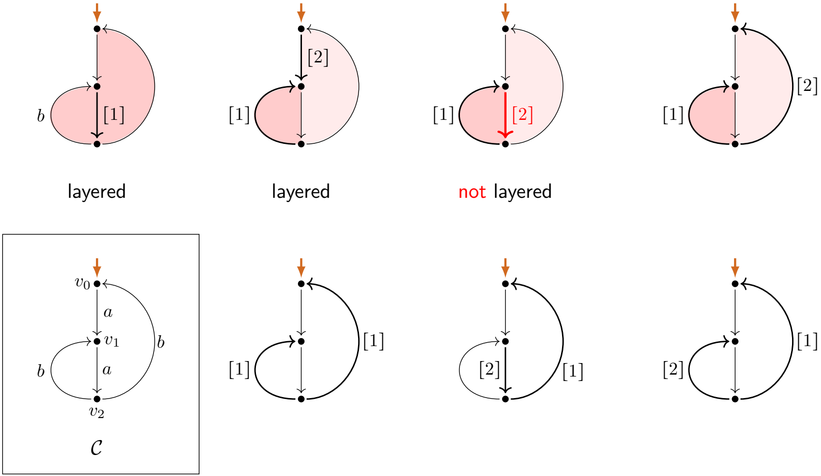

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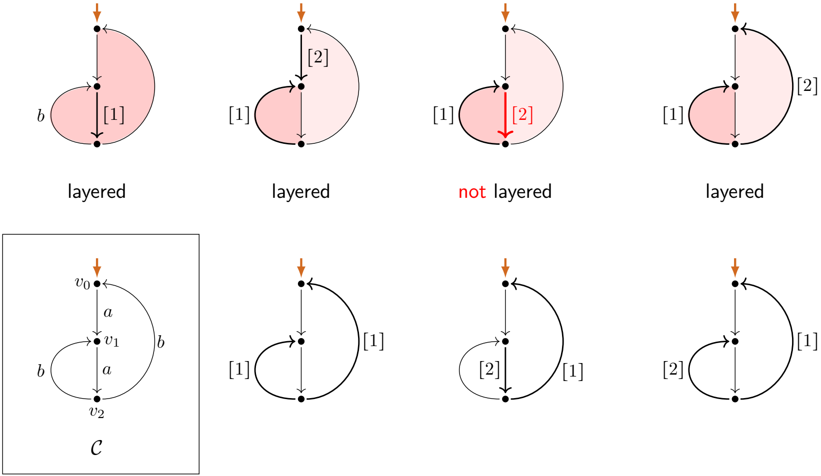

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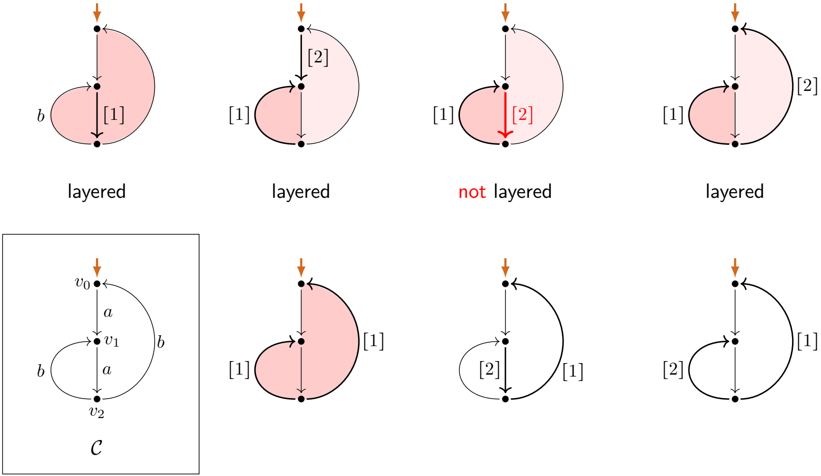

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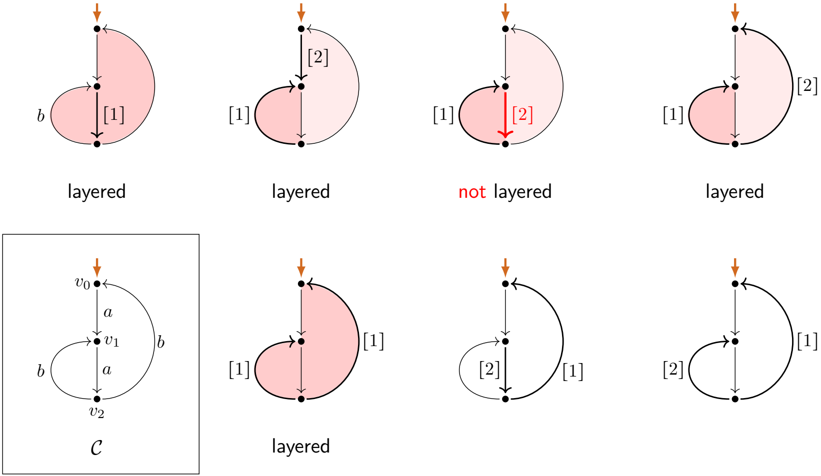
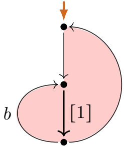


7 LEE-witnesses



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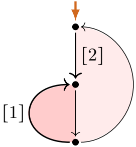

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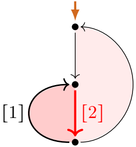

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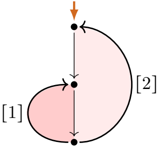

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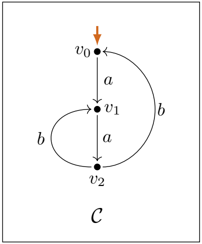

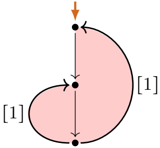

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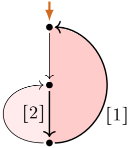
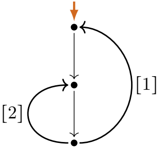
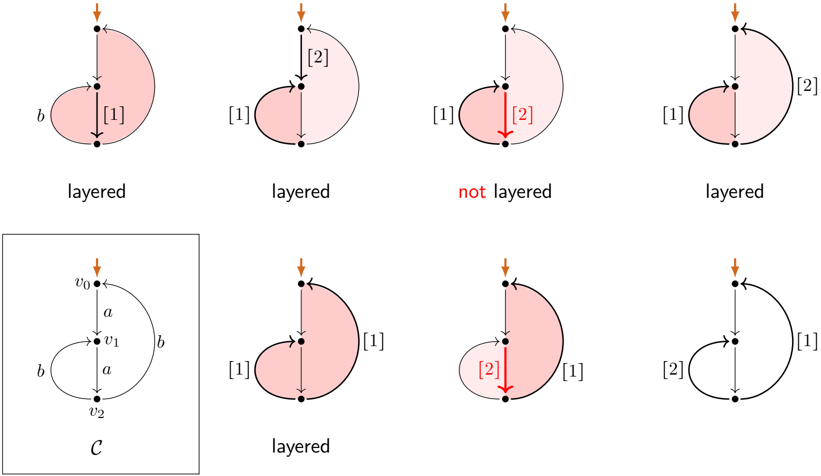
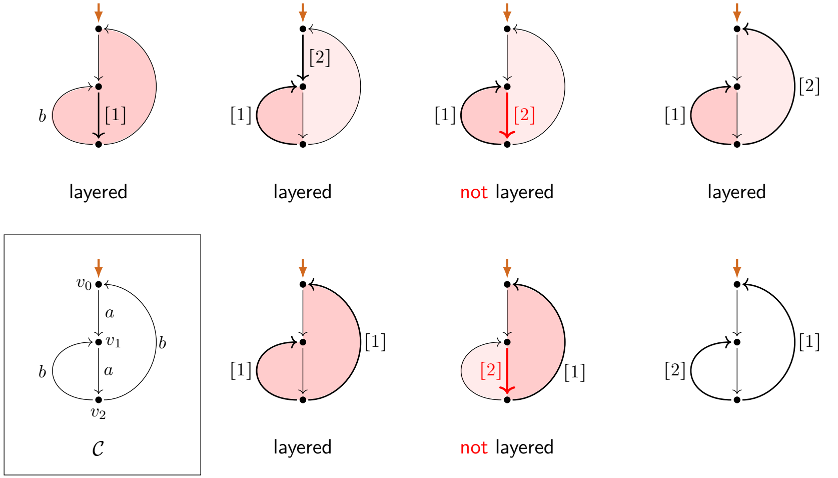




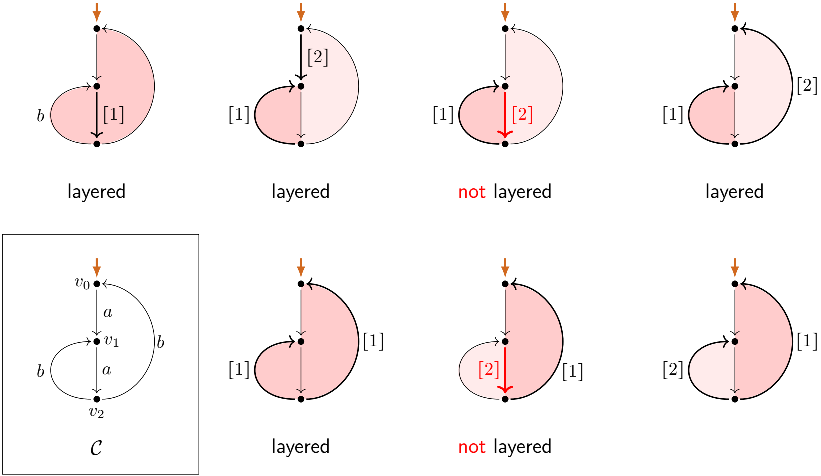
7 LEE-witnesses



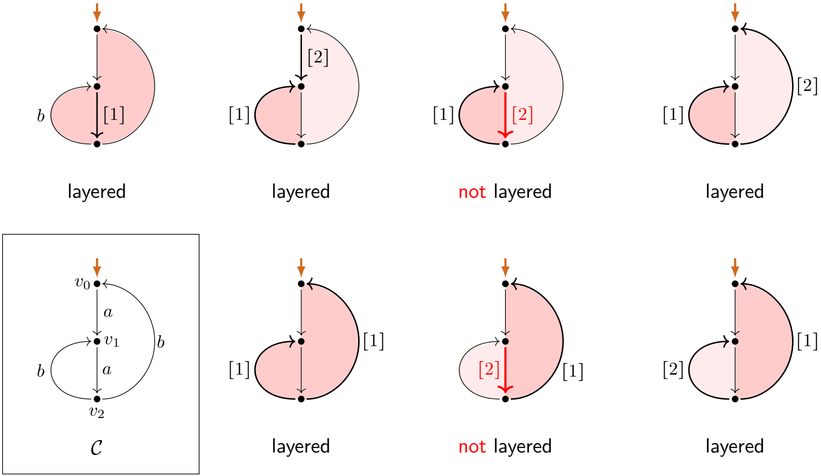
7 LEE-witnesses



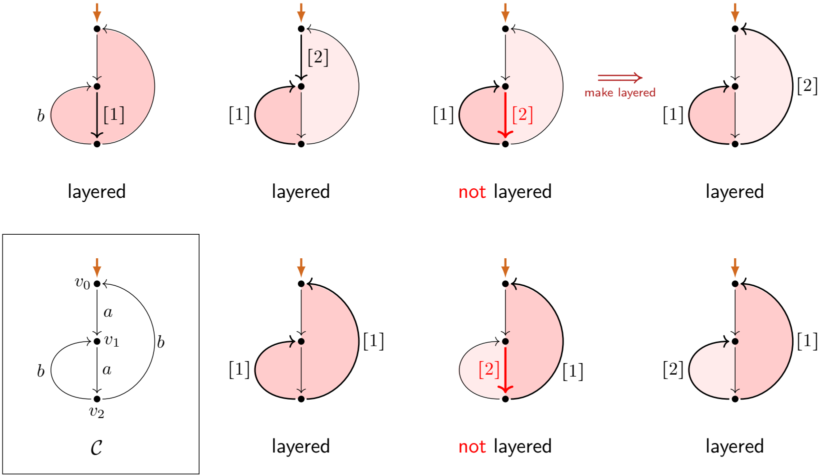
7 LEE-witnesses



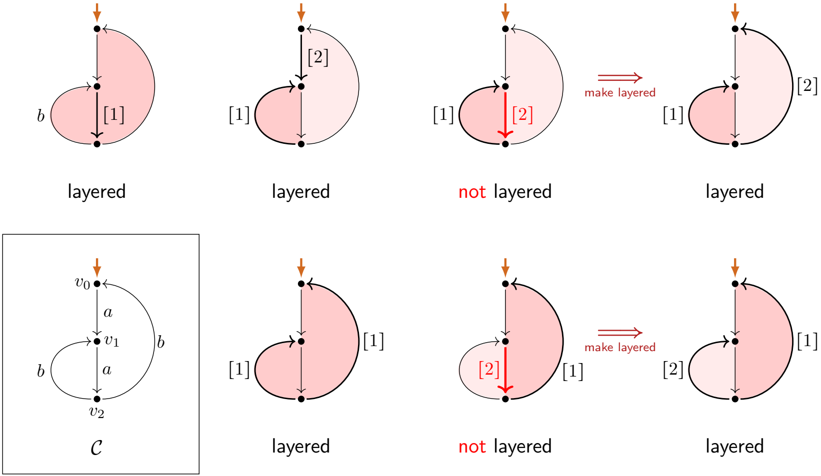
7 LEE-witnesses



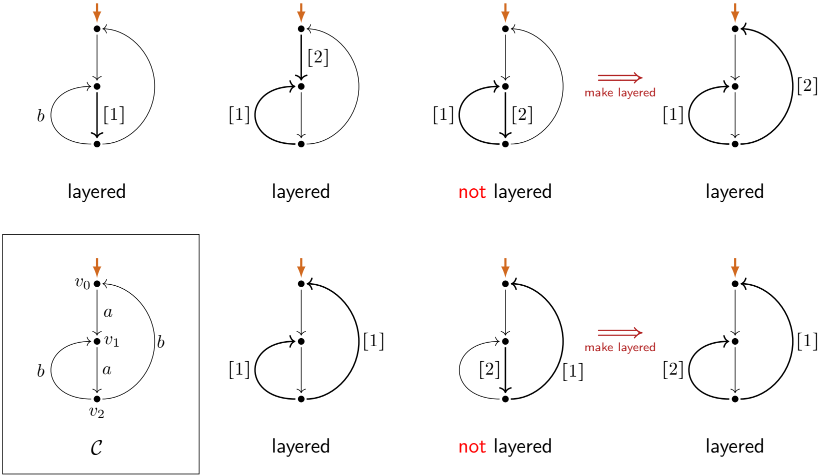
7 LEE-witnesses



7 LEE-witnesses



7 LEE-witnesses



Interpretation/extraction correspondences with LEE

(⇐ G/Fokkink 2020, G 2021)

(Int)$_{P}^{(*/\cancel{1})}$: *$P^{\bullet}$-$(*/\cancel{1})$-expressible graphs have structural property* LEE

Process **i**nterpretations $P(e)$
of $(*/\cancel{1})$ regular expressions e
are finite process graphs that satisfy LEE.

(Extr)$_{P}$: LEE *implies* $[\![\cdot]\!]_{P}$*-expressibility*

From every finite process graph G with LEE
a regular expression e can be **e**xtracted
such that $G \underline{\leftrightarrow} P(e)$.

Interpretation/extraction correspondences with LEE

($\Leftarrow$ G/Fokkink 2020, G 2021)

(Int)$_{P}^{(*/\underline{1})}$: $P^{\bullet}$-$(*/\underline{1})$*-expressible graphs have structural property* LEE

Process **i**nterpretations $P(e)$
of $(*/\underline{1})$ regular expressions e
are finite process graphs that satisfy LEE.

(Extr)$_{P}$: LEE *implies* $[\![\cdot]\!]_{P}$*-expressibility*

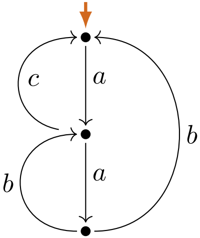
From every finite process graph G with LEE
a regular expression e can be **e**xtracted
such that $G \leftrightarrow P(e)$.

(Coll): LEE *is preserved under collapse*

The class of finite process graphs with LEE
is closed under bisimulation **c**ollapse.

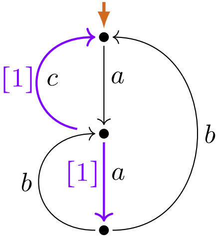
Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

G_4



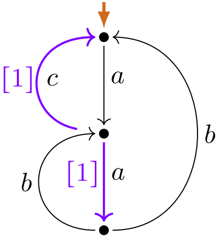
Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

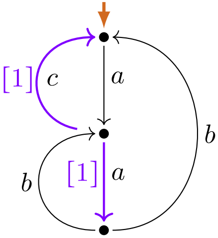
$\widehat{G_4}$



$$(\qquad\qquad\qquad\qquad\qquad)^* \cdot 0$$

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$

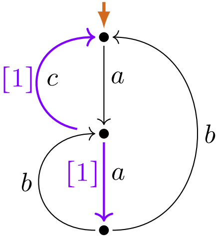


$\downarrow a$

$$\Big(\qquad\qquad \Big)^* \cdot 0$$

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



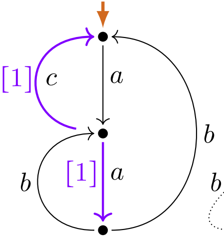
$$(a \cdot 1) \cdot ((\qquad\qquad\qquad)^* \cdot 0)$$

$$\downarrow a$$

$$(\qquad\qquad\qquad)^* \cdot 0$$

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$(a \cdot 1) \cdot ((\qquad)^{*} \cdot 0)$

a

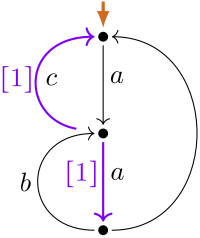
$(\qquad)^{*} \cdot 0$

b

b

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$(a \cdot 1) \cdot ((\qquad)^* \cdot 0)$

$\downarrow a$

$(\qquad)^* \cdot 0$

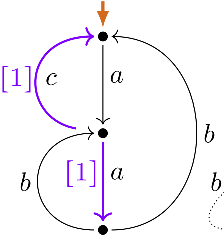
b

$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot ((\qquad)^*) \cdot 0)$

b

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$(a\cdot 1)\cdot((\qquad\qquad)^{*}\cdot 0)$

a $\quad$ c $\quad$ b

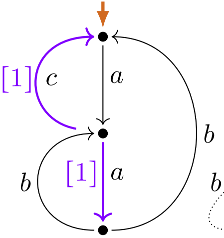
$(\qquad\qquad)^{*}\cdot 0$

a $\quad$ b

$(b\cdot 1 + b\cdot(a\cdot 1))\cdot((\qquad\qquad)^{*})\cdot 0)$

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$$(a \cdot 1) \cdot \Big(\Big(\qquad\qquad\qquad\qquad \Big)^* \cdot 0 \Big)$$

$$\downarrow a$$

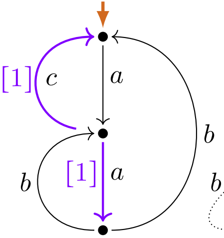
$$\Big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \Big)^* \cdot 0$$

$$\downarrow a$$

$$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot \Big(\Big(\qquad\qquad\qquad\qquad \Big)^* \Big) \cdot 0 \Big)$$

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$$(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0 \Big)$$

$$\downarrow a$$

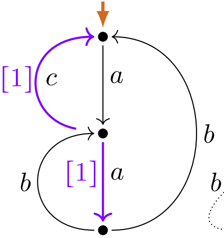
$$\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0$$

$$\downarrow a$$

$$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \Big) \cdot 0 \Big)$$

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$$\overbrace{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)}^{e}$$

$$\downarrow a$$

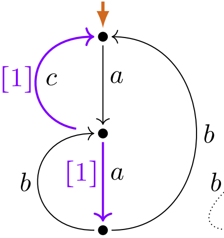
$$\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0$$

$$\downarrow a$$

$$(b\cdot 1+b\cdot(a\cdot 1))\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0\Big)$$

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$P(e) \rightleftharpoons G_4$

$$\overbrace{(a \cdot 1) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0 \big)}^{e}$$

$$\downarrow a$$

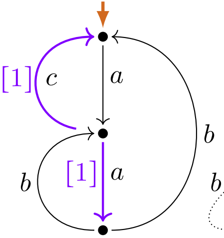
$$\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0$$

$$\downarrow a$$

$$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \big) \cdot 0 \big)$$

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$P(e) \mathrel{\underline{\to}} G_4$

$$\overbrace{(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0 \Big)}^{e}$$

$$\cancel{\downarrow}\, a$$

$$\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0$$

$$\downarrow a$$

$$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*}) \cdot 0 \Big)$$

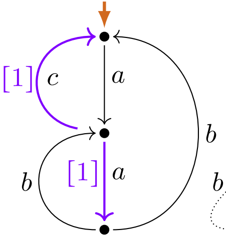
c

b

b

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$$\mathit{P}(e) \trianglerighteq G_4$$

$$\overbrace{(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \cdot 0 \Big)}^{e}$$

$$\downarrow a$$

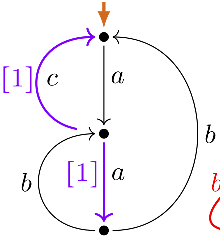
$$\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \cdot 0$$

$$\downarrow a$$

$$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \big) \cdot 0 \Big)$$

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$P(e) \rightleftharpoons G_4$

$$\overbrace{(a \cdot 1) \cdot \left(\left(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \right)^* \cdot 0 \right)}^{e}$$

$$\downarrow a$$

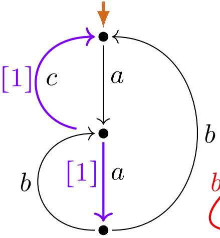
$$\left(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \right)^* \cdot 0$$

$$\downarrow a$$

$$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot \left(\left(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \right)^* \right) \cdot 0 \Big)$$

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_4}$



$$P(e) \mathrel{\underline{\to}} G_4 \mathrel{\not\underline{\leftrightarrow}} P(e)$$

$$\overbrace{(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^{*} \cdot 0\Big)}^{e}$$

$$\Big\downarrow a$$

$$\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^{*} \cdot 0$$

$$\Big\downarrow a$$

$$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^{*}\Big) \cdot 0\Big)$$

Interpretation of extracted expression

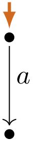
G_5

$$\mathsf{P}(e) = G_5$$

$$\overbrace{(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^{*} \cdot 0\Big)}^{e}$$

Interpretation of extracted expression

G_5



$$P(e) = G_5$$

$$\overbrace{(a \cdot 1) \cdot \left(\left(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \right)^* \cdot 0 \right)}^{e}$$

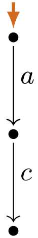
$$\downarrow a$$

$$(1 \cdot 1) \cdot \left(\left(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \right)^* \cdot 0 \right)$$

Interpretation of extracted expression

G_5

$P(e) = G_5$



$$\overbrace{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)}^{e}$$

$$\downarrow a$$

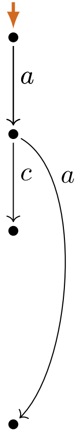
$$(1\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)$$

$$\downarrow c$$

$$\Big((1\cdot(a\cdot 1))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0$$

Interpretation of extracted expression

G_5



$$P(e) = G_5$$

$$\overbrace{(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0\Big)}^{e}$$

$$\downarrow a$$

$$(1 \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0\Big)$$

$$\downarrow c$$

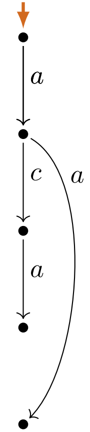
$$\Big((1 \cdot (a \cdot 1)) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^*\Big) \cdot 0$$

$$a$$

$$\Big((1 \cdot (b \cdot 1 + b \cdot (a \cdot 1))) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^*\Big) \cdot 0$$

Interpretation of extracted expression

G_5



$P(e) = G_5$

$$\overbrace{(a\cdot 1)\cdot\big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\big)}^{e}$$

$$\downarrow a$$

$$(1\cdot 1)\cdot\big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\big)$$

$$\downarrow c$$

$$\big((1\cdot(a\cdot 1))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0$$

$$\downarrow a$$

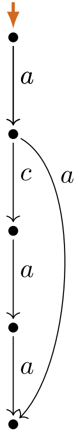
$$\big((1\cdot 1)\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0\big)$$

$$a$$

$$\big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0$$

Interpretation of extracted expression

G_5



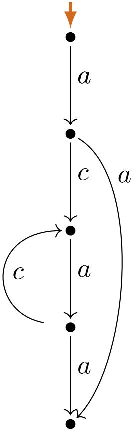
$$P(e) = G_5$$

$$
\begin{array}{c}
\overbrace{(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0 \Big)}^{e} \\
\downarrow a \\
(1 \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0 \Big) \\
\downarrow c \\
\Big((1 \cdot (a \cdot 1)) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \Big) \cdot 0 \\
\downarrow a \\
\Big((1 \cdot 1) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \Big) \cdot 0 \Big) \\
\downarrow a \\
\Big((1 \cdot (b \cdot 1 + b \cdot (a \cdot 1))) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \Big) \cdot 0
\end{array}
$$

a

Interpretation of extracted expression

G_5



$P(e) = G_5$

$$\overbrace{(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^{*} \cdot 0\Big)}^{e}$$

$$\downarrow a$$

$$(1 \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^{*} \cdot 0\Big)$$

$$\downarrow c$$

$$\Big((1 \cdot (a \cdot 1)) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^{*}\Big) \cdot 0$$

$$\downarrow a$$

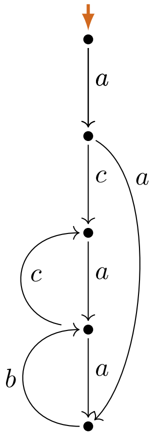
$$\Big((1 \cdot 1) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^{*}\Big) \cdot 0\Big)$$

$$\downarrow a$$

$$\Big((1 \cdot (b \cdot 1 + b \cdot (a \cdot 1))) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^{*}\Big) \cdot 0$$

Interpretation of extracted expression

G_5



$P(e) = G_5$

$$\overbrace{(a \cdot 1) \cdot \big((c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)))^{*} \cdot 0 \big)}^{e}$$

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$$(1 \cdot 1) \cdot \big((c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)))^{*} \cdot 0 \big)$$

$$\downarrow c$$

$$\big((1 \cdot (a \cdot 1)) \cdot (c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)))^{*} \big) \cdot 0$$

$$\downarrow a$$

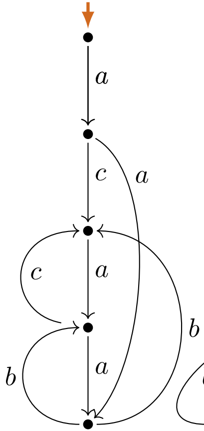
$$\big((1 \cdot 1) \cdot (c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)))^{*} \big) \cdot 0$$

$$\downarrow a$$

$$\big((1 \cdot (b \cdot 1 + b \cdot (a \cdot 1))) \cdot (c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)))^{*} \big) \cdot 0$$

Interpretation of extracted expression

G_5



$\textsf{P}(e) = G_5$

$$\overbrace{(a\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)}^{e}$$

$$\downarrow a$$

$$(1\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)$$

$$\downarrow c$$

$$\big((1\cdot(a\cdot 1))\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\big)\cdot 0$$

$$\downarrow a$$

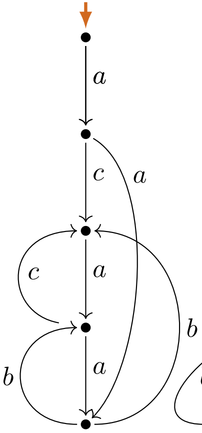
$$\big((1\cdot 1)\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\big)\cdot 0\big)$$

$$\downarrow a$$

$$\big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\big)\cdot 0$$

Interpretation of extracted expression

G_5



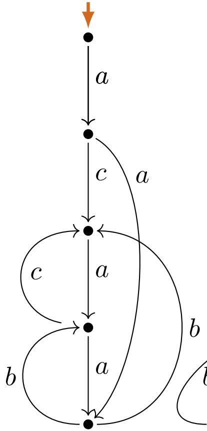
$$P(e) = G_5 \rightrightarrows G_4$$

$$\begin{array}{c}
\overbrace{(a\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)}^{e} \\
\big\downarrow a \\
(1\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big) \\
\big\downarrow c \\
\big((1\cdot(a\cdot 1))\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\big)\cdot 0 \\
\big\downarrow a \\
\big((1\cdot 1)\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\big)\cdot 0\big) \\
\big\downarrow a \\
\big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\big)\cdot 0
\end{array}$$

Interpretation of extracted expression

G_5

$P(e) = G_5 \Rightarrow G_4 \not\simeq G_5$



$$\overbrace{(a \cdot 1) \cdot \big((c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)))^* \cdot 0 \big)}^{e}$$

$$\downarrow a$$

$$(1 \cdot 1) \cdot \big((c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)))^* \cdot 0 \big)$$

$$\downarrow c$$

$$\big((1 \cdot (a \cdot 1)) \cdot (c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)))^* \big) \cdot 0$$

$$\downarrow a$$

$$\big((1 \cdot 1) \cdot (c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)))^* \big) \cdot 0 \big)$$

$$\downarrow a$$

$$\big((1 \cdot (b \cdot 1 + b \cdot (a \cdot 1))) \cdot (c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)))^* \big) \cdot 0$$

LEE under bisimulation?

LEE under bisimulation

Observation

- LEE is not invariant under bisimulation.

LEE under bisimulation

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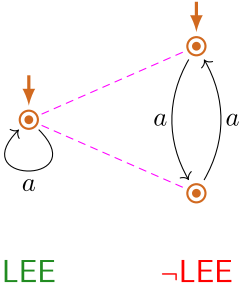
- LEE is **not** invariant under bisimulation.



LEE under bisimulation

Observation

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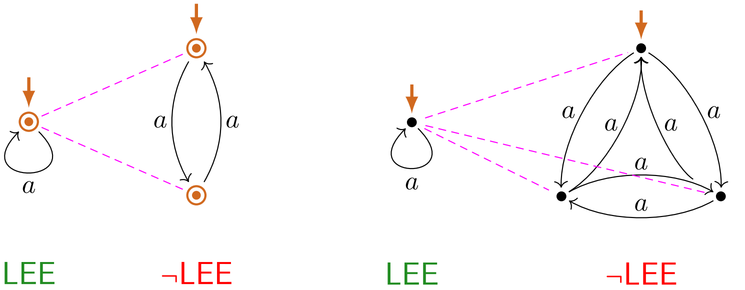
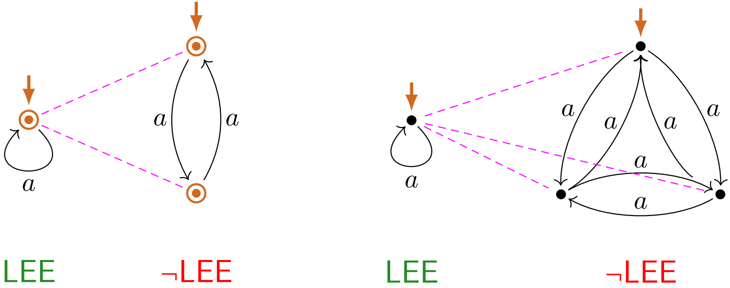


LEE under bisimulation

Observation

- LEE is not invariant under bisimulation.
- LEE is not preserved by converse functional bisimulation.



LEE under functional bisimulation

Lemma

(i) LEE *is preserved by functional bisimulations:*

$$\mathsf{LEE}(G_1) \wedge G_1 \mathrel{\underline{\rightarrow}} G_2 \implies \mathsf{LEE}(G_2)\,.$$

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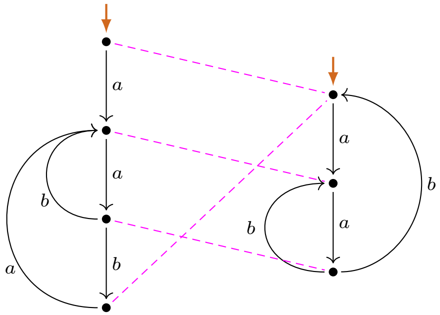
Proof (Idea).

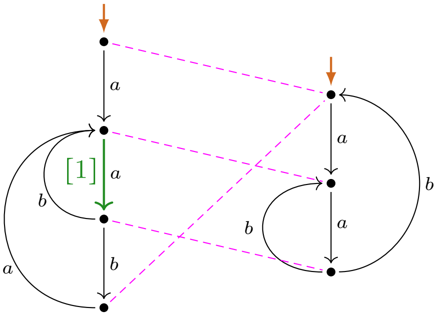
Use loop elimination in G_1 to carry out loop elimination in G_2.

Collapsing LEE-witnesses



$P(a(a(b+ba))^* \cdot 0)$

Collapsing LEE-witnesses



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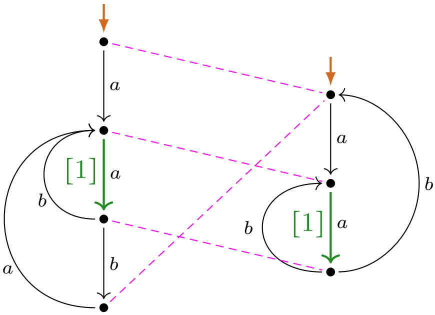

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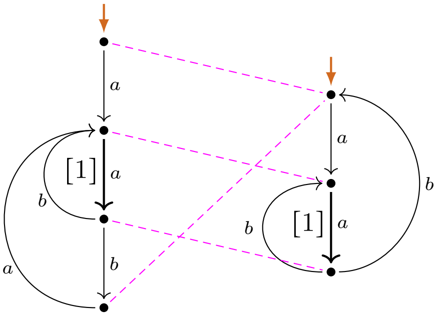

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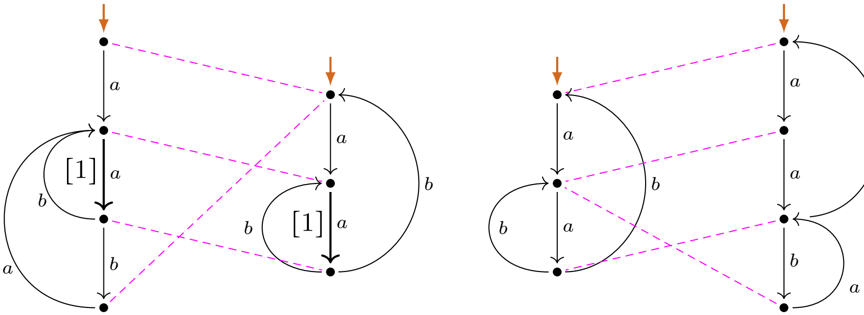

$P(a(a(b+ba))^* \cdot 0)$

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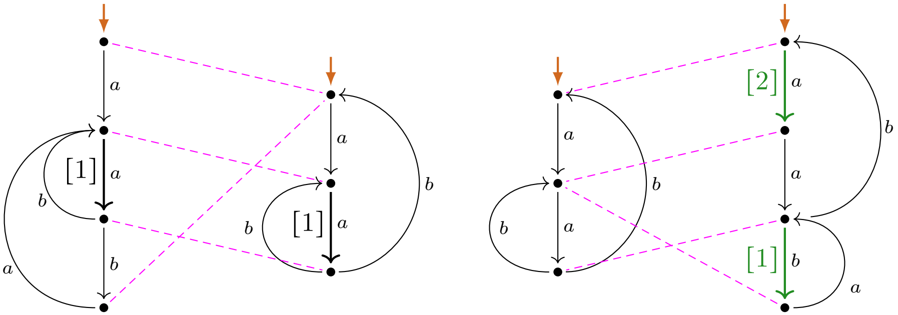
Collapsing LEE-witnesses



$\mathsf{P}(a(a(b+ba))^* \cdot 0)$

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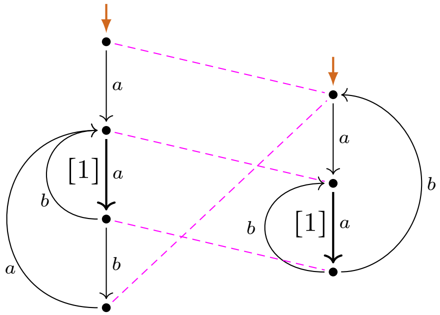

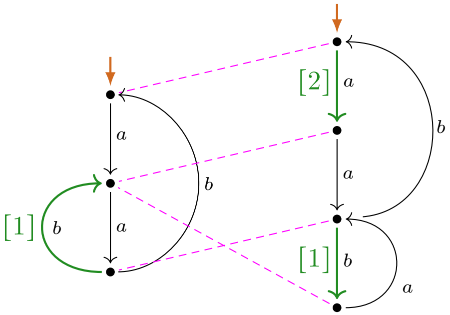

$P(a(a(b+ba))^* \cdot 0)$

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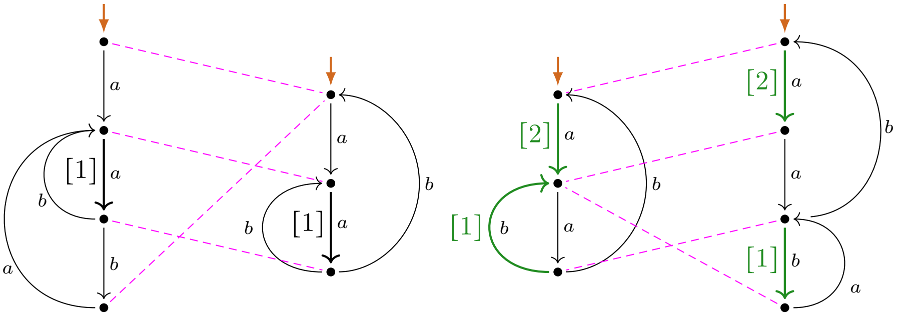

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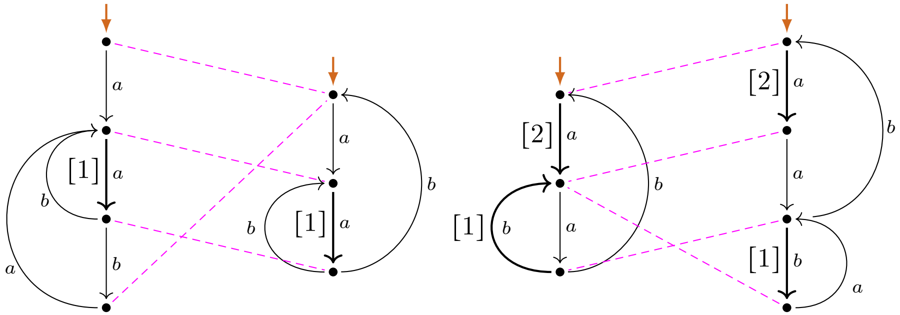

$P(a(a(b+ba))^{*} \cdot 0)$

$P((aa(ba)^{*} \cdot b)^{*} \cdot 0)$

LEE under functional bisimulation

Lemma

(i) LEE *is preserved by functional bisimulations:*

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Idea of Proof for (i)

Use loop elimination in G_1 to carry out loop elimination in G_2.

LEE under functional bisimulation / bisimulation collapse

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(i) LEE *is preserved by functional bisimulations:*

$$\mathsf{LEE}(G_1) \wedge G_1 \mathrel{\underline{\to}} G_2 \implies \mathsf{LEE}(G_2)\,.$$

(ii) LEE *is preserved from a process graph to its bisimulation collapse:*

$$\mathsf{LEE}(G) \wedge G \textit{ has bisimulation collapse } C \implies \mathsf{LEE}(C)\,.$$

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Use loop elimination in G_1 to carry out loop elimination in G_2.

- images of loop subcharts in G_1 under $\mathrel{\underline{\to}}$ are loop subcharts of G_2.

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- LEE is preserved by dropping transition-induced subgraphs.

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(i) LEE *is preserved by functional bisimulations:*

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Idea of Proof for (i)

Use loop elimination in G_1 to carry out loop elimination in G_2.

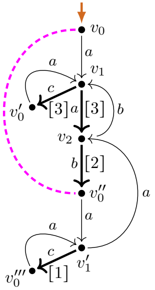
- images of loop subcharts in G_1 under $\mathrel{\underline{\rightarrow}}$ are loop subcharts of G_2.
- eliminating a loop subchart from G_2 amounts, via $\mathrel{\underline{\rightarrow}}$, to eliminating a transition induced subgraph from G_1.
- LEE is preserved by dropping transition-induced subgraphs.

Due to $\mathsf{LEE}(G_1)$, then such loop elimination in G_2 terminates in a graph without an infinite trace. This establishes $\mathsf{LEE}(G_2)$.

LLEE-preserving collapse (example, corollary)

Lemma (C)

The bisimulation collapse of a LLEE-graph is again a LLEE-graph.



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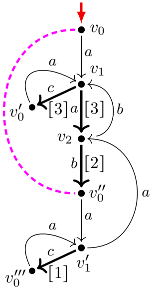

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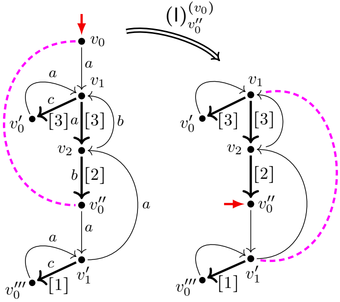

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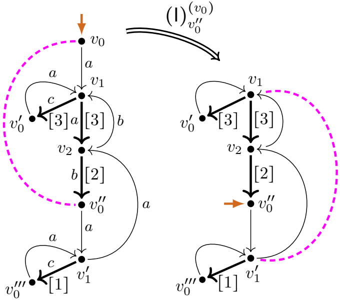

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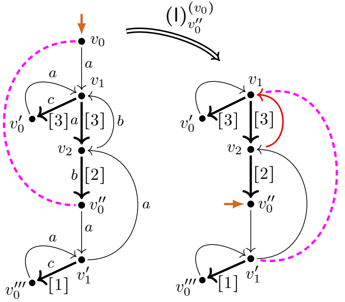

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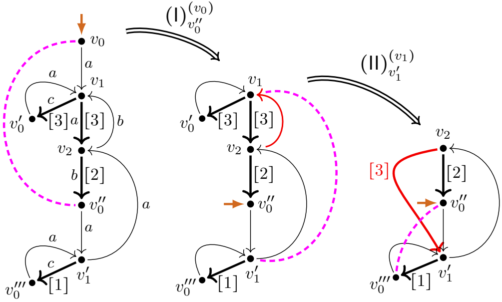

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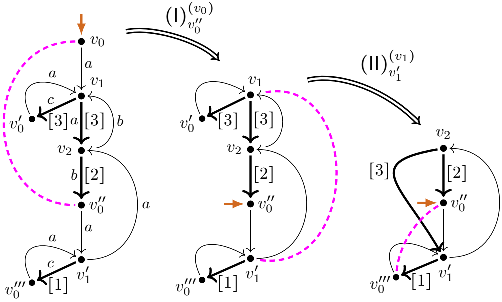

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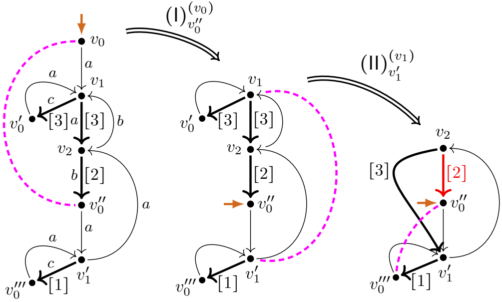

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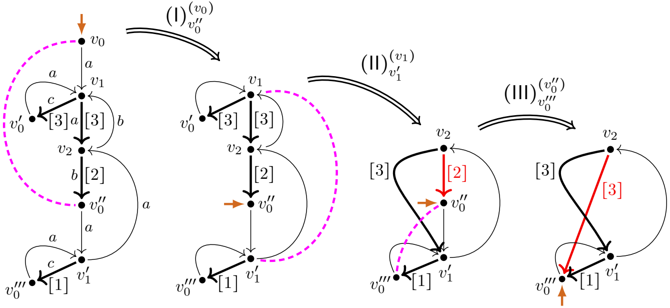

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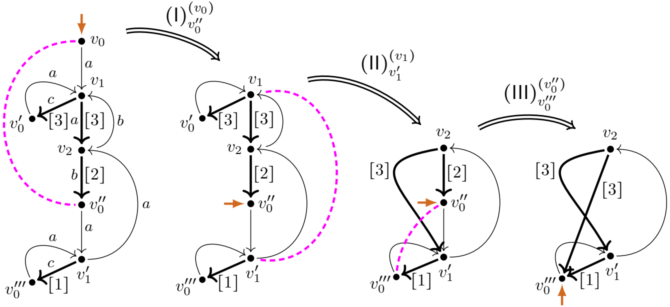

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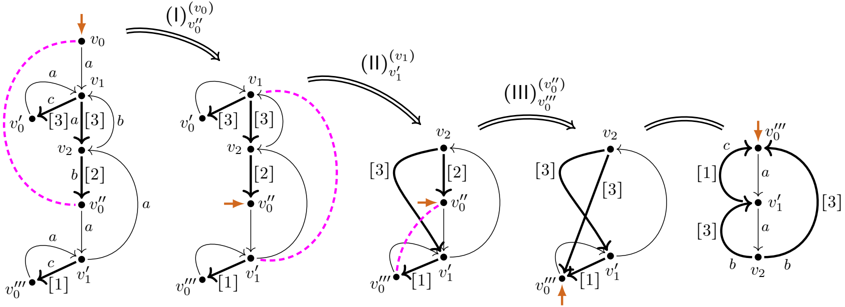

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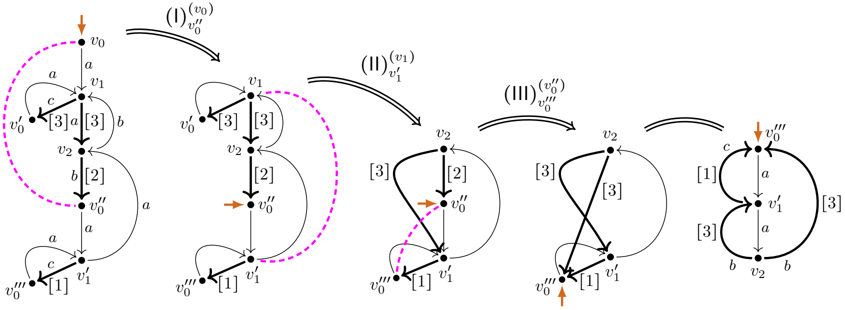

Corollary

A process graph is $[\![\cdot]\!]_P$-expressible by an $(*/\underline{1})$ regular expression
if and only if its bisimulation collapse is a LLEE-graph.

Image of P is **not** closed under bisimulation collapse

$P(uf)$

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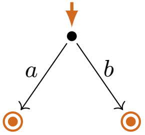
$$uf := a \cdot \overbrace{(a \cdot (a + a \cdot 0))^*}^{uf_a} + b \cdot \overbrace{(b \cdot (b + b \cdot 0))^*}^{uf_b}$$

Image of P is **not** closed under bisimulation collapse

$P(uf)$



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$$uf \xrightarrow{a} 1 \cdot uf_a \Downarrow \qquad uf \xrightarrow{b} \Downarrow 1 \cdot uf_b$$

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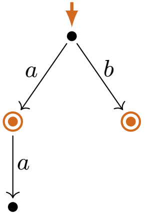

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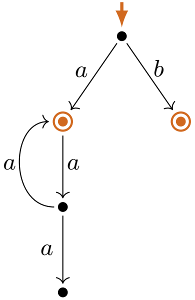
$$uf \xrightarrow{a} 1 \cdot uf_a \Downarrow \qquad\qquad uf \xrightarrow{b} \Downarrow 1 \cdot uf_b$$

$$1 \cdot uf_a \xrightarrow{a} (1 \cdot (a + a \cdot 0)) \cdot uf_a$$

Image of P is **not** closed under bisimulation collapse

$P(uf)$



$P(uf)$

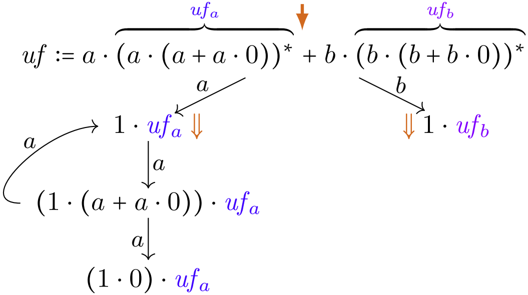
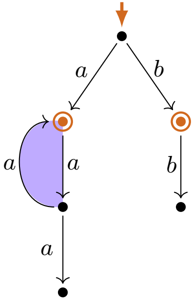


Image of P is **not** closed under bisimulation collapse

$P(uf)$



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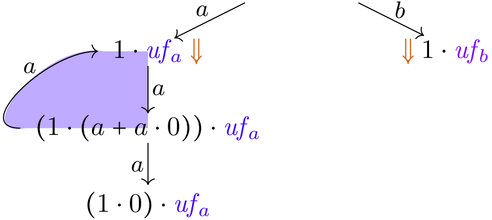
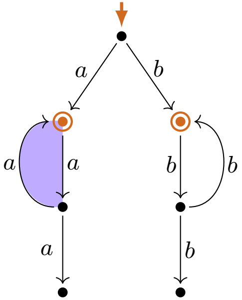


Image of *P* is **not** closed under bisimulation collapse

P(uf)



P(uf)

$$uf := a \cdot \overbrace{(a \cdot (a + a \cdot 0))^*}^{uf_a} + b \cdot \overbrace{(b \cdot (b + b \cdot 0))^*}^{uf_b}$$

$$uf \xrightarrow{a} 1 \cdot uf_a \Downarrow \qquad uf \xrightarrow{b} \Downarrow 1 \cdot uf_b$$

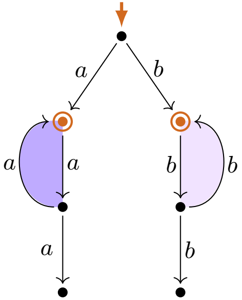
$$1 \cdot uf_a \xrightarrow{a} (1 \cdot (a + a \cdot 0)) \cdot uf_a \xrightarrow{a} (1 \cdot 0) \cdot uf_a$$

$$(1 \cdot (a + a \cdot 0)) \cdot uf_a \xrightarrow{a} 1 \cdot uf_a$$

$$1 \cdot uf_b \xrightarrow{b} (1 \cdot (b + b \cdot 0)) \cdot uf_b$$

Image of P is **not** closed under bisimulation collapse

$P(uf)$



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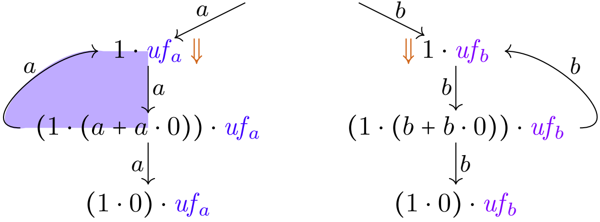

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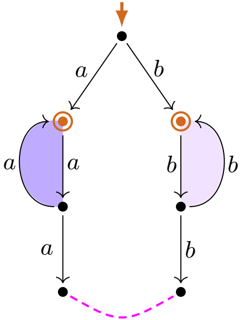

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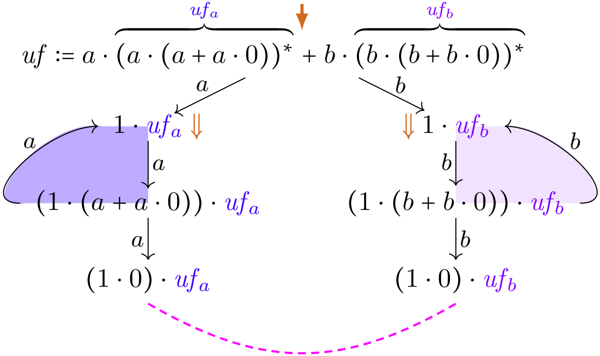

Compact process interpretation $P^{\bullet}$

Definition (Transition system specification $\mathcal{T}$)

$$\frac{}{1\Downarrow} \qquad \frac{e_i\Downarrow}{(e_1 + e_2)\Downarrow}\ (i \in \{1,2\}) \qquad \frac{e_1\Downarrow \qquad e_2\Downarrow}{(e_1 \cdot e_2)\Downarrow} \qquad \frac{}{(e^*)\Downarrow}$$

$$\frac{}{a \xrightarrow{a} 1} \qquad \frac{e_i \xrightarrow{a} e_i'}{e_1 + e_2 \xrightarrow{a} e_i'}\ (i \in \{1,2\})$$

$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2} \qquad \frac{e_1\Downarrow \qquad e_2 \xrightarrow{a} e_2'}{e_1 \cdot e_2 \xrightarrow{a} e_2'} \qquad \frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e' \cdot e^*}$$

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Compact process interpretation $P^{\bullet}$

Definition (Transition system specification $\mathcal{T}^{\bullet}$, changed rules w.r.t. $\mathcal{T}$)

$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2} \text{ (if } e_1' \text{ is normed)}$$

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$$\frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e' \cdot e^*} \text{ (if } e' \text{ is normed)} \qquad \frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e'} \text{ (if } e' \text{ is not normed)}$$

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Definition

The compact process (graph) interpretation $P^{\bullet}(e)$ of a reg. expr's e:

$P^{\bullet}(e) :=$ labeled transition graph generated by e by derivations in $\mathcal{T}^{\bullet}$.

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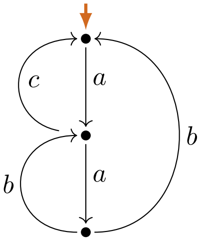
Lemma ($P^{\bullet}$ increases sharing; $P^{\bullet}$, P have same bisimulation semantics)

(i) $P(e) \Rightarrow P^{\bullet}(e)$ *for all regular expressions* e.

(ii) (G *is* $[\![\cdot]\!]_{P^{\bullet}}$*-expressible* $\iff$ G *is* $[\![\cdot]\!]_{P}$*-expressible) for all graphs* G.

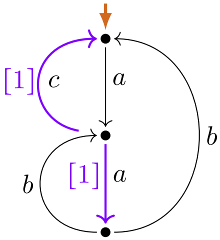
Refined extraction expression (example)

G_4



Refined extraction expression (example)

$\widehat{G_4}$



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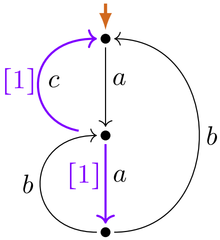

$(1 \cdot (\qquad\qquad\qquad)^*) \cdot 0$

Refined extraction expression (example)

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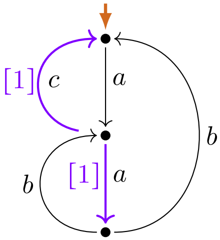
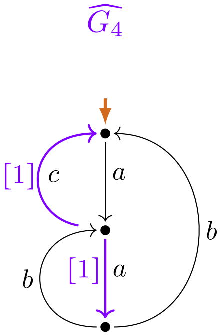


$$\overset{a}{\dashrightarrow}$$

$$(1 \cdot (\qquad\qquad)^*) \cdot 0$$

Refined extraction expression (example)



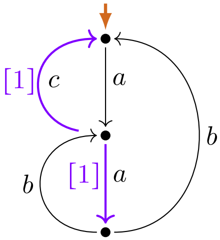
$$((1 \cdot a) \cdot (\qquad\qquad)^*) \cdot 0$$

$$\Big\downarrow a$$

$$(1 \cdot (\qquad\qquad)^*) \cdot 0$$

Refined extraction expression (example)

$\widehat{G_4}$



$((1 \cdot a) \cdot ($ $\quad)^*) \cdot 0$

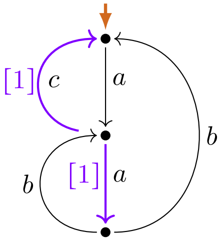
a

$(1 \cdot ($ $\quad)^*) \cdot 0$

b $\quad b$

Refined extraction expression (example)

$\widehat{G_4}$



$((1 \cdot a) \cdot (\qquad\qquad)^*) \cdot 0$

a

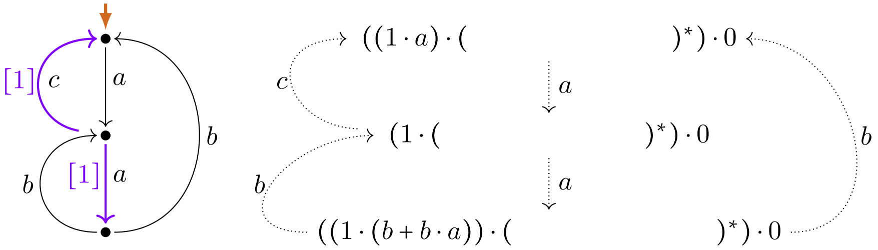
$(1 \cdot (\qquad\qquad)^*) \cdot 0$

$b \qquad b$

$((1 \cdot (b + b \cdot a)) \cdot (\qquad\qquad)^*) \cdot 0$

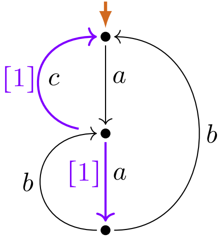
Refined extraction expression (example)

$\widehat{G_4}$



Refined extraction expression (example)

$\widehat{G_4}$



$$((1\cdot a)\cdot(\qquad\qquad\qquad\qquad)^*)\cdot 0$$

c $\quad$ $\downarrow a$ $\quad$ b

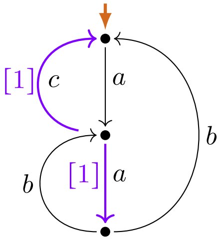
$$(1\cdot(c\cdot a + a\cdot(b+b\cdot a))^*)\cdot 0$$

b $\quad$ $\downarrow a$

$$((1\cdot(b+b\cdot a))\cdot(\qquad\qquad\qquad\qquad)^*)\cdot 0$$

Refined extraction expression (example)

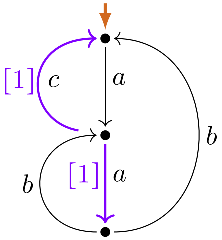
$\widehat{G_4}$



$$\begin{tikzcd}
((1 \cdot a) \cdot (c \cdot a + a \cdot (b + b \cdot a))^{*}) \cdot 0 \arrow[d, dotted, "a"] \\
(1 \cdot (c \cdot a + a \cdot (b + b \cdot a))^{*}) \cdot 0 \arrow[d, dotted, "a"] \arrow[u, dotted, bend left, "c"] \\
((1 \cdot (b + b \cdot a)) \cdot (c \cdot a + a \cdot (b + b \cdot a))^{*}) \cdot 0 \arrow[u, dotted, bend left, "b"] \arrow[uu, dotted, bend right, "b"']
\end{tikzcd}$$

Refined extraction expression (example)

$\widehat{G_4}$



$$\begin{tikzcd}
((1 \cdot a) \cdot (c \cdot a + a \cdot (b + b \cdot a))^*) \cdot 0 \arrow[d, "a"] \\
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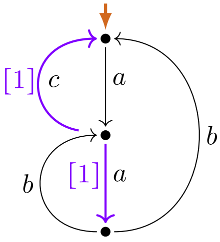
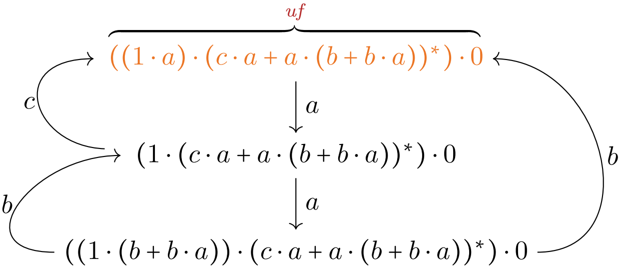
\end{tikzcd}$$

Refined extraction expression (example)

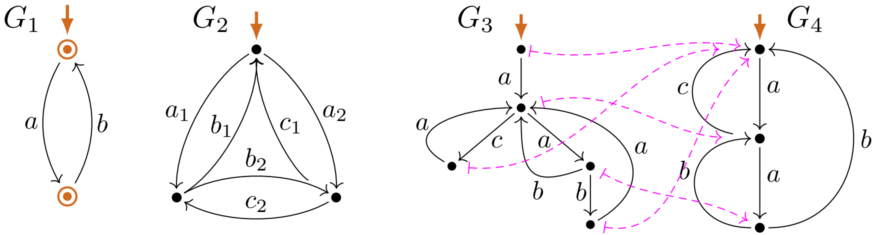
$\widehat{G_4}$

$P^{\bullet}(uf) = P(uf) \simeq G_4$





P-expressibility and $[\![\cdot]\!]_P$-expressibility (examples revisited)



not P-expressible P-/$P^{\bullet}$-expressible $P^{\bullet}$-expressible

not $[\![\cdot]\!]_P$-expressible $[\![\cdot]\!]_P$-expressible $[\![\cdot]\!]_P$-expressible

Summary and outlook

- 1-free/under-star-1-free $(\cancel{1}\backslash *)$ reg. expr'ss defined (also) with unary star
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Resources

- Slides/extended abstract on `clegra.github.io`
  - slides: `.../lf/TG-2024.pdf`
  - extended abstract: `.../lf/closing-bs-i-pi-us1f.pdf`

- CG, Wan Fokkink: A Complete Proof System for 1-Free Regular Expressions Modulo Bisimilarity,
  - LICS 2020, arXiv:2004.12740, video on youtube.
- CG: Modeling Terms by Graphs with Structure Constraints,
  - TERMGRAPH 2018, EPTCS 288, arXiv:1902.02010.
- CG: The Image of the Process Interpretation of Regular Expressions is Not Closed under Bisimulation Collapse,
  - arXiv:2303.08553.
- CG: Milner's Proof System for Regular Expressions Modulo Bisimilarity is Complete,
  - LICS 2022, arXiv:2209.12188, poster.

Language semantics $[\![\cdot]\!]_L$ of reg. expr's *(Copi–Elgot–Wright, 1958)*

$\mathbf{0} \xmapsto{L}$ empty language $\varnothing$

$\mathbf{1} \xmapsto{L} \{\epsilon\}$ (ϵ the empty word)

$a \xmapsto{L} \{a\}$

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$[\![e]\!]_L := L(e)$ (language defined by e)