Milner's Proof System for Regular Expressions Modulo Bisimilarity is Complete

Crystallization: Near-Collapsing Process Graph Interpretations of Regular Expressions

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Abstract

Milner (1984) defined a process semantics for regular expressions. He formulated a sound proof system for bisimilarity of process interpretations of regular expressions, and asked whether this system is complete.

We report conceptually on a proof that shows that Milner's system is complete, by motivating and describing all of its main steps. We substantially refine the completeness proof by Grabmayer and Fokkink (2020) for the restriction of Milner's system to '1-free' regular expressions. As a crucial complication we recognize that process graphs with empty-step transitions that satisfy the layered loop-existence/elimination property LLEE are not closed under bisimulation collapse (unlike process graphs with LLEE that only have proper-step transitions). We circumnavigate this obstacle by defining a LLEE-preserving 'crystallization procedure' for such process graphs. By that we obtain 'near-collapsed' process graphs with LLEE whose strongly connected components are either collapsed or of 'twin-crystal' shape. Such near-collapsed process graphs guarantee provable solutions for bisimulation collapses of process interpretations of regular expressions.

CCS Concepts: • **Theory of computation → Process calculi**; Regular languages.

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Kleene [14] (1951) introduced regular expressions, which are widely studied in formal language theory. In a typical formulation, they are constructed from constants 0, 1, letters a from some alphabet (interpreted as the formal languages $\varnothing$, $\{\epsilon\}$, and $\{a\}$, where ϵ is the empty word) and binary operators $+$ and $\cdot$, and the unary Kleene star * (which are interpreted as language union, concatentation, and iteration).

Milner [15] (1984) introduced a process semantics for regular expressions. He defined an interpretation $C(e)$ of regular expressions e as charts (finite process graphs): the interpretation of 0 is deadlock, of 1 is successful termination, letters a are atomic actions, the operators $+$ and $\cdot$ stand for choice and concatenation of processes, and (unary) Kleene star $(\cdot)^*$ represents iteration with the option to terminate successfully before each execution of the iteration body. He then defined the process semantics of 'star expressions' (regular expressions in this context) e as 'star behaviors' $[\![e]\!]_P := [C(e)]_{\underline{\leftrightarrow}}$, that is, as equivalence classes of chart interpretations with respect to bisimilarity $\underline{\leftrightarrow}$. Milner was interested in an axiomatization of equality of 'star behaviors'. For this purpose he adapted Salomaa's complete proof system [16] for language equivalence on regular expressions to a system Mil (see Def. 2.6) that is sound for equality of denoted star behaviors. Recognizing that Salomaa's proof strategy cannot be followed directly, he left completeness as an open question.

Over the past 38 years, completeness results have been obtained for restrictions of Milner's system to the following subclasses of star expressions: (a) without 0 and 1, but with binary star iteration $e_1{}^{\circledast}e_2$ instead of unary star [6], (b) with 0, with iterations restricted to exit-less ones $(\cdot)^* \cdot 0$, without 1 [5] and with 1 [4], (c) without 0, and with only restricted occurrences of 1 [3], and (d) '1-free' expressions formed with 0, without 1, but with binary instead of unary iteration [13]. By refining concepts developed in [13] for the proof of (d) we can finally establish completeness of Mil.

The aim of this article. We provide an outline of the completeness proof for Mil. Hereby our focus is on the main new concepts and results. For scrutiny of most of the crucial arguments of the proof we refer to the appendix, which is the kernel of a monograph on the proof that we are writing. While details are sometimes only hinted at in this article, we think that the crystallization technique we present opens up a wide space for other applications (we suggest one in Sect. 9). We want to communicate it in summarized form to the community in order to stimulate its further development.

1 Motivation for the chosen proof strategy

We explain the main obstacle we encountered for developing our proof strategy through explaining shortcomings of existing approaches. Finally we describe crucial new concepts that we use for adapting the collapse strategy from [12, 13].

Obstacle for the 'bisimulation chart' proof strategy. Milner [15] recognized that completeness of the proof system Mil cannot be established along the lines of Salomaa's completeness proof for his proof system $\mathbf{F}_1$ of language equivalence of regular expressions [16]. The reason is as follows. Adopting Salomaa's proof strategy would mean (i) to link given bisimilar chart interpretations $C(e_1)$ and $C(e_2)$ of star

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