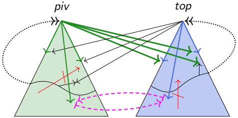
Milner's Proof System for Regular Expressions Modulo Bisimilarity is Complete

Crystallization: Near-Collapsing Process Graph Interpretations of Regular Expressions



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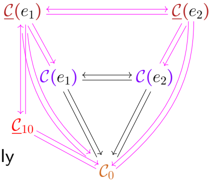


Computer Science Seminar

GSSI

July 27, 2022

Process semantics of regular expressions $[\![\cdot]\!]_{\mathbf{P}}$ *(Milner, 1984)*

$0 \xrightarrow{[\![\cdot]\!]_{\mathbf{P}}}$ deadlock δ, no termination

$1 \xrightarrow{[\![\cdot]\!]_{\mathbf{P}}}$ empty-step process ϵ, then terminate

$a \xrightarrow{[\![\cdot]\!]_{\mathbf{P}}}$ atomic action a, then terminate

$e + f \xrightarrow{[\![\cdot]\!]_{\mathbf{P}}}$ *(choice)* execute $[\![e]\!]_{\mathbf{P}}$ or $[\![f]\!]_{\mathbf{P}}$

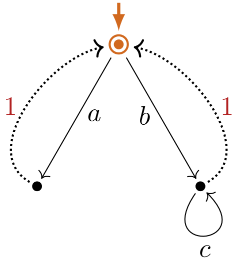
$e \cdot f \xrightarrow{[\![\cdot]\!]_{\mathbf{P}}}$ *(sequentialization)* execute $[\![e]\!]_{\mathbf{P}}$, then $[\![f]\!]_{\mathbf{P}}$

$e^* \xrightarrow{[\![\cdot]\!]_{\mathbf{P}}}$ *(iteration)* repeat (terminate or execute $[\![e]\!]_{\mathbf{P}}$)

$[\![e]\!]_{\mathbf{P}} := [\mathcal{C}(e)]_{\underline{\leftrightarrow}}$ (bisimilarity equivalence class of chart $\mathcal{C}(e)$)

Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)

1-chart



$\underline{\mathcal{C}}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$

Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)

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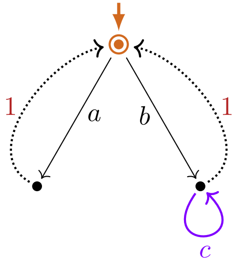

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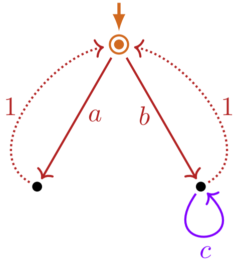

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Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)

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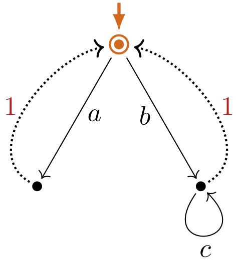
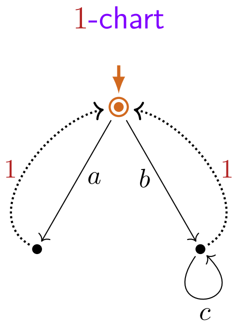
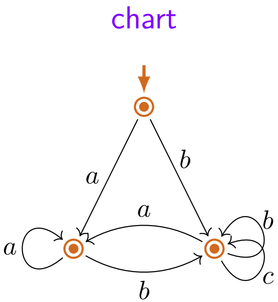


$\underline{\mathcal{C}}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$

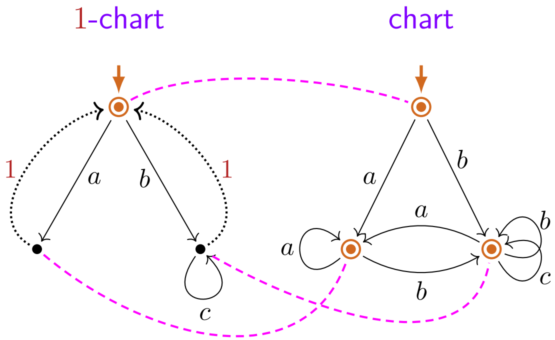
Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)





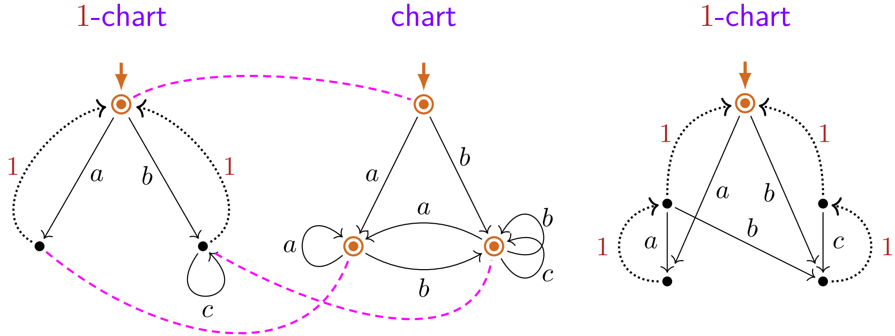
$\underline{\mathcal{C}}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$ $\mathcal{C}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$

Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)



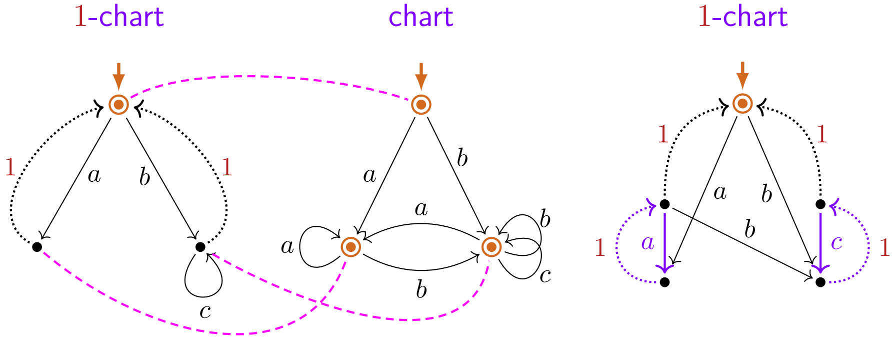
$\underline{\mathcal{C}}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$ $\quad$ $\mathcal{C}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$

Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)



$\underline{\mathcal{C}}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$ $\quad$ $\mathcal{C}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$ $\quad$ $\underline{\mathcal{C}}\big((a \cdot (1 \cdot ((a \cdot 1)^* \cdot 1)) \cdot b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1 + b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1\big)^*\big)$

Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)



$\underline{\mathcal{C}}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$

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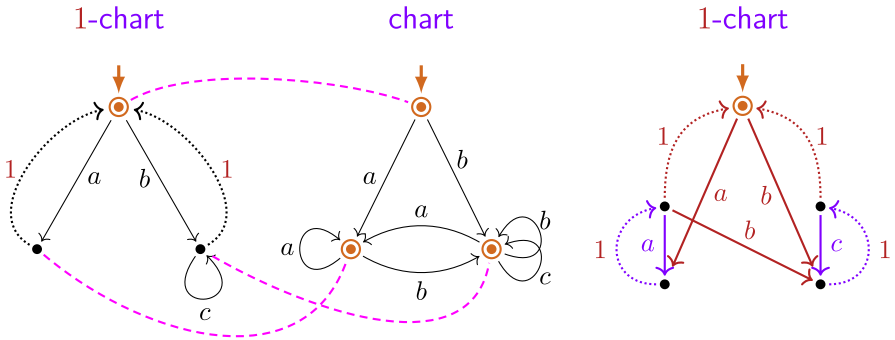
$$\underline{\mathcal{C}}\Big((a \cdot (1 \cdot ((a \cdot 1)^* \cdot 1)) \cdot b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1 + b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1\Big)^*\Big)$$

Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)



$\underline{\mathcal{C}}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$

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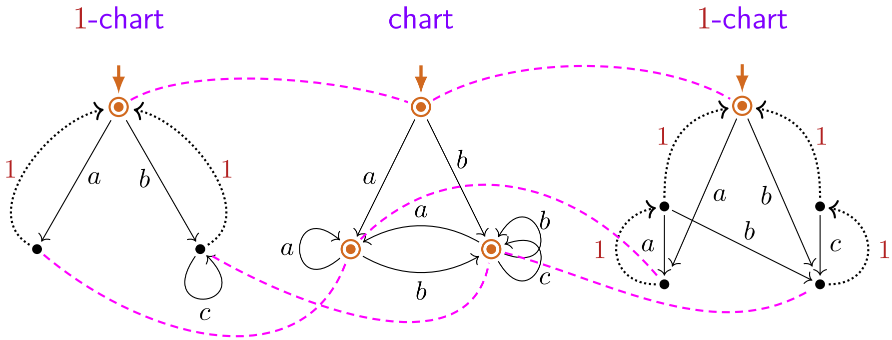
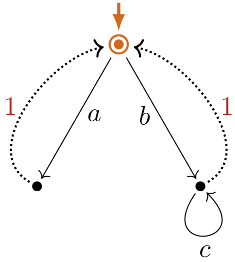
Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)



$\underline{\mathcal{C}}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$ $\mathcal{C}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$ $\underline{\mathcal{C}}\big((a \cdot (1 \cdot ((a \cdot 1)^* \cdot 1)) \cdot b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1 + b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1\big)^*\big)$

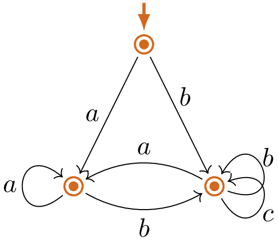
Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)

1-chart



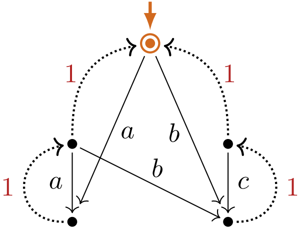
$\underline{\mathcal{C}}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$

chart



$\mathcal{C}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$

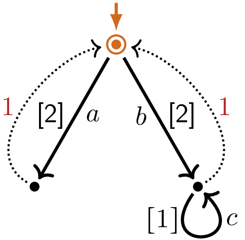
1-chart



$$\underline{\mathcal{C}}\Big(\big(a \cdot (1 \cdot ((a \cdot 1)^* \cdot 1)) \cdot b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1 + b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1\big)^*\Big)$$

Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)

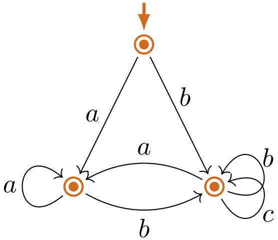
1-chart



LLEE

$\underline{\mathcal{C}}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$

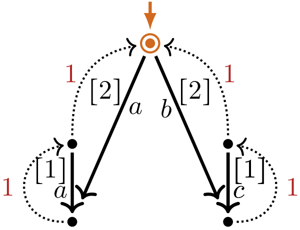
chart



~~LLEE~~

$\mathcal{C}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$

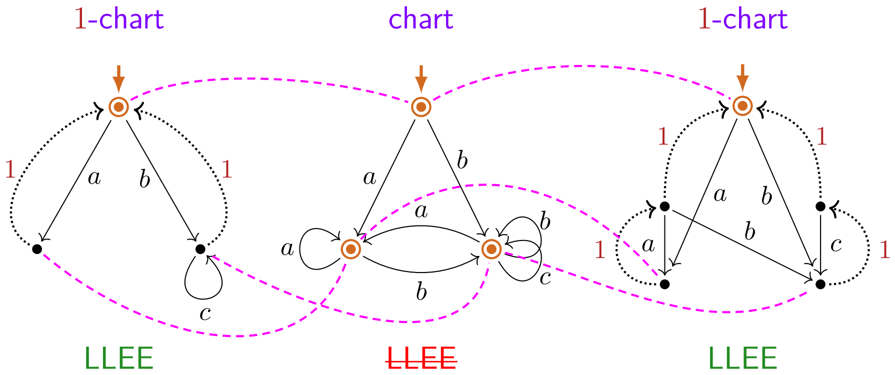
1-chart



LLEE

$$\underline{\mathcal{C}}\Big(\big(a \cdot (1 \cdot ((a \cdot 1)^* \cdot 1)) \cdot b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1 + b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1\big)^*\Big)$$

Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)



$\underline{\mathcal{C}}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$

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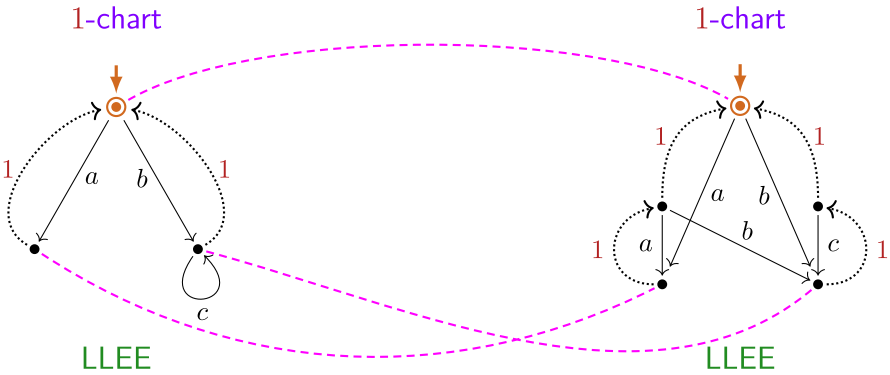
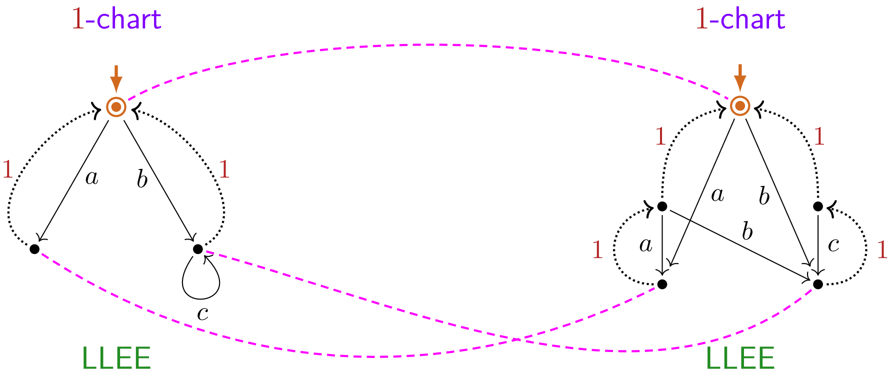
$$\underline{\mathcal{C}}\Big(\big(a \cdot (1 \cdot ((a \cdot 1)^* \cdot 1)) \cdot b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1 + b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1\big)^*\Big)$$

Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)



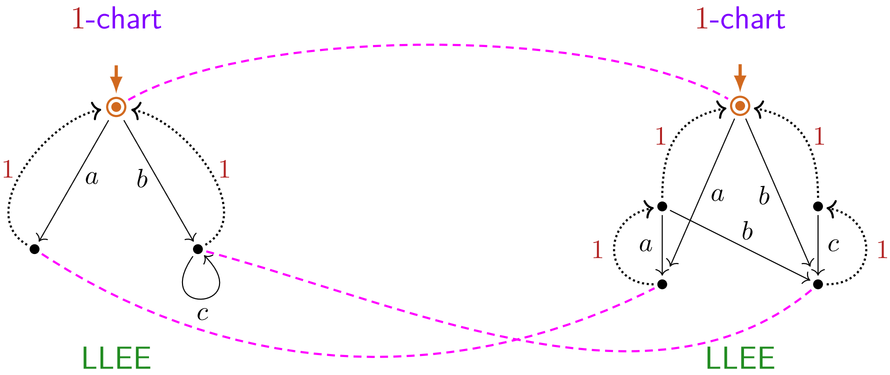
$\underline{\mathcal{C}}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$ $\quad \underline{\leftrightarrow} \quad$ $\underline{\mathcal{C}}\Big(\big(a \cdot (1 \cdot ((a \cdot 1)^* \cdot 1)) \cdot b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1 + b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1\big)^*\Big)$

Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)



$$[\![(a \cdot 1 + b \cdot (c^* \cdot 1))^*]\!]_P = [\![\big(a \cdot (1 \cdot ((a \cdot 1)^* \cdot 1)) \cdot b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1 + b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1\big)^*]\!]_P$$

Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)



$$(a \cdot 1 + b \cdot (c^* \cdot 1))^* \quad =_P \quad \Big(a \cdot (1 \cdot ((a \cdot 1)^* \cdot 1)) \cdot b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1 + b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1\Big)^*\Big)$$

Milner's proof system Mil

Axioms :

(A1)	$e + (f + g) = (e + f) + g$	(A7)	$e = 1 \cdot e$
(A2)	$e + 0 = e$	(A8)	$e = e \cdot 1$
(A3)	$e + f = f + e$	(A9)	$0 = 0 \cdot e$
(A4)	$e + e = e$	(A10)	$e^* = 1 + e \cdot e^*$
(A5)	$e \cdot (f \cdot g) = (e \cdot f) \cdot g$	(A11)	$e^* = (1 + e)^*$
(A6)	$(e + f) \cdot g = e \cdot g + f \cdot g$		

But: $e \cdot (f + g) \neq e \cdot f + e \cdot g$

But: $e \cdot 0 \neq 0$

Inference rules : rules of equational logic *plus*

$$\frac{e = f \cdot e + g}{e = f^* \cdot g}\ \mathsf{RSP}^*$$ (if f does not terminate immediately)

Milner's question (1984)

Is Mil complete with respect to $=_{\mathbf{P}}$? (Does $=_{\mathbf{P}} \subseteq =_{\mathsf{Mil}}$ hold?)

Bisimulation collapse proof strategy (G/Fokkink, LICS'20)

For 1-free regular expressions e_1 and e_2:

e_1 is solution $\quad \mathcal{C}(e_1) \xleftrightarrow{\text{(assm)}} \mathcal{C}(e_2) \quad$ e_2 is solution

$e_1 =_{\mathsf{Mil}} e_2$

Bisimulation collapse proof strategy (G/Fokkink, LICS'20)

For 1-free regular expressions e_1 and e_2:

e_1 is solution

$$\begin{tikzcd}
\mathcal{C}(e_1) \arrow[rr, Leftrightarrow, "\text{(assm)}"] \arrow[dr, Rightarrow] & & \mathcal{C}(e_2) \arrow[dl, Rightarrow] \\
& \mathcal{C}_0 &
\end{tikzcd}$$

e_2 is solution

$e_1 =_{\mathsf{Mil}} e_2$

Bisimulation collapse proof strategy (G/Fokkink, LICS'20)

For 1-free regular expressions e_1 and e_2:

e_1 is solution

$$\begin{tikzcd}
\overset{\text{LLEE}}{\mathcal{C}(e_1)} \arrow[rr, Leftrightarrow, "\text{(assm)}"] \arrow[dr, Rightarrow] & & \overset{\text{LLEE}}{\mathcal{C}(e_2)} \arrow[dl, Rightarrow] \\
& \mathcal{C}_0 &
\end{tikzcd}$$

e_2 is solution

$e_1 =_{\mathsf{Mil}} e_2$

Bisimulation collapse proof strategy (G/Fokkink, LICS'20)

For 1-free regular expressions e_1 and e_2:

e_1 is solution

$$
\begin{tikzcd}
\overset{\text{LLEE}}{\mathcal{C}(e_1)} \arrow[rr, Leftrightarrow, "\text{(assm)}"] \arrow[dr, Rightarrow] & & \overset{\text{LLEE}}{\mathcal{C}(e_2)} \arrow[dl, Rightarrow] \\
& \underset{\text{LLEE}}{\mathcal{C}_0} &
\end{tikzcd}
$$

e_2 is solution

$e_1 =_{\mathsf{Mil}} e_2$

Bisimulation collapse proof strategy (G/Fokkink, LICS'20)

For 1-free regular expressions e_1 and e_2:

e_1 is solution LLEE $\mathcal{C}(e_1)$ (assm) LLEE $\mathcal{C}(e_2)$ e_2 is solution

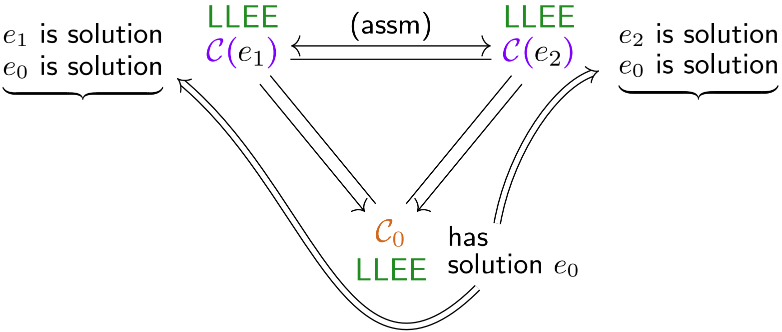
$$\begin{tikzcd} \mathcal{C}(e_1) \arrow[rr, Leftrightarrow, "\text{(assm)}"] \arrow[dr, Rightarrow] & & \mathcal{C}(e_2) \arrow[dl, Rightarrow] \\ & \mathcal{C}_0 & \end{tikzcd}$$

$\mathcal{C}_0$ LLEE has solution e_0

$e_1 =_{\mathsf{Mil}} e_2$

Bisimulation collapse proof strategy (G/Fokkink, LICS'20)

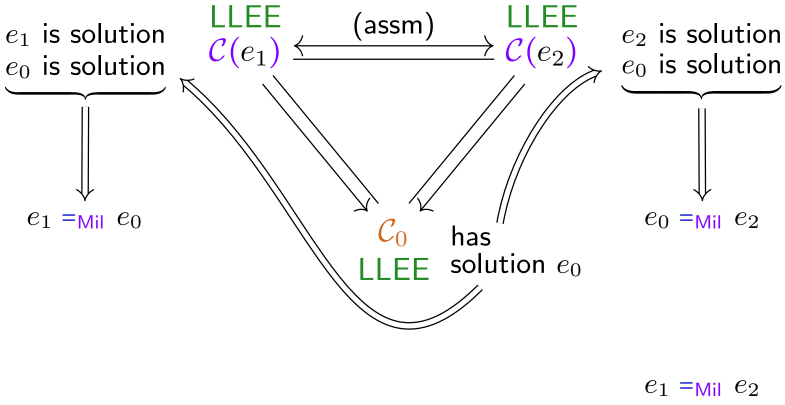
For 1-free regular expressions e_1 and e_2:



$e_1 =_{\mathsf{Mil}} e_2$

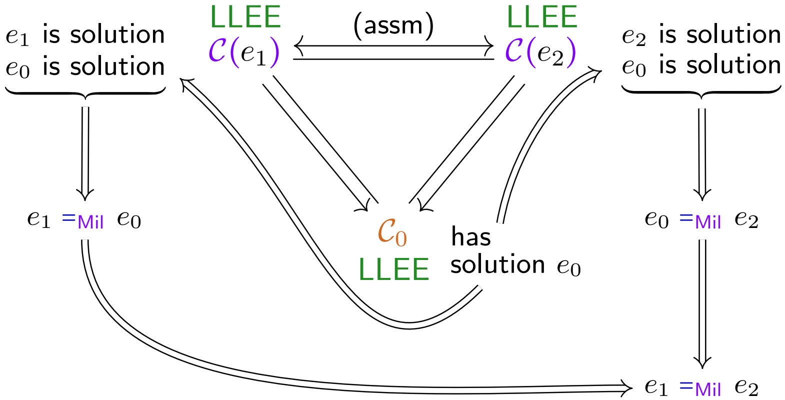
Bisimulation collapse proof strategy (G/Fokkink, LICS'20)

For 1-free regular expressions e_1 and e_2:



Bisimulation collapse proof strategy (G/Fokkink, LICS'20)

For 1-free regular expressions e_1 and e_2:



Bisimulation collapse proof strategy (general case)

Problem 1

chart interpretations ~~LLEE~~ $\mathcal{C}(e_1) \xleftrightarrow{\text{(assm)}} \mathcal{C}(e_2)$ ~~LLEE~~

Bisimulation collapse proof strategy (general case)

Remedy for Problem 1 *(G, TERMGRAPH 2020)*

chart interpretations ~~LLEE~~ $\mathcal{C}(e_1) \xleftrightarrow{\text{(assm)}} \mathcal{C}(e_2)$ ~~LLEE~~

Bisimulation collapse proof strategy (general case)

Remedy for Problem 1 *(G, TERMGRAPH 2020)*

1-*chart interpretations*

LLEE $\underline{\mathcal{C}}(e_1)$ $\underline{\mathcal{C}}(e_2)$ LLEE

chart interpretations ~~LLEE~~ $\mathcal{C}(e_1)$ $\mathcal{C}(e_2)$ ~~LLEE~~

$$\begin{tikzcd}
\underline{\mathcal{C}}(e_1) \arrow[d, Rightarrow, "\text{(IV)}"] & \underline{\mathcal{C}}(e_2) \arrow[d, Rightarrow, "\text{(IV)}"'] \\
\mathcal{C}(e_1) \arrow[r, leftrightarrow, "\text{(assm)}"] & \mathcal{C}(e_2)
\end{tikzcd}$$

Bisimulation collapse proof strategy (general case)

Remedy for Problem 1 *(G, TERMGRAPH 2020)*

1-*chart interpretations*

chart interpretations

$$
\begin{tikzcd}
\text{LLEE} & \underline{\mathcal{C}}(e_1) \arrow[rr, Leftrightarrow] \arrow[d, Rightarrow, "\text{(IV)}"] & & \underline{\mathcal{C}}(e_2) \arrow[d, Rightarrow, "\text{(IV)}"'] & \text{LLEE} \\
\cancel{\text{LLEE}} & \mathcal{C}(e_1) \arrow[rr, leftrightarrow, "\text{(assm)}"] & & \mathcal{C}(e_2) & \cancel{\text{LLEE}}
\end{tikzcd}
$$

Bisimulation collapse proof strategy (general case)

Remedy for Problem 1 *(G, TERMGRAPH 2020)*

1-chart interpretations

chart interpretations

bisimulation collapse

$$
\begin{tikzcd}
\text{LLEE} \;\; \underline{\mathcal{C}}(e_1) \arrow[rr, Leftrightarrow] \arrow[d, Rightarrow, "\text{(IV)}"] & & \underline{\mathcal{C}}(e_2) \;\; \text{LLEE} \arrow[d, Rightarrow, "\text{(IV)}"'] \\
\cancel{\text{LLEE}} \;\; \mathcal{C}(e_1) \arrow[rr, leftrightarrow, "\text{(assm)}"] \arrow[dr, Rightarrow] & & \mathcal{C}(e_2) \;\; \cancel{\text{LLEE}} \arrow[dl, Rightarrow] \\
& \mathcal{C}_0 &
\end{tikzcd}
$$

Bisimulation collapse proof strategy (general case)

Remedy for Problem 1 *(G, TERMGRAPH 2020)*

$$
\begin{tikzcd}[column sep=small]
1\text{-}\textit{chart interpretations} & \mathsf{LLEE} & \underline{\mathcal{C}}(e_1) \arrow[rrrr, Leftrightarrow] \arrow[dr, Rightarrow, "\text{(IV)}"] \arrow[ddrr, Rightarrow, bend right] & & & & \underline{\mathcal{C}}(e_2) \arrow[dl, Rightarrow, "\text{(IV)}"'] \arrow[ddll, Rightarrow, bend left] & \mathsf{LLEE} \\
\textit{chart interpretations} & \cancel{\mathsf{LLEE}} & & \mathcal{C}(e_1) \arrow[rr, leftrightarrow, "\text{(assm)}"] \arrow[dr, Rightarrow] & & \mathcal{C}(e_2) \arrow[dl, Rightarrow] & & \cancel{\mathsf{LLEE}} \\
\textit{bisimulation collapse} & & & & \mathcal{C}_0 & & &
\end{tikzcd}
$$

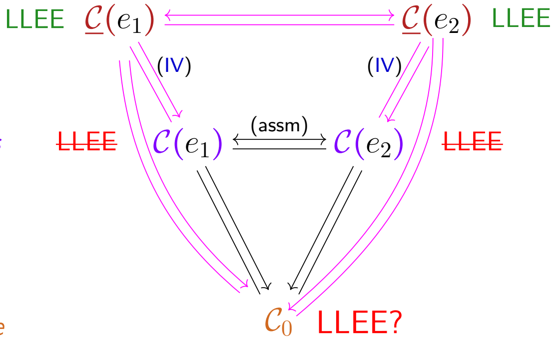
Bisimulation collapse proof strategy (general case)

Remedy for Problem 1 *(G, TERMGRAPH 2020)*

1-*chart interpretations*

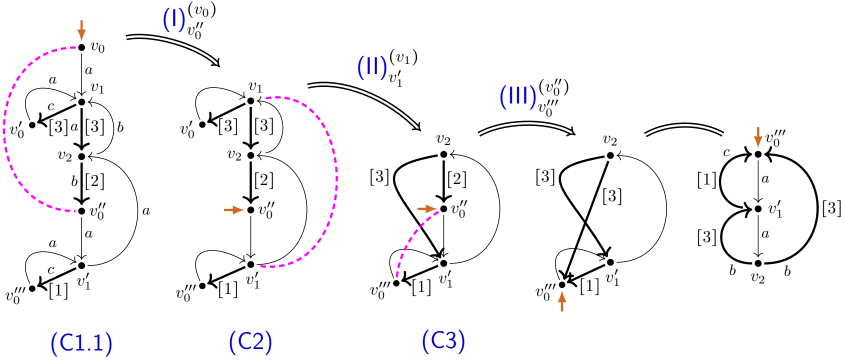
chart interpretations

bisimulation collapse



LLEE-preserving collapse of LLEE-charts (G/Fokkink, LICS'20)

(no 1-transitions!)



Lemma

The bisimulation collapse of a LLEE-chart is again a LLEE-chart.

Reduced bisimilarity redundancies in LLEE-charts (no 1-trans.!)

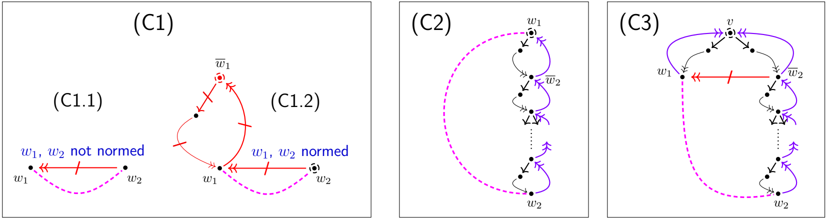
(G/Fokkink, LICS'20)

Lemma

Every not collapsed LLEE-chart contains bisimilar vertices $w_1 \neq w_2$ of kind (C1), (C2), or (C3) (a reduced bisimilarity redundancy $\langle w_1, w_2 \rangle$):

w_1, w_2 in different scc's

w_1, w_2 in the same scc



Lemma

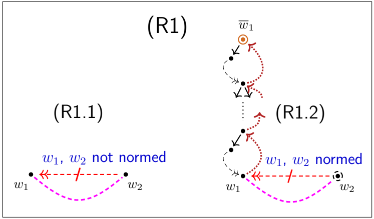
Every reduced bisimilarity redundancy in a LLEE-chart can be eliminated LLEE-preservingly.

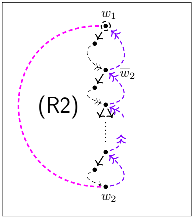
Reduced 1-bisimilarity redundancies in LLEE-1-charts

Lemma

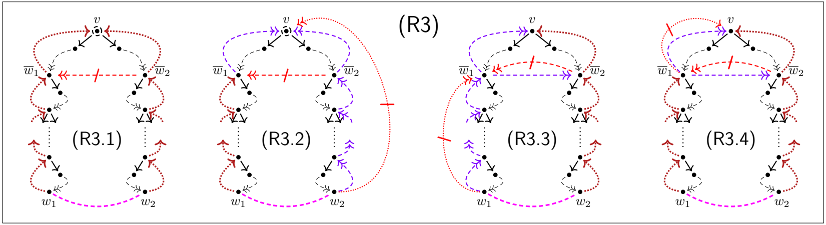
Every not collapsed LLEE-1-chart contains 1-bisimilar vertices $w_1 \neq w_2$ (a reduced 1-bisimilarity redundancy $\langle w_1, w_2 \rangle$) of kind (R1), (R2), (R3):

w_1, w_2 in different scc's





w_1, w_2 in the same scc

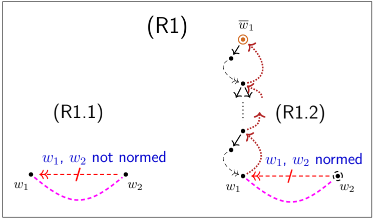


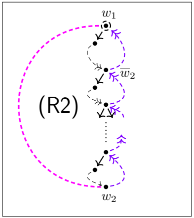
Reduced 1-bisimilarity redundancies in LLEE-1-charts

Lemma

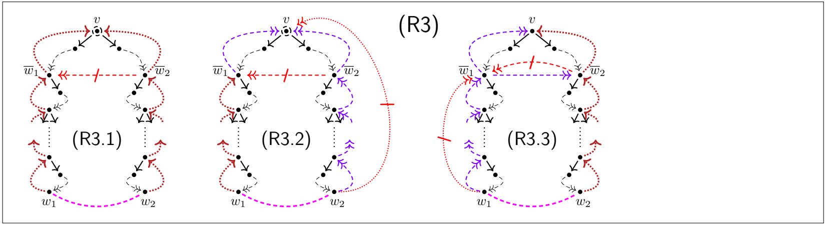
Every simple reduced 1-bisimilarity redundancies in a LLEE-1-chart can be eliminated LLEE-preservingly.

w_1, w_2 in different scc's





w_1, w_2 in the same scc



Reduced 1-bisimilarity redundancies in LLEE-1-charts

Stumbling Block

How to LLEE-preservingly eliminate
reduced 1-bisimilarity redundancies of kind (R3.4)?

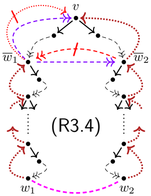
w_1, w_2 in different scc's

(R1)

(R2)

w_1, w_2 in the same scc

(R3)



Reduced 1-bisimilarity redundancies in LLEE-1-charts

Stumbling Block

How to LLEE-preservingly eliminate
precrystalline reduced 1-bisimilarity redundancies?

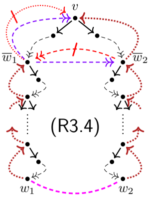
w_1, w_2 in different scc's

(R1)

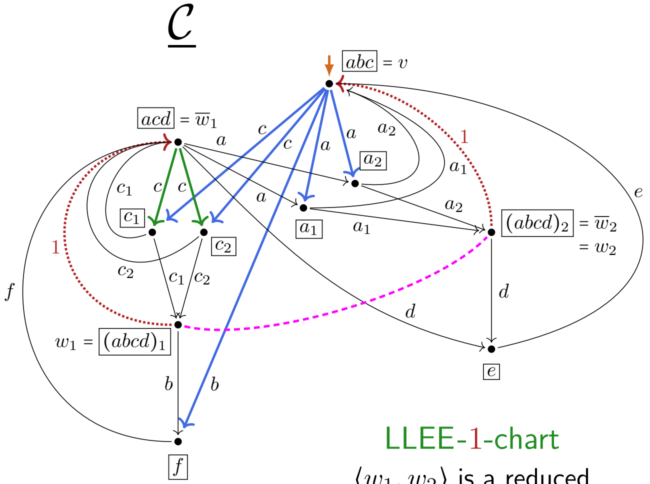
(R2)

w_1, w_2 in the same scc

(R3)



Counterexample LLEE-preserving collapse

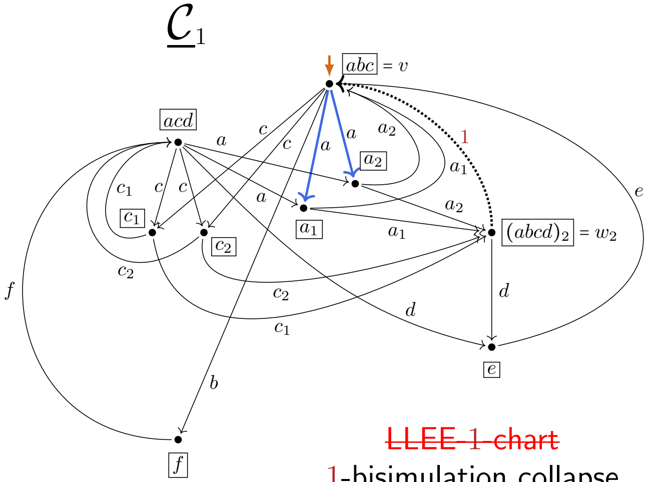


LLEE-1-chart

$\langle w_1, w_2 \rangle$ is a reduced

1-bisimilarity redundancy of kind (R3.4)

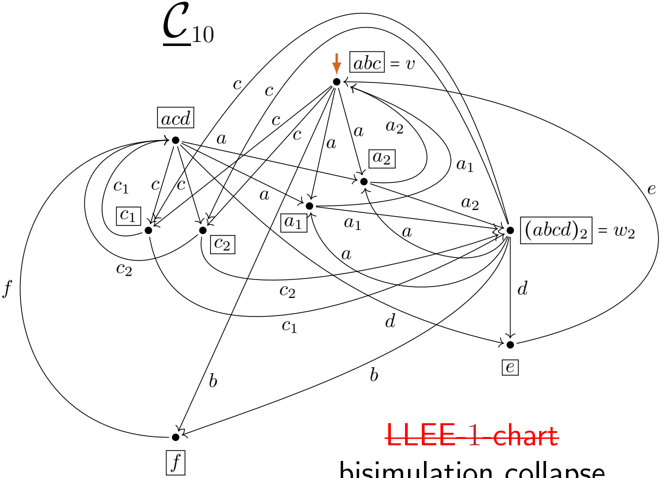
Counterexample LLEE-preserving collapse



~~LLEE-1-chart~~

1-bisimulation collapse

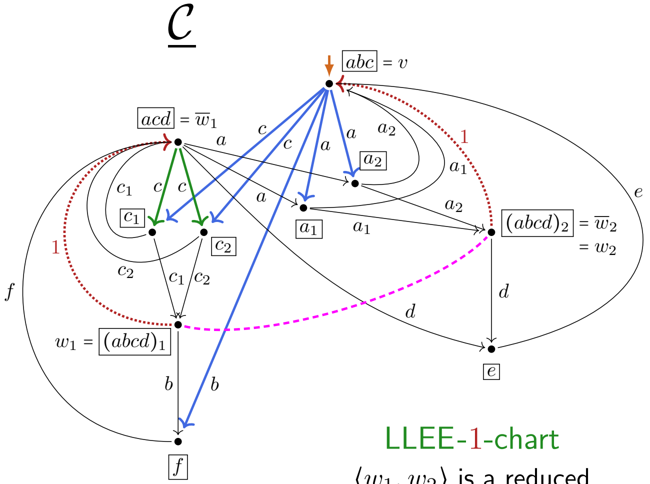
Counterexample LLEE-preserving collapse



~~LLEE-1-chart~~

bisimulation collapse

Counterexample LLEE-preserving collapse



LLEE-1-chart

$\langle w_1, w_2 \rangle$ is a reduced 1-bisimilarity redundancy of kind (R3.4)

Bisimulation collapse proof strategy (general case)

Problem 2: There are regular expressions e_1 and e_2 such that:

1-chart interpretations

chart interpretations

bisimulation collapse

$$
\begin{tikzcd}
\text{LLEE}\;\underline{\mathcal{C}}(e_1) \arrow[rrrr, Leftrightarrow] \arrow[dr, Rightarrow, "\text{(IV)}"] \arrow[ddrr, Rightarrow, bend right] & & & & \underline{\mathcal{C}}(e_2)\;\text{LLEE} \arrow[dl, Rightarrow, "\text{(IV)}"'] \arrow[ddll, Rightarrow, bend left] \\
& \cancel{\text{LLEE}}\;\mathcal{C}(e_1) \arrow[rr, Leftrightarrow, "\text{(assm)}"] \arrow[dr, Rightarrow] & & \mathcal{C}(e_2)\;\cancel{\text{LLEE}} \arrow[dl, Rightarrow] & \\
& & \mathcal{C}_0\;\cancel{\text{LLEE}} & &
\end{tikzcd}
$$

Bisimulation collapse proof strategy (general case)

Problem 2: There are regular expressions e_1 and e_2 such that:

1-*chart interpretations*

LLEE $\underline{\mathcal{C}}(e_1) \Leftrightarrow \underline{\mathcal{C}}(e_2)$ LLEE

(IV) (IV)

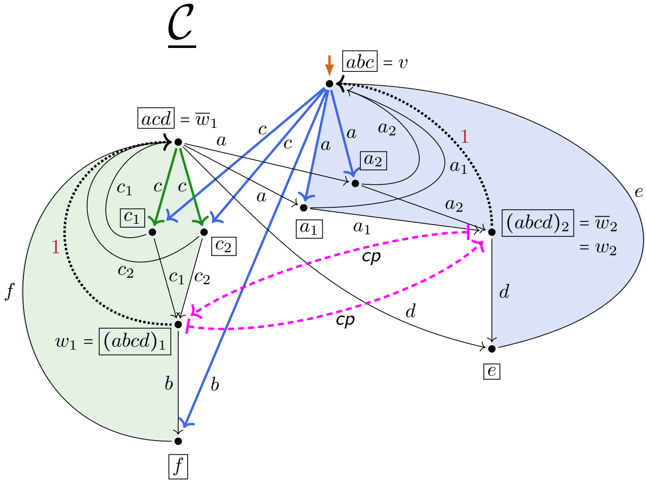
chart interpretations ~~LLEE~~ $\mathcal{C}(e_1) \xleftrightarrow{\text{(assm)}} \mathcal{C}(e_2)$ ~~LLEE~~

for all $\mathcal{C}_0$:

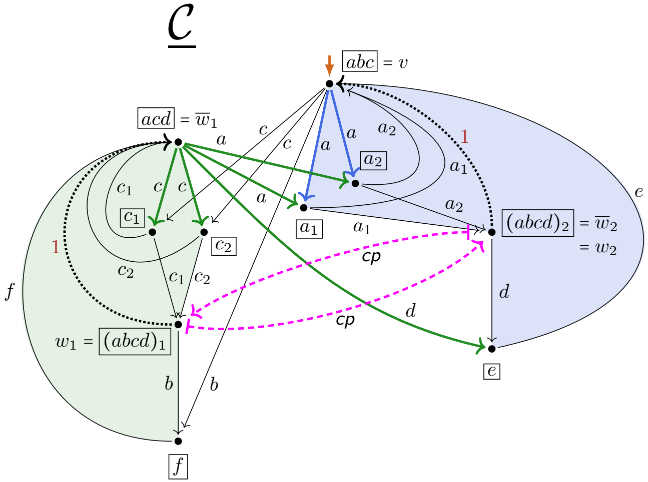
~~LLEE~~

$\mathcal{C}(e_1)$, $\mathcal{C}(e_2)$, $\underline{\mathcal{C}}(e_1)$ and $\underline{\mathcal{C}}(e_2)$ are **not** LLEE-preservingly jointly minimizable under bisimilarity.

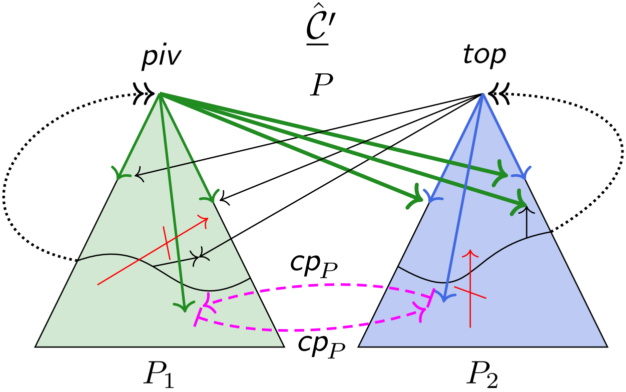
Twin-Crystal



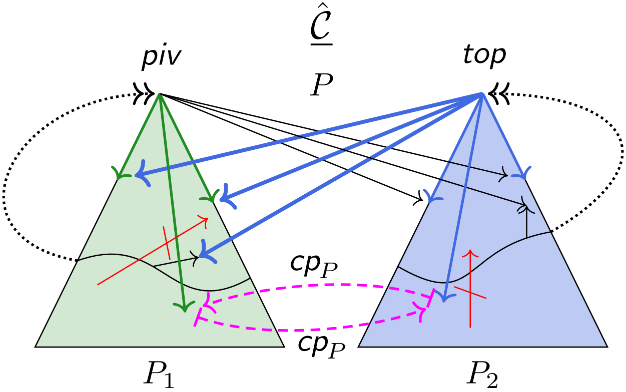
Twin-Crystal



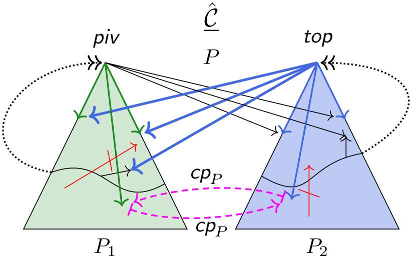
Twin-Crystal



Twin-Crystal



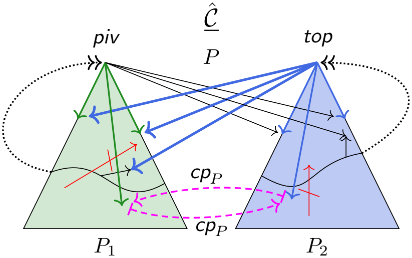
Crystallization



twin-crystal

Crystallized 1-charts = LLEE-1-charts that are collapsed apart from strongly connected components of twin-crystal form.

Crystallization

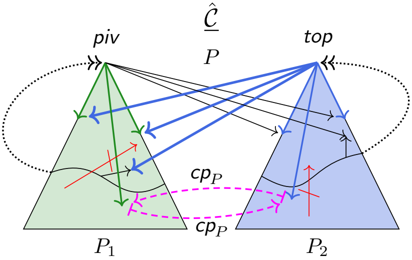


twin-crystal

Crystallized 1-charts = LLEE-1-charts that are collapsed apart from strongly connected components of twin-crystal form.

(CR) Crystallization: Every LLEE-1-chart can be reduced under bisimilarity to a 1-bisimilar crystallized 1-chart.

Crystallization



twin-crystal

Crystallized 1-charts = LLEE-1-charts that are collapsed apart from strongly connected components of twin-crystal form.

(CR) Crystallization: Every LLEE-1-chart can be reduced under bisimilarity to a 1-bisimilar crystallized 1-chart.

(CC) Every Mil-provable solution of a crystallized 1-chart give rise to Mil-provable solution on the bisimulation collapse.

Completeness proof of Mil (structure)

chart interpretations

$$\mathcal{C}(e_1) \xleftrightarrow{\text{(assm)}} \mathcal{C}(e_2)$$

$$\overset{?}{\Longrightarrow} \qquad e_1 =_{\mathsf{Mil}} e_2$$

Completeness proof of Mil (structure)

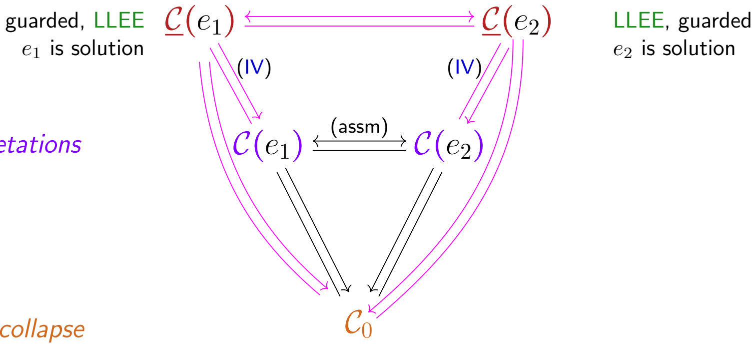
chart interpretations $\mathcal{C}(e_1) \xleftrightarrow{\text{(assm)}} \mathcal{C}(e_2)$

Completeness proof of Mil (structure)

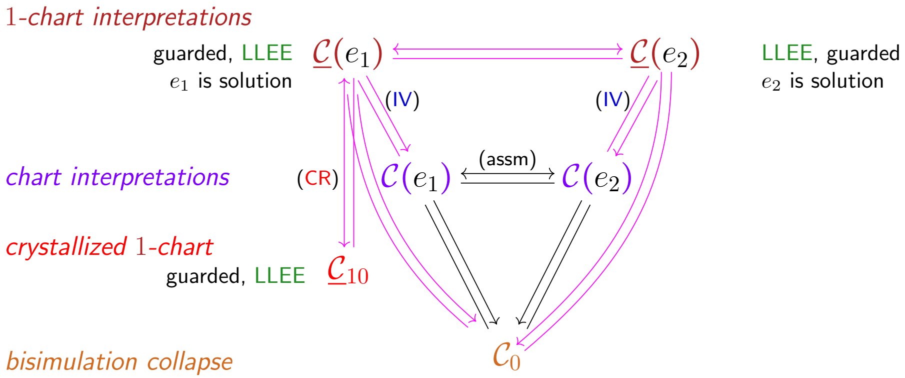
1-chart interpretations

chart interpretations

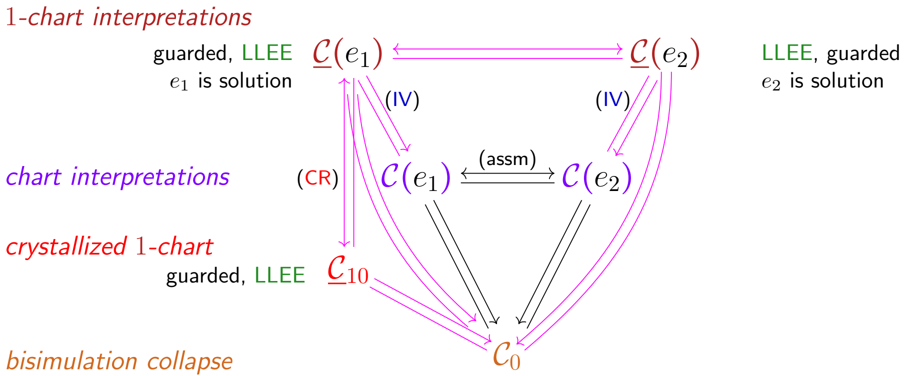
bisimulation collapse



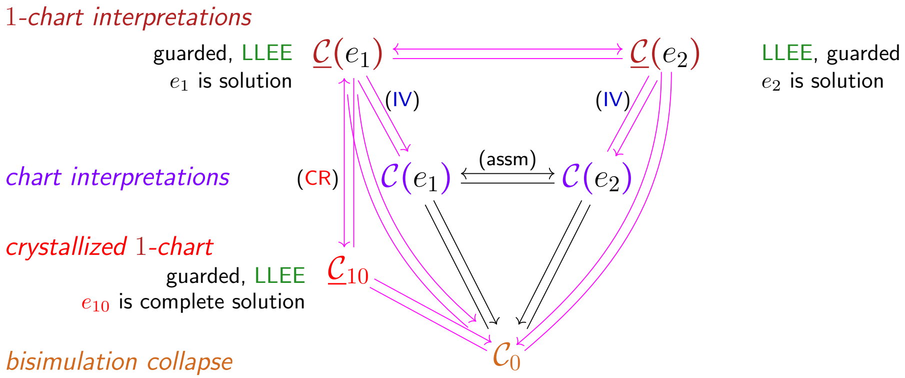
Completeness proof of Mil (structure)



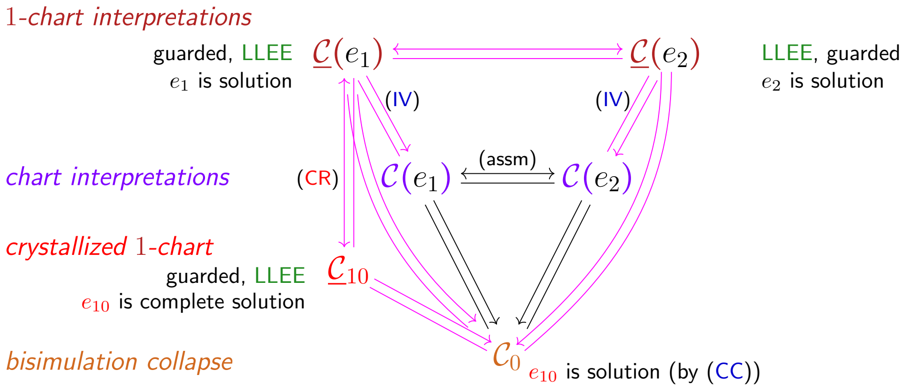
Completeness proof of Mil (structure)



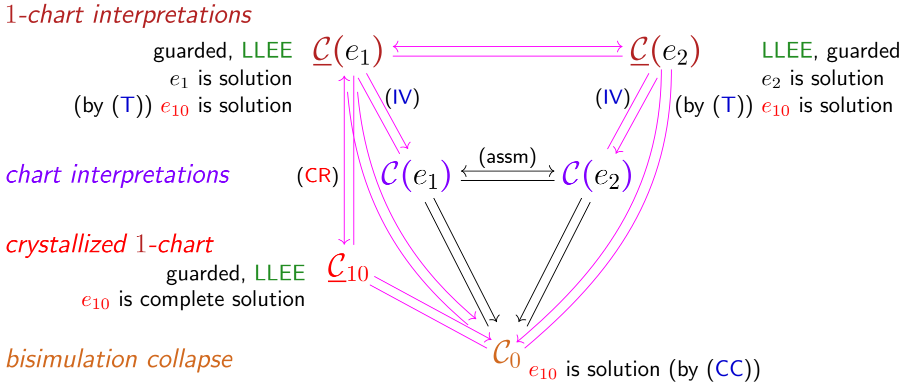
Completeness proof of Mil (structure)



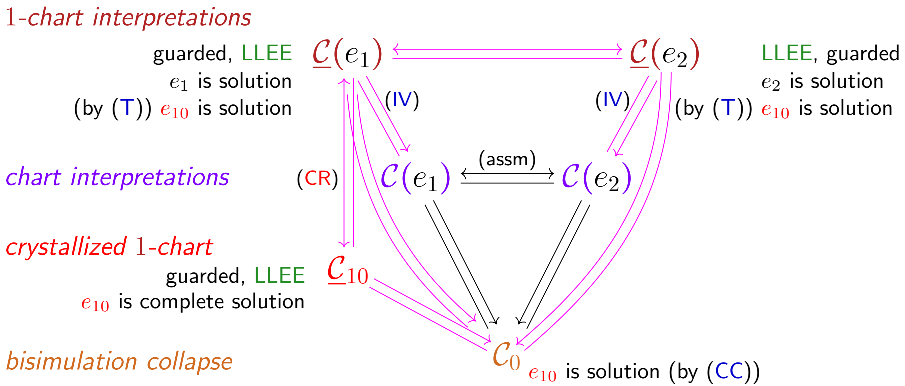
Completeness proof of Mil (structure)



Completeness proof of Mil (structure)



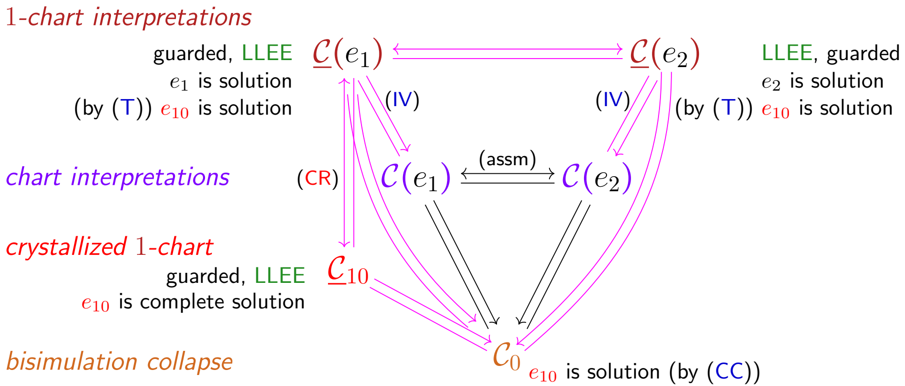
Completeness proof of Mil (structure)



$\mathcal{C}(e_1)$ guarded, LLEE
e_1 is solution
e_{10} is solution

$\mathcal{C}(e_2)$ guarded, LLEE
e_2 is solution
e_{10} is solution

Completeness proof of Mil (structure)



$$\left.\begin{array}{r} \mathcal{C}(e_1) \text{ guarded, LLEE} \\ e_1 \text{ is solution} \\ e_{10} \text{ is solution} \end{array}\right\} \underset{\text{(SE)}}{\Longrightarrow} e_1 =_{\mathsf{Mil}} e_{10}$$

$\mathcal{C}(e_2)$ guarded, LLEE

e_2 is solution

e_{10} is solution

Completeness proof of Mil (structure)

1-chart interpretations

guarded, LLEE
e_1 is solution
(by (T)) e_{10} is solution

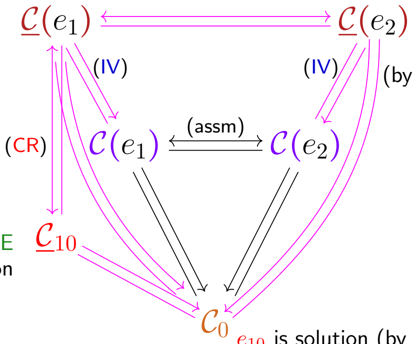
LLEE, guarded
e_2 is solution
(by (T)) e_{10} is solution

chart interpretations

crystallized 1-chart

guarded, LLEE
e_{10} is complete solution

bisimulation collapse



e_{10} is solution (by (CC))

$$\left.\begin{array}{r}\mathcal{C}(e_1)\text{ guarded, LLEE}\\ e_1\text{ is solution}\\ e_{10}\text{ is solution}\end{array}\right\}\xLongrightarrow[\text{(SE)}]{} e_1 =_{\text{Mil}} e_{10} \qquad e_{10} =_{\text{Mil}} e_2 \xLongleftarrow[\text{(SE)}]{} \left\{\begin{array}{l}\mathcal{C}(e_2)\text{ guarded, LLEE}\\ e_2\text{ is solution}\\ e_{10}\text{ is solution}\end{array}\right.$$

Completeness proof of Mil (structure)

1-chart interpretations

guarded, LLEE
e_1 is solution
(by (T)) e_{10} is solution

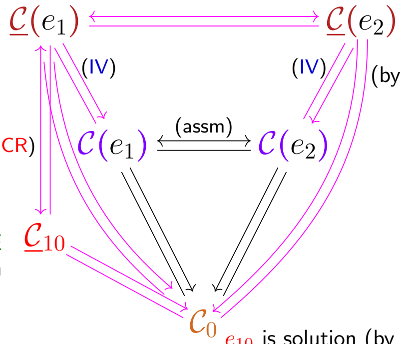
LLEE, guarded
e_2 is solution
(by (T)) e_{10} is solution

chart interpretations

crystallized 1-chart

guarded, LLEE
e_{10} is complete solution

bisimulation collapse

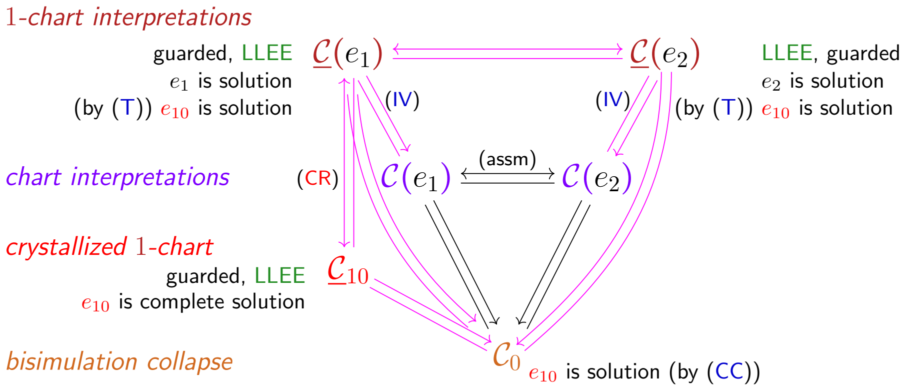


e_{10} is solution (by (CC))

$$\left.\begin{array}{r}\mathcal{C}(e_1)\ \text{guarded, LLEE}\\ e_1\ \text{is solution}\\ e_{10}\ \text{is solution}\end{array}\right\} \underset{\text{(SE)}}{\Longrightarrow} e_1 =_{\mathsf{Mil}} e_{10} \qquad e_{10} =_{\mathsf{Mil}} e_2 \underset{\text{(SE)}}{\Longleftarrow} \left\{\begin{array}{l}\mathcal{C}(e_2)\ \text{guarded, LLEE}\\ e_2\ \text{is solution}\\ e_{10}\ \text{is solution}\end{array}\right.$$

$$\Longrightarrow \qquad e_1 =_{\mathsf{Mil}} e_2$$

Completeness proof of Mil (structure)



Theorem

Milner's proof system Mil is complete
for process semantics equivalence $=_{\mathbf{P}}$ of regular expressions.

Since: $e_1 =_{\mathbf{P}} e_2 \Longrightarrow [\![e_1]\!]_{\mathbf{P}} = [\![e_2]\!]_{\mathbf{P}} \Longrightarrow \mathcal{C}(e_1) \leftrightarrows \mathcal{C}(e_1) \Longrightarrow e_1 =_{\mathsf{Mil}} e_2$.

Outlook

poster presentation

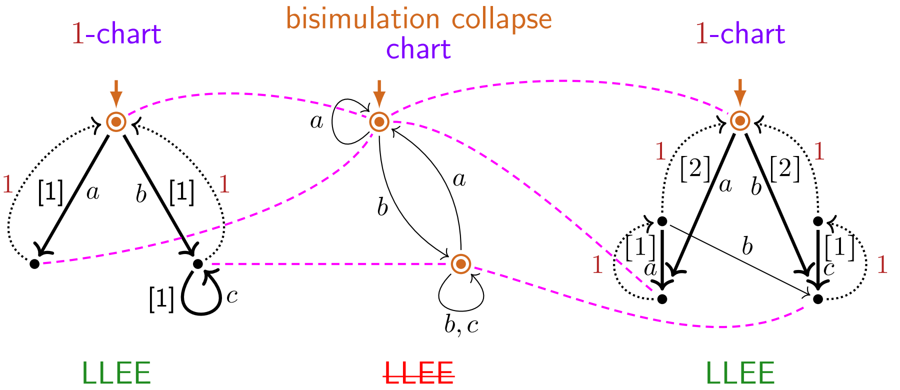
- (LICS, 5 August, 10 - 10.30)

next steps and projects

- monograph project: proof in fine-grained detail
- computation/animation tool for crystallization
- use crystallization for the recognition problem

report version of article (planned)

Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)



$\underline{\mathcal{C}}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$

$\mathcal{C}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$

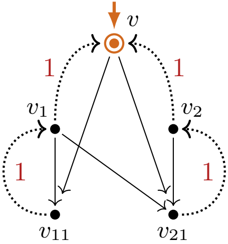
$$\underline{\mathcal{C}}\Big(\big(a \cdot (1 \cdot ((a \cdot 1)^* \cdot 1)) \cdot b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1 + b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1\big)^*\Big)$$

$\underline{\mathcal{C}}((a + b \cdot c^*)^*)$

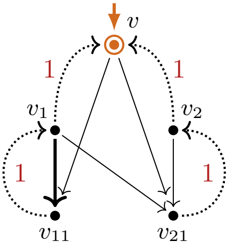
$\mathcal{C}((a + b \cdot c^*)^*)$

$\underline{\mathcal{C}}((a^* \cdot (1 + b \cdot c^*))^*)$

Layered loop existence/elimination and LLEE-witnesses



Layered loop existence/elimination and LLEE-witnesses



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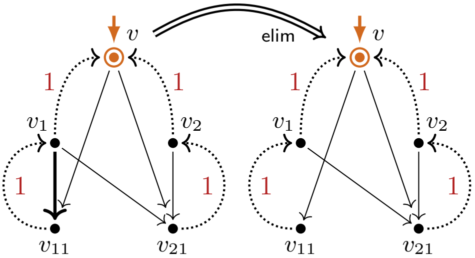

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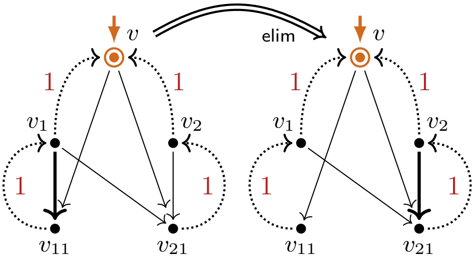
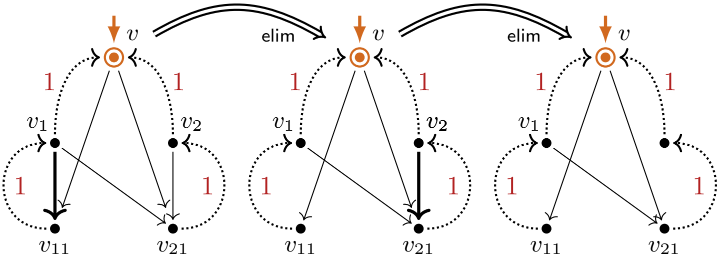
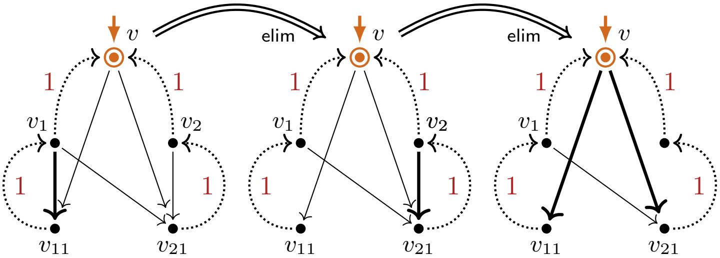


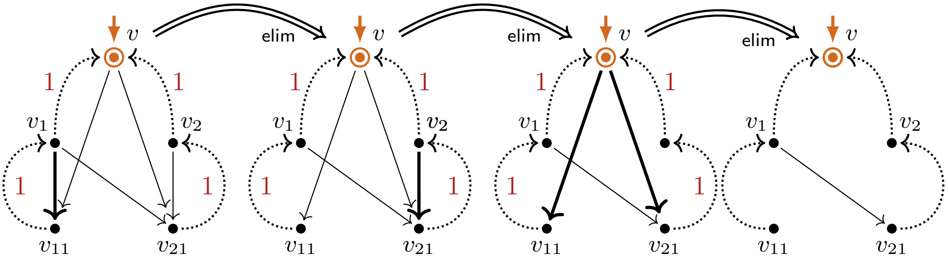
Layered loop existence/elimination and LLEE-witnesses



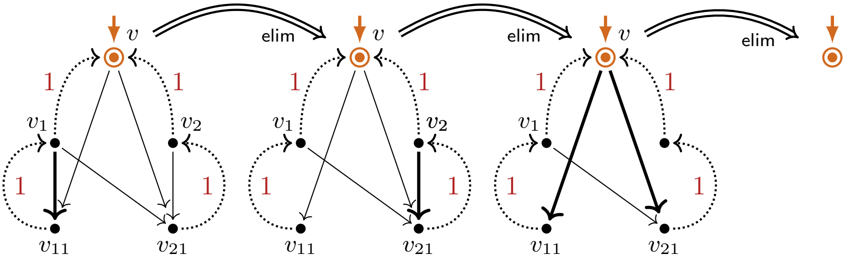
Layered loop existence/elimination and LLEE-witnesses



Layered loop existence/elimination and LLEE-witnesses



Layered loop existence/elimination and LLEE-witnesses



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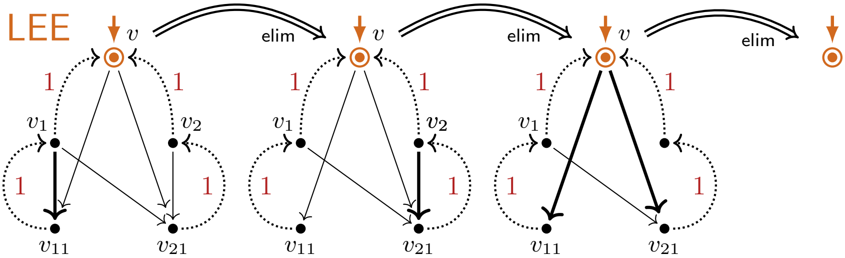
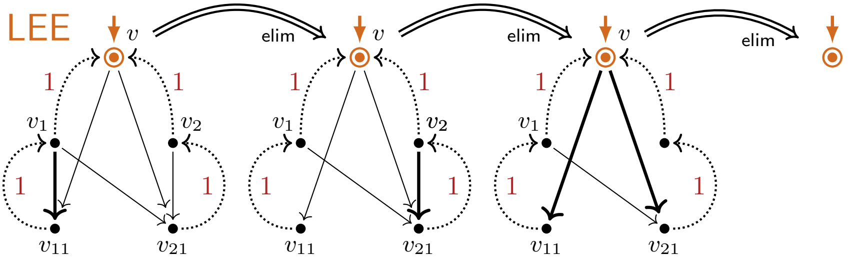
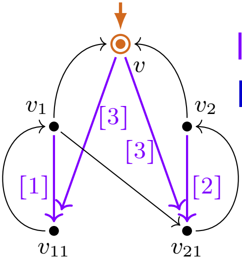


Layered loop existence/elimination and LLEE-witnesses

LEE



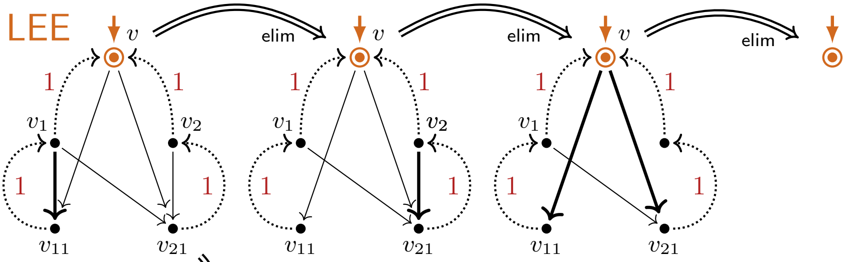


layered

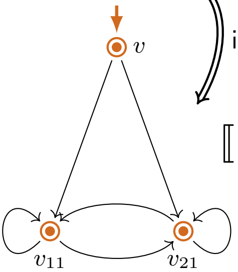
LEE-witness

Layered loop existence/elimination and LLEE-witnesses

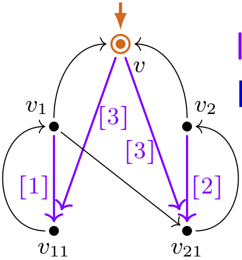
LEE



induces

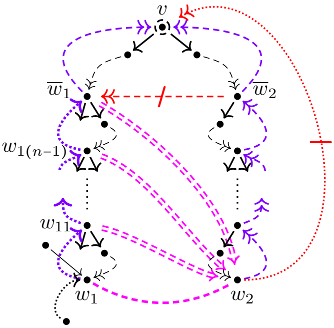


$[\![(a^* \cdot b^*)^*]\!]_{\mathbf{P}}$



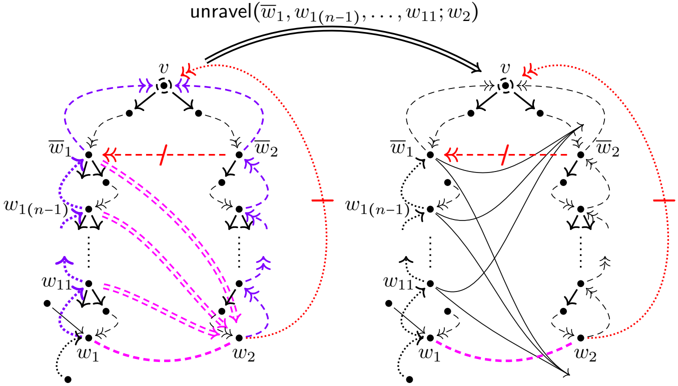
layered LEE-witness

Eliminating reduced 1-bisimilarity redundancies (example)



reduced 1-bisimilarity
redundancy of kind (R3.2)

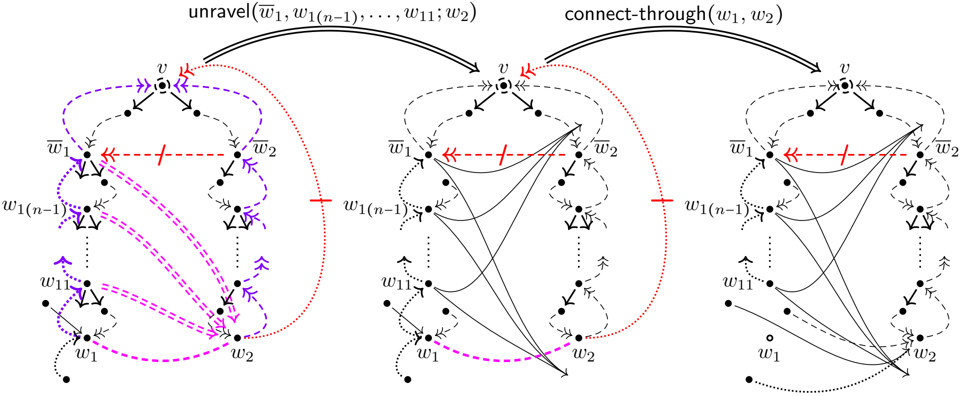
Eliminating reduced 1-bisimilarity redundancies (example)



reduced 1-bisimilarity redundancy of kind (R3.2)

result of unravelling loop vertices $\overline{w}_1, w_{1(n-1)}, \ldots, w_{11}$

Eliminating reduced 1-bisimilarity redundancies (example)



reduced 1-bisimilarity redundancy of kind (R3.2)

result of unravelling loop vertices $\overline{w}_1, w_{1(n-1)}, \ldots, w_{11}$

result of connecting through w_1 to w_2

Properties of LEE-charts

Theorem (G/Fokkink, LICS 2020)

A chart is expressible by a 1-free star expression modulo bisimilarity
if and only if
its bisimulation collapse is a LEE-chart.

Hence expressible | not expressible by 1-free star expressions: