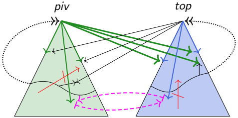
Milner's Proof System for Regular Expressions Modulo Bisimilarity is Complete

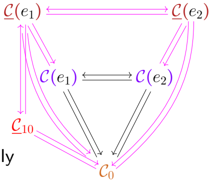
Crystallization: Near-Collapsing Process Graph Interpretations of Regular Expressions



Clemens Grabmayer

GRAN SASSO SCIENCE INSTITUTE
SCHOOL OF ADVANCED STUDIES
Scuola Universitaria Superiore

Department of Computer Science, GSSI, L'Aquila, Italy



LICS 2022

Technion, Haifa, Israel

August 4, 2022

Process semantics of regular expressions $[\![\cdot]\!]_{\mathbf{P}}$ *(Milner, 1984)*

$0 \xmapsto{[\![\cdot]\!]_{\mathbf{P}}}$ deadlock δ, no termination

$1 \xmapsto{[\![\cdot]\!]_{\mathbf{P}}}$ empty-step process ϵ, then terminate

$a \xmapsto{[\![\cdot]\!]_{\mathbf{P}}}$ atomic action a, then terminate

$e + f \xmapsto{[\![\cdot]\!]_{\mathbf{P}}}$ (*choice*) execute $[\![e]\!]_{\mathbf{P}}$ or $[\![f]\!]_{\mathbf{P}}$

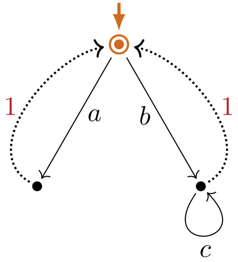
$e \cdot f \xmapsto{[\![\cdot]\!]_{\mathbf{P}}}$ (*sequentialization*) execute $[\![e]\!]_{\mathbf{P}}$, then $[\![f]\!]_{\mathbf{P}}$

$e^* \xmapsto{[\![\cdot]\!]_{\mathbf{P}}}$ (*iteration*) repeat (terminate or execute $[\![e]\!]_{\mathbf{P}}$)

$[\![e]\!]_{\mathbf{P}} := [\mathcal{C}(e)]_{\underline{\leftrightarrow}}$ (bisimilarity equivalence class of chart $\mathcal{C}(e)$)

Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)

1-chart



$\underline{\mathcal{C}}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$

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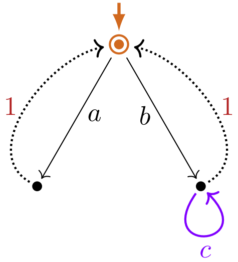

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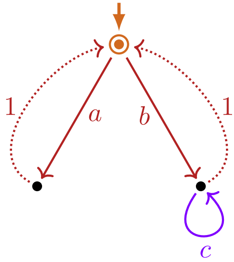

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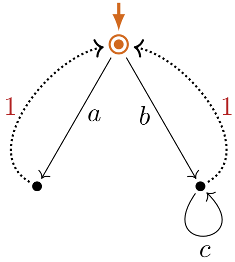

$\underline{\mathcal{C}}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$

Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)

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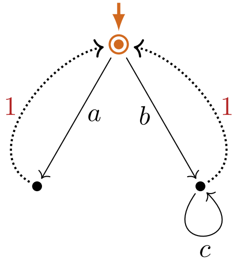
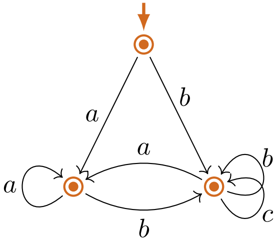
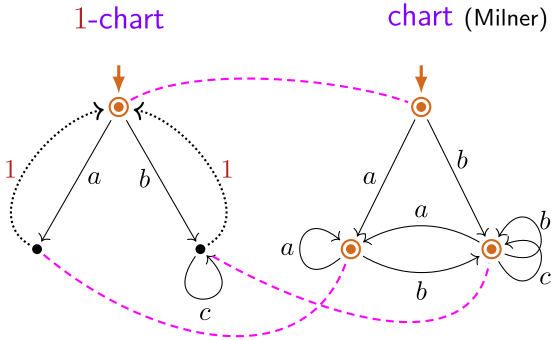


chart (Milner)



$\underline{\mathcal{C}}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$ $\mathcal{C}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$

Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)



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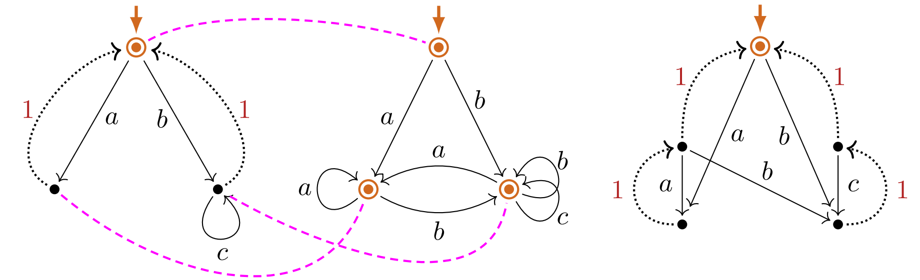
1-chart | chart (Milner) | 1-chart



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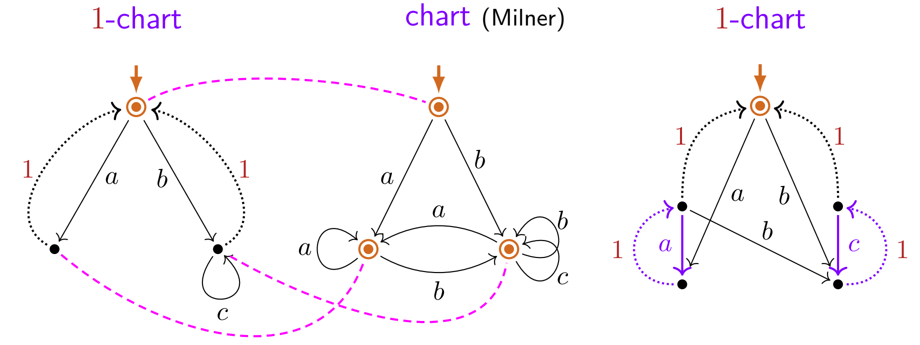
Process semantics $[\![\cdot]\!]_{P}$ (examples, bisimulation collapse)



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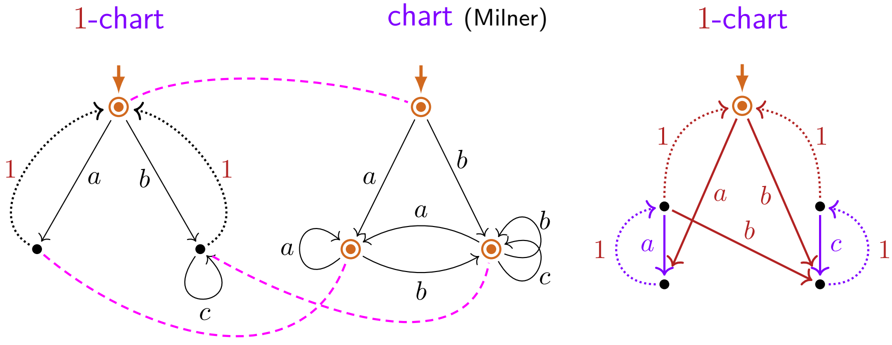

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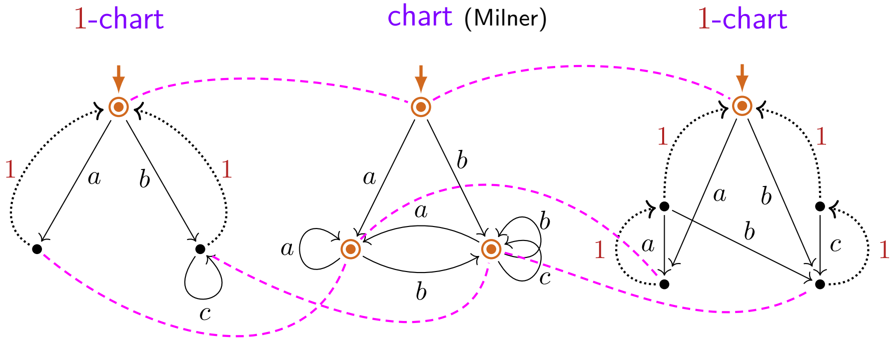

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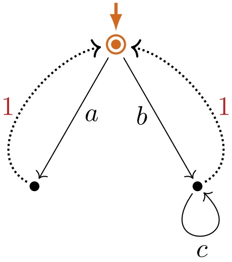
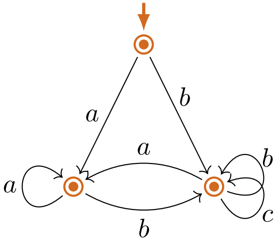
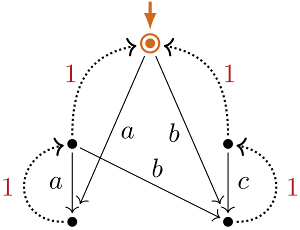
Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)

1-chart	chart (Milner)	1-chart
		

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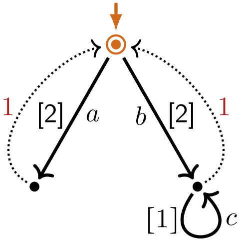
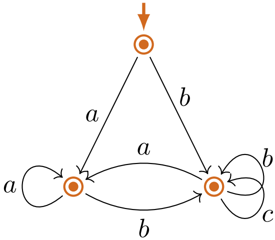
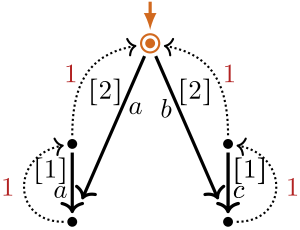
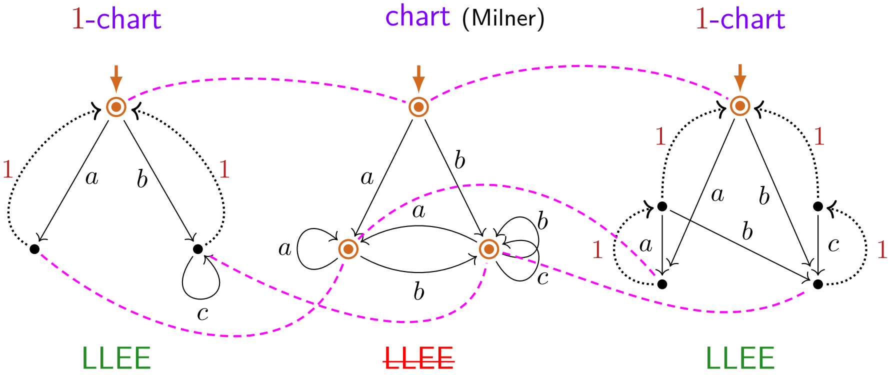
		
LLEE	~~LLEE~~	LLEE
$\underline{\mathcal{C}}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$	$\mathcal{C}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$	$\underline{\mathcal{C}}\big((a \cdot (1 \cdot ((a \cdot 1)^* \cdot 1)) \cdot b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1 + b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1\big)^*\big)$

Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)



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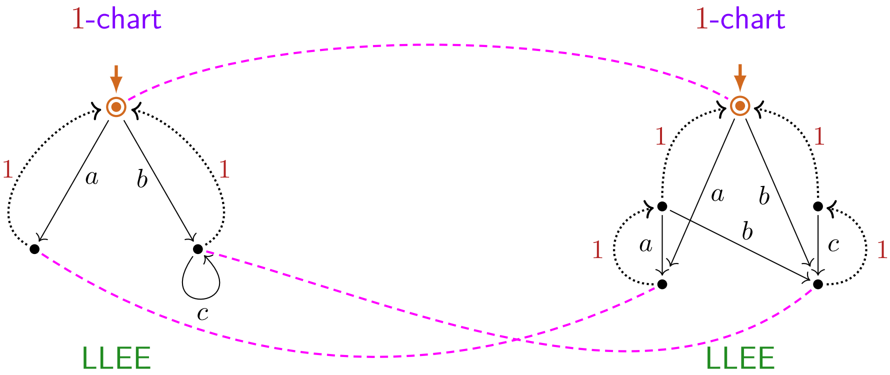
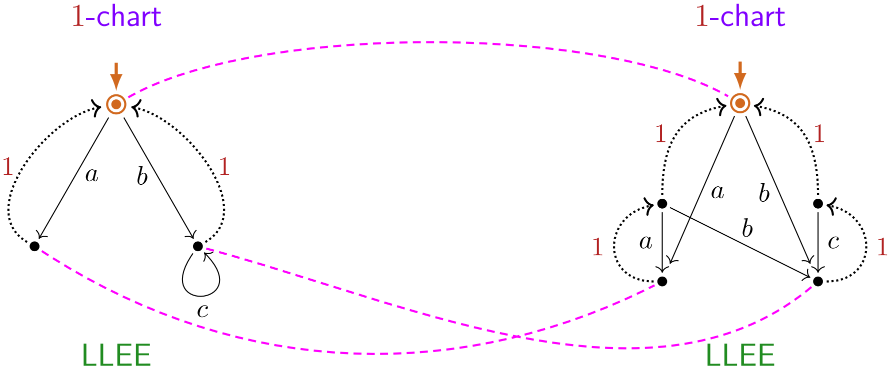
Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)



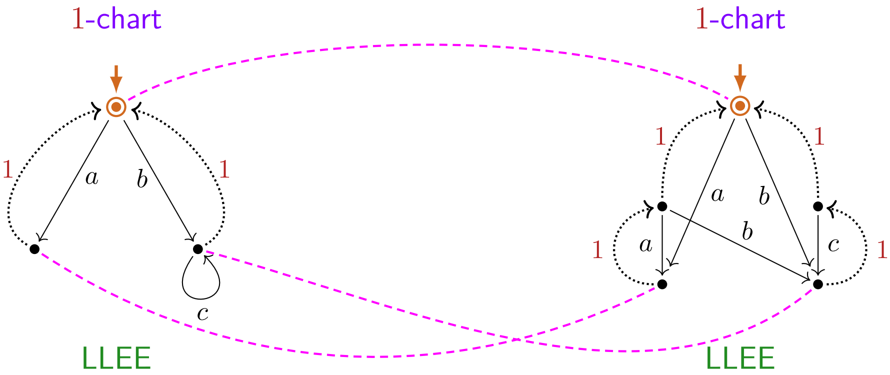
$\underline{\mathcal{C}}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$ $\quad \underline{\leftrightarrow} \quad$ $\underline{\mathcal{C}}\Big(\big(a \cdot (1 \cdot ((a \cdot 1)^* \cdot 1)) \cdot b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1 + b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1\big)^*\Big)$

Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)



$$[\![(a \cdot 1 + b \cdot (c^* \cdot 1))^*]\!]_P \;=\; [\![\big(a \cdot (1 \cdot ((a \cdot 1)^* \cdot 1)) \cdot b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1 + b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1 \big)^*]\!]_P$$

Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)



$(a \cdot 1 + b \cdot (c^* \cdot 1))^*$ $\equiv_P$ $\big(a \cdot (1 \cdot ((a \cdot 1)^* \cdot 1)) \cdot b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1 + b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1\big)^*$

Milner's proof system Mil

Axioms :

(A1) $e + (f + g) = (e + f) + g$

(A2) $e + 0 = e$

(A3) $e + f = f + e$

(A4) $e + e = e$

(A5) $e \cdot (f \cdot g) = (e \cdot f) \cdot g$

(A6) $(e + f) \cdot g = e \cdot g + f \cdot g$

(A7) $e = 1 \cdot e$

(A8) $e = e \cdot 1$

(A9) $0 = 0 \cdot e$

(A10) $e^* = 1 + e \cdot e^*$

(A11) $e^* = (1 + e)^*$

But: $e \cdot (f + g) \neq e \cdot f + e \cdot g$

But: $e \cdot 0 \neq 0$

Inference rules : rules of equational logic *plus*

$$\frac{e = f \cdot e + g}{e = f^* \cdot g}\ \mathsf{RSP}^*$$ (if f does not terminate immediately)

Milner's question (1984)

Is Mil complete with respect to $\equiv_{\mathsf{P}}$? (Does $e \equiv_{\mathsf{P}} f \Longrightarrow e =_{\mathsf{Mil}} f$ hold?)

Bisimulation collapse proof strategy (G/Fokkink, LICS'20)

For 1-free regular expressions e_1 and e_2:

e_1 is solution $\quad \mathcal{C}(e_1) \xleftrightarrow{\text{(assm)}} \mathcal{C}(e_2) \quad$ e_2 is solution

$e_1 =_{\mathsf{Mil}} e_2$

Bisimulation collapse proof strategy (G/Fokkink, LICS'20)

For 1-free regular expressions e_1 and e_2:

e_1 is solution

$$\begin{tikzcd}
\mathcal{C}(e_1) \arrow[rr, Leftrightarrow, "\text{(assm)}"] \arrow[dr, Rightarrow] & & \mathcal{C}(e_2) \arrow[dl, Rightarrow] \\
& \mathcal{C}_0 &
\end{tikzcd}$$

e_2 is solution

$e_1 =_{\mathsf{Mil}} e_2$

Bisimulation collapse proof strategy (G/Fokkink, LICS'20)

For 1-free regular expressions e_1 and e_2:

$$
\begin{tikzcd}
e_1 \text{ is solution} & \overset{\text{LLEE}}{\mathcal{C}(e_1)} \arrow[rr, Leftrightarrow, "\text{(assm)}"] \arrow[dr, Rightarrow] & & \overset{\text{LLEE}}{\mathcal{C}(e_2)} \arrow[dl, Rightarrow] & e_2 \text{ is solution} \\
& & \mathcal{C}_0 & &
\end{tikzcd}
$$

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& & \underset{\text{LLEE}}{\mathcal{C}_0} & &
\end{tikzcd}
$$

$e_1 =_{\mathsf{Mil}} e_2$

Bisimulation collapse proof strategy (G/Fokkink, LICS'20)

For 1-free regular expressions e_1 and e_2:

e_1 is solution

$$\begin{tikzcd}
\text{LLEE} \; \mathcal{C}(e_1) \arrow[rr, Leftrightarrow, "\text{(assm)}"] \arrow[dr, Rightarrow] & & \text{LLEE} \; \mathcal{C}(e_2) \arrow[dl, Rightarrow] \\
& \mathcal{C}_0 \; \text{LLEE} &
\end{tikzcd}$$

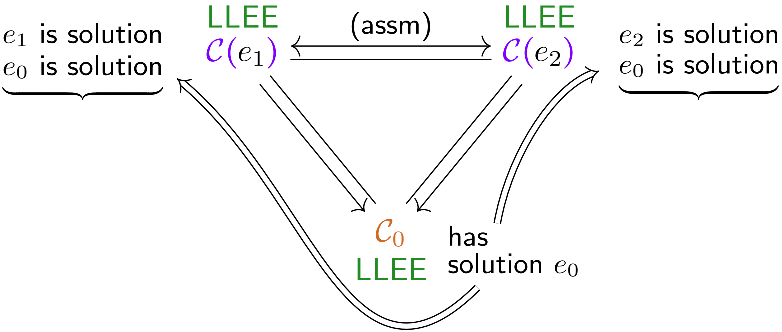
e_2 is solution

$\mathcal{C}_0$ has solution e_0

$e_1 =_{\text{Mil}} e_2$

Bisimulation collapse proof strategy (G/Fokkink, LICS'20)

For 1-free regular expressions e_1 and e_2:



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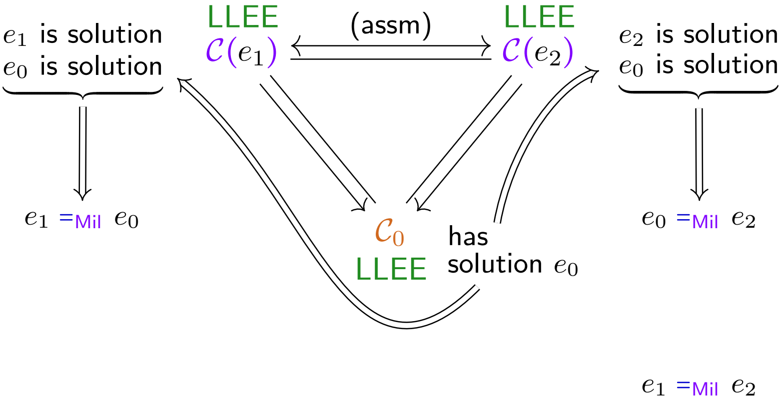

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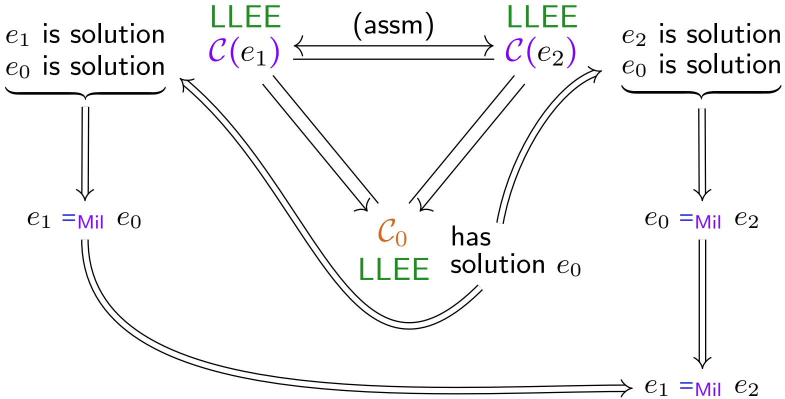


Bisimulation collapse proof strategy (general case)

Problem 1

chart interpretations

bisimulation collapse

$$\begin{tikzcd} \cancel{\text{LLEE}} & \mathcal{C}(e_1) \arrow[rr, leftrightarrow, "(\text{assm})"] \arrow[dr, Rightarrow] & & \mathcal{C}(e_2) \arrow[dl, Rightarrow] & \cancel{\text{LLEE}} \\ & & \mathcal{C}_0 & & \end{tikzcd}$$

Bisimulation collapse proof strategy (general case)

Remedy for Problem 1 *(G, TERMGRAPH 2020)*

chart interpretations ~~LLEE~~ ~~LLEE~~

$$\begin{tikzcd} \mathcal{C}(e_1) \arrow[rr, leftrightarrow, "\text{(assm)}"] \arrow[dr, Rightarrow] & & \mathcal{C}(e_2) \arrow[dl, Rightarrow] \\ & \mathcal{C}_0 & \end{tikzcd}$$

bisimulation collapse

Bisimulation collapse proof strategy (general case)

Remedy for Problem 1 *(G, TERMGRAPH 2020)*

1-*chart interpretations*

chart interpretations

bisimulation collapse

$$\begin{tikzcd}
\text{LLEE} & \underline{\mathcal{C}}(e_1) \arrow[d, Rightarrow, "\text{(IV)}"] & & \underline{\mathcal{C}}(e_2) \arrow[d, Rightarrow, "\text{(IV)}"'] & \text{LLEE} \\
\cancel{\text{LLEE}} & \mathcal{C}(e_1) \arrow[rr, leftrightarrow, "\text{(assm)}"] \arrow[dr, Rightarrow] & & \mathcal{C}(e_2) \arrow[dl, Rightarrow] & \cancel{\text{LLEE}} \\
& & \mathcal{C}_0 & &
\end{tikzcd}$$

Bisimulation collapse proof strategy (general case)

Remedy for Problem 1 *(G, TERMGRAPH 2020)*

1-chart interpretations

chart interpretations

bisimulation collapse

$$\begin{tikzcd}
\text{LLEE} & \underline{\mathcal{C}}(e_1) \arrow[rr, Leftrightarrow] \arrow[dr, Rightarrow, "\text{(IV)}"] & & \underline{\mathcal{C}}(e_2) \arrow[dl, Rightarrow, "\text{(IV)}"'] & \text{LLEE} \\
\cancel{\text{LLEE}} & & \mathcal{C}(e_1) \arrow[r, leftrightarrow, "\text{(assm)}"] \arrow[dr, Rightarrow] & \mathcal{C}(e_2) \arrow[d, Rightarrow] & \cancel{\text{LLEE}} \\
& & & \mathcal{C}_0 &
\end{tikzcd}$$

Bisimulation collapse proof strategy (general case)

Remedy for Problem 1 *(G, TERMGRAPH 2020)*

$$
\begin{tikzcd}
\textit{1-chart interpretations} & \mathsf{LLEE} & \underline{\mathcal{C}}(e_1) \arrow[rr, Leftrightarrow] \arrow[d, Rightarrow, "\text{(IV)}"] \arrow[dr, Rightarrow, bend right] & & \underline{\mathcal{C}}(e_2) \arrow[d, Rightarrow, "\text{(IV)}"'] \arrow[dl, Rightarrow, bend left] & \mathsf{LLEE} \\
\textit{chart interpretations} & \cancel{\mathsf{LLEE}} & \mathcal{C}(e_1) \arrow[rr, Leftrightarrow, "\text{(assm)}"] \arrow[dr, Rightarrow] & & \mathcal{C}(e_2) \arrow[dl, Rightarrow] & \cancel{\mathsf{LLEE}} \\
\textit{bisimulation collapse} & & & \mathcal{C}_0 & &
\end{tikzcd}
$$

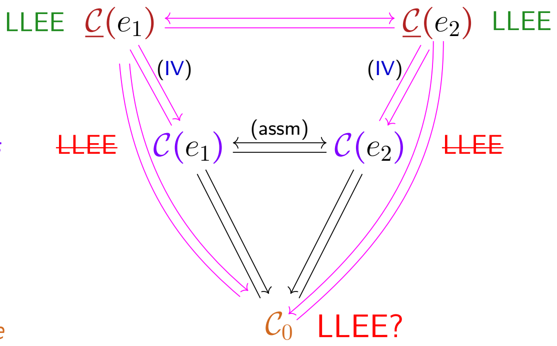
Bisimulation collapse proof strategy (general case)

Remedy for Problem 1 *(G, TERMGRAPH 2020)*

1-*chart interpretations*

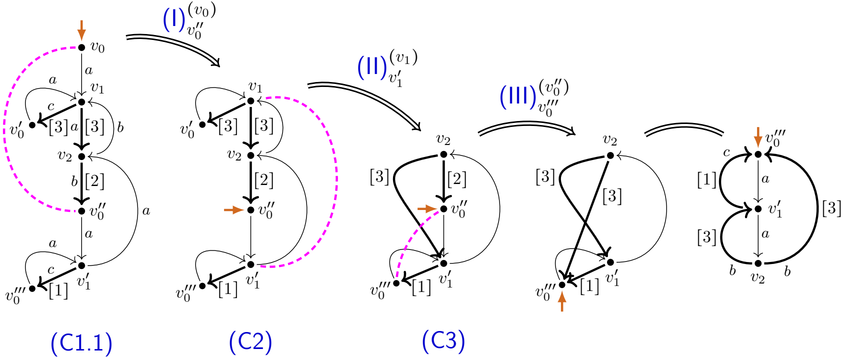
chart interpretations

bisimulation collapse



LLEE-preserving collapse of LLEE-charts (G/Fokkink, LICS'20)

(no 1-transitions!)



Lemma

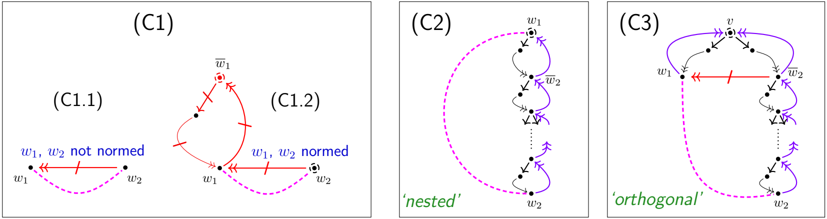
The bisimulation collapse of a LLEE-chart is again a LLEE-chart.

Reduced bisimilarity redundancies in LLEE-charts (no 1-trans.!)

(G/Fokkink, LICS'20)

w_1, w_2 in different scc's

w_1, w_2 in the same scc

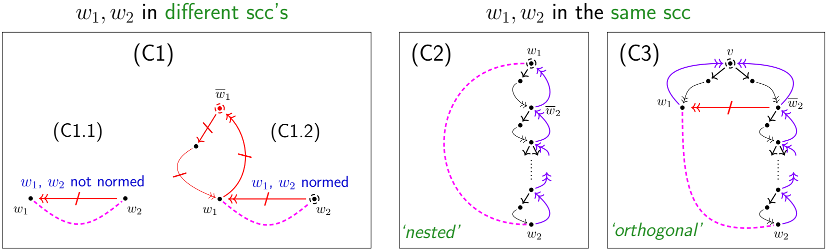


Lemma

Every not collapsed LLEE-chart contains bisimilar vertices $w_1 \neq w_2$ of kind (C1), (C2), or (C3) (a reduced bisimilarity redundancy $\langle w_1, w_2 \rangle$):

Reduced bisimilarity redundancies in LLEE-charts (no 1-trans.!)

(G/Fokkink, LICS'20)



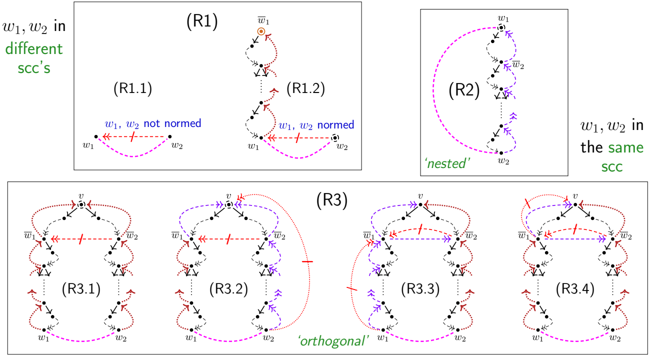
Lemma

Every not collapsed LLEE-chart contains bisimilar vertices $w_1 \neq w_2$ of kind (C1), (C2), or (C3) (a reduced bisimilarity redundancy $\langle w_1, w_2 \rangle$):

Lemma

Every reduced bisimilarity redundancy in a LLEE-chart can be eliminated LLEE-preservingly.

Reduced 1-bisimilarity redundancies in LLEE-1-charts

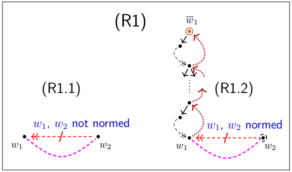


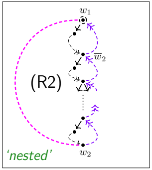
Lemma

Every not collapsed LLEE-1-*chart contains* 1*-bisimilar vertices* $w_1 \neq w_2$ *(a reduced* 1*-bisimilarity redundancy* $\langle w_1, w_2 \rangle$*) of kind* (R1), (R2), (R3).

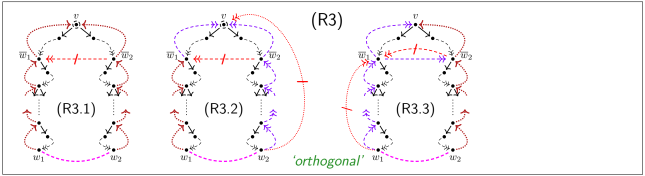
Reduced 1-bisimilarity redundancies in LLEE-1-charts

w_1, w_2 in different scc's





w_1, w_2 in the same scc



Lemma

Every not collapsed LLEE*-1-chart contains 1-bisimilar vertices $w_1 \neq w_2$ (a reduced 1-bisimilarity redundancy $\langle w_1, w_2 \rangle$) of kind* (R1), (R2), (R3).

Lemma

Every simple reduced 1-bisimilarity redundancies in a LLEE*-1-chart can be eliminated* LLEE*-preservingly.*

Reduced 1-bisimilarity redundancies in LLEE-1-charts

w_1, w_2 in different scc's

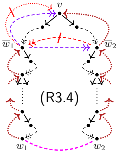
(R1)

(R2)

'nested'

w_1, w_2 in the same scc

(R3)



'orthogonal'

Lemma

Every not collapsed LLEE-1-*chart contains* 1*-bisimilar vertices* $w_1 \neq w_2$ *(a reduced* 1*-bisimilarity redundancy* $\langle w_1, w_2 \rangle$*) of kind* (R1), (R2), (R3).

Stumbling Block

How to LLEE-preservingly eliminate
reduced 1-bisimilarity redundancies of kind (R3.4)?

Reduced 1-bisimilarity redundancies in LLEE-1-charts

w_1, w_2 in different scc's

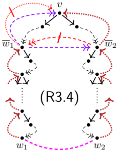
(R1)

(R2)

'nested'

w_1, w_2 in the same scc

(R3)



'orthogonal'

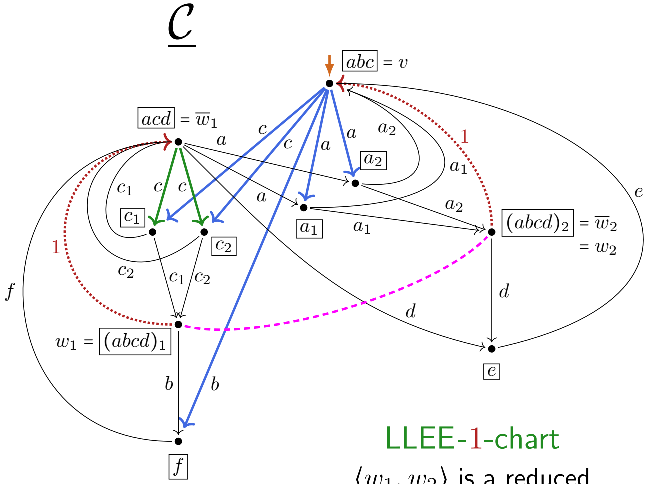
Lemma

Every not collapsed LLEE-1-chart contains 1-bisimilar vertices $w_1 \neq w_2$ (a reduced 1-bisimilarity redundancy $\langle w_1, w_2 \rangle$) of kind (R1), (R2), (R3).

Stumbling Block

How to LLEE-preservingly eliminate
precrystalline reduced 1-bisimilarity redundancies?

Counterexample LLEE-preserving collapse

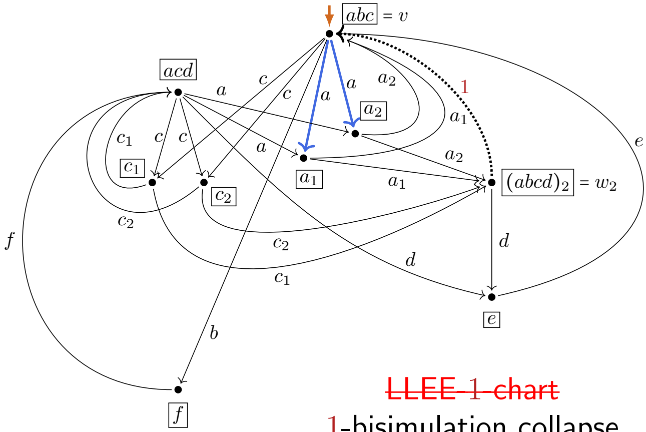


LLEE-1-chart

$\langle w_1, w_2 \rangle$ is a reduced 1-bisimilarity redundancy of kind (R3.4)

Counterexample LLEE-preserving collapse

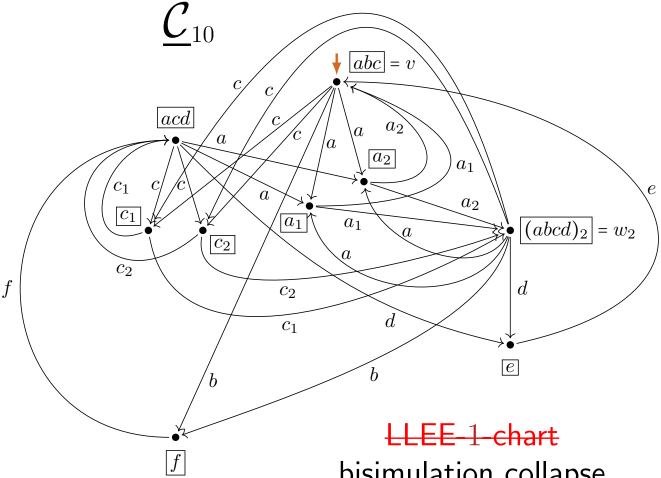
$\underline{\mathcal{C}}_1$



~~LLEE-1-chart~~

1-bisimulation collapse

Counterexample LLEE-preserving collapse



~~LLEE-1-chart~~

bisimulation collapse

Bisimulation collapse proof strategy (general case)

Problem 2: There are regular expressions e_1 and e_2 such that:

1-*chart interpretations*

chart interpretations

bisimulation collapse

$$
\begin{tikzcd}
\text{LLEE}\ \underline{\mathcal{C}}(e_1) \arrow[rr, Leftrightarrow] \arrow[d, Rightarrow, "\text{(IV)}"] \arrow[dr, Rightarrow, bend right] & & \underline{\mathcal{C}}(e_2)\ \text{LLEE} \arrow[d, Rightarrow, "\text{(IV)}"'] \arrow[dl, Rightarrow, bend left] \\
\cancel{\text{LLEE}}\ \mathcal{C}(e_1) \arrow[rr, Leftrightarrow, "\text{(assm)}"] \arrow[dr, Rightarrow] & & \mathcal{C}(e_2)\ \cancel{\text{LLEE}} \arrow[dl, Rightarrow] \\
& \mathcal{C}_0\ \cancel{\text{LLEE}} &
\end{tikzcd}
$$

Bisimulation collapse proof strategy (general case)

Problem 2: There are regular expressions e_1 and e_2 such that:

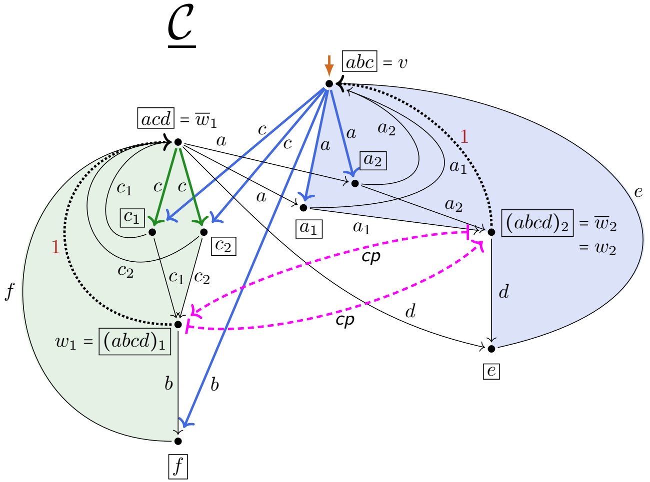
1-*chart interpretations*

chart interpretations

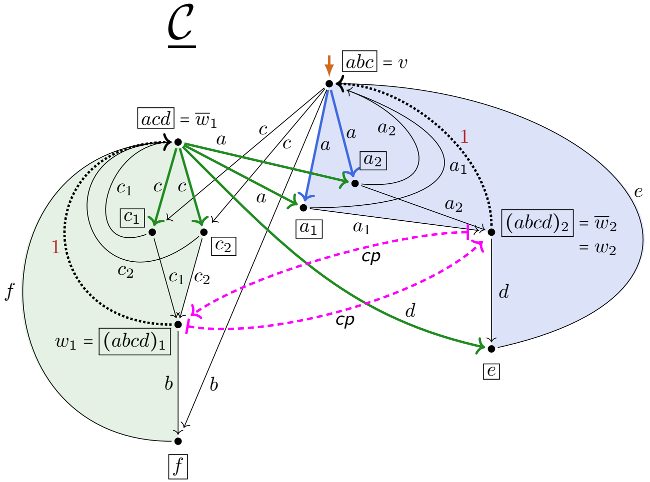
$$
\begin{tikzcd}
\text{LLEE} & \underline{\mathcal{C}}(e_1) \arrow[rrrr, Leftrightarrow] \arrow[dr, Rightarrow, "\text{(IV)}"] \arrow[ddrr, Rightarrow] & & & & \underline{\mathcal{C}}(e_2) \arrow[dl, Rightarrow, "\text{(IV)}"'] \arrow[ddll, Rightarrow] & \text{LLEE} \\
\cancel{\text{LLEE}} & & \mathcal{C}(e_1) \arrow[rr, Leftrightarrow, "\text{(assm)}"] \arrow[dr, Rightarrow] & & \mathcal{C}(e_2) \arrow[dl, Rightarrow] & & \cancel{\text{LLEE}} \\
& & & \text{for all } \mathcal{C}_0\text{: } \cancel{\text{LLEE}} & & &
\end{tikzcd}
$$

$\mathcal{C}(e_1)$, $\mathcal{C}(e_2)$, $\underline{\mathcal{C}}(e_1)$ and $\underline{\mathcal{C}}(e_2)$ are **not** LLEE-preserving jointly minimizable under bisimilarity.

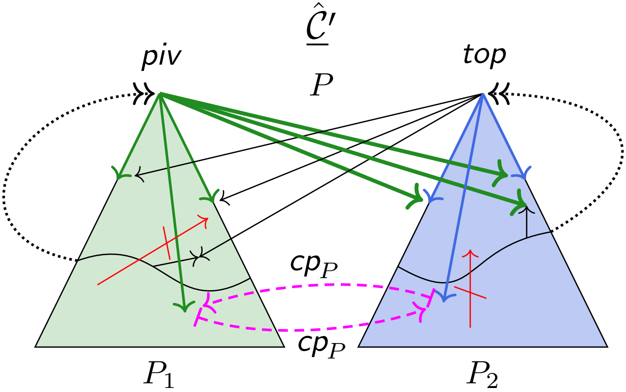
Twin-Crystal



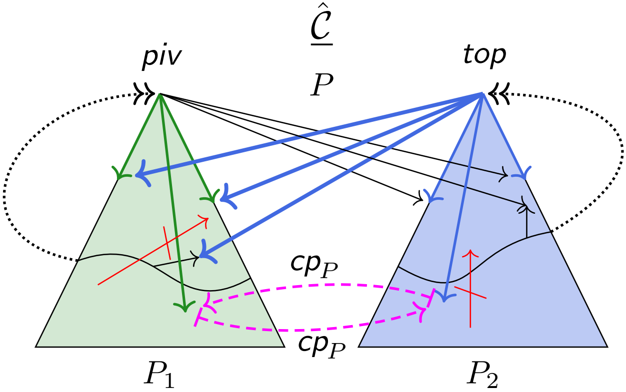
Twin-Crystal



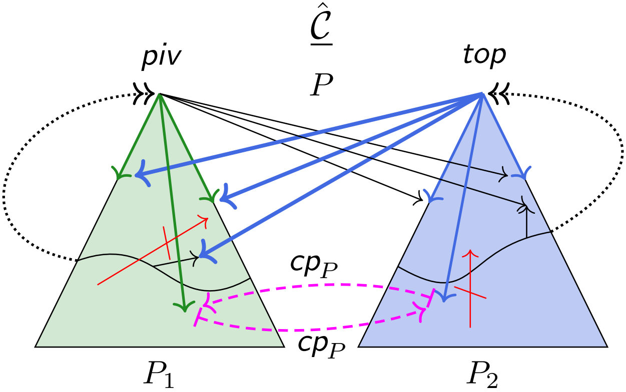
Twin-Crystal



Twin-Crystal

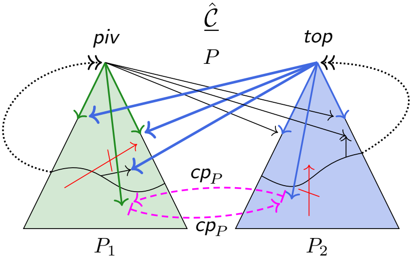


Twin-Crystal



- Mil-provable solutions of twin-crystals can be transferred to their bisimulation collapses

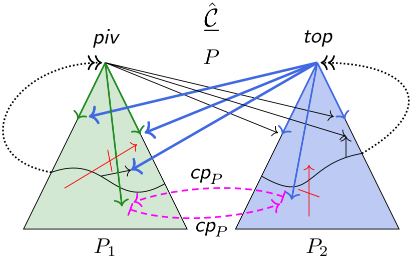
Crystallization



twin-crystal

Crystallized 1-charts = LLEE-1-charts that are collapsed apart from strongly connected components of twin-crystal form.

Crystallization

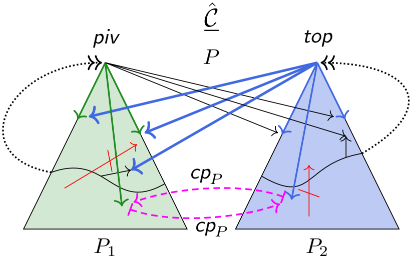


twin-crystal

Crystallized 1-charts = LLEE-1-charts that are collapsed apart from strongly connected components of twin-crystal form.

(CR) Crystallization: Every LLEE-1-chart can be reduced under bisimilarity to a 1-bisimilar crystallized 1-chart.

Crystallization



twin-crystal

Crystallized 1-charts = LLEE-1-charts that are collapsed
apart from strongly connected components of twin-crystal form.

(CR) Crystallization: Every LLEE-1-chart can be reduced
under bisimilarity to a 1-bisimilar crystallized 1-chart.

(CC) Every Mil-provable solution of a crystallized 1-chart
give rise to Mil-provable solution on the bisimulation collapse.

Completeness proof of Mil (structure)

chart interpretations $\mathcal{C}(e_1) \xleftrightarrow{\text{(assm)}} \mathcal{C}(e_2)$

$\overset{?}{\Longrightarrow}$ $e_1 =_{\text{Mil}} e_2$

Completeness proof of Mil (structure)

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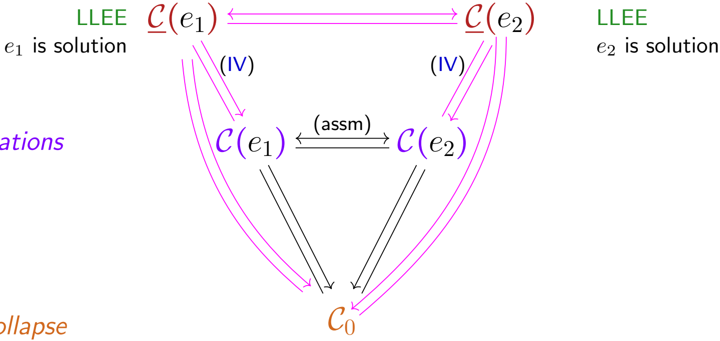
Completeness proof of Mil (structure)

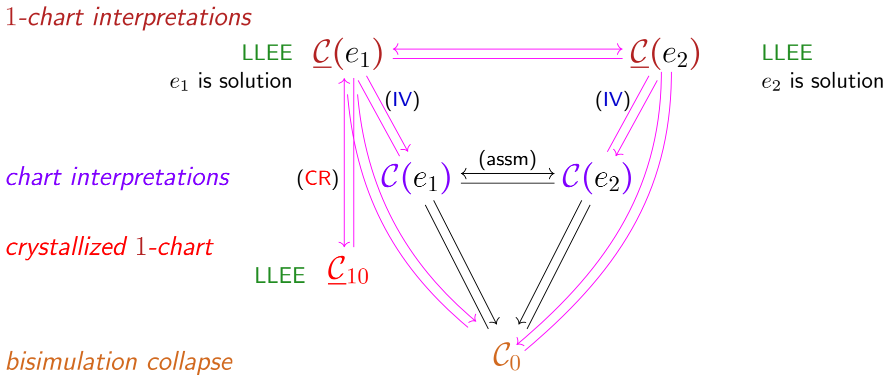
1-chart interpretations

chart interpretations

bisimulation collapse



Completeness proof of Mil (structure)



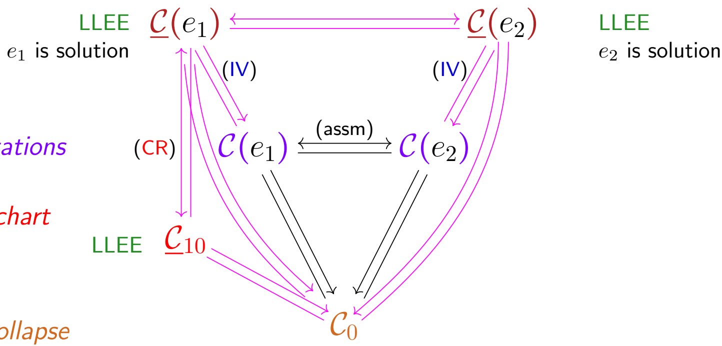
Completeness proof of Mil (structure)

1-chart interpretations

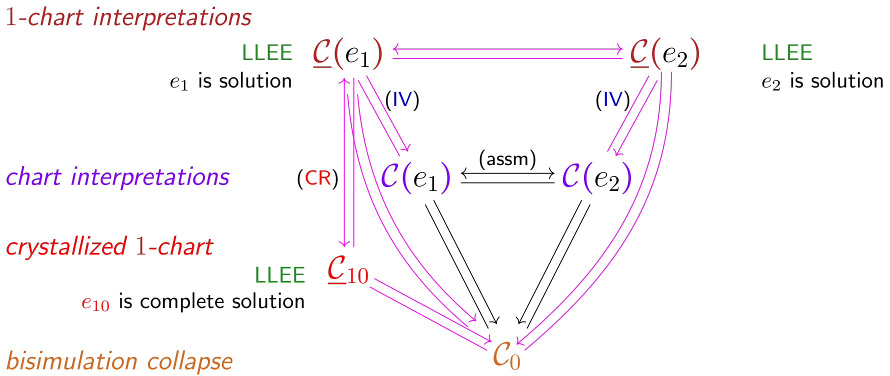
chart interpretations

crystallized 1-chart

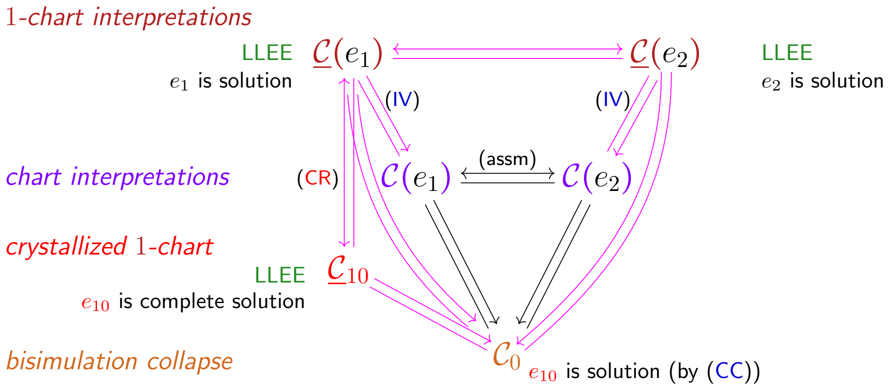
bisimulation collapse



Completeness proof of Mil (structure)



Completeness proof of Mil (structure)



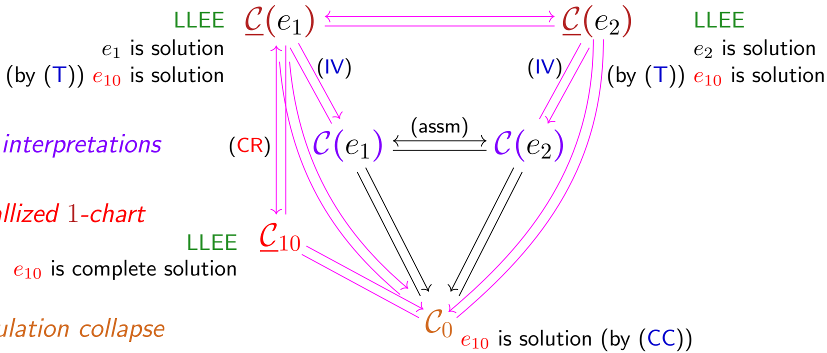
Completeness proof of Mil (structure)

1-chart interpretations

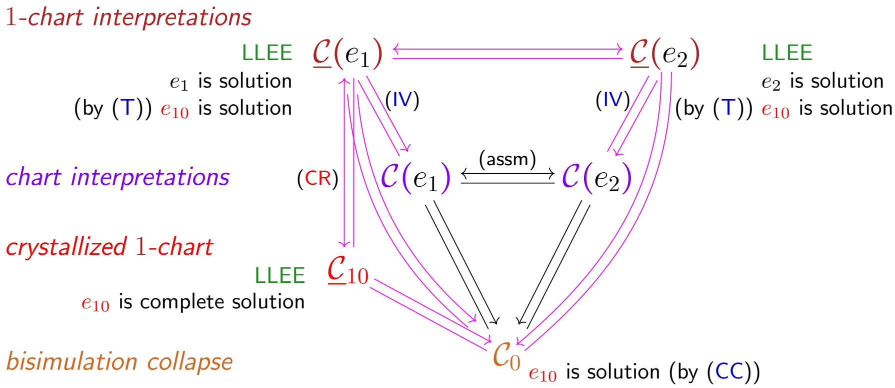
chart interpretations

crystallized 1-chart

bisimulation collapse



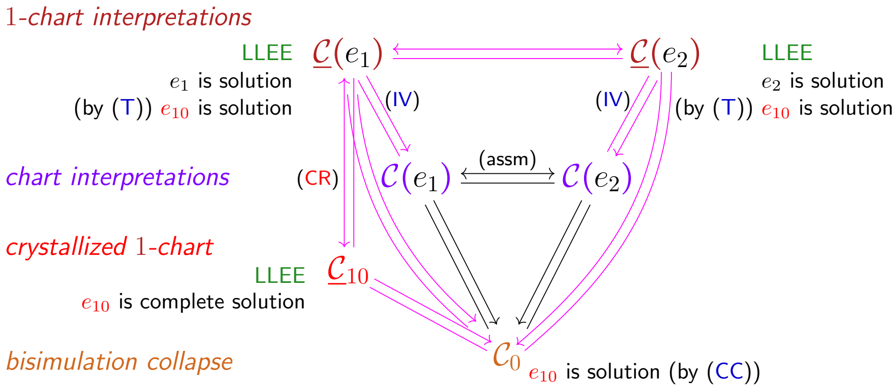
Completeness proof of Mil (structure)



$\mathcal{C}(e_1)$ LLEE
e_1 is solution
e_{10} is solution

$\mathcal{C}(e_2)$ LLEE
e_2 is solution
e_{10} is solution

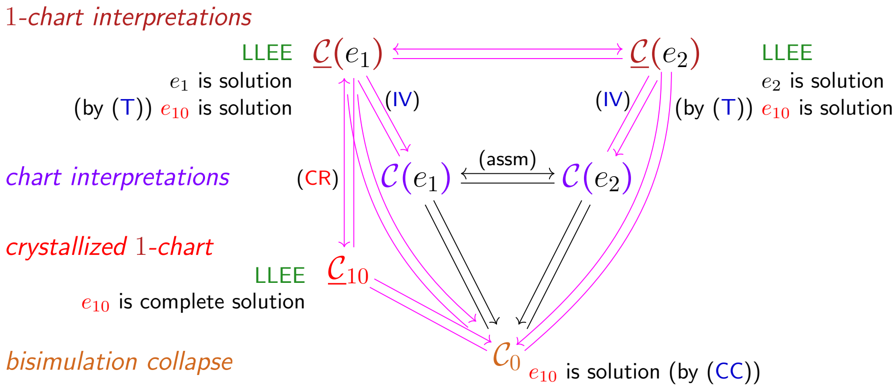
Completeness proof of Mil (structure)



$$\left.\begin{array}{r} \mathcal{C}(e_1)\ \text{LLEE} \\ e_1\ \text{is solution} \\ e_{10}\ \text{is solution} \end{array}\right\} \underset{\text{(SE)}}{\Longrightarrow} e_1 =_{\mathsf{Mil}} e_{10}$$

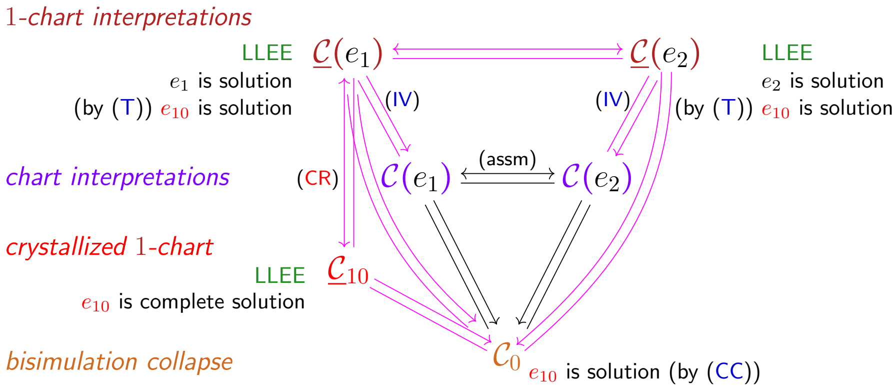
$\mathcal{C}(e_2)$ LLEE
e_2 is solution
e_{10} is solution

Completeness proof of Mil (structure)



$$\left.\begin{array}{r}\mathcal{C}(e_1)\ \text{LLEE}\\ e_1\ \text{is solution}\\ e_{10}\ \text{is solution}\end{array}\right\} \underset{\text{(SE)}}{\Longrightarrow} e_1 =_{\mathsf{Mil}} e_{10} \qquad e_{10} =_{\mathsf{Mil}} e_2 \underset{\text{(SE)}}{\Longleftarrow} \left\{\begin{array}{l}\mathcal{C}(e_2)\ \text{LLEE}\\ e_2\ \text{is solution}\\ e_{10}\ \text{is solution}\end{array}\right.$$

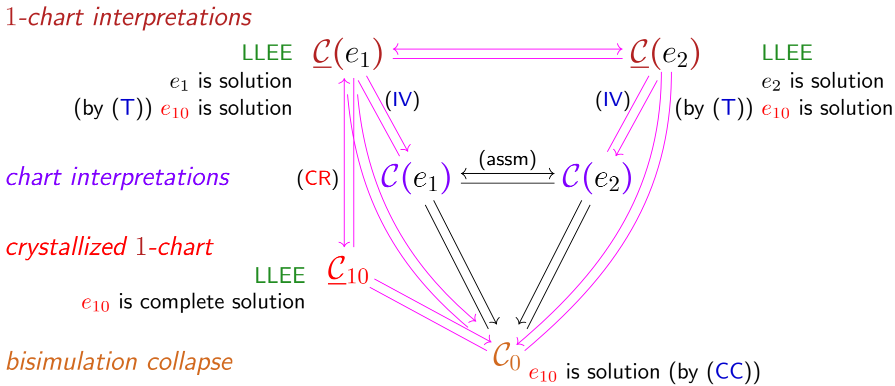
Completeness proof of Mil (structure)



$$\left.\begin{array}{r} \mathcal{C}(e_1)\ \text{LLEE} \\ e_1 \text{ is solution} \\ e_{10} \text{ is solution}\end{array}\right\} \underset{\text{(SE)}}{\Longrightarrow} e_1 =_{\mathsf{Mil}} e_{10} \qquad e_{10} =_{\mathsf{Mil}} e_2 \underset{\text{(SE)}}{\Longleftarrow} \left\{\begin{array}{l} \mathcal{C}(e_2)\ \text{LLEE} \\ e_2 \text{ is solution} \\ e_{10} \text{ is solution}\end{array}\right.$$

$$\Longrightarrow \qquad e_1 =_{\mathsf{Mil}} e_2$$

Completeness proof of Mil (structure)



Theorem

Milner's proof system Mil is complete for process semantics equivalence $\equiv_{\mathbf{P}}$ of regular expressions.

Since: $e_1 \equiv_{\mathbf{P}} e_2 \Longrightarrow [\![e_1]\!]_{\mathbf{P}} = [\![e_2]\!]_{\mathbf{P}} \Longrightarrow \mathcal{C}(e_1) \,\underline{\leftrightarrow}\, \mathcal{C}(e_1) \Longrightarrow e_1 =_{\mathsf{Mil}} e_2$.

Outlook

poster presentation

- tomorrow, 10–10.30

next steps and projects

- monograph project: proof in fine-grained detail
- computation/animation tool for crystallization
- use crystallization for recognition problem

resources on Github:

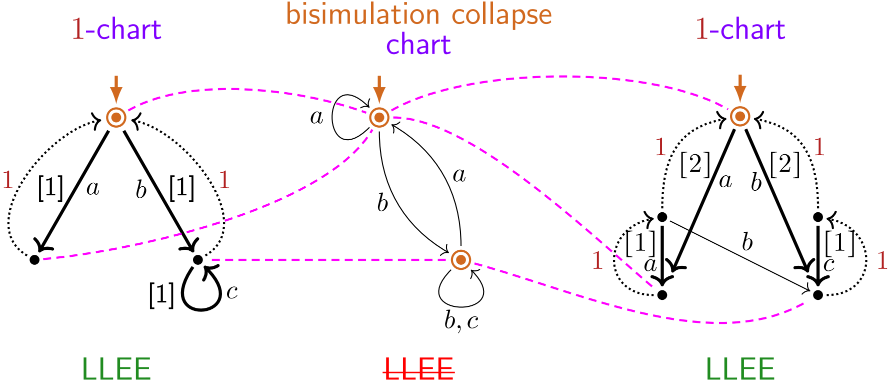
- `https://github.com/clegra/crystallization/blob/main`
  - article (after rebuttal): `/cryst-article.pdf`
  - poster: `/poster-lics2022.pdf`
  - presentation: `/presentation-lics2022.pdf`

acknowledgment & thanks to:

- Wan Fokkink (for long collaboration)

Thank you for your attention!

Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)



$\underline{\mathcal{C}}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$

$\underline{\mathcal{C}}((a + b \cdot c^*)^*)$

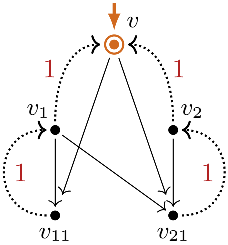
$\mathcal{C}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$

$\mathcal{C}((a + b \cdot c^*)^*)$

$$\underline{\mathcal{C}}\Big(\big(a \cdot (1 \cdot ((a \cdot 1)^* \cdot 1)) \cdot b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1 + b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1\big)^*\Big)$$

$\underline{\mathcal{C}}((a^* \cdot (1 + b \cdot c^*))^*)$

Layered loop existence/elimination and LLEE-witnesses



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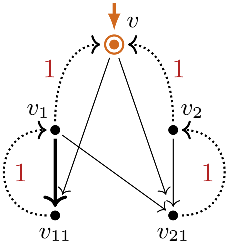

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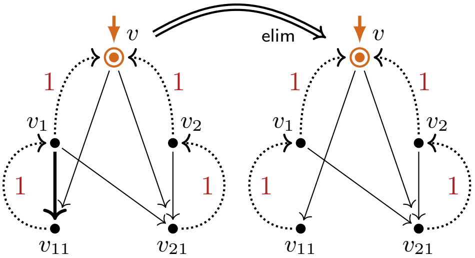

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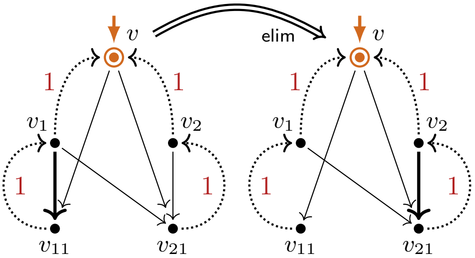

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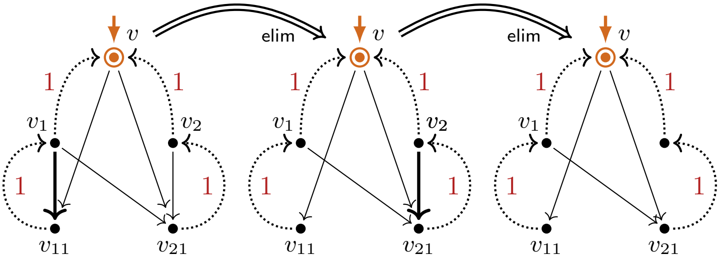
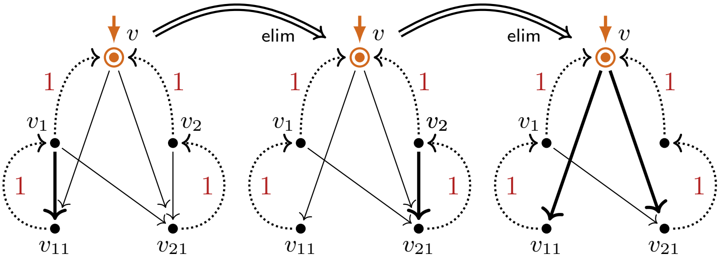


Layered loop existence/elimination and LLEE-witnesses



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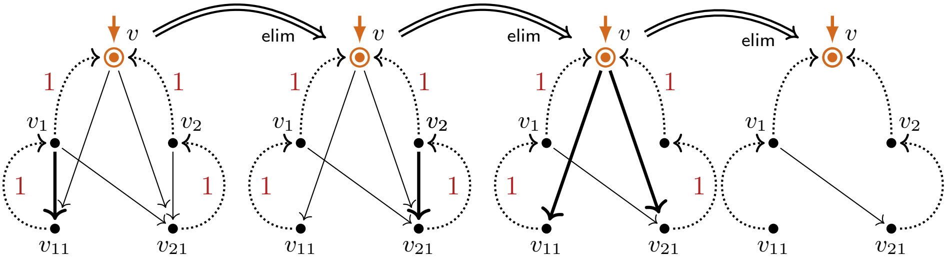
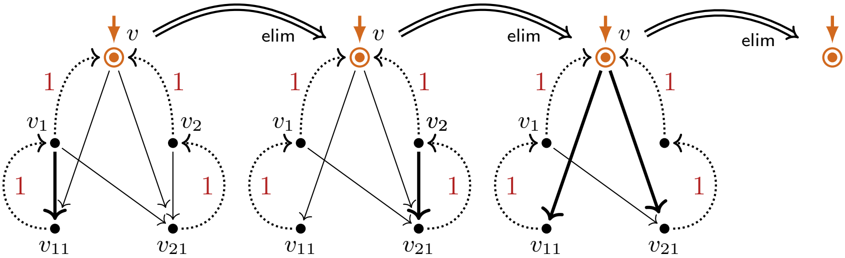
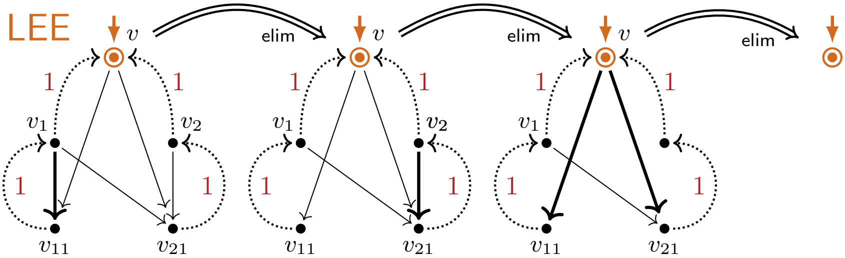


Layered loop existence/elimination and LLEE-witnesses



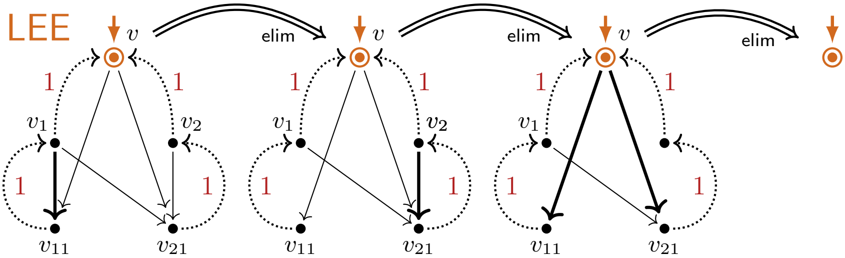
Layered loop existence/elimination and LLEE-witnesses

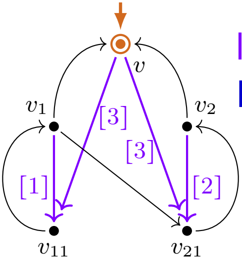
LEE



Layered loop existence/elimination and LLEE-witnesses

LEE

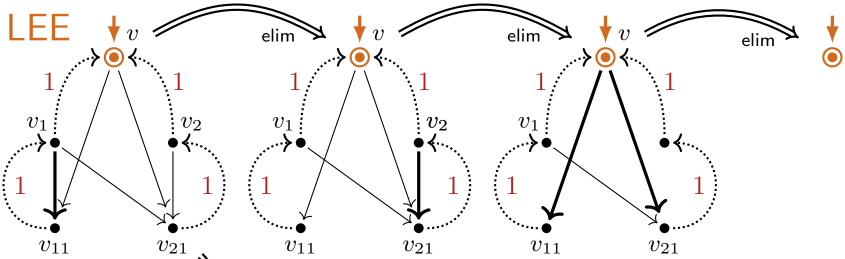




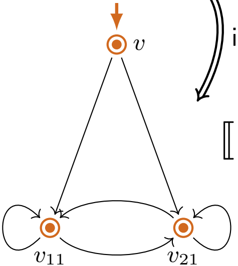
layered LEE-witness

Layered loop existence/elimination and LLEE-witnesses

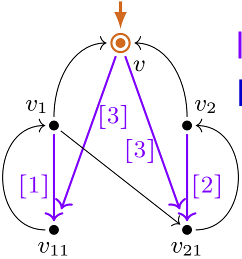
LEE



induces

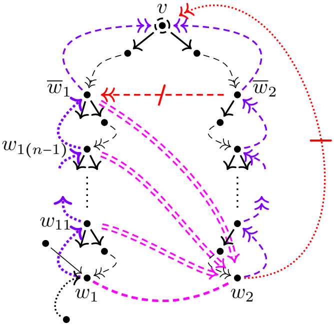


$[\![(a^* \cdot b^*)^*]\!]_{\mathbf{P}}$



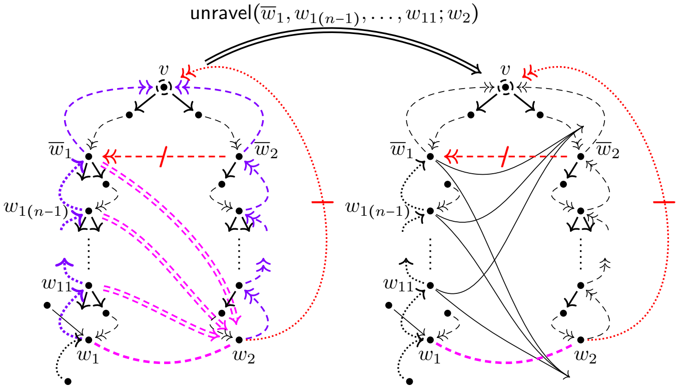
layered LEE-witness

Eliminating reduced 1-bisimilarity redundancies (example)



reduced 1-bisimilarity
redundancy of kind (R3.2)

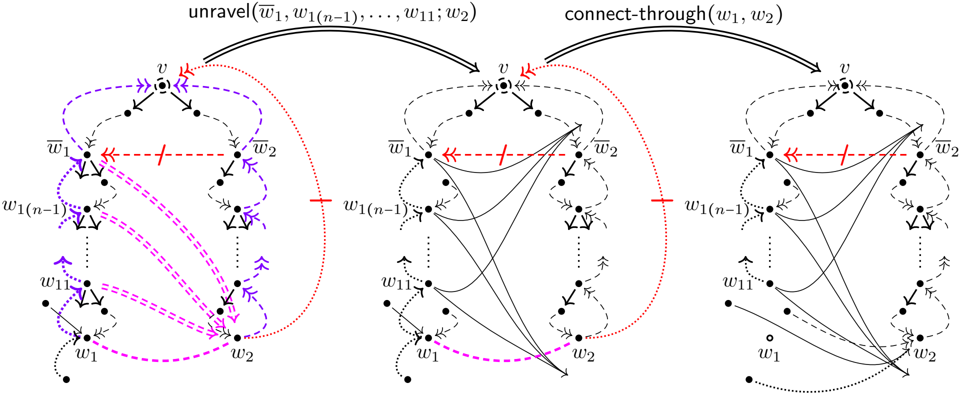
Eliminating reduced 1-bisimilarity redundancies (example)



reduced 1-bisimilarity redundancy of kind (R3.2)

result of unravelling loop vertices $\overline{w}_1, w_{1(n-1)}, \ldots, w_{11}$

Eliminating reduced 1-bisimilarity redundancies (example)



reduced 1-bisimilarity redundancy of kind (R3.2)

result of unravelling loop vertices $\overline{w}_1, w_{1(n-1)}, \ldots, w_{11}$

result of connecting through w_1 to w_2

Completeness proof of Mil (structure)

chart interpretations

$$\mathcal{C}(e_1) \xleftrightarrow{\text{(assm)}} \mathcal{C}(e_2)$$

$$\overset{?}{\Longrightarrow} \qquad e_1 =_{\mathsf{Mil}} e_2$$

Completeness proof of Mil (structure)

chart interpretations

$$\mathcal{C}(e_1) \xleftrightarrow{\text{(assm)}} \mathcal{C}(e_2)$$

Completeness proof of Mil (structure)

1-chart interpretations

guarded, LLEE
e_1 is solution

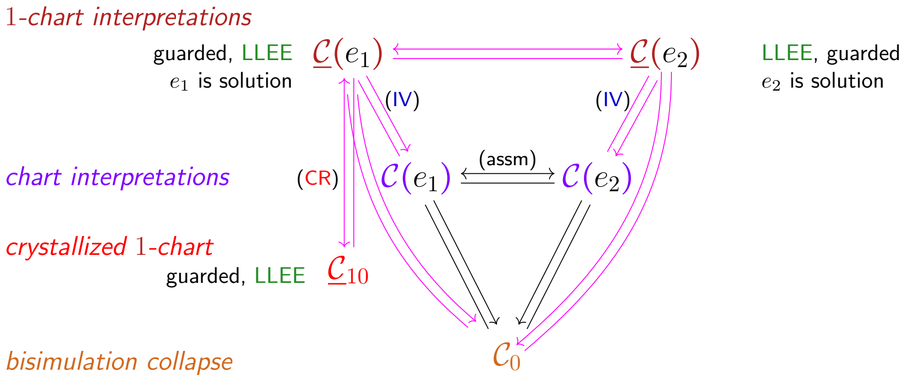
$$\begin{tikzcd}
\underline{\mathcal{C}}(e_1) \arrow[rrrr, Leftrightarrow] \arrow[dr, Rightarrow, "\text{(IV)}"] \arrow[ddrr, Rightarrow, bend right] & & & & \underline{\mathcal{C}}(e_2) \arrow[dl, Rightarrow, "\text{(IV)}"'] \arrow[ddll, Rightarrow, bend left] \\
& \mathcal{C}(e_1) \arrow[rr, leftrightarrow, "\text{(assm)}"] \arrow[dr, Rightarrow] & & \mathcal{C}(e_2) \arrow[dl, Rightarrow] & \\
& & \mathcal{C}_0 & &
\end{tikzcd}$$

LLEE, guarded
e_2 is solution

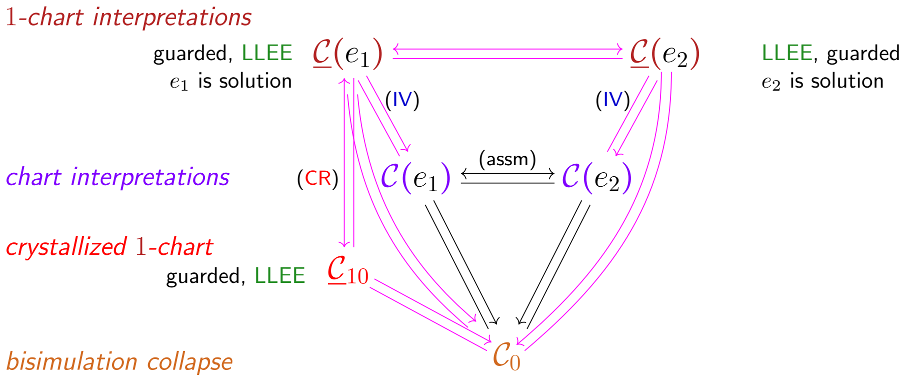
chart interpretations

bisimulation collapse

Completeness proof of Mil (structure)



Completeness proof of Mil (structure)



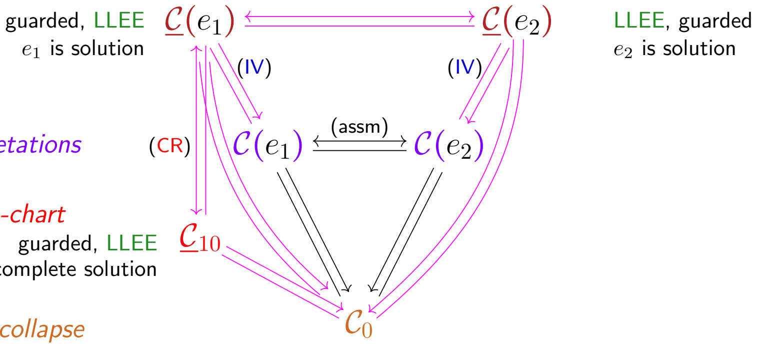
Completeness proof of Mil (structure)

1-chart interpretations

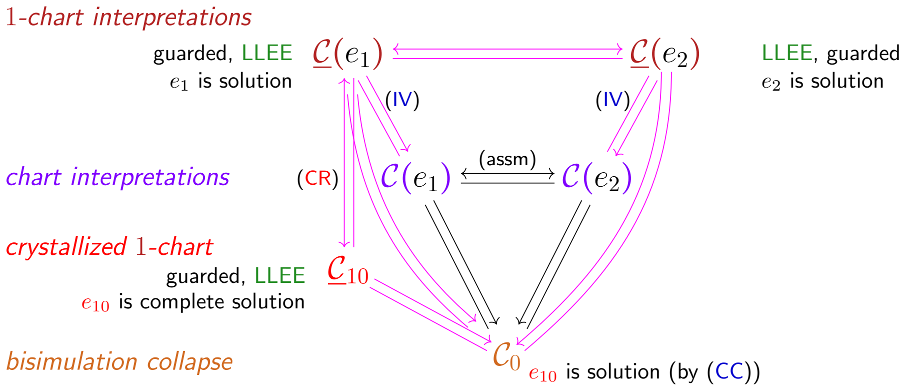
chart interpretations

crystallized 1-chart

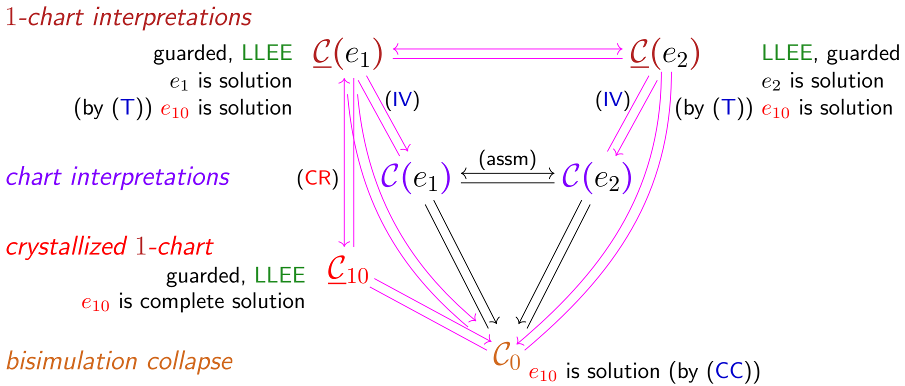
bisimulation collapse



Completeness proof of Mil (structure)



Completeness proof of Mil (structure)



Completeness proof of Mil (structure)

1-chart interpretations

guarded, LLEE
e_1 is solution
(by (T)) e_{10} is solution

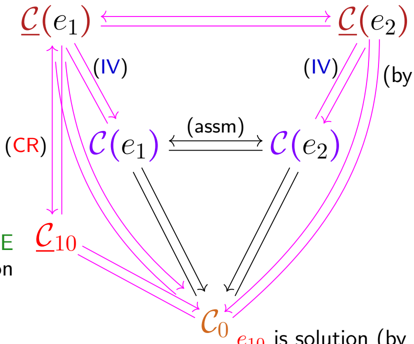
LLEE, guarded
e_2 is solution
(by (T)) e_{10} is solution

chart interpretations

crystallized 1-chart

guarded, LLEE
e_{10} is complete solution

bisimulation collapse

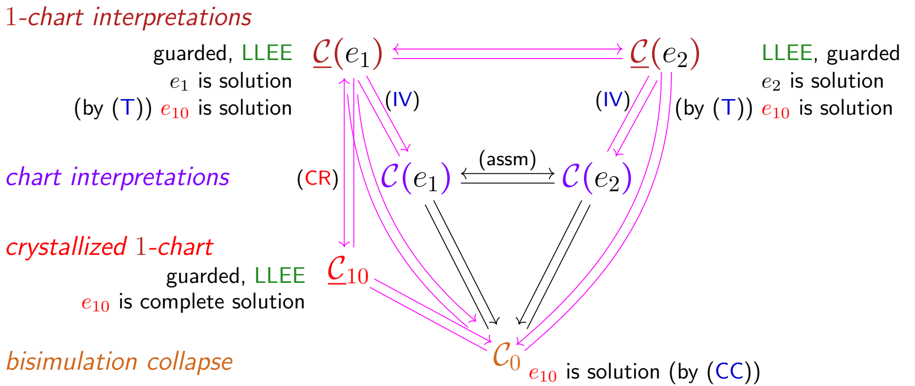


e_{10} is solution (by (CC))

$\mathcal{C}(e_1)$ guarded, LLEE
e_1 is solution
e_{10} is solution

$\mathcal{C}(e_2)$ guarded, LLEE
e_2 is solution
e_{10} is solution

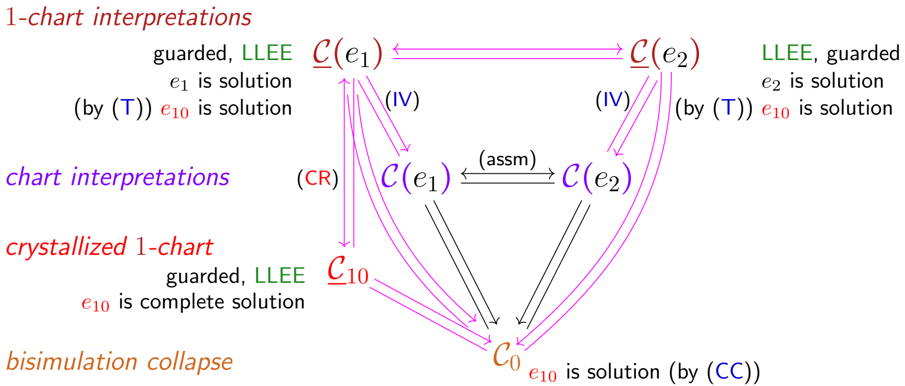
Completeness proof of Mil (structure)



$$\left.\begin{array}{r} \mathcal{C}(e_1) \text{ guarded, LLEE} \\ e_1 \text{ is solution} \\ e_{10} \text{ is solution}\end{array}\right\} \underset{\text{(SE)}}{\Longrightarrow} e_1 =_{\mathsf{Mil}} e_{10}$$

$\mathcal{C}(e_2)$ guarded, LLEE
e_2 is solution
e_{10} is solution

Completeness proof of Mil (structure)



$$\left.\begin{array}{r} \mathcal{C}(e_1) \text{ guarded, LLEE} \\ e_1 \text{ is solution} \\ e_{10} \text{ is solution} \end{array}\right\} \underset{\text{(SE)}}{\Longrightarrow} e_1 =_{\mathsf{Mil}} e_{10} \qquad e_{10} =_{\mathsf{Mil}} e_2 \underset{\text{(SE)}}{\Longleftarrow} \left\{\begin{array}{l} \mathcal{C}(e_2) \text{ guarded, LLEE} \\ e_2 \text{ is solution} \\ e_{10} \text{ is solution} \end{array}\right.$$

Completeness proof of Mil (structure)

1-chart interpretations

guarded, LLEE; e_1 is solution; (by (T)) e_{10} is solution — $\underline{\mathcal{C}}(e_1)$

$\underline{\mathcal{C}}(e_2)$ — LLEE, guarded; e_2 is solution; (by (T)) e_{10} is solution

chart interpretations

crystallized 1-chart

guarded, LLEE; e_{10} is complete solution — $\underline{\mathcal{C}}_{10}$

bisimulation collapse

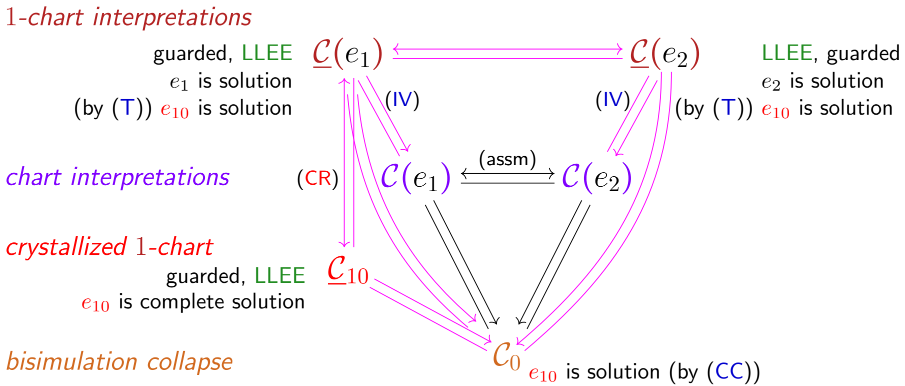
$\mathcal{C}_0$ — e_{10} is solution (by (CC))

$$
\begin{tikzcd}
\underline{\mathcal{C}}(e_1) \arrow[rrrr, Leftrightarrow] \arrow[dr, Rightarrow, "\text{(IV)}"] \arrow[dd, Leftrightarrow, "\text{(CR)}"'] \arrow[dddrr, Rightarrow, bend right] & & & & \underline{\mathcal{C}}(e_2) \arrow[dl, Rightarrow, "\text{(IV)}"'] \arrow[dddll, Rightarrow, bend left] \\
& \mathcal{C}(e_1) \arrow[rr, leftrightarrow, "\text{(assm)}"] \arrow[ddr, Rightarrow] & & \mathcal{C}(e_2) \arrow[ddl, Rightarrow] & \\
\underline{\mathcal{C}}_{10} \arrow[drr, Rightarrow] & & & & \\
& & \mathcal{C}_0 & &
\end{tikzcd}
$$

$$
\left.\begin{array}{r}\mathcal{C}(e_1) \text{ guarded, LLEE} \\ e_1 \text{ is solution} \\ e_{10} \text{ is solution}\end{array}\right\} \underset{\text{(SE)}}{\Longrightarrow} e_1 =_{\mathsf{Mil}} e_{10} \qquad e_{10} =_{\mathsf{Mil}} e_2 \underset{\text{(SE)}}{\Longleftarrow} \left\{\begin{array}{l}\mathcal{C}(e_2) \text{ guarded, LLEE} \\ e_2 \text{ is solution} \\ e_{10} \text{ is solution}\end{array}\right.
$$

$$
\Longrightarrow \quad e_1 =_{\mathsf{Mil}} e_2
$$

Completeness proof of Mil (structure)



Theorem

Milner's proof system Mil *is complete*
for process semantics equivalence $\equiv_{\mathbf{P}}$ *of regular expressions.*

Since: $e_1 \equiv_{\mathbf{P}} e_2 \Longrightarrow [\![e_1]\!]_{\mathbf{P}} = [\![e_2]\!]_{\mathbf{P}} \Longrightarrow \mathcal{C}(e_1) \leftrightarrow \mathcal{C}(e_1) \Longrightarrow e_1 =_{\mathsf{Mil}} e_2$.