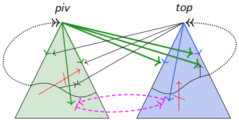
Milner's Proof System for Regular Expressions Modulo Bisimilarity is Complete

Crystallization: Near-Collapsing Process Graph Interpretations of Regular Expressions



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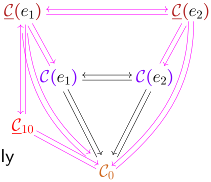


LICS 2022

Technion, Haifa, Israel

August 4, 2022

Process semantics of regular expressions $[\![\cdot]\!]_P$ *(Milner, 1984)*

$0 \xmapsto{[\![\cdot]\!]_P}$ deadlock δ, no termination

$1 \xmapsto{[\![\cdot]\!]_P}$ empty-step process ϵ, then terminate

$a \xmapsto{[\![\cdot]\!]_P}$ atomic action a, then terminate

$e + f \xmapsto{[\![\cdot]\!]_P}$ (*choice*) execute $[\![e]\!]_P$ or $[\![f]\!]_P$

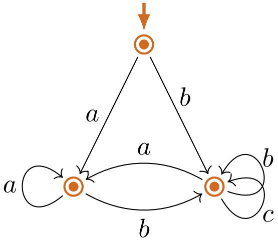
$e \cdot f \xmapsto{[\![\cdot]\!]_P}$ (*sequentialization*) execute $[\![e]\!]_P$, then $[\![f]\!]_P$

$e^* \xmapsto{[\![\cdot]\!]_P}$ (*iteration*) repeat (terminate or execute $[\![e]\!]_P$)

$[\![e]\!]_P := [\mathcal{C}(e)]_{\underline{\leftrightarrow}}$ (bisimilarity equivalence class of chart $\mathcal{C}(e)$)

Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)

chart (Milner)



$\mathcal{C}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$

Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)

1-chart

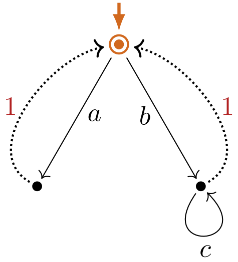
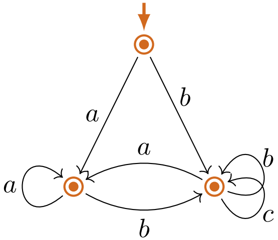


chart (Milner)



$\underline{\mathcal{C}}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$ $\mathcal{C}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$

Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)

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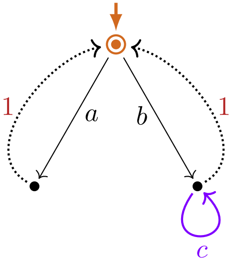

chart (Milner)

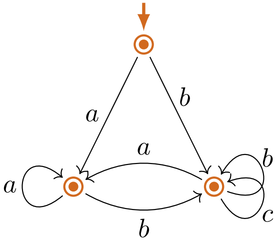

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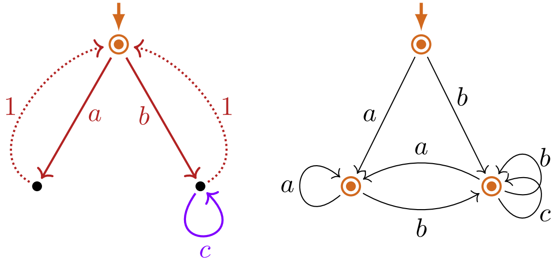
chart (Milner)



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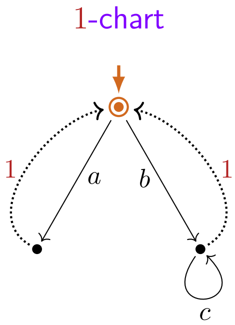

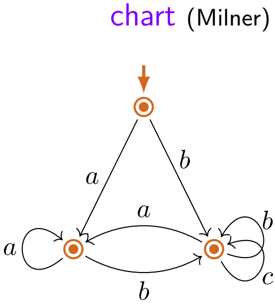

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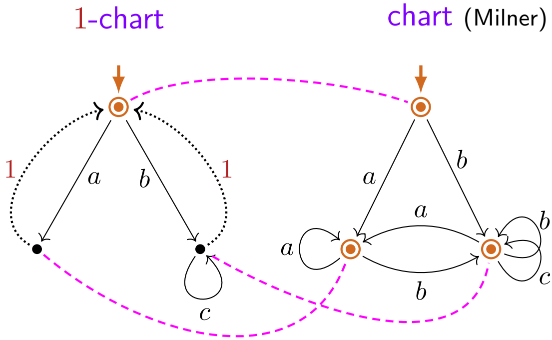

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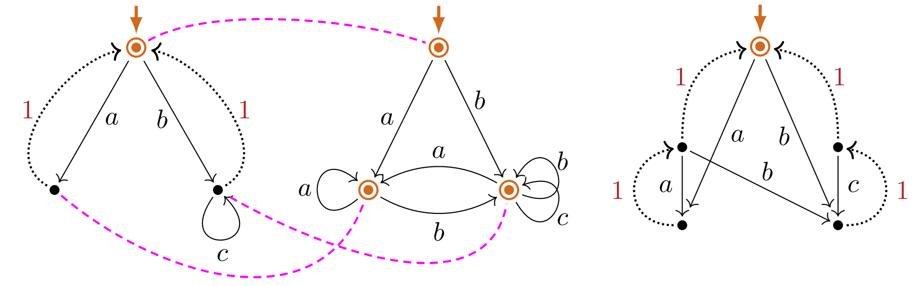
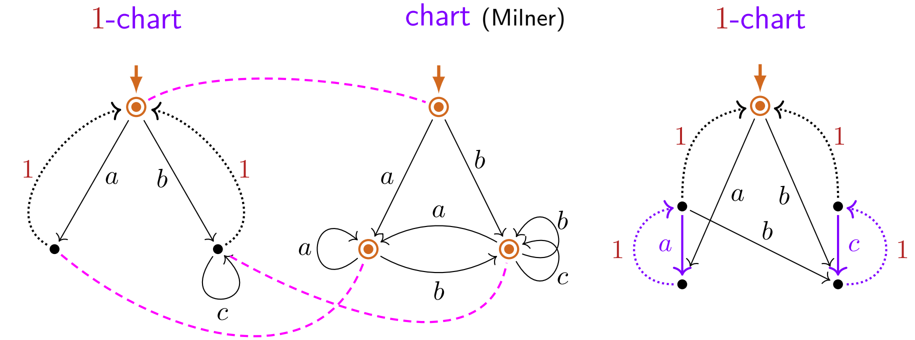
1-chart chart (Milner) 1-chart



$\underline{\mathcal{C}}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$ $\mathcal{C}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$ $\underline{\mathcal{C}}\big((a \cdot (1 \cdot ((a \cdot 1)^* \cdot 1)) \cdot b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1 + b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1\big)^*)$

Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)



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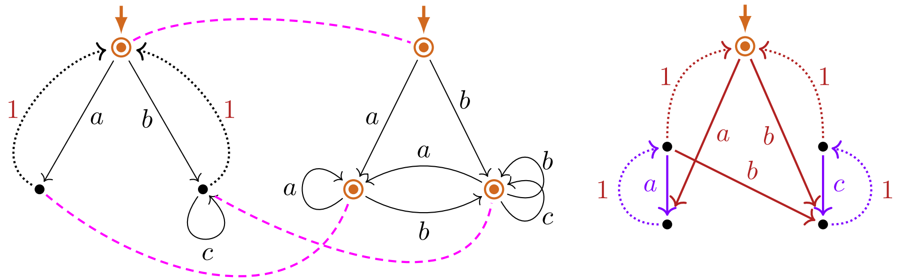

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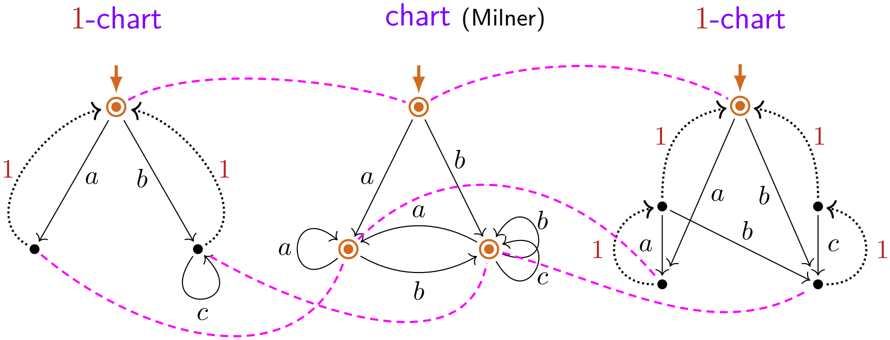
Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)



$\underline{\mathcal{C}}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$ $\qquad$ $\mathcal{C}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$ $\qquad$ $\underline{\mathcal{C}}\Big(\big(a \cdot (1 \cdot ((a \cdot 1)^* \cdot 1)) \cdot b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1 + b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1\big)^*\Big)$

Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)

1-chart

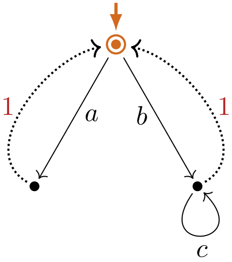
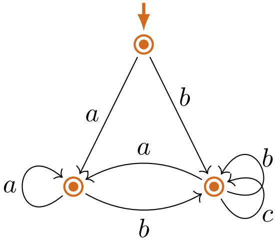
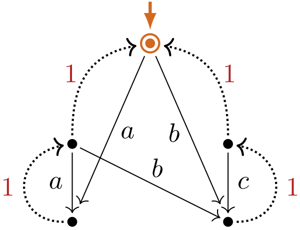


chart (Milner)



1-chart



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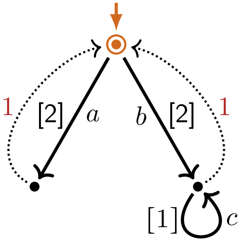
Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)

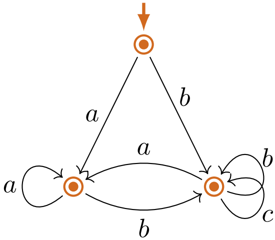
1-chart



LLEE

$\underline{\mathcal{C}}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$

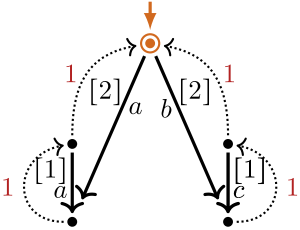
chart (Milner)



~~LLEE~~

$\mathcal{C}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$

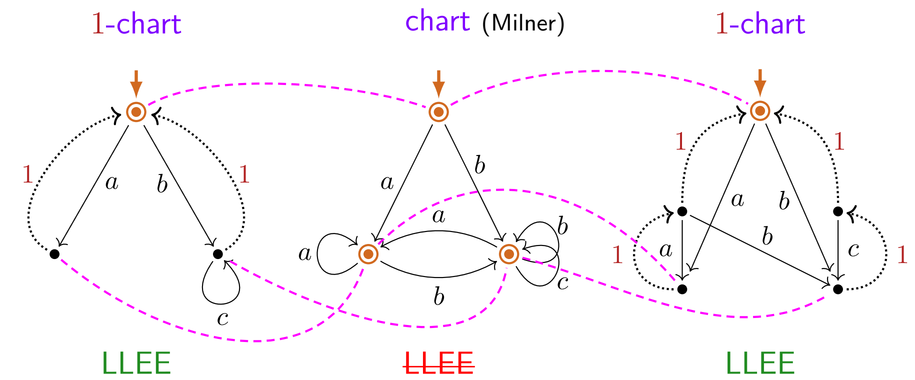
1-chart



LLEE

$$\underline{\mathcal{C}}\Big(\big(a \cdot (1 \cdot ((a \cdot 1)^* \cdot 1)) \cdot b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1 + b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1\big)^*\Big)$$

Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)



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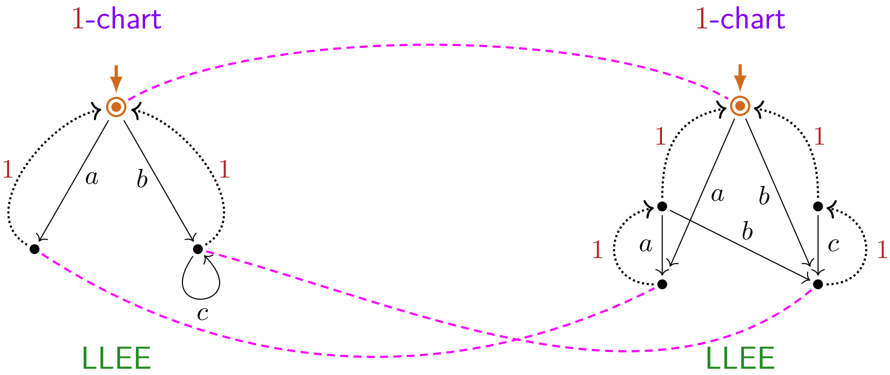
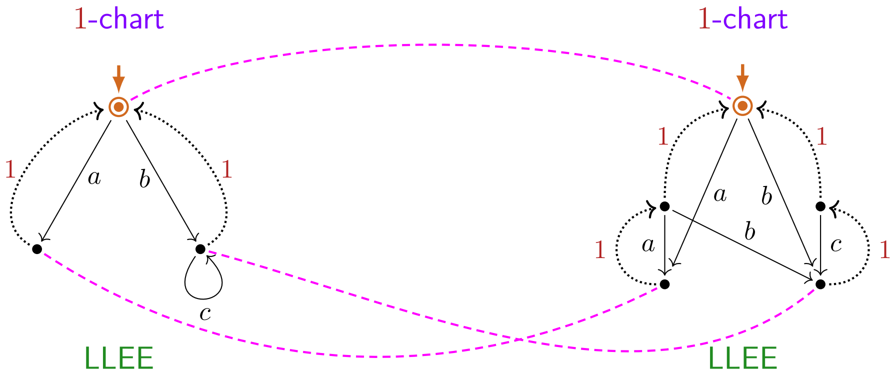
Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)



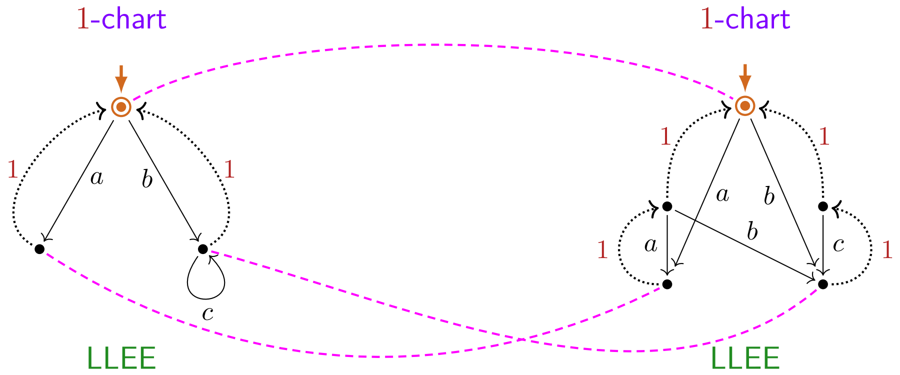
$\underline{\mathcal{C}}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$ $\underline{\leftrightarrow}$ $\underline{\mathcal{C}}\Big(\big(a \cdot (1 \cdot ((a \cdot 1)^* \cdot 1)) \cdot b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1 + b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1\big)^*\Big)$

Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)



$$[\![(a \cdot 1 + b \cdot (c^* \cdot 1))^*]\!]_P = [\![\big(a \cdot (1 \cdot ((a \cdot 1)^* \cdot 1)) \cdot b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1 + b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1\big)^*]\!]_P$$

Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)



$$(a \cdot 1 + b \cdot (c^* \cdot 1))^* \quad \equiv_P \quad \big(a \cdot (1 \cdot ((a \cdot 1)^* \cdot 1)) \cdot b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1 + b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1\big)^*\big)$$

Milner's proof system Mil

Axioms :

(A1) $e + (f + g) = (e + f) + g$

(A2) $e + 0 = e$

(A3) $e + f = f + e$

(A4) $e + e = e$

(A5) $e \cdot (f \cdot g) = (e \cdot f) \cdot g$

(A6) $(e + f) \cdot g = e \cdot g + f \cdot g$

(A7) $e = 1 \cdot e$

(A8) $e = e \cdot 1$

(A9) $0 = 0 \cdot e$

(A10) $e^* = 1 + e \cdot e^*$

(A11) $e^* = (1 + e)^*$

But: $e \cdot (f + g) \neq e \cdot f + e \cdot g$

But: $e \cdot 0 \neq 0$

Inference rules : rules of equational logic *plus*

$$\frac{e = f \cdot e + g}{e = f^* \cdot g}\ \mathsf{RSP}^*$$ (if f does not terminate immediately)

Milner's question (1984)

Is Mil complete with respect to $\equiv_{\mathbf{P}}$? (Does $e \equiv_{\mathbf{P}} f \Longrightarrow e =_{\mathsf{Mil}} f$ hold?)

Bisimulation collapse proof strategy (G/Fokkink, LICS'20)

For 1-free regular expressions e_1 and e_2:

e_1 is solution $\quad \mathcal{C}(e_1) \xleftrightarrow{\text{(assm)}} \mathcal{C}(e_2) \quad$ e_2 is solution

$e_1 =_{\mathsf{Mil}} e_2$

Bisimulation collapse proof strategy (G/Fokkink, LICS'20)

For 1-free regular expressions e_1 and e_2:

e_1 is solution

$$\begin{tikzcd} \mathcal{C}(e_1) \arrow[rr, Leftrightarrow, "\text{(assm)}"] \arrow[dr, Rightarrow] & & \mathcal{C}(e_2) \arrow[dl, Rightarrow] \\ & \mathcal{C}_0 & \end{tikzcd}$$

e_2 is solution

$e_1 =_{\mathsf{Mil}} e_2$

Bisimulation collapse proof strategy (G/Fokkink, LICS'20)

For 1-free regular expressions e_1 and e_2:

e_1 is solution

e_2 is solution

$$
\begin{tikzcd}
\overset{\text{LLEE}}{\mathcal{C}(e_1)} \arrow[rr, Leftrightarrow, "\text{(assm)}"] \arrow[dr, Rightarrow] & & \overset{\text{LLEE}}{\mathcal{C}(e_2)} \arrow[dl, Rightarrow] \\
& \mathcal{C}_0 &
\end{tikzcd}
$$

$e_1 =_{\mathsf{Mil}} e_2$

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For 1-free regular expressions e_1 and e_2:

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$$\begin{tikzcd}
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& \overset{\mathcal{C}_0}{\text{LLEE}} &
\end{tikzcd}$$

e_2 is solution

$e_1 =_{\text{Mil}} e_2$

Bisimulation collapse proof strategy (G/Fokkink, LICS'20)

For 1-free regular expressions e_1 and e_2:

e_1 is solution $\qquad$ e_2 is solution

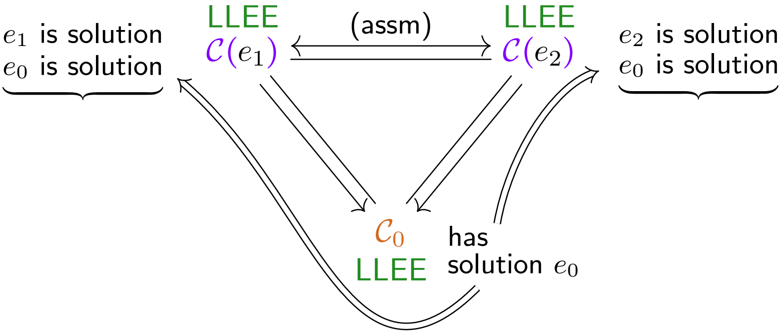
$$\begin{tikzcd}[column sep=large]
\text{LLEE } \mathcal{C}(e_1) \arrow[rr, Leftrightarrow, "\text{(assm)}"] \arrow[dr, Rightarrow] & & \text{LLEE } \mathcal{C}(e_2) \arrow[dl, Rightarrow] \\
& \mathcal{C}_0 \text{ LLEE} &
\end{tikzcd}$$

$\mathcal{C}_0$ has solution e_0

$e_1 =_{\mathsf{Mil}} e_2$

Bisimulation collapse proof strategy (G/Fokkink, LICS'20)

For 1-free regular expressions e_1 and e_2:



$e_1 =_{\mathsf{Mil}} e_2$

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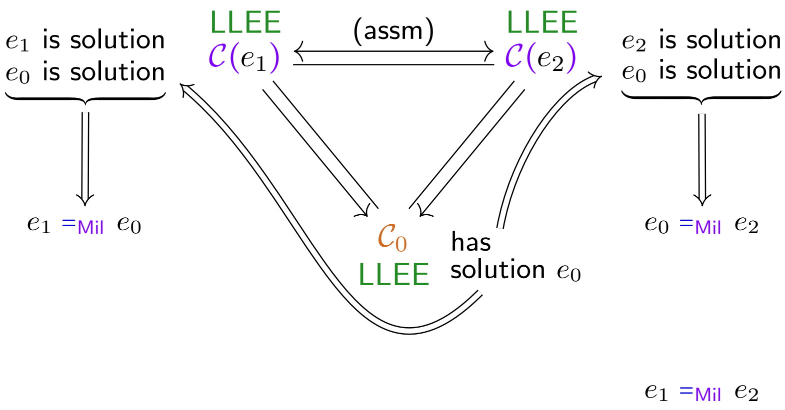

Bisimulation collapse proof strategy (G/Fokkink, LICS'20)

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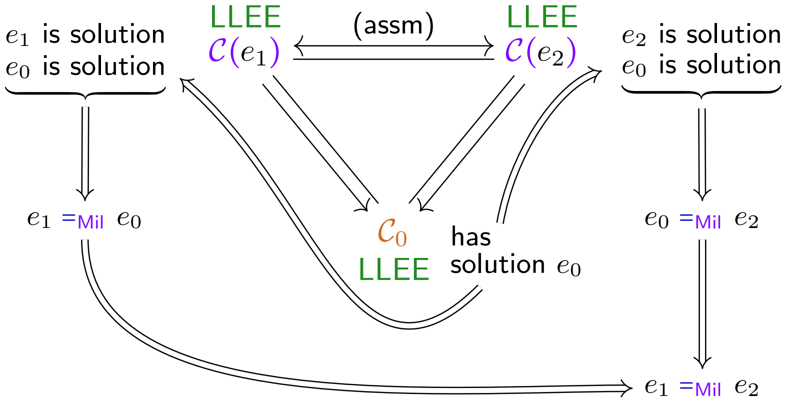


Bisimulation collapse proof strategy (general case)

Problem 1

$$\begin{tikzcd}
\textit{chart interpretations} & \cancel{\text{LLEE}} & \mathcal{C}(e_1) \arrow[rr, leftrightarrow, "\text{(assm)}"] \arrow[dr, Rightarrow] & & \mathcal{C}(e_2) \arrow[dl, Rightarrow] & \cancel{\text{LLEE}} \\
\textit{bisimulation collapse} & & & \mathcal{C}_0 & &
\end{tikzcd}$$

Bisimulation collapse proof strategy (general case)

Remedy for Problem 1 *(G, TERMGRAPH 2020)*

chart interpretations

$$\begin{tikzcd}
\cancel{\mathsf{LLEE}}\;\; \mathcal{C}(e_1) \arrow[rr, leftrightarrow, "(\mathrm{assm})"] \arrow[dr, Rightarrow] & & \mathcal{C}(e_2)\;\; \cancel{\mathsf{LLEE}} \arrow[dl, Rightarrow] \\
& \mathcal{C}_0 &
\end{tikzcd}$$

bisimulation collapse

Bisimulation collapse proof strategy (general case)

Remedy for Problem 1 *(G, TERMGRAPH 2020)*

1-chart interpretations

chart interpretations

bisimulation collapse

$$\begin{tikzcd}
\text{LLEE}\ \underline{\mathcal{C}}(e_1) \arrow[d, Rightarrow, "\text{(IV)}"] & & \underline{\mathcal{C}}(e_2)\ \text{LLEE} \arrow[d, Rightarrow, "\text{(IV)}"'] \\
\cancel{\text{LLEE}}\ \mathcal{C}(e_1) \arrow[rr, leftrightarrow, "\text{(assm)}"] \arrow[dr, Rightarrow] & & \mathcal{C}(e_2)\ \cancel{\text{LLEE}} \arrow[dl, Rightarrow] \\
& \mathcal{C}_0 &
\end{tikzcd}$$

Bisimulation collapse proof strategy (general case)

Remedy for Problem 1 *(G, TERMGRAPH 2020)*

$$
\begin{tikzcd}
\text{1-\textit{chart interpretations}} & \text{LLEE} & \underline{\mathcal{C}}(e_1) \arrow[rrrr, Leftrightarrow] \arrow[dr, Rightarrow, "\text{(IV)}"] & & & & \underline{\mathcal{C}}(e_2) \arrow[dl, Rightarrow, "\text{(IV)}"'] & \text{LLEE} \\
\text{\textit{chart interpretations}} & \cancel{\text{LLEE}} & & \mathcal{C}(e_1) \arrow[rr, leftrightarrow, "\text{(assm)}"] \arrow[dr, Rightarrow] & & \mathcal{C}(e_2) \arrow[dl, Rightarrow] & & \cancel{\text{LLEE}} \\
\text{\textit{bisimulation collapse}} & & & & \mathcal{C}_0 & & &
\end{tikzcd}
$$

Bisimulation collapse proof strategy (general case)

Remedy for Problem 1 *(G, TERMGRAPH 2020)*

$$\begin{tikzcd}
1\text{-}\textit{chart interpretations} & \text{LLEE} & \underline{\mathcal{C}}(e_1) \arrow[rr, Leftrightarrow] \arrow[d, Rightarrow, "\text{(IV)}"] \arrow[dr, Rightarrow, bend right] & & \underline{\mathcal{C}}(e_2) \arrow[d, Rightarrow, "\text{(IV)}"'] \arrow[dl, Rightarrow, bend left] & \text{LLEE} \\
\textit{chart interpretations} & \cancel{\text{LLEE}} & \mathcal{C}(e_1) \arrow[rr, Leftrightarrow, "\text{(assm)}"] \arrow[dr, Rightarrow] & & \mathcal{C}(e_2) \arrow[dl, Rightarrow] & \cancel{\text{LLEE}} \\
\textit{bisimulation collapse} & & & \mathcal{C}_0 & &
\end{tikzcd}$$

Bisimulation collapse proof strategy (general case)

Remedy for Problem 1 *(G, TERMGRAPH 2020)*

1-chart interpretations

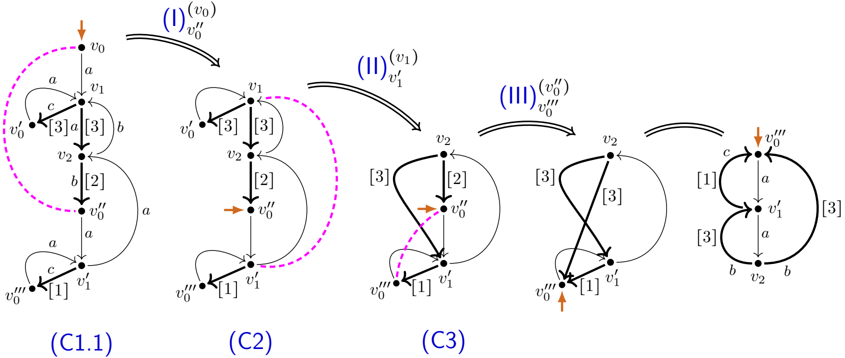
chart interpretations

bisimulation collapse

$$
\begin{tikzcd}
\text{LLEE}\ \underline{\mathcal{C}}(e_1) \arrow[rrrr, Leftrightarrow] \arrow[dr, Rightarrow, "\text{(IV)}"] \arrow[ddrr, Rightarrow, bend right] & & & & \underline{\mathcal{C}}(e_2)\ \text{LLEE} \arrow[dl, Rightarrow, "\text{(IV)}"'] \arrow[ddll, Rightarrow, bend left] \\
\cancel{\text{LLEE}} & \mathcal{C}(e_1) \arrow[rr, Leftrightarrow, "\text{(assm)}"] \arrow[dr, Rightarrow] & & \mathcal{C}(e_2) \arrow[dl, Rightarrow] & \cancel{\text{LLEE}} \\
& & \mathcal{C}_0\ \text{LLEE?} & &
\end{tikzcd}
$$

LLEE-preserving collapse of LLEE-charts (G/Fokkink, LICS'20)

(no 1-transitions!)



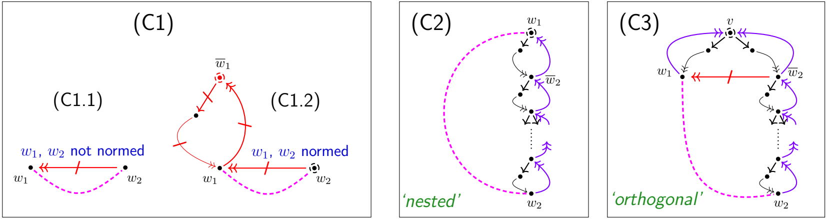
Lemma

The bisimulation collapse of a LLEE-chart is again a LLEE-chart.

Reduced bisimilarity redundancies in LLEE-charts (no 1-trans.!) (G/Fokkink, LICS'20)

w_1, w_2 in different scc's

w_1, w_2 in the same scc



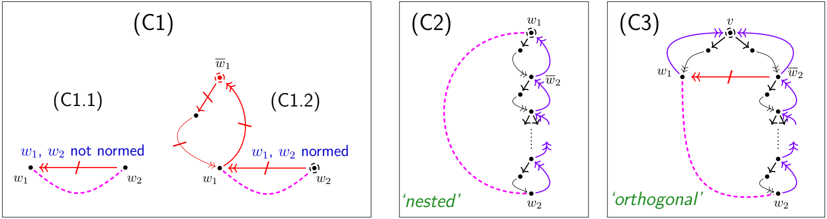
Lemma

Every not collapsed LLEE-chart contains bisimilar vertices $w_1 \neq w_2$ of kind (C1), (C2), or (C3) (a reduced bisimilarity redundancy $\langle w_1, w_2 \rangle$):

Reduced bisimilarity redundancies in LLEE-charts (no 1-trans.!)

(G/Fokkink, LICS'20)

w_1, w_2 in different scc's $\qquad$ w_1, w_2 in the same scc



Lemma

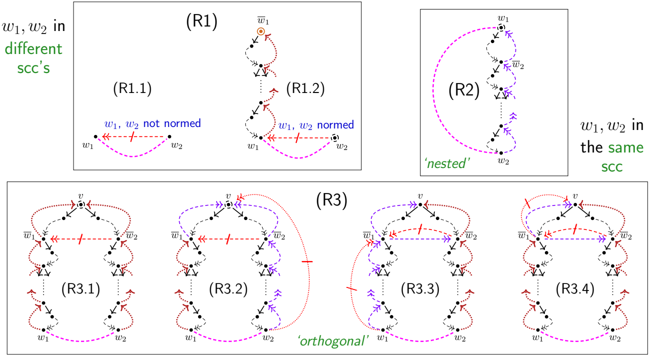
Every not collapsed LLEE-chart contains bisimilar vertices $w_1 \neq w_2$ of kind (C1), (C2), or (C3) (a reduced bisimilarity redundancy $\langle w_1, w_2 \rangle$):

Lemma

Every reduced bisimilarity redundancy in a LLEE-chart can be eliminated LLEE-preservingly.

Reduced 1-bisimilarity redundancies in LLEE-1-charts

w_1, w_2 in different scc's

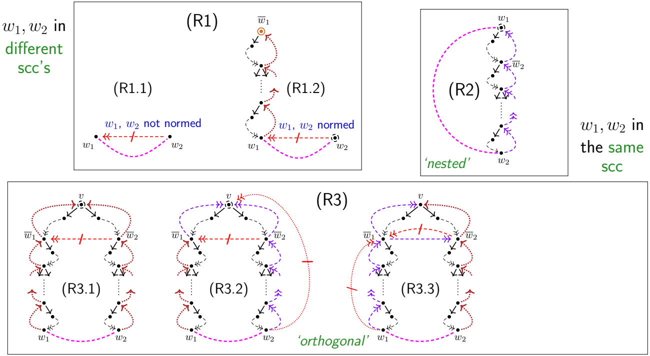


w_1, w_2 in the same scc

Lemma

Every not collapsed LLEE*-1-chart contains 1-bisimilar vertices* $w_1 \neq w_2$ *(a reduced 1-bisimilarity redundancy* $\langle w_1, w_2 \rangle$*) of kind* (R1), (R2), (R3).

Reduced 1-bisimilarity redundancies in LLEE-1-charts



Lemma

Every not collapsed LLEE-1-chart contains 1-bisimilar vertices $w_1 \neq w_2$ (a reduced 1-bisimilarity redundancy $\langle w_1, w_2 \rangle$) of kind (R1), (R2), (R3).

Lemma

Every simple reduced 1-bisimilarity redundancies in a LLEE-1-chart can be eliminated LLEE-preservingly.

Reduced 1-bisimilarity redundancies in LLEE-1-charts

w_1, w_2 in different scc's

(R1)

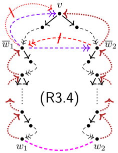
(R2)

'nested'

w_1, w_2 in the same scc

(R3)

'orthogonal'



Lemma

Every not collapsed LLEE-1-*chart contains* 1*-bisimilar vertices* $w_1 \neq w_2$ *(a reduced* 1*-bisimilarity redundancy* $\langle w_1, w_2 \rangle$*) of kind* (R1), (R2), (R3).

Stumbling Block

How to LLEE-preservingly eliminate
reduced 1-bisimilarity redundancies of kind (R3.4)?

Reduced 1-bisimilarity redundancies in LLEE-1-charts

w_1, w_2 in different scc's

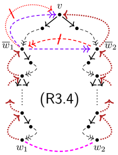
(R1)

(R2)

'nested'

w_1, w_2 in the same scc

(R3)



'orthogonal'

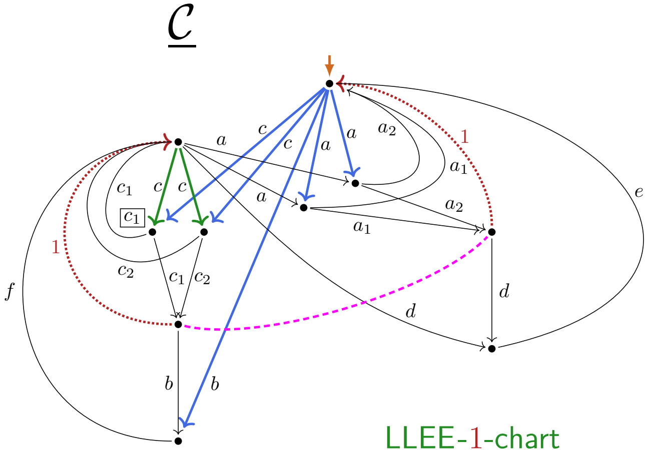
Lemma

Every not collapsed LLEE-1-chart contains 1-bisimilar vertices $w_1 \neq w_2$ (a reduced 1-bisimilarity redundancy $\langle w_1, w_2 \rangle$) of kind (R1), (R2), (R3).

Stumbling Block

How to LLEE-preservingly eliminate
precrystalline reduced 1-bisimilarity redundancies?

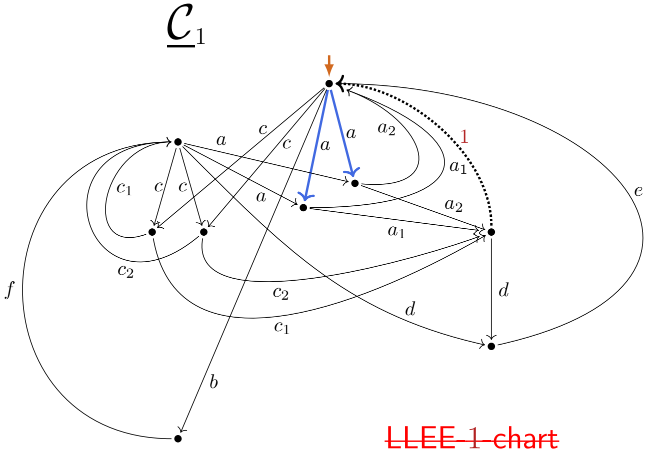
Counterexample LLEE-preserving collapse



LLEE-1-chart

$\langle w_1, w_2 \rangle$ is a reduced
1-bisimilarity redundancy of kind (R3.4)

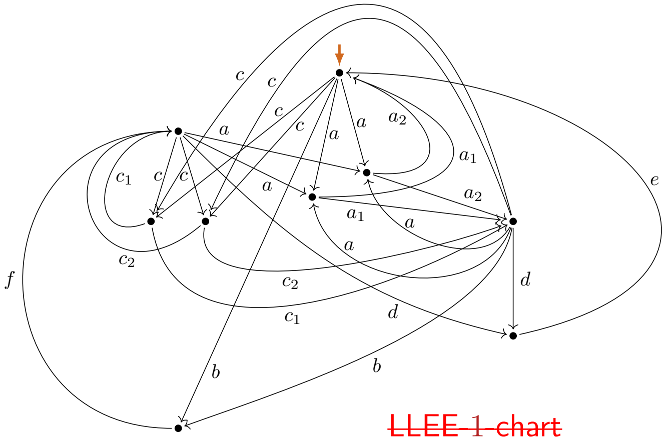
Counterexample LLEE-preserving collapse



~~LLEE-1-chart~~

1-bisimulation collapse

Counterexample LLEE-preserving collapse



~~LLEE-1-chart~~

bisimulation collapse

Bisimulation collapse proof strategy (general case)

Problem 2: There are regular expressions e_1 and e_2 such that:

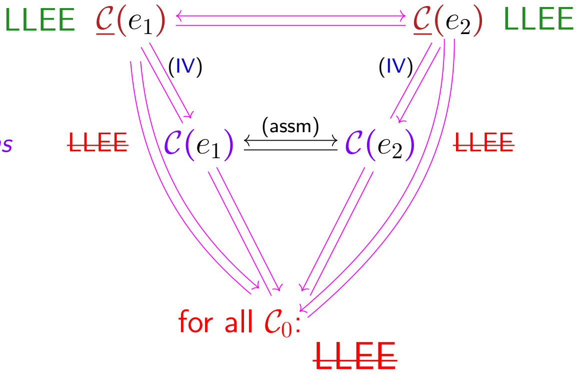
$$
\begin{tikzcd}
\text{1-\textit{chart interpretations}} & \text{LLEE} & \underline{\mathcal{C}}(e_1) \arrow[rr, Leftrightarrow] \arrow[d, Rightarrow, "\text{(IV)}"] \arrow[dr, Rightarrow, bend right] & & \underline{\mathcal{C}}(e_2) \arrow[d, Rightarrow, "\text{(IV)}"'] \arrow[dl, Rightarrow, bend left] & \text{LLEE} \\
\textit{chart interpretations} & \cancel{\text{LLEE}} & \mathcal{C}(e_1) \arrow[rr, Leftrightarrow, "\text{(assm)}"] \arrow[dr, Rightarrow] & & \mathcal{C}(e_2) \arrow[dl, Rightarrow] & \cancel{\text{LLEE}} \\
\textit{bisimulation collapse} & & & \mathcal{C}_0 & \cancel{\text{LLEE}} &
\end{tikzcd}
$$

Bisimulation collapse proof strategy (general case)

Problem 2: There are regular expressions e_1 and e_2 such that:

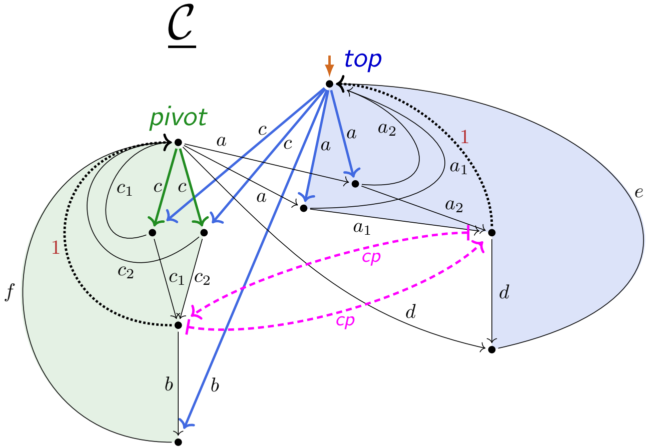
1-*chart interpretations*

chart interpretations

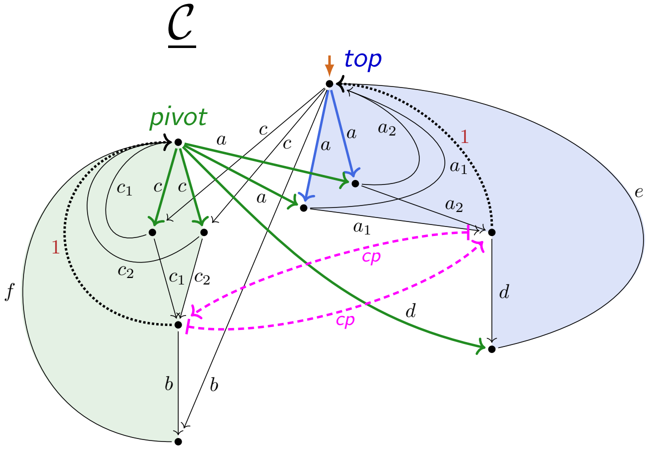


$\mathcal{C}(e_1)$, $\mathcal{C}(e_2)$, $\underline{\mathcal{C}}(e_1)$ and $\underline{\mathcal{C}}(e_2)$ are **not** LLEE-preservingly jointly minimizable under bisimilarity.

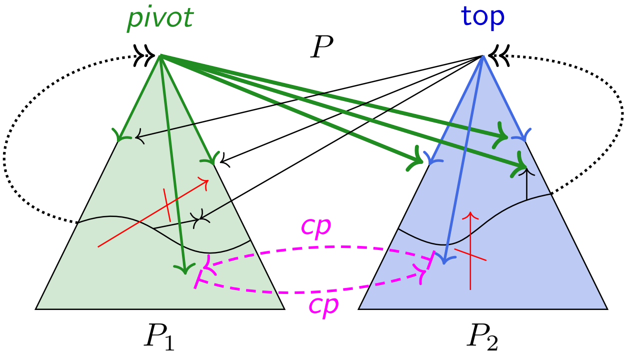
Twin-Crystal



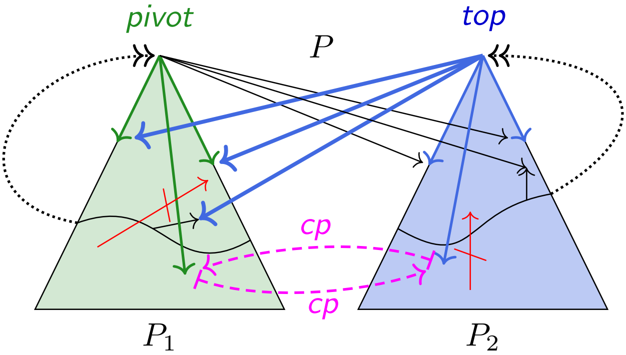
Twin-Crystal



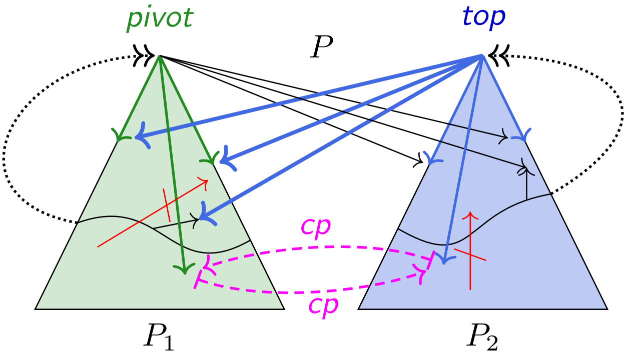
Twin-Crystal



Twin-Crystal

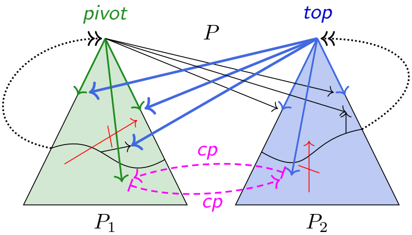


Twin-Crystal



- Provable solutions of twin-crystals are complete:
  they can be transferred to their bisimulation collapses

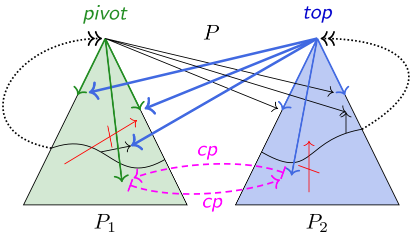
Crystallization



twin-crystal

Crystallized 1-charts = LLEE-1-charts that are collapsed apart from some scc's that are twin-crystals.

Crystallization

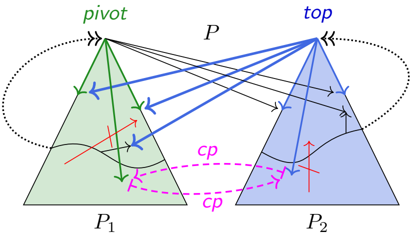


twin-crystal

Crystallized 1-charts = LLEE-1-charts that are collapsed apart from some scc's that are twin-crystals.

(CR) Crystallization: Every LLEE-1-chart can be reduced under bisimilarity to a bisimilar crystallized 1-chart.

Crystallization



twin-crystal

Crystallized 1-charts = LLEE-1-charts that are collapsed apart from some scc's that are twin-crystals.

(CR) Crystallization: Every LLEE-1-chart can be reduced under bisimilarity to a bisimilar crystallized 1-chart.

(CC) Every provable solution of a crystallized 1-chart gives rise to provable solution on the bisimulation collapse.

Completeness proof of Mil (structure)

chart interpretations $\mathcal{C}(e_1) \xleftrightarrow{\text{(assm)}} \mathcal{C}(e_2)$

$\overset{?}{\Longrightarrow}$ $e_1 =_{\mathsf{Mil}} e_2$

Completeness proof of Mil (structure)

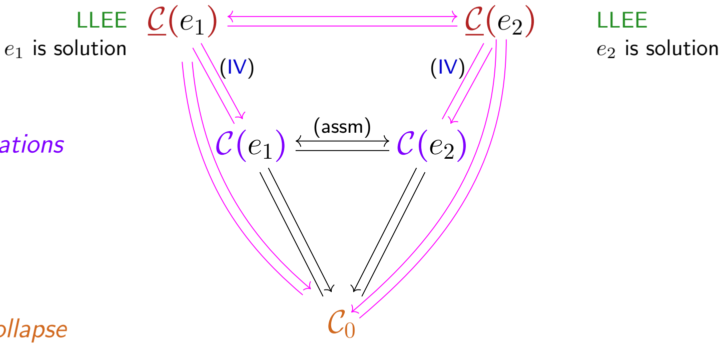
chart interpretations $\mathcal{C}(e_1) \xleftrightarrow{\text{(assm)}} \mathcal{C}(e_2)$

Completeness proof of Mil (structure)

1-chart interpretations

chart interpretations

bisimulation collapse



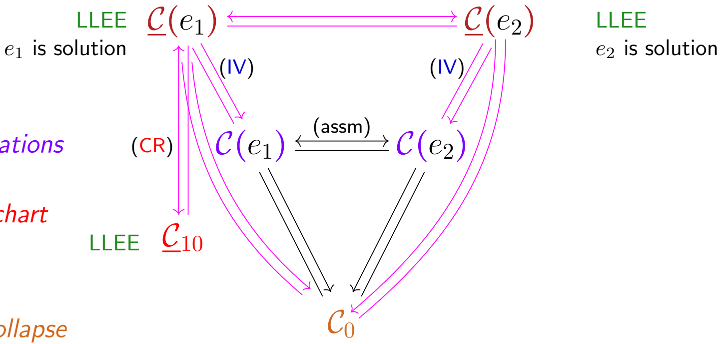
Completeness proof of Mil (structure)

1-chart interpretations

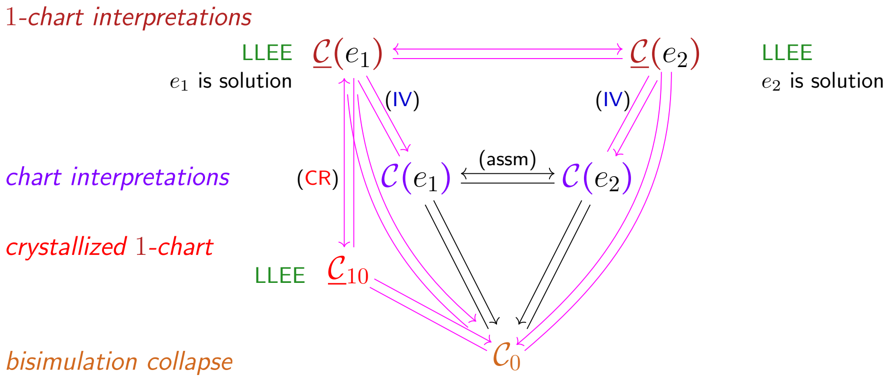
chart interpretations

crystallized 1-chart

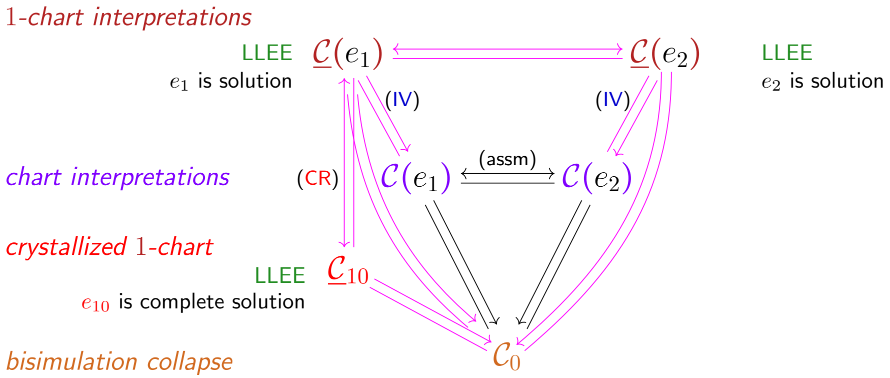
bisimulation collapse



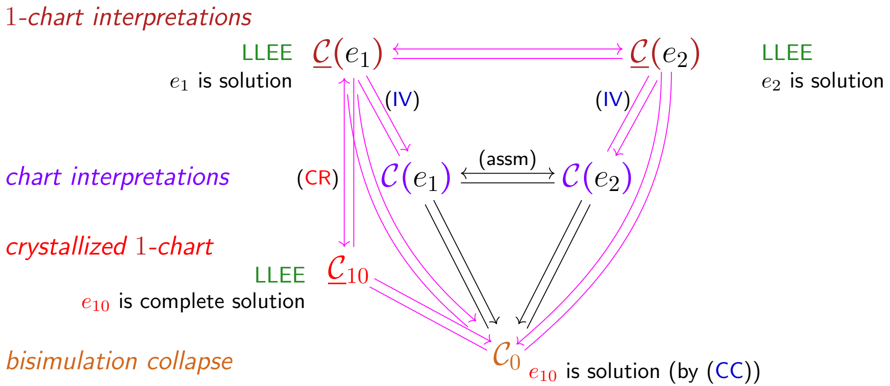
Completeness proof of Mil (structure)



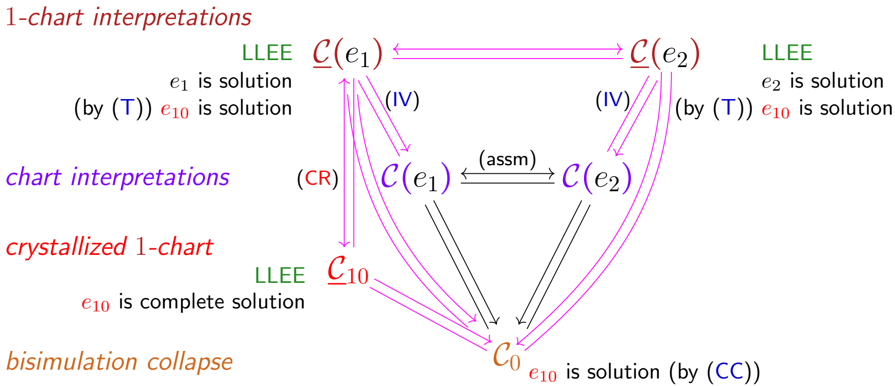
Completeness proof of Mil (structure)



Completeness proof of Mil (structure)



Completeness proof of Mil (structure)



Completeness proof of Mil (structure)

1-chart interpretations

LLEE
e_1 is solution
(by (T)) e_{10} is solution

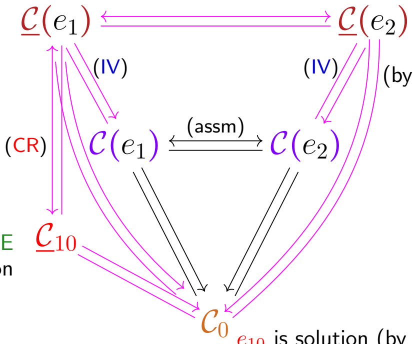
LLEE
e_2 is solution
(by (T)) e_{10} is solution

chart interpretations

crystallized 1-chart

LLEE
e_{10} is complete solution

bisimulation collapse



e_{10} is solution (by (CC))

$\mathcal{C}(e_1)$ LLEE
e_1 is solution
e_{10} is solution

$\mathcal{C}(e_2)$ LLEE
e_2 is solution
e_{10} is solution

Completeness proof of Mil (structure)

1-chart interpretations

LLEE
e_1 is solution
(by (T)) e_{10} is solution

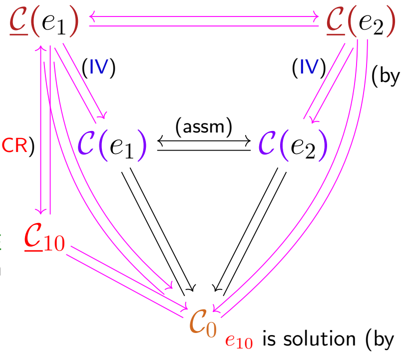
LLEE
e_2 is solution
(by (T)) e_{10} is solution

chart interpretations

crystallized 1-chart

LLEE
e_{10} is complete solution

bisimulation collapse

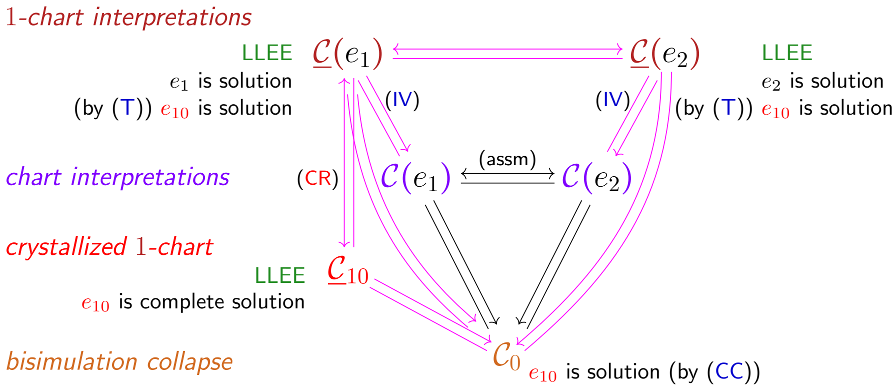


e_{10} is solution (by (CC))

$$\left.\begin{array}{r} \mathcal{C}(e_1) \text{ LLEE} \\ e_1 \text{ is solution} \\ e_{10} \text{ is solution} \end{array}\right\} \underset{\text{(SE)}}{\Longrightarrow} e_1 =_{\text{Mil}} e_{10}$$

$\mathcal{C}(e_2)$ LLEE
e_2 is solution
e_{10} is solution

Completeness proof of Mil (structure)



$$\left.\begin{array}{r}\mathcal{C}(e_1)\ \text{LLEE}\\ e_1\ \text{is solution}\\ e_{10}\ \text{is solution}\end{array}\right\} \xRightarrow[\text{(SE)}]{} e_1 =_{\text{Mil}} e_{10} \qquad e_{10} =_{\text{Mil}} e_2 \xLeftarrow[\text{(SE)}]{} \left\{\begin{array}{l}\mathcal{C}(e_2)\ \text{LLEE}\\ e_2\ \text{is solution}\\ e_{10}\ \text{is solution}\end{array}\right.$$

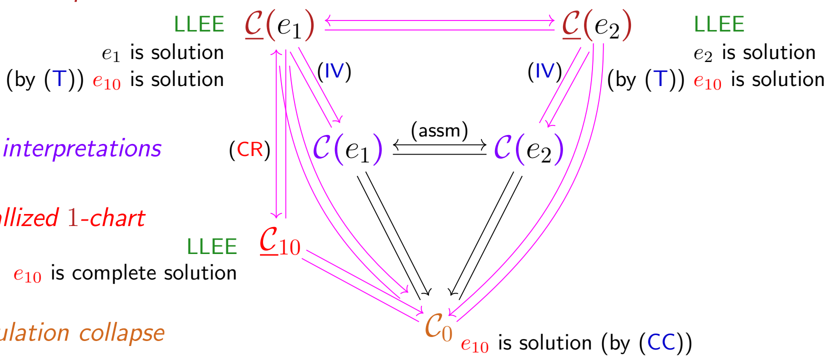
Completeness proof of Mil (structure)

1-chart interpretations

chart interpretations

crystallized 1-chart

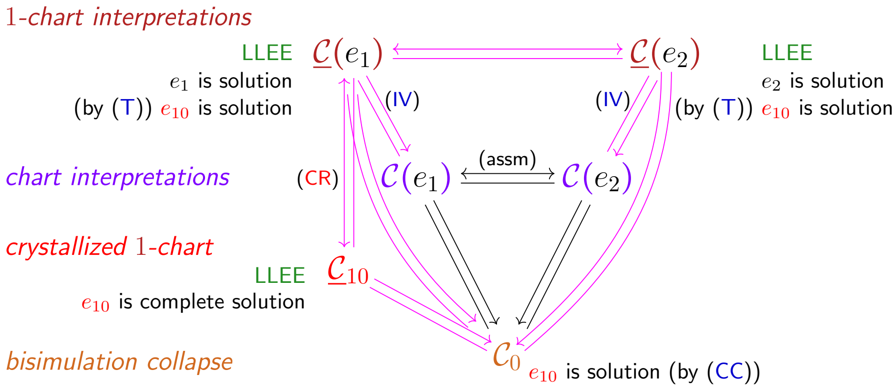
bisimulation collapse



$$\left.\begin{array}{r} \mathcal{C}(e_1) \text{ LLEE} \\ e_1 \text{ is solution} \\ e_{10} \text{ is solution} \end{array}\right\} \overset{}{\underset{\text{(SE)}}{\Longrightarrow}} e_1 =_{\text{Mil}} e_{10} \qquad e_{10} =_{\text{Mil}} e_2 \underset{\text{(SE)}}{\Longleftarrow} \left\{\begin{array}{l} \mathcal{C}(e_2) \text{ LLEE} \\ e_2 \text{ is solution} \\ e_{10} \text{ is solution} \end{array}\right.$$

$$\Longrightarrow \qquad e_1 =_{\text{Mil}} e_2$$

Completeness proof of Mil (structure)



Theorem

Milner's proof system Mil is complete for process semantics equivalence $\equiv_{\mathbf{P}}$ of regular expressions.

Since: $e_1 \equiv_{\mathbf{P}} e_2 \Longrightarrow [\![e_1]\!]_{\mathbf{P}} = [\![e_2]\!]_{\mathbf{P}} \Longrightarrow \mathcal{C}(e_1) \underline{\leftrightarrow} \mathcal{C}(e_1) \Longrightarrow e_1 =_{\mathsf{Mil}} e_2$.

Outlook

poster presentation

- tomorrow, 10–10.30

next steps and projects

- monograph project: proof in fine-grained detail
- computation/animation tool for crystallization
- use crystallization for recognition problem

resources on Github:

- https://github.com/clegra/crystallization/blob/main
  - article (after rebuttal): /cryst-article.pdf
  - poster: /poster-lics2022.pdf
  - presentation: /presentation-lics2022.pdf

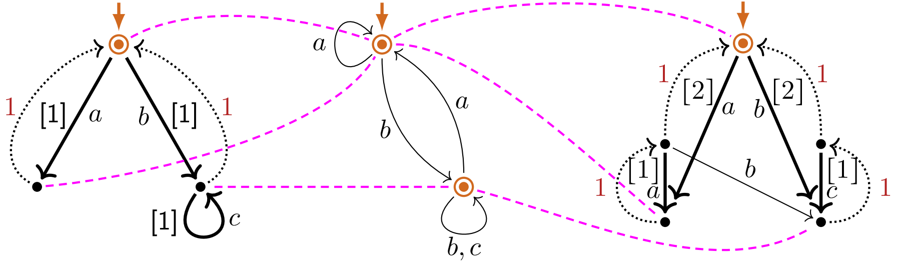
acknowledgment & thanks to:

- Wan Fokkink (for long collaboration)

Thank you for your attention!

Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)

1-chart | bisimulation collapse chart | 1-chart



LLEE | ~~LLEE~~ | LLEE

$\underline{\mathcal{C}}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$

$\mathcal{C}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$

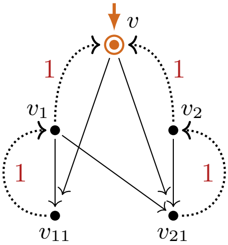
$$\underline{\mathcal{C}}\Big(\big(a \cdot (1 \cdot ((a \cdot 1)^* \cdot 1)) \cdot b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1 + b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1\big)^*\Big)$$

$\underline{\mathcal{C}}((a + b \cdot c^*)^*)$

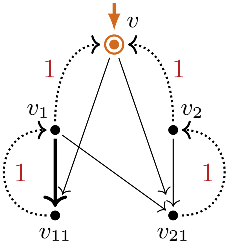
$\mathcal{C}((a + b \cdot c^*)^*)$

$\underline{\mathcal{C}}((a^* \cdot (1 + b \cdot c^*))^*)$

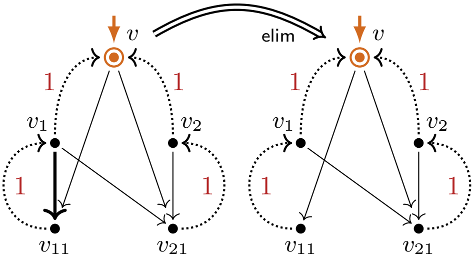
Layered loop existence/elimination and LLEE-witnesses



Layered loop existence/elimination and LLEE-witnesses



Layered loop existence/elimination and LLEE-witnesses



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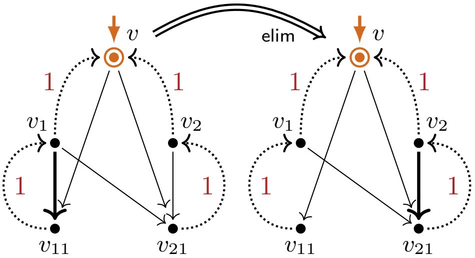
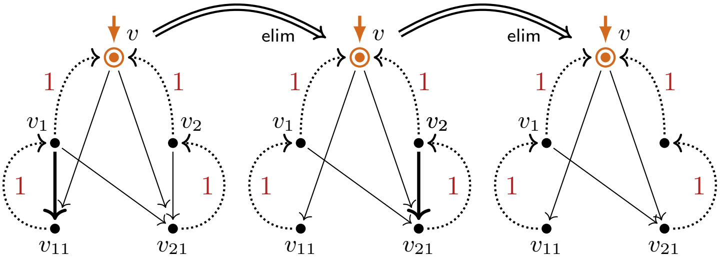


Layered loop existence/elimination and LLEE-witnesses



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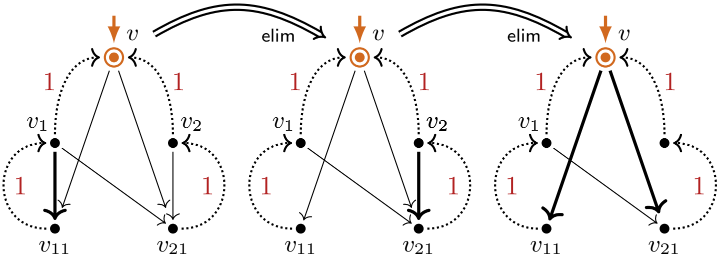
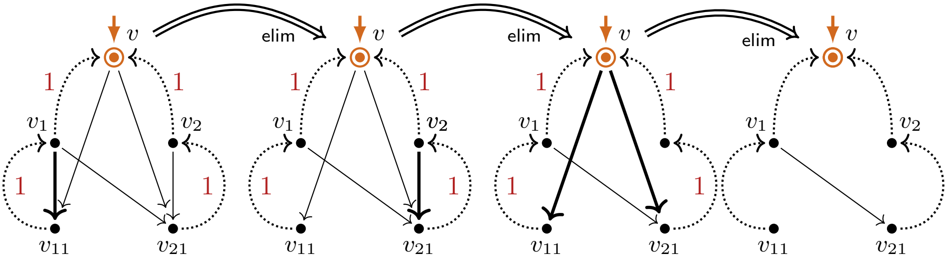
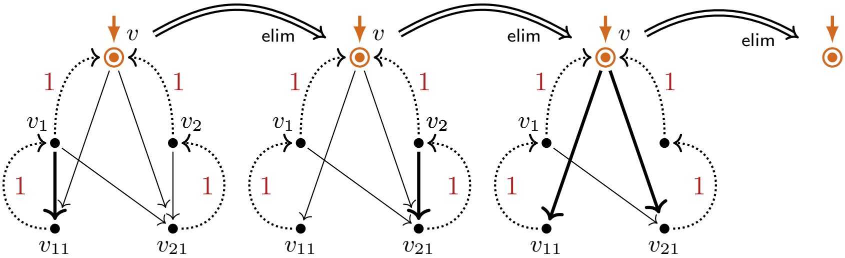


Layered loop existence/elimination and LLEE-witnesses



Layered loop existence/elimination and LLEE-witnesses



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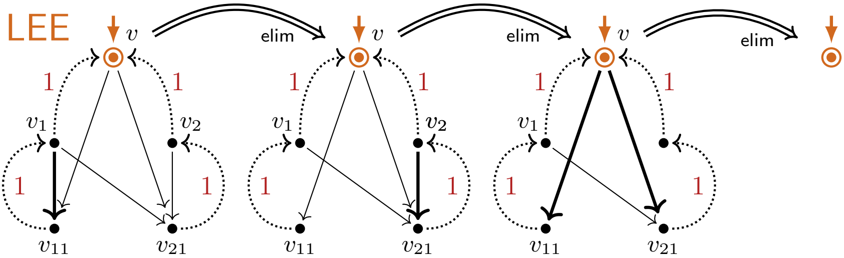
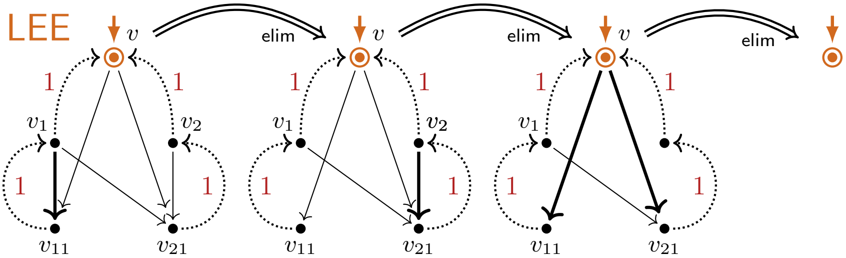
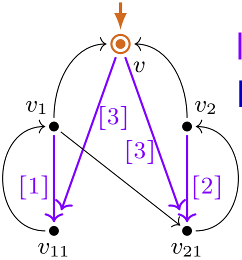


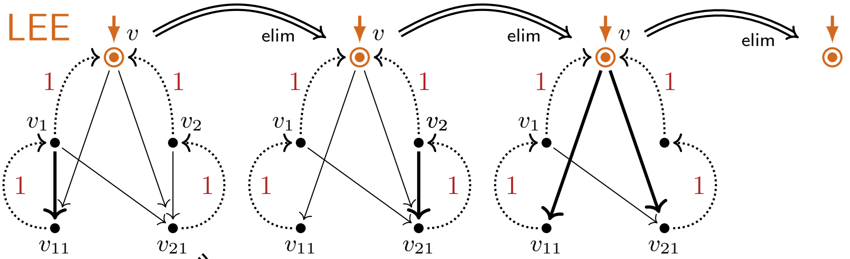
Layered loop existence/elimination and LLEE-witnesses

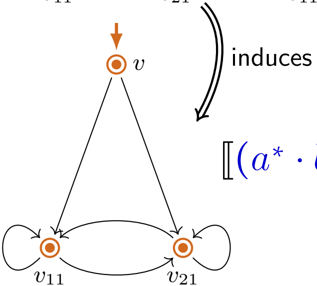




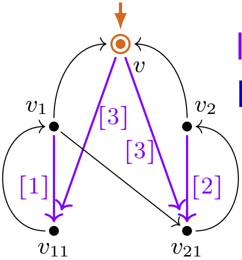
layered LEE-witness

Layered loop existence/elimination and LLEE-witnesses



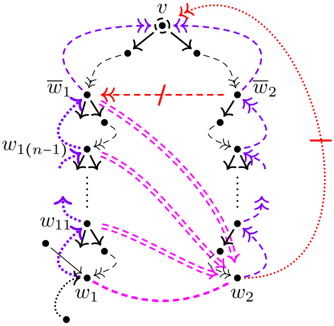


$[\![(a^* \cdot b^*)^*]\!]_{\mathsf{P}}$



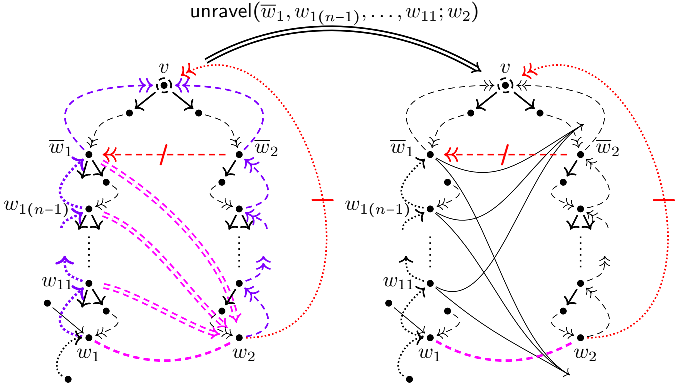
layered LEE-witness

Eliminating reduced 1-bisimilarity redundancies (example)



reduced 1-bisimilarity
redundancy of kind (R3.2)

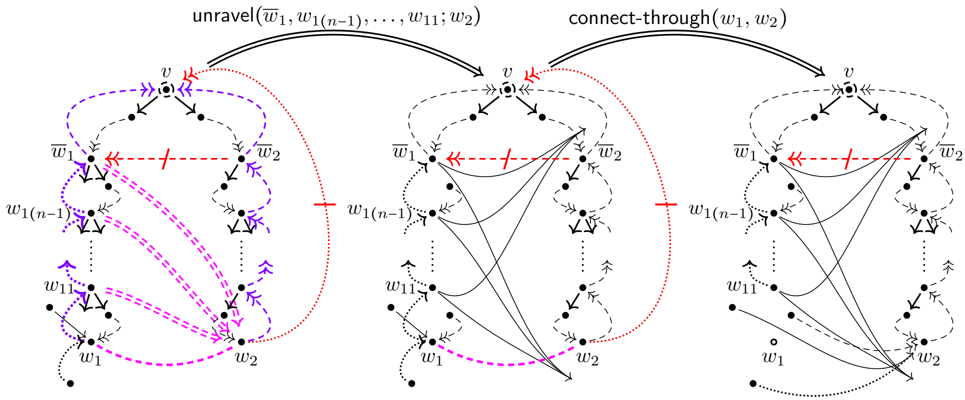
Eliminating reduced 1-bisimilarity redundancies (example)



reduced 1-bisimilarity redundancy of kind (R3.2)

result of unravelling loop vertices $\overline{w}_1, w_{1(n-1)}, \ldots, w_{11}$

Eliminating reduced 1-bisimilarity redundancies (example)



reduced 1-bisimilarity redundancy of kind (R3.2)

result of unravelling loop vertices $\overline{w}_1, w_{1(n-1)}, \ldots, w_{11}$

result of connecting through w_1 to w_2

Completeness proof of Mil (structure)

chart interpretations $\mathcal{C}(e_1) \xleftrightarrow{\text{(assm)}} \mathcal{C}(e_2)$

$\overset{?}{\Longrightarrow}$ $e_1 =_{\mathsf{Mil}} e_2$

Completeness proof of Mil (structure)

chart interpretations $\mathcal{C}(e_1) \overset{\text{(assm)}}{\leftrightarrow} \mathcal{C}(e_2)$

Completeness proof of Mil (structure)

1-chart interpretations

guarded, LLEE
e_1 is solution

LLEE, guarded
e_2 is solution

chart interpretations

bisimulation collapse

$$
\begin{tikzcd}
\underline{\mathcal{C}}(e_1) \arrow[rr, Leftrightarrow] \arrow[dr, Rightarrow, "\text{(IV)}"] \arrow[ddrr, Rightarrow, bend right] & & & & \underline{\mathcal{C}}(e_2) \arrow[dl, Rightarrow, "\text{(IV)}"'] \arrow[ddll, Rightarrow, bend left] \\
& \mathcal{C}(e_1) \arrow[rr, Leftrightarrow, "\text{(assm)}"] \arrow[dr, Rightarrow] & & \mathcal{C}(e_2) \arrow[dl, Rightarrow] & \\
& & \mathcal{C}_0 & &
\end{tikzcd}
$$

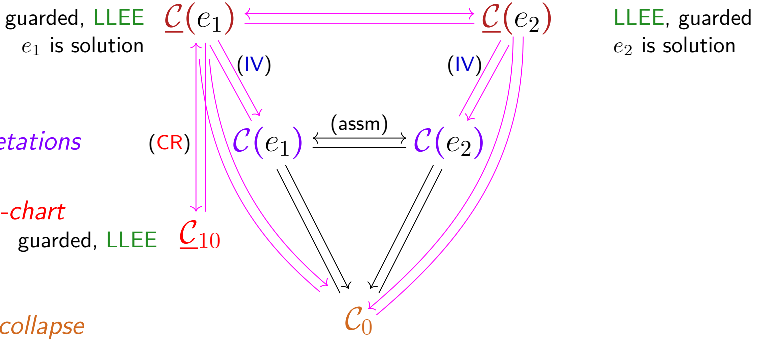
Completeness proof of Mil (structure)

1-chart interpretations

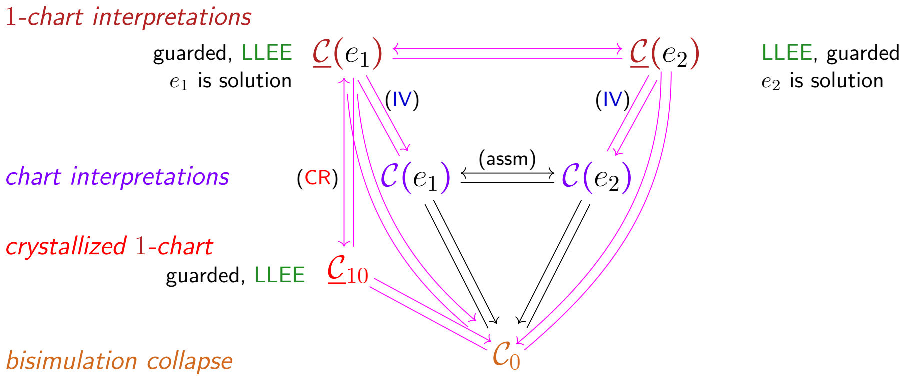
chart interpretations

crystallized 1-chart

bisimulation collapse



Completeness proof of Mil (structure)



Completeness proof of Mil (structure)

1-chart interpretations

guarded, LLEE
e_1 is solution

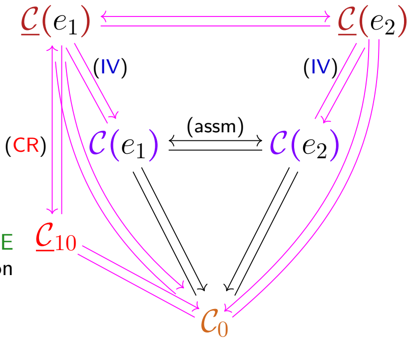
LLEE, guarded
e_2 is solution

chart interpretations

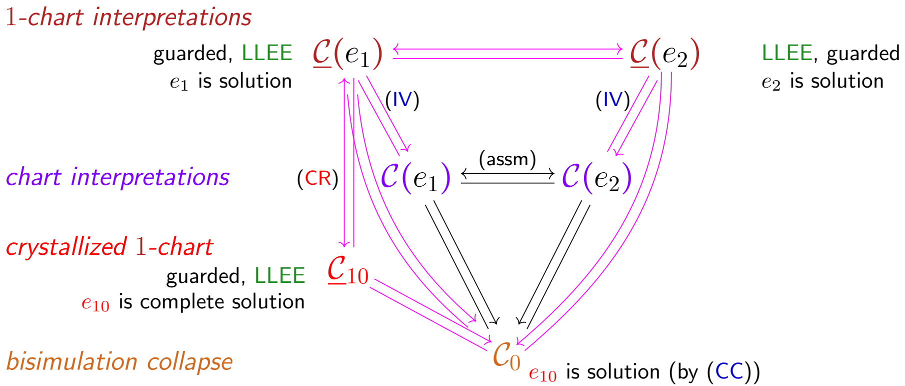
crystallized 1-chart

guarded, LLEE
e_{10} is complete solution

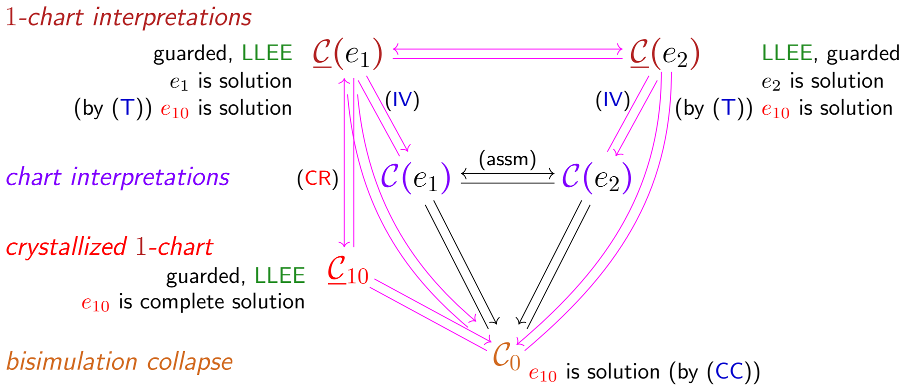
bisimulation collapse



Completeness proof of Mil (structure)



Completeness proof of Mil (structure)



Completeness proof of Mil (structure)

1-chart interpretations

guarded, LLEE
e_1 is solution
(by (T)) e_{10} is solution

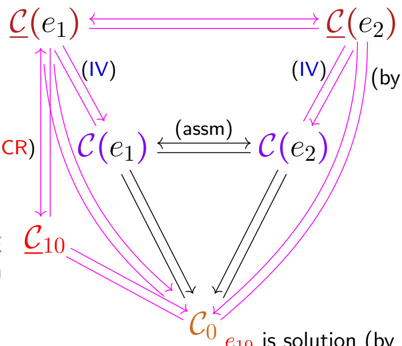
LLEE, guarded
e_2 is solution
(by (T)) e_{10} is solution

chart interpretations

crystallized 1-chart

guarded, LLEE
e_{10} is complete solution

bisimulation collapse



e_{10} is solution (by (CC))

$\mathcal{C}(e_1)$ guarded, LLEE
e_1 is solution
e_{10} is solution

$\mathcal{C}(e_2)$ guarded, LLEE
e_2 is solution
e_{10} is solution

Completeness proof of Mil (structure)

1-chart interpretations

guarded, LLEE
e_1 is solution
(by (T)) e_{10} is solution

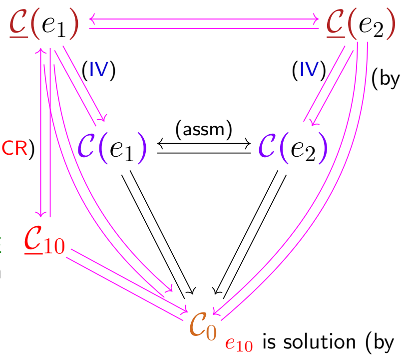
LLEE, guarded
e_2 is solution
(by (T)) e_{10} is solution

chart interpretations

crystallized 1-chart

guarded, LLEE
e_{10} is complete solution

bisimulation collapse

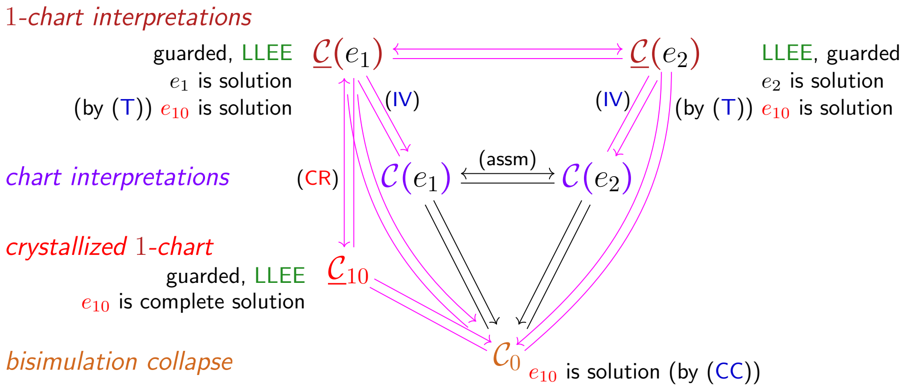


e_{10} is solution (by (CC))

$$\left.\begin{array}{r} \mathcal{C}(e_1) \text{ guarded, LLEE} \\ e_1 \text{ is solution} \\ e_{10} \text{ is solution} \end{array}\right\} \underset{\text{(SE)}}{\Longrightarrow} e_1 =_{\mathsf{Mil}} e_{10}$$

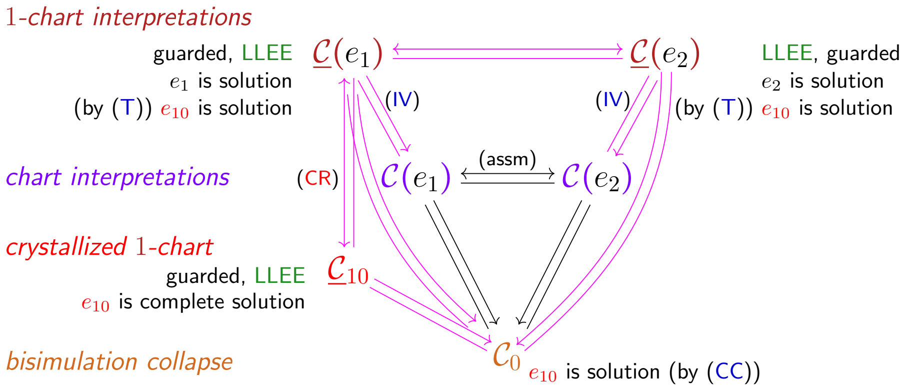
$\mathcal{C}(e_2)$ guarded, LLEE
e_2 is solution
e_{10} is solution

Completeness proof of Mil (structure)



$$\left.\begin{array}{r}\mathcal{C}(e_1) \text{ guarded, LLEE} \\ e_1 \text{ is solution} \\ e_{10} \text{ is solution}\end{array}\right\} \underset{\text{(SE)}}{\Longrightarrow} e_1 =_{\text{Mil}} e_{10} \qquad e_{10} =_{\text{Mil}} e_2 \underset{\text{(SE)}}{\Longleftarrow} \left\{\begin{array}{l}\mathcal{C}(e_2) \text{ guarded, LLEE} \\ e_2 \text{ is solution} \\ e_{10} \text{ is solution}\end{array}\right.$$

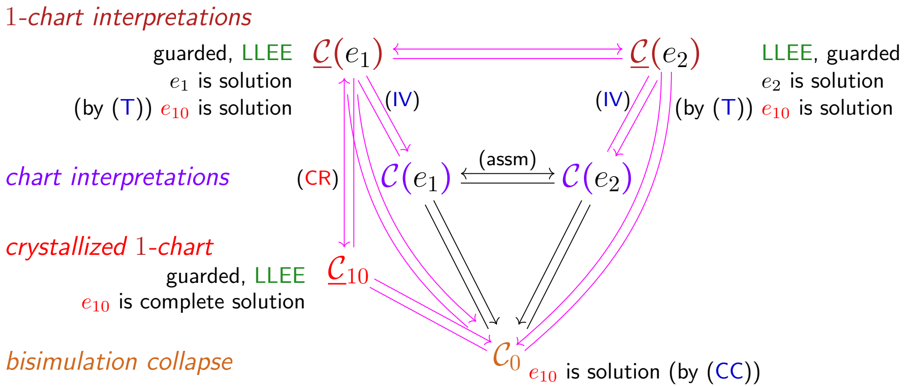
Completeness proof of Mil (structure)



$$\left.\begin{array}{r}\mathcal{C}(e_1) \text{ guarded, LLEE}\\ e_1 \text{ is solution}\\ e_{10} \text{ is solution}\end{array}\right\} \underset{\text{(SE)}}{\Longrightarrow} e_1 =_{\text{Mil}} e_{10} \qquad e_{10} =_{\text{Mil}} e_2 \underset{\text{(SE)}}{\Longleftarrow} \left\{\begin{array}{l}\mathcal{C}(e_2) \text{ guarded, LLEE}\\ e_2 \text{ is solution}\\ e_{10} \text{ is solution}\end{array}\right.$$

$$\Longrightarrow \qquad e_1 =_{\text{Mil}} e_2$$

Completeness proof of Mil (structure)



Theorem

Milner's proof system Mil is complete for process semantics equivalence $\equiv_{\mathbf{P}}$ of regular expressions.

Since: $e_1 \equiv_{\mathbf{P}} e_2 \implies [\![e_1]\!]_{\mathbf{P}} = [\![e_2]\!]_{\mathbf{P}} \implies \mathcal{C}(e_1) \leftrightarrows \mathcal{C}(e_1) \implies e_1 =_{\mathsf{Mil}} e_2$.