Course Proposal: A Short Introduction to Parameterized Complexity
(Algorithmic and Formal-Methods Viewpoints)

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Abstract We propose a short introductory course on parameterized complexity theory in the advanced courses block of the GSSI's Ph.D. program 2020/21 with 10/5 hours of lectures/practice sessions. We want to focus on fixed-parameter tractability results for graphs of bounded width (path-, tree-, clique-width) that can be obtained both by algorithmic techniques (dynamic programming) and by formal-method techniques (logical meta-theorems).

We want to combine our experience with algorithmic aspects of parameterized complexity theory (fpt-results for problems on graphs of bounded width, Alessandro), and formal-methods aspects (metatheorems for obtaining fpt-results, fpt-intractability, Clemens) for teaching a short, introductory, inter-pillar course.

1 Course structure (overview)

In the introductory part we want to provide motivation, definition, and examples of fixed-parameterized tractability (fpt) results.

Then we intend to focus on concepts of graph width such as path-width, tree-width, and clique-width, and the interrelation of fpt results with respect to different concepts of width. We will explain how fpt-results for problems on graphs of bounded width in one of these notions can be obtained by dynamic programming techniques, and by Courcelle's theorem, a monadic second-order logic based metatheorem for obtaining complexity bounds.

In a final lecture we will sketch the basics of fixed-parameter intractability results, which state that certain problems are not fixed-parameter tractable.

We propose a schedule with 10 hours of lectures (2 hours each morning over a 5-work-day week), and 5 hours of practice sessions (1 hour each afternoon), taught by Alessandro (algorithmics) and Clemens (formal methods).

For more details about the course structure, please see Section 4.

2 Comparison with last year's course

In comparison with last year's course by Hugo and Clemens this course is intended be shorter. We intend to move slower, and to treat less topics, but those in

more detail, more concretely, and supported by examples from research practice. We want to anchor the course in only a few selected topics on which Alessandro has worked [1,2]. In doing so we want to illuminate those results also by means of alternative proof approaches via logical metatheorems, and to motivate questions about how some problems can be shown not to be fixed-parameter tractable.

We will take very seriously signals that this year's students may not have a strong background in algorithmics and/or formal methods, and that they may find it difficult to digest some of the more complicated concepts in these fields. Therefore we want to contact the lecturers of immigration and core courses about their experience in order to take their assessment on board when developing our course. We need to adapt the level of our course to what we can reasonably expect the students to master within the available time.

3   Our cooperation

We first cooperated on a practice session in last year's course. In that session Clemens summarized a fixed-parameter tractability result [2] by Alessandro and Alfredo Navarra on 'coverage in multi-interface networks'. For its preparation we had regularly discussed this and other fpt results together. The easily explainable character of the coverage problem, and its proximity to real-world applications were very motivating for last year's students.

Starting from that experience, the wish to pick up our cooperation on topics that are of interest to both of us led us to the idea of a short course on selected topics from parameterized complexity theory. By teaching this course we also want to contribute to strengthening the ties in our department between the groups of people who work in algorithmics, and in formal methods, respectively.

4   Course structure (with more details, tentative)

Day 1: Introduction
- motivations, example
- definition of fixed-parameter tractability
- some easy fpt result explained, proof sketched
- practice session: explanatory examples

Day 2: Path-width and tree-width
- path-width (Robertson and Seymour, 1983)
- tree-width (Bertelè and Brioschi, 1972, Halin 1976)
- mention of spectrum of other notions of width
- interrelation of fpt-results for different versions of graph width
- practice session: example research Alessandro (path or tree-width)

Day 3: Clique-width, and logic preliminaries
- clique-width (Courcelle, Engelfriet, Rozenberg, 1993)

- first-order logic
- monadic second order logic
- practice session: example research Alessandro (clique-width)

Day 4: Courcelle's Theorem (tree-/clique-width), and applications

- Courcelle's theorem for tree-width (Courcelle 1990).
- Courcelle's theorem for clique-width
  - perhaps result by Engelfriet, van Oostrom (1997): a class of graphs has bounded clique-width if, and only if, it is interpretable in the class of coloured trees for some suitable set of colours
- some applications
- practice session: use of Courcelle's theorem for some results that have already been proved by other means

Day 5: Fpt-intractability

- motivation for fixed-parameter intractability results
- fixed-parameter tractable reductions
- XP: slicewise polynomical problems
- *W-Hierarchy and A-Hierarchy*

5 Literature and resources

▷ Book *Parameterized Complexity Theory* by Flum and Grohe, [4], available in the GSSI library.

▷ Book *Parameterized Algorithms* by Cygin et al., [3], available online via the link https://www.mimuw.edu.pl/~malcin/book/parameterized-algorithms.pdf .

▷ Course material [5] for last year's course by Hugo Gilbert and Clemens.

▷ Report [1] on the *fixed-parameter tractability of the maximum 2-edge-colorable subgraph problem* by Alessandro with Vahan Mkrtchyan,

▷ Paper on the *coverage in multi-interfaced networks* problem [2] by AA and Alfredo Navarra.

References

1. Alessandro Aloisio and Vahan Mkrtchyan. On the fixed-parameter tractability of the maximum 2-edge-colorable subgraph problem. *CoRR*, abs/1904.09246, 2019.
2. Alessandro Aloisio and Alfredo Navarra. Constrained connectivity in bounded x-width multi-interface networks. *Algorithms*, 13(2), 2020.
3. Marek Cygan, Fedor V. Fomin, Lukasz Kowalik, Daniel Lokshtanov, Daniel Marx, Marcin Pilipczuk, Michal Pilipczuk, and Saket Saurabh. *Parameterized Algorithms*. Springer, 1st edition, 2015.
4. Jörg Flum and Martin Grohe. *Parameterized Complexity Theory*. Springer, 2006.
5. Hugo Gilbert and Clemens Grabmayer. Parameterized Complexity Theory (Algorithmic and Formal-Methods Viewpoints). Course material for the course with this title, given in the Advanced Course's Period of the GSSI Ph.D. program, April 1–9, 2020, GSSI, L'Aquila, Italy. Available on schoology.com with the following access link: NX65-NNMD-9VCP3, 2020.