Question 1: Derivation

Comparison:

For an array of size n:

$C(n) = (n-1) + (n-2) + (n-3) + \ldots + 2 + 1$

Sum of arithmetic series formula

$C(n) = ((n-1)*n)/2$

Number of comparisons is:

$C(n) = (n(n-1))/2 =$ **O(n^2)**

Swaps:

When looking in reverse order (worst case), for it to be in order, every adjacent pair must be swapped. That is the most amount of swaps a bubble sort can have. Therefore, in the average case (random order) just about half of the pairs need to be swapped.

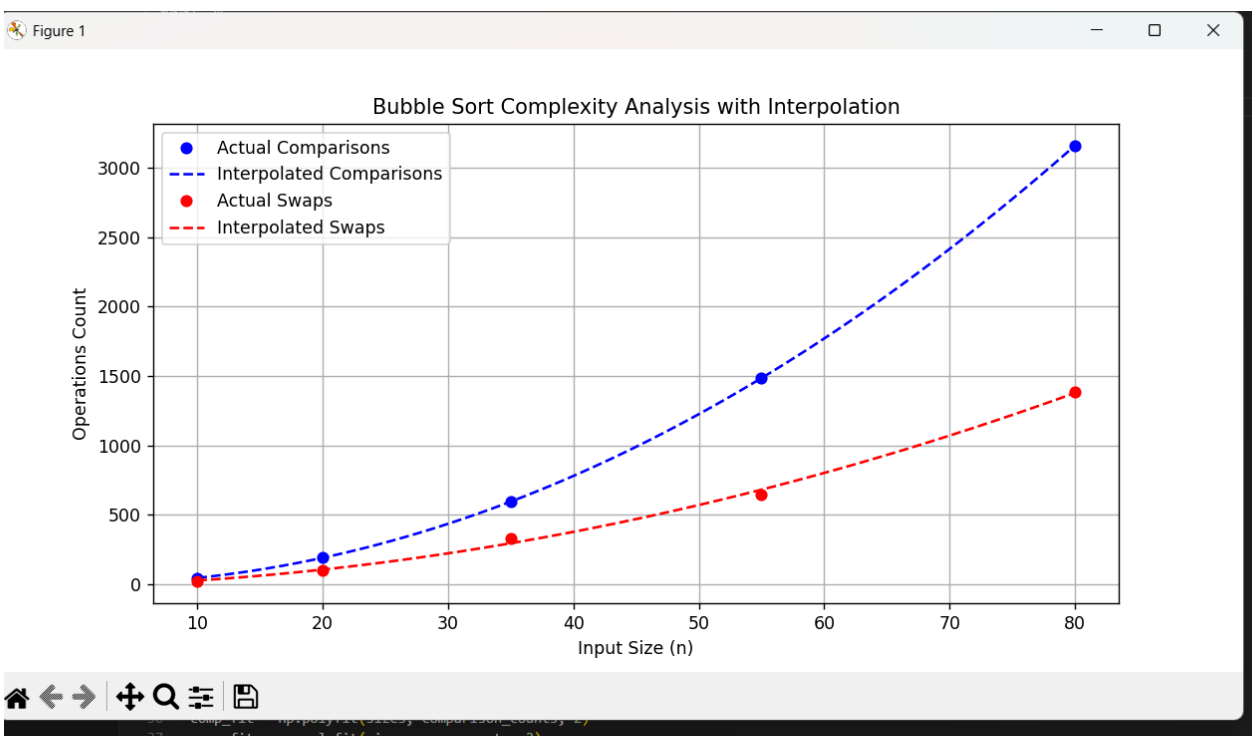
We take the reverse order, being:

$S(n) = (n(n-1))/2$ and divide by half

$S(n) = (n(n-1))/4$ this approximates to n^2/4

This However is still **O(n^2)** in complexity just with a lower constant factor compared to the worst case.

Question 4 – Plotting



It is clear that by looking at the plotted data, the actual comparisons counted within the coded Bubble sort algorithm, match very closely to the interpolated comparison values. Hence matching the complexity analysis. Similarly, the Actual swaps counted in the Bubble sort algorithm match very closely to the interpolated swap values; therefore, the results match the complexity analysis. Both plots clearly have an O(n^2) growth pattern just like the O(n^2) time complexity of Bubble Sort.