PLL 2024 - Linearização

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Equações do PLL no domínio do tempo:

$$pll_\alpha = A \sin\theta \tag{1}$$

$$pll_\beta = A \cos\theta \tag{2}$$

$$\varepsilon_\phi(t) = (v_\alpha - pll_\alpha)\cos\theta - (v_\beta - pll_\beta)\sin\theta \tag{3}$$

$$\varepsilon_A(t) = (v_\alpha - pll_\alpha)\sin\theta + (v_\beta - pll_\beta)\cos\theta \tag{4}$$

0.1 Linearização Harmônica por Perturbação

Para realizar a linearização por perturbação, devemos fazer as seguintes substituições:

$$\varepsilon_\phi = \varepsilon_{\phi,ss} + \Delta\varepsilon_\phi \tag{5}$$

$$\varepsilon_A = \varepsilon_{A,ss} + \Delta\varepsilon_A \tag{6}$$

$$v_\alpha = v_{\alpha,ss} + \Delta v_\alpha \tag{7}$$

$$v_\beta = v_{\beta,ss} + \Delta v_\beta \tag{8}$$

$$pll_\alpha = pll_{\alpha,ss} + \Delta pll_\alpha \tag{9}$$

$$pll_\beta = pll_{\beta,ss} + \Delta pll_\beta \tag{10}$$

$$\theta = \theta_{ss} + \Delta\theta \tag{11}$$

$$A = A_{ss} + \Delta A \tag{12}$$

O processo de linearização das equações (1)-(4) será apresentado na sequência. Por ora os resultados ainda conterão termos senoidais/cossenoidais.

Além disso, vou considerar:

$$\cos\Delta\theta \approx 1 \tag{13}$$

$$\sin\Delta\theta \approx \Delta\theta \tag{14}$$

0.1.1 Linearização da equação (1)

$$pll_\alpha = A \sin\theta \tag{15}$$

$$pll_{\alpha,ss} + \Delta pll_\alpha = (A_{ss} + \Delta A)\sin(\theta_{ss} + \Delta\theta) \tag{16}$$

$$pll_{\alpha,ss} + \Delta pll_\alpha = (A_{ss} + \Delta A)\left(\sin\theta_{ss}\cos\Delta\theta + \cos\theta_{ss}\sin\Delta\theta\right) \tag{17}$$

$$pll_{\alpha,ss} + \Delta pll_\alpha = (A_{ss} + \Delta A)\left(\sin\theta_{ss} + \cos\theta_{ss}\Delta\theta\right) \tag{18}$$

Coletando apenas os termos de primeira ordem:

$$\Delta pll_\alpha = A_{ss}\cos\theta_{ss}\Delta\theta + \sin\theta_{ss}\Delta A \tag{19}$$

0.1.2 Linearização da equação (2)

$$pll_\beta = A \cos\theta \tag{20}$$

$$pll_{\beta,ss} + \Delta pll_\beta = (A_{ss} + \Delta A)\cos(\theta_{ss} + \Delta\theta) \tag{21}$$

$$pll_{\beta,ss} + \Delta pll_\beta = (A_{ss} + \Delta A)\left(\cos\theta_{ss}\cos\Delta\theta - \sin\theta_{ss}\sin\Delta\theta\right) \tag{22}$$

$$pll_{\beta,ss} + \Delta pll_\beta = (A_{ss} + \Delta A)\left(\cos\theta_{ss} - \sin\theta_{ss}\Delta\theta\right) \tag{23}$$

Coletando os termos de primeira ordem:

$$\Delta pll_\beta = -A_{ss}\sin\theta_{ss}\Delta\theta + \cos\theta_{ss}\Delta A \tag{24}$$

0.1.3 Linearização da equação (3)

$$\varepsilon_\phi(t) = (v_\alpha - pll_\alpha)\cos\theta - (v_\beta - pll_\beta)\sin\theta \tag{25}$$

$$\begin{aligned}\varepsilon_{\phi,ss} + \Delta\varepsilon_\phi = {} & (v_{\alpha,ss} + \Delta v_\alpha - pll_{\alpha,ss} - \Delta pll_\alpha)\cos(\theta_{ss} + \Delta\theta) \\ & - (v_{\beta,ss} + \Delta v_\beta - pll_{\beta,ss} - \Delta pll_\beta)\sin(\theta_{ss} + \Delta\theta)\end{aligned} \tag{26}$$

$$\begin{aligned}\varepsilon_{\phi,ss} + \Delta\varepsilon_\phi = {} & (v_{\alpha,ss} + \Delta v_\alpha - pll_{\alpha,ss} - \Delta pll_\alpha)\left(\cos\theta_{ss}\cos\Delta\theta - \sin\theta_{ss}\sin\Delta\theta\right) \\ & - (v_{\beta,ss} + \Delta v_\beta - pll_{\beta,ss} - \Delta pll_\beta)\left(\sin\theta_{ss}\cos\Delta\theta + \cos\theta_{ss}\sin\Delta\theta\right)\end{aligned} \tag{27}$$

$$\begin{aligned}\varepsilon_{\phi,ss} + \Delta\varepsilon_\phi = {} & (v_{\alpha,ss} + \Delta v_\alpha - pll_{\alpha,ss} - \Delta pll_\alpha)(\cos\theta_{ss} - \sin\theta_{ss}\Delta\theta) \\ & - (v_{\beta,ss} + \Delta v_\beta - pll_{\beta,ss} - \Delta pll_\beta)(\sin\theta_{ss} + \cos\theta_{ss}\Delta\theta) \end{aligned} \quad (28)$$

Coletando os termos de primeira ordem:

$$\begin{aligned}\Delta\varepsilon_\phi = {} & -v_{\alpha,ss}\sin\theta_{ss}\Delta\theta + \cos\theta_{ss}\Delta v_\alpha \\ & + pll_{\alpha,ss}\sin\theta_{ss}\Delta\theta - \cos\theta_{ss}\Delta pll_\alpha \\ & - v_{\beta,ss}\cos\theta_{ss}\Delta\theta - \sin\theta_{ss}\Delta v_\beta \\ & + pll_{\beta,ss}\cos\theta_{ss}\Delta\theta + \sin\theta_{ss}\Delta pll_\beta \end{aligned} \quad (29)$$

0.1.4 Linearização da equação (4)

$$\varepsilon_A(t) = (v_\alpha - pll_\alpha)\sin\theta + (v_\beta - pll_\beta)\cos\theta \quad (30)$$

$$\begin{aligned}\varepsilon_{A,ss} + \Delta\varepsilon_A = {} & (v_{\alpha,ss} + \Delta v_\alpha - pll_{\alpha,ss} + \Delta pll_\alpha)\sin(\theta_{ss} + \Delta) + \\ & (v_{\beta,ss} + \Delta v_\beta - pll_{\beta,ss} + \Delta pll_\beta)\cos(\theta_{ss} + \Delta\theta) \end{aligned} \quad (31)$$

$$\begin{aligned}\varepsilon_{A,ss} + \Delta\varepsilon_A = {} & (v_{\alpha,ss} + \Delta v_\alpha - pll_{\alpha,ss} - \Delta pll_\alpha)(\sin\theta_{ss}\cos\Delta\theta + \cos\theta_{ss}\sin\Delta\theta) + \\ & (v_{\beta,ss} + \Delta v_\beta - pll_{\beta,ss} - \Delta pll_\beta)(\cos\theta_{ss}\cos\Delta\theta - \sin\theta_{ss}\sin\Delta\theta) \end{aligned} \quad (32)$$

$$\begin{aligned}\varepsilon_{A,ss} + \Delta\varepsilon_A = {} & (v_{\alpha,ss} + \Delta v_\alpha - pll_{\alpha,ss} - \Delta pll_\alpha)(\sin\theta_{ss} + \cos\theta_{ss}\Delta\theta) + \\ & (v_{\beta,ss} + \Delta v_\beta - pll_{\beta,ss} - \Delta pll_\beta)(\cos\theta_{ss} - \sin\theta_{ss}\Delta\theta) \end{aligned} \quad (33)$$

Coletando os termos de primeira ordem:

$$\begin{aligned}\Delta\varepsilon_A = {} & v_{\alpha,ss}\cos\theta_{ss}\Delta\theta + \sin\theta_{ss}\Delta v_\alpha \\ & - pll_{\alpha,ss}\cos\theta_{ss}\Delta\theta - \sin\theta_{ss}\Delta pll_\alpha \\ & - v_{\beta,ss}\sin\theta_{ss}\Delta\theta + \cos\theta_{ss}\Delta v_\beta \\ & + pll_{\beta,ss}\sin\theta_{ss}\Delta\theta - \cos\theta_{ss}\Delta pll_\beta \end{aligned} \quad (34)$$

0.1.5 Conjunto de equações linearizadas: versão inicial

$$\Delta pll_{\alpha} = A_{ss}\cos\theta_{ss}\Delta\theta + \sin\theta_{ss}\Delta A \tag{35}$$

$$\Delta pll_{\beta} = -A_{ss}\sin\theta_{ss}\Delta\theta + \cos\theta_{ss}\Delta A \tag{36}$$

$$\begin{aligned}\Delta\varepsilon_{\phi} = -v_{\alpha,ss}\sin\theta_{ss}\Delta\theta + \cos\theta_{ss}\Delta v_{\alpha} \\ + pll_{\alpha,ss}\sin\theta_{ss}\Delta\theta - \cos\theta_{ss}\Delta pll_{\alpha} \\ - v_{\beta,ss}\cos\theta_{ss}\Delta\theta - \sin\theta_{ss}\Delta v_{\beta} \\ + pll_{\beta,ss}\cos\theta_{ss}\Delta\theta + \sin\theta_{ss}\Delta pll_{\beta}\end{aligned} \tag{37}$$

$$\begin{aligned}\Delta\varepsilon_{A} = v_{\alpha,ss}\cos\theta_{ss}\Delta\theta + \sin\theta_{ss}\Delta v_{\alpha} \\ - pll_{\alpha,ss}\cos\theta_{ss}\Delta\theta - \sin\theta_{ss}\Delta pll_{\alpha} \\ - v_{\beta,ss}\sin\theta_{ss}\Delta\theta + \cos\theta_{ss}\Delta v_{\beta} \\ + pll_{\beta,ss}\sin\theta_{ss}\Delta\theta - \cos\theta_{ss}\Delta pll_{\beta}\end{aligned} \tag{38}$$

0.1.6 Substituindo $v_{\alpha\beta,ss}$ e $pll_{\alpha\beta,ss}$

N caso do PLL está sendo utilizado para extrair a componente de sequência negativa, podemos escrever:

$$v_{\alpha,ss} = A_{ss}\sin\theta_{ss} \tag{39}$$

$$v_{\beta,ss} = A_{ss}\cos\theta_{ss} \tag{40}$$

$$pll_{\alpha,ss} = A_{ss}\sin\theta_{ss} \tag{41}$$

$$pll_{\beta,ss} = A_{ss}\cos\theta_{ss} \tag{42}$$

Substituindo estes resultados em (29):

$$\begin{aligned}\Delta\varepsilon_{\phi} = -A_{ss}\sin\theta_{ss}\sin\theta_{ss}\Delta\theta + \cos\theta_{ss}\Delta v_{\alpha} \\ + A_{ss}\sin\theta_{ss}\sin\theta_{ss}\Delta\theta - \cos\theta_{ss}\Delta pll_{\alpha} \\ - A_{ss}\cos\theta_{ss}\cos\theta_{ss}\Delta\theta - \sin\theta_{ss}\Delta v_{\beta} \\ + A_{ss}\cos\theta_{ss}\cos\theta_{ss}\Delta\theta + \sin\theta_{ss}\Delta pll_{\beta}\end{aligned} \tag{43}$$

$$\begin{aligned}\Delta\varepsilon_{\phi} = -A_{ss}\sin^2\theta_{ss}\Delta\theta + \cos\theta_{ss}\Delta v_{\alpha} + A_{ss}\sin^2\theta_{ss}\Delta\theta - \cos\theta_{ss}\Delta pll_{\alpha}\\ - A_{ss}\cos^2\theta_{ss}\Delta\theta - \sin\theta_{ss}\Delta v_{\beta} + A_{ss}\cos^2\theta_{ss}\Delta\theta + \sin\theta_{ss}\Delta pll_{\beta}\end{aligned} \quad (44)$$

$$\Delta\varepsilon_{\phi} = \cos\theta_{ss}\Delta v_{\alpha} - \cos\theta_{ss}\Delta pll_{\alpha} - \sin\theta_{ss}\Delta v_{\beta} + \sin\theta_{ss}\Delta pll_{\beta} \quad (45)$$

Fazendo as substituições em (34):

$$\begin{aligned}\Delta\varepsilon_{A} = A_{ss}\sin\theta_{ss}\cos\theta_{ss}\Delta\theta + \sin\theta_{ss}\Delta v_{\alpha}\\ - A_{ss}\sin\theta_{ss}\cos\theta_{ss}\Delta\theta - \sin\theta_{ss}\Delta pll_{\alpha}\\ - A_{ss}\cos\theta_{ss}\sin\theta_{ss}\Delta\theta + \cos\theta_{ss}\Delta v_{\beta}\\ + A_{ss}\cos\theta_{ss}\sin\theta_{ss}\Delta\theta - \cos\theta_{ss}\Delta pll_{\beta}\end{aligned} \quad (46)$$

$$\Delta\varepsilon_{A} = \sin\theta_{ss}\Delta v_{\alpha} - \sin\theta_{ss}\Delta pll_{\alpha} + \cos\theta_{ss}\Delta v_{\beta} - \cos\theta_{ss}\Delta pll_{\beta} \quad (47)$$

0.1.7 Conjunto de equações linearizadas: versão simplificada

$$\Delta pll_{\alpha} = A_{ss}\cos\theta_{ss}\Delta\theta + \sin\theta_{ss}\Delta A \quad (48)$$

$$\Delta pll_{\beta} = -A_{ss}\sin\theta_{ss}\Delta\theta + \cos\theta_{ss}\Delta A \quad (49)$$

$$\Delta\varepsilon_{\phi} = \cos\theta_{ss}\Delta v_{\alpha} - \cos\theta_{ss}\Delta pll_{\alpha} - \sin\theta_{ss}\Delta v_{\beta} + \sin\theta_{ss}\Delta pll_{\beta} \quad (50)$$

$$\Delta\varepsilon_{A} = \sin\theta_{ss}\Delta v_{\alpha} - \sin\theta_{ss}\Delta pll_{\alpha} + \cos\theta_{ss}\Delta v_{\beta} - \cos\theta_{ss}\Delta pll_{\beta} \quad (51)$$

0.1.8 Conjunto de equações linearizadas: usando Euler

Antes de começar, temos que lembrar que $\theta_{ss} = \omega_1 t$. Então:

$$\Delta pll_{\alpha} = A_{ss}\left(\frac{e^{j\omega_1 t} + e^{-j\omega_1 t}}{2}\right)\Delta\theta + \left(\frac{e^{j\omega_1 t} - e^{-j\omega_1 t}}{2j}\right)\Delta A \quad (52)$$

$$\Delta pll_{\beta} = -A_{ss}\left(\frac{e^{j\omega_1 t} - e^{-j\omega_1 t}}{2j}\right)\Delta\theta + \left(\frac{e^{j\omega_1 t} + e^{-j\omega_1 t}}{2}\right)\Delta A \quad (53)$$

$$\Delta\varepsilon_{\phi} = \left(\frac{e^{j\omega_1 t} + e^{-j\omega_1 t}}{2}\right)\Delta v_{\alpha} - \left(\frac{e^{j\omega_1 t} + e^{-j\omega_1 t}}{2}\right)\Delta pll_{\alpha} - \left(\frac{e^{j\omega_1 t} - e^{-j\omega_1 t}}{2j}\right)\Delta v_{\beta} + \left(\frac{e^{j\omega_1 t} - e^{-j\omega_1 t}}{2j}\right)\Delta pll_{\beta} \quad (54)$$

$$\Delta\varepsilon_{A} = \left(\frac{e^{j\omega_1 t} - e^{-j\omega_1 t}}{2j}\right)\Delta v_{\alpha} - \left(\frac{e^{j\omega_1 t} - e^{-j\omega_1 t}}{2j}\right)\Delta pll_{\alpha} + \left(\frac{e^{j\omega_1 t} + e^{-j\omega_1 t}}{2}\right)\Delta v_{\beta} - \left(\frac{e^{j\omega_1 t} + e^{-j\omega_1 t}}{2}\right)\Delta pll_{\beta} \quad (55)$$

0.1.9 Conjunto de equações linearizadas: Representação usando vetores espaciais

O objetivo desta seção é reescrever as equações da seção anterior com base nas seguintes definições:

$$\Delta\vec{v}_{\alpha\beta}^{\,+} = \Delta v_{\alpha} + j\Delta v_{\beta} \quad (56)$$

$$\Delta\vec{v}_{\alpha\beta}^{\,-} = \Delta v_{\alpha} - j\Delta v_{\beta} \quad (57)$$

$$\Delta\vec{pll}_{\alpha\beta}^{\,+} = \Delta pll_{\alpha} + j\Delta pll_{\beta} \quad (58)$$

$$\Delta\vec{pll}_{\alpha\beta}^{\,-} = \Delta pll_{\alpha} - j\Delta pll_{\beta} \quad (59)$$

Primeiramente, podemos combinar as equações (52) e (53):

$$\Delta\vec{pll}_{\alpha\beta} = A_{ss}\left(\frac{e^{j\omega_1 t} + e^{-j\omega_1 t}}{2}\right)\Delta\theta + \left(\frac{e^{j\omega_1 t} - e^{-j\omega_1 t}}{2j}\right)\Delta A + j\left[-A_{ss}\left(\frac{e^{j\omega_1 t} - e^{-j\omega_1 t}}{2j}\right)\Delta\theta + \left(\frac{e^{j\omega_1 t} + e^{-j\omega_1 t}}{2}\right)\Delta A\right] \quad (60)$$

$$\Delta\vec{pll}_{\alpha\beta} = A_{ss}\left(\frac{e^{j\omega_1 t} + e^{-j\omega_1 t}}{2}\right)\Delta\theta + j\left(\frac{-e^{j\omega_1 t} + e^{-j\omega_1 t}}{2}\right)\Delta A + \left[A_{ss}\left(\frac{-e^{j\omega_1 t} + e^{-j\omega_1 t}}{2}\right)\Delta\theta + j\left(\frac{e^{j\omega_1 t} + e^{-j\omega_1 t}}{2}\right)\Delta A\right] \quad (61)$$

$$\Delta p\vec{l}l_{\alpha\beta} = A_{ss}e^{-j\omega_1 t}\Delta\theta + je^{-j\omega_1 t}\Delta A \tag{62}$$

Agora podemos processar a equação (54):

$$\begin{aligned}\Delta\varepsilon_\phi = &\left(\frac{e^{j\omega_1 t}+e^{-j\omega_1 t}}{2}\right)\Delta v_\alpha - \left(\frac{e^{j\omega_1 t}+e^{-j\omega_1 t}}{2}\right)\Delta pll_\alpha \\ &- \left(\frac{e^{j\omega_1 t}-e^{-j\omega_1 t}}{2j}\right)\Delta v_\beta + \left(\frac{e^{j\omega_1 t}-e^{-j\omega_1 t}}{2j}\right)\Delta pll_\beta\end{aligned} \tag{63}$$

$$\begin{aligned}\Delta\varepsilon_\phi = &\left(\frac{e^{j\omega_1 t}+e^{-j\omega_1 t}}{2}\right)\Delta v_\alpha - \left(\frac{e^{j\omega_1 t}+e^{-j\omega_1 t}}{2}\right)\Delta pll_\alpha \\ &+ j\left(\frac{e^{j\omega_1 t}-e^{-j\omega_1 t}}{2}\right)\Delta v_\beta - j\left(\frac{e^{j\omega_1 t}-e^{-j\omega_1 t}}{2}\right)\Delta pll_\beta\end{aligned} \tag{64}$$

$$\begin{aligned}\Delta\varepsilon_\phi = &\frac{1}{2}e^{j\omega_1 t}(\Delta v_\alpha + j\Delta v_\beta) + \frac{1}{2}e^{-j\omega_1 t}(\Delta v_\alpha - j\Delta v_\beta) \\ &- \frac{1}{2}e^{j\omega_1 t}(\Delta pll_\alpha + j\Delta pll_\beta) - \frac{1}{2}e^{-j\omega_1 t}(\Delta pll_\alpha - j\Delta pll_\beta)\end{aligned} \tag{65}$$

$$\Delta\varepsilon_\phi = \frac{1}{2}e^{j\omega_1 t}\Delta\vec{v}_{\alpha\beta}^{\,+} + \frac{1}{2}e^{-j\omega_1 t}\Delta\vec{v}_{\alpha\beta}^{\,-} - \frac{1}{2}e^{j\omega_1 t}\Delta p\vec{l}l_{\alpha\beta}^{\,+} - \frac{1}{2}e^{-j\omega_1 t}\Delta p\vec{l}l_{\alpha\beta}^{\,-} \tag{66}$$

Por último, temos que processar a equação (55):

$$\begin{aligned}\Delta\varepsilon_A = &\left(\frac{e^{j\omega_1 t}-e^{-j\omega_1 t}}{2j}\right)\Delta v_\alpha - \left(\frac{e^{j\omega_1 t}-e^{-j\omega_1 t}}{2j}\right)\Delta pll_\alpha \\ &+ \left(\frac{e^{j\omega_1 t}+e^{-j\omega_1 t}}{2}\right)\Delta v_\beta - \left(\frac{e^{j\omega_1 t}+e^{-j\omega_1 t}}{2}\right)\Delta pll_\beta\end{aligned} \tag{67}$$

$$\begin{aligned}\Delta\varepsilon_A = &-j\left(\frac{e^{j\omega_1 t}-e^{-j\omega_1 t}}{2}\right)\Delta v_\alpha + j\left(\frac{e^{j\omega_1 t}-e^{-j\omega_1 t}}{2}\right)\Delta pll_\alpha \\ &+ \left(\frac{e^{j\omega_1 t}+e^{-j\omega_1 t}}{2}\right)\Delta v_\beta - \left(\frac{e^{j\omega_1 t}+e^{-j\omega_1 t}}{2}\right)\Delta pll_\beta\end{aligned} \tag{68}$$

$$\Delta\varepsilon_A = \frac{1}{2}e^{j\omega_1 t}(-j\Delta v_\alpha + \Delta v_\beta) + \frac{1}{2}e^{-j\omega_1 t}(j\Delta v_\alpha + \Delta v_\beta)$$
$$+ \frac{1}{2}e^{j\omega_1 t}(j\Delta pll_\alpha - \Delta pll_\beta) + \frac{1}{2}e^{-j\omega_1 t}(-j\Delta pll_\alpha - \Delta pll_\beta) \quad (69)$$

$$\Delta\varepsilon_A = -\frac{j}{2}e^{j\omega_1 t}(\Delta v_\alpha + j\Delta v_\beta) + \frac{j}{2}e^{-j\omega_1 t}(\Delta v_\alpha - j\Delta v_\beta)$$
$$+ \frac{j}{2}e^{j\omega_1 t}(\Delta pll_\alpha + j\Delta pll_\beta) - \frac{j}{2}e^{-j\omega_1 t}(\Delta pll_\alpha - j\Delta pll_\beta) \quad (70)$$

$$\Delta\varepsilon_A = -\frac{j}{2}e^{j\omega_1 t}\Delta\vec{v}_{\alpha\beta}^{\,+} + \frac{j}{2}e^{-j\omega_1 t}\Delta\vec{v}_{\alpha\beta}^{\,-} + \frac{j}{2}e^{j\omega_1 t}\Delta p\vec{l}l_{\alpha\beta}^{\,+} - \frac{j}{2}e^{-j\omega_1 t}\Delta p\vec{l}l_{\alpha\beta}^{\,-} \quad (71)$$