

# PLL 2024 - Linearização

November 6, 2024

Equações do PLL no domínio do tempo:

$$pll_\alpha = A \sin \theta \quad (1)$$

$$pll_\beta = A \cos \theta \quad (2)$$

$$\varepsilon_\phi(t) = (v_\alpha - pll_\alpha) \cos \theta - (v_\beta - pll_\beta) \sin \theta \quad (3)$$

$$\varepsilon_A(t) = (v_\alpha - pll_\alpha) \sin \theta + (v_\beta - pll_\beta) \cos \theta \quad (4)$$

## 0.1 Linearização Harmônica por Perturbação

Para realizar a linearização por perturbação, devemos fazer as seguintes substituições:

$$\varepsilon_\phi = \varepsilon_{\phi,ss} + \Delta\varepsilon_\phi \quad (5)$$

$$\varepsilon_A = \varepsilon_{A,ss} + \Delta\varepsilon_A \quad (6)$$

$$v_\alpha = v_{\alpha,ss} + \Delta v_\alpha \quad (7)$$

$$v_\beta = v_{\beta,ss} + \Delta v_\beta \quad (8)$$

$$pll_\alpha = pll_{\alpha,ss} + \Delta pll_\alpha \quad (9)$$

$$pll_\beta = pll_{\beta,ss} + \Delta pll_\beta \quad (10)$$

$$\theta = \theta_{ss} + \Delta\theta \quad (11)$$

$$A = A_{ss} + \Delta A \quad (12)$$

O processo de linearização das equações (1)-(4) será apresentado na sequência. Por ora os resultados ainda conterão termos senoidais/cossenoidais.

Além disso, vou considerar:

$$\cos \Delta\theta \approx 1 \quad (13)$$

$$\sin \Delta\theta \approx \Delta\theta \quad (14)$$

### 0.1.1 Linearização da equação (1)

$$pll_{\alpha} = A \sin \theta \quad (15)$$

$$pll_{\alpha,ss} + \Delta pll_{\alpha} = (A_{ss} + \Delta A) \sin(\theta_{ss} + \Delta\theta) \quad (16)$$

$$pll_{\alpha,ss} + \Delta pll_{\alpha} = (A_{ss} + \Delta A) (\sin \theta_{ss} \cos \Delta\theta + \cos \theta_{ss} \sin \Delta\theta) \quad (17)$$

$$pll_{\alpha,ss} + \Delta pll_{\alpha} = (A_{ss} + \Delta A) (\sin \theta_{ss} + \cos \theta_{ss} \Delta\theta) \quad (18)$$

Coletando apenas os termos de primeira ordem:

$$\Delta pll_{\alpha} = A_{ss} \cos \theta_{ss} \Delta\theta + \sin \theta_{ss} \Delta A \quad (19)$$

### 0.1.2 Linearização da equação (2)

$$pll_{\beta} = A \cos \theta \quad (20)$$

$$pll_{\beta,ss} + \Delta pll_{\beta} = (A_{ss} + \Delta A) \cos(\theta_{ss} + \Delta\theta) \quad (21)$$

$$pll_{\beta,ss} + \Delta pll_{\beta} = (A_{ss} + \Delta A) (\cos \theta_{ss} \cos \Delta\theta - \sin \theta_{ss} \sin \Delta\theta) \quad (22)$$

$$pll_{\beta,ss} + \Delta pll_{\beta} = (A_{ss} + \Delta A) (\cos \theta_{ss} - \sin \theta_{ss} \Delta\theta) \quad (23)$$

Coletando os termos de primeira ordem:

$$\Delta pll_{\beta} = -A_{ss} \sin \theta_{ss} \Delta\theta + \cos \theta_{ss} \Delta A \quad (24)$$

### 0.1.3 Linearização da equação (3)

$$\varepsilon_{\phi}(t) = (v_{\alpha} - pll_{\alpha}) \cos \theta - (v_{\beta} - pll_{\beta}) \sin \theta \quad (25)$$

$$\begin{aligned} \varepsilon_{\phi,ss} + \Delta \varepsilon_{\phi} &= (v_{\alpha,ss} + \Delta v_{\alpha} - pll_{\alpha,ss} - \Delta pll_{\alpha}) \cos(\theta_{ss} + \Delta\theta) \\ &\quad - (v_{\beta,ss} + \Delta v_{\beta} - pll_{\beta,ss} - \Delta pll_{\beta}) \sin(\theta_{ss} + \Delta\theta) \end{aligned} \quad (26)$$

$$\begin{aligned} \varepsilon_{\phi,ss} + \Delta \varepsilon_{\phi} &= (v_{\alpha,ss} + \Delta v_{\alpha} - pll_{\alpha,ss} - \Delta pll_{\alpha}) (\cos \theta_{ss} \cos \Delta\theta - \sin \theta_{ss} \sin \Delta\theta) \\ &\quad - (v_{\beta,ss} + \Delta v_{\beta} - pll_{\beta,ss} - \Delta pll_{\beta}) (\sin \theta_{ss} \cos \Delta\theta + \cos \theta_{ss} \sin \Delta\theta) \end{aligned} \quad (27)$$

$$\begin{aligned}\varepsilon_{\phi,ss} + \Delta\varepsilon_{\phi} = & (v_{\alpha,ss} + \Delta v_{\alpha} - pll_{\alpha,ss} - \Delta pll_{\alpha}) (\cos \theta_{ss} - \sin \theta_{ss} \Delta\theta) \\ & - (v_{\beta,ss} + \Delta v_{\beta} - pll_{\beta,ss} - \Delta pll_{\beta}) (\sin \theta_{ss} + \cos \theta_{ss} \Delta\theta) \quad (28)\end{aligned}$$

Coletando os termos de primeira ordem:

$$\begin{aligned}\Delta\varepsilon_{\phi} = & -v_{\alpha,ss} \sin \theta_{ss} \Delta\theta + \cos \theta_{ss} \Delta v_{\alpha} \\ & + pll_{\alpha,ss} \sin \theta_{ss} \Delta\theta - \cos \theta_{ss} \Delta pll_{\alpha} \\ & - v_{\beta,ss} \cos \theta_{ss} \Delta\theta - \sin \theta_{ss} \Delta v_{\beta} \\ & + pll_{\beta,ss} \cos \theta_{ss} \Delta\theta + \sin \theta_{ss} \Delta pll_{\beta} \quad (29)\end{aligned}$$

#### 0.1.4 Linearização da equação (4)

$$\varepsilon_A(t) = (v_{\alpha} - pll_{\alpha}) \sin \theta + (v_{\beta} - pll_{\beta}) \cos \theta \quad (30)$$

$$\begin{aligned}\varepsilon_{A,ss} + \Delta\varepsilon_A = & (v_{\alpha,ss} + \Delta v_{\alpha} - pll_{\alpha,ss} + \Delta pll_{\alpha}) \sin(\theta_{ss} + \Delta) + \\ & (v_{\beta,ss} + \Delta v_{\beta} - pll_{\beta,ss} + \Delta pll_{\beta}) \cos(\theta_{ss} + \Delta\theta) \quad (31)\end{aligned}$$

$$\begin{aligned}\varepsilon_{A,ss} + \Delta\varepsilon_A = & (v_{\alpha,ss} + \Delta v_{\alpha} - pll_{\alpha,ss} - \Delta pll_{\alpha}) (\sin \theta_{ss} \cos \Delta\theta + \cos \theta_{ss} \sin \Delta\theta) + \\ & (v_{\beta,ss} + \Delta v_{\beta} - pll_{\beta,ss} - \Delta pll_{\beta}) (\cos \theta_{ss} \cos \Delta\theta - \sin \theta_{ss} \sin \Delta\theta) \quad (32)\end{aligned}$$

$$\begin{aligned}\varepsilon_{A,ss} + \Delta\varepsilon_A = & (v_{\alpha,ss} + \Delta v_{\alpha} - pll_{\alpha,ss} - \Delta pll_{\alpha}) (\sin \theta_{ss} + \cos \theta_{ss} \Delta\theta) + \\ & (v_{\beta,ss} + \Delta v_{\beta} - pll_{\beta,ss} - \Delta pll_{\beta}) (\cos \theta_{ss} - \sin \theta_{ss} \Delta\theta) \quad (33)\end{aligned}$$

Coletando os termos de primeira ordem:

$$\begin{aligned}\Delta\varepsilon_A = & v_{\alpha,ss} \cos \theta_{ss} \Delta\theta + \sin \theta_{ss} \Delta v_{\alpha} \\ & - pll_{\alpha,ss} \cos \theta_{ss} \Delta\theta - \sin \theta_{ss} \Delta pll_{\alpha} \\ & - v_{\beta,ss} \sin \theta_{ss} \Delta\theta + \cos \theta_{ss} \Delta v_{\beta} \\ & + pll_{\beta,ss} \sin \theta_{ss} \Delta\theta - \cos \theta_{ss} \Delta pll_{\beta} \quad (34)\end{aligned}$$

### 0.1.5 Conjunto de equações linearizadas: versão inicial

$$\Delta pll_\alpha = A_{ss} \cos \theta_{ss} \Delta \theta + \sin \theta_{ss} \Delta A \quad (35)$$

$$\Delta pll_\beta = -A_{ss} \sin \theta_{ss} \Delta \theta + \cos \theta_{ss} \Delta A \quad (36)$$

$$\begin{aligned} \Delta \varepsilon_\phi = & -v_{\alpha,ss} \sin \theta_{ss} \Delta \theta + \cos \theta_{ss} \Delta v_\alpha \\ & + pll_{\alpha,ss} \sin \theta_{ss} \Delta \theta - \cos \theta_{ss} \Delta pll_\alpha \\ & - v_{\beta,ss} \cos \theta_{ss} \Delta \theta - \sin \theta_{ss} \Delta v_\beta \\ & + pll_{\beta,ss} \cos \theta_{ss} \Delta \theta + \sin \theta_{ss} \Delta pll_\beta \end{aligned} \quad (37)$$

$$\begin{aligned} \Delta \varepsilon_A = & v_{\alpha,ss} \cos \theta_{ss} \Delta \theta + \sin \theta_{ss} \Delta v_\alpha \\ & - pll_{\alpha,ss} \cos \theta_{ss} \Delta \theta - \sin \theta_{ss} \Delta pll_\alpha \\ & - v_{\beta,ss} \sin \theta_{ss} \Delta \theta + \cos \theta_{ss} \Delta v_\beta \\ & + pll_{\beta,ss} \sin \theta_{ss} \Delta \theta - \cos \theta_{ss} \Delta pll_\beta \end{aligned} \quad (38)$$

### 0.1.6 Substituindo $v_{\alpha\beta,ss}$ e $pll_{\alpha\beta,ss}$

N caso do PLL está sendo utilizado para extrair a componente de sequência negativa, podemos escrever:

$$v_{\alpha,ss} = A_{ss} \sin \theta_{ss} \quad (39)$$

$$v_{\beta,ss} = A_{ss} \cos \theta_{ss} \quad (40)$$

$$pll_{\alpha,ss} = A_{ss} \sin \theta_{ss} \quad (41)$$

$$pll_{\beta,ss} = A_{ss} \cos \theta_{ss} \quad (42)$$

Substituindo estes resultados em (29):

$$\begin{aligned} \Delta \varepsilon_\phi = & -A_{ss} \sin \theta_{ss} \sin \theta_{ss} \Delta \theta + \cos \theta_{ss} \Delta v_\alpha \\ & + A_{ss} \sin \theta_{ss} \sin \theta_{ss} \Delta \theta - \cos \theta_{ss} \Delta pll_\alpha \\ & - A_{ss} \cos \theta_{ss} \cos \theta_{ss} \Delta \theta - \sin \theta_{ss} \Delta v_\beta \\ & + A_{ss} \cos \theta_{ss} \cos \theta_{ss} \Delta \theta + \sin \theta_{ss} \Delta pll_\beta \end{aligned} \quad (43)$$

$$\begin{aligned}\Delta\varepsilon_\phi = & -A_{ss}\sin^2\theta_{ss}\Delta\theta + \cos\theta_{ss}\Delta v_\alpha + A_{ss}\sin^2\theta_{ss}\Delta\theta - \cos\theta_{ss}\Delta pll_\alpha \\ & - A_{ss}\cos^2\theta_{ss}\Delta\theta - \sin\theta_{ss}\Delta v_\beta + A_{ss}\cos^2\theta_{ss}\Delta\theta + \sin\theta_{ss}\Delta pll_\beta\end{aligned}\quad (44)$$

$$\Delta\varepsilon_\phi = \cos\theta_{ss}\Delta v_\alpha - \cos\theta_{ss}\Delta pll_\alpha - \sin\theta_{ss}\Delta v_\beta + \sin\theta_{ss}\Delta pll_\beta \quad (45)$$

Fazendo as substituições em (34):

$$\begin{aligned}\Delta\varepsilon_A = & A_{ss}\sin\theta_{ss}\cos\theta_{ss}\Delta\theta + \sin\theta_{ss}\Delta v_\alpha \\ & - A_{ss}\sin\theta_{ss}\cos\theta_{ss}\Delta\theta - \sin\theta_{ss}\Delta pll_\alpha \\ & - A_{ss}\cos\theta_{ss}\sin\theta_{ss}\Delta\theta + \cos\theta_{ss}\Delta v_\beta \\ & + A_{ss}\cos\theta_{ss}\sin\theta_{ss}\Delta\theta - \cos\theta_{ss}\Delta pll_\beta\end{aligned}\quad (46)$$

$$\Delta\varepsilon_A = \sin\theta_{ss}\Delta v_\alpha - \sin\theta_{ss}\Delta pll_\alpha + \cos\theta_{ss}\Delta v_\beta - \cos\theta_{ss}\Delta pll_\beta \quad (47)$$

### 0.1.7 Conjunto de equações linearizadas: versão simplificada

$$\Delta pll_\alpha = A_{ss}\cos\theta_{ss}\Delta\theta + \sin\theta_{ss}\Delta A \quad (48)$$

$$\Delta pll_\beta = -A_{ss}\sin\theta_{ss}\Delta\theta + \cos\theta_{ss}\Delta A \quad (49)$$

$$\Delta\varepsilon_\phi = \cos\theta_{ss}\Delta v_\alpha - \cos\theta_{ss}\Delta pll_\alpha - \sin\theta_{ss}\Delta v_\beta + \sin\theta_{ss}\Delta pll_\beta \quad (50)$$

$$\Delta\varepsilon_A = \sin\theta_{ss}\Delta v_\alpha - \sin\theta_{ss}\Delta pll_\alpha + \cos\theta_{ss}\Delta v_\beta - \cos\theta_{ss}\Delta pll_\beta \quad (51)$$

### 0.1.8 Conjunto de equações linearizadas: usando Euler

Antes de começar, temos que lembrar que  $\theta_{ss} = \omega_1 t$ . Então:

$$\Delta pll_\alpha = A_{ss}\left(\frac{e^{j\omega_1 t} + e^{-j\omega_1 t}}{2}\right)\Delta\theta + \left(\frac{e^{j\omega_1 t} - e^{-j\omega_1 t}}{2j}\right)\Delta A \quad (52)$$

$$\Delta pll_\beta = -A_{ss}\left(\frac{e^{j\omega_1 t} - e^{-j\omega_1 t}}{2j}\right)\Delta\theta + \left(\frac{e^{j\omega_1 t} + e^{-j\omega_1 t}}{2}\right)\Delta A \quad (53)$$

$$\begin{aligned}\Delta\varepsilon_\phi = & \left( \frac{e^{j\omega_1 t} + e^{-j\omega_1 t}}{2} \right) \Delta v_\alpha - \left( \frac{e^{j\omega_1 t} + e^{-j\omega_1 t}}{2} \right) \Delta pll_\alpha \\ & - \left( \frac{e^{j\omega_1 t} - e^{-j\omega_1 t}}{2j} \right) \Delta v_\beta + \left( \frac{e^{j\omega_1 t} - e^{-j\omega_1 t}}{2j} \right) \Delta pll_\beta \quad (54)\end{aligned}$$

$$\begin{aligned}\Delta\varepsilon_A = & \left( \frac{e^{j\omega_1 t} - e^{-j\omega_1 t}}{2j} \right) \Delta v_\alpha - \left( \frac{e^{j\omega_1 t} - e^{-j\omega_1 t}}{2j} \right) \Delta pll_\alpha \\ & + \left( \frac{e^{j\omega_1 t} + e^{-j\omega_1 t}}{2} \right) \Delta v_\beta - \left( \frac{e^{j\omega_1 t} + e^{-j\omega_1 t}}{2} \right) \Delta pll_\beta \quad (55)\end{aligned}$$

### 0.1.9 Conjunto de equações linearizadas: Representação usando vetores espaciais

O objetivo desta seção é reescrever as equações da seção anterior com base nas seguintes definições:

$$\Delta\vec{v}_{\alpha\beta}^+ = \Delta v_\alpha + j\Delta v_\beta \quad (56)$$

$$\Delta\vec{v}_{\alpha\beta}^- = \Delta v_\alpha - j\Delta v_\beta \quad (57)$$

$$\Delta\vec{pll}_{\alpha\beta}^+ = \Delta pll_\alpha + j\Delta pll_\beta \quad (58)$$

$$\Delta\vec{pll}_{\alpha\beta}^- = \Delta pll_\alpha - j\Delta pll_\beta \quad (59)$$

Primeiramente, podemos combinar as equações (52) e (53):

$$\begin{aligned}\Delta\vec{pll}_{\alpha\beta} = & A_{ss} \left( \frac{e^{j\omega_1 t} + e^{-j\omega_1 t}}{2} \right) \Delta\theta + \left( \frac{e^{j\omega_1 t} - e^{-j\omega_1 t}}{2j} \right) \Delta A \\ & + j \left[ -A_{ss} \left( \frac{e^{j\omega_1 t} - e^{-j\omega_1 t}}{2j} \right) \Delta\theta + \left( \frac{e^{j\omega_1 t} + e^{-j\omega_1 t}}{2} \right) \Delta A \right] \quad (60)\end{aligned}$$

$$\begin{aligned}\Delta\vec{pll}_{\alpha\beta} = & A_{ss} \left( \frac{e^{j\omega_1 t} + e^{-j\omega_1 t}}{2} \right) \Delta\theta + j \left( \frac{-e^{j\omega_1 t} + e^{-j\omega_1 t}}{2} \right) \Delta A \\ & + \left[ A_{ss} \left( \frac{-e^{j\omega_1 t} + e^{-j\omega_1 t}}{2} \right) \Delta\theta + j \left( \frac{e^{j\omega_1 t} + e^{-j\omega_1 t}}{2} \right) \Delta A \right] \quad (61)\end{aligned}$$

$$\Delta p\vec{ll}_{\alpha\beta} = A_{ss}e^{-j\omega_1 t}\Delta\theta + je^{-j\omega_1 t}\Delta A \quad (62)$$

Agora podemos processar a equação (54):

$$\begin{aligned} \Delta\varepsilon_\phi = & \left(\frac{e^{j\omega_1 t} + e^{-j\omega_1 t}}{2}\right) \Delta v_\alpha - \left(\frac{e^{j\omega_1 t} + e^{-j\omega_1 t}}{2}\right) \Delta pll_\alpha \\ & - \left(\frac{e^{j\omega_1 t} - e^{-j\omega_1 t}}{2j}\right) \Delta v_\beta + \left(\frac{e^{j\omega_1 t} - e^{-j\omega_1 t}}{2j}\right) \Delta pll_\beta \end{aligned} \quad (63)$$

$$\begin{aligned} \Delta\varepsilon_\phi = & \left(\frac{e^{j\omega_1 t} + e^{-j\omega_1 t}}{2}\right) \Delta v_\alpha - \left(\frac{e^{j\omega_1 t} + e^{-j\omega_1 t}}{2}\right) \Delta pll_\alpha \\ & + j \left(\frac{e^{j\omega_1 t} - e^{-j\omega_1 t}}{2}\right) \Delta v_\beta - j \left(\frac{e^{j\omega_1 t} - e^{-j\omega_1 t}}{2}\right) \Delta pll_\beta \end{aligned} \quad (64)$$

$$\begin{aligned} \Delta\varepsilon_\phi = & \frac{1}{2}e^{j\omega_1 t}(\Delta v_\alpha + j\Delta v_\beta) + \frac{1}{2}e^{-j\omega_1 t}(\Delta v_\alpha - j\Delta v_\beta) \\ & - \frac{1}{2}e^{j\omega_1 t}(\Delta pll_\alpha + j\Delta pll_\beta) - \frac{1}{2}e^{-j\omega_1 t}(\Delta pll_\alpha - j\Delta pll_\beta) \end{aligned} \quad (65)$$

$$\Delta\varepsilon_\phi = \frac{1}{2}e^{j\omega_1 t}\Delta\vec{v}_{\alpha\beta}^+ + \frac{1}{2}e^{-j\omega_1 t}\Delta\vec{v}_{\alpha\beta}^- - \frac{1}{2}e^{j\omega_1 t}\Delta p\vec{ll}_{\alpha\beta}^+ - \frac{1}{2}e^{-j\omega_1 t}\Delta p\vec{ll}_{\alpha\beta}^- \quad (66)$$

Por último, temos que processar a equação (55):

$$\begin{aligned} \Delta\varepsilon_A = & \left(\frac{e^{j\omega_1 t} - e^{-j\omega_1 t}}{2j}\right) \Delta v_\alpha - \left(\frac{e^{j\omega_1 t} - e^{-j\omega_1 t}}{2j}\right) \Delta pll_\alpha \\ & + \left(\frac{e^{j\omega_1 t} + e^{-j\omega_1 t}}{2}\right) \Delta v_\beta - \left(\frac{e^{j\omega_1 t} + e^{-j\omega_1 t}}{2}\right) \Delta pll_\beta \end{aligned} \quad (67)$$

$$\begin{aligned} \Delta\varepsilon_A = & -j \left(\frac{e^{j\omega_1 t} - e^{-j\omega_1 t}}{2}\right) \Delta v_\alpha + j \left(\frac{e^{j\omega_1 t} - e^{-j\omega_1 t}}{2}\right) \Delta pll_\alpha \\ & + \left(\frac{e^{j\omega_1 t} + e^{-j\omega_1 t}}{2}\right) \Delta v_\beta - \left(\frac{e^{j\omega_1 t} + e^{-j\omega_1 t}}{2}\right) \Delta pll_\beta \end{aligned} \quad (68)$$



$$\begin{aligned}\Delta\varepsilon_A = & \frac{1}{2}e^{j\omega_1 t}(-j\Delta v_\alpha + \Delta v_\beta) + \frac{1}{2}e^{-j\omega_1 t}(j\Delta v_\alpha + \Delta v_\beta) \\ & + \frac{1}{2}e^{j\omega_1 t}(j\Delta pll_\alpha - \Delta pll_\beta) + \frac{1}{2}e^{-j\omega_1 t}(-j\Delta pll_\alpha - \Delta pll_\beta) \quad (69)\end{aligned}$$

$$\begin{aligned}\Delta\varepsilon_A = & -\frac{j}{2}e^{j\omega_1 t}(\Delta v_\alpha + j\Delta v_\beta) + \frac{j}{2}e^{-j\omega_1 t}(\Delta v_\alpha - j\Delta v_\beta) \\ & + \frac{j}{2}e^{j\omega_1 t}(\Delta pll_\alpha + j\Delta pll_\beta) - \frac{j}{2}e^{-j\omega_1 t}(\Delta pll_\alpha - j\Delta pll_\beta) \quad (70)\end{aligned}$$

$$\Delta\varepsilon_A = -\frac{j}{2}e^{j\omega_1 t}\Delta\vec{v}_{\alpha\beta}^+ + \frac{j}{2}e^{-j\omega_1 t}\Delta\vec{v}_{\alpha\beta}^- + \frac{j}{2}e^{j\omega_1 t}\Delta\vec{pll}_{\alpha\beta}^+ - \frac{j}{2}e^{-j\omega_1 t}\Delta\vec{pll}_{\alpha\beta}^- \quad (71)$$