

# **Course Theory of Complex Systems**

**Clélia de Mulatier**

**April 2 to May 31, 2024**

# Generalities

**Course Staff:** Clélia de Mulatier, lecturer  
Ebo Peerbooms, TA  
Maxim Zewe, TA  
Leander Post, TA

**Canvas:** **Home page:** general information + all the links

**Announcements**

**Modules:** will have all the material

**Assignments:** submit homework

**Quizzes**

**Grades**

**Timing???** **Lectures:** from 11h to 12h40 (Including 10min break)

**Tutorials:** from 13h10 to 14h45 (including 5min break)

**Contact:** If needed, contact us using the **messaging system in Canvas** (see “Inbox” section)  
This is better than by email.

**Students, you!:** Master Computational Science  
Master Physics & Astronomy  
other Masters (Math, AI,?)

**Advice:** **Make mixed working groups. Help each others!**



**campuswire**

[Link](#)

**Code: 4432**

- Ask questions about exercises or lectures;
- Report typos/mistakes in the questions of the exercises;
- Ask questions related to the organization of the course.

Don't hesitate to try replying to the questions! (when you can)

**Watch out!**  
Course overlap, **Exam overlap**

# **LO: Introduction**

**Theory of Complex Systems:  
What is it?**

# Theory of Complex Systems: What is it?

## Introductory Lecture

**Goals:** Examples of Complex Systems?  
What are Complex Systems?  
What is “the” Theory of Complex Systems”?

**Expectations:** Participate in the discussions

**Additional material:** [Outreach talk “What is a complex system?”, by Karoline Wiesner & James Ladyman \(here\)](#)  
[Website and booklet “Complexity explained” \(here\)](#)  
[Book: Chapter 1 of Introduction to the Theory of Complex Systems, by S. Thurner, R. Hanel, and P. Klimek \(2018\) \(link\)](#)

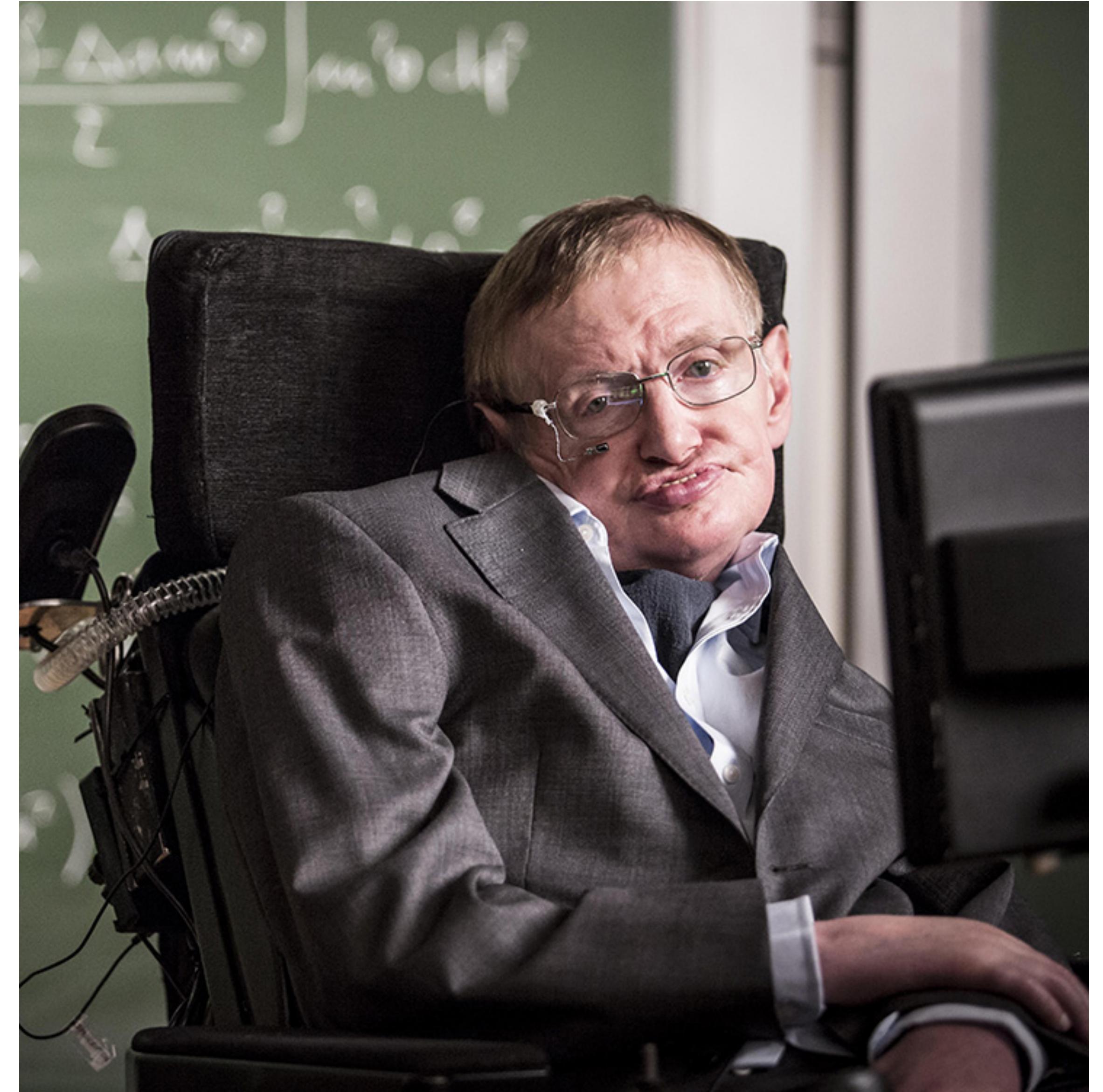
**Before we begin:** Complexity Science is a very young field compared to more traditional fields in physics/math. There are not a lot of general textbooks on Complex Systems. Currently available textbooks often approach Complex Systems from different fields and different points of view.

**“I think the next century will  
be the century of complexity.”**

— **Stephen Hawking**

*Unified Theory Is Getting Closer, Hawking Predicts,*  
Interview with San Jose Mercury News, January 23, 2000.

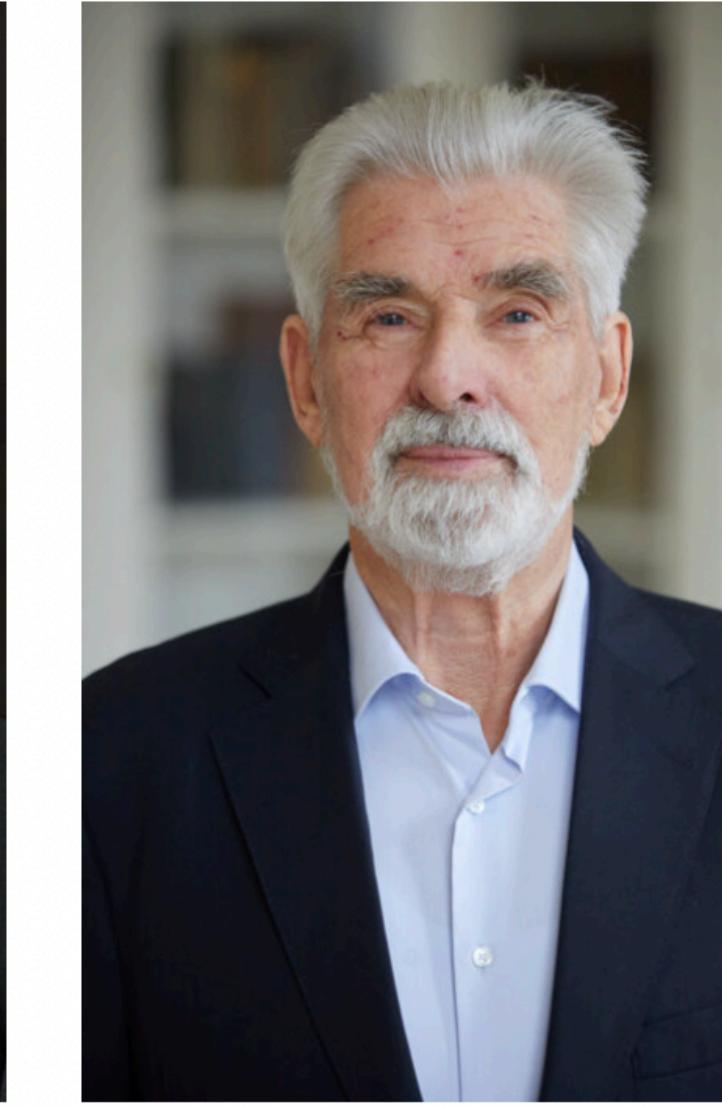
Pointing towards the increasing importance of a set of scientific ideas that emerged in the 1980s-90s that allow us to find order within highly complex, interconnected systems (e.g. weather, traffic movement, stock market, ecosystem).



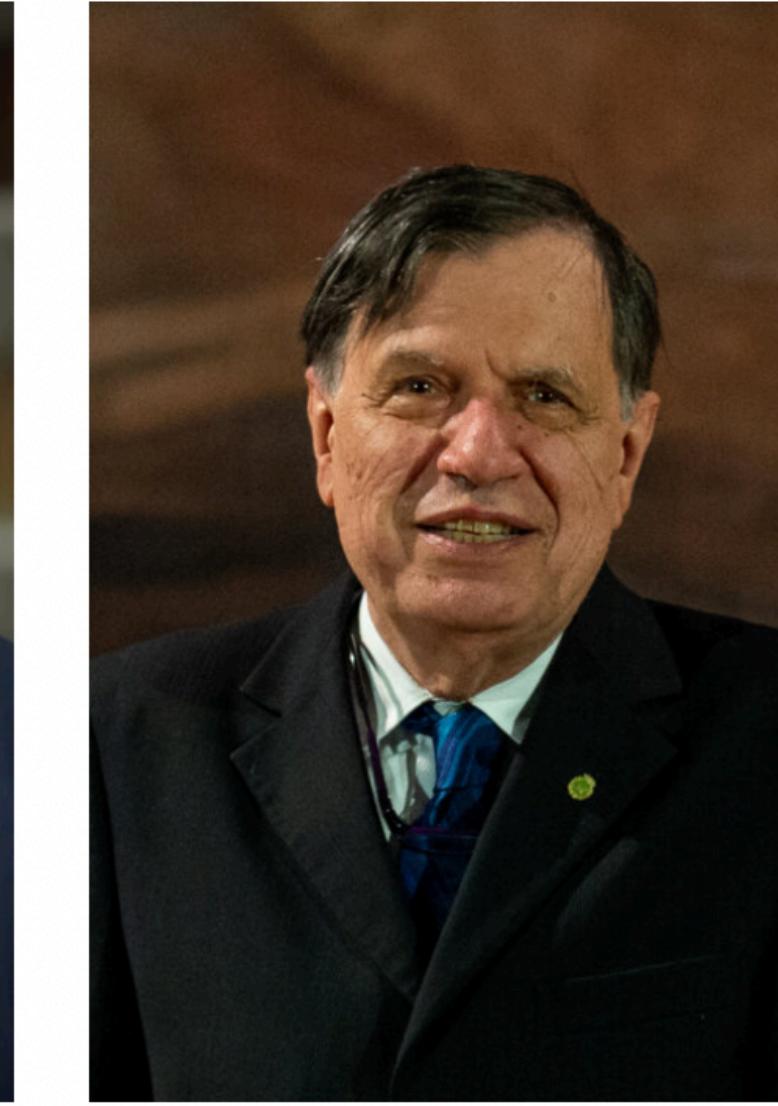
# The Nobel Prize in Physics 2021



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Risdon Photography  
**Syukuro Manabe**  
Prize share: 1/4



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Bernhard Ludewig  
**Klaus Hasselmann**  
Prize share: 1/4



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Laura Sbarbri  
**Giorgio Parisi**  
Prize share: 1/2

“ for groundbreaking contributions to our understanding of complex physical systems ”

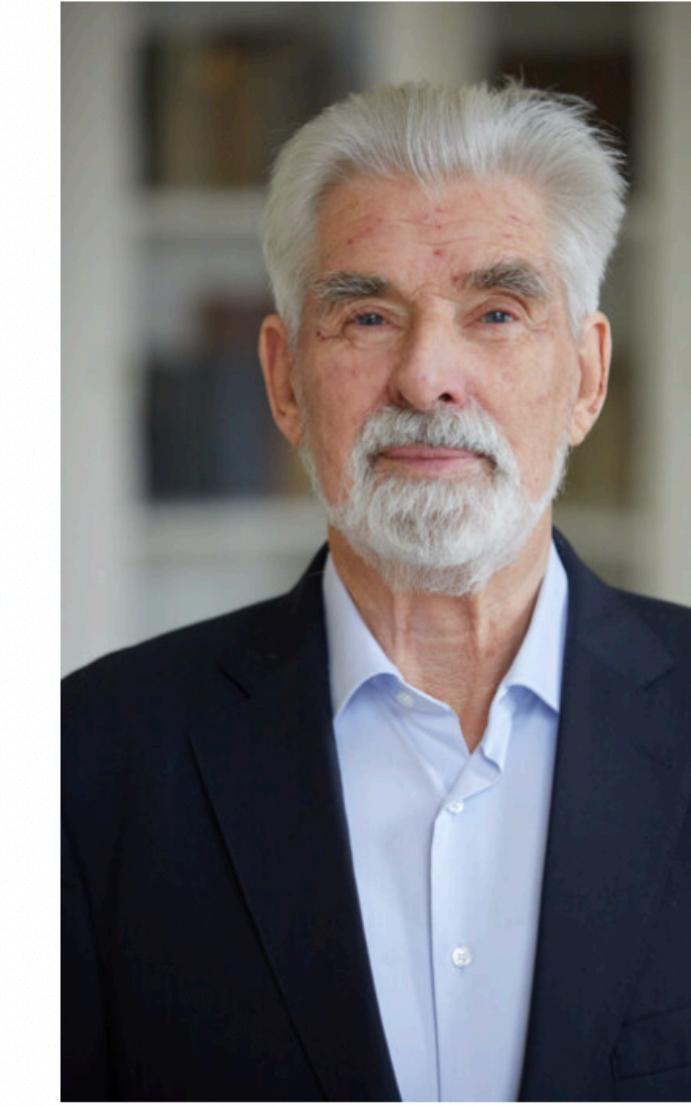
# The Nobel Prize in Physics 2021



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**Giorgio Parisi**

Prize share: 1/2

One half

*“for the physical modelling of Earth’s climate, quantifying variability and reliably predicting global warming”*

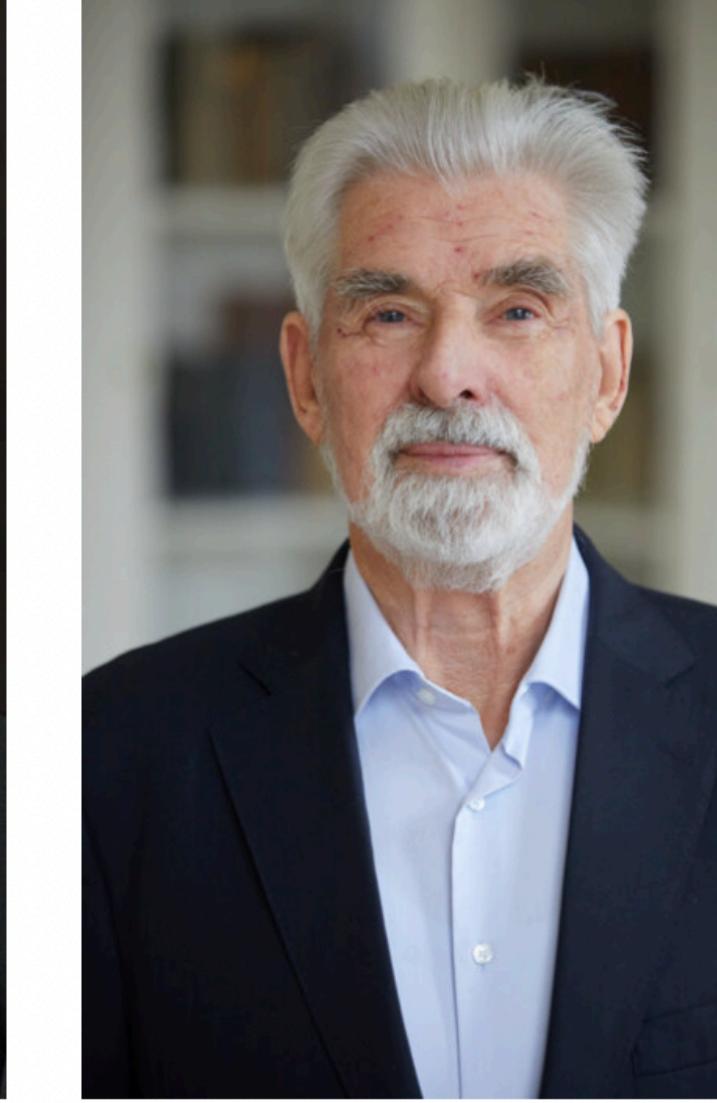
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**Syukuro Manabe**

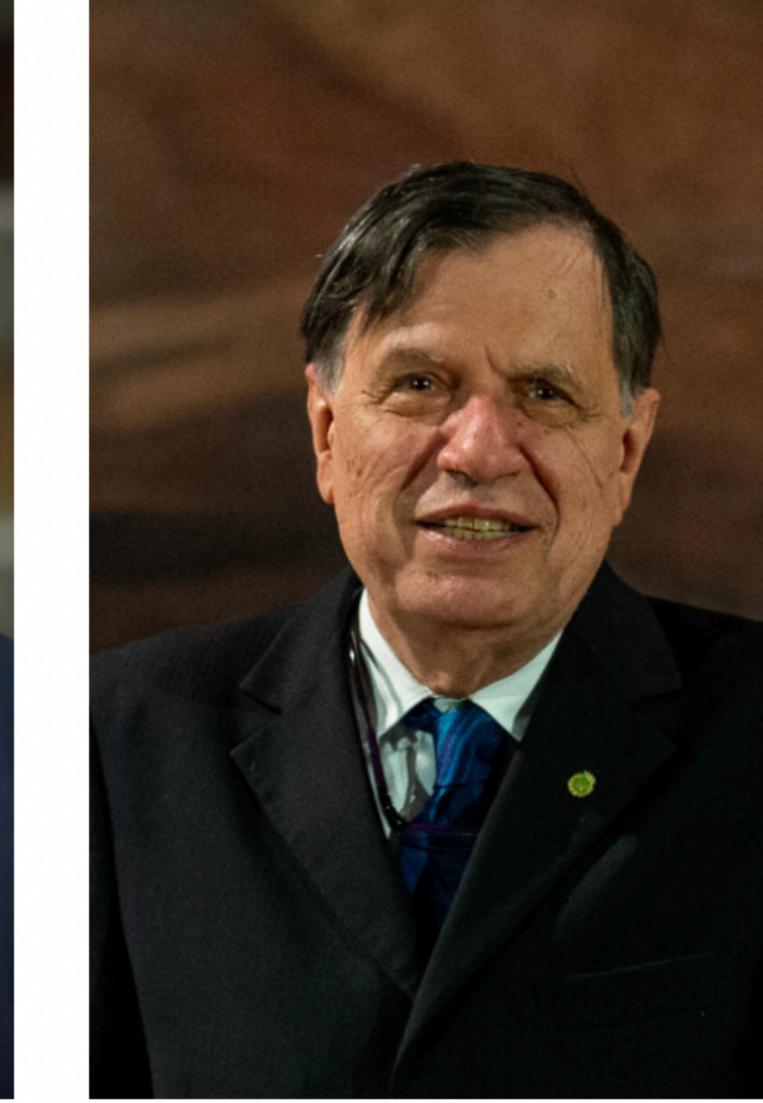
Prize share: 1/4



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**Klaus Hasselmann**

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Laura Sbarbri

**Giorgio Parisi**

Prize share: 1/2



and the other half

*“for the discovery of the interplay of disorder and fluctuations  
in physical systems from atomic to planetary scales”*

# Some of Giorgio Parisi's contributions

Spin Glasses

Optimisation algorithms

Turbulence

Kardar-Parisi-Zhang equation

Stochastic Resonance —> climate change

Quantum chromodynamics

Random Matrix Theory

Flock, emergent behaviour in active matter

Hard sphere glasses and jamming



See: Lecture on the Nobel Prize organised by the Physics & Astronomy  
Division of the KNAW By Said Rodriguez. —> in Canvas

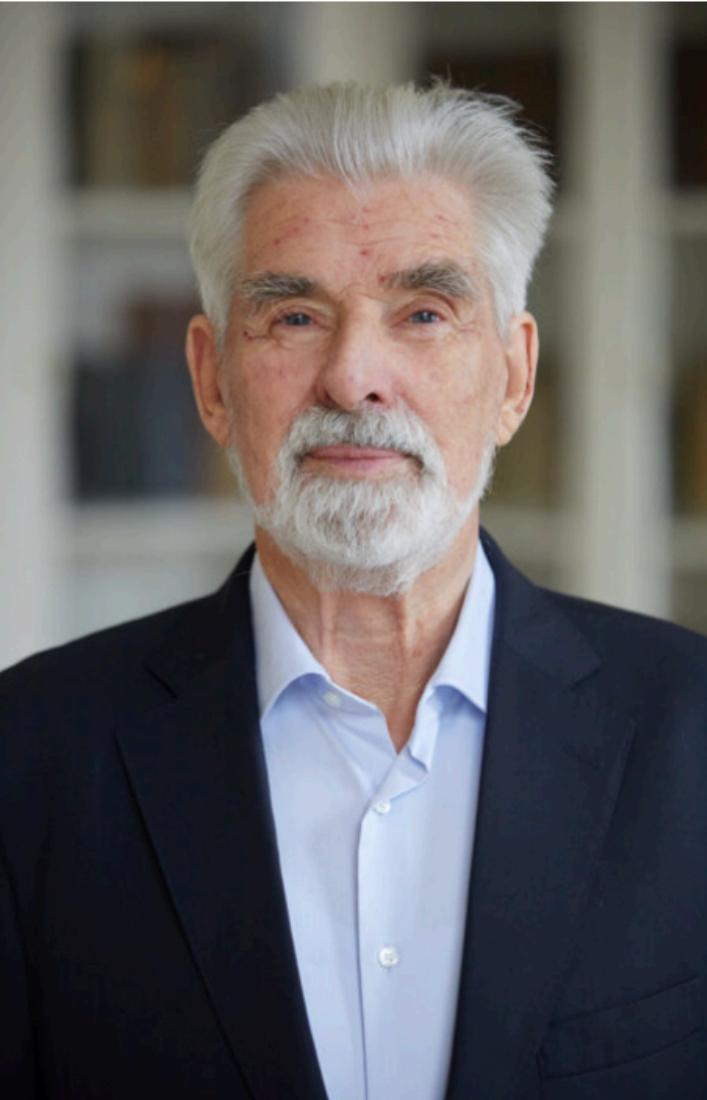
# The Nobel Prize in Physics 2021



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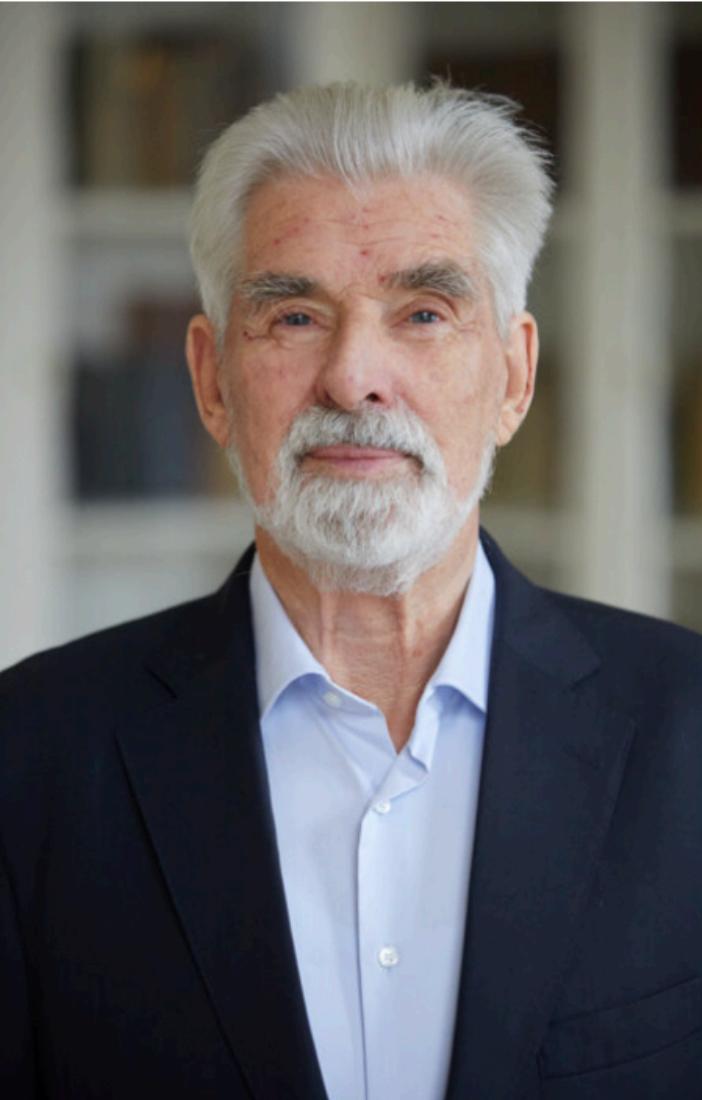
The prize is shared.  
What is the link between them?  
Answer [here](#) at 25:10 min

# The Nobel Prize in Physics 2021



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**The prize is shared.  
What is the link between them?**

**Common theme:**

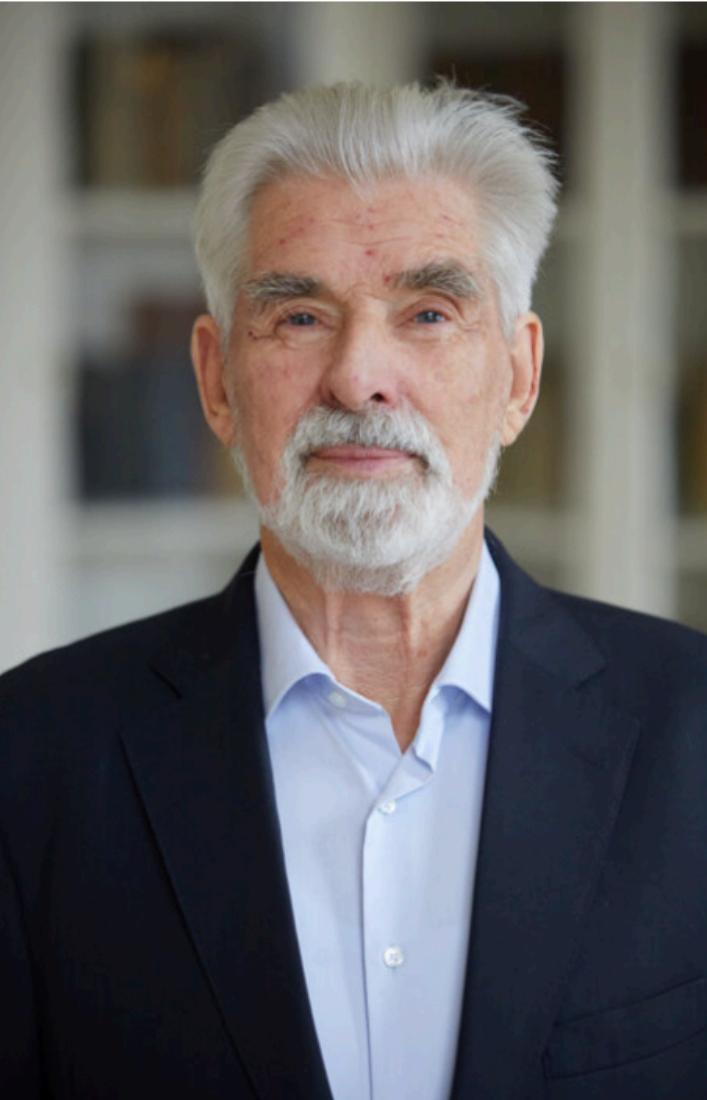
**“ How disorder and fluctuations, together,  
can give rise to something we can understand  
and something we can predict. ”**

# The Nobel Prize in Physics 2021



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**Giorgio Parisi**

Prize share: 1/2

**The prize is shared.**

**What is the link between them?**

**Common theme:**

“ How disorder and fluctuations, together, can give rise to something we can understand and something we can predict. ”

“Building from the disorder and fluctuations of a CS at their microscopic constituents and predicting the macroscopic behaviour”

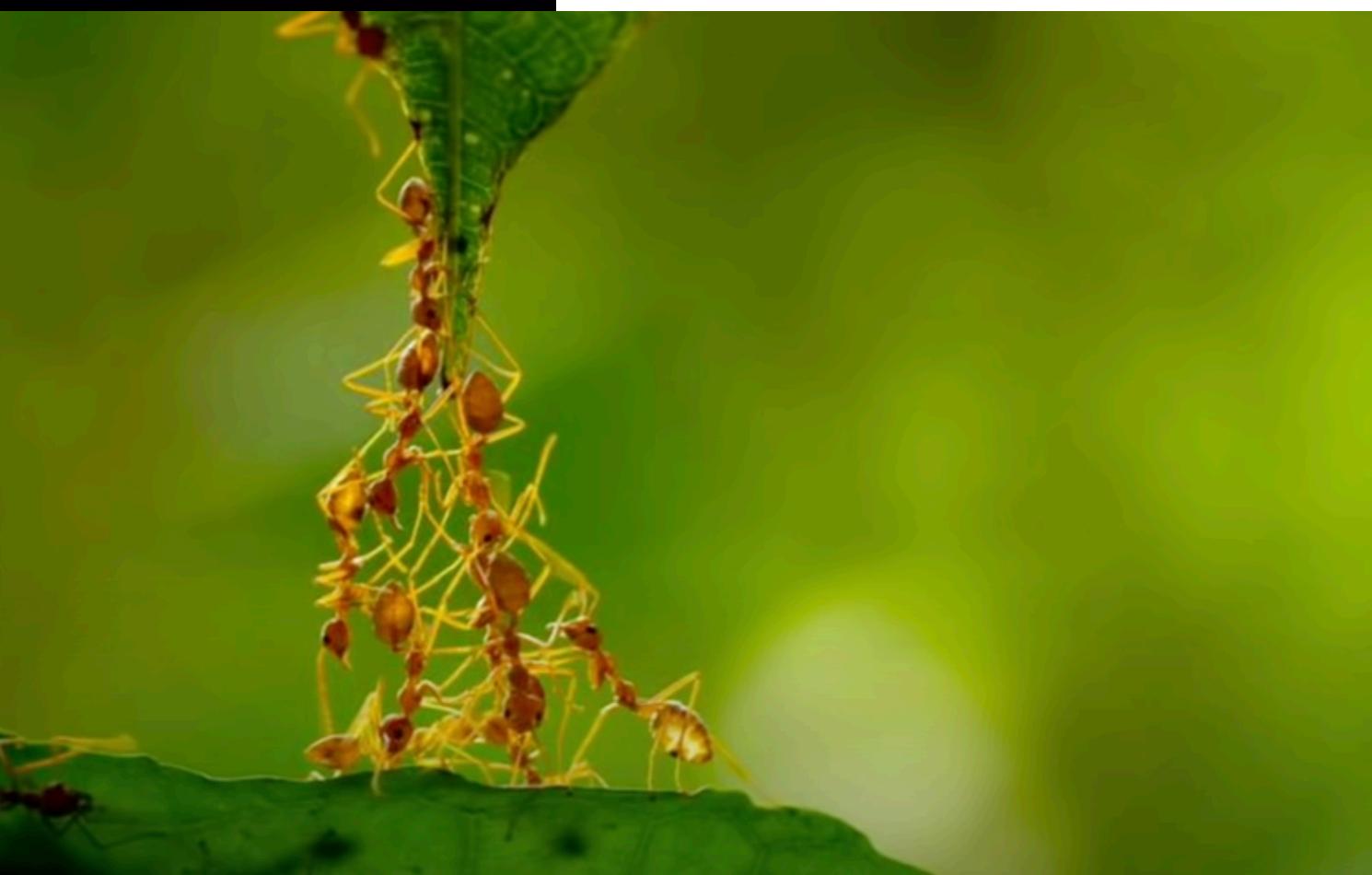
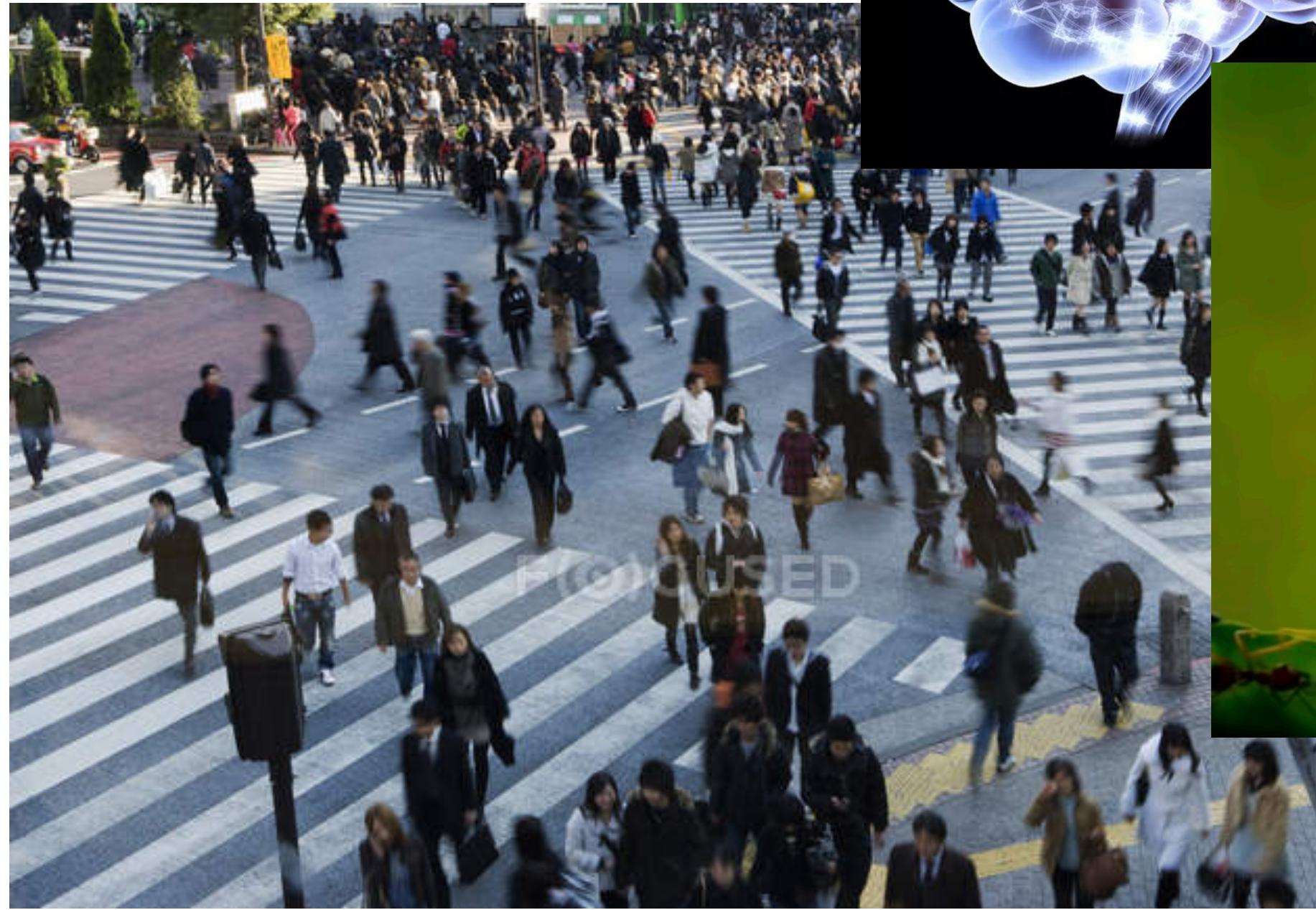
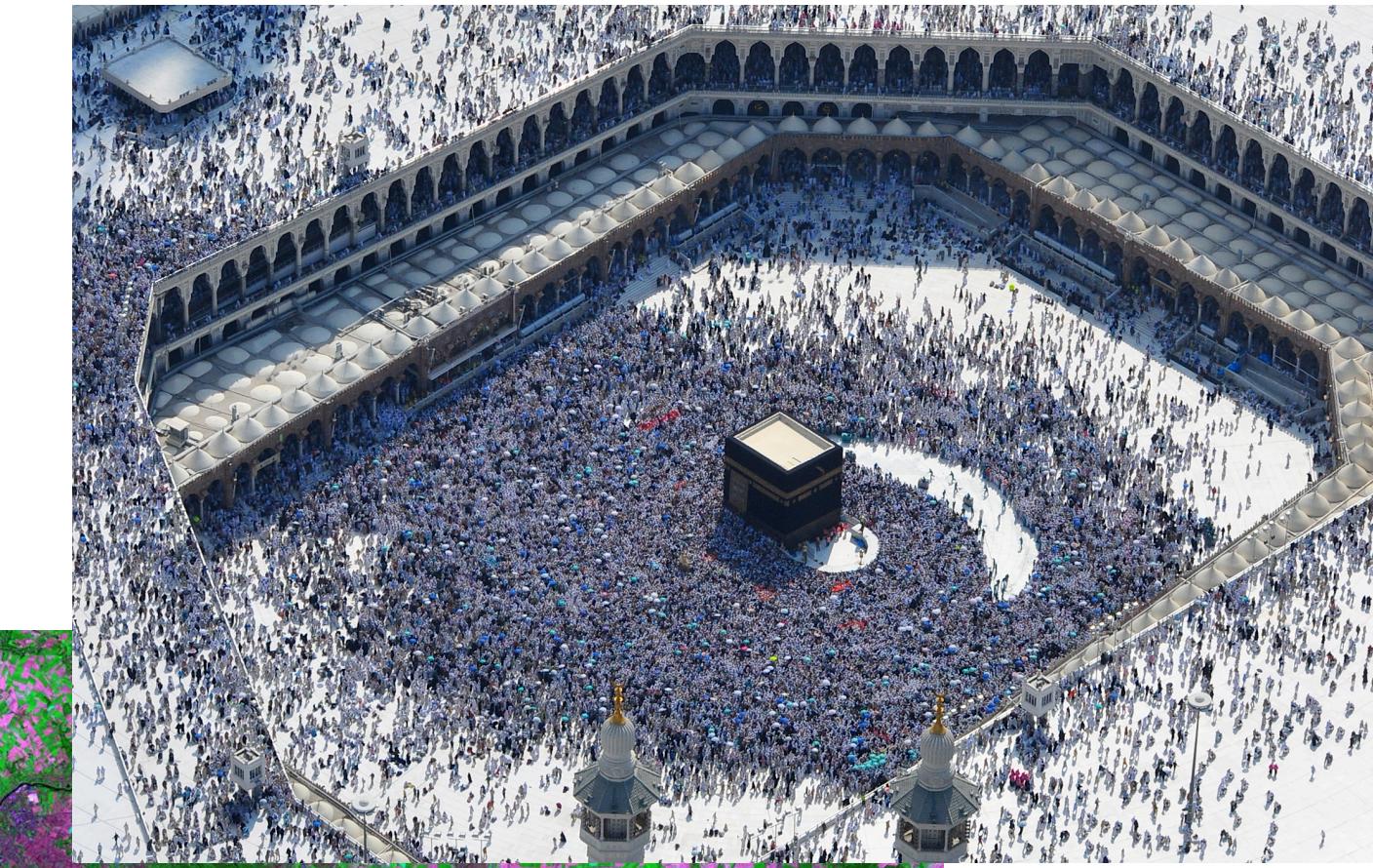
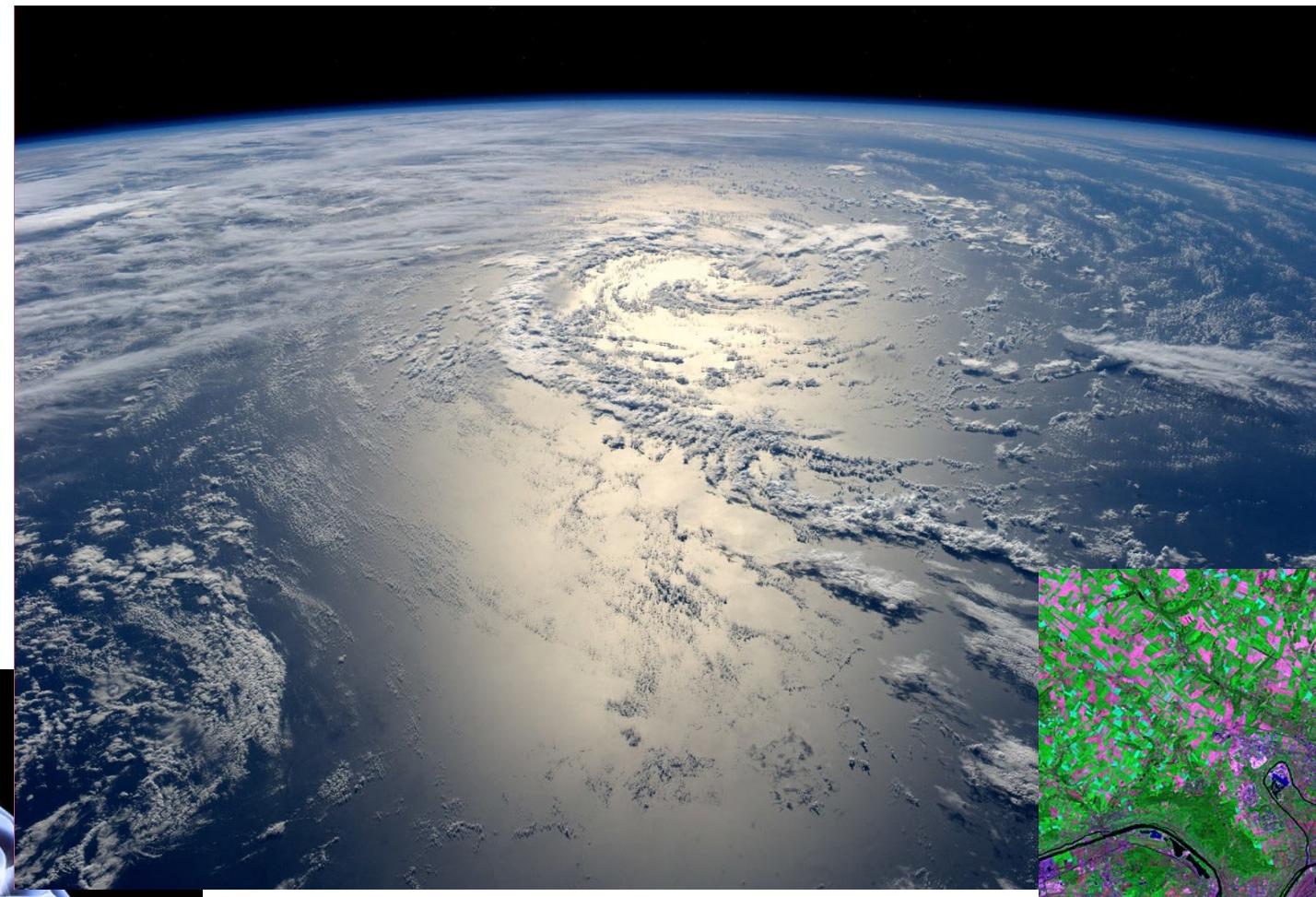
Uncovering the hidden structure.

# What is a “Complex System”?

# Examples of Complex Systems

<https://app.wooclap.com/CTAKGP>

# Examples of CS



# Examples of Complex Systems

## Examples

Earth climate

Brain

Immune system

Health

Psychological phenomena

Systems displaying collective behavior  
(Crowd movement, bird flock, school of fish,...)

traffic problems

Urban organisation

**Every** Living system (most...)

Social system

Economic system

Financial system

Ecosystem

**is a Complex System**

# What features characterise a complex system?

<https://app.wooclap.com/CTAKGP>

What are CS?

# What are characteristics of complex systems?

# What are characteristics of complex systems?

Interactions  
with environment

**Many  
interacting components**

Networks of interactions

**Organisation**

Properties of the system  
change with the size of the system

Interactions  
that changes in time

Adaptation  
**Dynamics**

Collective behaviour

Transition order/disorder

Self-organisation

**Emergent behaviour**

Scaling  
Organisation at large scale  
power law behaviour

Tipping points

Pattern formation

These are many aspects common to Complex Systems  
But...

What are Complex Systems?

# What are Complex Systems?

Loosely

Complex System

- composed of **many components interacting** with each other and their environment in multiple ways.
- that display **behaviours** that **cannot be understood by** the reductive **study of each of its individual component**.

The system is not just the “sum” of the behaviour of each part.

Ex.

Impossible to predict where traffic jams will occur by only studying the behaviour of individual drivers

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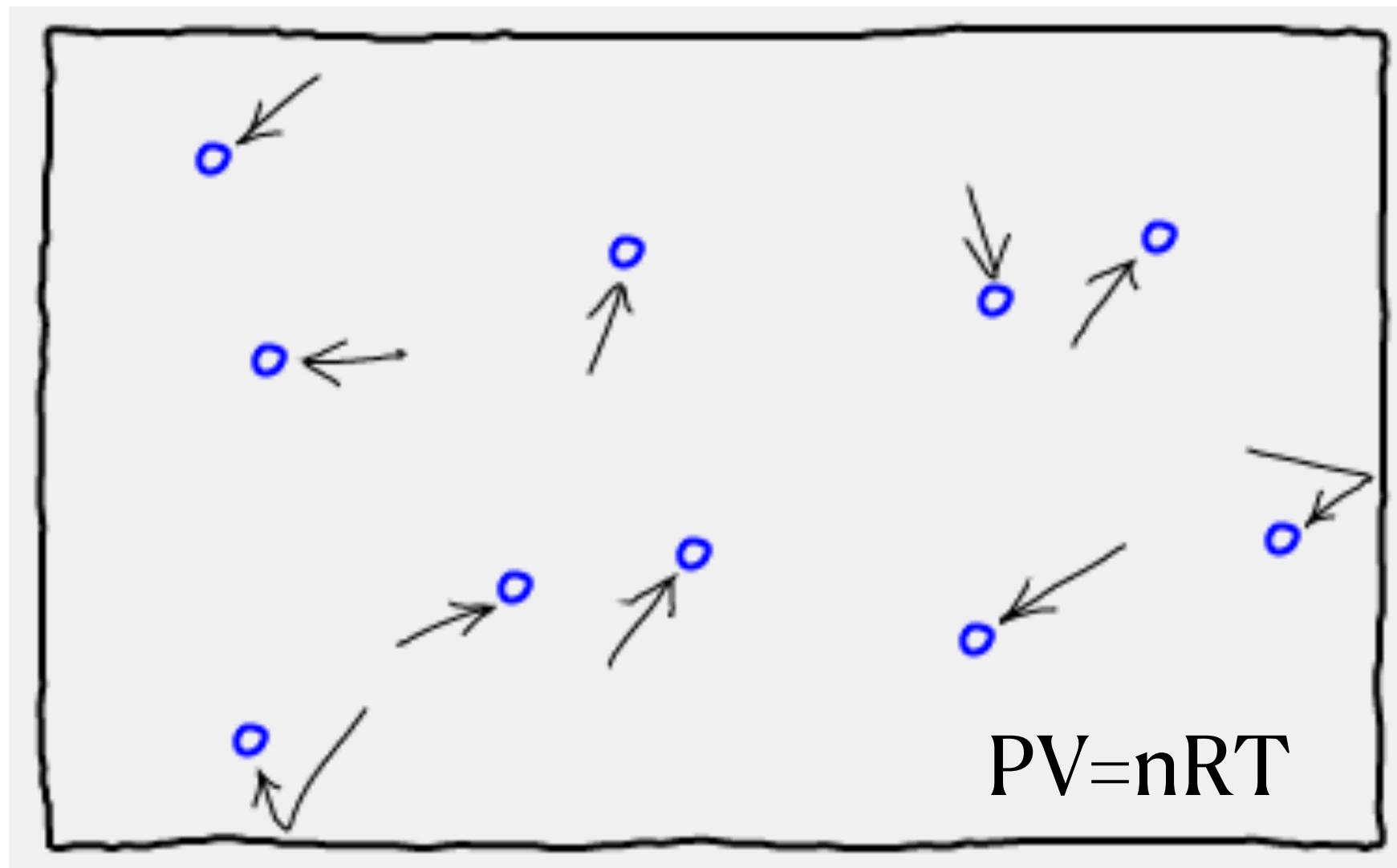
Impossible to predict where traffic jams will occur by only studying the behaviour of individual drivers

**Stephen Thurner:** “Complex systems are co-evolving multilayer networks.” See talk [here](#).

# What is not complex?

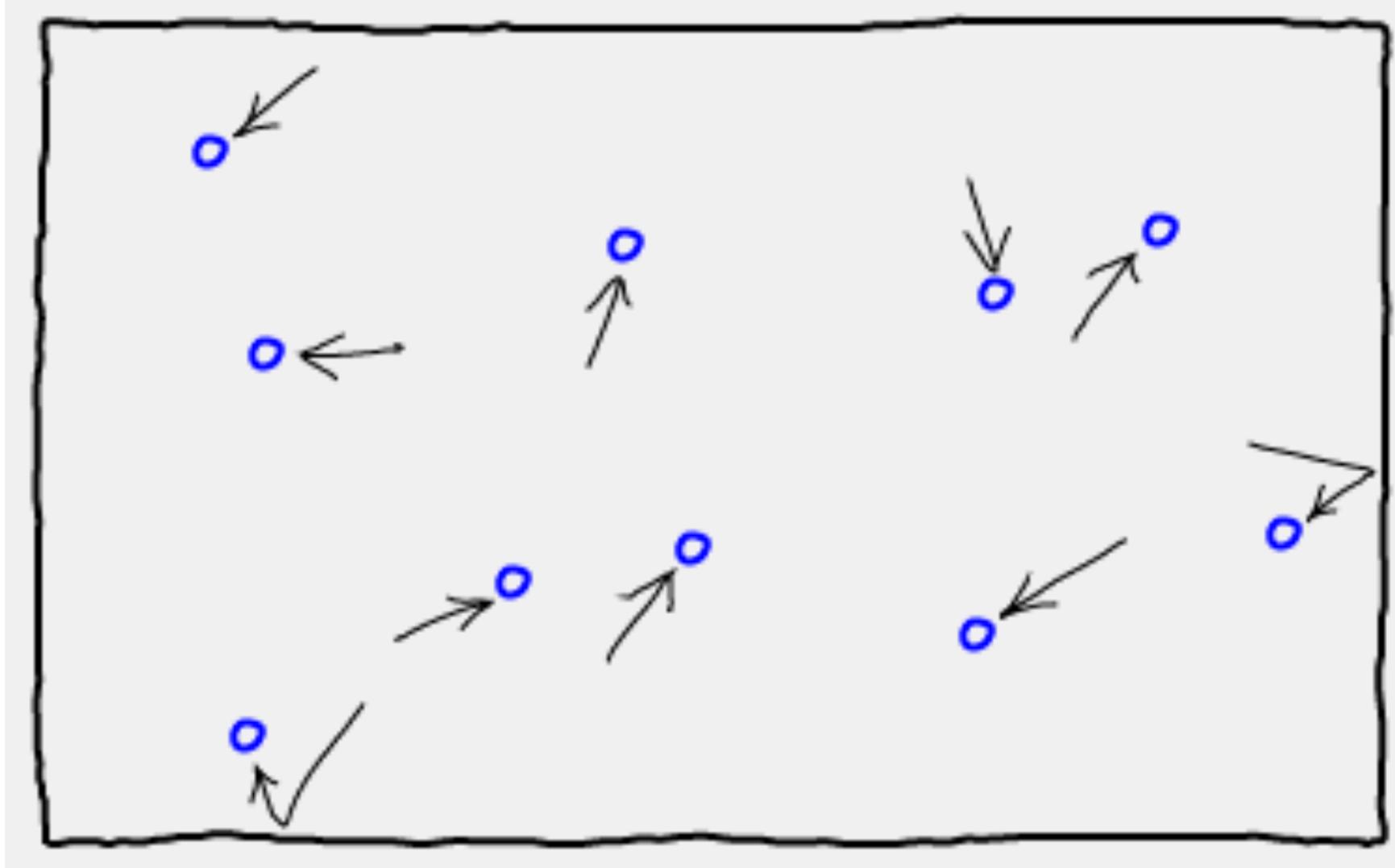
A complex system is **not just a complicated system**

A complicated system: has many interacting components,  
but we can predict its behavior by breaking it down into  
the behavior of each part



A **purely random system** is not a complex system

# What is not complex?



A **purely random** is not complex

**Warren Weaver** ([link video](#)) (American mathematician 1948)

- “disorganised complexity” → Randomness
- “organised complexity” → Regularity, pattern within the disorder => “Complexity”

A good measure of complexity captures the hidden organisation/structure behind the apparent disorder.

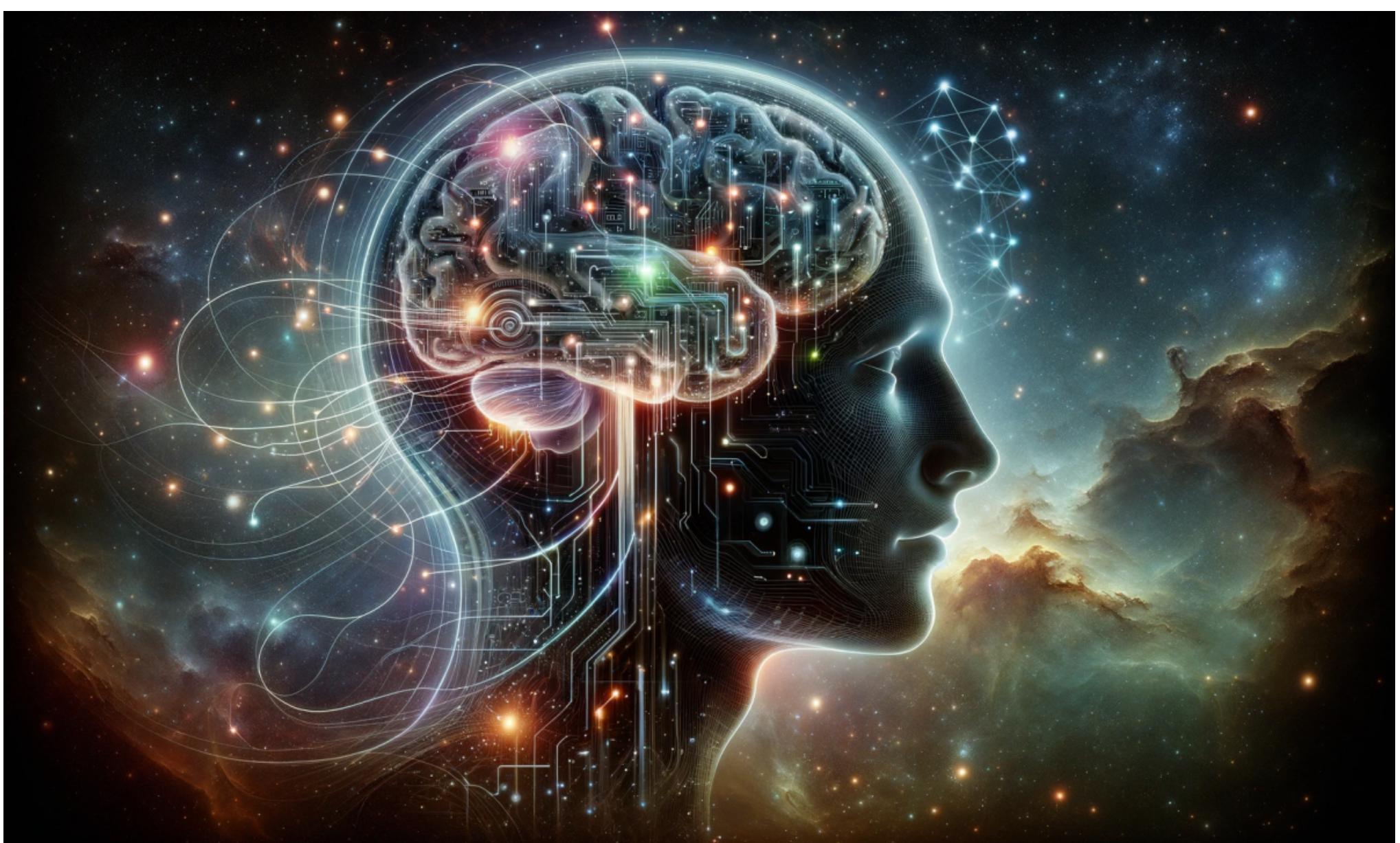
# Emergence in Complex Systems

## Emergence in Complex Systems

—> macroscopic behavior or property of a system that arises as a result of complex interactions between its many constituents, and that cannot be predicted from the behavior of the constituents alone.



Emergence of collective behaviors in animals, such as flocking



Emergence of cognitive phenomena  
(perception/memory/decision making)  
in the brain or in artificial neural networks

#ComplexityExplained

[Project](#)   [Explore Concepts](#)   [References](#)   [Booklet](#)   [Credits](#)   [Contact](#)

# What is Complexity Science?

*"I think the next [21st] century will be the century of complexity"* – Stephen Hawking

Complexity science, also called complex systems science, studies how a large collection of components – locally interacting with each other at small scales – can spontaneously self-organize to exhibit non-trivial global structures and behaviors at larger scales, often without external intervention, central authorities or leaders. The properties of the collection may not be understood or predicted from the full knowledge of its constituents alone. Such a collection is called a complex system and it requires new mathematical frameworks and scientific methodologies for its investigation.

# A Theory of Complex Systems?

“I expect we will find a complete unified theory sometime this century.”

Stephen Hawking

# Theory of complex systems?

**A Complex System to study: What do we want to know?**

# Theory of complex systems?

**A Complex System to study: What do we want to know?**

**Examples:**

**Link micro rules to macro behavior; Understand emergent behaviors**

Understand how interactions at microscopic level create macroscopic behaviour

**Predict macroscopic behaviour; Control macroscopic behaviour**

Will a traffic jam occur? Can we stop the spread of a disease?

**Study how stable is the system**

**Observations of the system: would like to uncover the hidden patterns, the structure**

# Reminder: What are characteristics of complex systems?

Interactions  
with environment

**Many  
interacting components**

Networks of interactions

**Organisation**

Properties of the system  
change with the size of the system

Interactions  
that changes in time

Individuals

**Collective**

Transition order/disorder

Organisation at large scale

Adaptation

**Dynamics**

Evolution/Co-evolution

Self-organisation

Tipping points

**Emergent behaviour**

Pattern formation

# What do you think would constitute a theory of complex systems?

<https://app.wooclap.com/CTAKGP>

What do you think would be the **key ingredients** for  
a **theory of complex systems**?

# Complex systems are studied from many different perspectives...

Dynamical Systems  
and Chaos

Information theory

Data analysis methods

Network Theory

Stochastic processes

Evolutionary processes

Critical Phenomena

Game Theory

Interdisciplinary research

# Complex systems are studied from many different perspectives...

**Dynamical Systems  
and Chaos**

**Information theory**

**Data analysis methods**

**Network Theory**

**Stochastic processes**

**Evolutionary processes**

**Critical Phenomena**

**Game Theory**

**Interdisciplinary research**

**There is not a single/unified theory (yet?)....**

# Complex systems are studied from many different perspectives...

**Dynamical Systems  
and Chaos**

**Information theory**

**Data analysis methods**

**Network Theory**

**Stochastic processes**

**Evolutionary processes**

**Critical Phenomena**

**Game Theory**

**Interdisciplinary research**

**Model-based approaches**

# Complex systems are studied from many different perspectives...

**Dynamical Systems  
and Chaos**

**Information theory**

**Stochastic processes**

**Critical Phenomena**

**Evolutionary processes**

**Game Theory**

**Data-driven approaches**

**Data analysis methods**

**Network Theory**

**Interdisciplinary research**

# Main focus of the course on Phase Transitions and Critical phenomena

**Dynamical Systems  
and Chaos**

**Information theory**

**Data analysis methods**

**Stochastic processes**

**Network Theory**

Phase transitions

Scaling, fractal structures

**Evolutionary processes**

**Critical Phenomena**

**Game Theory**

**Interdisciplinary research**

Emergence

Self-Organised Criticality

Pattern formation

# Main focus of the course on Phase Transitions and Critical phenomena

**Dynamical Systems  
and Chaos**

**Information theory**

**Data analysis methods**

Phase transitions

Scaling, fractal structures

**Network Theory**

**Stochastic processes**

**Evolutionary processes**

**Critical Phenomena**

Emergence

Self-Organised Criticality

Pattern formation

**Game Theory**

**Interdisciplinary research**

# What can you study at UvA if you are interested in complex systems?

**Good bases in probability theory and stochastic processes**  
**Out-of-equilibrium statistical physics**

**Interdisciplinary research**

**Dynamical systems and Chaos**

**Information Theory**

**Critical phenomena**

—>> **More information to be posted on Canvas  
towards the end of the course**

**Data-driven approaches to complex systems**

# Course Plan

**Introduction to the Theory of Complex Systems** with a main focus on **Critical Phenomena**

**Lectures General plan:**

- o. Introduction to Critical Phenomena
- 1+2. From micro to macro: Equilibrium descriptions of complex systems
3. Mean-field approximation
4. From micro to macro: complex systems out-of-equilibrium
5. Emergence; Criticality, Scale-invariance and Renormalization
6. Self-organised criticality; 7. Examples of collective behaviors

**Simple of models of Complex Systems:** Ising model, TASEP (traffic jam), percolation, SIS model (epidemic model), voter model, Vicsek model (bird flocks), Kuramoto model, ...

**The goal of the lecture** is that you learn general ideas and concepts.

# Course Plan

		Modules		Lecture 1	Lecture 2	Tutorial 1 and 2
April 1 - 5	Week 1	Module 0	Introduction	Introduction to Complex Systems	Introduction to Critical Phenomena	Poisson processes; Markov Processes
April 8 - 12	Week 2	Module 1	From Micro to Macro	From Micro- to Macroscopic description of complex systems	The Ising Model	Micro to Macro: from out-of-equilibrium to equilibrium description; Application to Metropolis algorithm
April 15 - 19	Week 3	Module 2	Example of Equilibrium Critical phenomena	Percolation problems	Properties of Critical Phenomena +Guest Lecture ?	Exo on Ising and Percolation
April 22 - 26	Week 4	Module 3	Mean Field Theory	Mean-field theory	Landau Theory (1h) + Guest lecture (1h)	Exo Mean-field
Break		Homework due before the break				
May 6 - 10	Week 6	Module 4	From Micro to Macro Complex Systems Out-of-Equilibrium	Epidemic Spreading / Wout Merbis?	Voters / Guest lecture Fernando Santos	Exo Epidemic models and voter models
May 13 - 17	Week 5	Module 5	Scale invariance, Criticality, Renormalization; Emergent Phenomena	Scale invariance, Criticality, Renormalization;	Discussion about Emergent Phenomena (1h) + Guest lecture (1h) Vítor Vasconcelos	Exo random walks, Fokker-Planck equation; renormalisation
May 20 - 24	Week 7	Module 6	Self-Organised Criticality + Collective Behavior	SOC	Collective Behavior	Exo SOC + collective behaviors
May 27 - 31	Week 8	Exam Week				

Tentative course plan, may change a little bit during the course.

# Course Plan

**The goal of the tutorials:** to learn concretely how to use specific tools/techniques to study complex systems.

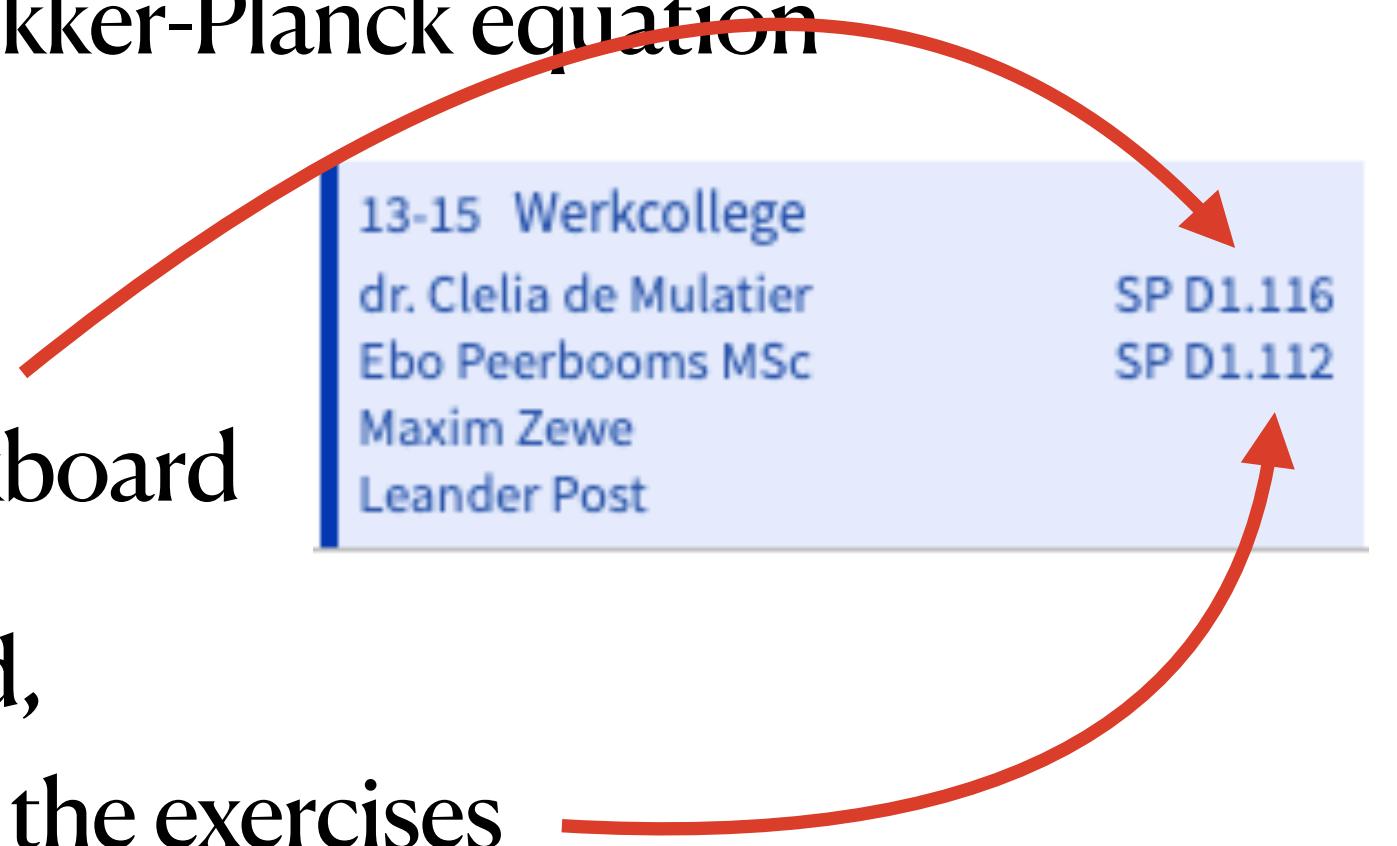
0. Poisson processes; Markov processes;
1. Markov Chain; Master equation; detailed balance; macroscopic observables
2. Complex systems at equilibrium: ex. Ising model and Percolation problem;  
Macroscopic observables, Partition function, free energy
3. Apply mean-field approximation, compute critical exponents
4. Complex systems out-of-equilibrium: ex. Epidemic models, voter model  
Master equation, Study stability of a stationary solution
5. Random walks, mean, standard deviation, sum of iid, continuous limit, Fokker-Planck equation
6. Final Examples

**Two Tutorial groups:**

1. More guided group; with corrections at the blackboard
2. Less guided group, you can go at your own speed,  
TAs will go from group to group to help you with the exercises  
(not all the exercises will be corrected at the blackboard)

13-15 Werkcollege  
dr. Clelia de Matalier  
Ebo Peerbooms MSc  
Maxim Zewe  
Leander Post

SP D1.116  
SP D1.112



# Course Plan

**Guest lectures by:** Fernando P. Santos, Vítor Vasconcelos, Greg Stephens (to be announced)

**Evaluation:** 3 in-class **quizzes (15%)** : every two weeks → count only the two best grades

1 **homework assignment (25%)** : due end of week 4 (just before the break)

1 **optional homework**

In class **Final written Exam (60%)** : week 8, on **Thursday 30 May 13h-16h** → overlap?

# Summary organization

**Lectures:** Introduce general ideas, new concepts, new tools

**Tutorials:** More technicals, understand how to use specific tools, practice

**Homework:** More practice!

**Final Exam:** Exercises will be inspired from the exercises seen during the lectures, tutorials and in the homework assignments.

**Advice for the course:** **work in groups 2-3 people max. with mixed background** (theory, computational, complex systems, ...)

**Teaching Assistants:** Maxim Zewe, Leander Post, Ebo Peerbooms

**Contact us** if you have any questions:



Use **campuswire** (**mostly!**) **Link** **Code: 4432**

Use messaging system on **Canvas** (better than by email)

**Questions?**

# References in connection with this lecture

## [Webpage of the 2021 Nobel Prize in Physics \(link\)](#)

To get a better idea of the scientific contributions that are rewarded

See the video of the prize announcement

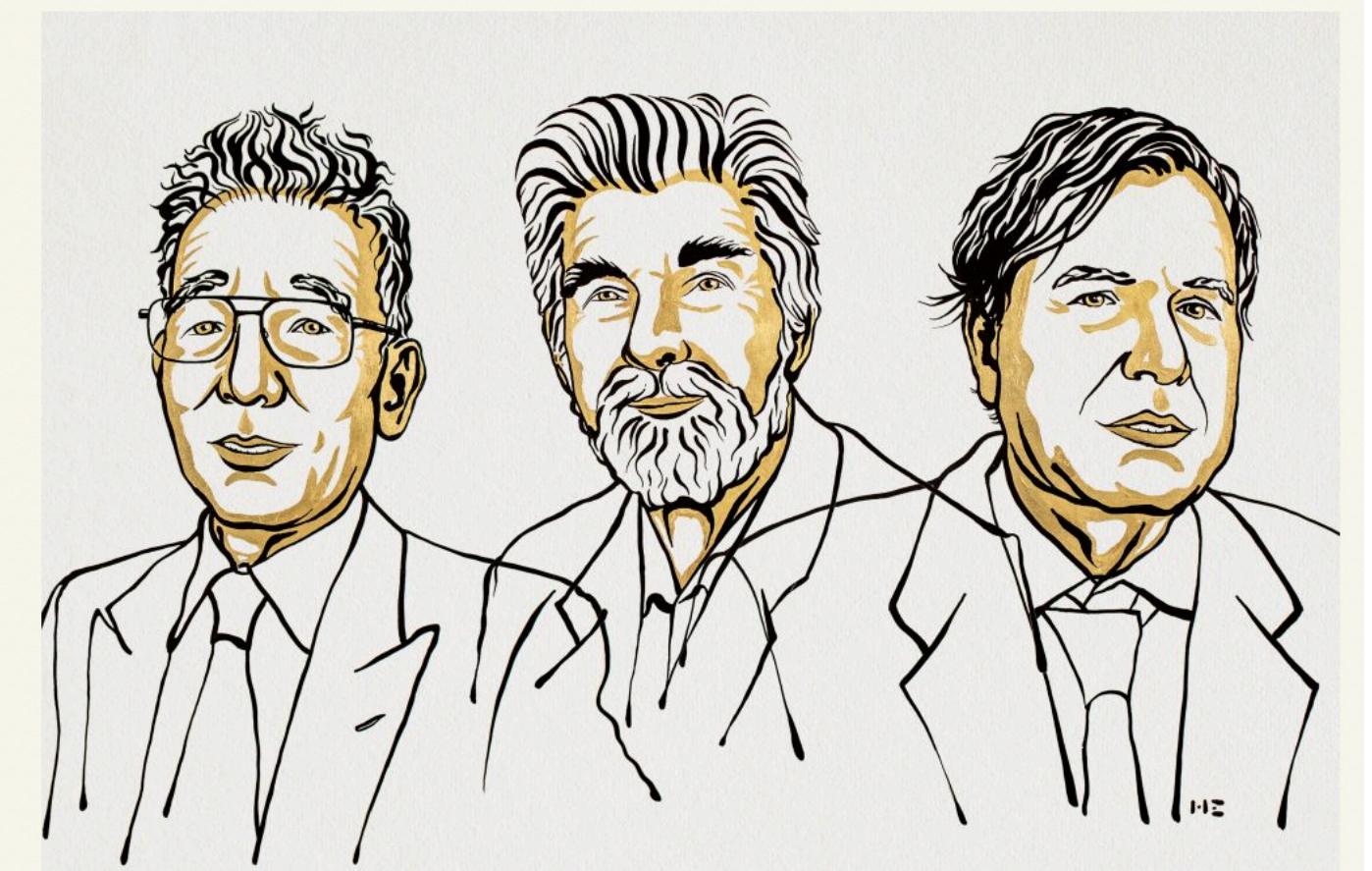
See the Lectures of each laureates

**Lecture on the Nobel Prize** organized by  
the Physics & Astronomy Division of the KNAW

**By Said Rodriguez:** → in Canvas

## The 2021 Nobel Prize laureates in Physics

This year's Nobel Prize in Physics is awarded with one half jointly to [Syukuro Manabe](#), [Klaus Hasselmann](#) "for the physical modelling of Earth's climate, quantifying variability and reliably predicting global warming" and the other half to [Giorgio Parisi](#) "for the discovery of the interplay of disorder and fluctuations in physical systems from atomic to planetary scales".



Ill. Niklas Elmehed © Nobel Prize Outreach.

**DIEP seminar** on the Nobel Prize 2021: [link](#)

[Outreach talk](#) “What is a complex system?”, by Karoline Wiesner & James Ladyman ([here](#))

[Website and booklet](#) “[Complexity explained](#)” ([here](#))

Book: Chapter 1 of [Introduction to the Theory of Complex Systems](#), by S. Thurner, R. Hanel, and P. Klimek (2018) ([link](#))