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# Theory Homework

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- Task 1
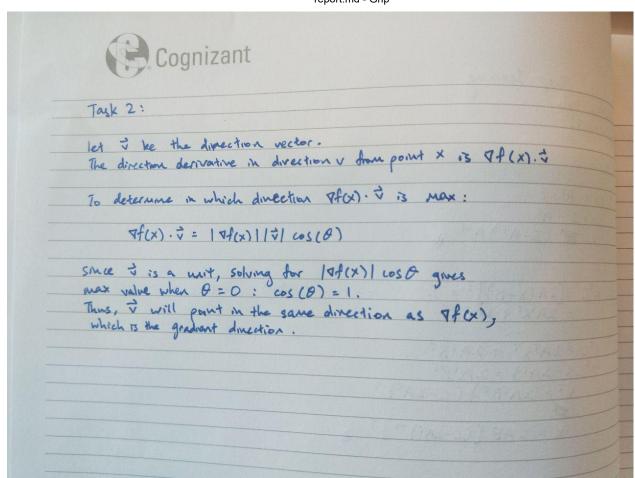
Cognizant

Deep Learning.

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Task 1: ① $XA + A^T = I$

$\qquad XA = I - A^T$

$\qquad X = (I - A^T) A^{-1}$  ※.

② $X^T C = [2A(X+B)]^T = I \qquad ---- \quad C - 2A^T$ invertible.

$\qquad\qquad = 2A^T X^T + 2A^T B^T$

$\qquad X^T C - 2A^T X^T = 2A^T B^T$

$\qquad X^T (C - 2A^T) = 2A^T B^T$

$\qquad\quad X^T = (2A^T B^T)(C - 2A^T)^{-1}$

$\qquad\quad X = 2AB [(C - 2A^T)^{-1}]^T$  ※.

③ $\quad (Ax - y)^T A = 0$.

$\qquad A^T x^T A - y^T A = 0$

$\qquad A^T x^T A = y^T A$

$\qquad (Ax)^T = y^T$  $\qquad --\cdot\rightarrow$ in order to solve this; A has to be invertible. ※.

$\qquad Ax = y$

$\qquad x = A^{-1} y$  ※.

④ $\quad (Ax - y)^T A + x^T B = 0$.

$\qquad (A^T x^T A - y^T A) = - x^T B$.

$\qquad A^T x^T A = y^T A - x^T B$.

$\qquad A^T x^T A + x^T B = y^T A$.

$\qquad x^T (A^T A + B) = y^T A$

$\qquad x^T = y^T A (A^T A + B)^{-1}$.

$\qquad x = y A^T [(A^T A + B)^{-1}]^T$  ※.

- Task 2

Cognizant

Task 2:

let $\vec{v}$ be the direction vector.
The direction derivative in direction v from point x is $\nabla f(x) \cdot \vec{v}$

To determine in which direction $\nabla f(x) \cdot \vec{v}$ is max:

$$\nabla f(x) \cdot \vec{v} = |\nabla f(x)| |\vec{v}| \cos(\theta)$$

Since $\vec{v}$ is a unit, solving for $|\nabla f(x)| \cos\theta$ gives
max value when $\theta = 0$ : $\cos(\theta) = 1$.
Thus, $\vec{v}$ will point in the same direction as $\nabla f(x)$,
which is the gradient direction.