Rendering Topics and Notes

(Compiled Discussion)

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Contents

1 BDPT vs. Photon Mapping

1.1 Basic Comparison

- **Both** Bidirectional Path Tracing (BDPT) and Photon Mapping (PM) are Monte Carlo techniques to solve the rendering equation.
- **BDPT**: Shoots subpaths from the camera and from the light sources, then connects these subpaths. Multiple Importance Sampling (MIS) is used to combine contributions from different connection strategies.
  - Caustics can be noisy under BDPT if they depend on rare connection events (small bright specular highlights).
  - Typically, BDPT improves upon naive unidirectional path tracing for difficult light transport scenarios.
- **PM**:
  1. *Photon Emission:* Emit photons from light sources, trace them through the scene. Photons are stored (position, flux, incoming direction) when they hit surfaces, commonly in a spatial data structure (like a kd-tree).
  2. *Gather:* For each shading point, look up nearby photons in the photon map. Sum the stored flux and divide by the local area to estimate incoming radiance, then multiply by the BSDF for outgoing radiance.
- PM typically requires large photon counts to reduce noise during the final gather.

1.2 Similarities and Differences

- **Similarities:** Both aim to solve the same integral (the rendering equation) using Monte Carlo.
- **Differences:**
  - BDPT does *not* store partial paths globally. It builds a few subpaths per pixel sample and connects them on the fly.
  - PM invests in a *global* photon map pass (often shooting millions of photons from the lights). Then, in a separate pass, we gather photons for shading.
  - PM handles caustics more efficiently if photons are densely deposited in sharp reflective/refractive regions.
  - BDPT can handle caustics but sometimes requires more careful sampling.

2 Two Distinct Photon Maps

- In many implementations, photon mapping uses **two** distinct photon maps:
  - *Caustics photon map*: dedicated to specular-to-diffuse paths (small, bright caustics).
  - *Global photon map*: for all other, more diffuse contributions.
- By storing caustic photons separately, one can search with a tighter radius for the small, high-frequency patterns, while the global map handles broader illumination.

3 Biasness of Median Cut for Environment Sampling

- **Median cut** for environment light sampling partitions an environment map into regions of approximately equal total energy.
- One representative (direction and intensity) is chosen per partition.
- This introduces a *bias*, because we approximate a continuous distribution by piecewise-constant clusters.
- *Consistency*: as the number of partitions grows, each cluster becomes smaller, and the approximation converges to the true distribution. The bias goes to zero in the limit, so it is *consistent*.

4 Ideal Distribution for Direct Illumination (DI)

- The **ideal sampling distribution** for direct illumination, $p^{\star}(\omega)$, is proportional to the integrand (i.e., the product of BRDF, incident radiance, and the geometric factor $\cos\theta$).
- If we could sample perfectly from $p^{\star}(\omega)$, variance would be zero.
- In practice, we sample from simpler distributions (light-based or BRDF-based) and use *Multiple Importance Sampling (MIS)* to combine them.
- **Sampling-Importance Resampling (SIR):** If direct inversion of $p^{\star}(\omega)$ is difficult, we can sample from a simpler proposal $q(\omega)$ and then apply an importance weight to each sample to approximate draws from $p^{\star}(\omega)$.

5 Diffuse-Specular Separation with Polarised Light and Spherical Harmonics

5.1 Polarised Illumination

- For a *dielectric* interface, specular reflection can preserve or enhance polarisation, whereas the diffuse component is mostly depolarised.
- By cross-polarising the light source and the camera (their polarisation axes are 90° apart), the specular reflection is largely blocked and the diffuse passes through.
- With parallel polarisation, the specular reflection is passed and can be observed. Combining these measurements allows *diffuse-specular separation*.

5.2 Using Spherical Harmonics (SH)

- Diffuse reflections vary more slowly with direction, while specular reflections show high-frequency variation.
- By projecting reflectance data into spherical harmonics (using different illumination bases), one can separate lower-order coefficients (diffuse) from the higher-order residuals (specular).

6 Why Dipole Diffusion Works Well for Wax but Not Skin

- **Dipole diffusion** assumes a homogeneous, highly scattering medium with relatively simple boundaries.
- Wax is reasonably homogeneous, strongly scattering, and has no complex layering or strong wavelength-dependent absorption peaks.
- **Human skin** has multiple layers (epidermis, dermis, subcutaneous fat), each with different properties. Pigments like *melanin* and *hemoglobin* show strong wavelength dependence.
- **RGB vs. Spectral**:
  - An RGB-only approach lumps the entire visible spectrum into 3 broad channels, missing fine absorption features.
  - A *spectral* approach (more than 3 channels) can capture the subtle variations important for accurate skin rendering.

7 Edge Preserving Filter in Local Tone Mapping

- Local tone mapping operators often *decompose* the image into a *base* (low-frequency) layer and a *detail* (high-frequency) layer.
- An *edge-preserving filter* extracts the large-scale luminance while preserving edges, so that detail edges do not get blurred.
- Then the base layer is compressed aggressively, while the detail layer is only lightly scaled, preserving local contrast.
- **Global Tone Mapping:** Sometimes a global (logarithmic or Reinhard-type) curve is used as a first pass, to bring extreme HDR intensities into a manageable range before the local step.
- Separating luminance from color: one often operates on luminance in *log space*, compress it, and re-apply the original chromaticities at the end.

8 Minimum Number of Mirror Ball Photographs

- To create a high-quality light probe in *angular map* format from mirror-ball captures, a minimum of **2** shots from different viewpoints is typically used.
- This helps fill in the areas occluded by the camera, reduces strong camera reflection artifacts, and recovers the environment behind the ball.

9 Advantage of Dual Light Stage Over LED Sphere

- A standard LED sphere places many point lights on a geodesic dome. Capturing high-frequency reflectance (sharp specular lobes) requires a very high density of controllable LEDs.
- A *dual* light stage uses two stages (or a stage that moves the subject and camera):
  - Lights can move in a dense sampling of directions *and* the camera can also move, sampling many view directions.
  - Hemholtz reciprocity arguments allow more thorough sampling of the 4D reflectance function.
  - This setup effectively increases the "Nyquist rate"—one can capture finer details of reflectance without requiring an impossibly dense LED array.

10 Neural Networks and Nyquist in Image-Based Relighting (IBRL)

- High-frequency reflections require extremely dense sampling to avoid aliasing if done purely by classical measurement.
- A *neural network* can learn a prior over possible reflectance functions from a training set, then fill in or extrapolate intermediate viewpoints/illumination directions.
- This can reconstruct or "hallucinate" plausible sharp highlights even from a relatively sparse sampling, effectively beating a naive Nyquist sampling requirement by leveraging learned reflectance priors.

11 NeRF vs. 3D Gaussian Splatting (3DGS)

- **NeRF**: A continuous volumetric representation learned by a small MLP. For any 3D point and viewing direction, it predicts color and density. Rendering involves ray marching with many samples per ray.
- **3D Gaussian Splatting**: A large set of point-based primitives (Gaussians) in 3D. Each has position, covariance (shape), color, and opacity. Rendering is done by splatting these Gaussians onto the image plane.
- **Advantages/Disadvantages**:
  - NeRF typically yields high-fidelity and smooth geometry but can be slow to render (many samples per ray).
  - 3DGS can be very fast to render once you have the splats, but it may use more memory (storing many Gaussians). It does not enforce a strict continuity of topology as a mesh might.

12 Albedo ρ_d for a Diffuse Surface

- Albedo ρ_d is the fraction of incident light that a surface reflects.
- For a Lambertian surface, the BRDF is $f_r(\omega_i, \omega_o) = \frac{\rho_d}{\pi}$.

13 MC Rendering Weights and Halfway Vector Notation

- Often, the weight of a sample in direct illumination can involve a term like

$$\frac{p(\omega_h)}{4\,(\omega_h \cdot \omega_i)},$$

depending on the specific parameterization (e.g. halfway-vector sampling).

- Directions on the unit sphere have differential solid angle $d\omega = \sin\theta \, d\theta \, d\phi$.

14 Vignetting in Light Probe Photography

- When capturing a light probe with a wide-angle or fisheye lens, **lens falloff** or vignetting can cause angular-dependent attenuation in the measured intensities.
- This must be measured and corrected, to divide out the known falloff function so that the final environment map matches the true scene radiance.

15 PFM vs. Radiance (`.HDR`) Format

- A **PFM** file can store 32-bit floats per channel. At 3 channels (RGB) that is 96 bits/pixel.
- A **Radiance** `.HDR` file uses a shared exponent and 8-bit mantissas (per channel), totaling 32 bits/pixel, and thus uses less space for HDR data.

16 4D Light Field Definition and Parameterization

- A 4D light field is the radiance along every ray in free space, ignoring time and wavelength: $L(u, v, s, t)$.
- Commonly, the two-plane parameterization uses (u, v) on one plane, (s, t) on a parallel plane; each ray is identified by the intersection points with these planes.

17 Dual Photography with a Single Photodiode

- **Dual photography** exploits the *linearity of light transport*. By projecting a carefully chosen set of illumination patterns onto a scene, and measuring the *total* brightness with a single photodiode, one can reconstruct what a full 2D camera sensor would have captured.
- This hinges on inverting a light transport matrix T where

$$\mathbf{I} = T\,\mathbf{x}.$$

- $\mathbf{I}$ is the measured intensities (one scalar per pattern), $\mathbf{x}$ represents the image pixel values, and T is the transport operator.

18 IBL vs. IBL Reproduction

- **IBL (Image-Based Lighting)** uses a captured environment map to illuminate synthetic objects in a renderer.
- **IBL Reproduction** physically recreates that measured lighting distribution in the real world, typically with a dome of controllable light sources or projectors that match the intensities and directions from the environment map.

18.1 Spectral Reproduction

- By using multiple narrow-band light sources (e.g. 6-channel or 8-channel LED arrays), one can approximate the full spectral power distribution (SPD) of the target environment.
- Helps match color appearances (e.g. skin tones) under the original scene's lighting spectrum.

19 Sensor Resolution for a Light-Field Camera

- Suppose you want a *spatial resolution* of 250×250 and an *angular resolution* of 20×20.
- Then you need
$$250 \times 250 = 62{,}500 \quad \text{microlenses,}$$
and each microlens must capture a 20×20 angular grid of rays, requiring
$$20 \times 20 = 400 \quad \text{pixels per microlens.}$$
- Overall sensor resolution:
$$62{,}500 \times 400 = 25{,}000{,}000 \quad \text{(25 MP).}$$

20 Physically-Based (PB) vs. Data-Driven (DD) Microfacet BRDF

- Both interpret reflection in terms of micro-geometry (*microfacet distribution* $D(h)$, Fresnel F, geometry term G).
- **Physically-Based (PB)**: uses an *analytic* function for $D(h)$ (e.g. Beckmann, GGX) with a few parameters (roughness, index of refraction).
- **Data-Driven (DD)**: obtains part or all of these terms by fitting to measured reflectance data (e.g. from the MERL BRDF database). Possibly uses a more flexible function or interpolation scheme for D.

21 MERL BRDF Database Parameterization

- MERL uses the **halfway-difference** (Rusinkiewicz) parameterization for *isotropic* materials.
- This effectively reduces the BRDF from 4D to 3D by exploiting isotropy (no dependence on the absolute azimuth around the normal, only on the difference of azimuth angles).
- Captures high-frequency specular lobes more compactly and avoids redundant dimensions.

22 Satin vs. Velvet Reflectance Lobe

- **Satin**: often has elongated, *anisotropic* highlights aligned with the weave, producing stretched specular lobes.
- **Velvet**: made of dense short fibers, giving a soft *retroreflective* look and sometimes a strong backscattering component.

23 Generating MC Samples from a Uniform Disk

- To sample uniformly over a disk of radius R, the cumulative distribution for the radius is
$$F_r(r) \;=\; \frac{\pi\, r^2}{\pi\, R^2} \;=\; \left(\frac{r}{R}\right)^2.$$
- If $u \sim \mathrm{Uniform}[0,1]$, solve $F_r(r) = u$ to get
$$r \;=\; R\,\sqrt{u}.$$
- For the angular coordinate, if $v \sim [0,1]$, then
$$\theta = 2\pi\, v.$$
- Hence a uniform sample (x, y) in the disk is
$$x = r\,\cos(\theta), \quad y = r\,\sin(\theta) \quad \text{where} \quad r = R\,\sqrt{u},\ \theta = 2\pi\, v.$$

24 Catadioptric Setups for BRDF Measurement

- *Catadioptric* systems use mirrors (catoptric) plus lenses (dioptric) to capture many incoming/outgoing directions in one shot.
- **Spherical mirror dome or ellipsoidal reflector**: the sample and camera are placed at or near focal points, so that many directions are mapped onto the camera sensor simultaneously.
- This avoids mechanically rotating lights/cameras for each angle, speeding up data capture.

25 Number of Measurements for Planar SVBRDF

- **Point light source:** each capture covers a single incident direction, so one must sweep the light across many angles, requiring many captures.
- **Linear light source:** each capture covers a 1D slice of incident angles, so fewer captures are needed compared to a single point light.
- **Spherical illumination:** could illuminate from *all* directions at once, drastically reducing the capture count (but possibly increasing dynamic range or requiring multiplexed decoding).

26 Irradiance Caching vs. Photon Mapping

- Both approximate multi-bounce (often diffuse) indirect light to reduce rendering cost.
- **Irradiance Caching**: stores irradiance values at sparse points on surfaces and interpolates them. Good for smooth, diffuse scenes, not ideal for sharp caustics.
- **Photon Mapping**: explicitly stores photon hits (positions, flux) and performs a density estimate for radiance. Good for capturing high-frequency effects like caustics.

27 IBRL as a Linear System and the Light Transport Matrix

- Image-Based Relighting (IBRL) can be expressed as

$$\mathbf{p} = T\,\mathbf{l},$$

  where $\mathbf{p}$ is the output image (pixel intensities), $\mathbf{l}$ is a vector of light-basis coefficients, and T is the light transport matrix.

- **Matrix Interpretation**:
  - Each row of T corresponds to a pixel in the output image.
  - Each column of T corresponds to a basis light (or illumination pattern), describing how that light contributes to all pixels.

28 Homogeneous Spherical Sample for BRDF Measurement

- One can measure a *homogeneous material* by taking a sphere of that material. A single photograph (for a known light direction) yields a range of surface normals (the sphere covers all orientations).

- Using an LCD screen as illumination for a planar sample allows *multiplexed* capture (varying patterns on the screen) to measure spatially varying BRDFs more efficiently than moving a single point light source.

29 Angular Map Format vs. Mirror Ball Format

- **Angular map**: each pixel directly corresponds to a unique direction. This can be convenient for sampling or convolving.
- **Mirror ball photo**: one must handle significant distortions near the edges and correct for perspective, making it more complicated to retrieve a uniform sampling of directions.
- Angular maps offer more even coverage and simpler lookups for environment lighting.

30 MIS for Direct Illumination vs. BDPT

- **Direct Illumination** uses MIS to combine two sampling strategies: sample directions from the BSDF vs. sample points on the light source. Their estimates are weighted to reduce variance.
- **BDPT** extends this idea to the entire path: each complete path can be constructed in multiple ways (light-to-camera or camera-to-light subpath connections). All such strategies' contributions are combined using MIS.

31 SH for Efficient Irradiance and Radiance Caching

- By projecting environment lighting into spherical harmonics (SH), one can quickly compute diffuse irradiance at a surface orientation using a *low-dimensional dot product* with SH basis coefficients.
- In radiance caching, each cache point stores SH coefficients of the incoming lighting, which can be quickly sampled to shade nearby points.

32 Diffusion Approximation for BSSRDF

- In highly scattering, relatively low absorbing media, multiple scattering *randomizes* directions, so the problem can be approximated by a *diffusion* equation.
- Instead of $S(\mathbf{x}_i, \omega_i; \mathbf{x}_o, \omega_o)$ (8D), one reduces to a radial profile $R_d(\|\mathbf{x}_i - \mathbf{x}_o\|)$ in a dipole or multipole approximation.
- Spectral models help capture strong wavelength dependences, particularly in realistic skin rendering.

33 Secondary Highlights in Hair vs. Fur

- **Secondary highlights in hair** occur due to internal reflections inside the cylindrical hair shaft (light refracts in, reflects off the back interface, then refracts out).
- **Fur** often scatters more because it can be *thicker*, *rougher*, or less transparent, leading to a lot of volumetric scattering.

34 Single vs. Multiple Scattering in Participating Media

- **Single scattering dominates** in *optically thin* media (e.g. light haze, smoke) where the probability of multiple collisions is small.
- **Multiple scattering dominates** in *optically thick* media (clouds, dense fog, milk, skin).
- **Diffusion approximation**: in strongly scattering media, photon directions become randomized; the BSSRDF can simplify to a radial diffusion profile.

35 Vignettes on Mirror Balls and Normals (Misc. Equations)

- For a mirror ball reflection, if R is the reflection direction, N is the surface normal, and V is the view vector,
$$R \;=\; 2\,(N \cdot V)\,N \;-\; V.$$
- Latitude-longitude mapping often uses:
$$\theta = \arccos(z), \quad \phi = \operatorname{atan2}(y, x).$$

36 Image-Based Lighting vs. Image-Based Relighting

- **IBL**: We use a *captured environment map* to illuminate synthetic objects in CG.
- **IBRL** (Image-Based Re-Lighting): We *change* the illumination of a *real* captured scene after the fact by combining multiple measured images or factoring out reflectance.

37 4D Light Field vs. 4D Reflectance Field

- **4D Light Field**: Radiance as a function of position and direction in free space ($L(u, v, s, t)$). Enables viewpoint changes without geometry.

- **4D Reflectance Field**: For an object or scene, how it reflects incoming light from every possible direction to every possible viewing direction. Used for re-illumination or view changes of a real object.

38 2D View Interpolation vs. Neural Radiance Fields (NeRF)

- **2D View Interpolation**: With a camera array, we can warp/blend nearby camera views to synthesize an in-between view. Requires depth or disparity estimation and can fail with complex occlusions.
- **NeRF**: Learns a continuous volumetric representation via an MLP. For any 3D point and direction, it predicts radiance/density. Rendering is done by ray marching. Often yields more robust and higher-fidelity novel-view synthesis, especially with fewer physical cameras.

39 Reflectance Trace Peaks for SVBRDF Measurements

- **Linear light source on a planar sample**:
  - *Diffuse* reflection tends to have a broad lobe peaked around the surface normal direction (for Lambertian-like materials).
  - *Specular* reflection is peaked around the mirror angle (the angle equal and opposite to the incoming light direction).
- **Backscattering with a phone (camera + flash co-located)**:
  - Illumination and viewing directions are nearly the same, so the specular highlight is observed close to the normal if the surface is facing the camera.
  - The *specular peak* collapses near the backscattering direction.
- From such reflectance traces, one can fit:
  - *Diffuse albedo* (base color),
  - *Specular amplitude* (reflectivity),
  - *Surface roughness* (width of the specular lobe), etc.

40 Photon Mapping Bias and Progressive Photon Mapping (PPM)

- Standard photon mapping uses a fixed gather radius to compute radiance from nearby photons. This introduces *bias*, especially for small bright features (caustics) that get blurred if the radius is large.

- **Progressive Photon Mapping (PPM)**: iteratively shrinks the gather radius while sending more photons. This method reduces bias for caustic-intensive scenes and converges to the correct solution in the limit.

41 Startup Bias in Metropolis Light Transport (MLT)

- **Startup bias** arises when the Markov chain initializes from an unrepresentative path distribution and takes time to *mix* into the true equilibrium.
- One can mitigate this by discarding a burn-in portion of the samples or running multiple independent chains with different starts.

42 Differential Radiance Scattering (Example)

- An example integral for scattering in participating media might look like

$$dL_o(x,\omega) = \int_0^1 \Big(-\,\sigma_t\, L_i(x,-\omega) + \sigma_s \int_\Omega p(x,-\omega' \to \omega)\, L_i(x,\omega')\, d\omega' \Big)\, dt.$$

- One can simplify in strongly scattering cases or under assumptions about isotropic phase functions, etc., but that is the general idea.