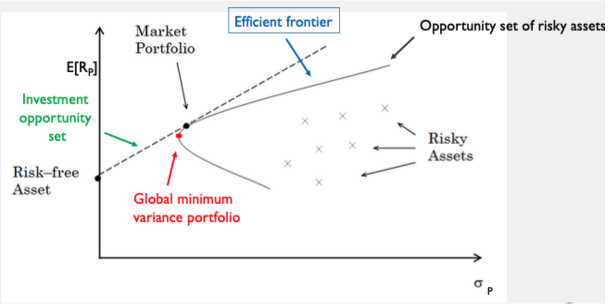
ST24 Finance and Financial Management (BUSI60043 @ Imperial College London) by ck21

Time value of money

Assets and Interest. Sequence of Cashflows. Present Value (PV) - earlier money on a timeline. Future Value (FV) - later money on a timeline. Interest Rate (R) - exchange rate between earlier and later money; discount rate; cost of capital; opportunity cost of capital; yield. Simple Interest - $FV_t = P_0 \star (I + t \star R)$ Compound - interest on interest. Interest can be credited/charged more than once a year. Need Effective Annual Rate r_{EAR}. If annual rate r is compounded n times a year then for r_{EAR} we have $r_{EAR} = (1 + \frac{r}{n})^n - 1$.
Discounting. If future value FT refers to T periods from today - $PV = \frac{FV_T}{(1+R)^T}$. Finding the present value of a future amount is Discounting. For a given interest rate, the longer the time period, the lower the present value cet. par.. For a given time period, the higher the interest rate, the smaller the present value cet. par.. Discount Factor - present value of £1 over T periods at rate R is the discount factor. $\frac{1}{(1+R)^T}$. Markets, outside options, arbitrage determines discount factor. Discount Rate (Yield) - $R = (\frac{FV}{PV})^{1/T} - 1$.
Annuities and Perpetuities. Annuity - finite series of equal payments that occur at regular intervals. Ordinary annuity if the first payment occurs at the end of the period. Annuity due if the first payment occurs at the beginning of the period. $PV = CF_1 \times (\frac{1}{R} - \frac{1}{R(1+R)^T})$, $FV = CF_1 \times (\frac{(1+R)^T - 1}{R})$. Perpetuity - infinite series of equal payments or payments growing at g% occurring at regular intervals. $PV_{CONSTANT} = \frac{CF_1}{R}$. $PV_{GROWING} = \frac{CF_1}{R-g}$.

Capital asset pricing model

Returns and Definitions. Actual return of investing in an asset is the relative prcie change plus any interim payments (dividends) the asset might give rise to. $R_t = \frac{P_t - P_{t-1} + div_t}{P_{t-1}}$. Return next period (expected return) is modelled as random variable $R_1, R_2 ... R_S$. Probability weighted average of outcomes $E(R) = \mu = \Sigma_{i=1}^{S} R_i Prob(R_i)$. Variance is the fluctuation of the variable around its mean - $Var[R] = \sigma^2 = E[R^2] - (E[R])^2$. Std Dev is square root of variance. Volatility and Risk are other terms for Std Dev. Covariance - degree to which two variables move in the same direction at the same time. $Cov(R_A, R_B) = \sigma_{AB} = \Sigma_{i=1}^{S} Prob(i)[(R_A(i) - E[R_A])(R_B(i) - E[R_B])]$. Correlation similar to covariance but normalised by Std Dev. Another measure of co-movement. $Corr(R_A, R_B) = \rho_{A,B} = \frac{Cov(R_A, R_B}{\sigma_A \sigma_B)}$. Always between -1 and $+1$. Same sign as covariance.
Portfolios. Collection of assets. N assets with returns $R_1, ..., R_N$. Weights $x_1, ..., x_N$ defined as total value of asset i position in portfolio divided by total value of portfolio. Compare assets via Risk-Free Asset. Characterised by $E[R_F] = R_F, Var[R_F] = 0, Cov[R_F, R_i] = 0$. Assume R_F constant over time. Suppose we invest x in risky asset and $1 - x$ in risk-free asset. Expected return is $E[R_P] = xE[R_i] + (1 - x)R_F = R_F + xE[R_i - R_F]$, where $E[R_i - R_F]$ is the risk premium and $R_i - R_F$ is the excess return. Variance on such a portfolio is $Var[R_P] = x^2\sigma_i^2$. Std Dev $\sigma_p = |x|\sigma_i$. Sharpe Ratio is $\frac{E[R_i] - R_F}{\sigma_i}$. Return premium per unit of risk. Used to construct Capital Allocation Line (CAL).
Indifference Curves. Higher return preferred to lower return cet. par.. Lower risk preferred to higher risk cet. par.. Max $E[R_P]$, min $\sigma^2 p$. Indifference curves express sets of portfolios that would make investors equally happy. Slope upwards, convex, cannot intersect, increase in value as they move towards upper left-hand corner. CAL - what they can get. Mean Variance Utility Function $E[U(R_P)] = E[R_P] - 0.5A \cdot var(R_p)$. Coefficient A measures degree of risk aversion. Standard deviation of portfolio equals weighted average of std dev of assets in it only if assets are perfectly correlated.
Minimum Variance Portfolios. If assets are only partially correlated (correlation $\rho < 1$, portfolio std dev is less than weighted average of std dev of assets. Lower the correlation coefficient - stronger this diversification effect. Investors can obtain the same level of expected return with lower risk. Each correlation structure comes with a minimum variance portfolio. Lower the correlation coeffcient, the lower the minimum variance. Higher payoff from diversification/ Minimum variance reaches zero when the correlation $\rho = -1$. Find portfolio weights where the first derivative of variance is zero.



Assume an equally weighted portfolio of independent assets. The risk decreases with the number of assets - $\sigma_P^2 = \frac{1}{N} E[\sigma_i^2]$. Compensate for diversifiable (idiosyncratic) risk, not systematic.
CAPM. Talks about systematic risk. How do we measure riskiness of a security in a portfolio? What is the expected return of a security? What are α and β? How do we determine a company's cost of capital (discount rate in calculations that check the viability of business projects, aka NPV)?
Market Portfolio. Since all rational investors will hold the same risky portfolio (in combination with the riskfree asset), the portfolio must be the portfolio of all risky assets. This portfolio is the market portfolio in equilibrium. Includes not just stocks but all assets - risky bonds, real estate, commodities etc. Cannot observe its return, so we use a proxy for it such as a stock market index containing a large number of securities. Market value of an asset = Price × Number of shares (bonds etc). Investors choose a portfolio on the Capital Market Line (CML). Connects risk-free asset with the market portfolio. Risk-averse investors hold more of the risk-free asset. Risk-tolerant investors hold more of the market portfolio.
Deriving the CAPM. Using the formula for the variance of a portfolio, show how the risk of the market portfolio depends on each asset. $Var[R_M] = var[\Sigma w_i R_i] = \Sigma_{i=1}^{N} \Sigma_{j=1}^{N} w_i w_j Cov(R_i R_j) = \Sigma_{i=1}^{N} w_i \Sigma_{j=1}^{N} Cov(R_i, w_j R_j) = \Sigma_{i=1}^{N} w_i Cov(R_i, R_M)$
Risk-Reward Ratio. Must be the same for all assets and portfolios. $\frac{E[R_i] - R_f}{Cov(R_M, R_i)} = \frac{E[R_M] - R_F}{Var(R_M)}$ $E[R_i] = (E[R_M] - R_f)\frac{Cov(R_M, R_i}{Var(R_M)} + R_f$. CAPM Equation is $E[R_i] = R_f = \beta_i[E[R_M] - R_f]$. Beta is $\beta = \frac{Cov(R_M, R_i}{Var(R_M}$ - the measure of the systematic risk of an asset. CAPM says that only systematic risk is priced and that there exists a positive linear relationship between the expected return of an asset and its beta. Idiosyncratic risk can be and is diversified away so it is not priced. Investors want remuneration for bearing the systematic risk, not the total risk.
Alpha. Stocks that have returns above what CAPM prescribes are undervalued and should be bought. Difference between actual return and CAPM - $\alpha = E[R_i] - (R_f + \beta_i(E[R_M] - R_f))$. Unexpected deviation from fair return; excess performance above CAPM return.
Index Model. $R_i - R_f = \alpha_i + \beta_i(R_M - R_f) + \epsilon_i$. α is the abnormal return while ϵ is the firm-specific risk. Uses an actual stock index to proxy this theoretical market. CAPM makes forward looking predictions; index models use historical data.

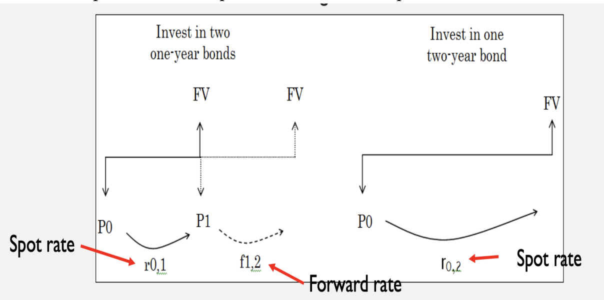
Arbitrage

Pure Arbitrage. Trading strategy that requires no initial investment, has no negative cashflows at any time, has a positive cashflow at least one time. Possible when there is mispricing. Arbitrage forces prices to converge. Requires both long and short positions. Dominance Argument - numerous small investors marginally adjusting their holdings. Dependent on degree of risk aversion. Arbitrage argument - few large investors needed. Each taking as large a position as possible. Degree of risk aversion need not matter.
Arbitrage-Free Pricing. True arbitrage cannot exist. Prices would adjust and eliminate. No-arb condition a cornerstone of asset pricing (derivatives). Arb opps are rare, ephemeral, entail tiny gains per unit, subtle, complicated, can come with risk of interim adverse price movements, and are eroded by frictions (transaction costs, limitations on borrowing/shorting).
Bid-Ask Spread. Bid - price at which the MM buys an asset. Ask - price at which MM sells an asset. Mid is $\frac{Bid+Ask}{2}$. Used to value holdings. Bid-ask spread is the transaction cost investors have to pay in order to trade (and is the profit for the MM). Competitive markets depend on liquidity. In illiquid markets, prices are no longer a measure of value - loss of info. Need to balance what MMs are willing to bear and what investors are willing to trade. MMs care about liquidity risk → trading frequency of assets - more compensation (wider spread) required to handle assets with low trading volume. Also, volatility risk - more tumultous times - thicker safety net demanded (wider spread). Internal guidelines on inventory, opportunity cost of funds tied up, Adverse selection - informed counterparty risk.
Trade Types. Market Order - executed immediately at current market prices. Complication - what if order is larger than max size for the price? Quick but may get worse price. Limit Order - trade at best prices avail if no worse than specified limit price. May get good price but may not execute. Stop order - triggered when price reaches specified stop price. Limits losses in trader absence but could be worst time to trade. Fill or Kill - expires immediately following submission, no partial trading.
Implications of No Arbitrage. Law of One Price (LOOP) - if two securities have the same payoffs, they have the same price. Portfolio replication - if a portfolio has the same payoff as a security, it must have the same price. Dynamic Hedging strategy - if a self-financing strategy has the same payoff as a security, it must have the same price.
NPV of Trading Securities. Trading a security as an investment decision. If it is a normal market (no arb opps) then no arb price of security is the PV(all cashflows paid by the security). NPV(buying) is the PV(all cashflows due to security holder - Price(Security) = 0. NPV(selling)=Price(security) - PV(all cashflows due to security holder) = 0.
Zero-Sum Game of Trading. Competitive markets trading comes with NPV > 0 or NPV < 0. Trading on its own should neither create nor destroy value. Value should be created by real investment projects.

Bonds

Introduction. Contractual Obligation of issuer (borrower) to make to the bondholder (lender) periodic fixed or floating (but deterministic) interest payments (coupons) and to repay the bond's principal (face or par value) at maturity. Bonds issued by a corporate entity (in contrast to government bonds issued in the domestic currency) have default risk. The firm's creditworthiness is the most important determinant of the interest rate the company will pay.
Bond Prices and Yields. Bond prices equal to the present value of the coupons C_t and face value FV_T discounted at the rate $r_{0,t}$ (may be different for different maturities). $P_0 = \Sigma_{t=1}^{T} \frac{C_t}{(1+r_{0,t})^t} + \frac{FV_T}{(1+r+0,T)^T}$. The interest rate y (YTM) equating the bond price to the present value of its cash flows is the yield to maturity (gross redemption yield). $-P_0 + \Sigma_{t=1}^{T} \frac{C_t}{(1+y)^t} + \frac{FV_T}{(1+y)^T} = 0$. The coupon payment by the current price gives the current yield $C.Y. = \frac{Coupon}{P_0}$.
Accrued Interest. If the bonds are bought on a day between coupon payments, the seller is entitled to receive a pro-rata part of the next coupon. The sale (or invoice) price that the buyer would pay would equal the stated (or flat) price plus the accrued interest.

$Interest = \frac{AnnualCouponPayment}{2} \times \frac{D_{LastCoupon}}{D_{BetweenCoupon}}$. D is days.
Example - coupon rate 8%. Face value \$1000. Semi-annual coupons. Quoted price \$990. 30 Days since last coupon, 182 days between coupons. Semi-annual coupons so coupon is \$40. Accrued interest is $40 \times 30/182 = \$6.59$; $P(invoice) = 990 + 6.59 = \$996.59$.
Characterising Bonds with Prices. $P_0 > FV$ Bond sells at premium so $y < c$. $P_0 = FV$ bond sells at par so $y = c$. $P_0 < FV$ bond sells at discount so $y > c$.
Yield to Maturity. Measure of total return that the investor obtains when buying the bond today at the price P_0 and keeping it till maturity (like a holding period return). Measure not perfect as it assumes that the investor will be able to reinvest each coupon received at the rate y throughout the life of the bond. Current Yield is an approx to the bond's YTM. Bonds at different maturities may have different yields. The yield as a function of time to maturity is the yield curve. Need the YTM $r_{0,T}$ on risk-free zero-coupon bonds - spot rates. The YTM of a straight bond (plain vanilla bond) with maturity in T years is a non-linear average of the spot rates up to maturity T. $\frac{C}{(1+y)^1} + \frac{C}{(1+y)^2} + ... + \frac{C}{(1+y)^T} + \frac{FV}{(1+y)^T}$. Graph of the risk-free zero-coupon yields for each maturity is called the term structure of interest rates (pure yield curve).
Forward Rates. Observed term structure contains implied forward rates. They make investors indifferent between investing in bonds of different maturities. Forward rates solve equations of the type $(1 + r_{0,1}) \times (1 + f_{1,2} = (1 + r_{0,2})^2$ and $(1 + rr_{0,2})^2 \times (1 + f_{2,3}) = (1 + r_{0,3})^3$. Forward rates can be seen as the market expectation of spot rates that will prevail in the future.



Market Expectations Hypothesis. Long run rates are a geometric average of current and future short rates. $(1 + r0, 2)^2 = (1 + r_{0,1}) \times (1 + E[r_{1,2}])$. The forward rate is the unbiased estimate of future short rates. $f^t_{1,2} = E_t(r^{t+1}_{0,1})$. We assume that bonds of different maturities are treated as perfect substitutes. Other assumptions - risk neutral investors, no transaction costs, no coupon payments, no default. The fact that the term structure is upward-sloping most of the time implies that the forward rate seems to overestimate the expected spot rate.
Liquidity Preference Theory. Investors have short horizons and are risk-averse. Lenders demand a liquidity premium to hold long-term bonds. Long-term borrowers are willing to compensate - a gain for them as by locking in a rate, they reduce future risk. Liquidity premium on long-term bonds - $(1 + r_{0.2})^2 = (1 + r_{0,1}) \times (1 + E[r_{1,2}] + \pi_{1,2})$, $f^t_{1,2} = E_t(r^{t+1}_{0,1} + \pi^t_{1,2}$.
Segmented Markets Theory. Bonds of different maturities are in fact distinct and unconnected markets. Each of these markets finds its equilibrium separately. This is in contrast to the preceding two theories that treat bonds as perfect substitutes. Some investors may indeed face binding limitations in what maturities to invest. (MMF - very short maturities do appear in a segmented market). AKA preferred habitat theory. Pair of expectations hypothesis and LPT is the currently accepted theory of the term structure.
Interpreting Term Structure. Expectations of increases in rates can result in a rising yield curve. This does not imply expectations of higher future rates. Very steep yield curves are interpreted as warning of impending rate increases. Long-term rates tend to rise in expectation of an expansion phase. A falling yield curve can be a strong indication that yields are likely to fall. Declining rates are taken as a sign of a coming recession. Interest rates fall due to falling real rates and falling inflation.
Sensitivity to interest rate changes. Prices of long maturity bonds are more sensitive to interest rate changes (more terms in y raised to higher powers). A first-order approximation of the impact on the price is the slope of the bond price function w.r.t. the interest rate. The slope is sort of average maturity calculated using the present value of the cash flow as a weight for each date t. The curvature of the bond price function depends on how the cash flows are distributed over the life of the bond.
Interest rate risk.
$P = \frac{CF_1}{(1+y} + \frac{CF_2}{(1+y)^2} + ... + \frac{CF_T}{(1+y)^T}$, $\frac{dP}{dy} = -\frac{CF_1}{(1+y)^2} - 2\frac{CF_2}{(1+y)^3} - ... - T\frac{CF_T}{(1+y)^{T+1}}$, $-\frac{dP}{dy} \times \frac{1+y}{P} = 1\frac{CF_1}{P(1+y)^1} + 2\frac{CF_2}{P(1+y)^2} + ... + T\frac{CF_T}{P(1+y)^T}$. In general - bonds with longer duration react more to changes in interest rates.
Duration. Negative to the elasticity of the price to (ONE PLUS) the yield. $D = -\frac{dP}{dy} \times \frac{1+y}{p} = \Sigma^T_{t=1} t \times w_t$ where $w_t = \frac{cashflow}{P(1+y)^t}$. Average of cashflow maturities weighted by their contribution to the price $Volatility = \frac{Duration}{1+y}$.
Parallel Shifts. Need to hedge based on the risk you fear. Steepeners - gain from widening spread between short-term and long-term YTM. Combine "long" short-dated bond position with a "short" long-dated bond position. Flatteners - gain from a shrinking spread (flattening curve) between short- and long-term YTM. Sell short-term bonds and purchase long-term bonds.
Convexity. Duration gives the approximate change in the bond price given a change in yield. Duration is a straight-line approximation of a non-linear (curvey) yield/price relationship. The more curvey the yield/price relationship, the worse the approximation. Convexity accounts for the curvature.
$C = \frac{\frac{d^2V}{dy^2}}{V} = \frac{1}{(1+y)^2}[\Sigma^T_{t=1}(t + t^2)\frac{PV(C_t)}{V} + (T + T^2)\frac{PV(FV_T)}{V}]$. A valuable feature - drop in bond prices following an increase in the yield is smaller than the increase in the price following a drop in the yield. Percentage changes in the value of a bond can be estimated using both duration and convexity. $\frac{\Delta V}{V} \approx -\frac{1}{1+y} \times D \times \Delta y + \frac{1}{2} \times C \times (\Delta y)^2$.
Duration and DV01. Duration of a portfolio is the weighted sum of component directions. An equally weighted portfolio of four assets with durations 2.4, 4.1, 8.5 and 12.6 - $\bar{D} = \frac{1}{4}(2.4 + 4.1 + 8.5 + 12.6) = 6.9$. Parallel to β for stocks. DV01 is the dollar value of a basis point - it relies on duration.
Takeaways. Bonds and yields move inversely. Duration and convexity are two ways of thinking about interest rate risk. Higher duration - higher the sensitivity to interest rate changes. Yield curve is generally upward sloping and allows us to speak about forward rates.

Options

Options Contracts. Call Options - right (but not obligation) to buy a given quantity of an asset at a predetermined price (strike price exercise price) before or on a given date (expiry expiration date). Put Options - right (but not obligation) to sell a given quantity of an asset at a predetermined price (strike price exercise price) before or on a given date (expiry expiration date). European Options may only be exercised at expiration, while American Options may be exercised at any time before or at expiration.
Long and Short Positions. The counterparty to the buyer (holder) of an option is the writer of an option. This is the party with the short position. The writer has an obligation to sell (buy) the underlying asset if the call option holder (put option holder) chooses to explore the option. The holder of the option is the party with the long position. In exchange, as a compensation for having the obligation, at time zero, the writer of the option receives a non-refundable premium.
Notation. S_t is spot price (price at which the asset is trading at time t). X is the strike or exercise price (often denoted as K). T is the strike or expiration date. C_t is the price (premium) of a call option at time t. P_t is the price (premium) of a put option at time t. $C_T = max\{S_T - X; 0\}$, $P_T = max\{X - S_T; 0\}$. Calls - $S = X$ ATM, $S > X$ ITM, $S < X$ OOTM. Puts - flip sign.
Options Strategies. Covered Calls - write a call while owning the underlying. Protective Put - buying a put option combined with owning the underlying. Bull Spread - spread is a combination of two or more call options (or two or more puts) on the same underlying with different strike prices or expiration dates. Bull Spread - buy a call with strike X_1 and write a call with strike $X_2, X_1 < X_2$. Bear Spread - buy a call with strike X_2 and write a call with strike $X_1, X_1 < X_2$. Butterfly Spread - buy a call with strike X, buy another call with a much higher strike X_3 and write two calls with a strike X_2 in-between. $X_1 < X_2 < X_3$. Straddle - buy a call and a put with same strike. Strangle - buy a call and put with different strikes. Strip - buy one call and 2 puts with same strike. Strap - buy 2 calls and a put with same strike.
Put-Call Parity. Evaluate terminal payoff from two portfolios. Portfolio 1 - Long call with strike X and bond paying X at expiry. Cost of portfolio today is $C_0 + PV(X)$. Portfolio 2 - A share with a long put with strike X and X same as before. Cost of portfolio today - $P_0 + S_0$. Both have same final payoffs at expiry, same strike X. Two portfolios must be equally expensive at time 0. Put-Call Parity: price of call $+PV(X) = S_0+$ price of put. So, $P_0 = C_0 + e^{-rT} * X - S_0$. r is risk free rate, T is time to expiry. We only need to price the call.
Taking Advantage. $Put = Call + Bond - Stock$. But we have $P_0 = £12$. Put is too cheap at £12 while call is relatively expensive.

Instrument	Today	$S_T \geq 56$	$S_T < 56$
Call	+10.00	56 - S_T	0
Bond	+52.74	-56	-56
Put	-12.00	0	56 - S_T
Stock	-48.00	S_T	S_T
Net position	**+£2.74**	**0**	**0**

Binomial Models. It is safe to assume that each period the price of a stock can go either up by a factor u or down by a factor d. If u and d are selected correctly, then as we shorten the length of the time period ($\Delta t \to 0$), we approach a continuous model of stock prices where returns are normally distributed. Options and other derivatives are priced by no arbitrage. We need to find a portfolio replicating the payoff of the option.
One Period Binomial Pricing. $S_0 = 20, u = 1.1, d = 0.9$. Find price of call expiring at $t = 1$ with $X = 20$ when $r = 3\%$.
Creating the Replicating Portfolio. Suppose traders can borrow and lend at the risk-free rate r. Find a portfolio replicating the payoff of the call composed of (1) the underlying stock (buy Δ shares) and riskless lending / borrowing (borrow B_0). Value of derivative today must equal value of the replicating portfolio.
Risk Neutral Probabilities. With risk-neutral traders we would have $S_0 = \frac{\pi S_H + (1-\pi)S_L}{1+r_f}$ where π is the probability that the next period the payoff is S_H. The current share price would be the discounted value of the expected future share price $S_0 = \frac{\pi \times u \times S_0 + 91 - \pi) \times d \times S_0}{1+r_f} \implies \pi = \frac{1=r_f - d}{u-d}$. Earlier we derived the call price independent of the trader's risk preferences. The call price will be the same with risk-neutral traders. But if the traders were risk-neutral, it would also be the case that $C_0 = \frac{\pi C_H + (1-\pi)C_L}{1+r_f}$. When investors are not risk neutral, π is not a real probability of the stock price going up from S_0 to S_H.