ALBERT-LUDWIGS-UNIVERSITÄT FREIBURG IM BREISGAU

Fachprüfungsausschuss Volkswirtschaftslehre (B. Sc.)

Predicting Foreign Exchange Rates with Machine Learning

Bachelor's Thesis

Advisor
Prof. Dr. Dirk Neumann

Author
Clemens Philipp Rockinger
4310834
Berlin

Timeframe
Mar, 29 2020 – Jul, 29 2020

Contents

Abstract

The main question of this thesis is whether the Random Walk is an accurate forecasting model for foreign exchange rates and if not how to improve on it with simple statistical and machine learning models. The foreign exchange market is considered one the biggest and most liquid of all financial markets. With researchers trying to model and forecast foreign exchange rates for decades, the assumption long stood that the movement is random and not dependant on past values. Recent research however suggests, that underlying patterns and dependencies can be discovered and consequently achieving a higher forecasting accuracy than the Random Walk is possible. With the use of an Auto Regressive Integrated Moving Average model (ARIMA) and a k-Nearest Neighbour Regression model two univariate methods are chosen to forecast foreign exchange rates. These models convince through their simple implementation and have been proven in applications on time series analysis and forecasting. Using the EURUSD, EURGBP and EURJPY currency pairs, forecasting is done in a single-step ahead approach using the stationary logarithmic return values of the foreign exchange rates. The results are expressed in measures derived from the forecasting errors and then compared to each other. From the results it can be seen that the Random Walk model is less accurate in predicting foreign exchange rates than the k-Nearest Neighbour or ARIMA model. This disproves the Random Walk theory derived from the efficient market hypothesis. The differences between the k-NN and the ARIMA model are small with the ARIMA slightly ahead in the case of all currency pairs. Further research is needed to determine whether more inputs, a different model design or entirely different models would improve the forecasting results.

Zusammenfassung

Zentrale Frage dieser Bachelorarbeit ist, ob der Random Walk ein genaues Prognosemodell für Devisenkurse ist und wenn nicht, wie man mit einfachen statistischen- und Machine Learning- Modellen Vorhersagen mit besserer Genauigkeit treffen kann. Der Devisenmarkt gilt als einer der größten und liquidesten aller Finanzmärkte. Da Forscher jahrzehntelang versuchten, Devisenkurse zu modellieren und vorherzusagen, ging man lange Zeit davon aus, dass die Bewegungen zufällig und nicht von vorhergegangenen Werten abhängig sind. Neuere Forschungen legen jedoch nahe, dass zugrunde liegende Bewegungsmuster und zeitliche Abhängigkeiten entdeckt werden können und daher eine höhere Vorhersagegenauigkeit als bei dem Random-Walk-Modell möglich ist. Dafür wurden in dieser Arbeit ein Auto Regressive Integrated Moving Average Modell (ARIMA) und ein k-Nearest Neighbour Regressionsmodell gewählt. Die Modelle überzeugen durch ihre einfache Implementierung und haben sich in Anwendungen zur Zeitreihenanalyse und -prognose bewährt. Unter Verwendung der Währungspaare EURUSD, EURGBP und EURJPY erfolgt die Vorhersage in einem single-step-ahead Ansatz unter Verwendung der stationären log-return Werte. Die Genauigkeit der Vorhersagen wurde durch, von den Prognosefehler abgeleiteten, Maße verglichen. Aus den Ergebnissen ist ersichtlich, dass das Random-Walk-Modell bei der Vorhersage von Wechselkursen weniger genau ist als das k-Nearest Neighbour- oder das ARIMA-Modell. Dies widerlegt die aus der Effizienzmarkthypothese abgeleitete Random-Walk-Theorie. Die Unterschiede zwischen dem k-NNN- und dem ARIMA-Modell sind verschwindend gering, wobei das ARIMA-Modell bei allen Währungspaaren leicht bessere Ergebnisse liefert. Es sind weitere Untersuchungen erforderlich, um festzustellen, ob mehr Inputs, anders Design der Modelle oder gänzlich andere Modelle die Prognoseergebnisse weiter verbessern könnten.

1 Introduction

Money as a medium of exchange first started to evolve from the discrepancy of time between the process of buying and selling goods in the wake of increasing division of labour. Trading goods against goods was no longer necessary as the value of it could be converted into a universally valid form of money. Through parallel evolution of different societies, many different kinds of currencies emerged, making trading between these societies difficult. The values of currencies were long pegged to the gold standard, until the Bretton-Woods System was introduced which meant all participating currencies were fixed to the US-Dollar. Only from the 1960s when capital controls were started to be lifted, currencies were allowed to float freely, influenced only by market dynamics (Baldwin and Wyplosz, 2015).

In recent years the total volume of trading in foreign exchange markets increased drastically. According to the most recent report of the International Bank of Settlements from 2019, the average daily turnover in April of that year was $6.6 trillion. This shows an increase of nearly 20% over the last three years from $5.3 trillion (*BIS Triennial Central Bank Survey: Foreign exchange turnover in April 2019*, 2019). It is undoubtedly considered the biggest and most liquid of all financial markets, with trading going on nearly 24 hours a day.

Due to the ongoing internationalisation of businesses many commercial and financial entities are at least partly involved in the foreign exchange market. As a result these entities are exposed to foreign exchange risk when operating outside their domestic currency area. This makes understanding and predicting the movements of foreign exchange rates essential for market participants.

The movements of foreign exchange rates have been a highly debated topic in economic research. Over the decades different approaches have been made to find underlying patterns in exchange rate fluctuations in order to improve forecastability. With fundamental models only able to predict long term trends in foreign exchange rate movement, the applications in a real life day to day use case are very limited.

It was long assumed that exchange rate movements followed a random pattern, meaning future values were independent of their past values. This resulted in the assumption of the Random Walk. However recent research indicates that dependencies between past and future values do indeed exist, which would justify trying to forecast foreign exchange rates as these dependencies can be modelled. Although it has been up to debate whether these dependencies are of linear or non-linear fashion, recent evidence suggest non-linearity and thus a plenitude of forecasting models have been taken into consideration when trying to forecast exchange rate movements. Lastly, through the wide and easy access to large amounts of financial data and with growing capabilities of computational power, computer-assisted analysis of foreign exchange rates has considerably grown in relevancy.

In this thesis I want to explore whether a simple statistical or machine learning model can improve on the short term forecasting accuracy of the Random Walk model. For that I will employ a linear Autoregressive Integrated Moving Average model (ARIMA) and a non-linear k-Nearest Neighbour Regression model. With the help of these two techniques, I will try to forecast the daily changes of foreign exchange rates in a univariate one-step ahead approach. This research will also try to give insight into the differences of forecasting accuracy between the statistical model and the machine learning model. The algorithms will be trained with fourteen years of daily foreign exchange rate price data of the EURUSD, EURGBP and EURJPY currency pairs and the results will be compared using accuracy scores derived from observed forecasting errors.

The remainder of this thesis is structured as follows. Chapter 2 highlights the existing literature on exchange rate and financial forecasting and introduces basic concepts of time series analysis and modelling. Chapter 3 establishes the data and approach taken for the empirical part of this thesis. Further, the requirements for the forecasting models are introduced, as well as the models themselves. Chapter 4 shows the forecasting results and the corresponding accuracy scores. The forecasts are plotted against the actual values to visualize the forecasting performances. Further, an interpretation of the results is given. Chapter 5 reiterates the results and puts them into context with the research question. On top of that, the limitations of the research are examined and possible ways to improve on them are introduced. Finally, Chapter 6 concludes the study with a summary of the results and the implications and limitations of the thesis. It also provides some suggestions and outlook for future research.

2 Fundamentals and Related Work

This chapter highlights the theories and assumptions made by other researchers and studies on various attempts to explain, model and forecast the movement of foreign exchange rates, with a focus on the Random Walk theory and ways to improve on it. A introduction to time series analysis is given by focusing on the modelling of stochastic processes with the use of Autoregressive and Moving Average processes. A brief introduction to the k-Nearest Neighbour Regression model is also included, with a short background of its previous applications in financial time series forecasting.

2.1 Fundamentals of Foreign Exchange Rates

In this section the main fundamental determinants of foreign exchange rates used in economic research are described, as well as the possible risks in the foreign exchange market.

Purchasing Power Parity (PPP)

The principle of Purchasing Power Parity is probably the most basic model in determining foreign exchange rates. It is assumed that if the same identical product or service can be sold in two different markets and no constraints exist on the sale or transport of said good, the product's price should be the same in both markets(Moffett et al., 2014). Assuming the so called law of one price holds up, we can calculate the long term exchange rates between two currencies, by comparing the prices of the same goods between two countries with different currencies.

Relative Purchasing Power Parity relaxes the assumptions of PPP and states that if a spot exchange rate of two countries starts in an equilibrium, the changes in the inflation rate between them are cancelled out by an equal but opposite shift in the spot exchange rate in the long run (Moffett et al., 2014, pp. 140-141).

The PPP is calculated by

$$\frac{S_{t+1}}{S_t} = \frac{1+\pi^{\$}}{1+\pi^{€}}. \tag{2.1.1}$$

Where S_t is the spot exchange rate in time period t and S_{t+1} the exchange rate in time period $t+1$. $\pi^{\$}$ and $\pi^{€}$ describe the rate of inflation of the U.S. Dollar and Euro respectively.

As an example, if the inflation rate of the United States is 3% higher than in Japan, relative PPP predicts that the yen will appreciate by 3% per year in respect to the U.S. Dollar.

It should be noted that, for the most part the model does hold up quite well for long term predictions and for countries with relative high rates of inflation and underdeveloped capital markets but not for short term predictions (Rogoff, 1996).

International Fisher Effect (IFE)

The International Fisher Effect (IFE), developed by Irving Fisher, shows a correlation between national interest rates and foreign exchange rates. He argues that the real interest rate tends to stay the same over time. This means that when inflation rises or falls, so do nominal interest rates in a way that the real interest rates stays the same. Thus, foreign exchange rates are expected to only variate by the expected change in nominal national interest rates. Assuming unrestricted capital flow, investors invest their money in the markets where interest rates are higher, therefore increasing demand in that currency. This results in an increase in value of the currency and a devaluation of the 'opposing' currency. Therefore, differences in interest rates between national markets will be erased by adjusting exchange rates (Collinson et al., 2017).

The exact formulation of the international Fisher Effect looks like this:

$$\frac{S_{t+1}}{S_t} = \frac{1+i^{\$}}{1+i^{€}}. \tag{2.1.2}$$

Where $i^{\$}$ describes the national interest rate of the U.S. and $i^{€}$ the national interest rate of the Eurozone.

2.1.1 Foreign Exchange Risk

Companies and investors conducting business in an international environment are almost always exposed to foreign exchange risk. Being able to plan is crucial in a business environment as decisions have to be made ahead of time. Being exposed to exchange rate risk creates the challenge to offset losses created by exchange rate fluctuation. There are three different types of exchange risk exposure: ’Transaction exposure’ describes the potential risk of a shift of exchange rates between the date of origin and the due date of a financial obligation, therefore affecting cash flows. ’Translation exposure’ describes the potential risk which occurs when multinational entities have to prepare consolidated financial statements. As the value of equity owned by a foreign subsidiary, which is most likely denominated in a different currency, has to be translated into a single reporting currency. ’Operating exposure’ describes the potential risk of a change in the present value of a firm caused by exchange rate changes in the future. This can have a negative influence on future sales volume, prices and costs. (Moffett et al., 2014, pp. 224 - 225). In summary, exchange rate risk can be defined as the possibility that a company will not be able to adjust its prices and costs to exactly offset variations in the exchange rate (Collinson et al., 2017).

One widely used method to counteract these exposures is called ”hedging”. Hedging is an attempt to minimize foreign exchange exposure by taking a position - an asset, a contract, or a derivative - that will move in the opposite direction than the exposed value. The result is a reduction of the potential variance of expected cash flows for an entitiy. However this also means that any advantages that could arise from varying exchange rates are eliminated as exchange rates can also move in a way that would potentially increase the value of the exposed position (Moffett et al., 2014).

To conclude, actors on the foreign exchange market have a big incentive to minimize exchange risk. This can be accomplished by hedging the exposed positions or trying to forecast future exchange rate movements and act accordingly.

2.2 Efficient Market Hypothesis and Random Walk

In the 1960s Eugene Fama was the first person to suggest the ’Efficient Market Hypothesis’ (EMH). He proposed that in financial markets, security prices fully reflect all available information. (Fama, 1965*a*). Following that, the conditions for the EFM in the foreign exchange market are: 1) All relevant information can be accessed fast and easy; 2) Transaction costs are low; and 3) Financial instruments denominated in different currencies are perfect substitutes for one another (Moffett et al., 2014, p. 154)

> ” An ’efficient’ market is defined as a market where there are large numbers of rational, profit-maximizers actively competing, with each trying to predict future market values of individual securities, and where important current information is almost freely available to all participants. [...] In an efficient market at any point in time the actual price of a security will be a good estimate of its intrinsic value” (Fama, 1965*b*, p. 56)

Although Famas research had the stock market as underlying data, his assumptions can also be applied to the foreign exchange market as the fashion of the movement is quite similar (Andrieș et al., 2014; Bahmani-Oskooee and Sohrabian, 1992). As a result, it can be argued that prices evolve randomly. This is called the ”Random Walk Theory” which states that the movement of foreign exchange rates do not adhere to any patterns. Fama suggests that the changes in market rates have no memory, thus the past history of the time series cannot predict the future accurately (Fama, 1965*b*).

This idea was further developed and popularized by Malkiel (1973) in his book ”A random walk down Wall Street”, in which he argues that the technical analysis of price movements of stock does not work and compares the movement to being as predictable as flipping a coin. Further work was done by Meese and Rogoff (1983*a*) in which they try to beat the Random Walk forecast with structural models like the flexible-price and the sticky-price models. As a result they could not outperform the Random Walk forecast for the short term movements of exchange rates. In a later paper of the same authors they could only show that structural models could outperform a Random Walk model for horizons beyond twelve months (Meese and Rogoff, 1983*b*). This has led to the Random Walk model often being the benchmark for verifying and examining the accuracy of various foreign exchange rate forecasting models (Taylor, 1995).

In contrast to the assumptions of the Efficient Market Hypothesis made above, we have to look at the behaviour of the market participants who, in the end, set the price by asking and bidding. If investors and other market participants would think that the market adheres to the EHM, they would not be investing, as the possibility of turning a profit becomes very slim. In reality markets are neither completely efficient nor completely inefficient, thus more knowledgeable investors can strive to outperform less knowledgeable ones.

This is further supported by the plenitude of foreign exchange analysts who favour 'chartists' methods over a random walk model, by trying to identify supposedly reoccurring patterns in the paths of exchange rate movements (Taylor, 1995; Goodhart, 1988). The sheer amount of technical analysts , trying to model exchange rate movements, alone can be used as an argument in favour of the possibility to outperform the Random Walk model.

Another method to explain exchange rate movement is the fundamental approach. Fundamental methods incorporate macroeconomic models and variables, like the ones mentioned in Section 2.1. Despite not showing great success in short term exchange rate forecasting, they can hold up for identifying long term trends (Mussa, 1984; Taylor, 1995; Meese and Rogoff, 1983*a*).

To conclude, in the last two decades the assumption that foreign exchange rates follow a Random Walk has more or less been dismissed by empirical work as there is strong evidence that exchange rate movement is not independent of past changes. With the rising popularity of more sophisticated models that can analyse and model non-linear patterns in foreign exchange rates recent evidence has clearly shown that the assumption of independence can be rejected (Tenti, 1996)

2.3 Forecasting Models

The foreign exchange market is as a highly dynamic and complex system, with exchange rates considered to be inherently noisy and non-linear (Abu-Mostafa and Atiya, 1996). These characteristics, combined with the correlation of economic, political and psychological factors, has made foreign exchange rate prediction one of the most challenging applications in financial forecasting (Fernandez-Rodrıguez et al., 1999).

Statistical and regressional models have been used to forecast financial data since their early development. With financial data often being accumulated in a discrete timely manner, models built to work with time series data were favoured. In the 1970s Box & Jenkins introduced their so called Box Jenkins model (Box and Jenkins, 1970) which included Autoregressive (AR) and Moving Average (MA) models or a combination of both (ARMA). For non stationary time series they proposed the ARIMA model which is an 'integrated' ARMA model. Examples of ots applications include forecasting stock prices (Mondal et al., 2014) and forecasting the price of gold (Guha and Bandyopadhyay, 2016).

Especially the last three decades have seen an increase in the use of Autoregressive Moving Average models in exchange rate forecasting. The general approach is often similar in the way that foreign exchange rate data of one or a few currency pairs is fit to an ARIMA model and then a forecast is made which accuracy is then evaluated. A similar approach as mine in this thesis was made by Pathirana (2015) where ARIMA models are compared to the k-NN model. His findings are that they both perform better than the Random Walk and the k-NN better than the ARIMA model. These distinctions will also be examined in the remainder of this thesis. Further, some papers try to focus on the Box Jenkins approach and 'model mine' in order to find the best ARIMA order (Meyler et al., 1998; Tlegenova, 2015; Appiah and Adetunde, 2011). While others focus on applying many different statistical models on the data in order to find the one which makes the best forecast (Akincilar et al., 2011; Shittu and Yaya, 2009). Further, ARIMA models have been used in combination with other forecasting methods with the resulting approach often called 'hybrid ARIMA' (Pai and Lin, 2005). Especially promising have been the results of studies which incorporated an ARIMA model combined with machine learning models (Kapila Tharanga Rathnayaka et al., 2015; Kia et al., 2012).

The k - Nearest Neighbour (k-NN) algorithm is one of the simplest and most widely applied algorithms in machine learning. It can be used as a classification method with discrete labels and as a regression model with continuous labels. In this thesis I am going to focus on the k-NN Regression algorithm as it serves a better fit for time series forecasting. The idea of using the k-NN method in a regression context was first introduced by Stone (1977) and further developed by Yakowitz (1987). Although its simplicity, the k-NN method has been shown to have good performance when forecasting foreign exchange rates, especially compared to linear forecasting models (Fernandez-Rodrıguez et al., 1999) as it is able to more accurately depict non-linear dependencies. It was also used in particular for short term forecasting by Meade (2002), where it showed mixed results.

Lastly, due to rapid developments in the field of Machine Learning, applications in financial forecasting have become quite popular. Good results could be achieved in areas that have been seen as difficult. Especially Artificial Neural Networks are strong in modelling non-linear time series and therefore almost always outperform statistical models (Zhang and Hu, 1998; Wu, 1995; Tenti, 1996), as well as the random walk model (Weigend et al., 1991) when it comes to forecasting foreign exchange rates. However, these algorithms are quite complex and difficult to administer therefore incorporating them into the research would go beyond the scope of this thesis.

2.4 Fundamentals of Time Series

A time series can be described as a collection of observations made sequentially through time (Chatfield, 2000, p. 6) and finds application in many different fields such as agriculture, tourism, sales and economics. In time series data, the order of the obtained observations is taken into account as they are measured with time as a second describing variable.

In order to better understand the analysis of time series it has to be understood, that future data that can be predicted *exactly* from its past values is called deterministic. However in this chapter I am focusing on stochastic and random series, meaning that future values are only partly determined by past ones. Thus, this model for time series is often called a stochastic process (Brockwell and Davis, 2009):

$$\{X\} = X_1, X_2, X_3, \ldots, X_t, \ldots \text{ or } \{X_t, t \in T\} \tag{2.4.1}$$

Where each X_t is a random variable and each observation x_t is a realization of it at time t. This means that the time series of observed values is a realization of the stochastic process. Therefore it can be argued that the goal of time series data analysis and forecasting is to discover and estimate the behaviour of the underlying stochastic process. For this, a class of possible processes has to be selected *a priori*. More on that in section 2.5

In classical Time Series analysis we can decompose these variations into different components:

Trend

A trend describes the steady upward or downward movement of a time series over at least several successive time periods and can be loosely defined as ”long-term change in the mean level” (Chatfield, 2000, p. 18). In mathematical terms a local (as opposed to global) linear trend is defined as

$$\mu_t = \alpha_t + \beta_t t \tag{2.4.2}$$

where α_t is the local intercept, and β_t the slope at timepoint t.

Seasonal Variation

This variation describes a pattern of behaviour that occurs at a particular time of the year and repeats itself every year. An example would be that the sales of skiing equipment generally goes up in winter. We can distinguish between additive and multiplicative seasonality. If seasonal variation becomes larger over time, it is usually multiplicative. Differentiating between these two is important when trying to remove seasonality.

Irregular Fluctuations

This describes any residual variation which is left over after seasonality, trend and other systematic effects have been removed. They often act in a random way so that they cannot be forecast.

2.5 Stochastic Processes

As mentioned before, in order to be able to describe, analyse and forecast a time series, the underlying process has to be understood. To facilitate this, it makes sense to choose a process where each X_t has a similar distribution, meaning the process is *stable* or more precise, *sationary* over time. Generally we can say a stochastic process $\{X_t\}$ is second order stationary if

$$Cov[X_t, X_{t+k}] = E[(X_t - \mu)(X_{t+k} - \mu)] = \gamma_k. \tag{2.5.1}$$

Where the autocovariance γ_k only depends on the lag k and not on t and where μ is a finite constant equal to $E[X_t]$. Thus, second-order stationarity is sometimes called covariance stationarity. The set of autocovariance coefficients $\{\gamma_k\}$, with k = 0, 1, 2, …, form the *autocovariance function* of the stochastic process.

Standardizing the autocovariance function, leads to a *autocorrelation function* which consists of a set of *autocorrelation coefficents*, $\{\rho_k\}$, given by:

$$\rho_k = \frac{\gamma_k}{\gamma_0} \tag{2.5.2}$$

for k = 0, 1, 2, …with the property $| \rho_k | \leq 1$ (Chatfield, 2000).

Autocorrelation refers to the correlation between its own past and future values. As a result stationary processes have great value because only if the stationarity conditions apply, are we able to estimate the first and second moments of a time series(Schira, 2016).

2.5.1 Moving Average Processes

In order to understand Moving Average (MA) Processes we have to introduce the White Noise processes first, as it is often the foundation of other stochastic processes. It consists of a sequence of uncorrelated, identically distributed random variables ε_t:

$$\{\varepsilon_t\} = \varepsilon_1, \varepsilon_2, \varepsilon_3, \cdots, \varepsilon_t, \cdots . \tag{2.5.3}$$

It is stationary with constant variance functions, and mean and autocovariances being zero.

A moving average process is defined by forming the Moving Average over a White Noise process. In contrast to the white noise process, random variables that follow each other are not independent in a MA process. A moving average process $\{X_t\}$ of the order q is defined as:

$$X_t = \alpha_0 + \varepsilon_t + \alpha_1 \varepsilon_{t-1} + \ldots + \alpha_q \varepsilon_{t-q} \tag{2.5.4}$$

Where α_i are the parameters and $\{\varepsilon_t\}$ a white noise process with constant variance $V(\varepsilon_t) = \sigma_\varepsilon^2$. As we can see, the expected value is constant and equal to α_0. Therefore the expected value and the variance are not dependant on the time resulting in the MA(q) process being stationary as the sequence of autocovariances terminates for lags $k > q$ (Chatfield, 2000; Schira, 2016)

2.5.2 Autoregressive Processes

Autoregressive (AR) Processes form a class of models quite important for forecasting time series data. They are constructed from the weighted linear sum of the past p values and a white noise process. A process $\{X_t\}$ is called an autoregressive process of order p (AR(p)) if

$$X_t = \beta_0 + \beta_1 X_{t-1} + \beta_2 X_{t-2} + \cdots + \beta_p X_{t-p} + \varepsilon_t. \tag{2.5.5}$$

Where β_i are constant parameters or ”weights” and $\{\varepsilon_t\}$ a white noise process. Therefore the value at time t depends linearly on the last p values. When checking for stationarity we assume the following equation:

$$\lambda^0 = \beta_1 \lambda^1 + \beta_2 \lambda^2 + \cdots + \beta_p \lambda^p \tag{2.5.6}$$

This is a p-th order polynomial which consists of the unknown value λ and is modelled after the original AR(p) process equation. Parameter β_0 is left out, as it is not important when examining for stationarity. This polynomial has p roots and it can be shown that the corersponding AR(p) process is stationary if all roots $\mid \lambda \mid > 1$ lie outside the unit circle (Chatfield, 2000; Schira, 2016). As an example an AR(1) process with $\beta_1 = 1$ can be viewed as a random walk. The process reduces to

$$X_t = \beta + X_{t-1} + \varepsilon_t \tag{2.5.7}$$

hence it is a series of random variables. With $\beta \neq 0$ it is considered a random walk *with drift*, which means the expected value will grow (sink) with positive (negative) β without limit. As a result we can argue a Random Walk with drift is not stationary. Equation 2.5.7 with $\beta = 0$ can also be called a *Brownian motion*, named after botanist Robert Brown. Although its mean is stationary, its variance increases in linear fashion with the distance from the beginning of the process, therefore also not being stationary.

2.5.3 ARMA Processes

The Autoregressive Moving Average Process (ARMA(p, q)) is a combination of an Autoregressive process of the order p and a Moving Average process of the order q and is denoted with:

$$\begin{aligned} X_t = \beta_0 + \beta_1 X_{t-1} + \beta_2 X_{t-2} + \cdots + \beta_p X_{t-p} + \varepsilon_t \\ + \alpha_1 \varepsilon_{t-1} + \alpha_2 \varepsilon_{t-2} + \cdots + \alpha_q \varepsilon_{t-q} \end{aligned} \tag{2.5.8}$$

Basically it consists of equations 2.5.4 and 2.5.5 combined. Thus it can be described as an Autoregressive process whose White Noise process is not uncorrelated (Schira, 2016). As a result, an ARMA(p, q) process is stationary, if its AR part is stationary, since its MA part is anyway.

AR, MA, and ARMA models are all models that can give forecasts with good accuracy but only for stationary time series. In practice, time series are rarely stationary. Especially when looking at financial data, as they are often subject to certain variations explained at the beginning of this section. In order to be able to make accurate forecasts using non stationary time series, we have to introduce another model.

2.5.4 ARIMA Processes

Autoregressive Integrated Moving Average (ARIMA) processes describe a stochastic process which makes it possible to handle non stationary time series data. The foundation for it is the ARMA(p, q) process and its AR(p) and MA(q) parts. In order to achieve stationarity, the data is *differenced* with $(X_t - X_{t-1}) = (1 - B)X_t$ being the first difference. Then the second difference is taken meaning the difference of the difference. This process goes on until stationarity is achived. The d-th difference is written as $(1 - B)^d X_t$ and is then fitted to an ARMA(p, q). Therefore the model for the undifferenced series becomes an ARIMA(p, d, q) process where d is the order of differencing (Box and Jenkins, 1970).

A special case of ARIMA processes is the ARIMA(0,1,0). It is called the Random Walk process and boils down to the same equation as Equation 2.5.7. It will be used as a benchmark in the empirical part of this thesis.

2.6 K - Nearest Neighbour Regression

The intuition behind the k-NN Regression approach is that patterns of behaviour generated by the underlying process are expected to repeat themselves and can therefore be identified by looking at neighbouring observations. If a previous pattern of behaviour is similar to current behaviour, then the previous subsequent behaviour can be used to forecast the behaviour in the immediate future. More formally:

> "In the nearest neighbour approach, the time series, x_t, is viewed as a sequence of d histories, X_t^d. These are vectors of d consecutive observations where d is a chosen integer." (Meade, 2002, p. 73)

The algorithm is considered a non-parametric method, meaning it does not make an assumptions on the distribution of the underlying data as well as being labelled as a lazy learning method, meaning the learning done by the k-NN model, is only made - in theory - when confronted with a query.

Without going into too much detail how the k-NN algorithm works, a few basic assumptions have to be made. In general, as for many machine learning models, each data point has one or more features, which can be seen as 'input' variables and are sometimes called predictors. Each data point also has a label, which can be seen as 'output' variable, this is what we want to forecast. In a neighbours-based regression a label assigned to a query point is calculated by taking the mean of the labels of its neighbours while taking the distance to other points and their corresponding features into account.

There are three main ways to calculate these distances:

Euclidean Distance

One of the most widely used distance metric is the euclidean distance.

$$D_{Eu}(x, y) = \sqrt{\sum_{i=1}^{k}(x_i - y_i)^2} \tag{2.6.1}$$

Where k is the number of neighbours and x and y are the data points with i features of which the distance D is being calculated.

Manhattan Distance

The manhattan distance is a distance metric where the distance of two points is the sum of the absolute differences.

$$D_{Man}(x, y) = \sum_{i=1}^{k} \mid x_i - y_i \mid \tag{2.6.2}$$

Minkowski Distance

The Minkowski distance is a generalization of the two distances mentioned above.

$$D_{Mink}(x, y) = (\sum_{i=1}^{k}(\mid x_i - y_i \mid)^q)^{1/q} \tag{2.6.3}$$

where $q > 0$. When $q = 2$ it is equal to the Euclidean distance and when $q = 1$ it is equal to the Manhattan distance.

3 Methodology

In this chapter I want to discuss the methods I apply in my empirical research. This includes an overview of the preparation and preprocessing of my data followed by the introduction to the assumptions I make. Further an explanation of the programming will be given.

3.1 Source of Data

The data for the nominal exchange rates in the following analysis was downloaded from Bloomberg.

The data set contains the closing prices for the currency pairs EURUSD, EURGP and EURJPY in the period from 09.01.2006 to 09.01.2020. These are depicted by 3654 data points per currency pair with a daily sample rate. It has to be noted, that the sample rate represent every trading day which is 5 days a week, thus on weekends there is always a gap of two days. This should not be too big of a concern as trading is just 'halted' over the weekend and resumed on Mondays therefore no interference can be expected in the data. The foreign exchange rate data is the only data used in this thesis which means that an univariate approach is taken on the forecasting.

3.2 Box Jenkins Method

For the further preparation and analysis of the data and for finding the best fitting ARIMA model, I am going to loosely follow the so called Box Jenkins method as seen in Figure 3.1 introduced by Box and Jenkins (1970). This ensures the comprehensibility and reproducibility of the analysis and research.

We start our analysis by plotting the EURUSD exchange rate in Figure 3.2. The plots for the EURGBP and EURJPY exchange rates can be found in the appendix in Figures A.1 and A.2. In the following analysis I will generally only show plots and figures of the EURUSD exchange rate as it would

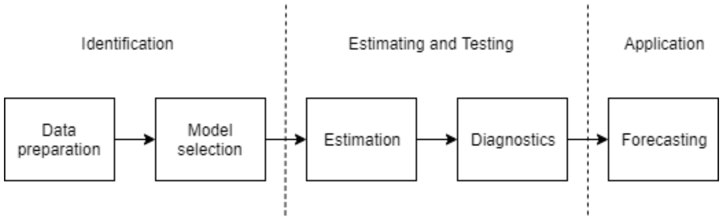

Figure 3.1: The Box Jenkins method

otherwise take up too much space. Subsequently all other following plots and figures for EURGBP and EURJPY can be found in the appendix.

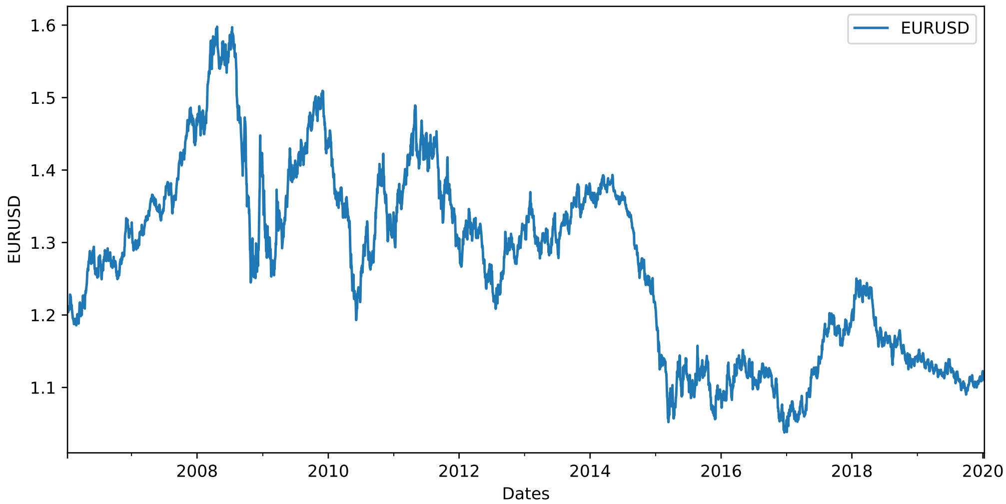

Figure 3.2: The daily EURUSD exchange rate from 09.01.2006 to 09.01.2020.

When observing the plot, we can see that the exchange rate fluctuates considerably and seems to show non-stationarity. In order to get a better first look and start understanding the underlying data we can decompose the time series into its different components discussed in Section 2.4. We can see in Figure 3.3 that there is next to no underlying seasonality but a trend which looks like the original plot and residual fluctuations. Thus supporting the assumption that the data is non-stationary.

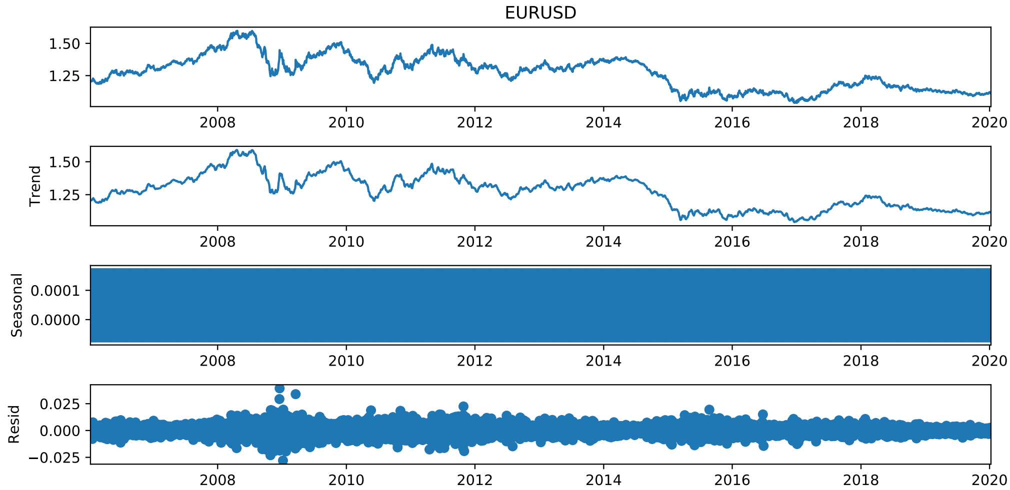

Figure 3.3: Plots showing the EURUSD exchange rate decomposed into trend, seasonal variation, and residual.

3.3 Autocorrelation and Partial Autocorrelation

Following the Box Jenkins method laid out in section 3.2 we have to ensure that the data is compatible with the chosen models. We can look at the autocorrelation and partial autocorrelation functions in order to better understand the underlying data and its dependencies especially regarding stationarity.

The autocorrelation function (ACF) is used when the dependencies between X_t and its past values X_{t-i} are of interest. The autocorrelation coefficient between X_t and X_{t-j} is denoted as ρ_j where j is the number of lags and is defined as:

$$\rho_j = \frac{Cov(X_t, X_{t-j})}{\sqrt{Var(X_t)Var(X_{t-j})}} \tag{3.3.1}$$

Where under the assumption of weak stationarity $\rho_0 = 1$, $\rho_j = \rho_{-j}$ and $-1 \leq \rho_j \leq 1$. In addition, a weak stationary series is not serially correlated if $\rho_j = 0$ for all $j > 0$.

The partial autocorrelation function (PACF) is considered conditional correlation as it adjusts the ACF for the presence of all other terms of shorter lags $(X_{t-1}, X_{t-2}, \cdots, X_{t-k-1})$. Since the ACF is comprised of both a direct correlation and a indirect correlation. The PACF seeks to remove the indirect correlation which is a linear function between the correlation of the observation and the observations at time steps in between (Tsay, 2005).

Observing the ACF and PACF plots in Figure 3.4, it can be seen that the autocorrelation function decreases very slowly and all observed values of lags j are within the 95% significance level. This means that there are so many 'significant' values, that modelling the data - as is - will be problematic and is more evidence, that the time series is not stationary (Chatfield, 2000). Although this also indicates the possibility of the underlying data simply being a random walk, we want to continue the analysis and will see in Chapter 4 whether a Random Walk process accurately describes the data.

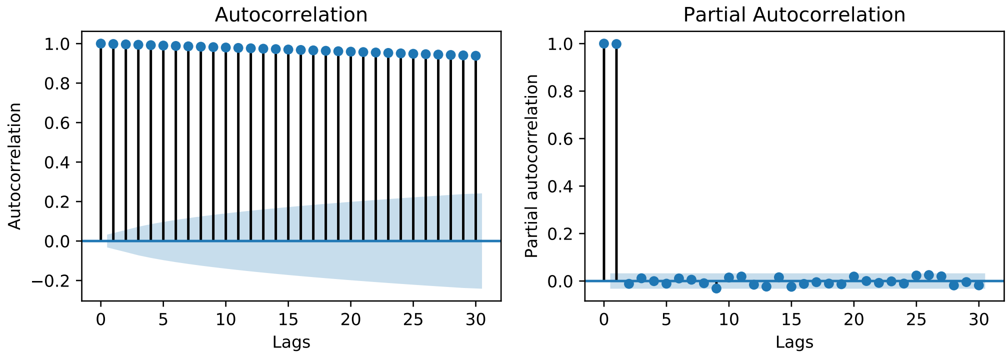

Figure 3.4: The autocorrelation and partial autocorrelation plots of the EURUSD exchange rate with lags $j = 30$ and the corresponding 95% confidence interval (blue)

3.4 Augmented Dickey-Fuller Test

In order to make a definite statement on stationarity we apply the augmented Dickey-Fuller (ADF) test. An augmented Dickey-Fuller test tests the null hypothesis whether a unit root is present in a time series. It belongs to the category of tests called "Unit Root Test" which was briefly mentioned in section 2.5.2. The null hypothesis of the test is H_0: $\beta = 1$ which is tested against the alternative H_1: $\beta < 1$ with the regression

$$X_t = c_t + \beta X_{t-1} + \sum_{i=1}^{p-1} \Delta X_{t-i} + \varepsilon_t. \tag{3.4.1}$$

Where c_t is a deterministic function of time t and $\Delta X_j = X_j - X_{j-1}$ is the differenced series of X_t. Therefore, the ADF-test is the t-ratio of $\hat{\beta} - 1$ which is expressed as

$$ADF - test = \frac{\hat{\beta} - 1}{\hat{\sigma}_\beta}. \tag{3.4.2}$$

Where $\hat{\beta}$ is the least-squares estimate of β. If the null hypothesis is rejected, the time series is stationary (Tsay, 2005). When applied to the exchange rate data set, it can be seen in Figure 3.1 that the p-value is greater than the 5

% significance level, thus the null hypothesis can not be rejected and we conclude that the time series is not stationary.

Augmented Dickey-Fuller test	
Test Statistic	-1.591
p - value	0.488
Critical value (1%)	-3.432
Critical value (5%)	-2.862
Critical value (10%)	-2.567

Table 3.1: Augmented Dickey Fuller test of the daily EURUSD exchange rates.

3.5 Data Transformation

As discussed in Chapter 2 the accuracy of most forecasting models increases when they are confronted with stationary time series. This is not only the case for the statistical forecasting models but also for the k-NN Regression. Since the forecasts of the k-NN Regression are the average target values in the historical data, the model is not able to predict values that lie outside the range of historical observations. To make the data stationary the log-return r_t is taken, which is defined as:

$$r_t = ln\frac{X_t}{X_{t-1}} \tag{3.5.1}$$

where X is the exchange rate in period t or $t-1$. This also makes the series easier to handle, as return series have more attractive statistical properties (Tsay, 2005).

Plotting the autocorrelation and partial autocorrelation functions of the log return series in Figure 3.5 we can see that both the ACF plot and PACF plot have a spike at lag $j = 1$. They then cut off for lags $j > 1$ with values not being greater than the 5% significance interval thus not being significantly different from zero. Comparing this to the plots in Figure 3.4, it can be seen that the slow decay of the ACF is removed, thus the series of log returns seems stationary. To confirm this assumption, the ADF test is applied again.

When testing the log return time series for stationarity with the ADF test we can see in Table 3.2 that the null hypothesis can be rejected with the p-value smaller than the 5% significance level, thus the time series of log returns is de-trended and stationary. This will be the base time series for further analysis and forecasting.

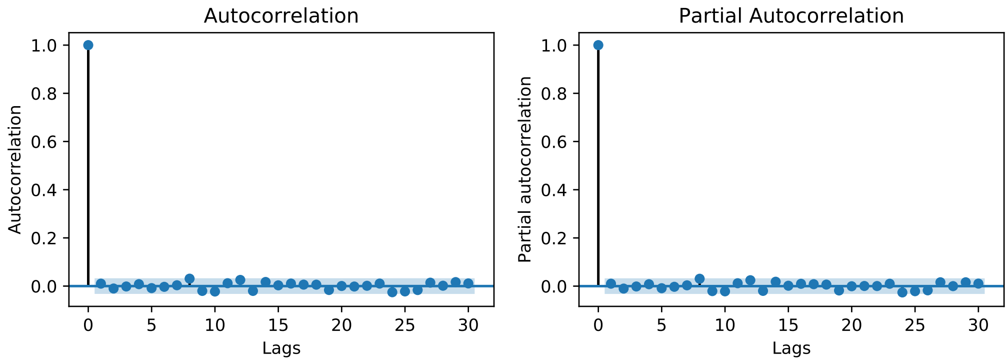

Figure 3.5: The autocorrelation and partial autocorrelation plots of the log return EURUSD exchange rate with lags $j = 30$ and the corresponding 95% confidence interval (blue)

Augmented Dickey-Fuller test	
Test Statistic	-59.806
p - value	0.000
Critical value (1%)	-3.432
Critical value (5%)	-2.862
Critical value (10%)	-2.567

Table 3.2: Augmented Dickey Fuller test of the log return EURUSD exchange rates.

3.6 Implementation

All data analysis, visualization and forecasting is done in `Python`. For the data analysis the libraries `pandas`, `numpy`, `statsmodels`, and `sklearn.preprocessing`, for visualization the libary `matplotlib` and for forecasting the libraries `sklearn`, `pmdarima` and `statsmodels` are used.

3.7 Model Selection and Preparation

This section focuses on the implementation of the forecasting models and it is explained how their parameters are chosen.

3.7.1 Data Preparation and Forecasting Implementation

When using any statistical or machine learning forecasting model, the intuition is that for the model to make a forecast it has to 'learn' the potential patterns of the underlying data. This includes splitting the data set into a *training* and a *testing* set. The training set is fit to the model, so that the algorithm can learn, whereas the test set is for testing the accuracy of the model by comparing it to the forecasting results. For this research the training set is 80% and the testing set is 20% of the complete data set. The data is not randomized during splitting as the data is time dependent. This ratio is chosen as it gives a lot of data to the model to train with and limits the time that the forecasting process takes, while not infringing on the forecasting accuracy.

The forecasts are done in a *single-step* or *one-step* ahead fashion, meaning the forecast value $\hat{X}_t$ is understood as a prediction of X_t, given all the information up to t. In this case, the forecast is always only made for the next day while the forecast for the day after that already incorporates the true value of the last day. This ensures that the one-step ahead forecast uses all available data up to that point. This reflects a possible real life application in short term exchange rate forecasting, by being able to apply the results to problems of exchange rate risks discussed in Section 2.1.1.

3.7.2 Random Walk

The Random Walk is the most basic of the models applied in this research and is explained in detail in Sections 2.2 and 2.5.2. As mentioned in Section 3.3 the plot of the autocorrelation function in Figure 3.4 already provides a strong indication that the underlying process could be a random walk. Thus, the accuracy of the forecasts made by the Random Walk model will be used as a benchmark and compared to the forecasts of the ARIMA and k-Nearest Neighbour model. As explained in Section 2.5.4 the random walk process can be designed as a ARIMA$(0, 1, 0)$ process. This makes it very easy to apply, since no other AR(p) or MA(q) orders have to be found.

3.7.3 ARIMA Model

Following the Box Jenkins method finding the best ARIMA(p, d, q) order is an essential part in achieving good forecasting accuracy. Finding the right values for p, d and q follows the principle of parsimony (Box and Jenkins, 1970). This reflects the intuition of Occam's razor, meaning the simplest explanation is most likely the right one. Thus it is desired to find the best fitting ARIMA process with the smallest values for p, d and q.

This search can be made relatively easy by applying the `auto_arima` function of the `pmdarima` library. The function searches for the ARIMA order with the lowest *Akaike information criterion* (AIC) which is an estimator of the prediction error and is calculated relative to the models of different order and deals with the trade-off between goodness of fit and simplicity of the model (Akaike, 1973). In general the AIC is calculated as

$$AIC = \frac{-2}{T} \times \ln(\text{likelihood}) + \frac{2}{T} \times (\text{number of parameters}) \tag{3.7.1}$$

where the likelihood function is evaluated at the maximum-liklihood estimates and T is the sample size. The number of parameters denotes the sum of the (p, d, q) order (Tsay, 2005). For the EURSD logarithmic return series the search results in a ARIMA(0,1,3) model as seen in Figure 3.6.

```
                                ARIMA Results
==============================================================================
Dep. Variable:                      y   No. Observations:                 2922
Model:               ARIMA(0, 1, 3)   Log Likelihood               10619.515
Date:                Mon, 20 Jul 2020   AIC                         -21229.030
Time:                        22:42:08   BIC                         -21199.132
Sample:                             0   HQIC                        -21218.261
                               - 2922
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
intercept  -2.216e-06   8.74e-06     -0.254      0.800   -1.93e-05    1.49e-05
ma.L1         -0.8779      0.017    -51.003      0.000      -0.912      -0.844
ma.L2         -0.0324      0.021     -1.548      0.122      -0.073       0.009
ma.L3         -0.0218      0.017     -1.307      0.191      -0.055       0.011
sigma2       4.35e-05   7.73e-07     56.274      0.000     4.2e-05     4.5e-05
===================================================================================
```

Figure 3.6: Result of the `auto_arima` search for the best ARIMA(p, d, q) order for the logarithmic return EURUSD exchange rates.

This implies that there is no AR(p) term in the best fitting ARIMA process. Following the Box Jenkins method, the validity of the model can also be verified by looking at the residuals of the chosen model. If they behave like white noise, all correct AR(p), I(d) and MA(q) terms have been found. The results for the other currency pairs can be found in the Appendix.

3.7.4 k-NN Regression

In order to apply k-Nearest Neighbour Regression on a univariate time series we have to define the explanatory variables. In this thesis I follow the method suggested by Martínez et al. (2019) where lagged values of the time series are used. I have chosen the lags 1–5 as there are five data points per trading week. Using the lagged series we can find previous patters similar to the current structure of the series and use their subsequent patterns to possibly predict future behavior.

Further, the number of neighbors k has to be chosen. There are two main approaches in accomplishing that. The first solution would be to apply some heuristic, for example it is recommended to set k to the square root of the number of training points $k = \sqrt{N}$. This would result in $k = 54$. The other method is to select k using an optimization tool which finds the value of k with the smallest *Root Mean Squared Error* (RMSE). The result of such search can be seen in Figure 3.7 where the RMSE falls in a exponential fashion with increasing k and leveling off at around $k = 40$ indicating no further accuracy improvement with values higher than that. Taking into consideration the danger of over fitting the model and thus loosing accuracy, we choose the number of neighbours with $k = 40$.

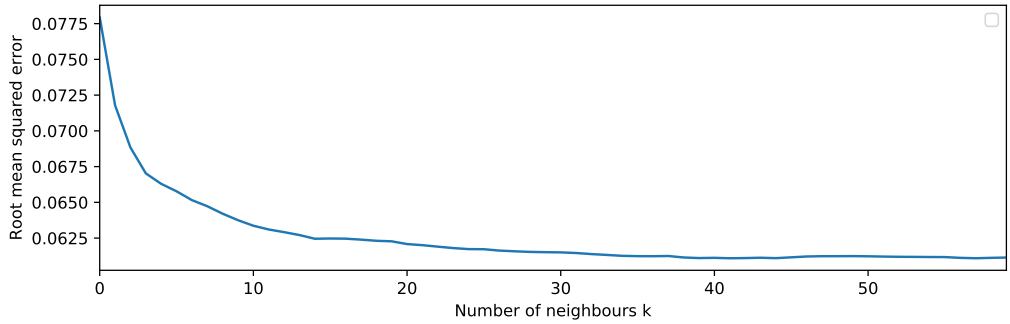

Figure 3.7: Root Mean Squared Error of the k-NN fit with $k = \{1, 60\}$

Lastly the distance metric for the k-NN model has to be chosen. As mentioned in Section 2.6 it plays a crucial role in the nearest neighbor algorithm since it determines the way the nearest neighbors are chosen. For the following forecasts, the Euclidian distance is chosen since it is suggested by Martínez et al. (2019) and the most poppular distance metric in literature.

3.8 Evaluating Forecasting Performance

In order to compare the quality of forecasts made, we have to establish ways in which the forecasting accuracy can be measured and compared. Generally, we take a look at the forecast errors which is the difference between the observed value in the testing set and the corresponding forecast:

$$e_t = X_t - \hat{X}_t \tag{3.8.1}$$

Where X_t is the actual observation in time period t and $\hat{X}_t$ is the forecast value for the same period so that e_t is the error.

This error can be modelled in different ways. In this analysis I want to focus on three main ways to compare the forecasts (Montgomery et al., 2015).

Mean Absolute Error

The Mean Absolute Error (MAE) or sometimes called the Mean Absolute Deviation, measures the variability in forecast errors and is calculated by taking the mean of the absolute values of all errors.

$$MAE = \frac{1}{n}\sum_{t=1}^{n} \mid e_t \mid \tag{3.8.2}$$

Where n is the number of observations in the test set and observed forecasts. The MAE is popular as it is both easy to understand and to compute.

Root Mean Squared Error

The Root Mean Squared Error (RMSE) also measures the variability by taking the mean of the squared error values.

$$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(e_t)^2} \tag{3.8.3}$$

The MAE and the RMSE are both scale-dependent measures, meaning their values are expressed in terms of the original units of measurement. As the input data is a logarithmic return series, the interpretability of these values is limited especially when comparing different exchange rates. In order to be able to measure the accuracy independent of the input currency we have to introduce a relative error measurement.

Mean Absolute Percentage Error

The Mean Absolute Percentage Error (MAPE), a relative - as opposed to scale-dependent - measurement, is used to measure the forecasting error based on percentage errors p_t

$$p_t = 100 \times (\frac{e_t}{X_t}). \tag{3.8.4}$$

Which results in the MAPE being

$$MAPE = \frac{1}{n}\sum_{t=1}^{n} p_t. \tag{3.8.5}$$

In order for the MAPE to give a correct measurement we have to make sure that the time series does not contain zero or near zero values as it would skew the measurements. This is not the case with the logarithmic return series, used as input for the forecasting models. Thus, we have to convert it back to the 'original' exchange rate scale. This is accomplished with

$$P_t = e^{r_t} P_{t-1} \tag{3.8.6}$$

where P_t is the value in the original scale and r_t is the value of the return series in period t. P_{t-1} is taken from the test data set as an actual observed value.

This reversion has the added advantage of increasing the interpretability and the possible real life use of the forecasting results.

4 Results

Presented in this chapter are the results attained from the forecasting models. The goal is to compare the results from the Random Walk process to the results of the ARIMA and k-NN Regression model and also the results of the ARIMA and k-NN Regression model between each other. Having applied the statistical tests on stationarity in the last chapter, the stationary logarithmic return time series is used as an input for the models for all three currency pairs. The model comparisons are then done by applying the measurements of forecasting accuracy presented at the end of the previous chapter. The MAE and RMSE are calculated from the direct results, whereas the MAPE is calculated from the results reverted into original scale.

4.1 EURUSD

The results of the accuracy scores for the EURUSD exchange rate are listed in Table 4.1. It can be seen that the ARIMA$(0, 1, 3)$ model and the k-Nearest Neighbour Regression model outperform the Random Walk in every accuracy measure. The Mean Average Error and Root Mean Square Error of the Random Walk are approximately 30 % higher than that of the comparing models. The difference can also be seen in the Mean Average Percentage Error where it is over twenty percentage points. Further, comparing the scores of the ARIMA(0,1,3) and the k-Nearest Neighbour Regression we can see that the MAE and RMSE are quite similar but the ARIMA$(0, 1, 3)$ always performs slightly better. When looking at the MAPE the difference is better illustrated with the linear ARIMA performing approximately four percentage points better than the non-linear k-Nearest Neighbour Regression

Reverting the values back to their original scale improves interpretability and comparability. Thus in Figure 4.1 the last twenty reverted forecast values are plotted against the corresponding real observations. Here the difference is even clearer, as the Random Walk deviates quite a bit from the path of real observations. Especially noticeable are the high amplitudes compared to the actual observations which do not exist for the other two models. Further, the slight performance superiority of the ARIMA$(0, 1, 3)$ over the k-NN can

	Random Walk	**ARIMA $(0, 1, 3)$**	**k-Nearest Neighbour Regression**
MAE	0.00377972	0.002859522	0.002944062
RMSE	0.00498926	0.003699132	0.003752473
MAPE	0.00286045	0.000147952	0.000556816

Table 4.1: Mean Absolute Error, Root Mean Squared Error and Mean Absolute Percentage Error calculated from the EURUSD forecasts made by the Random Walk, ARIMA$(0, 1, 3)$ and k-NN Regression models.

be better observed. The plot shows that the k-NN graph has slightly more deviations than the ARIMA graph.

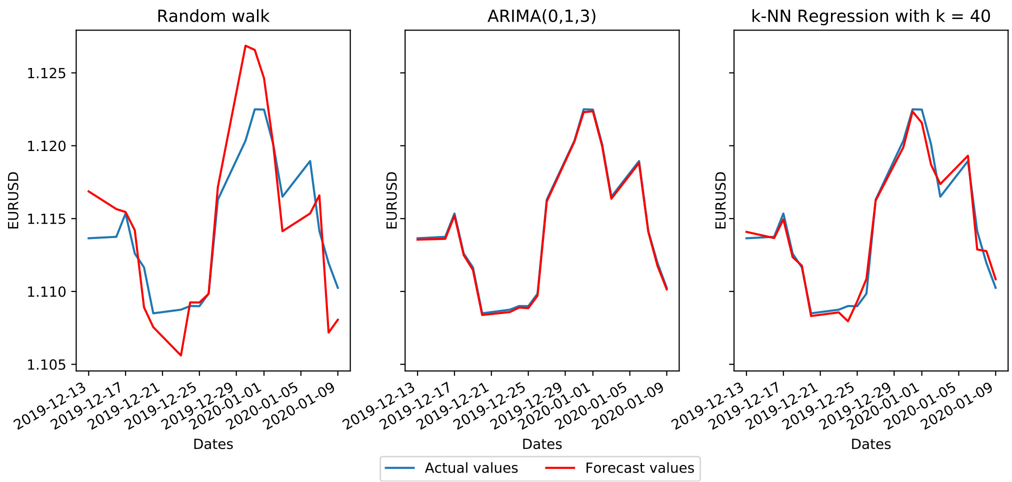

Figure 4.1: Original scale forecast values of the last twenty periods plotted against the actual EURUSD rates of the last twenty periods.

4.2 EURGBP

A similar picture to that of the EURUSD results is painted for the EURGBP results. As Table 4.1 shows, the Random Walk performs worse than the ARIMA$(1, 1, 2)$ and k-NN Regression model in every accuracy score. The difference is even greater, with the MAE and RMSE of the Random Walk approximately 50 % higher than of the comparing models. The differences in the ARIMA and k-NN model are very small but the ARIMA outperforms the k-NN again in every measure.

The original scale forecasting results for the last twenty periods are plotted against the real observations in Figure 4.2. A similar behaviour can be seen compared to Figure 4.1. Again the Random Walk deviates quite a bit from the actual values with high amplitudes being observed. To add to that the differences between the two more sophisticated models are better high-

	Random Walk	**ARIMA $(1, 1, 2)$**	**k-Nearest Neighbour Regression**
MAE	0.004926379	0.003332027	0.003348528
RMSE	0.006679597	0.004585522	0.004598941
MAPE	0.003312705	0.000266724	0.000527595

Table 4.2: Mean Absolute Error, Root Mean Squared Error and Mean Absolute Percentage Error calculated from the EURGBP forecasts made by the Random Walk, ARIMA$(1, 1, 2)$ and k-NN Regression models.

lighted, with the k-NN having larger variances from the actual values than the ARIMA(1,1,2).

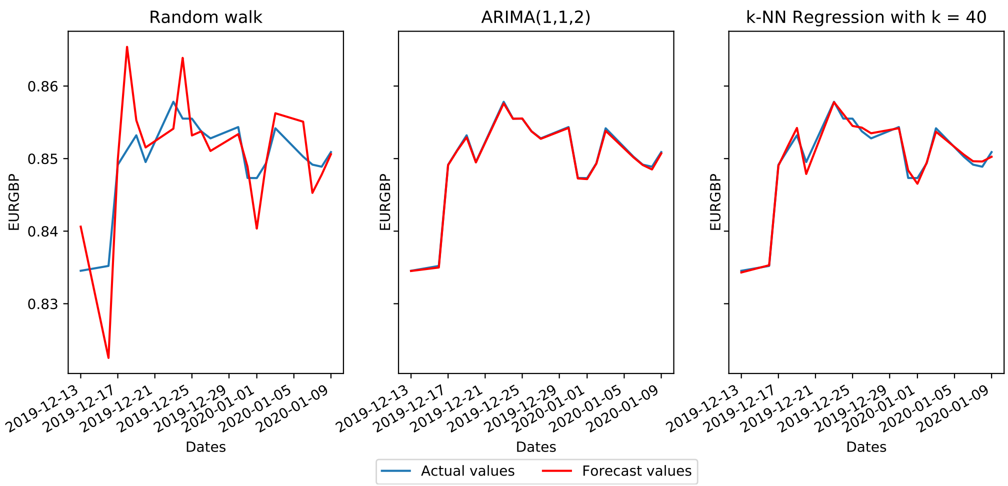

Figure 4.2: Original scale forecast values of the last twenty periods plotted against the actual EURGBP rates of the last twenty periods.

4.3 EURJPY

The EURJPY results behave very similar to the EURUSD and EURGBP results. The accuracy scores are listed in Table 4.3 where it is apparent that the Random Walk again performs worse than the ARIMA$(2, 1, 2)$ and the k-NN Regression model. With MAE and RMSE of the Random Walk a good 30 % higher. The scores for ARIMA and k-NN are very close with the ARIMA model performing slightly better again.

The same behaviour can be seen in Figure 4.3 with the Random Walk deviating with a high amplitude from the actual values and the ARIMA having a better fit than the k-NN results.

	Random Walk	**ARIMA $(2, 1, 2)$**	**k-Nearest Neighbour Regression**
MAE	0.004745888	0.003416747	0.003505657
RMSE	0.006221984	0.004520421	0.004590752
MAPE	0.003403121	0.000232314	0.000640997

Table 4.3: Mean Absolute Error, Root Mean Squared Error and Mean Absolute Percentage Error calculated from the EURJPY forecasts made by the Random Walk, ARIMA$(2, 1, 2)$ and k-NN Regression models.

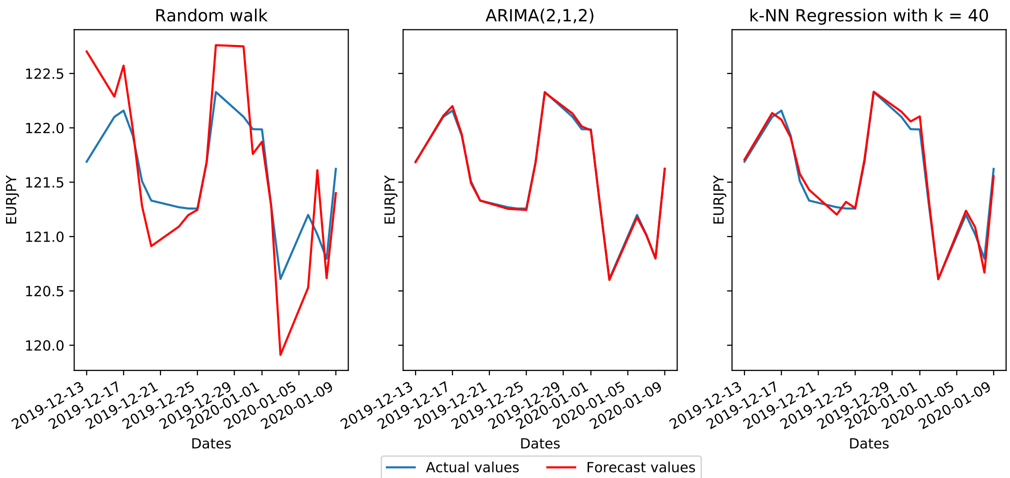

Figure 4.3: Original scale forecast values of the last twenty periods plotted against the actual EURJPY rates of the last twenty periods.

4.4 Interpretation

The results indicate that, for all three currency pairs - EURUSD, EURGBP and EURJPY - the error scores of the random walk model are higher than the ones of the more sophisticated models. This implies that the ARIMA model with the respective orders and the k-Nearest Neighbour Regression model perform better single-step forecasts with univariate inputs than the Random Walk process. This disproves the assumption that the foreign exchange rate movement is independent of its past values as suggested in the Random Walk theory and proves the existence of underlying dependencies and patterns that can be forecast.

These observations are also reflected in the plotted data, where the Random Walk strongly deviates from the actual values. Especially interesting is the amplitude of these deviations as the curve often ”overshoots” the real values. An explanation for this could be the missing MA(q) term in the ARIMA$(0, 1, 0)$ model, which is used to depict the Random Walk. The Moving Average term provides a certain amount of smoothing to the values which would prevent overshooting. This could also be one explanation why the ARIMA models outperform the Random Walk.

Further, it can be seen that the accuracy scores of the respective ARIMA models and the k-Nearest Neighbour Regression model are mostly very similar. This behaviour can also be observed in the plotted data where the curves of the forecasts and actual values are very close together and in parts overlapping. This indicates a very similar forecasting performance, but the ARIMA has a slightly higher accuracy in every case. This is an interesting observation, as exchange rates are considered non-linear so the intuition would be that the non-linear model outperforms the linear model. Considering the results presented in this thesis this intuition can not be confirmed. How this could be explained would need further research. Though it has to be noted that the differences of the ARIMA and k-NN model are so small it has to be proven whether the differences are significant enough to make assumptions on which model is actually more accurate.

With the use of the MAPE it is possible to compare the forecasting accuracy across the different currency pairs. It can be observed that the behaviour of the accuracy results of the MAPE are very similar for all pairs. This could indicate that both models perform well on any type of exchange rates that have similar properties. Whether that is only the case for currency pairs with Euros or also for combinations of other currencies would have to be proven in future research.

5 Discussion

In this chapter the results produced in the previous chapter are put into the context of the research question, whether a statistical model or a basic machine learning model can improve on the on-step ahead forecasts by a Random Walk model.

Comparing the results of the individual accuracy scores, it can be seen that the Random Walk model *always* performs worse than the two more sophisticated models. This contradicts the assumptions of the 'Efficient Market Hypothesis' and its related literature, for example Meese and Rogoff (1983*a*), which states that the movements of foreign exchange rates do not adhere to any patterns and are best described with the movements of a Random Walk. Subsequently, the findings are in line with the results of Pathirana (2015) mentioned in Chapter 2, who also found that an ARIMA and k-NN model have greater forecasting accuracy of foreign exchange rates than the Random Walk.

Further, it can be seen that the forecasting accuracy of the respective ARIMA models and the k-Nearest Neighbour Regression model are mostly very similar, but the ARIMA slightly outperforms the k-NN Regression model in every case. These findings do not represent the findings by Pathirana (2015) where the k-NN outperformed the ARIMA model. It has to be noted that his research used a different distance metric which could be the cause of the discrepancy. This gives room for further research whether using a different distance metric leads to better results for the k-Nearest Neighbour Regression.

On the other hand, the differences between the error scores of the k-NN and ARIMA model are very small. So small in fact, that it can be discussed whether the differences are significant enough to conclude that one model performs better than the other. Further inaccuracies can be suspected when looking at the scale of the error scores. This stems from the logarithmic return series, which itself has small values. As a result the accuracy of statistical models can diminish when handling very small values. Whether this is the case here could be examined by using a different method for the data transformation which does not yield such small figures.

To add to that, forecasting accuracy could possibly be improved when switching from a univariate to a multi-variate forecasting method. By including variables that are thought to have an effect on the exchange rates, like interest rates or the rate of inflation, the accuracy of the forecasting models could be enhanced, although they would have to be adjusted for multi-variate input.

It would also be interesting to see whether more complicated models, like the Artificial Neural Network models mentioned in Chapter 2, lead to better forecasts or do simple models have similar results with the benefit of an easier application.

Lastly, the forecasting approach taken in this thesis, only makes it possible to estimate the exchange rate price for the next day but not for a longer period. Thus, this single-step ahead method may hold limited value in real life applications, as most financial exchange rate products have longer terms than that. It would be interesting to see how the models presented in this thesis would perform when forecasting for longer horizons of time. Subsequently, the use case of the methods in this thesis seems minimal for hedging foreign exchange risks, but it might benefit speculative actors in the foreign exchange market that trade in significantly shorter periods.

6 Conclusion and Outlook

This thesis aimed to identify whether certain statistical and machine learning models could improve on the short term forecasting accuracy of foreign exchange rates compared to that of the Random Walk model.

The analysis was made from a univariate perspective, using only the time series data from the currency pairs EURUSD, EURGBP and EURJPY as input data. Firstly, by using ACF plots and the ADF-test it was concluded that the exchange rates were non stationary which makes statistical time series modelling difficult. Stationarity then was achieved by taking the logarithmic returns of the time series. For the statistical modelling, the right order ARIMA(p, d, q) model was found by following the basic principles of the Box Jenkins method. The k-Nearest Neighbour Regression algorithm was trained with 1-5 period lagged values from the time series. All forecasting was done for the next day using all available data up to that point, which corresponds to a one-step ahead forecast. Lastly, the accuracy scores were calculated from the forecasting errors in order to be able to compare the results to the ones made by the Random Walk model.

The results indicate, that the forecasting accuracy of the ARIMA(p, d, q) and the k-Nearest Neighbour Regression model is greater than that of the Random Walk. This is true for all examined currency pairs. It can also be seen that the scores of the respective ARIMA model and the k-NN model are very similar, but the ARIMA model always holds a slight advantage over the k-NN model. It has to be noted that the small values of the logarithmic return series can cause statistical complications and skew the accuracy results. By reverting the logarithmic return series back to the original currency price scale we were able to find that the forecasts made by the sophisticated models were quite accurate in following the movement of the actual values of the exchange rates. This also made for a better interpretability and further usability of the forecasts.

Based on these conclusions, the applicability in foreign exchange risk hedging has to be stated as limited because of the short forecasting horizon. Although they might be of interest for speculative actors on the foreign exchange market that operate in a shorter action horizon.

In order to improve on the findings of this thesis, the inclusion of more input variables seems promising as that could help better describe and analyse underlying patterns in the exchange rate movement. Especially for longer forecasting horizons variables like the interest rate or the rate of inflation seem plausible. Further, changing parameters like the distance metric or the number of lags of the input in the k-NN Regression model could result in improvements. Lastly, the application of different and more sophisticated machine learning models could also improve the forecasting results. As mentioned in Chapter 2 especially Artificial Neural Network models have seen a rise in popularity forecasting financial time series. Due to their strong ability to model non-linear correlations, they might be able to find patterns that the models used in this thesis were not able to.

Bibliography

Abu-Mostafa, Y. S. and Atiya, A. F. (1996), ‘Introduction to financial forecasting’, *Applied intelligence* **6**(3), 205–213.

Akaike, H. (1973), ‘Maximum likelihood identification of gaussian autoregressive moving average models’, *Biometrika* **60**(2), 255–265.

Akincilar, A., Temız, İ. and Şahin, E. (2011), ‘An application of exchange rate forecasting in turkey.’, *Gazi University Journal of Science* **24**(4).

Andrieș, A. M., Ihnatov, I. and Tiwari, A. K. (2014), ‘Analyzing time–frequency relationship between interest rate, stock price and exchange rate through continuous wavelet’, *Economic Modelling* **41**, 227 – 238.

Appiah, S. and Adetunde, I. (2011), ‘Forecasting exchange rate between the ghana cedi and the us dollar using time series analysis’, *Current research journal of economic theory* **3**(2), 76–83.

Bahmani-Oskooee, M. and Sohrabian, A. (1992), ‘Stock prices and the effective exchange rate of the dollar’, *Applied Economics* **24**(4), 459–464.

Baldwin, R. E. and Wyplosz, C. (2015), *The economics of European integration*, 5th edn, McGraw-Hill Education, London.

BIS Triennial Central Bank Survey: Foreign exchange turnover in April 2019 (2019), Technical report, Bank of International Settlements.

Box, G. E. P. and Jenkins, G. (1970), *Time Series Analysis, Forecasting and Control*, Holden-Day, Inc.

Brockwell, P. J. and Davis, R. A. (2009), *Time Series: Theory and Methods*, 2nd edn, Springer.

Chatfield, C. (2000), *Time-series forecasting*, CRC press.

Collinson, S., Narula, R. and Rugman, A. M. (2017), *International Business*, 7th edn, Pearson, Harlow, United Kingdom.

Fama, E. F. (1965*a*), ‘The behavior of stock-market prices’, *The Journal of Business* **38**(1), 34–105.

Fama, E. F. (1965*b*), ‘Random walks in stock market prices’, *Financial Analysts Journal* **21**(5), 55–59.

Fernandez-Rodrıguez, F., Sosvilla-Rivero, S. and Andrada-Felix, J. (1999), ‘Exchange-rate forecasts with simultaneous nearest-neighbour methods: evidence from the ems’, *International Journal of Forecasting* **15**(4), 383–392.

Goodhart, C. (1988), 'The foreign exchange market: a random walk with a dragging anchor', *Economica* pp. 437–460.

Guha, B. and Bandyopadhyay, G. (2016), 'Gold price forecasting using arima model', *Journal of Advanced Management Science* **4**(2).

Kapila Tharanga Rathnayaka, R. M., Seneviratna, D. M. K. N., Jianguo, W. and Arumawadu, H. I. (2015), A hybrid statistical approach for stock market forecasting based on artificial neural network and arima time series models, *in* '2015 International Conference on Behavioral, Economic and Socio-cultural Computing (BESC)', pp. 54–60.

Kia, A. N., Fathian, M. and Gholamian, M. (2012), 'Using mlp and rbf neural networks to improve the prediction of exchange rate time series with arima', *International Journal of Information and Electronics Engineering* **2**(4), 543–546.

Malkiel, B. G. (1973), *A random walk down Wall Street*, Norton, New York.

Martínez, F., Frías, M. P., Pérez, M. D. and Rivera, A. J. (2019), 'A methodology for applying k-nearest neighbor to time series forecasting', *Artificial Intelligence Review* **52**(3).

Meade, N. (2002), 'A comparison of the accuracy of short term foreign exchange forecasting methods', *International Journal of forecasting* **18**(1), 67–83.

Meese, R. A. and Rogoff, K. (1983*a*), 'Empirical exchange rate models of the seventies: Do they fit out of sample?', *Journal of international economics* **14**(1-2), 3–24.

Meese, R. and Rogoff, K. (1983*b*), 'The out-of-sample failure of empirical exchange rate models: sampling error or misspecification?', pp. 67–112.

Meyler, A., Kenny, G. and Quinn, T. (1998), 'Forecasting irish inflation using arima models'.

Moffett, M. H., Stonehill, A. I. and Eiteman, D. K. (2014), *Fundamentals of multinational finance*, Pearson always learning, 4th edn, Pearson.

Mondal, P., Shit, L. and Goswami, S. (2014), 'Study of effectiveness of time series modeling (arima) in forecasting stock prices', *International Journal of Computer Science, Engineering and Applications* **4**(2), 13.

Montgomery, D. C., Jennings, C. L. and Kulahci, M. (2015), *Introduction to time series analysis and forecasting*, John Wiley & Sons.

Mussa, M. L. (1984), The theory of exchange rate determination, *in* 'Exchange rate theory and practice', University of Chicago Press, pp. 13–78.

Pai, P.-F. and Lin, C.-S. (2005), 'A hybrid arima and support vector machines model in stock price forecasting', *Omega* **33**(6), 497 – 505.

Pathirana, V. K. (2015), Nearest neighbor foreign exchange rate forecasting with Mahalanobis distance, PhD thesis.

Rogoff, K. (1996), ‘The purchasing power parity puzzle’, *Journal of Economic literature* **34**(2), 647–668.

Schira, J. (2016), *Statistische Methoden der VWL und BWL*, wi-Wirtschaft, 5th edn, Pearson, Hallbergmoos.

Shittu, O. I. and Yaya, O. S. (2009), ‘Measuring forecast performance of arma and arfima models: An application to us dollar/uk pound foreign exchange rate’, *European journal of scientific research* **32**(2), 167–176.

Stone, C. J. (1977), ‘Consistent nonparametric regression’, *The annals of statistics* pp. 595–620.

Taylor, M. P. (1995), ‘The economics of exchange rates’, *Journal of Economic Literature* **33**(1), 13–47.

Tenti, P. (1996), ‘Forecasting foreign exchange rates using recurrent neural networks’, *Applied Artificial Intelligence* **10**(6), 567–582.

Tlegenova, D. (2015), Forecasting exchange rates using time series analysis: The sample of the currency of kazakhstan, Papers, arXiv.org.

Tsay, R. S. (2005), *Analysis of financial time series*, Vol. 543, John Wiley & Sons.

Weigend, A. S., Rumelhart, D. E. and Huberman, B. A. (1991), Generalization by weight-elimination with application to forecasting, *in* ‘Advances in neural information processing systems’, pp. 875–882.

Wu, B. (1995), ‘Model-free forecasting for nonlinear time series (with application to exchange rates)’, *Computational Statistics & Data Analysis* **19**(4), 433 – 459.

Yakowitz, S. (1987), ‘Nearest-neighbour methods for time series analysis’, *Journal of time series analysis* **8**(2), 235–247.

Zhang, G. and Hu, M. Y. (1998), ‘Neural network forecasting of the british pound/us dollar exchange rate’, *Omega* **26**(4), 495 – 506.

Appendix

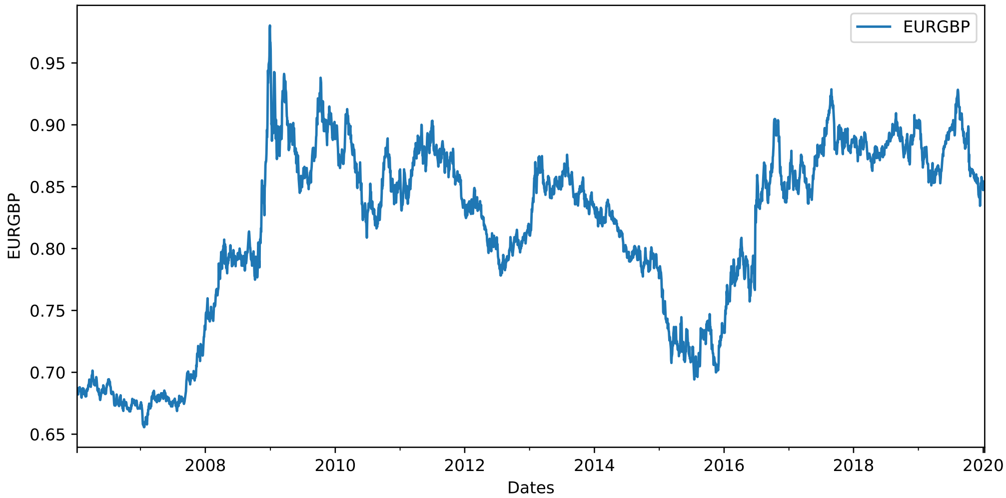

Figure A.1: The daily EURGBP exchange rates from 09.01.2006 to 09.01.2020

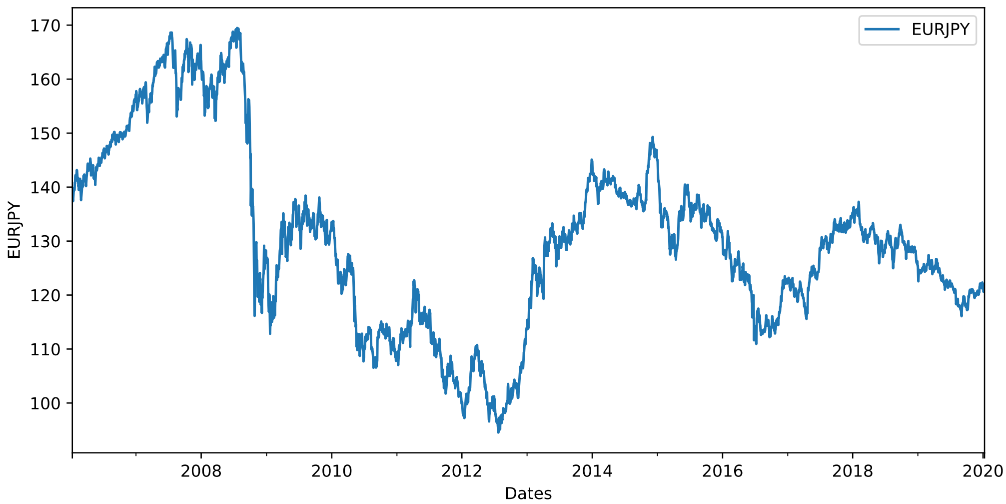

Figure A.2: The daily EURJPY exchange rates from 09.01.2006 to 09.01.2020

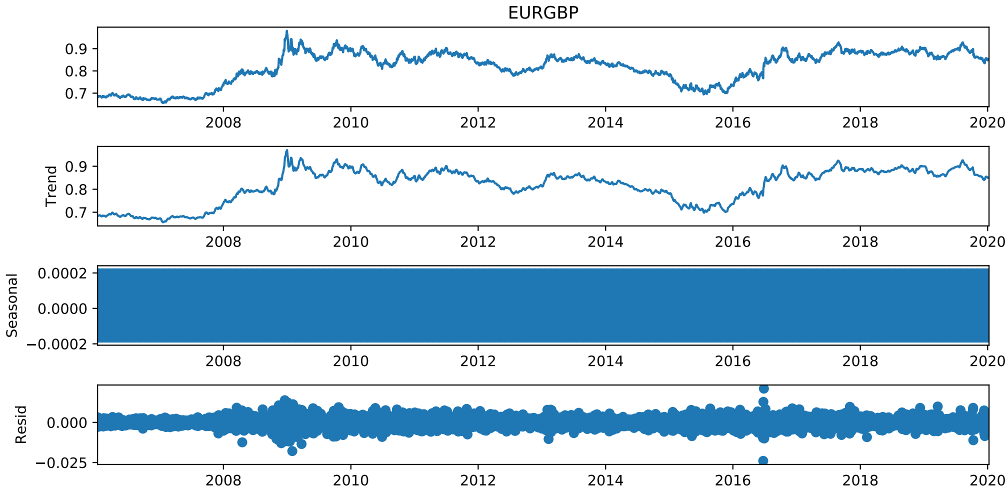

Figure A.3: Plots showing the EURGBP exchange rates decomposed into trend, seasonal fluctuation, and residual

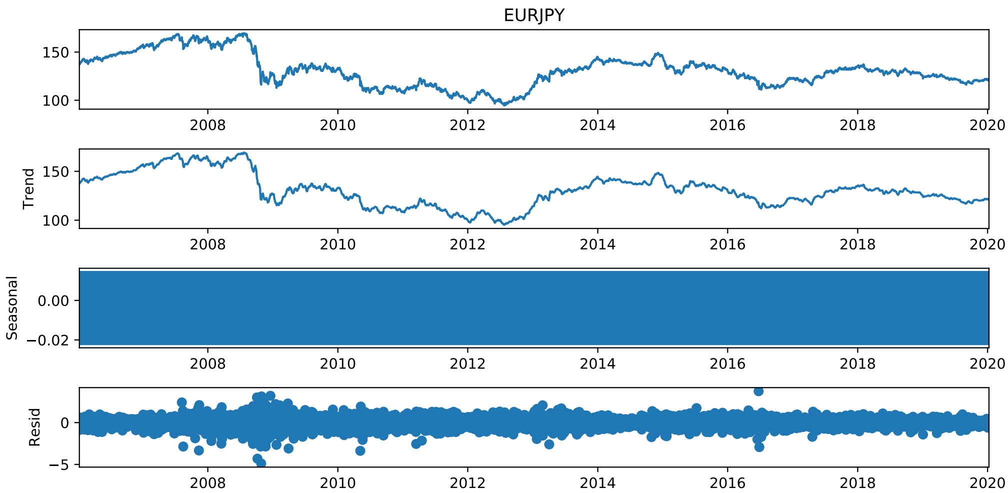

Figure A.4: Plots showing the EURJPY exchange rates decomposed into trend, seasonal fluctuation, and residual

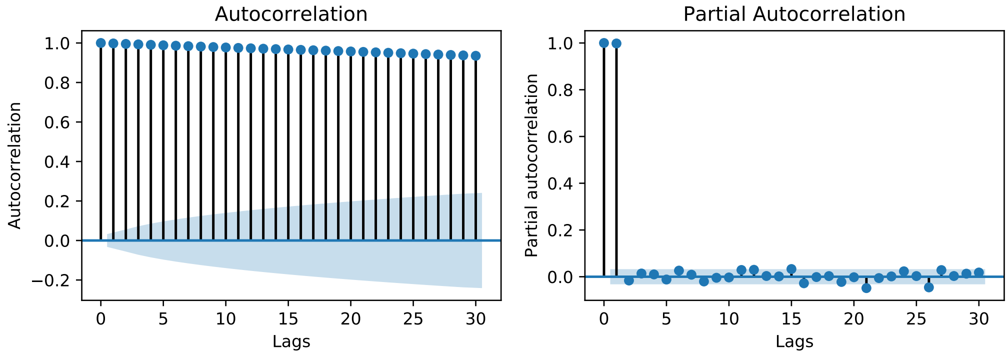

Figure A.5: The autocorrelation and partial autocorrelation plots of the EURGBP exchange rate with lags $j = 30$ and the corresponding 95% confidence interval (blue)

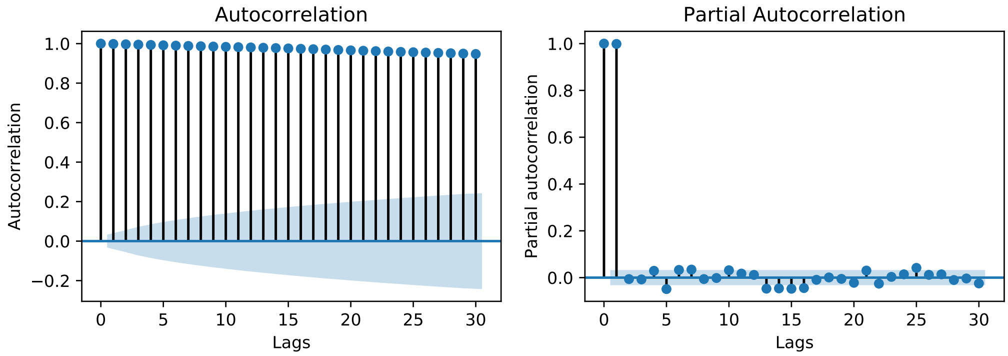

Figure A.6: The autocorrelation and partial autocorrelation plots of the EURJPY exchange rate with lags $j = 30$ and the corresponding 95% confidence interval (blue)

Augmented Dickey-Fuller test	
Test Statistic	-1.591
p - value	0.186
Critical value (1%)	-3.432
Critical value (5%)	-2.862
Critical value (10%)	-2.567

Table A.1: Augmented Dickey Fuller test of the daily EURGBP exchange rates.

Augmented Dickey-Fuller test	
Test Statistic	-1.873
p - value	0.345
Critical value (1%)	-3.432
Critical value (5%)	-2.862
Critical value (10%)	-2.567

Table A.2: Augmented Dickey Fuller test of the daily EURJPY exchange rates.

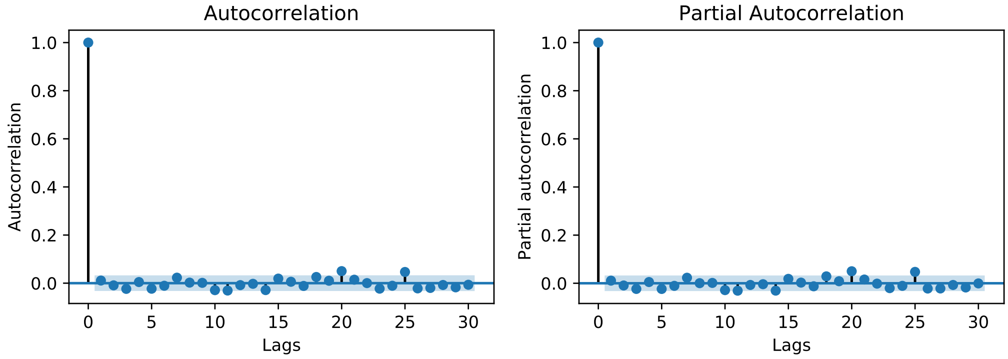

Figure A.7: The autocorrelation and partial autocorrelation plots of the log return EURGBP exchange rate with lags $j = 30$ and the corresponding 95% confidence interval (blue)

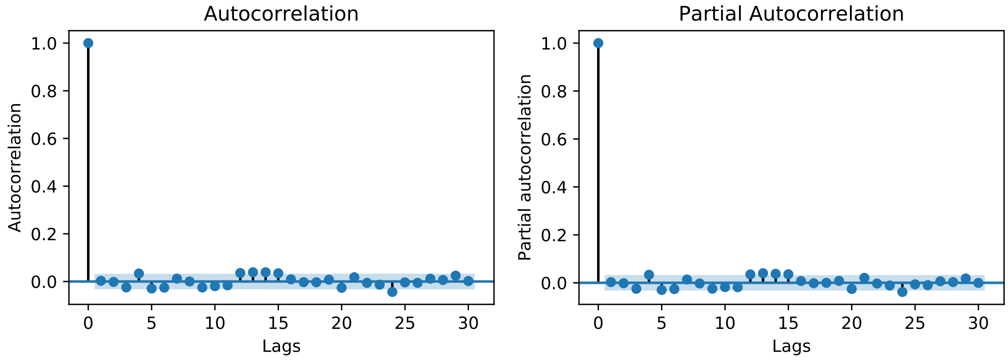

Figure A.8: The autocorrelation and partial autocorrelation plots of the log return EURJPY exchange rate with lags $j = 30$ and the corresponding 95% confidence interval (blue)

Augmented Dickey-Fuller test	
Test Statistic	-59.757
p - value	0.000
Critical value (1%)	-3.432
Critical value (5%)	-2.862
Critical value (10%)	-2.567

Table A.3: Augmented Dickey Fuller test of the log return EURGBP exchange rates.

Augmented Dickey-Fuller test	
Test Statistic	-14.336
p - value	0.000
Critical value (1%)	-3.432
Critical value (5%)	-2.862
Critical value (10%)	-2.567

Table A.4: Augmented Dickey Fuller test of the log return EURJPY exchange rates.

```
                               ARIMA Results
==============================================================================
Dep. Variable:                      y   No. Observations:                 2922
Model:             ARIMA(1, 1, 2)   Log Likelihood               11043.848
Date:              Mon, 20 Jul 2020   AIC                         -22077.697
Time:                      22:39:23   BIC                         -22047.798
Sample:                           0   HQIC                        -22066.928
                             - 2922
Covariance Type:                opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
intercept  -3.422e-06   2.85e-06     -1.199      0.231   -9.02e-06    2.17e-06
ar.L1         -0.8032      0.610     -1.316      0.188      -1.999       0.393
ma.L1         -0.1815      0.605     -0.300      0.764      -1.368       1.005
ma.L2         -0.7994      0.599     -1.335      0.182      -1.973       0.374
sigma2      3.027e-05   3.79e-07     79.777      0.000    2.95e-05     3.1e-05
===================================================================================
```

Figure A.9: Result of the `auto_arima` search for the best ARIMA(p, d, q) order for the log return EURGBP exchange rates.

```
                               ARIMA Results
==============================================================================
Dep. Variable:                      y   No. Observations:                 2922
Model:             ARIMA(2, 1, 2)   Log Likelihood                9957.705
Date:              Mon, 20 Jul 2020   AIC                         -19903.410
Time:                      22:45:41   BIC                         -19867.532
Sample:                           0   HQIC                        -19890.487
                             - 2922
Covariance Type:                opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
intercept  -3.909e-06   2.32e-06     -1.682      0.093   -8.47e-06    6.47e-07
ar.L1         -0.8179      0.314     -2.601      0.009      -1.434      -0.201
ar.L2          0.0040      0.015      0.273      0.785      -0.025       0.033
ma.L1         -0.1782      0.313     -0.569      0.569      -0.792       0.436
ma.L2         -0.8070      0.310     -2.603      0.009      -1.415      -0.199
sigma2       6.22e-05   8.31e-07     74.856      0.000    6.06e-05    6.38e-05
===================================================================================
```

Figure A.10: Result of the `auto_arima` search for the best ARIMA(p, d, q) order for the log return EURJPY exchange rates.

List of Figures

List of Tables

Eidesstattliche Erklärung

Hiermit versichere ich, die vorliegende Arbeit ohne unerlaubte Hilfe und ohne Benutzung anderer als der angegebenen Hilfsmittel angefertigt zu haben. Alle Stellen, die wörtlich oder sinngemäß aus Veröffentlichungen entnommen sind, habe ich als solche kenntlich gemacht. Die eingereichte Bachelorarbeit wurde weder vollständig noch in wesentlichen Teilen Gegenstand eines anderen Prüfungsverfahrens. Die elektronische Version der eingereichten Bachelorarbeit stimmt in Inhalt und Formatierung mit den auf Papier ausgedruckten Exemplaren überein.

Freiburg im Breisgau, July 27, 2020

. . . .
(Signature Clemens Philipp Rockinger)