Game Theory for Image Segmentation

Controversies in Game Theory IX

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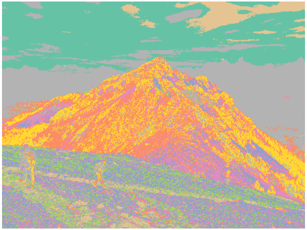
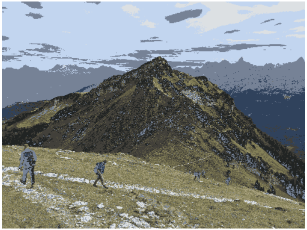
June 13, 2023





Overview

- Image segmentation has been extensively studied in the image processing community for decades.
- We propose to build on the PhD dissertation of Samuel Rota Bulò[1] and the publication of Shen et al. [5] and reproduce their results.
- To do so, we formulate image segmentation as a clustering game, which we simulate through discrete-time evolutionary dynamics.
- More precisely, we investigate and evaluate three approaches: *Best response dynamics*, *Replicator dynamics* and *Infection and immunization dynamics* [4].
- Additionally, we extend their results to semantic segmentation by making use of state-of-the-art self-supervised features, namely DINO [2] features.

Image Segmentation as a clustering game

Image segmentation can be defined as a clustering game where:

- Pixels $\{1, \ldots, n\}$ are represented as a mixed strategy $(x_1, \ldots, x_n)$ in the simplex Δ i.e. $\sum_{i=1}^{n} x_i = 1$.
- The payoff of a pixel i against another pixel j is given by a similarity matrix $A \in \mathbb{R}^{n \times n}$ that quantify how close two pixels are from one another.
- In practice, we choose a Gaussian kernel $A_{i,j} = \exp\left(-\frac{\|C(i)-C(j)\|^2}{\sigma^2}\right)$ where $C(i)$ is a 3-dimensional vector representing the value of pixel i.
- The goal of the clustering game is to find a Nash equilibrium w.r.t. this setting.

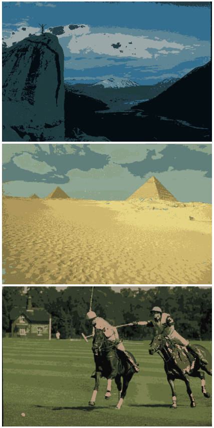
Evolutionary dynamics

In order to solve such a high dimensional problem, we use evolutionary dynamics and three different discrete-time dynamics:

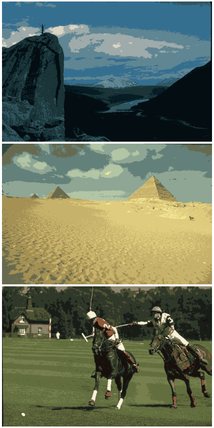
1. *Best response dynamics* which injects in the population, at each time step, the individual that would yield the best response w.r.t. the payoff matrix.
2. *Replicator dynamics* that can be seen as a form of *Darwin selection* whereby the "fittest" populations are more inclined to survive.
3. *Infection and immunization dynamics* where "infectious" individuals are iteratively introduced in a population to yield immunity.

We refer the reader to our report for more rigorous definitions and our Python notebook for a practical implementation of these approaches.

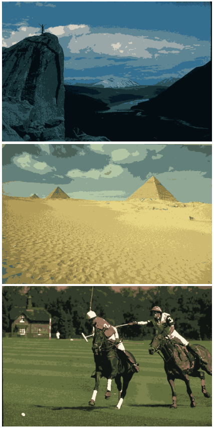
Results



(a) Best Response dynamics



(b) Replicator dynamics



(c) Pure InImDyn

Figure: Segmentations obtained with our implementations of the three dynamics. Per-vertex average colors are shown here.

Results

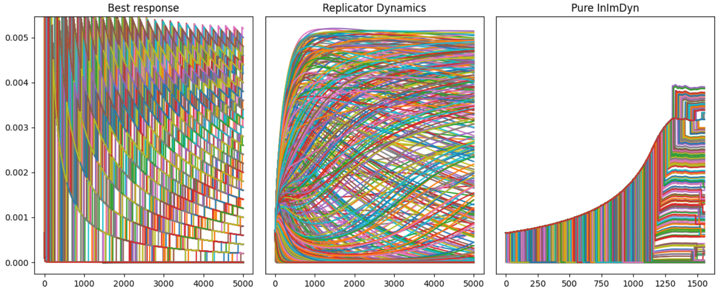
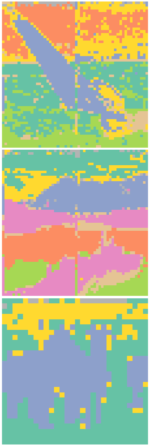


Figure: Trajectories of the mixed strategy entries on the first segment of two images from the BSDS300 dataset

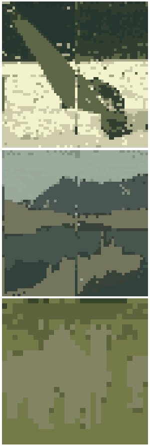
The figure above shows that with the same number of iterations, the *Infection and immunization dynamics* reaches a stable state faster than the two other strategies.

Semantic segmentation with deep self-supervised DINO features

(a) Original



(b) Clusters



(c) Average colors

Implementation

We provide all the sources to reproduce our results on the BSDS300 [3] dataset as a Jupyter Notebook. Feel free to consult it at:

`https://github.com/clementjambon/evolutionary-segmentation`

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