‘MATH+ECON+CODE’ MASTERCLASS ON EQUILIBRIUM TRANSPORT AND MATCHING MODELS IN ECONOMICS

Alfred Galichon (NYU+ScPo)

Day 1: competitive equilibrium with gross substitutes
Getting started

Preliminary stuff

- Schedule: June 21-25, 2021. Classes meet 2pm-6pm Paris time / 8am-noon New York time. In addition, five advanced lectures will be taught online on a monthly basis, July - November 2021. Precise days of the lectures will be announced later.
- Format: online.
- Course description: https://www.math-econ-code.org/june2021
- Course material: https://github.com/math-econ-code/mec_equil

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Introducting the team

- **Alfred Galichon**, instructor: professor of economics and of mathematics at NYU, and principal investigator, ERC grant EQUIPRICE, SciencesPo (ag133@nyu.edu)
- **Gabriele Buontempo**, graduate assistant: graduate student at SciencesPo (gabriele.buontempo@sciencespo.fr)
- **Akshaya Devasia**, graduate assistant: graduate student at SciencesPo
- **Loan Tricot**, graduate assistant: graduate student at HEC
- Last but not least, **you** are now part of the team!

A group photo

- Put on your best smile anyone!

Knowing you

- Please fill out the following poll: https://bit.ly/3qdM334

Remarks on this course

- Targeting Math and Econ students. Self-contained for both audiences
- Teaching format: 5 days; each day = four hours, 1/2 economics modeling, 1/2 Python coding. Plus 5 "advanced lectures" to deepen the participants' mathematical understanding.
- Programming: our demos will be done Python and the support will be in these languages only, but you are welcome to use the language of your choice e.g. R, Matlab, C++, Julia...
- Parts of this course may be recorded, and the recording may be publicly broadcasted in part or in full. As a participant to the masterclass, you have consented that your voice and image should be broadcasted.
- To stay engaged with the course and interact optimally with other participants, we recommend you keep your camera on and turn your microphone off except when you wish to speak. You have the option to turn your camera and your microphone off at any time.
- Questions?

This course

- is focused on the computation of competitive equilibrium, which is at the core of surge pricing engines and allocation mechanisms. It will investigate diverse applications such as network congestion, surge pricing, and matching platforms. It provides a bridge between theory, empirics and computation and will introduce tools from economics, mathematical and computer science.
- will provide the conceptual basis of competitive equilibrium with gross substitutes, along with various computational techniques (optimization problems, equilibrium problems). It will apply parallel computation is adapted to the computation of equilibrium. Applications to hedonic equilibrium, multinomial choice with peer effects, and congested traffic equilibrium on networks.
- will introduce tools from economic theory, mathematics, econometrics and computing, on a needs basis, without any particular prerequisite other than the equivalent of a first year graduate sequence in econ or in applied math.
- is complementary to the m+e+c_optim masterclass (taught in January), but can be taken without it as a prerequisite. Overlap is limited.

Outline: main week

- D1: competitive equilibrium with gross substitutes
  - Walrasian equilibrium and gross substitutes. Hedonic pricing. Jacobi algorithm.
- D2: matching models with fully transferable utility
  - Optimization and equilibrium formulation. Duality. Computation by descent methods. Sinkhorn's algorithm.
- D3: matching models with imperfectly transferable utility
  - Distance-to-frontier function, matching function equilibrium. Matching models with taxes. Collective models with public consumption.
- D4: matching models with nontransferable utility
  - Gale and Shapley's deferred acceptance algorithm. Adachi's algorithm and Tarski's fixed point theorem. Aggregate stable matchings.
- D5: one-to-many matching models
  - Kelso and Crawford's deferred acceptance algorithm. Hatfield and Milgrom's model of matching with contracts.

Outline: advanced lectures

- AL1: Special matrices
  - Z-, P- and M-matrices, diagonally dominant matrices, Stieltjes matrices, and Perron-Froebenius theory. M-maps
- AL2: submodularity
  - Lattices, submodularity and Topkis' theorem. Veinott's order. Monotone comparative statics. Strategic complementarities. Uniform Gross Substitutes.
- AL3: gross substitutes
  - Polymatroids, exchangeability. Lovasz extensions. Discrete convex analysis, L / L# / M / M# convexity.
- AL4: networks
  - Equilibrium pricing on networks. Dynamic programming and Bellman-Ford's algorithm. The min-cut-max-flow theorem. The main network algorithm.
- AL5: parallel computing
  - Principles of parallelization; computing on GCP and AWS; introduction to TensorFlow and Keras.

Setup

- See Gabriele's setup instructions in the github repository: `https://github.com/math-econ-code/mec_equil/blob/master/D0_docker-tutorial.pdf`

Assessment

- Students taking this class for credit have a choice between:
  - Either a take-home exam (24 hours), to be taken over a 24 hour period, July 2, 10am NY time - July 3, 10am NY time (to be confirmed).
  - Or a short paper (12 pages or more), to be discussed with the instructor. The paper will bear some connections, in a broad sense, with the topics of the course. Many papers are considered acceptable: original research paper, survey paper, report on numerical experiments, replication of existing empirical results... are all acceptable. The requirement is to be innovative on a theoretical, empirical, or computational level. This work should be submitted before December 31, 2021.
- Email me to indicate which of these you are opting for.

GETTING STARTED

- From the command line, after installing docker, pull the image using:

  ```
  docker pull alfredgalichon/mec_equil:2021-06
  ```

- Create a local repository (here /Users/alfre/Desktop/docker-tmp)
- Next, run the container using:

  ```
  docker run -it --rm -p 8888:8888 -v
  /Users/alfre/Desktop/docker-tmp:/home mec_equil
  ```

- Copy the URL and paste it on in your browser.

Other ways to run the notebooks

- Locally and without the container, using `anaconda` and `git`.
  - Simple, but some may run into version issues
- On Google Colab:
  https://colab.research.google.com/
  - The simplest options, but with some limitations (limited lifetime and memory, some dependencies may not install)
- Run the container on GCP / AWS:
  - More powerful and flexible option, but also more involved. We will demonstrate this option later this week.